(*September 17, 2014*)

Complex analysis examples 02

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[This document is http://www.math.umn.edu/~garrett/m/complex/examples_2014-15/cx_ex_02.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Monday, Sept 22, preferably as a PDF emailed to me.

[02.1] Parametrize counter-clockwise a circle γ of radius $r > 0$ centered at z_o, and *directly* compute $\int_\gamma (z - z_o)^n \, dz$ for all positive and negative integers n.

[02.2] Using only geometric series expansions, determine the Laurent expansion of $f(z) = 1/(z-1)(z-2)$ in the annulus $1 < |z| < 2$, and also in the annulus $|z| > 2$.

[02.3] Determine the Laurent expansion of $f(z) = 1/(z-1)^4$ in the annulus $|z| > 1$, and also in the annulus $|z - 1| > 0$.

[02.4] Show that an entire function f satisfying $|f(z)| \leq C \cdot (1 + |z|)^{1/2}$ for some constant C is *constant*.

[02.5] Compute $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$

[02.6] Compute $\int_{-\infty}^{\infty} \frac{e^{itx}\, dx}{x^4+1}$ with real t.

[02.7] Compute $\int_0^{\infty} \frac{x\, dx}{1+x^3}$

[02.8] Compute $\frac{1}{1} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$