(*December 11, 2005*)

Exercises 9

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[9.1] Show that d/dx is a continuous map $H_s(S^1) \longrightarrow H_{s-1}(S^1)$ for all real s.

[9.2] For a norm $v \longrightarrow |v|$ that arises from a hermitian inner product (as $|v| = \langle v, v\rangle^{1/2}$), show that the hermitian inner product can be recovered from the norm. (*Hint:* To begin, we have $|v+w|^2 - |v-w|^2 = 4\mathrm{Re}\langle v, w\rangle$.)

[9.3] Classify the *closed* subgroups of the circle S^1.

[9.4] Classify the *closed* subgroups of $\mathbb{R}^n$.

[9.5] Show that any *continuous* group homomorphism $\varphi : \mathbb{Z}_p \longrightarrow \mathbb{C}^\times$ has *finite* image.

[9.6] For positive integer n let T_n be an operator on functions on the circle defined by

$$(T_n f)(x) = \frac{f(\frac{x}{n} + f(\frac{x+1}{n} + f(\frac{x+2}{n} + \ldots + f(\frac{x+n-1}{n}}{n}$$

Describe all eigenfunctions for T_n. Do the T_n for varying n commute? What are common eigenvectors? *(One might also reflect on the fact that the summands in the expression for $T_n f$ are not well-defined on the circle, but live on the n-fold cover of the given circle. Thus, the operators T_n are best described as operators on functions on the ur-solenoid.)*