"Ghastly exercise in notations and definitions"

<u>19th century</u>

$\Theta(z) = \sum_{n \in \mathbb{Z}} e^{2\pi i n^2 z}$

$E_k = \sum_{(c,d)=1} \frac{1}{(cz+d)^{2k}}$

Δ fn, j-invariant   elliptic fn.

Jacobi

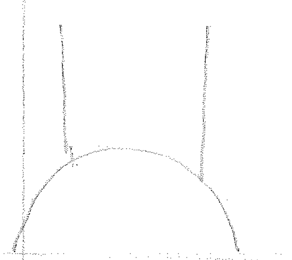
Poincaré : uniformization of R.S. $\sum_{\gamma \in \Gamma} f(\gamma \cdot g)$

<u>Felix Klein</u> (1880s)

Γ: $SL_2(\mathbb{Z}) \curvearrowright \mathfrak{h}$ · general mod form. (holomorphic)

hyperbolic geometry.

Γ = fundamental group.



- The usual counting result. (Hurwitz.)

- ~~holomorphic~~

<u>Typical results.</u> - fnl eqn. of θ-functions

1916 Ramanujan

$$\mathcal{T}(s) = \sum \frac{\tau(n)}{n^s} = \prod_p \left(\frac{1}{1 - \tau(p) p^{-s} + p^{11-2s}} \right)$$

mellin transform

$\sum_{n \geq 1} \tau(n) e^{2\pi i n z}$ is a mod form

$|\tau(p)|^2 \leq 4 \cdot p^{11}$

1917: Mordell proves using Hecke operators

1936* Hecke generalizes to mod forms in general & also congruence subgroups

$f(z) = \sum a_n e^{2\pi i n z}$

$\sum \frac{a_n}{n^s}$ has Euler product

~~1944~~ ~~Maap~~ (1939, 40) $L(s, f \times g)$ R.S.

Rep. theory

before 1920s -30s: Frobenius, Schur, Weyl

- rep of finite & compact Lie groups (f.d)

- 1930s Relativistic Quantum mech.. infinite dimen rep of SL_2 $\widetilde{SO(3,1)} \cong SL_2(\mathbb{C})$ space-time

  - $\exists \mathcal{H}$ - h.s. $\hookleftarrow$ irrep of $SL_2\mathbb{C}$ | has to be inf-dim for commutation.

1939, Wigner rep.

Physics!

1940s. Bargmann $L^2(\underline{Sl_2(\mathbb{R})})$ w.r.t Casimir

Δ

<u>1950</u>, Complex s.s Lie gps e.g. $Sl_n(\mathbb{C})$
not $Sl_n(\mathbb{R})$

<u>1949</u> Maaß... "Theta correspondence" $\underline{\Delta}$. e'g. cong. subgps.

<u>1950s</u> $\Delta|$ $\mathfrak{h} = Sl_2(\mathbb{R}) / SO(2)$

(Bourbaki's influence) Δ..

notion of cusp form $\int_0^1 f(x)\, dx = 0$ $(x+iy)$

$f =$ cusp $\oplus$ ~~const~~ $\oplus$ "Eis"
E_s
$\mathbb{I}$ (mero cont.)

(Godement's remark 1966 Boulder)

<u>Sel</u>: "Trace formula": ∞ cusp for $Sl_2(\mathbb{Z})$

1950

<u>Algebraic</u> <u>side</u>:

- 1940s Chevalley idèles reformulate class field thy
- 1950 Iwasawa Tate $\mathbb{I}/k^{\times}$ cpt
modernized Hecke $L(s,\chi) = \sum \frac{\chi(n)}{n^s}$

1952: Gelfand-Fomin note Hecke op $\leftrightarrow$ $Sl_2(\mathbb{Q}_p)$ rep^{s}

1958 & -60 : Adeles & alg groups & mod forms as functions on $Z^+ G_k \backslash G_{\mathbb{A}}$.

1960 Gelfand - PS : discrete decomposition of cusp forms. $c_p f = 0$

Langlands enters picture

1962-64 : Δ decomposition

- meas contin of Eis series

- "residues" poles of Eis.

1967

$$E_s \rightsquigarrow c_p E_s \sim \int_{N_k \backslash N_{\mathbb{A}}} E_s(ng)\, dn = (\;) + c_s(\;)$$

$c_s \sim \prod_p$ suggests defn of L-group.

1963 : Satake rep. of $G(\mathbb{Q}_p)$ spherical rep

$f \rightsquigarrow L(s,f)$

- Langlands - Shahidi c_s.

- Rankin - Selberg

- Trace formula.

Maass

Theta correspondence.