

(September 11, 2010)

## Modular forms and number theory exercises 01

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[mfms 01.1] Prove the Euler product expansion of the zeta function, namely, for  $\operatorname{Re}(s) > 1$

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}$$

A useful point is that

$$\frac{1}{1 - p^{-s}} = 1 + p^{-s} + (p^2)^{-s} + (p^3)^{-s} + \dots$$

Often this Euler product expansion is interpreted as a slightly analytic manifestation of the *unique factorization* in  $\mathbb{Z}$ .

Proper care for *convergence* is a non-trivial task, but worth doing once in one's life.

Part of the burden is merely notational, but the risks of bad notation are considerable.