Camera Calibration

Real-time Facial Reenactment

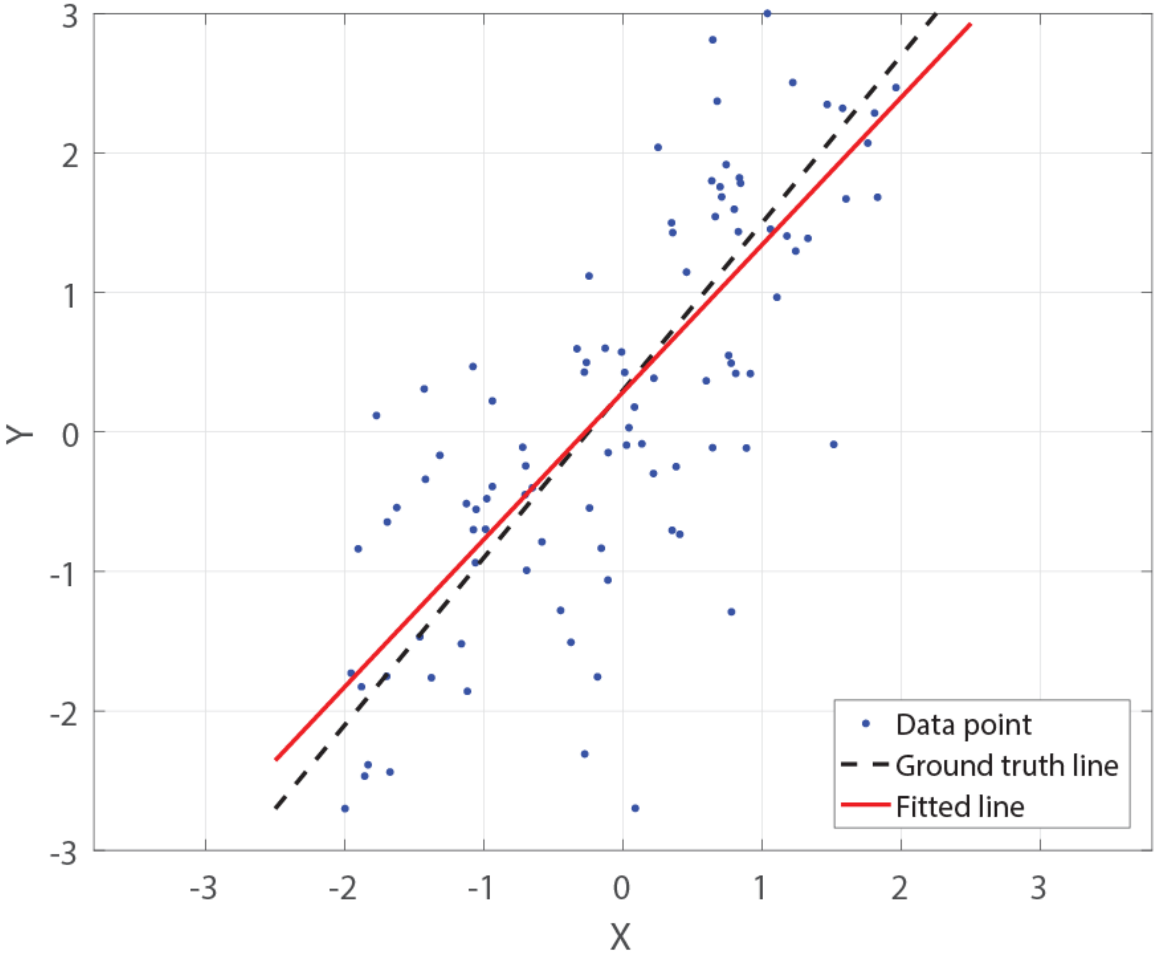
Live capture using a commodity webcam

Thies et al. “Face2Face: Realtime Face Capture and Reenactment of RGB Videos”

Announcement

- HW #2 due today
- HW #2 short presentation on next Thursday (share your panorama!)
- HW #3 will be out next Tuesday

Line Fitting (Ax=b)



Total error: $$E=\sum_{i=1}^{n}\left(v_i-(mu_i+d)\right)^2$$

$$\begin{bmatrix} u_1 & 1 \\ u_2 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix}\begin{bmatrix} m \\ d \end{bmatrix}\approx\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$\mathbf{A}\mathbf{x}\approx\mathbf{b}$$

$$E=(\mathbf{Ax}-\mathbf{b})^{\mathrm{T}}(\mathbf{Ax}-\mathbf{b})=\|\mathbf{Ax}-\mathbf{b}\|^2$$

$$=\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{Ax}-2\mathbf{x}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}\mathbf{b}+\mathbf{b}^{\mathrm{T}}\mathbf{b}$$

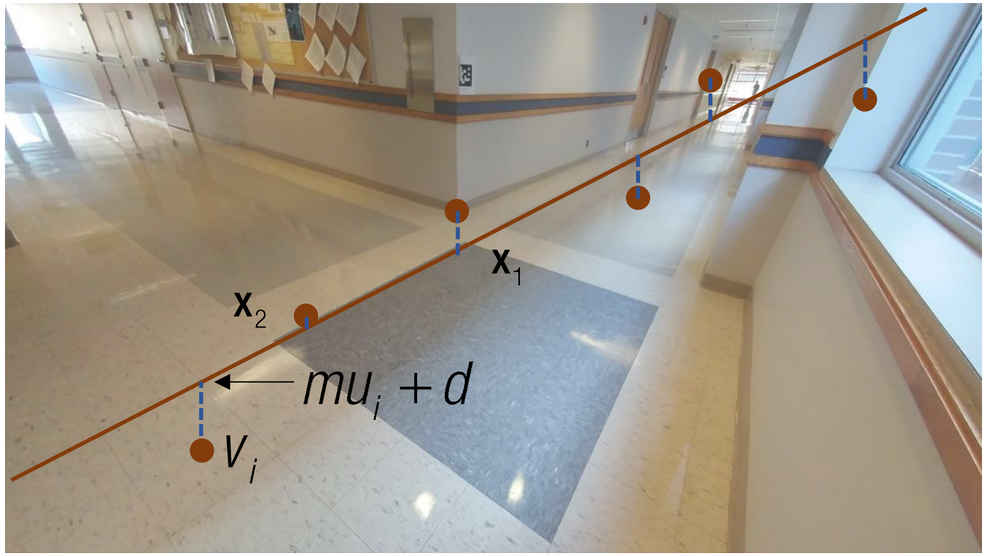
$$\frac{\partial E}{\partial \mathbf{x}}=2\mathbf{A}^{\mathrm{T}}\mathbf{Ax}-2\mathbf{A}^{\mathrm{T}}\mathbf{b}=\mathbf{0}$$

$$\longrightarrow \mathbf{A}^{\mathrm{T}}\mathbf{Ax}=\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

$$\mathbf{A}^{\mathrm{T}}\ \mathbf{A}\ \mathbf{x}=\mathbf{A}^{\mathrm{T}}\ \mathbf{b}$$

$$\mathbf{x}=\left[\mathbf{A}^{\mathrm{T}}\ \mathbf{A}\right]^{-1}\mathbf{A}^{\mathrm{T}}\ \mathbf{b}$$

Line Fitting (Ax=b)



Given points: $(u_1, v_1), (u_2, v_2), \cdots, (u_n, v_n)$

Find the best line: $(a, b, c) \longrightarrow (\underline{m}, \underline{d})$

slope y-intercept

$$\begin{aligned} au_1 + bv_1 + c &\approx 0 \\ au_2 + bv_2 + c &\approx 0 \\ &\vdots \\ au_n + bv_n + c &\approx 0 \end{aligned} \longrightarrow \begin{aligned} v_1 &\approx mu_1 + d \\ v_2 &\approx mu_2 + d \\ &\vdots \\ v_n &\approx mu_n + d \end{aligned}$$

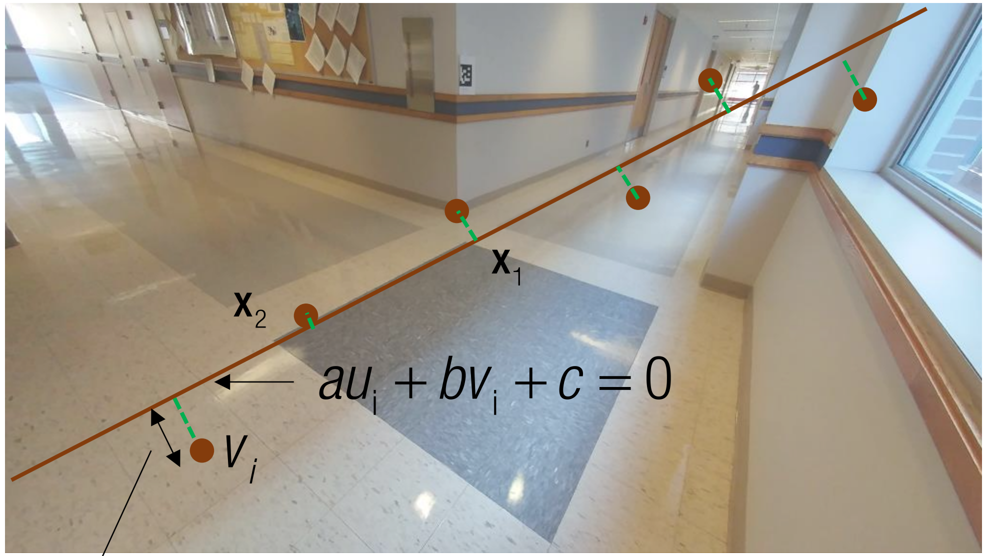
$$\mathbf{Ax} = \mathbf{b}$$

What is different?

Line Fitting (Ax=0)

Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

Find the best line: (a,b,c)



$$d_i = \frac{|au_i + bv_i + c|}{\sqrt{a^2+b^2}}$$

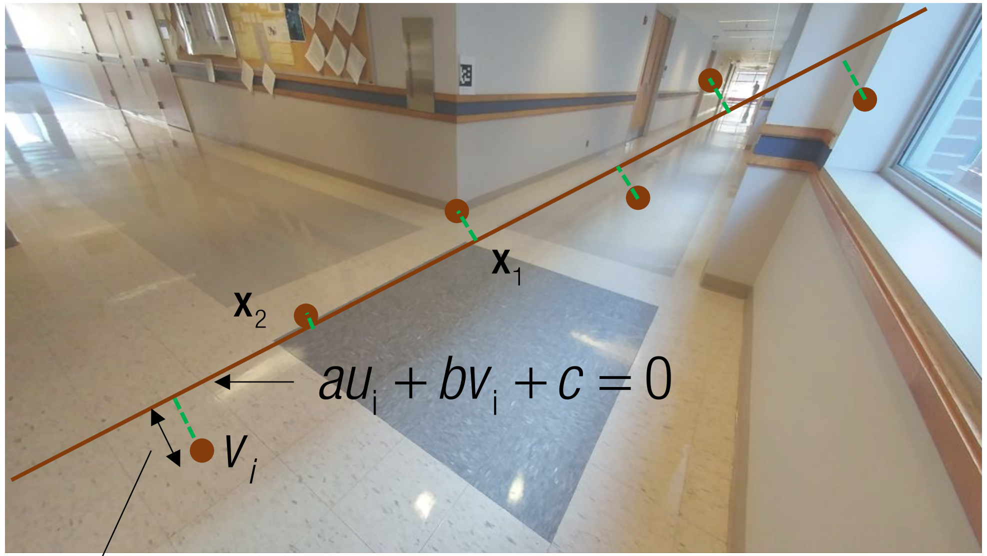
$$\begin{aligned} au_1 + bv_1 + c &\approx 0 \\ au_2 + bv_2 + c &\approx 0 \\ &\vdots \\ au_n + bv_n + c &\approx 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} v_1 &\approx mu_1 + d \\ v_2 &\approx mu_2 + d \\ &\vdots \\ v_n &\approx mu_n + d \end{aligned}$$

$\mathbf{Ax} = \mathbf{0}$ $\qquad$ $\mathbf{Ax} = \mathbf{b}$

Line Fitting (Ax=0)

Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

Find the best line: (a,b,c)



$$d_\mathrm{i} = \frac{|au_\mathrm{i} + bv_\mathrm{i} + c|}{\sqrt{a^2 + b^2}}$$

$$\begin{aligned} au_1 + bv_1 + c &\approx 0 \\ au_2 + bv_2 + c &\approx 0 \\ &\vdots \\ au_n + bv_n + c &\approx 0 \end{aligned} \longrightarrow \begin{aligned} v_1 &\approx mu_1 + d \\ v_2 &\approx mu_2 + d \\ &\vdots \\ v_n &\approx mu_n + d \end{aligned}$$

$\mathbf{Ax} = \mathbf{0}$

$\mathbf{Ax} = \mathbf{b}$

Trivial solution: $\mathbf{x} = \mathbf{0}$

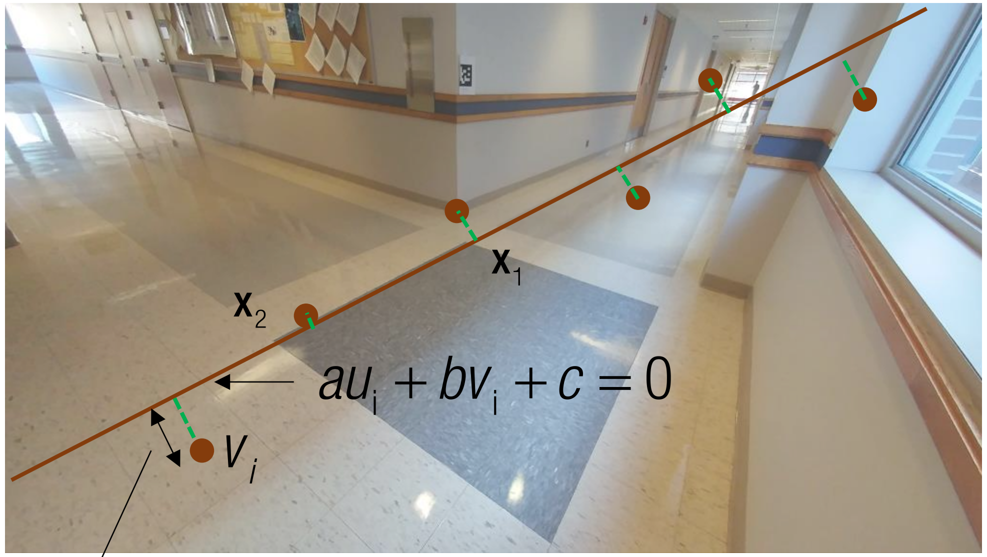
$\underset{\mathbf{x}}{\text{minimize}} \|\mathbf{Ax}\|^2$ subject to $\|\mathbf{x}\| = 1$

Condition to avoid the trivial solution

Line Fitting (Ax=0)

Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

Find the best line: (a,b,c)



$$d_i = \frac{|au_i + bv_i + c|}{\sqrt{a^2 + b^2}}$$

$$\begin{aligned} au_1 + bv_1 + c &\approx 0 \\ au_2 + bv_2 + c &\approx 0 \\ &\vdots \\ au_n + bv_n + c &\approx 0 \end{aligned} \longrightarrow \begin{aligned} v_1 &\approx mu_1 + d \\ v_2 &\approx mu_2 + d \\ &\vdots \\ v_n &\approx mu_n + d \end{aligned}$$

$\mathbf{Ax} = \mathbf{0}$ $\qquad$ $\mathbf{Ax} = \mathbf{b}$

Trivial solution: $\mathbf{x} = \mathbf{0}$

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If A is mxn matrix where m < n,

Line Fitting (Ax=0)

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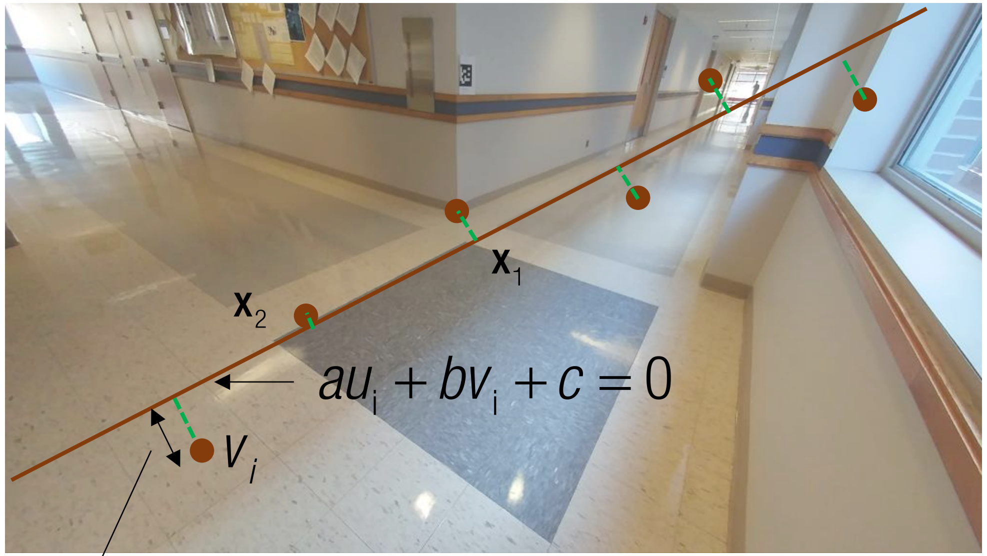

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$\mathbf{Ax} = \mathbf{0}$ $\qquad$ $\mathbf{Ax} = \mathbf{b}$

Trivial solution: $\mathbf{x} = \mathbf{0}$

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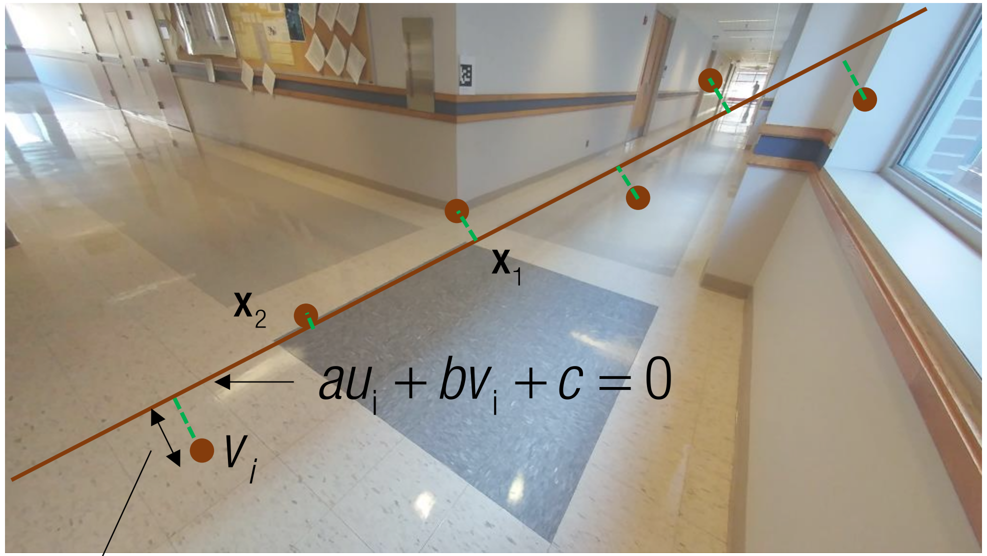
If A is mxn matrix where m < n, $\quad = \text{null}(\mathbf{A})$

Line Fitting (Ax=0)

Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

Find the best line: (a,b,c)



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If A is mxn matrix where m > n,

Line Fitting (Ax=0)

Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

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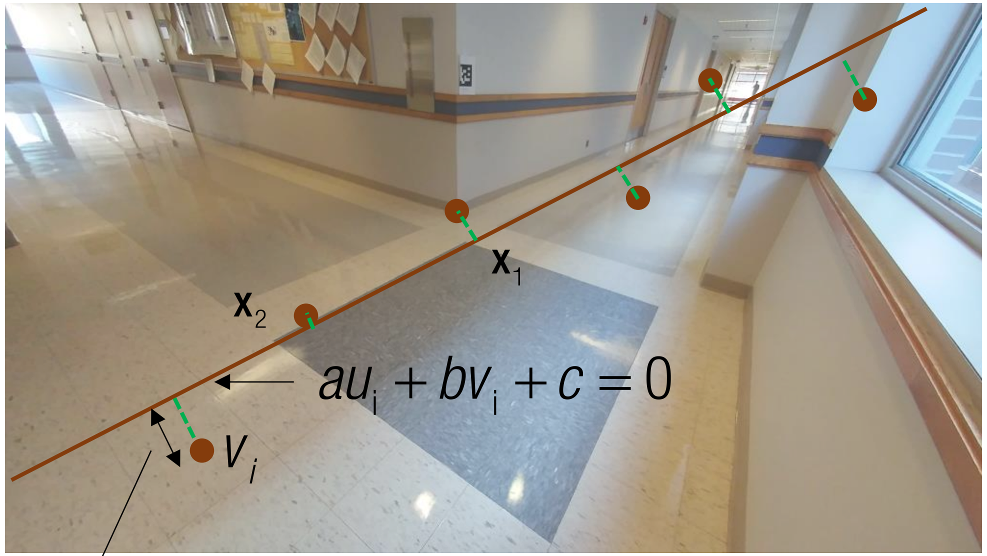

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$\mathbf{Ax} = \mathbf{0}$ $\qquad$ $\mathbf{Ax} = \mathbf{b}$

Trivial solution: $\mathbf{x} = \mathbf{0}$

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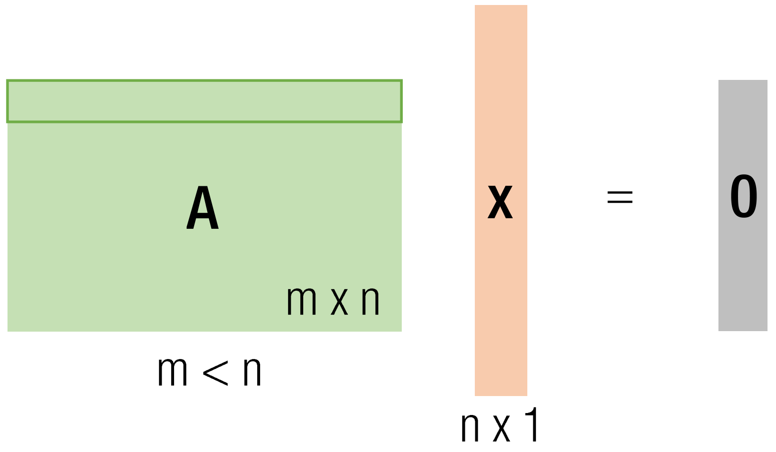
If A is mxn matrix where m > n, $\blacksquare = \text{approx.null}(\mathbf{A})$

Nullspace

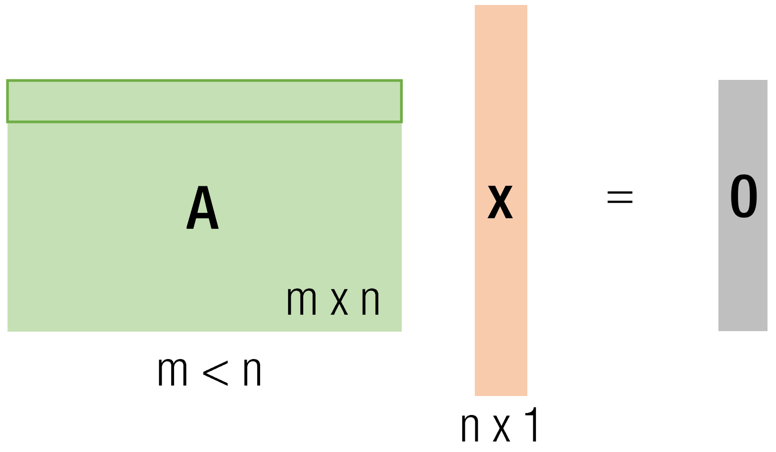
eqs < # unknowns



Nullspace

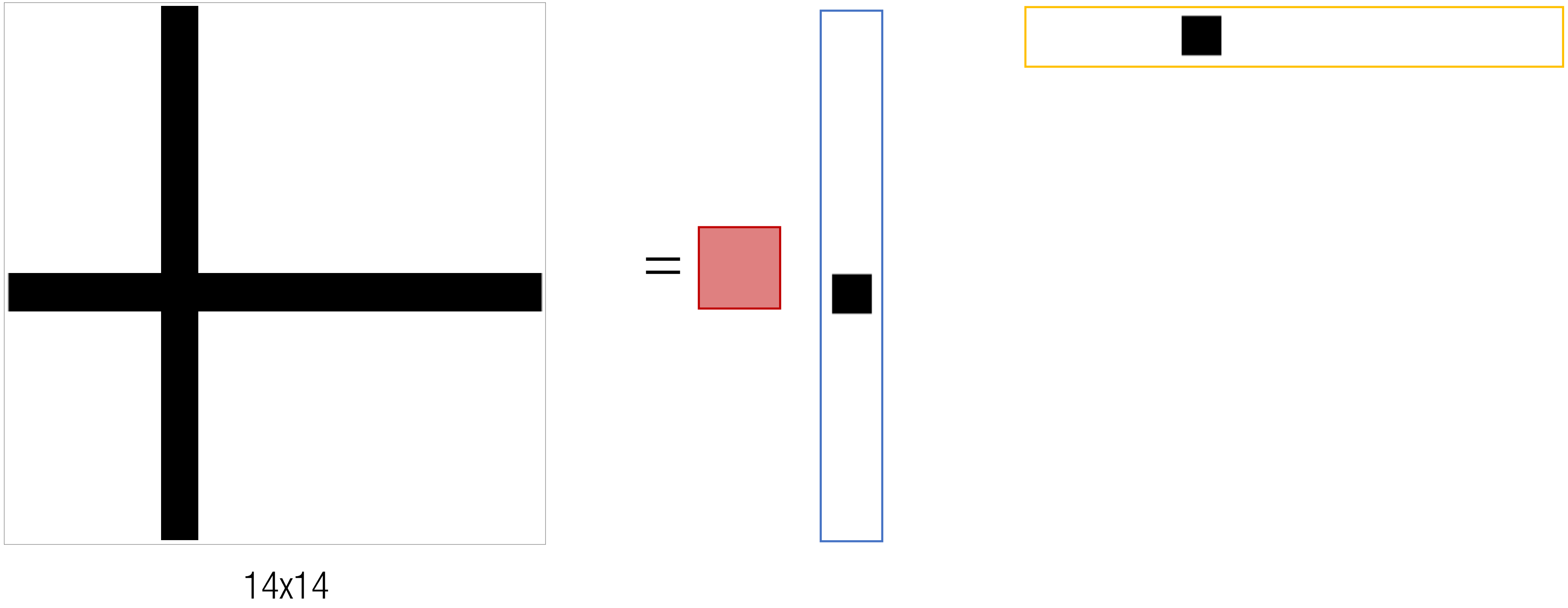
\# eqs < # unknowns

There exist at least n-m vectors that makes **Ax**=**0**.

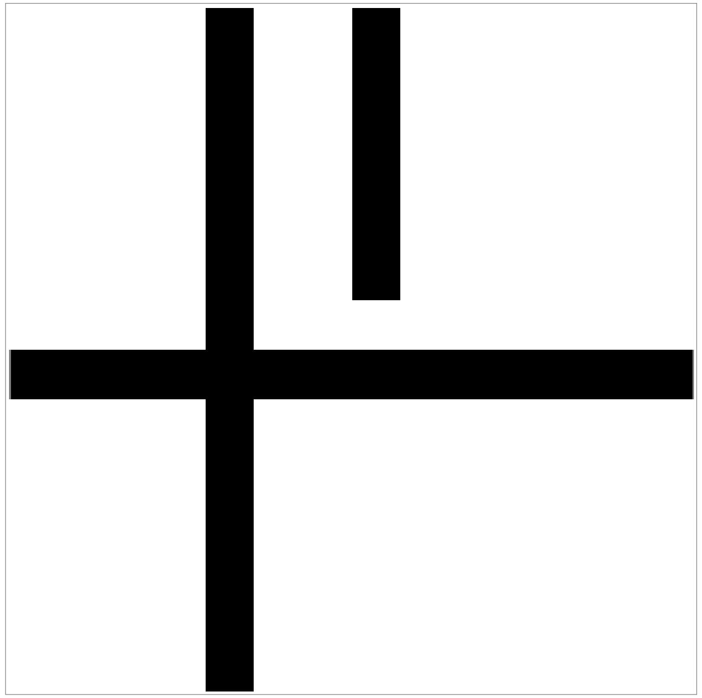


$\longrightarrow \mathbf{x} = \text{null}(\mathbf{A})$

More SVD



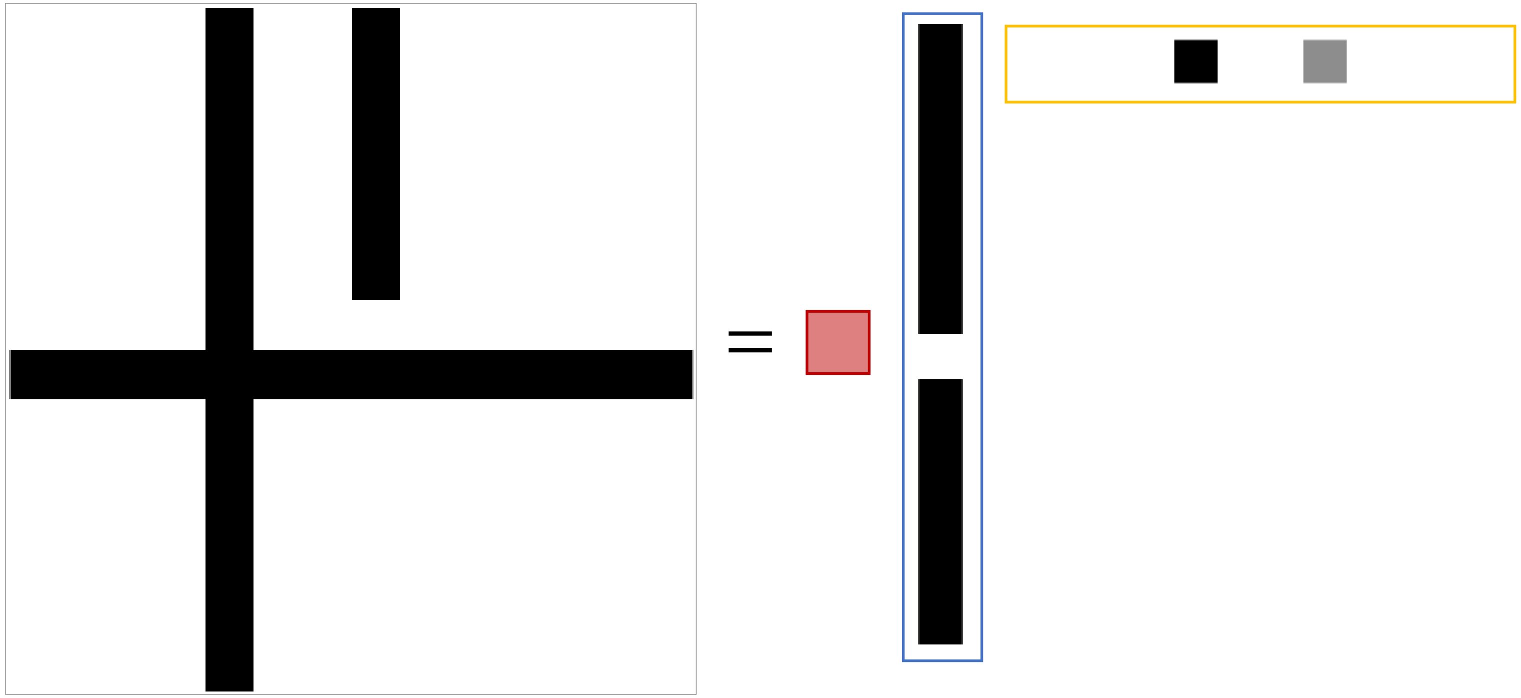
More SVD



14x14

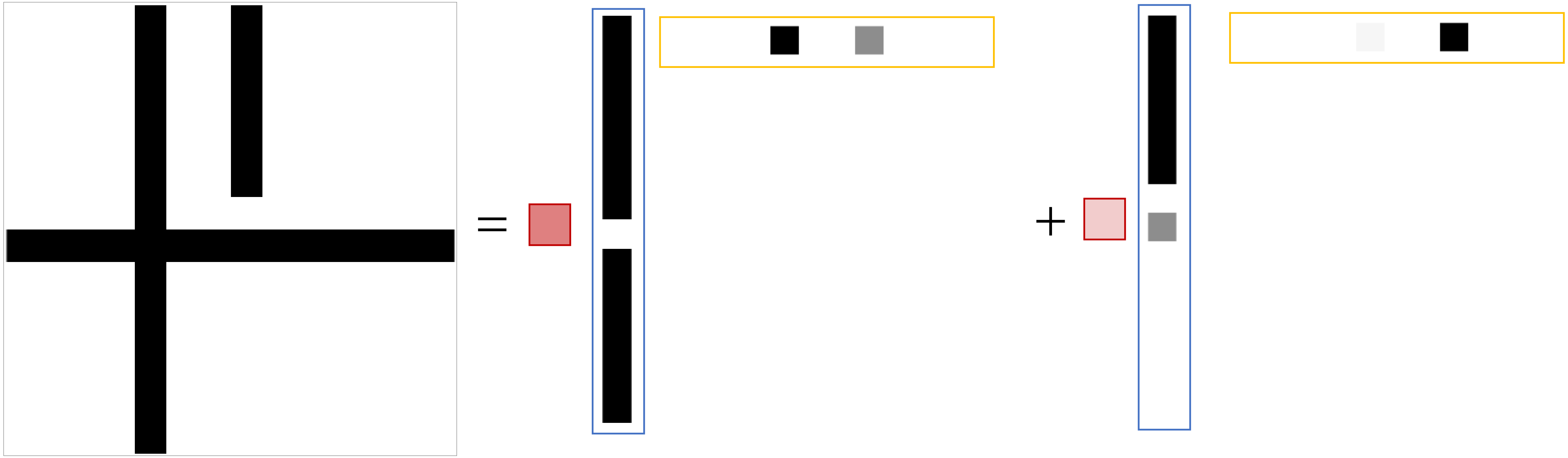
=

More SVD



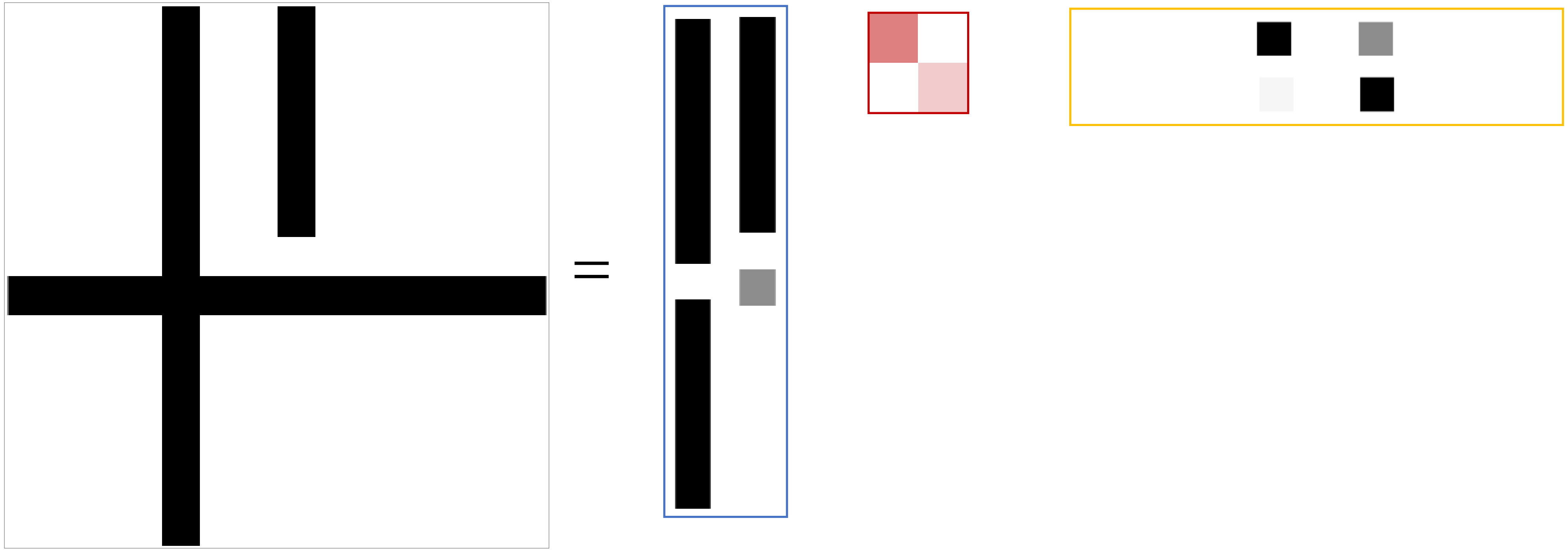
14x14

More SVD



14x14

More SVD

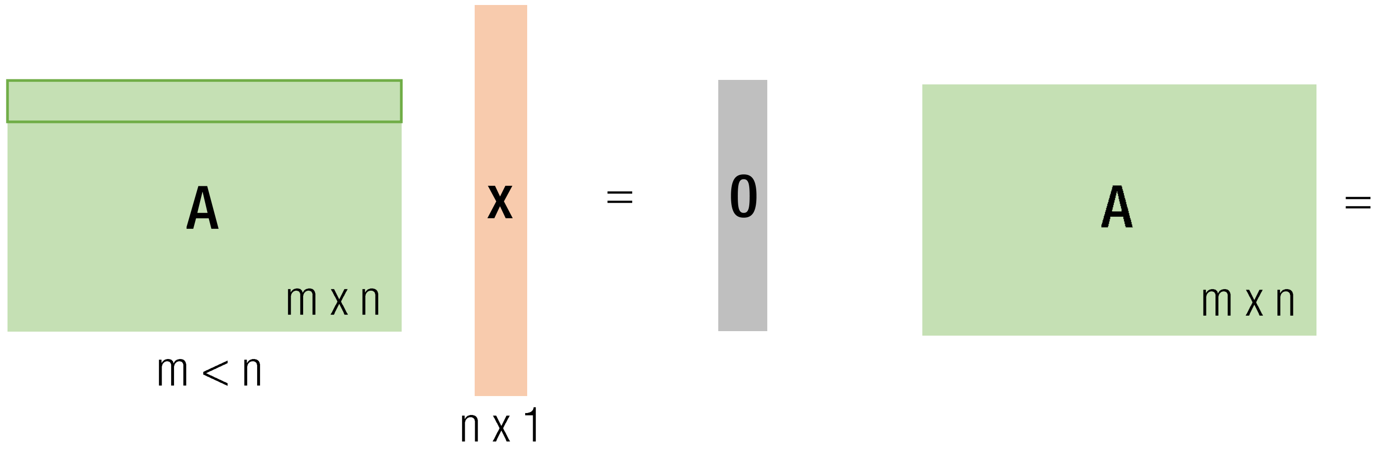


14x14

Singular Value Decomposition (SVD)

\# eqs < # unknowns

There exist at least n-m vectors that makes **Ax**=**0**.



$\longrightarrow \mathbf{x} = \mathrm{null}(\mathbf{A})$

Singular Value Decomposition (SVD)

eqs < # unknowns

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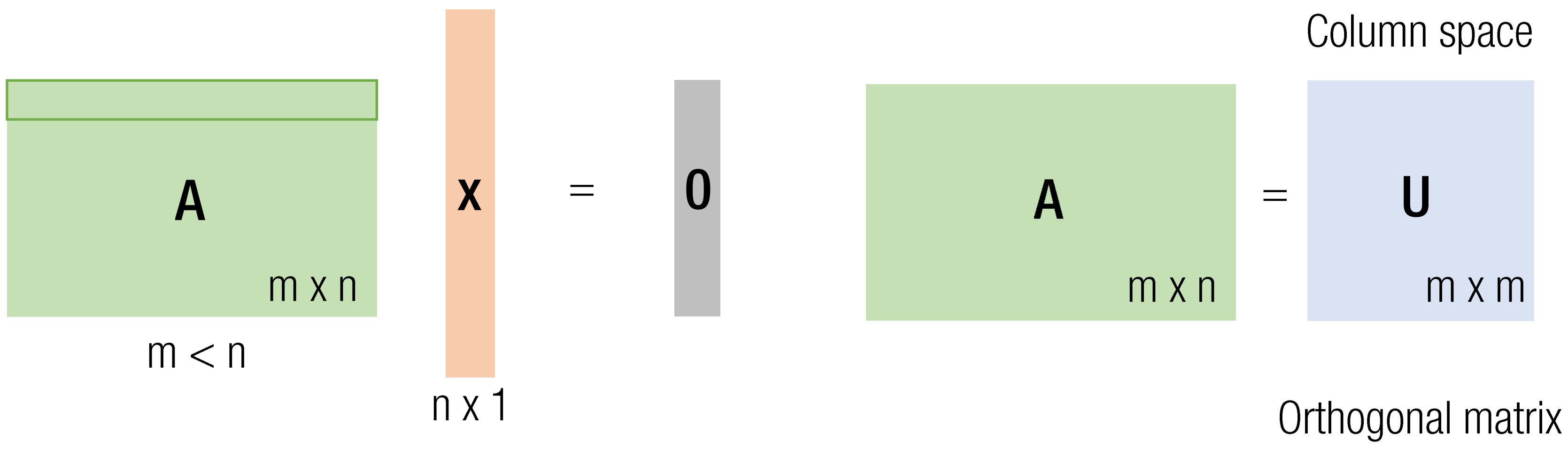

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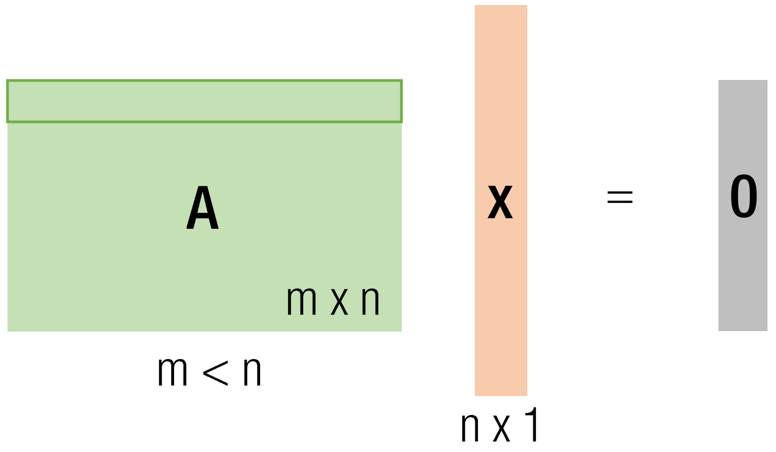

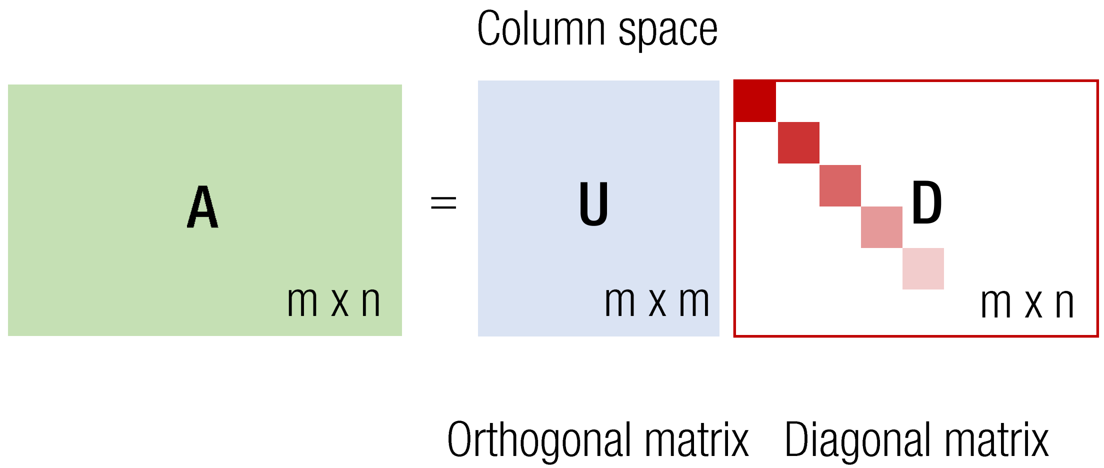

$$\longrightarrow \mathbf{x} = \text{null}\left(\mathbf{A}\right)$$

Singular Value Decomposition (SVD)

eqs < # unknowns

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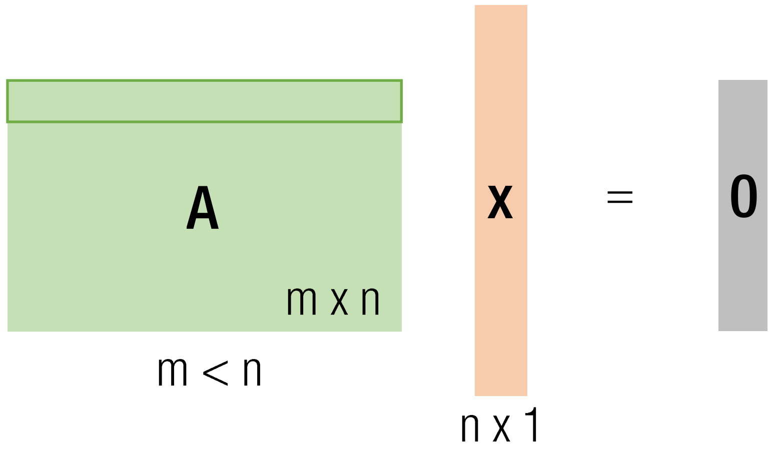

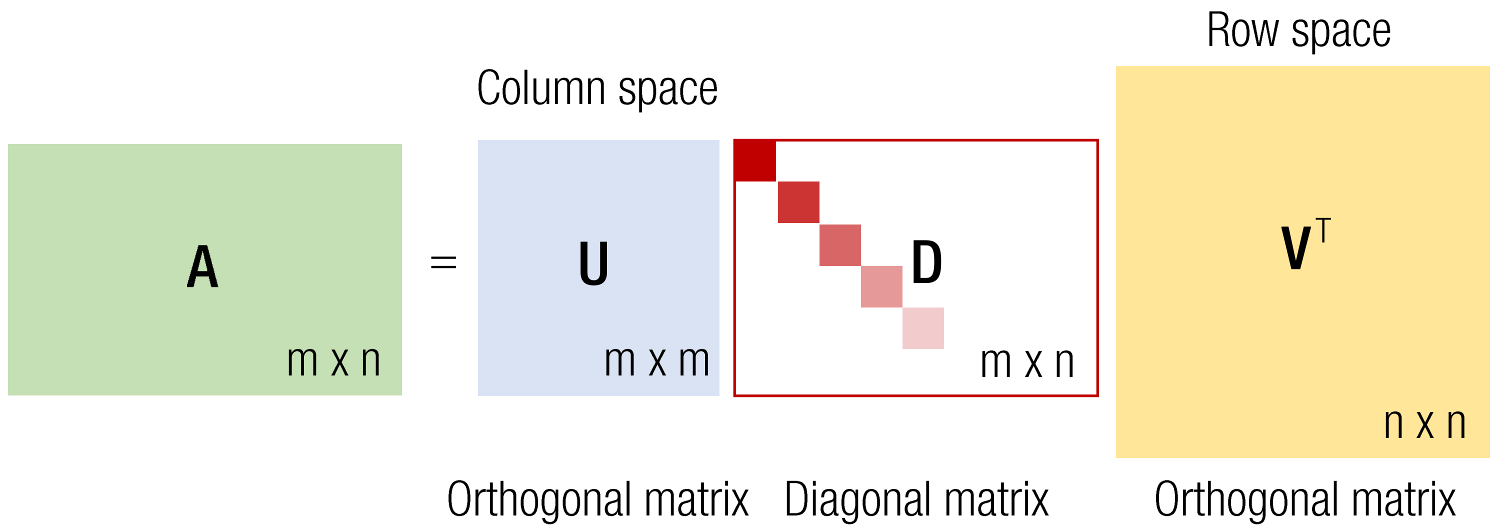

$\longrightarrow \mathbf{x} = \mathrm{null}(\mathbf{A})$

Singular Value Decomposition (SVD)

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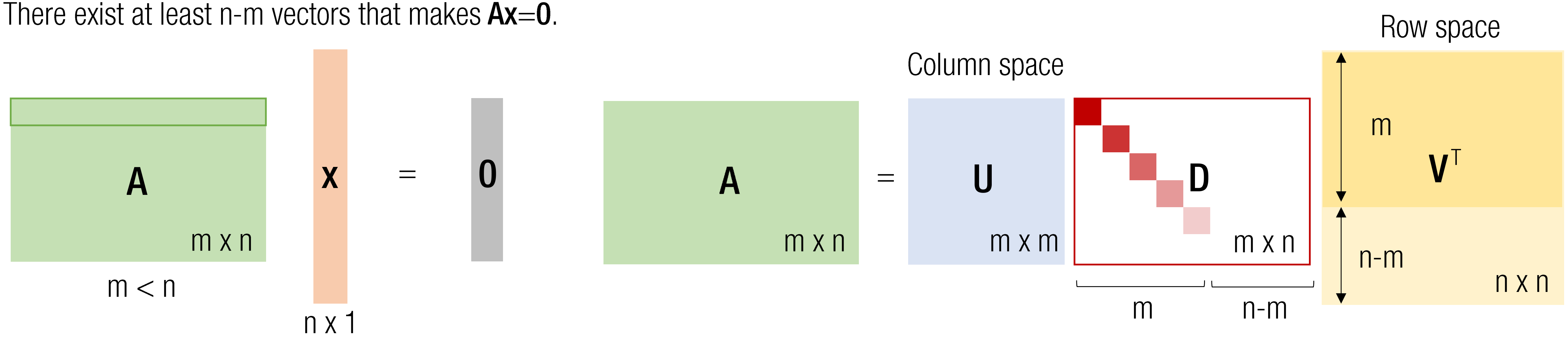

$$\longrightarrow \mathbf{x} = \text{null}(\mathbf{A})$$

Singular Value Decomposition (SVD)

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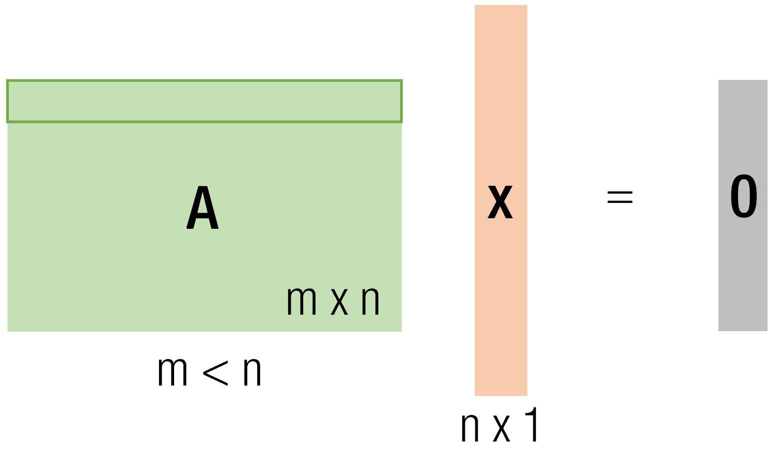
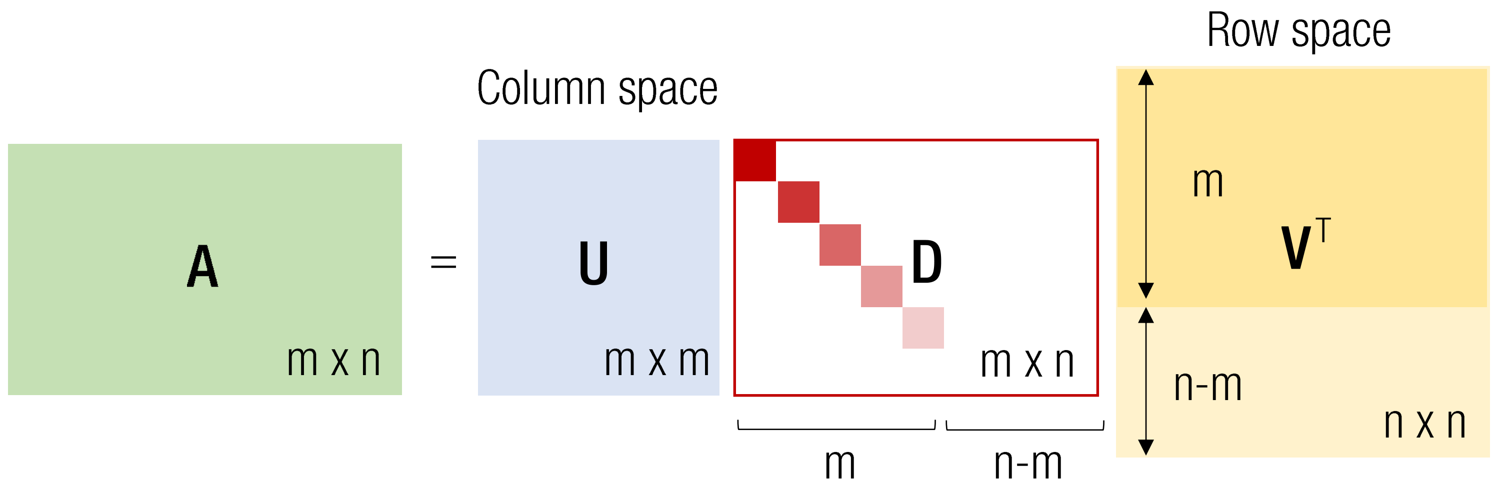
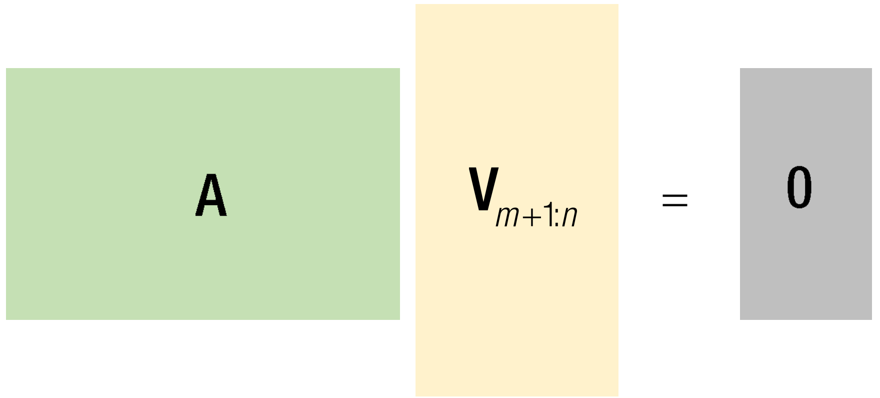




$$\longrightarrow \ \mathbf{x} = \mathrm{null}(\mathbf{A})$$

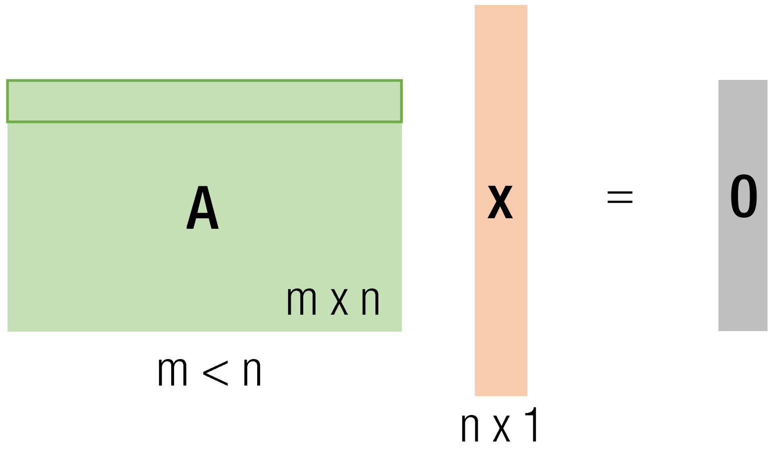


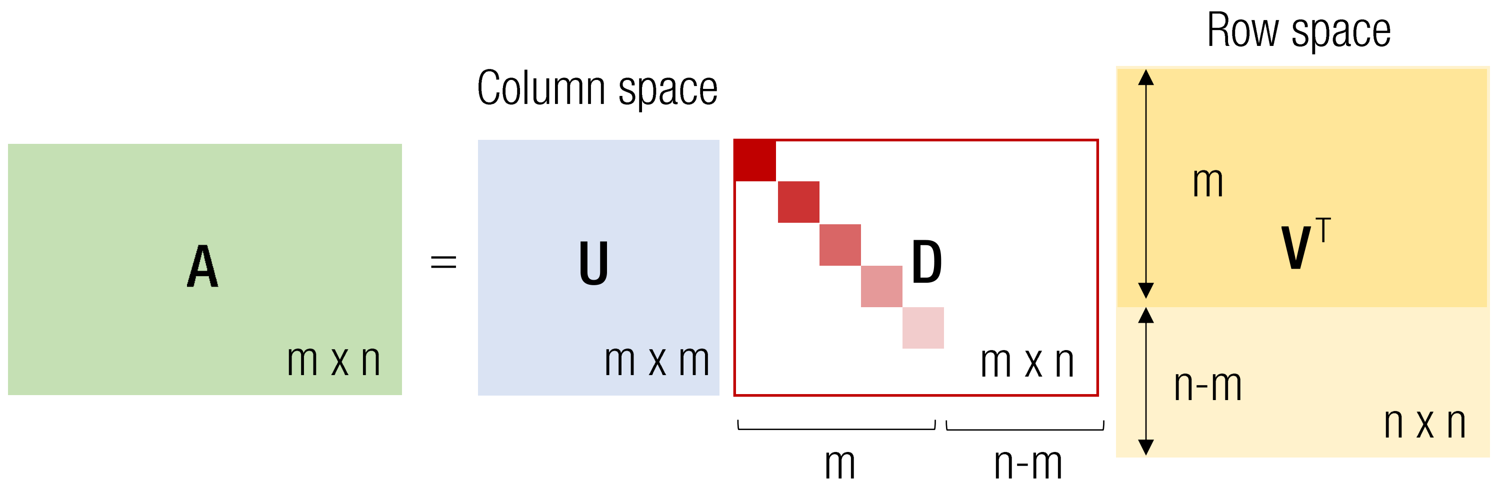
$$\mathbf{V}_{m+1:n} = \mathrm{null}(\mathbf{A})$$

Singular Value Decomposition (SVD)

eqs < # unknowns

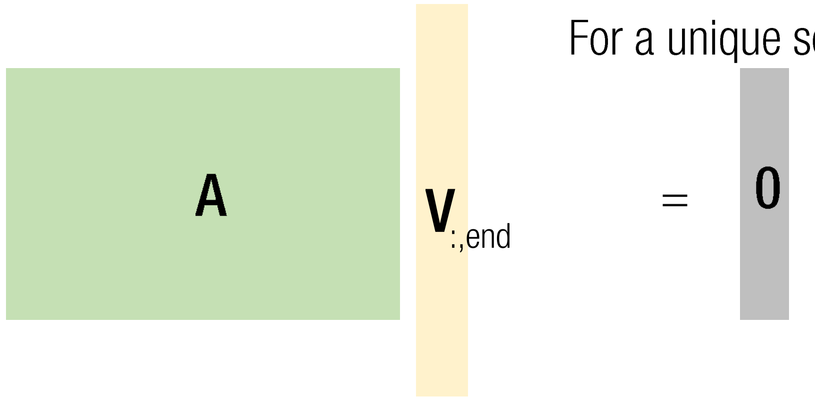
There exist at least n-m vectors that makes **Ax**=**0**.





$\longrightarrow \mathbf{x} = \text{null}(\mathbf{A})$

For a unique solution, **A** should be m x (m+1)

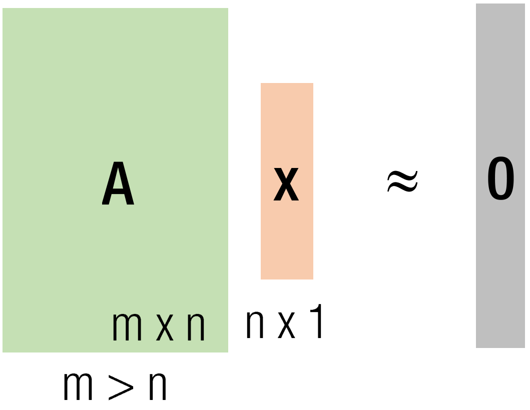


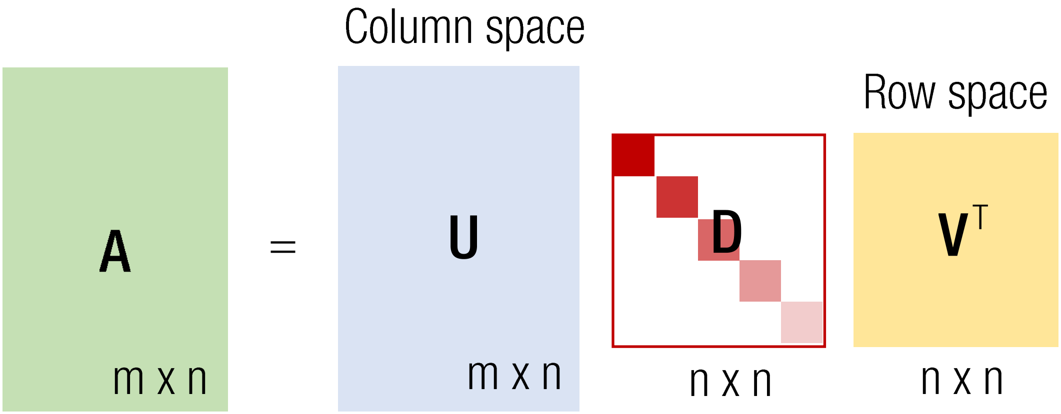
$\mathbf{V}_{:end} = \text{null}(\mathbf{A})$

Singular Value Decomposition (SVD)

\# eqs > # unknowns

There exist no nullspace of **A**.



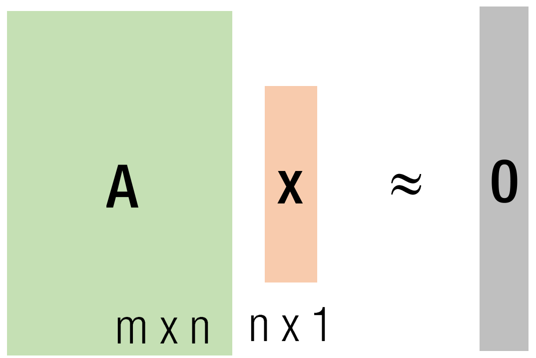


$$\underset{\mathbf{x}}{\text{minimize}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$

Singular Value Decomposition (SVD)

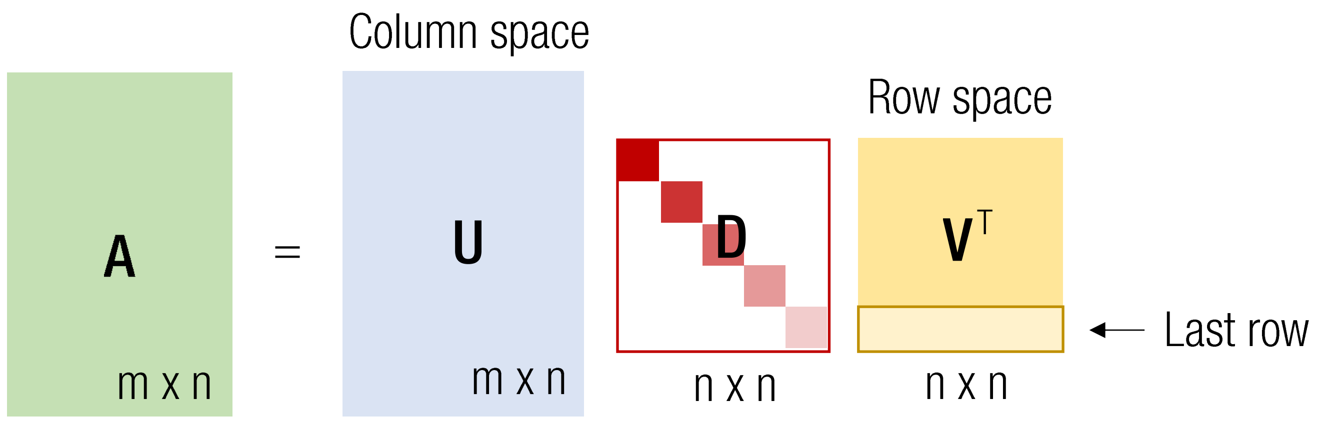
eqs > # unknowns

There exist no nullspace of **A**.

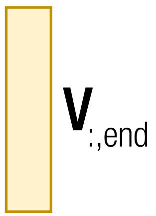


m > n

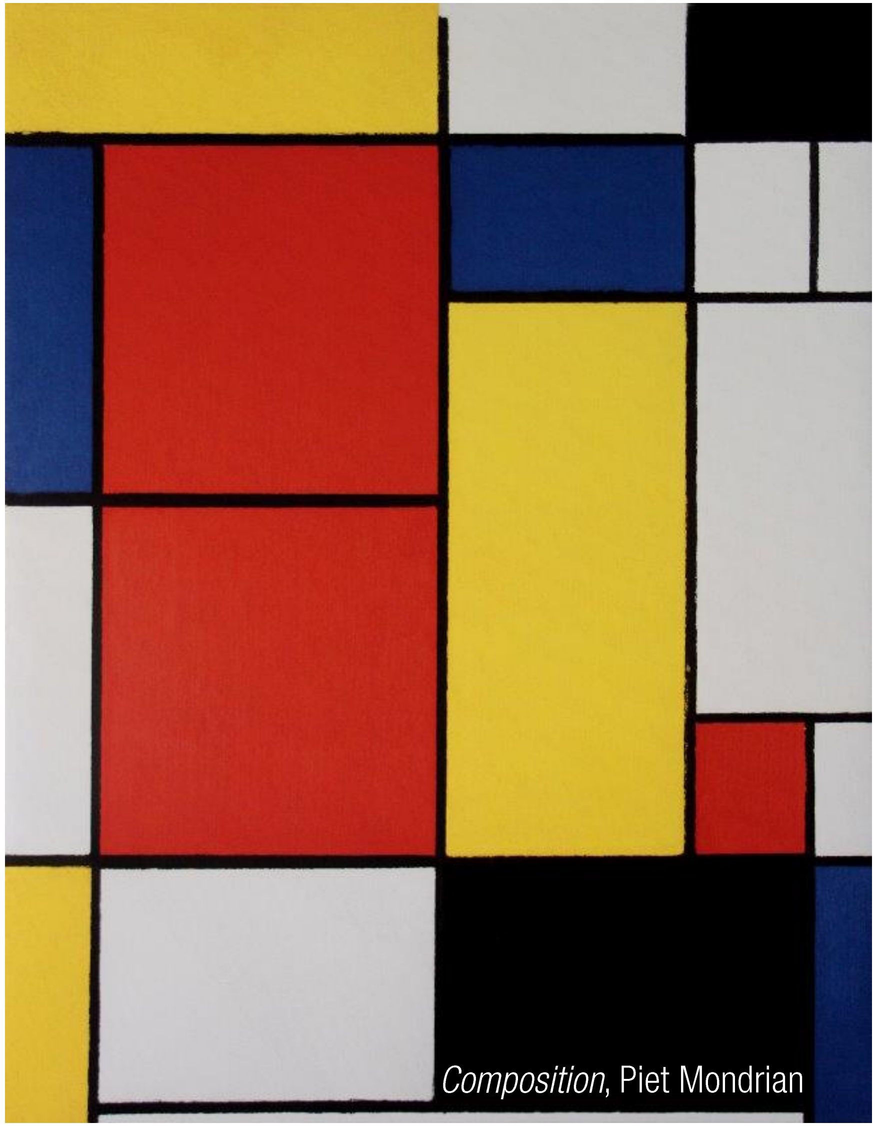
$$\underset{\mathbf{x}}{\text{minimize}} \|\mathbf{A}\mathbf{x}\|^2 \text{ subject to } \|\mathbf{x}\| = 1$$



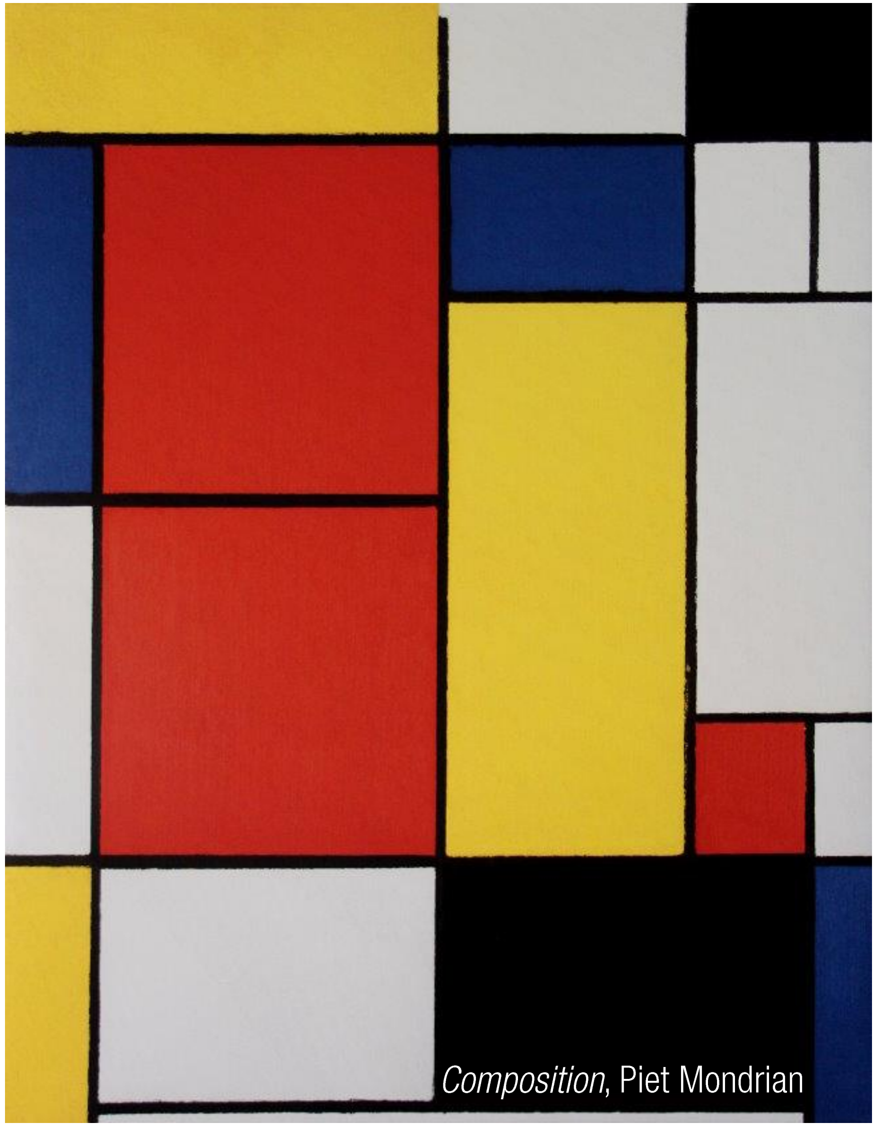
Approximated nullspace of **A**:



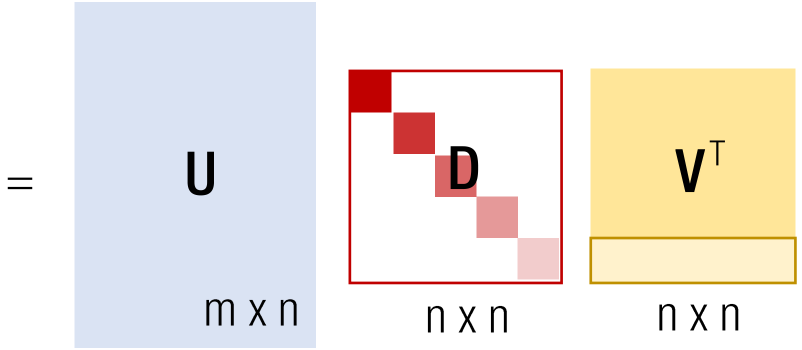
Mondrian Painting SVD


Composition, Piet Mondrian

Mondrian Painting SVD

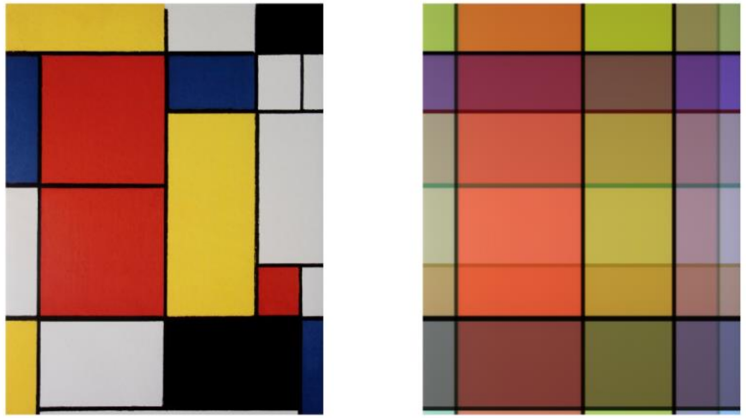


Composition, Piet Mondrian



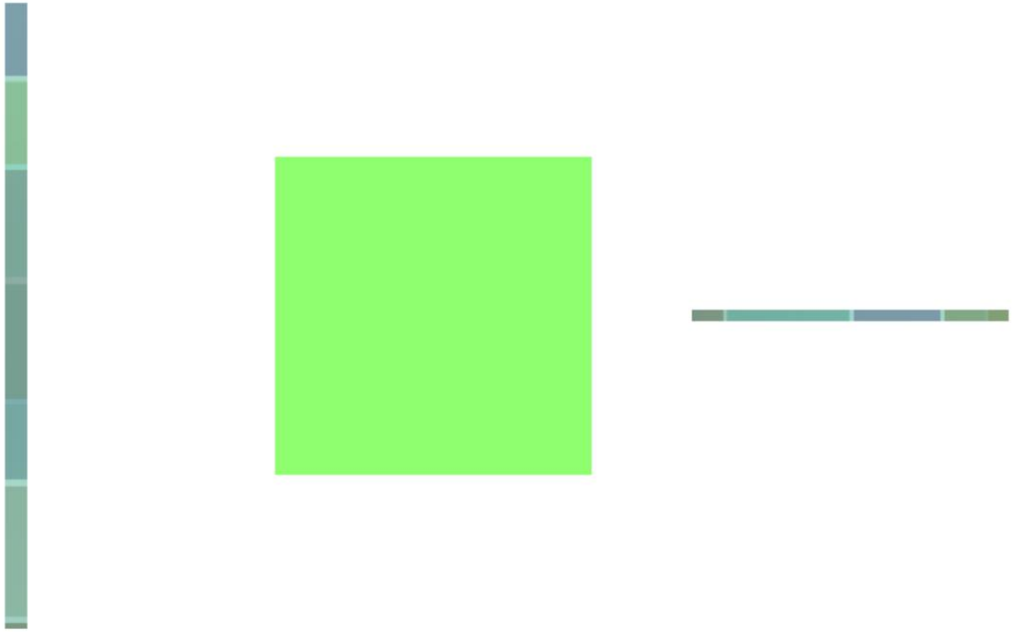
Mondrian Painting SVD Approximation

MondrianSVD.m



Ground truth

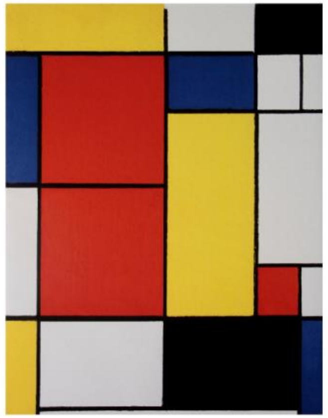
Number of basis: 1



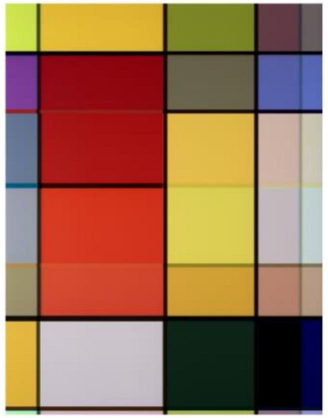
$$A = U \quad D \quad V^T$$

Mondrian Painting SVD Approximation

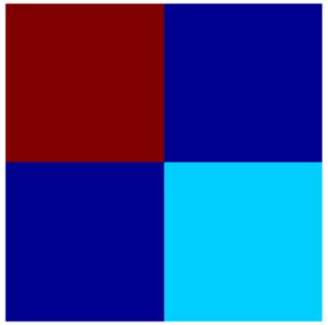
MondrianSVD.m



Ground truth



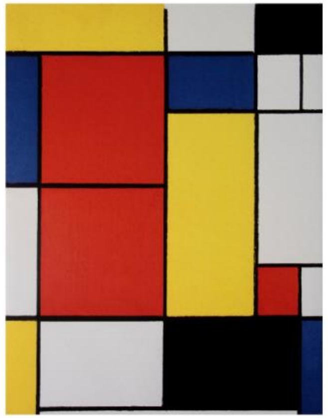
Number of basis: 2



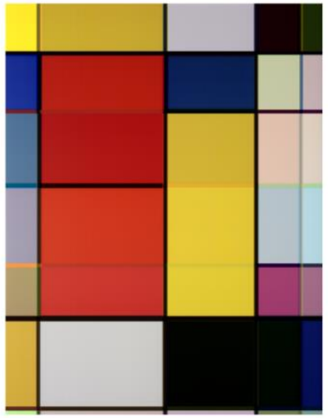
$$A = U \quad D \quad V^T$$

Mondrian Painting SVD Approximation

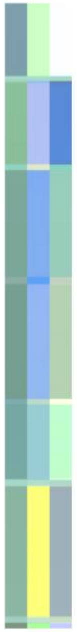
MondrianSVD.m

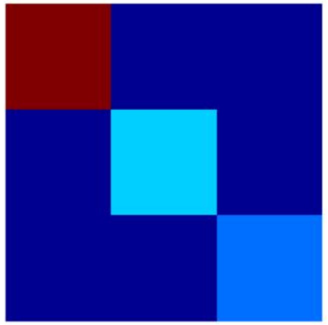


Ground truth



Number of basis: 3

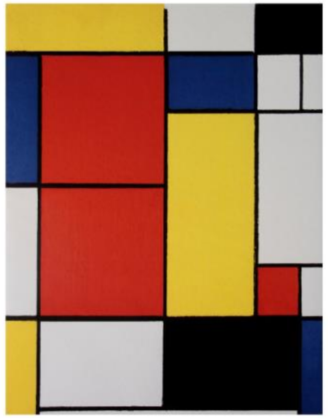




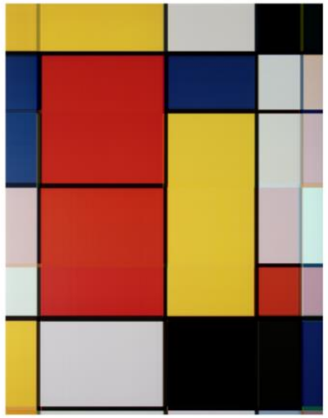
$$A = U \quad D \quad V^T$$

Mondrian Painting SVD Approximation

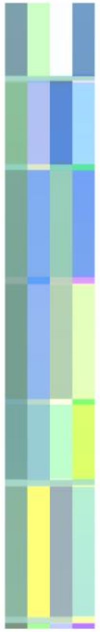
MondrianSVD.m

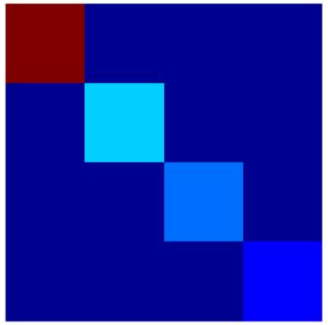


Ground truth



Number of basis: 4

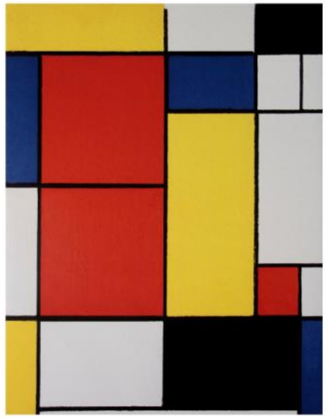




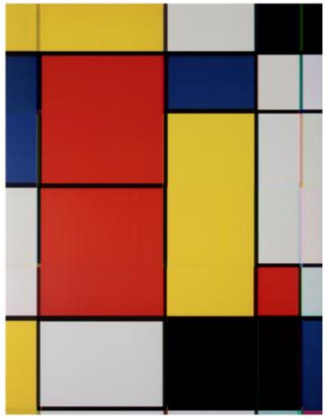
$$A = U \quad D \quad V^T$$

Mondrian Painting SVD Approximation

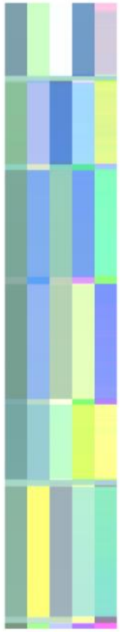
MondrianSVD.m

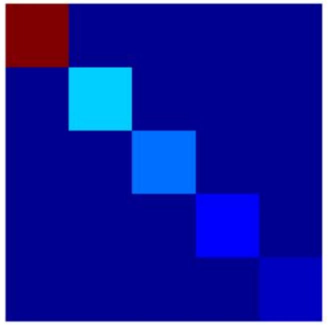


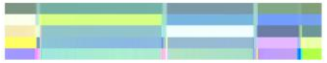
Ground truth



Number of basis: 5



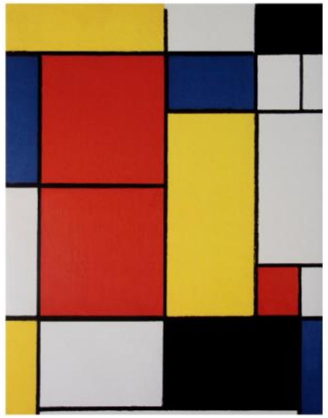




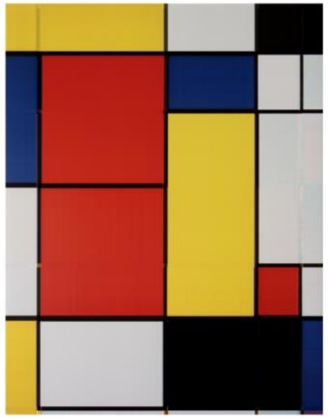
$$A = U \quad D \quad V^T$$

Mondrian Painting SVD Approximation

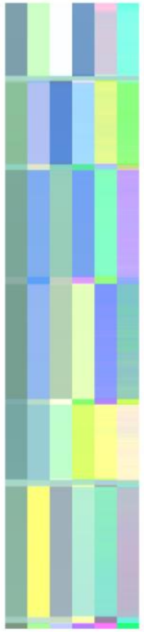
MondrianSVD.m

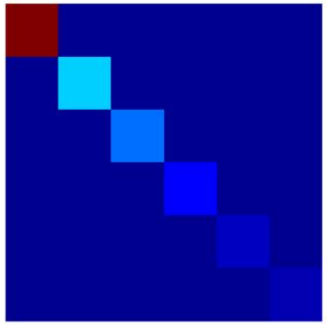


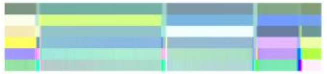
Ground truth



Number of basis: 6



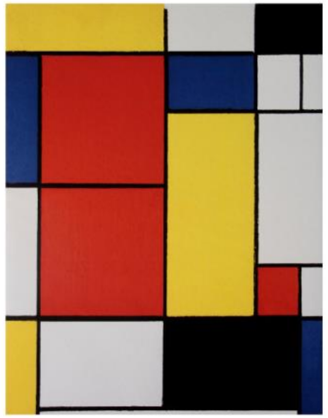


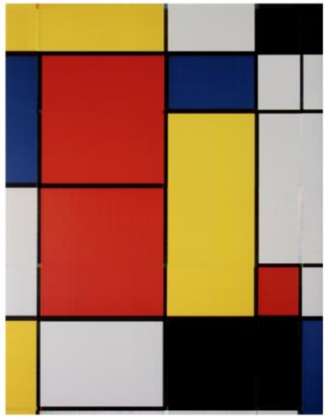


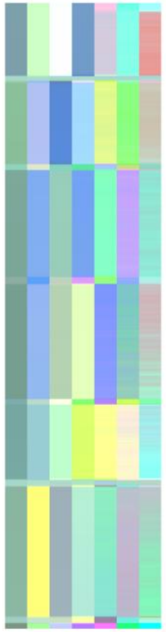
$$A = U \quad D \quad V^T$$

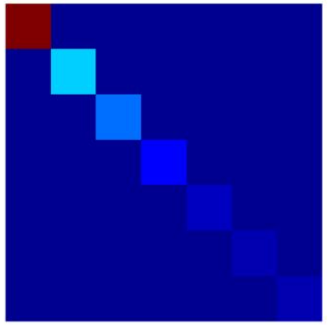
Mondrian Painting SVD Approximation

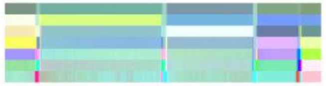
MondrianSVD.m


Ground truth


Number of basis: 7



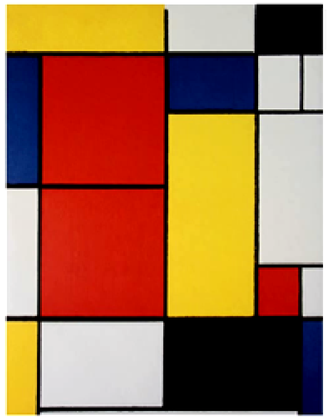




$$A = U D V^T$$

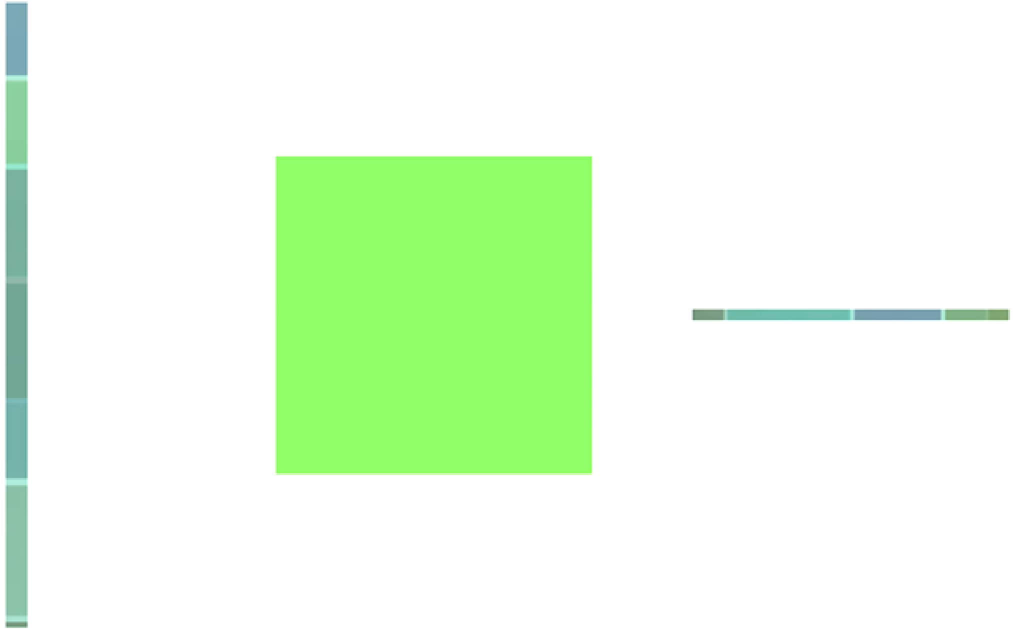
Mondrian Painting SVD Approximation

MondrianSVD.m



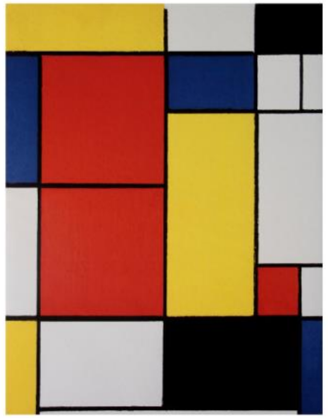
Ground truth

Number of basis: 1

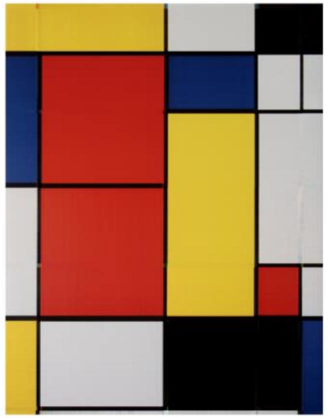


$$A = U \quad D \quad V^T$$

Reconstruction Error



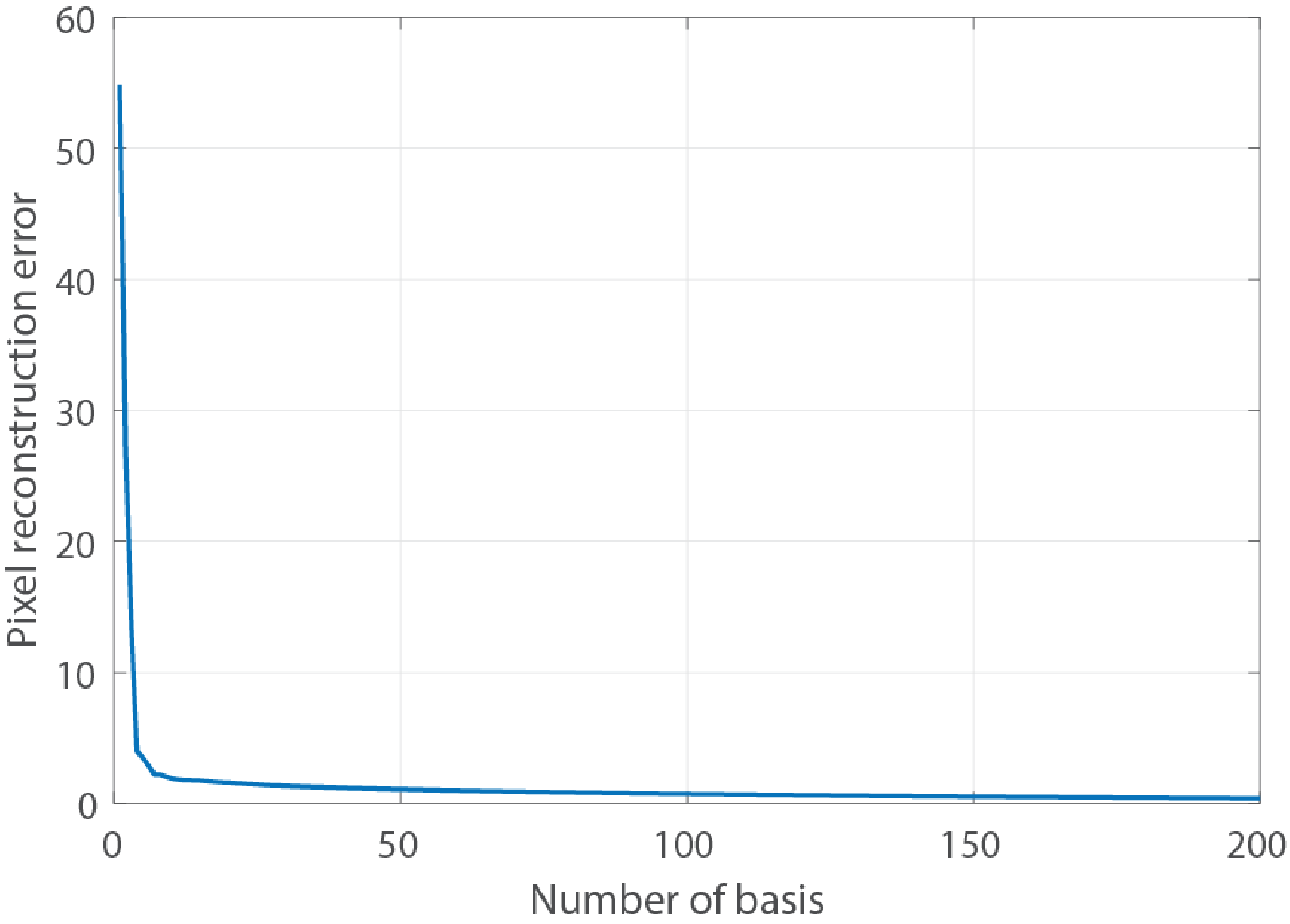
Ground truth



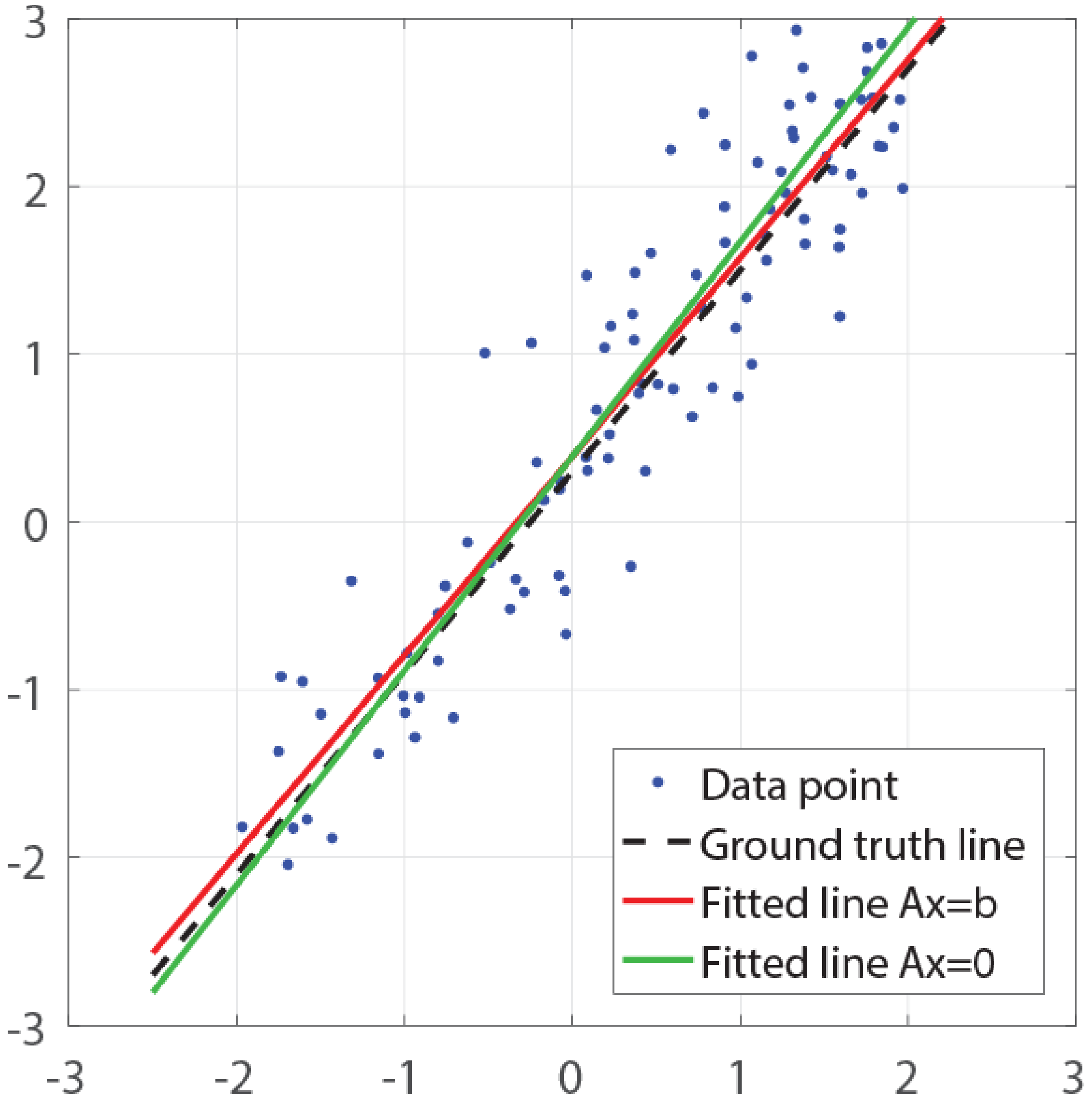
Number of basis: 7

A

MondrianSVD.m



Line Fitting (Ax=b)



Given points: $(u_1,v_1),(u_2,v_2),\cdots,(u_n,v_n)$

Find the best line: (a,b,c)

$$\begin{aligned} au_1+bv_1+c &\approx 0 \\ au_2+bv_2+c &\approx 0 \\ &\vdots \\ au_n+bv_n+c &\approx 0 \end{aligned} \longrightarrow \begin{aligned} v_1 &\approx mu_1+d \\ v_2 &\approx mu_2+d \\ &\vdots \\ v_n &\approx mu_n+d \end{aligned}$$

$$\underbrace{\begin{bmatrix} u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \\ \vdots & \vdots & 1 \\ u_n & v_n & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\mathbf{x}} \approx \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\mathbf{0}} \qquad \underbrace{\begin{bmatrix} u_1 & 1 \\ u_2 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} m \\ d \end{bmatrix}}_{\mathbf{x}} \approx \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}}_{\mathbf{b}}$$

How to compute homography?

Homography Computation

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Homography Computation

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\rightarrow \quad h_{11}u_x + h_{12}u_y + h_{13} + h_{31}u_x v_x + h_{32}u_y v_x + h_{33}v_x = 0$$

$$h_{21}u_x + h_{22}u_y + h_{23} + h_{31}u_x v_y + h_{32}u_y v_y + h_{33}v_y = 0$$

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Unknowns: $h_{11}, \cdots, h_{33}$

Equations: 2 per correspondence

Homography Computation

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\rightarrow \quad \begin{aligned} h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_x v_x - h_{32}u_y v_x - h_{33}v_x &= 0 \\ h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_x v_y - h_{32}u_y v_y - h_{33}v_y &= 0 \end{aligned}$$

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} u_x & u_y & 1 & & & & -u_x v_x & -u_y v_x & -v_x \\ & & & u_x & u_y & 1 & -u_x v_y & -u_y v_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Homography Computation

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$\rightarrow \quad h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0$$

$$\rightarrow \begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & 1 & -u_xv_y & -u_yv_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{0}$$

2x9

Homography Computation

How many correspondences are needed?

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & 1 & -u_xv_y & -u_yv_y & -v_y \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\mathbf{x} = \mathbf{0}$$

2x9

Homography Computation

How many correspondences are needed? 4

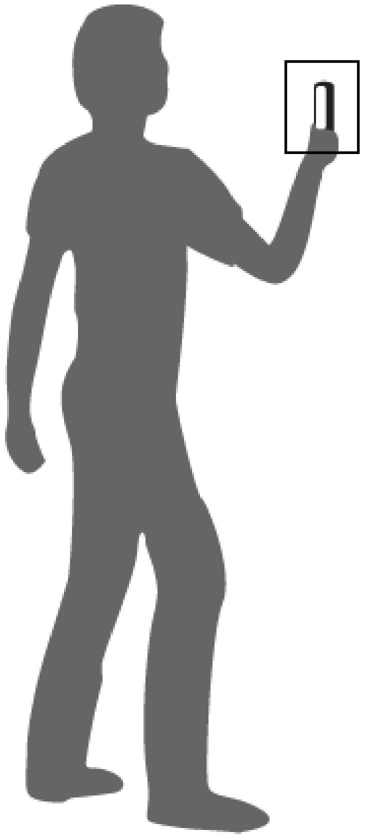
$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} u_x & u_y & 1 & & & & -u_xv_x & -u_yv_x & -v_x \\ & & & u_x & u_y & & -u_xv_y & -u_yv_y & -v_y \end{bmatrix}}_{\mathbf{A}\ (8\text{x}9)} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

$$\mathbf{x} = \mathbf{V}_{:end} = \text{null}(\mathbf{A})$$

Camera Calibration

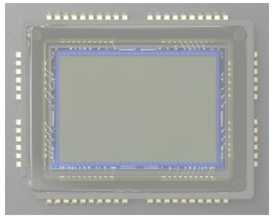
Camera Intrinsic Parameter



Pixel space Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

+

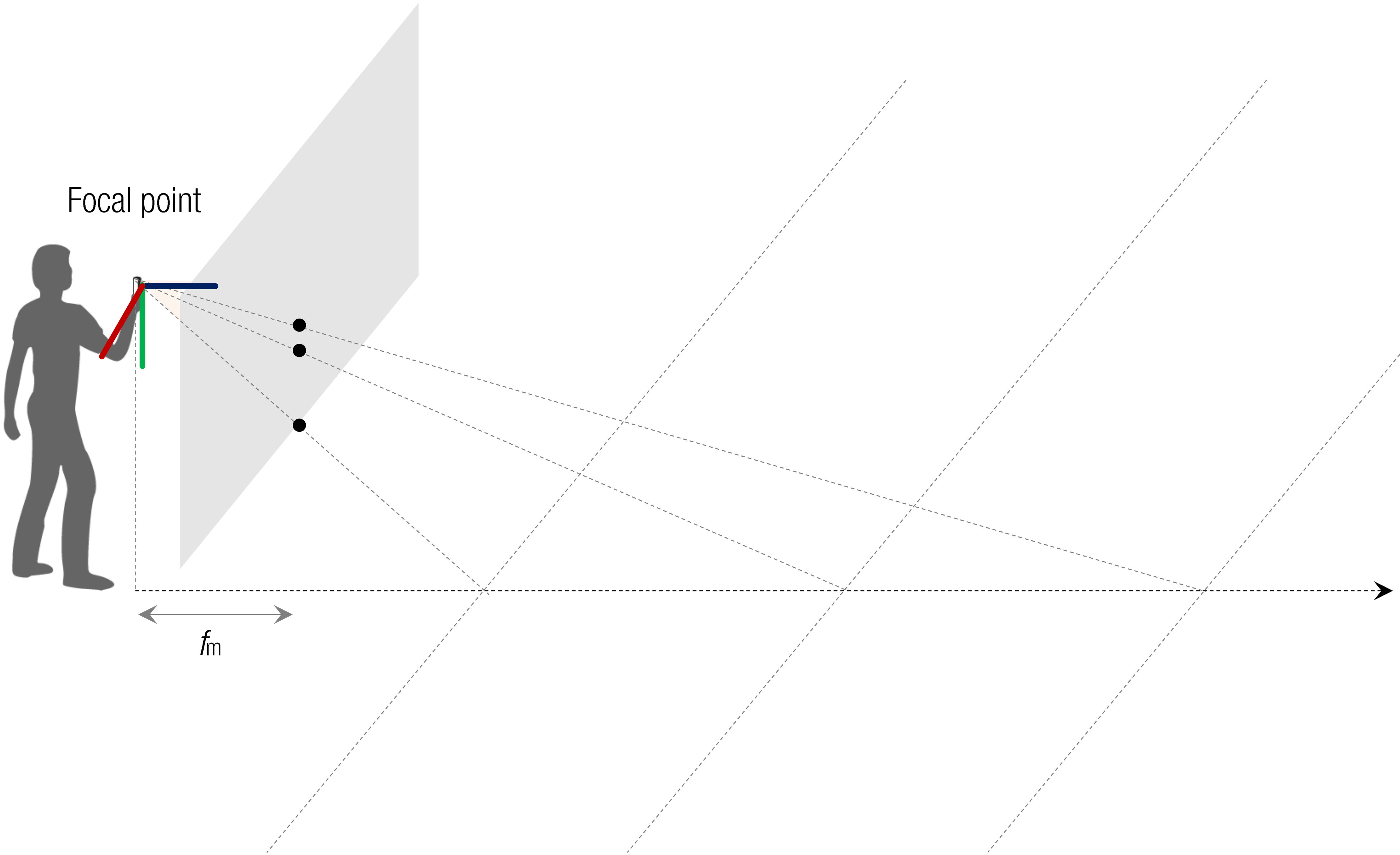


Camera intrinsic parameter
: metric space to pixel space

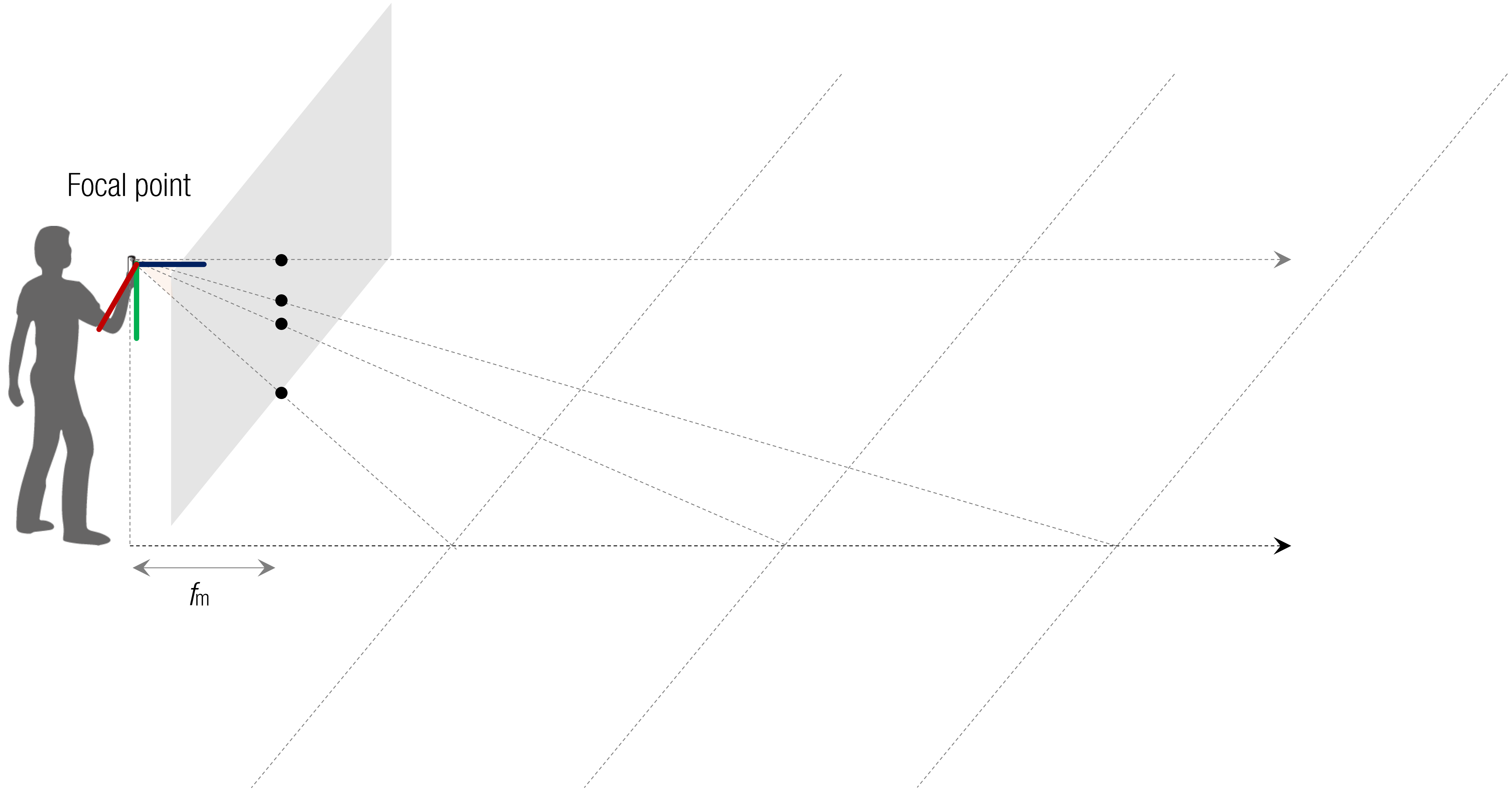
Physical Focal Point

Physical Focal Point

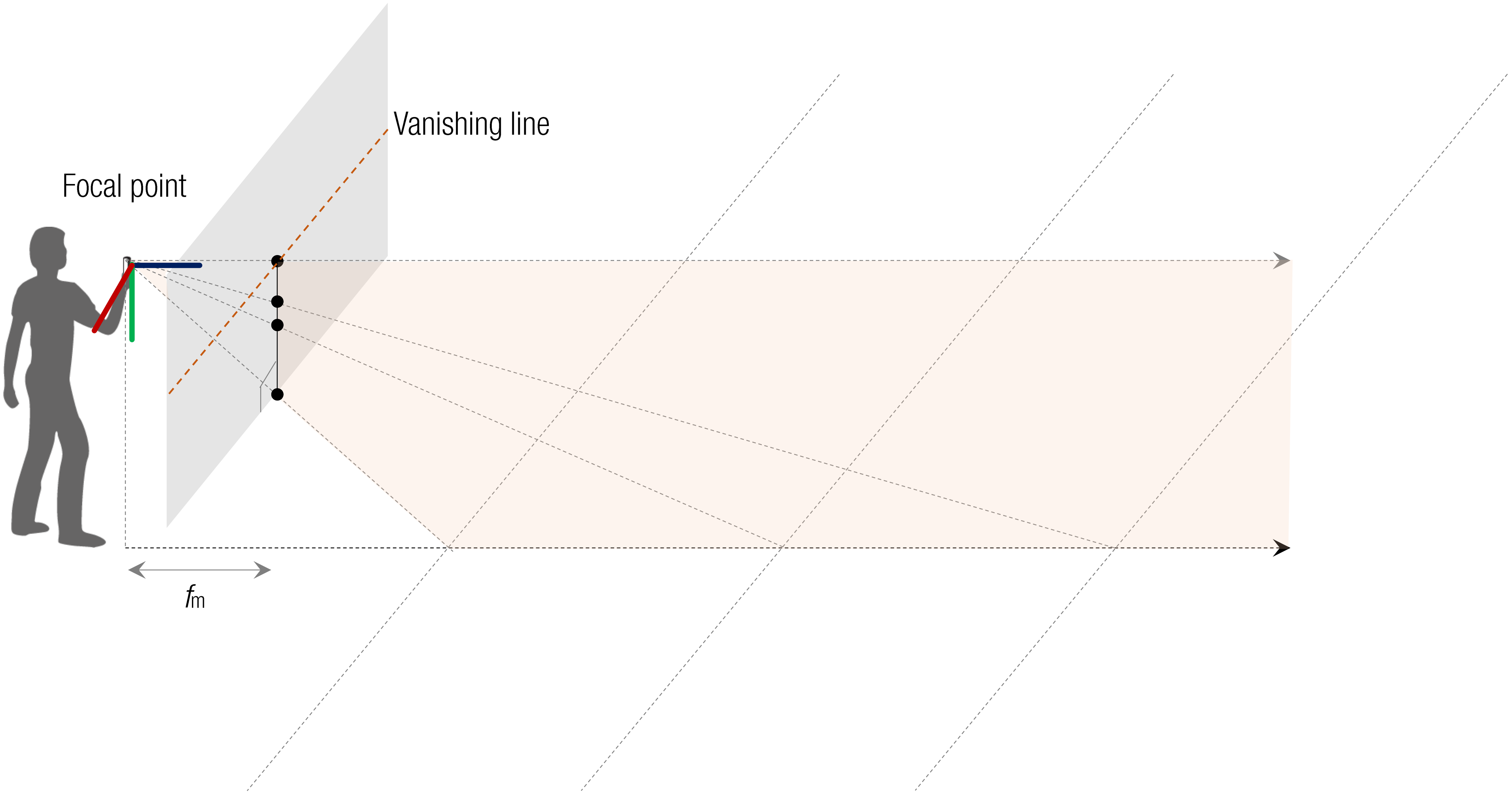
Physical Focal Point



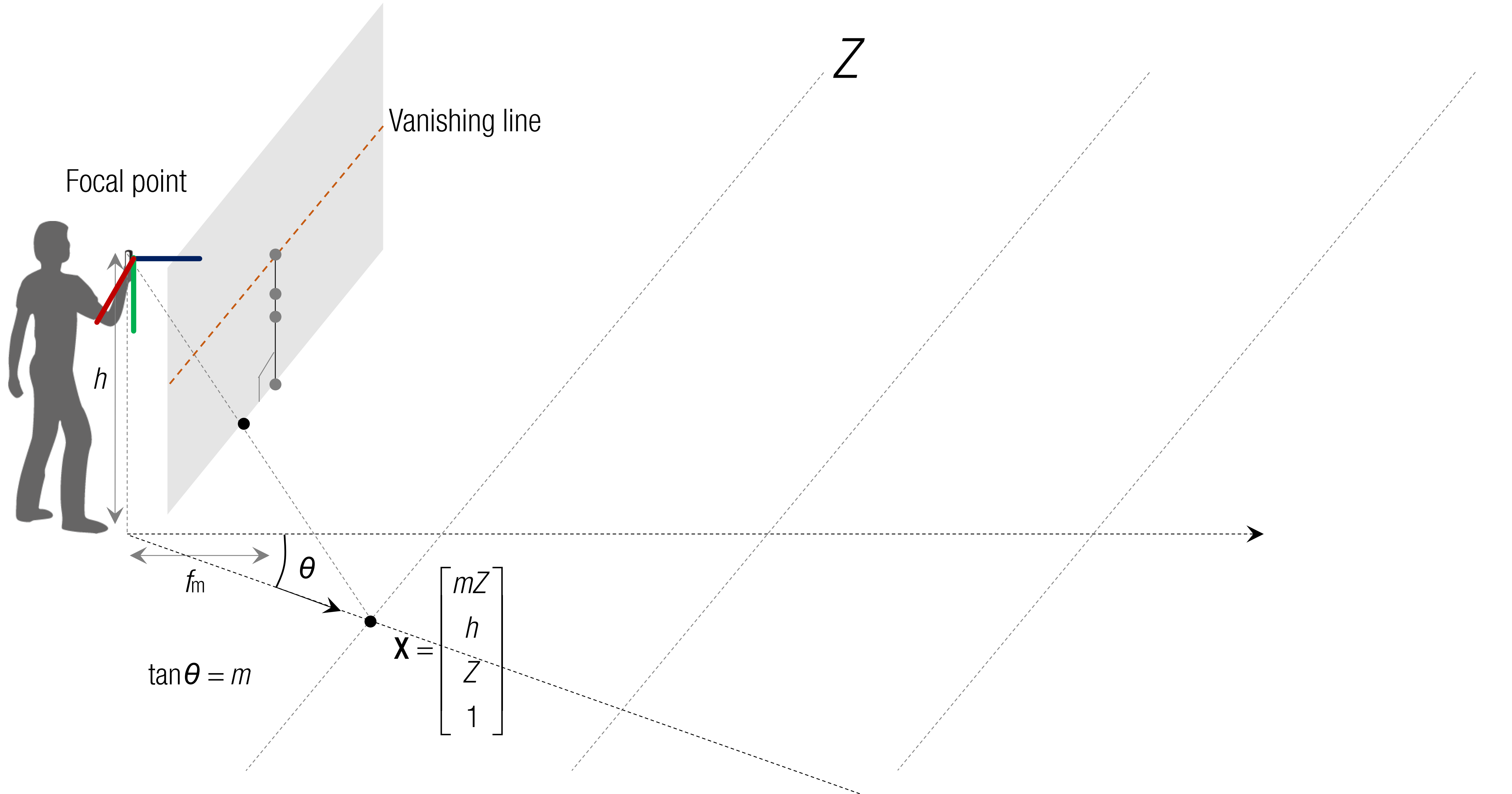
Physical Focal Point



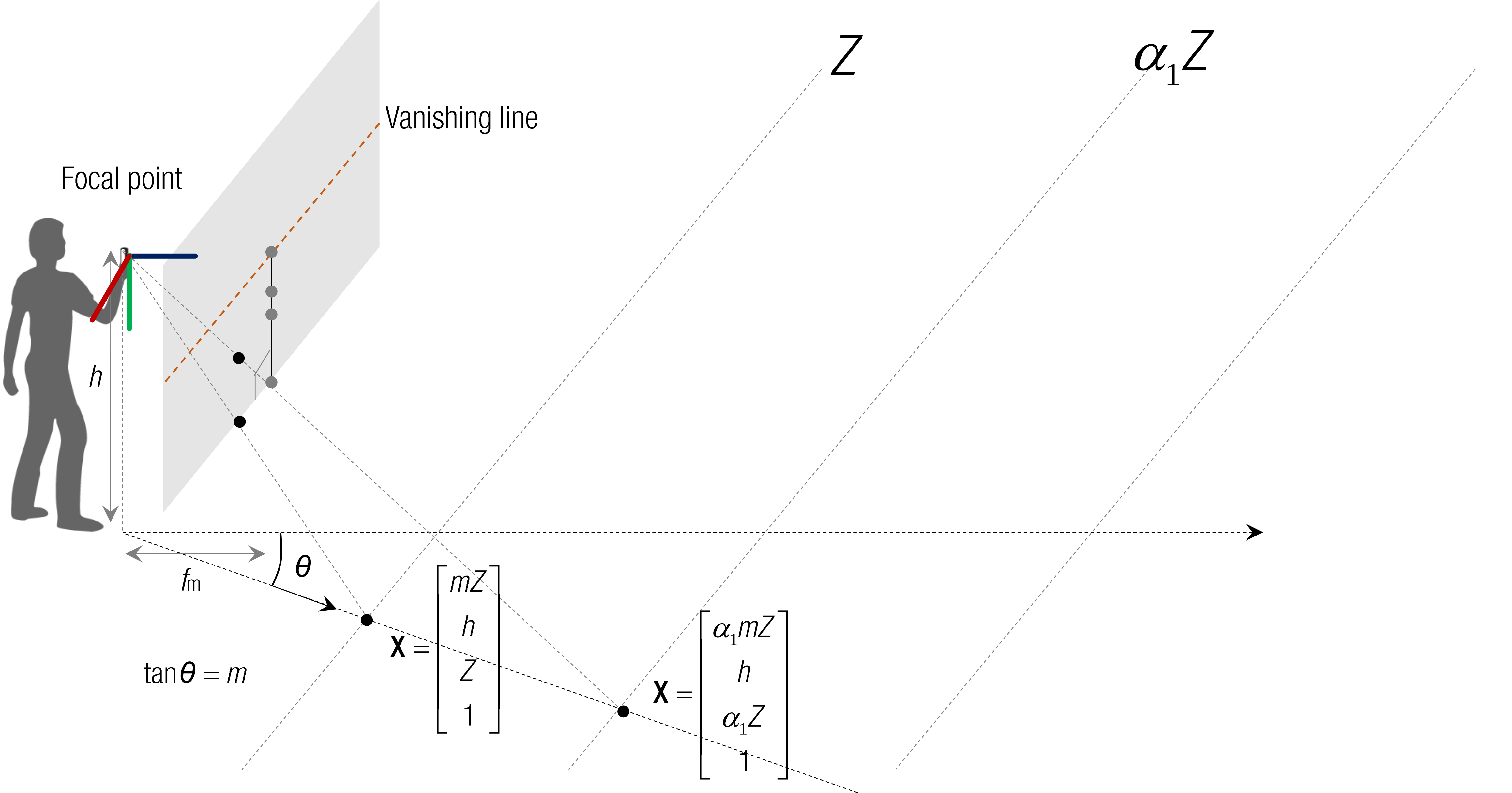
Physical Focal Point



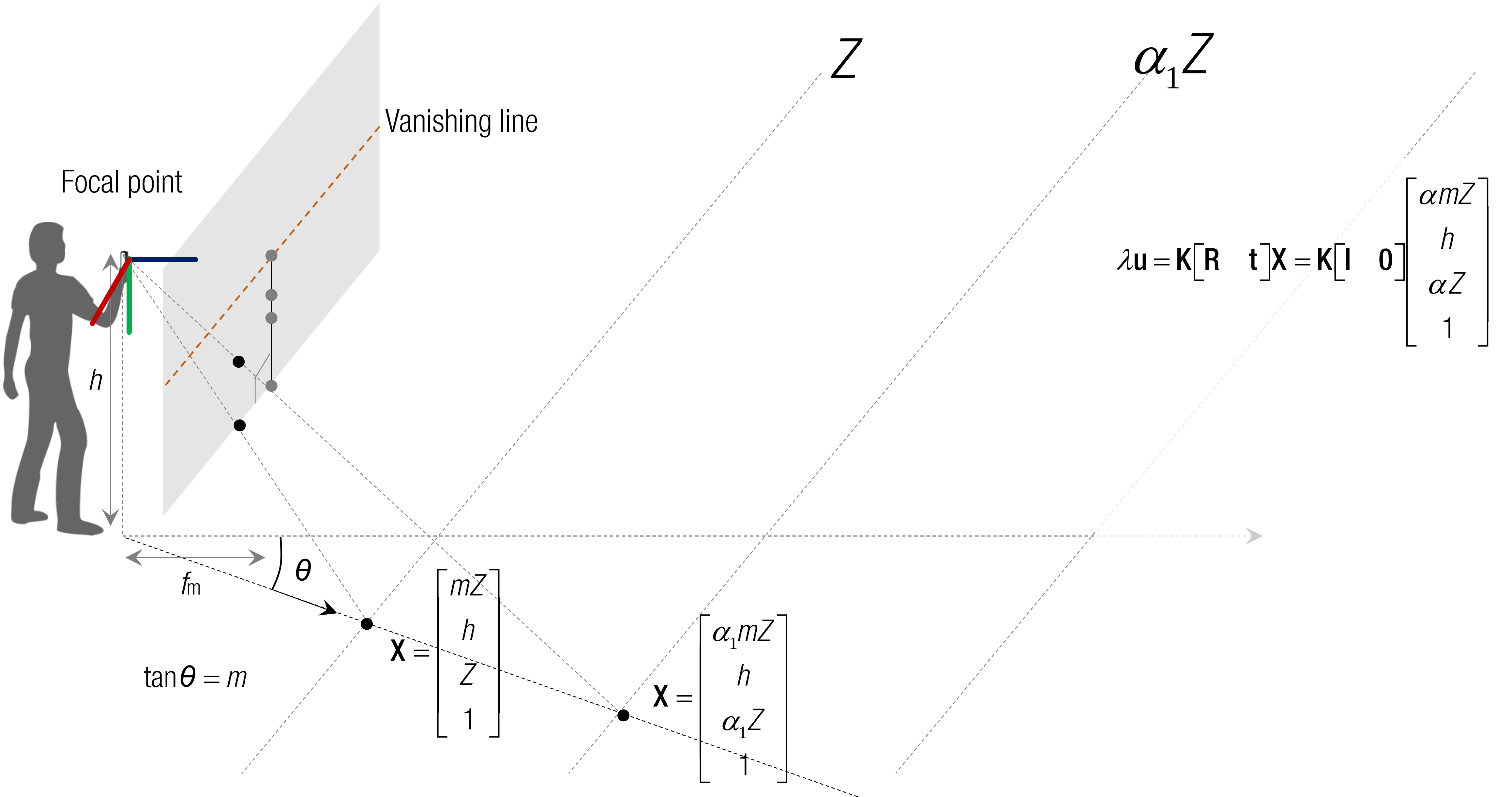
Physical Focal Point



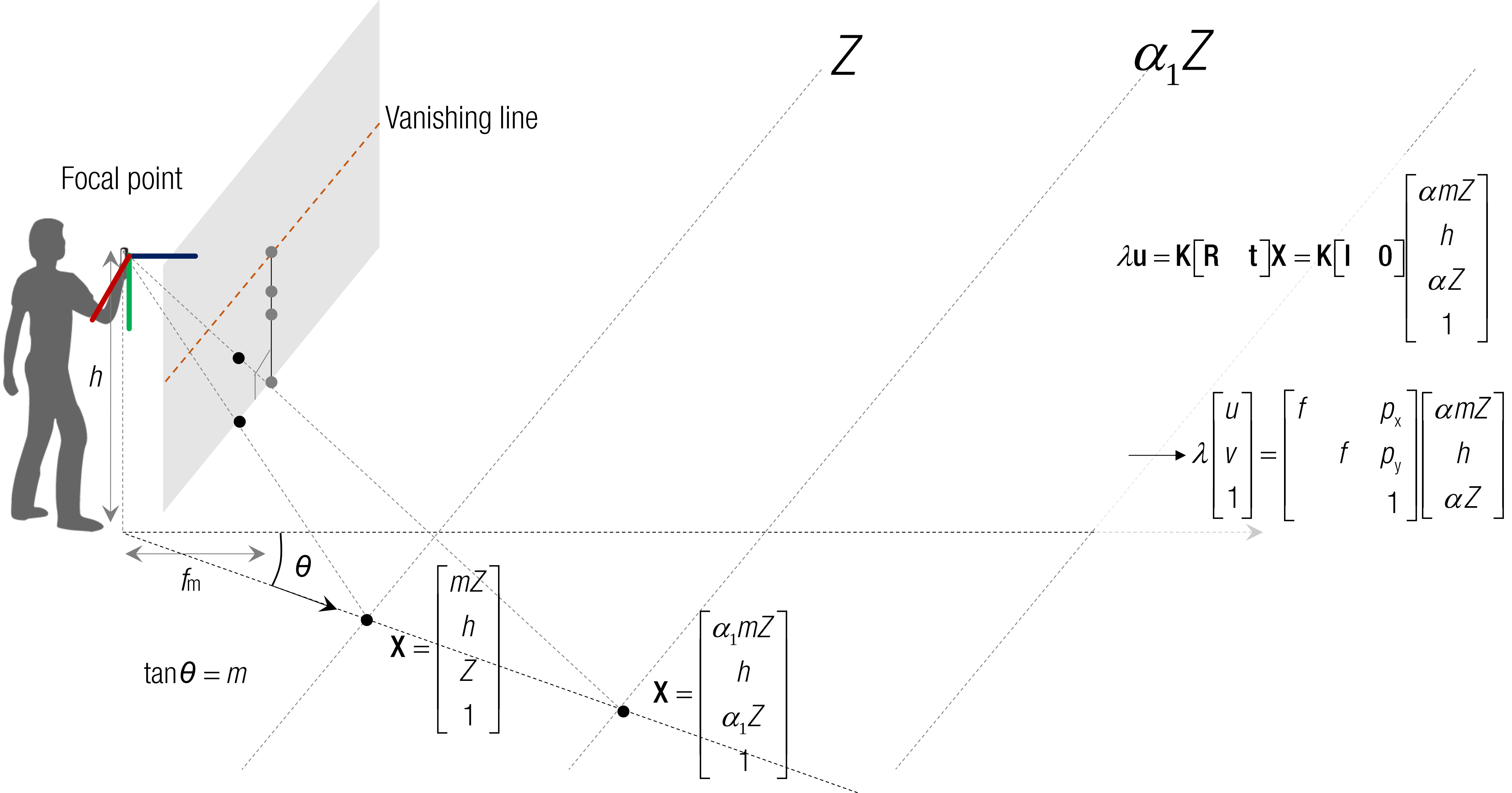
Physical Focal Point



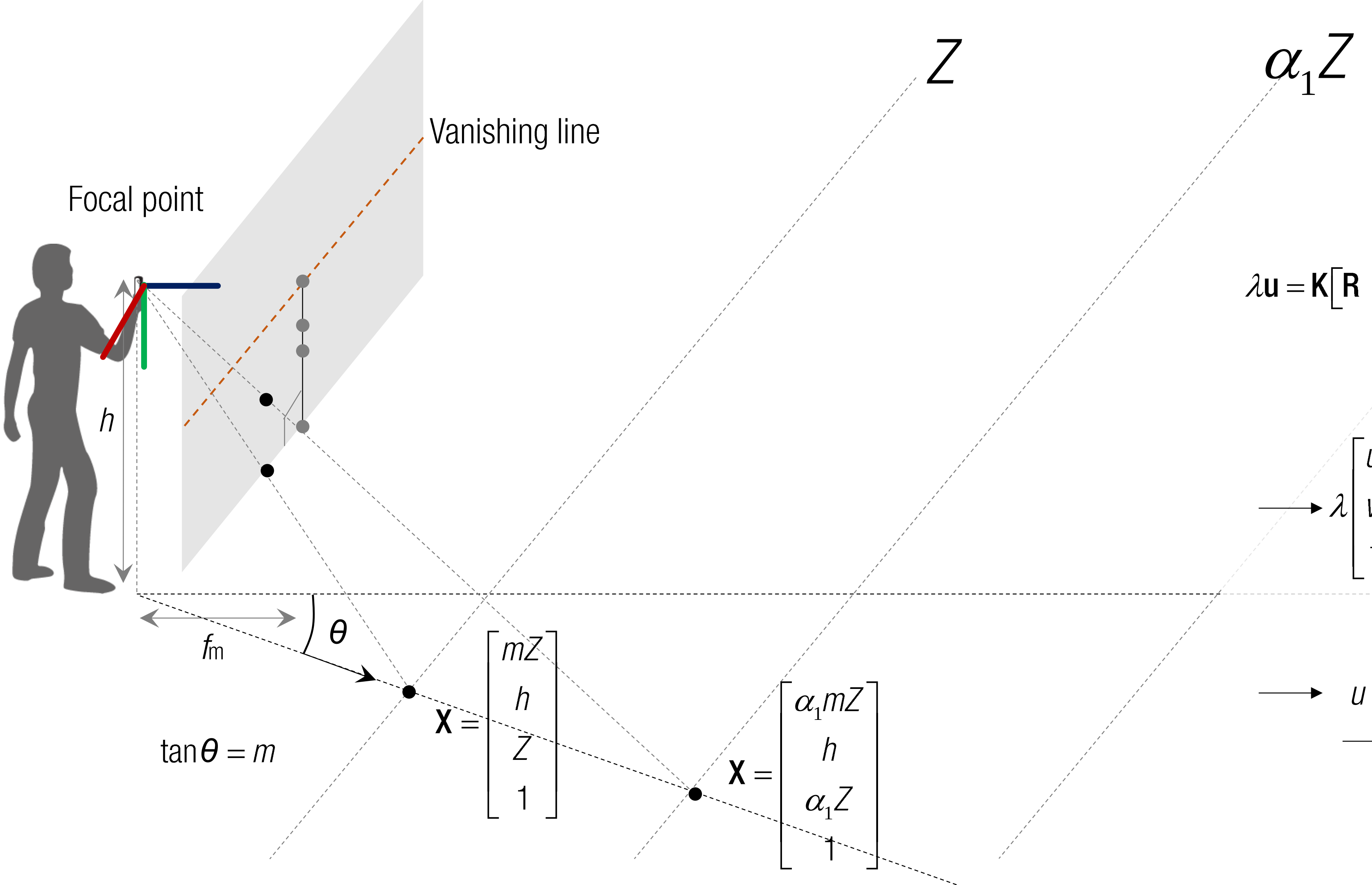
Physical Focal Point



Physical Focal Point



Physical Focal Point



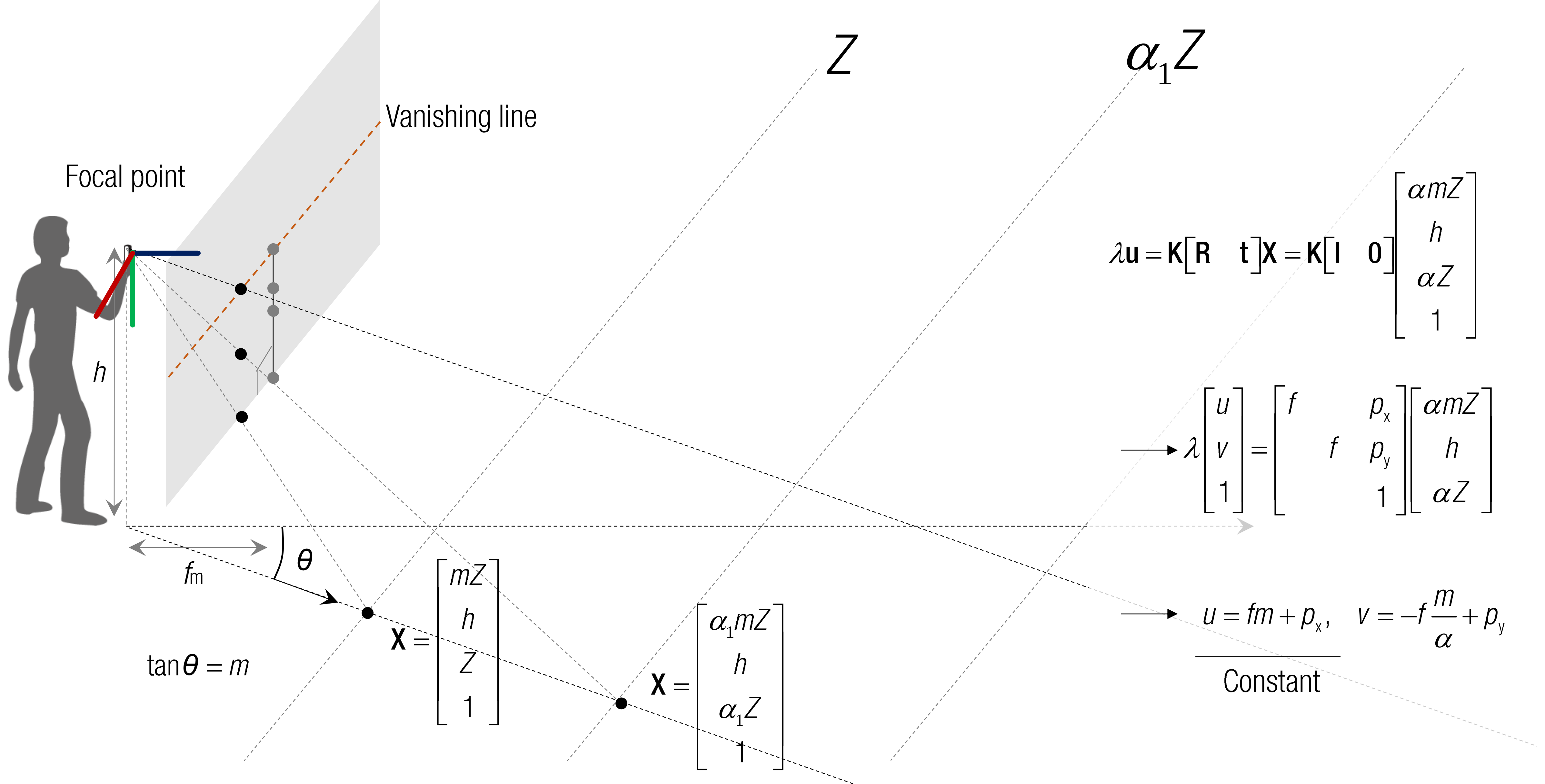
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z \\ 1\end{bmatrix}$$

$$\rightarrow \lambda\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}f & & p_x \\ & f & p_y \\ & & 1\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z\end{bmatrix}$$

$$\rightarrow u = fm + p_x, \quad v = -f\frac{m}{\alpha} + p_y$$

Constant

Physical Focal Point



$$\tan\theta = m$$

$$\mathbf{X} = \begin{bmatrix} mZ \\ h \\ Z \\ 1 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} \alpha_1 mZ \\ h \\ \alpha_1 Z \\ 1 \end{bmatrix}$$

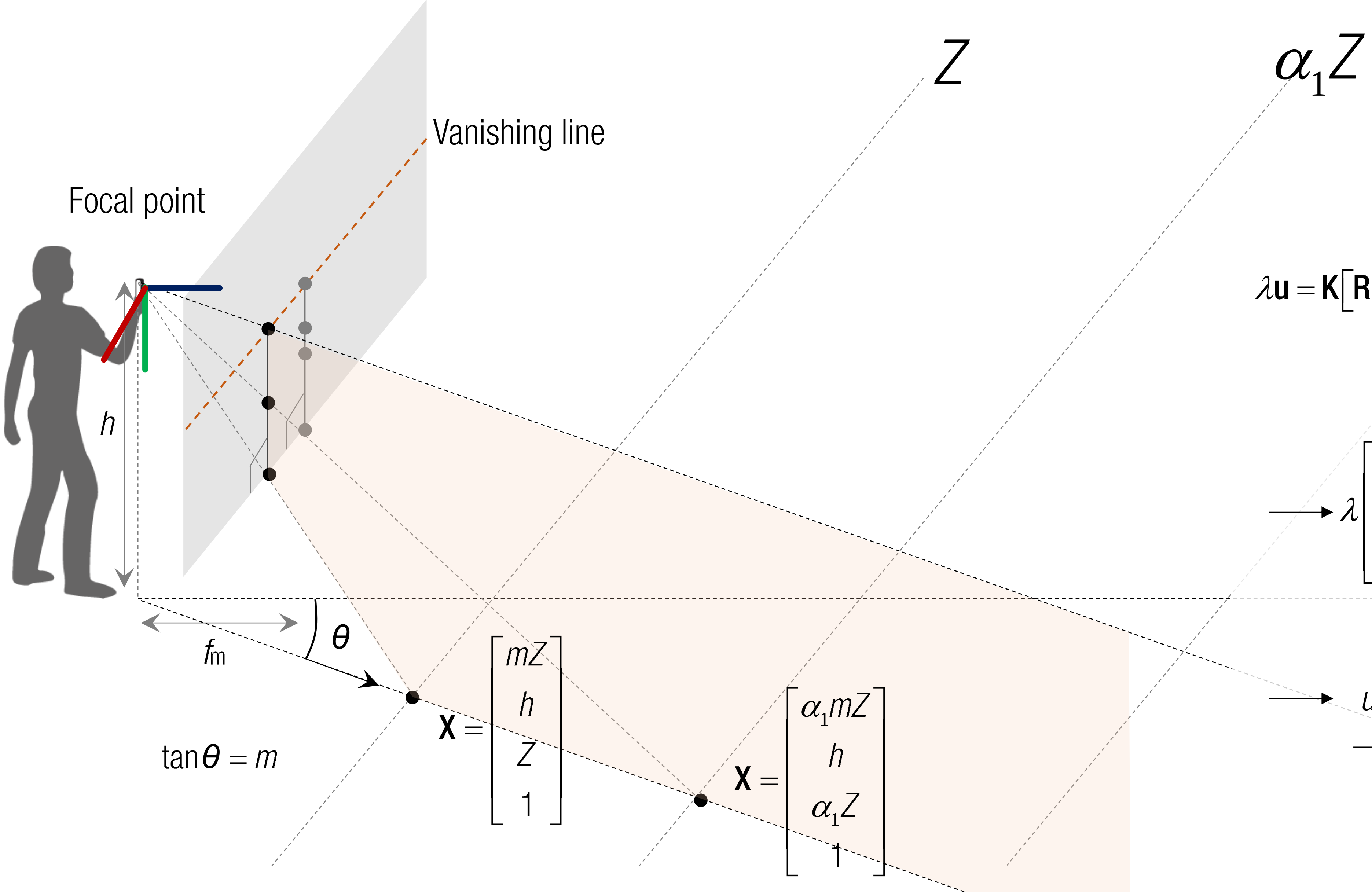
$$\lambda\mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \\ 1 \end{bmatrix}$$

$$\rightarrow \lambda\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \end{bmatrix}$$

$$\rightarrow u = fm + p_x, \quad v = -f\frac{m}{\alpha} + p_y$$

Constant

Physical Focal Point



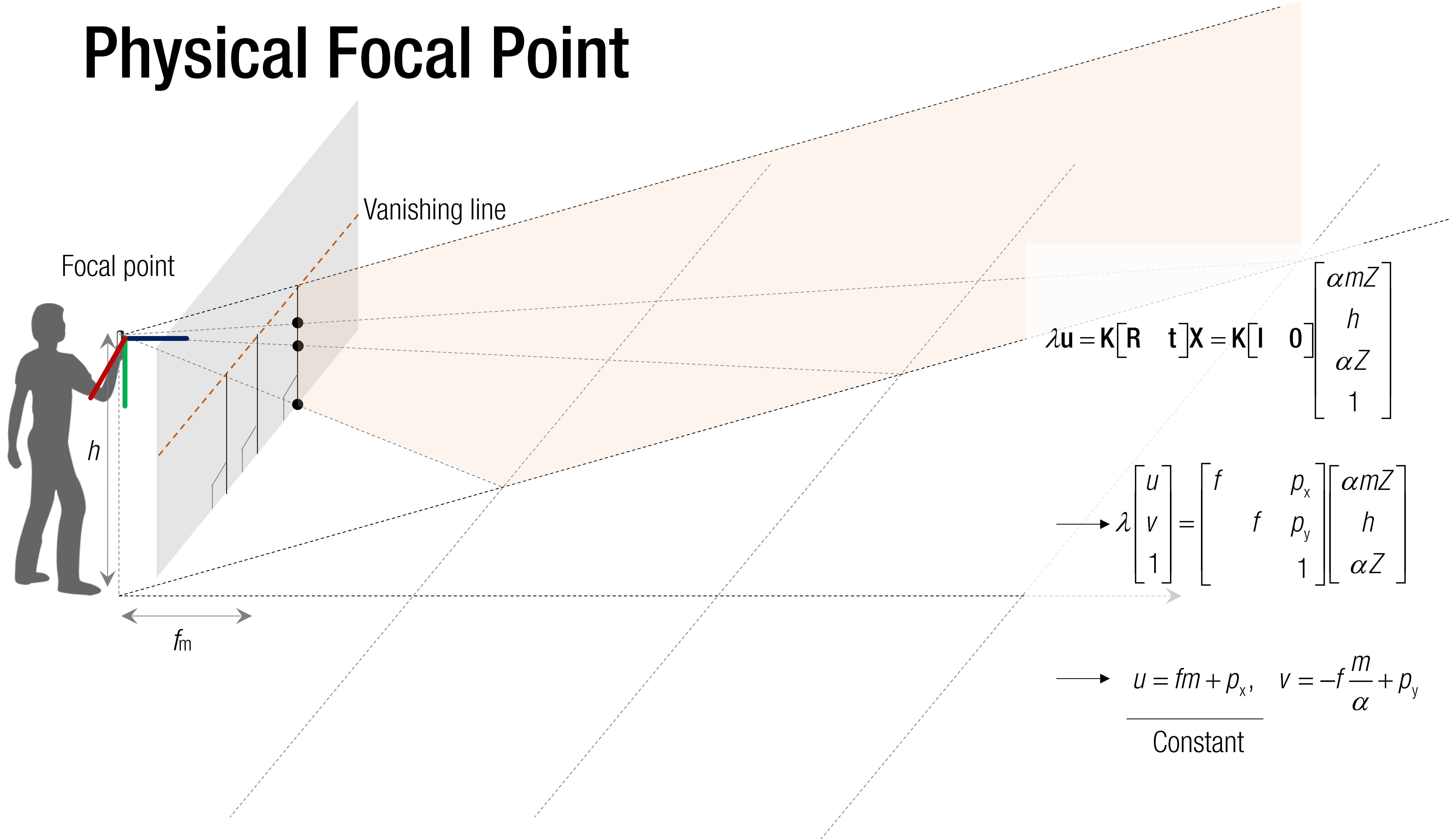
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \\ 1 \end{bmatrix}$$

$$\rightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \alpha mZ \\ h \\ \alpha Z \end{bmatrix}$$

$$\rightarrow u = fm + p_x, \quad v = -f\frac{m}{\alpha} + p_y$$

Constant

Physical Focal Point

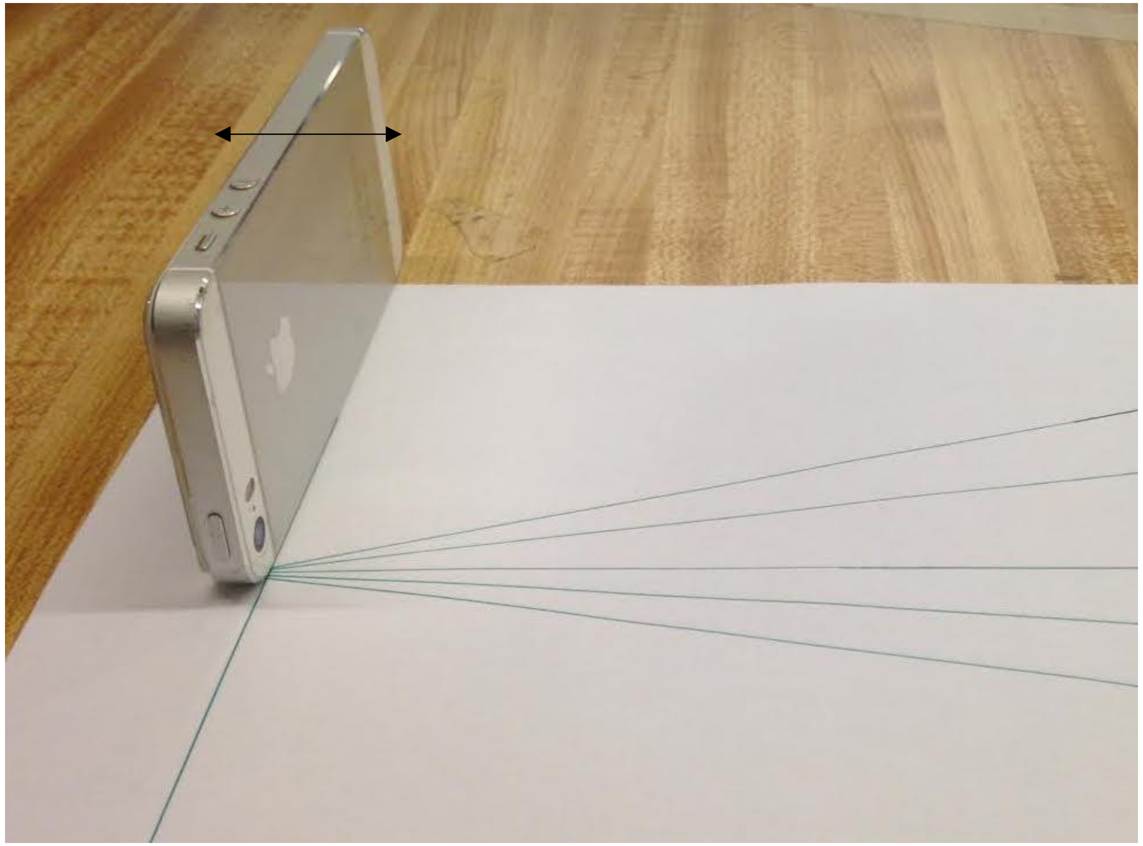


$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z \\ 1\end{bmatrix}$$

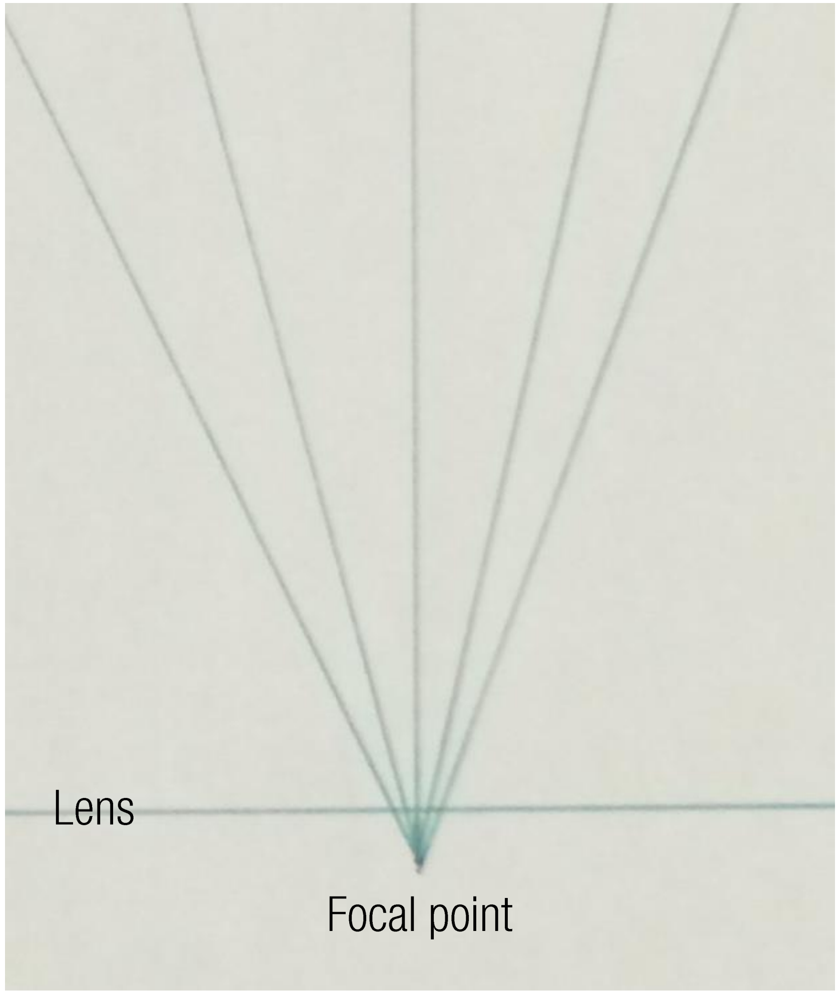
$$\rightarrow \lambda\begin{bmatrix}u \\ v \\ 1\end{bmatrix} = \begin{bmatrix}f & & p_x \\ & f & p_y \\ & & 1\end{bmatrix}\begin{bmatrix}\alpha m Z \\ h \\ \alpha Z\end{bmatrix}$$

$$\rightarrow \underbrace{u = fm + p_x}_{\text{Constant}}, \quad v = -f\frac{m}{\alpha} + p_y$$

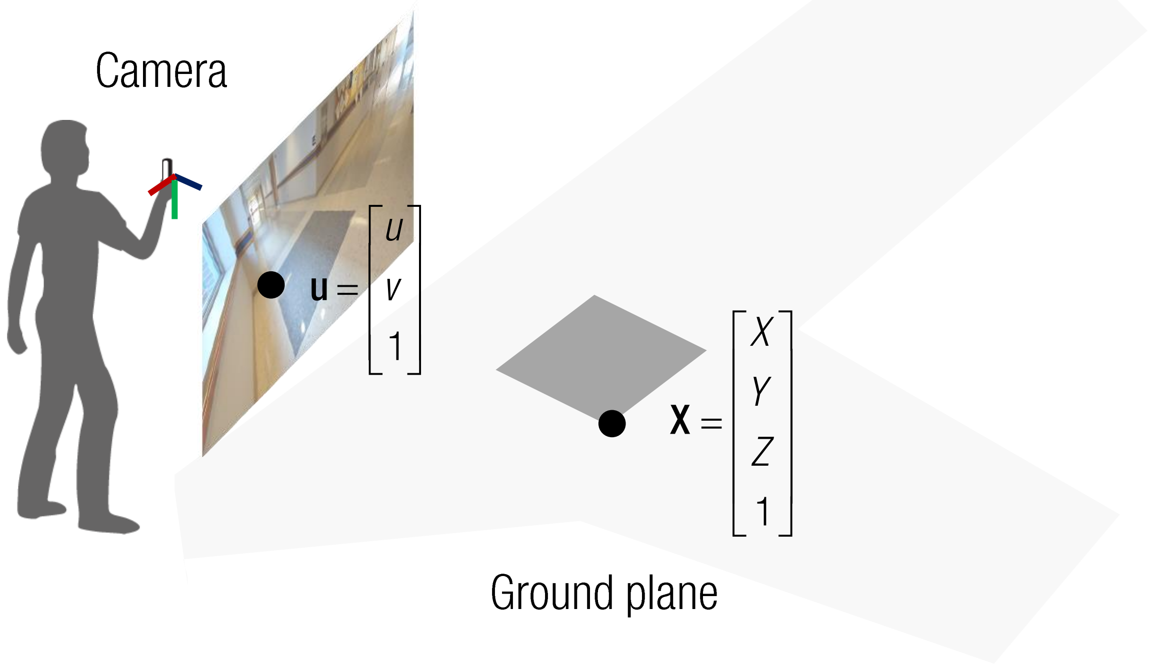
Where am I? (Focal Point)



Where am I? (Focal Point)



Camera Calibration in Pixel Space

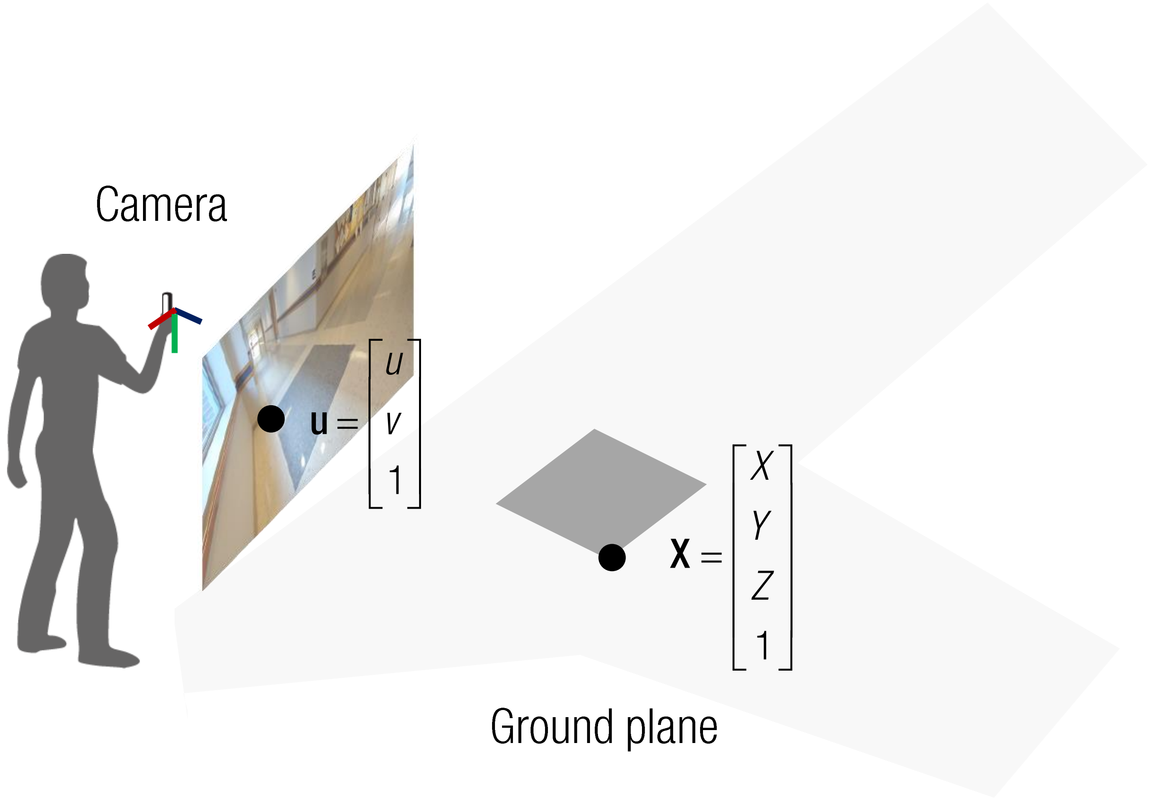


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns:

# of equations:

Camera Calibration in Pixel Space

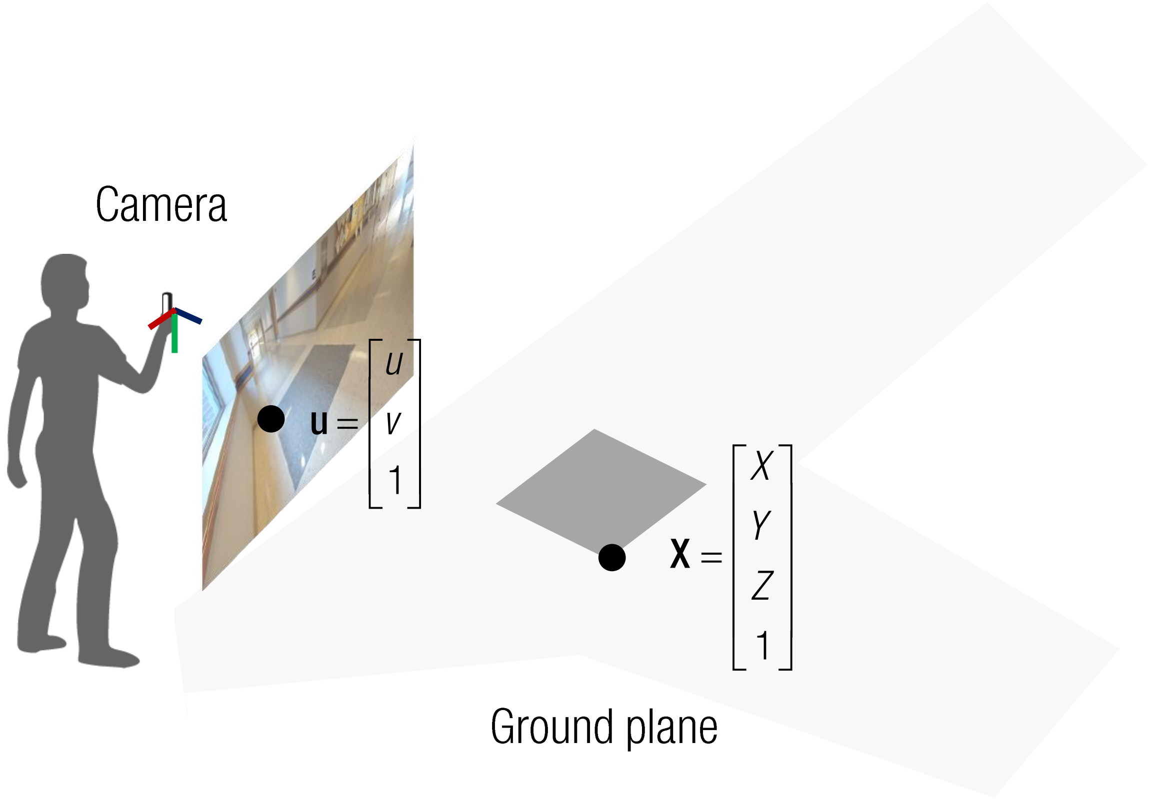


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6 (**R** and **t**) + 3 (**X**)

# of equations:

Camera Calibration in Pixel Space

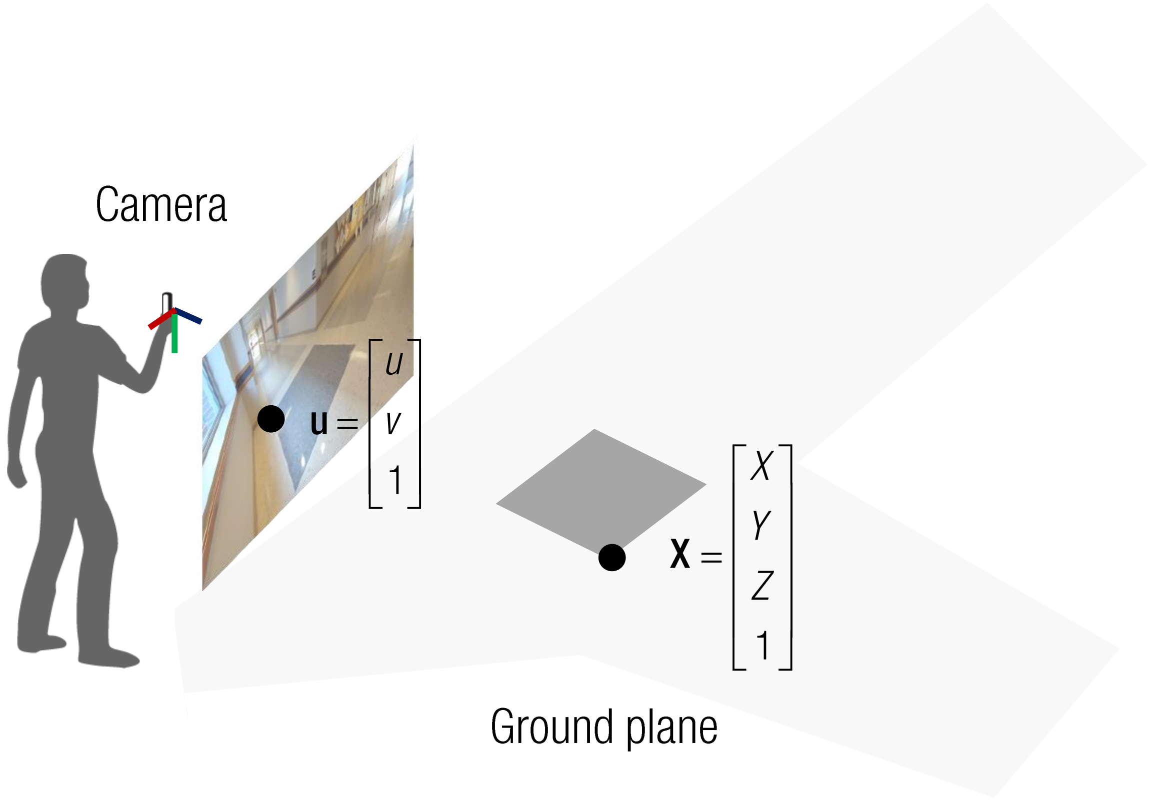


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6 (**R** and **t**) + 3 (**X**)

# of equations: 2

Camera Calibration in Pixel Space



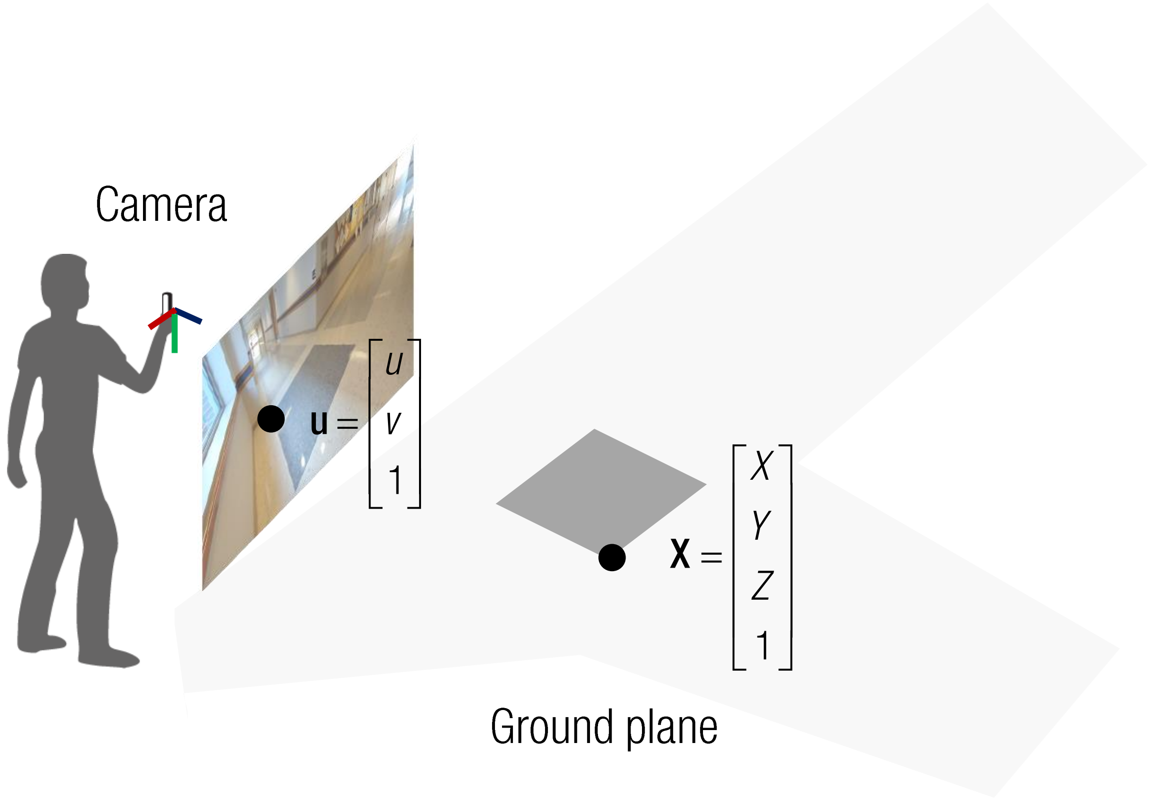
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6F (**R** and **t**) + 3P (**X**)

# of equations: 2P

where F is # of images and P is # of points.

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

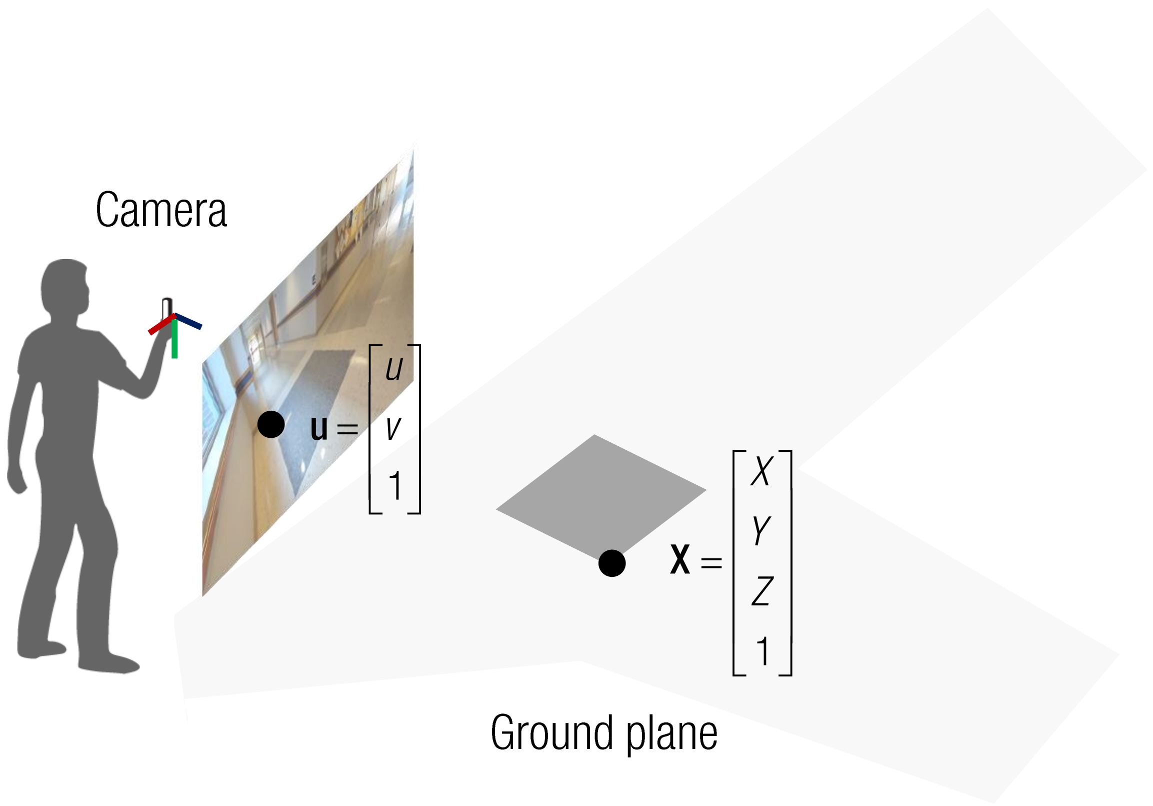
# of unknowns: 3 (**K**) + 6F (**R** and **t**) + 3P (**X**)

# of equations: 2P

where F is # of images and P is # of points.

# of unknowns > # of equations

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (**K**) + 6F (**R** and **t**) + 3P (**X**)

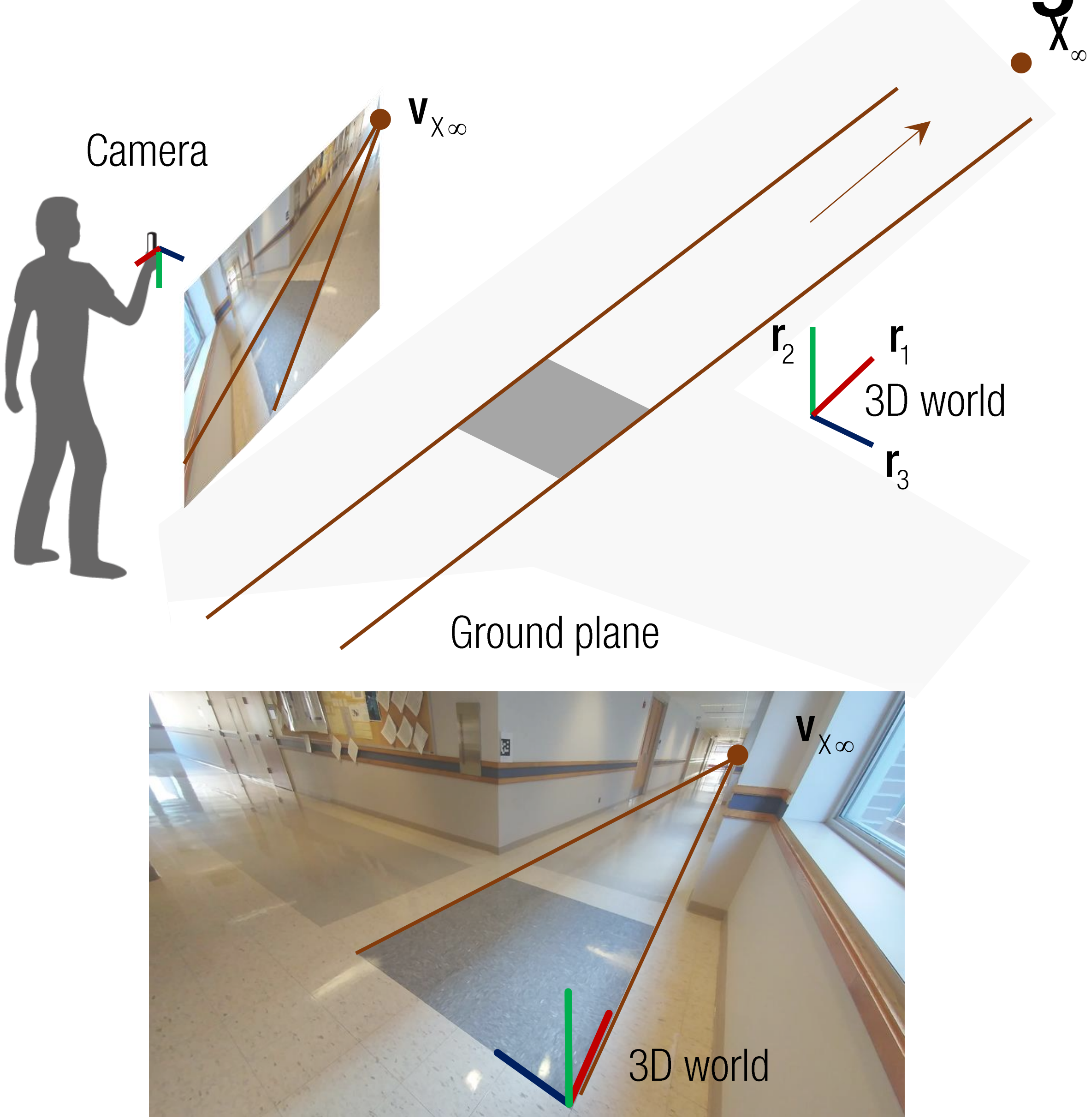
# of equations: 2P

where F is # of images and P is # of points.

# of unknowns > # of equations

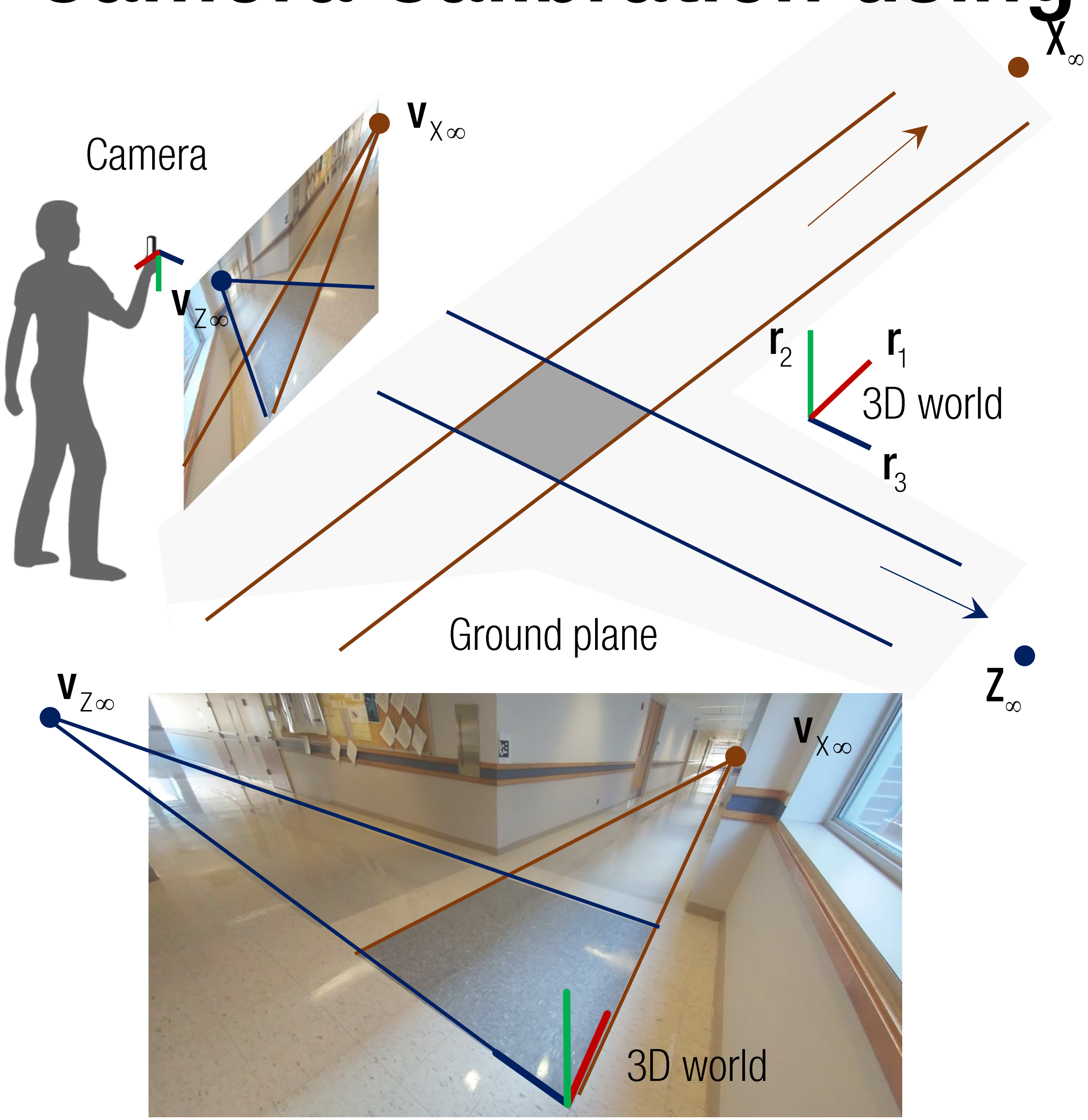
What do we know about the scene?

Camera Calibration using Vanishing Points



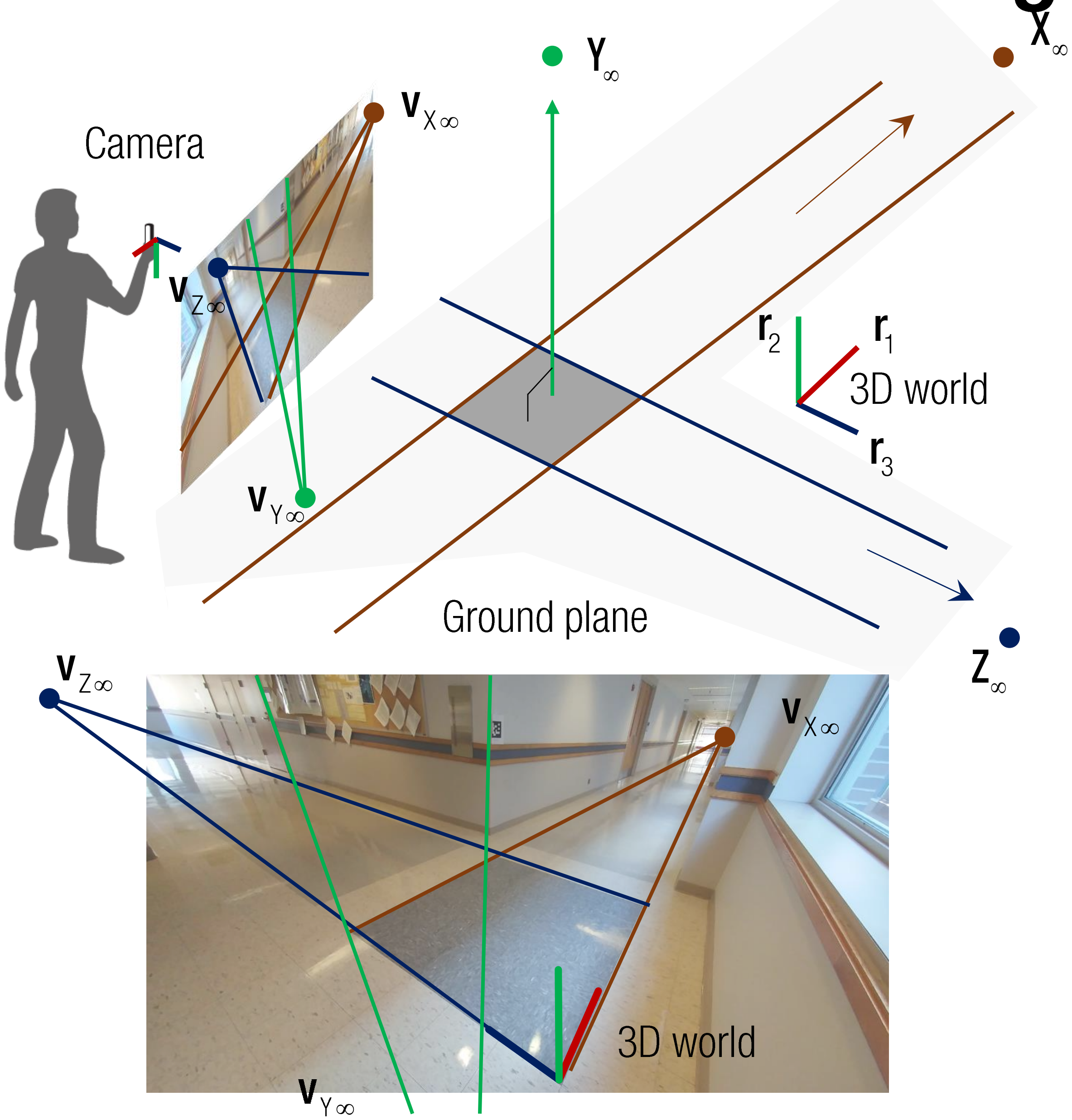
$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty$$

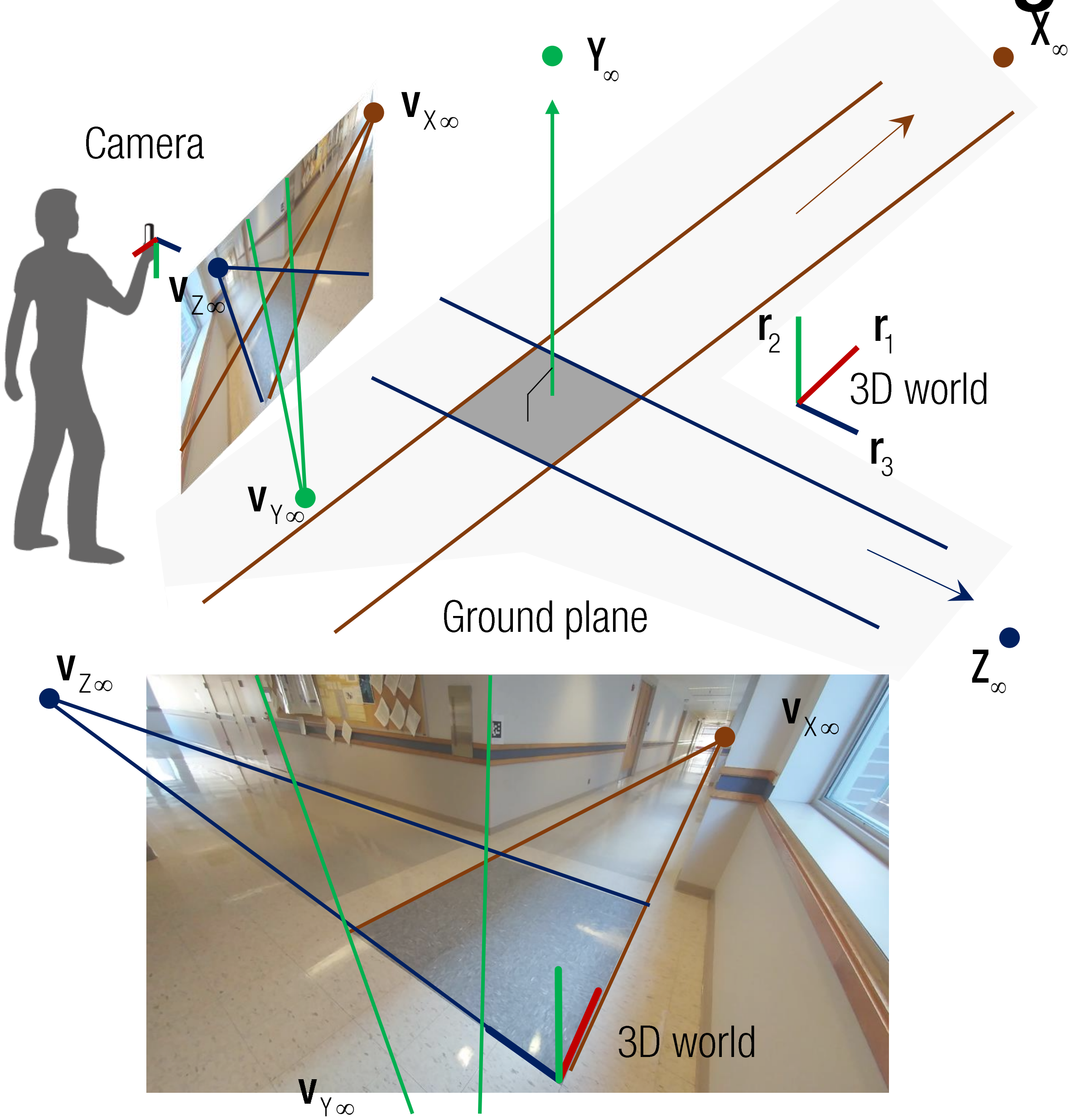
Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

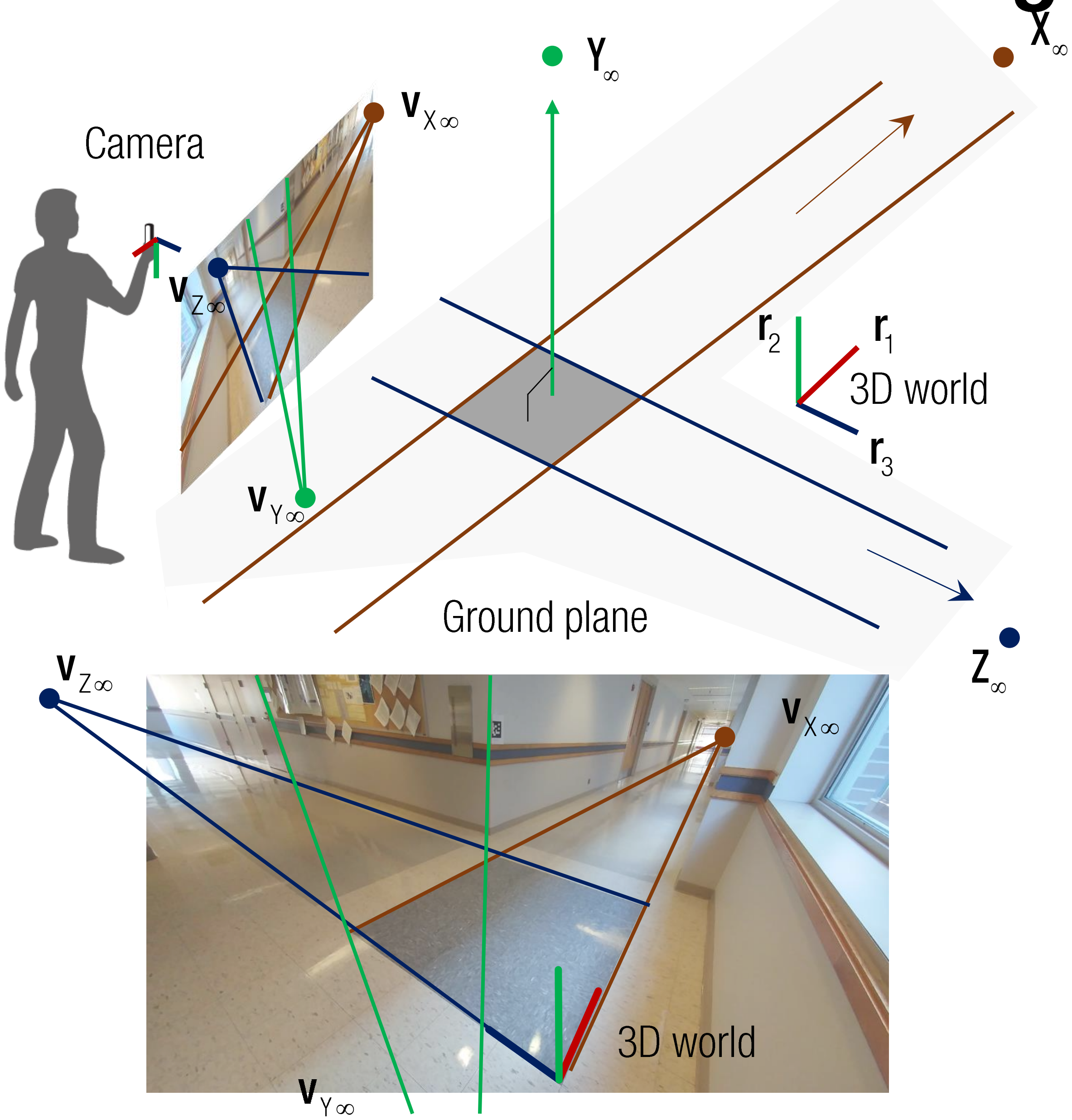
Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0$$
$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0$$
$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0$$

These axes are perpendicular to each other.

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

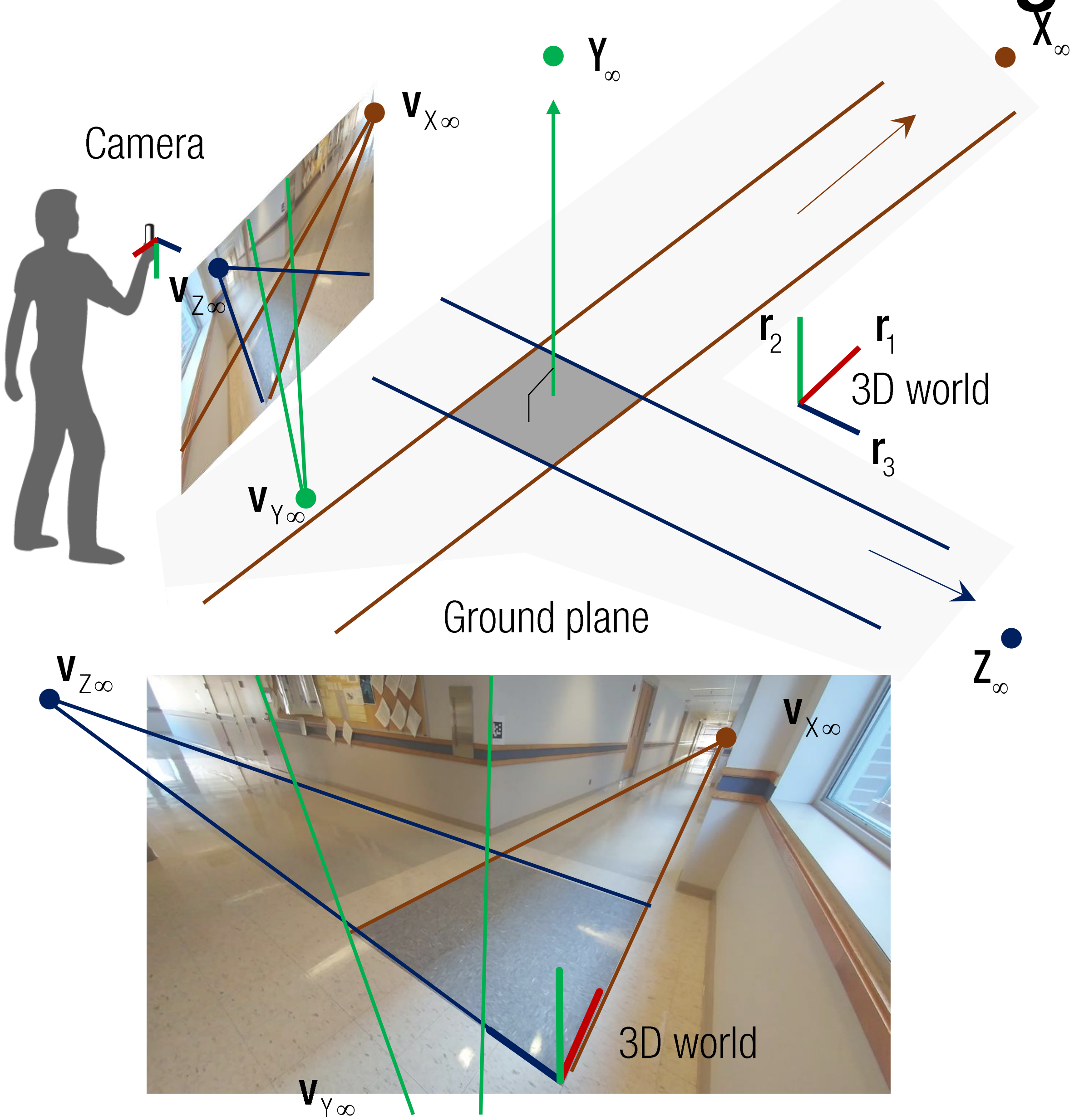
Known property of points at infinity:

$$(\mathbf{X}_\infty)^T(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^T(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^T(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^T(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^T(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^T(\mathbf{R}\mathbf{X}_\infty) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \quad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

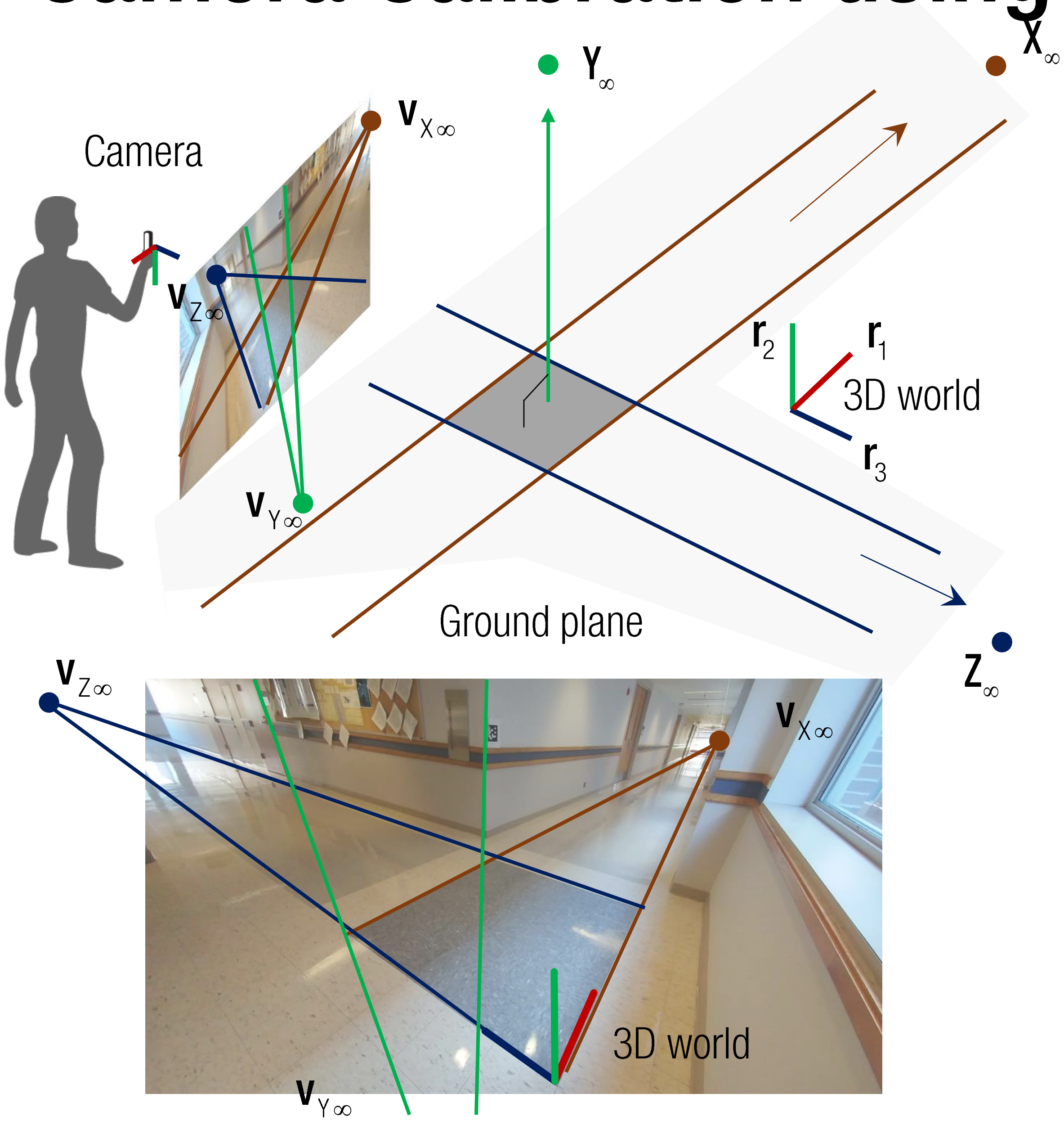
Known property of points at infinity:

$$(\mathbf{X}_\infty)^\mathsf{T}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^\mathsf{T}(\mathbf{R}\mathbf{X}_\infty) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

Known property of points at infinity:

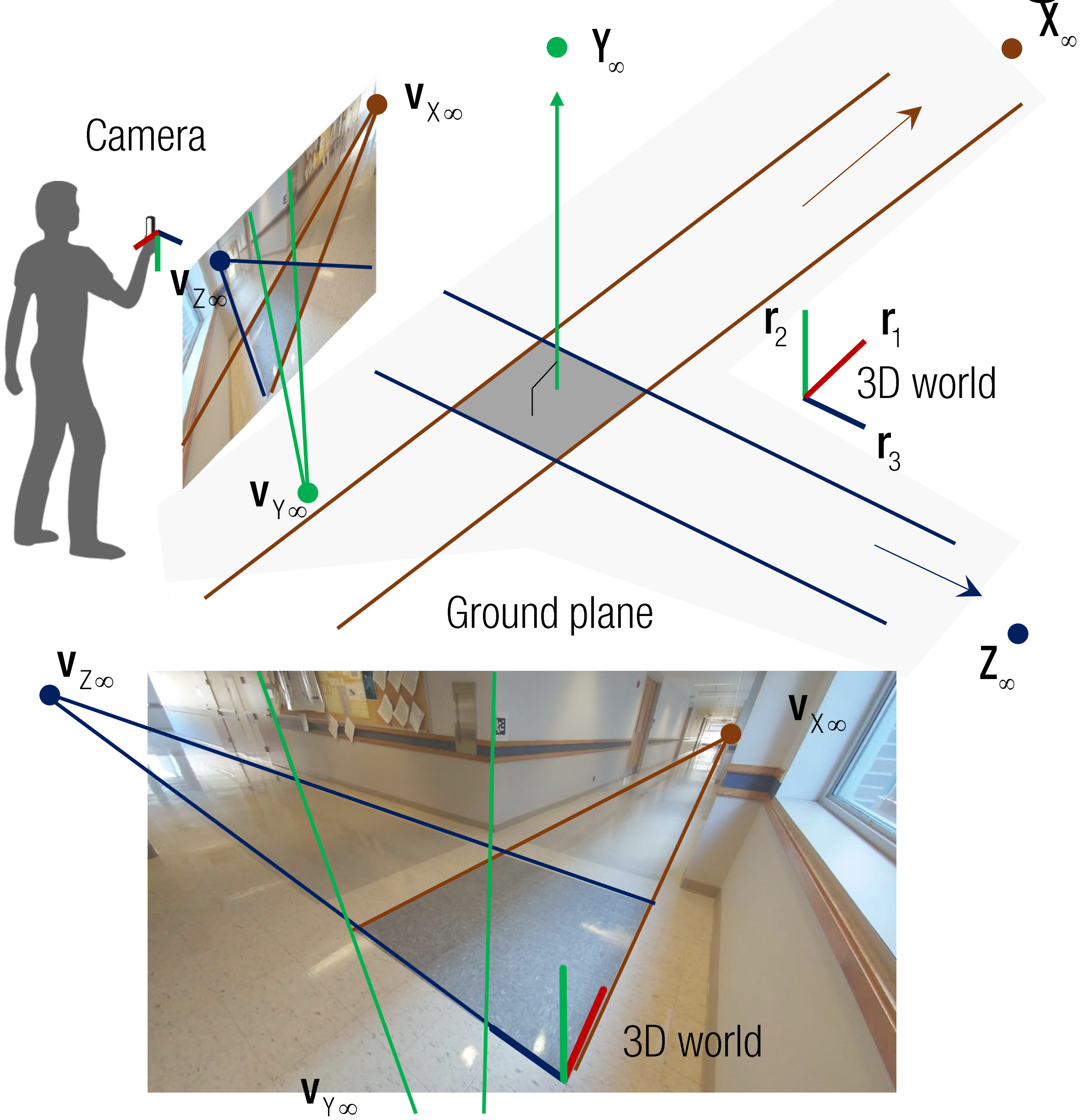
$$(\mathbf{X}_\infty)^{\mathsf{T}}(\mathbf{Y}_\infty) = 0 \qquad (\mathbf{R}\mathbf{X}_\infty)^{\mathsf{T}}(\mathbf{R}\mathbf{Y}_\infty) = 0$$

$$(\mathbf{Y}_\infty)^{\mathsf{T}}(\mathbf{Z}_\infty) = 0 \quad \longleftrightarrow \quad (\mathbf{R}\mathbf{Y}_\infty)^{\mathsf{T}}(\mathbf{R}\mathbf{Z}_\infty) = 0$$

$$(\mathbf{Z}_\infty)^{\mathsf{T}}(\mathbf{X}_\infty) = 0 \qquad (\mathbf{R}\mathbf{Z}_\infty)^{\mathsf{T}}(\mathbf{R}\mathbf{X}_\infty) = 0$$

$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right) = \left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right) = \left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right) = 0$$

Camera Calibration using Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_{\infty} \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_{\infty} \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_{\infty}$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_{\infty} \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_{\infty} \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_{\infty}$$

Note that the camera extrinsic is still unknown (**R** and **t**).

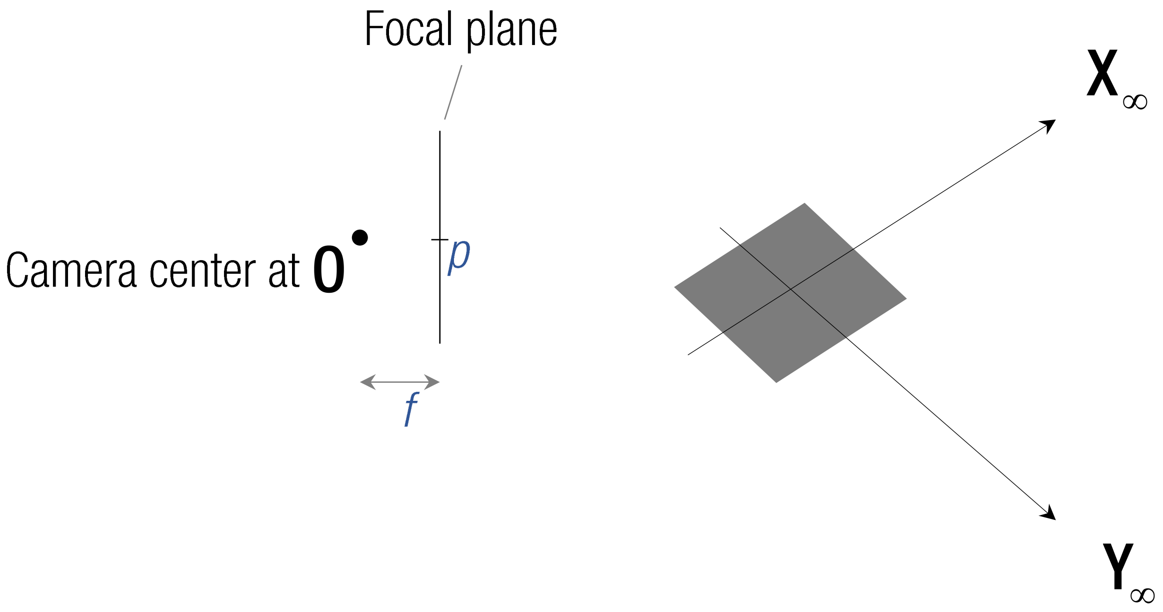
Known property of points at infinity:

$$\begin{aligned} (\mathbf{X}_{\infty})^{\mathsf{T}}(\mathbf{Y}_{\infty}) &= 0 & & (\mathbf{R}\mathbf{X}_{\infty})^{\mathsf{T}}(\mathbf{R}\mathbf{Y}_{\infty}) = 0 \\ (\mathbf{Y}_{\infty})^{\mathsf{T}}(\mathbf{Z}_{\infty}) &= 0 & \longleftrightarrow \quad & (\mathbf{R}\mathbf{Y}_{\infty})^{\mathsf{T}}(\mathbf{R}\mathbf{Z}_{\infty}) = 0 \\ (\mathbf{Z}_{\infty})^{\mathsf{T}}(\mathbf{X}_{\infty}) &= 0 & & (\mathbf{R}\mathbf{Z}_{\infty})^{\mathsf{T}}(\mathbf{R}\mathbf{X}_{\infty}) = 0 \end{aligned}$$

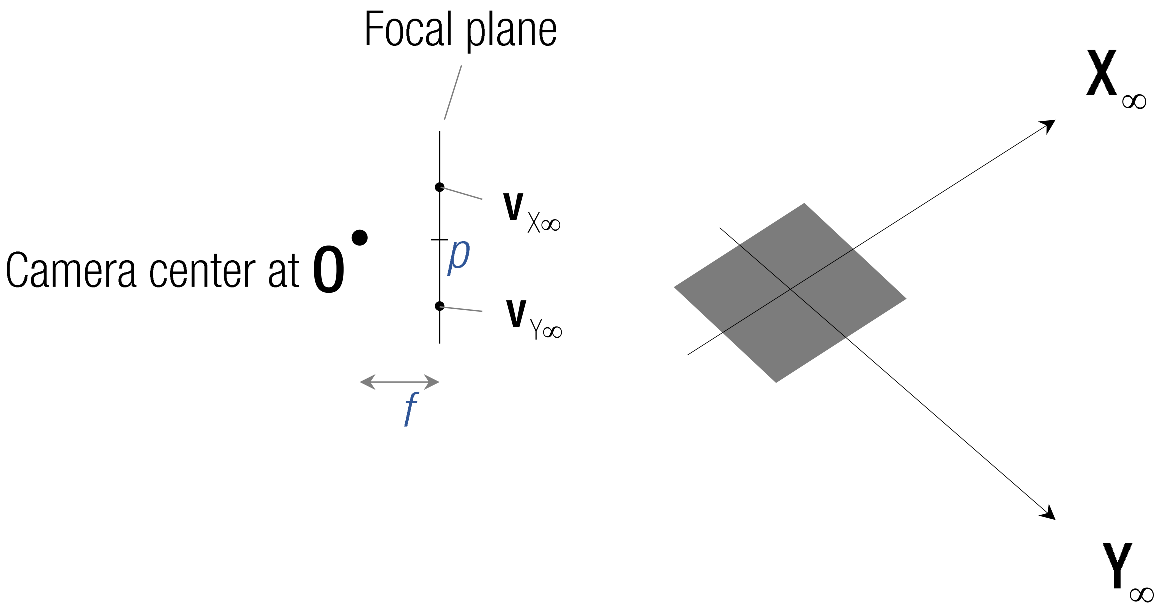
$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^{\mathsf{T}}(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^{\mathsf{T}}(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^{\mathsf{T}}(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

: 3 unknowns and 3 equations

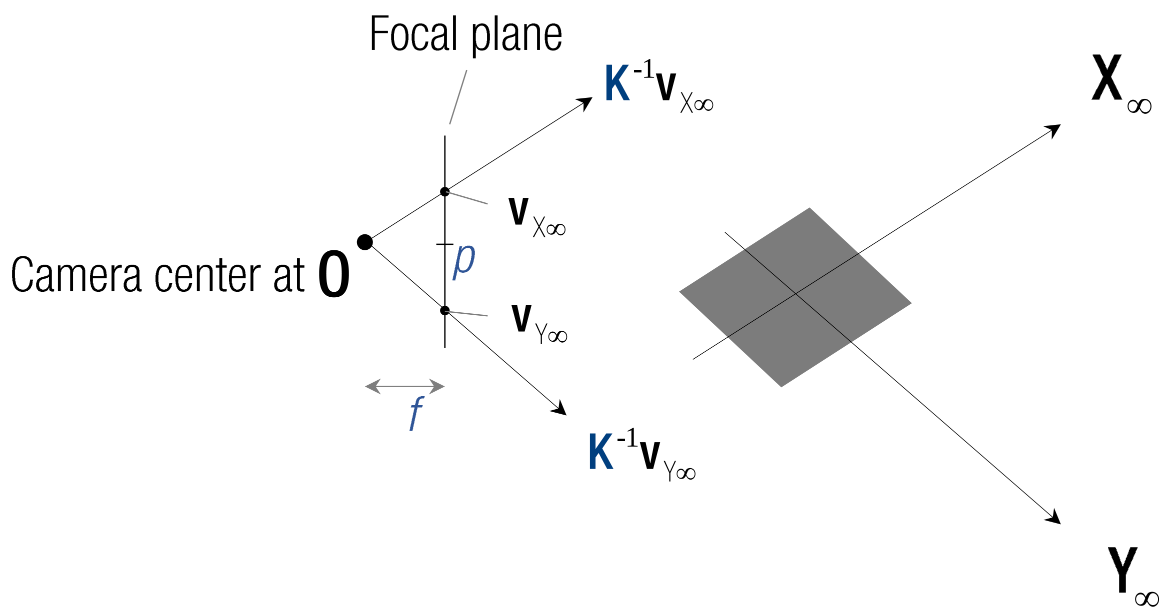
Geometric Interpretation with 1D Camera



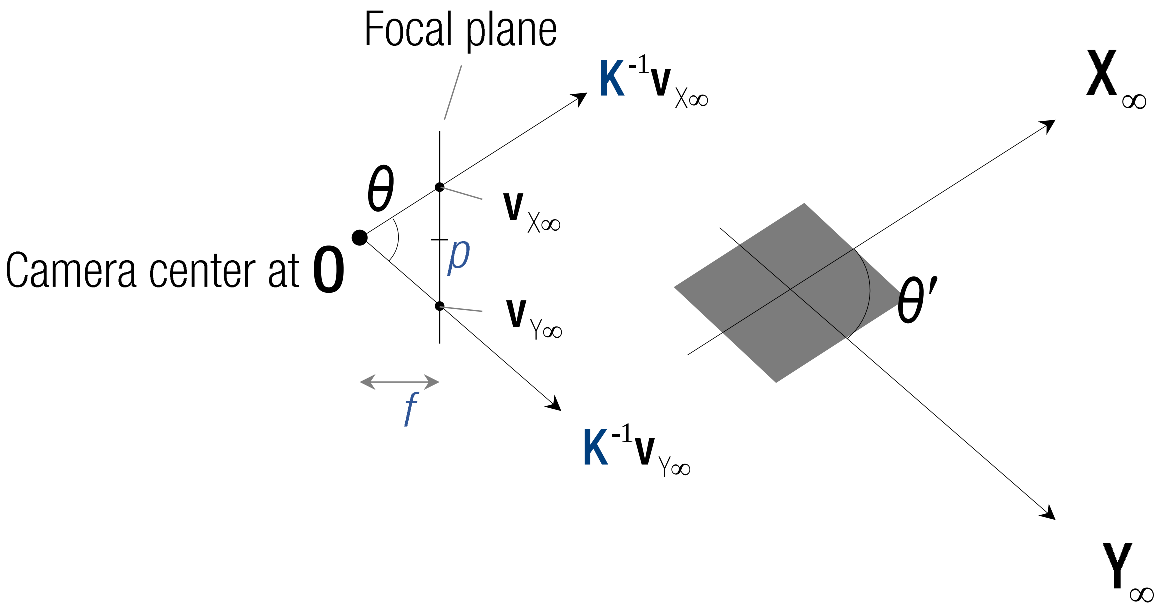
Geometric Interpretation with 1D Camera



Geometric Interpretation with 1D Camera

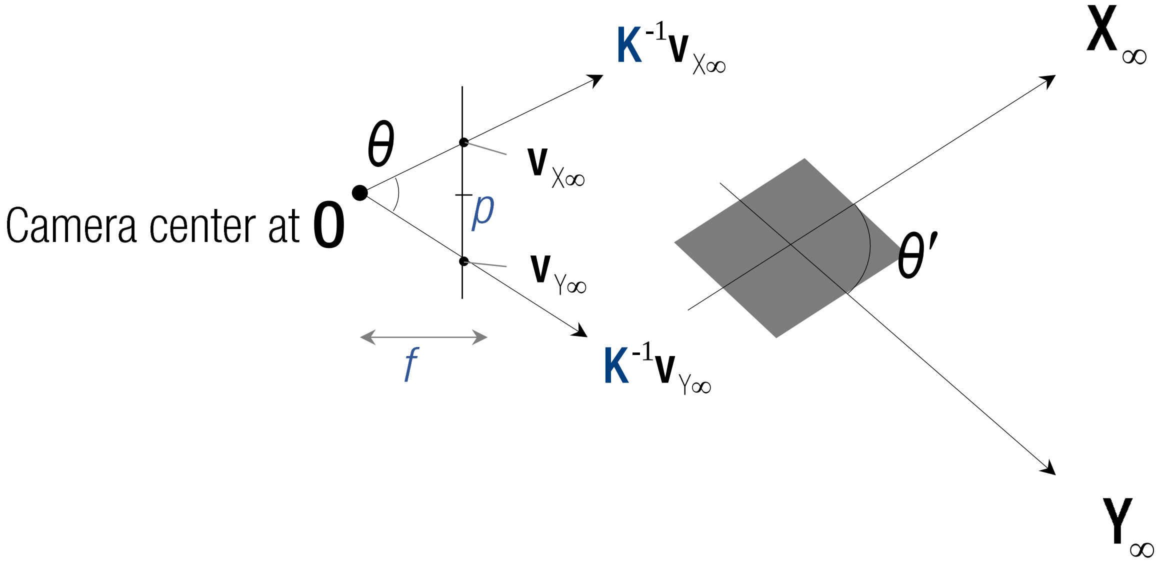


Geometric Interpretation with 1D Camera



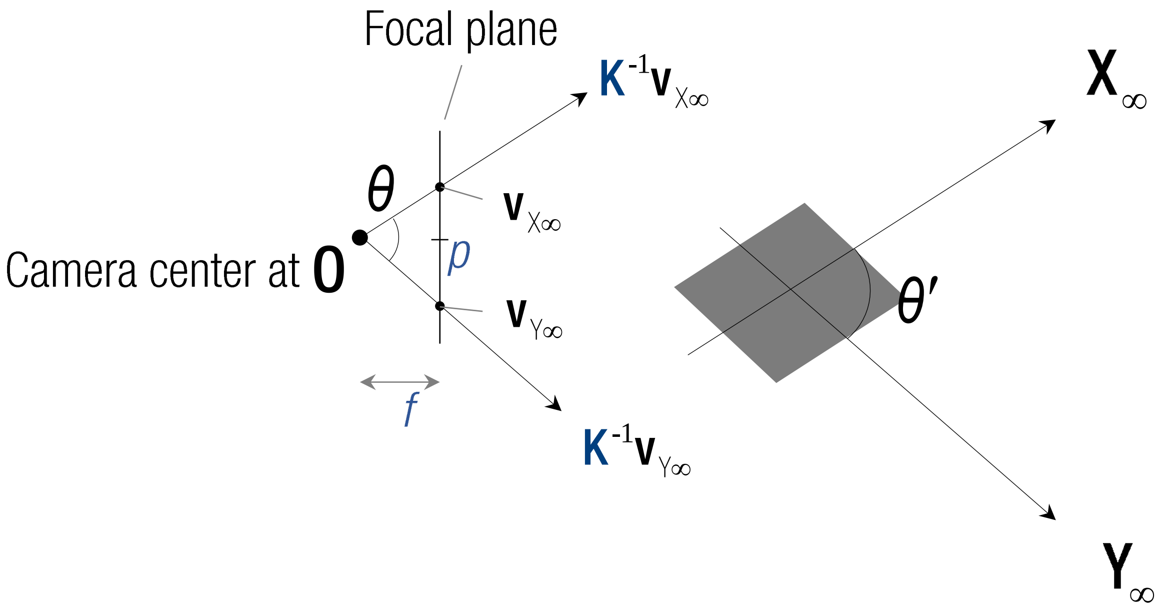
$\theta = \theta'$

Geometric Interpretation with 1D Camera



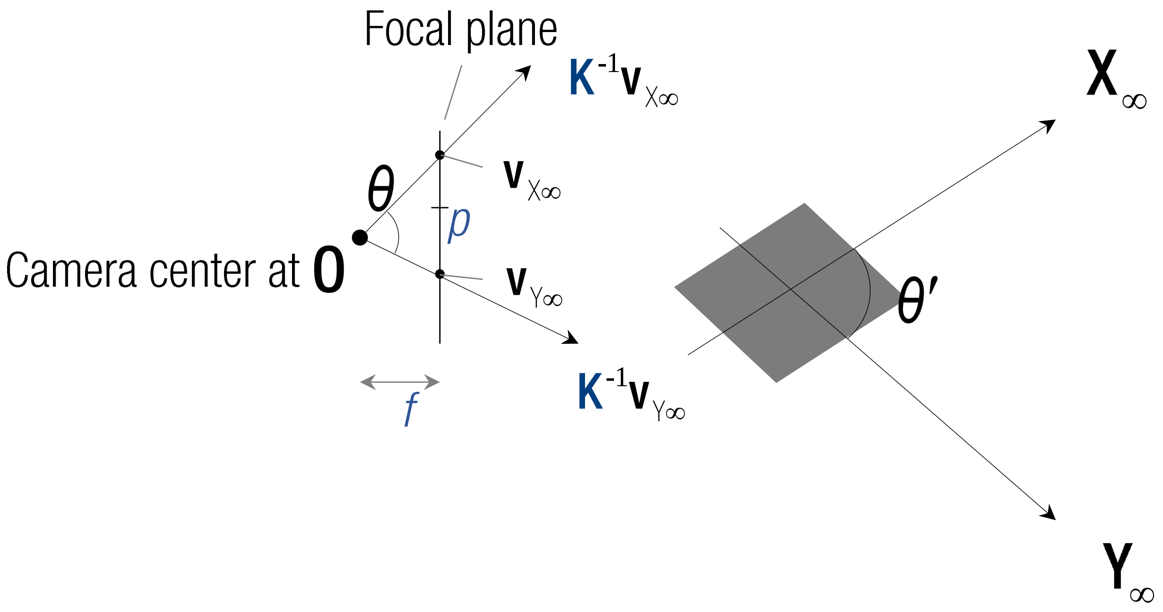
$\theta \neq \theta'$

Geometric Interpretation with 1D Camera



$\theta = \theta'$

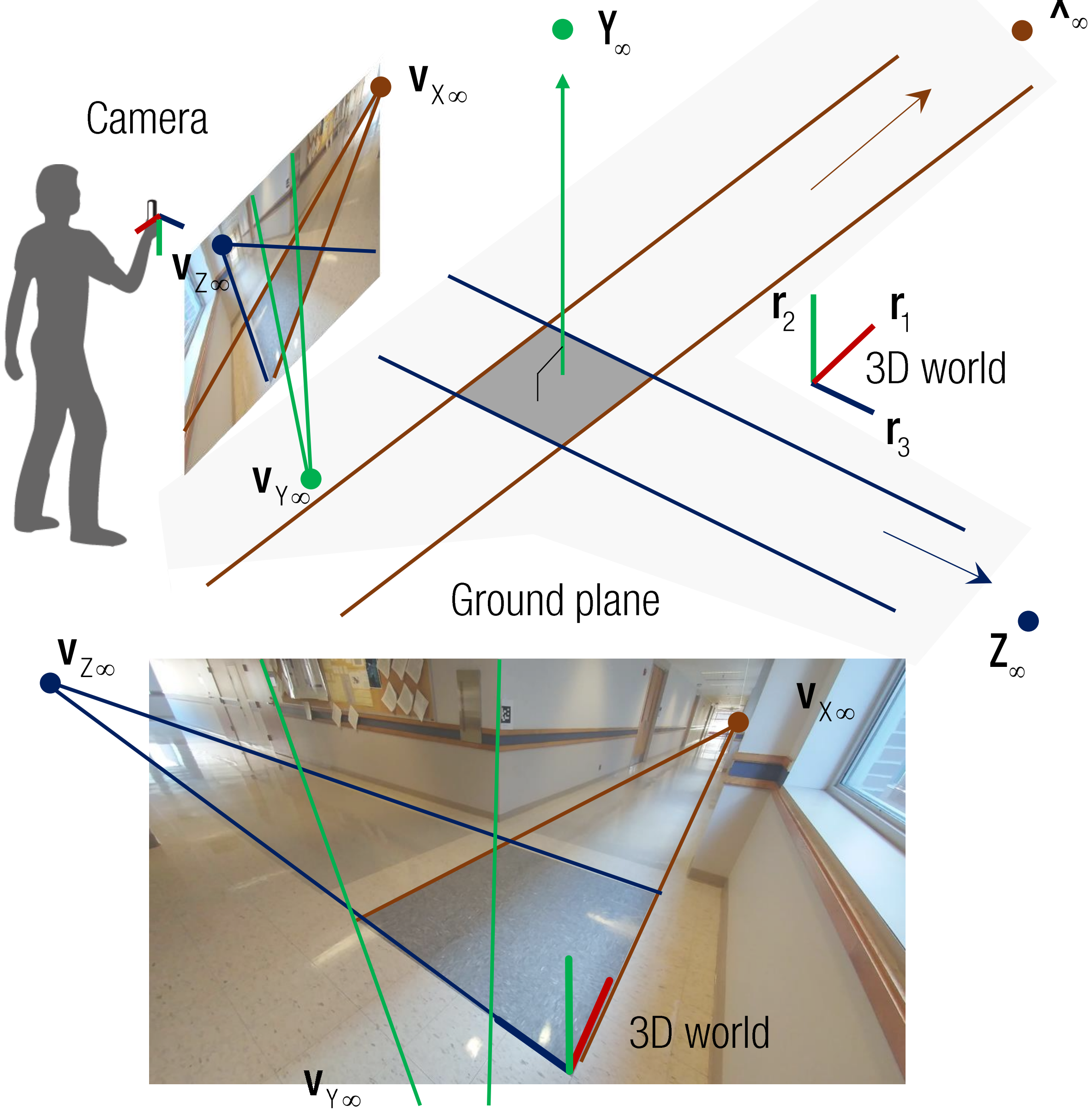
Geometric Interpretation with 1D Camera



$\theta \neq \theta'$

Given two vanishing points, the focal length and principal point are uniquely defined.
For the 2D camera case, another vanishing point is needed to uniquely define f, p_x, and p_y.

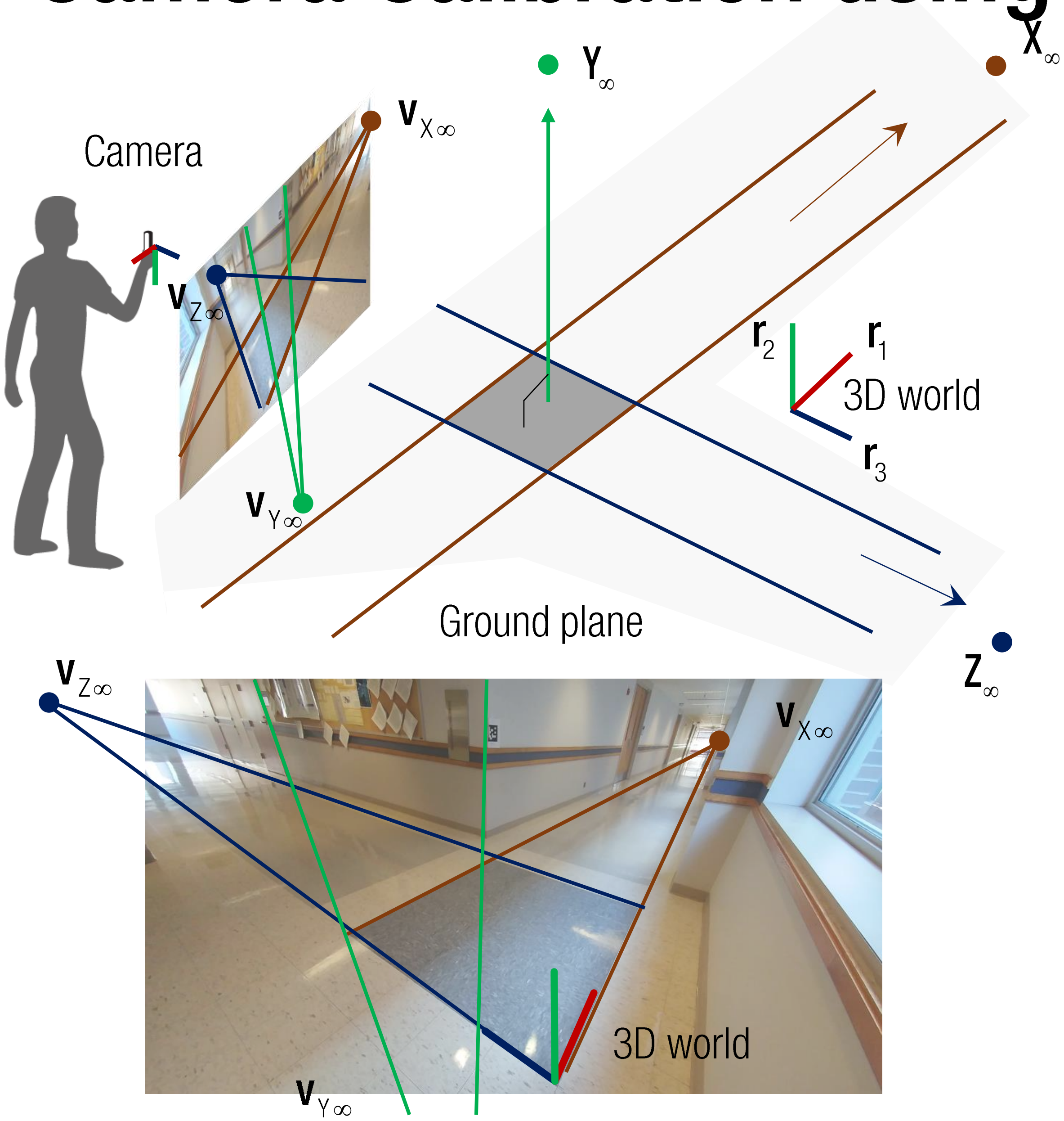
Camera Calibration using Vanishing Points



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

Camera Calibration using Vanishing Points

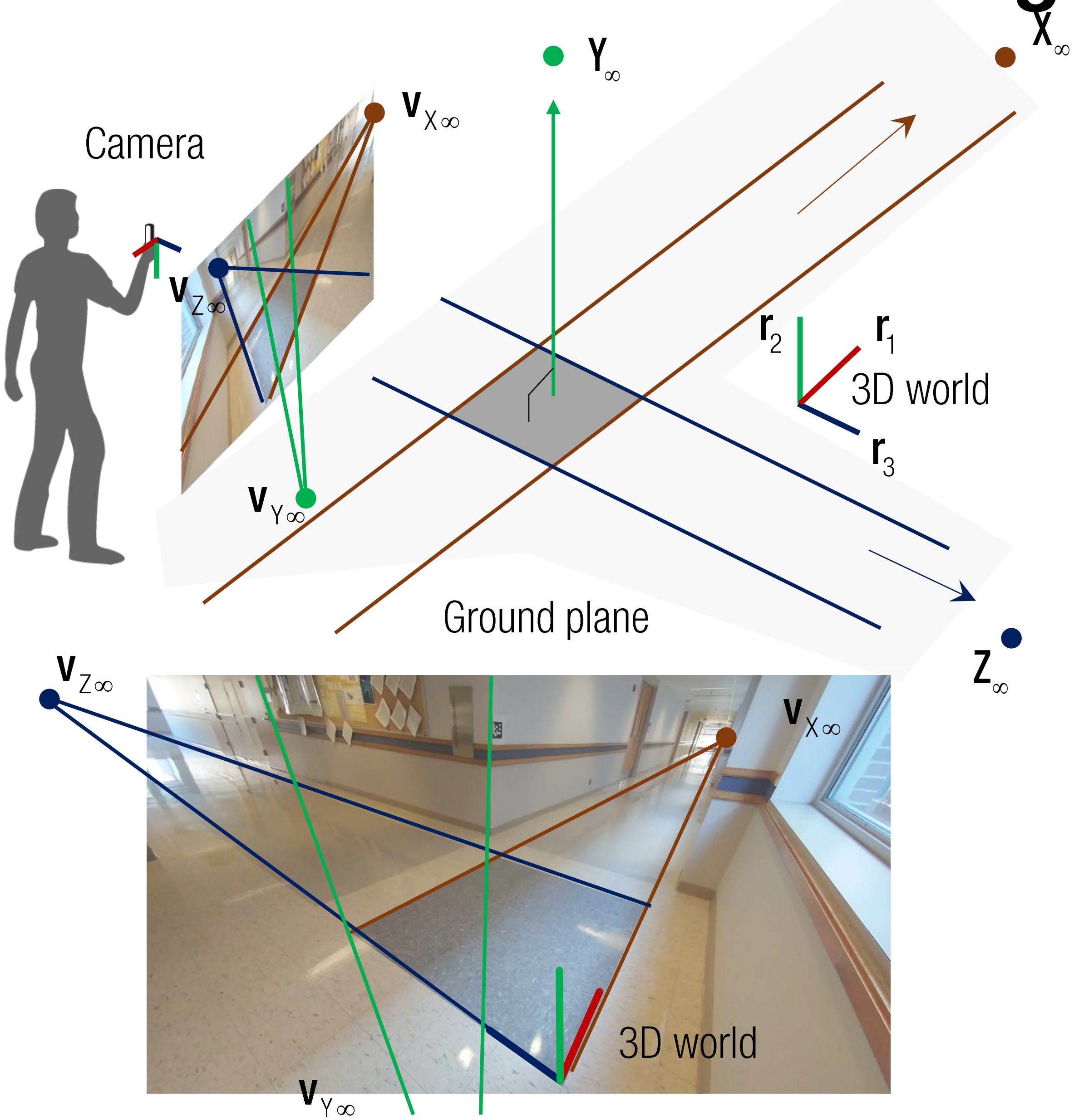


$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j = X,Y,Z, \text{ and } i \neq j$$

Camera Calibration using Vanishing Points



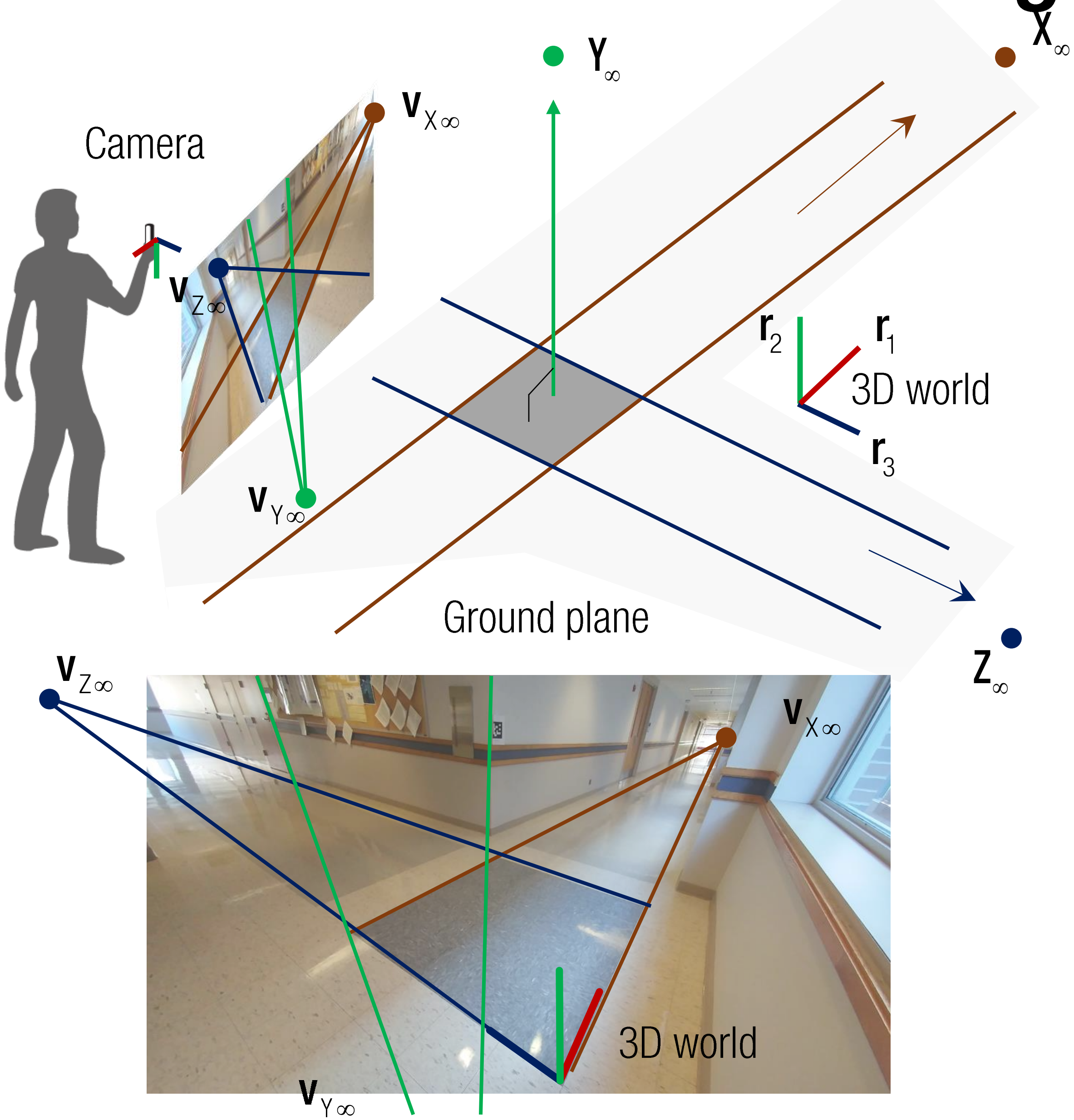
$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_i^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j = X,Y,Z, \text{ and } i \neq j$$

$$\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}=\begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

Camera Calibration using Vanishing Points



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

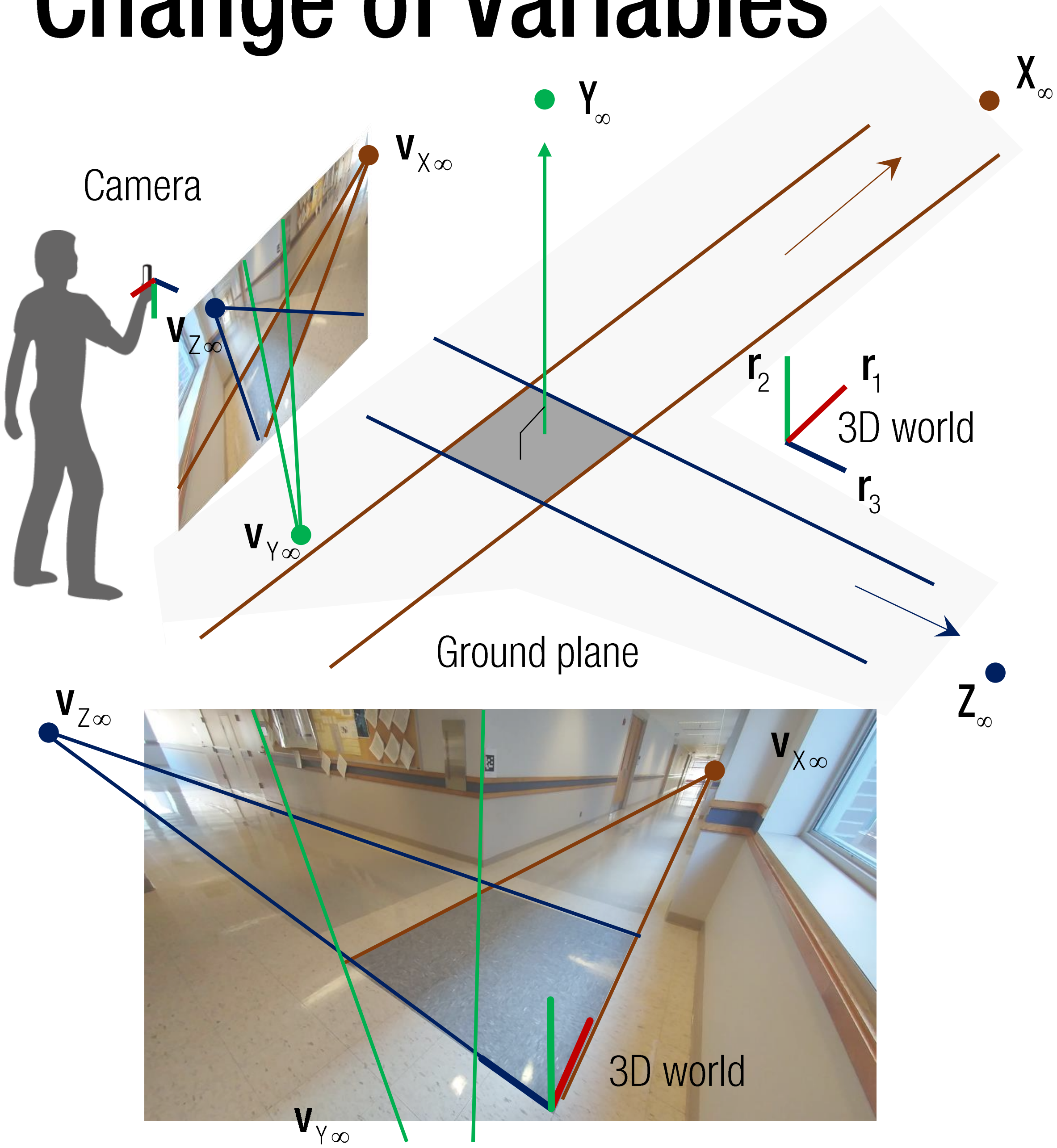
: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_{i}^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_{j} \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1\end{bmatrix}$$

Change of Variables



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

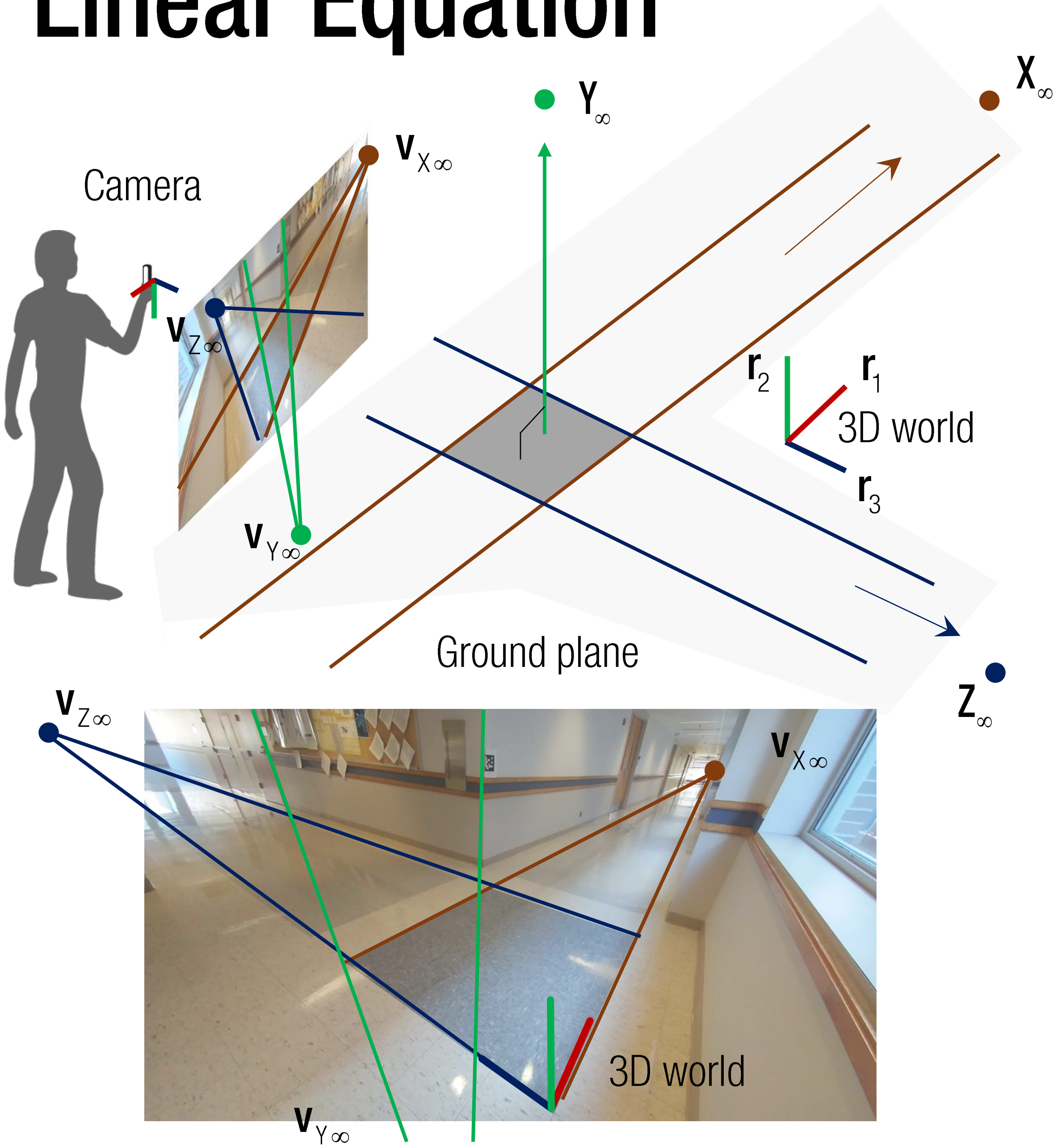
$$\longrightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_{i}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{j}\right)=\mathbf{v}_{i}^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_{j} \qquad i,j=X,Y,Z, \text{ and } i\neq j$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1\end{bmatrix}=\begin{bmatrix}b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4\end{bmatrix}$$

where $b_1=\frac{1}{f^2}, \quad b_2=-\frac{p_x}{f^2}, \quad b_3=-\frac{p_y}{f^2}, \quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

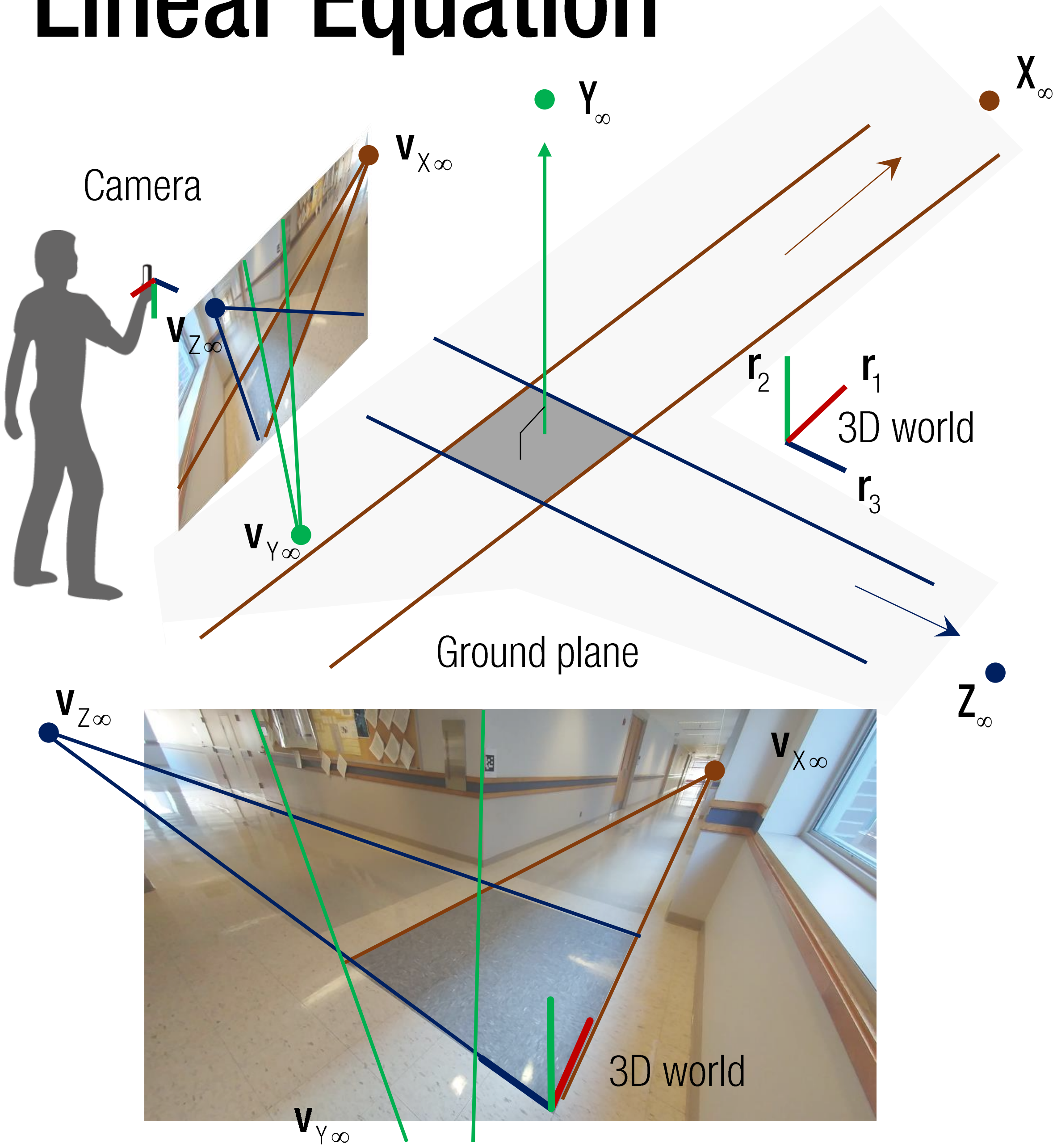
: 3 unknowns and 3 equations

$\rightarrow$ $\left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j$ $\quad i,j=X,Y,Z,$ and $i\neq j$

$\rightarrow$ $\mathbf{v}_i^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j$:

Linear in b

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

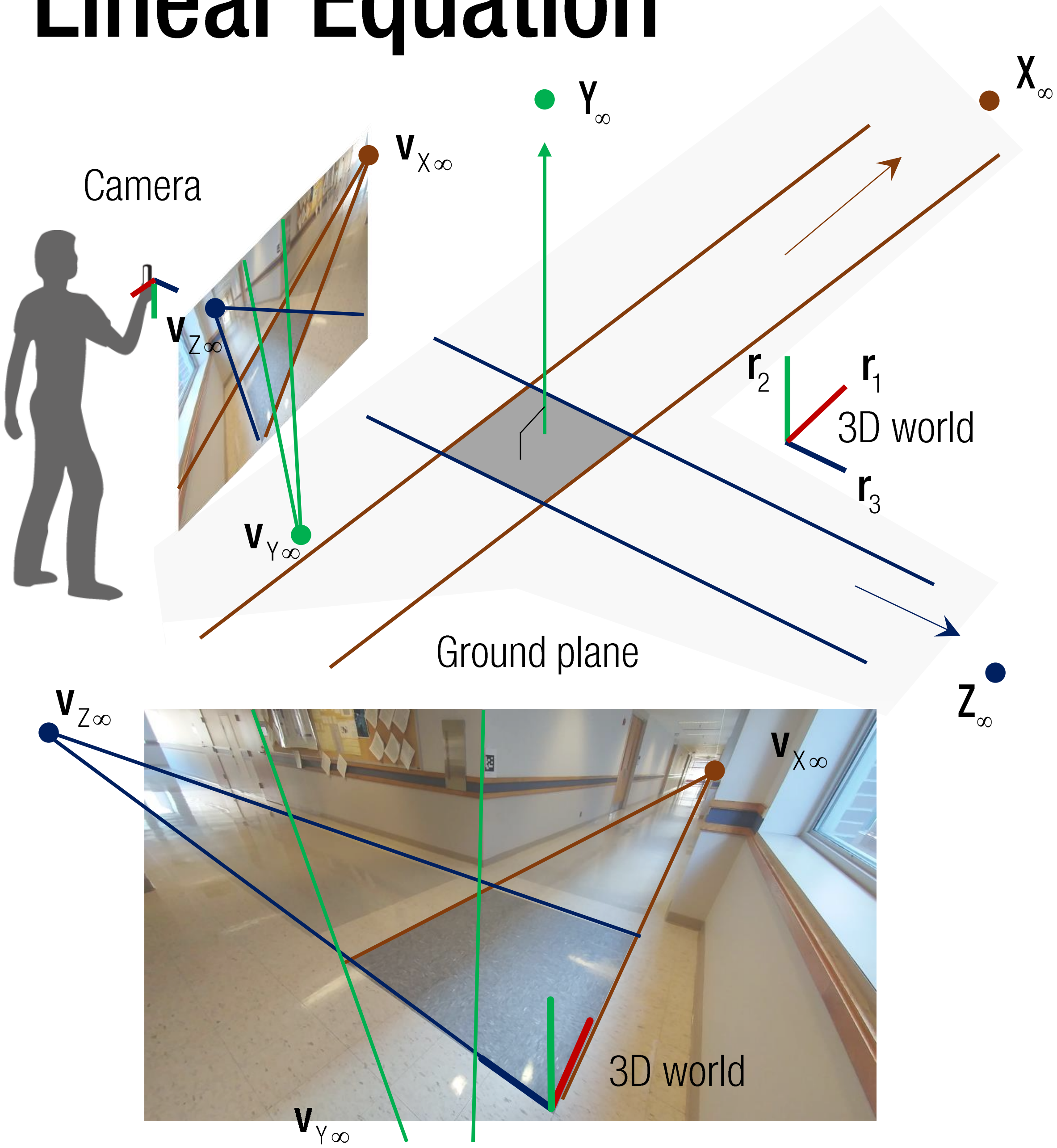
: 3 unknowns and 3 equations

$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j \qquad i, j = X, Y, Z, \text{ and } i \neq j$$

$$\rightarrow \quad \underbrace{\mathbf{v}_i^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j}_{\text{Linear in b}} = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

Linear in b

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

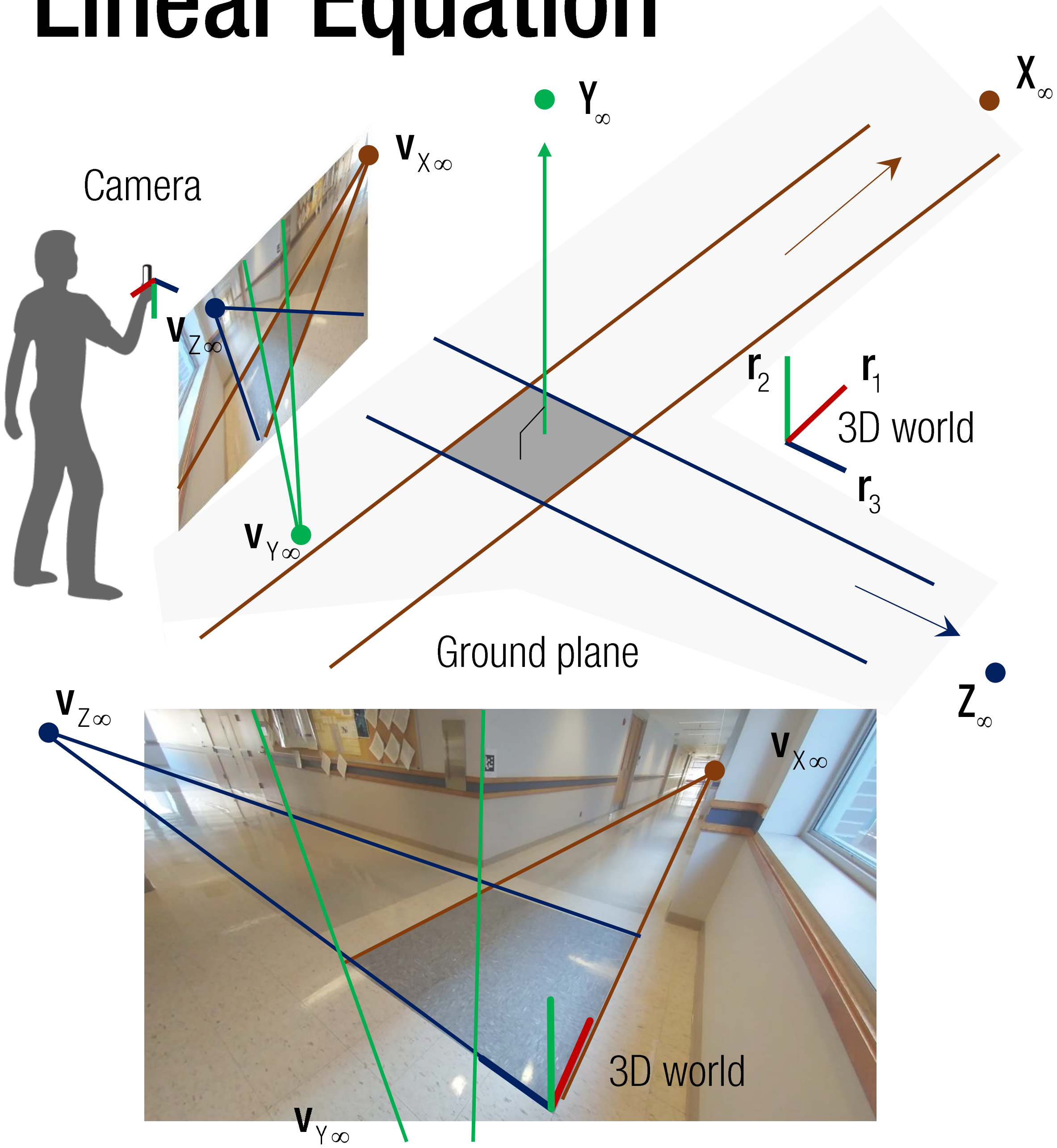
$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j=\mathrm{X,Y,Z},\ \text{and}\ i\neq j$$

$$\rightarrow \quad \mathbf{v}_i^{\mathsf{T}}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j=\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{\mathsf{T}}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

Linear in b

$$\rightarrow \quad \begin{bmatrix} u_i u_j + v_i v_j & u_i + u_j & v_i + v_j & 1 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}=0$$

Linear Equation



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

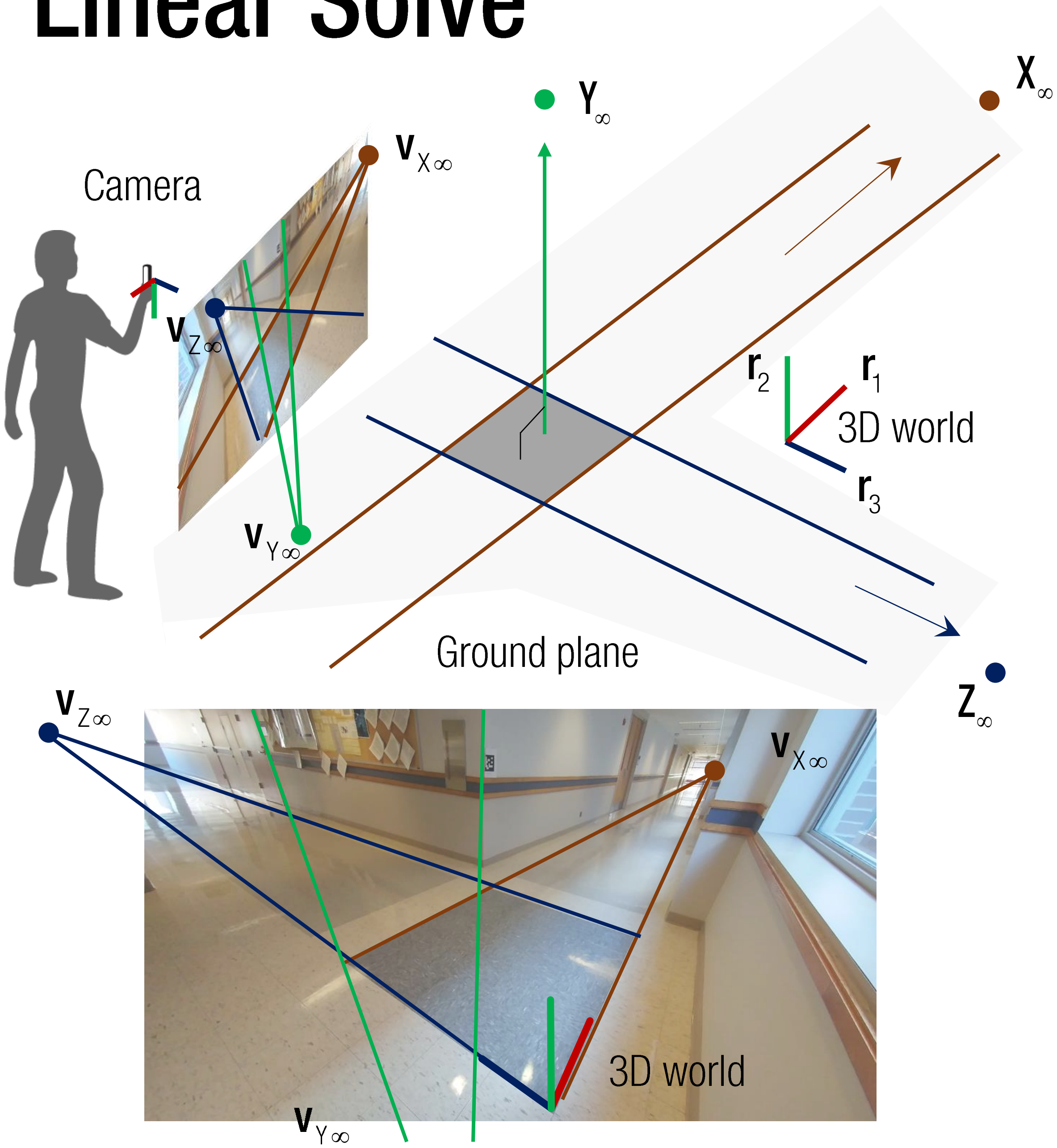
: 3 unknowns and 3 equations

$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$\rightarrow \quad \underbrace{\mathbf{v}_i^{\mathsf{T}}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j}_{\text{Linear in b}}=\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{\mathsf{T}}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix}=0$$

$$\rightarrow \quad \underbrace{\begin{bmatrix} u_1u_2+v_1v_2 & u_1+u_2 & v_1+v_2 & 1 \\ u_3u_2+v_3v_2 & u_3+u_2 & v_3+v_2 & 1 \\ u_1u_3+v_1v_3 & u_1+u_3 & v_1+v_3 & 1 \end{bmatrix}}_{3\text{x}4}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}=0$$

Linear Solve



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\rightarrow \quad \left(\mathbf{K}^{-1}\mathbf{v}_i\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_j\right)=\mathbf{v}_i^{T}\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{v}_j \qquad i,j = X,Y,Z, \text{ and } i \neq j$$

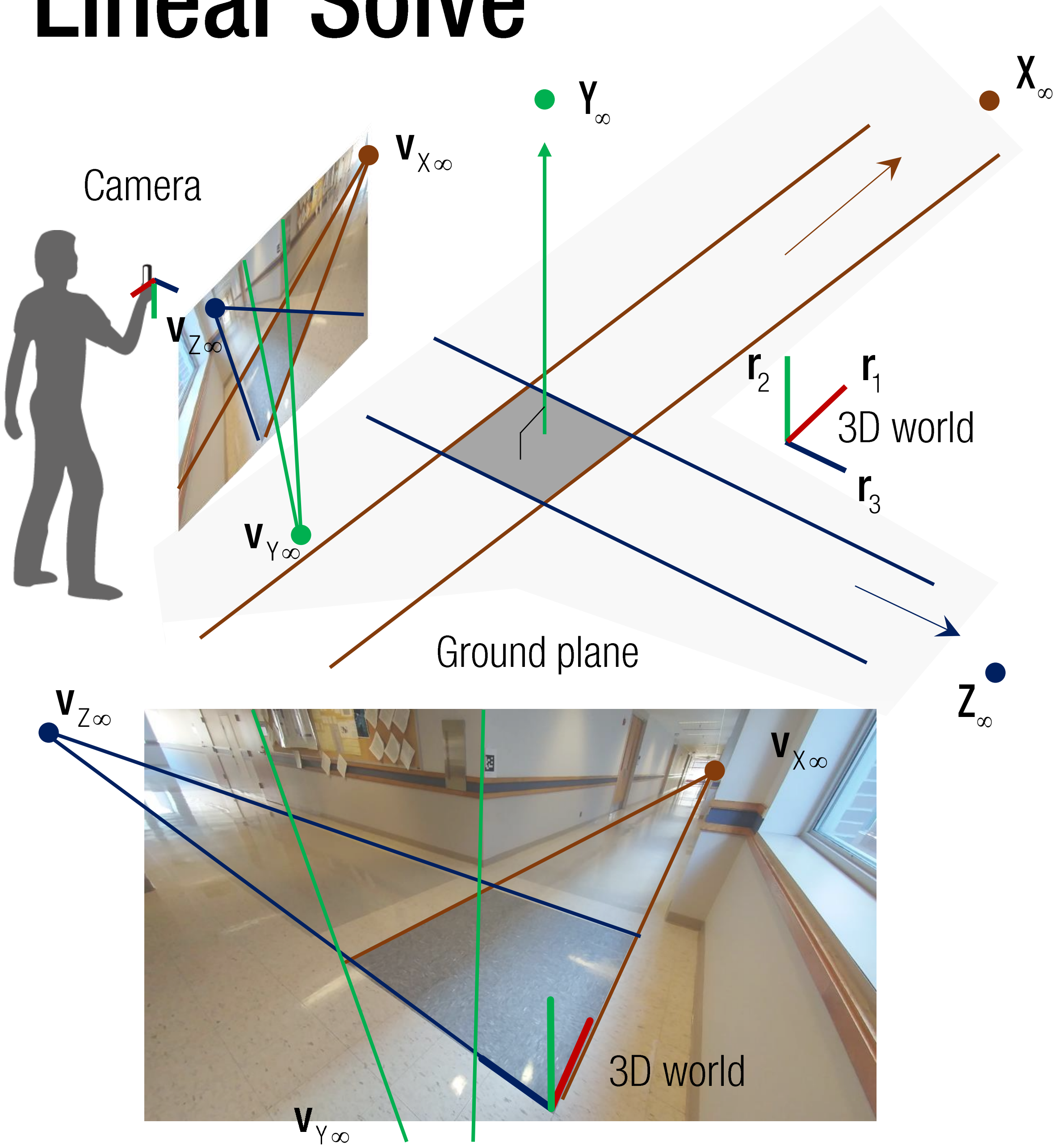
$$\rightarrow \quad \underbrace{\mathbf{v}_i^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\mathbf{v}_j}_{\text{Linear in b}} = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^{T}\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}\begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

Linear in b

$$\rightarrow \quad \underbrace{\begin{bmatrix} u_1u_2+v_1v_2 & u_1+u_2 & v_1+v_2 & 1 \\ u_3u_2+v_3v_2 & u_3+u_2 & v_3+v_2 & 1 \\ u_1u_3+v_1v_3 & u_1+u_3 & v_1+v_3 & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

3x4

Linear Solve



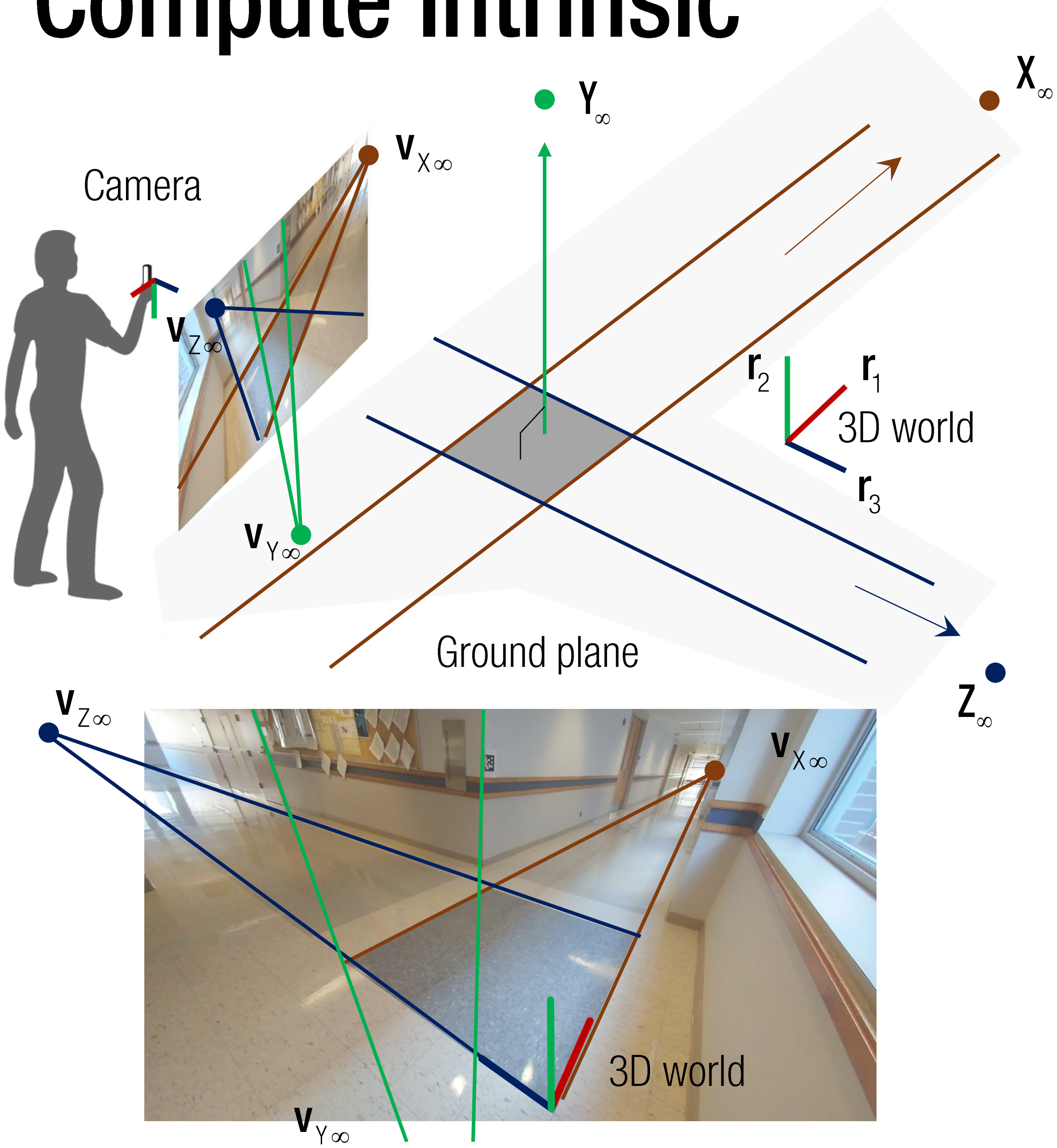
$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{T}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\rightarrow \quad \mathbf{x}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^{\,2}}{f^2}+\frac{p_y^2}{f^2}+1$

Compute Intrinsic



$$\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)=\left(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{v}_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

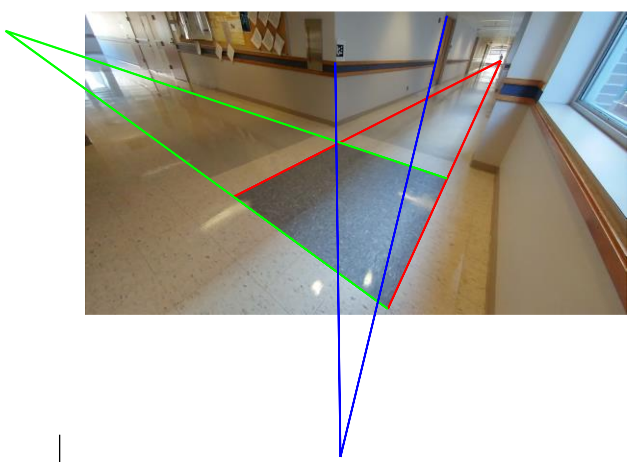
$$\rightarrow \quad \mathbf{x}=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

$$\rightarrow \quad p_x=-\frac{b_2}{b_1},\quad p_y=-\frac{b_3}{b_1},\quad f=\sqrt{\frac{b_4}{b_1}-(p_x^2+p_y^2)}$$

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
z21 = [2564; 31; 1];z22 = [2439; 551; 1];

m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```

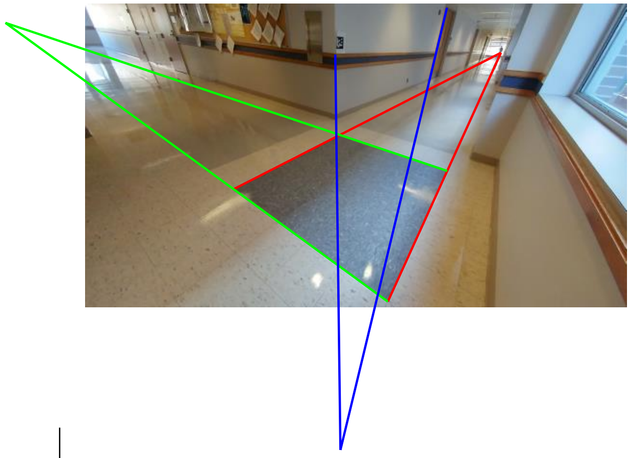


Vanishing points

CameraCalibration.m

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
z21 = [2564; 31; 1];z22 = [2439; 551; 1];

m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```



Vanishing points

CameraCalibration.m

$$\mathbf{A} = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);

px = -x(2)/x(1);
py = -x(3)/x(1);
f = sqrt(x(4)/x(1)-px^2-py^2);

K = [f 0 px;
    0 f py;
    0 0 1]
```

Linear solve using SVD

```
K =

   1317.2        0   1931.8
        0   1317.2   1146.1
        0        0        1
```

$f = 1500$

$p_x = \text{size(im,2)}/2 = 1920$

$p_y = \text{size(im,1)}/2 = 1080$

Previous manual estimate

Camera Calibration

$$\mathbf{A} = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];
```

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
z21 = [2564; 31; 1];z22 = [2439; 551; 1];

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l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```

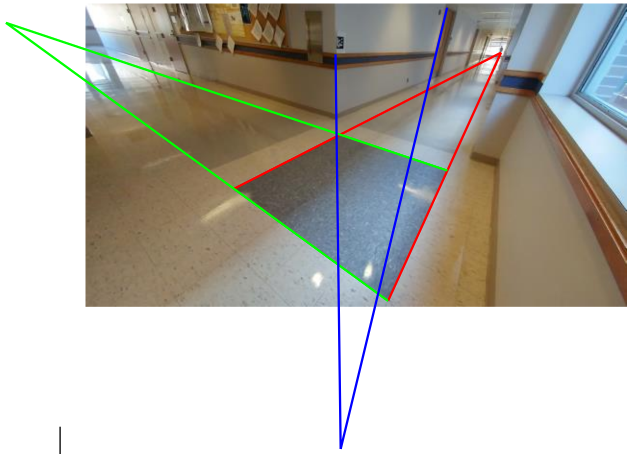

Vanishing points

CameraCalibration.m

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```

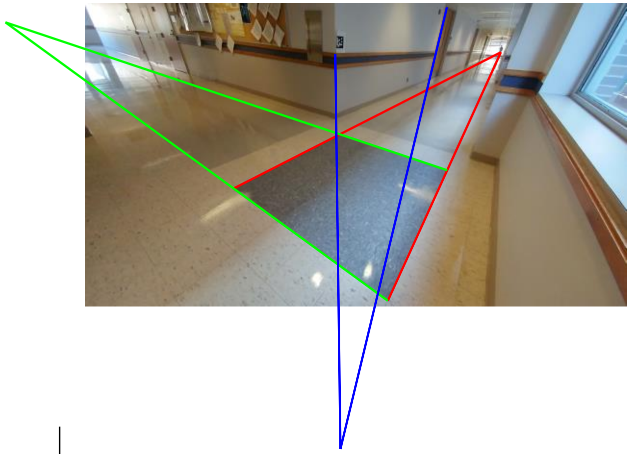

Vanishing points

CameraCalibration.m

$$\mathbf{A} = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
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l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

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l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```

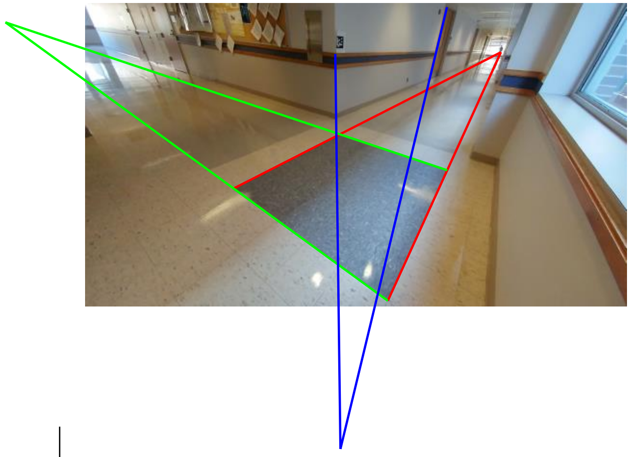

Vanishing points

$$\mathbf{A} = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

```
px = -x(2)/x(1);
py = -x(3)/x(1);
f = sqrt(x(4)/x(1)-px^2-py^2);

K = [f 0 px;
    0 f py;
    0 0 1]
```

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

CameraCalibration.m

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
z21 = [2564; 31; 1];z22 = [2439; 551; 1];

m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

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y = GetPointFromTwoLines(l21,l22);
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```
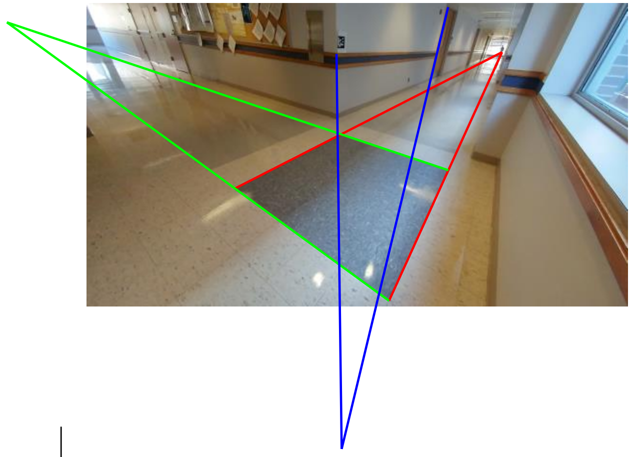



Vanishing points

CameraCalibration.m

$$\mathbf{A} = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

```
px = -x(2)/x(1);
py = -x(3)/x(1);
f = sqrt(x(4)/x(1)-px^2-py^2);

K = [f 0 px;
    0 f py;
    0 0 1]
```

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

```
K =

   1317.2        0   1931.8
        0   1317.2   1146.1
        0        0        1
```

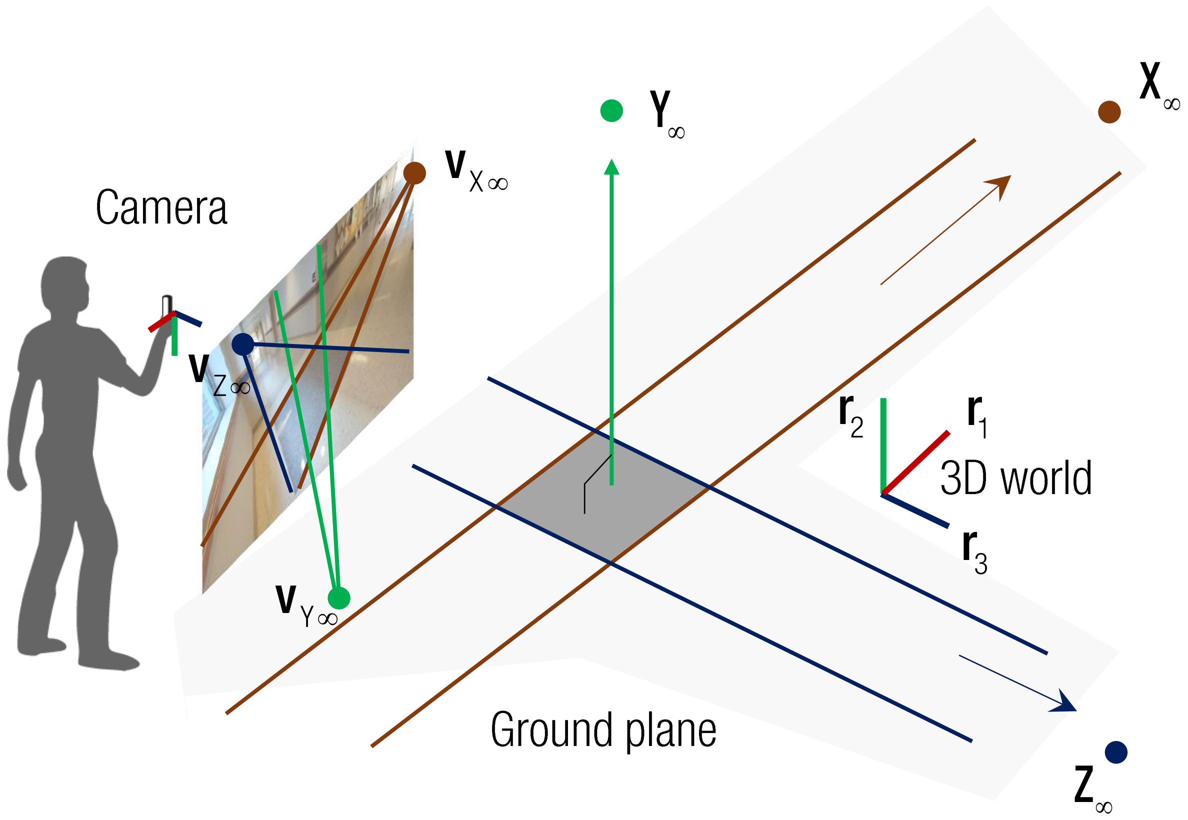
$f = 1224$

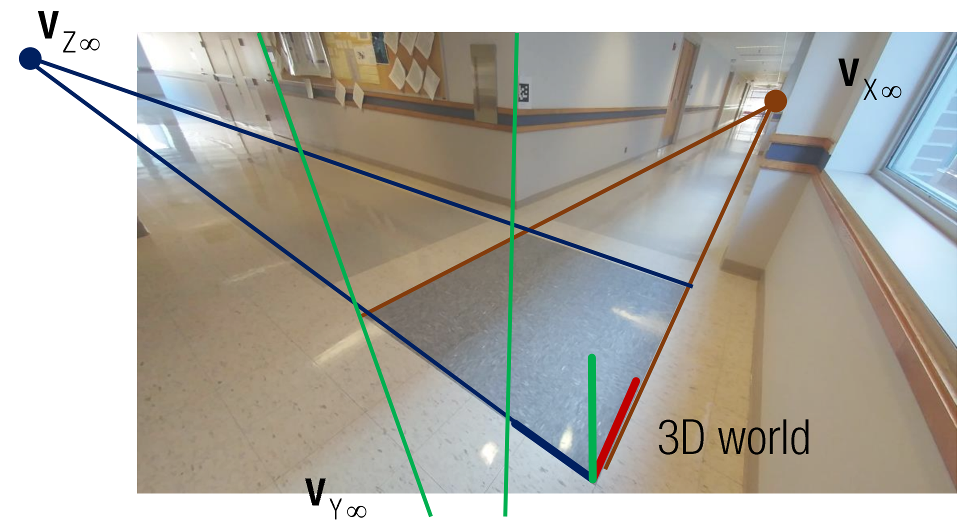
p_x = size(im,2)/2 = 1920

p_y = size(im,1)/2 = 1080

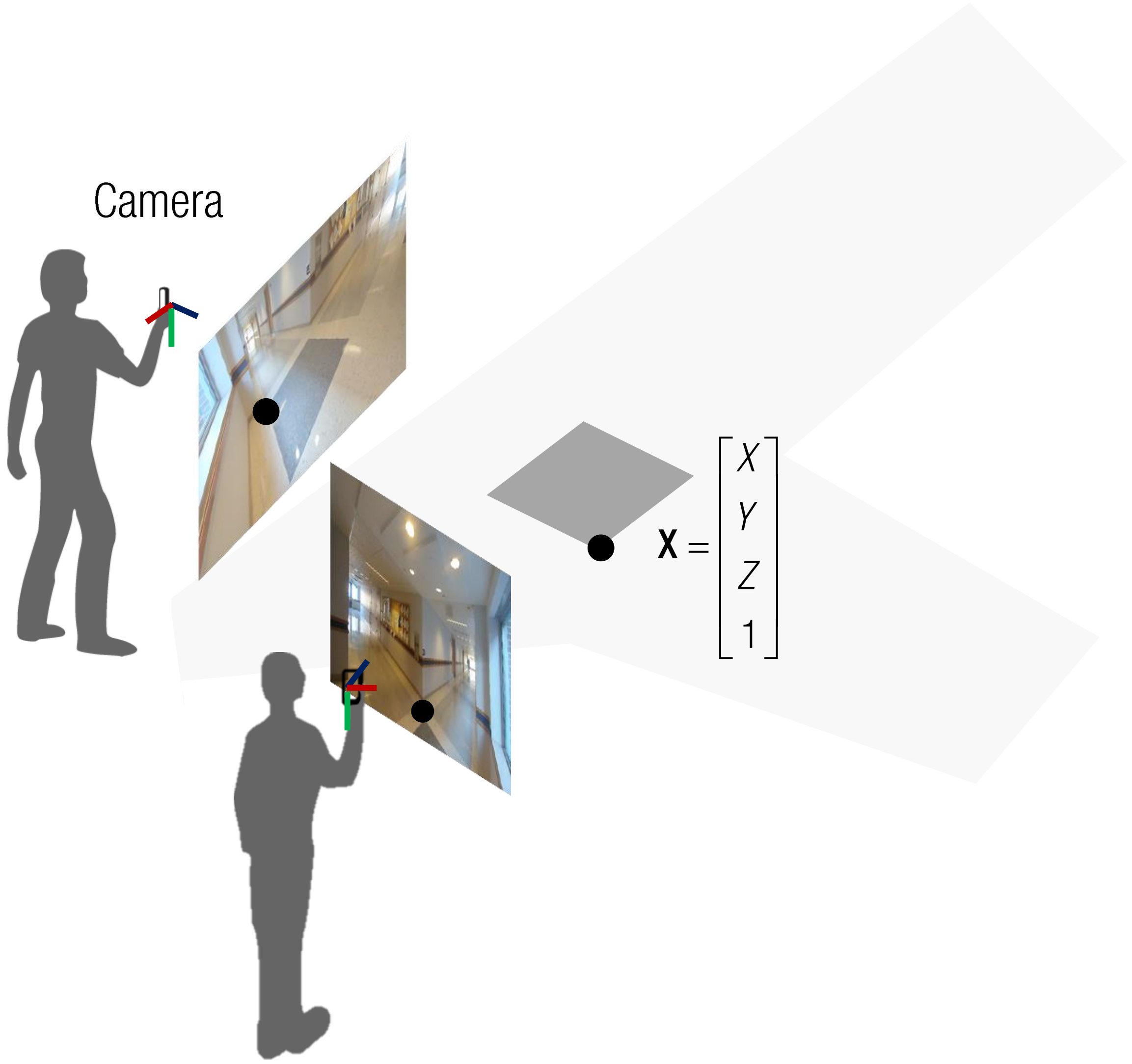
Previous manual estimate

Camera Calibration via Vanishing Points



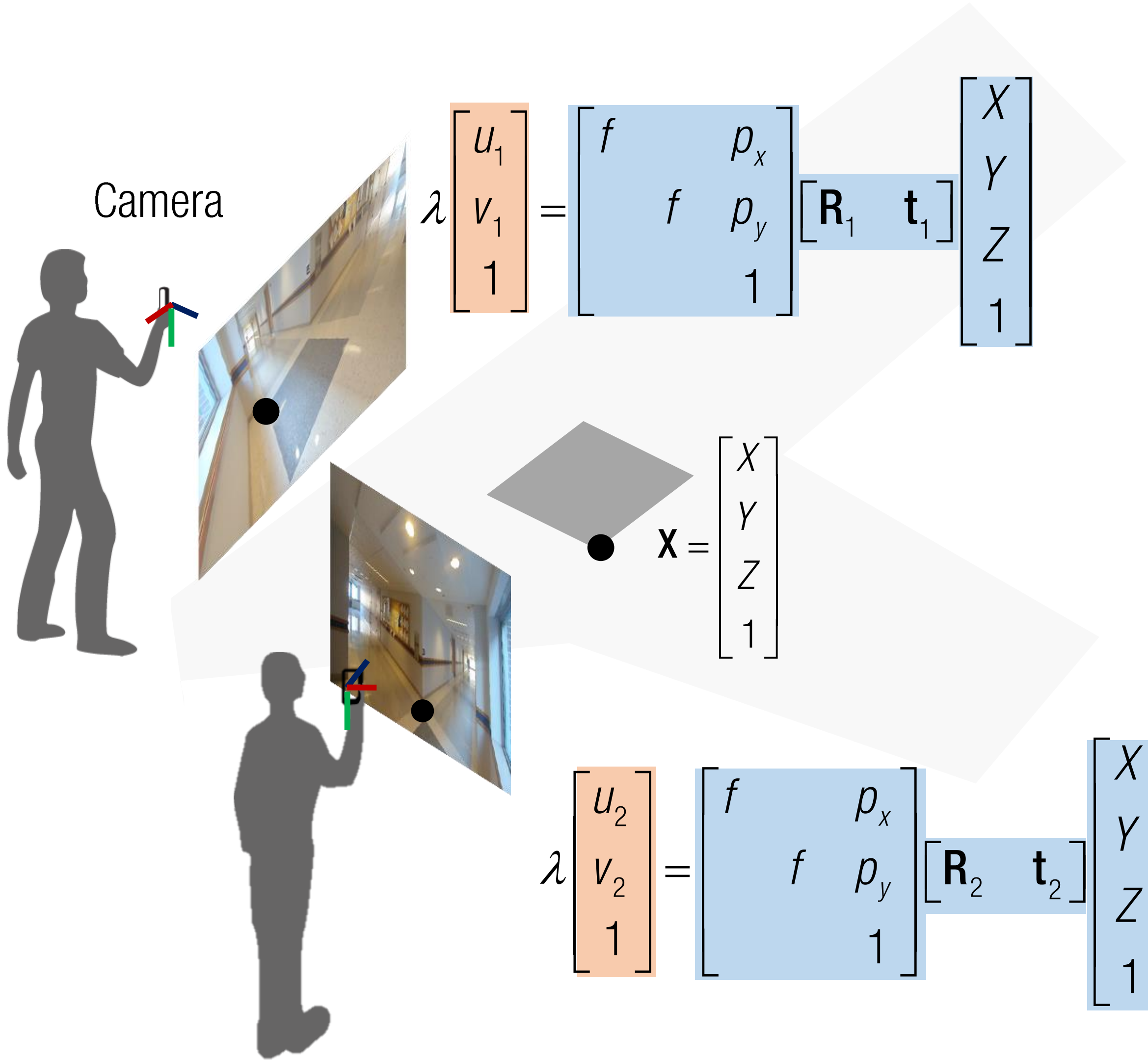


Multiview Camera Calibration



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

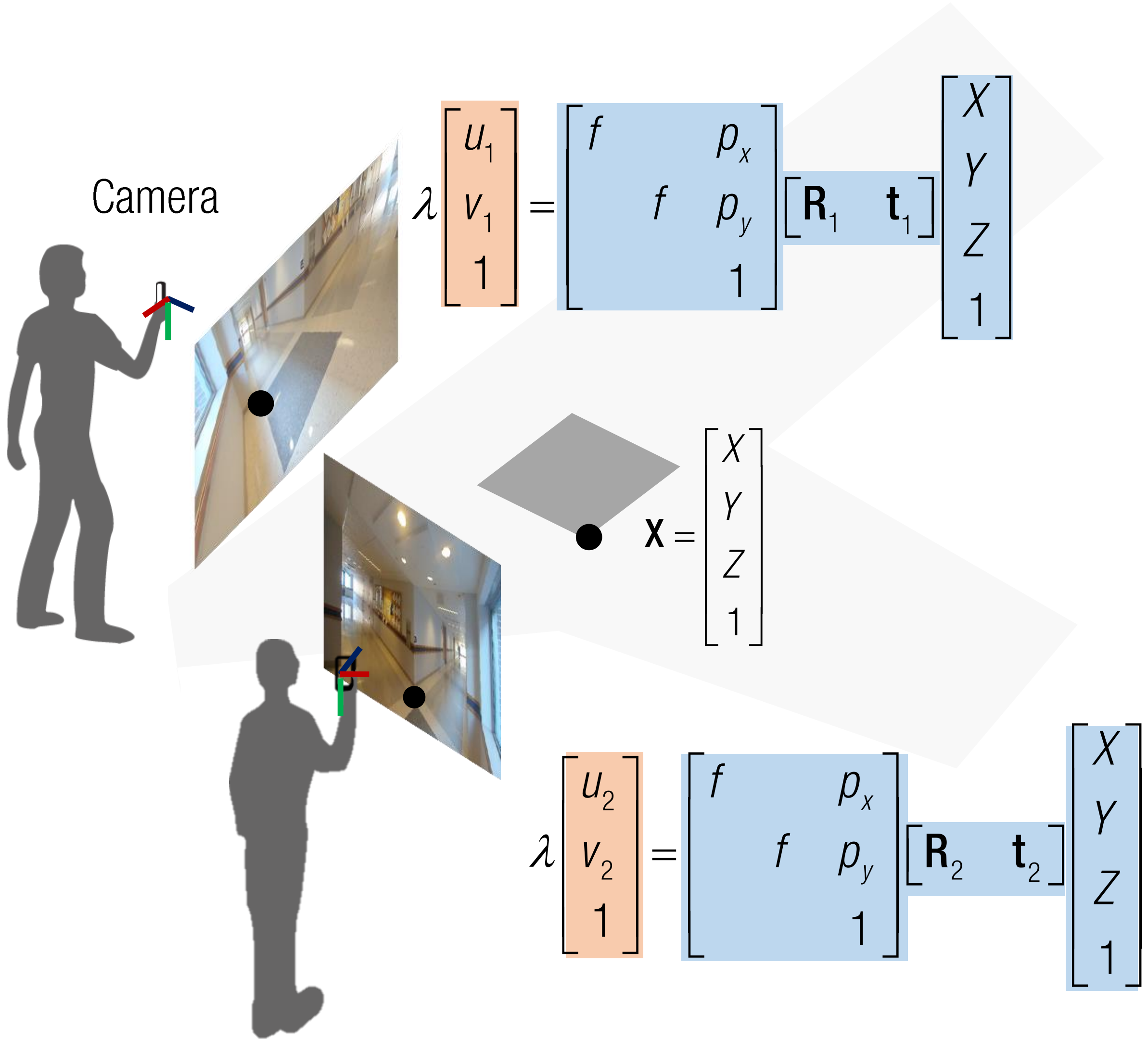
$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{t}_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{t}_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns:

n: the number of images

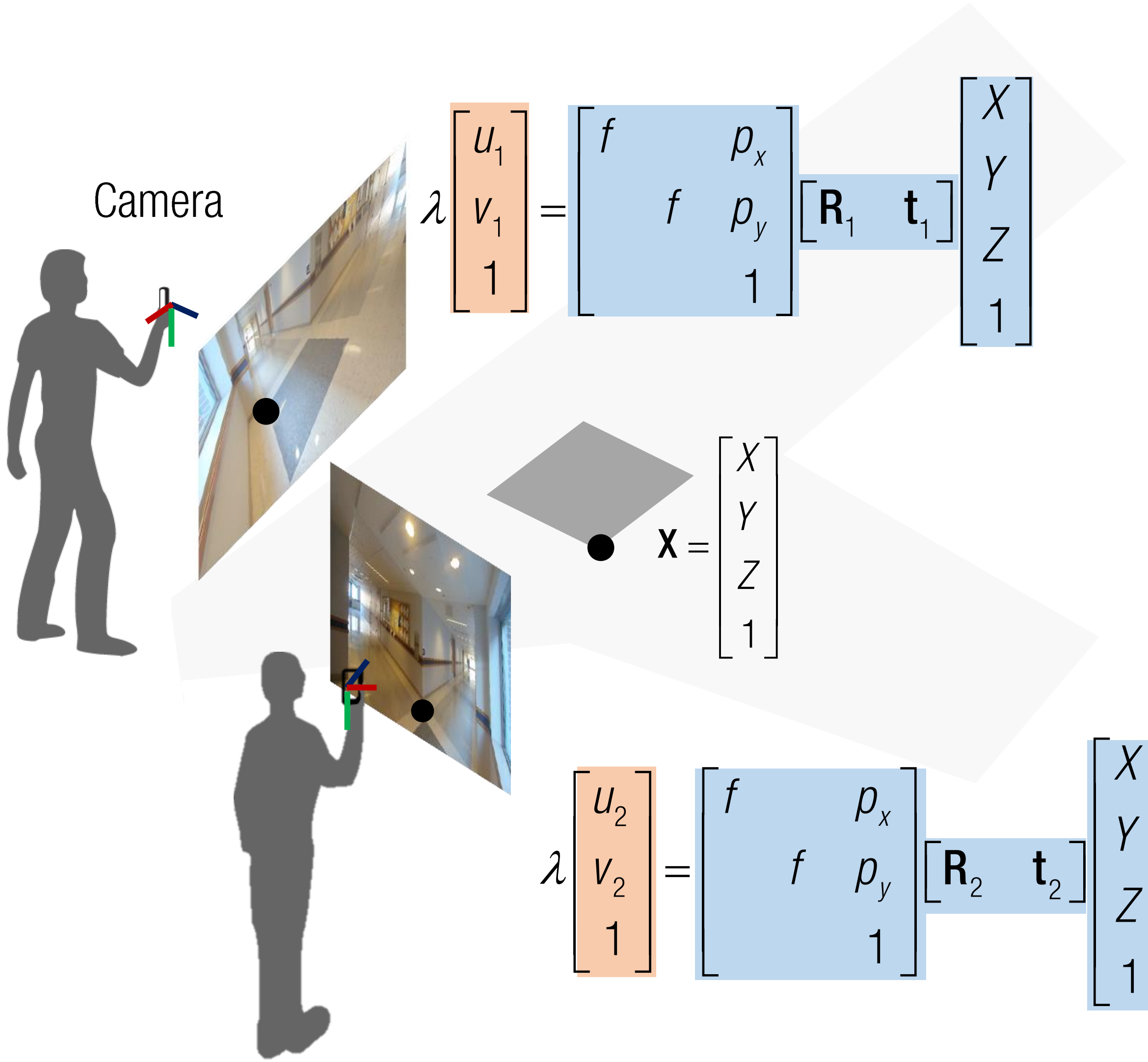
p: the number of points

# of equations:

: Knowns

: Unknowns

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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# of unknowns: 3 (**K**) + 6*n* (**R** and **t**) + 3*p* (**X**)

n: the number of images

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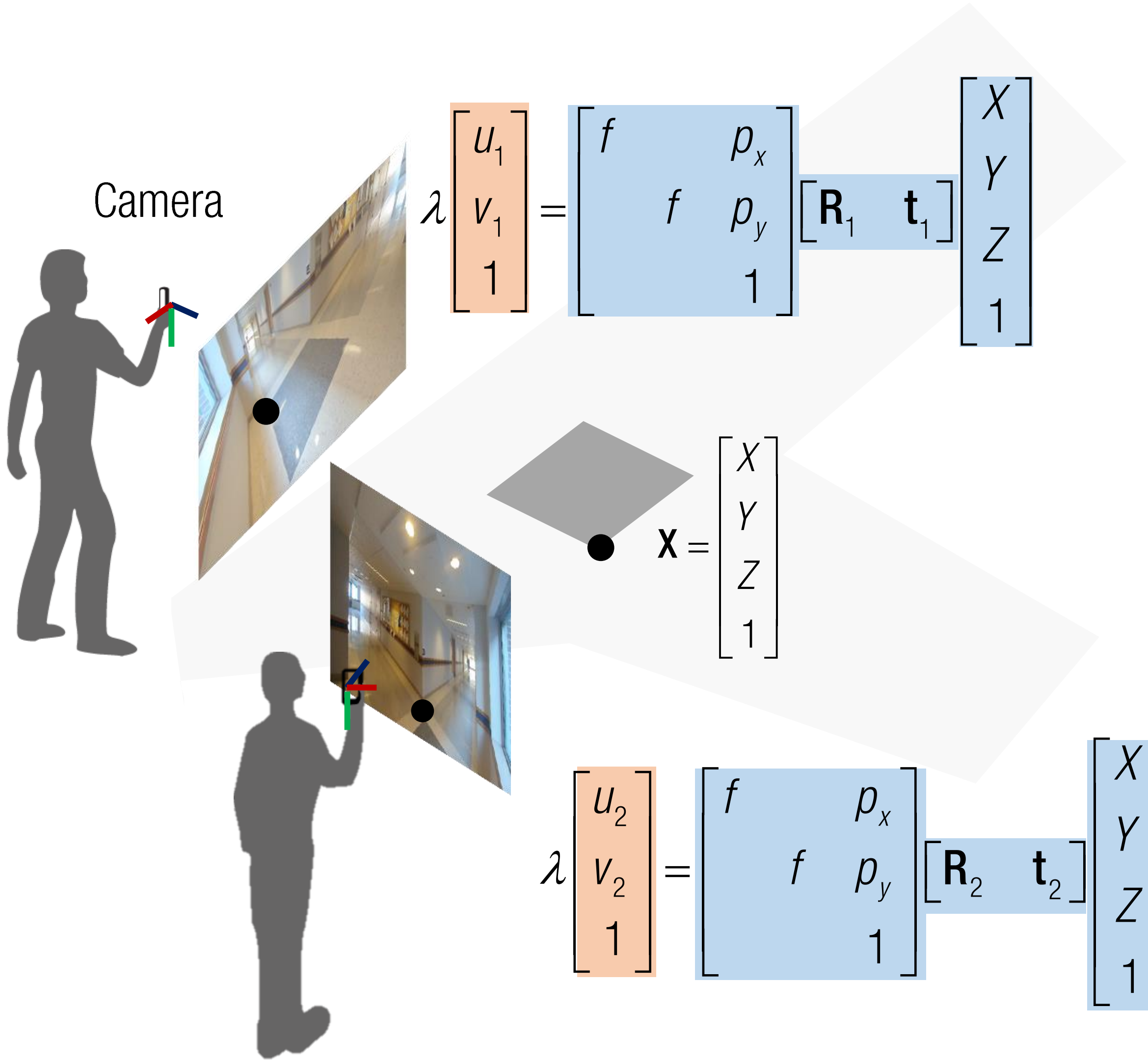
# of equations:

: Knowns

: Unknowns

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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# of unknowns: 3 (**K**) + 6n (**R** and **t**) + 3p (**X**)

n: the number of images

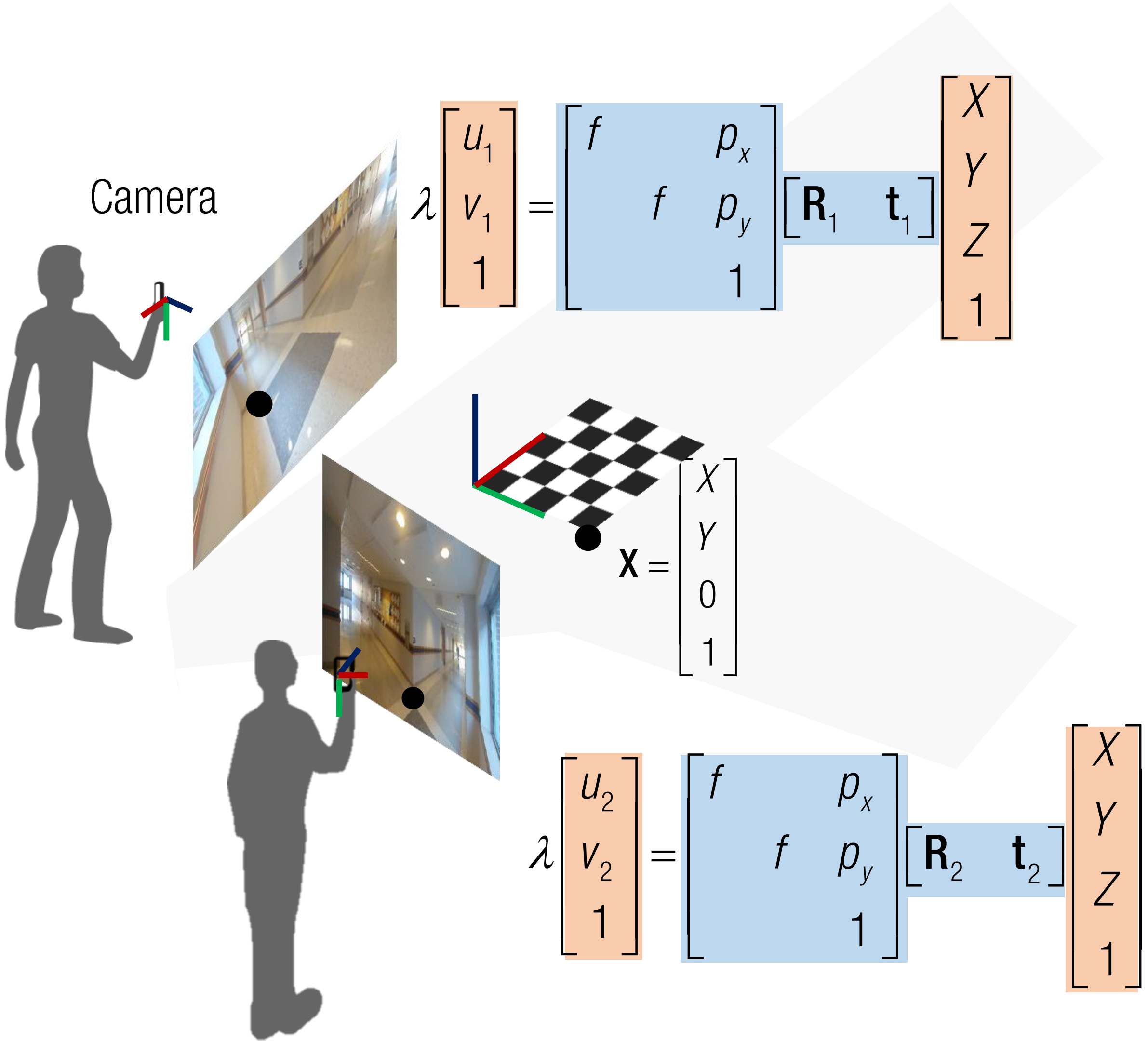
p: the number of points

# of equations: 2np

: Knowns

: Unknowns

Insight: Known Common 3D Points



# of unknowns: 3 (**K**) + 6n (**R** and **t**)

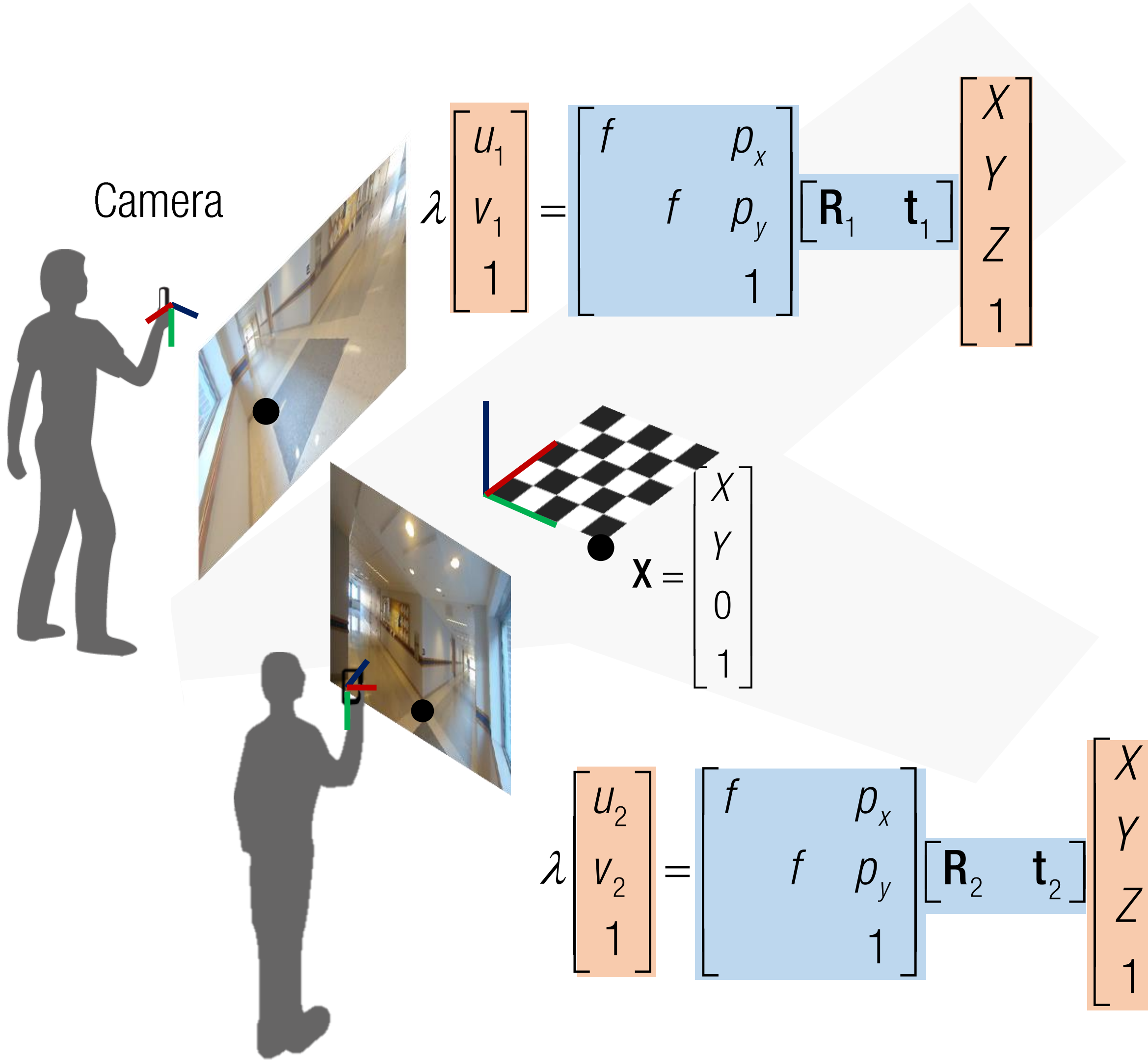
n: the number of images
p: the number of points

# of equations: 2np

: Knowns

: Unknowns

Insight: Known Common 3D Points



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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# of unknowns: 3 (**K**) + 6n (**R** and **t**)

n: the number of images

p: the number of points

# of equations: 2np

We can solve for **K**, **R**, **t** if $3 + 6n < 2\ nm$

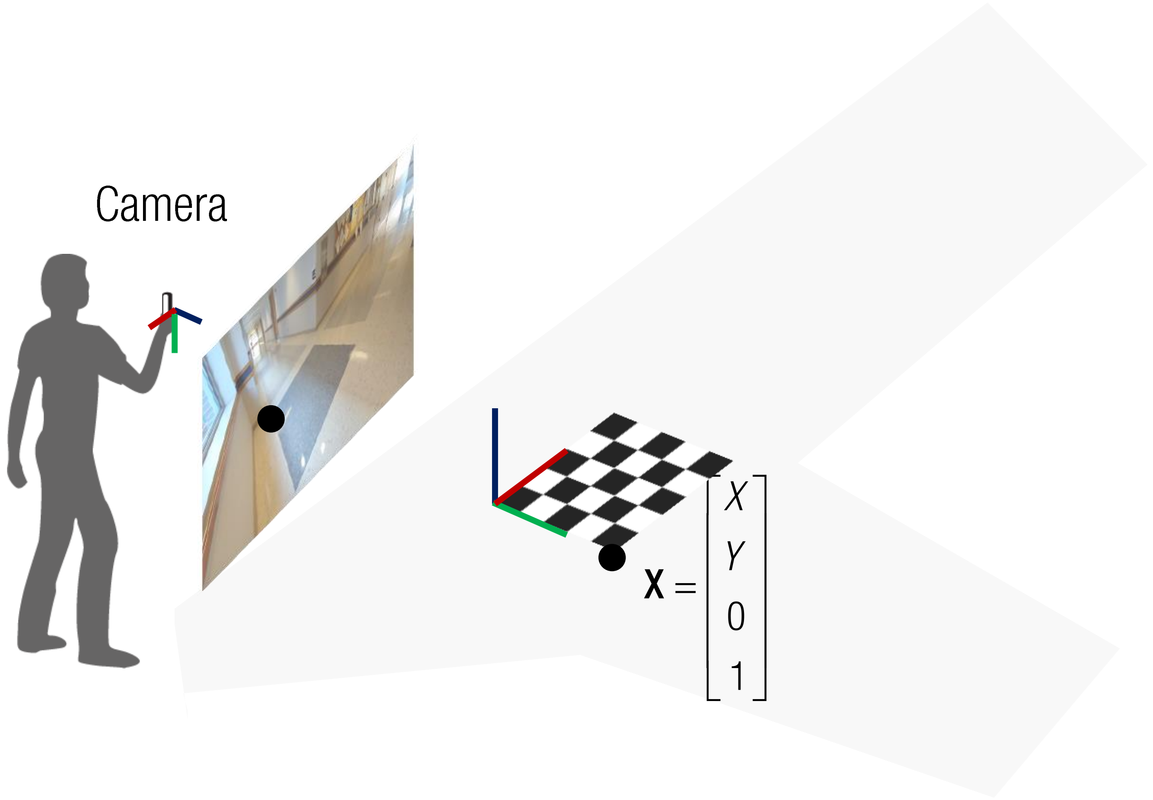
: Knowns

: Unknowns

Homography

Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Homography

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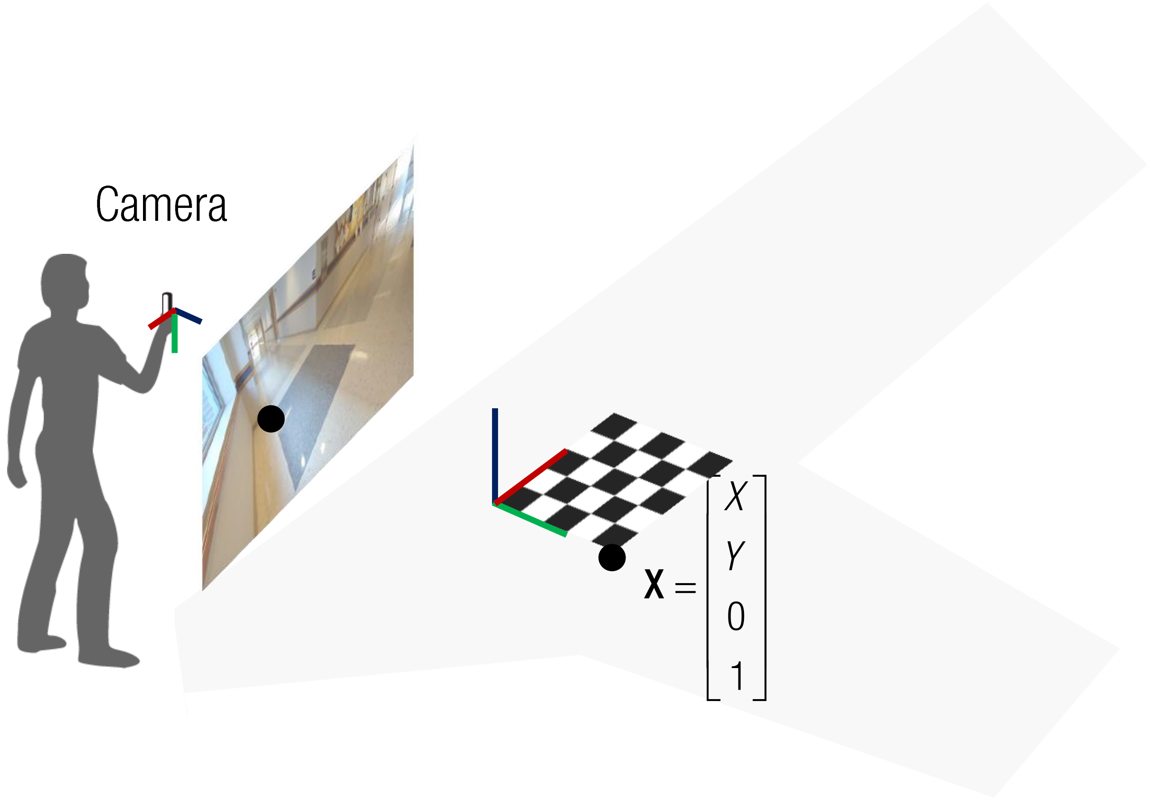
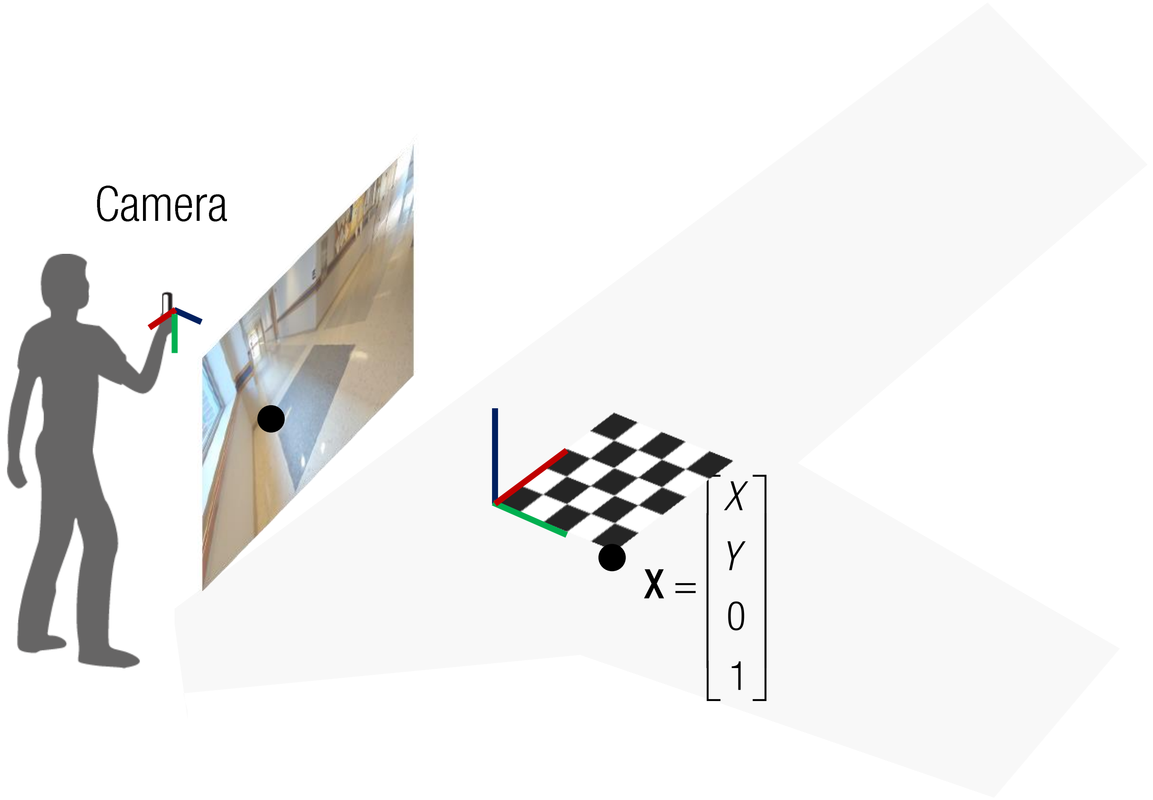
$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



Homography



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Points in 2D plane are mapped to an image with homography:

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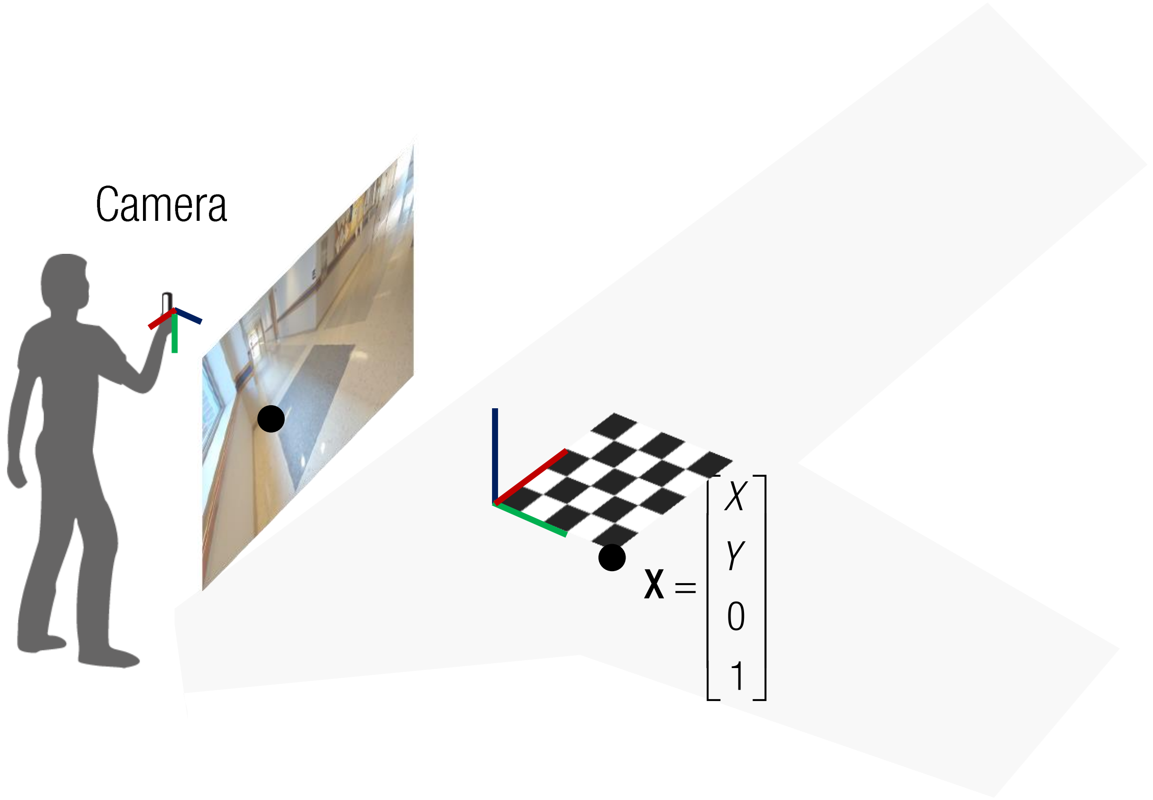
Homography



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Points in 2D plane are mapped to an image with homography:

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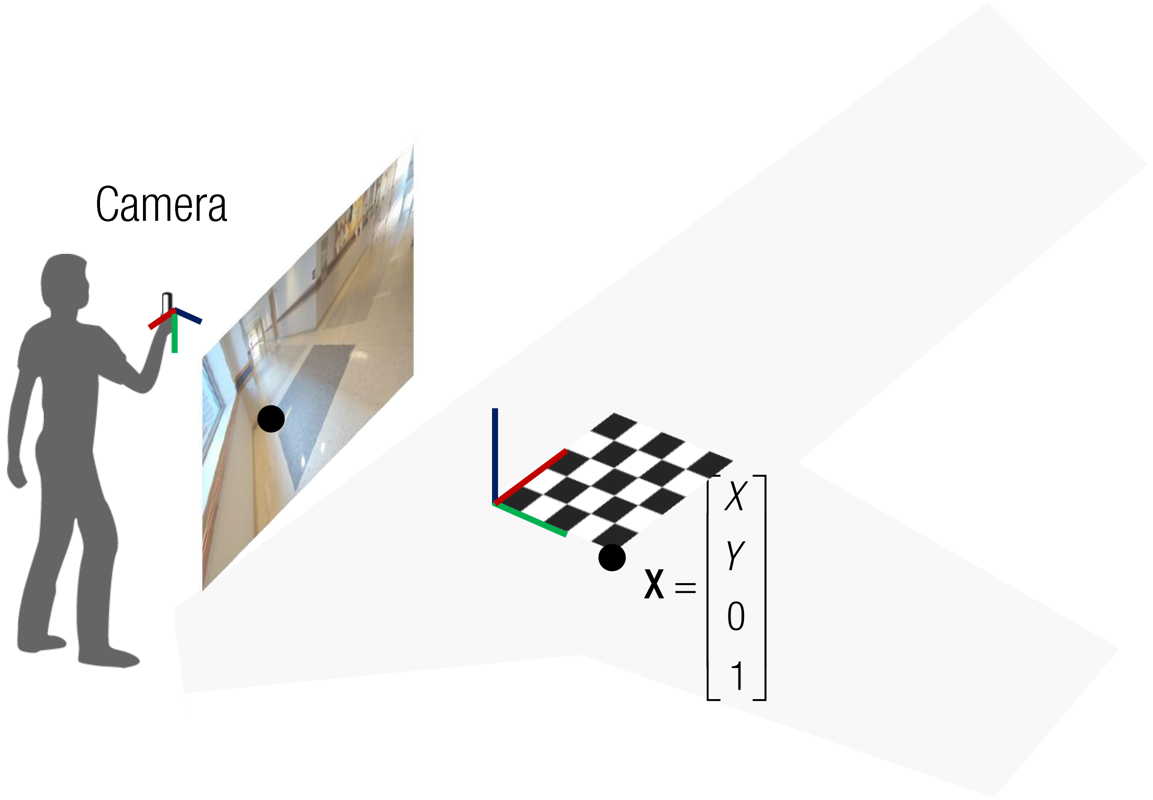
1. Compute homography

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography



Points in 2D plane are mapped to an image with homography:

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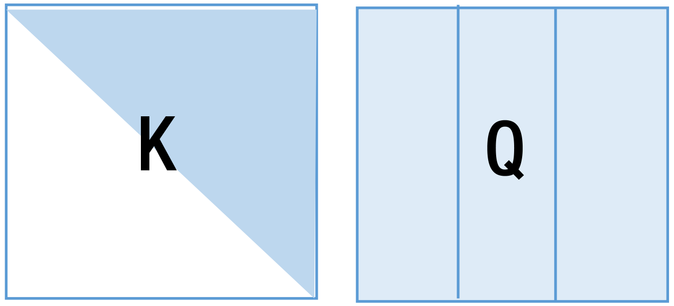
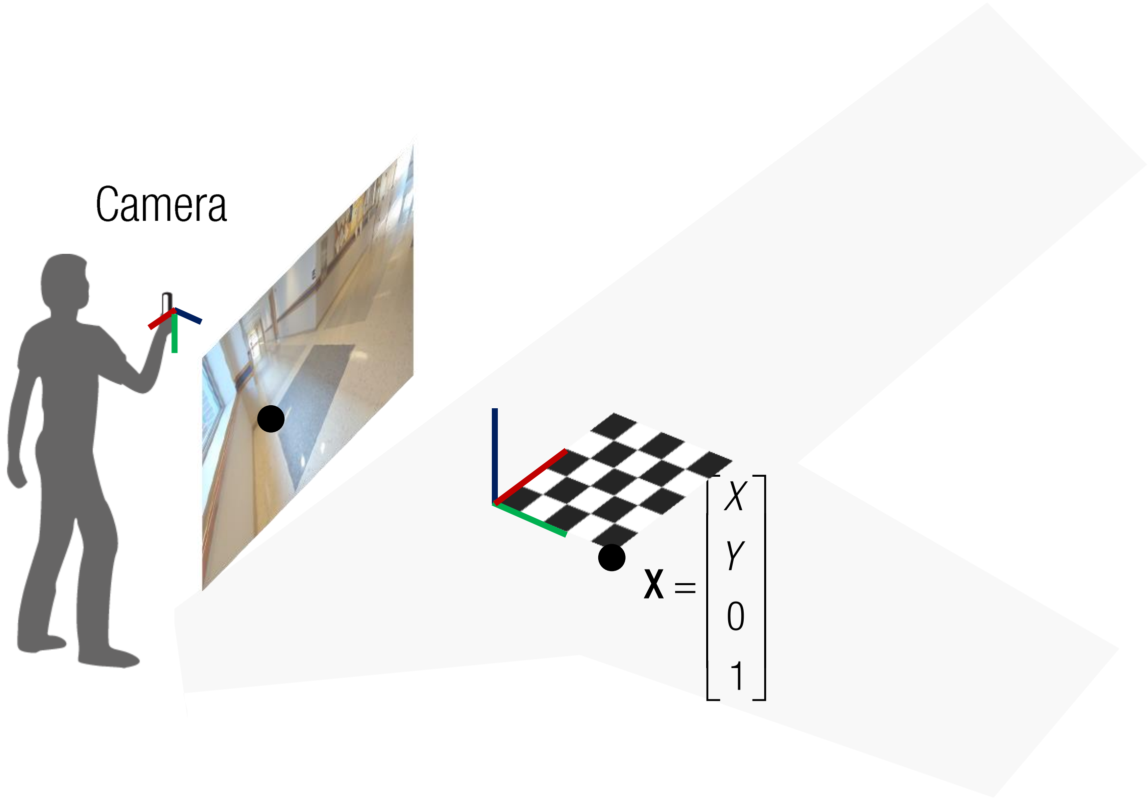
$\mathbf{H} =$



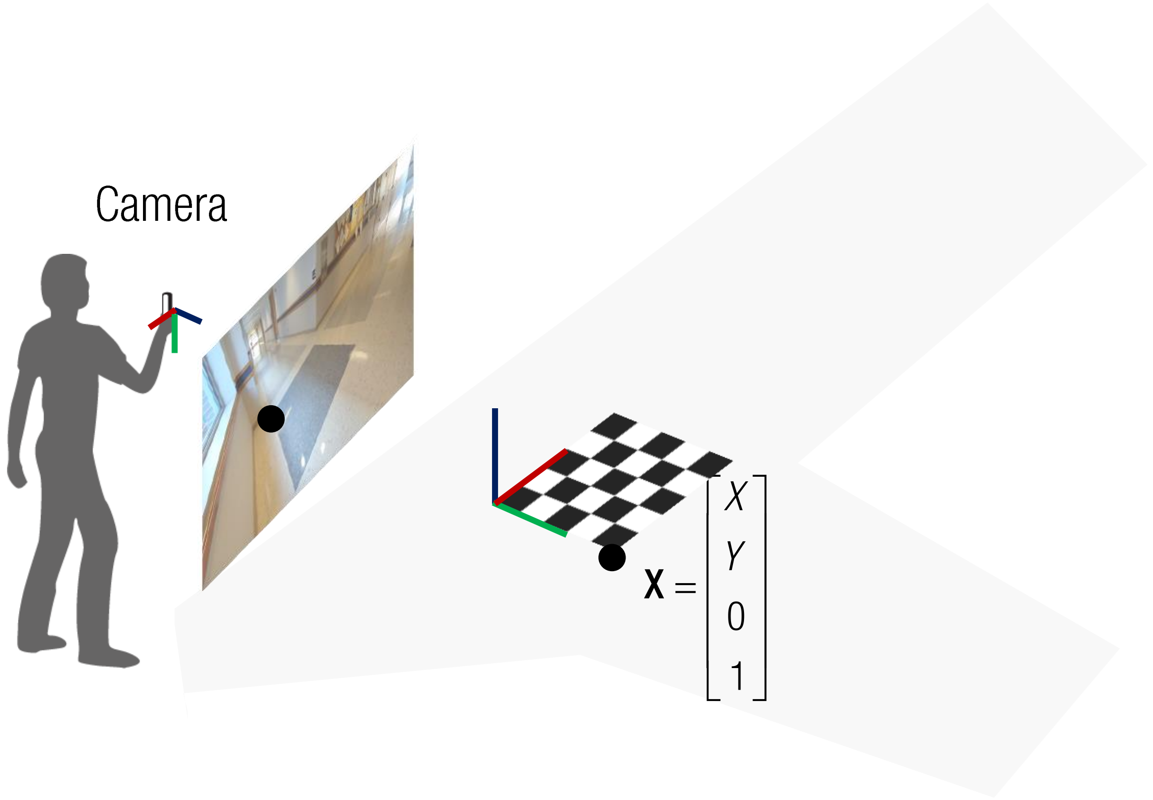
Method1: RQ Decomposition

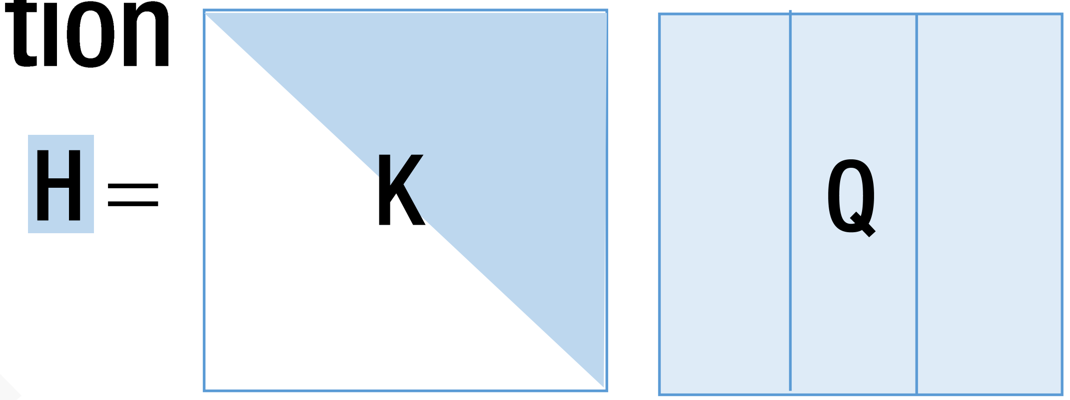
$$H = KQ$$

Upper triangle matrix (K) | Othogonal matrix (Q)



Method1: RQ Decomposition

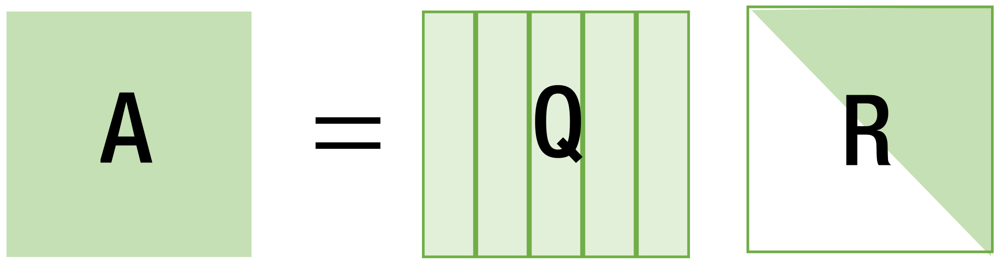




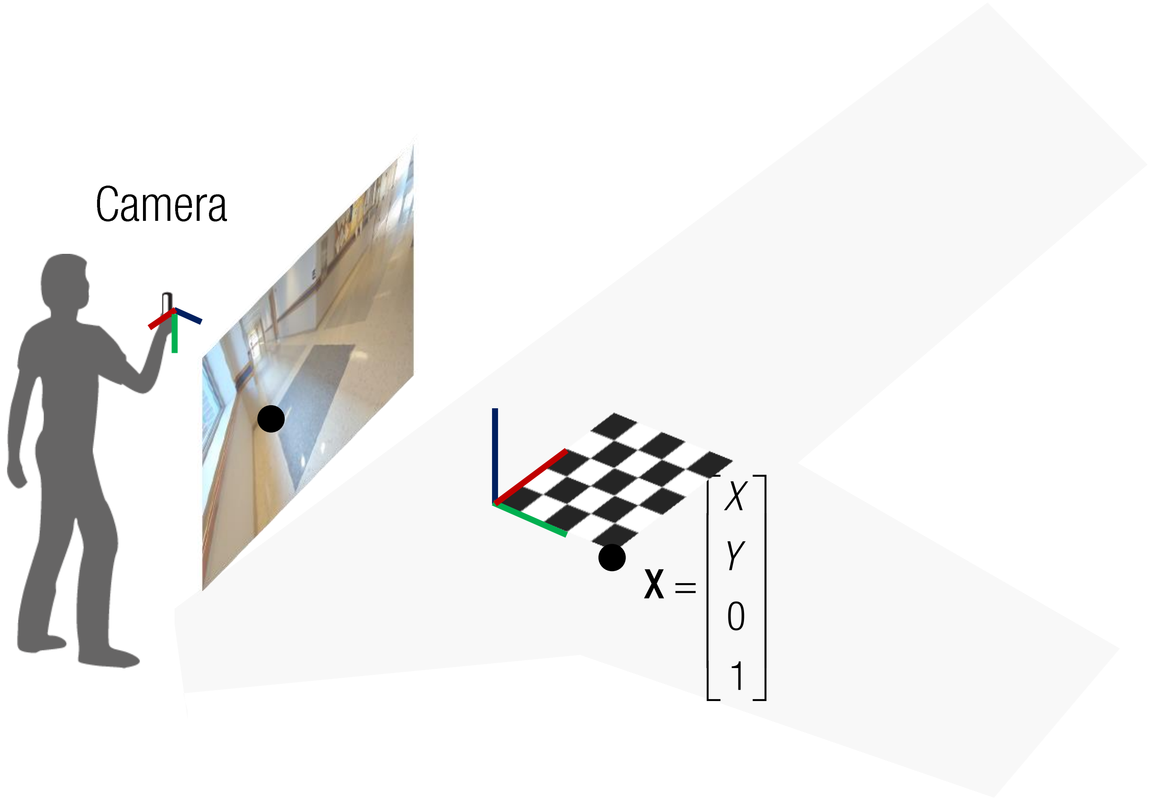
Upper triangle matrix

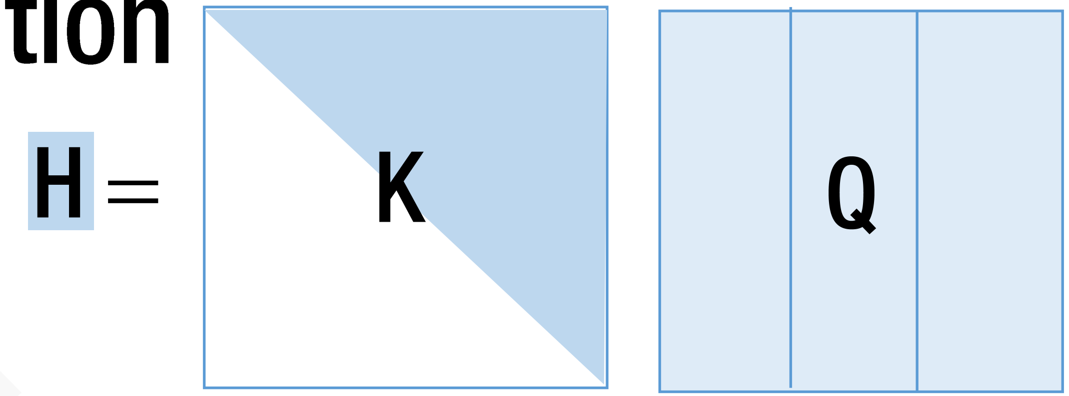
Othogonal matrix

QR decomposition:



Method1: RQ Decomposition

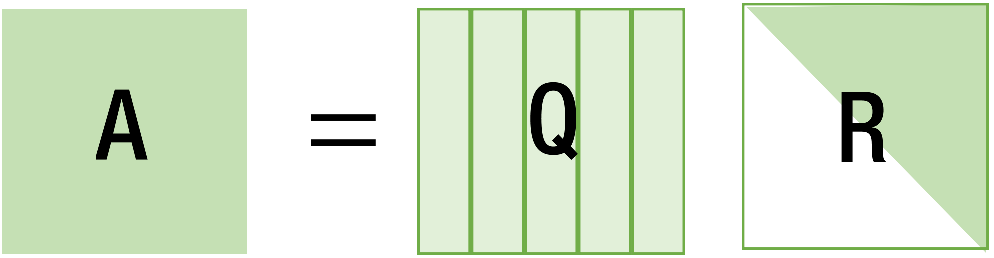




Upper triangle matrix

Othogonal matrix

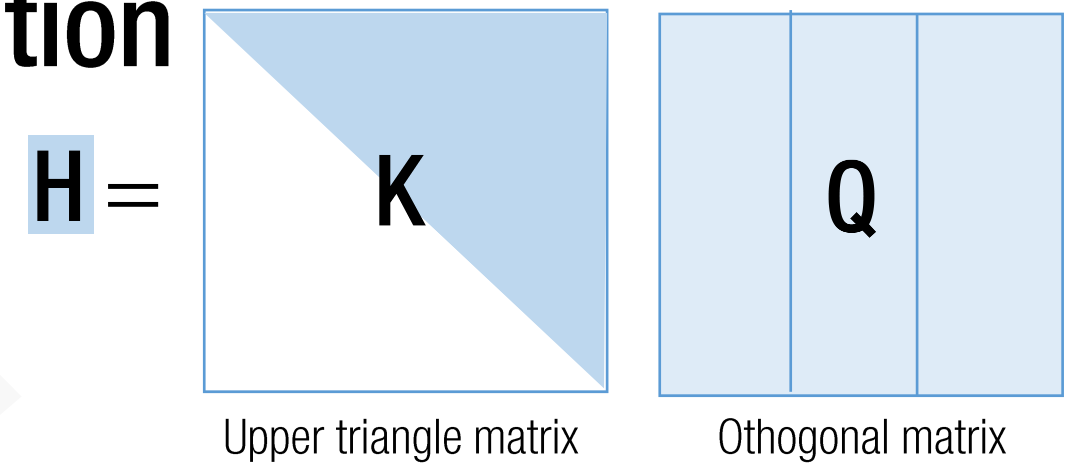
QR decomposition:

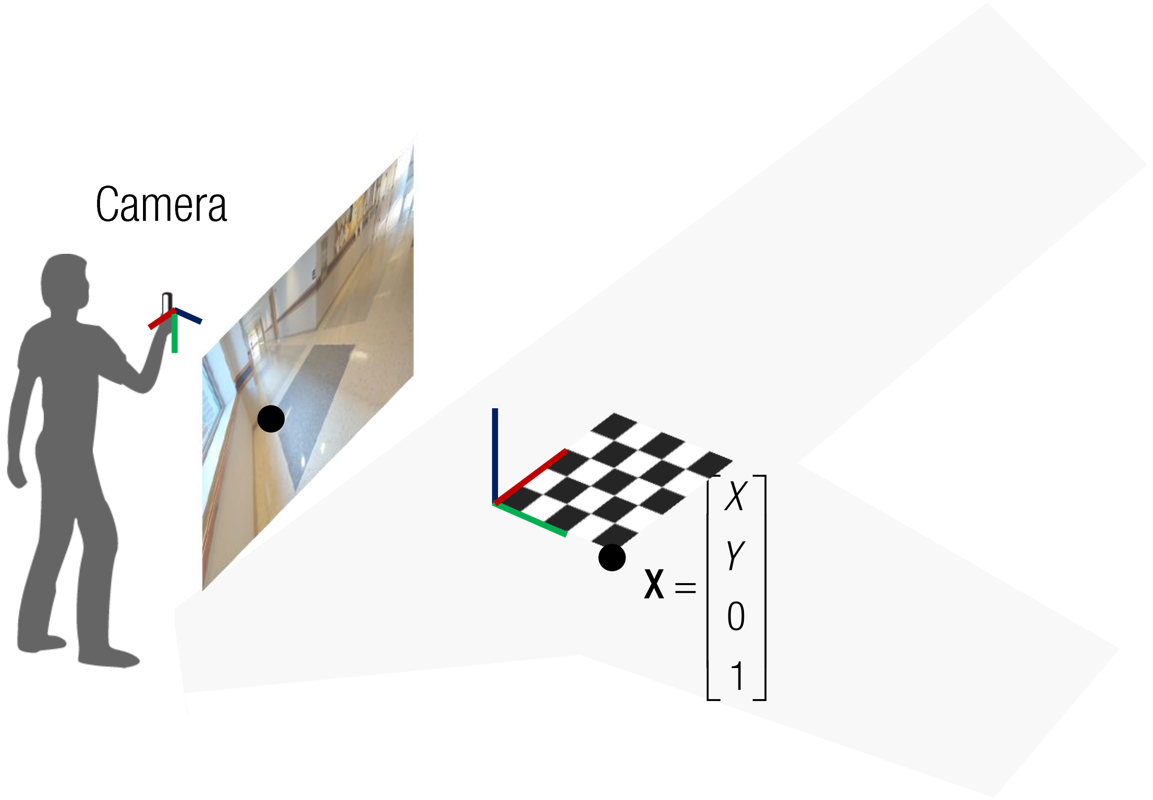


MATLAB

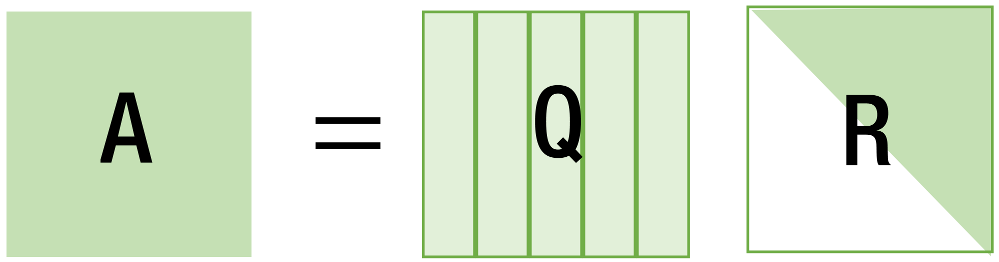
[Q R] = qr(A)

Method1: RQ Decomposition





QR decomposition:

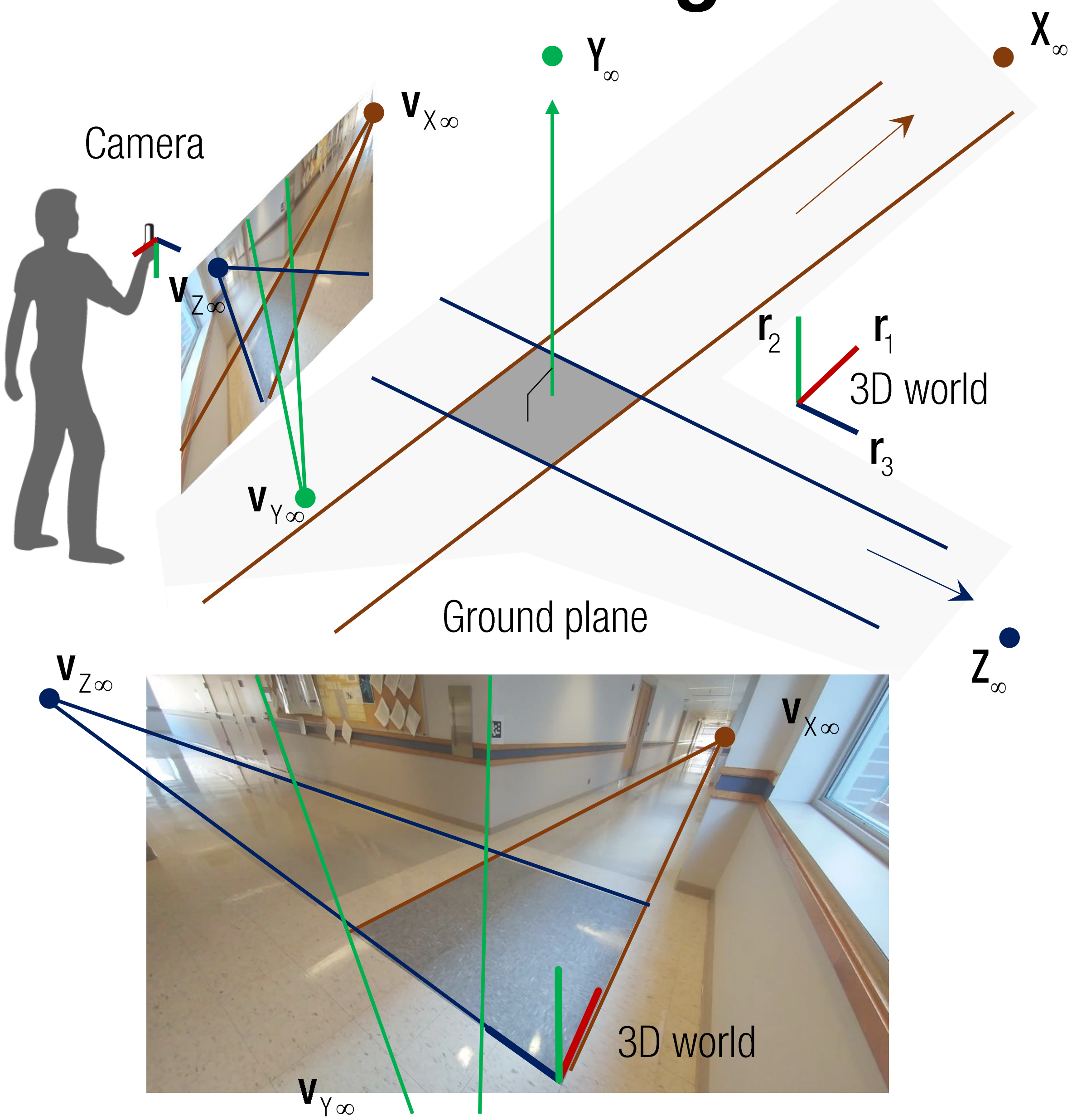


MATLAB

[Q R] = qr(A)

HW: How to convert **QR** to **RQ**?

Recall: Vanishing Points



$$\lambda \mathbf{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{X}_\infty \quad \lambda \mathbf{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Z}_\infty \quad \lambda \mathbf{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{R}\mathbf{Y}_\infty$$

$$\lambda \mathbf{K}^{-1}\mathbf{v}_{X\infty} = \mathbf{R}\mathbf{X}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Y\infty} = \mathbf{R}\mathbf{Y}_\infty \qquad \lambda \mathbf{K}^{-1}\mathbf{v}_{Z\infty} = \mathbf{R}\mathbf{Z}_\infty$$

Note that the camera extrinsic is still unknown (**R** and **t**).

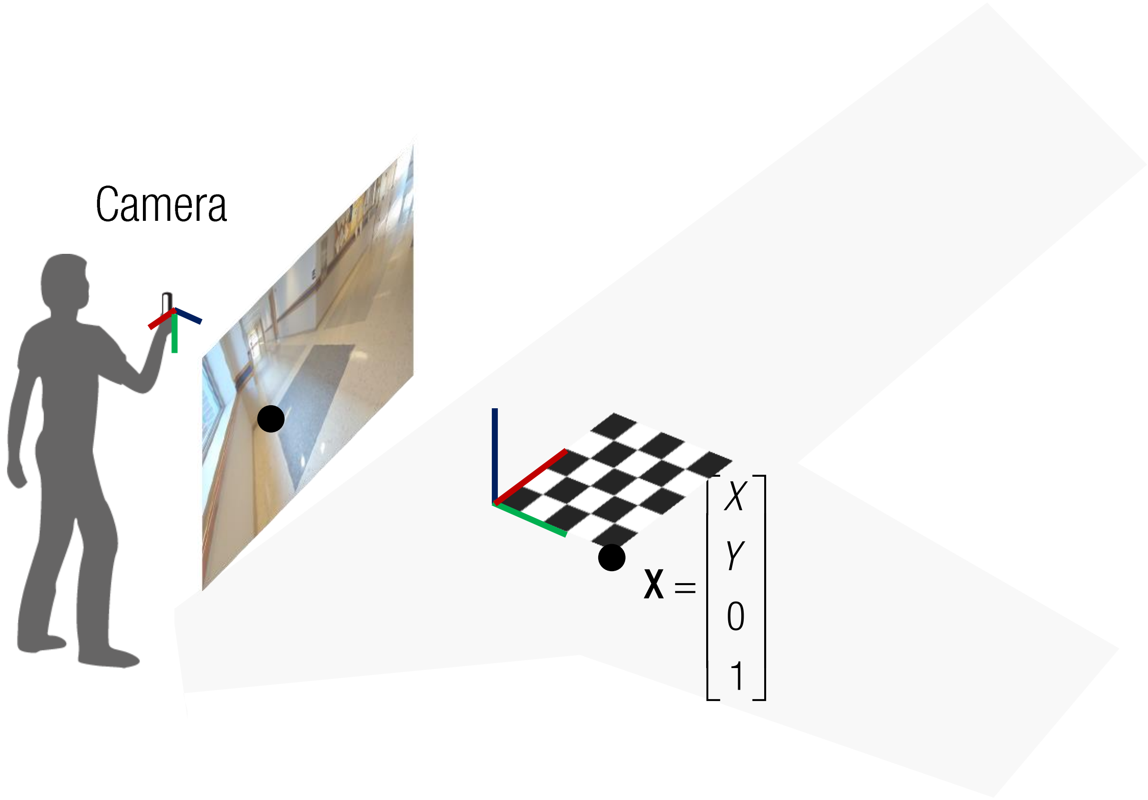
Known property of points at infinity:

$$\begin{aligned} (\mathbf{X}_\infty)^T(\mathbf{Y}_\infty) &= 0 \\ (\mathbf{Y}_\infty)^T(\mathbf{Z}_\infty) &= 0 \\ (\mathbf{Z}_\infty)^T(\mathbf{X}_\infty) &= 0 \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} (\mathbf{R}\mathbf{X}_\infty)^T(\mathbf{R}\mathbf{Y}_\infty) &= 0 \\ (\mathbf{R}\mathbf{Y}_\infty)^T(\mathbf{R}\mathbf{Z}_\infty) &= 0 \\ (\mathbf{R}\mathbf{Z}_\infty)^T(\mathbf{R}\mathbf{X}_\infty) &= 0 \end{aligned}$$

$$(\mathbf{K}^{-1}\mathbf{v}_{X\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Y\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Y\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{Z\infty}) = (\mathbf{K}^{-1}\mathbf{v}_{Z\infty})^T(\mathbf{K}^{-1}\mathbf{v}_{X\infty}) = 0$$

: 3 unknowns and 3 equations

Method2: Rotation



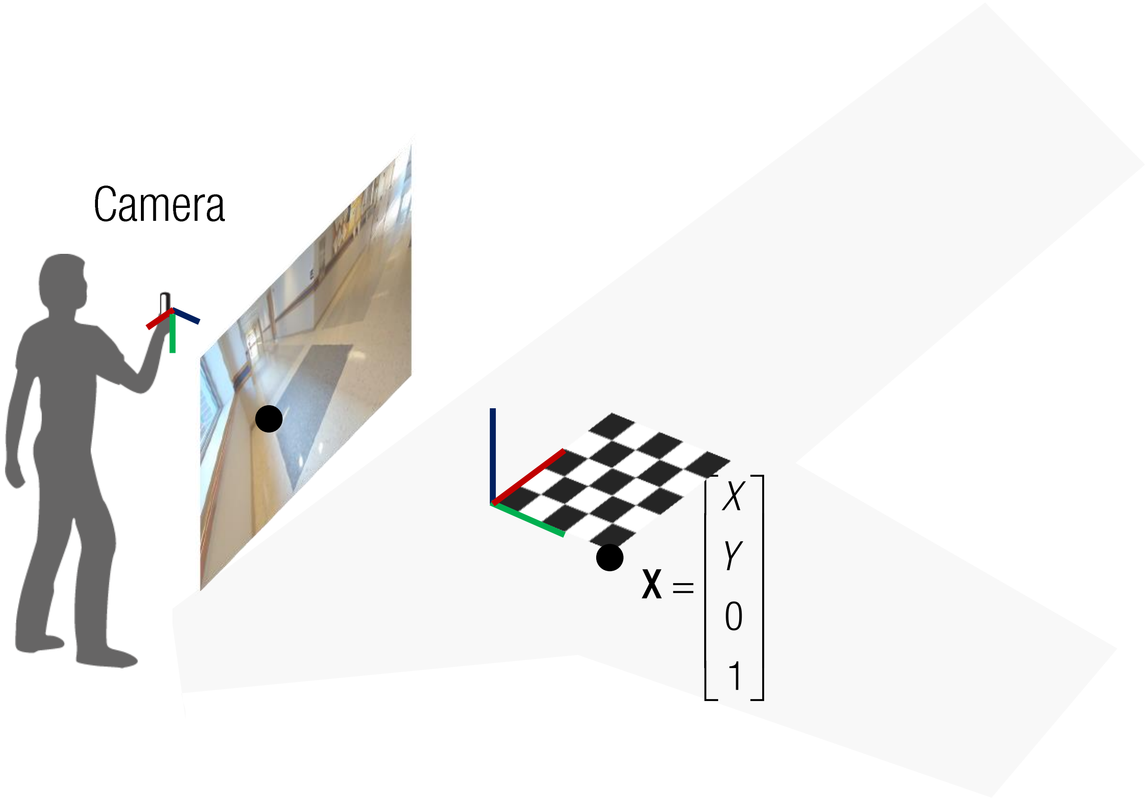
Homography factorization:

: Knowns

: Unknowns

$$\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

Method2: Rotation



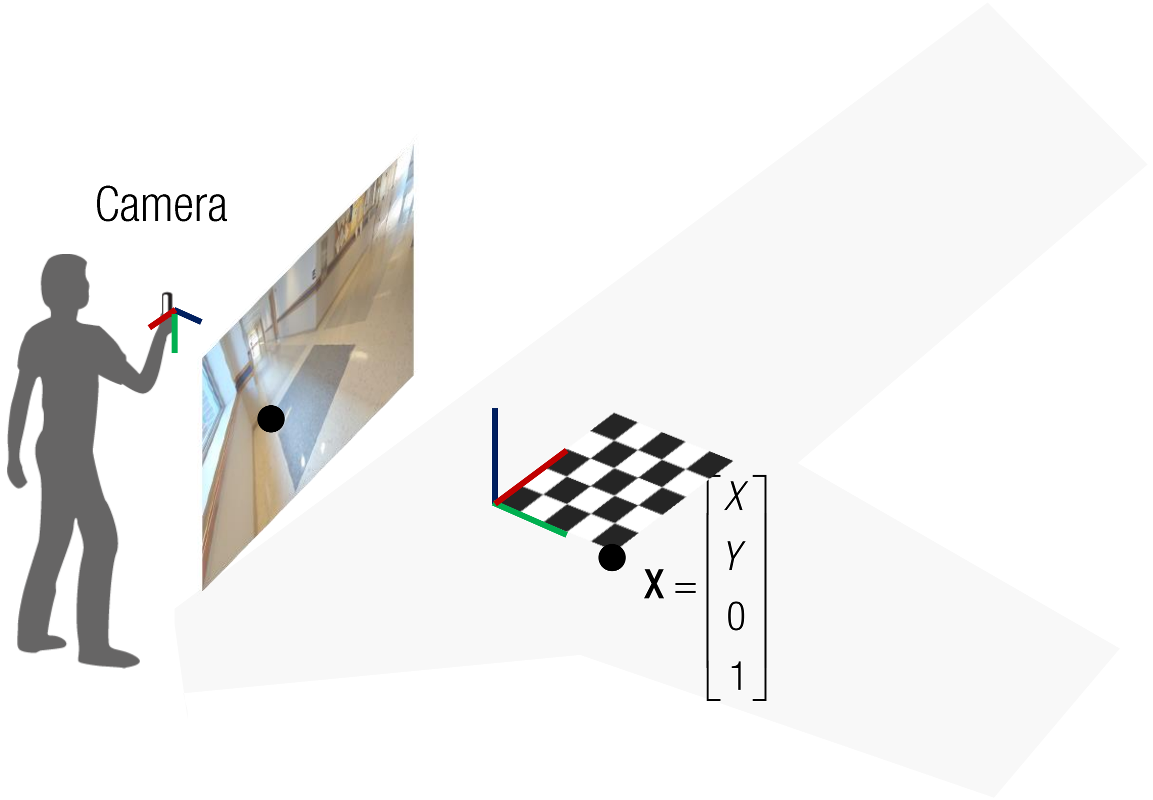
: Knowns

: Unknowns

Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

: Knowns

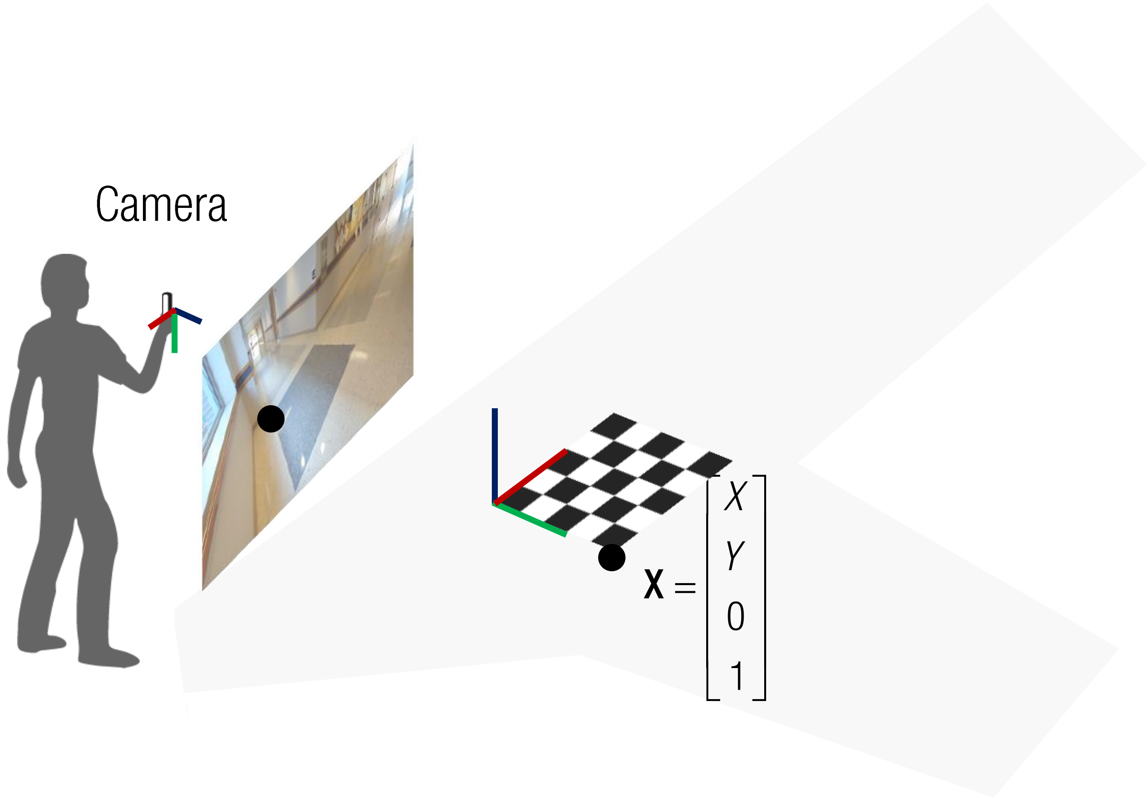
: Unknowns

Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^T\mathbf{r}_2 = 0 \qquad \left\|\mathbf{r}_1\right\| = 1 \qquad \left\|\mathbf{r}_2\right\| = 1$$

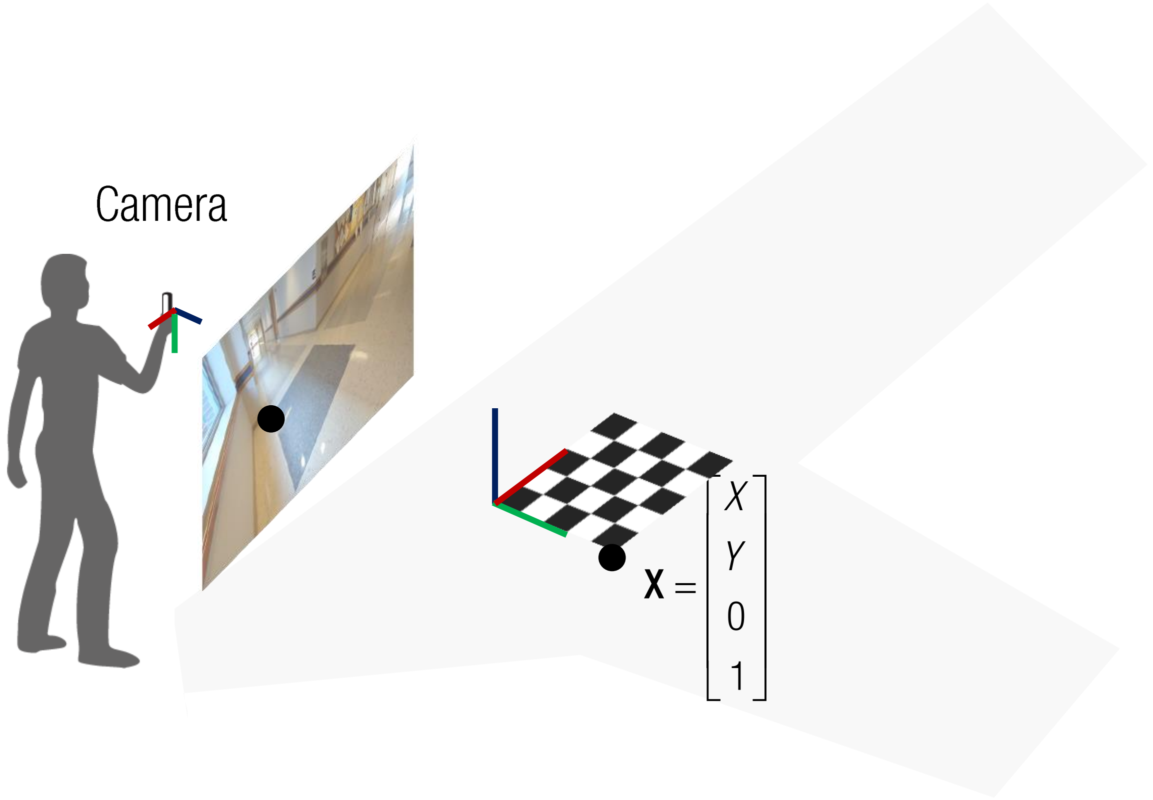
Method2: Rotation

$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$

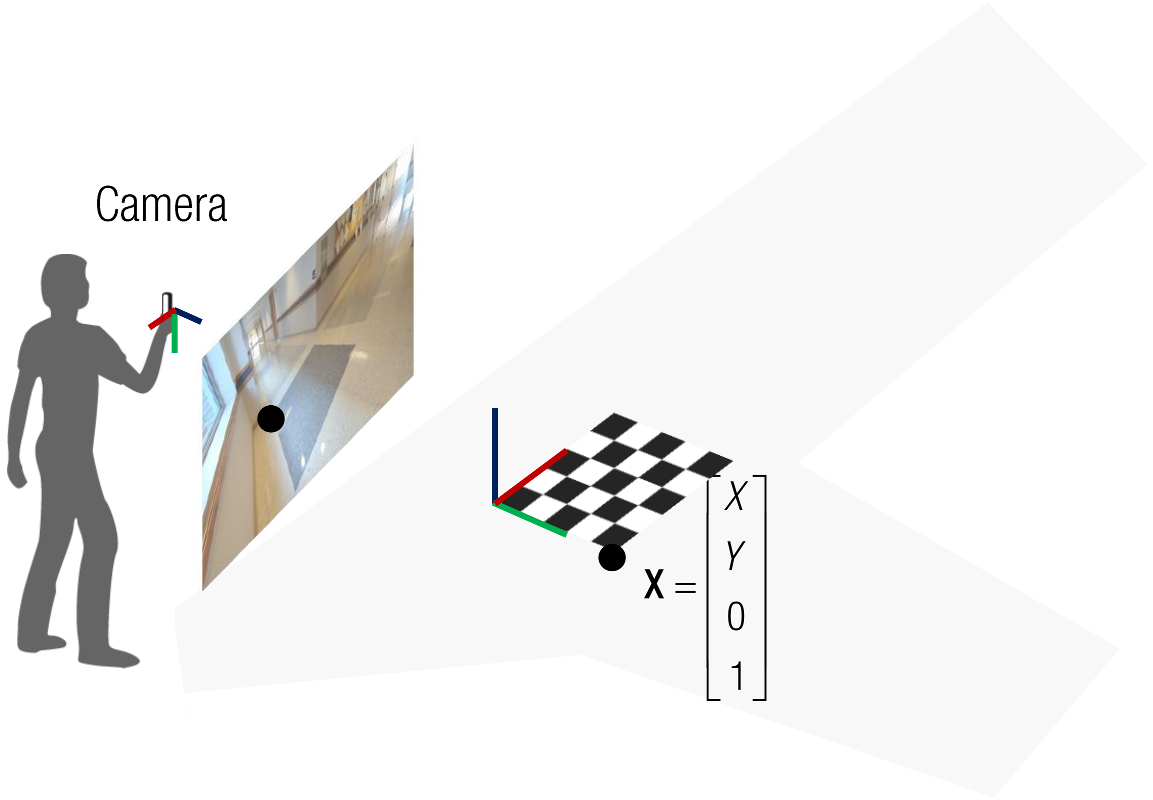
Orthogonality of rotation matrix property:

$\mathbf{r}_1^{\mathsf{T}}\mathbf{r}_2 = 0 \qquad \|\mathbf{r}_1\| = 1 \qquad \|\mathbf{r}_2\| = 1$

$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$



Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

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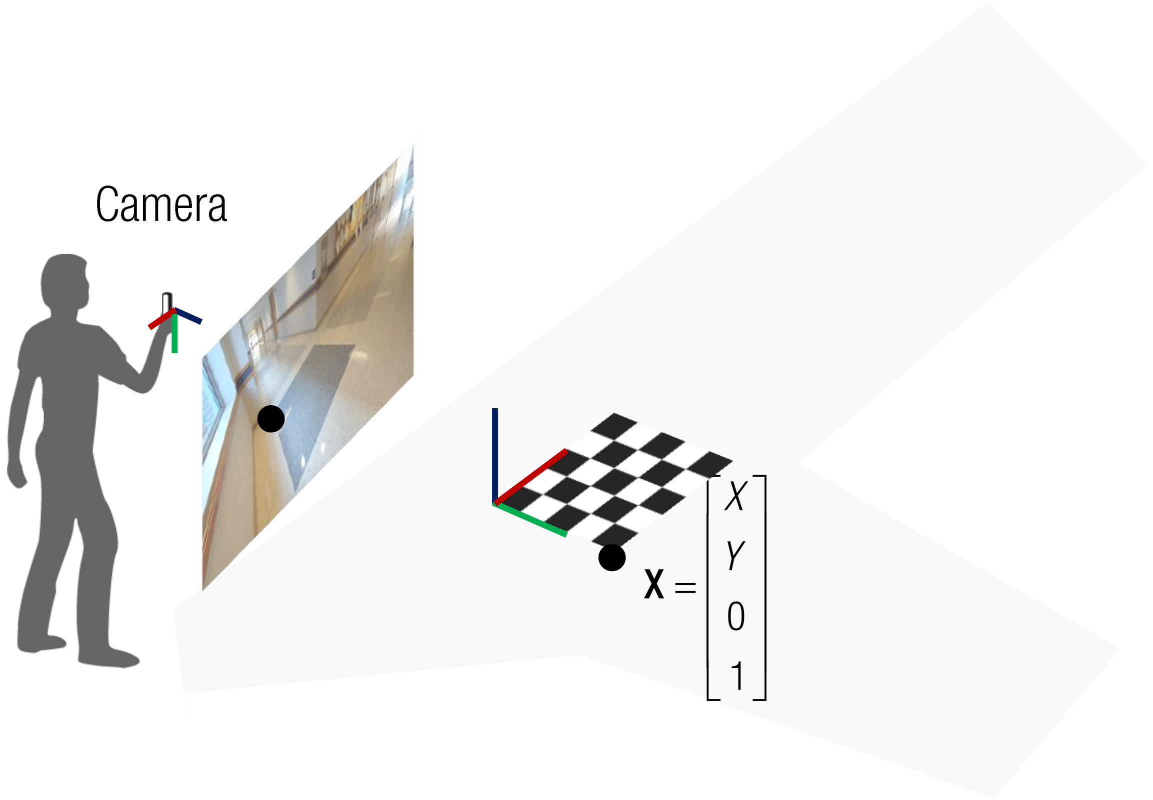
$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

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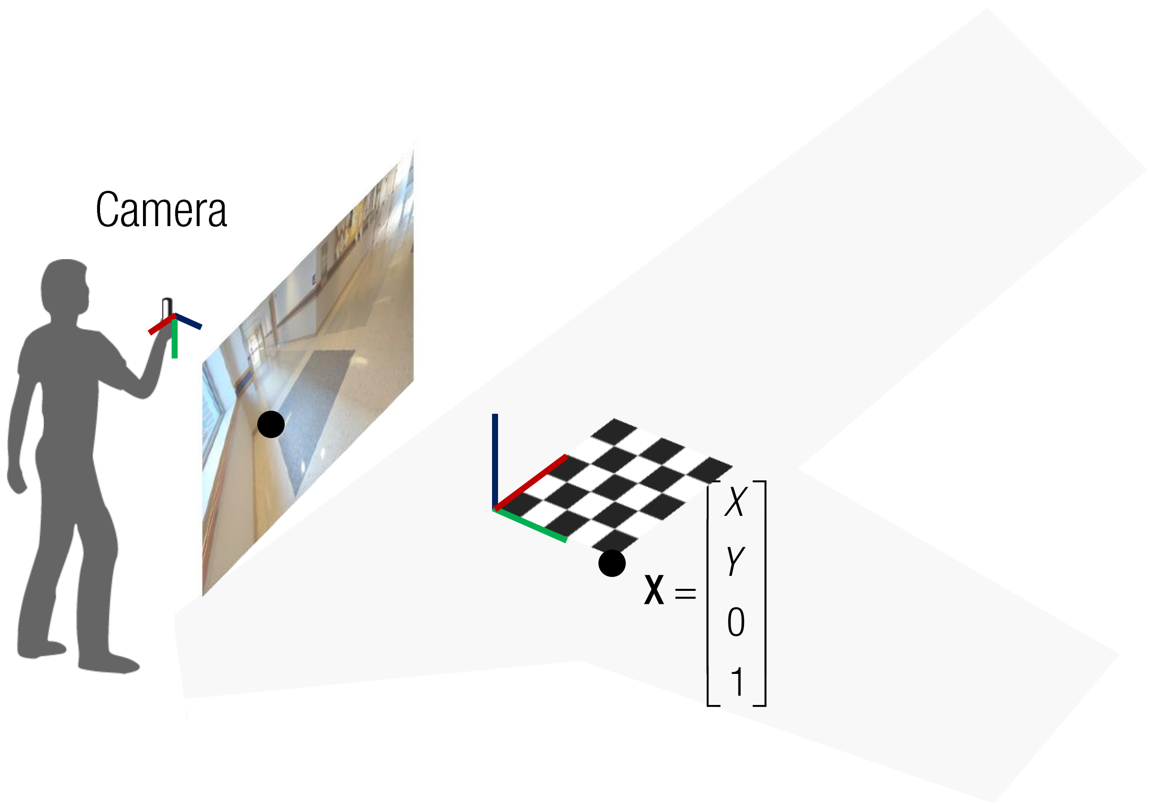
$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-T}\mathbf{K}^{-1} =$$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

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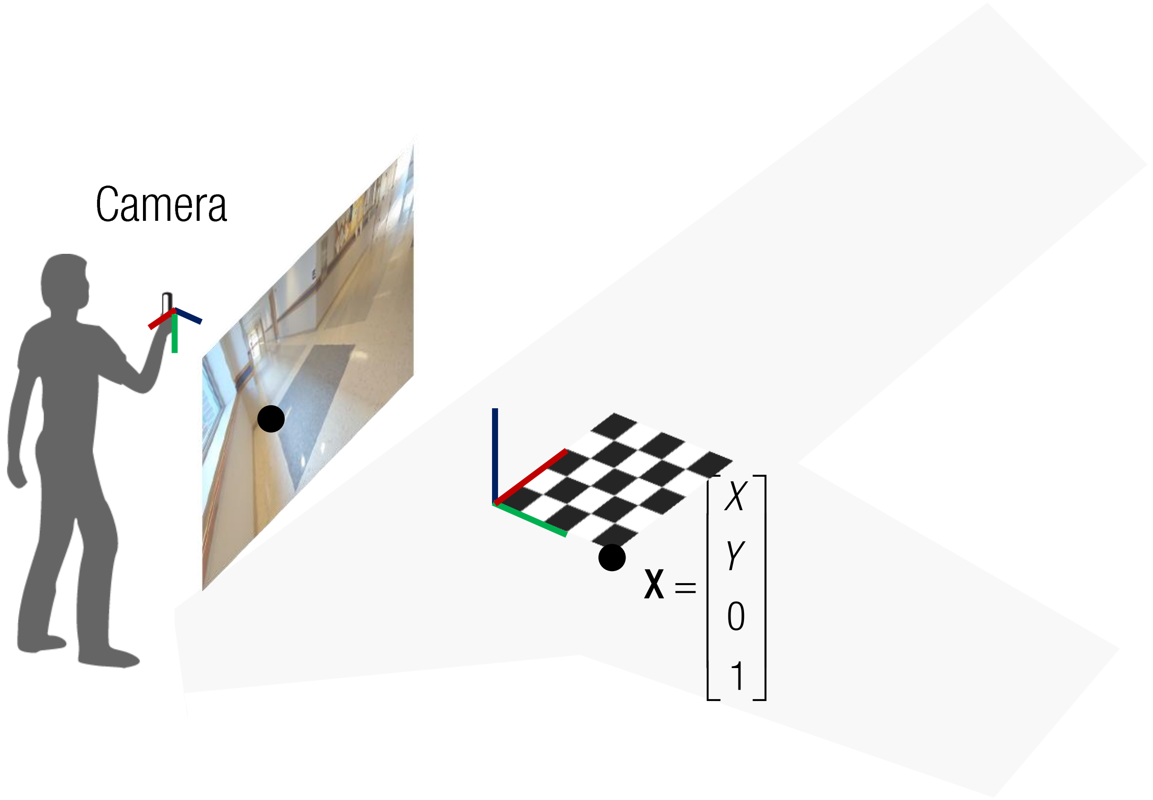
$$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-\mathsf{T}}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^{\mathrm{T}}\mathbf{r}_2 = 0 \qquad \left\|\mathbf{r}_1\right\| = 1 \qquad \left\|\mathbf{r}_2\right\| = 1$$

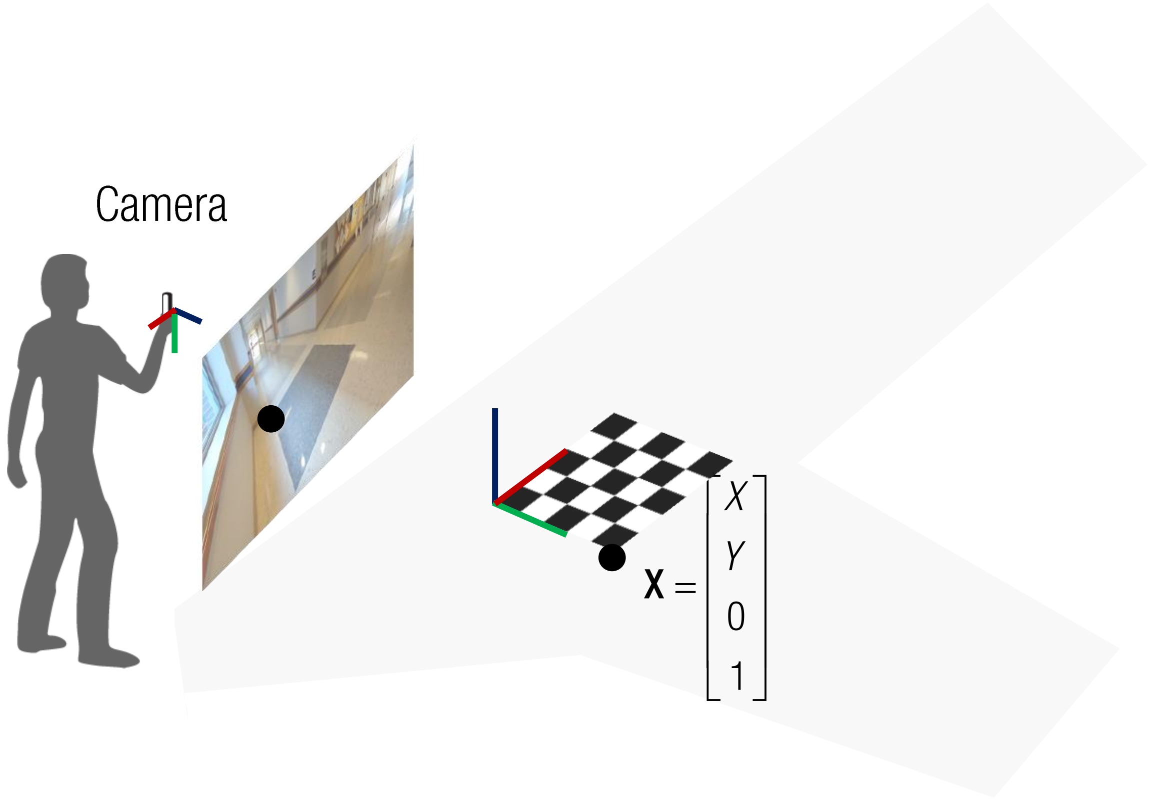
$$\longrightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\| = \left\|\mathbf{K}^{-1}\mathbf{h}_2\right\| \quad \text{or,} \quad \mathbf{h}_1^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

Method2: Rotation



$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Orthogonality of rotation matrix property:

$$\mathbf{r}_1^{\mathrm{T}}\mathbf{r}_2 = 0 \qquad \|\mathbf{r}_1\| = 1 \qquad \|\mathbf{r}_2\| = 1$$

$$\rightarrow \left(\mathbf{K}^{-1}\mathbf{h}_1\right)^{\mathrm{T}}\left(\mathbf{K}^{-1}\mathbf{h}_2\right) = \mathbf{h}_1^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

$$\|\mathbf{K}^{-1}\mathbf{h}_1\| = \|\mathbf{K}^{-1}\mathbf{h}_2\| \quad \text{or,} \quad \mathbf{h}_1^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_1 = \mathbf{h}_2^{\mathrm{T}}\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1}\mathbf{h}_2$$

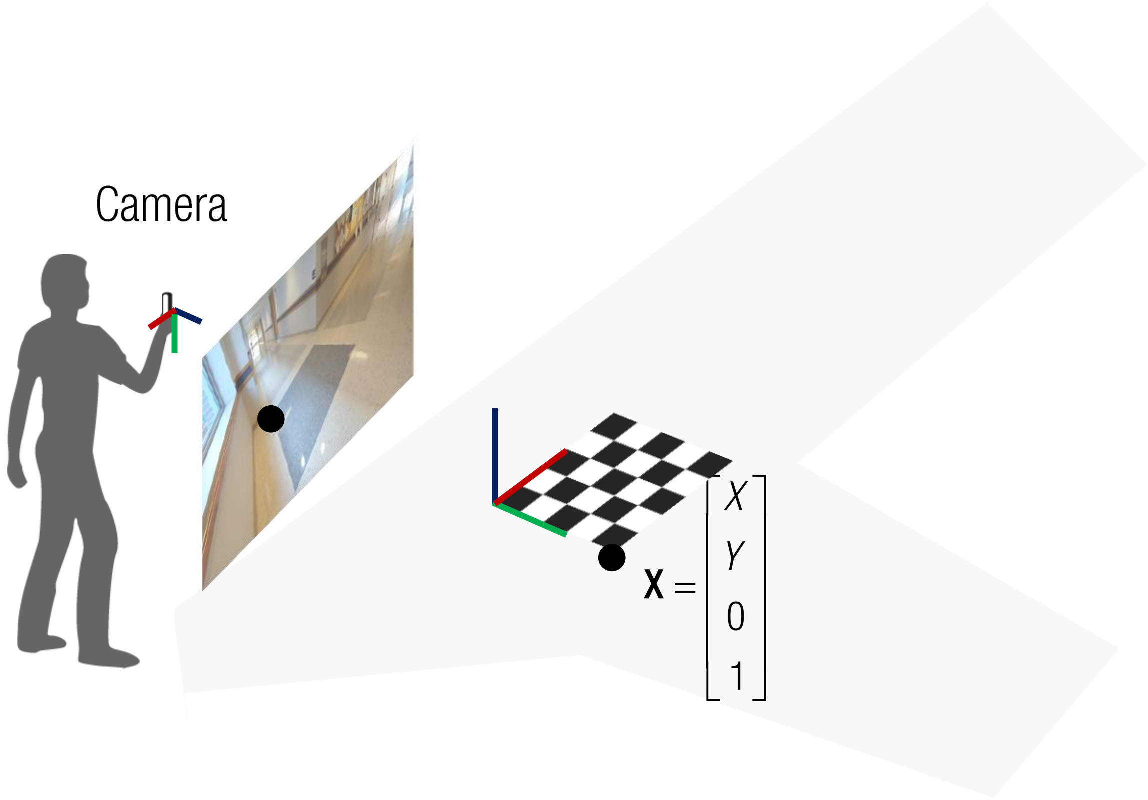
$$\mathbf{K}^{-\mathrm{T}}\mathbf{K}^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}}_{\mathbf{B}}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

Linear in **B**: $\mathbf{h}_1^{\mathrm{T}}\mathbf{B}\mathbf{h}_2 = 0 \qquad \mathbf{h}_1^{\mathrm{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathrm{T}}\mathbf{B}\mathbf{h}_2$

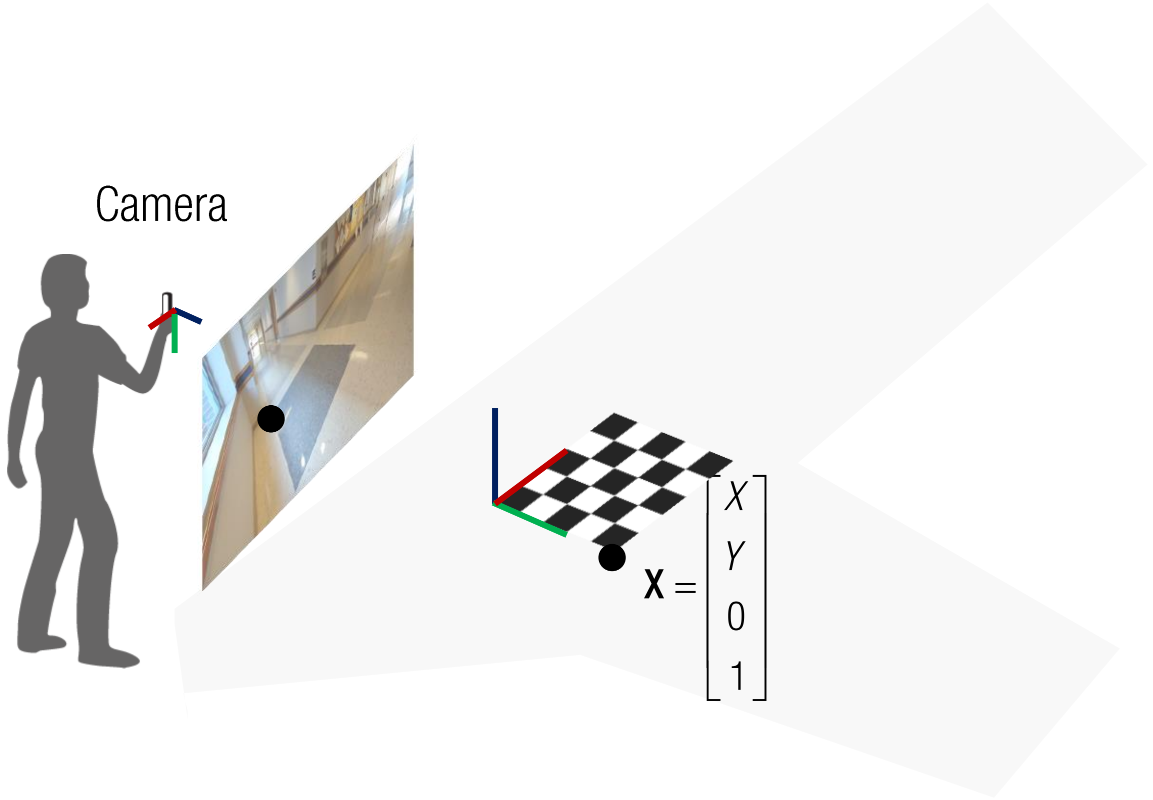
Method2: Rotation

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Method2: Rotation

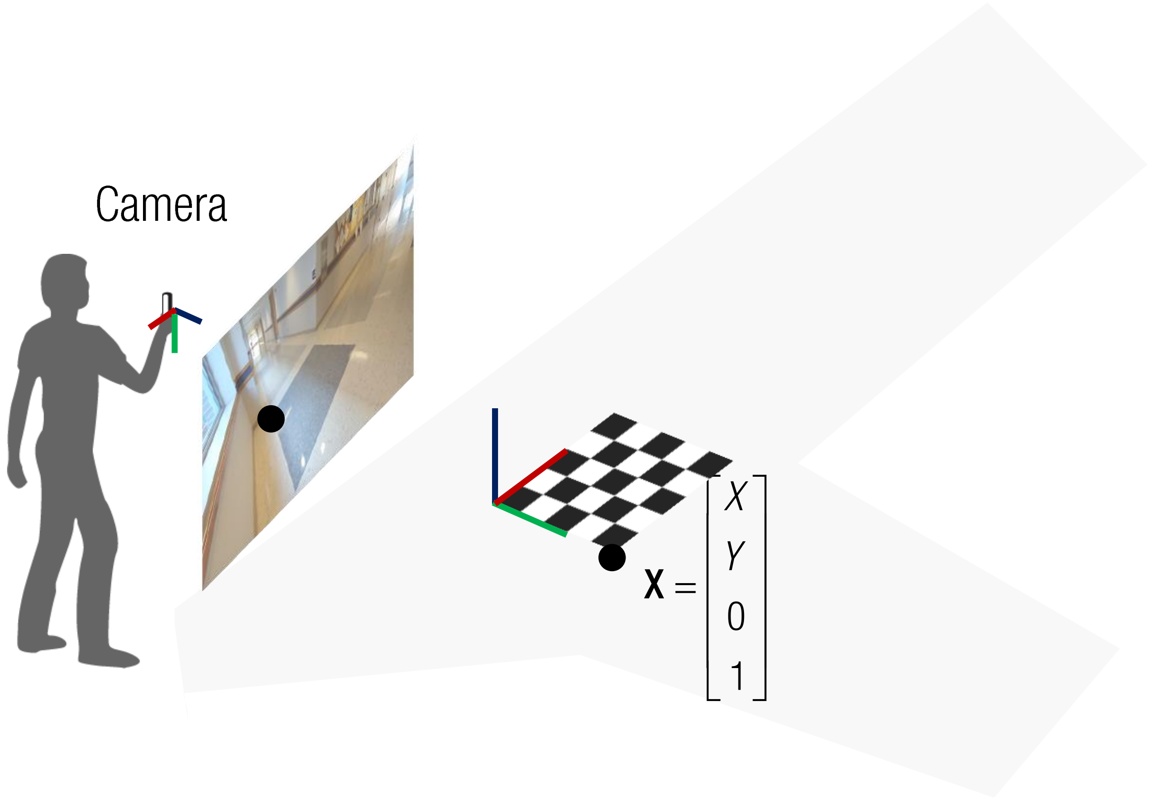


$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

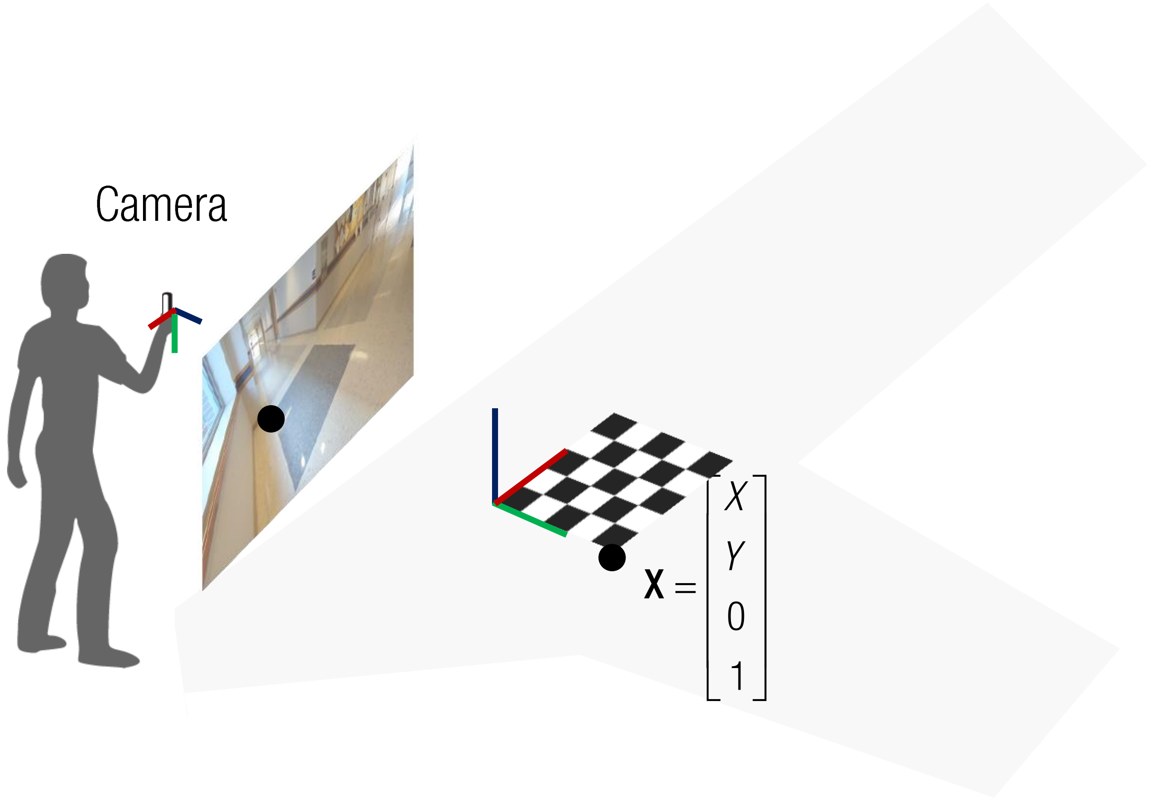
$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

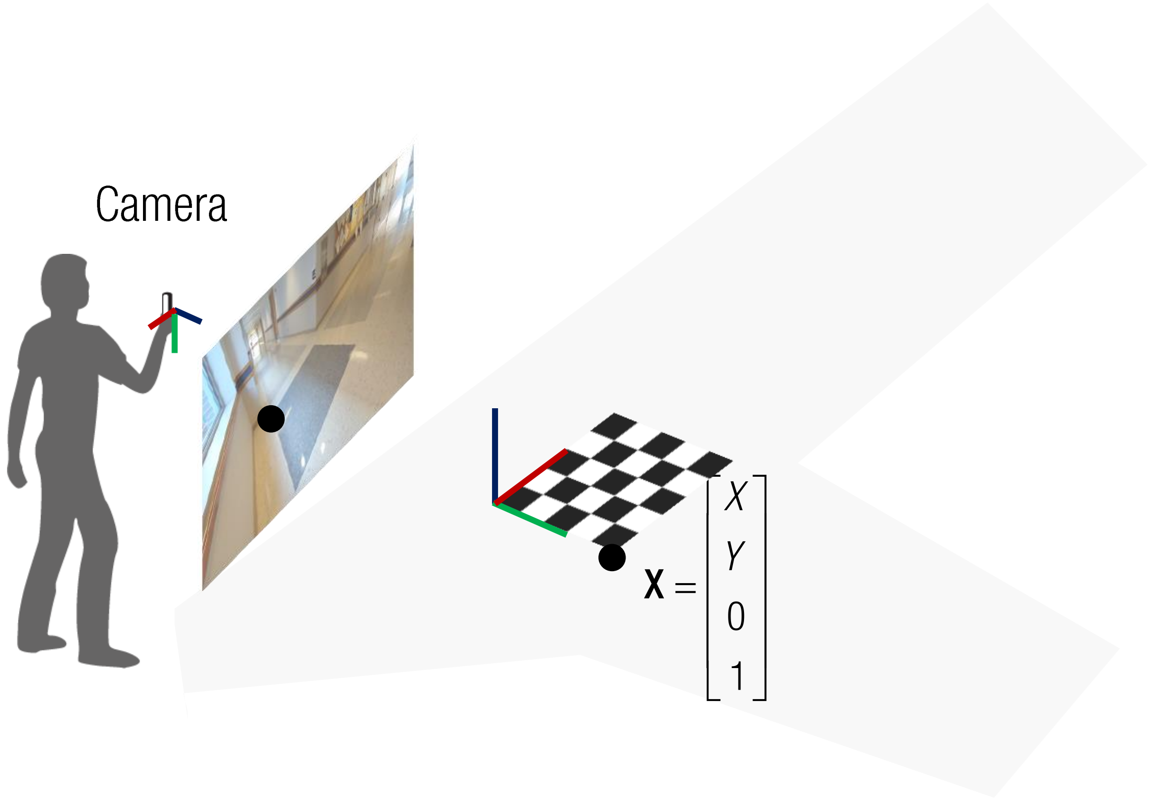
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

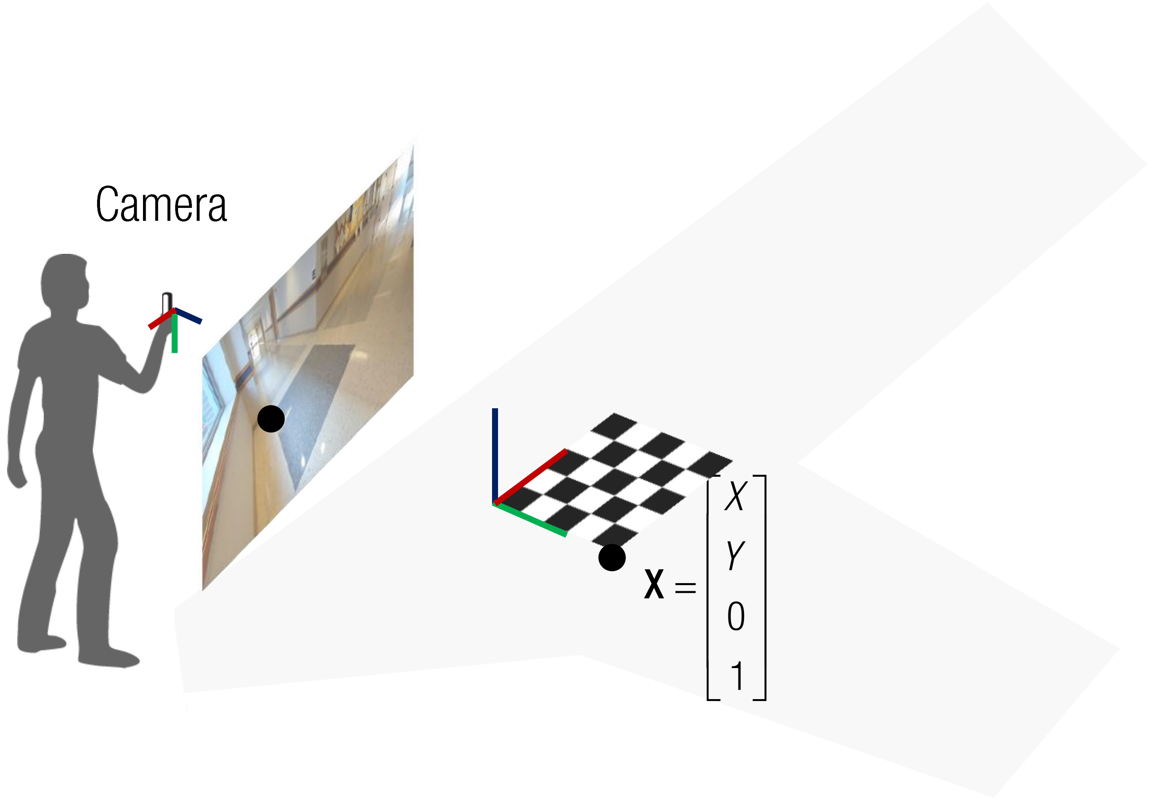
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

2x4

Each image produces 2 equations and therefore, **x** can be computed with minimum 2 images.

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

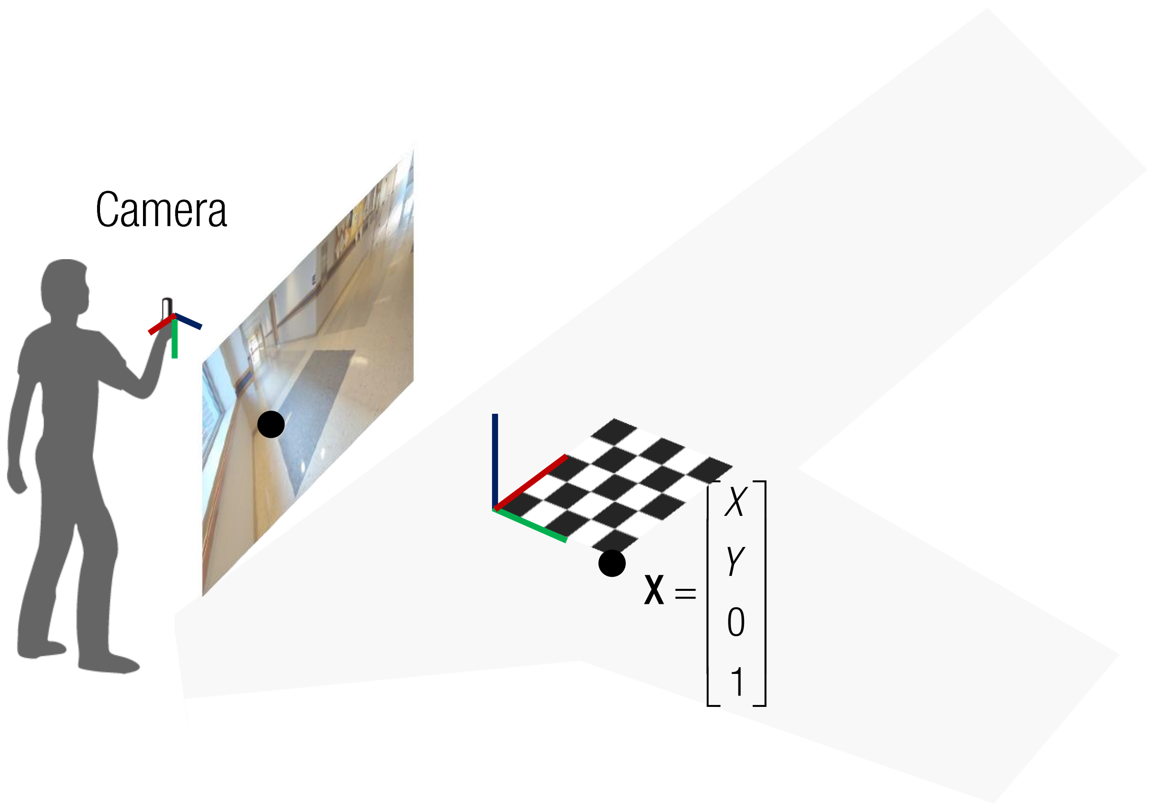
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{X}} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_1 = \mathbf{h}_2^{\mathsf{T}}\mathbf{B}\mathbf{h}_2$$

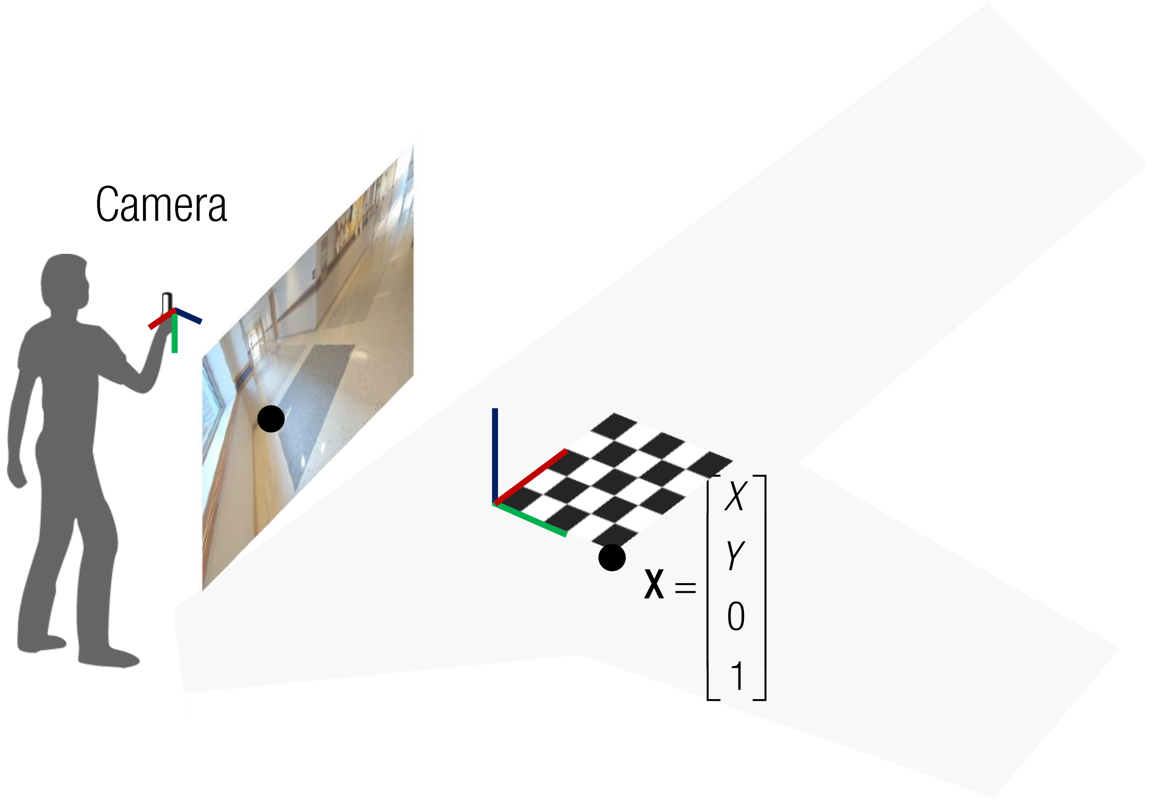
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11} + h_{12} & h_{21} + h_{22} & 1 \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11} - h_{12}) & 2(h_{21} - h_{22}) & 0 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{X}} = \mathbf{0}$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography factorization:

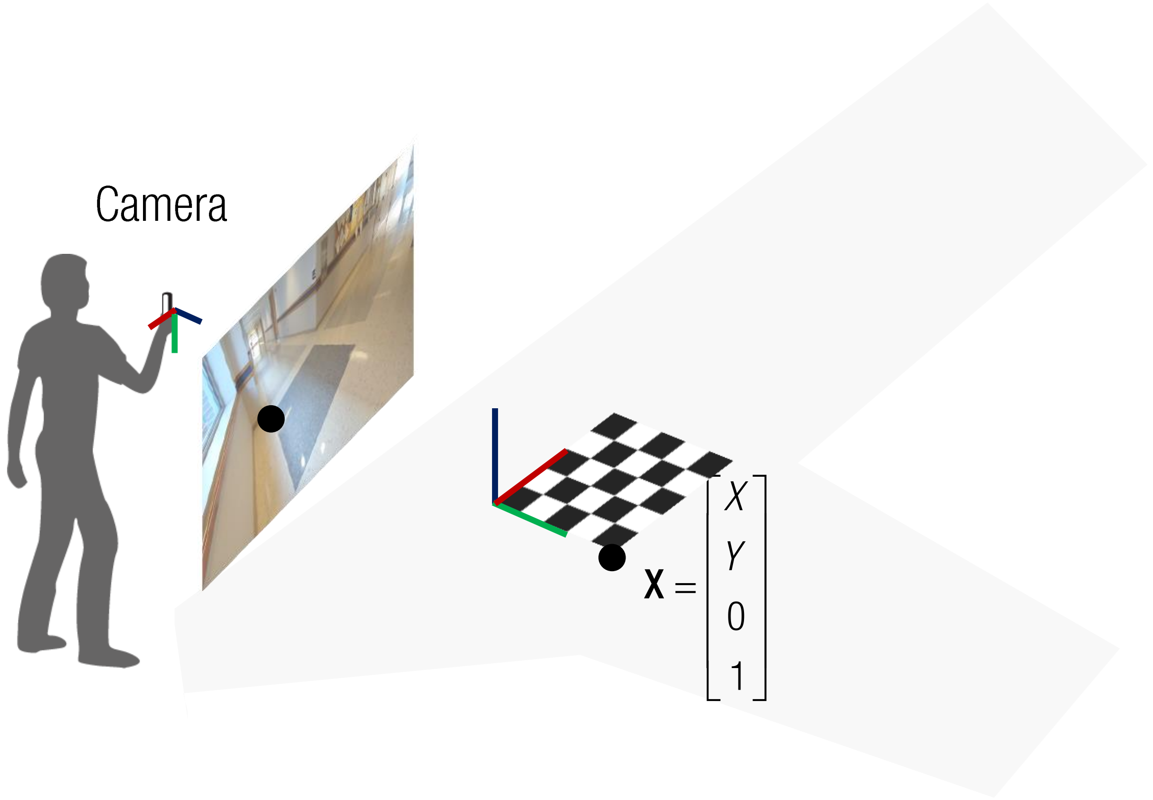
$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

Method2: Rotation

: Knowns

: Unknowns



Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}$$

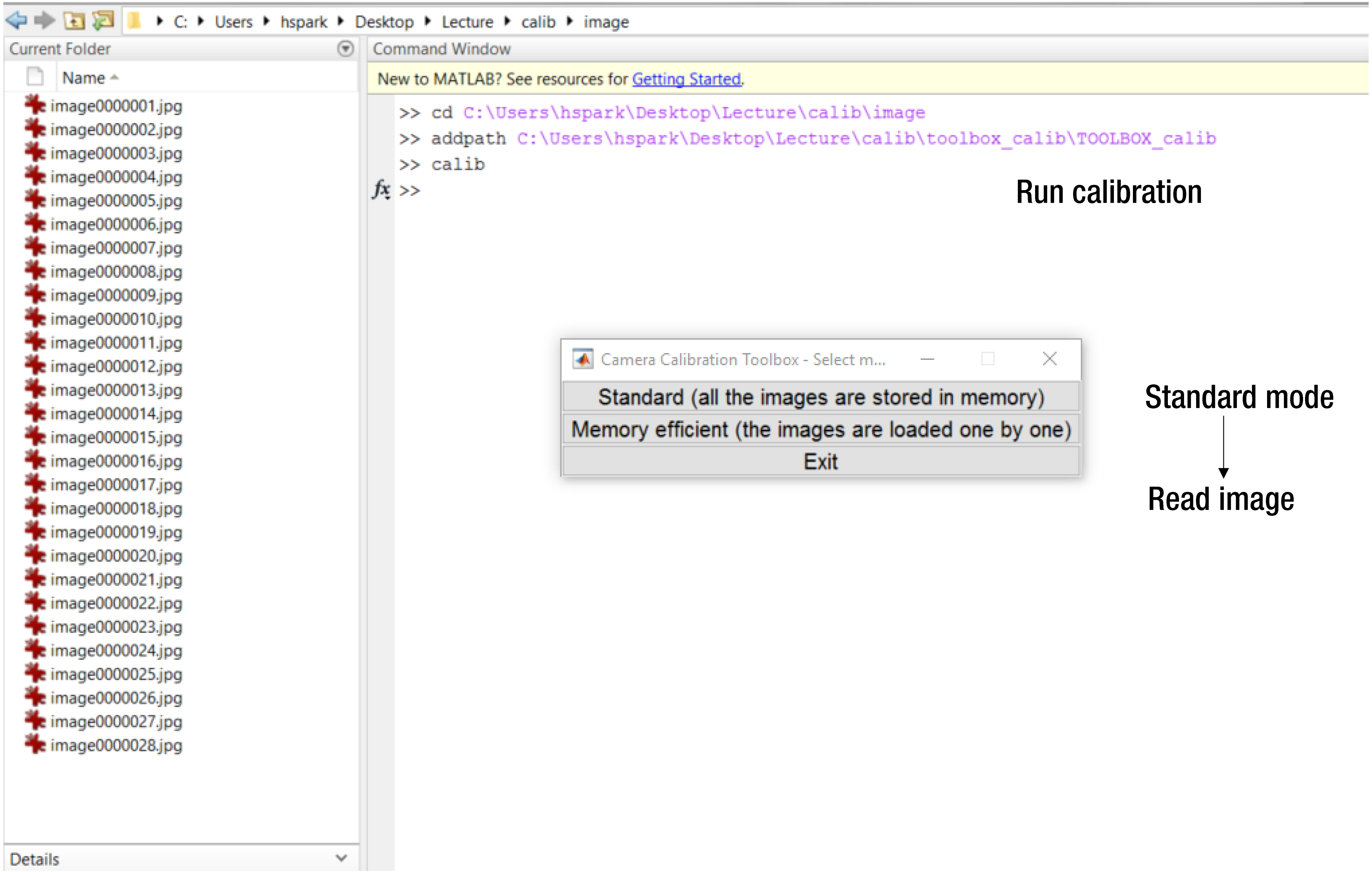
$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1 \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2 \qquad \mathbf{r}_3 = \mathbf{K}^{-1}\mathbf{h}_3$$

$$\mathbf{r}_1 = \frac{\mathbf{K}^{-1}\mathbf{h}_1}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_2 = \frac{\mathbf{K}^{-1}\mathbf{h}_2}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{h}_3}{\left\|\mathbf{K}^{-1}\mathbf{h}_1\right\|}, \quad \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Divided by constant factor

MATLAB Calibration Toolbox Demo

https://www.vision.caltech.edu/bouguetj/calib_doc/



Run calibration

Standard mode

↓

Read image

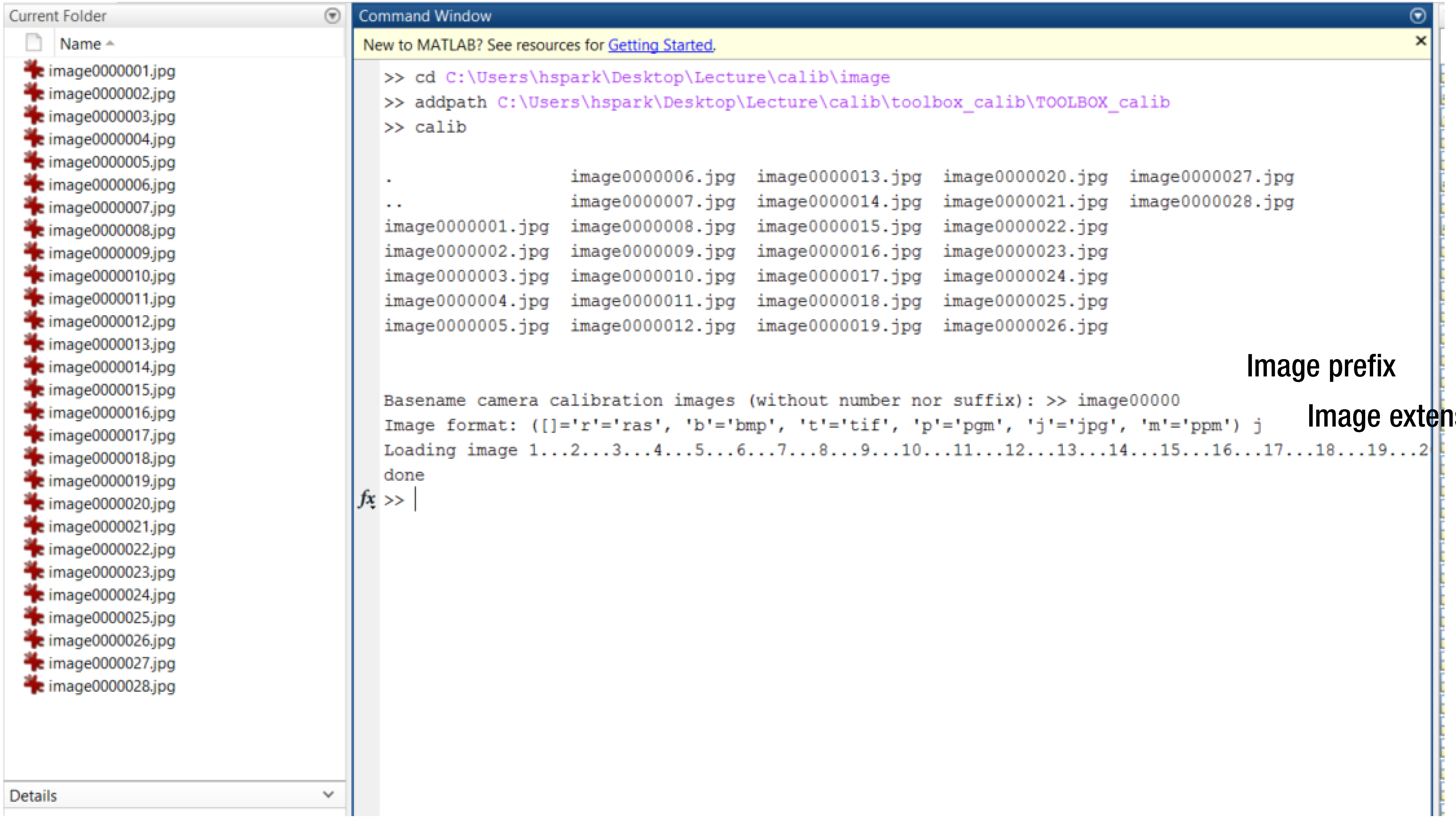
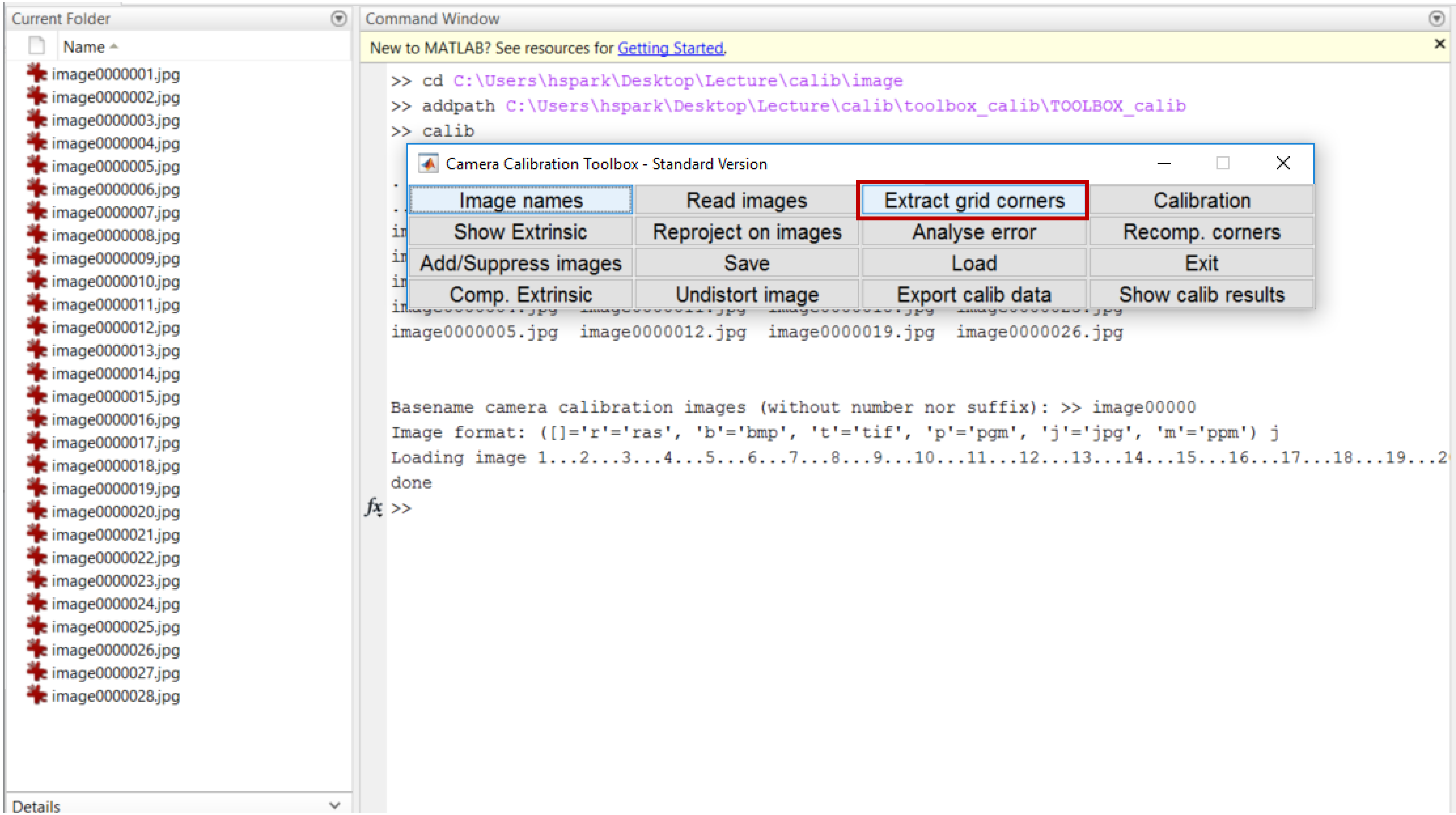
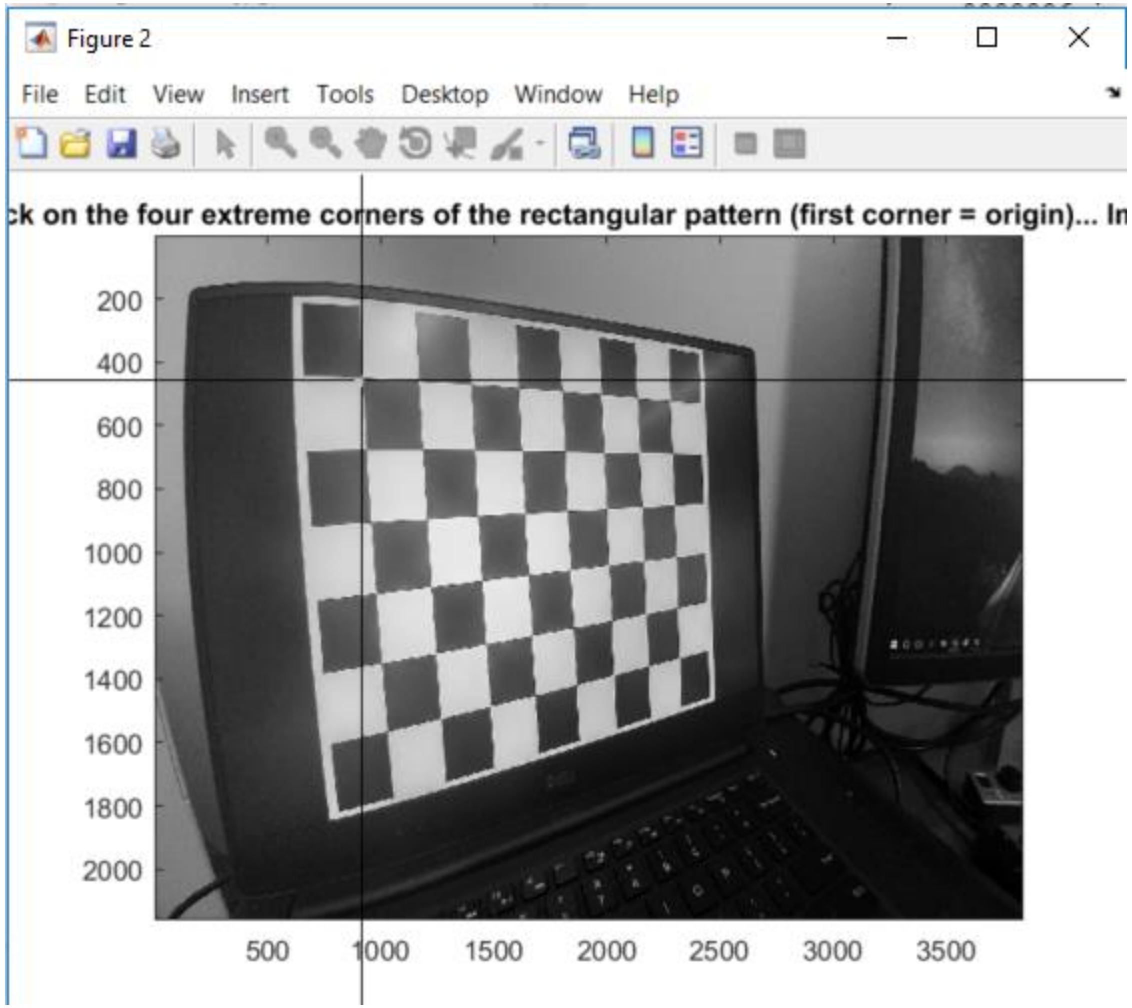


Image prefix

Image extension

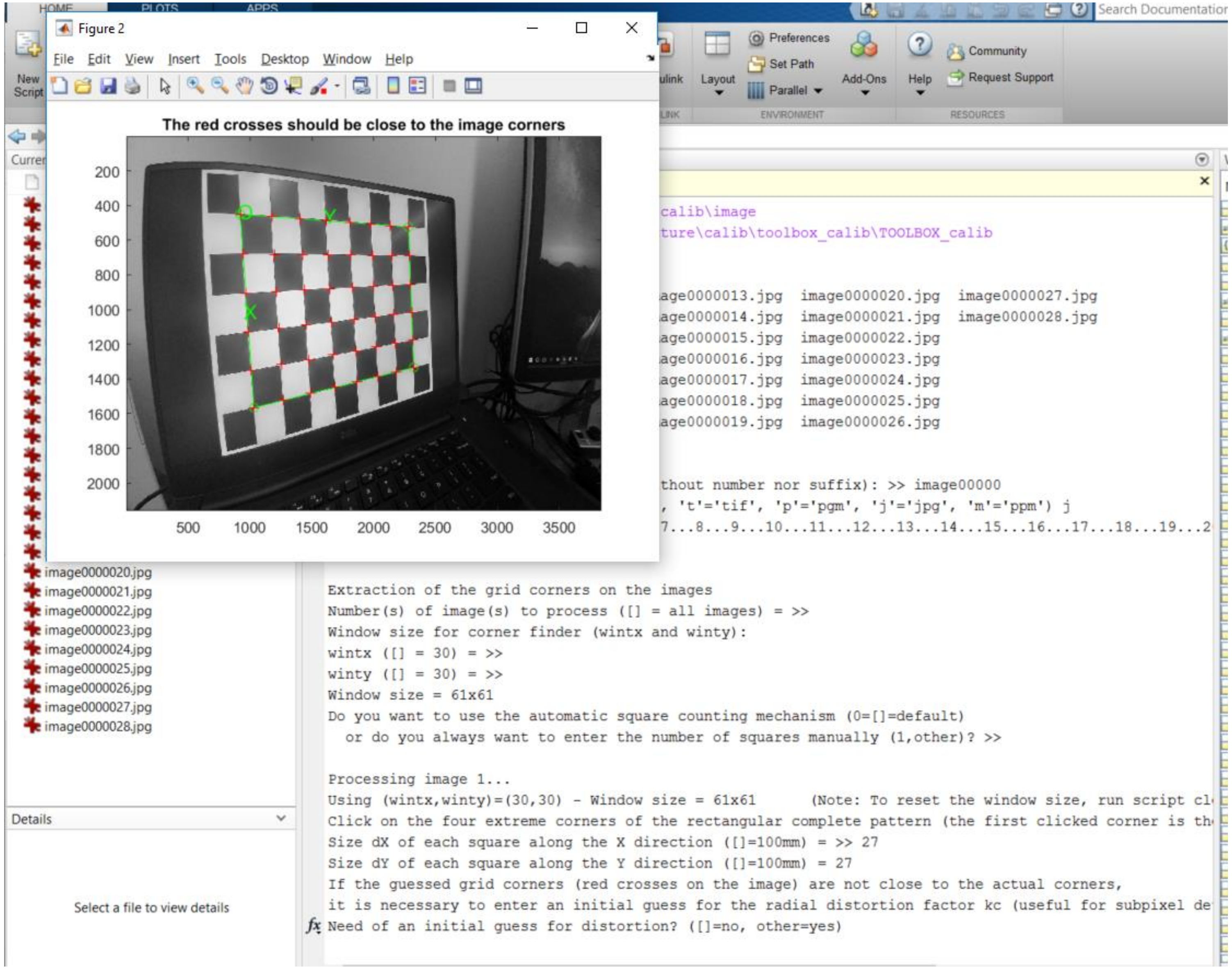


Current Folder
Name
image0000001.jpg
image0000002.jpg
image0000003.jpg
image0000004.jpg
image0000005.jpg
image0000006.jpg
image0000007.jpg
image0000008.jpg
image0000009.jpg
image0000010.jpg
image0000011.jpg
image0000012.jpg
image0000013.jpg
image0000014.jpg
image0000015.jpg
image0000016.jpg
image0000017.jpg
image0000018.jpg
image0000019.jpg
image0000020.jpg
image0000021.jpg
image0000022.jpg
image0000023.jpg
image0000024.jpg
image0000025.jpg
image0000026.jpg
image0000027.jpg
image0000028.jpg
Details
Command Window
New to MATLAB? See resources for Getting Started.
>> cd C:\Users\hspark\Desktop\Lecture\calib\image
>> addpath C:\Users\hspark\Desktop\Lecture\calib\toolbox_calib\TOOLBOX_calib
>> calib
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
image0000005.jpg image0000012.jpg image0000019.jpg image0000026.jpg
Basename camera calibration images (without number nor suffix): >> image00000
Image format: ([]='r'='ras', 'b'='bmp', 't'='tif', 'p'='pgm', 'j'='jpg', 'm'='ppm') j
Loading image 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...2
done
>>



Click four corner in the following order:

1. Top left
2. Top right
3. Bottom right
4. Bottom left



Default mode (press Enter)

Set grid size (27mm)

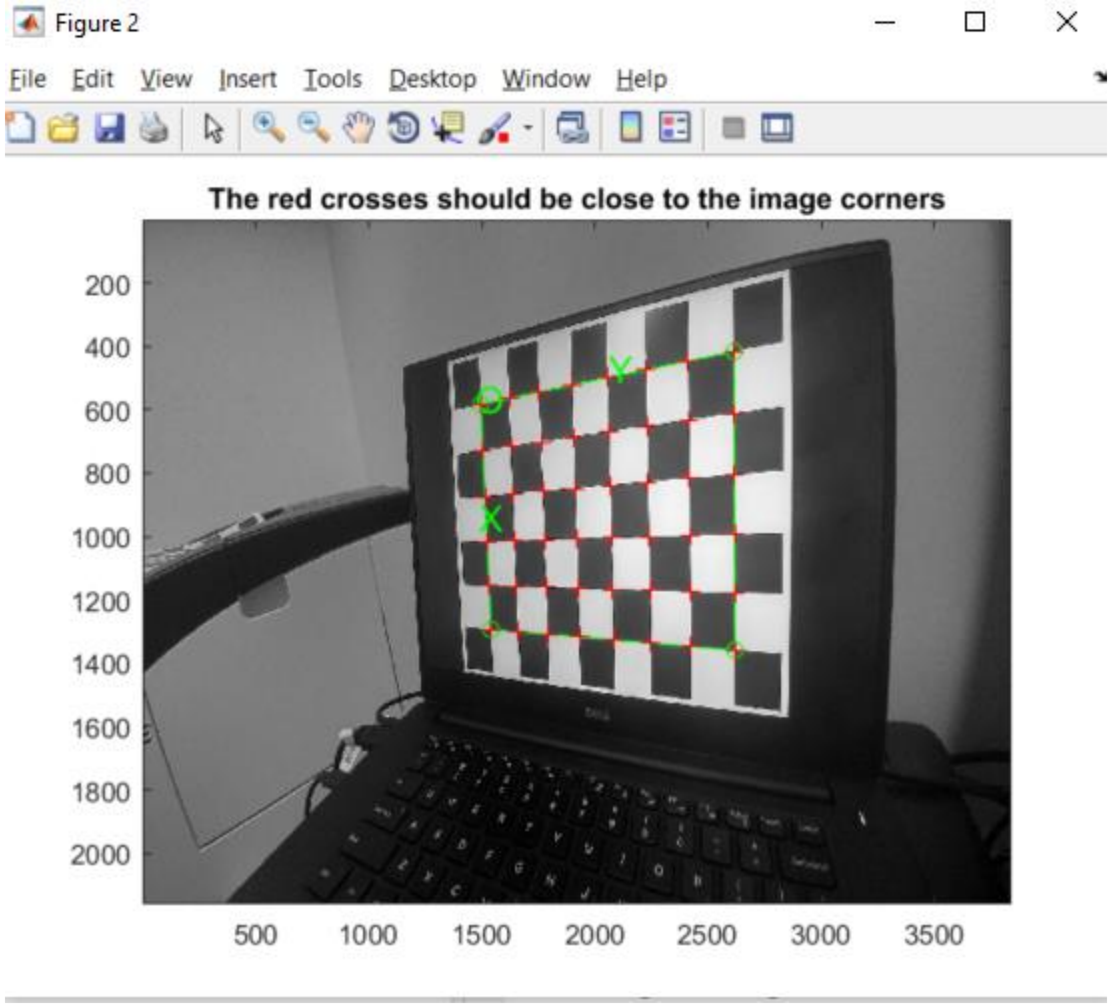
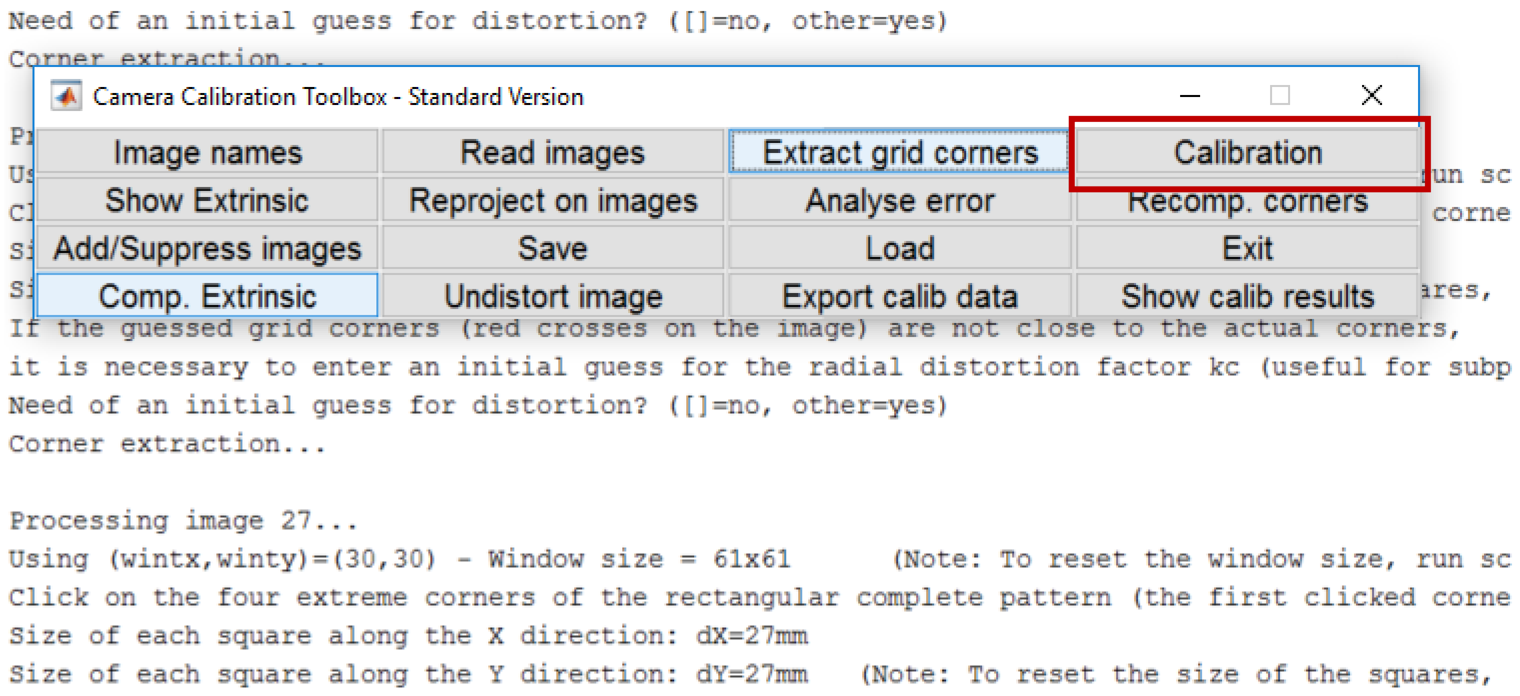
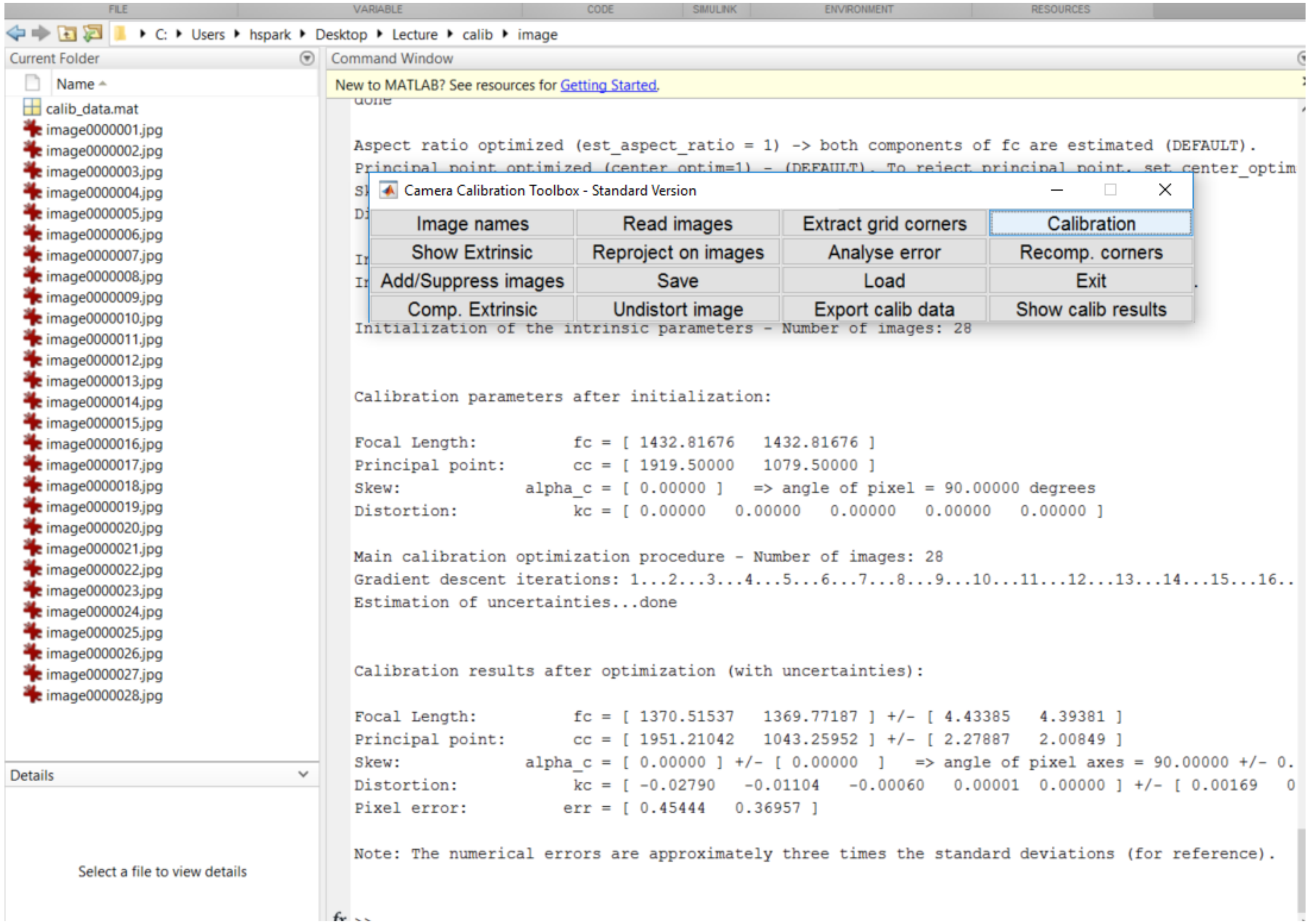


Figure 2
File Edit View Insert Tools Desktop Window Help
The red crosses should be close to the image corners
200
400
600
800
1000
1200
1400
1600
1800
2000
500 1000 1500 2000 2500 3000 3500
O
X
Y

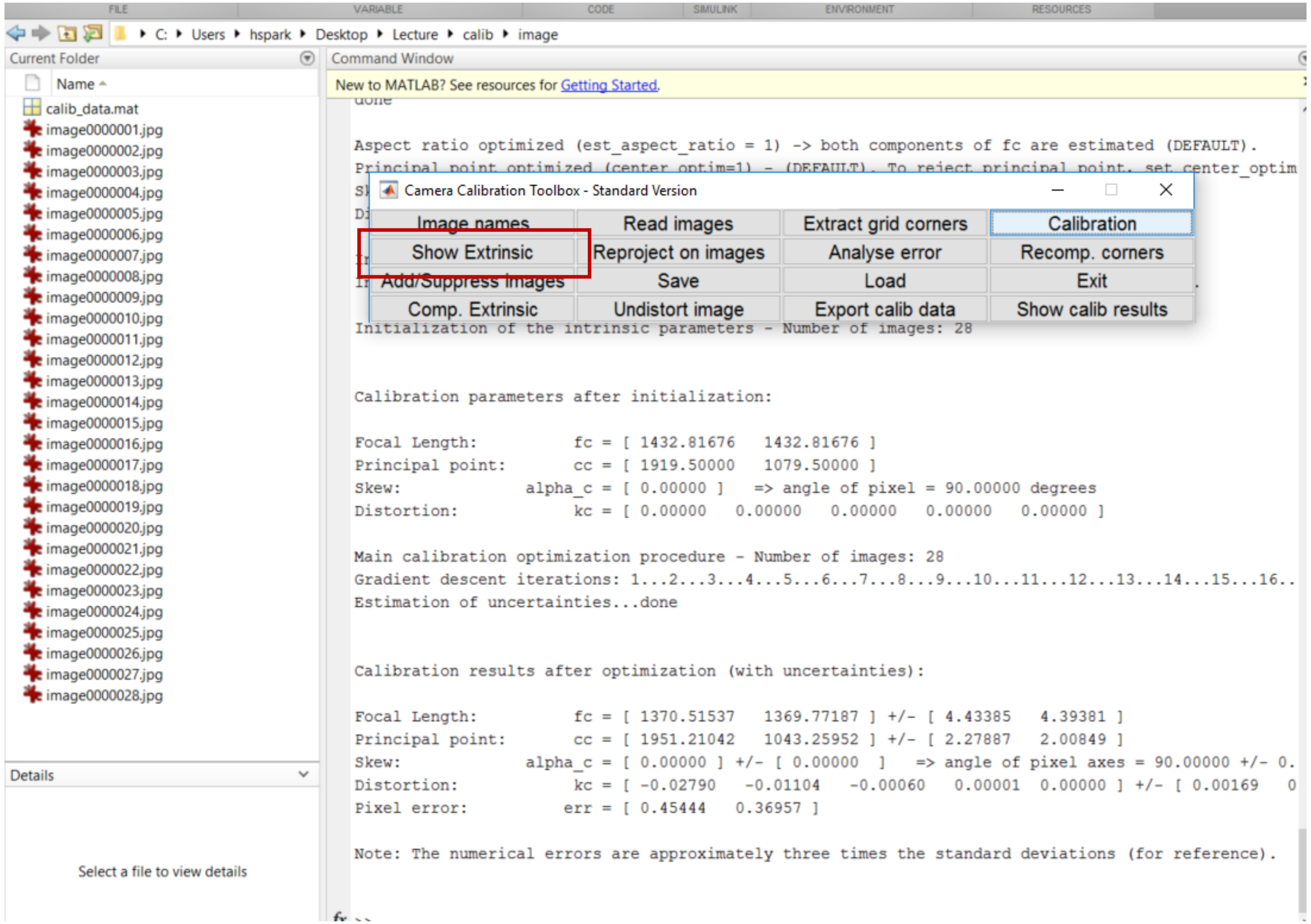

Need of an initial guess for distortion? ([]=no, other=yes)
Corner extraction...
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
If the guessed grid corners (red crosses on the image) are not close to the actual corners,
it is necessary to enter an initial guess for the radial distortion factor kc (useful for subp
Need of an initial guess for distortion? ([]=no, other=yes)
Corner extraction...
Processing image 27...
Using (wintx,winty)=(30,30) - Window size = 61x61 (Note: To reset the window size, run sc
Click on the four extreme corners of the rectangular complete pattern (the first clicked corne
Size of each square along the X direction: dX=27mm
Size of each square along the Y direction: dY=27mm (Note: To reset the size of the squares,

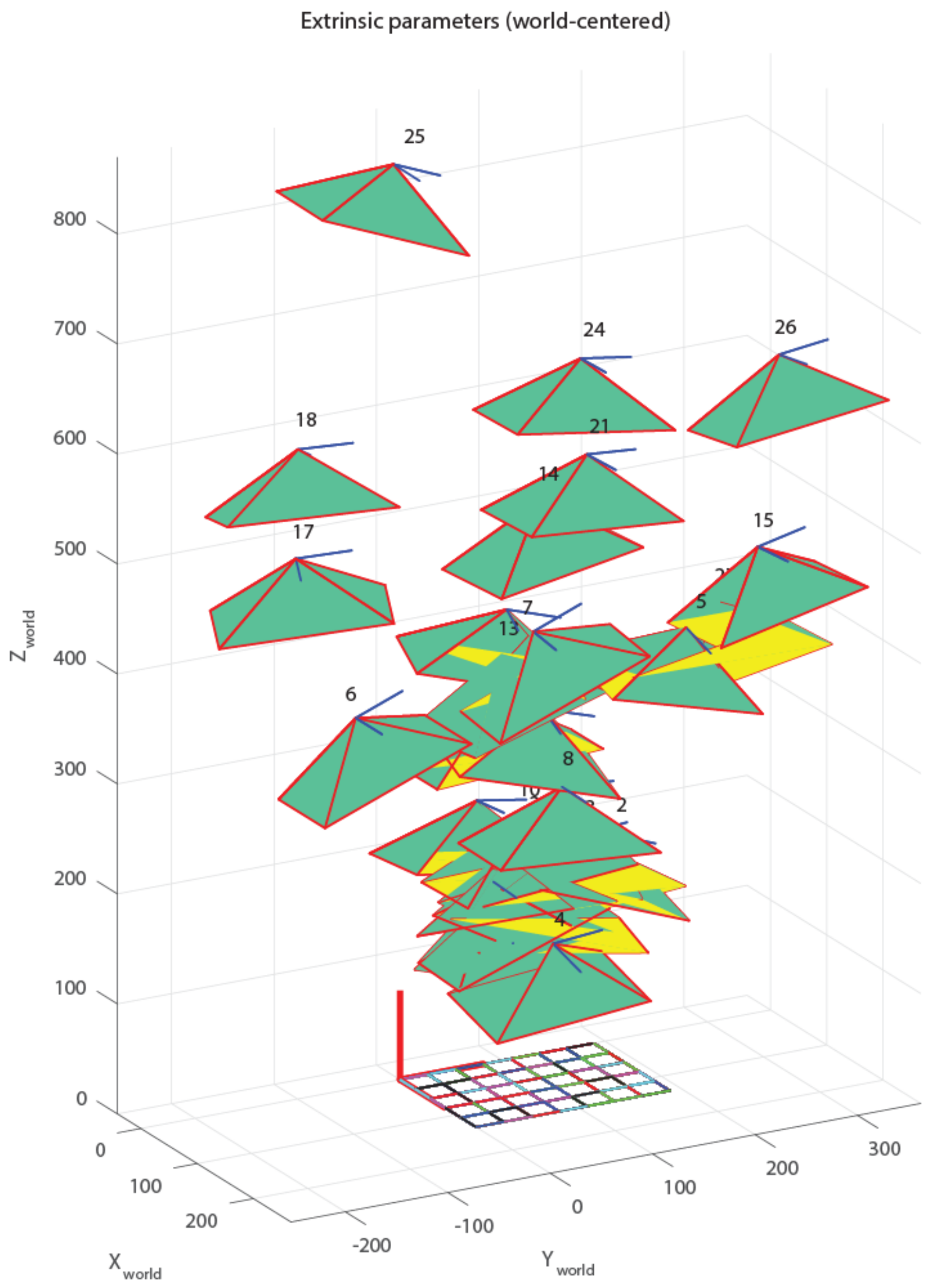


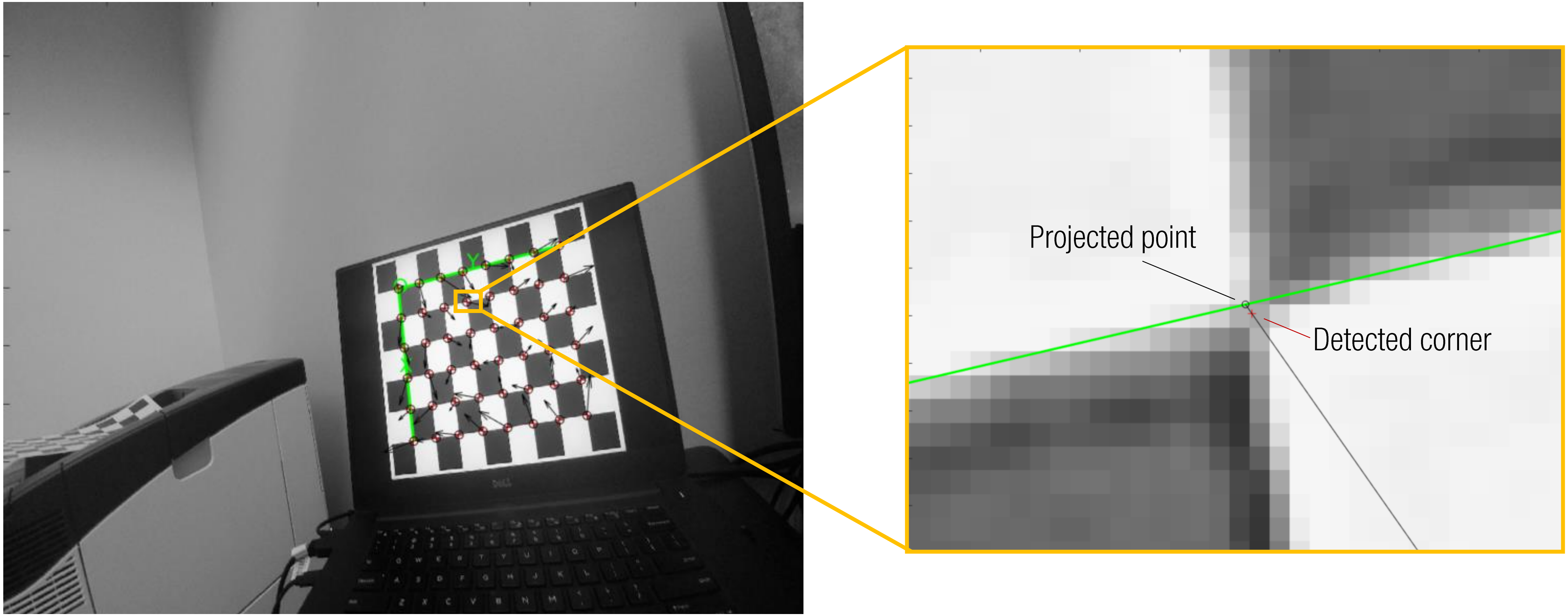
Cf) calibration with vanishing points

K =

1317.2	0	1931.8
0	1317.2	1146.1
0	0	1


FILE
VARIABLE
CODE
SIMULINK
ENVIRONMENT
RESOURCES
C: ▸ Users ▸ hspark ▸ Desktop ▸ Lecture ▸ calib ▸ image
Current Folder
Name
calib_data.mat
image0000001.jpg
image0000002.jpg
image0000003.jpg
image0000004.jpg
image0000005.jpg
image0000006.jpg
image0000007.jpg
image0000008.jpg
image0000009.jpg
image0000010.jpg
image0000011.jpg
image0000012.jpg
image0000013.jpg
image0000014.jpg
image0000015.jpg
image0000016.jpg
image0000017.jpg
image0000018.jpg
image0000019.jpg
image0000020.jpg
image0000021.jpg
image0000022.jpg
image0000023.jpg
image0000024.jpg
image0000025.jpg
image0000026.jpg
image0000027.jpg
image0000028.jpg
Details
Select a file to view details
Command Window
New to MATLAB? See resources for Getting Started.
Aspect ratio optimized (est_aspect_ratio = 1) -> both components of fc are estimated (DEFAULT).
Principal point optimized (center_optim=1) - (DEFAULT). To reject principal point, set center_optim
Camera Calibration Toolbox - Standard Version
Image names
Read images
Extract grid corners
Calibration
Show Extrinsic
Reproject on images
Analyse error
Recomp. corners
Add/Suppress images
Save
Load
Exit
Comp. Extrinsic
Undistort image
Export calib data
Show calib results
Initialization of the intrinsic parameters - Number of images: 28
Calibration parameters after initialization:
Focal Length: fc = [1432.81676 1432.81676]
Principal point: cc = [1919.50000 1079.50000]
Skew: alpha_c = [0.00000] => angle of pixel = 90.00000 degrees
Distortion: kc = [0.00000 0.00000 0.00000 0.00000 0.00000]
Main calibration optimization procedure - Number of images: 28
Gradient descent iterations: 1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16..
Estimation of uncertainties...done
Calibration results after optimization (with uncertainties):
Focal Length: fc = [1370.51537 1369.77187] +/- [4.43385 4.39381]
Principal point: cc = [1951.21042 1043.25952] +/- [2.27887 2.00849]
Skew: alpha_c = [0.00000] +/- [0.00000] => angle of pixel axes = 90.00000 +/- 0.
Distortion: kc = [-0.02790 -0.01104 -0.00060 0.00001 0.00000] +/- [0.00169 0
Pixel error: err = [0.45444 0.36957]
Note: The numerical errors are approximately three times the standard deviations (for reference).


Extrinsic parameters (world-centered)
Z world
X world
Y world
800
700
600
500
400
300
200
100
0
25
24
26
18
21
14
17
15
7
13
5
6
8
2
4

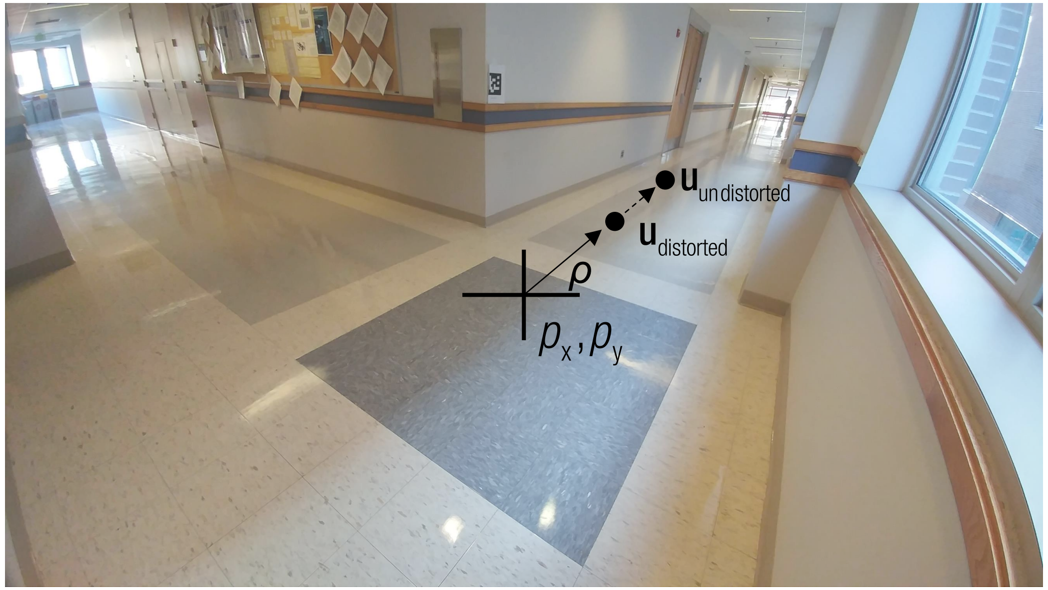

Projected point
Detected corner


Lens Radial Distortion

Lens Radial Distortion Correction

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

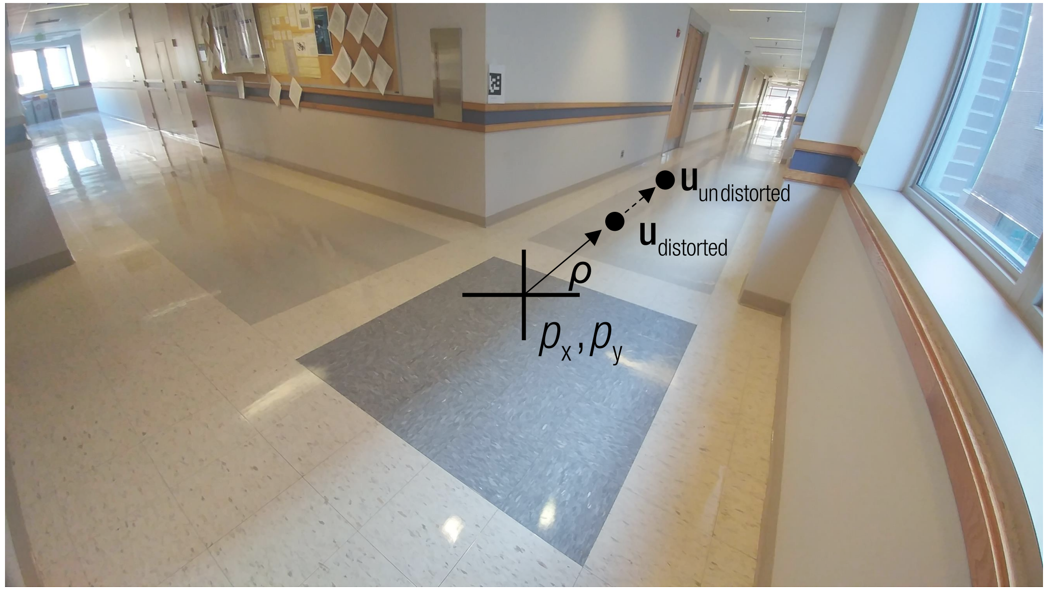


Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



Normalized point:

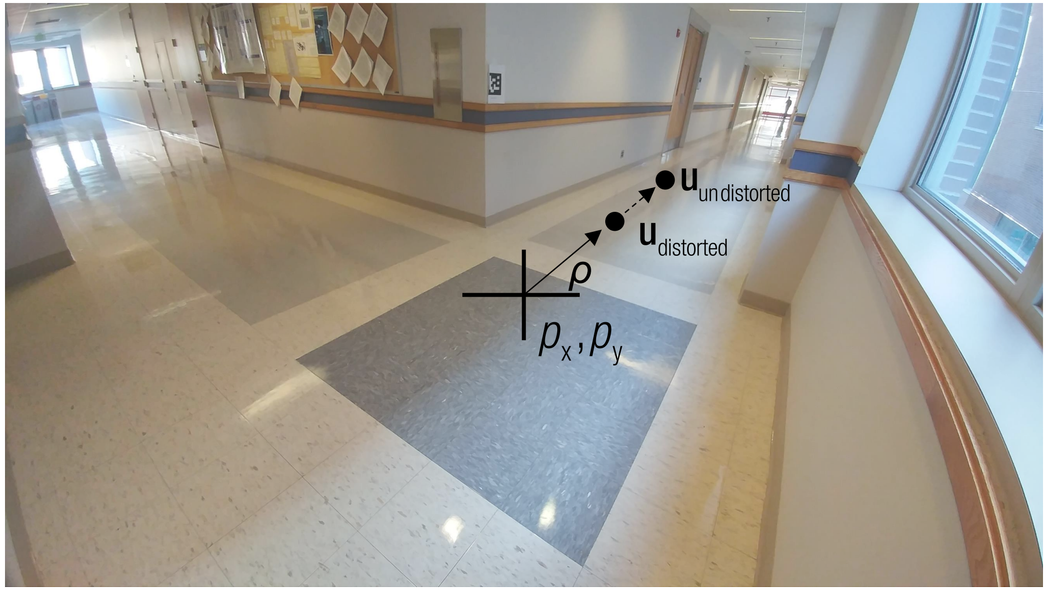
$$\bar{\mathbf{u}}_{distorted} = \mathbf{K}^{-1}\mathbf{u}_{distorted}, \quad \bar{\mathbf{u}}_{undistorted} = \mathbf{K}^{-1}\mathbf{u}_{undistorted}$$

$$\bar{\mathbf{u}}_{distorted} = L(\rho)\bar{\mathbf{u}}_{undistorted}$$

where $\rho = \left\| \mathbf{K}^{-1}\bar{\mathbf{u}}_{distorted} \right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Parameter Estimation (2nd order)

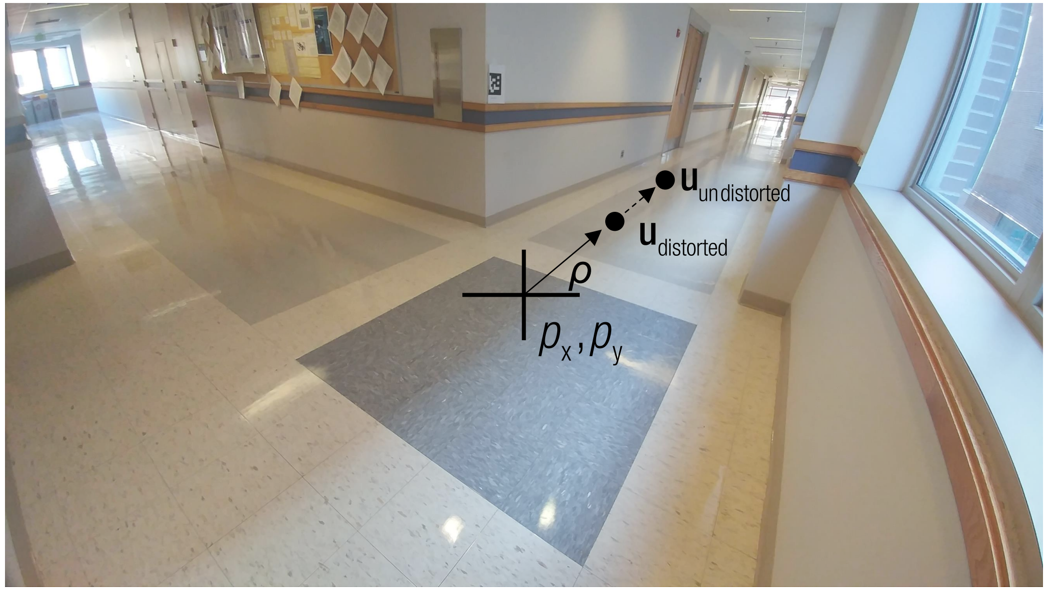


Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

Radial Distortion Parameter Estimation (2nd order)



Normalized point:

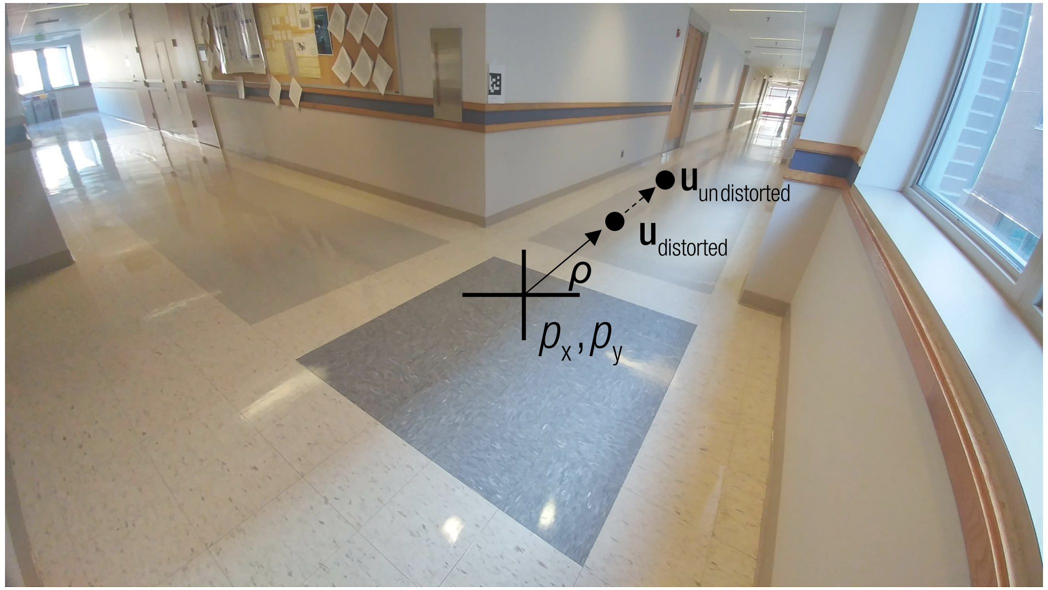
$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

$$\begin{bmatrix} \rho^2\bar{\mathbf{u}}^1_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2\bar{\mathbf{u}}^m_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{u}}^1_{\text{distorted}} - \bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots \\ \bar{\mathbf{u}}^m_{\text{distorted}} - \bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}$$

m: # of points

Radial Distortion Parameter Estimation (2nd order)



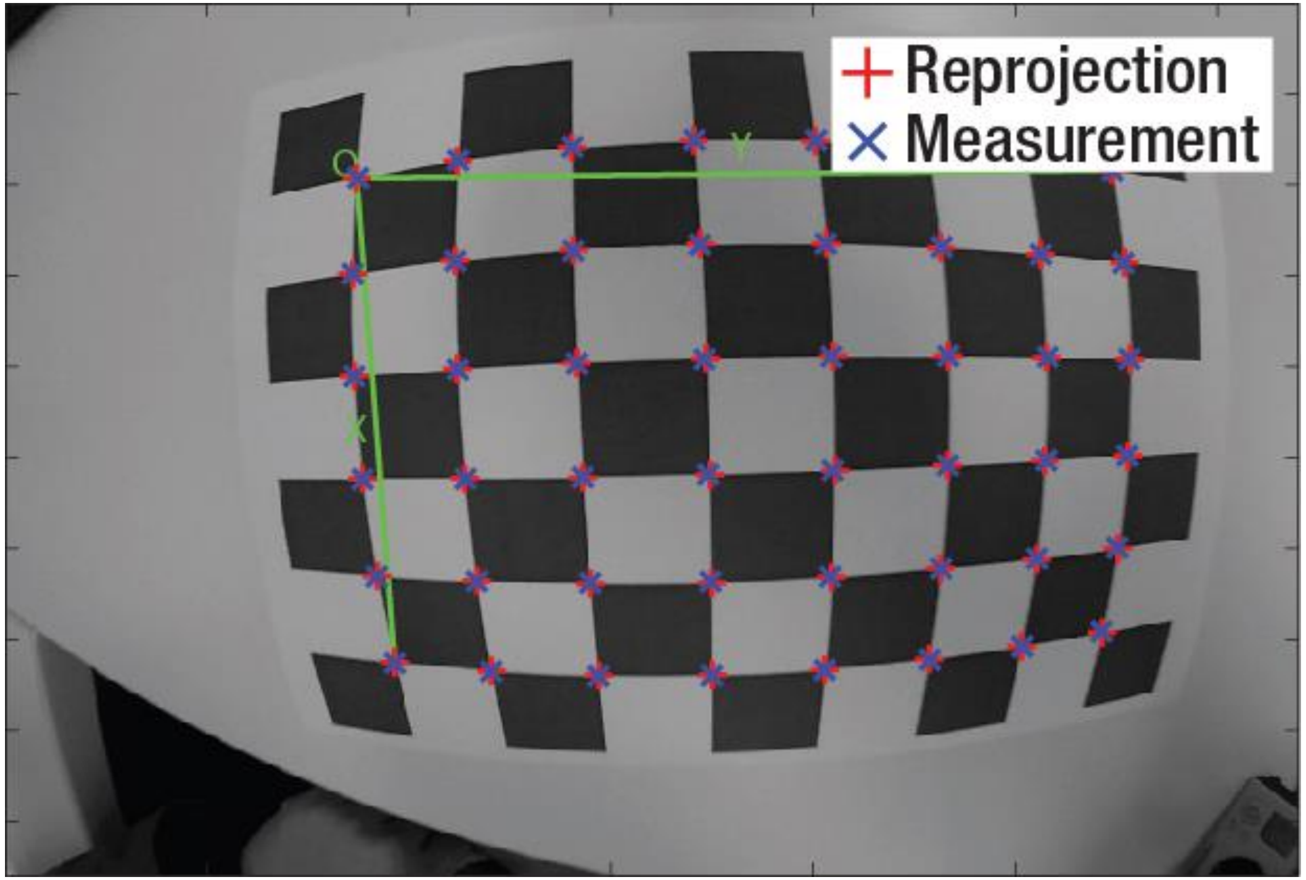
Normalized point:

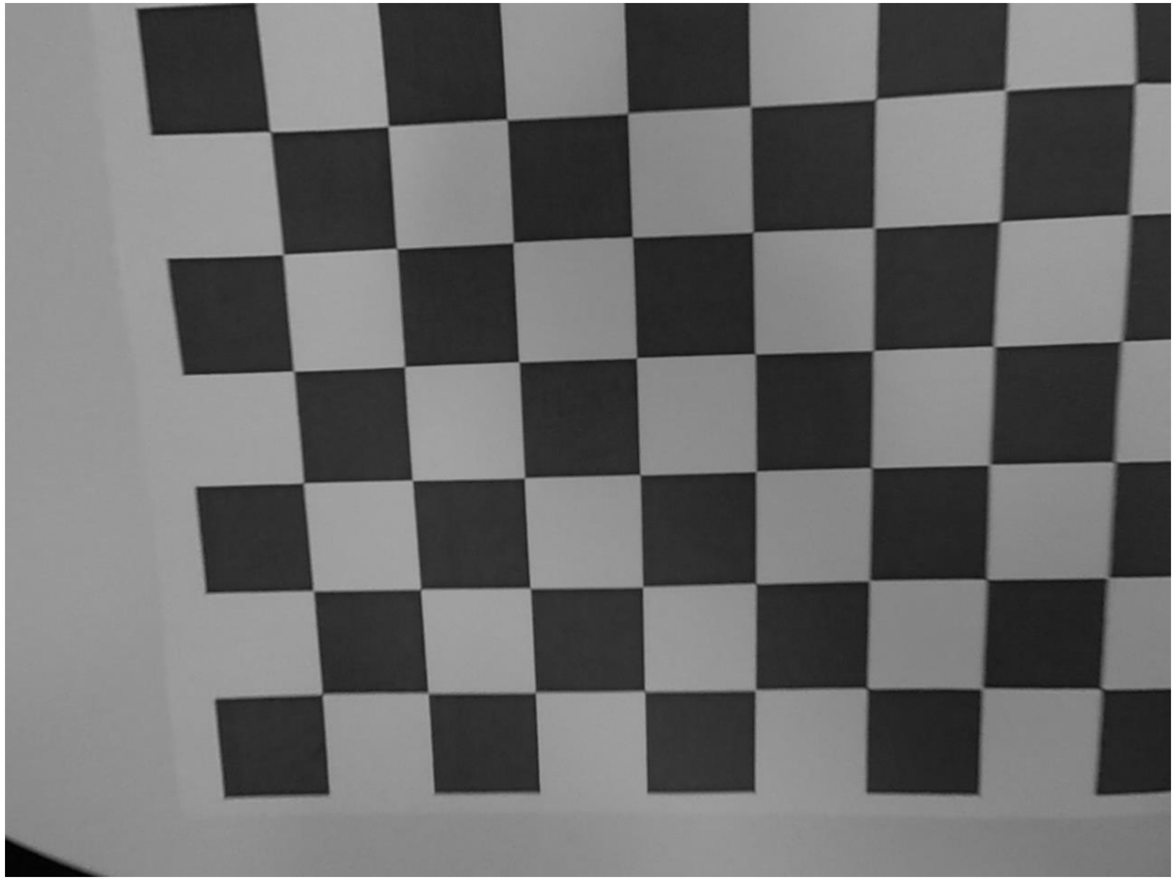
$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

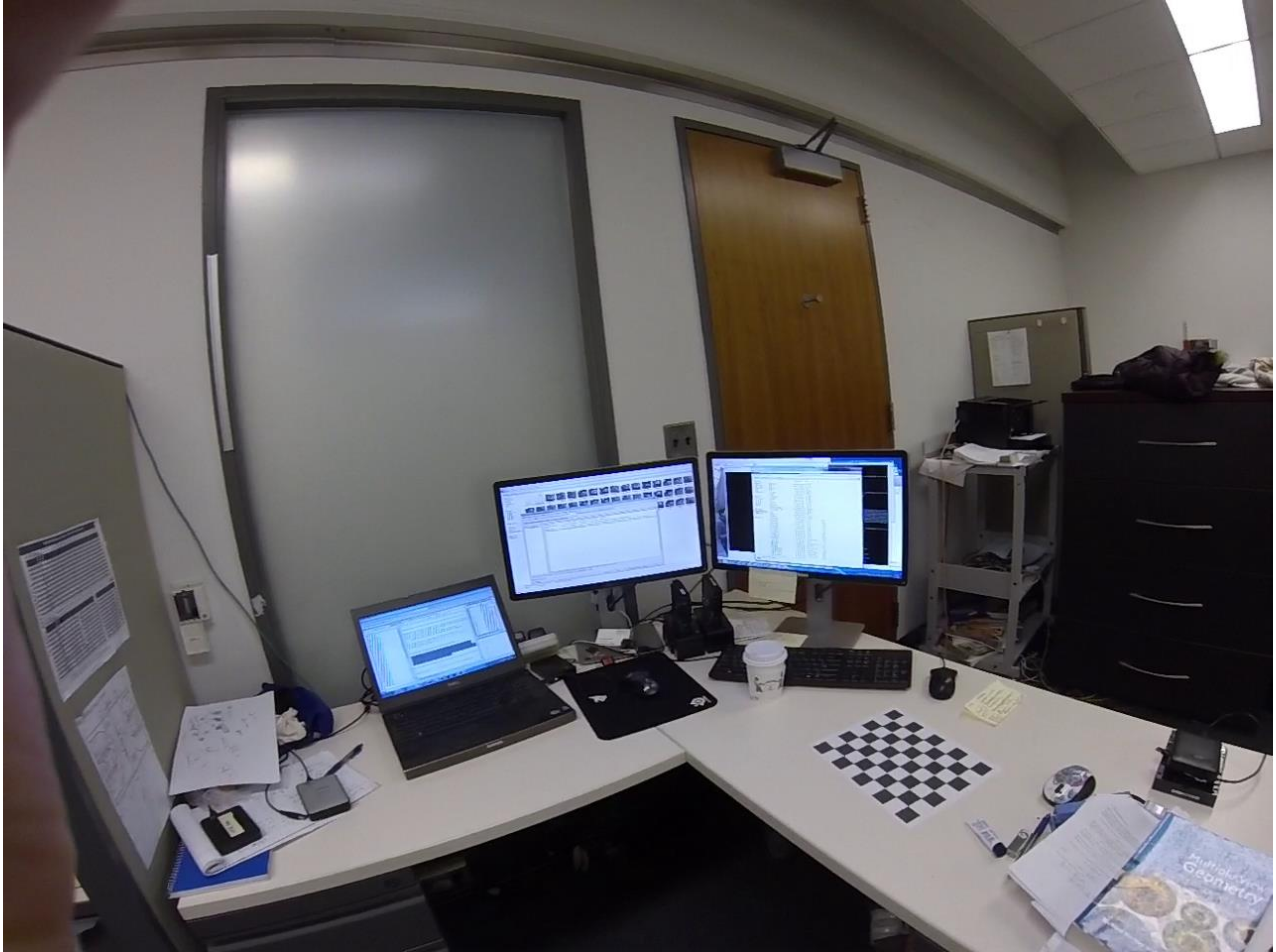
$$\bar{\mathbf{u}}_{\text{distorted}} = (1 + k_1\rho^2 + k_2\rho^4)\bar{\mathbf{u}}_{\text{undistorted}}$$

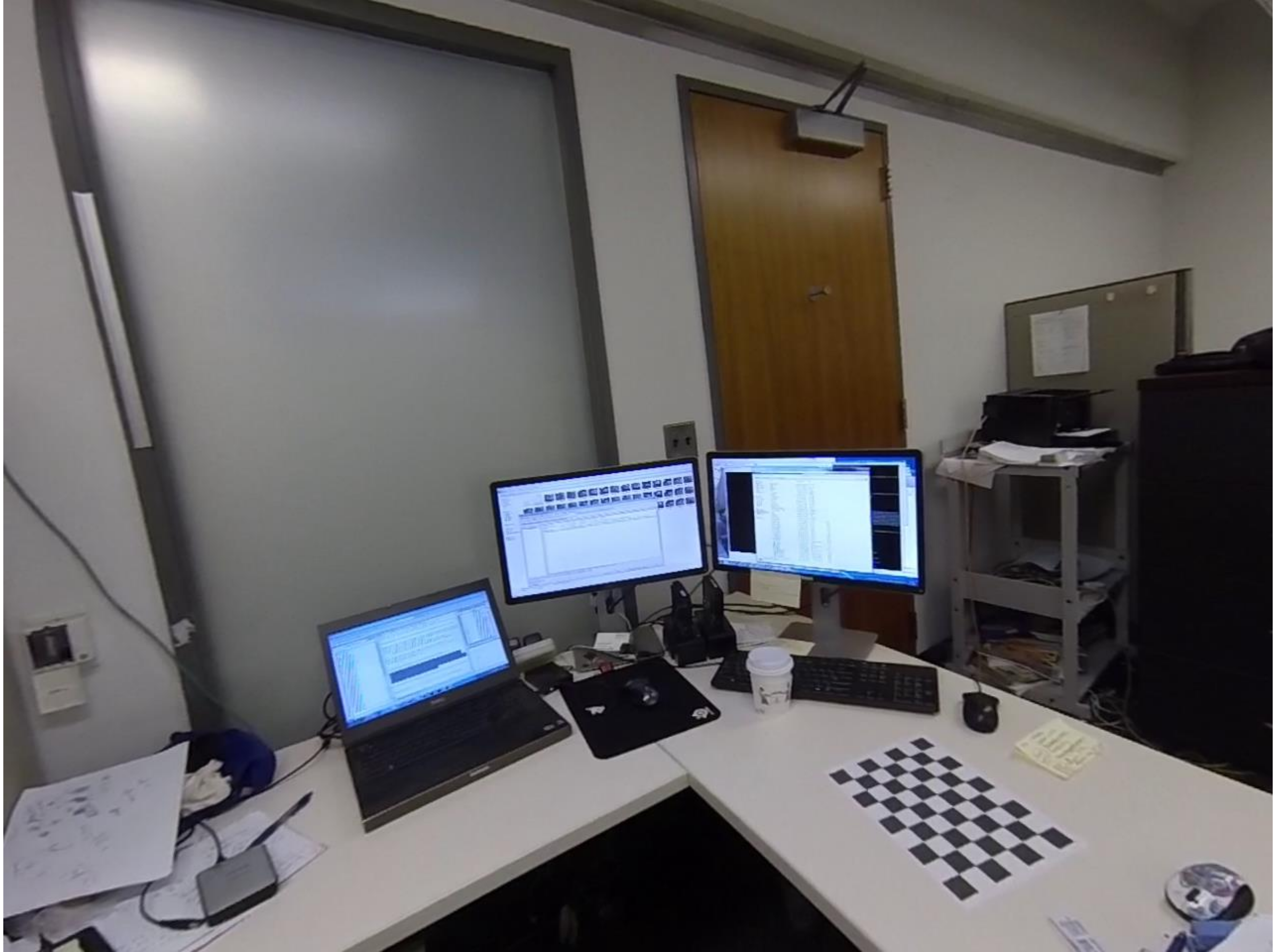
$$\underbrace{\begin{bmatrix} \rho^2\bar{\mathbf{u}}^1_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^1_{\text{undistorted}} \\ \vdots & \vdots \\ \rho^2\bar{\mathbf{u}}^m_{\text{undistorted}} & \rho^4\bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} k_1 \\ k_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} \bar{\mathbf{u}}^1_{\text{distorted}} - \bar{\mathbf{u}}^1_{\text{undistorted}} \\ \bar{\mathbf{u}}^m_{\text{distorted}} - \bar{\mathbf{u}}^m_{\text{undistorted}} \end{bmatrix}}_{\mathbf{b}}$$

m: # of points


Reprojection
Measurement
Y
X




Geometry



Fisheye Lens Distortion

Fisheye Lens Distortion