Complex Analysis Prelim Written Exam Spring 2015

Questions are equally weighted. Give essential explanations and justifications: a large part of each question is demonstration that you understand the context and understand which issues are primary. Do not choose assumptions or contexts making the problems silly. Coherent writing is essential: your paper should not be a puzzle for the grader.

Write your codename, not actual name, on each booklet. No notes, books, calculators, computers, cell phones, wireless, bluetooth, or other communication devices may be used during the exam.

[1] Describe all the values of \((-1)^i\), where \(i = \sqrt{-1}\).

[2] Write three terms of the Laurent expansion of \(f(z) = \frac{1}{z(z-1)(z-2)}\) in the annulus \(1 < |z| < 2\).

[3] Give an explicit conformal mapping from the half-disk \(\{z : |z| < 1, \text{Re}(z) > 0\}\) to the unit disk \(\{z : |z| < 1\}\).

[4] Determine the radius of convergence of the power series for \(\sqrt{z}\) expanded at \(-4 + 3i\).

[5] Evaluate \(\int_{-\infty}^{\infty} \frac{e^{i\xi x}}{1 + x^2} \, dx\) for real \(\xi\).

[6] Show that a holomorphic function \(f\) on \(\mathbb{C}\) satisfying \(|f(z)| \leq \sqrt{1 + |z|}\) for all \(z \in \mathbb{C}\) is a constant.

[7] Show that \(4z^5 - z + 2\) has all its zeros in the unit disk.

[8] Show that there is a holomorphic function \(f(z)\) on a neighborhood of 0 so that \(f(z)^2 = \frac{\sin z}{z}\), and determine radius of convergence of the power series at 0.

[9] Describe all complex-valued harmonic functions on the annulus \(1 < |z| < 2\) which extend continuously to the circle \(|z| = 2\) and take value 0 on that circle.