Complex analysis midterm 05

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Please write on one side of a page, with your name on every page.

Please restate the respective questions, and respond in complete sentences, in standard English, legibly. The goal is *explanation* and also *persuasion*, not crypticness or telegraphic-ness.

Responses should be intelligible *without* definitive prior expertise. That is, the message(s) should be intelligible without knowing the message(s) in advance.

Questions are equally weighted.

[05.1] Exhibit a change of variables so that

$$\int_a^b \frac{dx}{\sqrt{x^3 - 1}} \;=\; \int_A^B \frac{dy}{\sqrt{\text{quartic in } y}}$$

[05.2] For lattice $\Lambda \subset \mathbb{C}$, express $\sum_{\lambda \in \Lambda} \frac{1}{(z-\lambda)^5}$ in terms of $\wp(z)$ and $\wp'(z)$. Do not worry about explicit determination of constants, but explication of them will get extra credit.

[05.3] Let ω_1, ω_2 be a basis for a lattice $\Lambda \subset \mathbb{C}$. Express $\wp(z + \frac{\omega_1}{2})$ as a rational function of $\wp(z)$. Do not worry about explicit determination of constants, but explication of them will get extra credit.