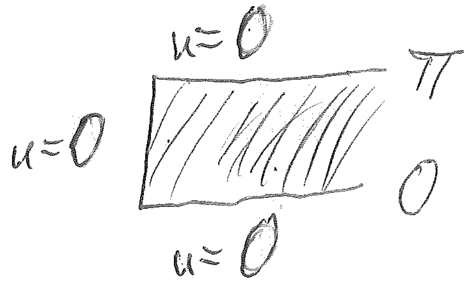
V. of P. Example

rectangular

135 prob 4



Given $|u| \leq C$

pde $\quad u_{xx} + u_{yy} + q(y) = 0$

$$u = \sum_{n=1}^{\infty} I_n(x) \sin(ny) \qquad I_n(0) = 0$$

$$\sum_{n=1}^{\infty} \left[I_n''(x) - n^2 I_n(x) \right] \sin(ny)$$

$$= -\sum_{n=1}^{\infty} q_n \sin(ny) = -q(y)$$

so

$$I_n''(x) - n^2 I_n(x) = -q_n \qquad (\text{ODE}) \quad n \geq 1$$

$$I_n(x) = \frac{q_n}{n^2} + C_1 e^{nx} + C_2 e^{-nx}$$

$C_1 \neq 0 \Rightarrow I_n(x)$ blows up as $x \to +\infty$ $\quad \boxed{\text{bad!}}$

Contradicts $|u(x,y)| \leq \sigma$.

$$I_n(x) = \frac{2}{\pi}\int_0^{\pi} u(x,y)\sin(ny)\,dy$$

$$|I_n(x)| \leq \frac{2}{\pi}\sigma\cdot\pi = 2\sigma$$

(50) $I_n(x) = \frac{q_n}{n^2} + C_2 e^{-nx}$ $\quad x \geq 0$

use $I_n(0) = 0$

$$I_n(x) = \frac{q_n}{n^2} - \frac{q_n}{n^2}e^{-nx}$$

$$\boxed{u = \sum_{1}^{\infty} \frac{q_n}{n^2}\left(1-e^{-nx}\right)\sin(ny)}$$

Easy!