"2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, ...

14.134725+, 21.022039+, 25.010857+, ...

$$\pi(x) \sim \frac{x}{\log x}$$

AND ALL THAT !!"

COURSE ANNOUNCEMENT – MATH 8280 – SPRING 2016
(Topics in Number Theory)
AN INTRODUCTION TO ANALYTIC NUMBER THEORY
Instructor: D. A. Hejhal

Beginning in the 19th century, it began to be realized (by Riemann, among others) that certain fundamental questions involving the ordinary integers, more specifically the primes, were amenable to study by bringing to bear on them methods of analysis, especially *complex* analysis.

In very loose terms, *analytic number theory* is that part of number theory whose results are obtained principally with the aid of constructs and techniques having at least one foot in some aspect of (either classical or modern) analysis. Today, for instance, in addition to complex, both harmonic and spectral analysis have begun to be used.

The purpose of this course, which should probably carry a number closer to 8001 (8009 would be apt!) is to offer students conversant in the standard advanced undergraduate courses in real, complex, and Fourier analysis, plus a bit of modern algebra, a kind of "gentle" introduction to analytic number theory, by coming in chiefly from the *multiplicative* side — that is to say, primes and constructs like the Riemann zeta function, $\zeta(s)$.

Analytic number theory is a subject steeped in history. One of its main theorems is the celebrated Prime Number Theorem from 1896 (and its counterpart for primes in arithmetic progressions). Several approaches to the PNT are now known. One of the best ways of getting a feel for analytic number theory, and what makes it tick, is to simply make a careful study of the various approaches to the PNT.

The basic plan of the first 2/3 of the course is to do exactly this – taking the time, where need be, to develop some interesting cognate material involving, e.g., aspects of complex (and real!) analysis pertinent for $\zeta(s)$ and the study of its zeros. Connections with the Riemann Hypothesis and (so-called) "explicit formula" for the prime counting function $\pi(x)$ will arise here.

Following that, as time permits, a few topics further afield (but still relatively gentle) will touched on. One possibility: some "nitty-gritty" numerical calculation of a few Riemann zeros and related zeros of $L(s, \chi)$, where χ is a multiplicative character.

The course format will primarily be lectures, guided in part by the classic books of Ingham and Davenport. Some unpublished course notes by A. Selberg and a recent AMS volume by Iwaniec/Kowalski will also prove useful.

To facilitate fixing a class time, any students interested in taking this course (or simply desiring further information) should contact the instructor in the very near future. {Email: hejhal@math.umn.edu}

Students interested in the course, but lacking some of the prerequisites, may, *after* a discussion with the instructor, be allowed to join the class and receive Math 5990 credit for it. (A slightly modified syllabus would then apply; early contact with the instructor is again encouraged.)