Rules: 4 exams, in class
(see http://www.math.umn.edu/~garrett/m/real)

Calendar for ex/hwk + exams not sacred

3 notes

No final

1) Snow days → Zoom
2) Medium-sick days → Zoom

Exams by email, on honor system
(keep those files small?)
Brief recap about distributions / a.k.a. "generalized functions."

$D \subset C^0(R)$, Schwartz functions $\subset L^2(R)$.

want? $\mathcal{S}^*$ map $\rightarrow D^*$

$cpt$-sp$^+$ tempered distributions $\rightarrow \mathcal{S}^*$

In general $V \overset{\subset}{\otimes} V^*$

$D \overset{?}{\rightarrow} D^*$ by $\varphi \rightarrow \text{integrad-against-}\varphi$

View $D^*$ as ext'n of $D$!

Define ops on $D^*$ (consistently w/)

by $\varphi'(\varphi) = -\varphi(\varphi') - \text{i.b.p.}$
\( \Delta \) for \( u \in \mathcal{D}'^+ \) (\( \ri : \mathcal{D}^+ \to \mathcal{D} \))

\[
\hat{u}(\phi) = u(\check{\phi}) \check{\varepsilon}
\]

+ weak-dual top on duals, \( \mathcal{D} \)

"+ "

(Thm) \( \mathcal{D} \) dense in all of these, etc.

So also for \( u \in \mathcal{D}'^+ \), \( \Delta \) \( u_n \xrightarrow{\Delta} u \)

\[ \Delta \text{ as well} \]

\[ \frac{d}{dx} u = \lim_{n} \frac{d}{dx} (w^+ - \lim_{n} u_n) \]

\[ \hat{u} \text{ distible} \]

\[ \hat{u} = w^+ - \lim_{n} \left( \frac{d}{dx} u_n \right) \]

"usual"

can eval by "classic"

\[ \lim \text{ of diff quot} \]

\[ \Delta \text{ of } \theta \]

\[ \Delta \]
\[ \left( \frac{\sin x}{x} \right)^\pm = ? \quad \text{(is it \textit{cvgf}, not \textit{abs})} \]

But, "know" \[ \left( \frac{1}{x} \right)^\pm \approx \frac{\sin x}{x} \]

all even, so \( \wedge \equiv \vee \)

not by \( \text{Sh} \), but

by \( F \), inversion.

Next: "Soboler spaces" + \( \text{Besov} \) \( \text{Lem} \) 1906
\( \text{G}, \text{FvBo} \) 1907

(True) version of Dirichlet
Minimum principle
(not in \( C^0, C^1, \ldots \))

differentialability
via Hilbert sp's?
First, $H^k$ on $T = \text{circle} = \mathbb{R}/\mathbb{Z} \cong [0, 2\pi]$.

On $T$, order $L^p$ regular Borel measures.

\[ D \hookrightarrow C^\infty \hookrightarrow C^0 \hookrightarrow L^2 \hookrightarrow (C^0)^* \hookrightarrow (C^1)^* \hookrightarrow \cdots \]

Markov–Riesz–Kahane–Riener

\[ \frac{d}{dx} (C^0)^* \subseteq (C^0)^* \text{?!} \]

In this sense, Yilun. \( \vdash \)

Not $H^p$'s.

No min princ.

No $\perp$.

No $\perp$ bases.
Instead, want Hsps grading range of funs

$\mathbb{D} \rightarrow C^1 \rightarrow H^1 \rightarrow C^0 \rightarrow L^2 \rightarrow (C^0)^* \rightarrow H^{-1} \rightarrow (C^1)^* \rightarrow H^2 \rightarrow \cdots \rightarrow H^*$

Hsps!

Sobolev imb. thm: $H^k \subseteq C^k \cap C^\infty$

+ more: F. - ego of dishes