(*September 28, 2014*)

Complex analysis examples 03

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[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_03.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Monday, Oct 6, preferably as a PDF emailed to me.

[03.1] For a bounded sequence of complex numbers c_n, prove that $\sum_{n=0}^{\infty} c_n \frac{z^n}{z^n+1}$ converges to a holomorphic function on $|z| < 1$.

[03.2] Prove that $f(z) = \int_0^1 \frac{e^{tz}\,dt}{t^2+1}$ is holomorphic.

[03.3] Prove that $f(z) = \int_0^\infty \frac{e^{-tz}\,dt}{t^2+1}$ is holomorphic for $\mathrm{Re}(z) > 0$.

[03.4] Let f be a continuous, bounded real-valued function on $\mathbb{R}$, extending to a bounded, holomorphic function on the upper half-plane $\mathfrak{H}$. Show f is constant.

[03.5] Evaluate the Fourier transform $\int_{-\infty}^{\infty} e^{-itx} \cdot \frac{1}{(x+i)^s}\,dx$ for complex s with $\mathrm{Re}(s) > 1$, using the Gamma function.

[03.6] Show that $f(z) = \int_0^1 \frac{dt}{t \cdot z + (1-t)\cdot z_o}$ is holomorphic at any z_1 such that 0 is *not* on the straight line segment with endpoints z_o and z_1. Find the radius of convergence of its power series expanded at $z_o = -4+3i$.

[03.7] Show that there is a holomorphic function $f(z) = \sqrt{z^5-1}$ near any point z_o with $z_o^5 \neq 1$. Determine the radius of convergence of the power series for $f(z)$ expanded at 0.

[03.8] With real b, the function $f(z) = 1 + e^z + e^{bz}$ does not vanish on the real line. Estimate the number of its zeros in $|z| < R$ for large R.
