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Examples 13

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[This document is http://www.math.umn.edu/˜garrett/m/real/examples_2016-17/real-ex-13.pdf]

For feedback on these examples, please get your write-ups to me by Monday, April 24, 2017.

[13.1] On $\mathbb{R}^2$, compute the Fourier transform of $(x \pm iy)^n \cdot e^{-\pi(x^2+y^2)}$ for $n = 0, 1, 2, \ldots$. (*Hint:* Re-express things, including Fourier transform, in terms of $z = x + iy$ and $\overline{z} = x - iy$, $w = u + iv$, and $\overline{w} = u - iv$.)

[13.2] Let S, T be two compact, self-adjoint operators on a Hilbert space, and $ST = TS$. Show that there is an orthonormal basis for V consisting of simultaneous eigenfunctions for S, T.

[13.3] Show that $\varphi \to \int_{\mathbb{R}} e^{x^2} \varphi(x)\, dx$ is a distribution.

[13.4] Show that $\varphi \to \sum_{0 \le n \in \mathbb{Z}} \varphi^{(n)}(n)$ is a distribution.

[13.5] (Without invoking classification of distributions supported at a point) show that $\varphi \to \sum_{0 \le n \in \mathbb{Z}} \varphi^{(n)}(0)$ is *not* a distribution.

[13.6] Let T be a continuous/bounded self-adjoint operator on a Hilbert space V, with spectrum consisting of just two points $\lambda \ne \mu$. Show that the isomorphism $C^o(\{\lambda, \mu\}) \approx \overline{\mathbb{R}[T]}$ implies that V is the direct sum of λ and μ eigenspaces.
