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Modular forms and number theory exercises 11

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[mfms 11.1] Fill in the sketches from class:

Identify the irreducible (complex) representations of the dihedral group G with $2n$ elements, in terms of the one-dimensional irreducibles of the normal subgroup N of order n. Be careful about n odd versus even, and about special cases.

Determine the decompositions into irreducible representations of $\sigma \otimes \tau$ for all irreducible representations σ, τ of G.

[mfms 11.2] * Let k be a finite field with q elements, and let G be the group

$$G \;=\; \{\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \;:\; a \in k^\times,\, b \in k\}$$

There is the normal subgroup

$$N \;=\; \{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \;:\; b \in k\} \;\approx\; k \;\text{ (with addition)}$$

and subgroup

$$A \;=\; \{\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \;:\; a \in k^\times\} \;\approx\; k^\times \;\text{ (with multiplication)}$$

Classify the irrreducibles of G in terms of the (one-dimensional) irreducibles of N which appear, and the action of A on these subrepresentations. Describe the decomposition of tensor products of two irreducibles into irreducibles.