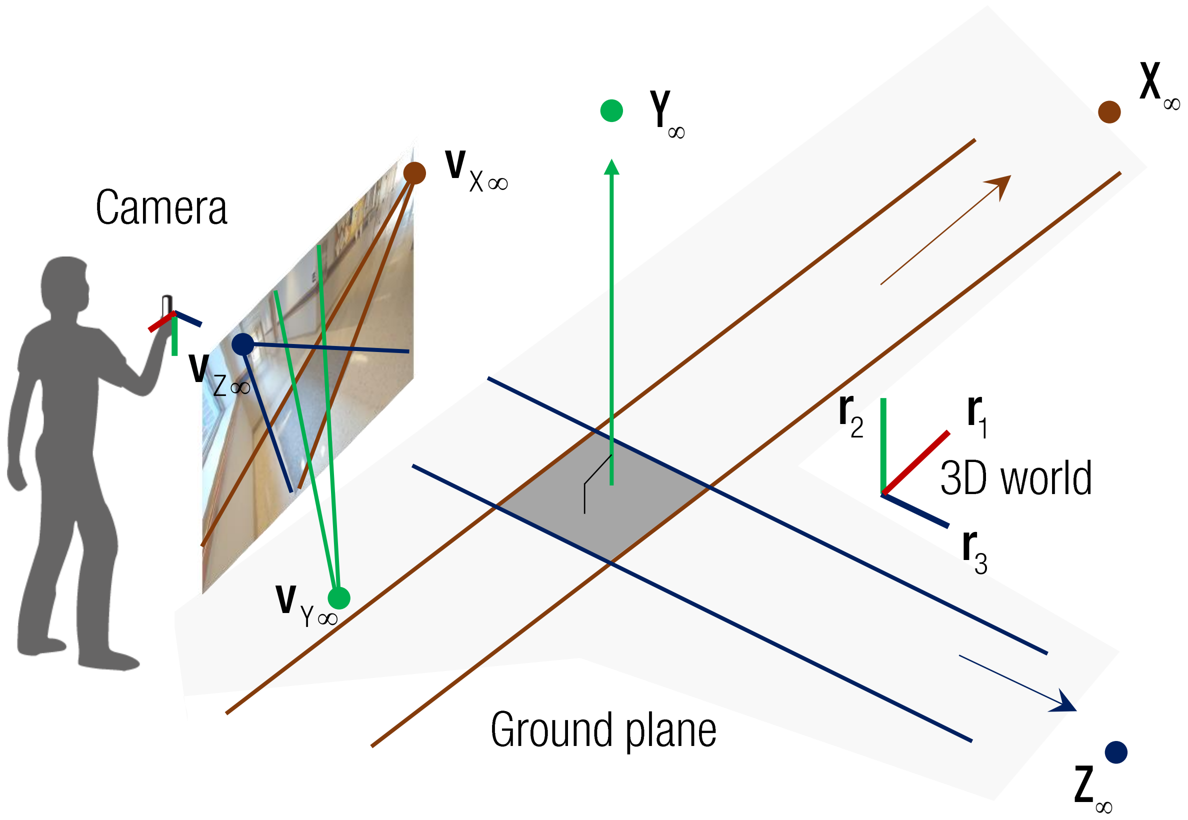
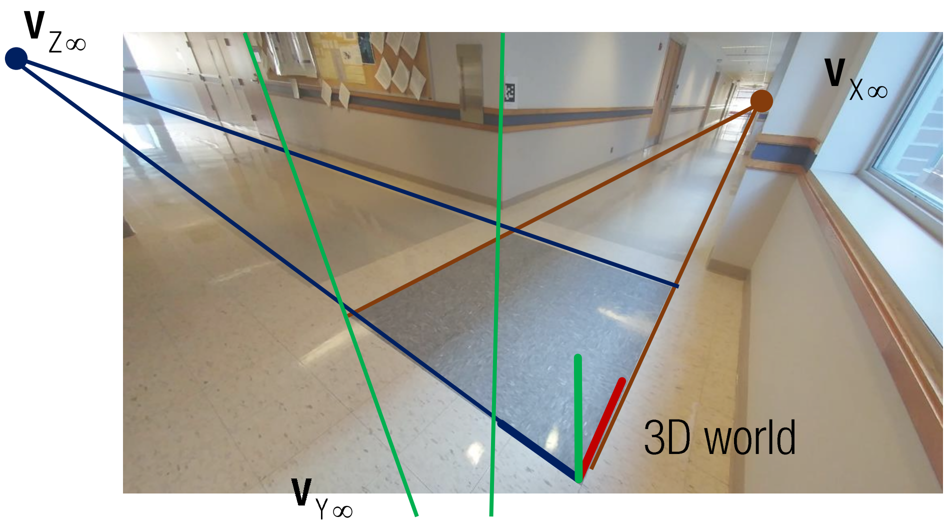
Camera Calibration

Announcement

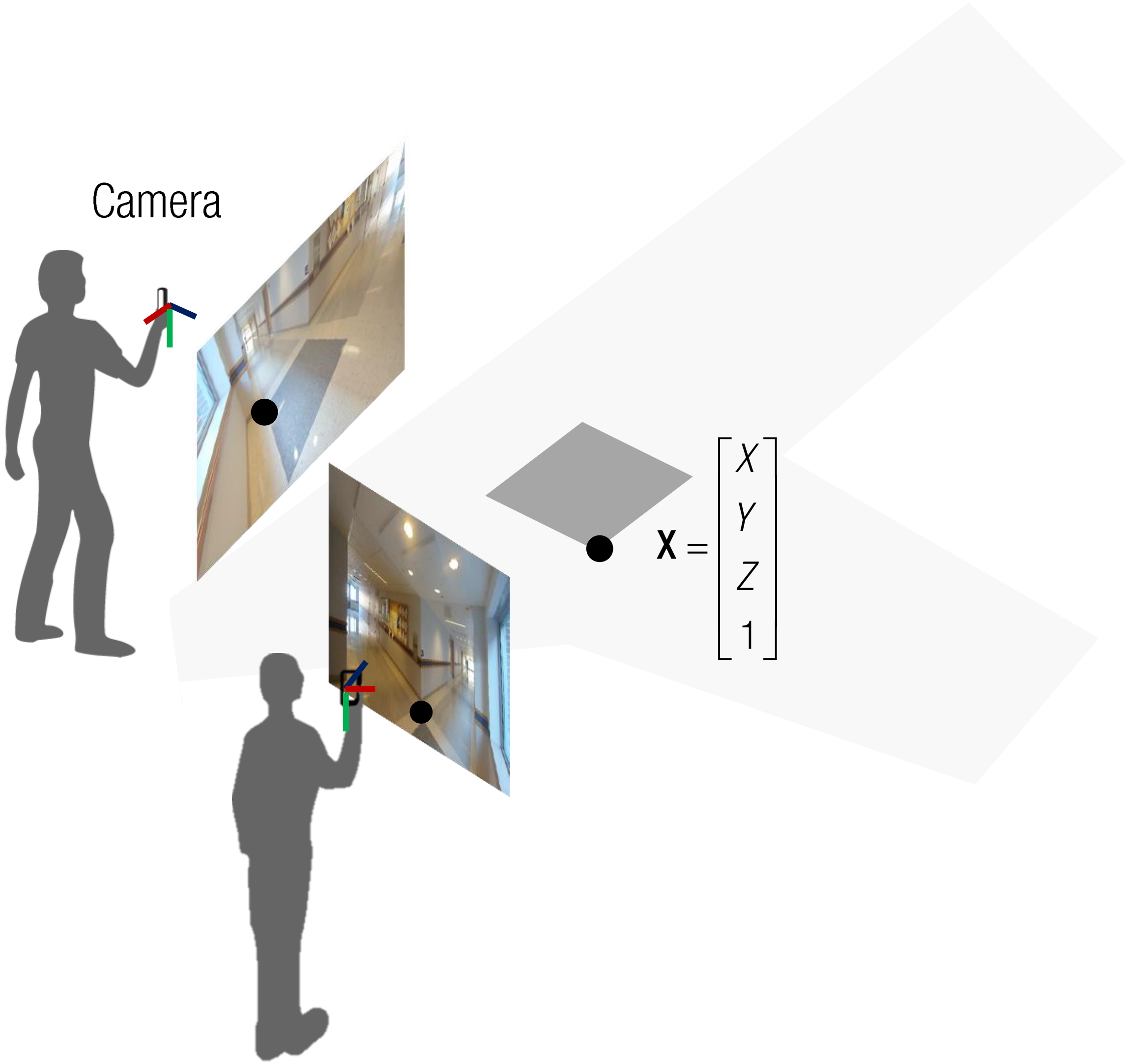
- HW #3 out today (start early!!!)
- HW #3 short presentation on Thursday (share your panorama!)
- HW #2 grading will be done by Friday
- Paper selection by next Tuesday (Feb 28)

Camera Calibration via Vanishing Points



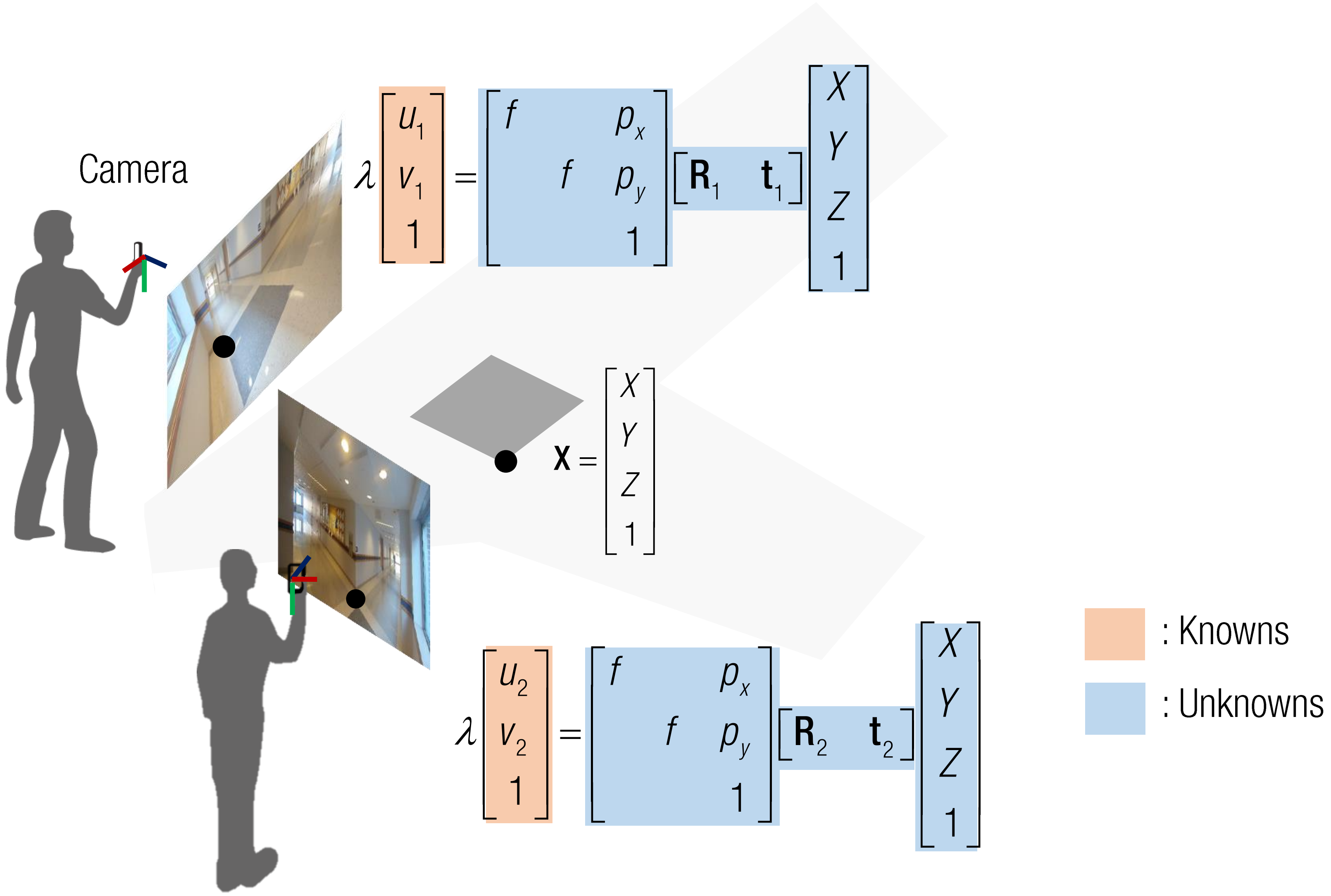


Multiview Camera Calibration



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Multiview Camera Calibration

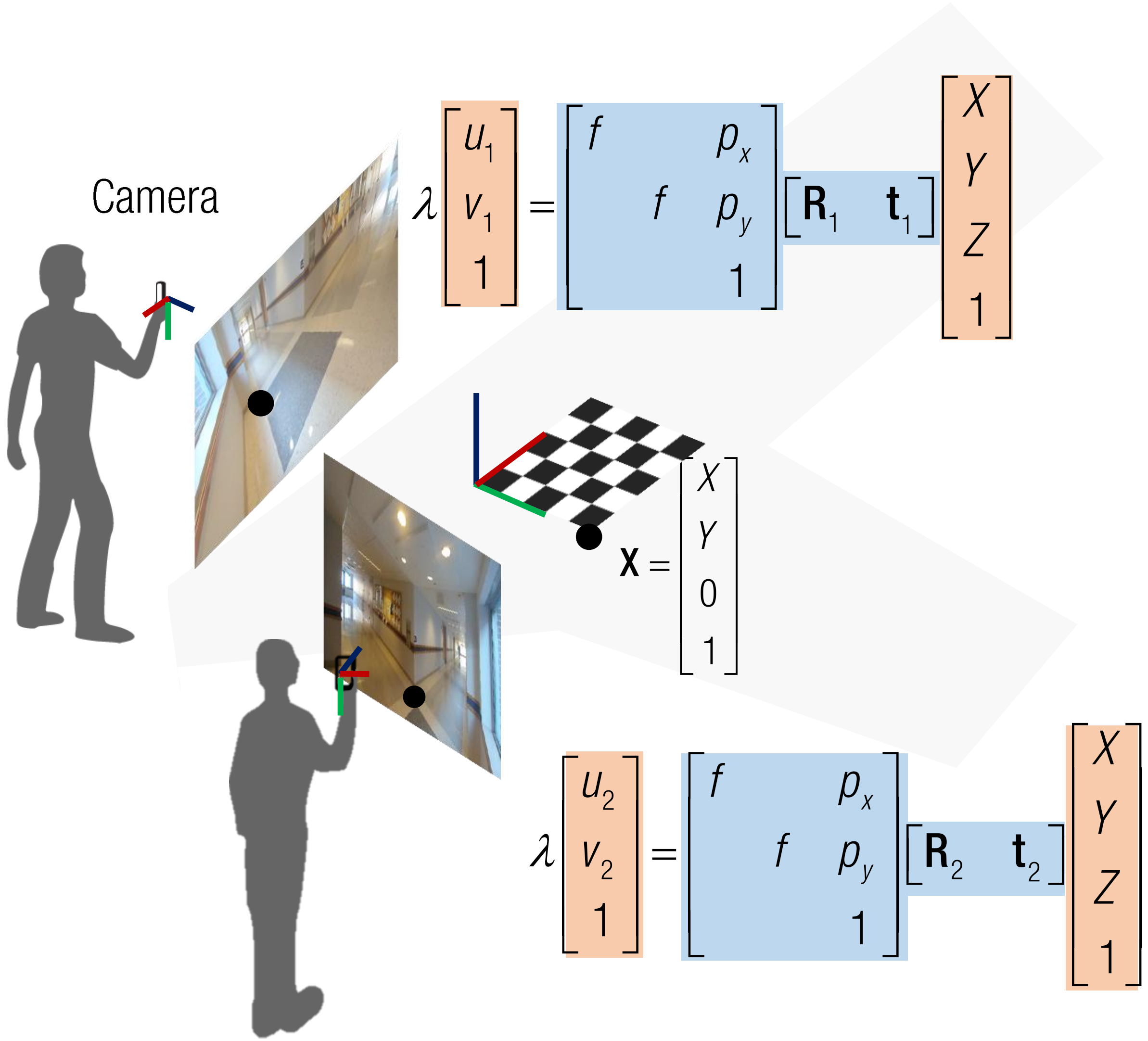


$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{t}_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Insight: Known Common 3D Points



# of unknowns: 3 (**K**) + 6n (**R** and **t**)

n: the number of images

p: the number of points

# of equations: 2np

We can solve for **K**, **R**, **t** if $3 + 6n < 2\,nm$

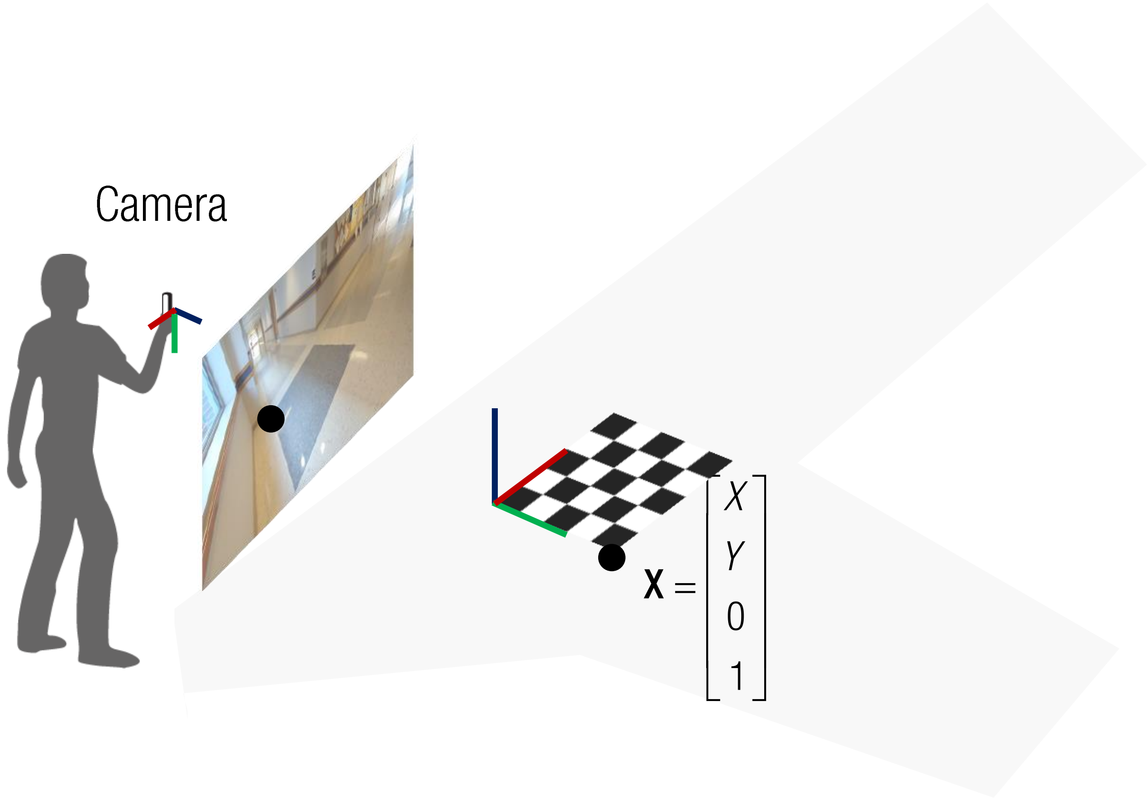
: Knowns

: Unknowns

Homography

Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$



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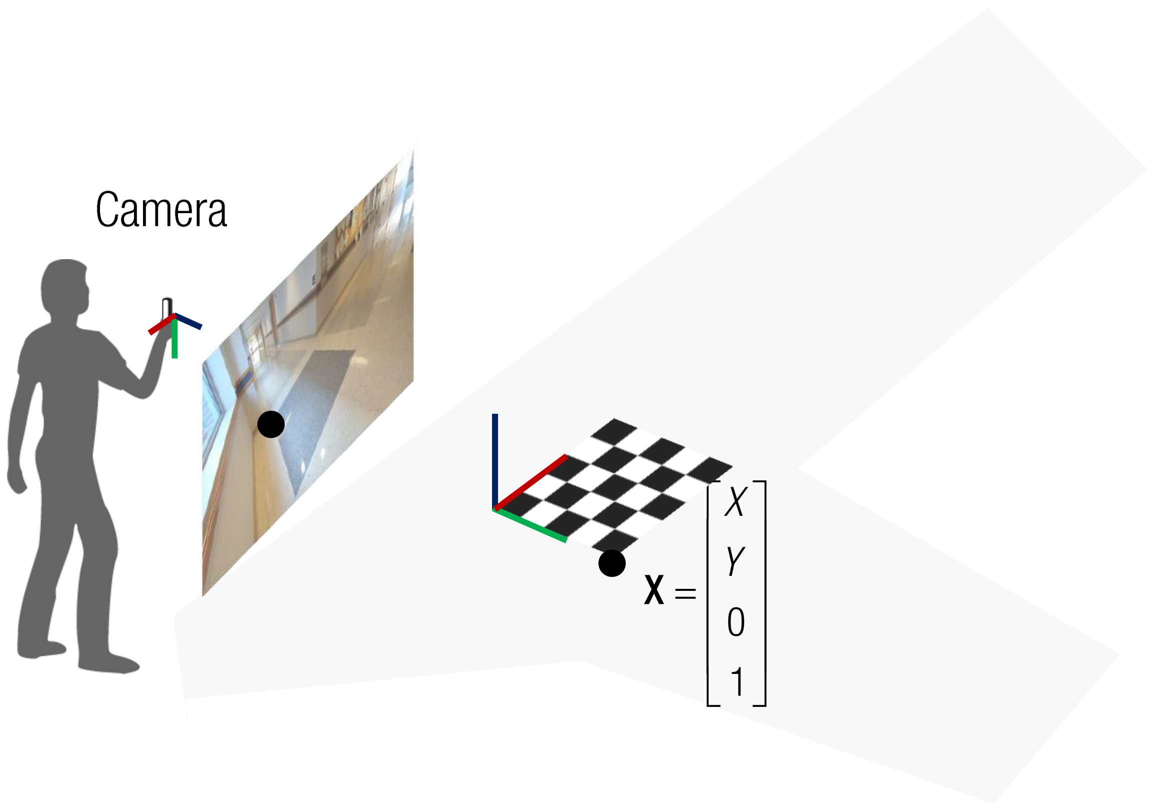
$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



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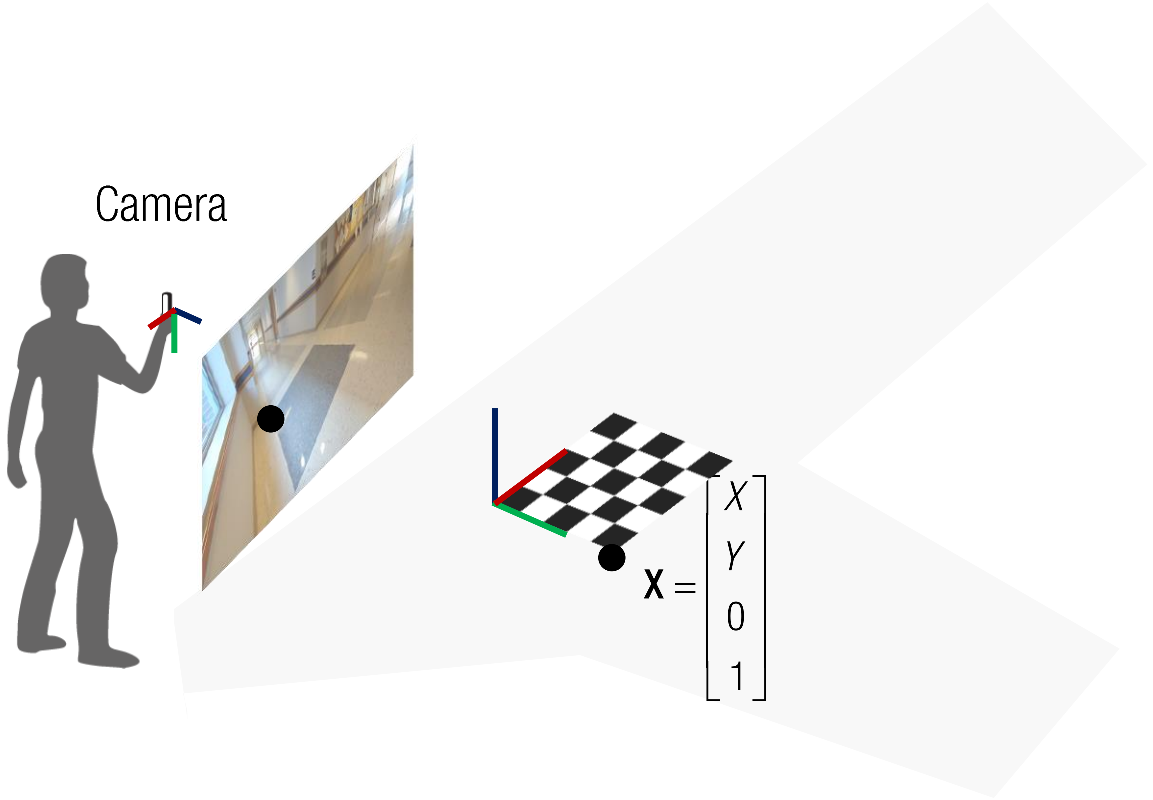
Homography



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

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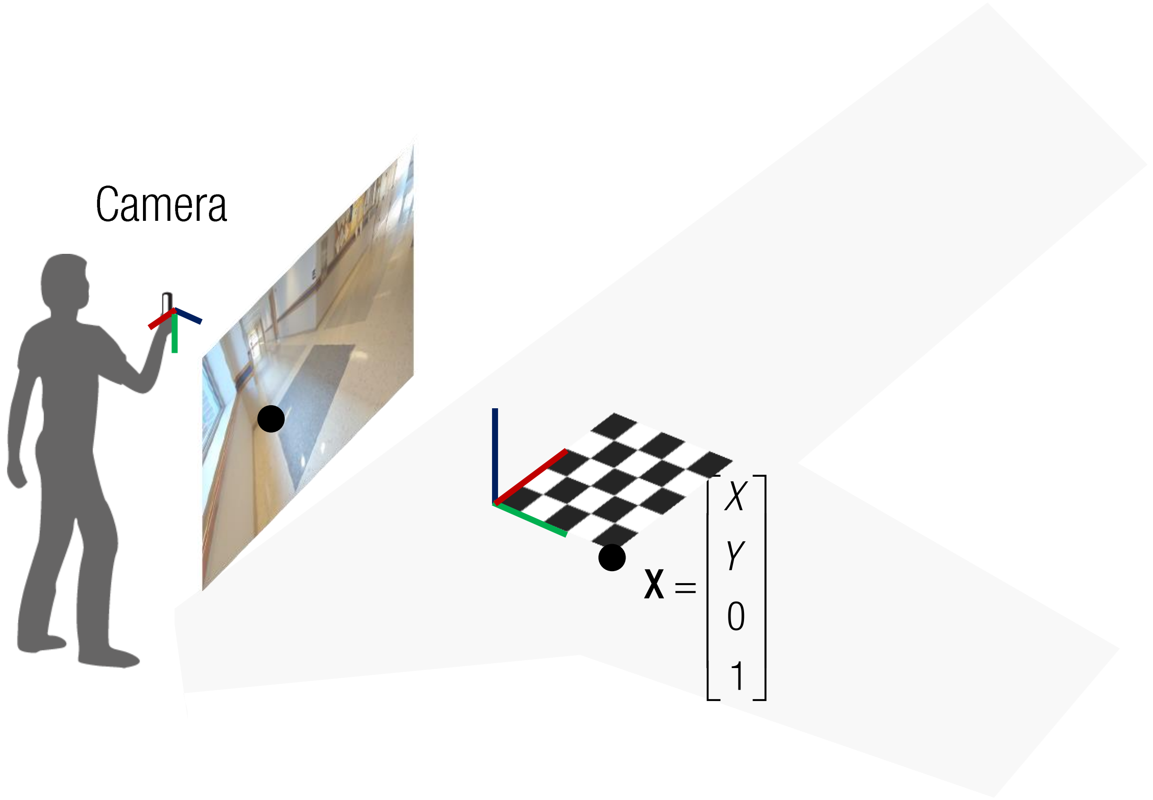
$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography



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1. Compute homography

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography

Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

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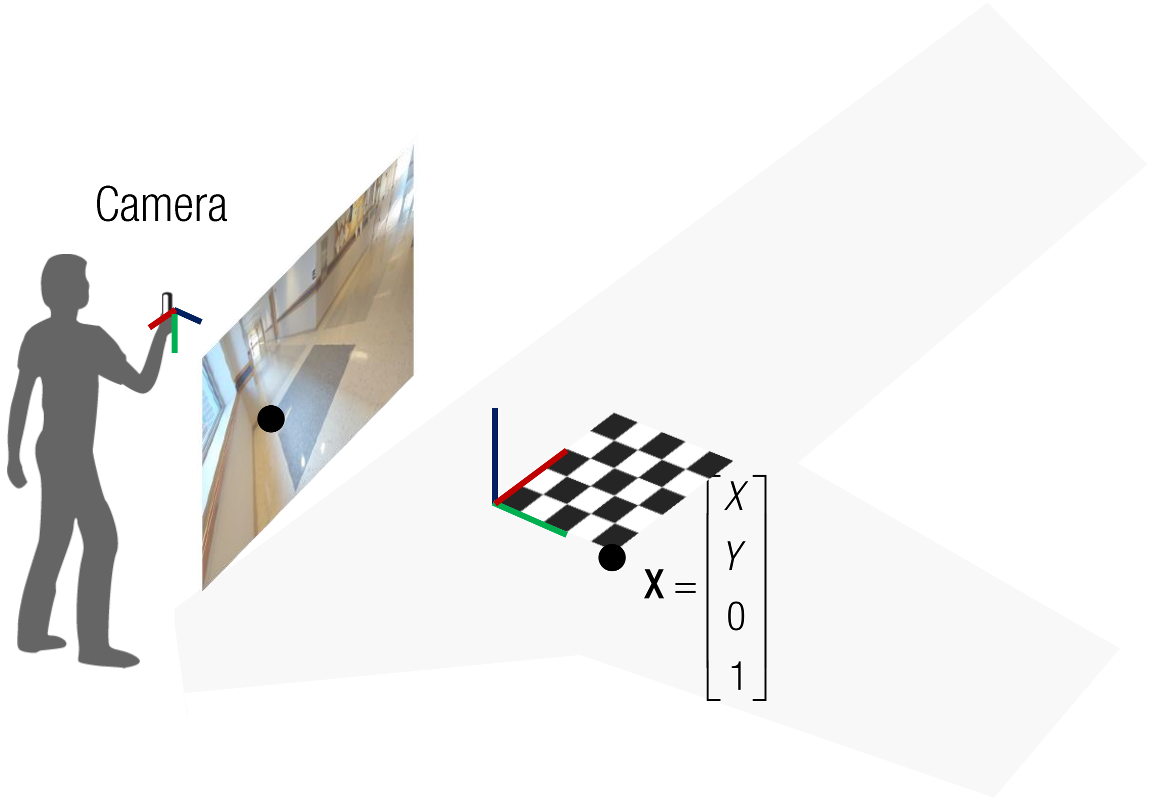
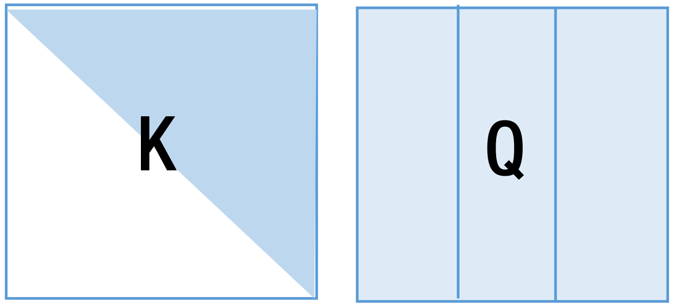
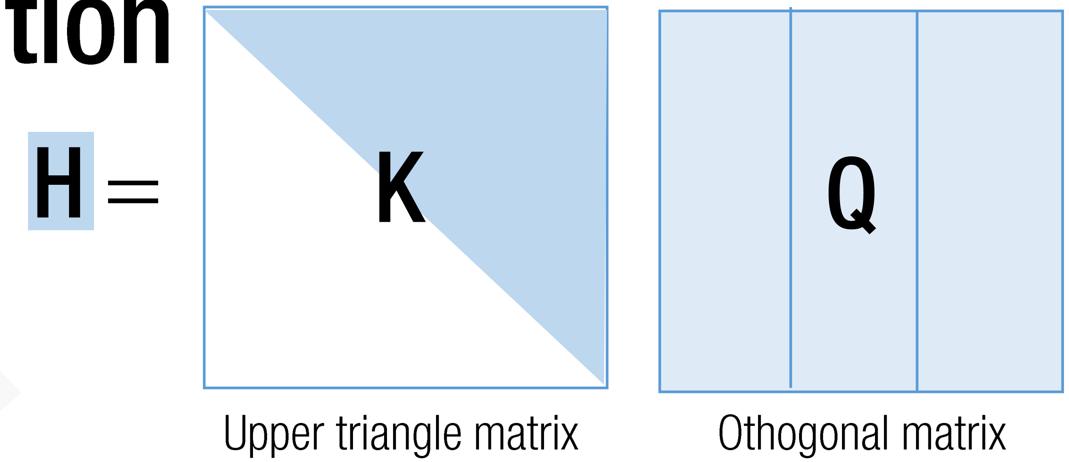
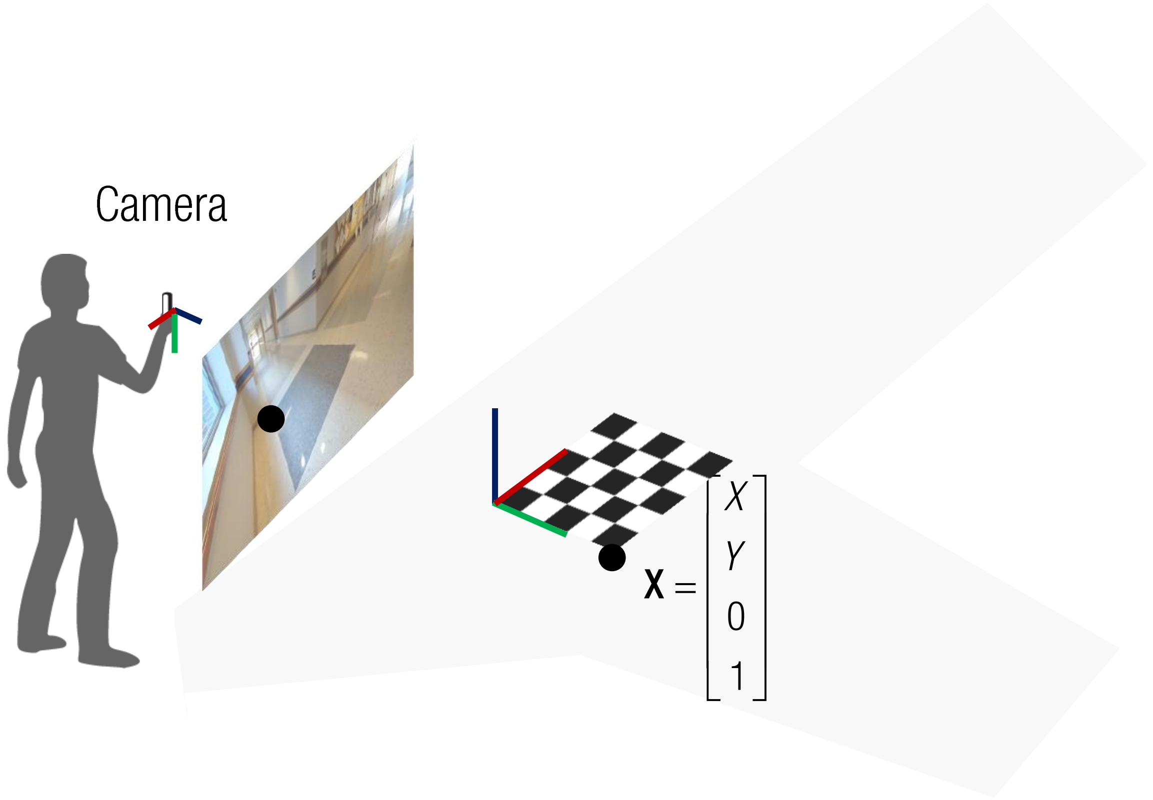


$\mathbf{H} =$

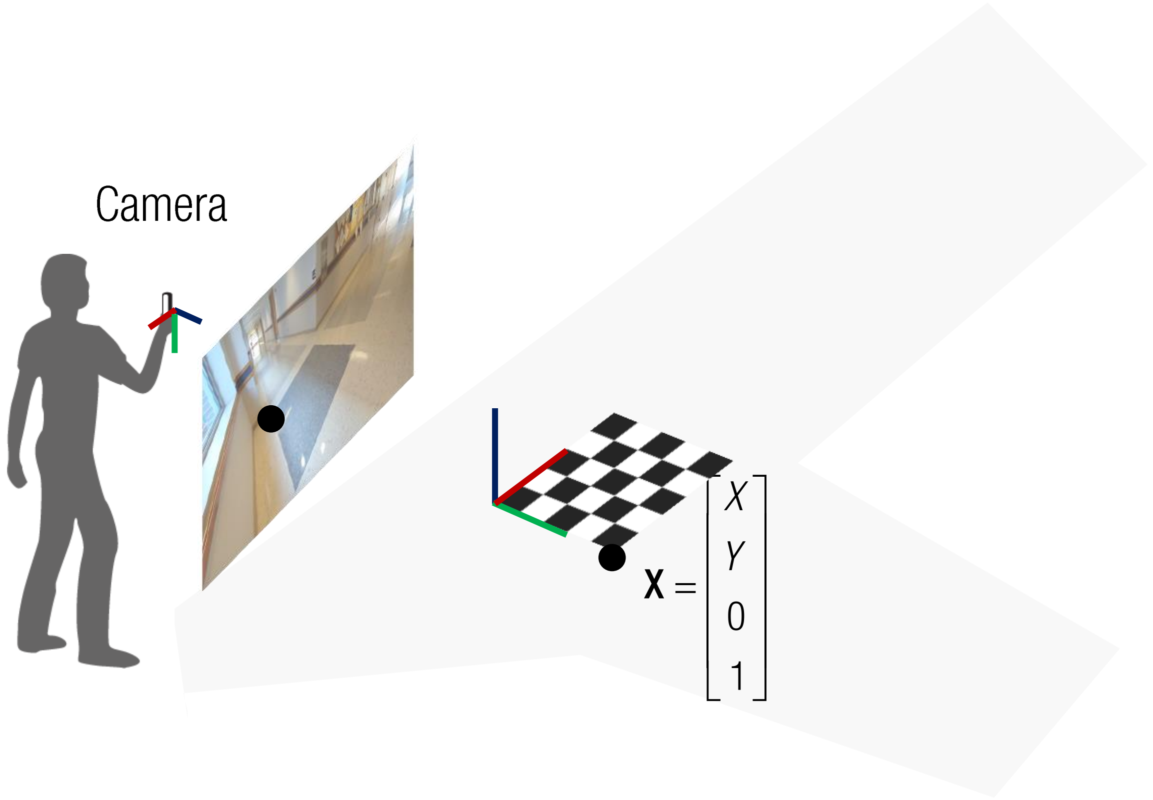


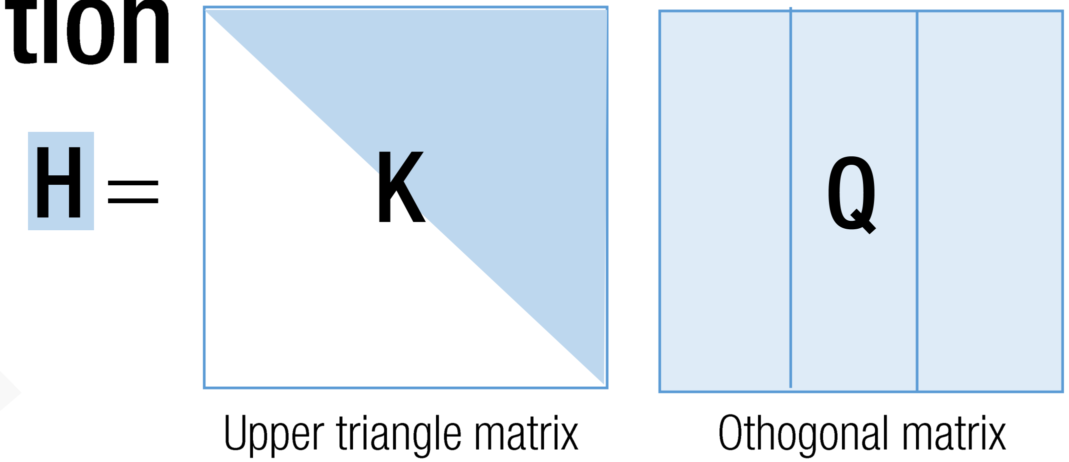
Method1: RQ Decomposition



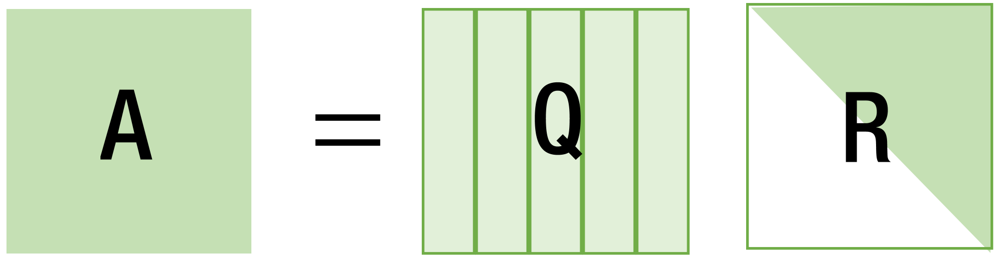


Method1: RQ Decomposition

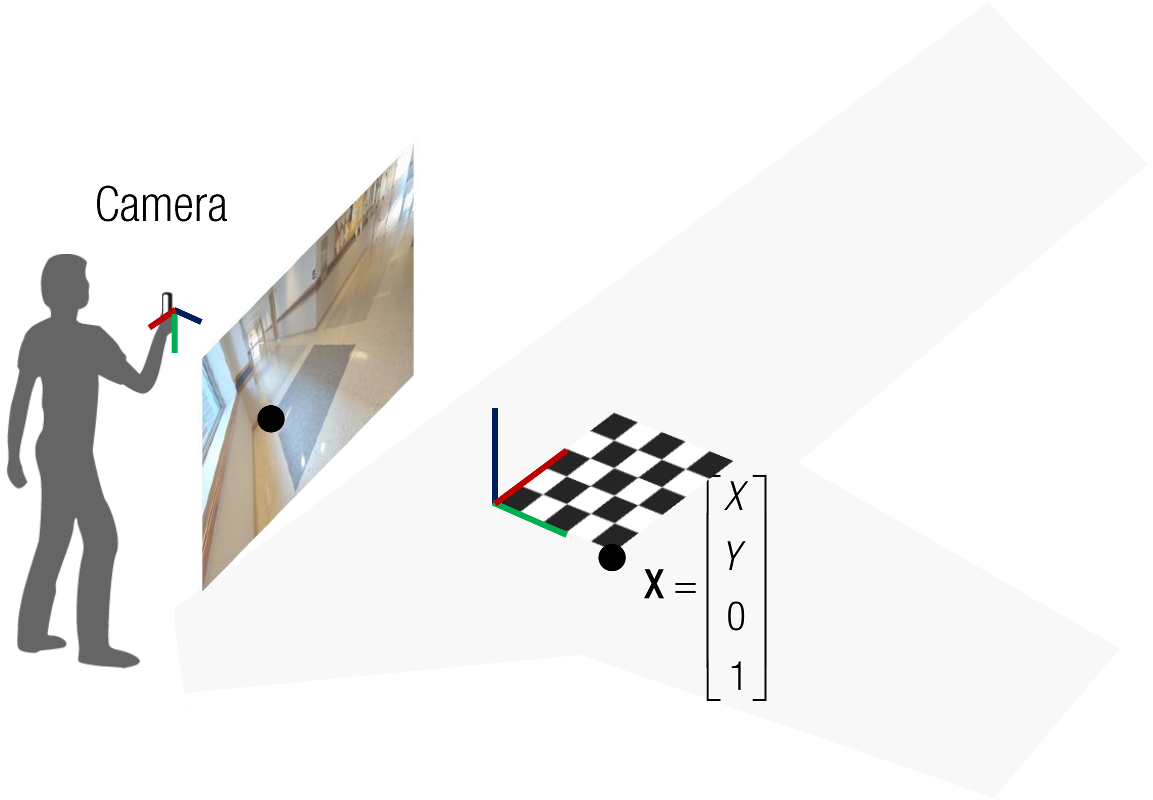


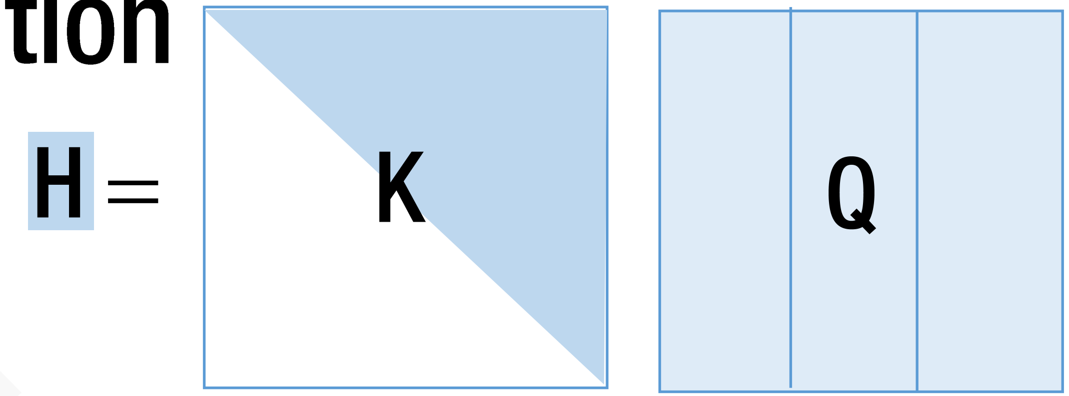


QR decomposition:



Method1: RQ Decomposition

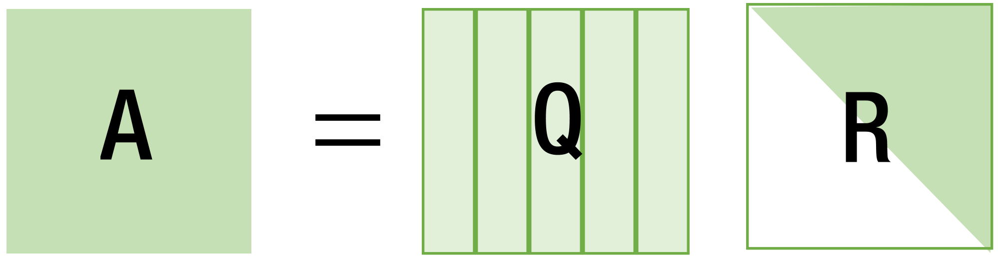




Upper triangle matrix

Othogonal matrix

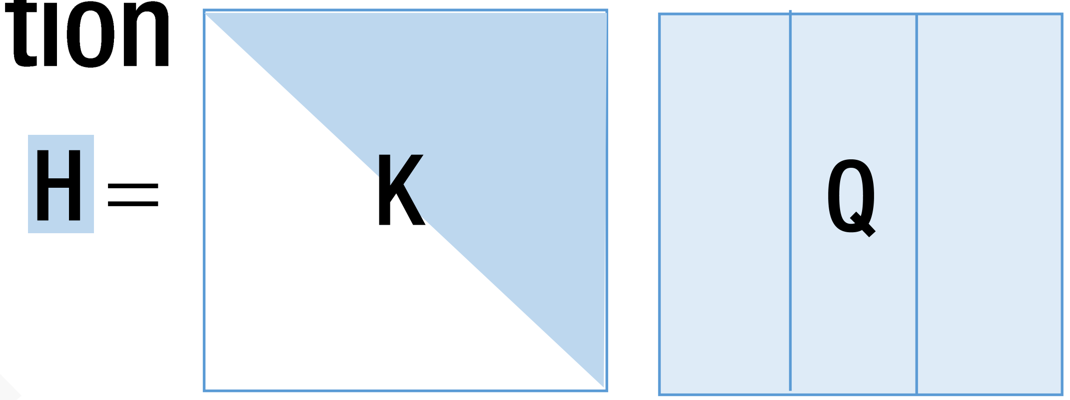
QR decomposition:

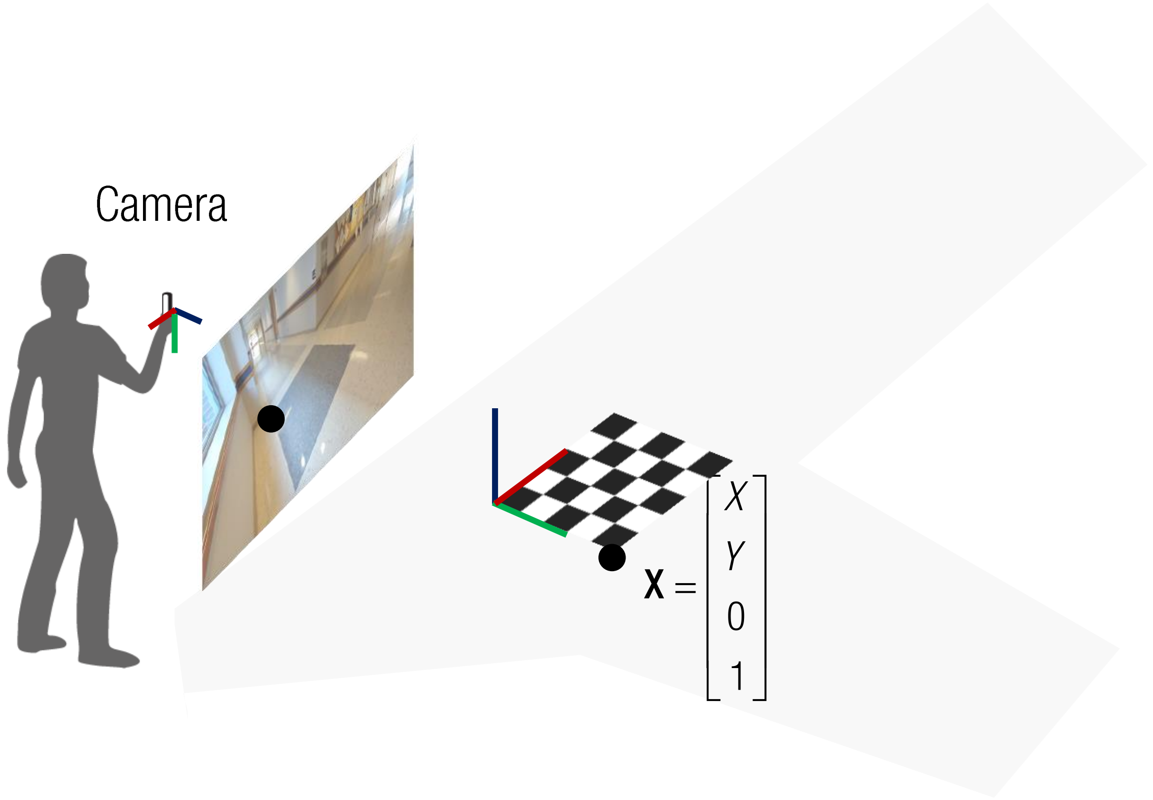


MATLAB

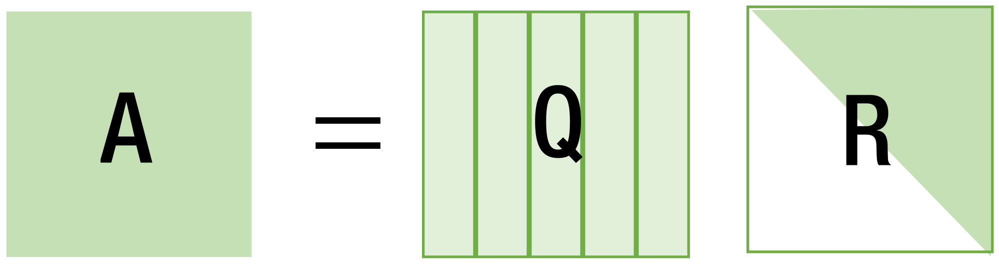
[Q R] = qr(A)

Method1: RQ Decomposition





QR decomposition:

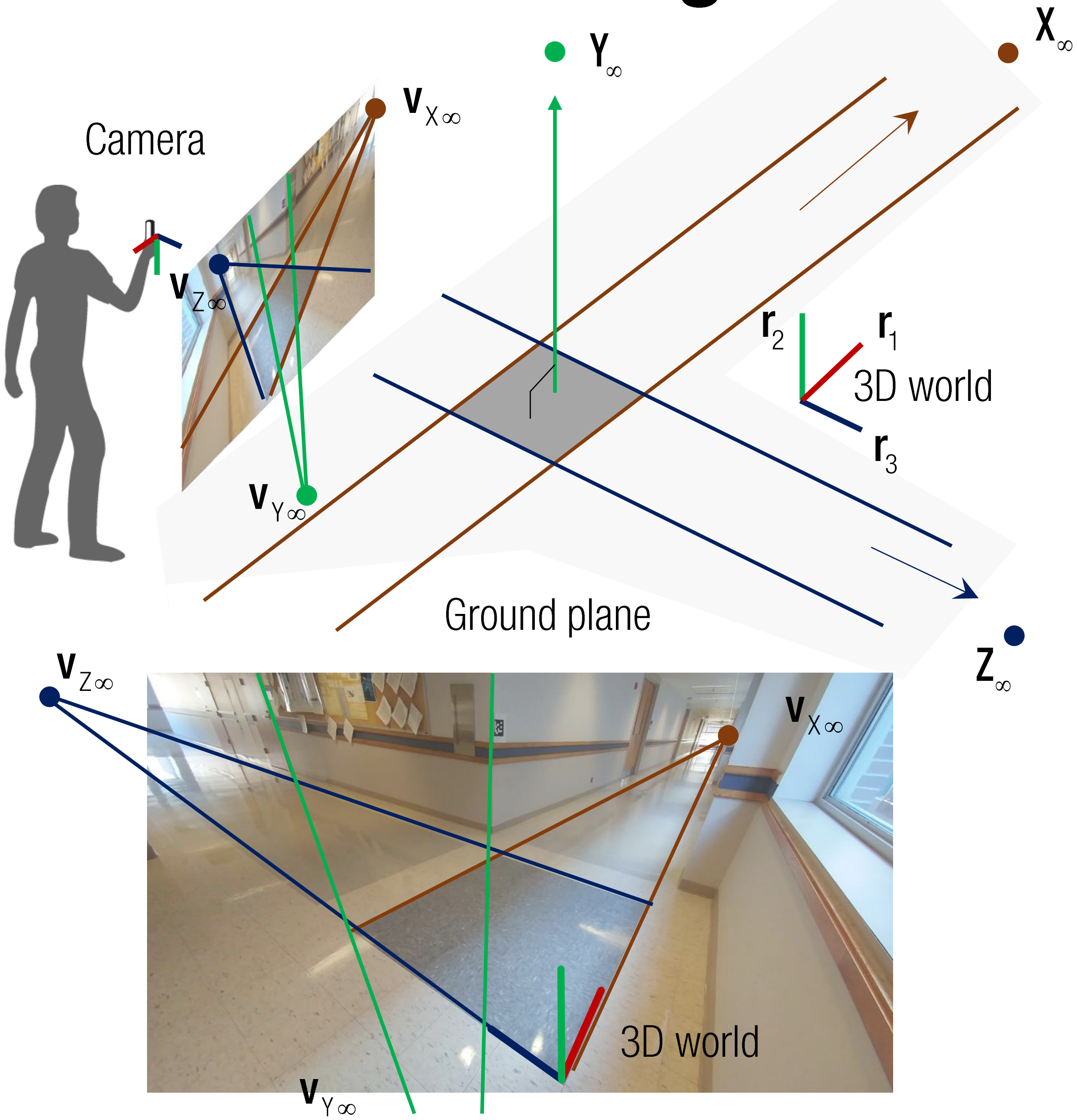


MATLAB

[Q R] = qr(A)

HW: How to convert **QR** to **RQ**?

Recall: Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{X}_\infty) = 0$$

$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^\mathsf{T}(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

: 3 unknowns and 3 equations

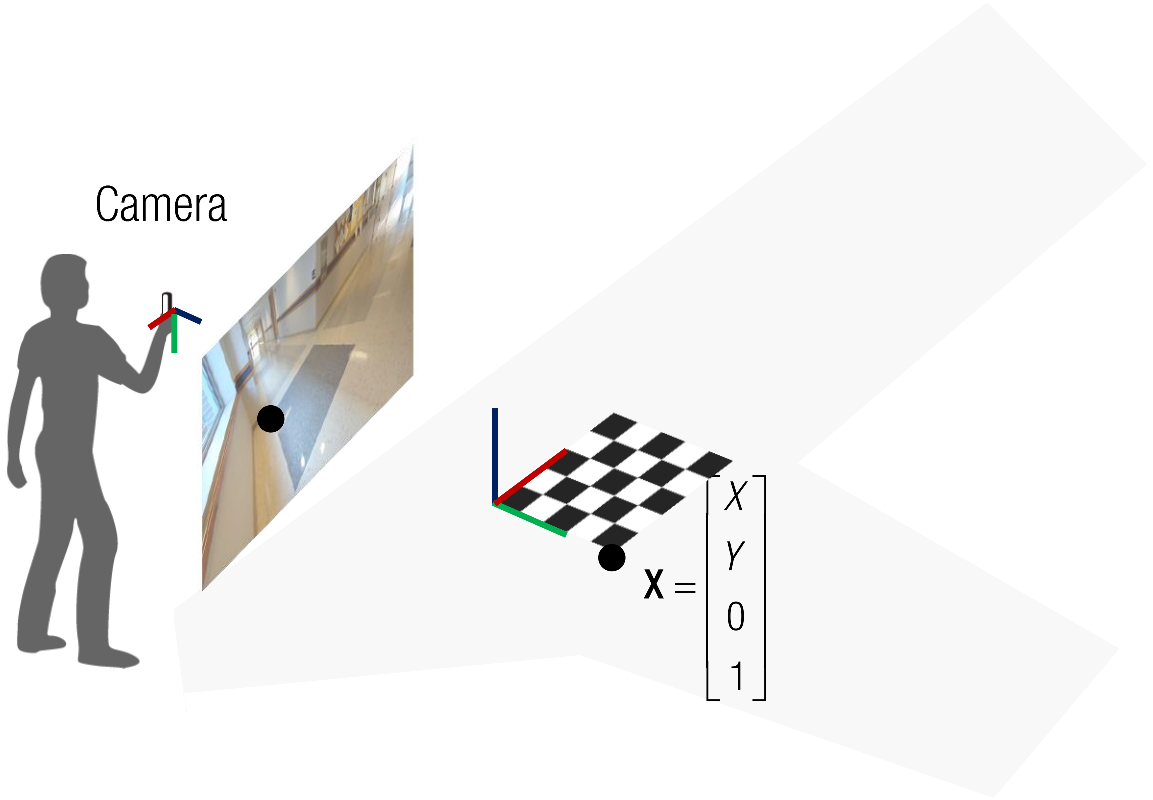
Method2: Rotation

: Knowns

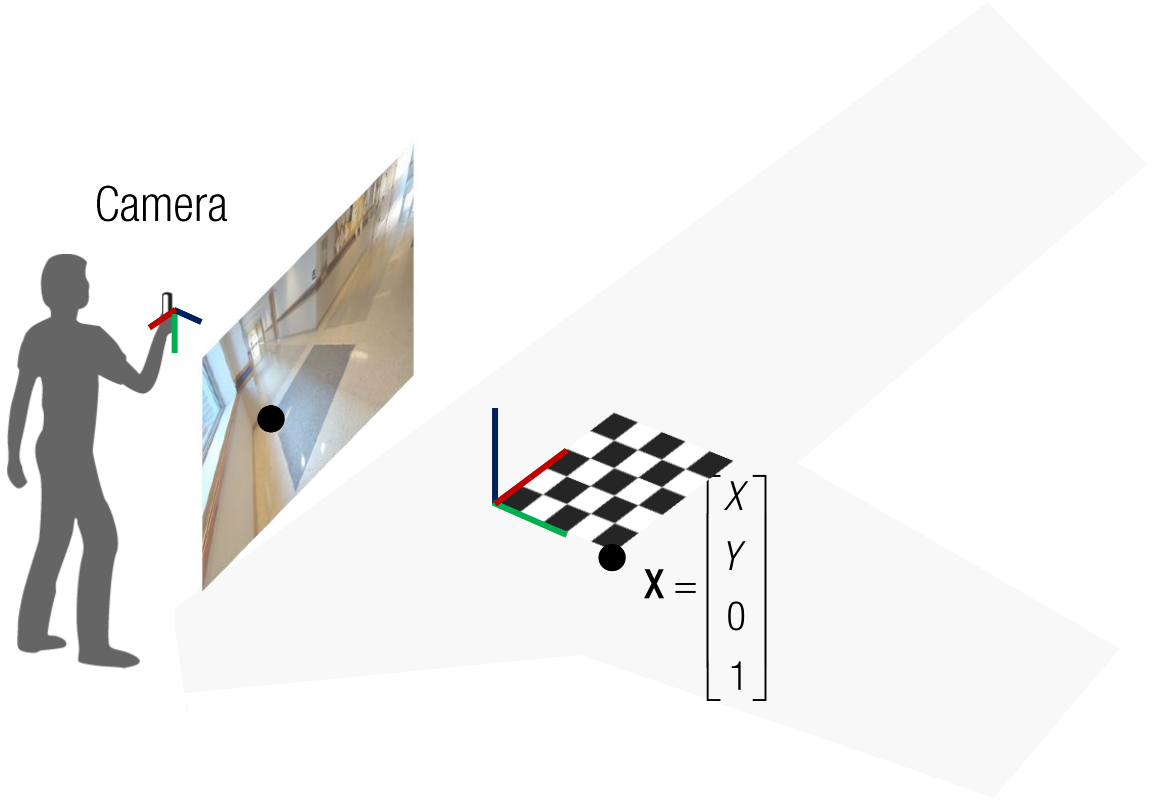
: Unknowns

Homography factorization:

$$\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$



Method2: Rotation



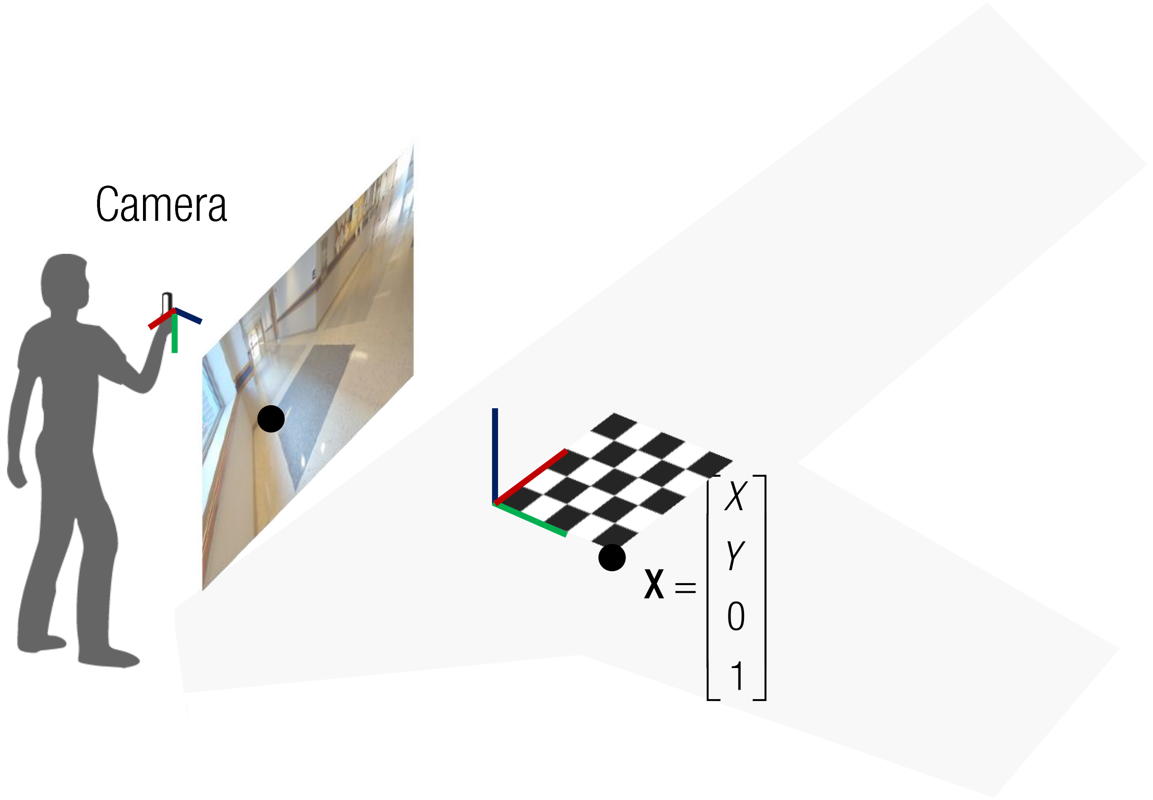
: Knowns

: Unknowns

Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

: Knowns

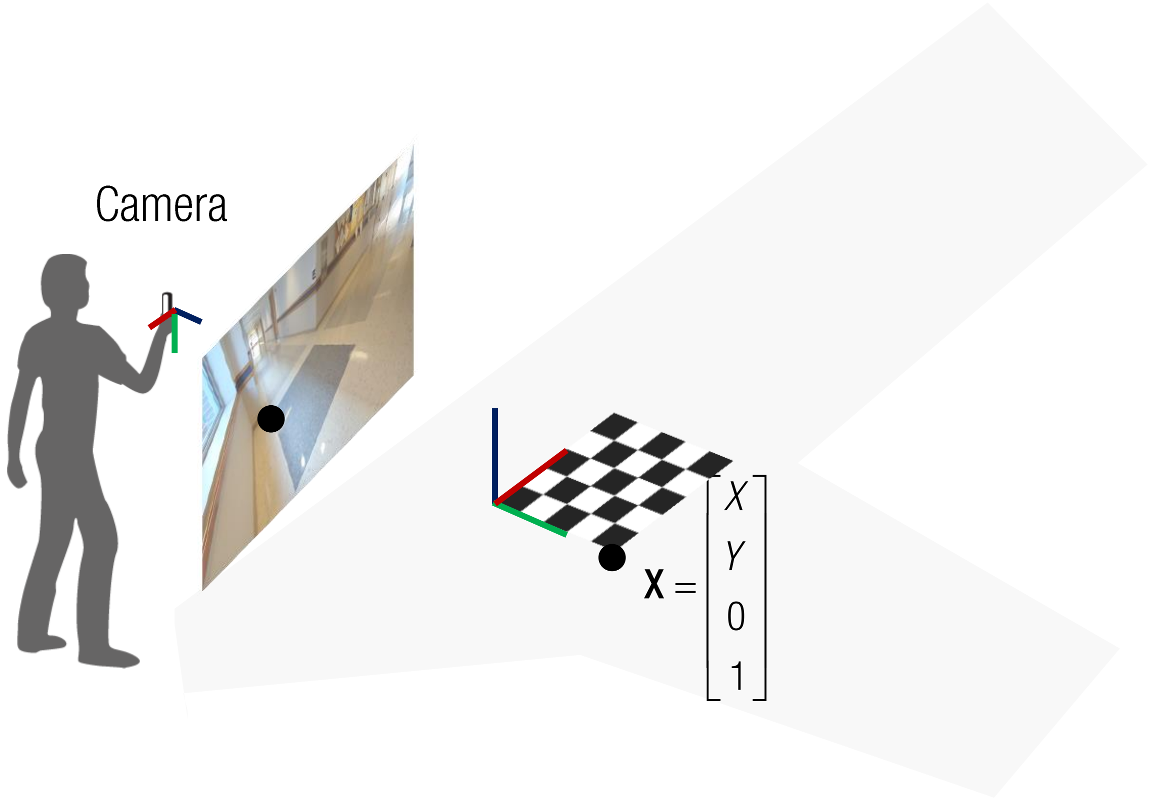
: Unknowns

Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T\mathbf{r}_2 = 0 \qquad \left\|\mathbf{r}_1\right\| = 1 \qquad \left\|\mathbf{r}_2\right\| = 1$$

Method2: Rotation

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Orthogonality of rotation matrix property:

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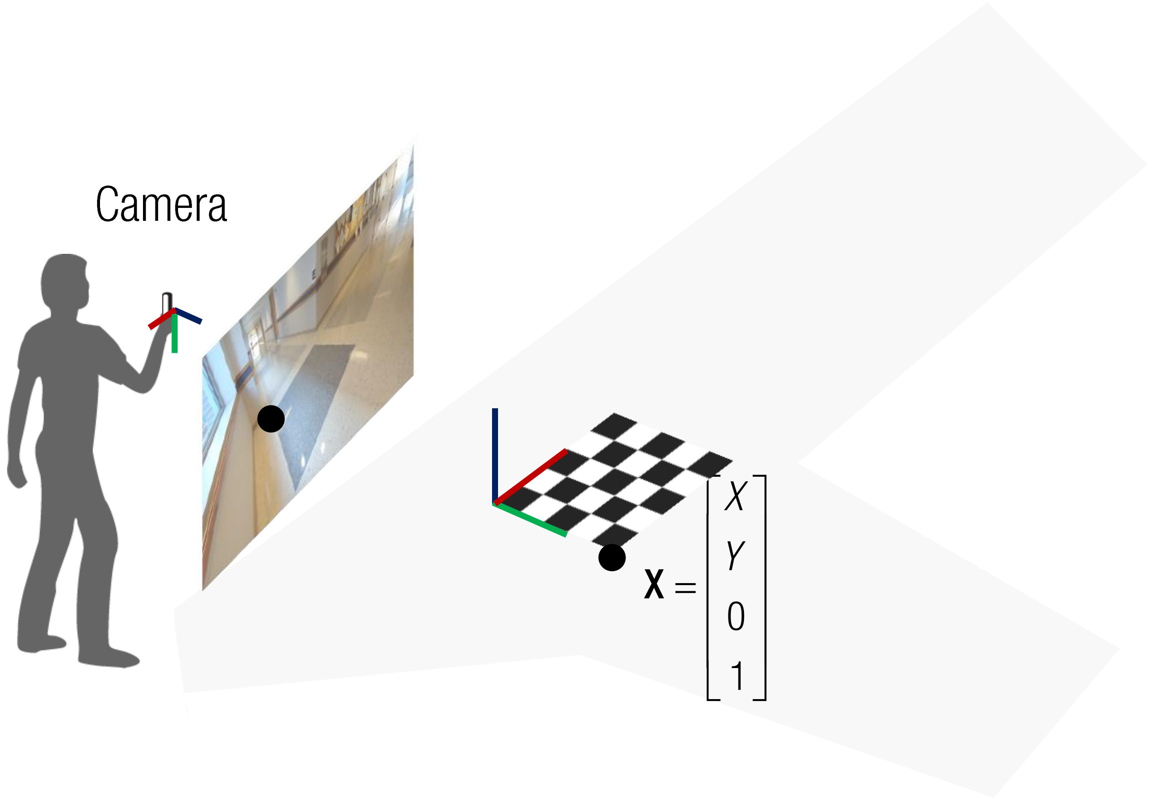
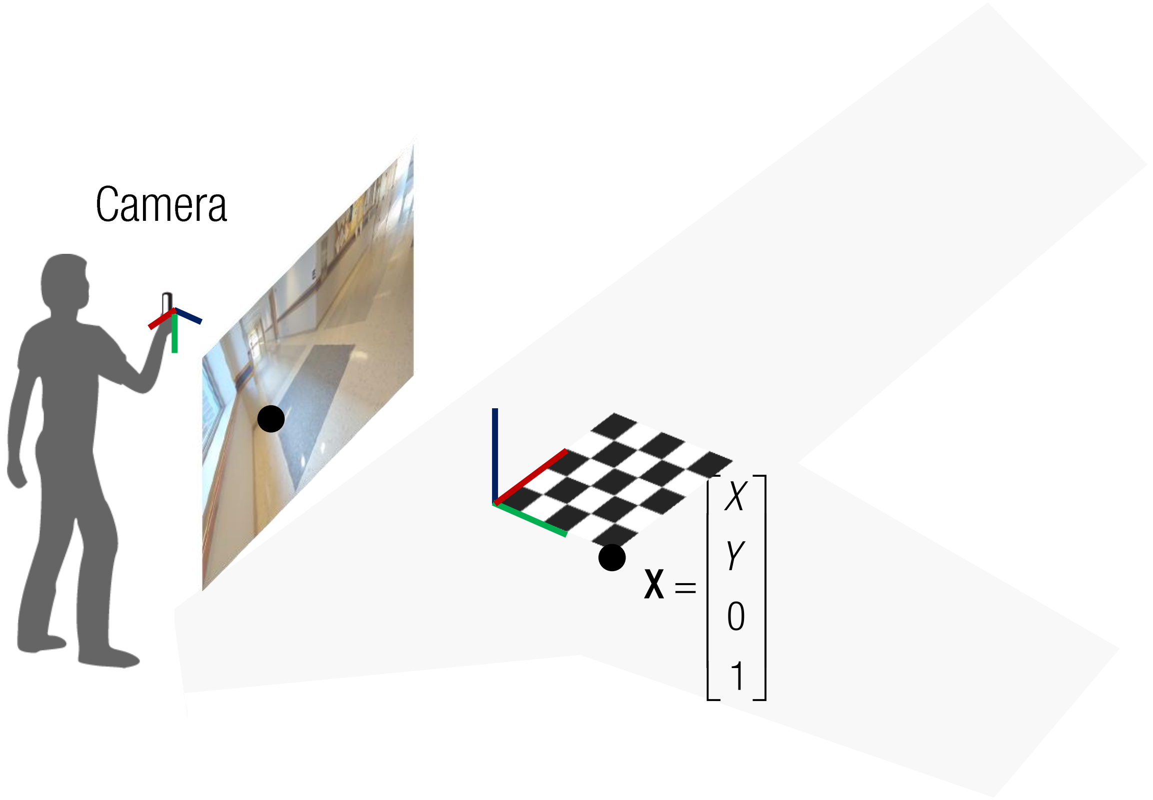
$$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^T\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$



Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

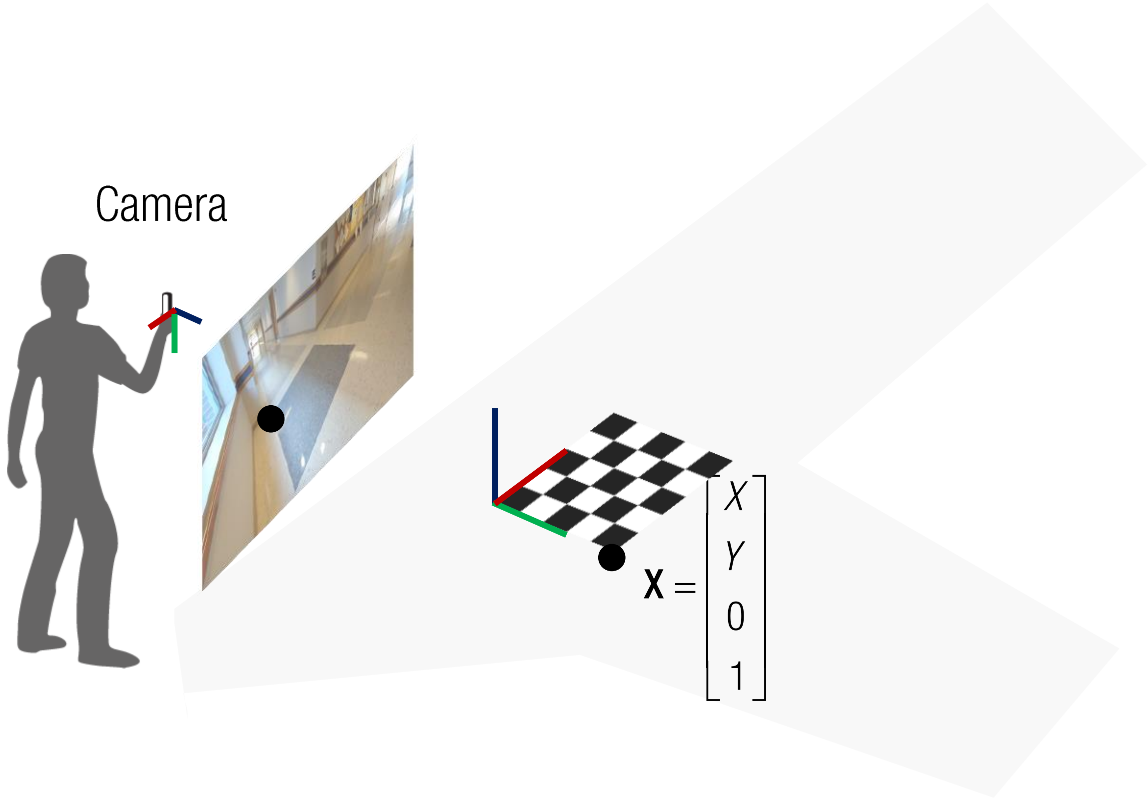
Orthogonality of rotation matrix property:

$$\mathbf{r}_1^{\mathsf{T}}\mathbf{r}_2 = 0 \qquad \|\mathbf{r}_1\| = 1 \qquad \|\mathbf{r}_2\| = 1$$

$$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\| = \left\|\mathbf{K}^{-1}\mathbf{h}_2\right\| \quad \text{or,} \quad \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

Method2: Rotation



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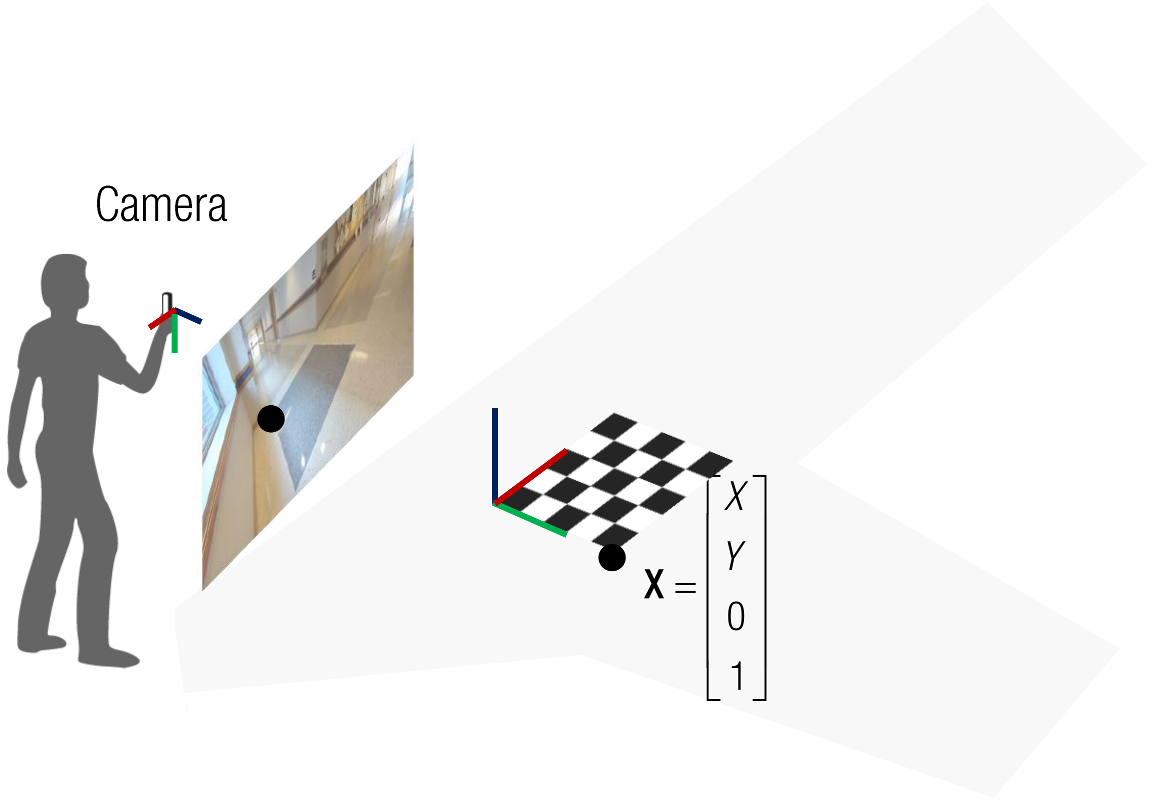
$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1} =$$

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

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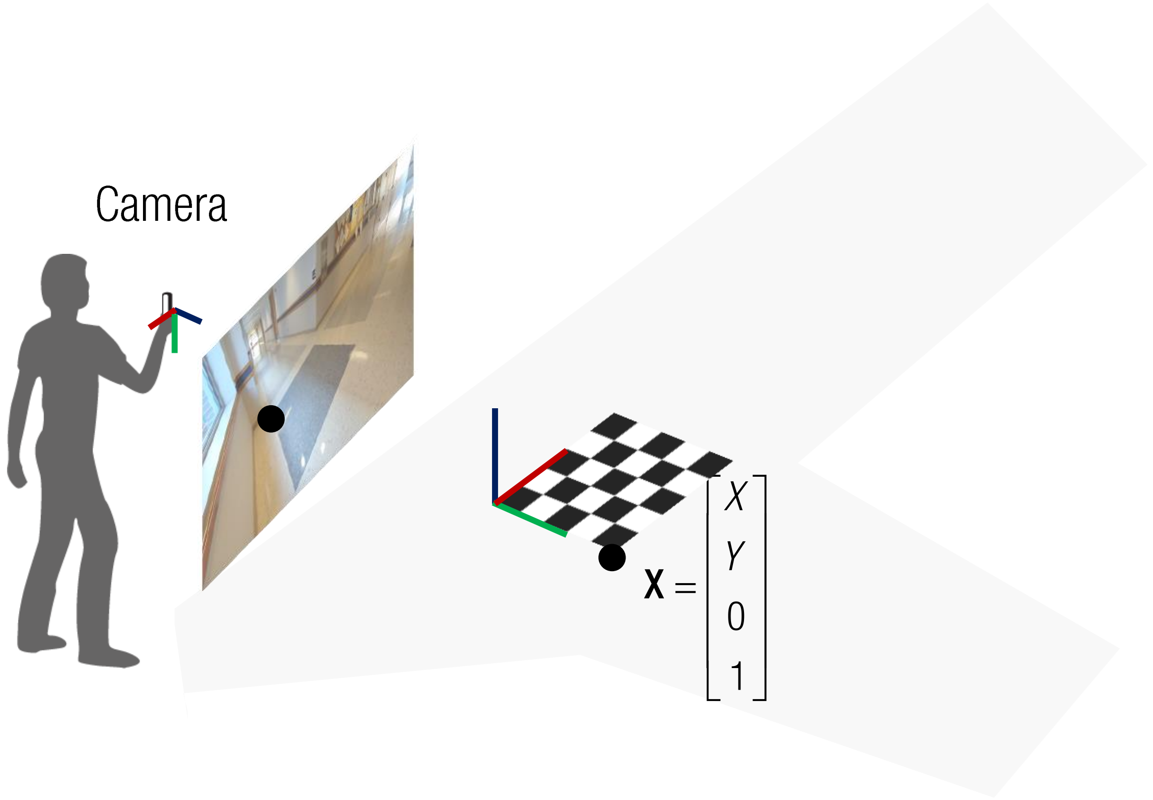
$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

Method2: Rotation



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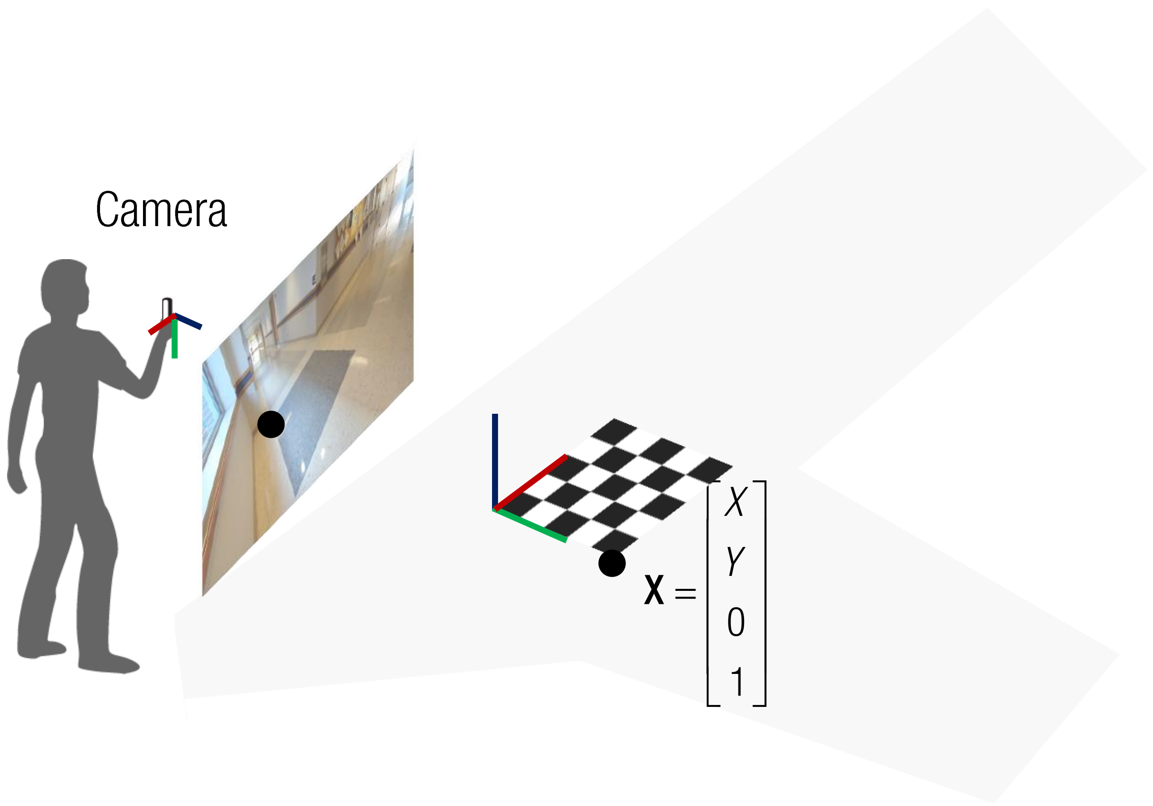
$$\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\| = \left\|\mathbf{K}^{-1}\mathbf{h}_2\right\| \quad \text{or,} \quad \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

Method2: Rotation



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Orthogonality of rotation matrix property:

$$\mathbf{r}_1^{\mathsf{T}}\mathbf{r}_2 = 0 \qquad \left\|\mathbf{r}_1\right\| = 1 \qquad \left\|\mathbf{r}_2\right\| = 1$$

$$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\| = \left\|\mathbf{K}^{-1}\mathbf{h}_2\right\| \quad \text{or,} \quad \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

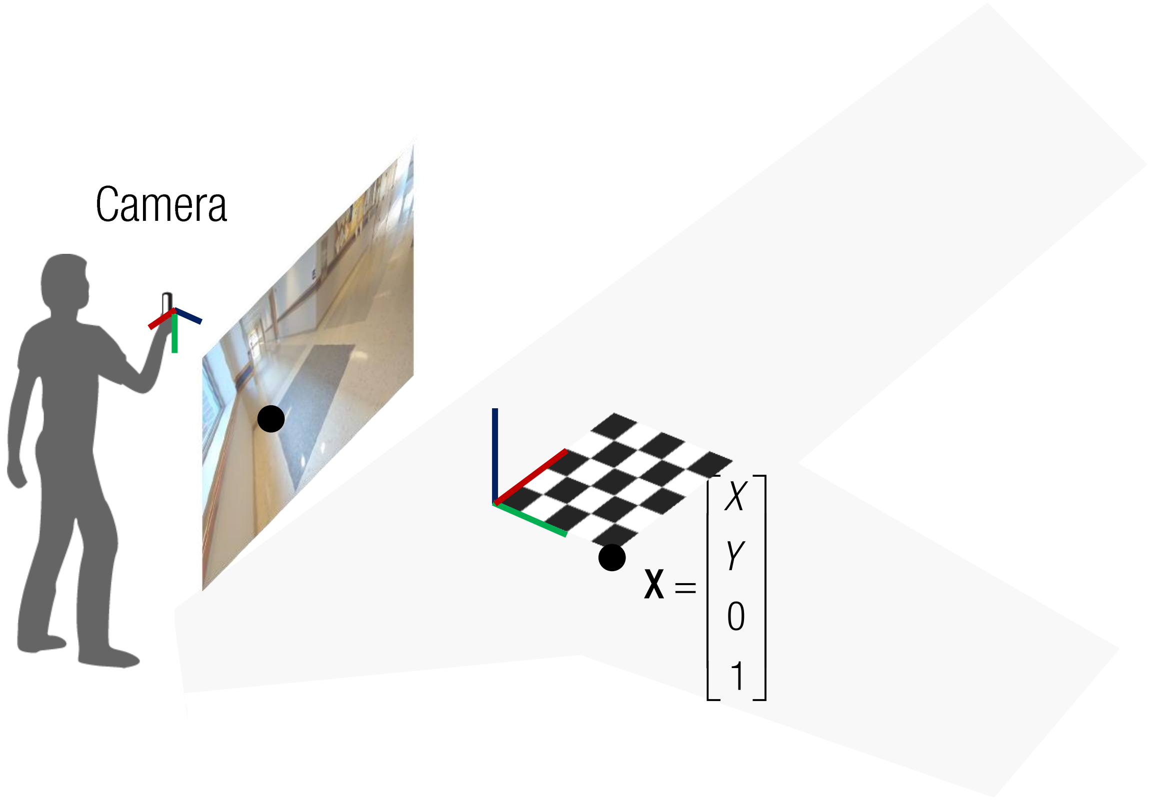
$$\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}}_{\mathbf{B}}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

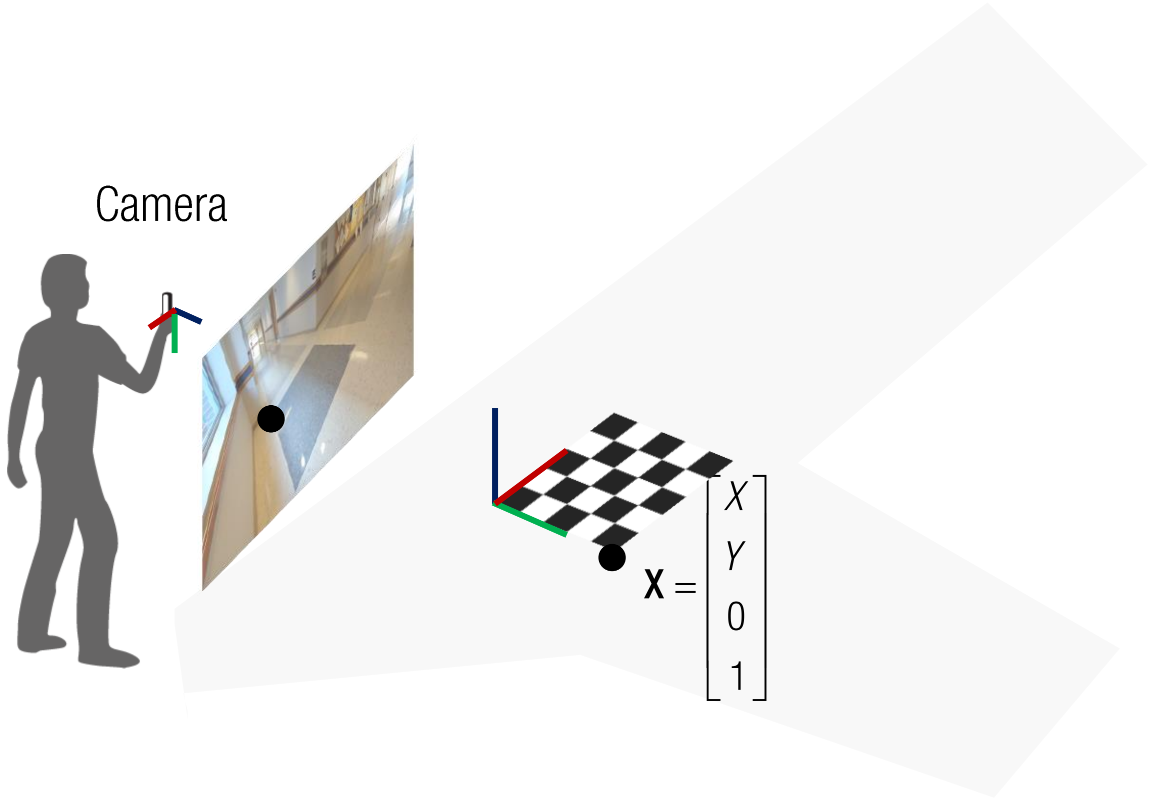
Linear in **B**: $\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = 0 \qquad \mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$

Method2: Rotation

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$



Method2: Rotation



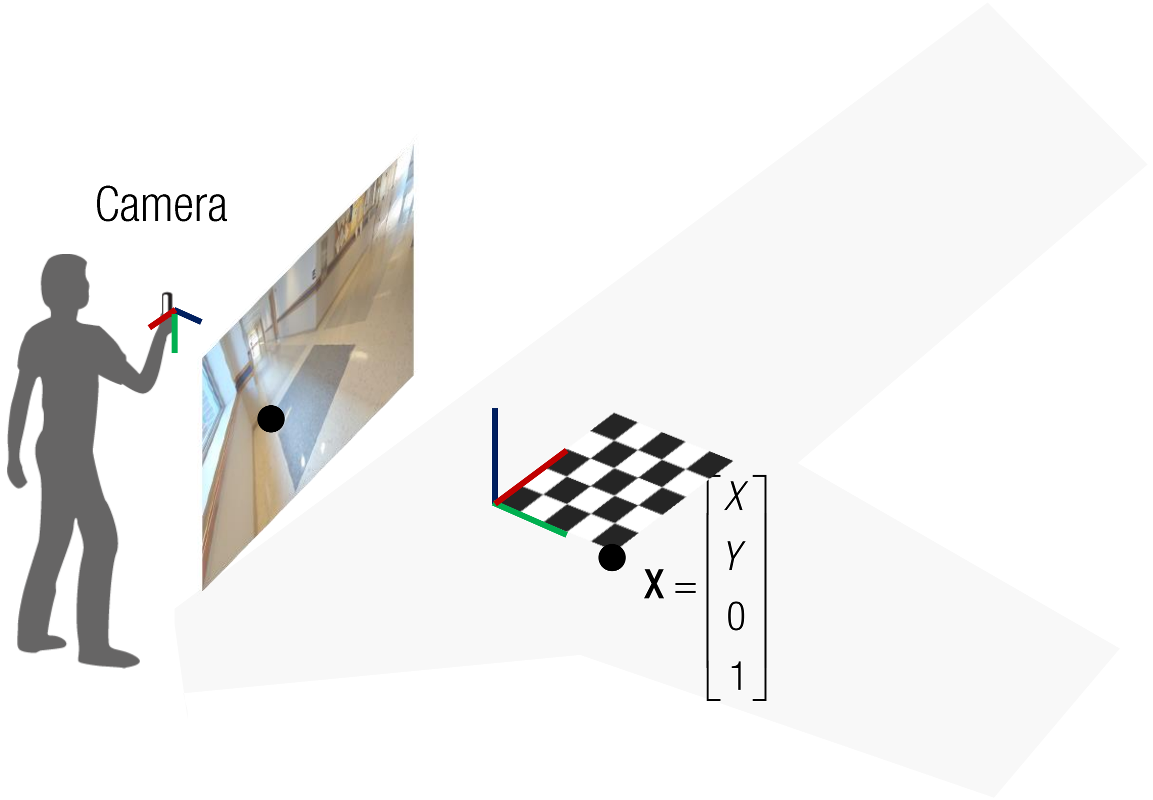
$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

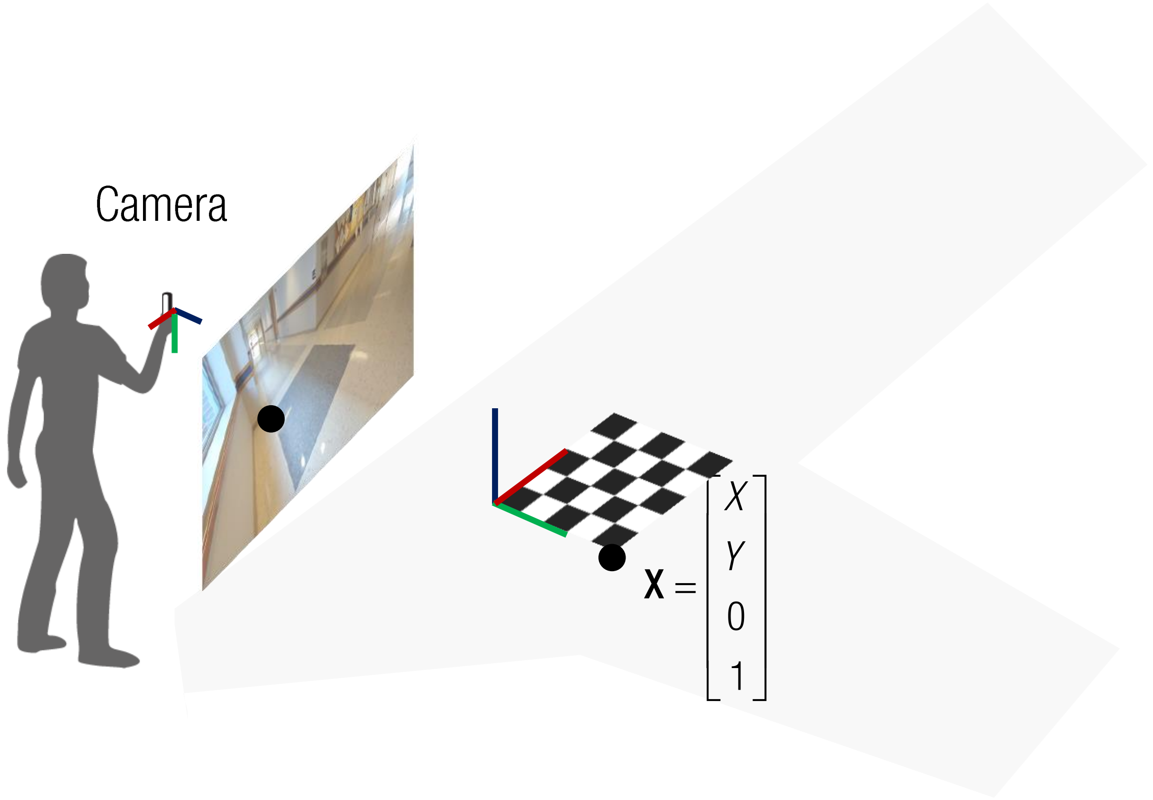
$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

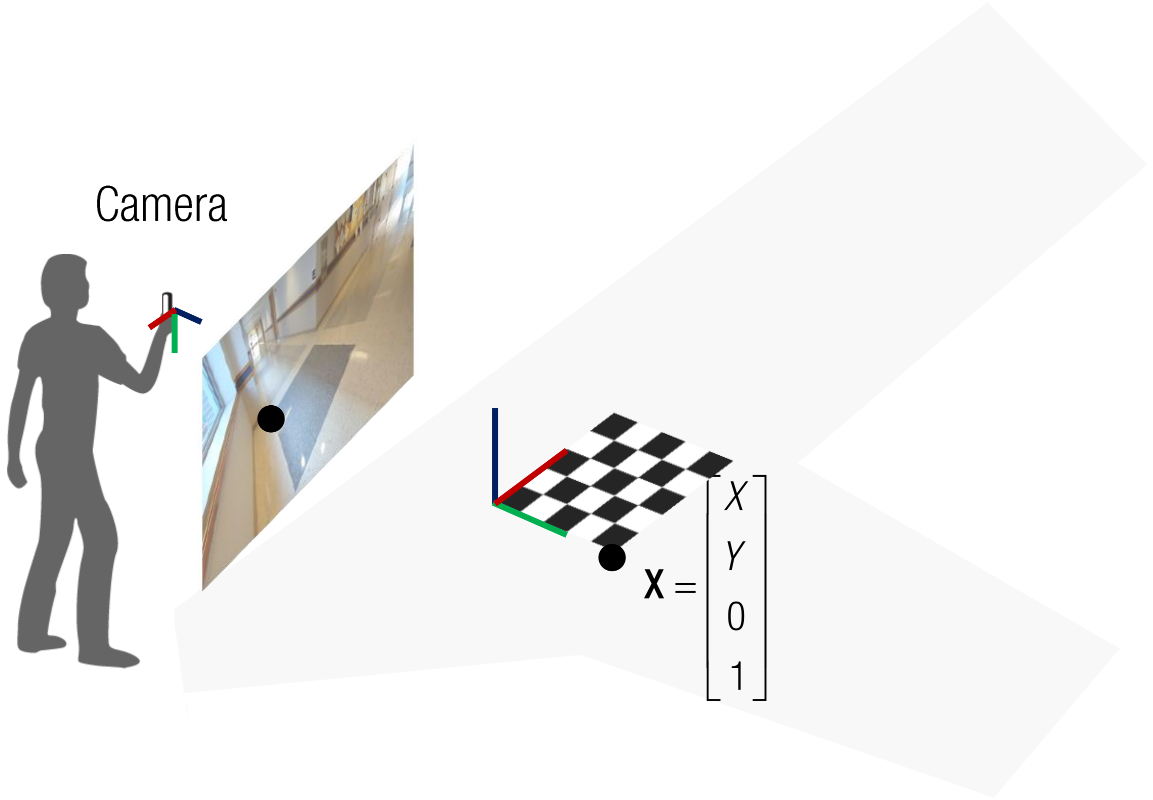
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$\mathbf{x} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

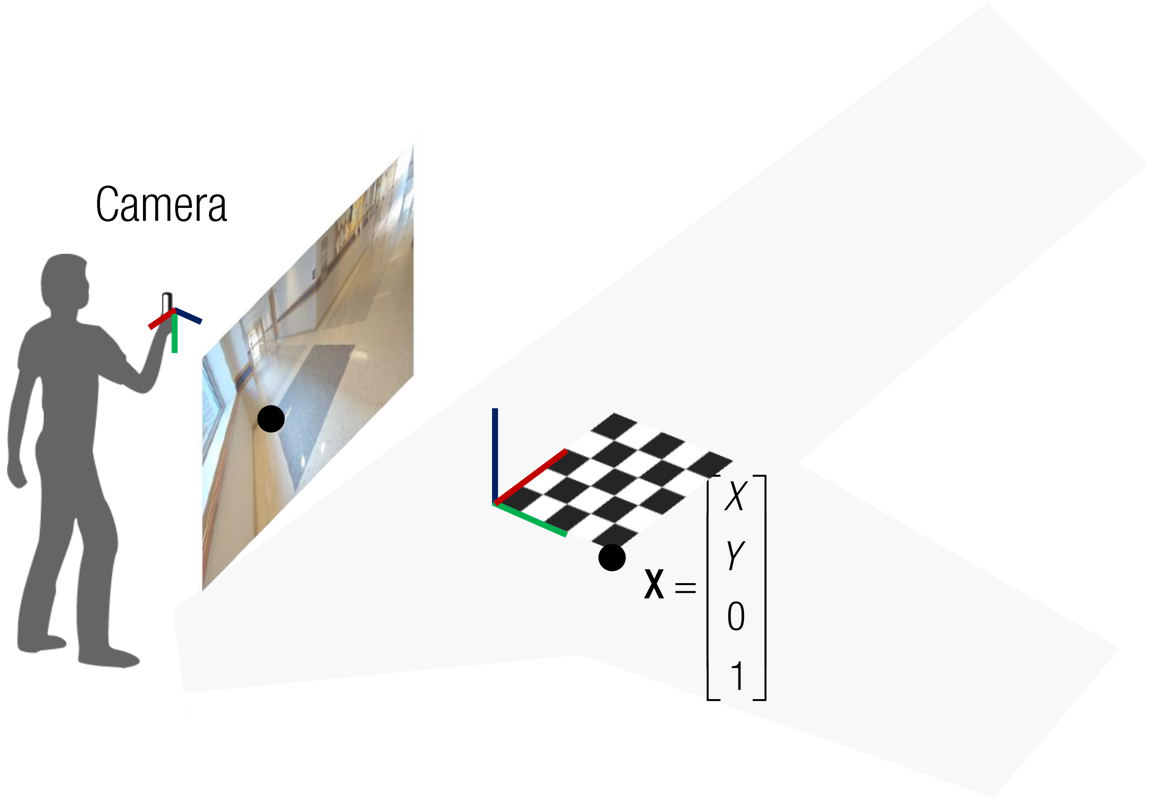
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

2x4

Each image produces 2 equations and therefore,
x can be computed with minimum 2 images.

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11}\\ h_{21}\\ h_{31}\end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2\\ & b_1 & b_3\\ b_2 & b_3 & b_4\end{bmatrix} \begin{bmatrix} h_{12}\\ h_{22}\\ h_{32}\end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

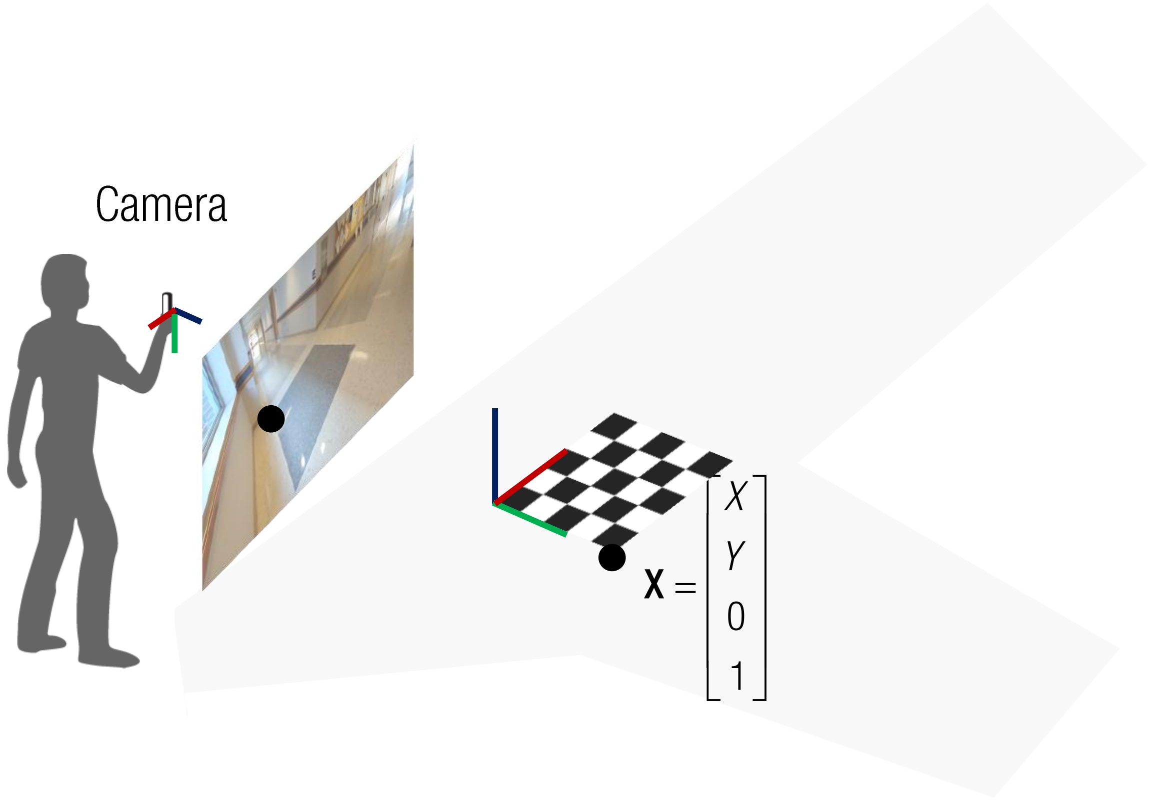
$$\begin{bmatrix} h_{11}\\ h_{21}\\ h_{31}\end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2\\ & b_1 & b_3\\ b_2 & b_3 & b_4\end{bmatrix} \begin{bmatrix} h_{11}\\ h_{21}\\ h_{31}\end{bmatrix} = \begin{bmatrix} h_{12}\\ h_{22}\\ h_{32}\end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2\\ & b_1 & b_3\\ b_2 & b_3 & b_4\end{bmatrix} \begin{bmatrix} h_{12}\\ h_{22}\\ h_{32}\end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12}+h_{21}h_{22} & h_{11}+h_{12} & h_{21}+h_{22} & 1\\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}-h_{12}) & 2(h_{21}-h_{22}) & 0\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1\\ b_2\\ b_3\\ b_4\end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

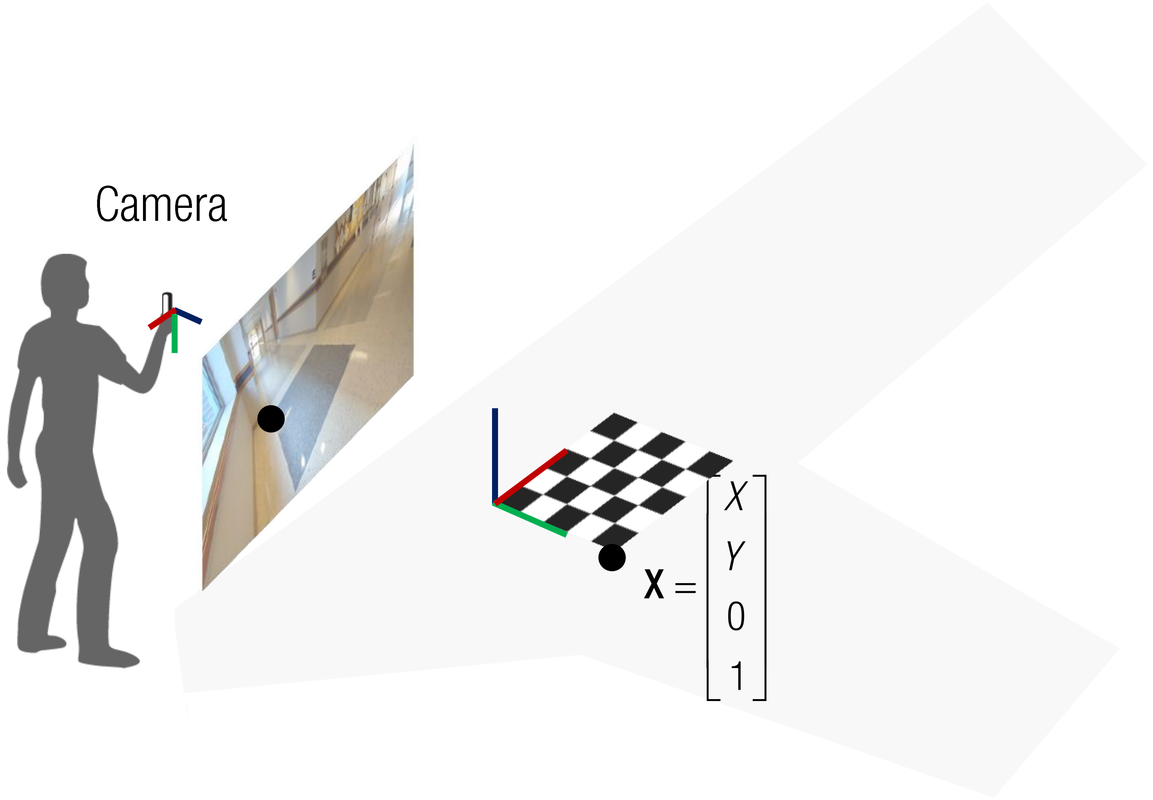
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{X}} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



: Knowns

: Unknowns

Homography factorization:

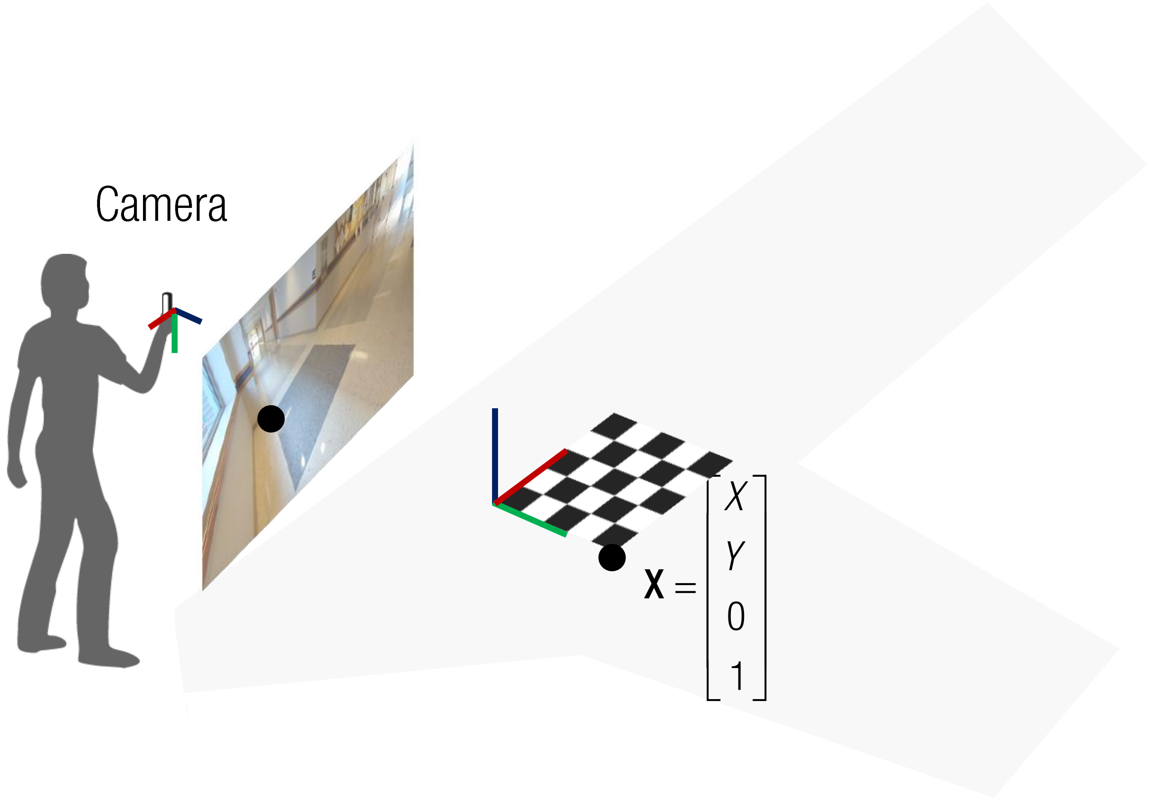
$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Method2: Rotation

: Knowns

: Unknowns



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

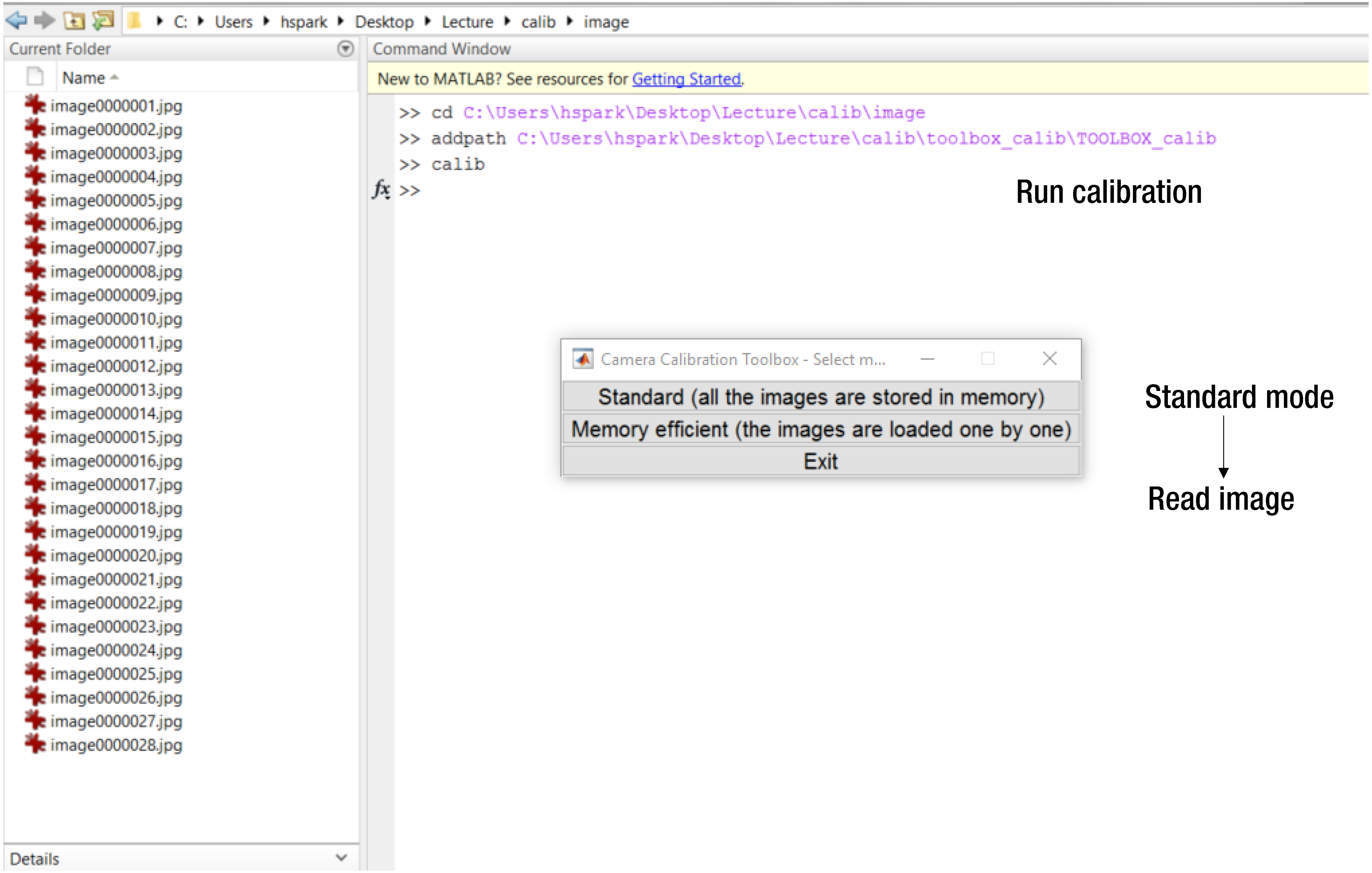
$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

$$\mathbf{r}_1 = \frac{\mathbf{K}^{-1}\mathbf{h}_1}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_2 = \frac{\mathbf{K}^{-1}\mathbf{h}_2}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{h}_3}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Divided by constant factor

MATLAB Calibration Toolbox Demo

https://www.vision.caltech.edu/bouguetj/calib_doc/

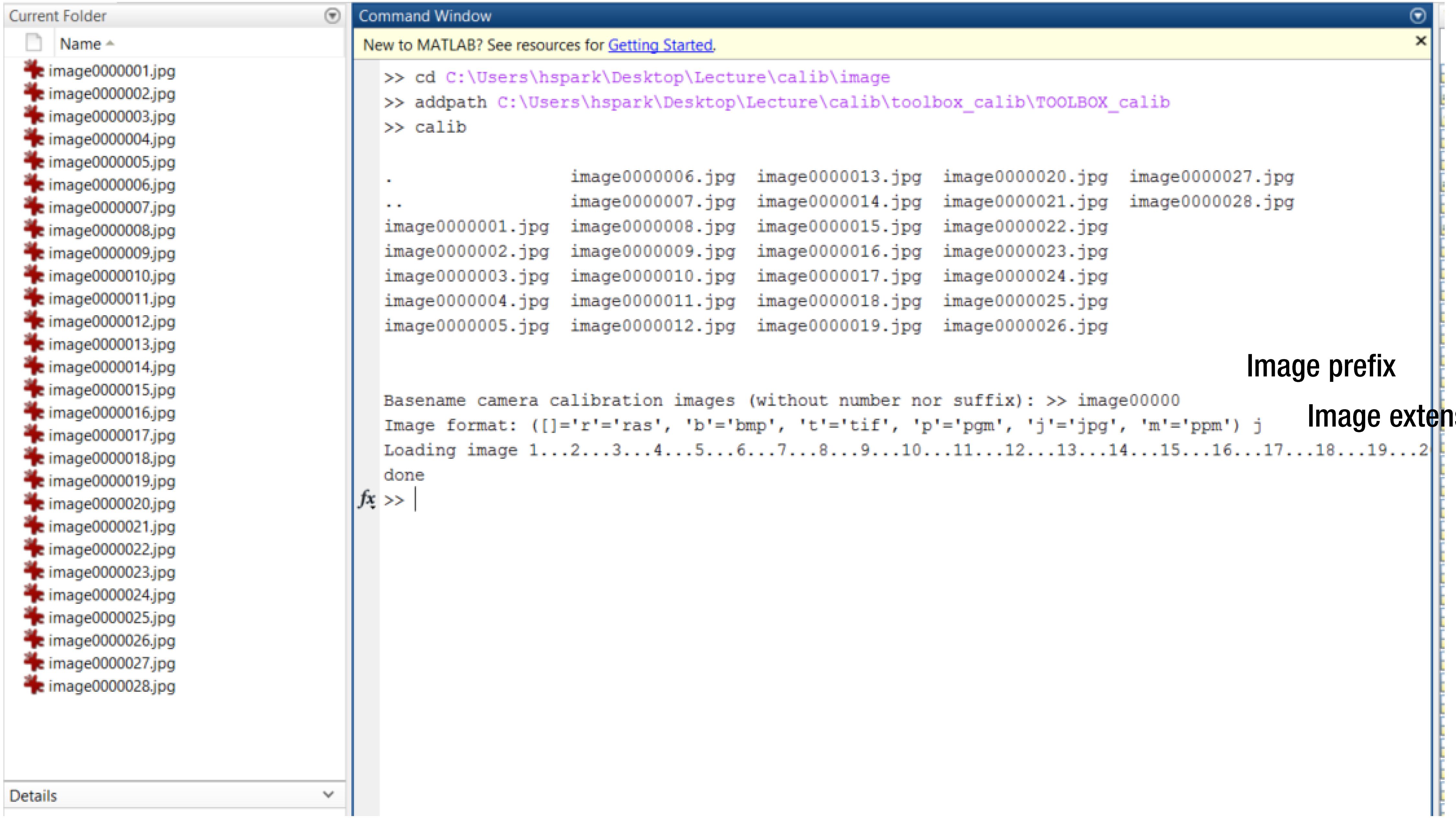


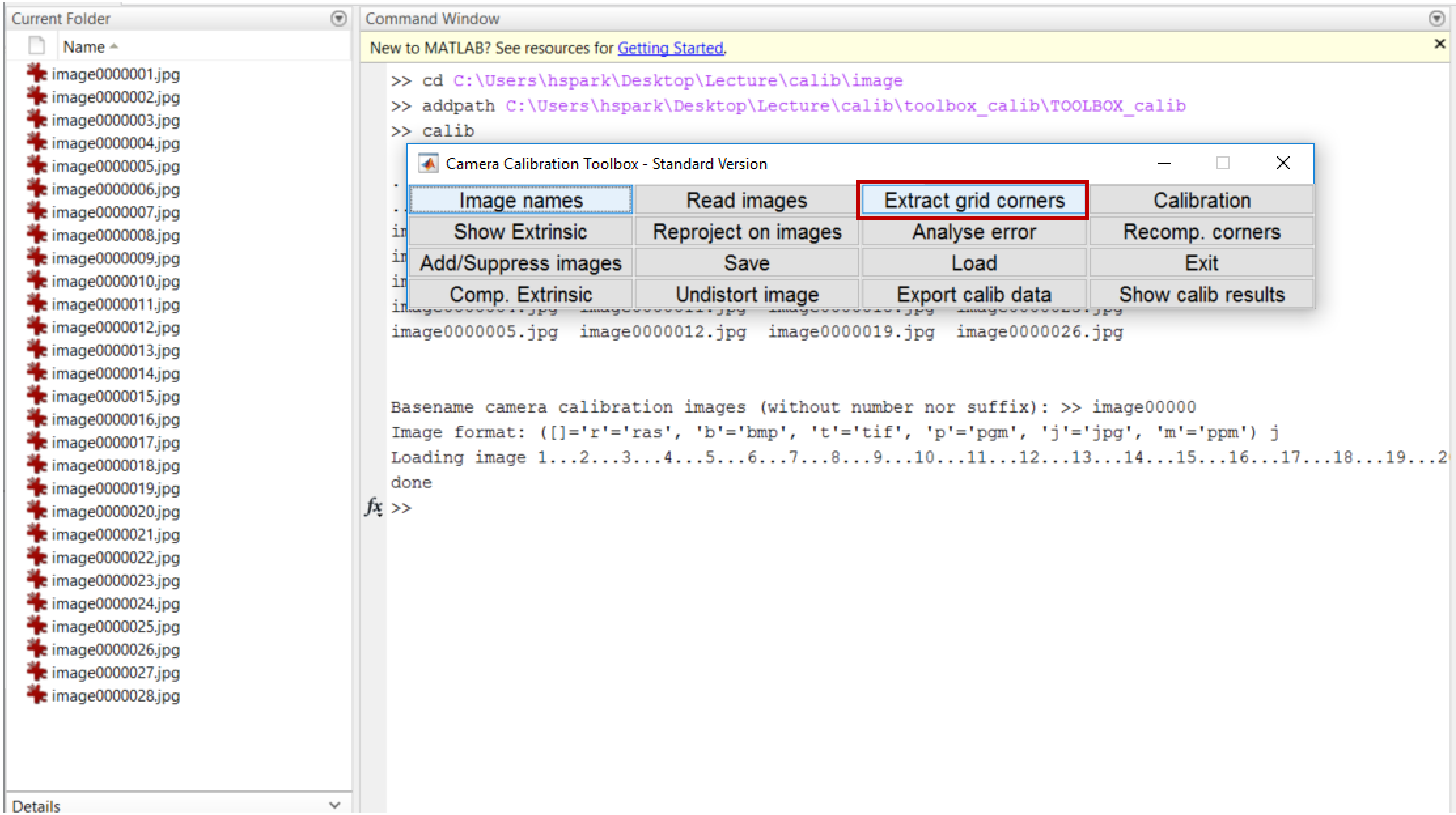
Run calibration

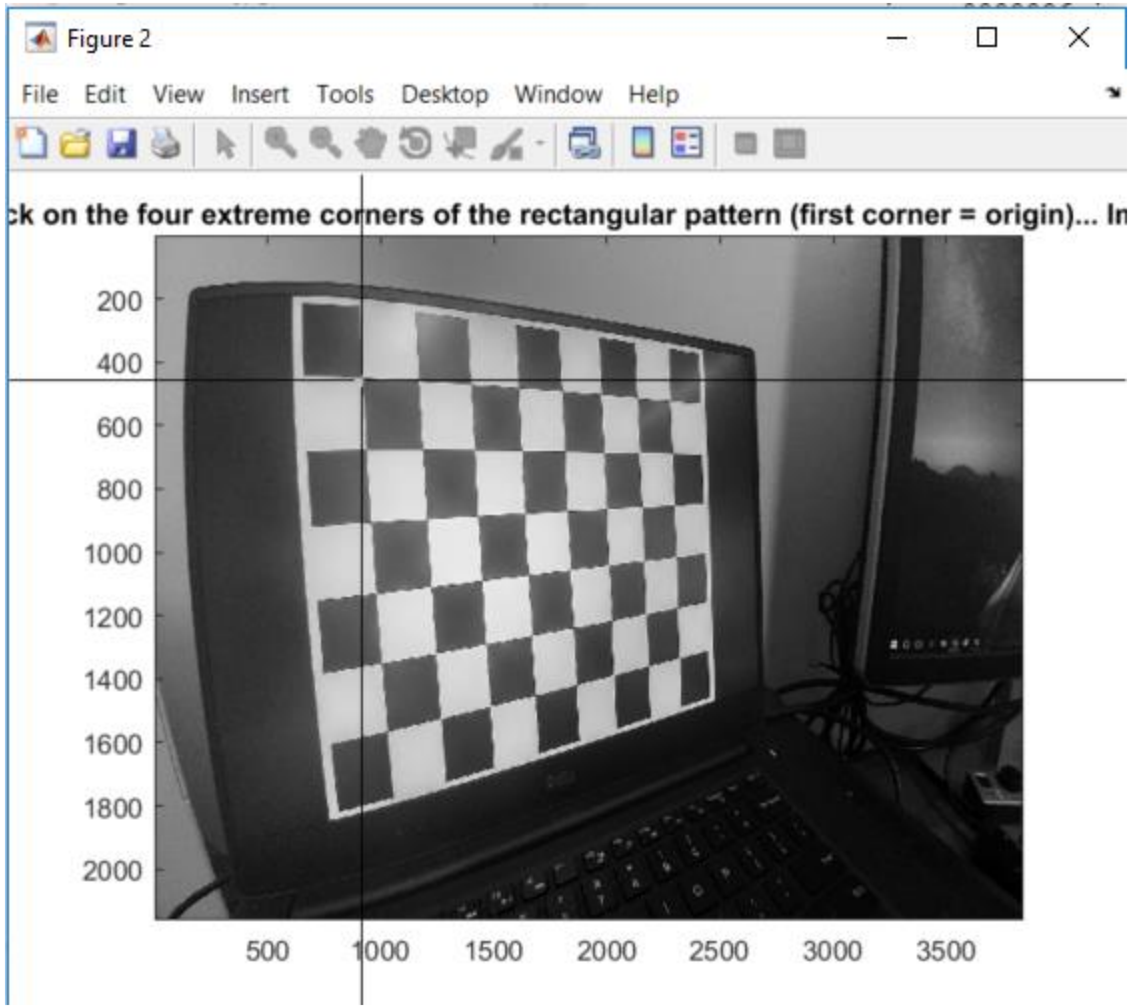
Standard mode

↓

Read image

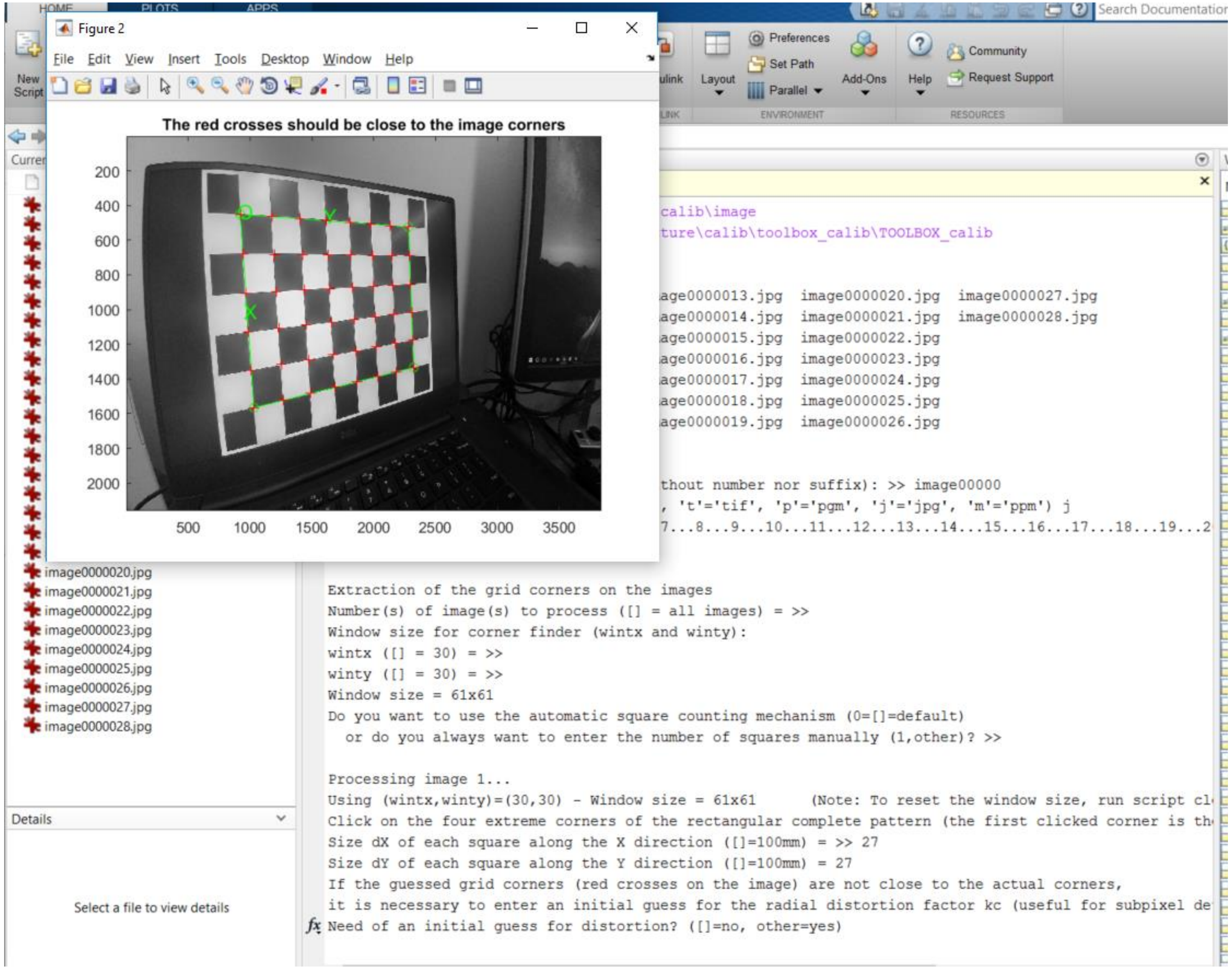

Current Folder
Name
image0000001.jpg
image0000002.jpg
image0000003.jpg
image0000004.jpg
image0000005.jpg
image0000006.jpg
image0000007.jpg
image0000008.jpg
image0000009.jpg
image0000010.jpg
image0000011.jpg
image0000012.jpg
image0000013.jpg
image0000014.jpg
image0000015.jpg
image0000016.jpg
image0000017.jpg
image0000018.jpg
image0000019.jpg
image0000020.jpg
image0000021.jpg
image0000022.jpg
image0000023.jpg
image0000024.jpg
image0000025.jpg
image0000026.jpg
image0000027.jpg
image0000028.jpg
Details
Command Window
New to MATLAB? See resources for Getting Started.
>> cd C:\Users\hspark\Desktop\Lecture\calib\image
>> addpath C:\Users\hspark\Desktop\Lecture\calib\toolbox_calib\TOOLBOX_calib
>> calib
. image0000006.jpg image0000013.jpg image0000020.jpg image0000027.jpg
.. image0000007.jpg image0000014.jpg image0000021.jpg image0000028.jpg
image0000001.jpg image0000008.jpg image0000015.jpg image0000022.jpg
image0000002.jpg image0000009.jpg image0000016.jpg image0000023.jpg
image0000003.jpg image0000010.jpg image0000017.jpg image0000024.jpg
image0000004.jpg image0000011.jpg image0000018.jpg image0000025.jpg
image0000005.jpg image0000012.jpg image0000019.jpg image0000026.jpg
Basename camera calibration images (without number nor suffix): >> image00000
Image format: ([]='r'='ras', 'b'='bmp', 't'='tif', 'p'='pgm', 'j'='jpg', 'm'='ppm') j
Loading image 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...2
done
>>
Image prefix
Image extension


Current Folder
Name
image0000001.jpg
image0000002.jpg
image0000003.jpg
image0000004.jpg
image0000005.jpg
image0000006.jpg
image0000007.jpg
image0000008.jpg
image0000009.jpg
image0000010.jpg
image0000011.jpg
image0000012.jpg
image0000013.jpg
image0000014.jpg
image0000015.jpg
image0000016.jpg
image0000017.jpg
image0000018.jpg
image0000019.jpg
image0000020.jpg
image0000021.jpg
image0000022.jpg
image0000023.jpg
image0000024.jpg
image0000025.jpg
image0000026.jpg
image0000027.jpg
image0000028.jpg
Details
Command Window
New to MATLAB? See resources for Getting Started.
>> cd C:\Users\hspark\Desktop\Lecture\calib\image
>> addpath C:\Users\hspark\Desktop\Lecture\calib\toolbox_calib\TOOLBOX_calib
>> calib
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
image0000005.jpg image0000012.jpg image0000019.jpg image0000026.jpg
Basename camera calibration images (without number nor suffix): >> image00000
Image format: ([]='r'='ras', 'b'='bmp', 't'='tif', 'p'='pgm', 'j'='jpg', 'm'='ppm') j
Loading image 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...2
done
>>



Click four corner in the following order:

1. Top left
2. Top right
3. Bottom right
4. Bottom left



Default mode (press Enter)

Set grid size (27mm)

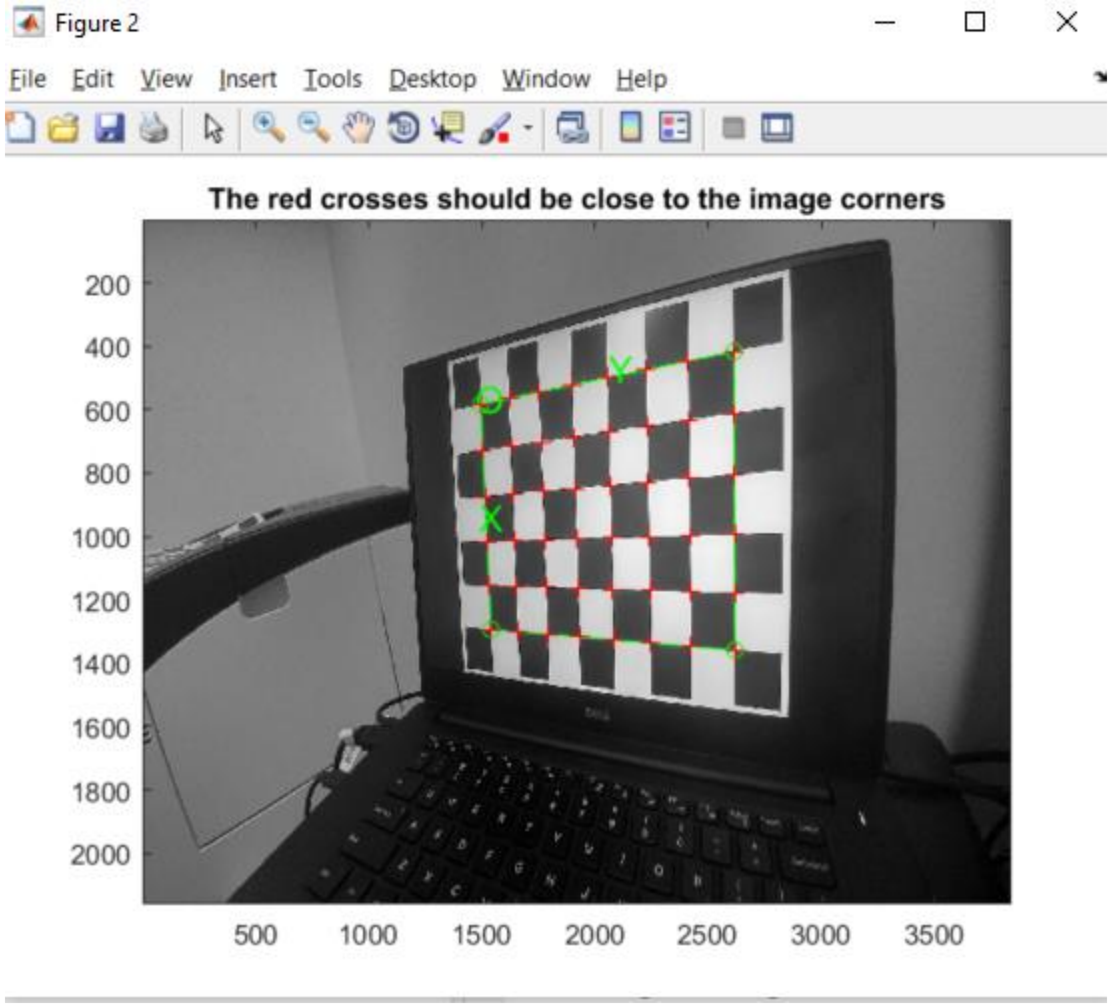
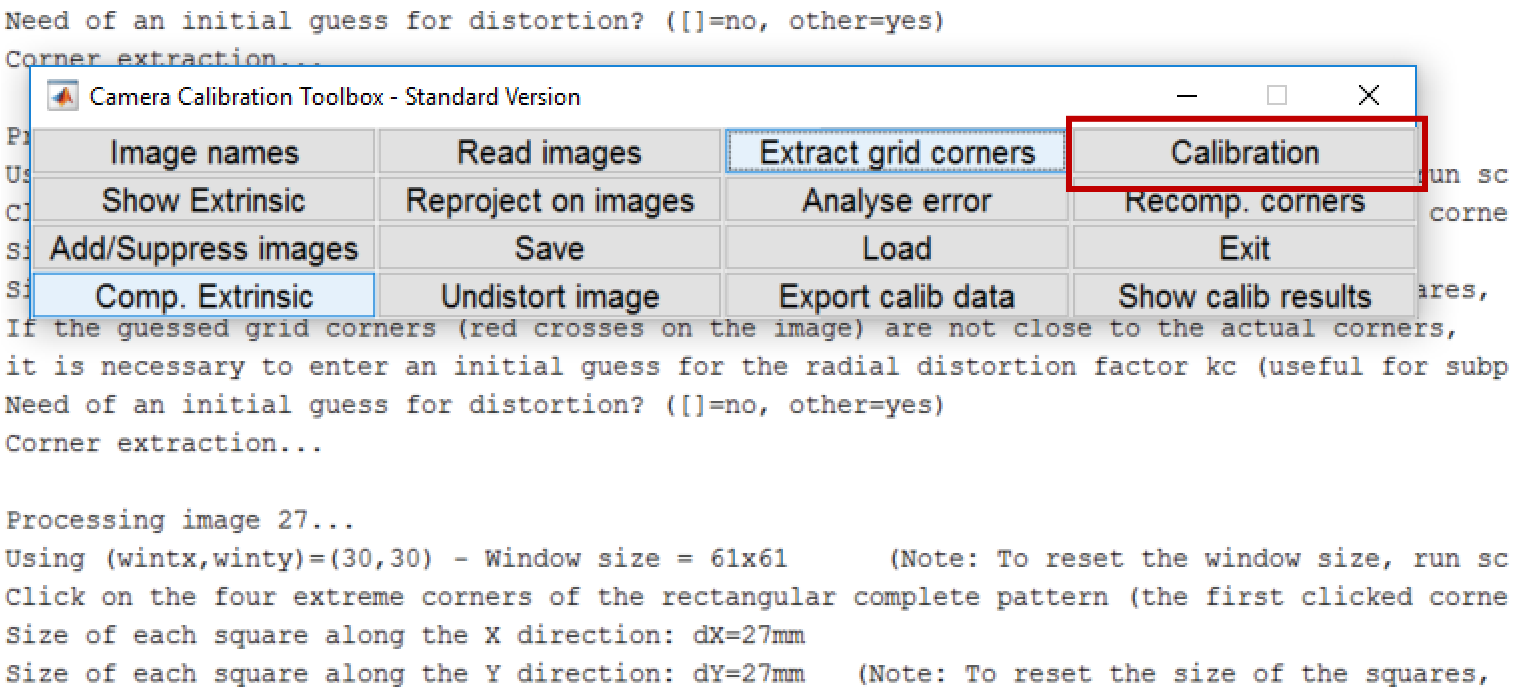
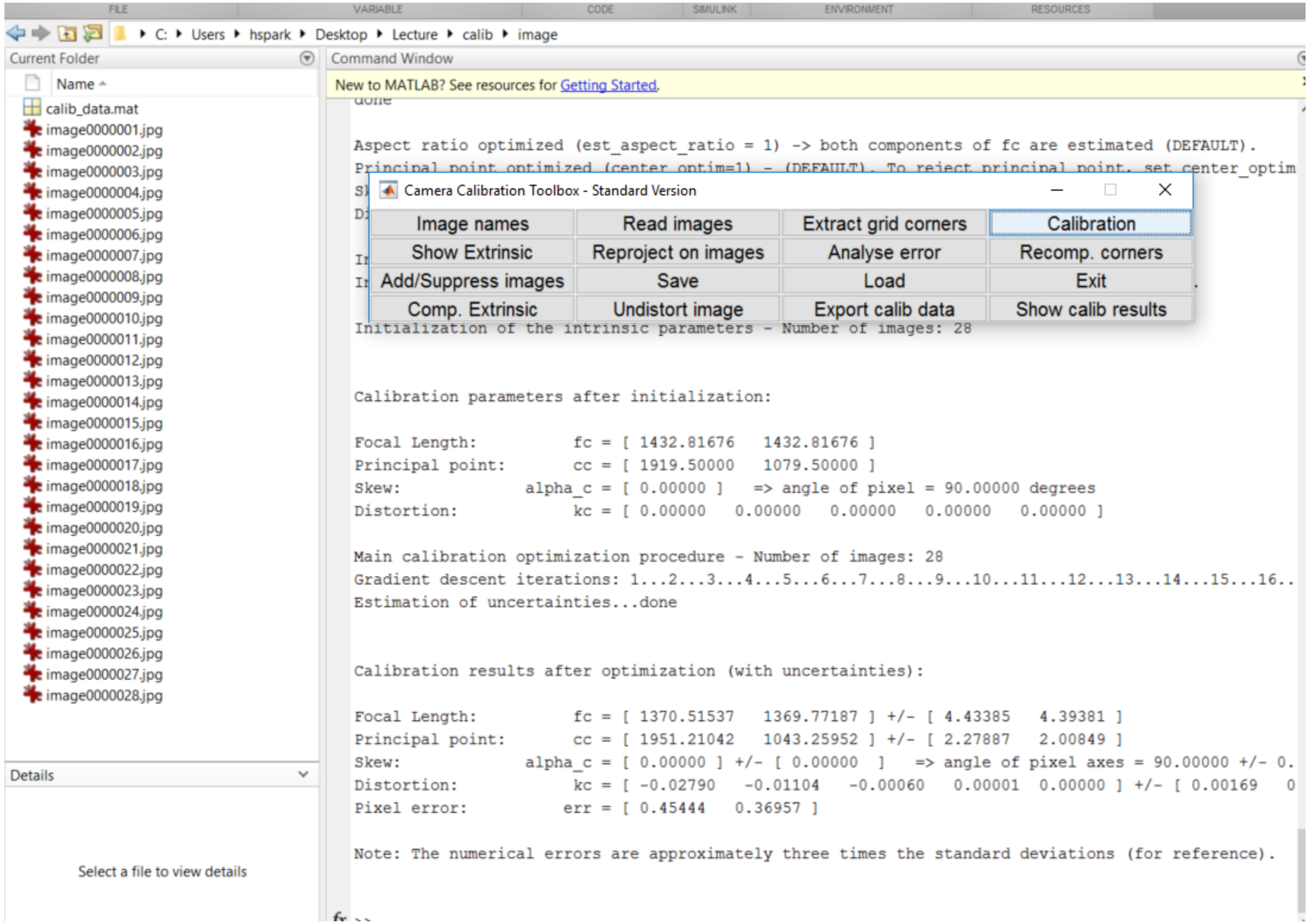


Figure 2
File Edit View Insert Tools Desktop Window Help
The red crosses should be close to the image corners
200
400
600
800
1000
1200
1400
1600
1800
2000
500
1000
1500
2000
2500
3000
3500
O
Y
X

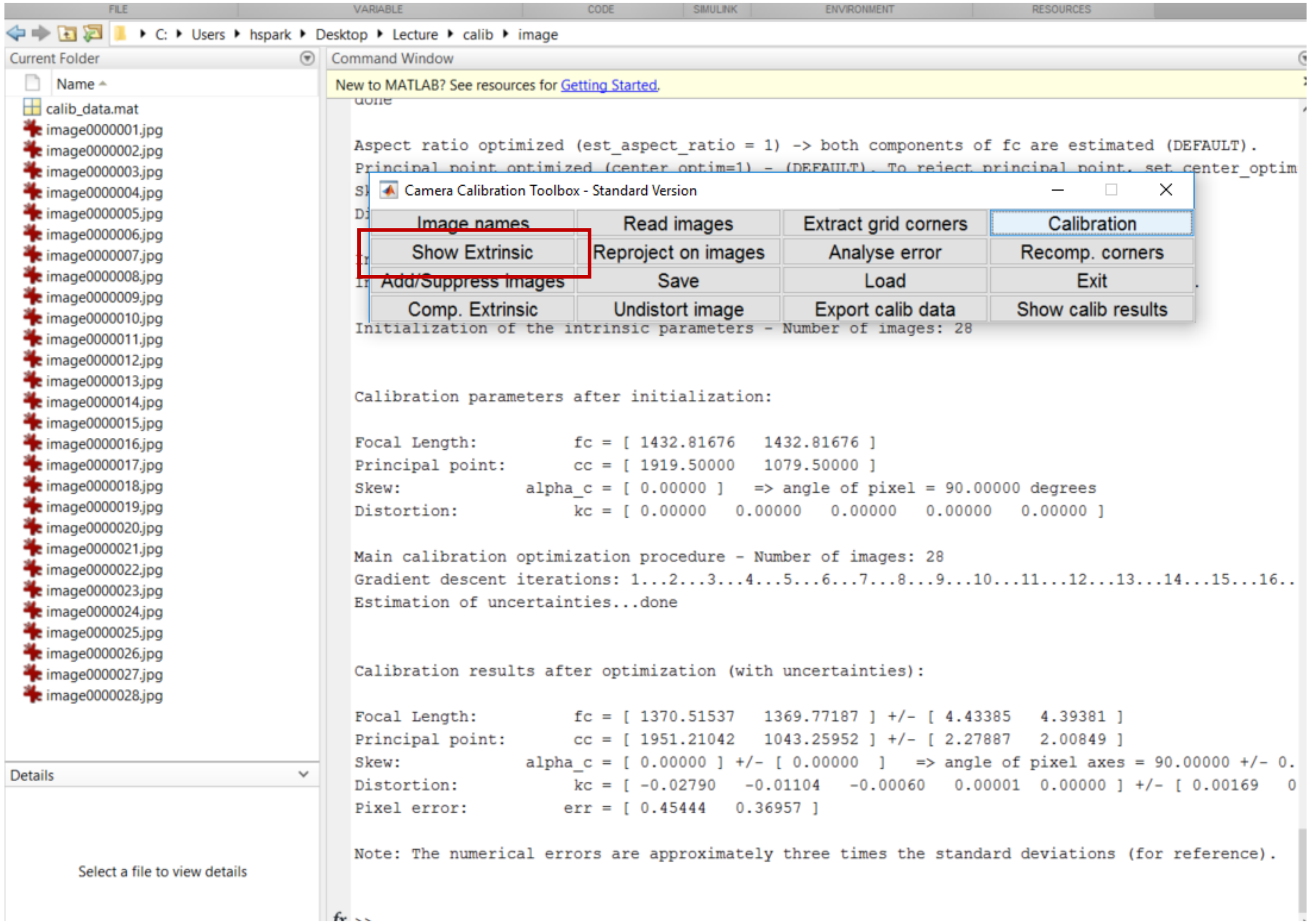

Need of an initial guess for distortion? ([]=no, other=yes)
Corner extraction...
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
If the guessed grid corners (red crosses on the image) are not close to the actual corners,
it is necessary to enter an initial guess for the radial distortion factor kc (useful for subp
Need of an initial guess for distortion? ([]=no, other=yes)
Corner extraction...
Processing image 27...
Using (wintx,winty)=(30,30) - Window size = 61x61 (Note: To reset the window size, run sc
Click on the four extreme corners of the rectangular complete pattern (the first clicked corne
Size of each square along the X direction: dX=27mm
Size of each square along the Y direction: dY=27mm (Note: To reset the size of the squares,

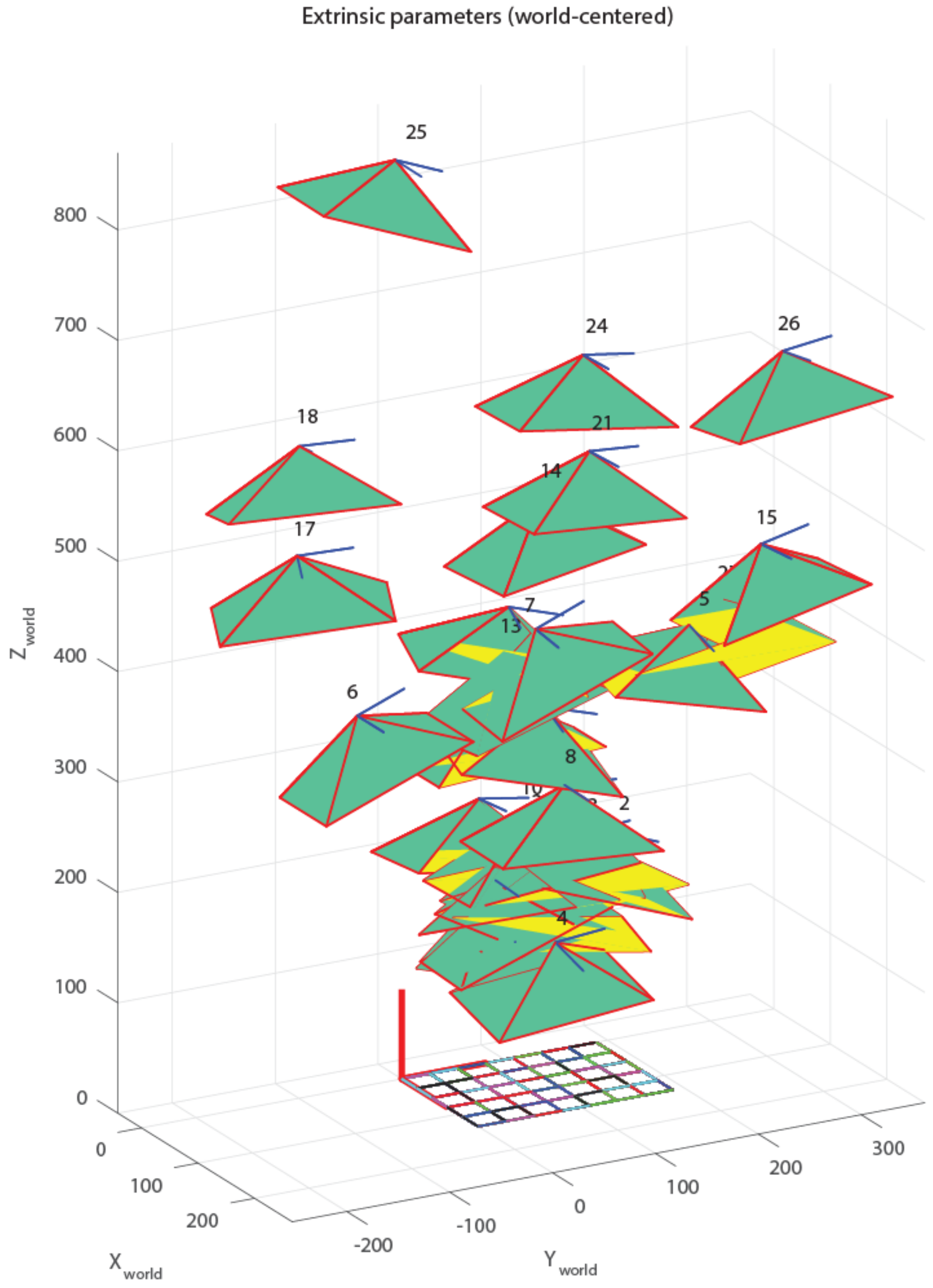


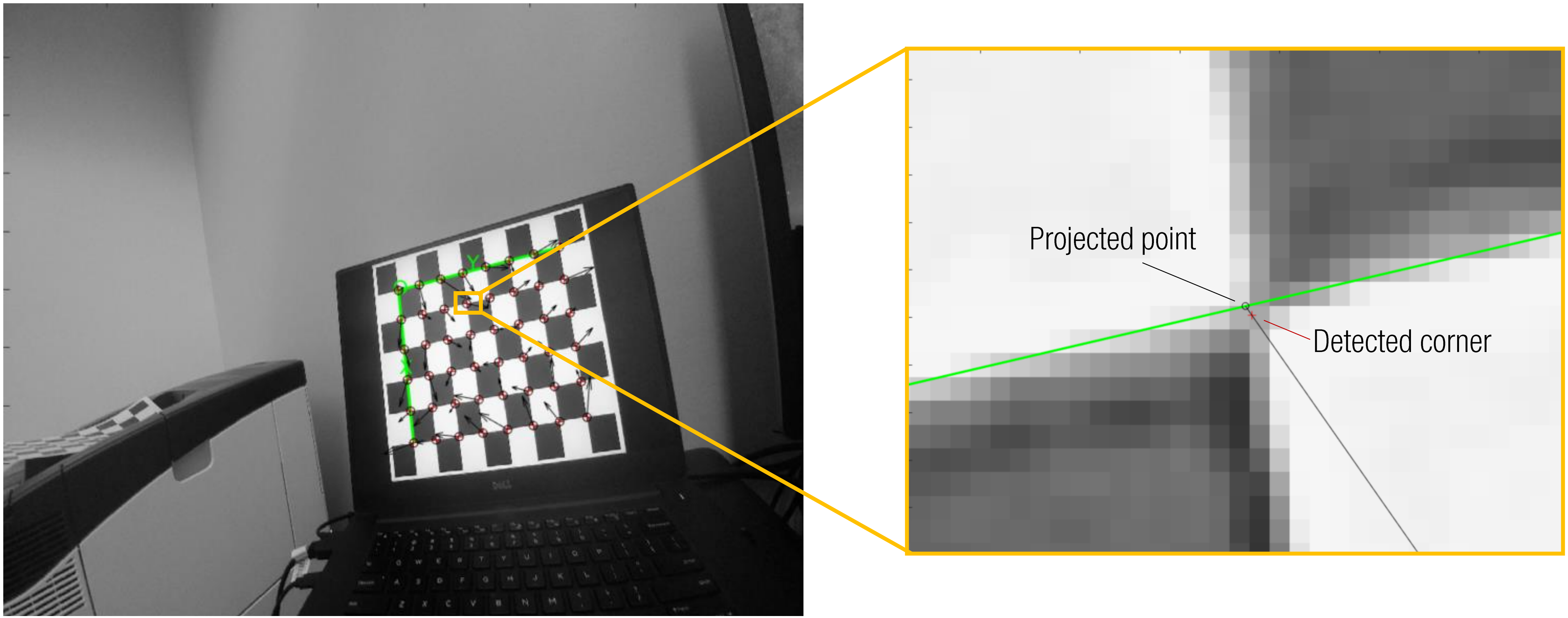
Cf) calibration with vanishing points

K =

1317.2	0	1931.8
0	1317.2	1146.1
0	0	1


Current Folder
Name
calib_data.mat
image0000001.jpg
image0000002.jpg
image0000003.jpg
image0000004.jpg
image0000005.jpg
image0000006.jpg
image0000007.jpg
image0000008.jpg
image0000009.jpg
image0000010.jpg
image0000011.jpg
image0000012.jpg
image0000013.jpg
image0000014.jpg
image0000015.jpg
image0000016.jpg
image0000017.jpg
image0000018.jpg
image0000019.jpg
image0000020.jpg
image0000021.jpg
image0000022.jpg
image0000023.jpg
image0000024.jpg
image0000025.jpg
image0000026.jpg
image0000027.jpg
image0000028.jpg
Details
Select a file to view details
C: Users hspark Desktop Lecture calib image
Command Window
New to MATLAB? See resources for Getting Started.
Aspect ratio optimized (est_aspect_ratio = 1) -> both components of fc are estimated (DEFAULT).
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
Initialization of the intrinsic parameters - Number of images: 28
Calibration parameters after initialization:
Focal Length: fc = [1432.81676 1432.81676]
Principal point: cc = [1919.50000 1079.50000]
Skew: alpha_c = [0.00000] => angle of pixel = 90.00000 degrees
Distortion: kc = [0.00000 0.00000 0.00000 0.00000 0.00000]
Main calibration optimization procedure - Number of images: 28
Gradient descent iterations: 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16..
Estimation of uncertainties...done
Calibration results after optimization (with uncertainties):
Focal Length: fc = [1370.51537 1369.77187] +/- [4.43385 4.39381]
Principal point: cc = [1951.21042 1043.25952] +/- [2.27887 2.00849]
Skew: alpha_c = [0.00000] +/- [0.00000] => angle of pixel axes = 90.00000 +/- 0.
Distortion: kc = [-0.02790 -0.01104 -0.00060 0.00001 0.00000] +/- [0.00169 0
Pixel error: err = [0.45444 0.36957]
Note: The numerical errors are approximately three times the standard deviations (for reference).


Extrinsic parameters (world-centered)
Z world
X world
Y world
800
700
600
500
400
300
200
100
0
25
24
26
18
21
14
17
15
6
7
13
5
8
2
4
0
100
200
-200
-100
0
100
200
300

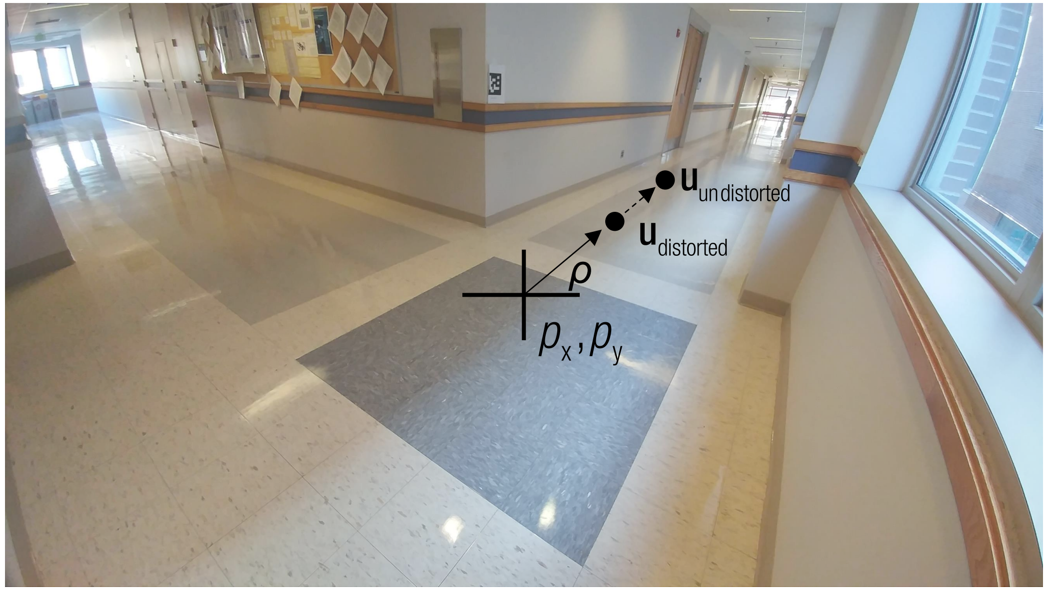

Projected point
Detected corner


Lens Radial Distortion


Lens Radial Distortion Correction

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

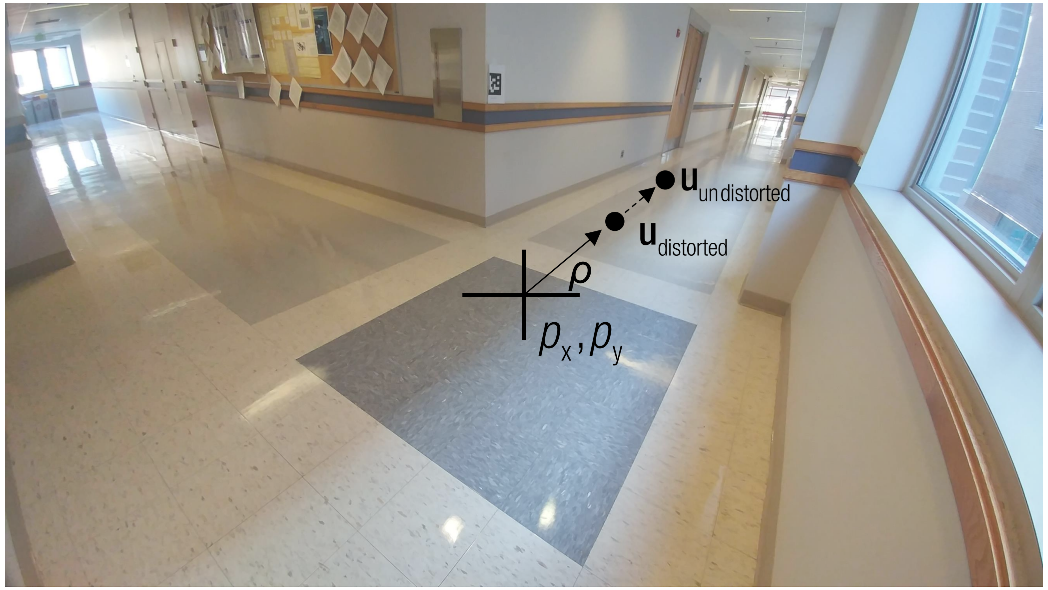


Normalized point:

$$\bar{\mathbf{u}}_{distorted} = \mathbf{K}^{-1}\mathbf{u}_{distorted}, \quad \bar{\mathbf{u}}_{undistorted} = \mathbf{K}^{-1}\mathbf{u}_{undistorted}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



Normalized point:

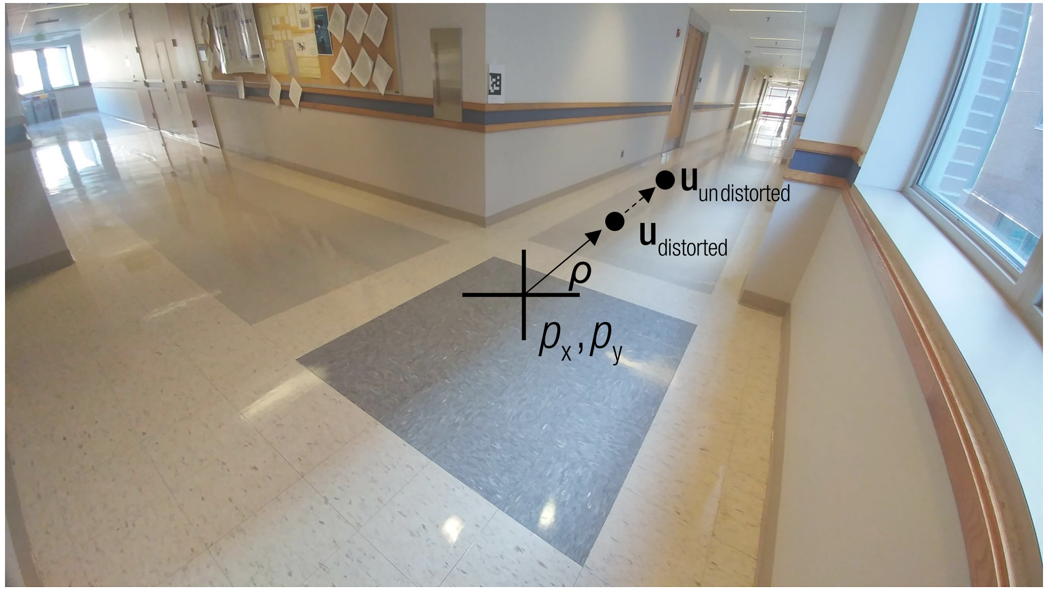
$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

where $\rho = \left\| \mathbf{K}^{-1}\bar{\mathbf{u}}_{\text{distorted}} \right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Parameter Estimation (2nd order)

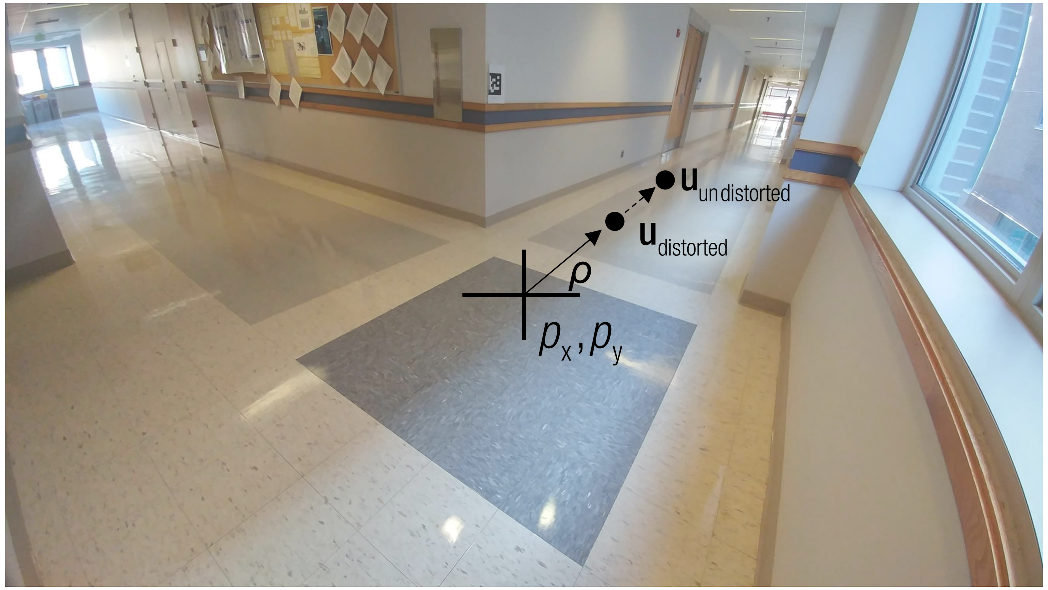


Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

Radial Distortion Parameter Estimation (2nd order)



Normalized point:

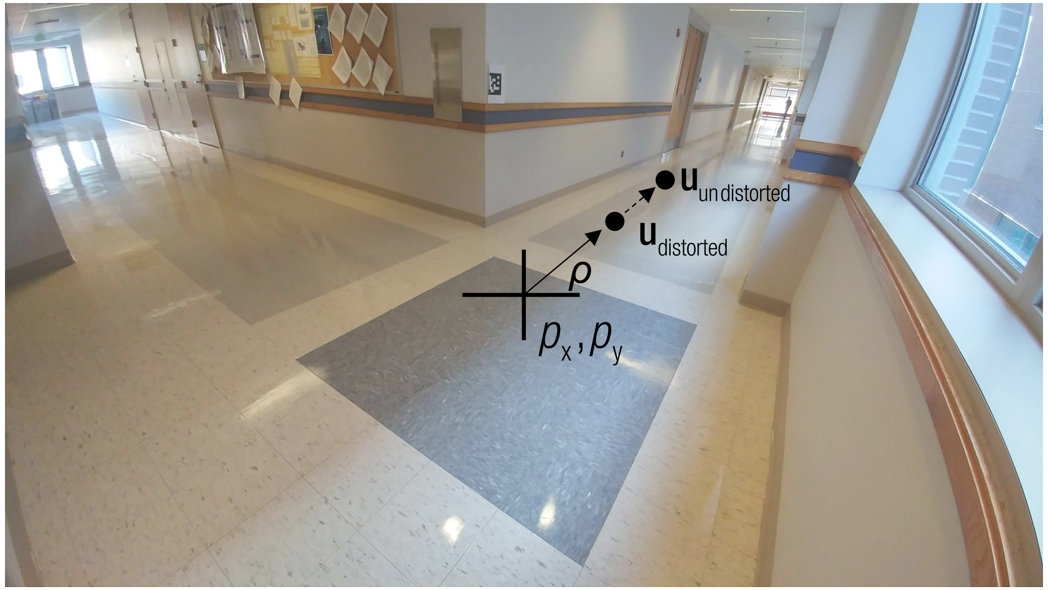
$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

$$\begin{bmatrix} \rho^2\bar{\mathbf{u}}^1_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2\bar{\mathbf{u}}^m_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{u}}^1_{\text{distorted}} - \bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots \\ \bar{\mathbf{u}}^m_{\text{distorted}} - \bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}$$

m: # of points

Radial Distortion Parameter Estimation (2nd order)



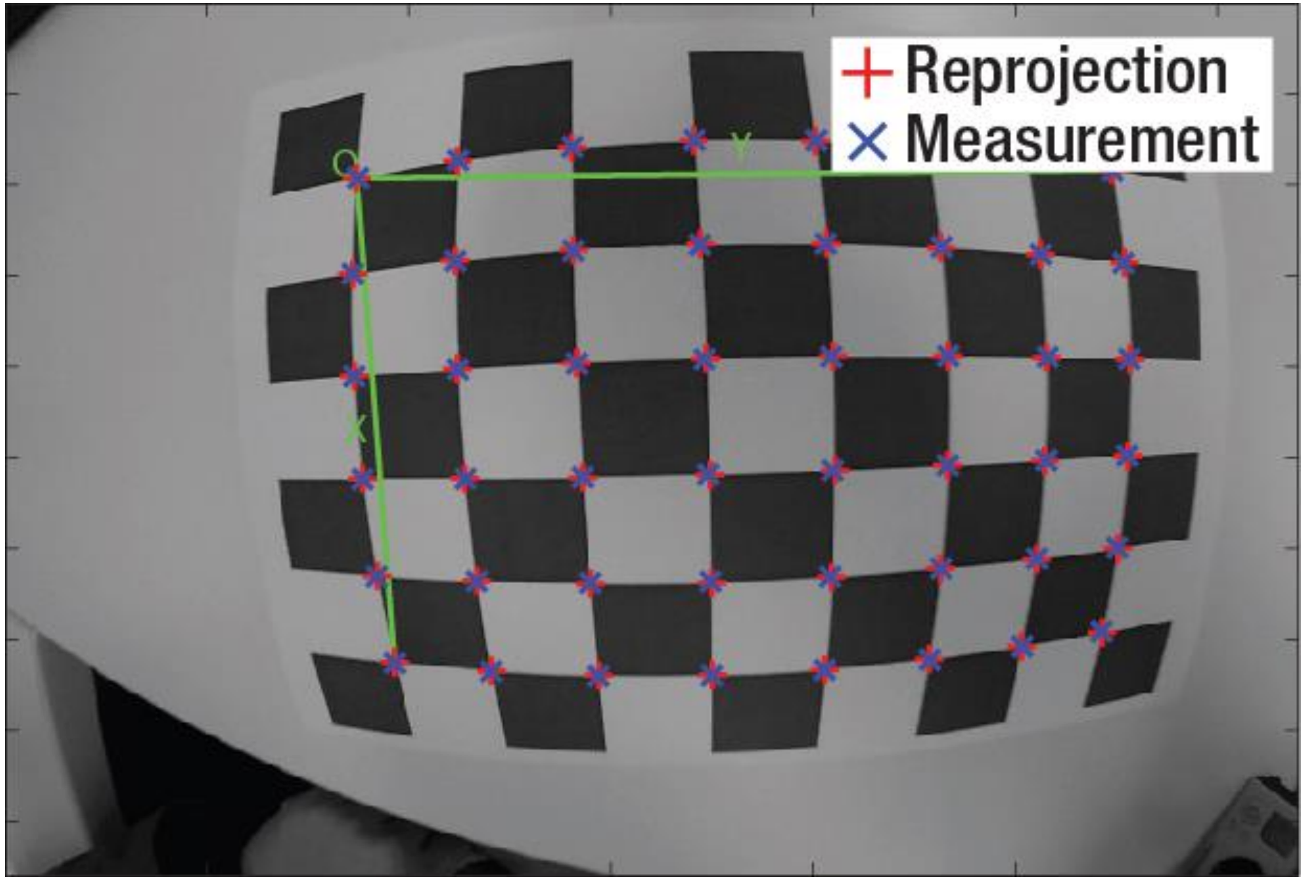
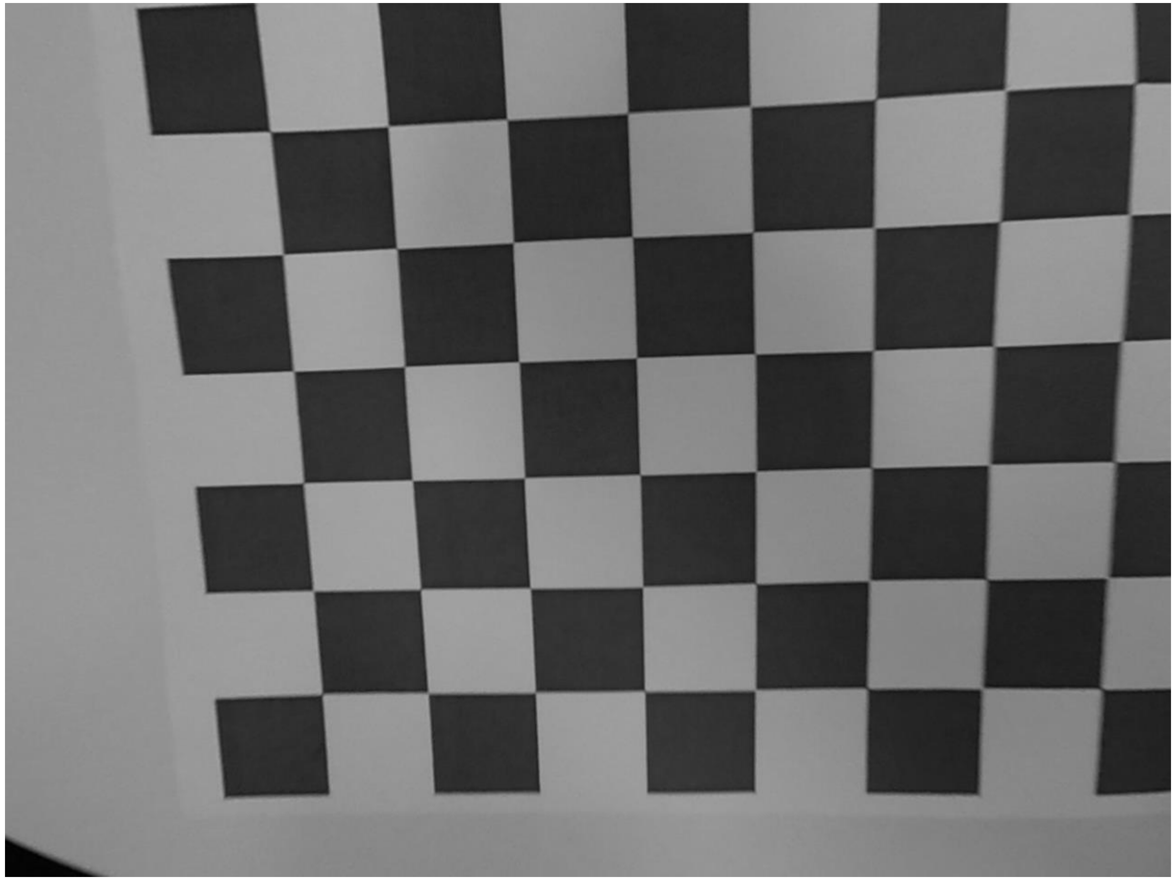
Normalized point:

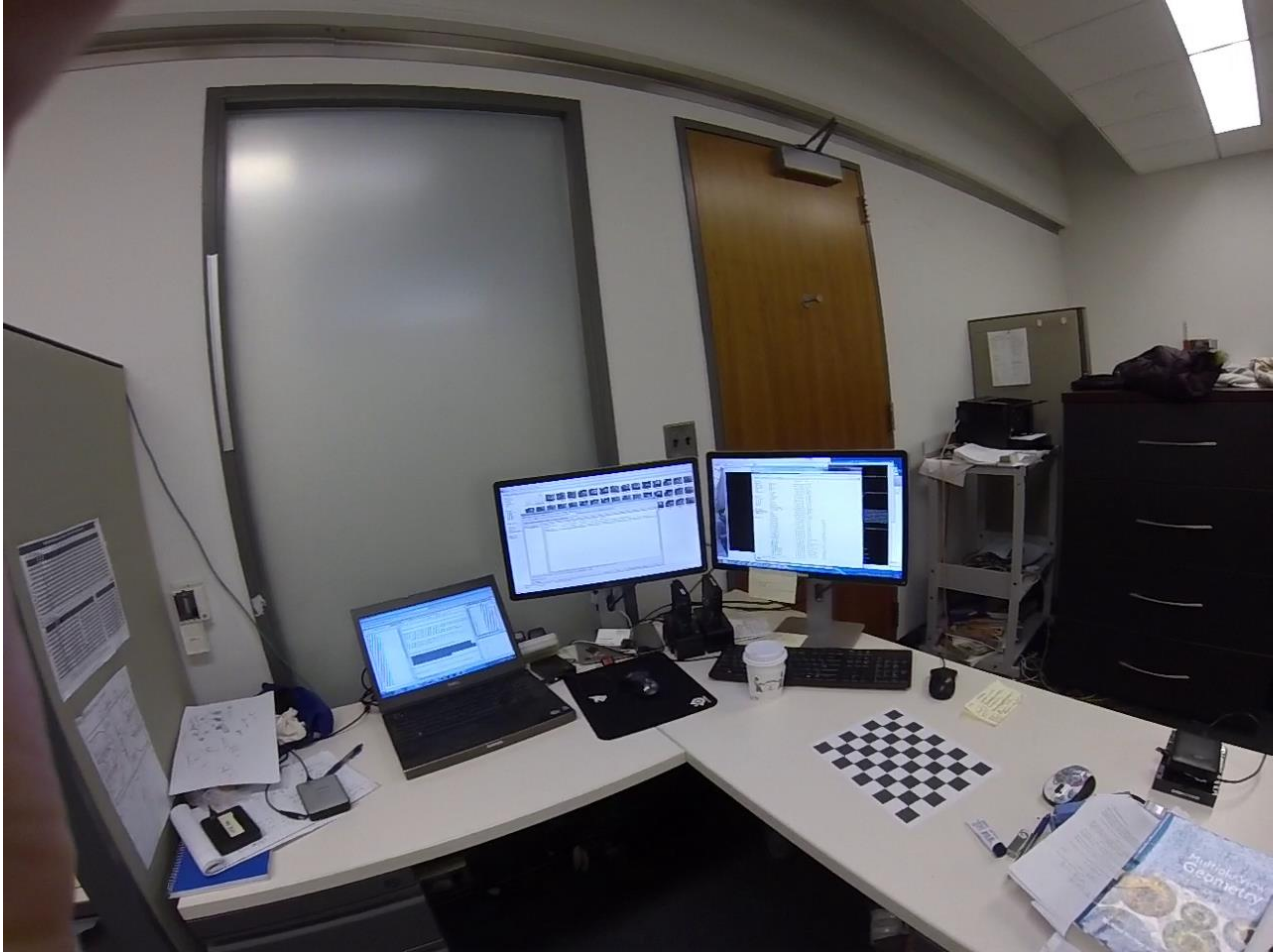
$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

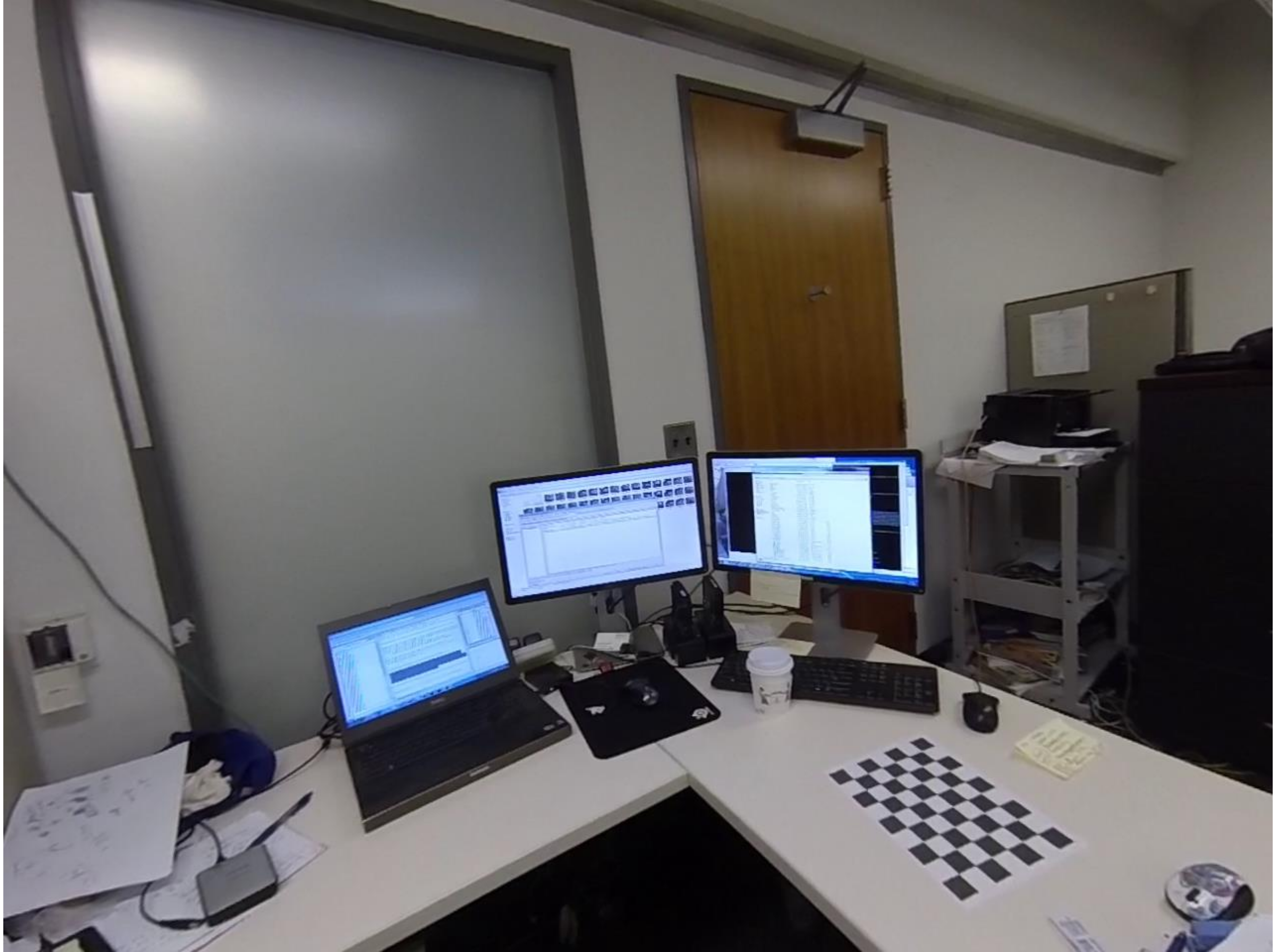
$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

$$\underbrace{\begin{bmatrix} \rho^2\bar{\mathbf{u}}^1_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2\bar{\mathbf{u}}^m_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} k_1 \\ k_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \bar{\mathbf{u}}^1_{\text{distorted}} - \bar{\mathbf{u}}^1_{\text{undistorted}} \\ \bar{\mathbf{u}}^m_{\text{distorted}} - \bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}}_{\mathbf{b}}$$

m: # of points


Reprojection
Measurement



Geometry

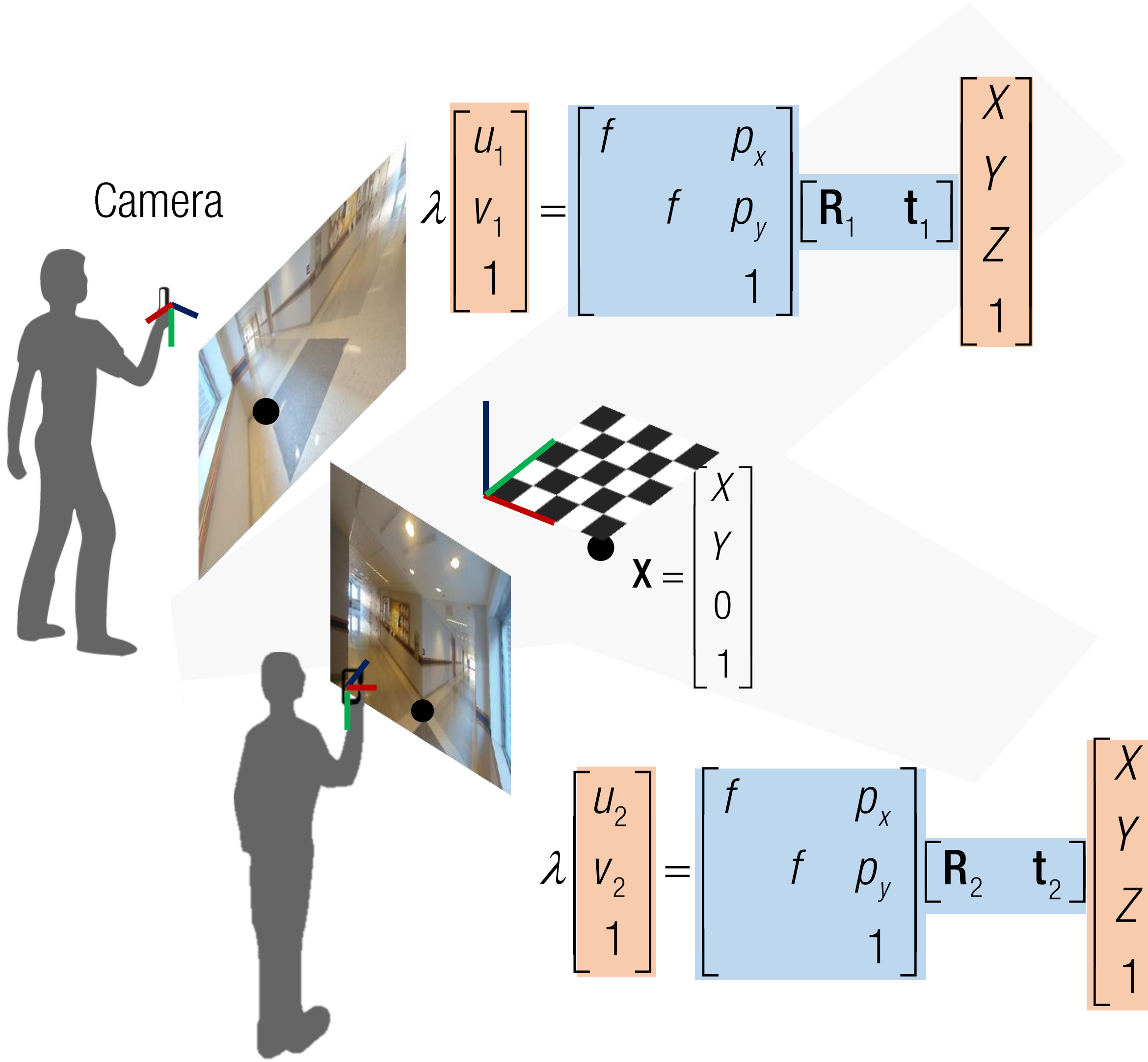


Fisheye Lens Distortion

Fisheye Lens Distortion

Where am I via Homography?

Recall: Camera Calibration from Multiple Imags



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{t}_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6n (**R** and **t**)

n: the number of images

# of equations: 2nm (**X**)

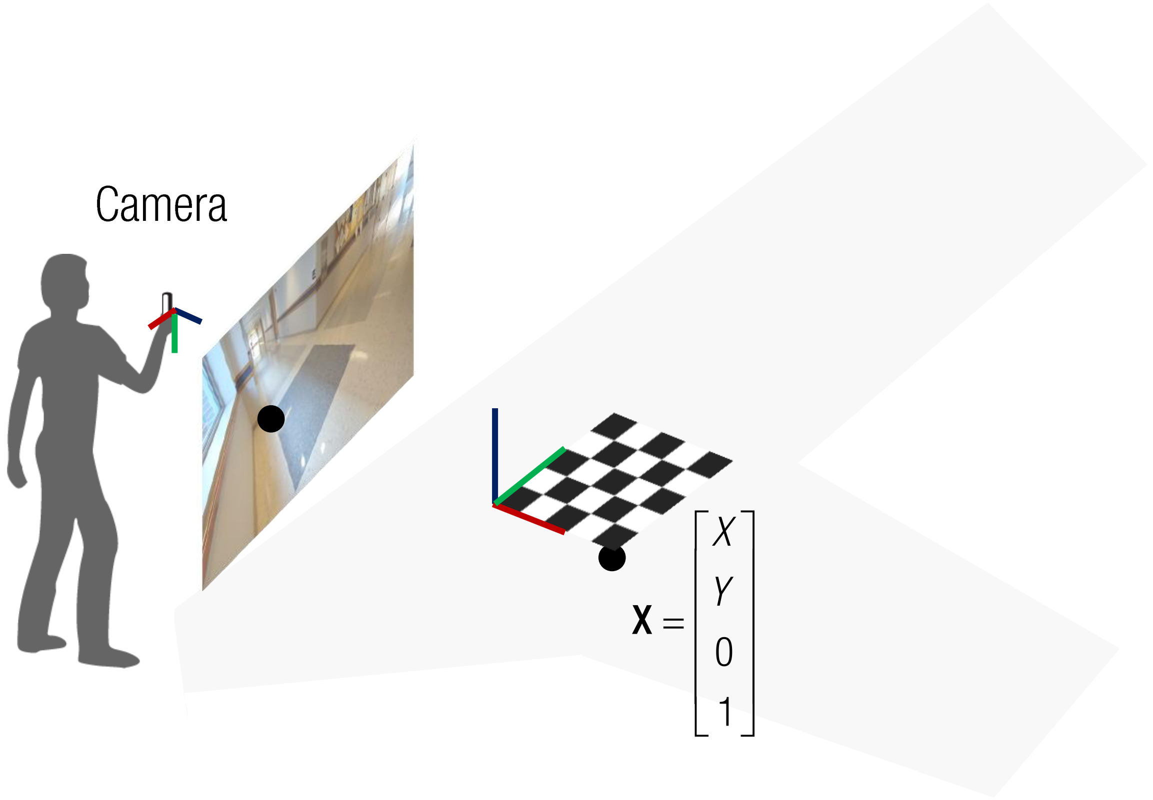
m: the number of known 3D points

We can solve for **K**, **R**, **t** if $3 + 6n < 2\,nm$

(orange) : Knowns

(blue) : Unknowns

Homography Mapping



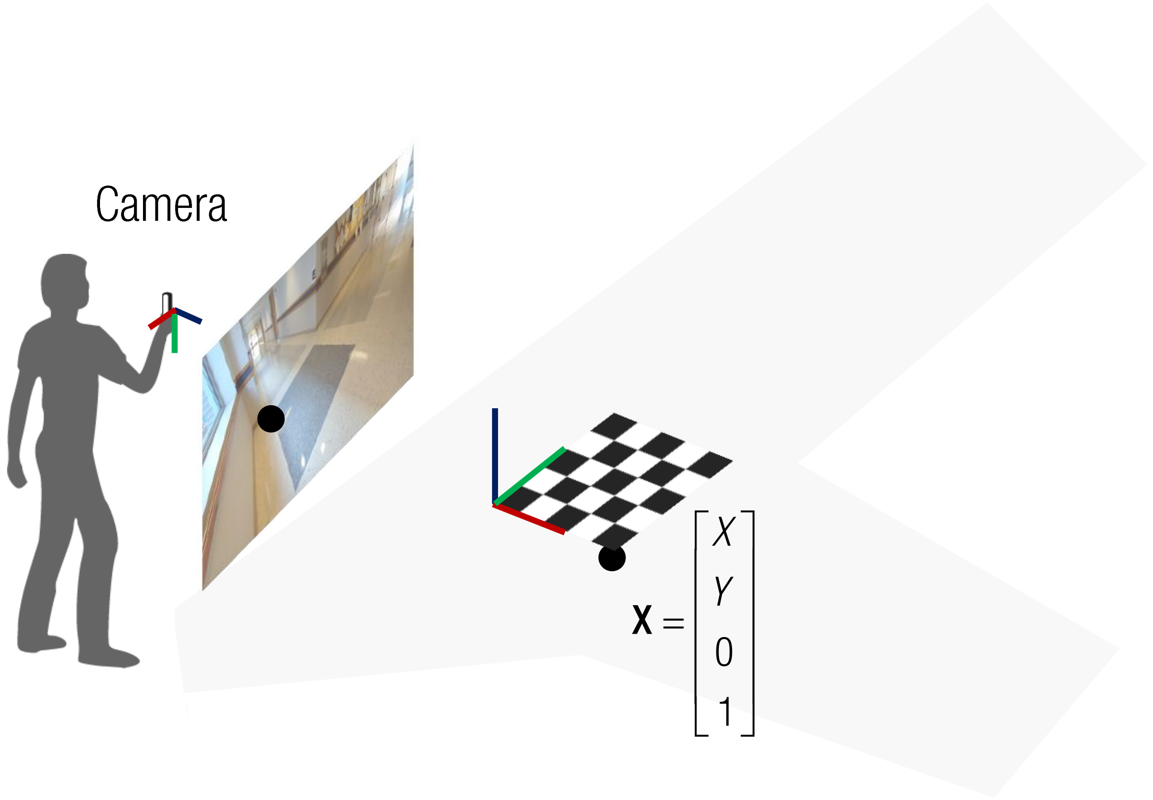
Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



Points in 2D plane are mapped to an image with homography:

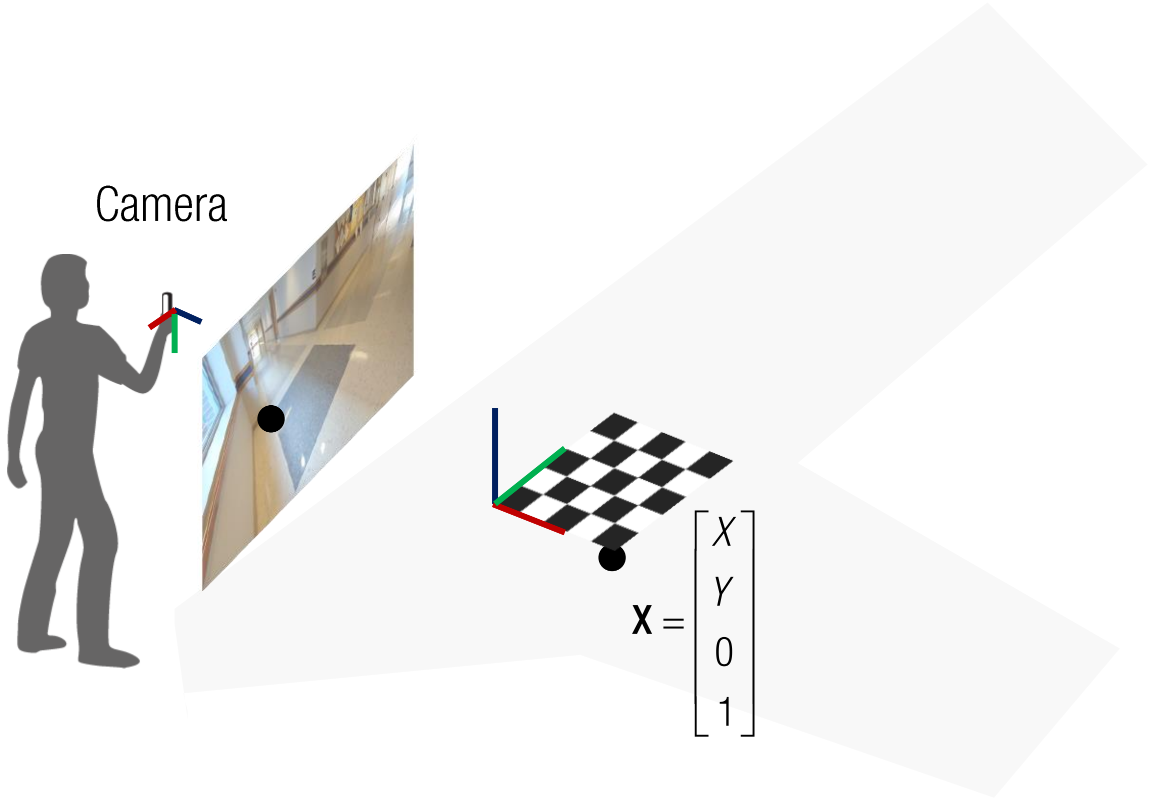
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}}_{\mathbf{Q}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \underbrace{\mathbf{H}}_{3\text{x}3} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



Points in 2D plane are mapped to an image with homography:

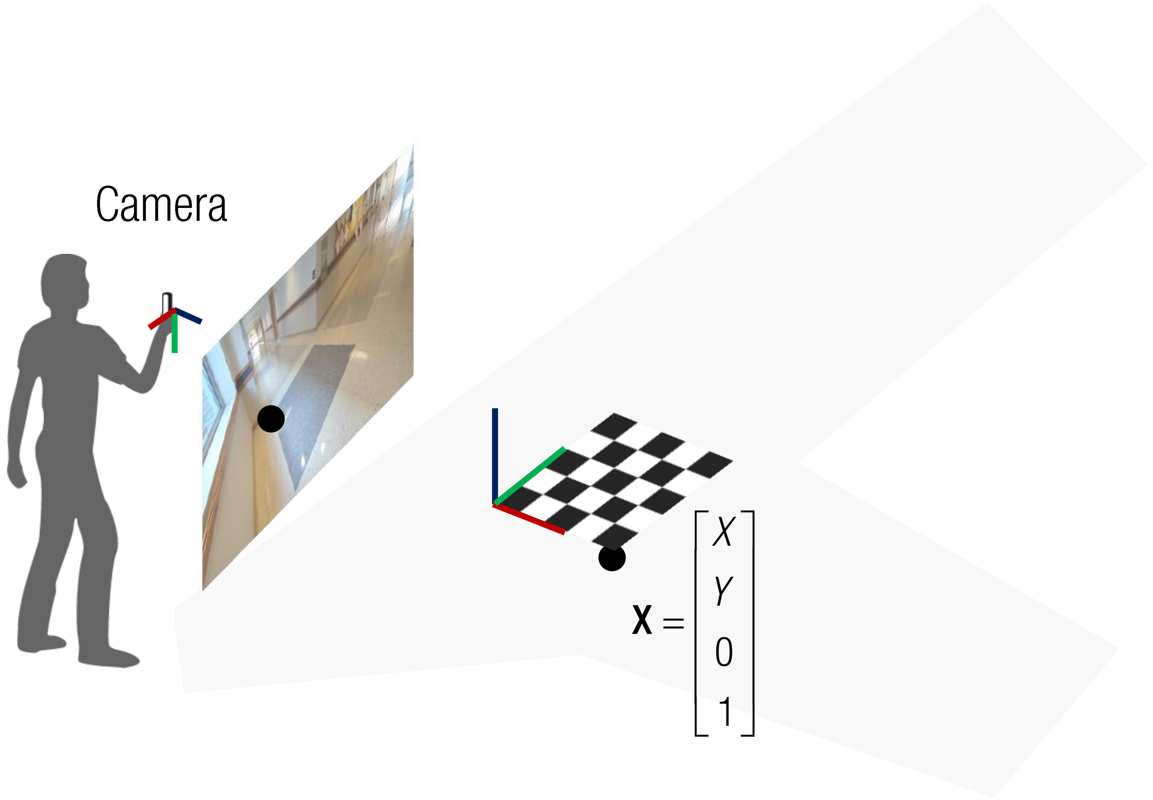
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}}_{\mathbf{Q}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \underbrace{\mathbf{H}}_{3\text{x}3} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

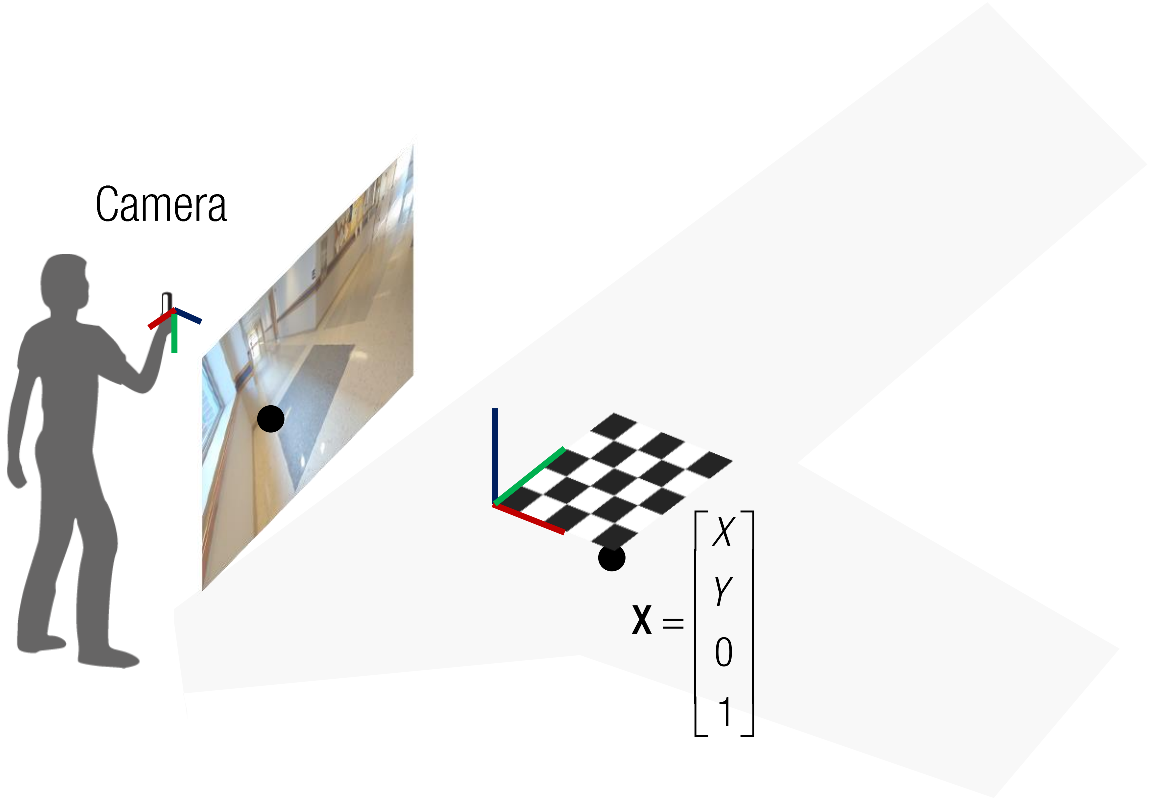
Points in 2D plane are mapped to an image with homography:

$$\mathbf{K}^{-1}\mathbf{H} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

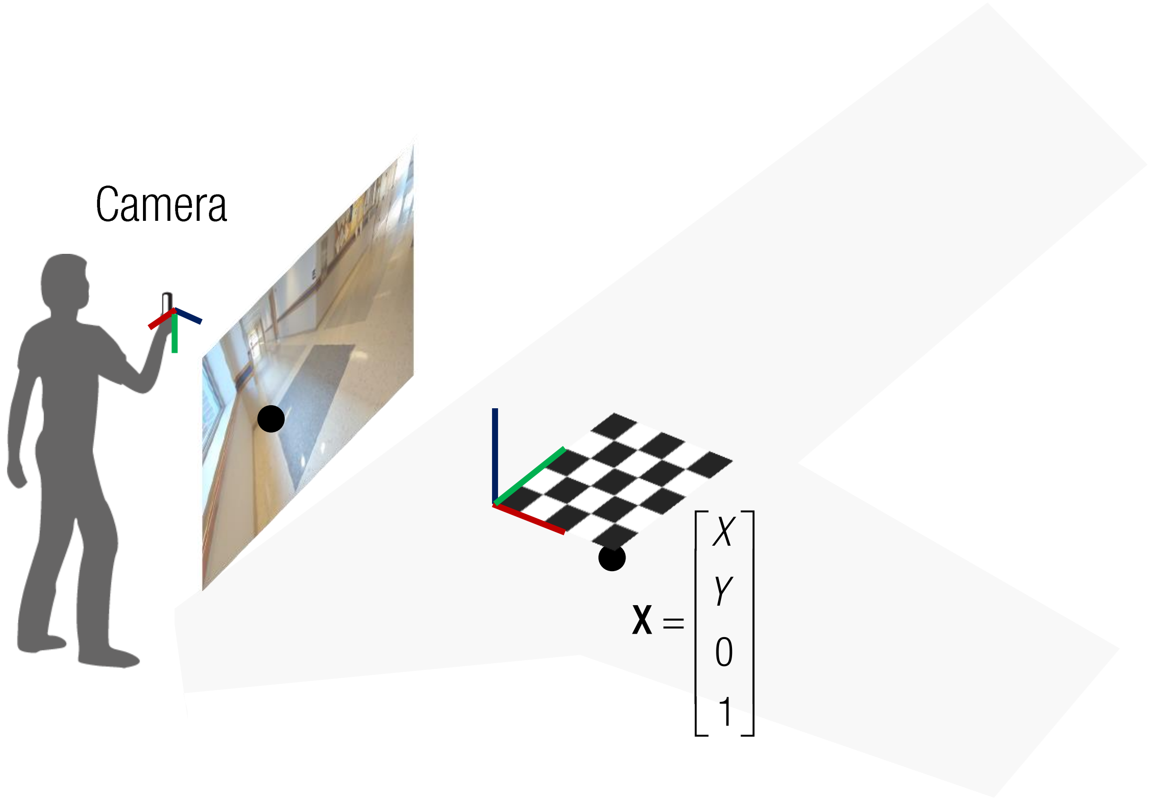
Points in 2D plane are mapped to an image with homography:

$$\mathbf{K}^{-1}\mathbf{H} = \mathbf{K}^{-1}\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\rightarrow \mathbf{r}_1 = \frac{\mathbf{K}^{-1}\mathbf{h}_1}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_2 = \frac{\mathbf{K}^{-1}\mathbf{h}_2}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{h}_3}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}$$

Common denominator

Homography Mapping



Points in 2D plane are mapped to an image with homography:

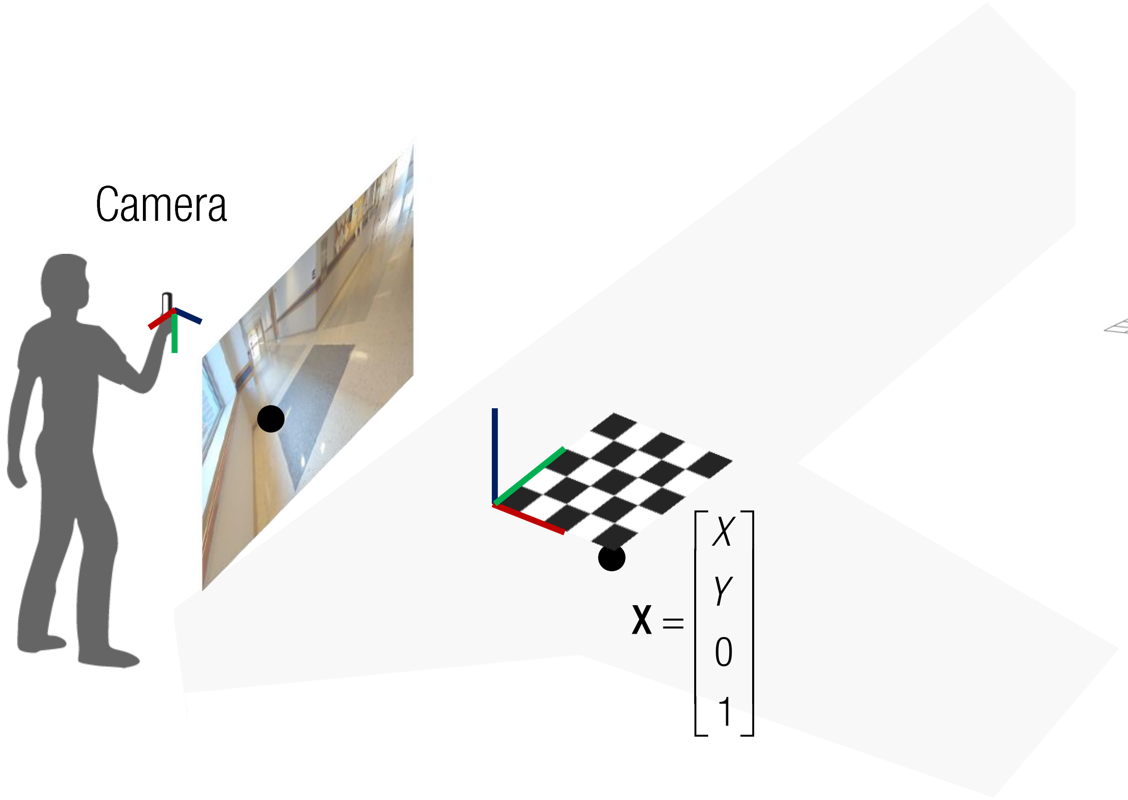
$$\mathbf{K}^{-1}\mathbf{H} = \mathbf{K}^{-1}\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

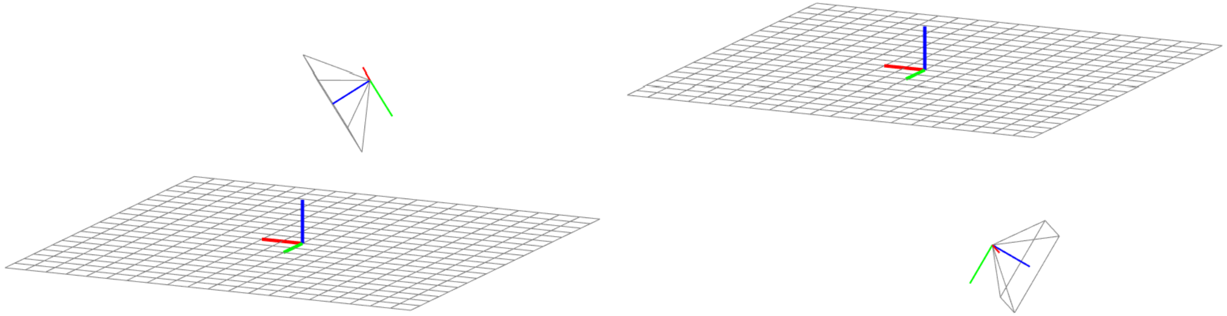
$$\rightarrow \mathbf{r}_1 = \frac{\mathbf{K}^{-1}\mathbf{h}_1}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_2 = \frac{\mathbf{K}^{-1}\mathbf{h}_2}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{h}_3}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}$$

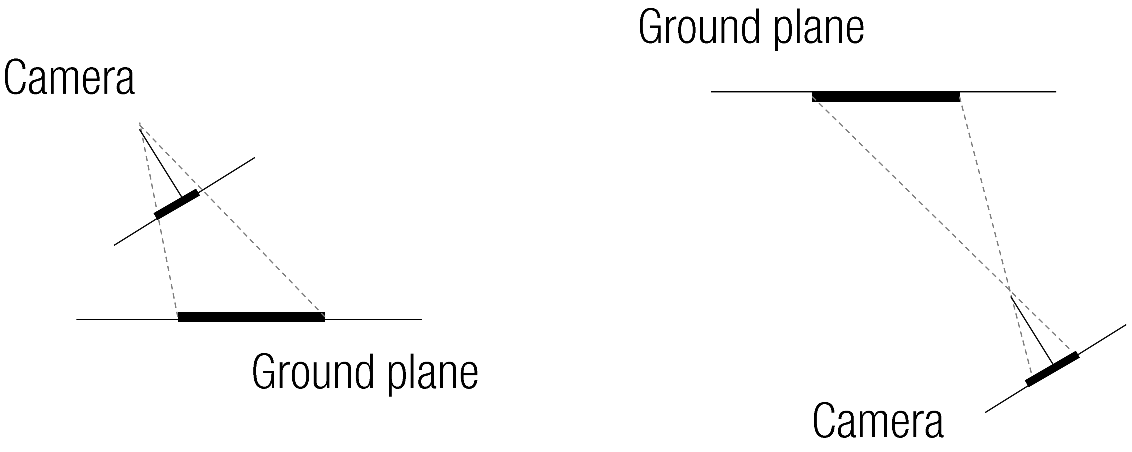
Common denominator

$$\rightarrow \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Geometric Ambiguity

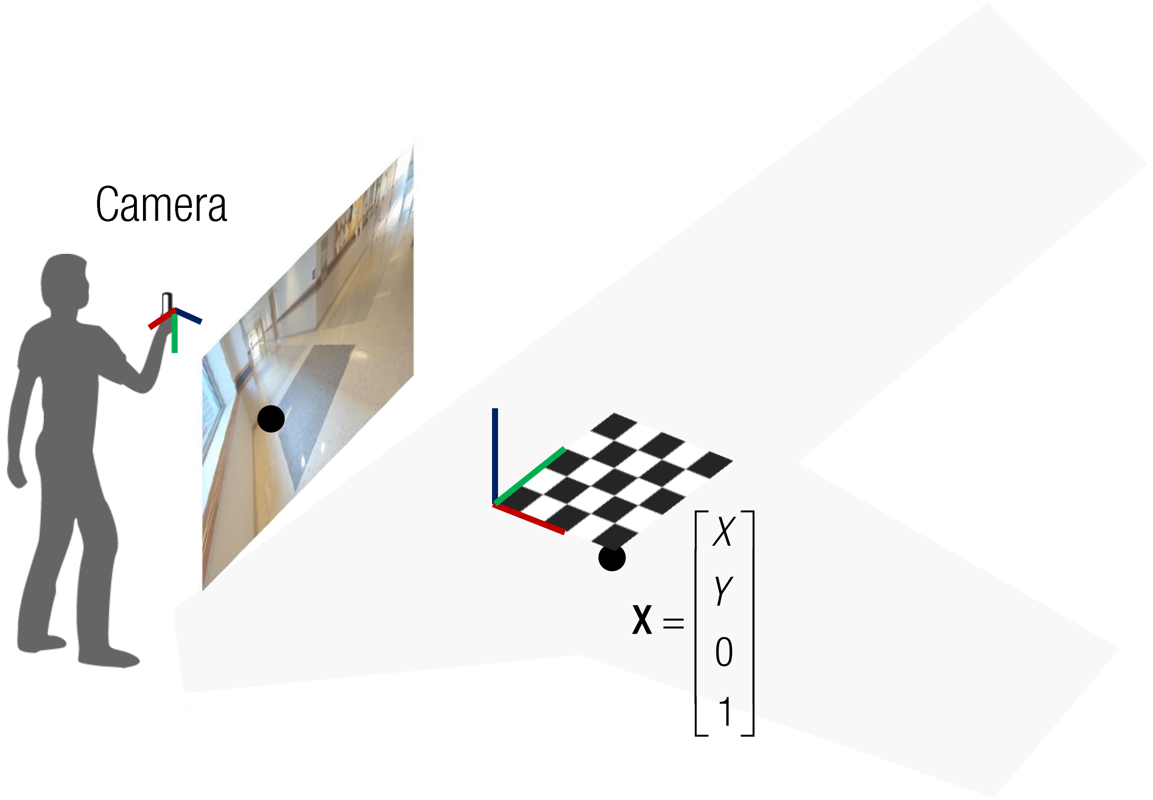






Two configurations

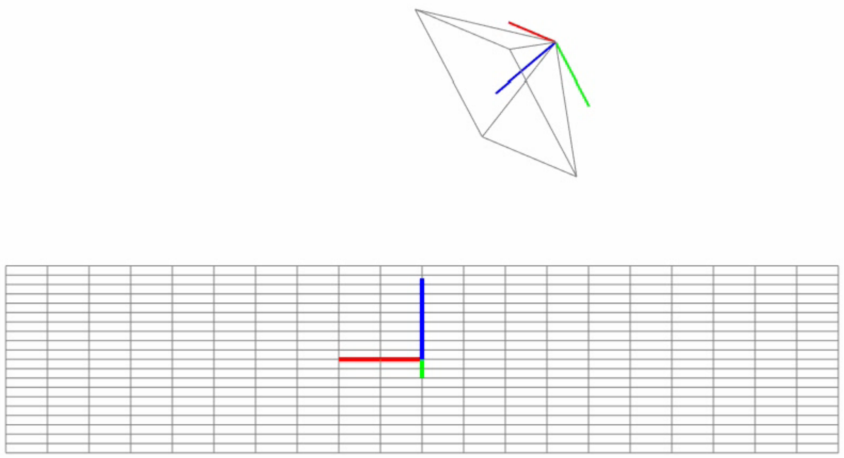
Homography Mapping



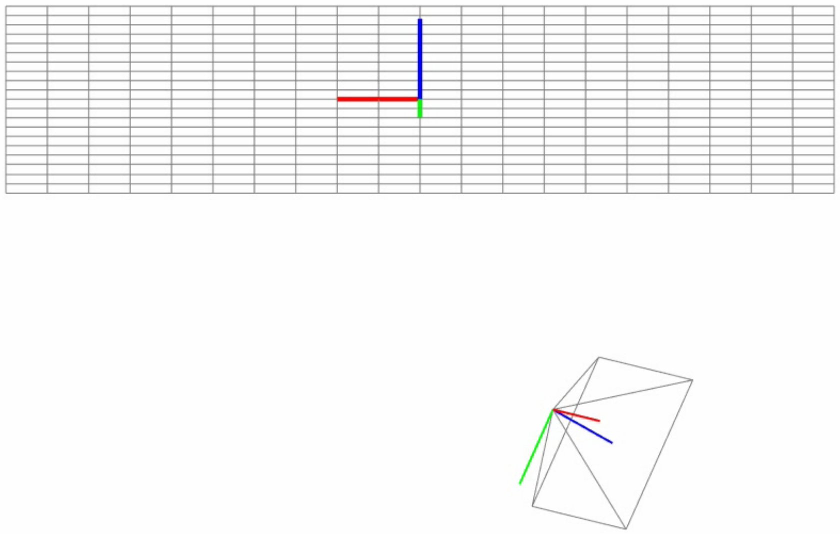
ComputeCameraFromHomography.m

```
function ComputeCameraFromHomography

f = 1300;
K = [f 0 size(im,2)/2;
    0 f size(im,1)/2;
    0 0 1];

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

u = [m11(1:2)';m12(1:2)';m13(1:2)'; m14(1:2)'];
X = [0 0;1 0;1 1;0 1];
X = [X ones(4,1)]; % homogeneous coordinate

H = ComputeHomography(u, X)

denom = norm(inv(K)*H(:,1));
r1 = inv(K)*H(:,1)/denom;
r2 = inv(K)*H(:,2)/denom;
t = inv(K)*H(:,3)/denom;

r3 = Vec2Skew(r1)*r2;
```

Homography Mapping

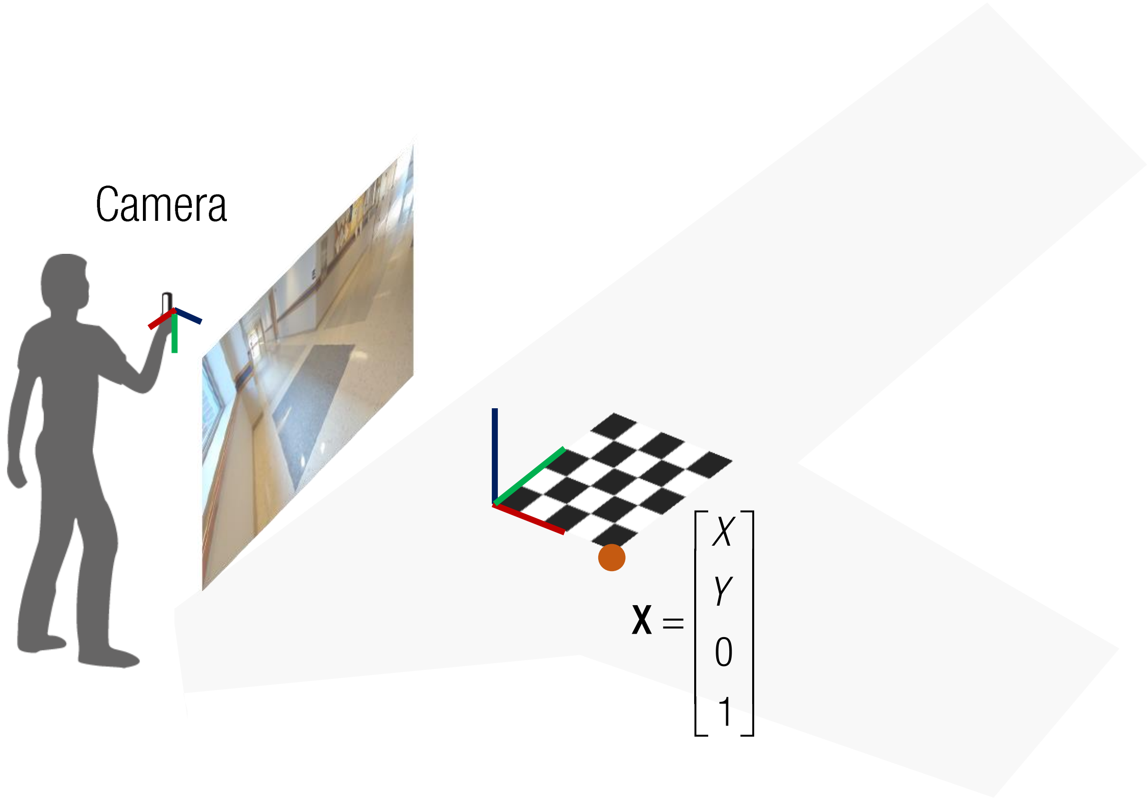


H



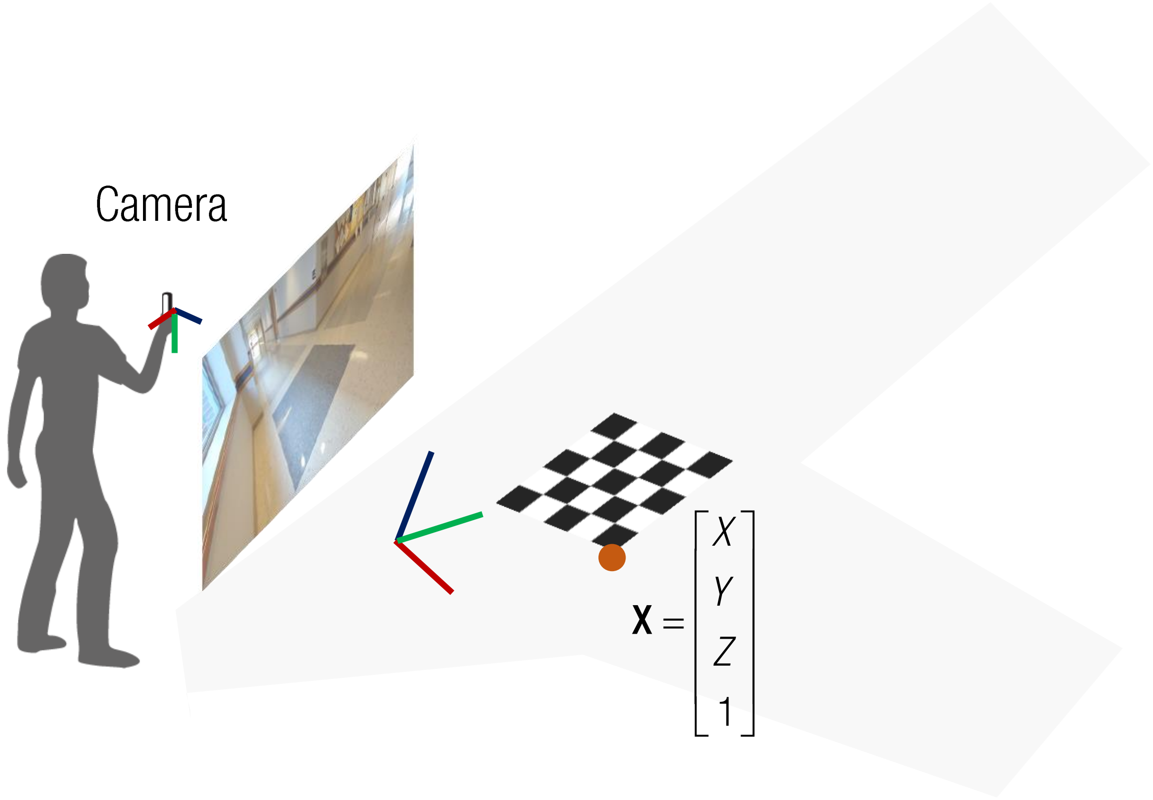
-H

Plane Representation



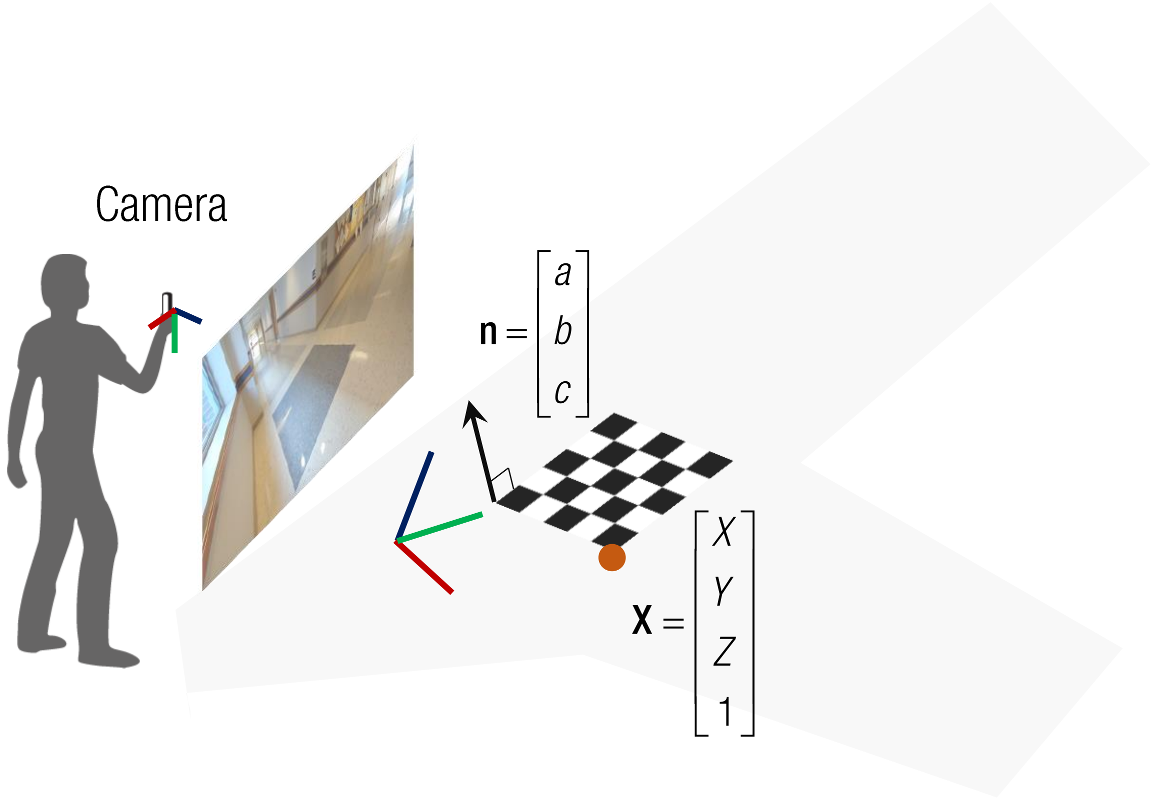
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Plane Representation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Plane Representation



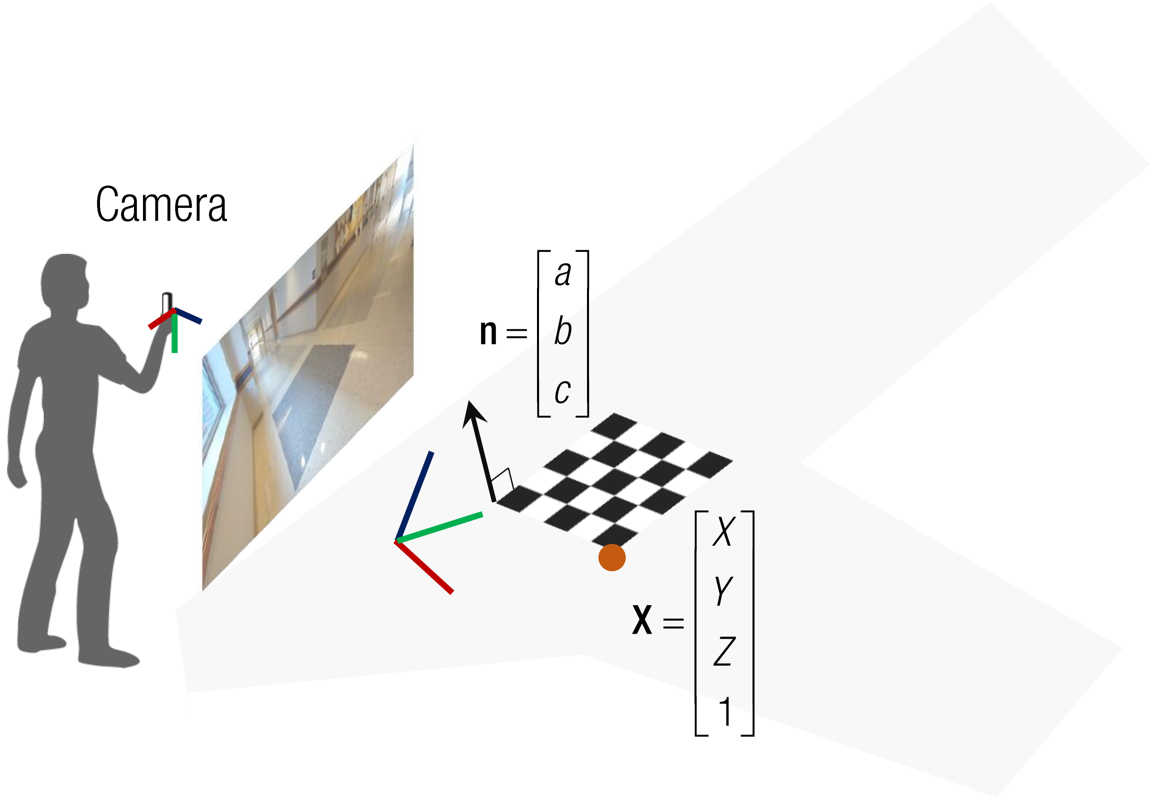
Plane equation:

$$aX + bY + cZ + d = 0$$

Surface normal:

$$\mathbf{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

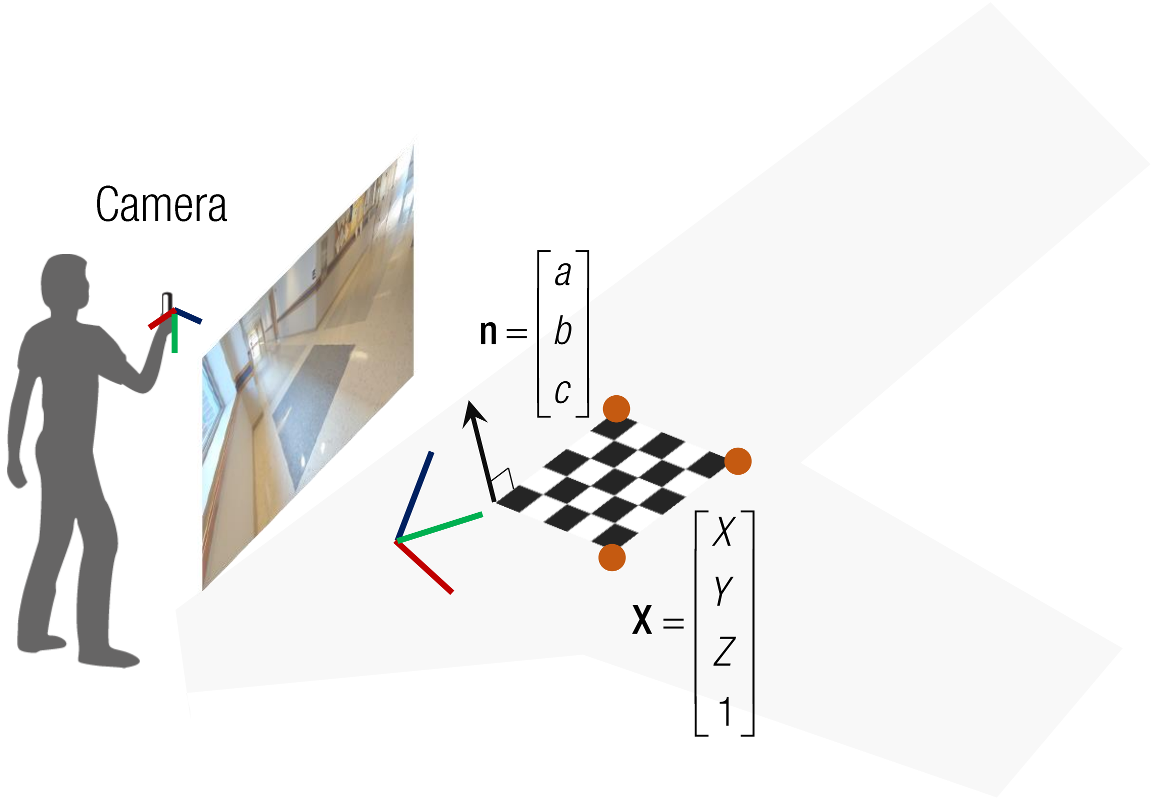
Surface normal:

$$\mathbf{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$aX + bY + cZ + d = 0$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

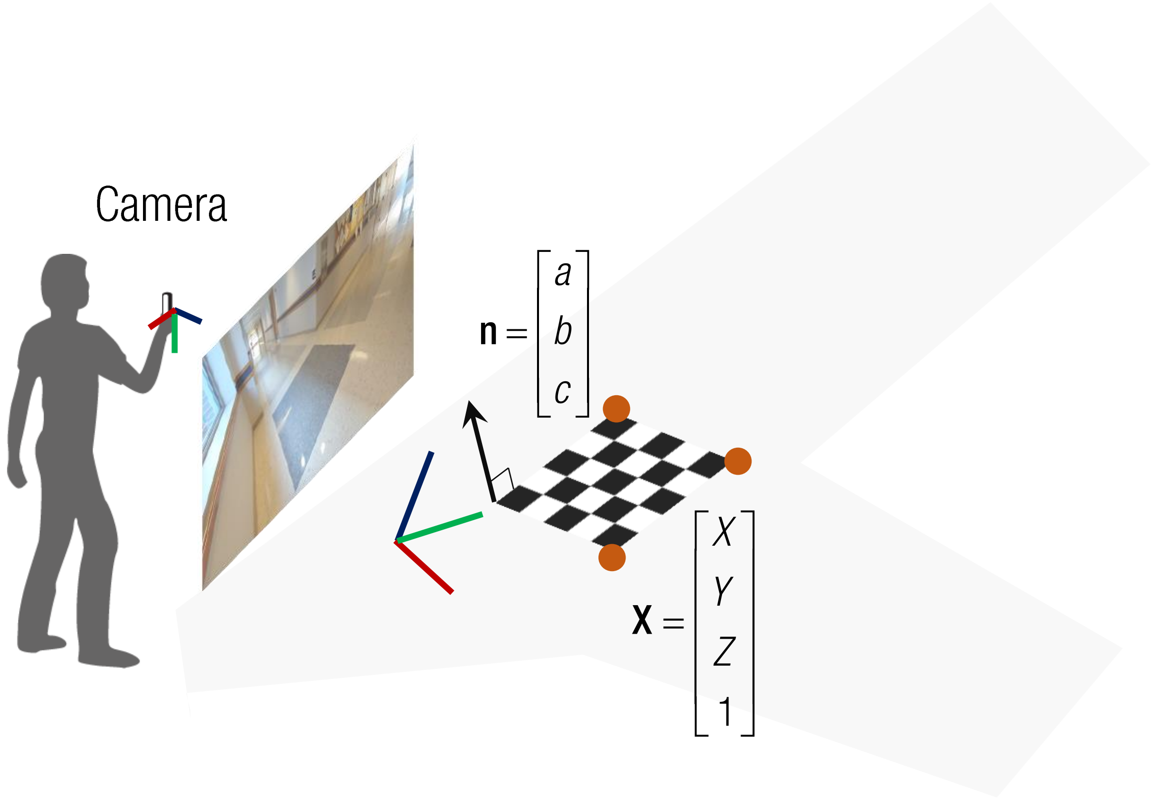
Surface normal:

$$\mathbf{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$aX_1 + bY_1 + cZ_1 + d = 0$$
$$aX_2 + bY_2 + cZ_2 + d = 0$$
$$aX_3 + bY_3 + cZ_3 + d = 0$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

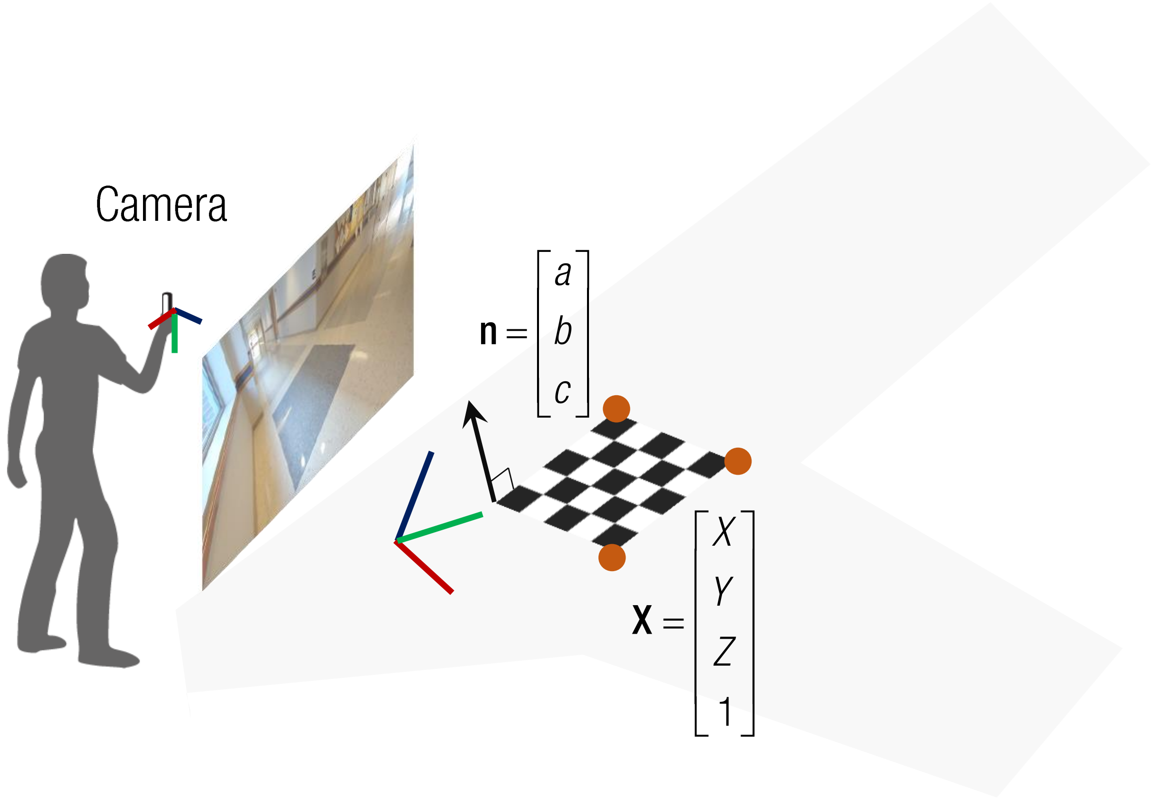
Surface normal:

$$\mathbf{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

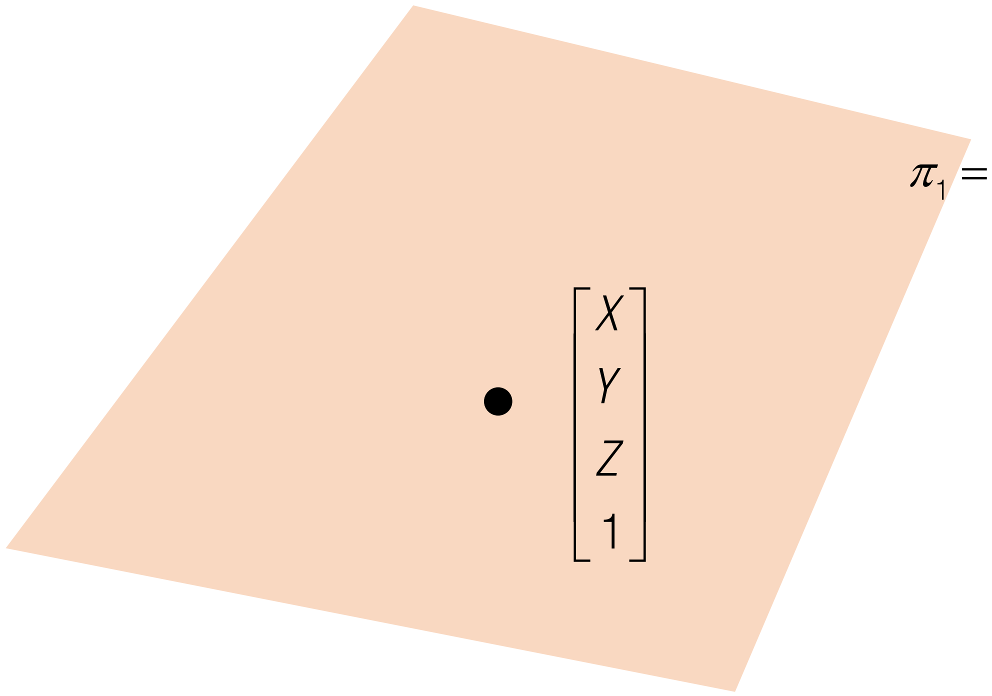
Surface normal:

$$\mathbf{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{\mathbf{X}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

Plane-Point Duality



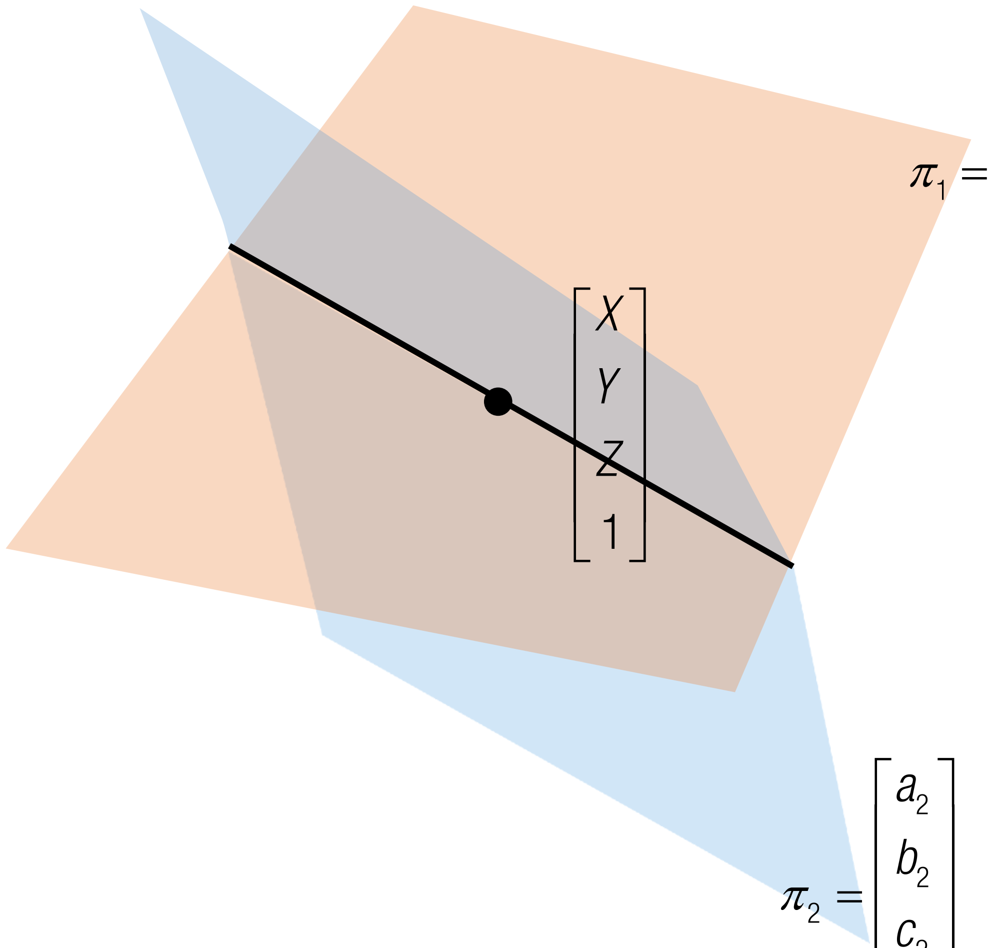
$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\pi_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix}$$

Plane equation:

$$a_1X + b_1Y + c_1Z + d_1 = 0$$

Plane-Point Duality



$$\pi_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

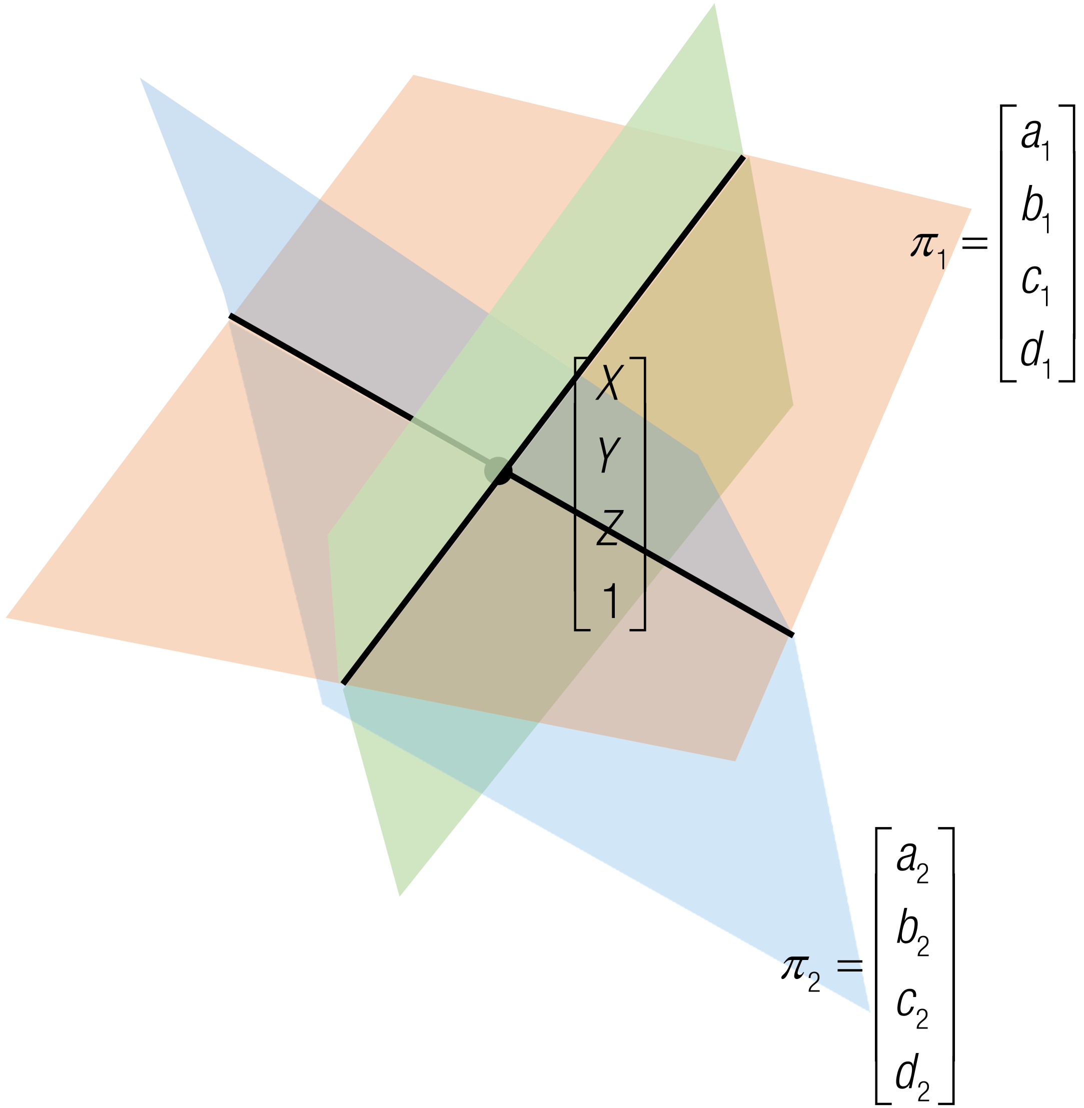
$$\pi_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix}$$

Plane equation:

$$a_1 X + b_1 Y + c_1 Z + d_1 = 0$$

$$a_2 X + b_2 Y + c_2 Z + d_2 = 0$$

Plane-Point Duality



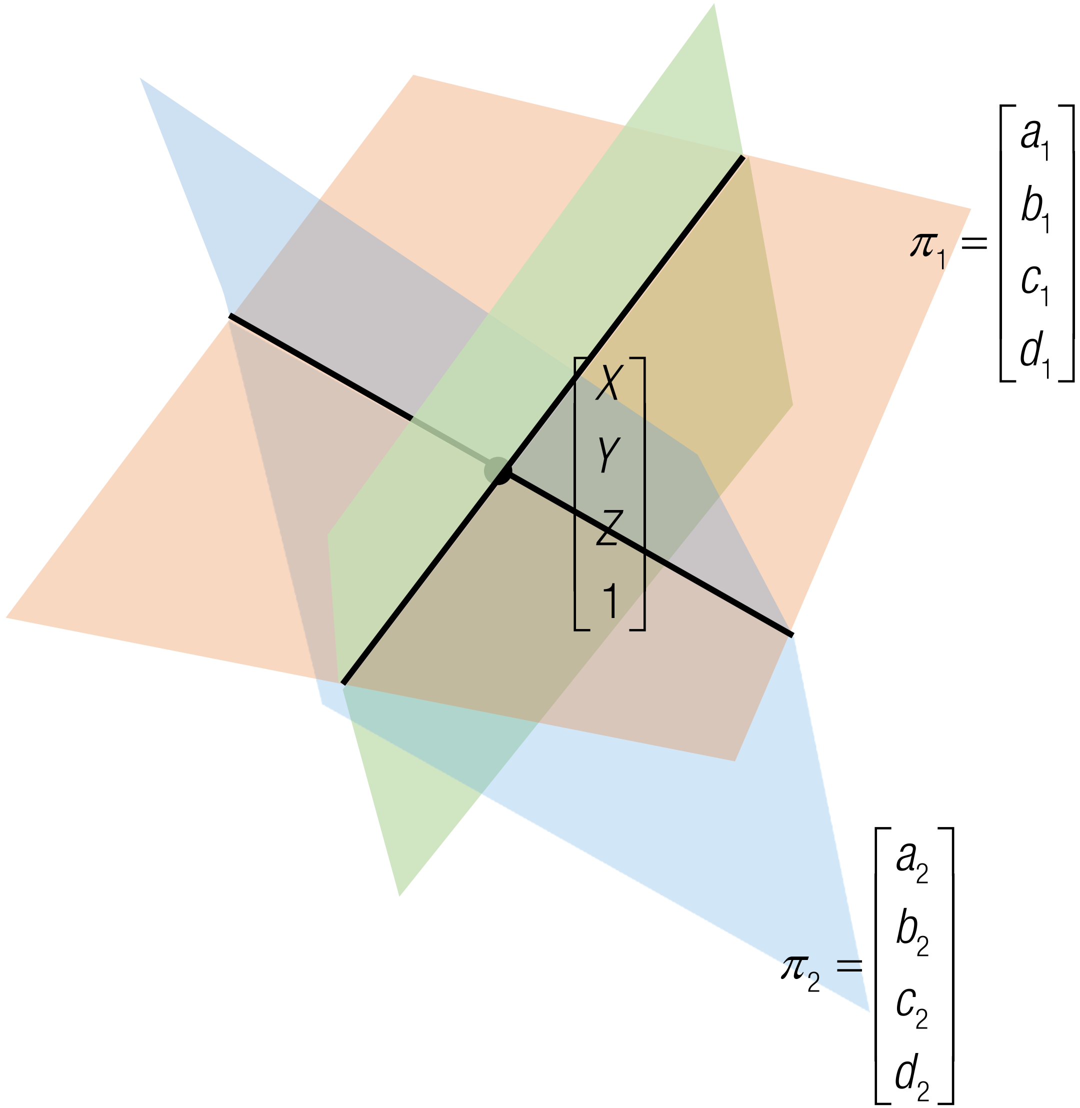
Plane equation:

$$a_1X + b_1Y + c_1Z + d_1 = 0$$

$$a_2X + b_2Y + c_2Z + d_2 = 0$$

$$a_3X + b_3Y + c_3Z + d_3 = 0$$

Plane-Point Duality



Plane equation:

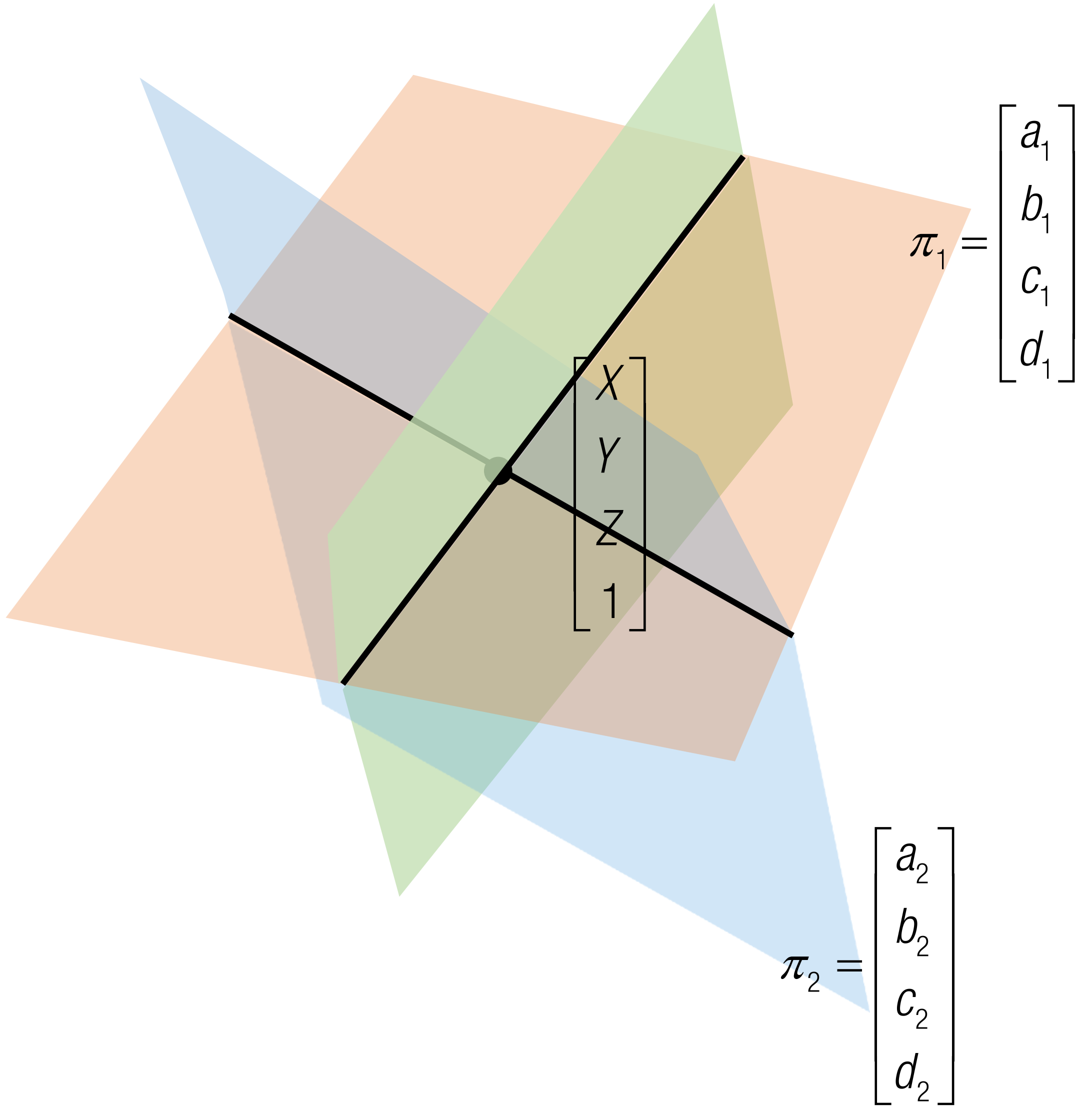
$$a_1X + b_1Y + c_1Z + d_1 = 0$$

$$a_2X + b_2Y + c_2Z + d_2 = 0$$

$$a_3X + b_3Y + c_3Z + d_3 = 0$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Plane-Point Duality



Plane equation:

$$a_1X + b_1Y + c_1Z + d_1 = 0$$

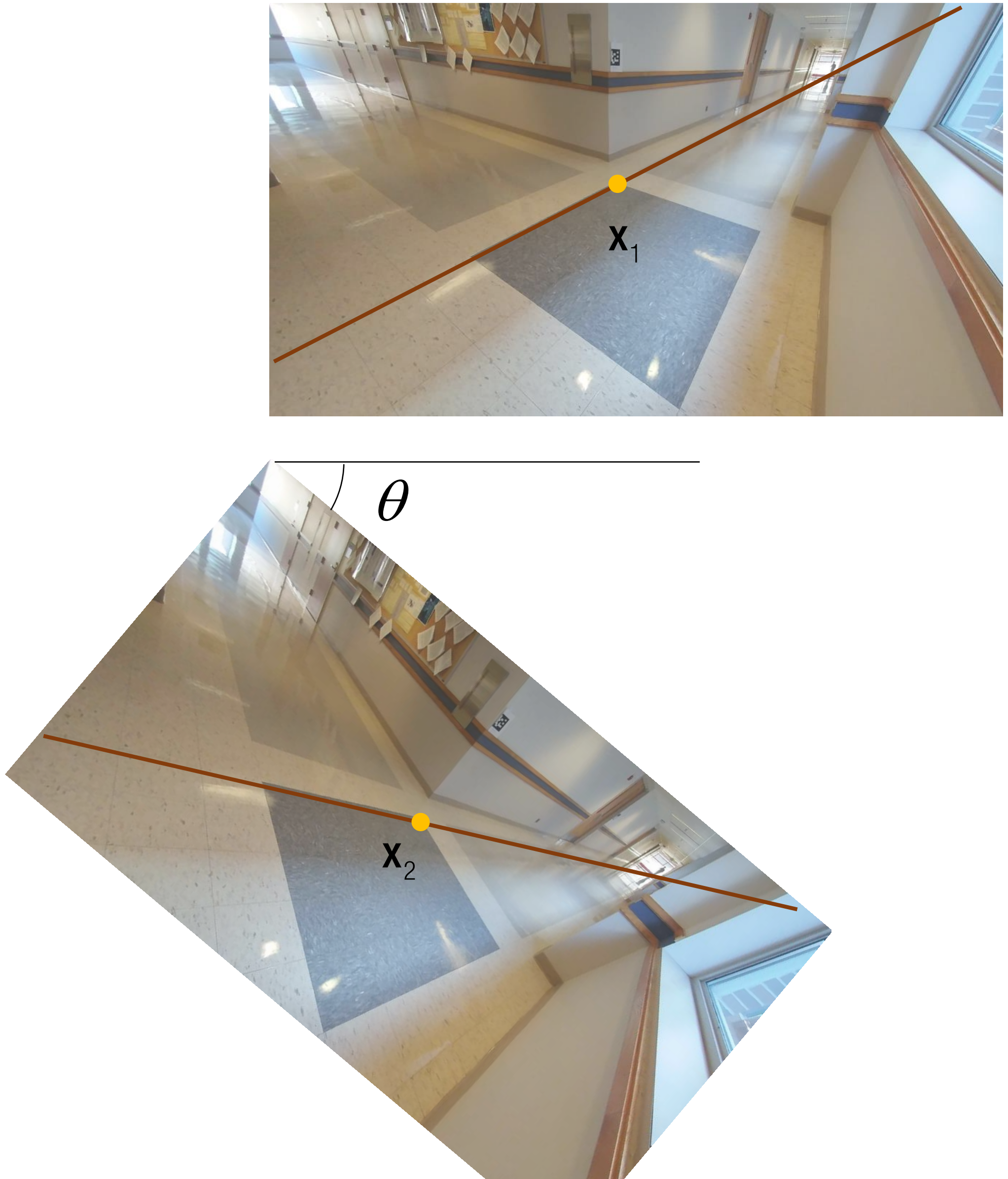
$$a_2X + b_2Y + c_2Z + d_2 = 0$$

$$a_3X + b_3Y + c_3Z + d_3 = 0$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\mathbf{X} = \mathbf{0}$$

Recall: 2D Point and Line Duality



The 2D line joining two points:

$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

The intersection between two lines:

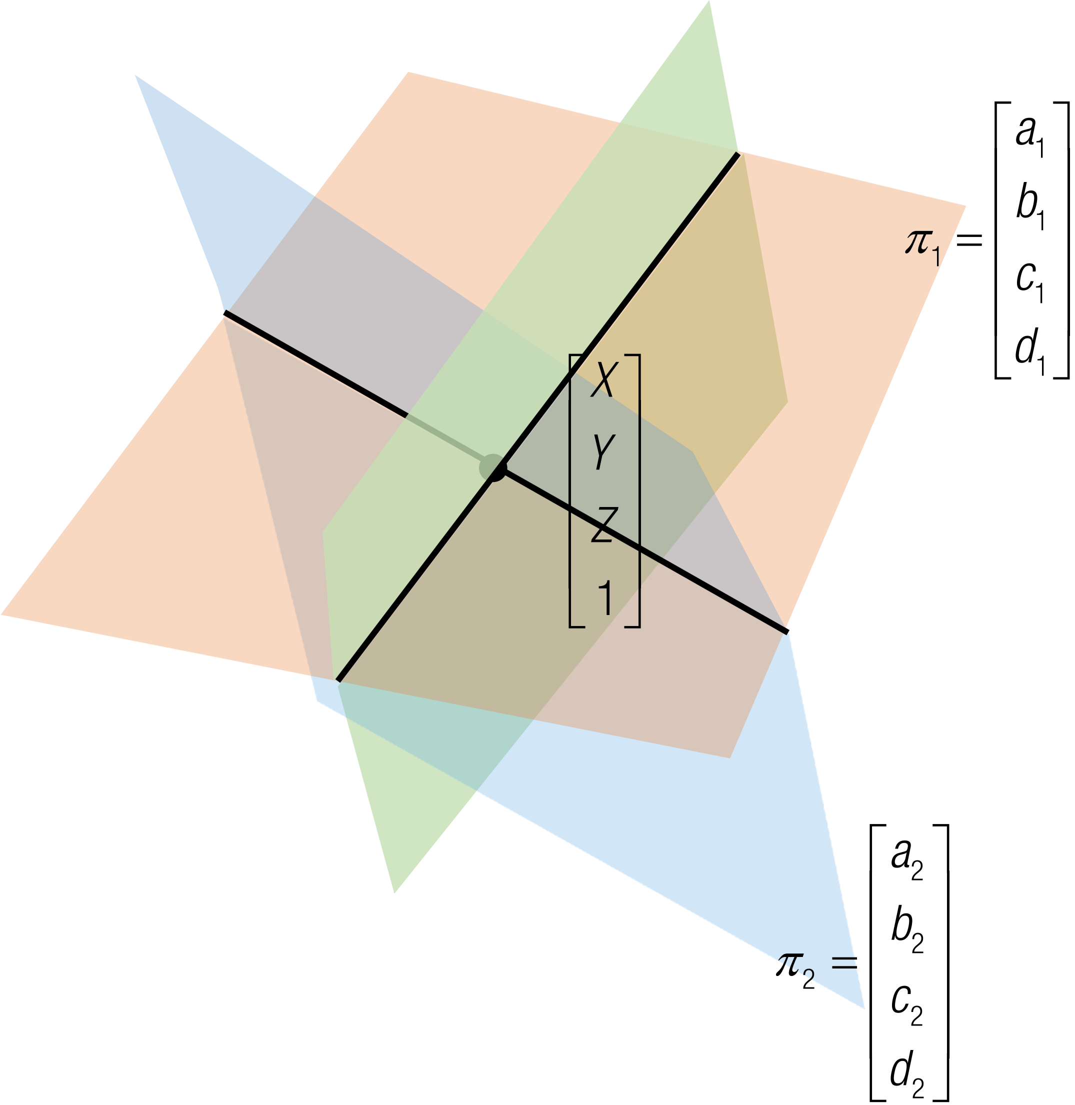
$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$$

Given any formula, we can switch the meaning of point and line to get another formula.

$$\mathbf{x}_2 = \mathbf{T}\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^{-T}\mathbf{l}_1$$

T: Transformation

Plane-Point Duality

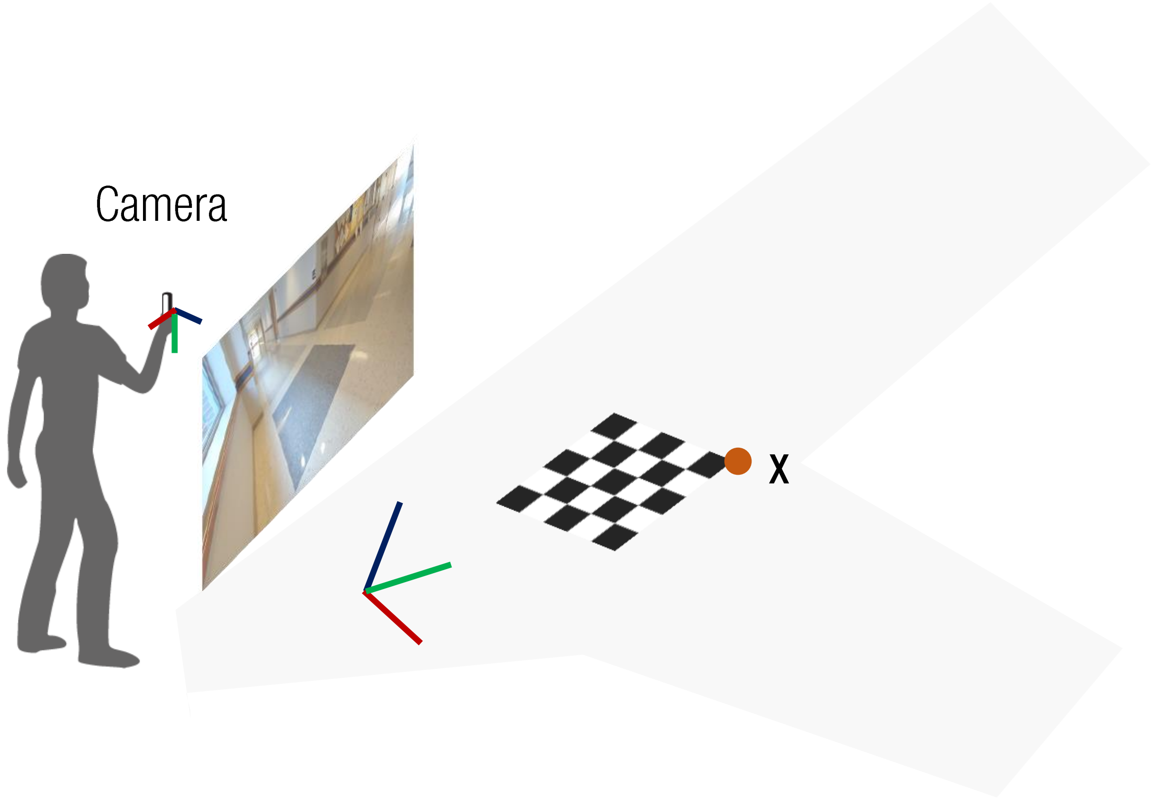


$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Given any formula, we can switch the meaning of point and plane to get another formula.

Plane Representation



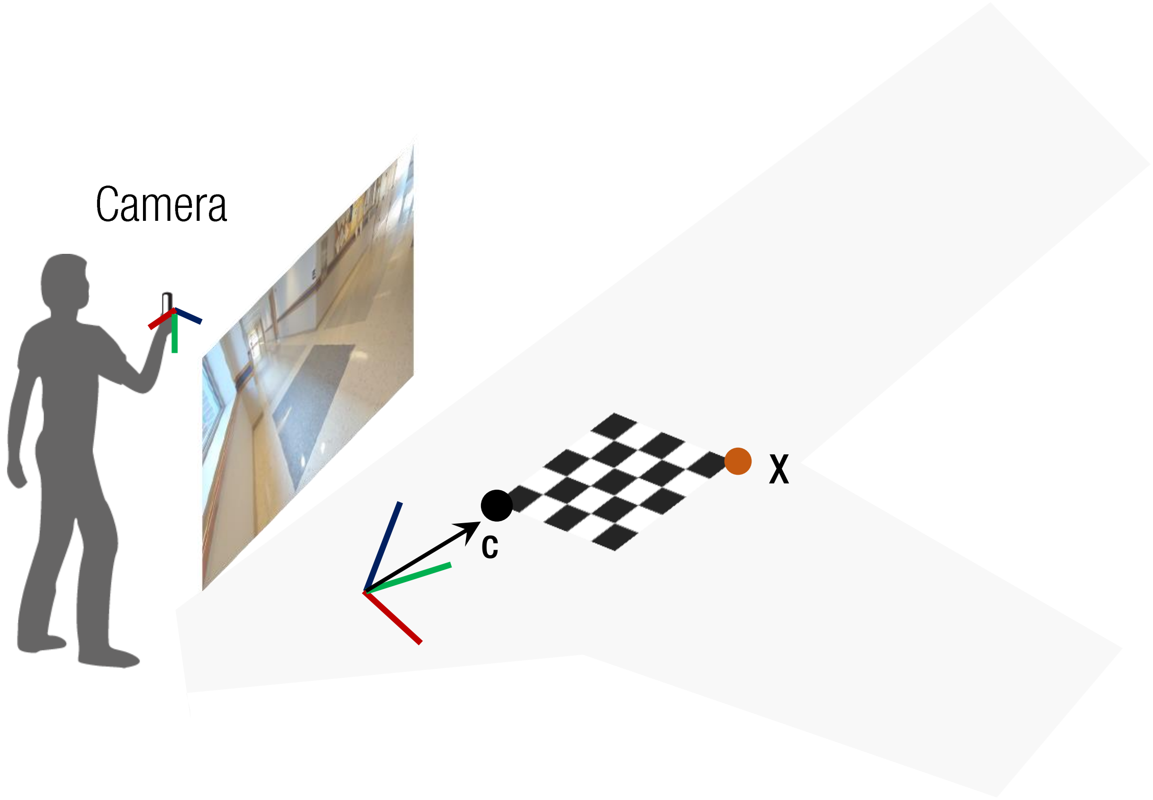
How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3DOF

Plane Representation



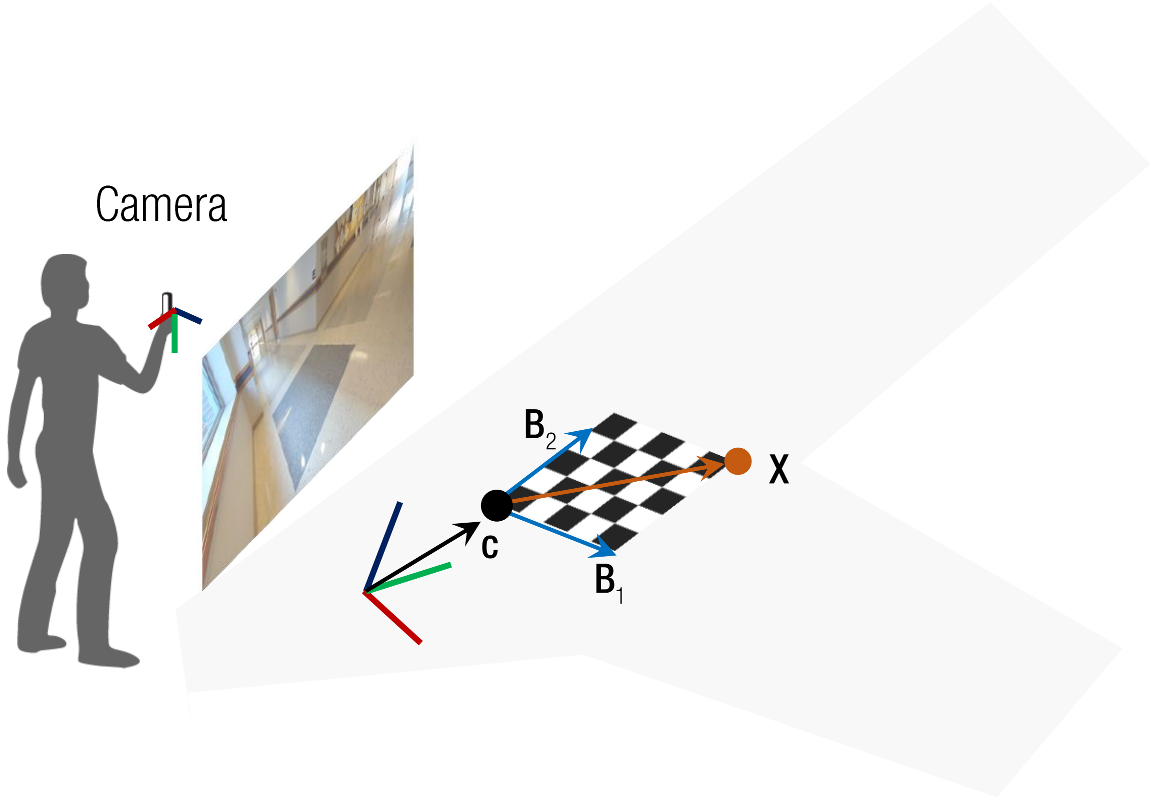
How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{c} +$$

3DOF

Plane Representation



How to parametrize a point in the plane?

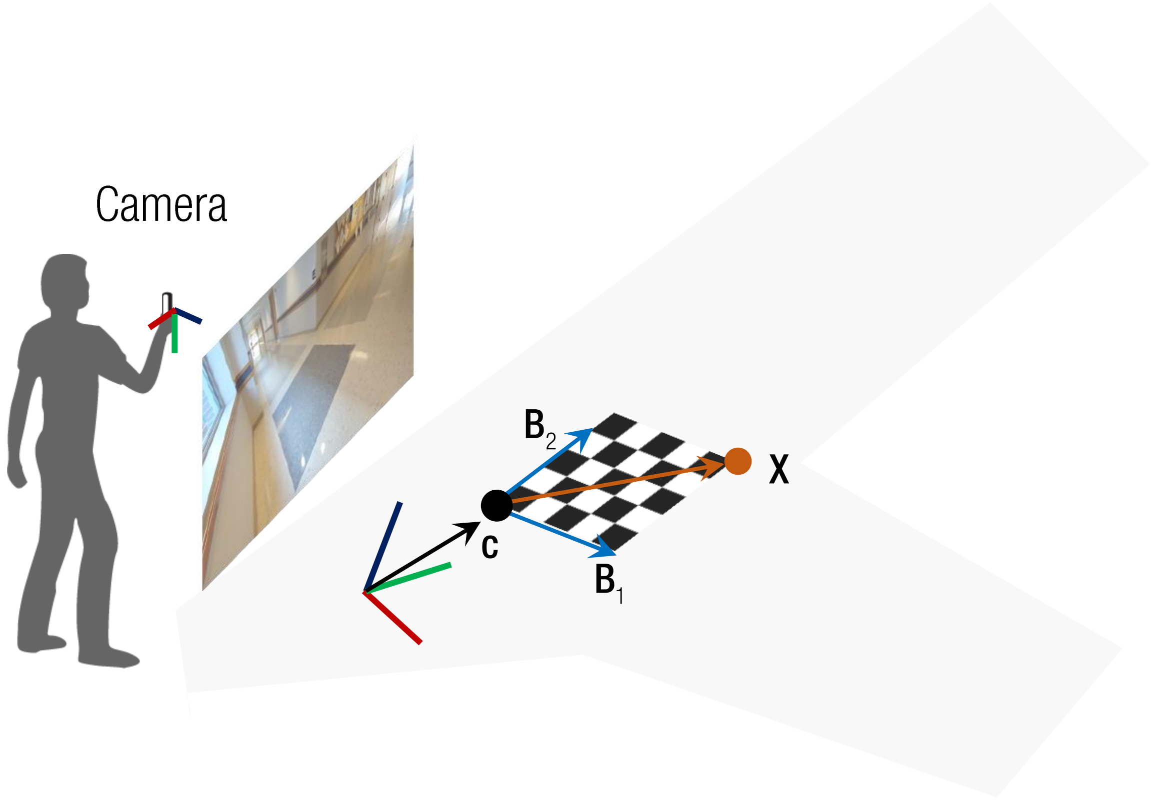
$$aX + bY + cZ + d = 0$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{c} + \mu_1 \mathbf{B}_1 + \mu_2 \mathbf{B}_2$$

Basis

3DOF

Plane Representation



How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

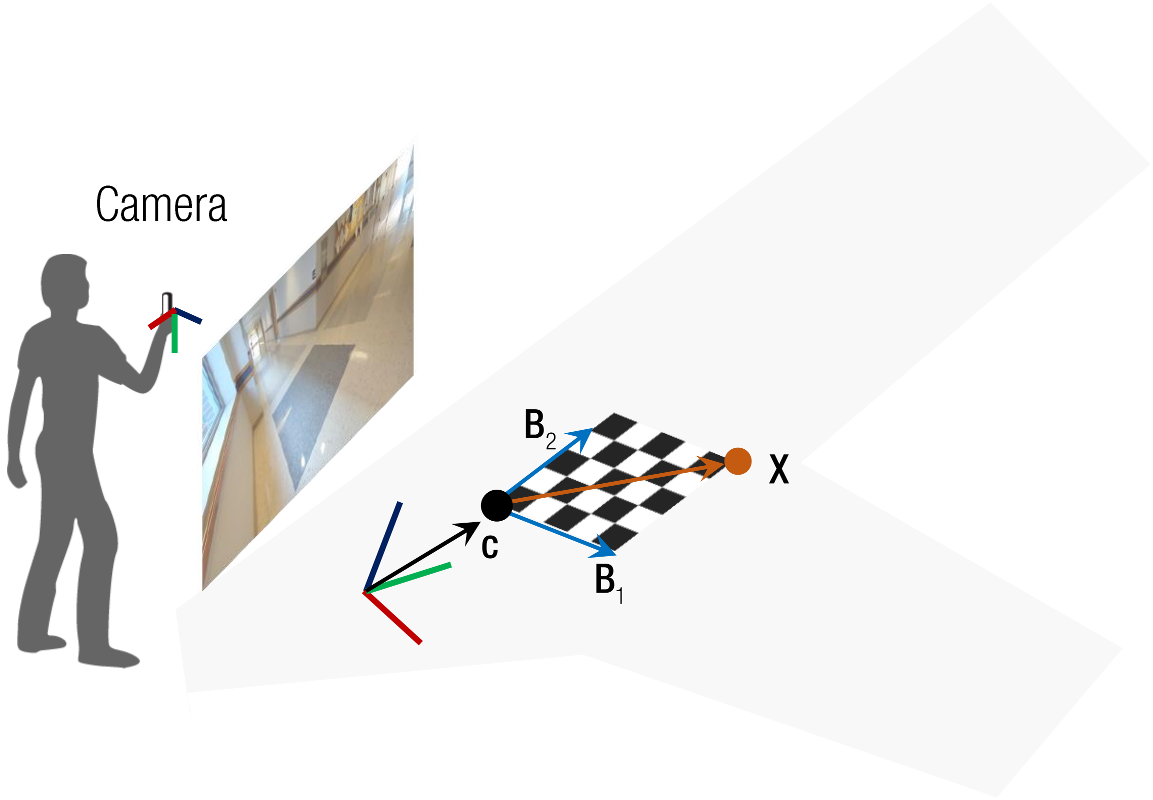
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{c} + \mu_1 \mathbf{B}_1 + \mu_2 \mathbf{B}_2 = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Basis

3DOF

2DOF

Plane Representation



How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{c} + \mu_1 \mathbf{B}_1 + \mu_2 \mathbf{B}_2 = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Basis

3DOF

2DOF

Plane projection:

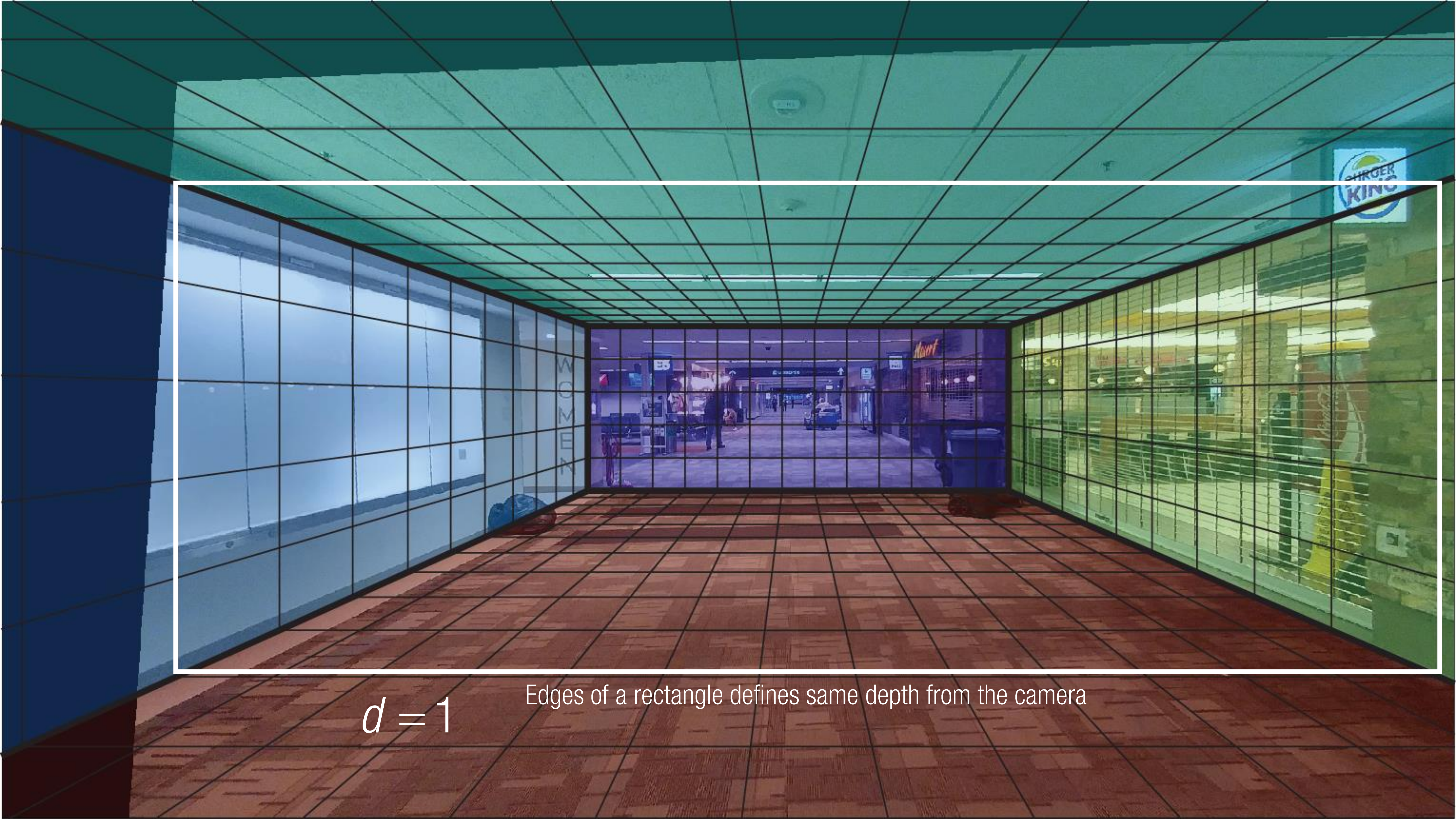
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} \\ 1 \end{bmatrix}$$

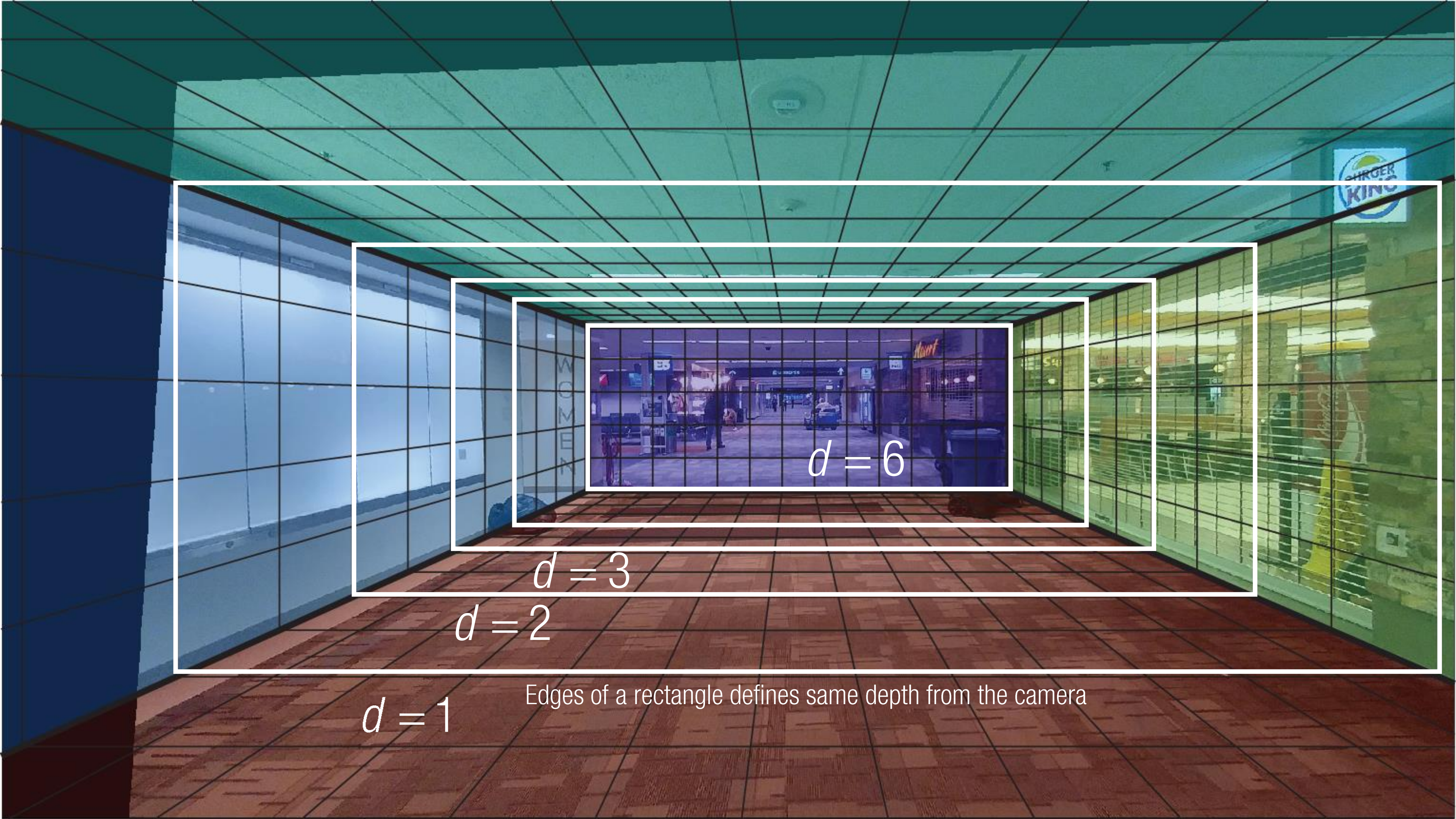
$$= \mathbf{K}\begin{bmatrix} \mathbf{R}\mathbf{B}_1 & \mathbf{R}\mathbf{B}_2 & \mathbf{R}\mathbf{c} + \mathbf{t} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

WOMEN
E5
BURGER KING

HW #3 Tour into your photo

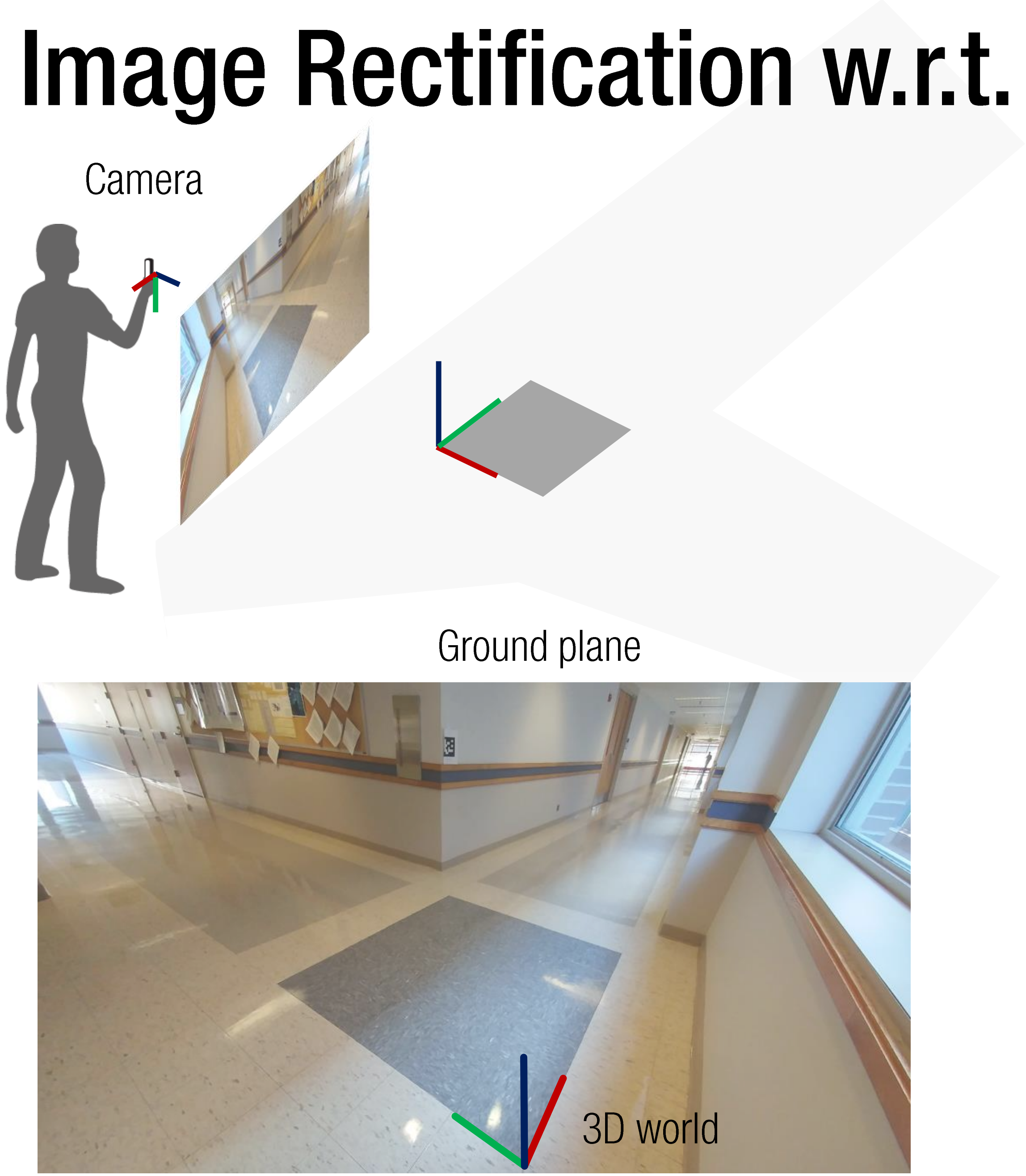

BURGER KING
WOMEN


Edges of a rectangle defines same depth from the camera
$d = 1$



d = 6
d = 3
d = 2
Edges of a rectangle defines same depth from the camera
d = 1

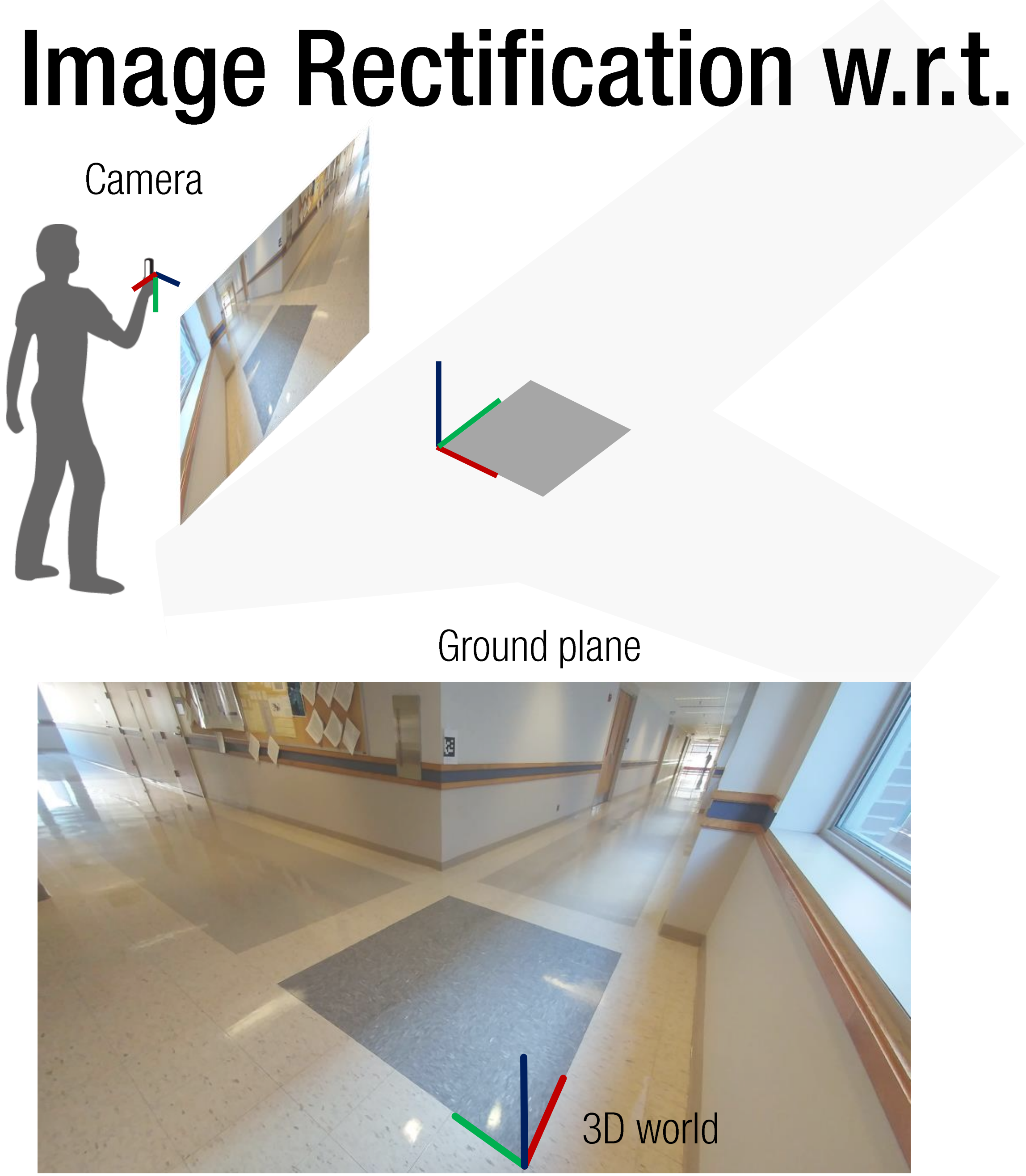
Image Rectification w.r.t. Ground Plane



How can I make my image upright?

⟶ Y axis of camera // Surface normal of ground plane

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

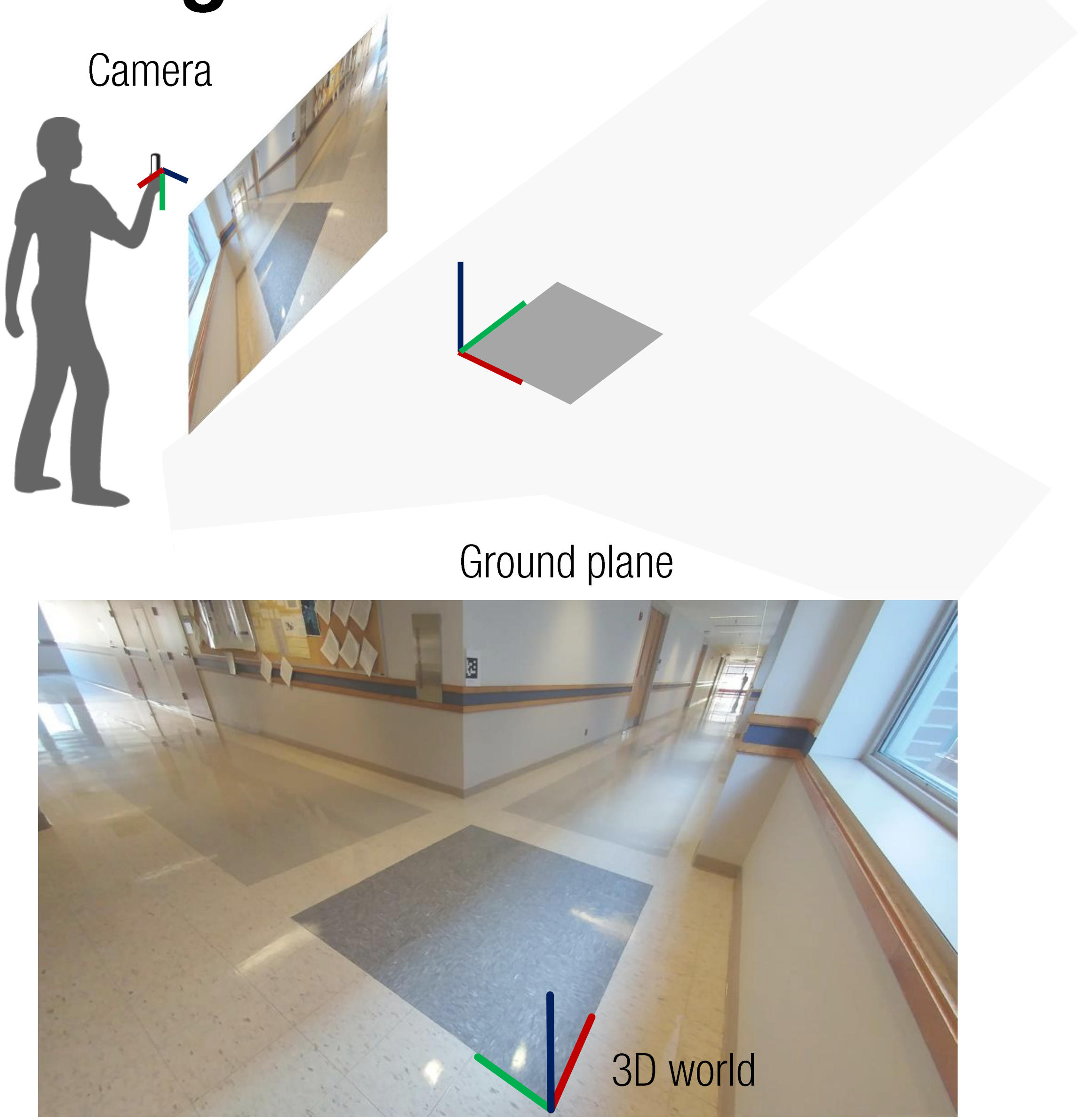
→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Camera pose from homography

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

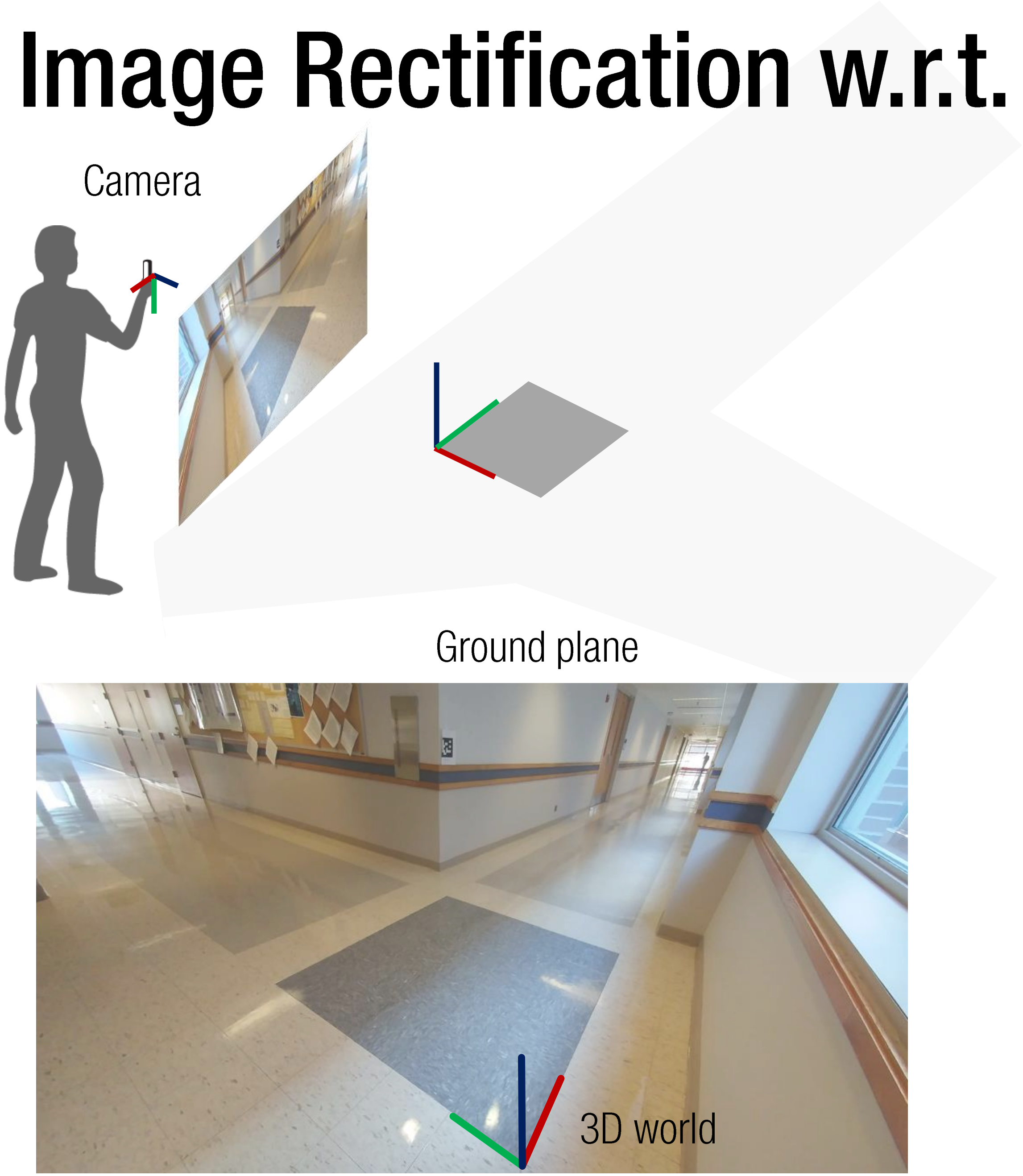
$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Camera pose from homography

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Image rotation

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

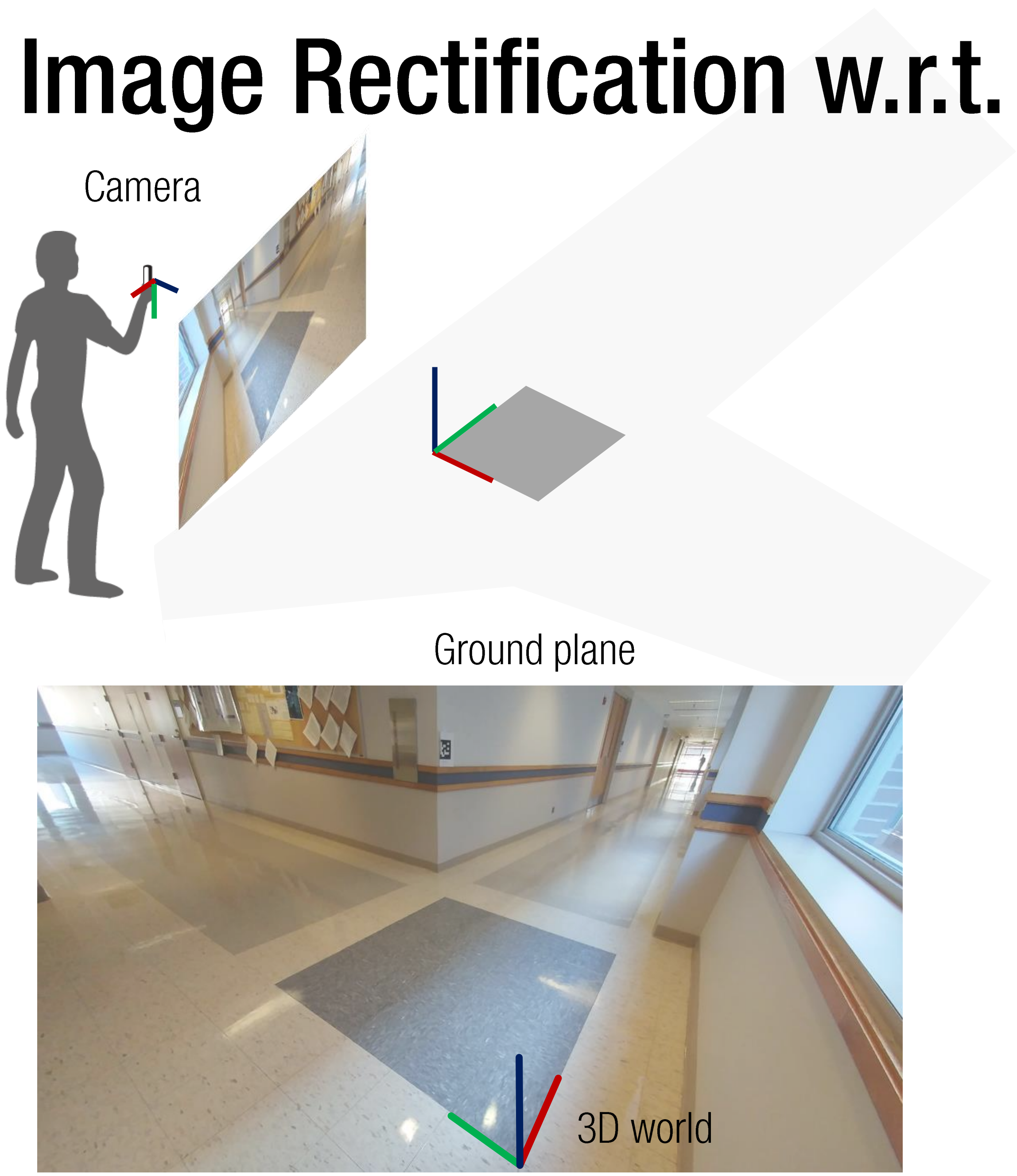
Camera pose from homography

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

Rectified rotation

Image Rectification w.r.t. Ground Plane



$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

Rectified rotation

$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - \left(\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y\right)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - \left(\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y\right)\tilde{\mathbf{r}}_y \right\|}$$

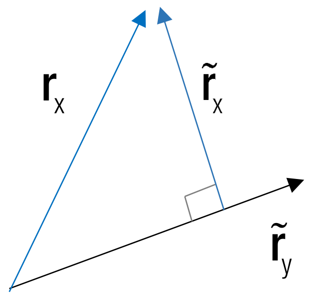
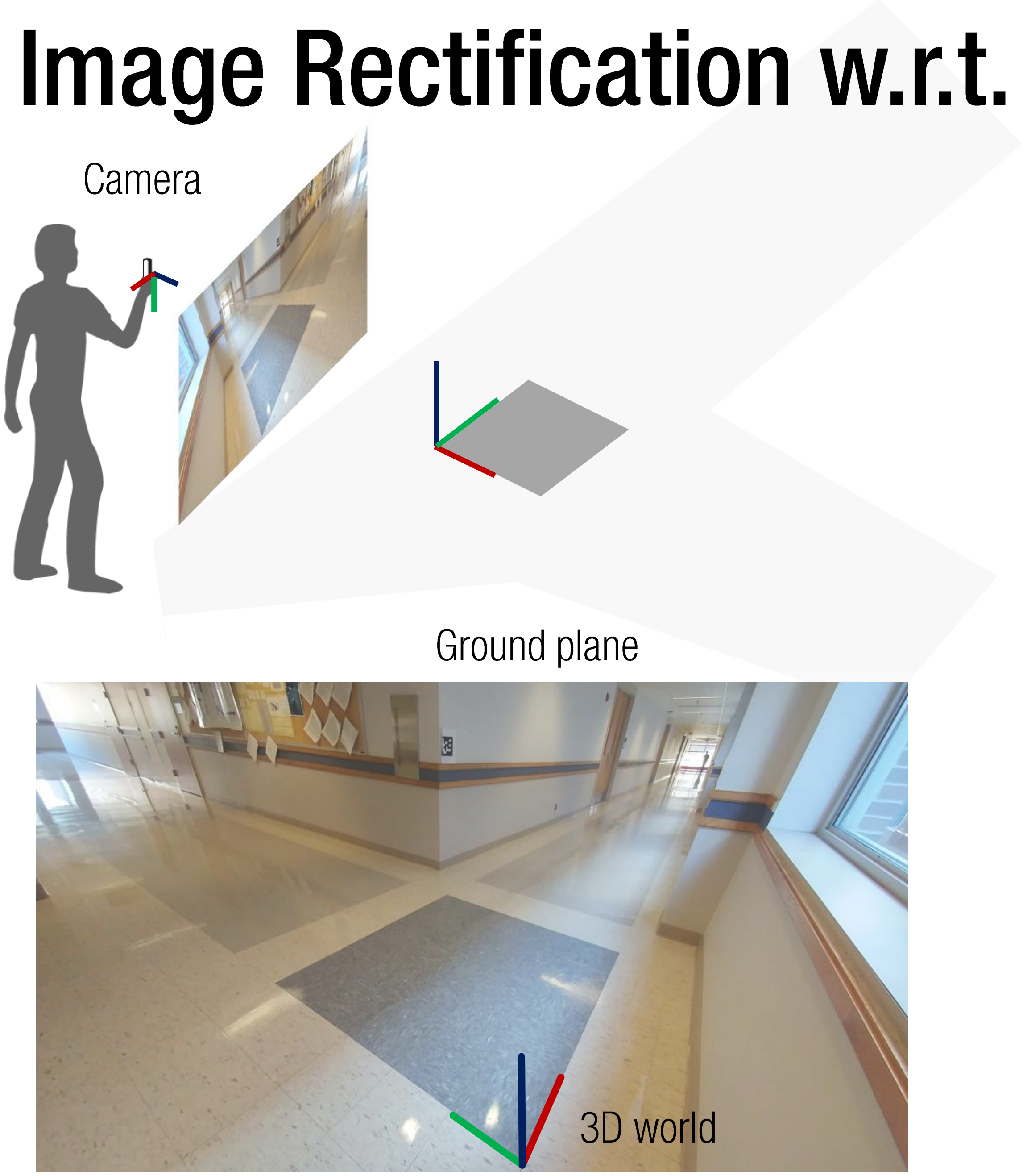


Image Rectification w.r.t. Ground Plane

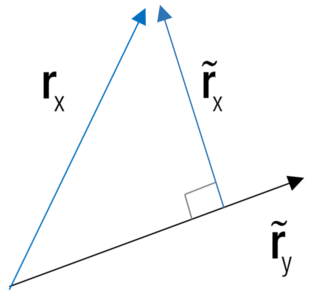


$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

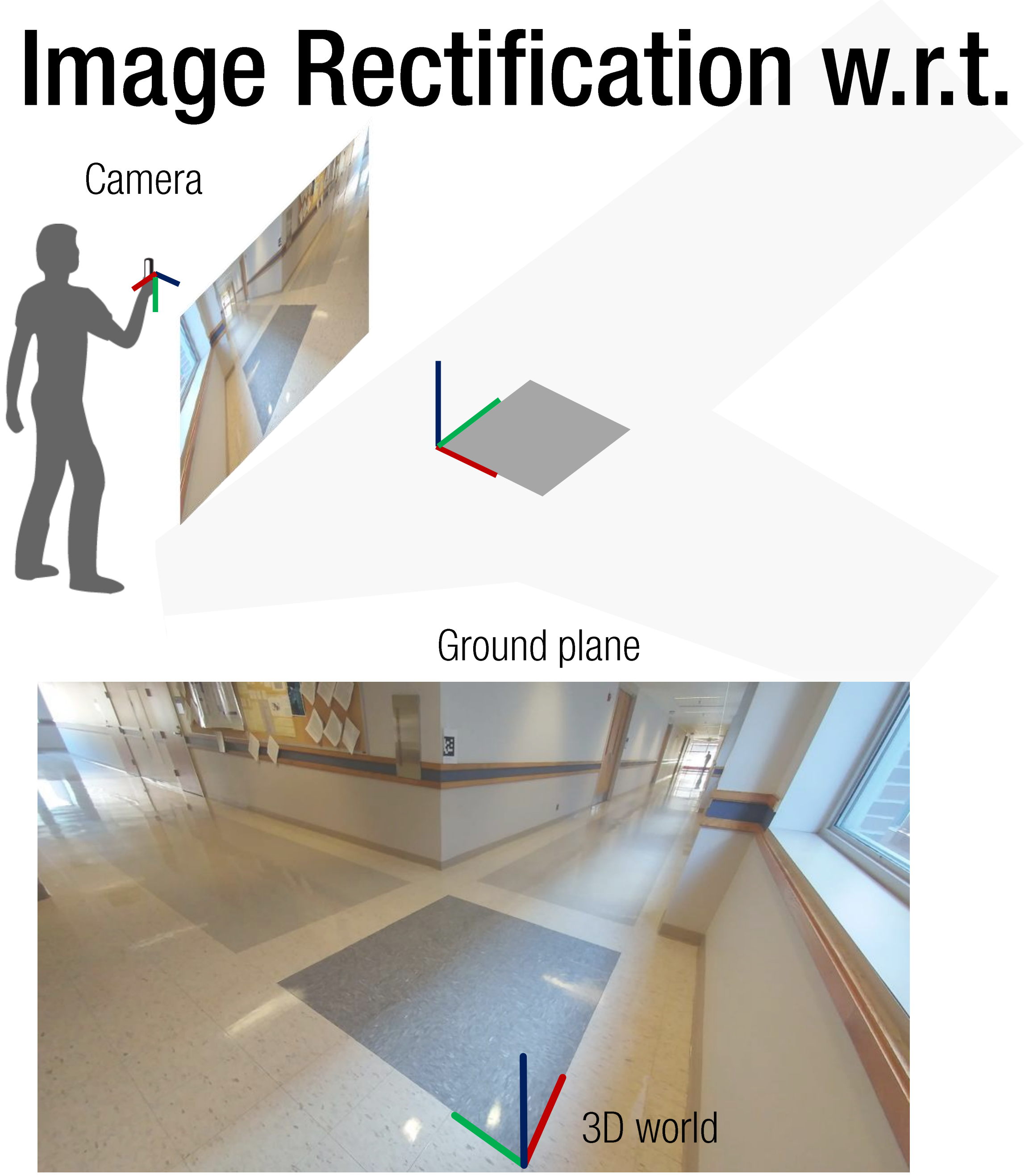
Rectified rotation

$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - \left(\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y\right)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - \left(\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y\right)\tilde{\mathbf{r}}_y \right\|}$$



$$\tilde{\mathbf{r}}_z = \tilde{\mathbf{r}}_x \times \tilde{\mathbf{r}}_y$$

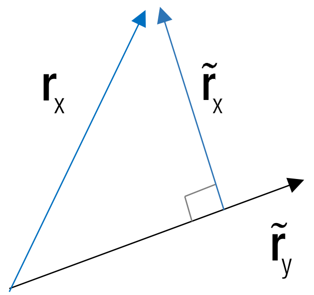
Image Rectification w.r.t. Ground Plane



$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

Rectified rotation



$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y \right\|}$$

$$\tilde{\mathbf{r}}_z = \tilde{\mathbf{r}}_x \times \tilde{\mathbf{r}}_y$$

$$\lambda \tilde{\mathbf{R}}^{\mathsf{T}} \mathbf{K}^{-1} \tilde{\mathbf{u}} = \mathbf{R}^{\mathsf{T}} \mathbf{K}^{-1} \mathbf{u} \longrightarrow \lambda \tilde{\mathbf{u}} = \mathbf{K} \tilde{\mathbf{R}} \mathbf{R}^{\mathsf{T}} \mathbf{K}^{-1} \mathbf{u}$$

Image Rectification w.r.t. Ground Plane

Image Rectification w.r.t. Ground Plane

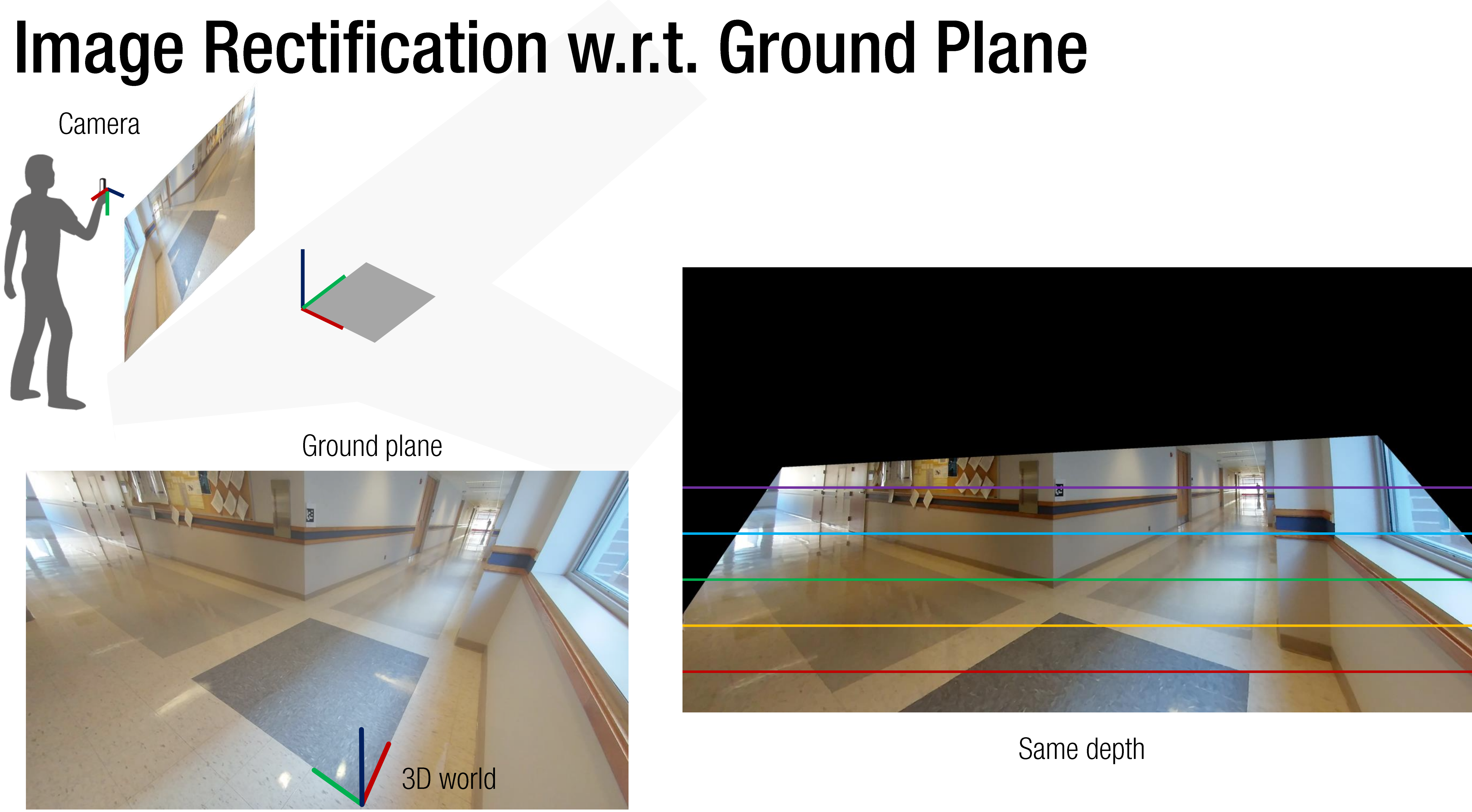
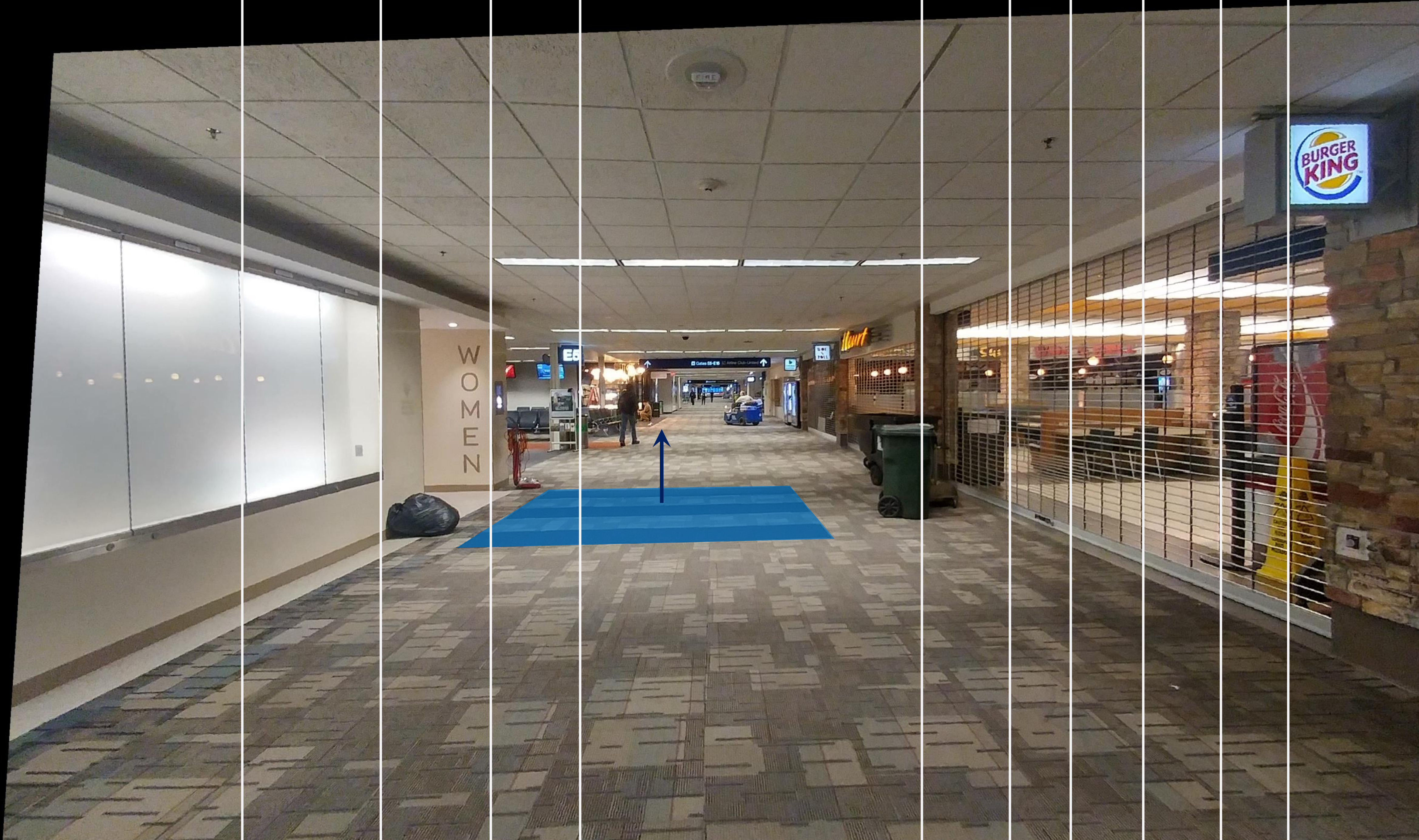


Image Rectification w.r.t. Ground Plane


WOMEN
E5
BURGER KING

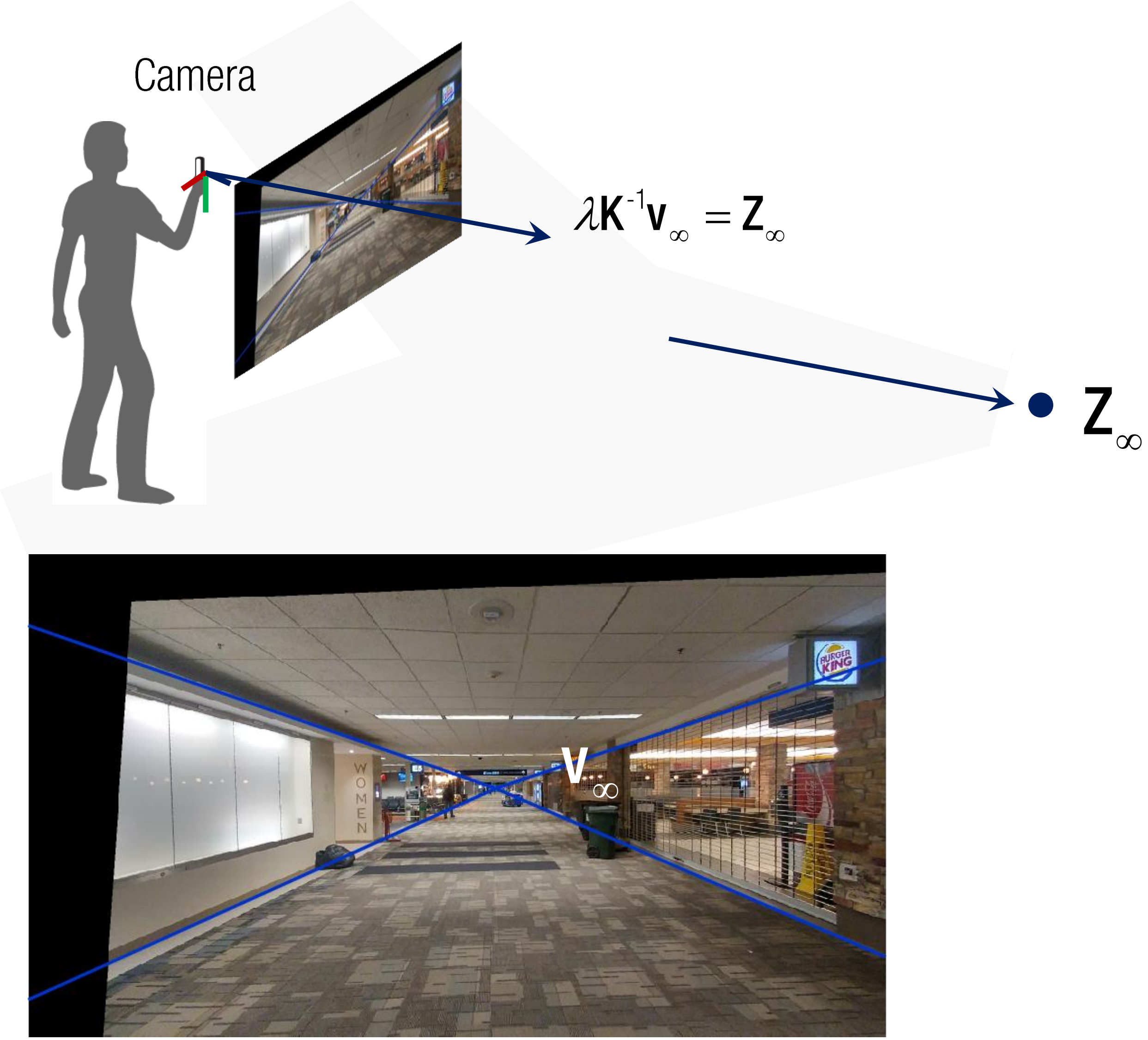

BURGER KING
WOMEN
E5


BURGER KING
WOMEN
E5


WOMEN
BURGER KING


Vanishing point
WOMEN
BURGER KING

Vanishing Point

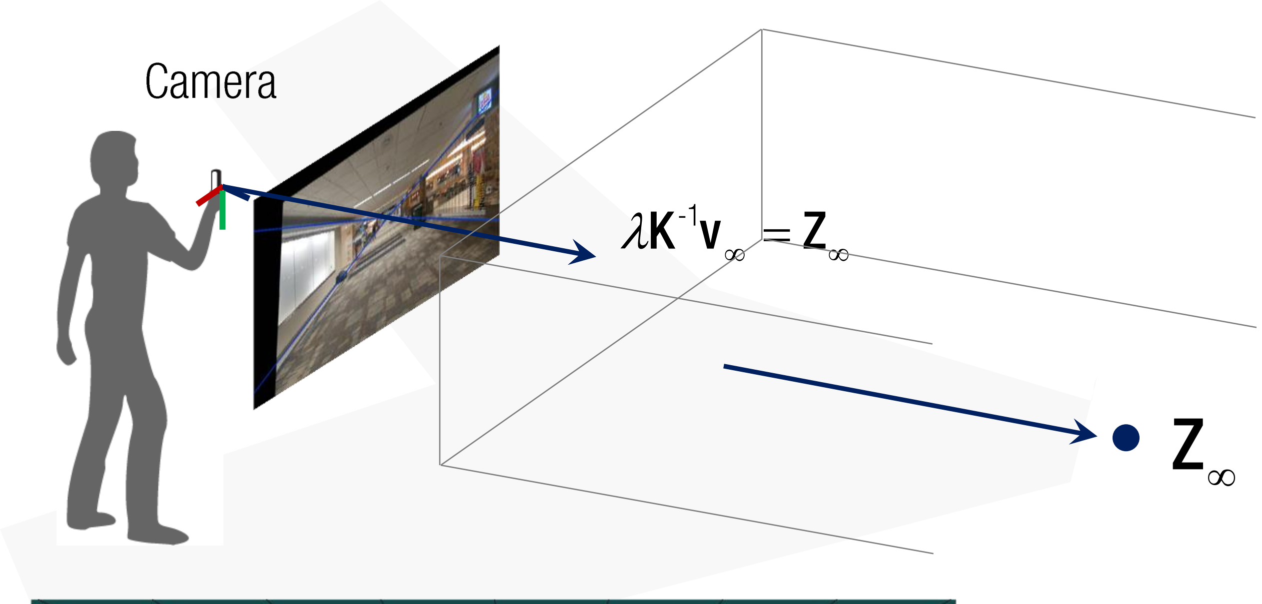


Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Box Representation

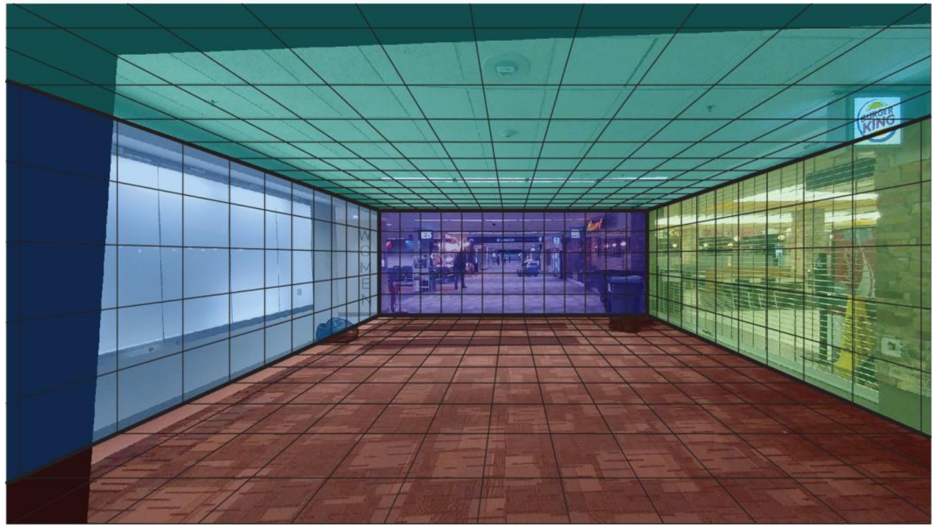


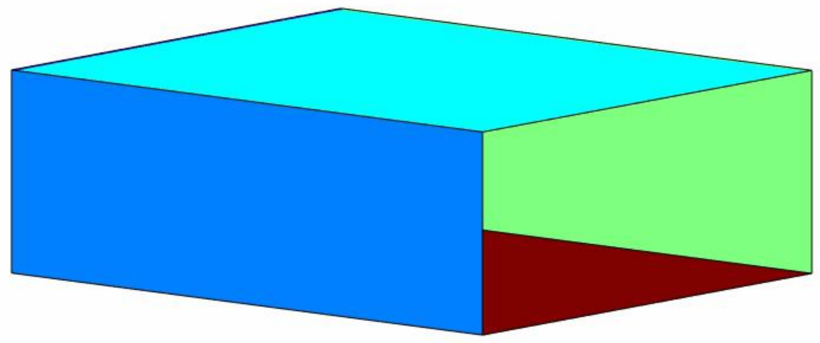
Vanishing point projection:

$$\lambda v_\infty = K Z_\infty$$

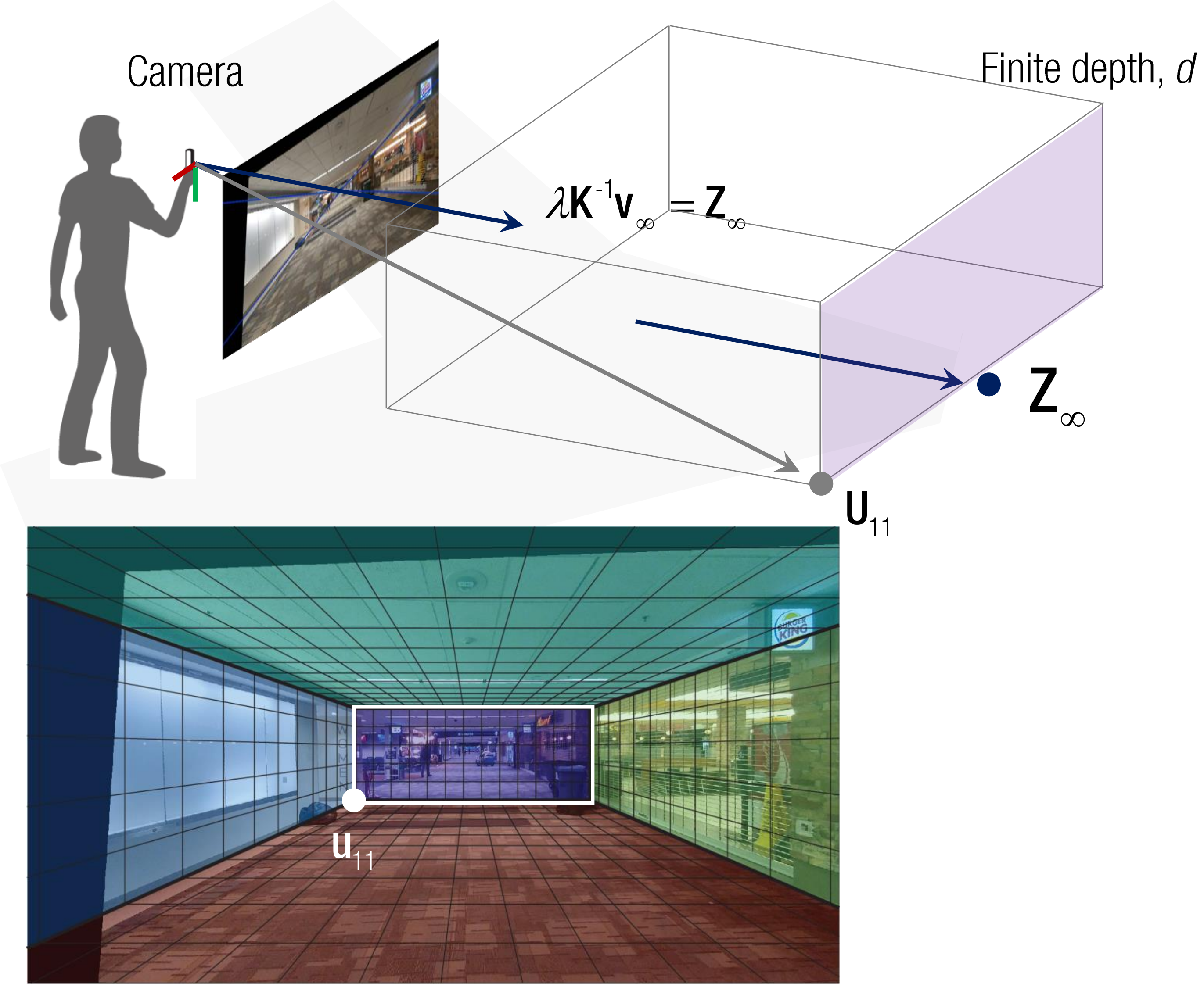
$$\longrightarrow \lambda K^{-1} v_\infty = Z_\infty$$

Define the direction of the box





Box Representation



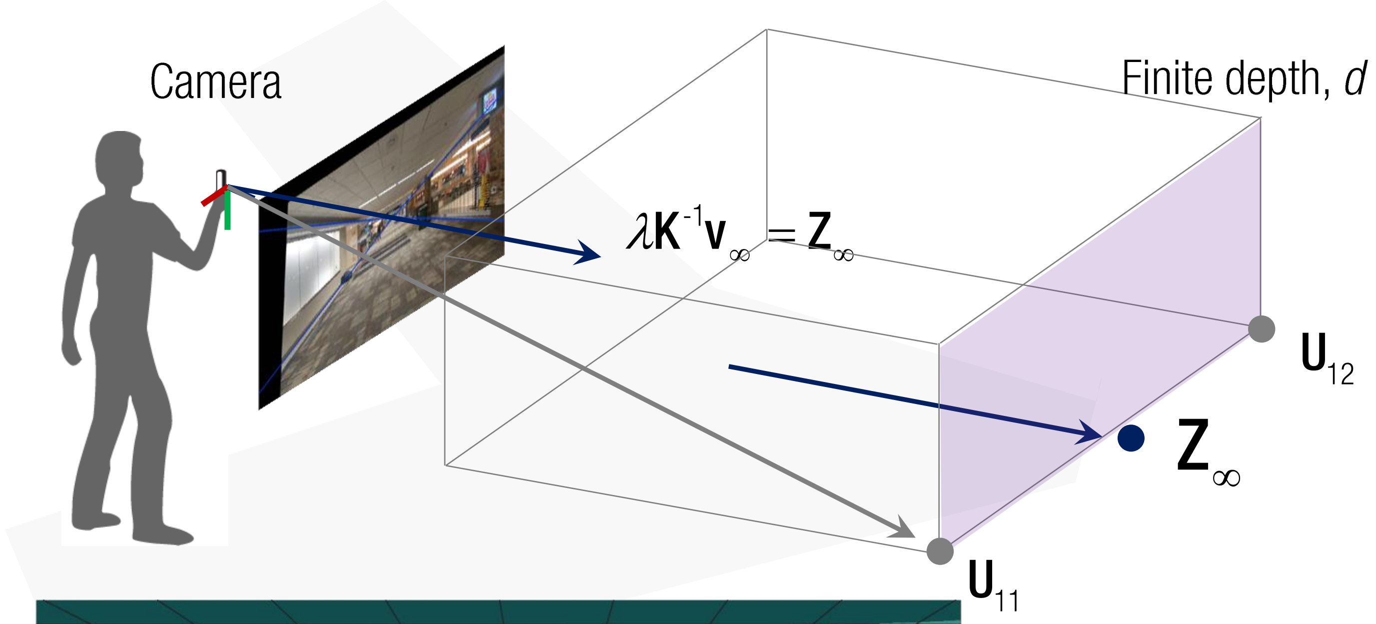
Vanishing point projection:

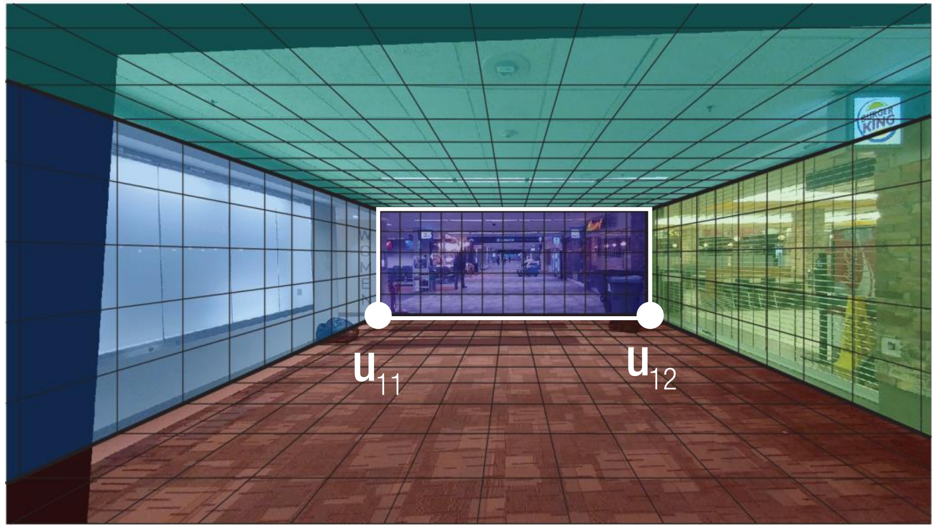
$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

$$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

Box Representation





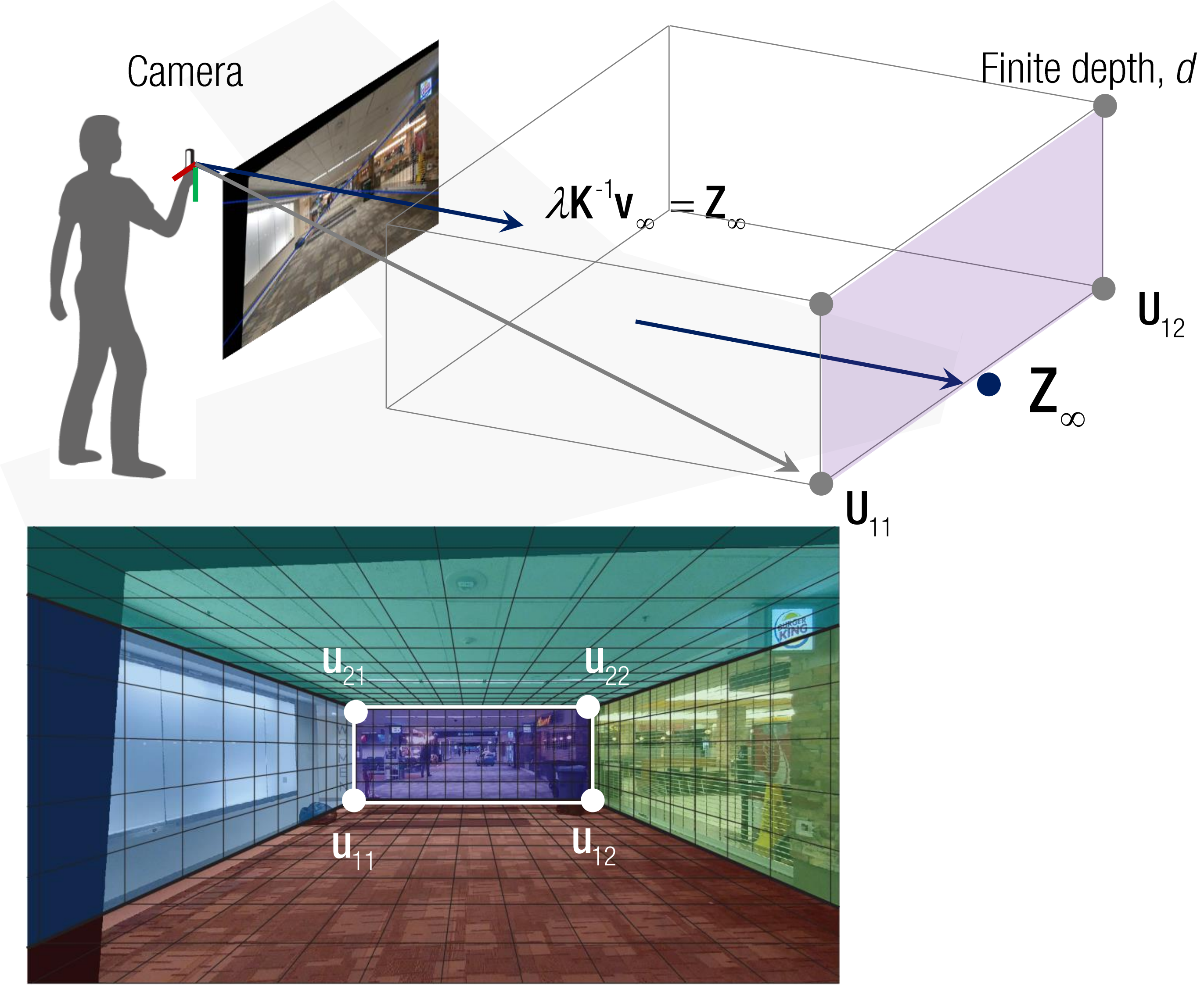
Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$ $\quad$ $\mathbf{U}_{12} = d\mathbf{K}^{-1}\mathbf{u}_{12}$ $\quad$: Same x coord.

Box Representation



Vanishing point projection:

$$\lambda \mathbf{v}_{\infty} = \mathbf{K}\mathbf{Z}_{\infty}$$

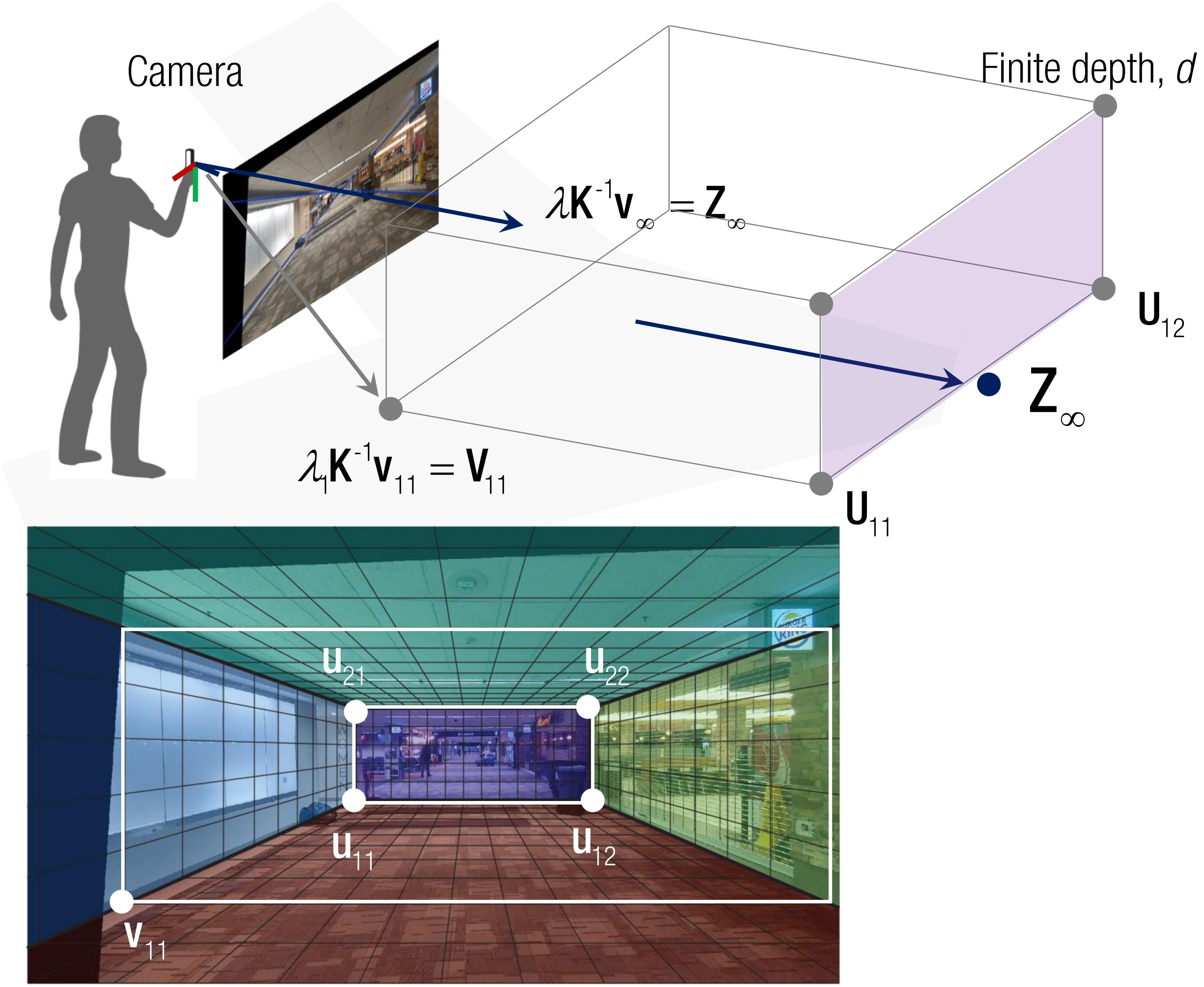
$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_{\infty} = \mathbf{Z}_{\infty}$$

$$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11} \qquad \mathbf{U}_{12} = d\mathbf{K}^{-1}\mathbf{u}_{12}$$: Same x coord.

$$\mathbf{U}_{21} = d\mathbf{K}^{-1}\mathbf{u}_{21} \qquad \mathbf{U}_{22} = d\mathbf{K}^{-1}\mathbf{u}_{22}$$

Same y coord.

Box Representation



Vanishing point projection:

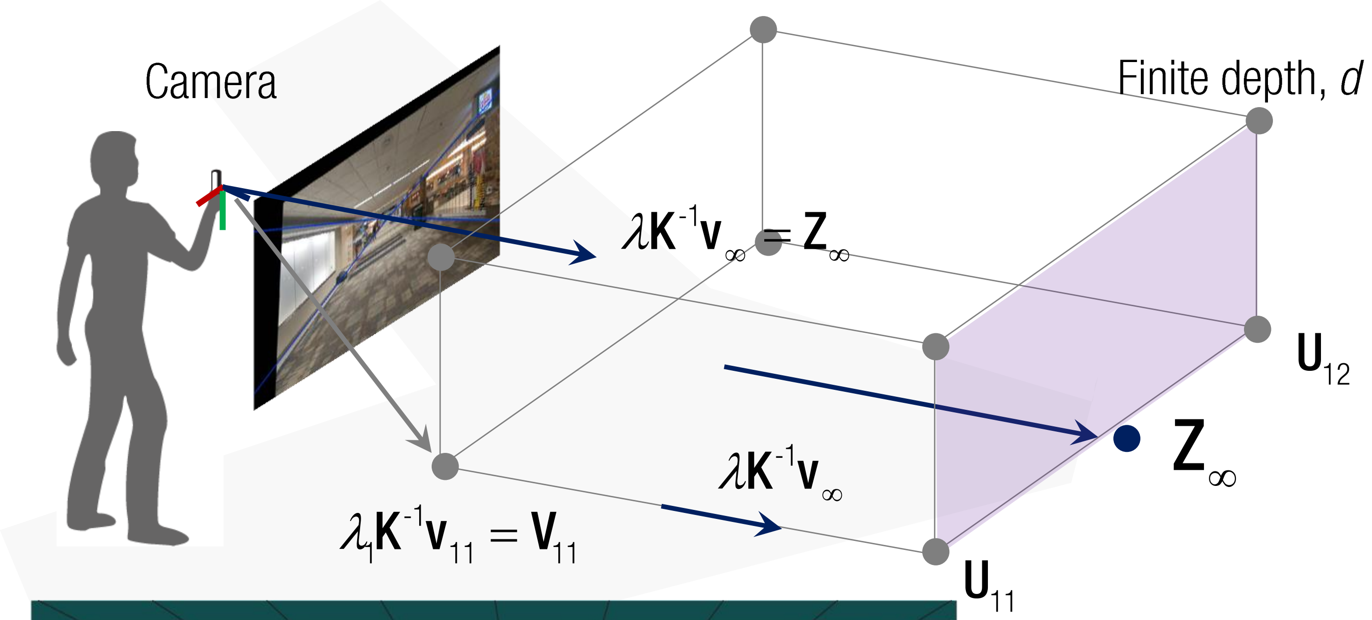
$$\lambda v_\infty = K Z_\infty$$

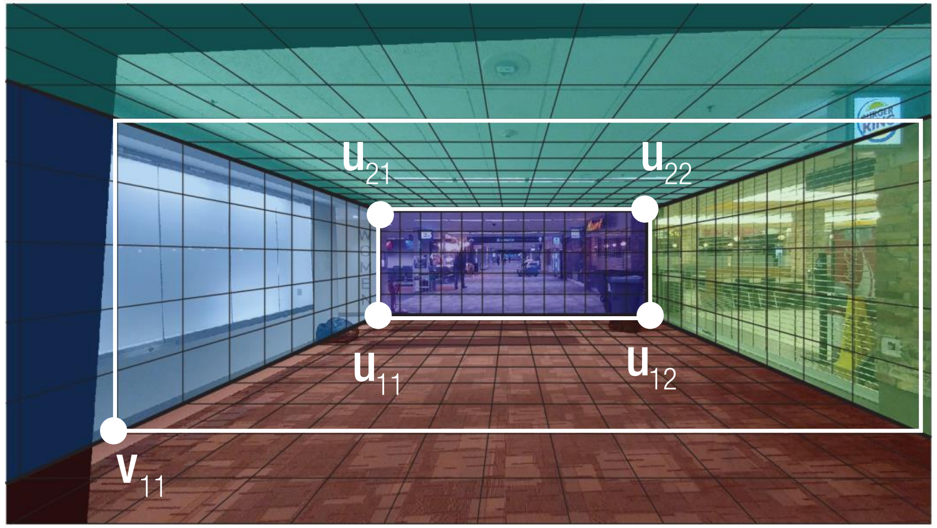
$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

Depth of frontal surface?

$$\underline{\lambda_1} K^{-1} v_{11} = V_{11}$$

Box Representation





Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Depth of frontal surface?

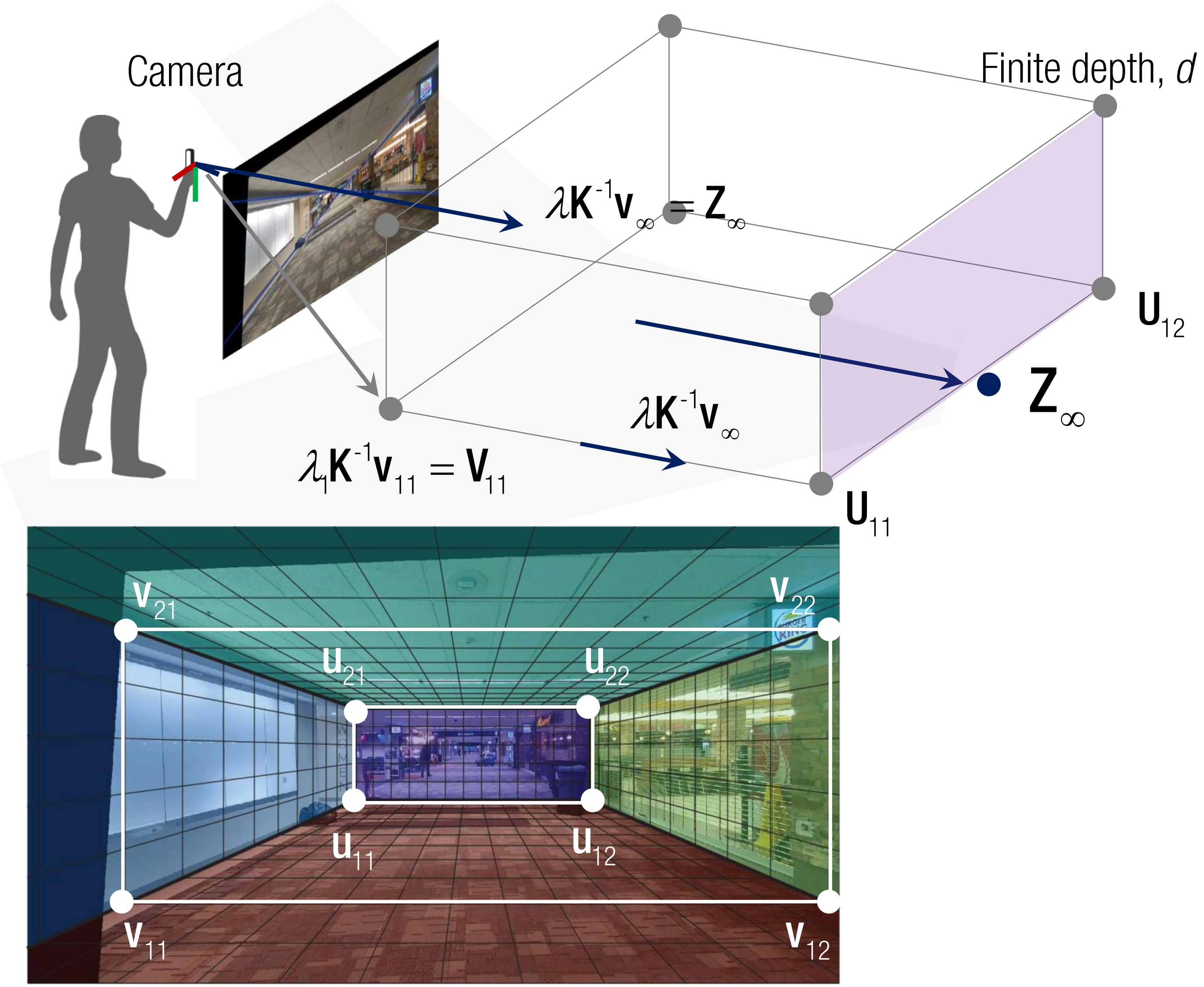
$$\underline{\lambda_1} \mathbf{K}^{-1}\mathbf{v}_{11} = \mathbf{V}_{11}$$

Line between $\mathbf{U}_{11}$ and $\mathbf{V}_{11}$ is parallel to the vanishing point direction.

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_{11} + \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

HW: express λ_1 using d.

Box Representation



Vanishing point projection:

$$\lambda \mathbf{v}_{\infty} = \mathbf{K}\mathbf{Z}_{\infty}$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_{\infty} = \mathbf{Z}_{\infty}$$

Depth of frontal surface?

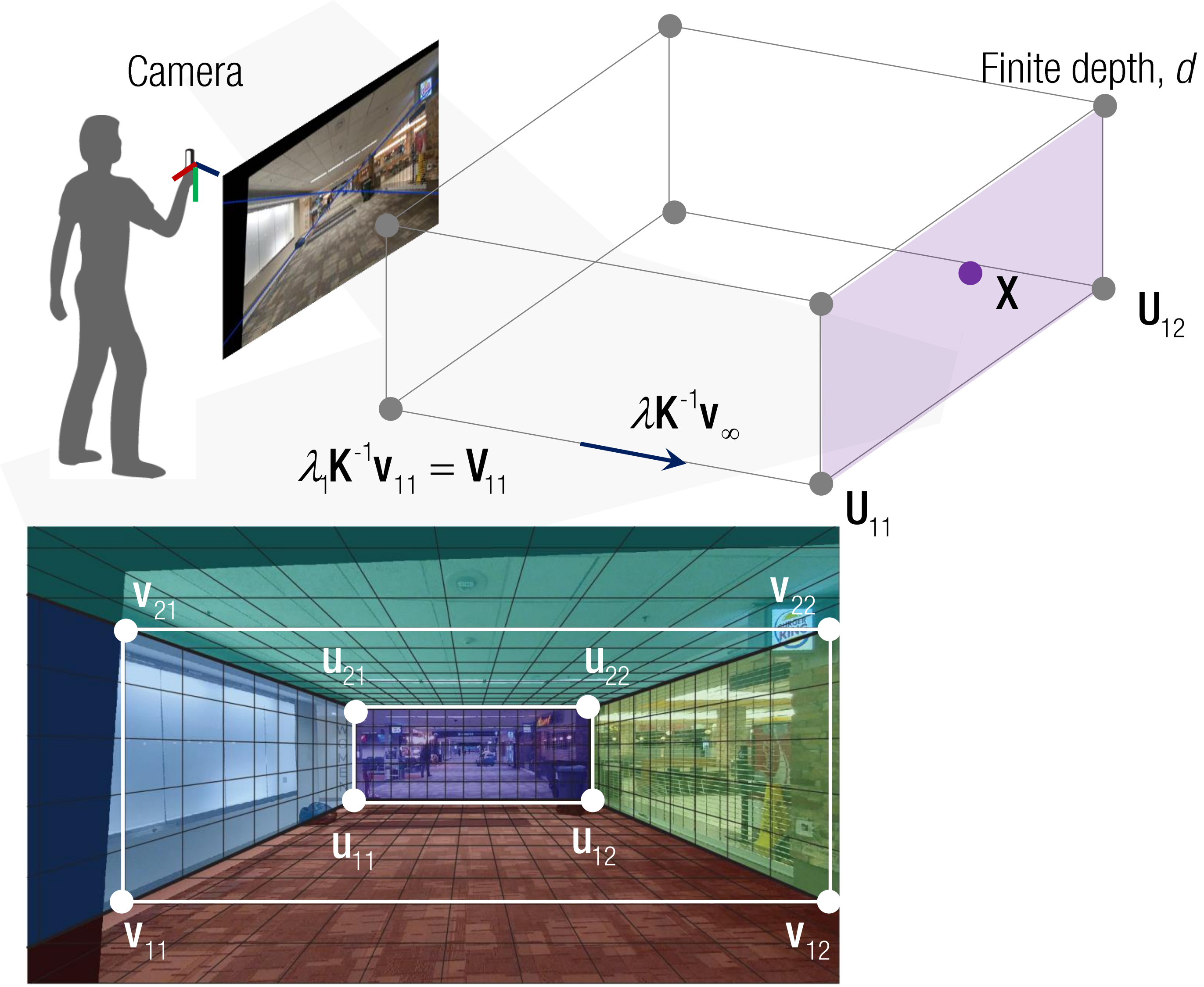
$$\underline{\lambda_1}\mathbf{K}^{-1}\mathbf{v}_{11} = \mathbf{V}_{11}$$

Line between $\mathbf{U}_{11}$ and $\mathbf{V}_{11}$ is parallel to the vanishing point direction.

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_{11} + \lambda \mathbf{K}^{-1}\mathbf{v}_{\infty} = \mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

HW: express λ_1 using d.

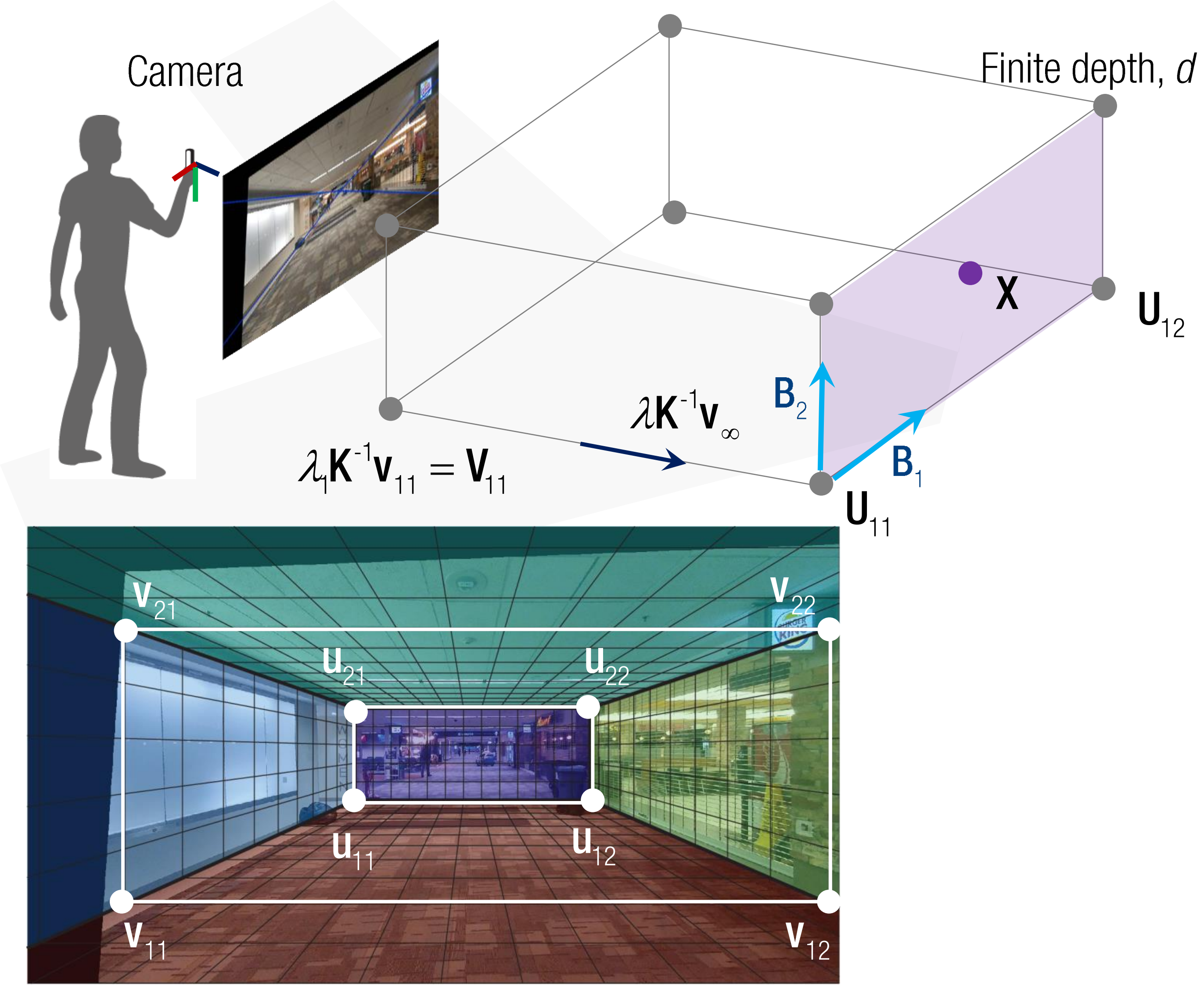
Box Representation



Point in a plane:

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Box Representation



Point in a plane:

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{U}_{11} + \mu_1 \mathbf{B}_1 + \mu_2 \mathbf{B}_2$$

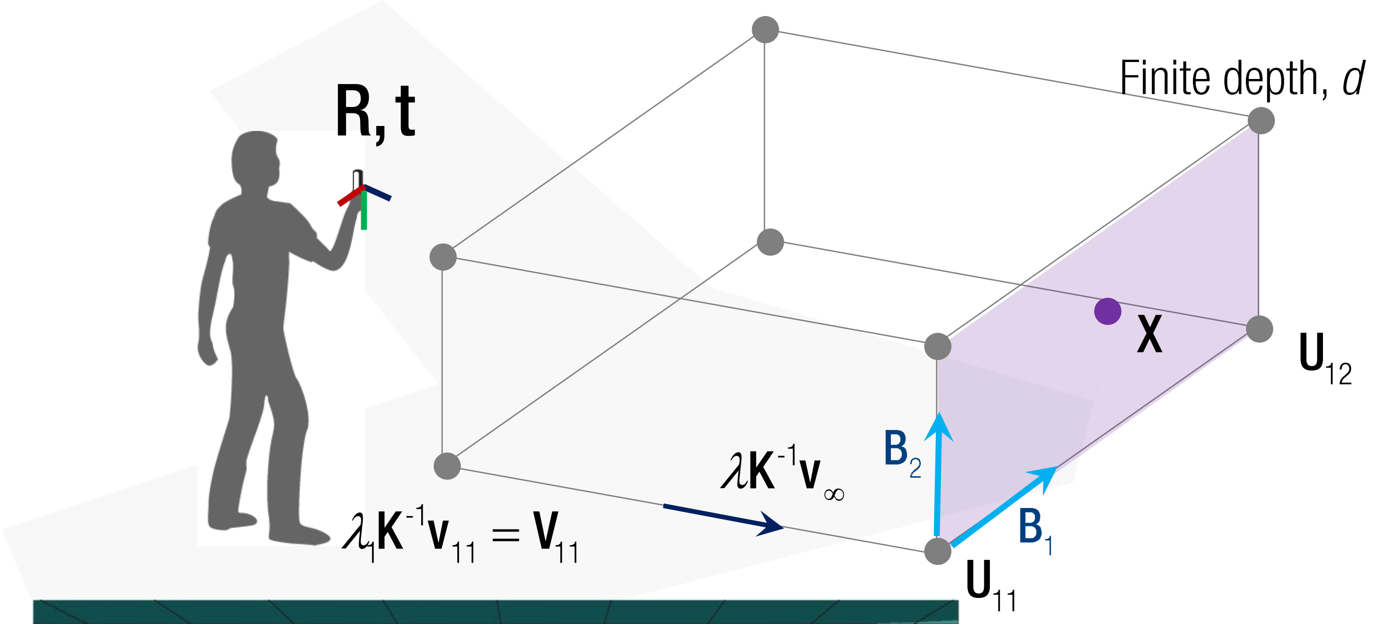
$$= \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

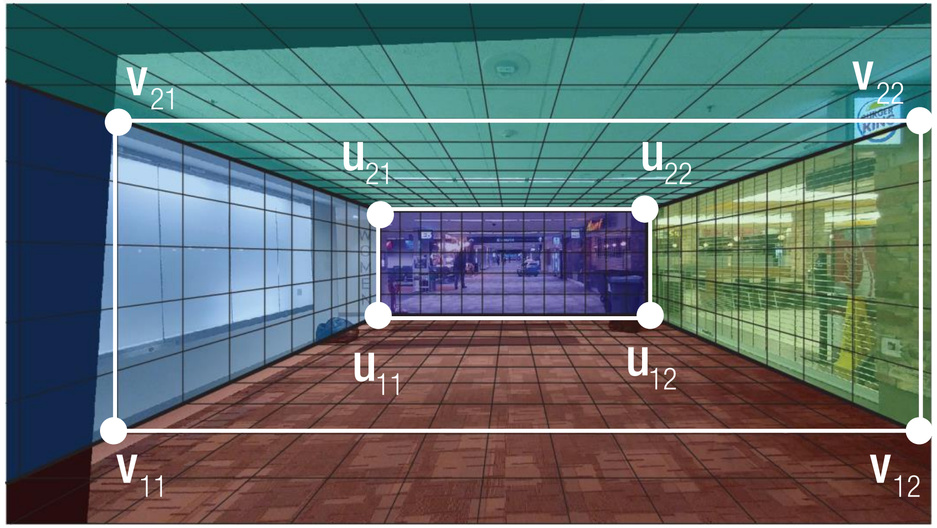
2DOF

Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K} \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Box Representation



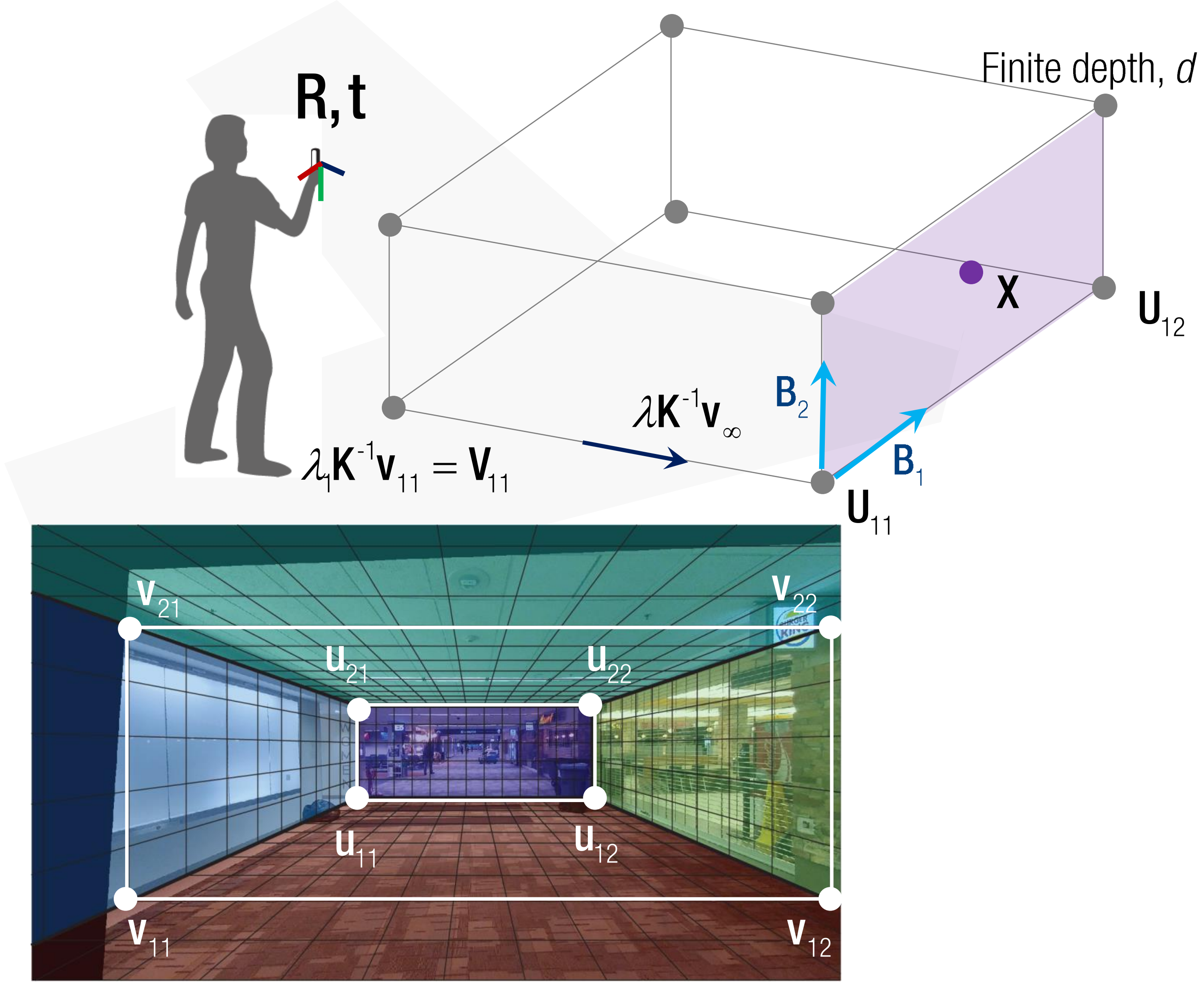


Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1 \\ \mu_2 \\ 1\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mu_1 \\ \mu_2 \\ 1\end{bmatrix}$$

Texture Mapping

Homography



Homography mapping from 3D plane to image:

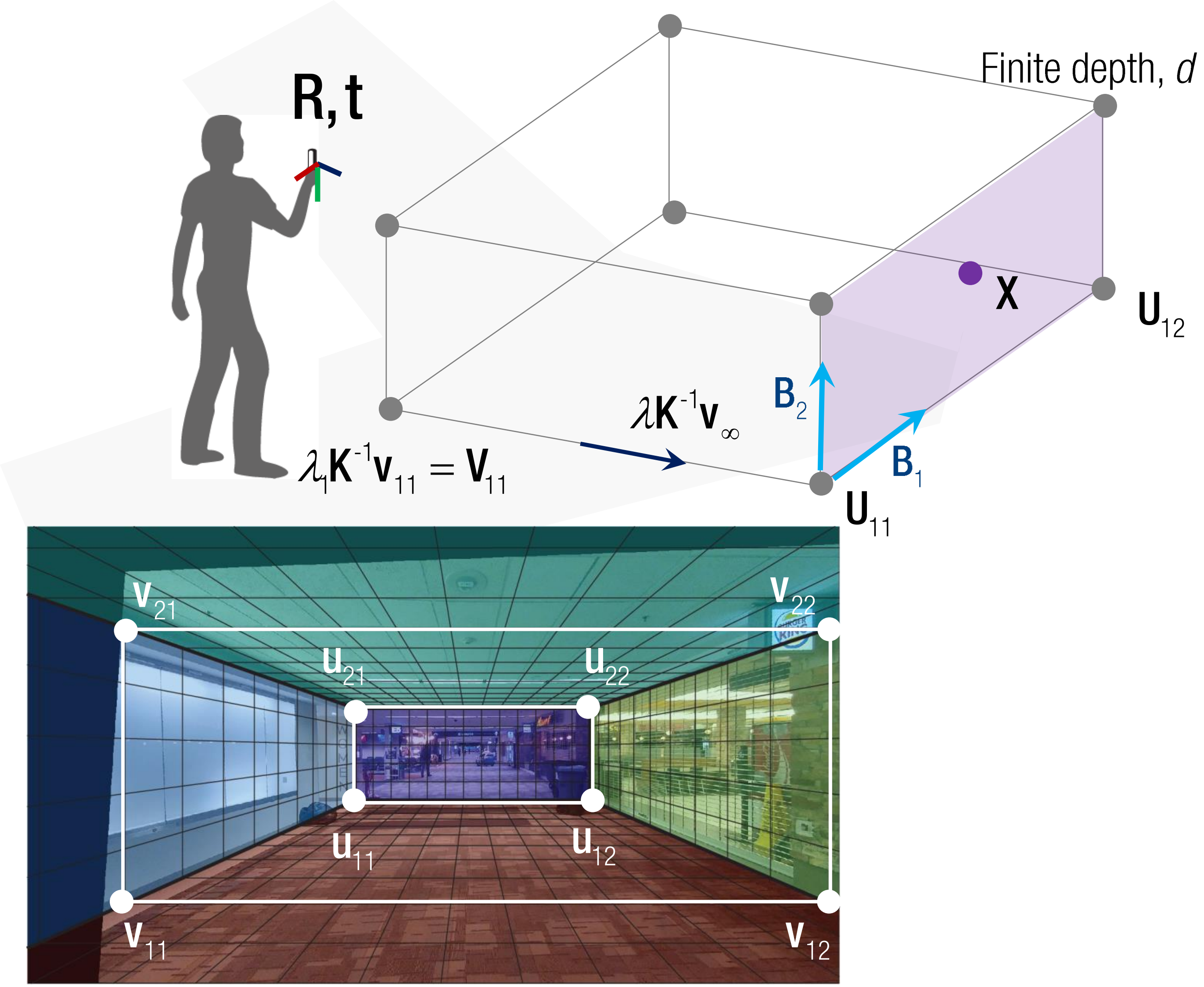
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}\\ 1\end{bmatrix}$$

$$= \mathbf{K}\begin{bmatrix}\mathbf{R}\mathbf{B}_1 & \mathbf{R}\mathbf{B}_2 & \mathbf{R}\mathbf{c}+\mathbf{t}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \tilde{\mathbf{H}}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography



Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix} \\ & & 1\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

$$= \mathbf{K}\begin{bmatrix}\mathbf{R}\mathbf{B}_1 & \mathbf{R}\mathbf{B}_2 & \mathbf{R}\mathbf{c}+\mathbf{t}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \tilde{\mathbf{H}}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

$$\rightarrow \lambda \tilde{\mathbf{H}}^{-1}\tilde{\mathbf{u}} = \begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \lambda_1 \mathbf{H}^{-1}\mathbf{u} \quad \rightarrow \quad \lambda \tilde{\mathbf{u}} = \tilde{\mathbf{H}}\mathbf{H}^{-1}\mathbf{u}$$

HW #3 Tour into your photo