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Examples Diagnostic

Paul Garrett garrett@math.umn.edu <http://www.math.umn.edu/~garrett/>

[This document is http://www.math.umn.edu/~garrett/m/real/examples_2018-19/Diagnostic.pdf]

[Diagnostic.1] What does it mean for a real-valued function on a subset Ω of \mathbb{R}^n to be *continuous*?

[Diagnostic.2] What does it mean for a real-valued function on a subset Ω of \mathbb{R}^n to be *uniformly continuous*?

[Diagnostic.3] What does it mean for a sequence $\{f_n\}$ of real-valued functions on a subset Ω of \mathbb{R}^n to approach another \mathbb{R} -valued function on Ω *pointwise*?

[Diagnostic.4] What does it mean for a sequence $\{f_n\}$ of real-valued functions on a subset Ω of \mathbb{R}^n to approach another \mathbb{R} -valued function on Ω *uniformly (pointwise)*?

[Diagnostic.5] For a sequence $\{f_n\}$ of continuous functions on an interval $[a, b]$, approaching a function f *uniformly* pointwise, why is the limit f continuous?

[Diagnostic.6] For a sequence $\{f_n\}$ of continuous functions on an interval $[a, b]$, approaching a function f *uniformly* pointwise, why is $\lim_n \int_a^b f_n(x) dx = \int_a^b f(x) dx$?

[Diagnostic.7] Give several examples of sequences $\{f_n\}$ of continuous real-valued functions on an interval $[a, b]$, converging pointwise, such that $\lim_n \int_a^b f_n(x) dx \neq \int_a^b \lim_n f_n(x) dx$.

[Diagnostic.8] What does it mean for a subset of \mathbb{R}^n to be *compact*?

[Diagnostic.9] Why is a continuous real-valued function on a compact subset of \mathbb{R}^n *uniformly* continuous?

[Diagnostic.10] What is a metric space?

[Diagnostic.11] What does it mean for a real-valued function on a metric space to be *continuous*?

[Diagnostic.12] What does it mean for a real-valued function on a metric space to be *uniformly continuous*?

[Diagnostic.13] What does it mean for a metric space to be *complete*?

[Diagnostic.14] What does it mean for a subset of a metric space to be *compact*?

[Diagnostic.15] What does it mean that \mathbb{Q} is *dense* in \mathbb{R} ?

[Diagnostic.16] What does it mean that a subset Y of a metric space X is *dense* in X ?

[Diagnostic.17] Why is \mathbb{Q} *countable*? Why is \mathbb{R} *uncountable*?
