Linear ODE: $D = \frac{d}{dx}$, $P(x) = x^n + c_1 x^{n-1} + \cdots + c_n$

$$P(D)y = e^{ax} P_d(x) \left\{ \begin{matrix} \cos bx \\ \sin bx \end{matrix} \right\}$$

($P_d(x)$: degree d polynomial)

$$P(r) = \prod (r - r_j)^{m_j} \quad \text{by algebra.}$$

Look at these factors!! Types:

$$(r-a)^m \;, \quad a \text{ real}$$

$$(r-a-\beta i)^m (r-a+\beta i)^m \;, \quad \left\{ \begin{matrix} a, \beta \text{ real} \\ \beta \neq 0 \end{matrix} \right\}$$

Let:

$\{$Multiplicity of $a \pm bi\} = M$, say.

$$\Downarrow$$

$$y_p = x^M e^{ax} \left[P_d(x) \cos bx + \tilde{P}_d(x) \sin bx \right]$$

Undetermined coefficient polynomials degree d