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Modular forms and number theory exercises 07

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[mfms 07.1] Prove that $\mathbb{Z}[\sqrt{3}]$ is Euclidean. Letting $\gamma = \frac{1+\sqrt{5}}{2}$, prove that $\mathbb{Z}[\gamma]$ is Euclidean.

[mfms 07.2] (*) Classify the (topologically) closed subgroups of $\mathbb{R}$ with addition: there are the obvious $\{0\}$ and $\mathbb{R}$ itself, and apart from these there are exactly subgroups $\mathbb{Z} \cdot \tau$, for $0 \neq \tau \in \mathbb{R}$.

[mfms 07.3] (*) Use the latter result to show that the group of integer solutions to $m^2 - Dn^2 = 1$ (with integer non-square $D > 0$) has *rank* at most 1. (Hint, use exponential/logarithm.)

[mfms 07.4] (**) Classify the closed subgroups of $\mathbb{R}^n$.