Exam on Fri (hour system, email....)

If you want delay (+ examples!) tell me...

Next week = spring break! :)

For given lattice \( \Lambda \), \( f \) & \( f' \) relations:

\[
\delta(2) = \frac{1}{2^2} + \sum_{\lambda \in \Lambda} \left( \frac{1}{(z^2 - \lambda)^2} - \frac{1}{\lambda^2} \right)
\]

\[
P \begin{cases}
\delta'(2)^2 = 4\delta(2)^3 - g_2 \delta(2) - g_3 \\
g_2 = \sum_{\lambda \in \Lambda} \frac{1}{\lambda^4} \\
g_3 = \frac{140}{3} \sum_{\lambda \in \Lambda} \frac{1}{\lambda^6}
\end{cases}
\]

"attached to" lattice

"frens of lattice" if

"= a thing"
(\(f = \text{note}\)): look at Laurent coeff of \(f\) \((f_0',)^2 \& f^3 \& p\), etc. @ 0.

After analyzing to cancel pole part

(\& can't \(\sim g_3\))

What remains is an integral duality process

\[
\text{const} \left( \sim \Lambda \right)
\]

\(\delta_{\text{var}} = 0 \Rightarrow \text{it's 0.}

\exists \text{computing } \Lambda \text{ straightforward, but messy to execute.}

\exists \text{many cases (s) more relations among}

(temp. note \(h\), "e" for "Einstein")

\[
\sum_{\theta \neq \lambda \in \Lambda} \frac{1}{\gamma_{2n}}
\]

\(E_{2n}(\Lambda)\)-convention, normalized

\(\lambda \leftrightarrow -\lambda \ldots \)

(\(\lambda \equiv 0 \text{ for } 2n \in 2\mathbb{Z}\))
As in pg of W's cubic equation, explicate
Laurent exp of \( f' @ 0 \)!

(not completely docs that \( f' \) is \( \Lambda - p' \),
then \( f' \) is -- need lemma)

\[ f(2) = \frac{1}{2^2} + \sum_{\nu \neq \lambda \in \Lambda} \left( \frac{1}{(2-\lambda)^2} - \frac{1}{\lambda^2} \right) \]

So
\[ f(2) - \frac{1}{2^2} = \text{holo } @ 2 = 0 \]

= p.s. exprn

(which, when added to \( \frac{1}{2^2} \),
serves Laurent exprn)

Uniqueness of p.s. & of Laurent

expn!

No matter tricky answer is same.
Example of "self-deception" (?!) 

$$\sum_{m \in \mathbb{Z}} \frac{1}{2^m} = \left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots \right)$$

$$+ \left( \frac{2}{2} + \frac{2^2}{2^2} + \frac{2^3}{2^3} + \ldots \right)$$

$$= \frac{1}{1 - \frac{1}{2}} + \frac{2}{1 - 2}$$

$$= \frac{2}{2 - 1} + \frac{2}{1 - 2} = 0$$
Can compute p.s. coef of \( f(z) - \frac{1}{2^2} \) by Taylor-Maclaurin (not Cauchy integral form)

\[ n^{th} \text{ p.s. coef is } (f_0(z) - \frac{1}{2^2})^{(m)} \]

\[ 2^0 \]

\[ m! \]

\[ f_0(z) - \frac{1}{2^2} \text{ is even, so odd-order coef are 0} \]

\[ \frac{(f_0(z) - \frac{1}{2^2})^{(2n)}}{2^0} = (n=0, \text{ its 0; count term} \text{ 00}) \]

\[ \sum \frac{(-2)(-3)(-4)\ldots(-2n-1)}{(2n)!} (z^2 - \lambda)^{2n+2} \]

\[ z = 0 \]
\[ f(z) = \frac{1}{z^2} + O(1) + 2\epsilon_4 z^2 + 5\epsilon_6 z^4 + 7\epsilon_8 z^6 + \ldots \]

\[ \text{An odd-order term} = 0 \]

\[ \text{Uniform in } \lambda \]

\[ f' = 4f^3 - g_2 f - g_3, \quad (\text{no info about } e_2\text{'s}) \]

\[ 60e_4 \quad 140e_6 \]

\[ \text{Entails many relations among } e_{2n}\text{'s} \]

In fact, \( e_7 \) and \( e_6 \) determine all others?
Eg. diff W-reln:

\[ 2 \beta \beta'' = 12 \beta^2 \beta' - g_2 \beta' \]

\[ \dot{\beta}'' = 6 \beta - \frac{g_2}{2} \]

\[ \frac{d}{dz}: \boxed{\dot{\beta}''' = 12 \beta \beta'} \quad (g's \_gone\_\_\_\_\_\_\_\_) \]

expand laurent expn &

get recursion for \( e \)

\( 2k > 4, 6 \)

in terms of \( e_t, e_\xi \)

\( e \)'s are values of \( \xi \)

Not idle; \( e \)'s are values of \( \xi \)

L-seqs...

\( \xi \) values of \( \xi \)

fam. L-seqs dep. on \( \xi \)-many

"new" numbers (transcendental?!)
Change of pace: show a discrete subgroup of a top. $G$ is closed.

For subset $X$ of top $G$, $Y$ to be discrete is $\forall x \in X$, $\exists$ open $U \ni x$ in $Y$.

Let $X \cap U = \{x\}$.

Example, $\{\frac{1}{n} : n=1,2,3,\ldots\}$ is discrete in $\mathbb{R}$.

-- Not closed (in $\mathbb{R}$):

$O$ (a set) is a limit pt., not a pathology.
A top gp is

\[ \begin{cases} G \times G \to G \text{ by mult is cont.} \\ \text{top} \\ \text{op} \\ G \to G \text{ by inverse is cont.} \end{cases} \]

(???) \( G \) is loc. cnt. \& Hausdorff (??)

Hence !!!

(all metric gps are H. v. c.)

if \( y_1 \neq y_2 \) in \( Y \), then \( \exists \) open 

\( U_1 \) \& \( y_1 \in U_1 \) \&

\( U_2 \) \& \( y_2 \in U_2 \) \&

\( U_1 \cap U_2 = \emptyset \)
Silly-tiny example of (non-medical!)

$\exists \text{top, sp, not } H$.

$Y$ is $Y + \text{ a set of subsets of } Y$, "open sets".

At $\emptyset$ is open, $Y$ is open.

Arb $U$'s of opens are open.

Finite $U$'s.

Why this is good/useful?

$Y = \{0, 1\}$ & opens are $Y \& \emptyset$.

"Indiscrète top" $\exists$; illustrates scope of $\Rightarrow$.

$\Rightarrow$ ADD ADJECTIVES, defn/axioms.
$M$: discrete subsp of $\mathcal{G}$ top sp $G$

cop. det. opt?

is (top) closed:

let $g \in G$, $g \neq 1$, but $g \in \mathcal{I}$.

Then $\forall U \in \mathcal{I}$, $g^{-1}U$ is open in $g$,

$s \in g^{-1} \cap \Gamma_0$

$\Rightarrow$

$s \in \Gamma_0 \cap (gu)$

$\Rightarrow$

$s \in \Gamma_0 \cap gu$

$\Rightarrow$

$s \not\in \mathcal{I} \Rightarrow$ (exists many)

So $\frac{-1}{\frac{-1}{(gu)\cdot (gu)}}$

$\cap \mathcal{G}^0 gu = \bar{u}^{-1} u$;

$\{\bar{u}', u e u\}$
Cart. of mult & inverse \[ \exists \text{id. of } G \]

\[ \forall \text{ open } U \ni 1 \quad \text{ or open } U \ni 1 \]

\[ \exists \overline{U} \ni U \leq \overline{U} \]

That is, \[ \forall \text{ open } U \ni 1 \quad \exists \overline{U} \quad \text{choice } \{ \overline{U} \} \in G \quad \text{open } \overline{U} \ni U \]

\[ \tilde{g}_1 \neq 1 \quad \text{in } U \quad \Rightarrow \quad \#_G \text{ has } \tilde{g}_1 \quad \tilde{g}_2 \quad \text{not equal} \]

\[ U \cap \Gamma = \{ \tilde{g}, \tilde{g} \} \quad \text{discretize.} \]

So cannot be \[ \exists g \in \Gamma \quad \text{but} \]

\[ g \neq \Gamma \]