Exam by email Fri, Feb 04

(or at other times by arrangement — just let me know)

Sorry again about Zoom glitch today — "It's not my fault!"

(At least) 2 things that deserve a bit of follow-up attention:

- Convergence of Fourier series
- Chain rule — with $\partial F$

I don't want to write much formulaic stuff about Fourier series here, but, rather, re-emphasize that the general idea pre-dates Fourier by 100+ years, but it was Fourier who optimistically/progressively promoted the universality of such expressibility.

Also, his first paper was blocked due to the "Scandalousness" ... so Dirichlet inaccurately gets credit for some basic theorems, I for the "Dirichlet" Fourier kernel.
About a chain rule for \( \frac{\partial}{\partial z} = \frac{2}{2z} \) and \( \frac{\partial}{\partial \bar{z}} = \frac{2}{2\bar{z}} \).

Yes, on one hand, we can just \[\text{\textit{unwind}}\] the literal, real variables versus

\[ \frac{2}{\partial z} = \left( \frac{2}{2x} - i \frac{2}{2y} \right), \text{ etc.} \]

But, there is some interest in seeing how to \[\text{\textit{prove}}\] that harmonic stuff is harmonic.

Correctly spell the \( \partial, \bar{\partial} \) "machine":

(\text{This is also typeset in "discussion 05"})

Tenderly

To begin, of course "the chain rule" is useful thought of as about \textit{causality/change}, so, beyond notational persiflage (!).

\( \Delta \) change in \( f(g(x), h(x)) \) due to change in \( x \), is

\( \Delta \) change in \( f \) due to first argument \( x \) change in \( g \) due to change in \( x \)

+ \( \Delta \) change in \( f \) due to second \( \text{\textit{second}} \) \( g \) \( x \) due to change in \( x \).
The correct manifestation of this of the mysteries $z, \overline{z}$ & $\overline{z}, \overline{\overline{z}}$ is not immediately clear.

As in the typeset discussion OS, using $h_1$ & $h_2$ for partial derivs w.r.t. $\overline{z}$ & the 2nd arguments is better than joining the vastly arguments supposedly irrevocable names, such as $z, \overline{z}$. So, for example, \[ \frac{\partial}{\partial \overline{z}} (f \circ g) = (\partial f / \partial g) \cdot dg + (\partial f / \partial \overline{g}) \cdot d\overline{g} \]

\[ \text{(-valued on } \mathbb{C}) \]

because the two arguments to $f$ are $\overline{g}$ & $\overline{f}$ of course.

Lesson: the chain rule is well worth thinking about. \(\square\)