Different Flavors of Pi

π arises in many different contexts. There are many different methods to estimate the actual value of π. In this activity, we will use three different methods, and see which one gets us the closest!

Method 1: Infinite Series

Take $\frac{1}{1}$. Then add $\frac{1}{4}$. Then $\frac{1}{9}$. You should have something between 1 and 2. Turns out if you add $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}$, and keep going on forever, adding 1 over each square number, you end up with $\frac{\pi^2}{6}$. The fancy way of writing this is $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$. We can't go on forever, but if we want to get close, we can just go partway!

Here's how to estimate π:

- Add up the first 10 terms of the series (everything up to $\frac{1}{100}$). Let's call this number s.
- We're pretending we don't know π already. So let's write x for our estimate of π. So the equation we have is $s=\frac{x^2}{6}$.
- Plug in your value of s, and solve for x. That's our estimate for π!

x was our estimate for π when we added up the first 10 terms. So let's call it x_{10}. Now find x_{20} the same way. x_{30}? How far do we have to go to make x start with 3.1? What about 3.14?

Method 2: Parallelogram

Cut out a big circle. Measure the radius r. Then fold the circle into 8 equal-sized wedges (or more!), like pizza slices. Cut the circle into those 8 wedges. Then put them together head to toe to form an almost-parallelogram. It's not quite a parallelogram, but the more wedges you use, the closer you get!

Measure the base b and the height h of the "parallelogram". Then the area is $A=bh$. But the area of the circle, which is the same number, is $A=\pi r^2$. So again let's have x be our estimate. Then $bh=xr^2$. Plug in b,h, and r, and solve for x.

Method 3: Buffon's Needle

A toothpick is 2.625 inches long. The lines on the following page are the same distance apart. Toss a toothpick randomly onto the sheet of paper. It turns out that the probability of the toothpick landing on a line is $\frac{2}{\pi}$.

- Toss 10 toothpicks. How many landed on a line?
- What was the probability p that each of the toothpicks you threw landed on a line? (Divide the previous number by 10)
- Let our estimate for π be x. We have $p = \frac{2}{x}$. Plug in your value for p, and solve for x. Was it close?
- Try it again with 100 tosses (maybe group up with your table for this).