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Examples 03

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[This document is http://www.math.umn.edu/~garrett/m/real/examples_2016-17/real-ex-03.pdf]

For feedback on these examples, please get your write-ups to me by Wednesday, 12 Oct 2016.

[03.1] Show that ℓ^2 is *complete* as a metric space.

[03.2] Show that the characteristic function χ_E of a measurable set E is measurable.

[03.3] Show that the product of two \mathbb{R} -valued measurable functions on \mathbb{R} is measurable.

[03.4] Show that the set $[0, +\infty] = [0, +\infty) \cup \{\infty\}$ with basis consisting of intervals $[0, b)$, (a, b) for all real $0 < a < b$, and of sets $(b, +\infty) \cup \{\infty\}$ gives a *topology* on $[0, +\infty]$. Show that with this topology $[0, +\infty]$ is Hausdorff, compact, and countably-based.

[03.5] Let f be a $[0, +\infty]$ -valued measurable function on \mathbb{R} . Show that there is a sequence of non-negative real-valued *simple* functions f_n approaching f pointwise almost-everywhere, so that $\int f_n \rightarrow \int f$.

[03.6] Use Urysohn's lemma to prove that $C^o[a, b]$ is dense in $L^1[a, b]$.
