Questions are equally weighted. Give essential explanations and justifications: a large part of each question is demonstration that you understand the context and understand which issues are primary. Do not choose assumptions or contexts making the problems silly. Coherent writing is essential: your paper should not be a puzzle for the grader.

Write your codename, not actual name, on each booklet. No notes, books, calculators, computers, cell phones, wireless, bluetooth, or other communication devices may be used during the exam.

[1] Write three terms of the Laurent expansion of \( f(z) = \frac{1}{z(z-1)(z+1)} \) in the annulus \( 0 < |z| < 1 \).

[2] Determine the radius of convergence of the power series for \( \log z \) expanded at \( -4 + 3i \).

[3] Show that a real-valued holomorphic function is constant.

[4] Give an explicit conformal mapping from \( X = \{ z : |z - 1| < \sqrt{2}, |z + 1| < \sqrt{2} \} \) to the unit disk \( \{ z : |z| < 1 \} \).

[5] Evaluate \( \int_{0}^{\infty} \frac{\sqrt{x}}{1 + x^2} \, dx \).

[6] Let \( f \) be an entire function such that \( f(z + 1) = f(z) = f(z + i) \) for all \( z \). Show that \( f \) is constant.

[7] Show that \( z^{10} - z^7 + 4z^2 - 1 \) has exactly two zeros in \( |z| \leq 1 \).

[8] Show that \( \frac{\pi^2}{\sin^2 \pi z} = \sum_{n \in \mathbb{Z}} \frac{1}{(z - n)^2} \).

[9] Let \( f \) be a harmonic function on \( \mathbb{C} \), such that \( |f(z)| \leq \sqrt{1 + |z|} \). Show \( f \) is constant.