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Review examples 00

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[This document is http://www.math.umn.edu/˜garrett/m/real/examples_2016-17/real-ex-00.pdf]

If you want feedback on your write-ups on any of these examples, please get your write-ups by Monday, 12 Sept, 2016.

[0.1] (There is not much hope in making sense of the outcome of an uncountable number of non-zero operations:) Let Ω be an *uncountable* collection of positive real numbers. Letting F range over all finite subsets of Ω, show that $\sup_F \sum_{\alpha\in F} \alpha = +\infty$.

[0.2] (The *archimedean property* of the real numbers:) Using the characterization of $\mathbb{R}$ as the metric-space completion of $\mathbb{Q}$, show that a real number x with $|x| < \frac{1}{n}$ for $n = 1, 2, 3, \ldots$ must be 0.

[0.3] Prove carefully that the *inf* of a *finite* set of (strictly) positive real numbers is (strictly) positive.

[0.4] Prove (or review the proof) that intervals $[a, b] \subset \mathbb{R}$ (with $-\infty < a < b < \infty$) are *connected* in the sense that they cannot be written as a disjoint union of two non-empty (relatively) open subsets. Use this to prove the intermediate value theorem for continuous functions.

[0.5] Prove (or review the proof) that a continuous real-valued function f on a finite interval $[a, b] \subset \mathbb{R}$ assumes its *inf*. That is, there is a point $x_o \in [a, b]$ such that $f(x_o) = \inf_{x\in[a,b]} f(x)$.

[0.6] Prove (or review the proof) that a continuous real-valued function f on a finite closed interval $[a, b] \subset \mathbb{R}$ is *uniformly* continuous: for all $\varepsilon > 0$ there is $\delta > 0$ such that, for all $x, y \in [a, b]$, $|x - y| < \delta$ implies $|f(x) - f(y)| < \varepsilon$.
