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Complex analysis examples 04

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[This document is http://www.math.umn.edu/˜garrett/m/complex/examples_2014-15/cx_ex_04.pdf]

If you want feedback from me on your treatment of these examples, please get your work to me by Monday, Oct 20, preferably as a PDF emailed to me.

[04.1] Compute $\displaystyle\int_0^\infty \frac{x^s\, dx}{1+x^2}$

[04.2] Compute $\displaystyle\int_0^1 \frac{\big(x(1-x)\big)^s}{1+x^3}\, dx$

[04.3] Compute $\displaystyle\int_0^\infty e^{-i\xi x}\, x^s\, e^{-x}\, dx$ with $\mathrm{Re}(s) > -1$.

[04.4] Compute $\displaystyle\int_{-\infty}^\infty e^{-i\xi x}\, xe^{-x^2}\, dx$

[04.5] For continuous φ on the unit circle $|z| = 1$, define

$$f_\varphi(z) \;=\; \int_0^{2\pi} \frac{\varphi(e^{i\theta})}{e^{i\theta} - z}\, d\theta \qquad\qquad (\text{for } |z| < 1)$$

Show that $f(z)$ is holomorphic. Give an example of φ *not* identically 0 so that f_φ *is* identically 0.

[04.6] Let f be an entire function such that $f(z+1) = f(z)$ and $f(z+i) = f(z)$ for all z. Show that f is constant.

[04.7] Show that a *real-valued* holomorphic function is constant.

[04.8] The *Bergmann kernel* of the unit disk is

$$K(z,w) \;=\; \frac{1}{\pi}\, \frac{1}{(1-\overline{w}\, z)^2}$$

For f holomorphic on the open unit disk and extending continuously to a continuous function on the *closed* unit disk, show that

$$f(w) \;=\; \int\int_{x^2+y^2\le 1} f(x+iy)\, \overline{K(z,w)}\, dx\, dy$$
