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Complex analysis examples 01

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[This document is http://www.math.umn.edu/~garrett/m/complex/examples_2020-21/cx_ex_01.pdf]

If you want feedback from me on your treatment of these examples, please email me your work by Friday, Sept 18, as a PDF.

[01.1] Express the two values for \sqrt{i} in terms of radicals.

[01.2] Determine all values of i^i .

[01.3] Derive the usual formula for $\sin(z+w)$ by using e^z .

[01.4] Express $\cos 5x$ as a polynomial in $\cos x$ and $\sin x$.

[01.5] By *mere algebra*, write a power series expansion near $z = 0$ for

$$f(z) = \frac{1}{(z-1)(z-2)}$$

[01.6] Determine the radius of convergence of $\sum_{n \geq 1} \frac{3^n}{n(n+1)(n+2)} z^n$.

[01.7] Determine the radius of convergence of $\sum_{n \geq 1} \frac{n!}{n^n} z^n$.

[01.8] For two complex numbers a, b , with b not a non-positive integer, show that the radius of convergence of

$$\sum_{n \geq 0} \frac{a(a+1)(a+2) \dots (a+n-1)(a+n)}{b(b+1)(b+2) \dots (b+n-1)(b+n)} z^n$$

is at least 1.

[01.9] From the very definition of convergence, show that when the partial sums of a series $a_1 + a_2 + \dots$ are *bounded*, and when the elements of the sequence $\{b_n\}$ are *positive* (real) and *go to 0 monotonically*, then the series $\sum a_n b_n$ *converges*.

[01.10] Show that the function $f(z) = \sum z^n/n^2$ on the open disk $|z| < 1$ extends to a *continuous* function on the *closed* unit disc.
