Complex Analysis Prelim Written Exam Fall 2018

Questions are equally weighted. Give essential explanations and justifications: a large part of each question is demonstration that you understand the context and understand which issues are primary. Do not choose assumptions or contexts making the problems silly. Coherent writing is essential: your paper should not be a puzzle for the grader.

Write your codename, not actual name, on each booklet. No notes, books, calculators, computers, cell phones, wireless, bluetooth, or other communication devices may be used during the exam.

[1] Tell the values of $i^i$.

[2] Write three terms of the Laurent expansion of $f(z) = \frac{1}{z^4 - 1}$ centered at 0 and convergent in $1 < |z|$.

[3] Let $f$ be an entire function such that $\Re f(z)$ is non-negative for all $z \in \mathbb{C}$. Show $f$ is constant.

[4] Evaluate $\int_0^{\infty} \frac{x^{1/5}}{1 + x^2} \, dx$.

[5] Determine the radius of convergence of the power series for $\log z$ at $z_0 = -4 + 3i$.

[6] Show that there is a holomorphic function $f$ on the region $1 < |z| < 2$ such that $f(z)^2 = (z^2 - 1)(z^2 - 4)$.

[7] Prove that $w^2 = z^4 + 1$ defines an elliptic curve.

[8] Try to define $w$ locally as a holomorphic function of $z$, defined by the relation $w^5 - 5zw + 1 = 0$. What are the branch points?