

(December 1, 2014)

Complex analysis midterm 03

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Please write on one side of a page, with your name on every page.

Please restate the respective questions, and respond in complete sentences, in standard English, legibly. The goal is *explanation* and also *persuasion*, not crypticness or telegraphic-ness.

Responses should be intelligible *without* definitive prior expertise. That is, the message(s) should be intelligible without knowing the message(s) in advance.

Questions are equally weighted.

[03.1] Give an explicit conformal map of the half-disk $\{z = x + iy : |z| < 1, x > 0\}$ to the unit disk $\{z : |z| < 1\}$.

[03.2] Determine a finite set $S \subset \mathbb{C}$ of points such that for $w_o \notin S$ there is a holomorphic function $f(w)$ near w_o such that $z = f(w)$ gives a solution to the equation $z^5 - 5z - w = 0$. (*Hint*: holomorphic inverse function theorem.)

[03.3] Show that $f(z) = e^{iz} - z$ has at least one complex zero.
