Exam by <u>email</u> Fri, Feb 04 02 Feb 2022

(or at other times by arrangement — just let me know)

Sorry again about Zoom glitch today — "It's not my fault!"

(At least) 2 things that deserve a bit of follow-up attention:

- ((convergence of) Fourier series
- chain rule ... with ∂ & ∂

I don't want to write much formulaic stuff about F.-series here, but, rather, re-emphasize that the general idea pre-dates Fourier by 100+ years, but it was Fourier who optimistically/aggressively promoted the <u>universality</u> of such expressibility. Also, his first paper was blocked due to the "scandalousness" ... so Dirichlet inaccurately gets credit for some basic theorems, & for the "~~Dirichlet~~" Fourier kernel.

About a chain rule for $\partial \left(= \frac{\partial}{\partial z}\right)$ and $\bar{\partial} \left(= \frac{\partial}{\partial \bar{z}}\right)$.

Yes, on one hand, we can just ~~revert~~ unwind to the literal, real variables versions

— to prove that harmonic $\circ$ holo is harmonic

$$\frac{\partial}{\partial z} = \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y}\right), \text{ etc.}$$

But, there is some interest/utility in seeing how to

Correctly operate the $\partial, \bar{\partial}$ "machine"!

(This is also typeset, tersely, in "discussion 05")

To begin, of course "the chain rule" is usefully thought of as about causality/change, so, beyond notational persiflage (!),

"change in $f(g(x), h(x))$ due to change in x is

change-in-f-due-to-first-argument $\times$ change in g due to change in x

$+$ change-in-f-due-to-second arg $\times$ change in h due to change in x"

The correct manifestation of this of the mysterious $z, \bar{z}$ & $\partial, \bar{\partial}$ is not immediately clear!

☺

As in the typeset discussion 05, using h_1 & h_2 for partial derivs w.r.t. 1st & 2nd arguments is vastly better than giving the arguments supposedly inviolate names, such as $z, \bar{z}$. ☺ So, for example, (with general f, g $\mathbb{C}$-valued on $\mathbb{C}$)

$$\frac{\partial}{\partial z}(f \circ g) = ((\partial f) \circ g) \times \partial g$$

$$+ (\bar{\partial} f \circ g) \times \partial g$$

because the derivatives with respect to the two arguments to f are ∂f & $\bar{\partial} f$.

Lesson: the chain rule is well worth thinking about! ☺