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Complex analysis examples 08

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[This document is http://www.math.umn.edu/~garrett/m/complex/examples_2020-21/cx_ex_08.pdf]

If you want feedback from me on your treatment of these examples, please email your work to me by Friday, April 23.

- [08.1] Determine the genus of the curve $y^2 = x^5 - 1$.
- [08.2] Show a change of variables to convert $y^2 = x^6 - 1$ to something of the form $y^2 = \text{quintic in } x$.
- [08.3] Determine the genus of the curve $y^3 = x^3 - 1$.
- [08.4] Determine the genus of the curve $y^3 = x^4 - 1$.
- [08.5] Determine the ramification above $x = 0$ in the ramified cover $(x, y) \rightarrow x \in \mathbb{P}^1$ where $y^5 + xy^2 + x^2 = 0$.
- [08.6] Determine the ramification above $x = 0$ in the ramified cover $(x, y) \rightarrow x \in \mathbb{P}^1$ where $y^5 + x^2y^2 + x^2 = 0$.
- [08.7] Show that a ramified cover $f : E_1 \rightarrow E_2$ of elliptic curves E_j must actually be *unramified*, that is, not ramified at any point.
- [08.8] Show that in a ramified cover $C_1 \rightarrow C_2$ of compact connected Riemann surfaces, the genus of C_1 must be at least the genus of C_2 .
- [08.9] Determine the points z such that there is ramification over z in the ramified covering $(z, w) \rightarrow z$ from the curve $w^5 + 5zw + z^3 = 0$.
- [08.10] Let z_1, \dots, z_n be points in \mathbb{P}^1 . Determine the dimension of the space of meromorphic functions on \mathbb{P}^1 with poles at most at $\{z_1, \dots, z_n\}$, counting multiplicities.
- [08.11] Let ζ_1, \dots, ζ_m and z_1, \dots, z_n be points in \mathbb{P}^1 . Determine the dimension of the space of meromorphic functions on \mathbb{P}^1 with poles at most at $\{z_1, \dots, z_n\}$, counting multiplicities, and zeros (at least) at ζ_1, \dots, ζ_m .
- [08.12] Let z_1, \dots, z_n be points on an elliptic curve $E = \mathbb{C}/\Lambda$. Determine the dimension of the space of meromorphic functions on E with poles at most at $\{z_1, \dots, z_n\}$, counting multiplicities.
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