Questions are equally weighted. Give essential explanations and justifications: a large part of each question is demonstration of understanding of the context and of which issues are primary. Do not make assumptions or choose contexts making the problems silly. Coherent writing is essential: your paper should not be a puzzle for the grader.

Put your codename, not your actual name, on your paper. Please follow the special rules for this exam, as explained by the proctor and the office of the Director of Graduate Studies in Mathematics.

[1] Enumerate the finite abelian groups of order 1024.

[2] Enumerate the conjugacy classes in the group $GL_2(\mathbb{F}_q)$ of multiplicatively invertible two-by-two matrices with entries in the finite field $\mathbb{F}_q$ with $q$ elements.

[3] Show that the ideal $I$ in $\mathbb{Z}[x]$ generated by $17$ and $x^2 + 1$ is not maximal.

[4] Show that two linear operators $S,T$ on a finite-dimensional complex vectorspace with $ST = TS$ have a common eigenvector.

[5] Show that $x^7 + y^{11} + z^{13}$ is irreducible in $\mathbb{C}[x,y,z]$.


[7] Describe in terms of radicals all intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\zeta)$, where $\zeta$ is a primitive eighth root of unity.

[8] Describe the prime ideals in the formal power series ring $k[[x]]$, with field $k$. 