1.11. How many injective functions are there from $\{1,2,3\}$ to $\{1,2,3,4,5\}$?

Solution. Let f be such a function. Then $f(1)$ can take 5 values, $f(2)$ can then take only 4 values and $f(3)$ - only 3. Hence the total number of functions is $5 \times 4 \times 3 = 60$.

1.13. How many surjective functions are there from $\{1,2,3,4,5\}$ to $\{1,2,3,4\}$?

Solution. Every surjective function f sends some two elements of $\{1,2,3,4,5\}$ to the same element of $\{1,2,3,4\}$. There are $\binom{5}{2} = 10$ such pairs of elements. For a given pair $\{i,j\} \subset \{1,2,3,4,5\}$ there are 4!=24 surjective functions f such that $f(i) = f(j)$. Hence there are a total of $24 \times 10 = 240$ surjective functions.

1.18. Show that for a surjective function $f : A \to B$ there is a right inverse $g : B \to A$ so that $f \circ g = \mathrm{id}_B$.

Solution. For each $b \in B$ we can set $g(b)$ to be any element $a \in A$ such that $f(a) = b$. Since f is surjective, there is such an $a \in A$ for each $b \in B$. Then $f \circ g(b) = f(g(b)) = f(a) = b$, i.e. $f \circ g = \mathrm{id}_B$.

1.19. Show that for an injective function $f : A \to B$ there is a left inverse $g : B \to A$ such that g$\circ f = \mathrm{id}_A$.

Solution. For each $b \in B$ such that $b = f(a)$ for some $a \in A$, we set $g(b) = a$. This is well-defined since for each $b \in B$ there is at most one such a. Now pick some element $\alpha \in A$ and for each $b \in B$ such that there does not exist an $a \in A$ with $f(A) = b$ set $g(b) = \alpha$.

1.21. Verify that $\sum_{k=0}^{k=n} (-1)^k \binom{n}{k} = 0$.

Solution. Setting $x = 1$ and $y = -1$ in the formula $(x+y)^n = \sum_{k=0}^{k=n} \binom{n}{k} x^{n-k} y^k$ yields the result.