

A black and white photograph showing a group of approximately ten people gathered on the deck of a boat. In the foreground, a large astronomical telescope is mounted on a tripod, its eyepiece and body clearly visible. Several people are standing behind the telescope, looking through its eyepiece or adjusting its controls. One man in the center, wearing a light-colored t-shirt and glasses, is leaning over the telescope. To his right, another man is seated, also looking at the instrument. The background shows the open sea with some distant land or structures visible on the horizon. The overall atmosphere suggests a scientific or amateur astronomical observation session.

Camera Model

Website

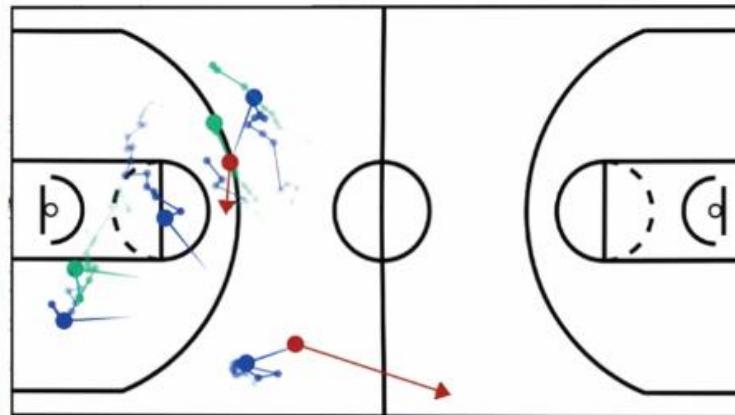
- http://www-users.cs.umn.edu/~hspark/CSci5980/csci5980_3dvision.html



Description



(a) Head-mounted cameras



(b) 3D reconstruction of players

Paper Presentation

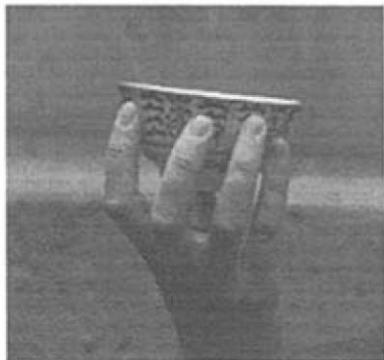
- Format
 - 20 min presentation with 15+ min questions
 - One presenter: this person defends the paper.
 - One committee: this person attacks the paper.
- You are free to choose a paper that
 - Is not written by you or your research group
 - Has a strong relevance to 3D vision (theory and application)
 - Is approved by me
- Or, you can choose a paper from my list.

Paper List

- Tomasi and Kanade, Shape and Motion from Image Streams under Orthography: a Factoriaztion Method, IJCV, 1992



1



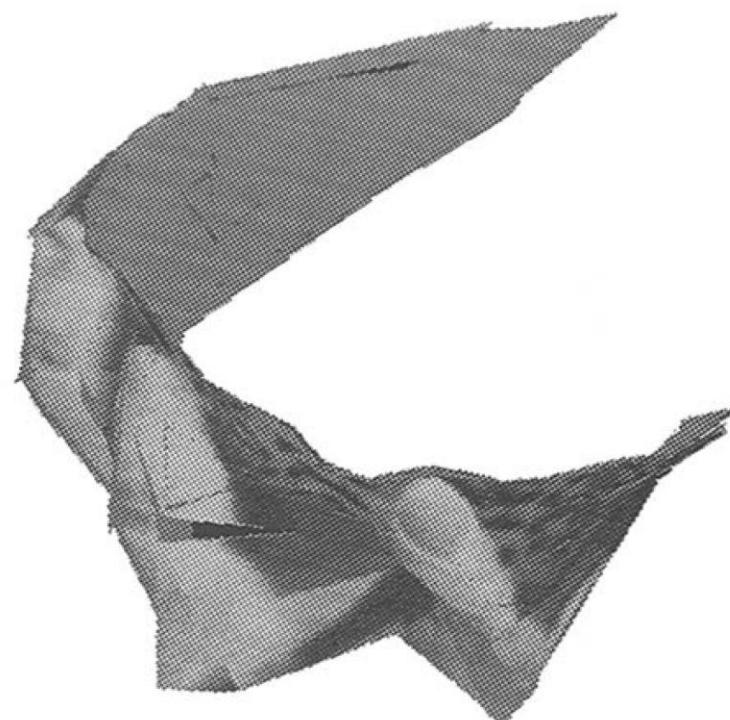
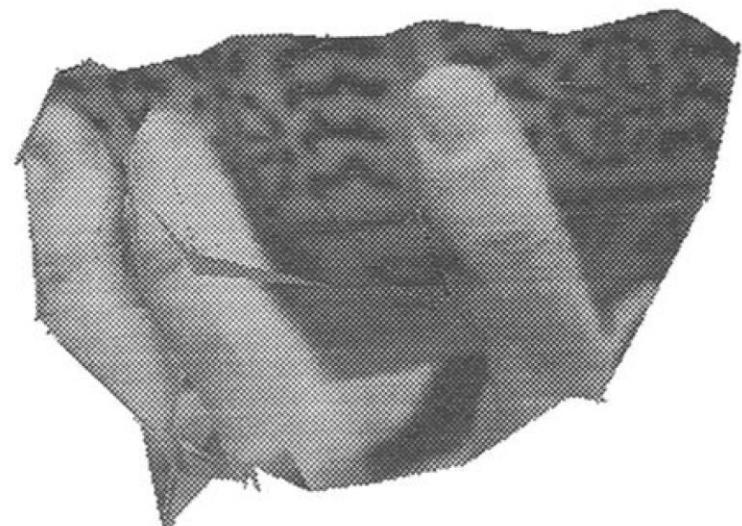
100



150



210



Paper List

- Reid and Zisserman, Goal-directed Video Metrology, ECCV, 1996



Paper List

- Zhang, A Flexible New Technique for Camera Calibration, PAMI, 2000

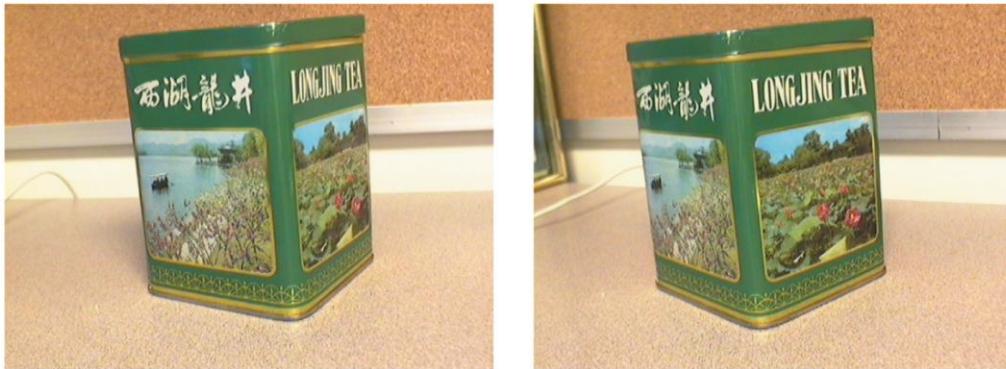


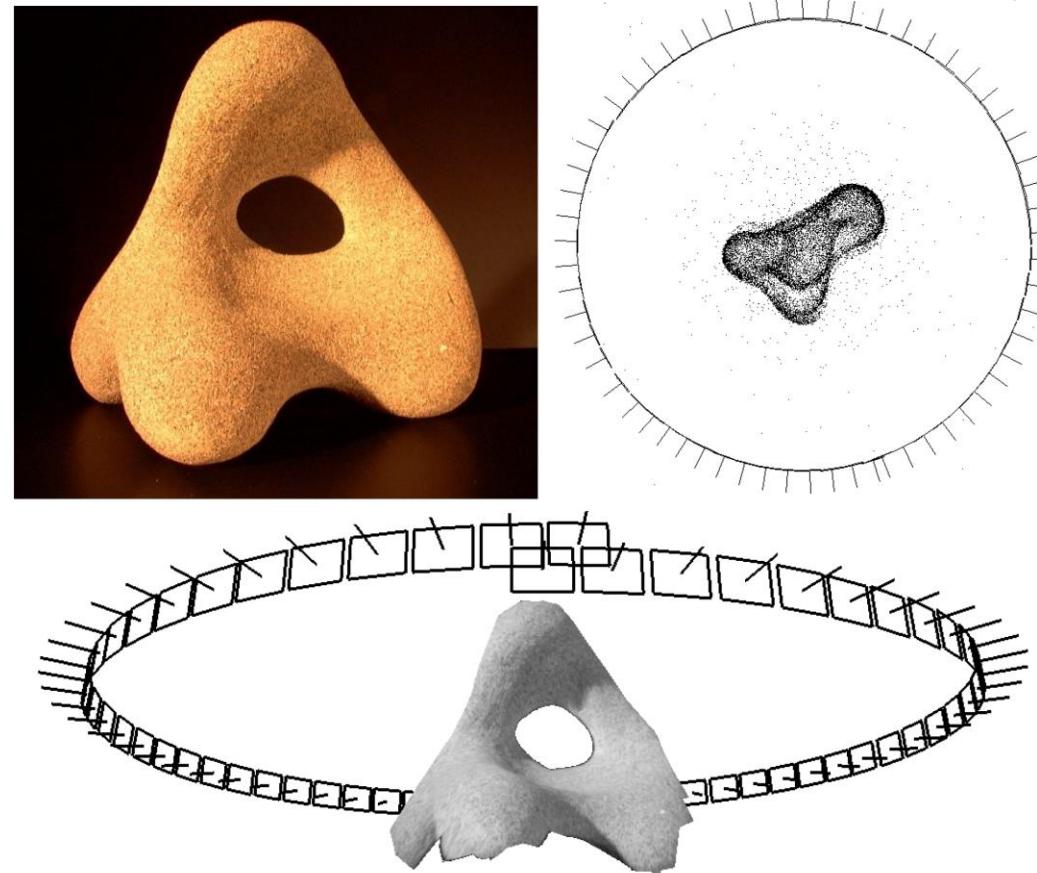
Figure 6: Two images of a tea tin



Figure 7: Three rendered views of the reconstructed tea tin

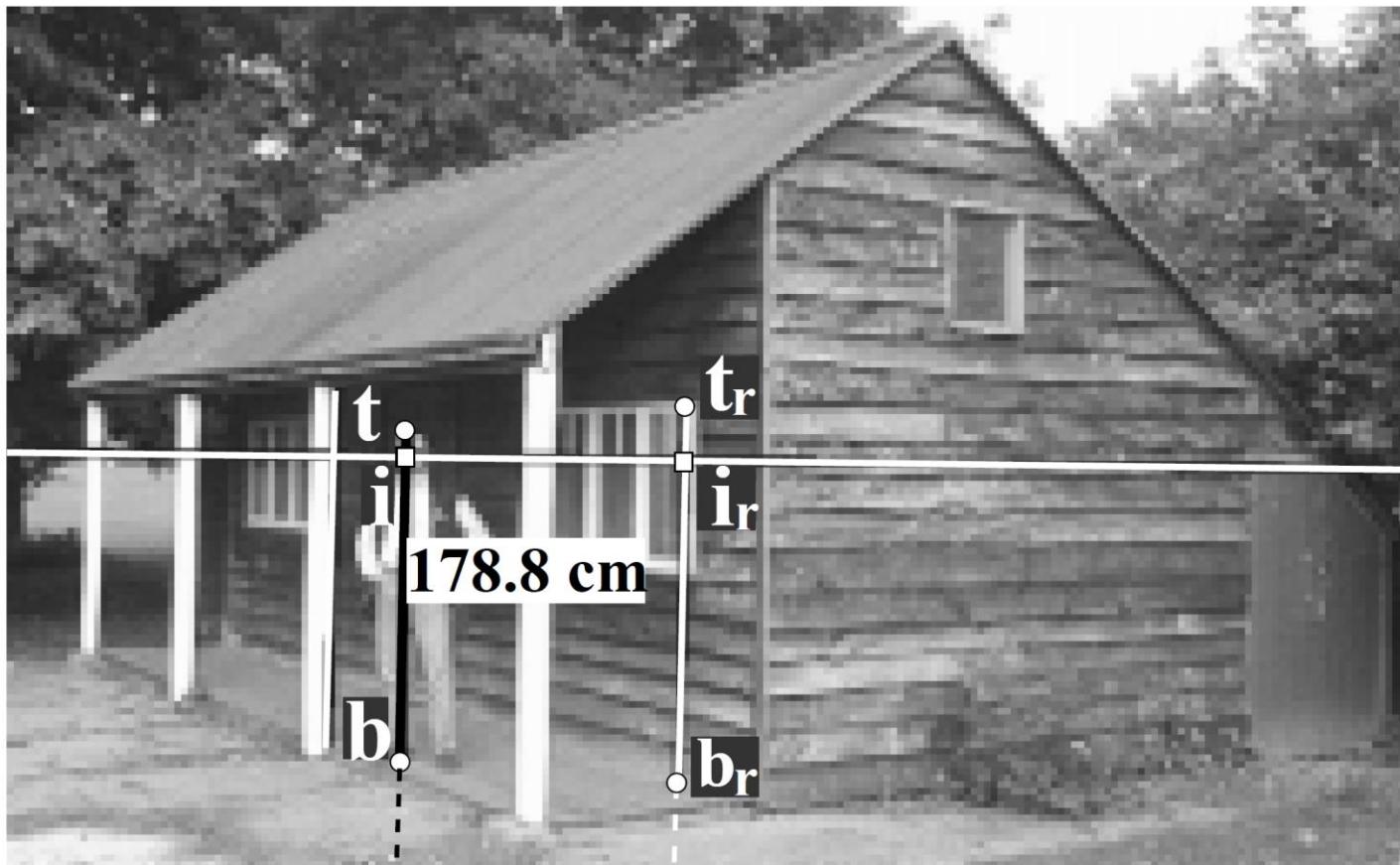
Paper List

- Nister, An Efficient Solution to the Five-Point Relative Pose Problem, PAMI, 2004



Paper List

- Criminisi, Reid, and Zisserman, Single View Metrology, IJCV, 2000



Paper List

- Xiao and Furukawa, Reconstructing the World's Museum, ECCV, 2012



Paper List

- Izadi et al., KinectFusion, UIST, 2011



A black and white photograph showing a group of approximately ten people gathered on the deck of a boat. In the foreground, a large astronomical telescope is mounted on a tripod, its eyepiece and body clearly visible. Several people are standing behind the telescope, looking through its eyepiece or adjusting its controls. One man in the center, wearing a light-colored t-shirt and glasses, is leaning over the telescope. To his right, another man is seated, also looking at the instrument. The background shows the open sea with some distant land or structures visible on the horizon. The overall atmosphere suggests a scientific or amateur astronomical observation session.

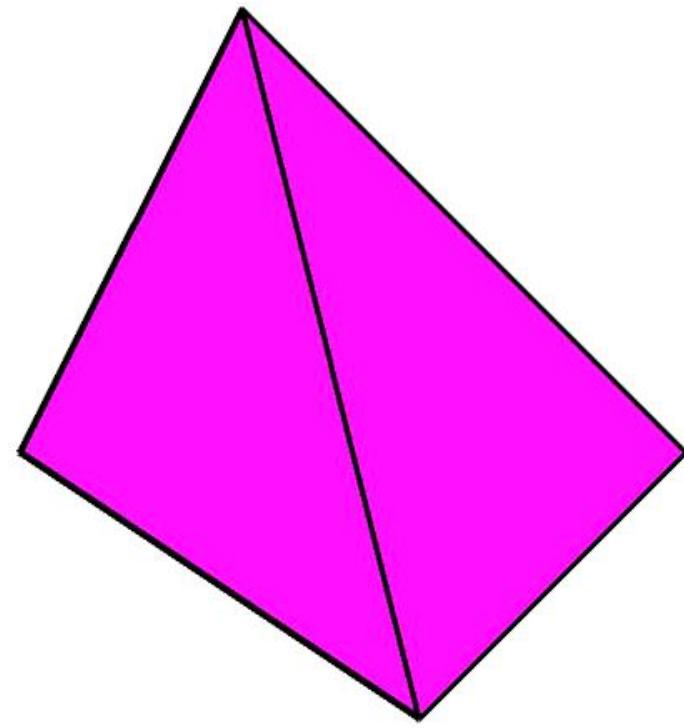
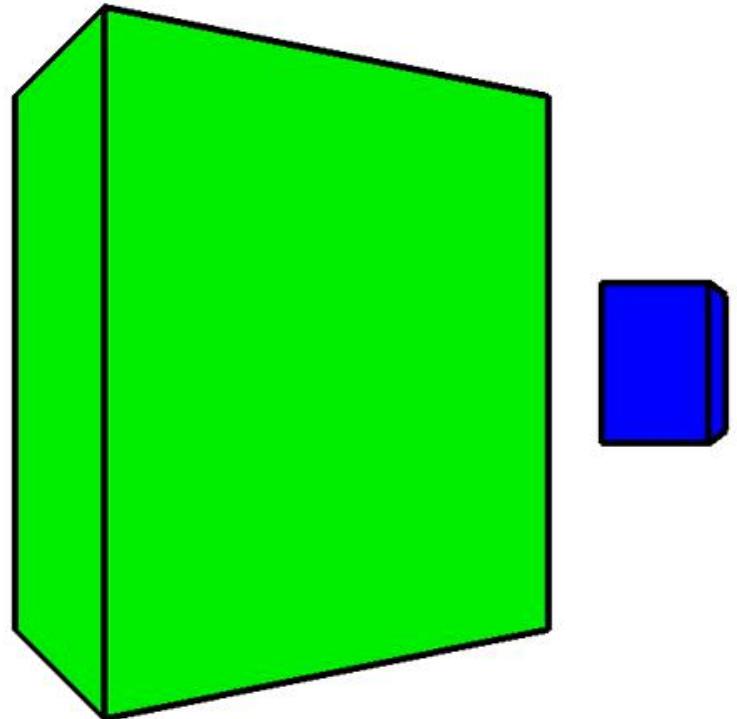
Camera Model

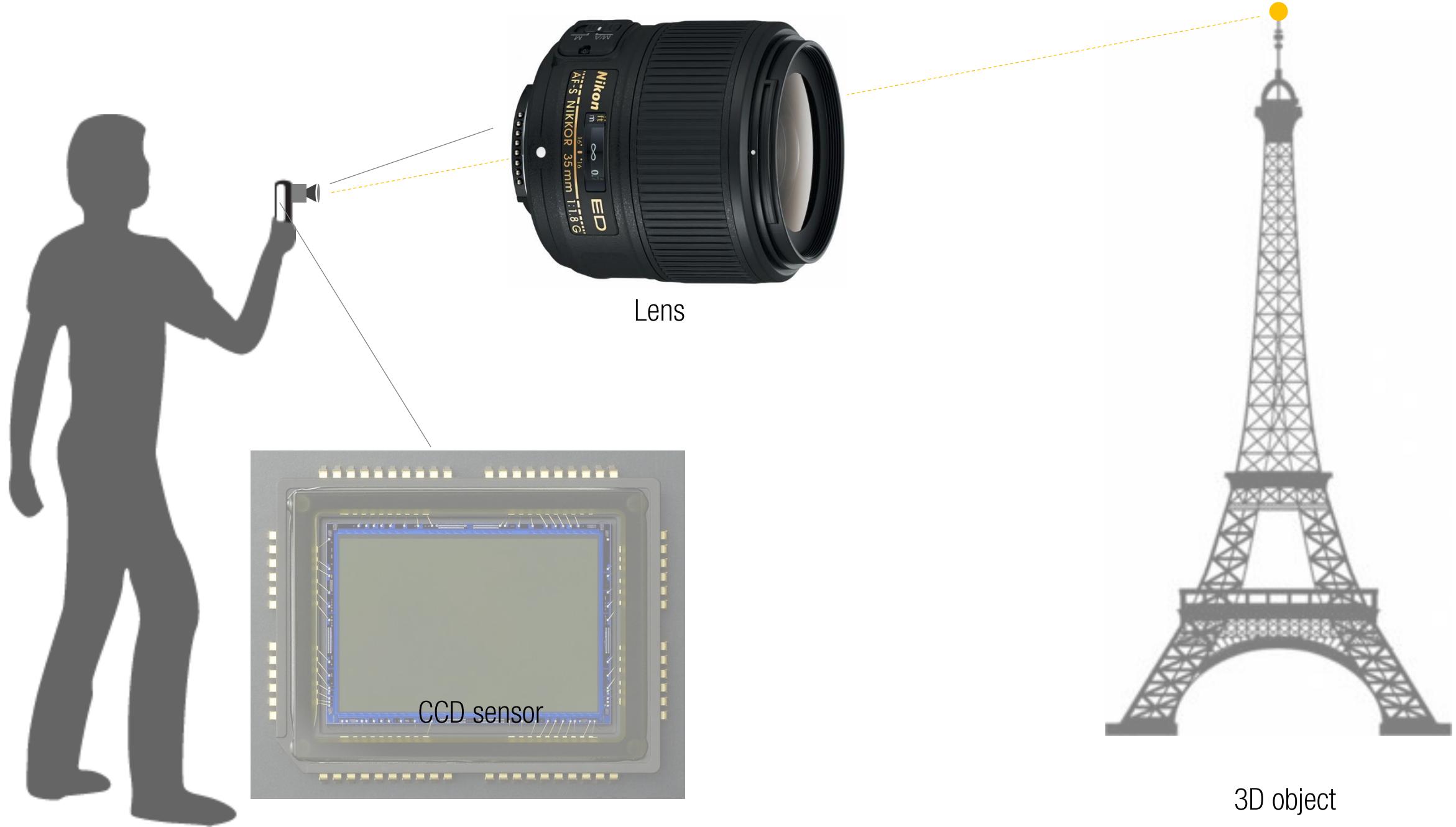


VERTIGO (1958)

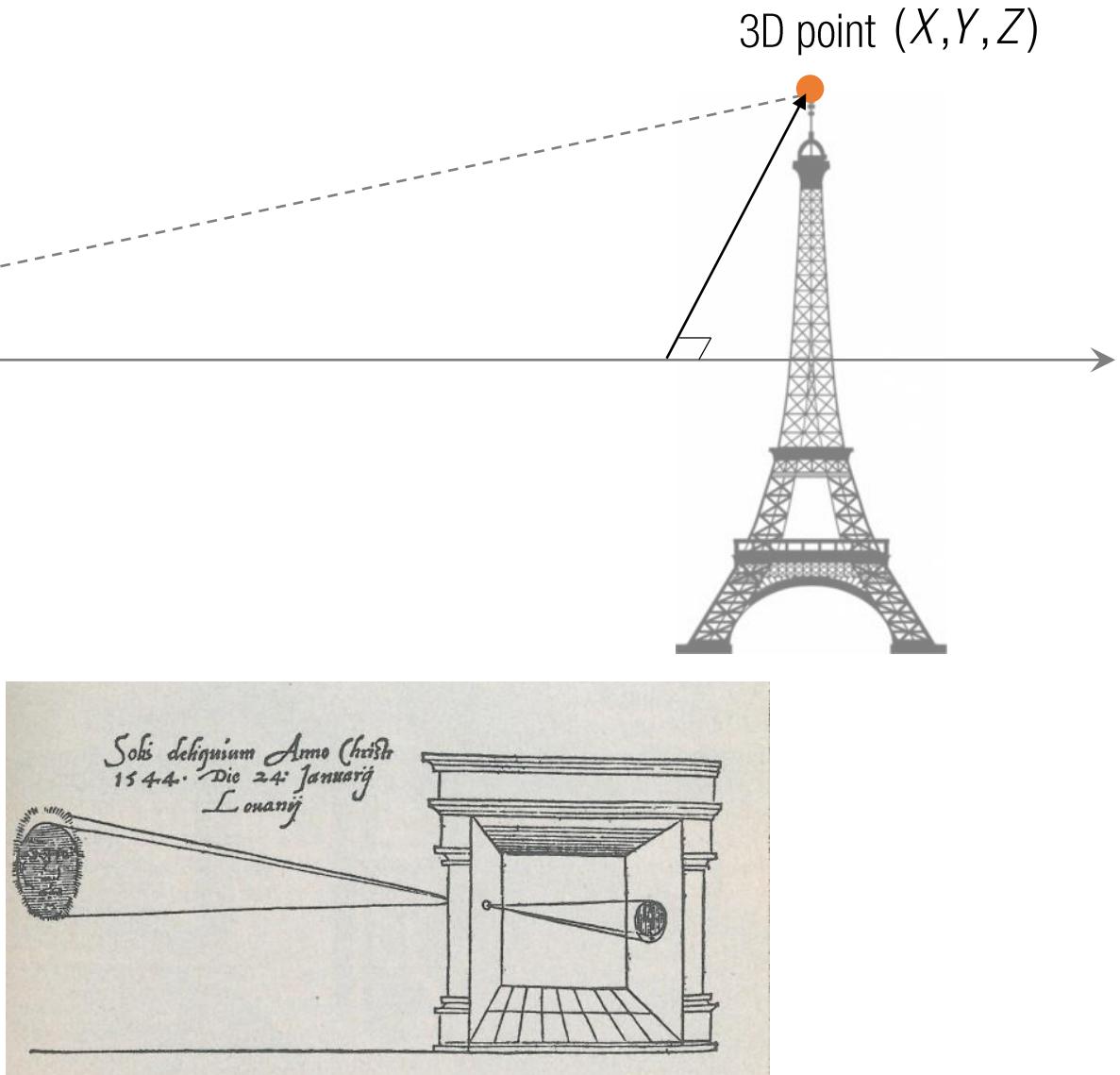
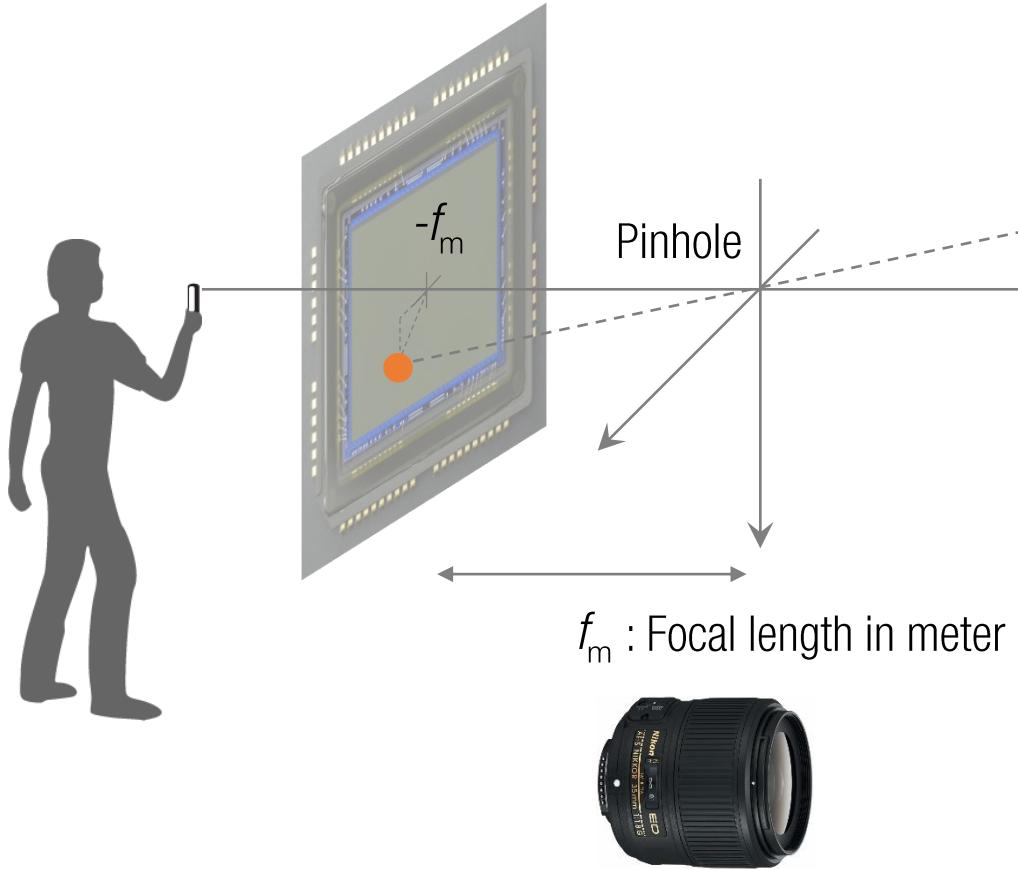


VERTIGO (1958)

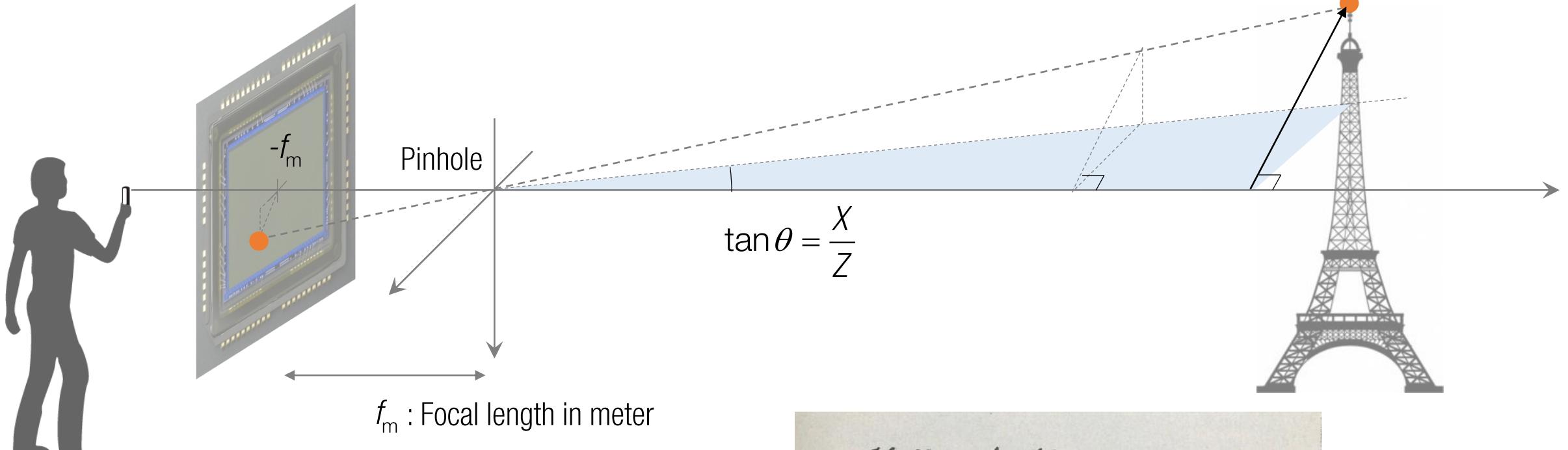




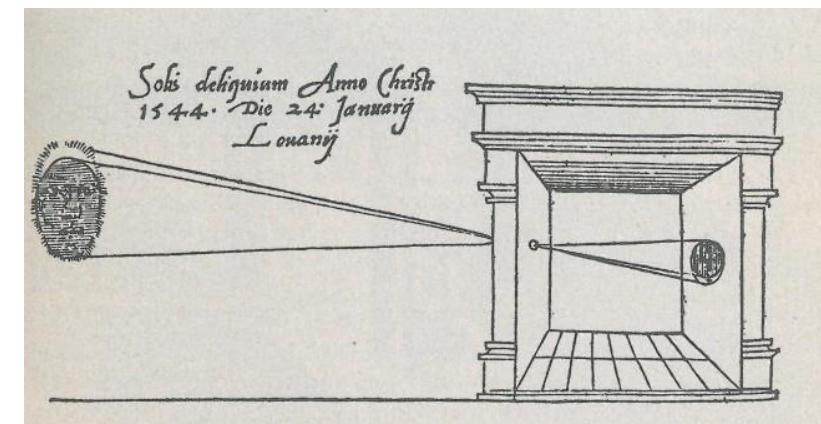
3D Point Projection (Metric Space)



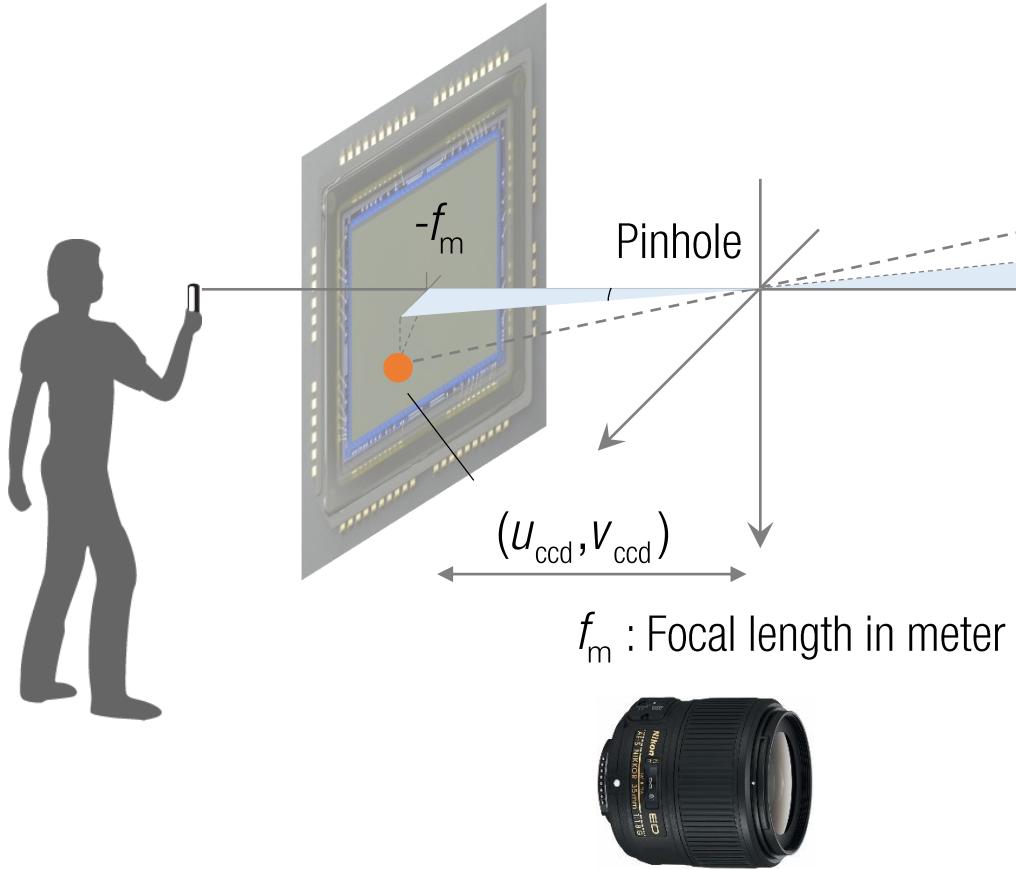
3D Point Projection (Metric Space)



f_m : Focal length in meter

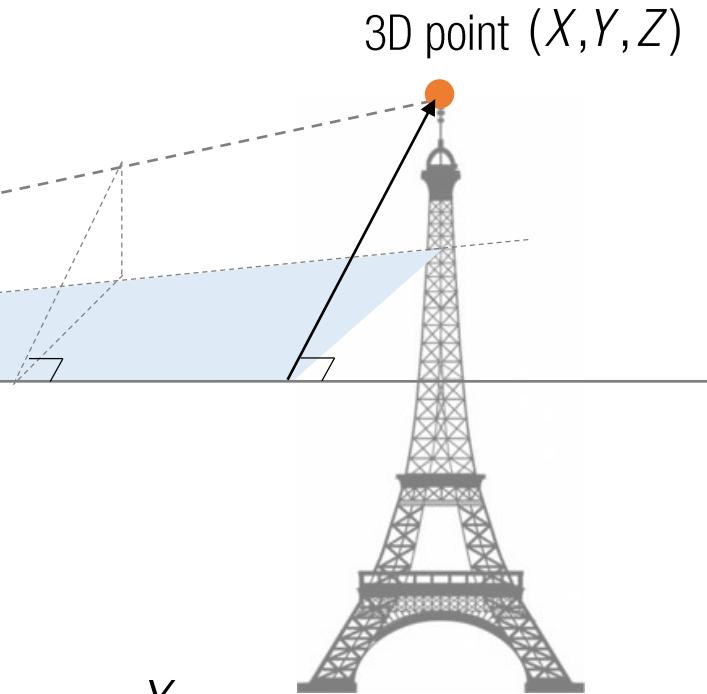


3D Point Projection (Metric Space)

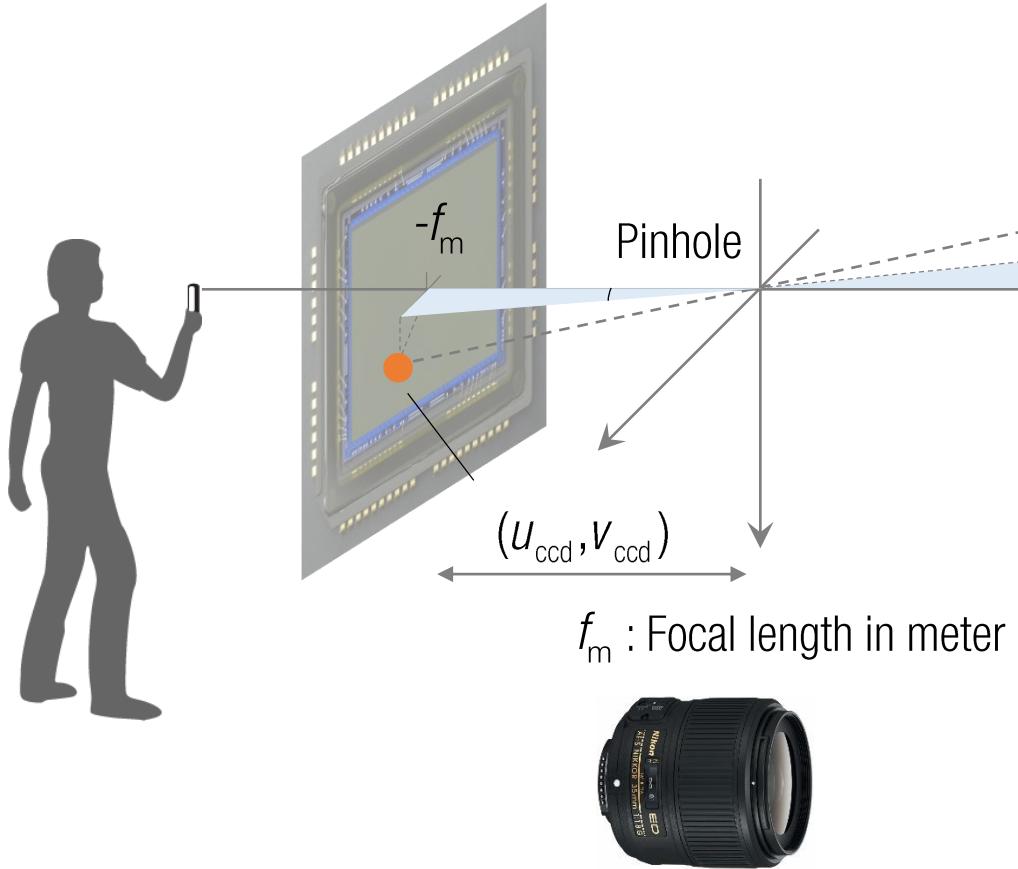


$$\tan \theta = \frac{X}{Z}$$

$$u_{\text{ccd}} = -f_m \tan \theta = -f_m \frac{X}{Z}$$



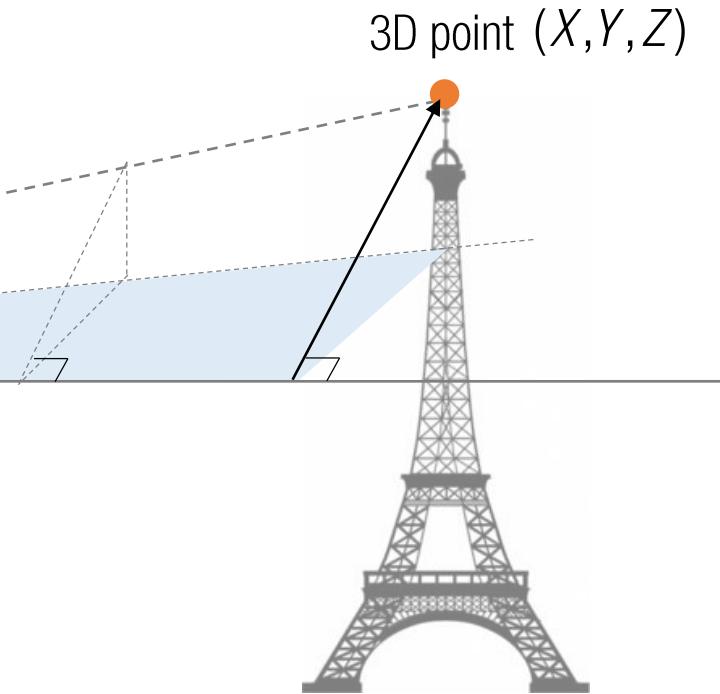
3D Point Projection (Metric Space)



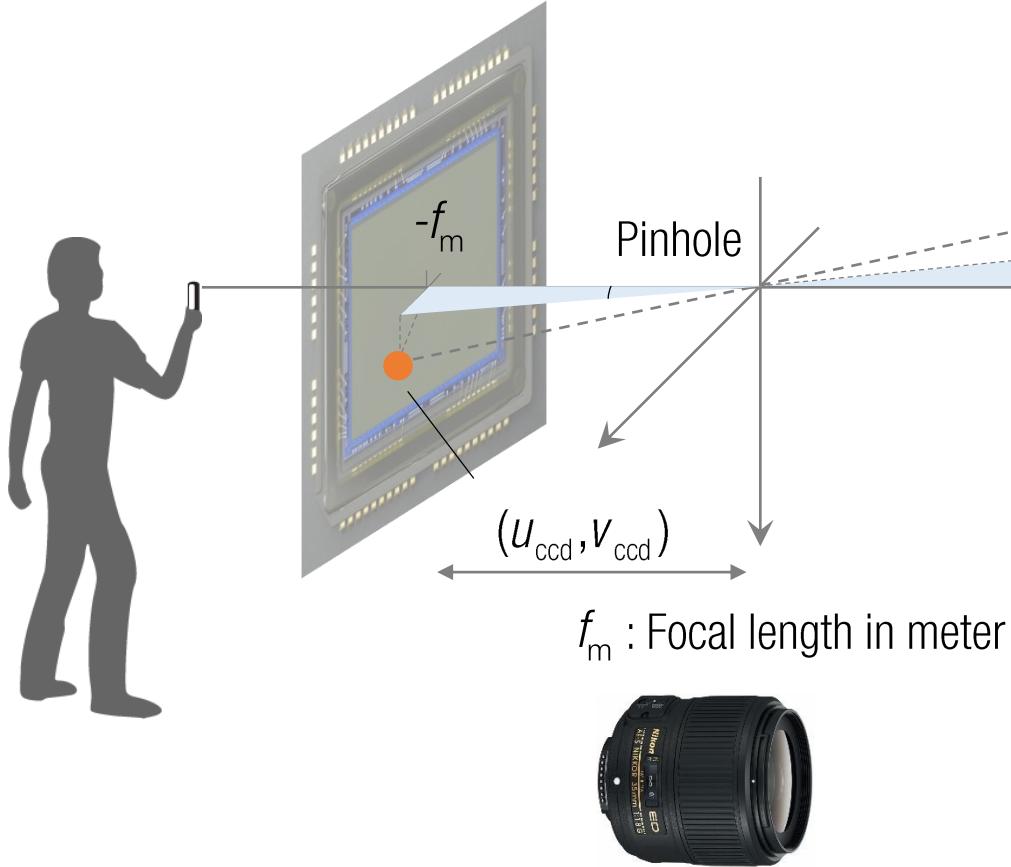
$$\tan \theta = \frac{X}{Z}$$

$$u_{\text{ccd}} = -f_m \frac{X}{Z}$$

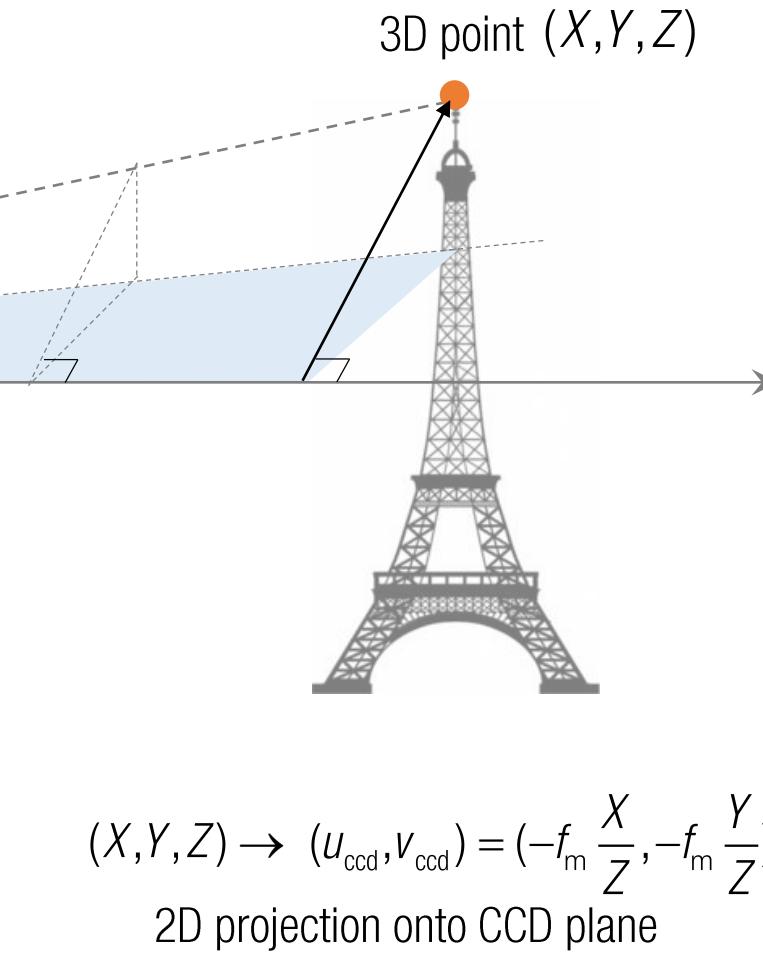
$$v_{\text{ccd}} = -f_m \frac{Y}{Z}$$



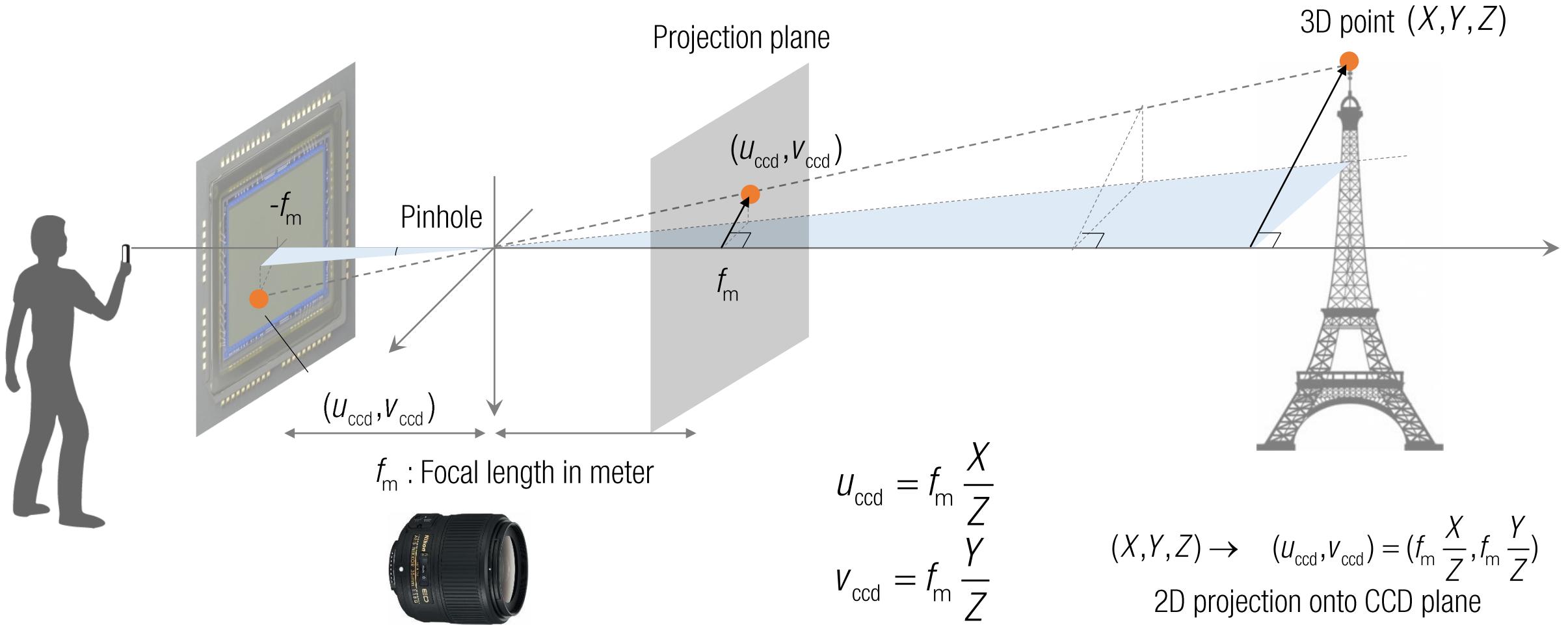
3D Point Projection (Metric Space)



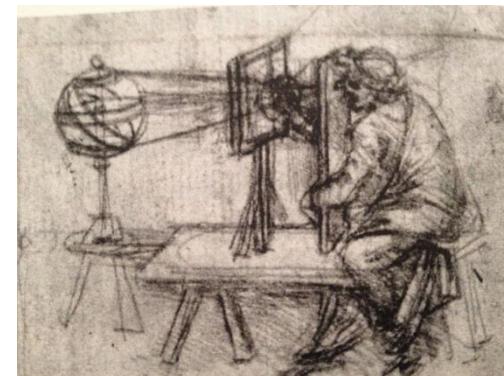
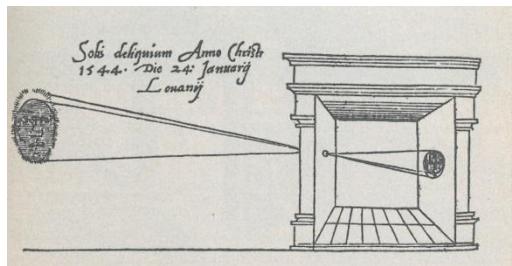
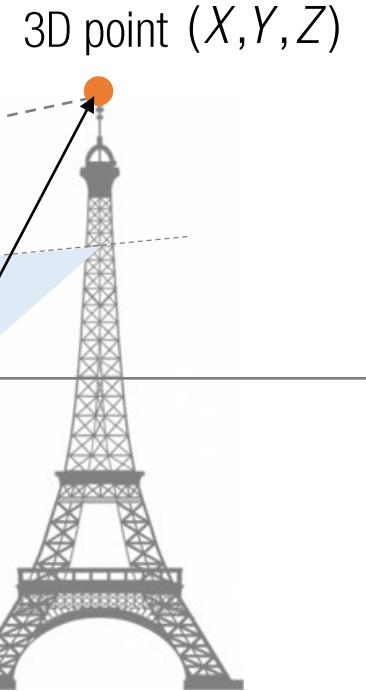
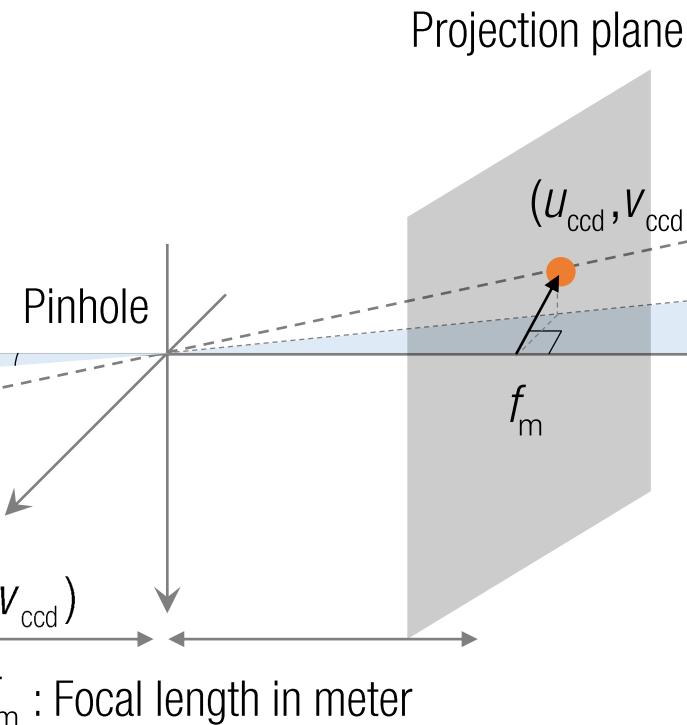
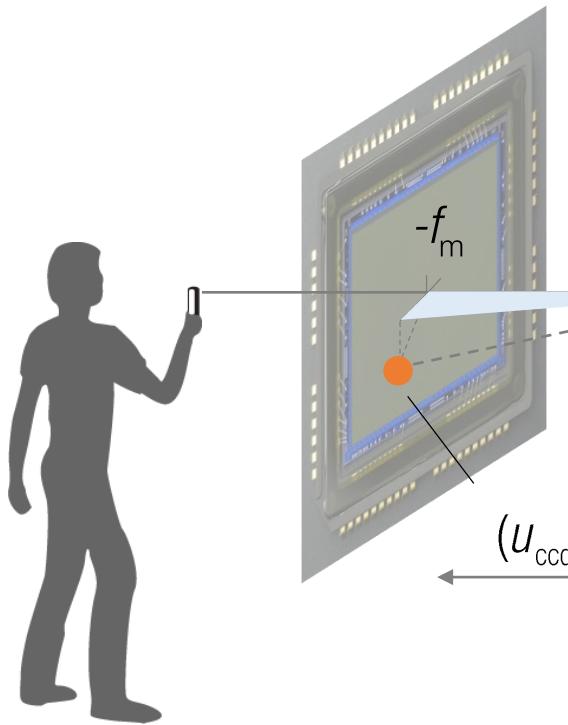
$$\tan \theta = \frac{X}{Z}$$



3D Point Projection (Metric Space)



3D Point Projection (Metric Space)



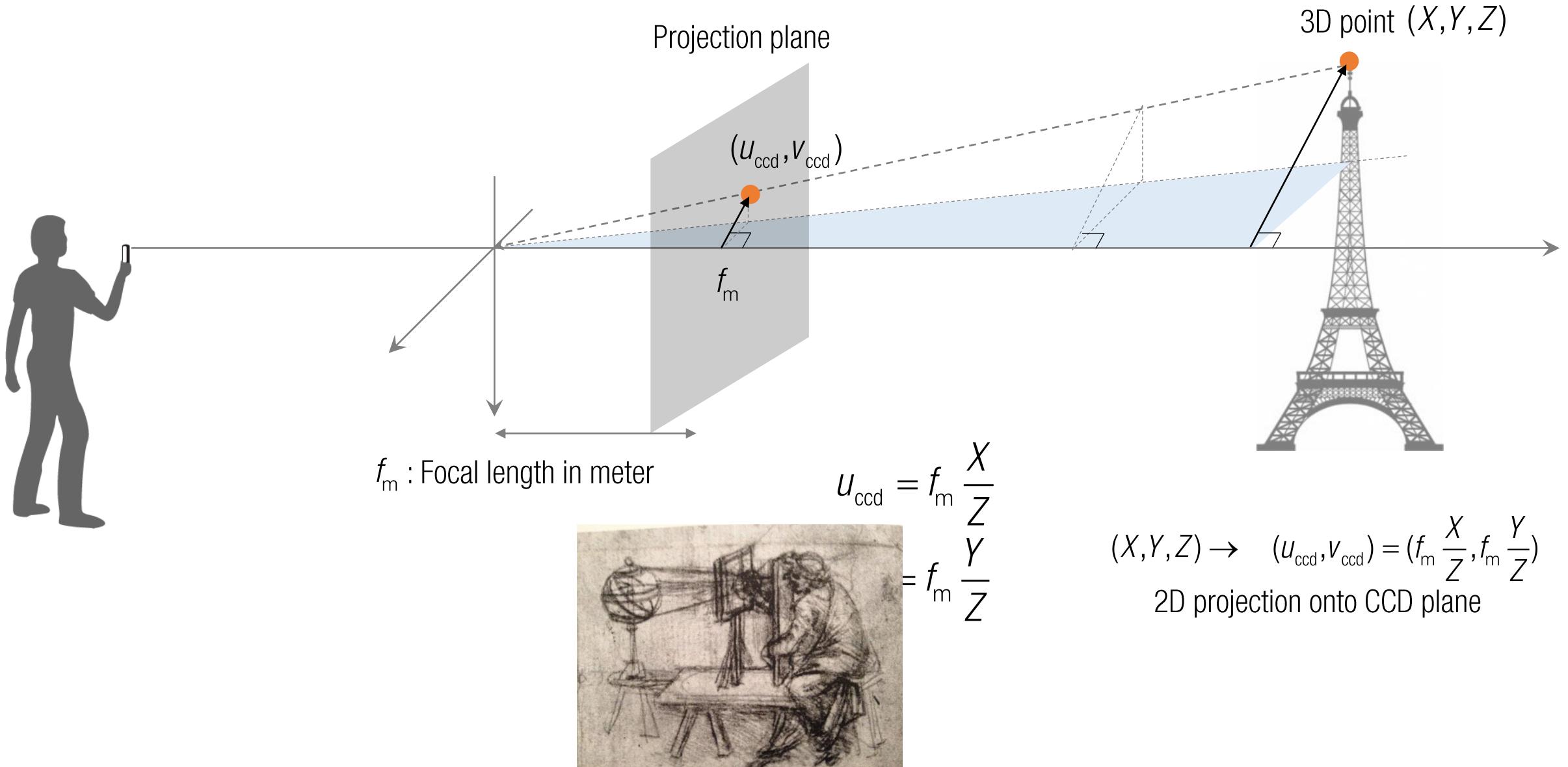
$$u_{\text{ccd}} = f_m \frac{X}{Z}$$

$$= f_m \frac{Y}{Z}$$

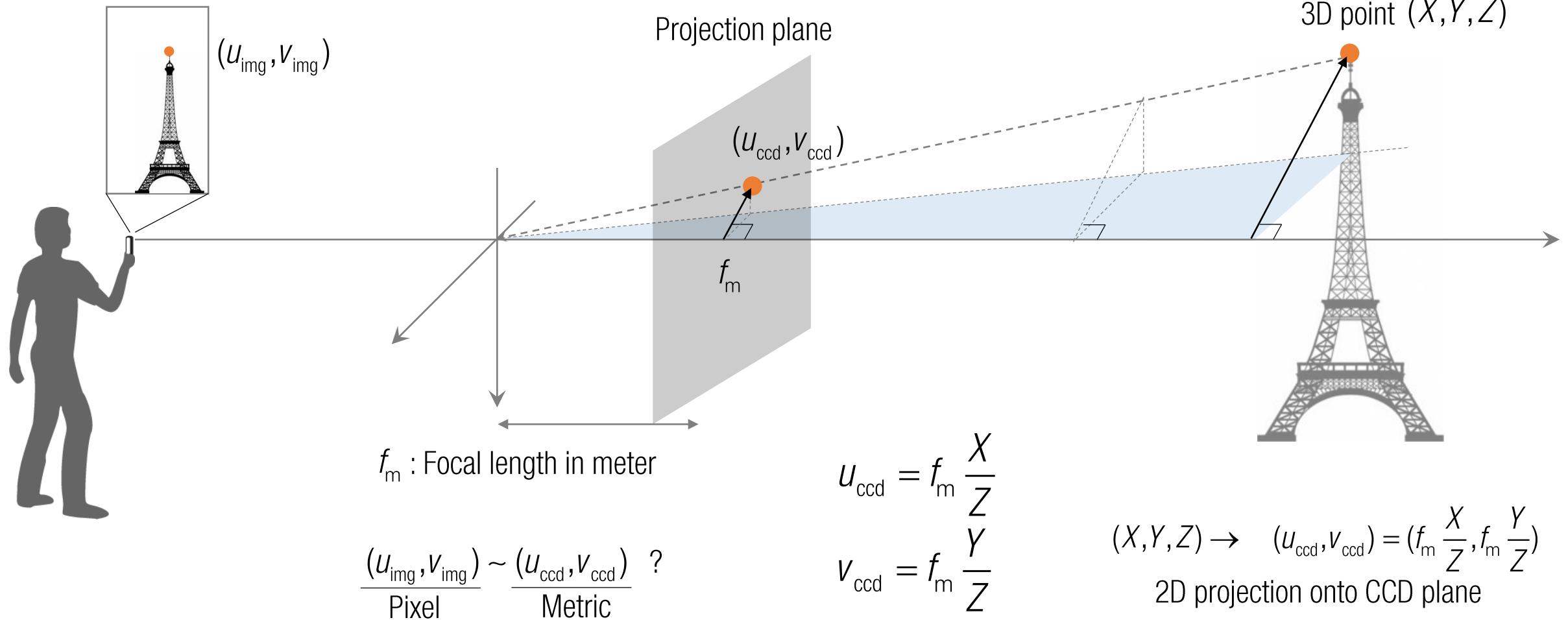
$$(X, Y, Z) \rightarrow (u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right)$$

2D projection onto CCD plane

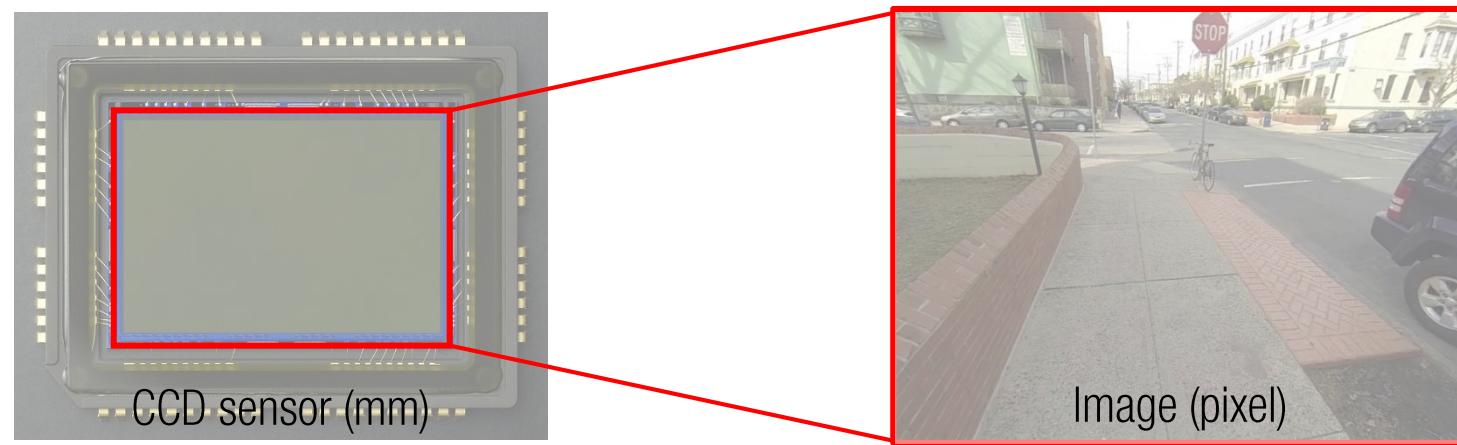
3D Point Projection (Metric Space)



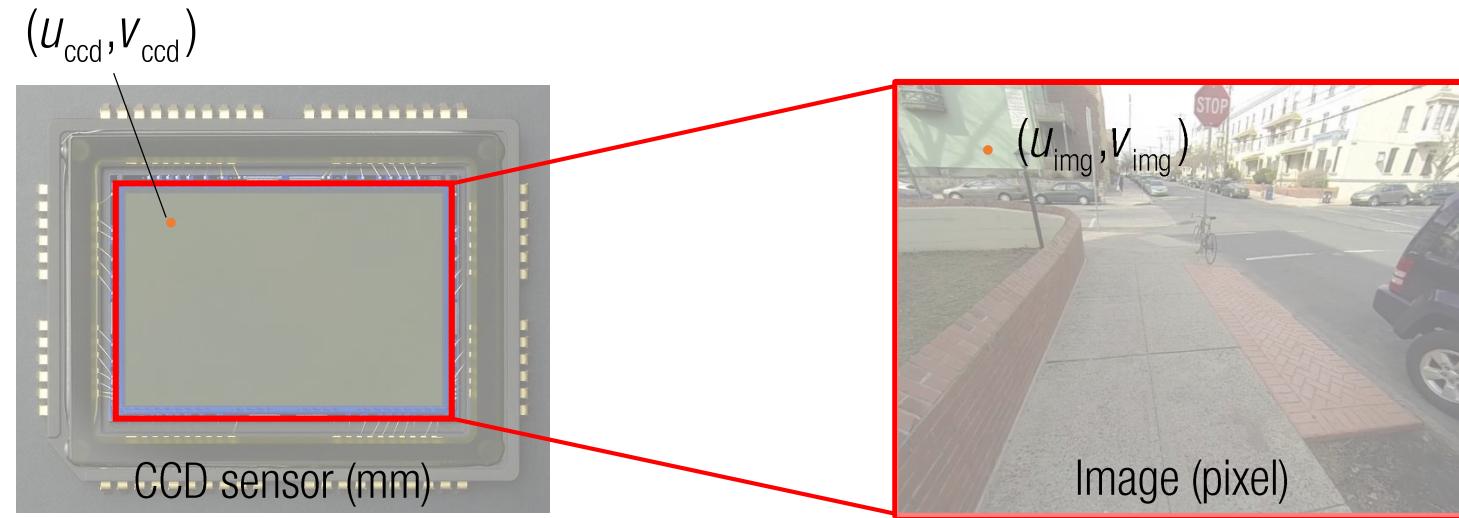
3D Point Projection (Metric Space)



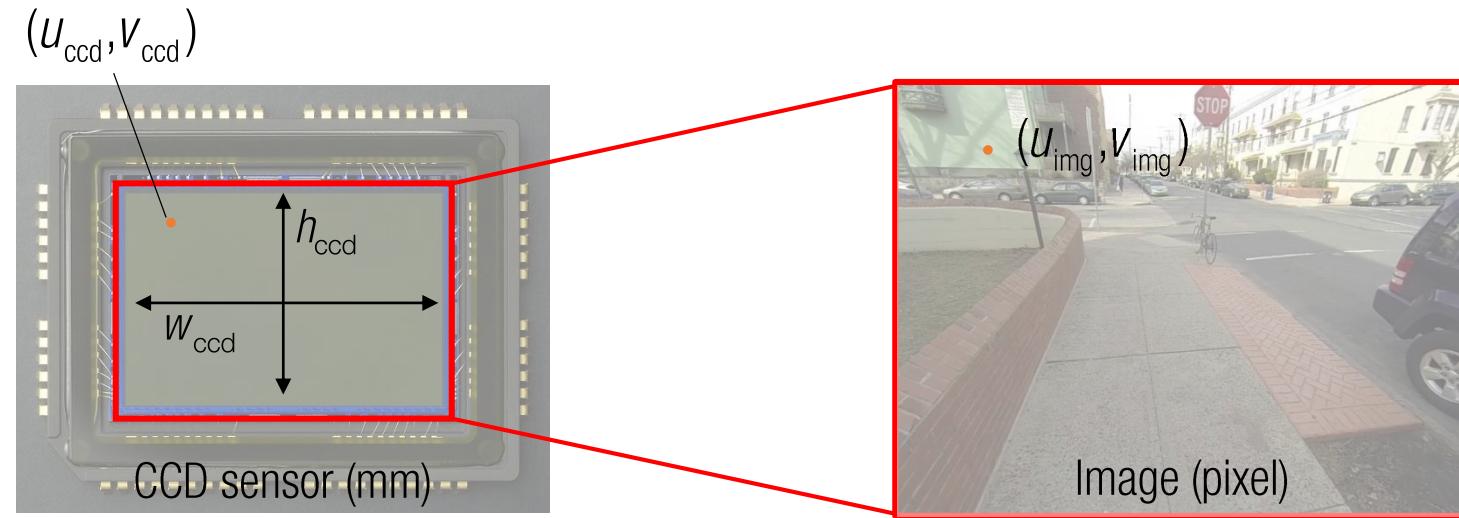
3D Point Projection (Pixel Space)



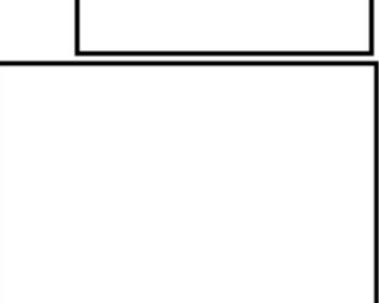
3D Point Projection (Pixel Space)



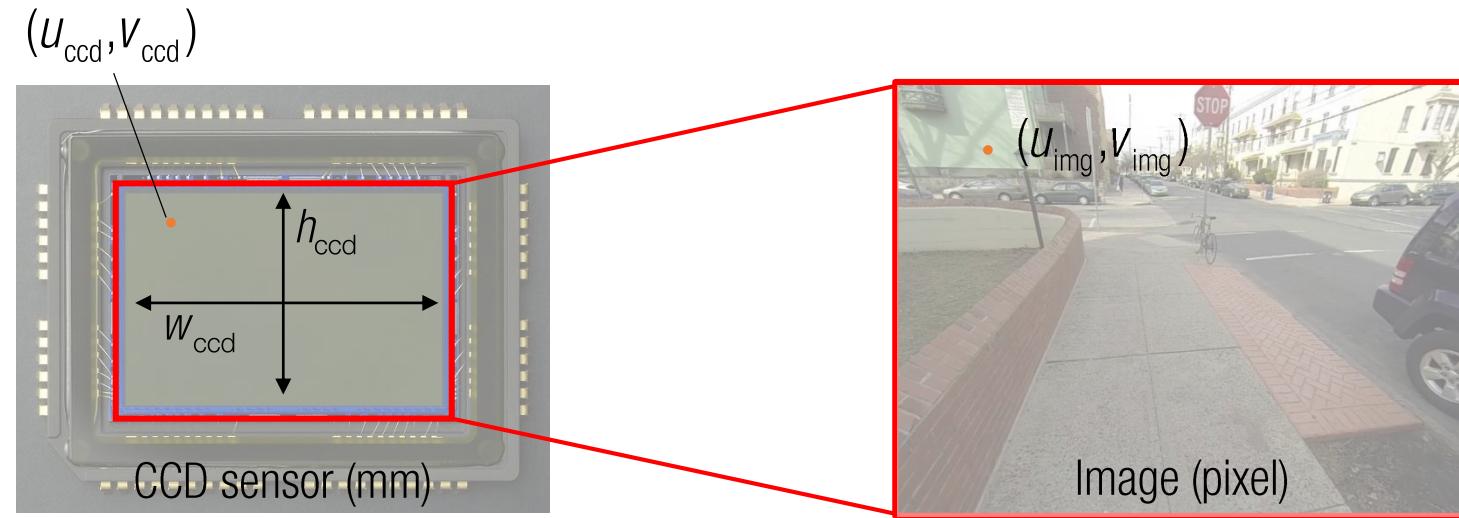
3D Point Projection (Pixel Space)



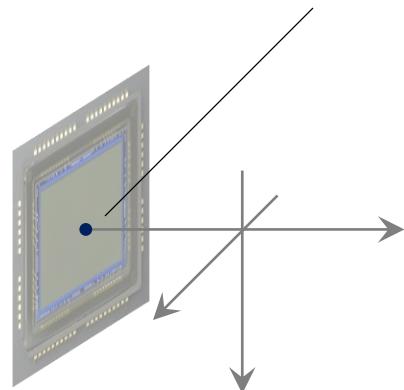
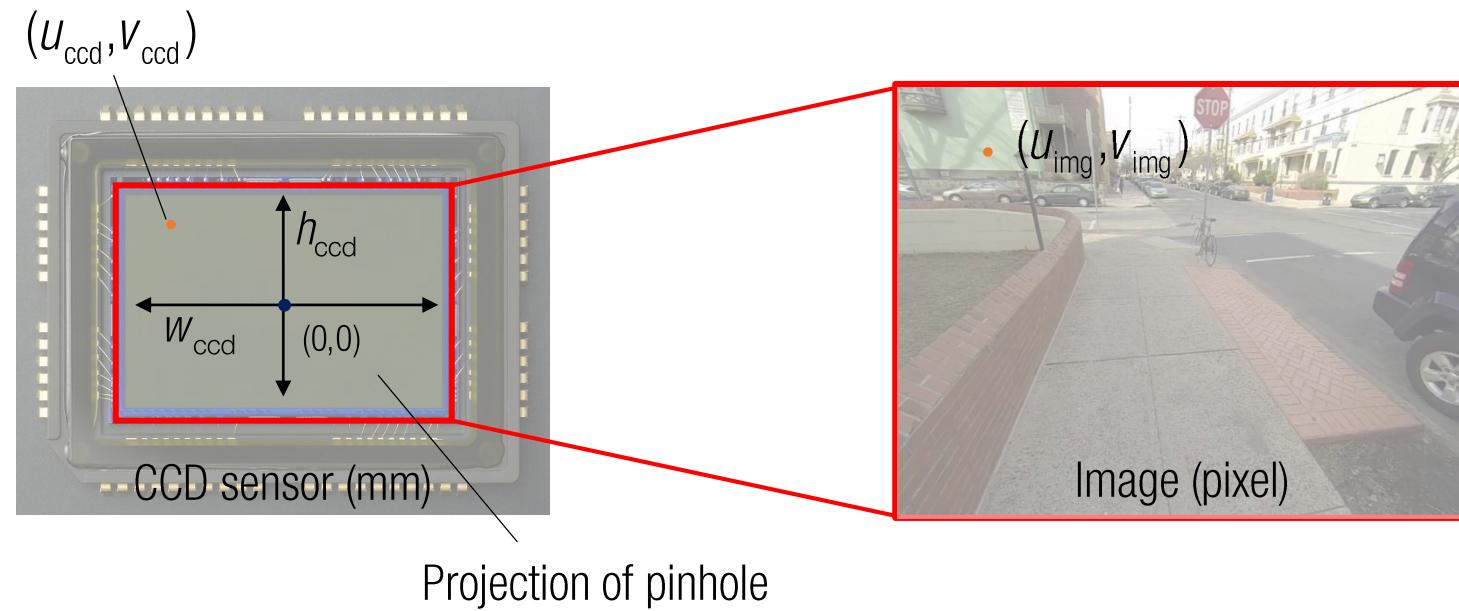
| Imager Sizes | Formats (Type) | ~Diag. | Uses |
|--------------|--|--------|---|
| | □ 1/7" - 1.85 x 1.39mm | 2.3 | Cell phones, web cams, etc.... |
| | □ 1/6" - 2.15 x 1.61mm | 2.7 | Cell phones, web cams, etc.... |
| | □ 1/5" - 2.55 x 1.91mm | 3.2 | Cell phones, web cams, etc.... |
| | □ 1/4" - 3.2 x 2.4mm | 4.0 | Cell phones, web cams, etc.... |
| | □ 1/3.6" - 4.0 x 3.0mm | 5.0 | P&S DSC |
| | □ 1/3.2" - 4.536 x 3.416mm | 5.678 | P&S DSC |
| | □ 1/3" - 4.8 x 3.6mm | 6.0 | Casio QV-8000SX (1.2MP), Epson PhotoPC 700 (1.2MP) |
| | □ 1/2.7" - 5.27 x 3.96mm | 6.592 | Canon PowerShot A20 (1.92MP), HP PhotoSmart C618 (1.92) |
| | □ 1/2" - 6.4 x 4.8mm | 8.0 | Olympus C-2100Z (1.92MP), Epson PhotoPC 850Z (1.92) |
| | □ 1/1.8" - 7.176 x 5.319mm | 8.932 | Nikon Coolpix 995 (3.14MP), Olympus C-4040Z (3.9MP), Canon PowerShot G2 (3.8MP), Sony DSC-S85 (3.8MP) |
| | □ 2/3" - 8.8 x 6.6mm | 11.0 | Nikon Coolpix 5000 (4.92MP), Sony DSC-F707 (4.92MP), Olympus E-10 (3.7MP), Minolta DiMAGE 7 (4.92MP) |
| | □ 1" - 12.8 x 9.6mm | 16.0 | Not used in DSCs. Used in some high-end video cameras |
| | Kodak KAF-5100CE CCD 17.8 x 13.4mm (4/3") | 22.28 | Olympus announced development of a new camera and new lenses for this 4/3" size. 2614 x 1966 = 5.1MP - 6.8µm pixel |
| | Foveon X3 F7-35X3-A25B 20.7 x 13.8mm | 24.9 | Sigma SD9 (X3) 2268 x 1512 = 3.43MP - 9.12µm pixel 1.74x Focal Length Multiplier (35mm film) |

| | | | |
|---|--|--------|--|
|  | Canon D30 CMOS 21.8 x 14.5mm | 26.2 | Canon D30 2160 x 1440 = 3.11MP - 10.1µm pixel 1.65x Focal Length Multiplier (35mm film) |
|  | Canon D60 CMOS 22.7 x 15.1mm | 27.3 | Canon D60 3072 x 2048 = 6.3MP - 7.4µm pixel 1.59x Focal Length Multiplier (35mm film) |
|  | Nikon D100 CCD Nikon D1x CCD 23.7 x 15.6mm | 28.2 | Nikon D100 - 3008 x 2000 = 6.1MP - 7.8µm pixel Nikon D1x - 4024 x 1324 = 5.24MP - 5.9 x 11.7µm pixel 1.52x Focal Length Multiplier (35mm film) |
|  | APS Film 25.1 x 16.7mm | 30.148 | APS cameras 1.44x Focal Length Multiplier (35mm film) |
|  | Canon EOS-1D CCD 27.0 x 17.8mm | 32.3 | Canon EOS-1D 2464 x 1648 = 4.06MP - 10.8µm pixel 1.34x Focal Length Multiplier (35mm Film) |
|  | Kodak KAF-6303CE CCD 27.8 x 18.5mm | 33.4 | Kodak 760 3088 x 2056 = 6.35MP - 9.0µm pixel 1.30x Focal Length Multiplier (35mm film) |
|  | 35mm Film Canon 1Ds Kodak 14n 36.0 x 24.0mm | 43.27 | 35mm film cameras Canon 1Ds - 4064 x 2704 = 10.99MP - 8.85µm pixel Kodak DCS Pro 14n - 4536 x 3024 = 13.7MP - 7.94µm pixel |

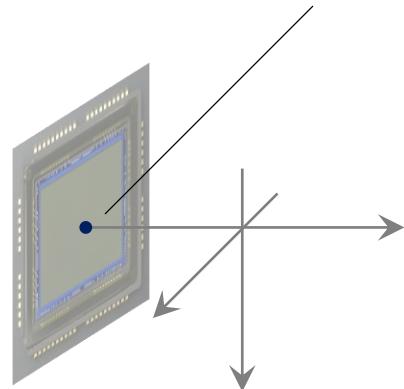
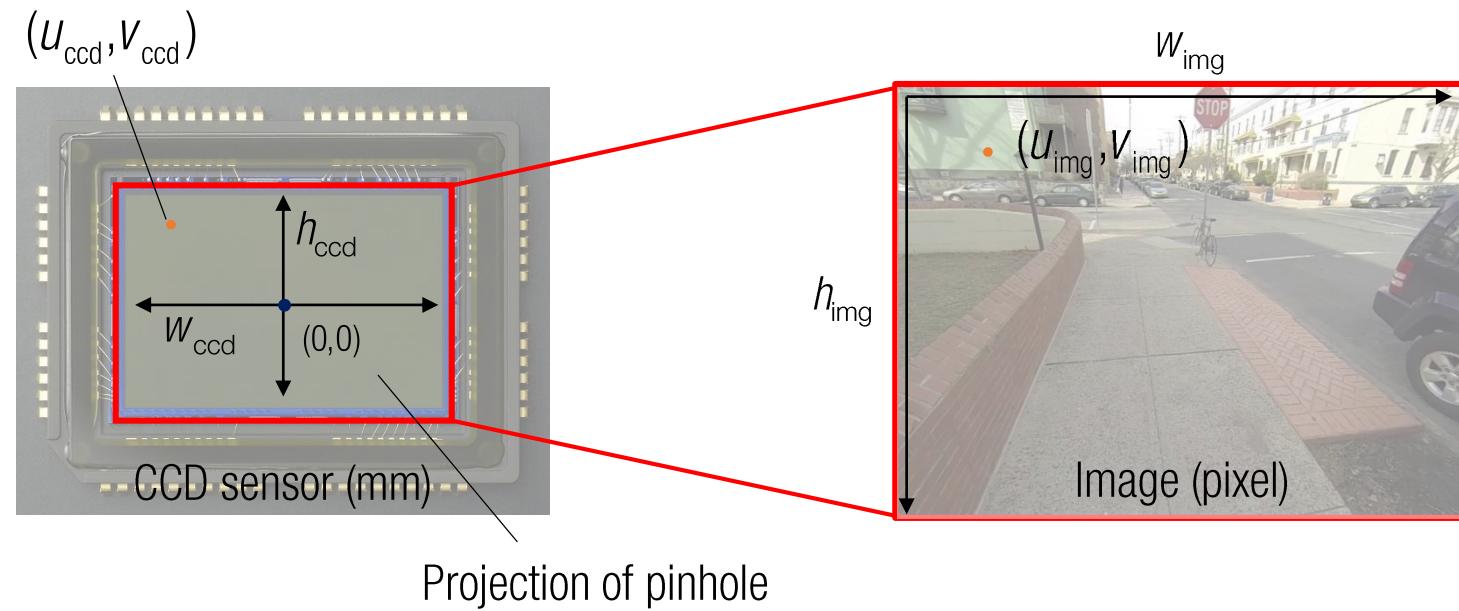
3D Point Projection (Pixel Space)



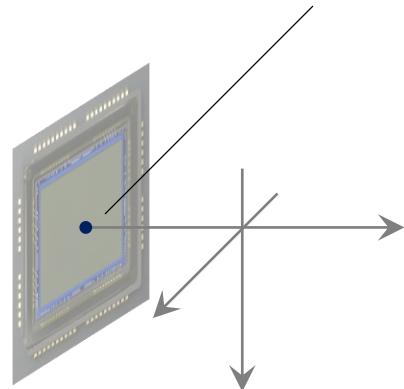
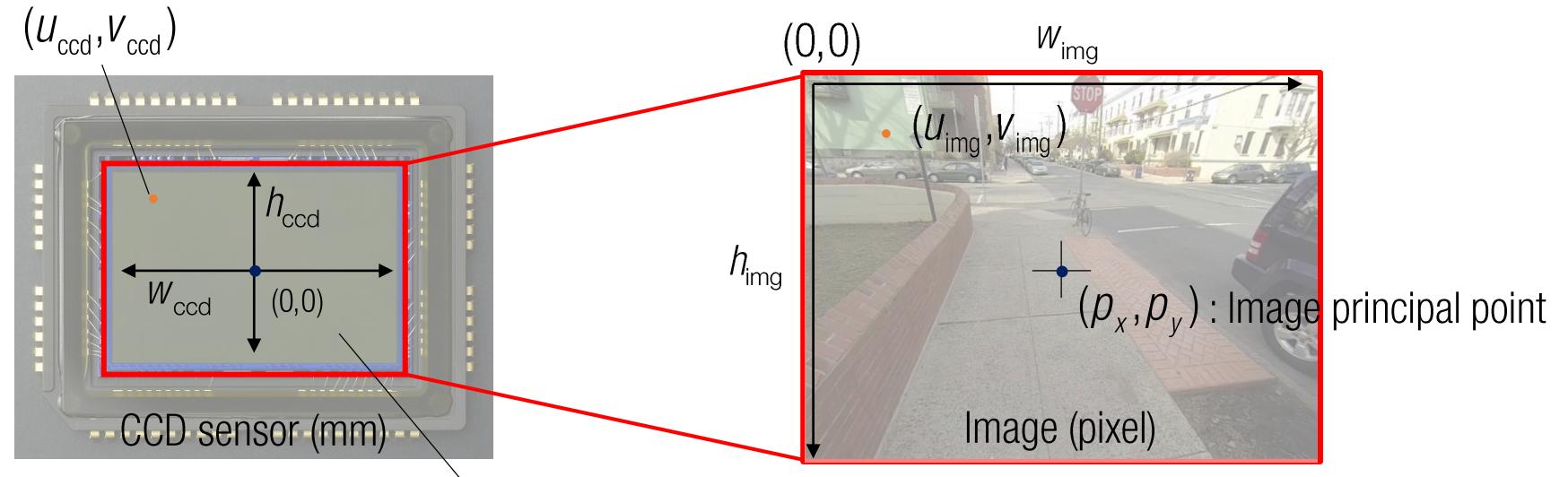
3D Point Projection (Pixel Space)



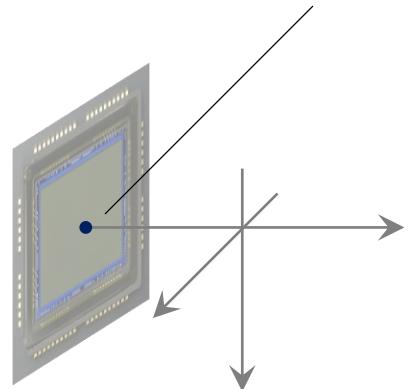
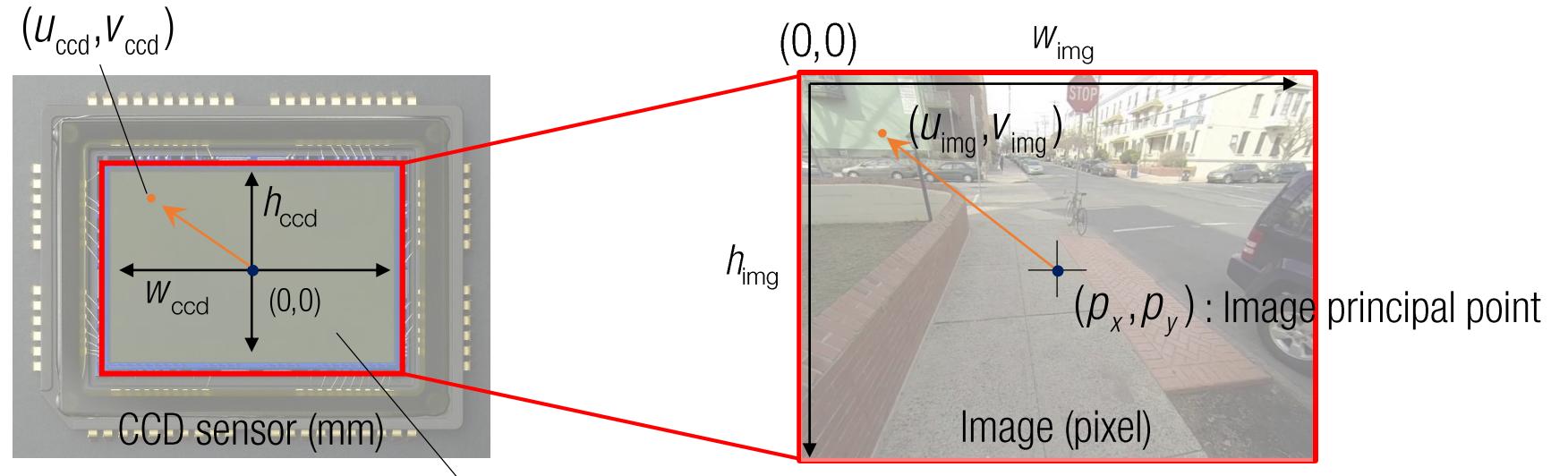
3D Point Projection (Pixel Space)



3D Point Projection (Pixel Space)

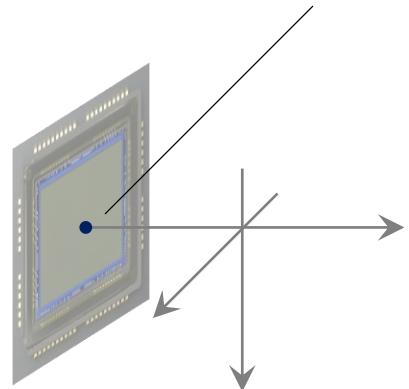
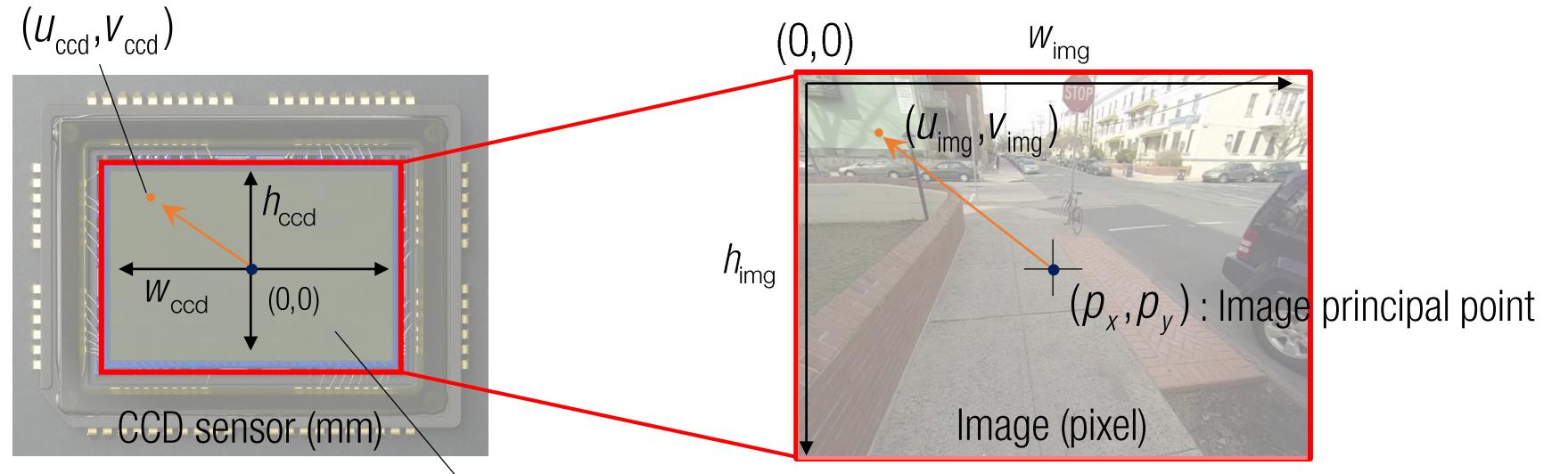


3D Point Projection (Pixel Space)



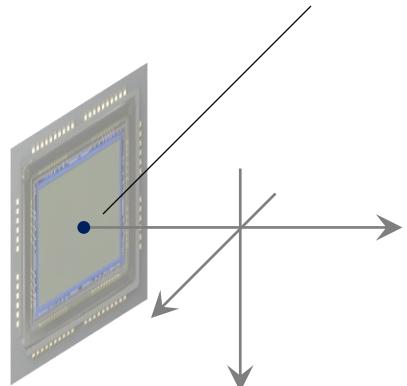
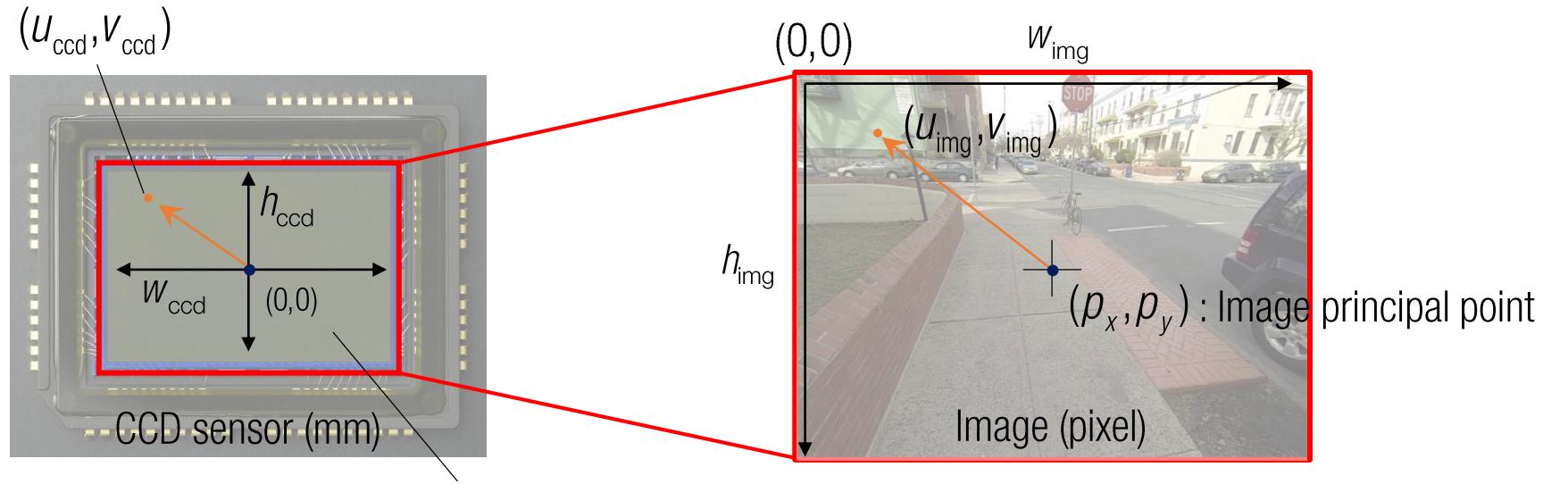
$$\frac{u_{\text{ccd}}}{W_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{W_{\text{img}}}$$

3D Point Projection (Pixel Space)



$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$

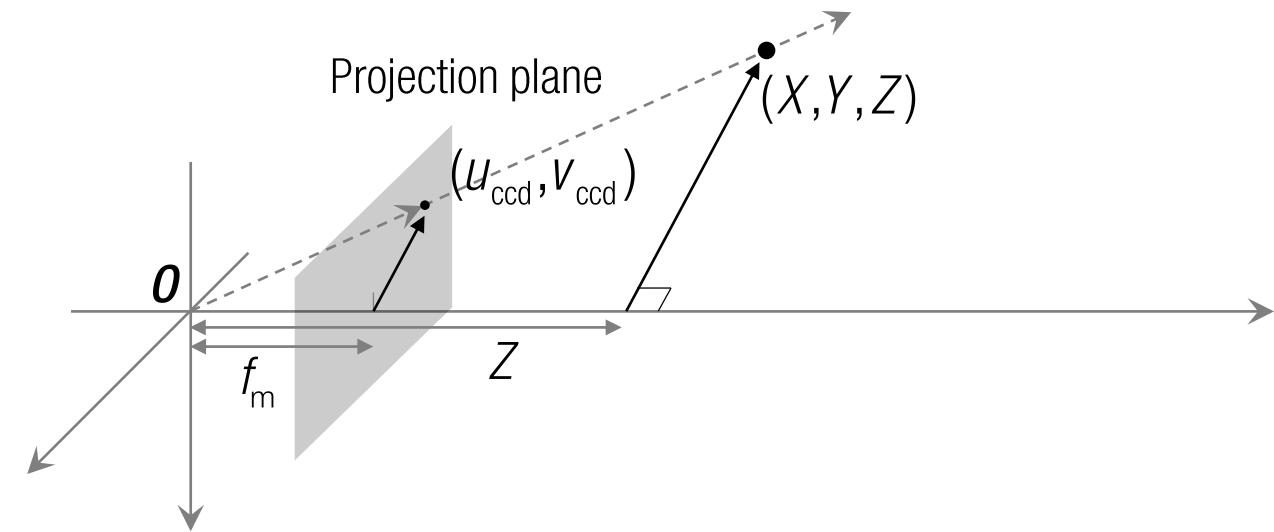
3D Point Projection (Pixel Space)



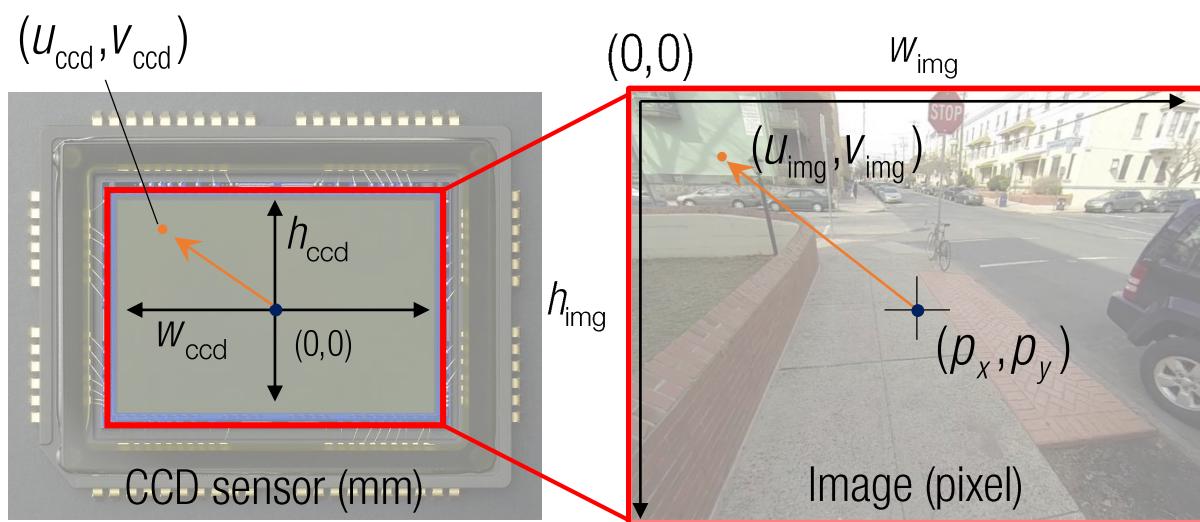
$$\frac{u_{\text{ccd}}}{w_{\text{ccd}}} = \frac{u_{\text{img}} - p_x}{w_{\text{img}}} \quad \frac{v_{\text{ccd}}}{h_{\text{ccd}}} = \frac{v_{\text{img}} - p_y}{h_{\text{img}}}$$

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x \quad v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y$$

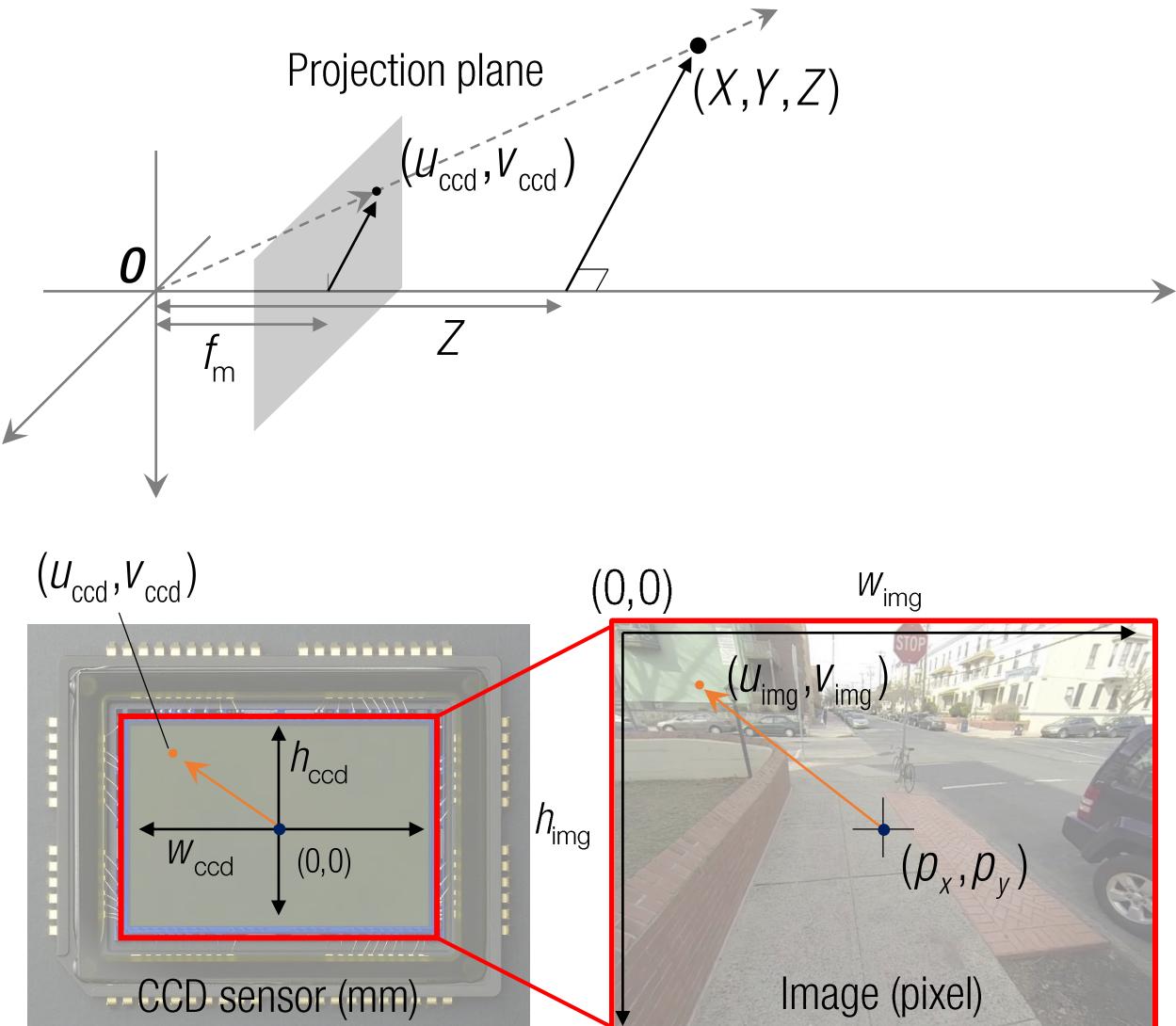
3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$



3D Point Projection (Pixel Space)



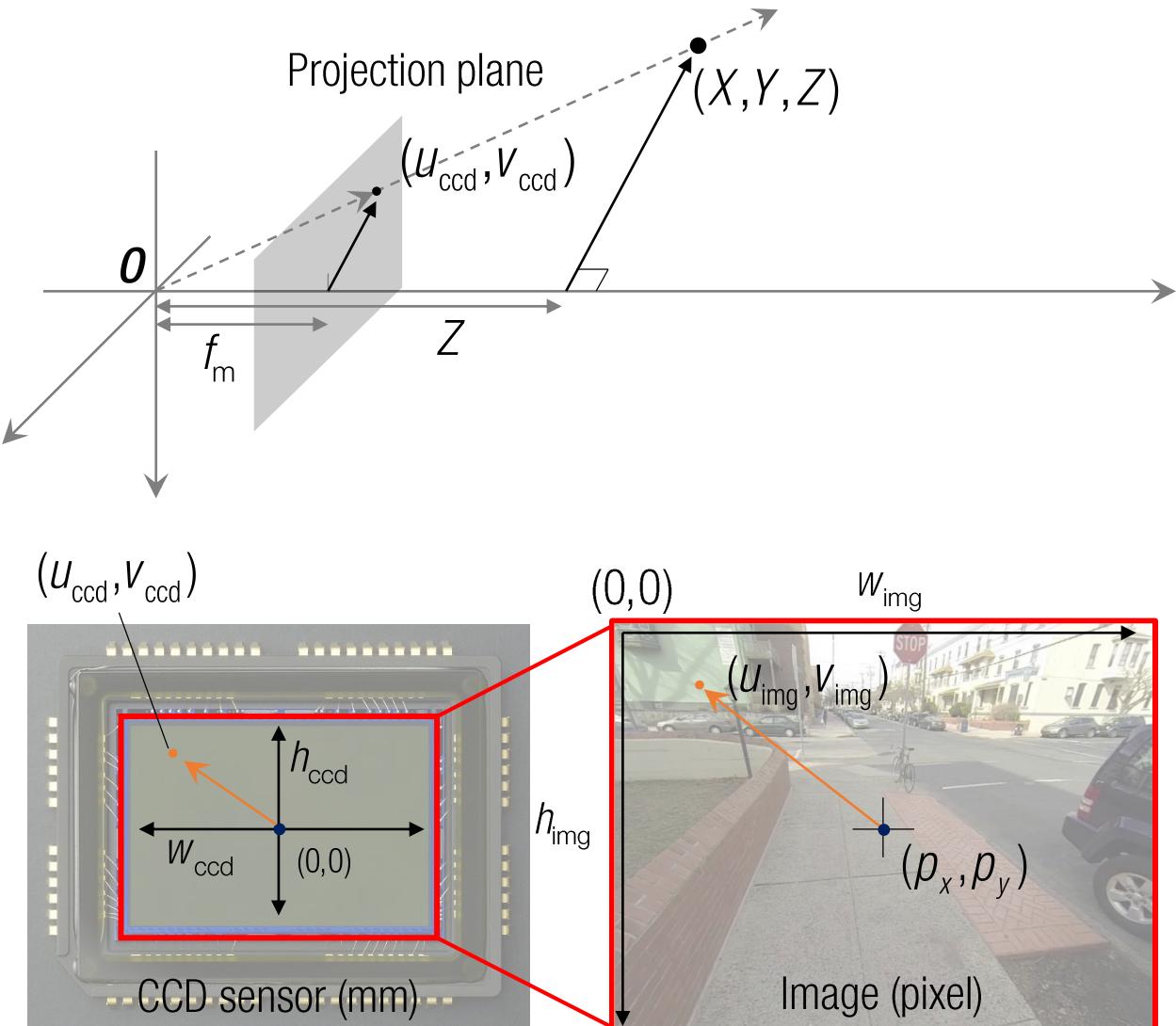
$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{W_{\text{img}}}{W_{\text{ccd}}} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y$$

3D Point Projection (Pixel Space)



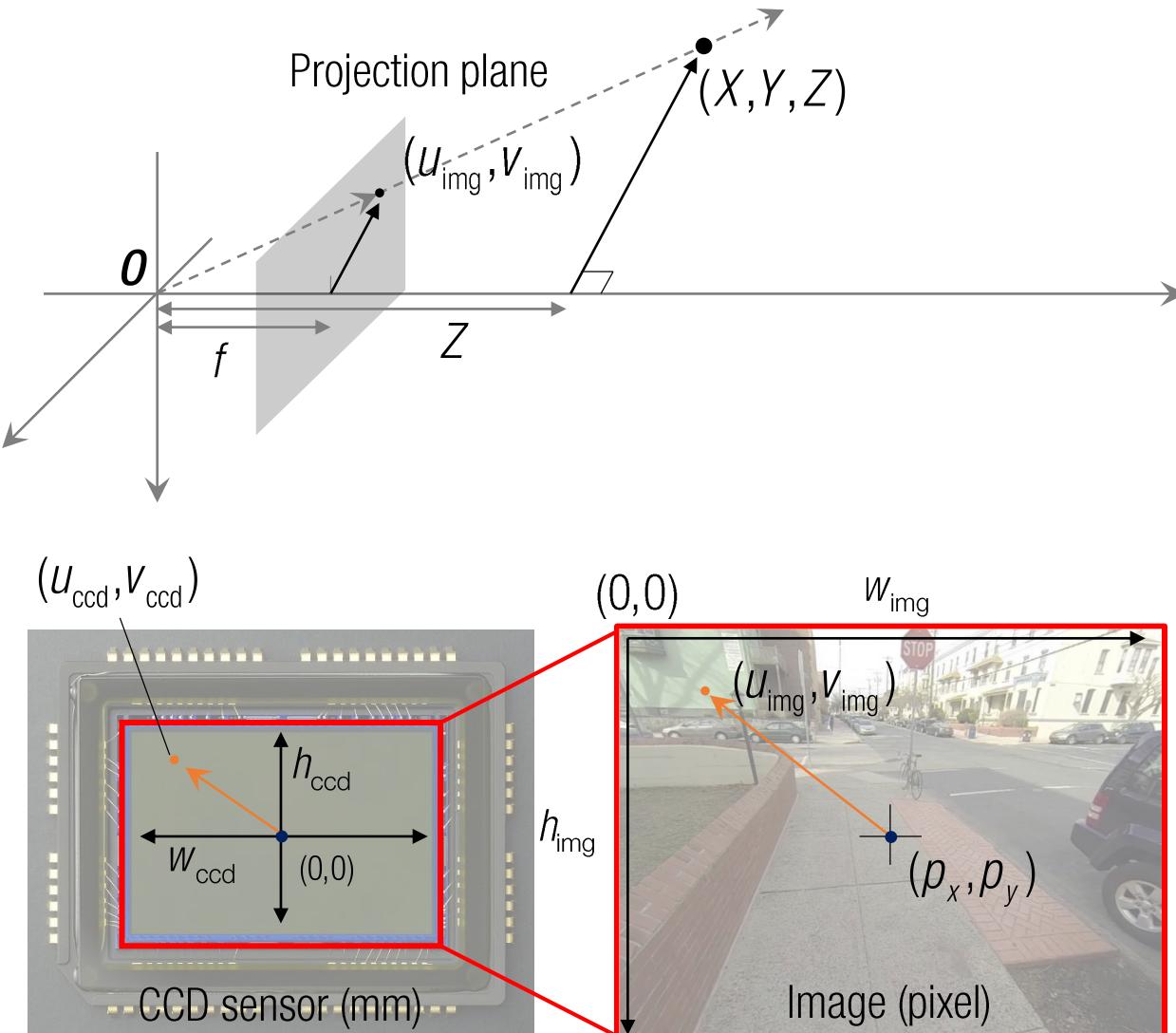
$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = f_m \frac{w_{\text{img}}}{w_{\text{ccd}}} \frac{X}{Z} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z} + p_y$$

3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

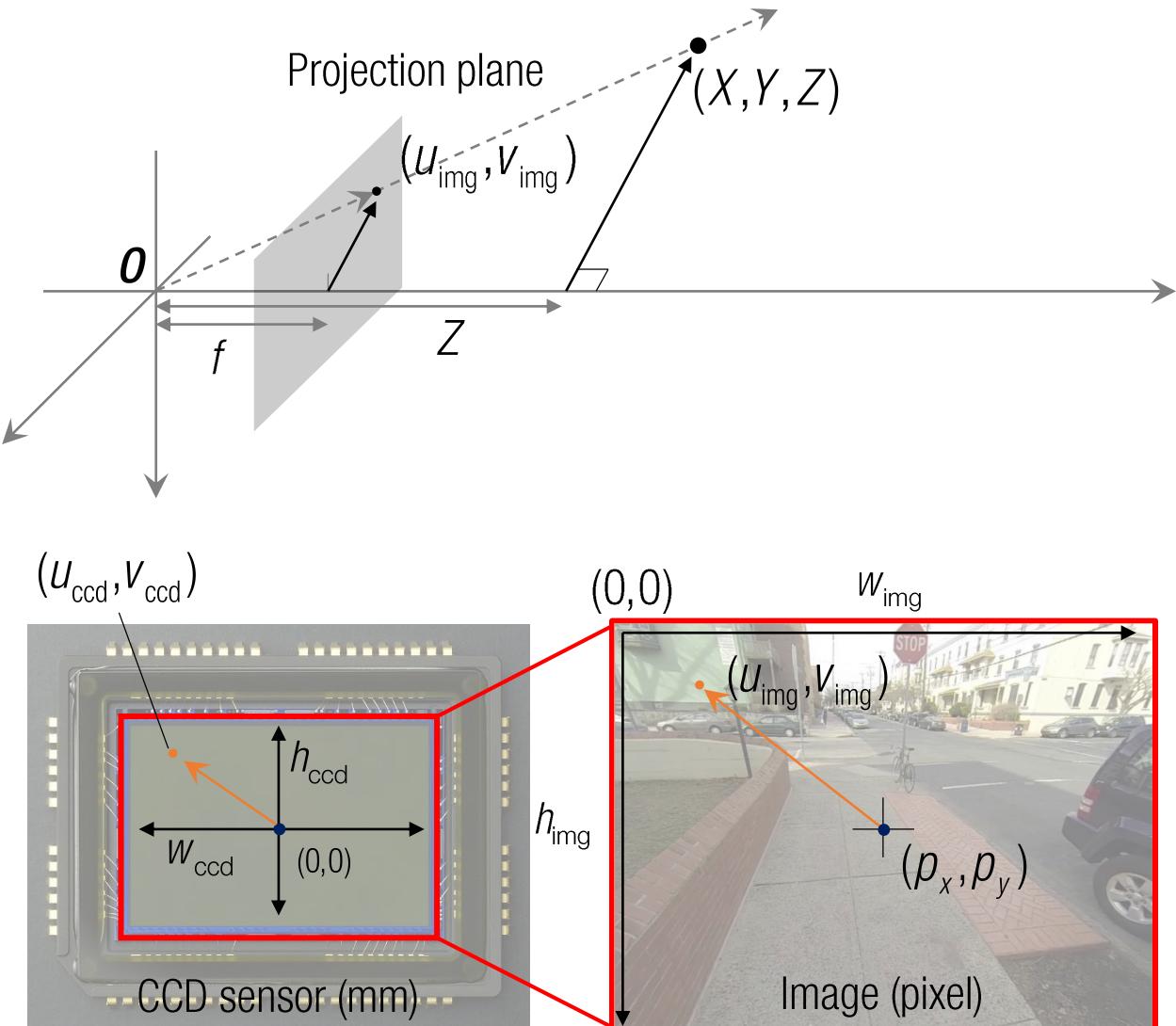
Pixel projection

$$\rightarrow u_{\text{img}} = U_{\text{ccd}} \frac{W_{\text{img}}}{W_{\text{ccd}}} + p_x = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} \frac{X}{Z} + p_x$$

$$v_{\text{img}} = V_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z} + p_y$$

Focal length in pixel

3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

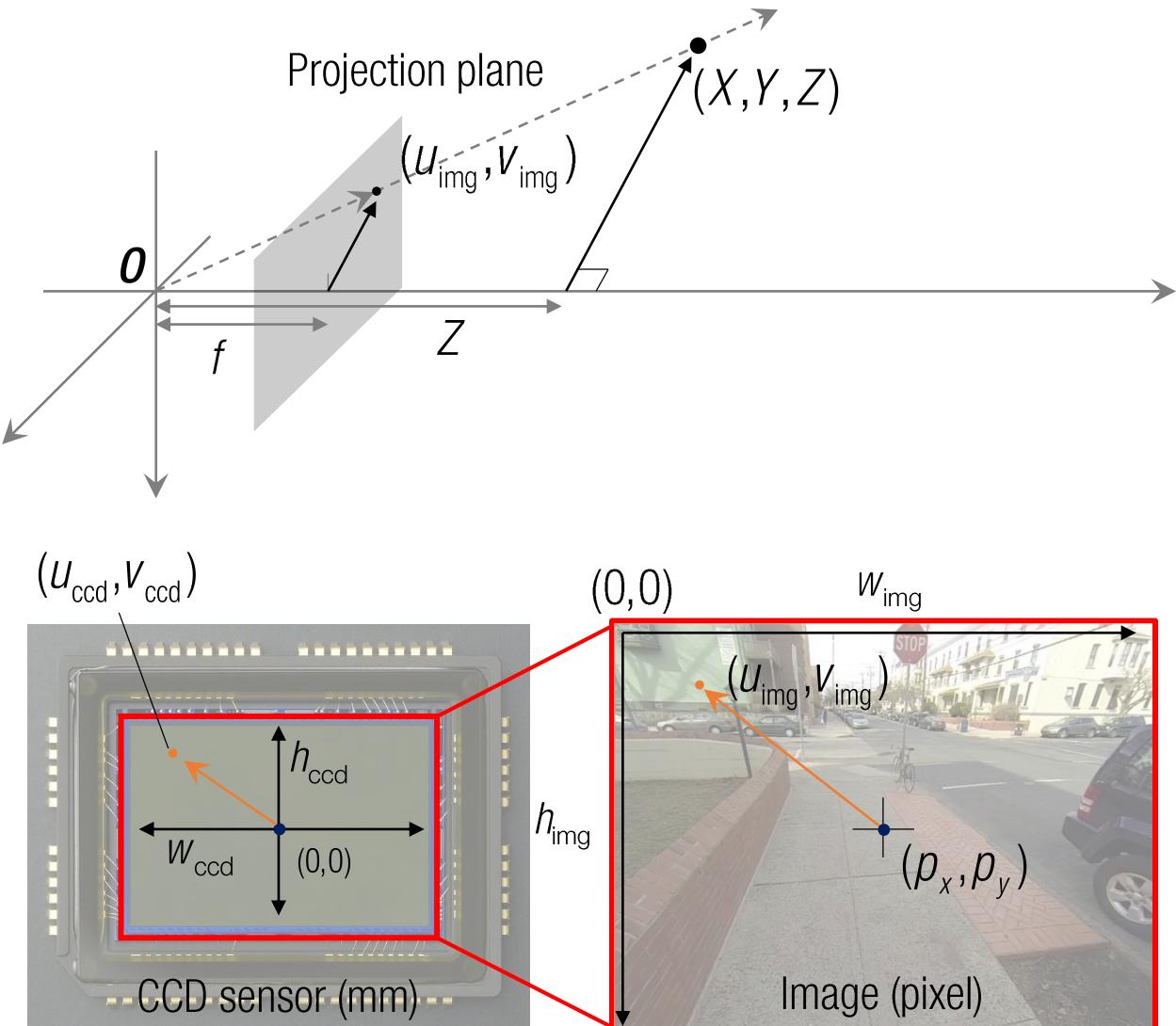
$$\rightarrow u_{\text{img}} = u_{\text{ccd}} \frac{w_{\text{img}}}{w_{\text{ccd}}} + p_x = f_x \frac{X}{Z} + p_x$$

$$v_{\text{img}} = v_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f_y \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } f_x = f_m \frac{w_{\text{img}}}{w_{\text{ccd}}} \quad f_y = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}}$$

3D Point Projection (Pixel Space)



$$(u_{\text{ccd}}, v_{\text{ccd}}) = \left(f_m \frac{X}{Z}, f_m \frac{Y}{Z} \right) \quad : \text{Metric projection}$$

Pixel projection

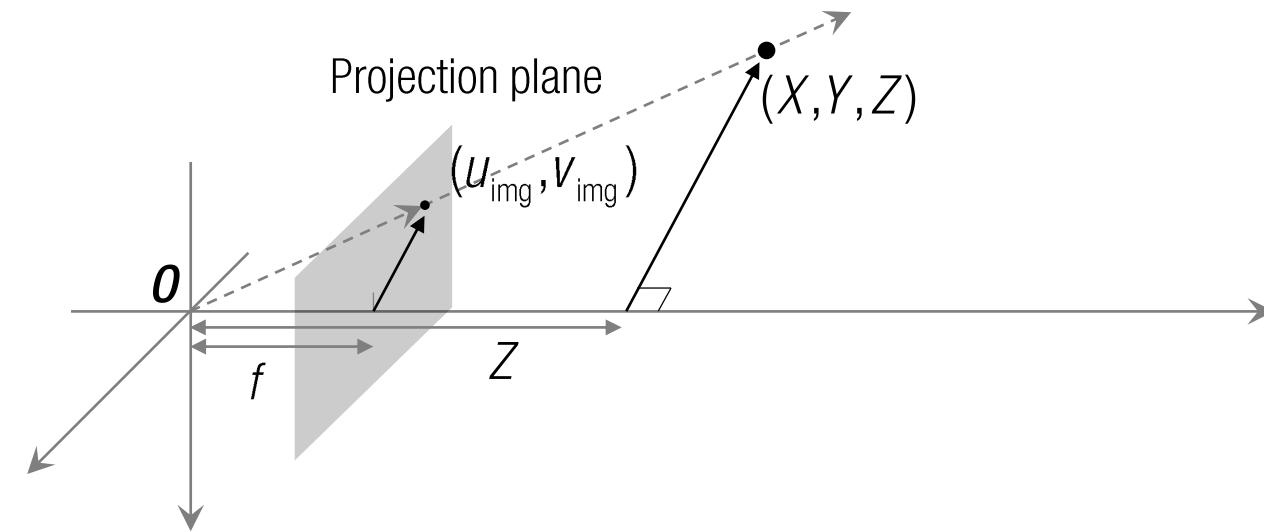
$$\rightarrow u_{\text{img}} = U_{\text{ccd}} \frac{W_{\text{img}}}{W_{\text{ccd}}} + p_x = f \frac{X}{Z} + p_x$$

$$v_{\text{img}} = V_{\text{ccd}} \frac{h_{\text{img}}}{h_{\text{ccd}}} + p_y = f \frac{Y}{Z} + p_y$$

Focal length in pixel

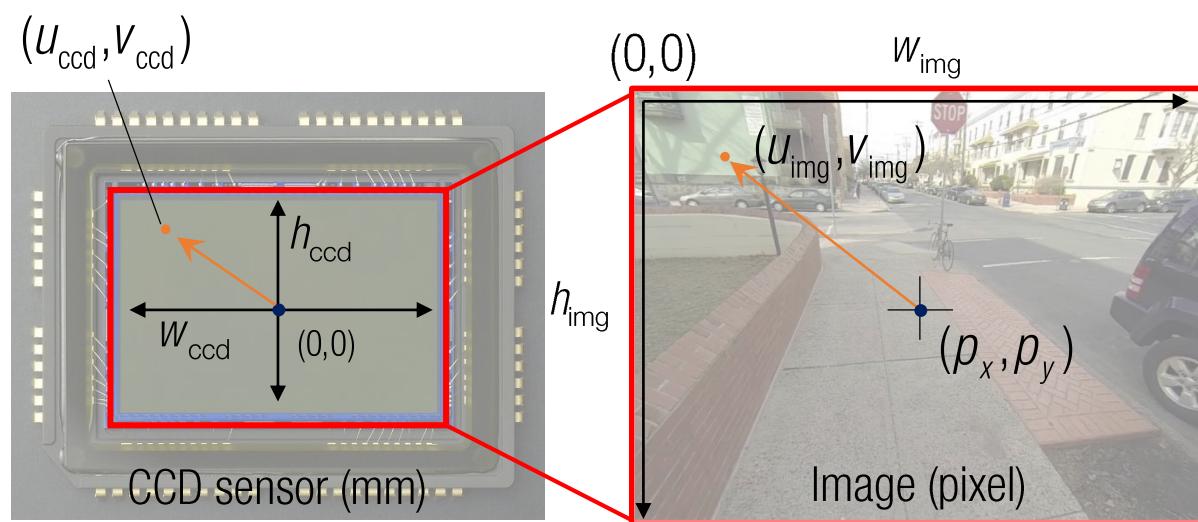
$$\text{where } f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \quad \text{if } \frac{W_{\text{img}}}{W_{\text{ccd}}} = \frac{h_{\text{img}}}{h_{\text{ccd}}}$$

3D Point Projection (Pixel Space)

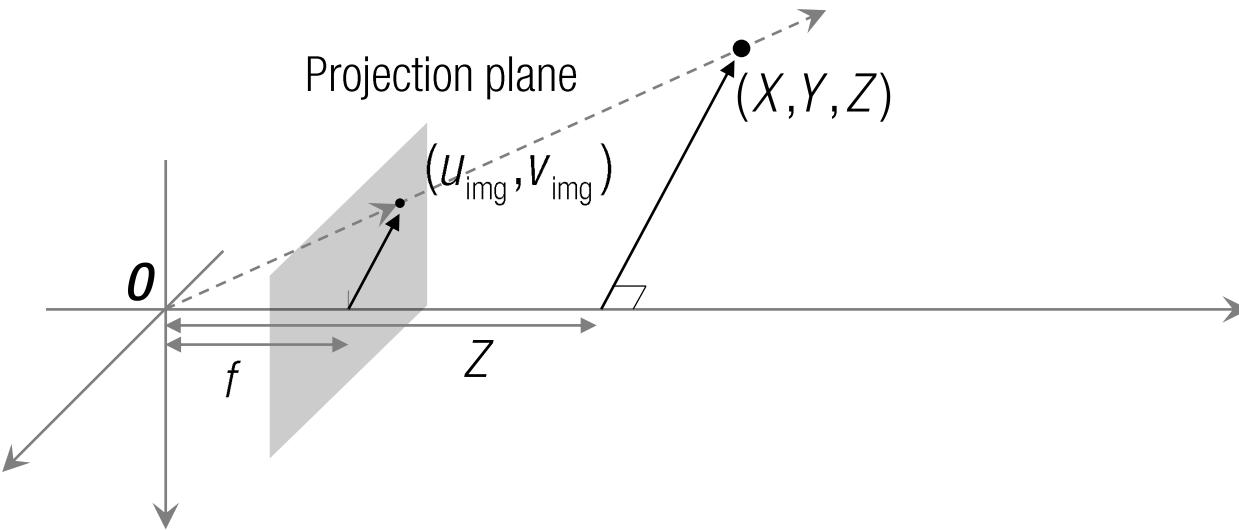


$$u_{\text{img}} = f \frac{X}{Z} + p_x$$

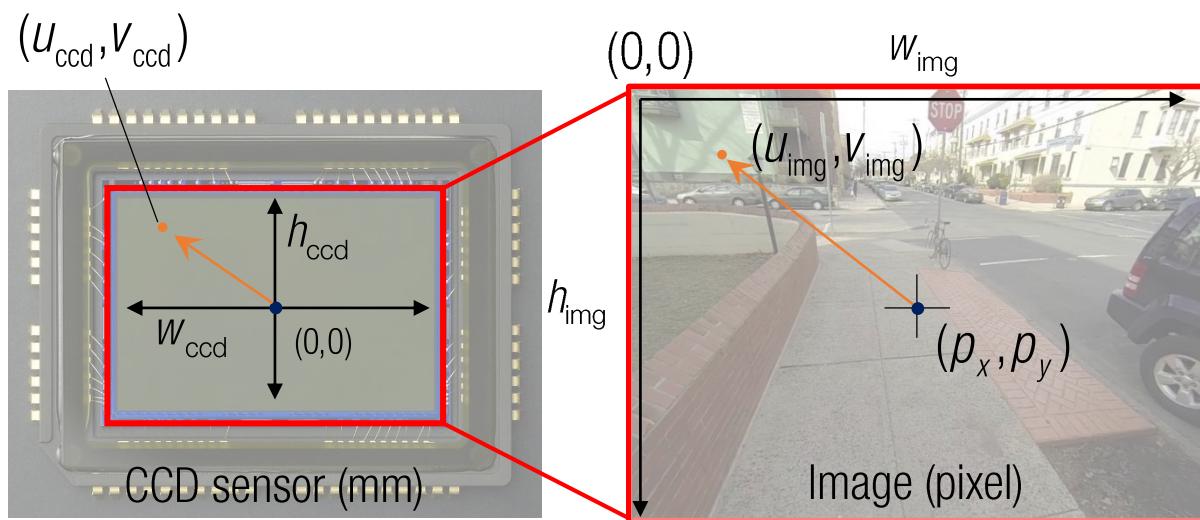
$$v_{\text{img}} = f \frac{Y}{Z} + p_y$$



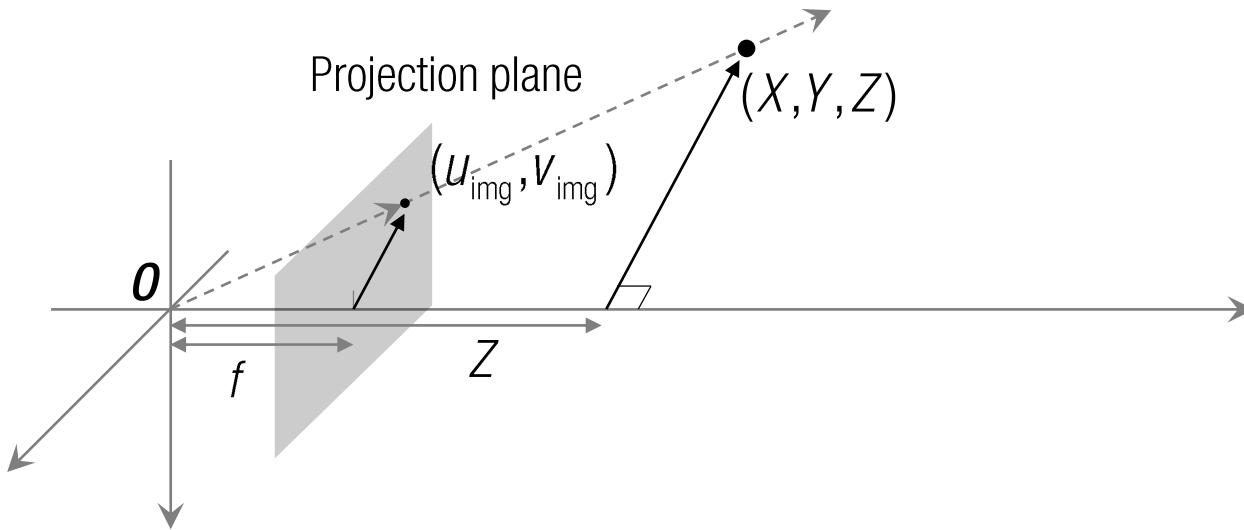
3D Point Projection (Pixel Space)



$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$
$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$



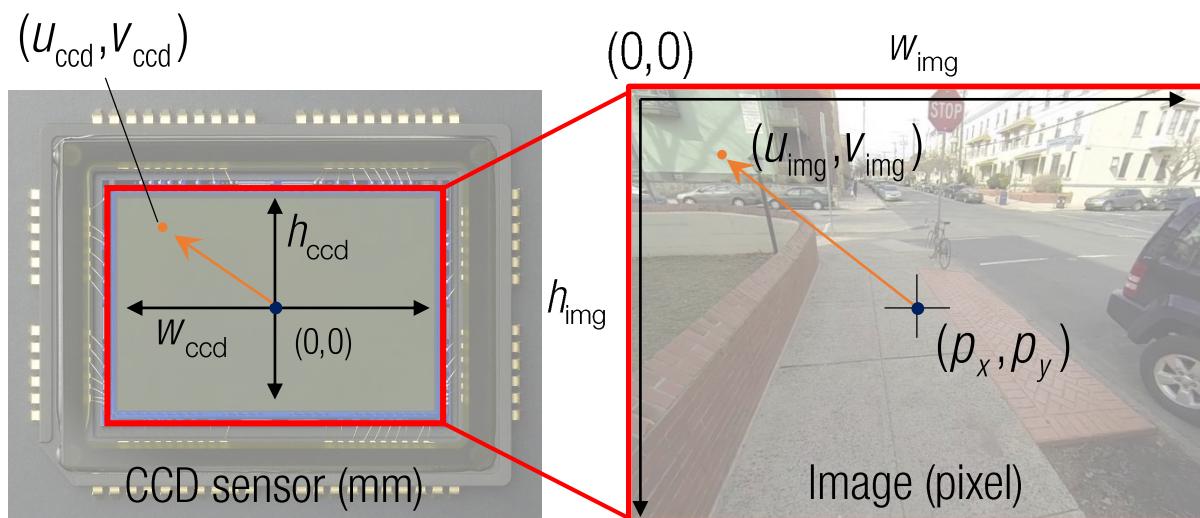
3D Point Projection (Pixel Space)



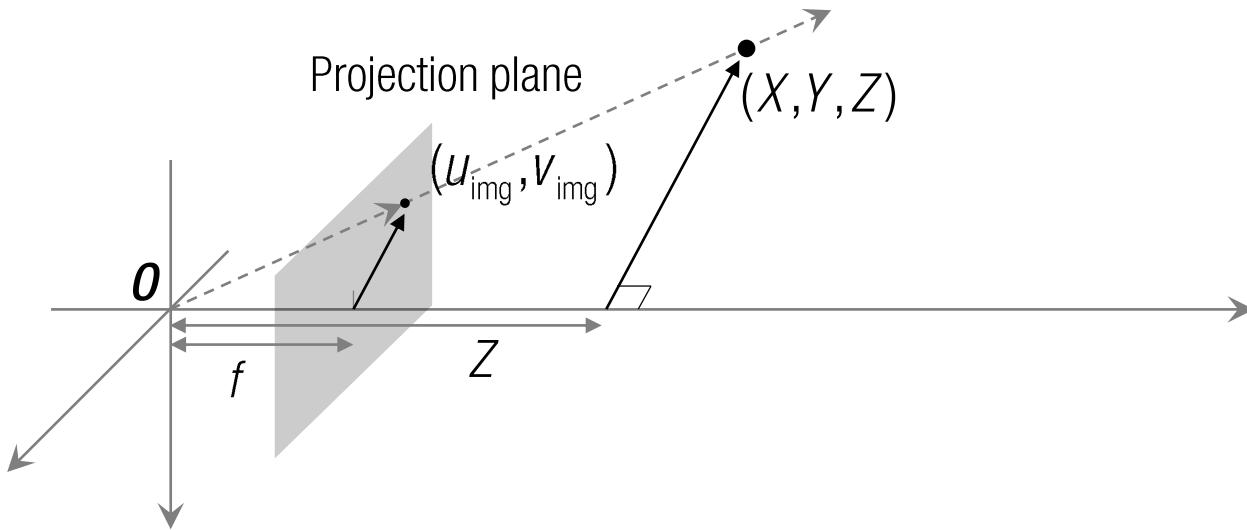
$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

$$Z \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



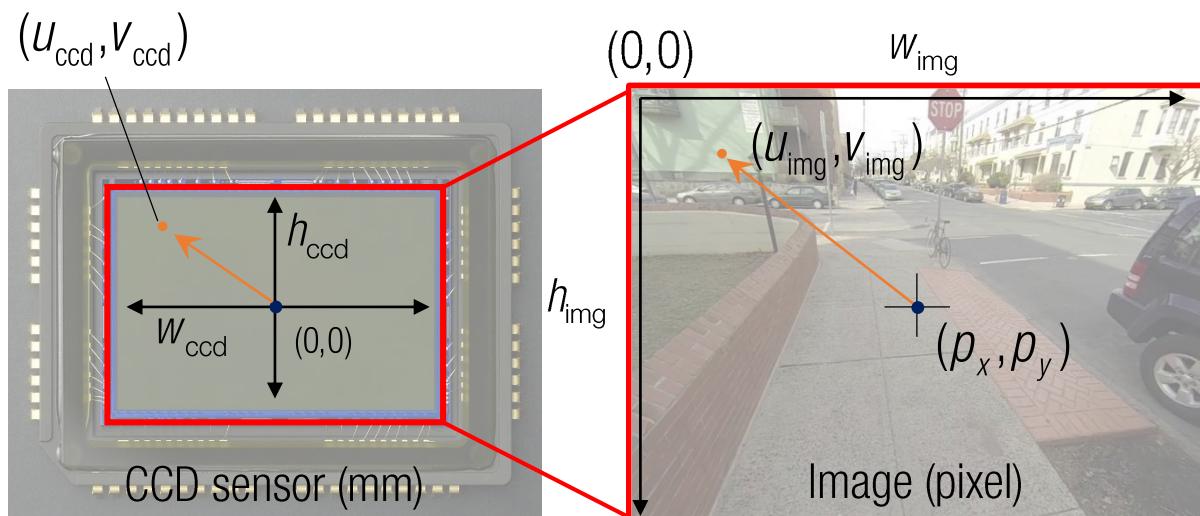
3D Point Projection (Pixel Space)



$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



Computer Graphics



$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Z u_{\text{img}} = f X + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Z v_{\text{img}} = f Y + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

←
Graphics

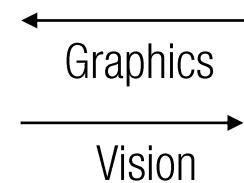
Computer Vision = inv(Computer Graphics)



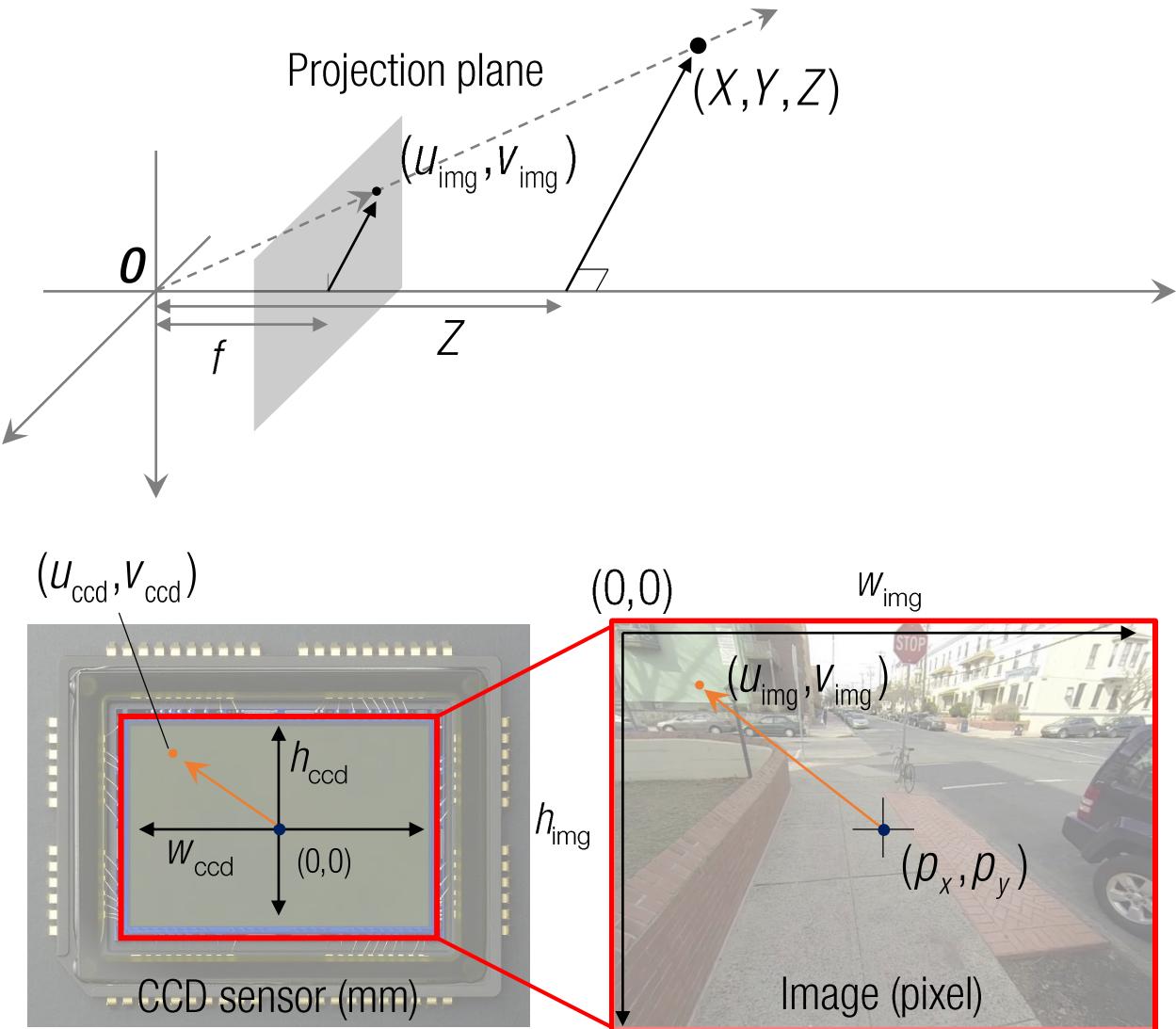
$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Z u_{\text{img}} = f X + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Z v_{\text{img}} = f Y + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



3D Point Projection (Pixel Space)



$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

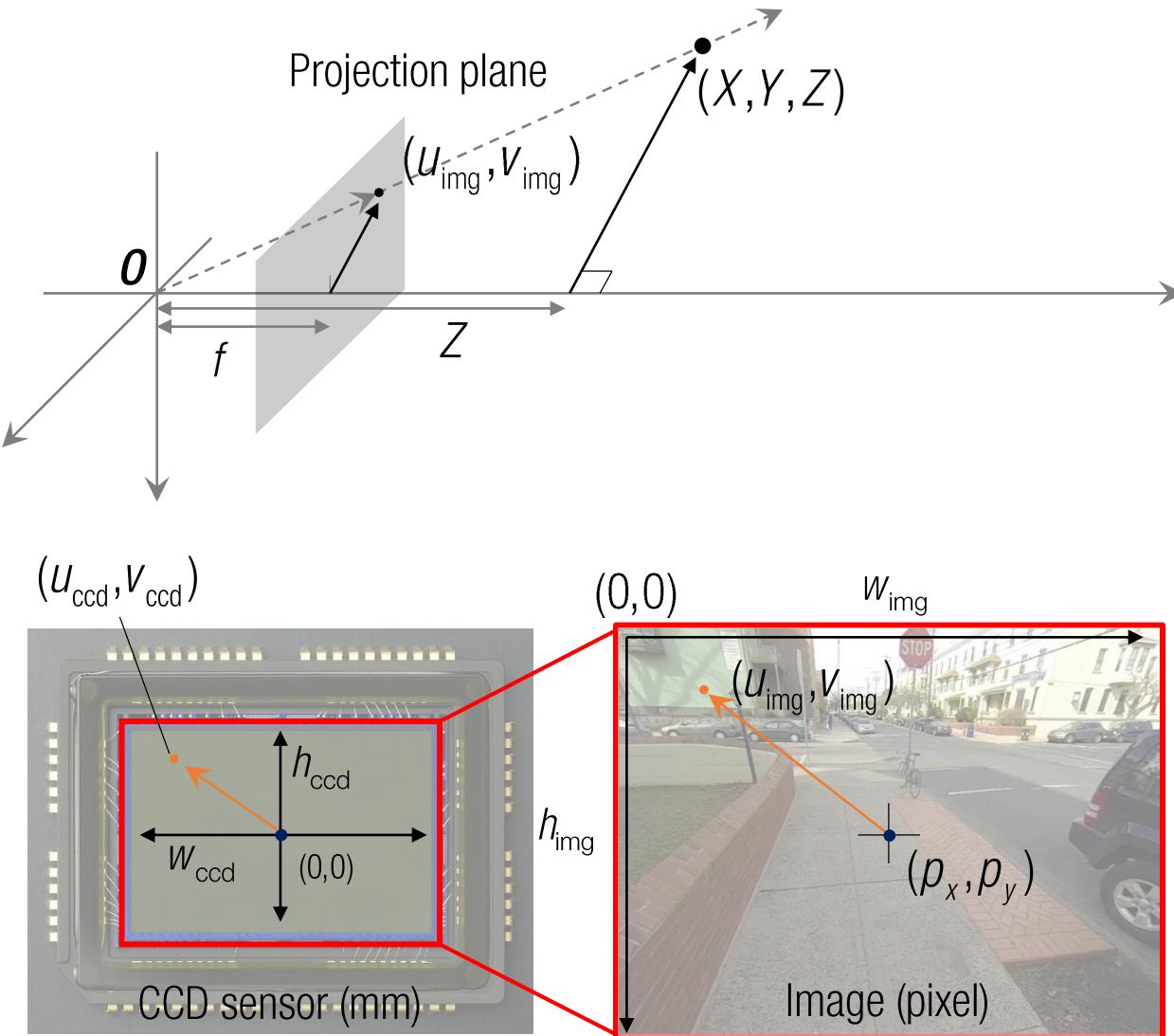
$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & X \\ & f & Y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Metric space

3D Point Projection (Pixel Space)



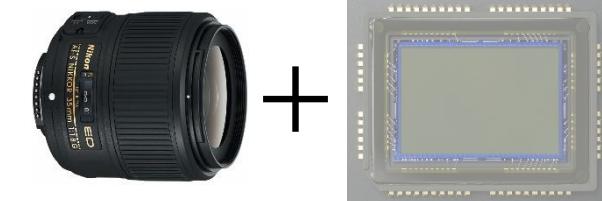
$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

Pixel space

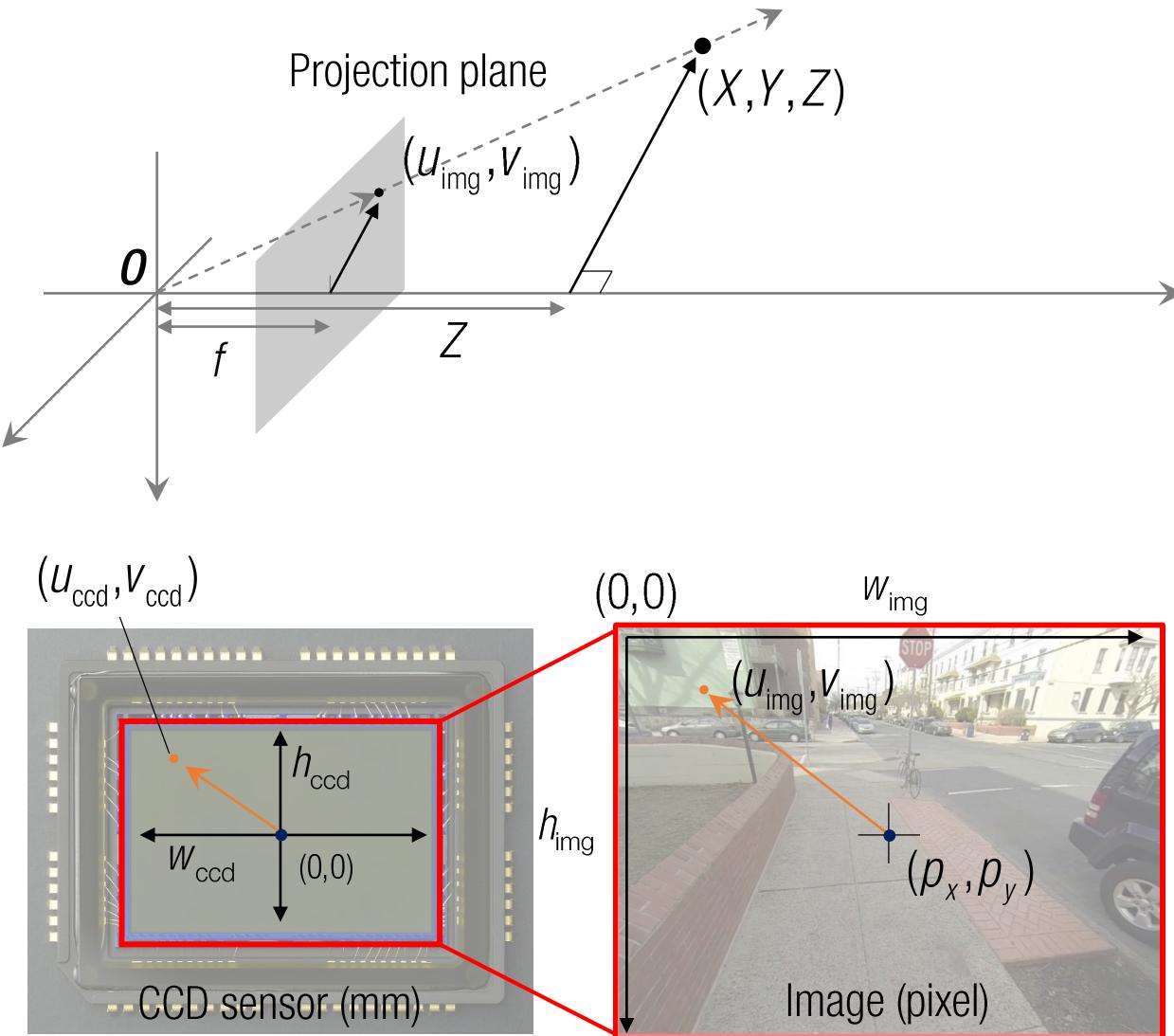
$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Metric space



Camera intrinsic parameter
: metric space to pixel space

3D Point Projection (Pixel Space)



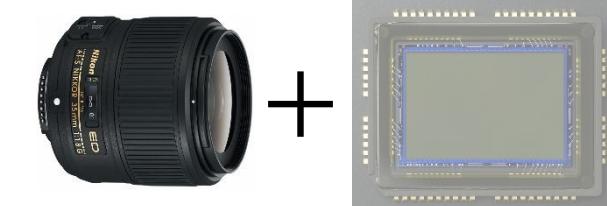
$$u_{\text{img}} = f \frac{X}{Z} + p_x \quad \longrightarrow \quad Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f \frac{Y}{Z} + p_y \quad \longrightarrow \quad Zv_{\text{img}} = fY + p_y Z$$

Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & p_x \\ K & p_y \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Metric space

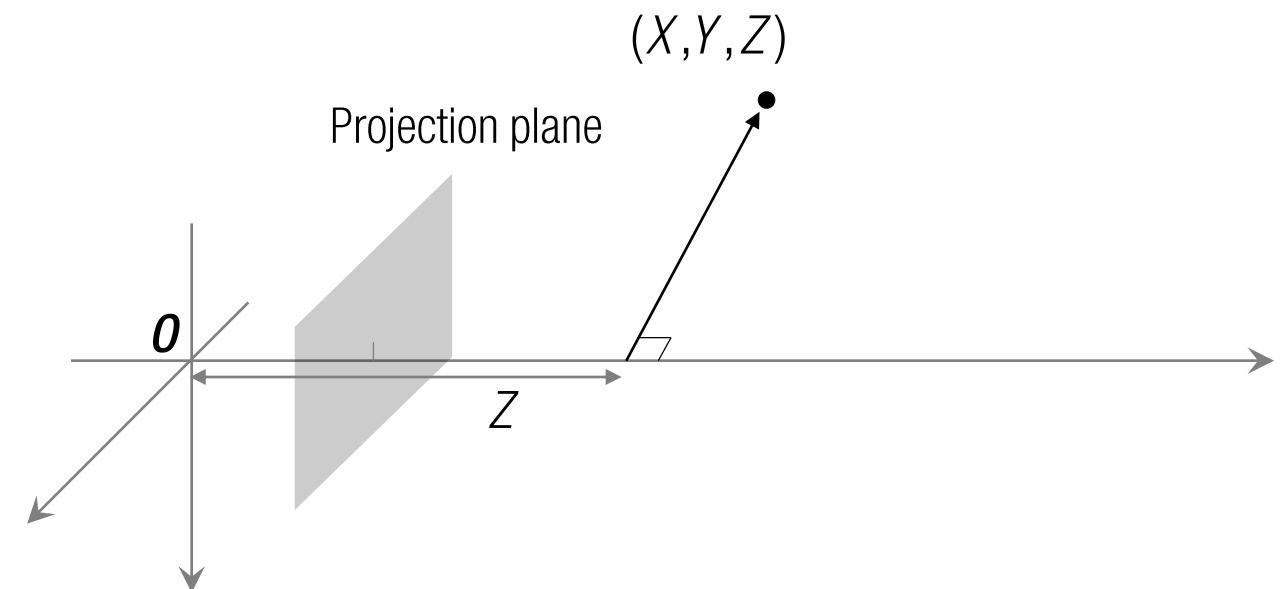


Camera intrinsic parameter
: metric space to pixel space

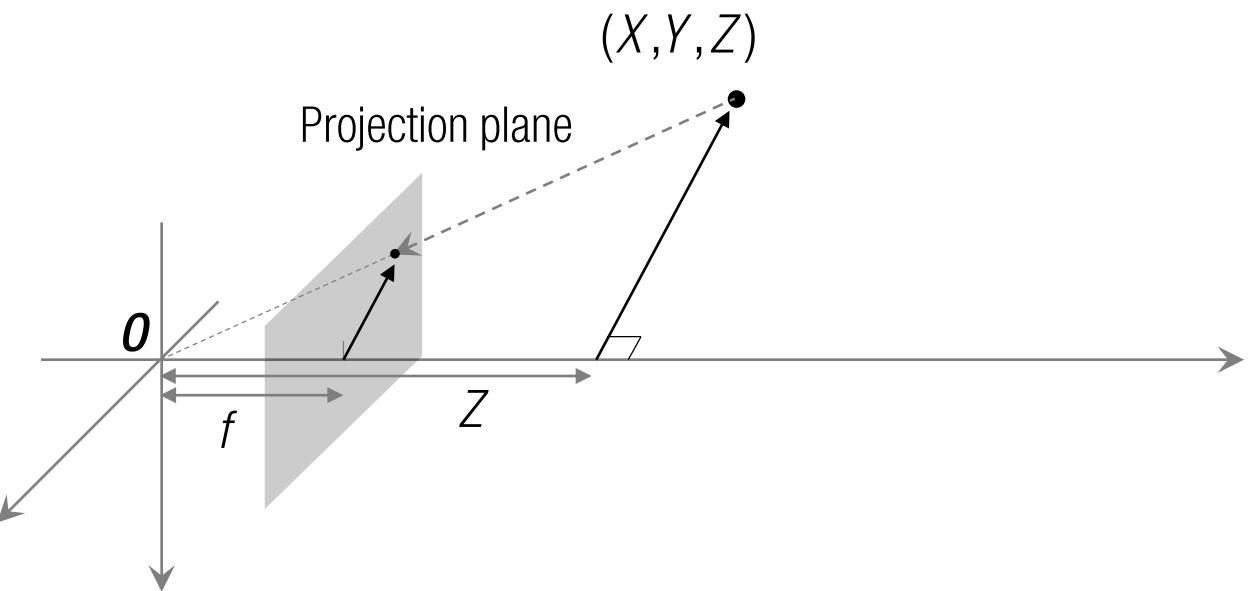
Camera Intrinsic Parameter

Metric space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



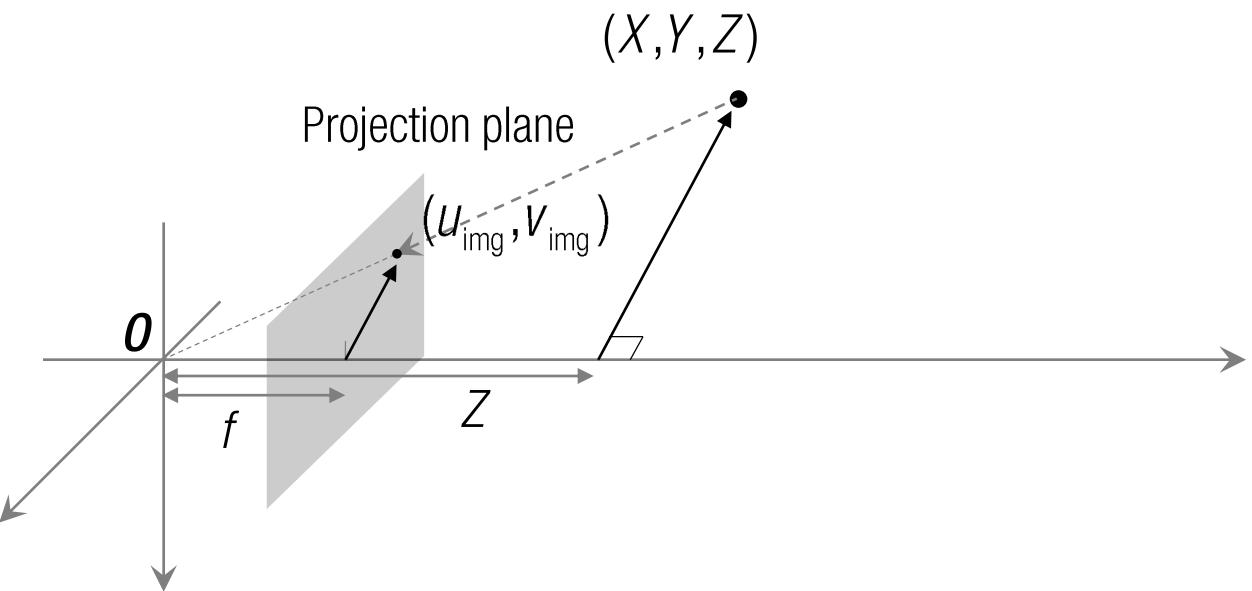
Camera Intrinsic Parameter



Metric space

$$\mathbf{K} \begin{bmatrix} f & p_x \\ & p_y \\ & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

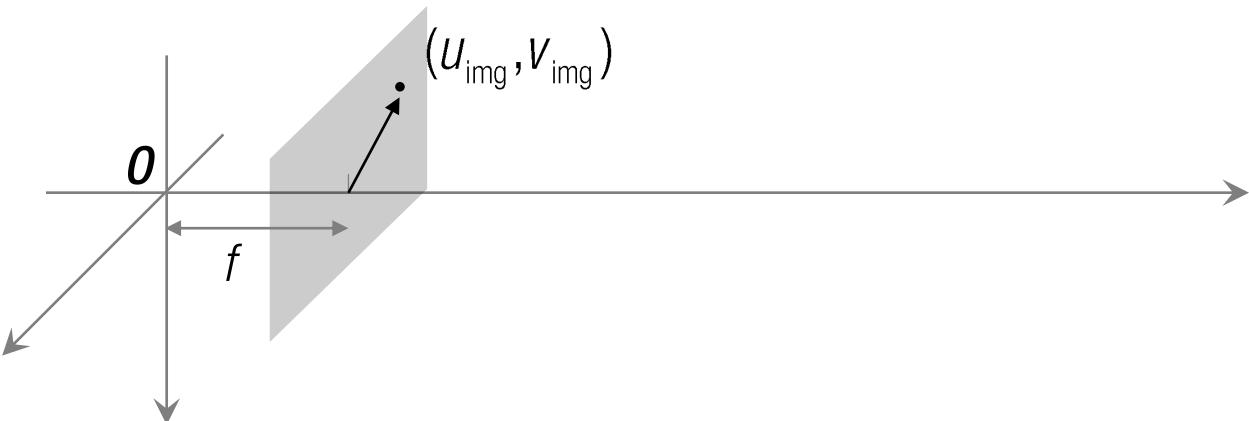
Camera Intrinsic Parameter



$$\text{Pixel space} \quad \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \mathbf{K} \\ 1 \end{bmatrix} \text{Metric space} \quad \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2D Inverse Projection

Projection plane



Pixel space

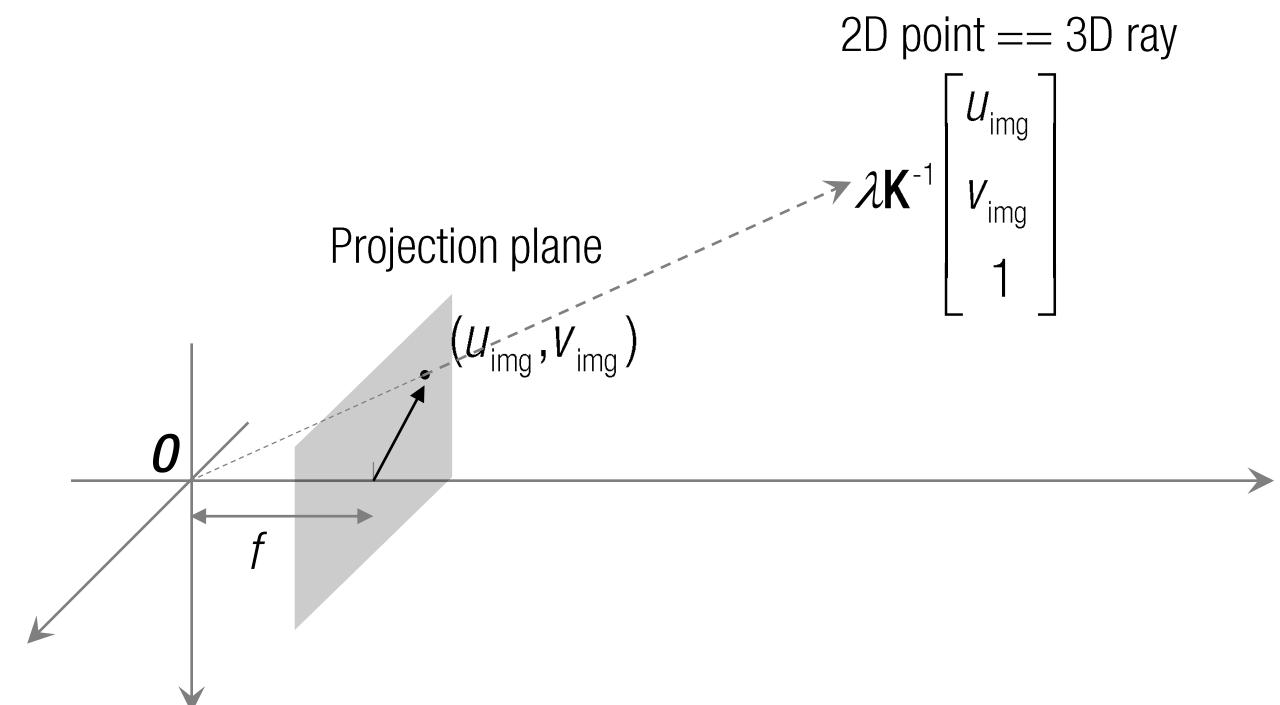
$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ K \\ 1 \end{bmatrix}$$

Metric space

$$\begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

2D Inverse Projection



2D point == 3D ray

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \mathbf{K} \end{bmatrix}$$

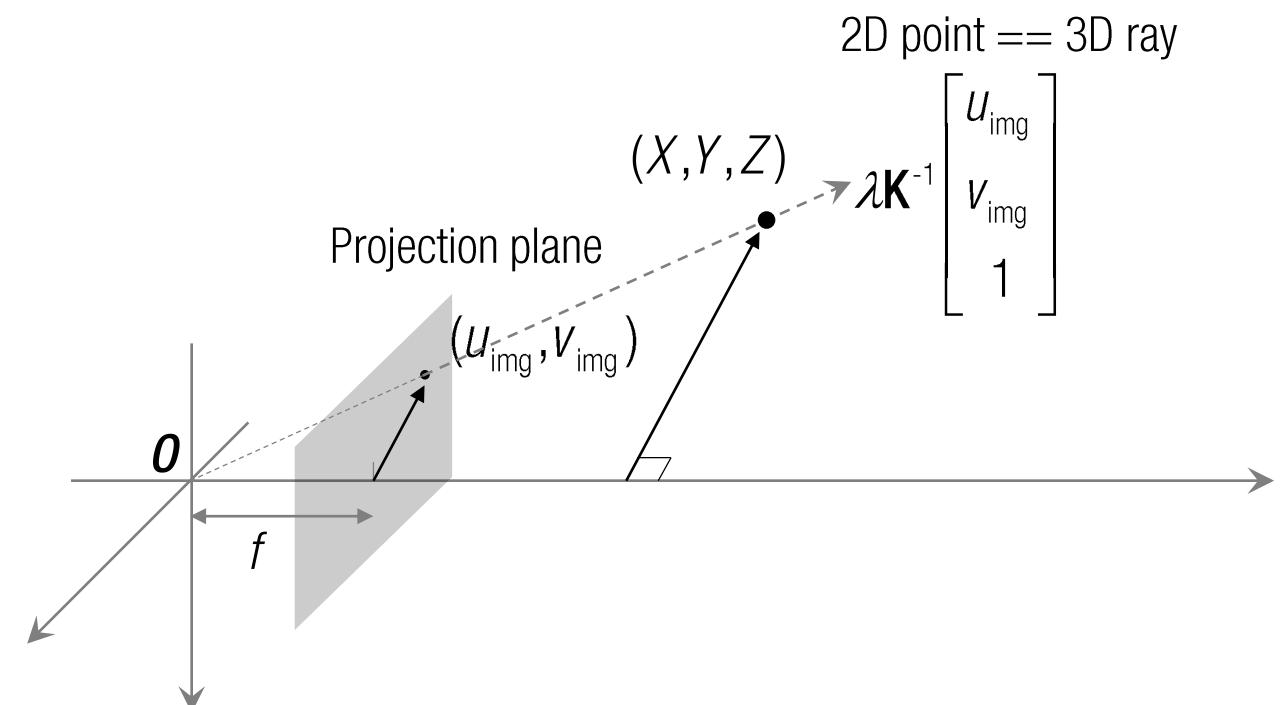
Metric space

$$p_x \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

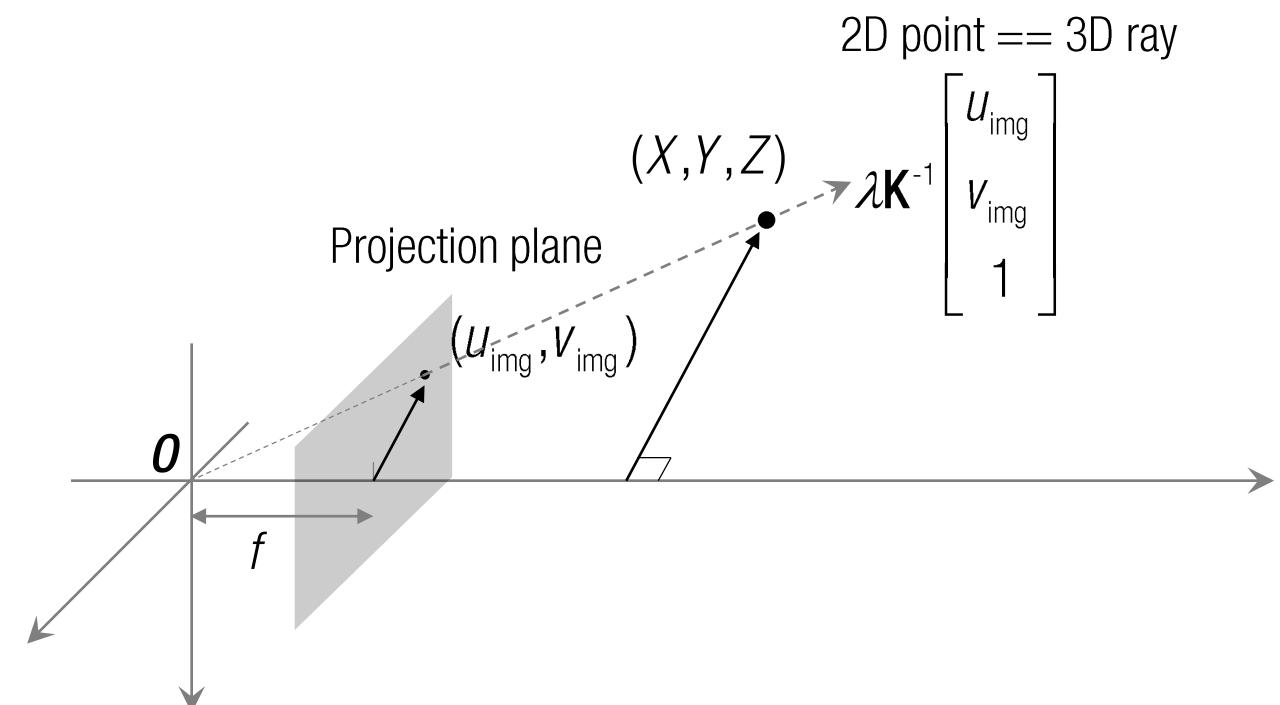
3D ray

2D Inverse Projection



$$\begin{array}{c} \text{Pixel space} \\ \lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f \\ \mathbf{K} \\ p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\ \hline \text{3D ray} \\ \lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{array}$$

2D Inverse Projection



2D point == 3D ray

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix}$$

Pixel space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \mathbf{K} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

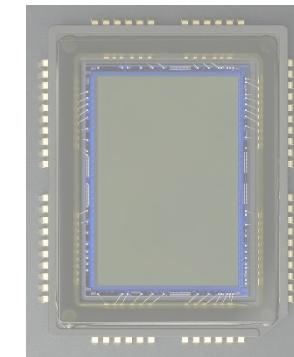
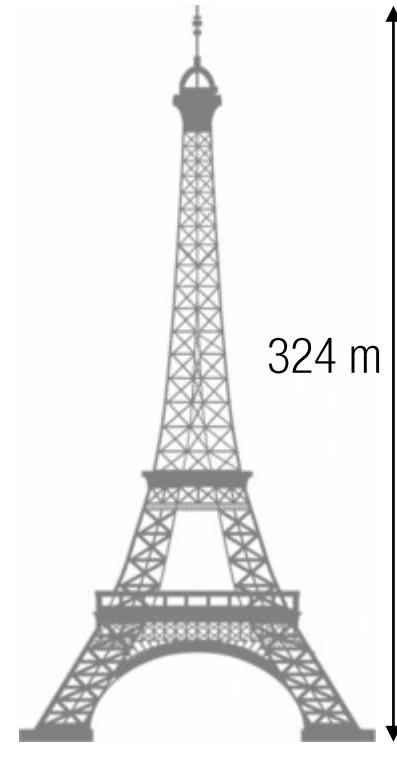
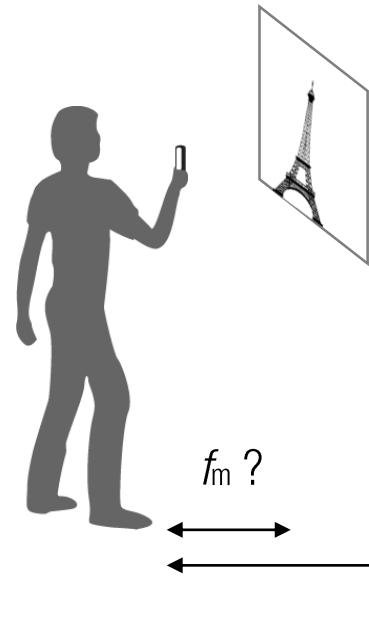
Metric space

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

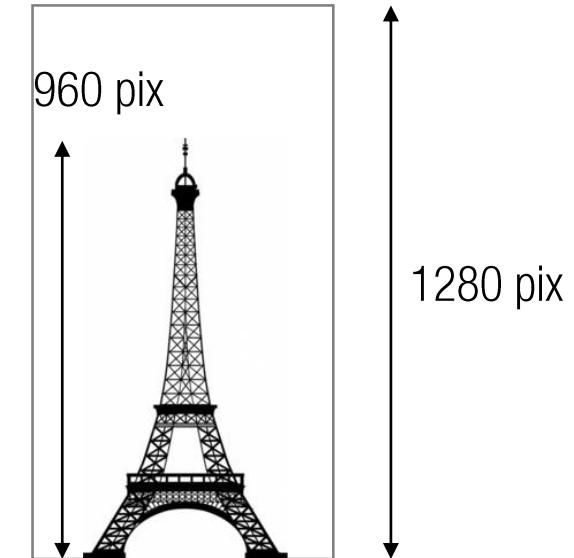
The 3D point must lie in
the 3D ray passing through the origin and 2D image point.

Exercise

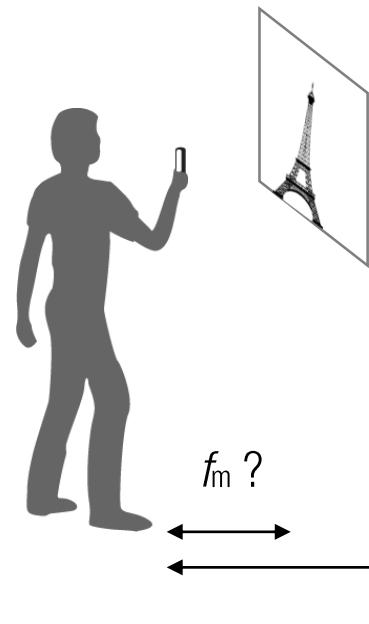
What f to make the height of Eifel tower appear 960 pixel distance?



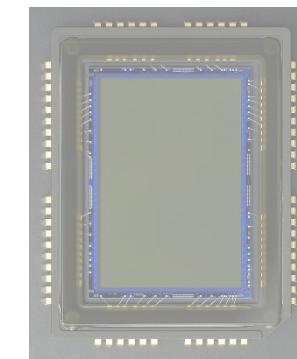
ccd size



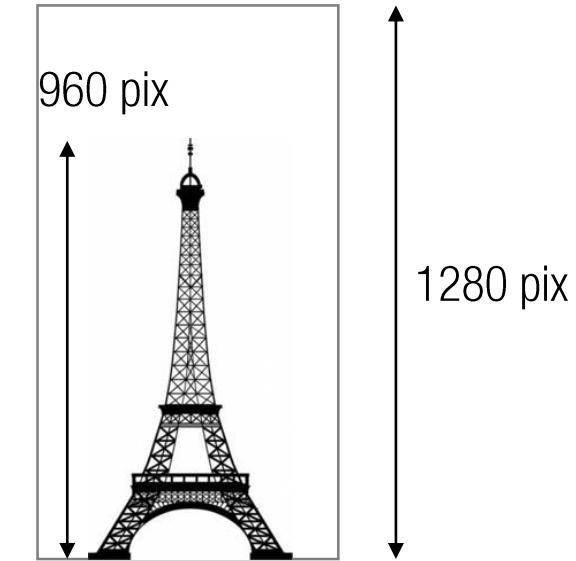
Exercise



What f to make the height of Eifel tower appear 960 pixel distance?

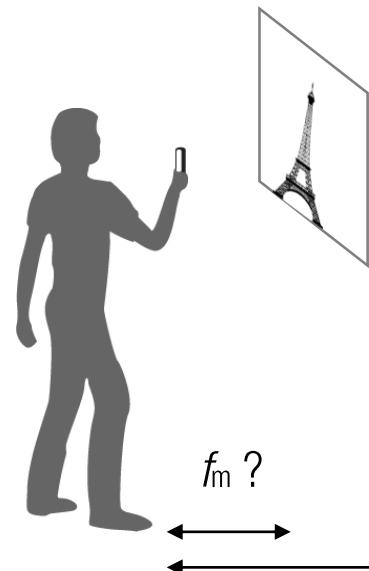


ccd size

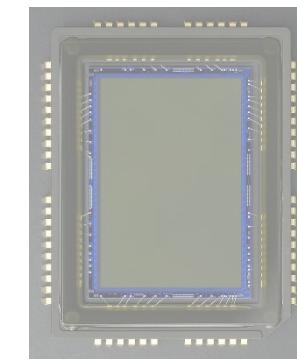


$$y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

Exercise

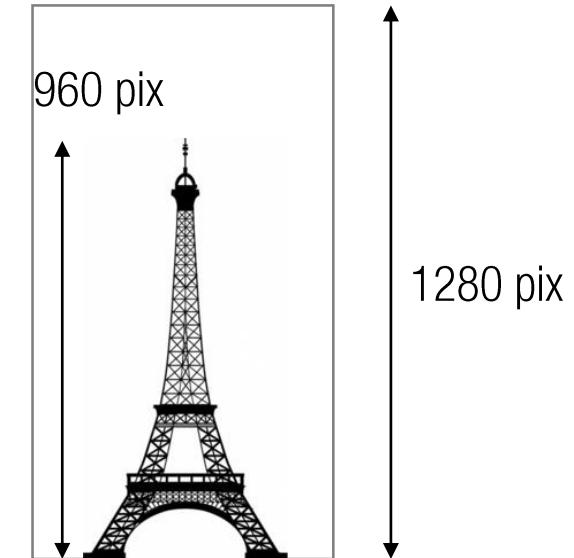


What f to make the height of Eifel tower appear 960 pixel distance?

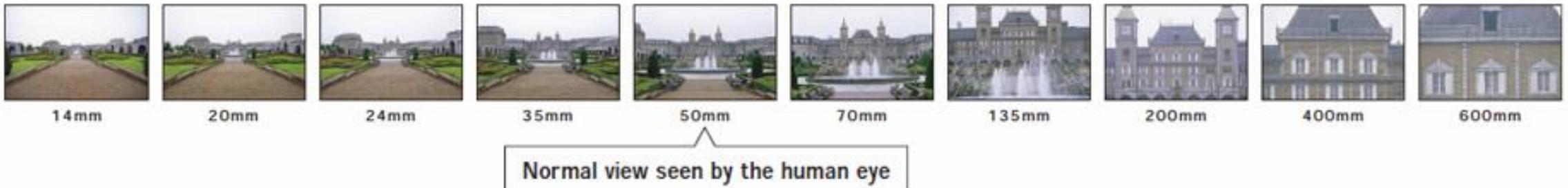
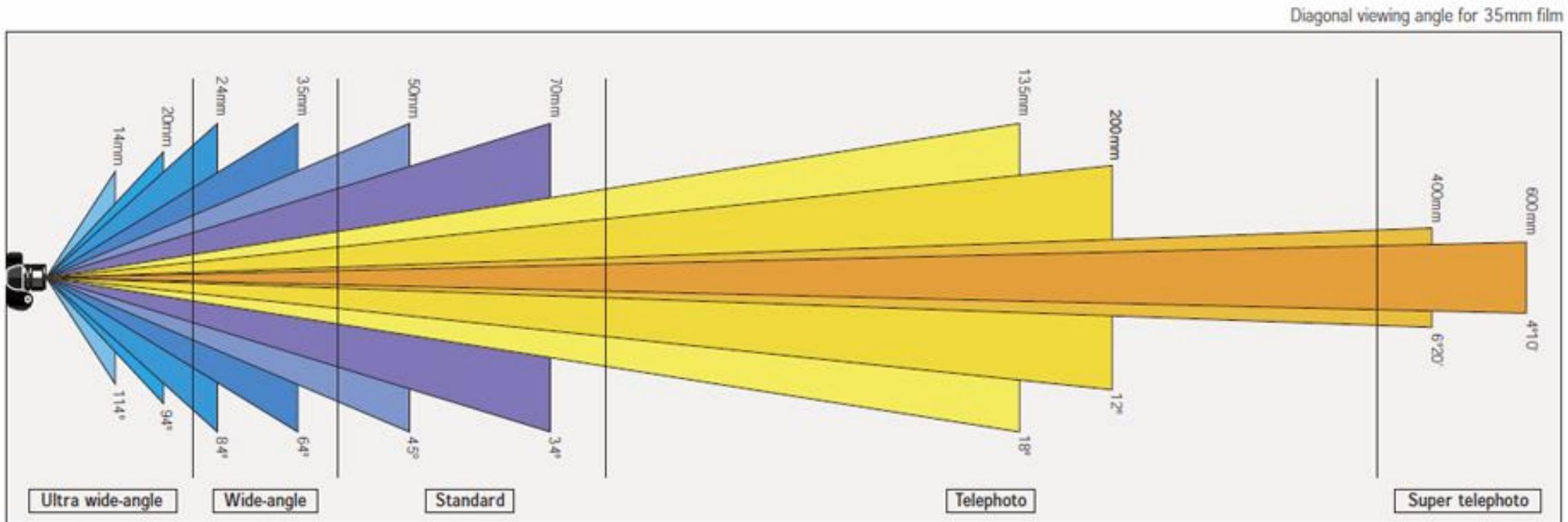


$$y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

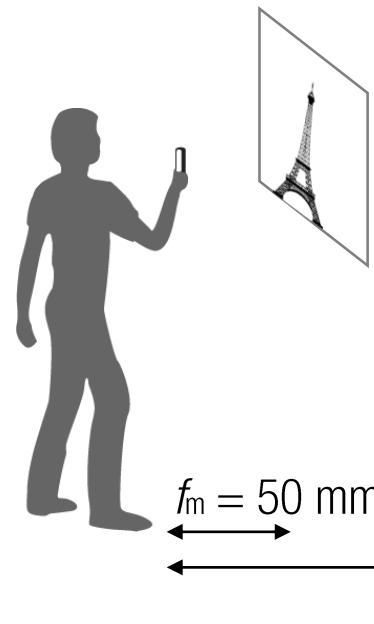
$$960 = f_m \frac{1280}{0.0218} \frac{324}{1500} \rightarrow f_m = 0.0757 \text{m}$$



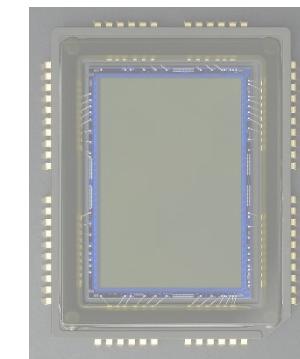
Focal Length



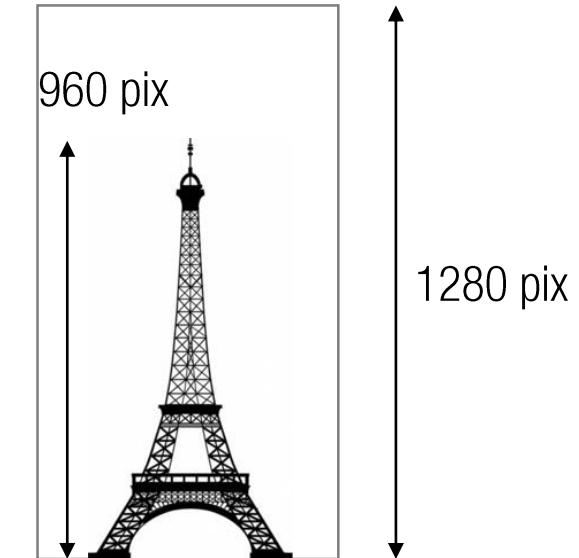
Exercise



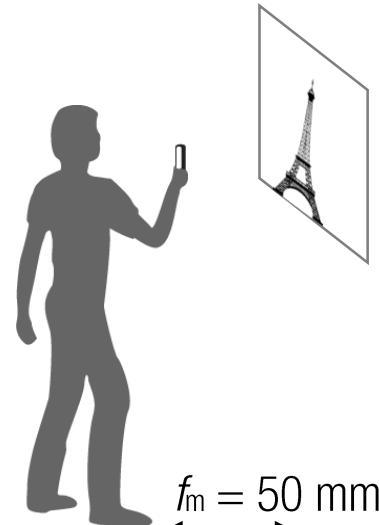
What Z to make the height of Eifel tower appear 960 pixel distance?



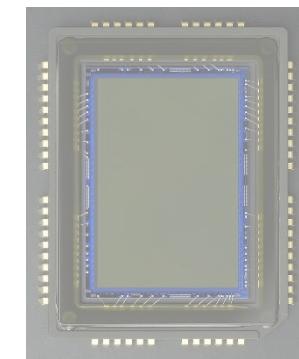
$$y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$



Exercise

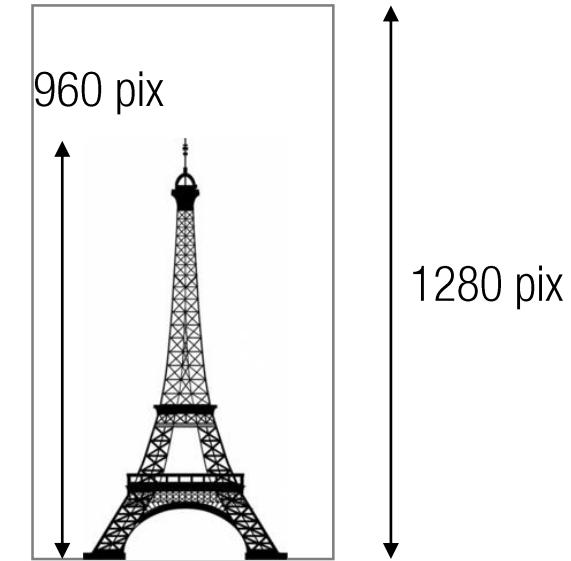


What Z to make the height of Eifel tower appear 960 pixel distance?



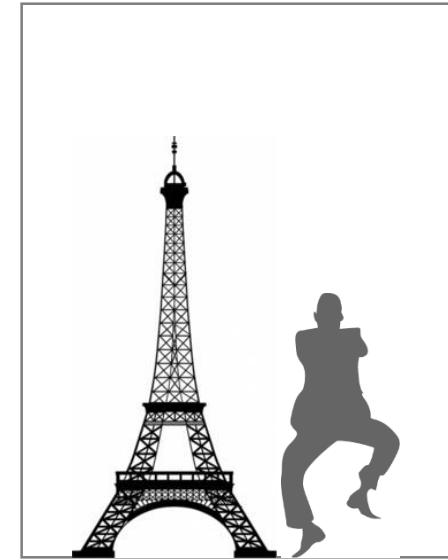
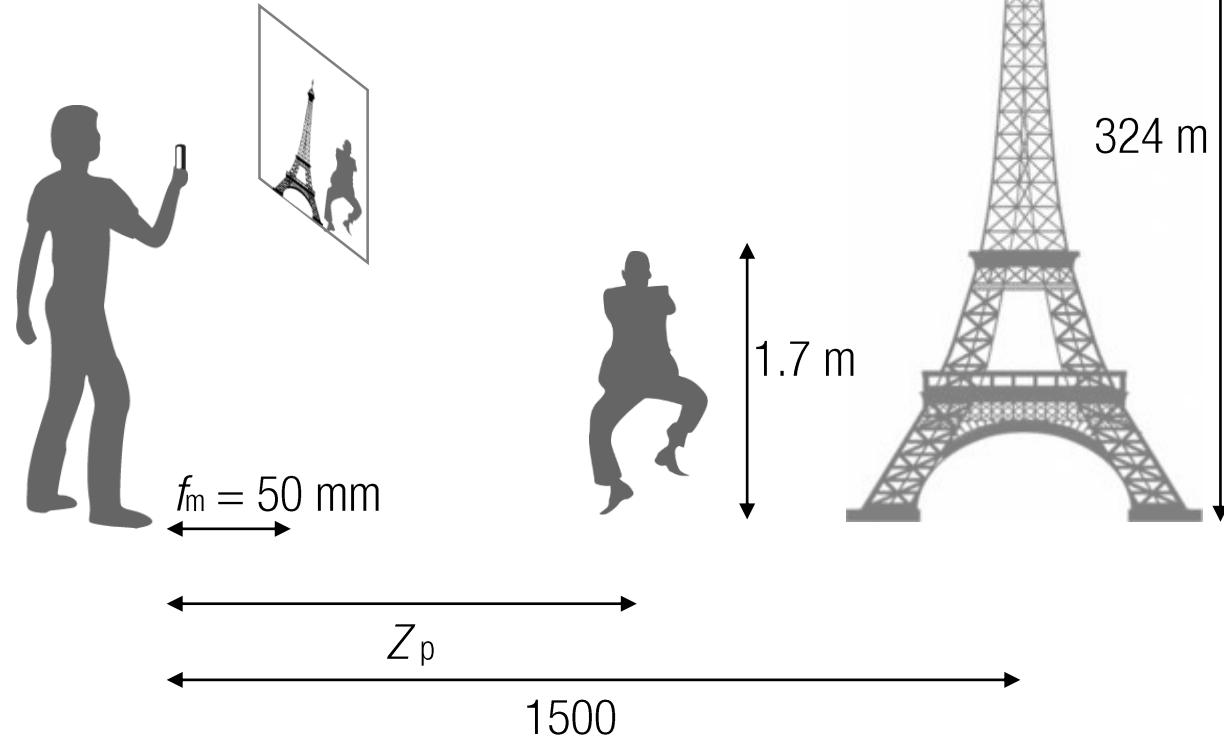
$$y_{\text{img}} = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

$$960 = 0.05 \frac{1280}{0.0218} \frac{324}{Z} \rightarrow Z = 990.826 \text{ m}$$



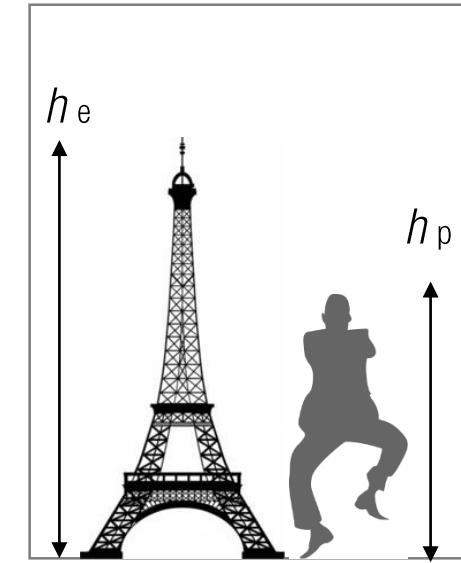
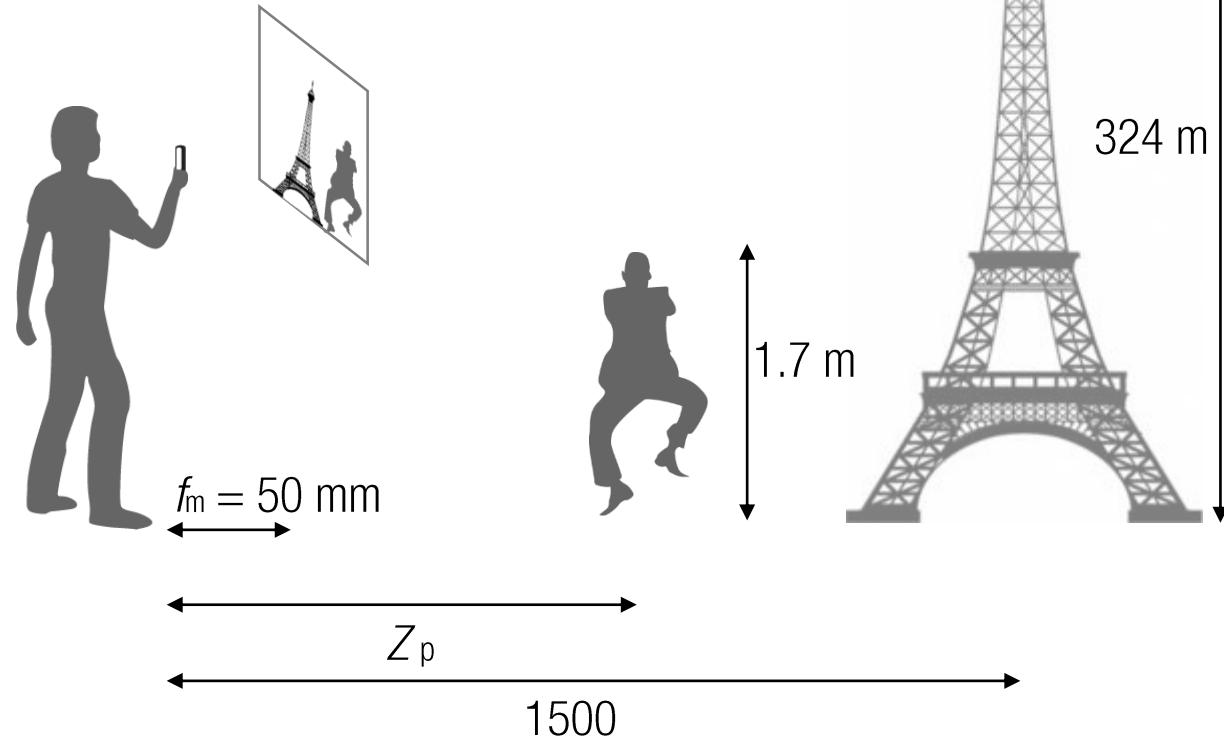
Exercise

What Z_p to make the height of Eifel tower appear twice of the person?



Exercise

What Z_p to make the height of Eifel tower appear twice of the person?

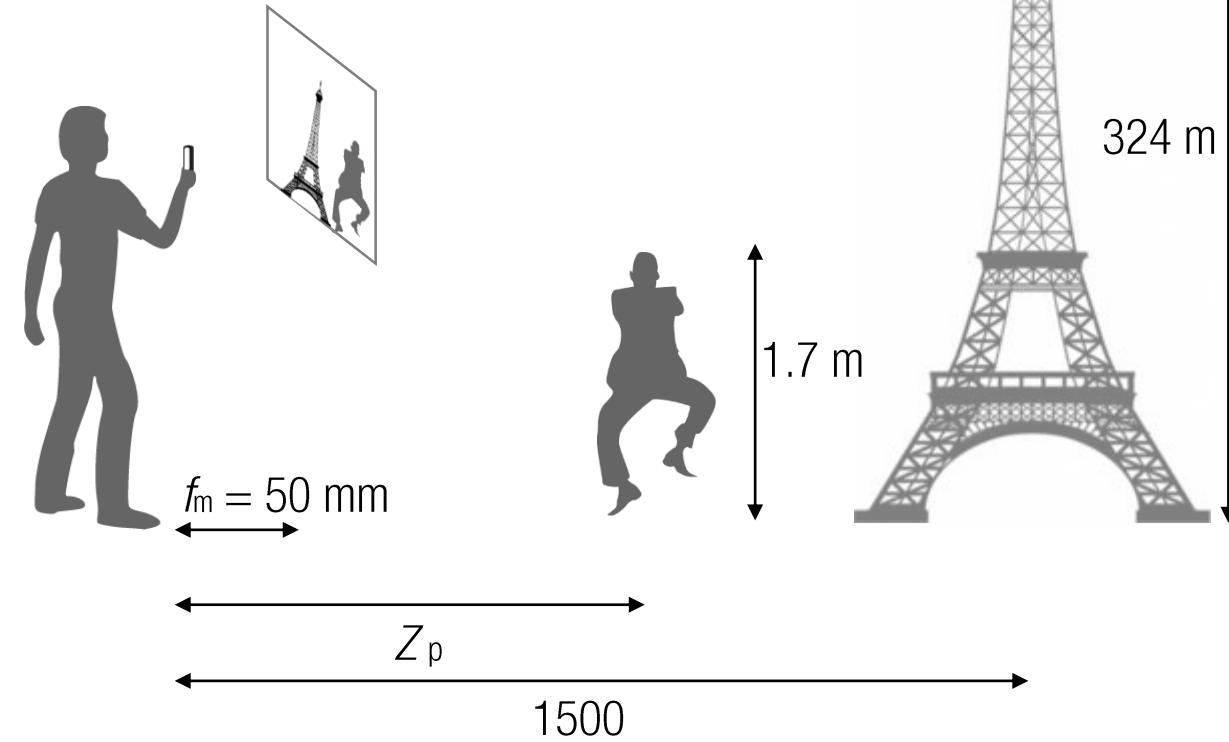


$$h_e = f \frac{Y}{Z}$$

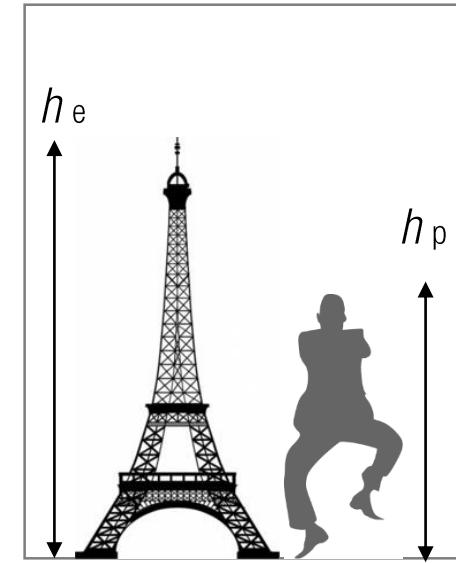
$$h_p = f \frac{Y_p}{Z_p}$$

$$\text{s.t. } h_p = \frac{h_e}{2}$$

Exercise



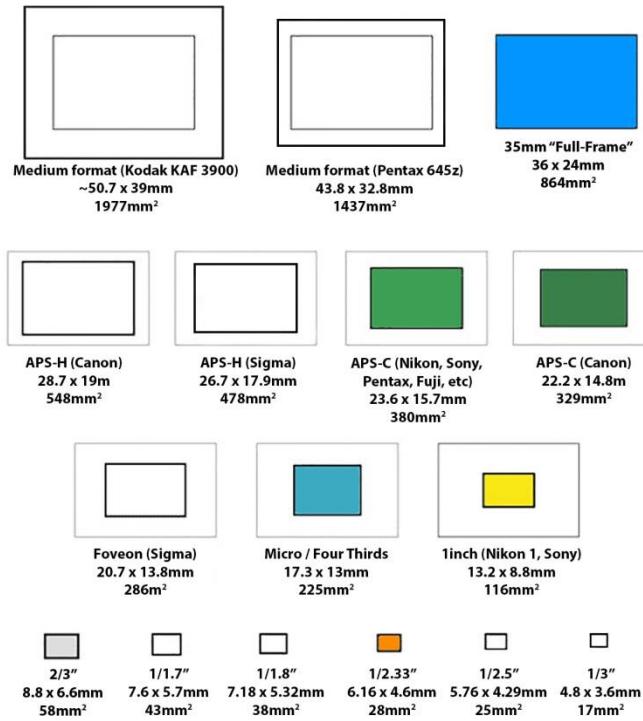
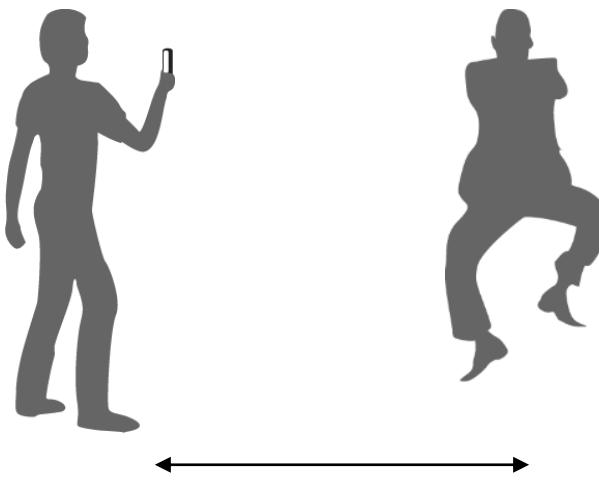
What Z_p to make the height of Eifel tower appear twice of the person?



$$h_e = f \frac{Y}{Z} \quad h_p = f \frac{Y_p}{Z_p} \quad \text{s.t.} \quad h_p = \frac{h_e}{2}$$

$$f \frac{Y_p}{Z_p} = f \frac{Y}{2Z} \rightarrow \quad Z_p = 2 \cdot 1500 \frac{1.7}{324} = 15.74 \text{ m}$$

Where Was I?

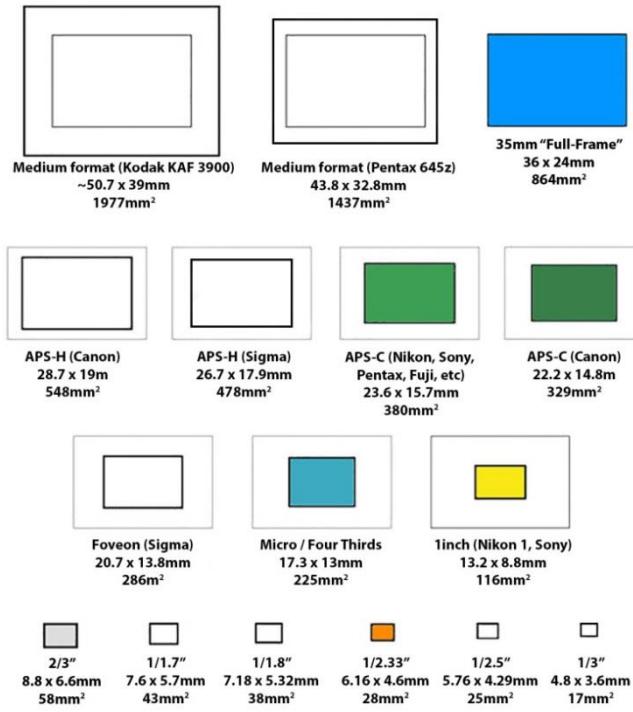
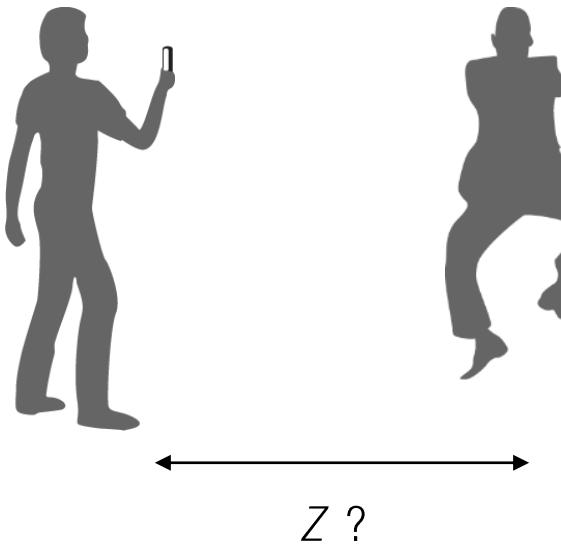


| Camera Architecture | | |
|----------------------------|--|---|
| | LG G4 | LG G5 |
| Front Camera: Resolution | | 8MP |
| Front Camera: Sensor | | Toshiba T4KA3 (1.12µm, 1/4") |
| Front Camera: Focal Length | | 2.6mm |
| Front Camera: Aperture | | f/2.0 |
| Rear Camera: Resolution | 16MP | Main: 16MP Wide Angle: 8MP |
| Rear Camera: Sensor | Sony IMX234 Exmor RS (1.12µm, 1/2.6") | Main: Sony IMX234 Exmor RS (1.12µm, 1/2.6") Wide Angle: Sony IMX268 (1.4µm, 1/3.2")? |
| Rear Camera: Focal Length | 4.42mm (29mm equivalent) | Main: 4.42mm (29mm equivalent) Wide Angle: 1.53mm (12mm equivalent)? |
| Rear Camera: Aperture | f/1.8 | Main: f/1.8 Wide Angle: f/2.4 |

Sensor spec

$$y = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

Where Was I?



Sensor size

$$y = f \frac{Y}{Z} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y}{Z}$$

| Apple iPhone Cameras | | |
|-----------------------------|---|---|
| | Apple iPhone 6s Apple iPhone 6s Plus | Apple iPhone 7 Apple iPhone 7 Plus |
| Front Camera | 5.0MP | 7MP |
| Front Camera - Sensor | Sony ? (1.12 µm, 1/5") | Sony Exmor RS (1.0 µm, 1/5") |
| Front Camera - Focal Length | 2.65mm (31mm eff) | 2.87mm (32mm eff) |
| Front Camera - Max Aperture | F/2.2 | F/2.2 |
| Rear Camera | 12MP | 12MP |
| Rear Camera - Sensor | Sony Exmor RS (1.22 µm, 1/3") | Sony Exmor RS (1.22 µm, 1/3") |
| Rear Camera - Focal Length | 4.15mm (29mm eff) | 4mm (28mm eff) 8mm (56mm eff) Secondary |
| Rear Camera - Max Aperture | F/2.2 | F/1.8 F/2.8 Secondary |

Sensor spec

Where Was I?



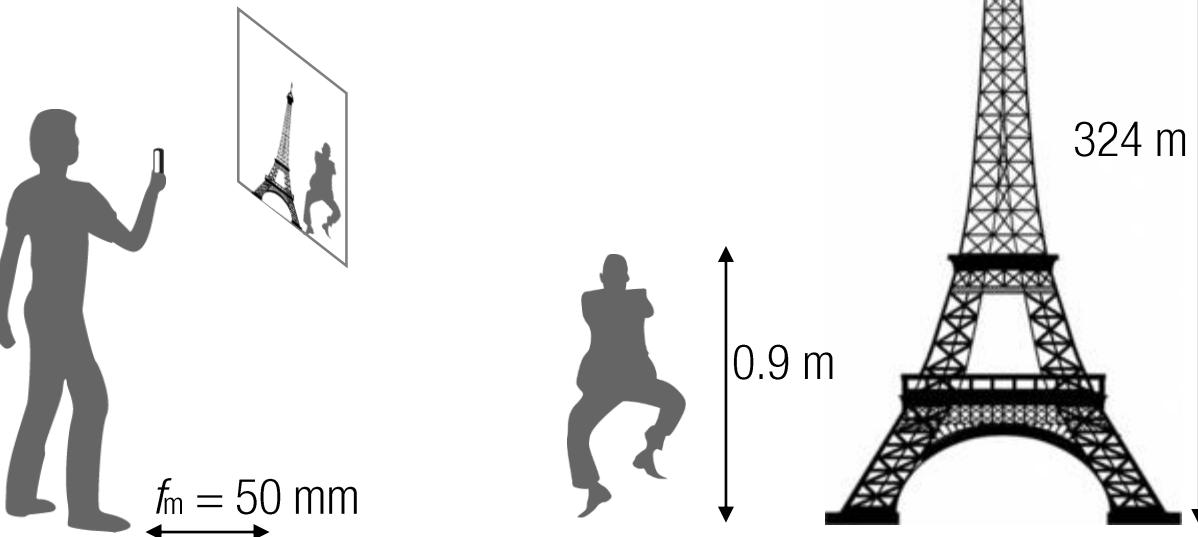
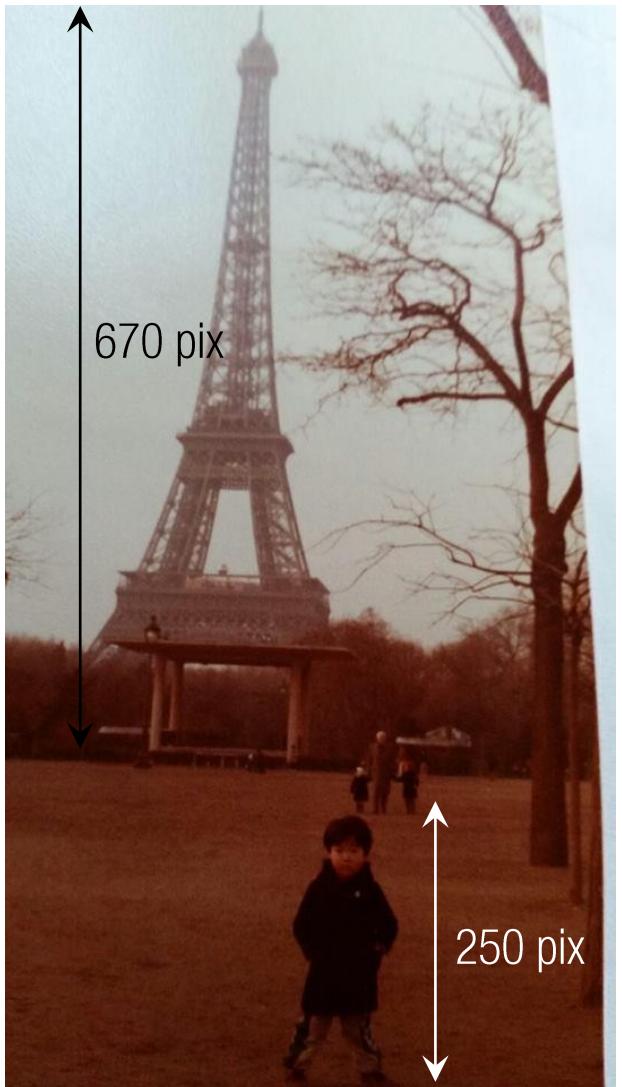
Circa 1984

Where Was I?

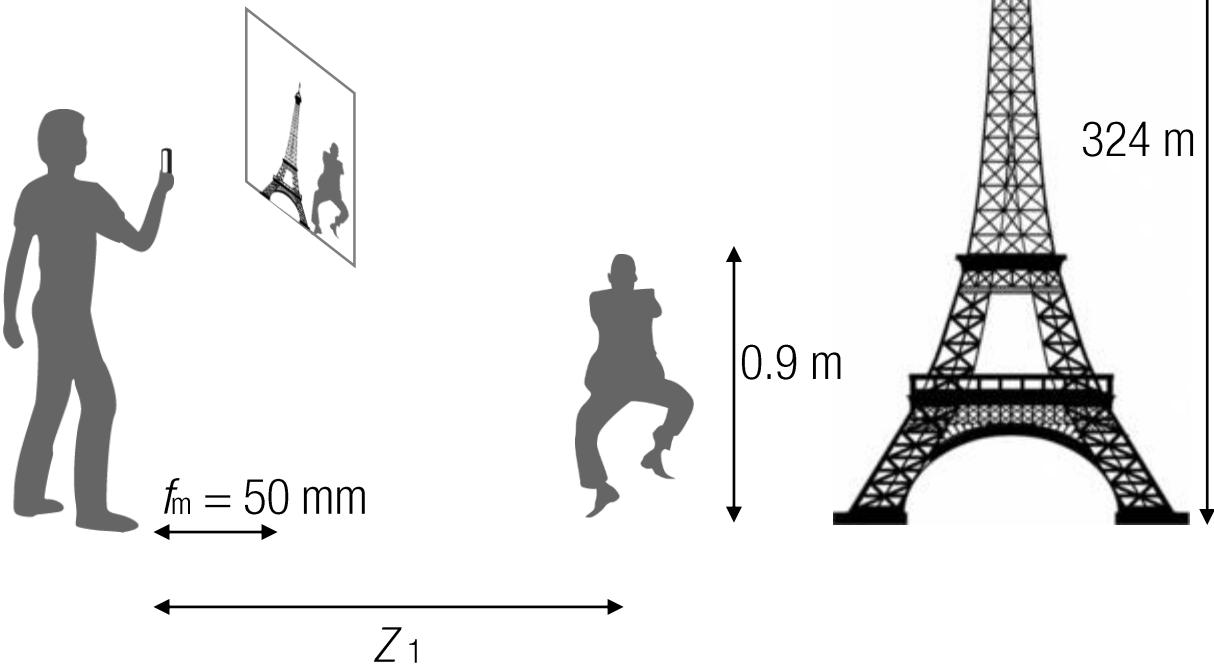
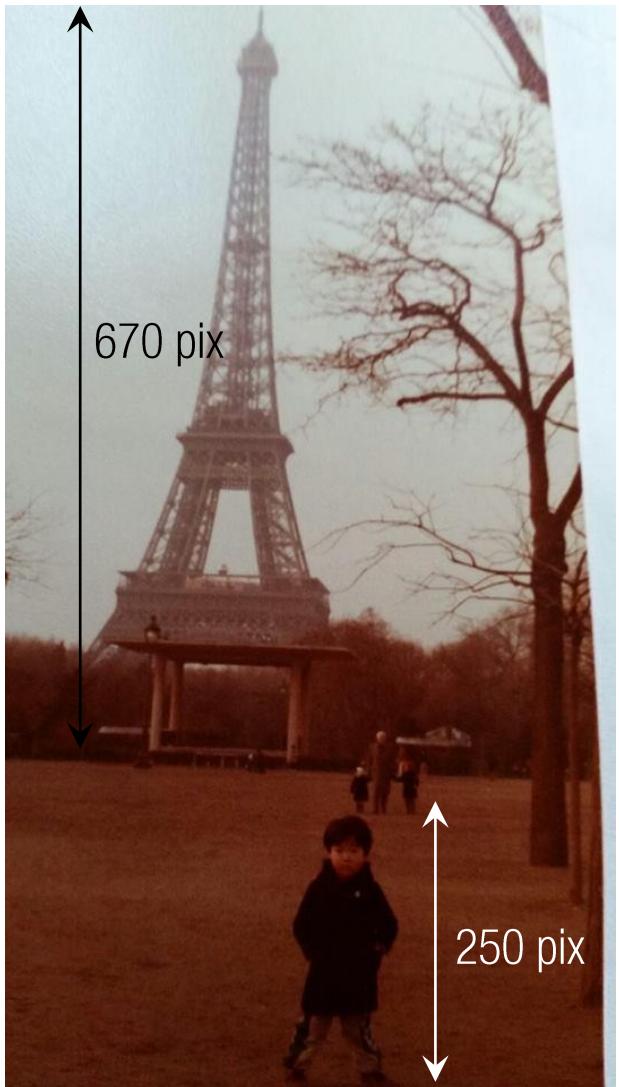


Circa 1984

Where Was I?

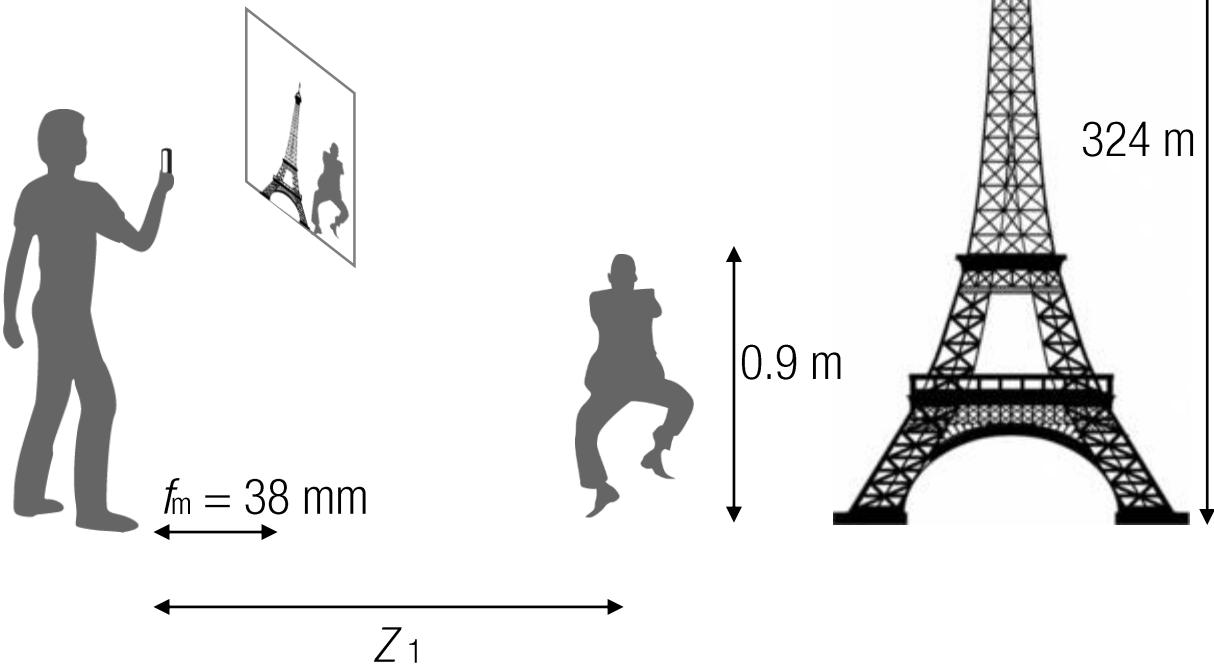
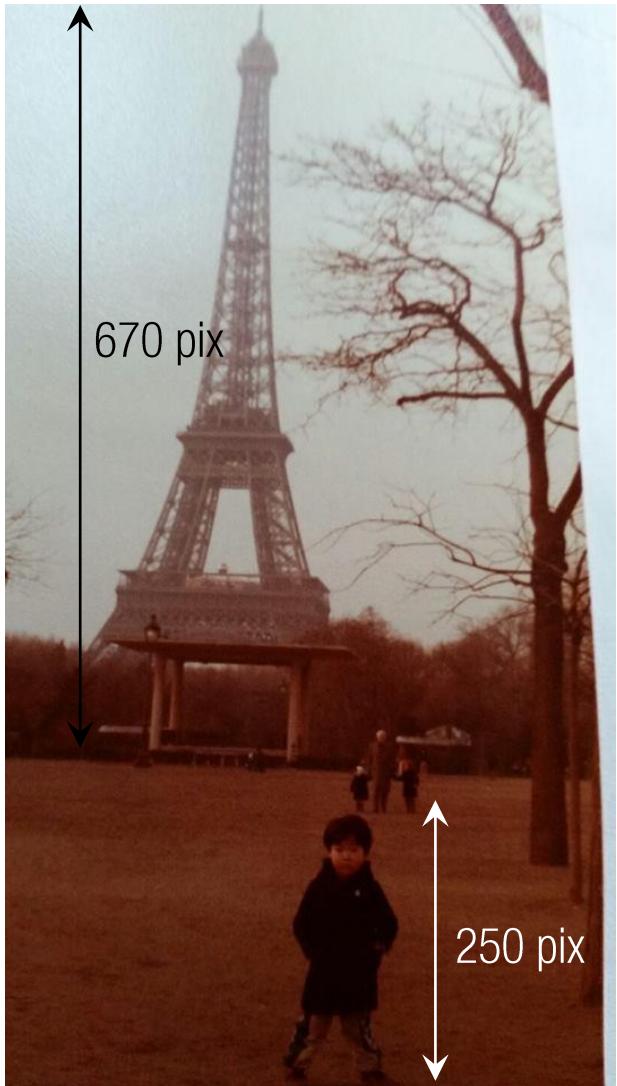


Where Was I?



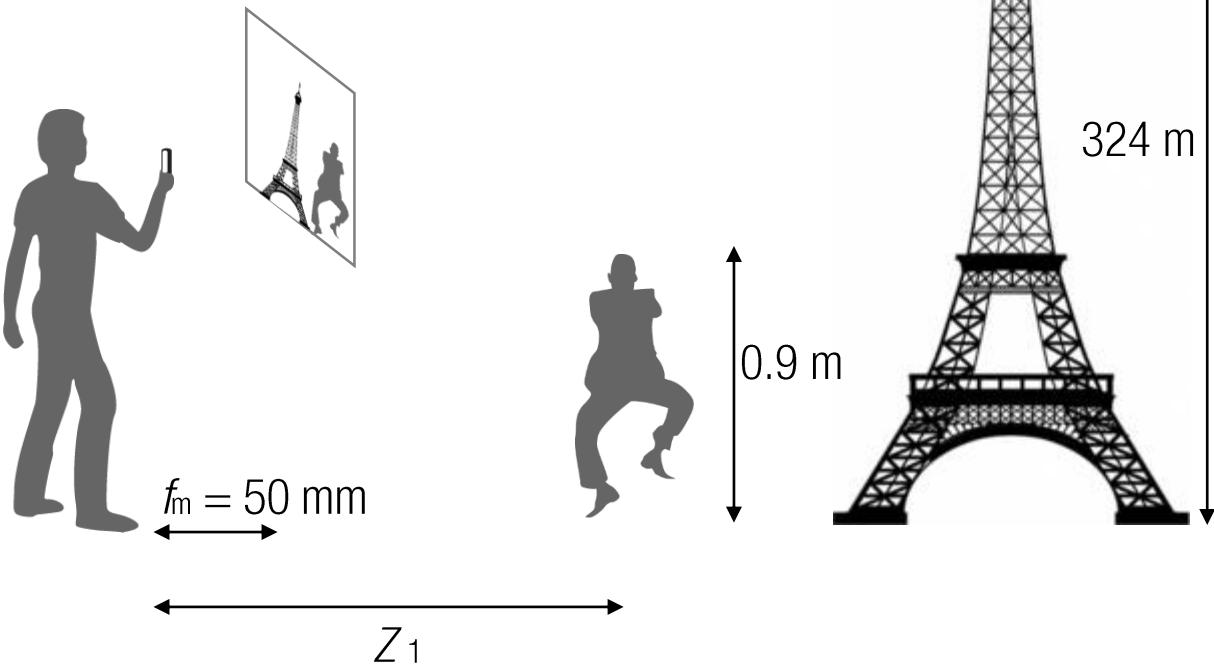
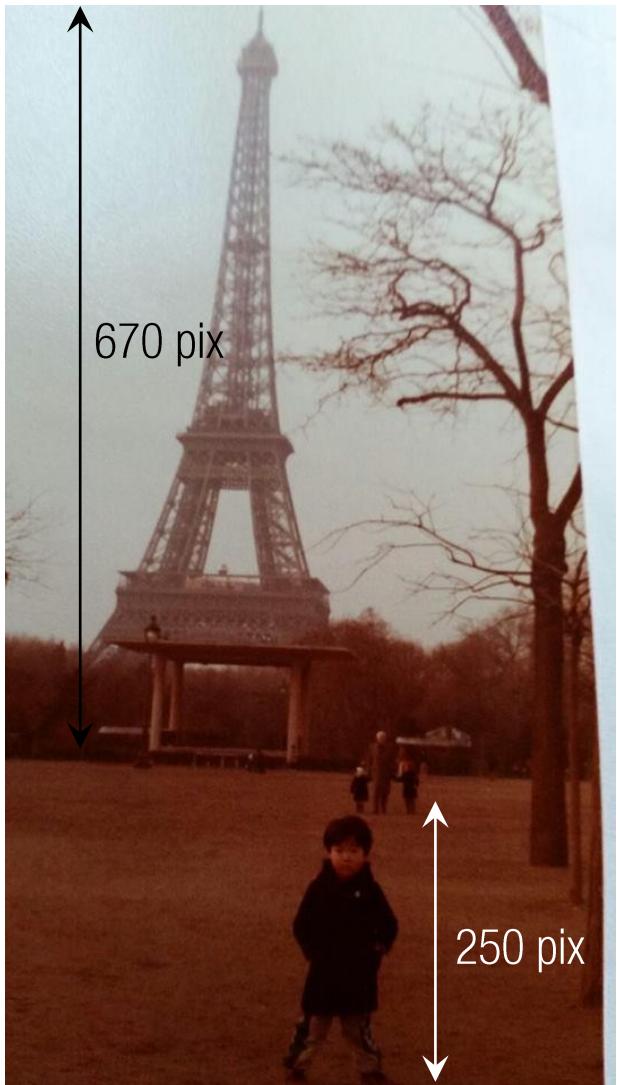
$$y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1}$$

Where Was I?



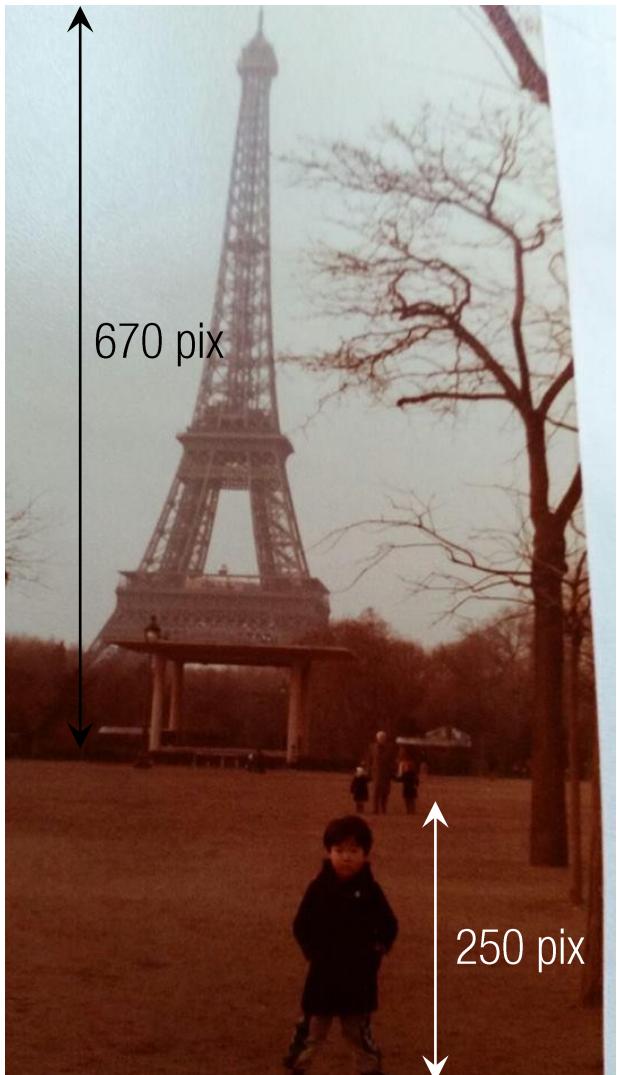
$$y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{y_1}$$

Where Was I?

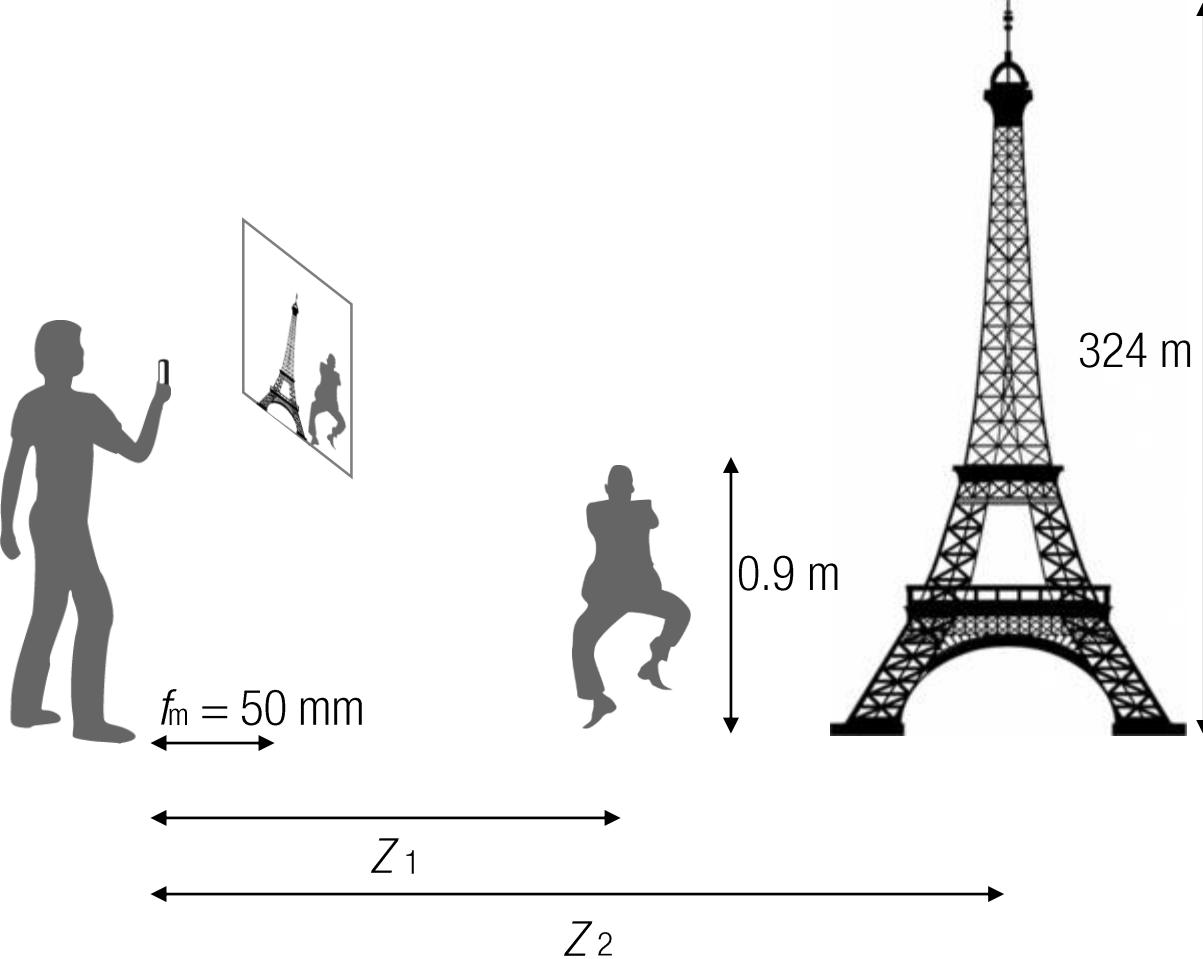


$$y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{m}$$

Where Was I?



Circa 1984



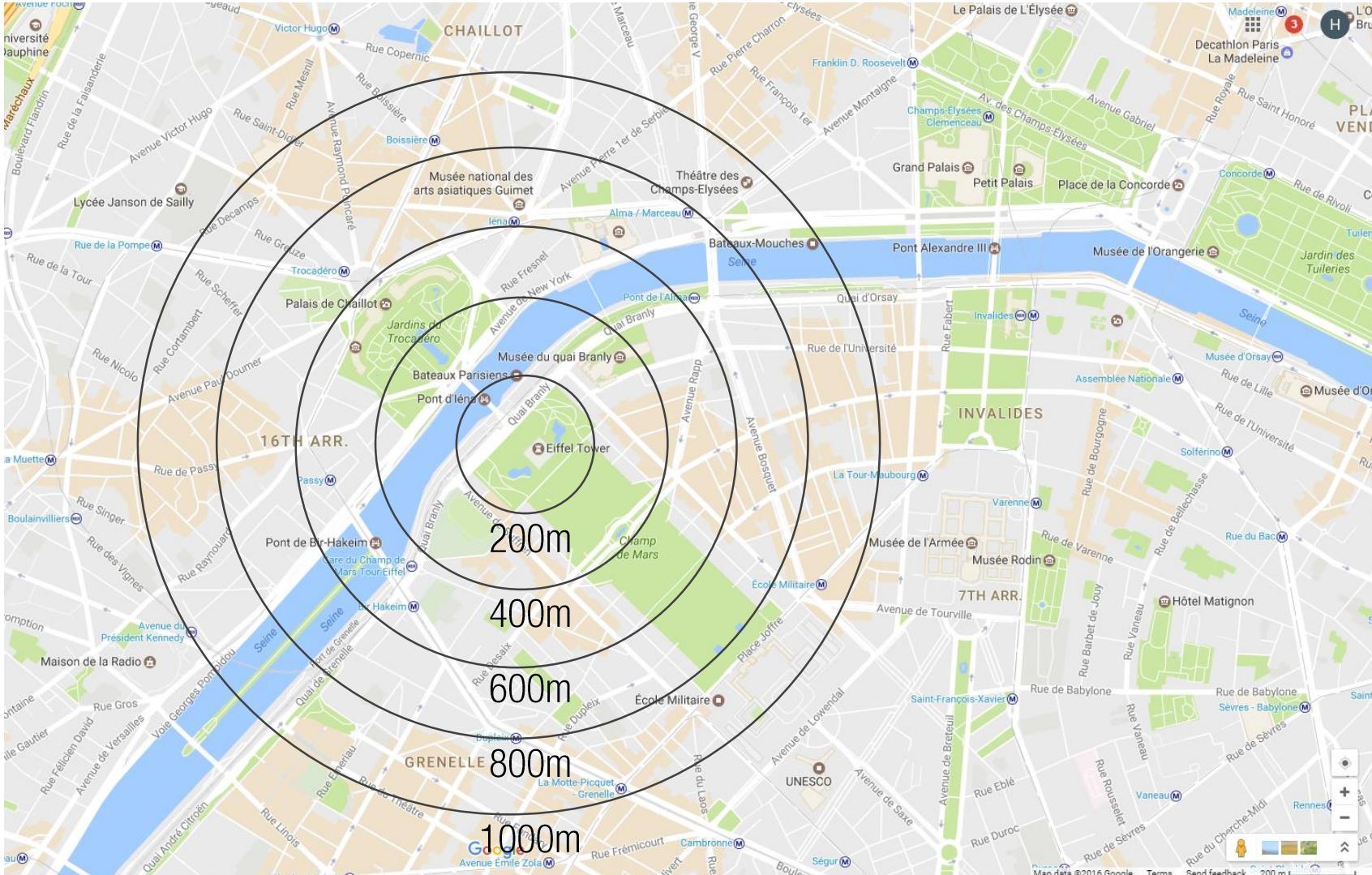
$$y_1 = f \frac{Y_1}{Z_1} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{Z_1} \rightarrow Z_1 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_1}{y_1} = 0.05 \frac{1280}{0.0218} \frac{0.9}{250} = 8.03 \text{ m}$$

$$y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079 \text{ m}$$

Where Was I?



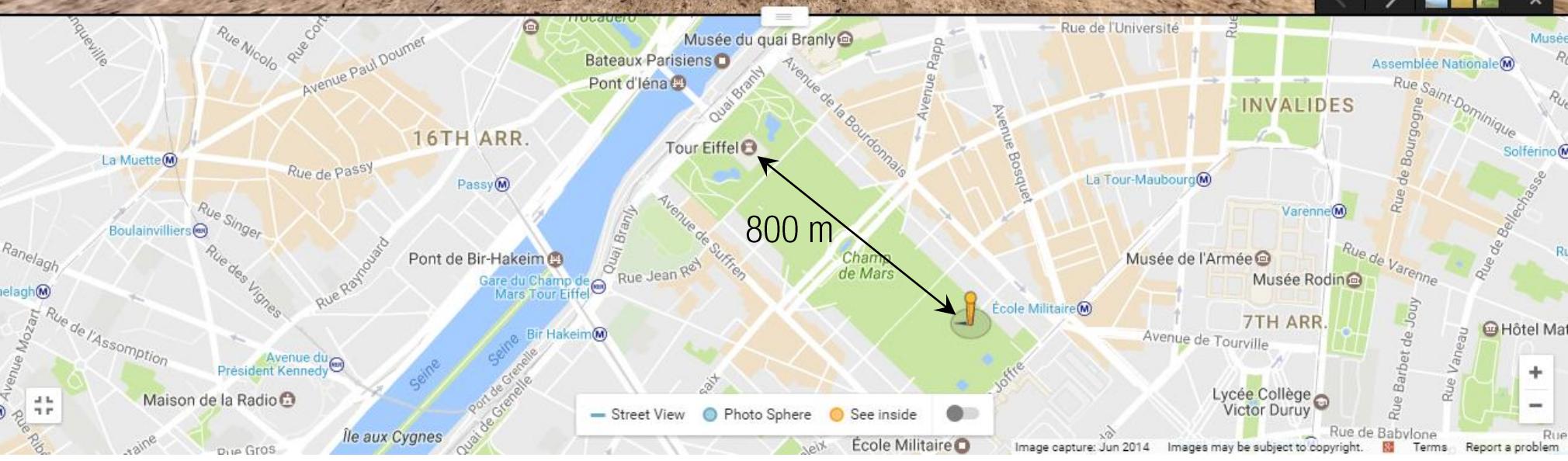
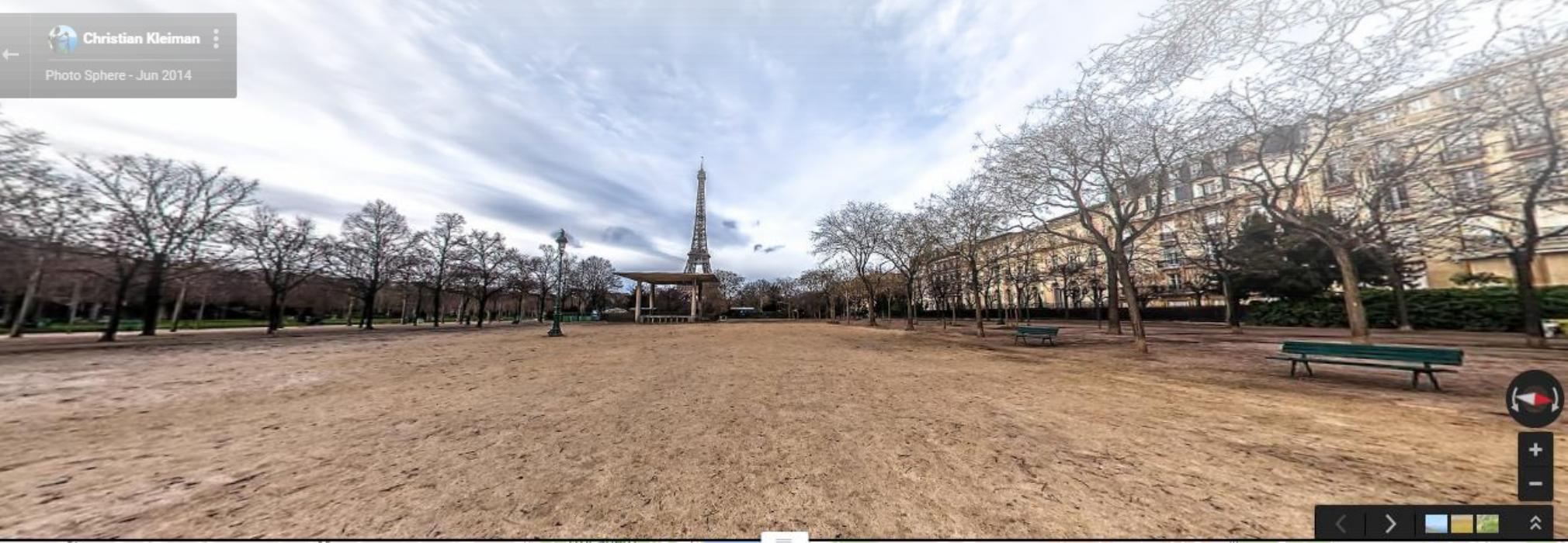
$$y_2 = f \frac{Y_2}{Z_2} = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{Z_2} \rightarrow Z_2 = f_m \frac{h_{\text{img}}}{h_{\text{ccd}}} \frac{Y_2}{y_2} = 0.05 \frac{1280}{0.0218} \frac{324}{670} = 1079 \text{m}$$



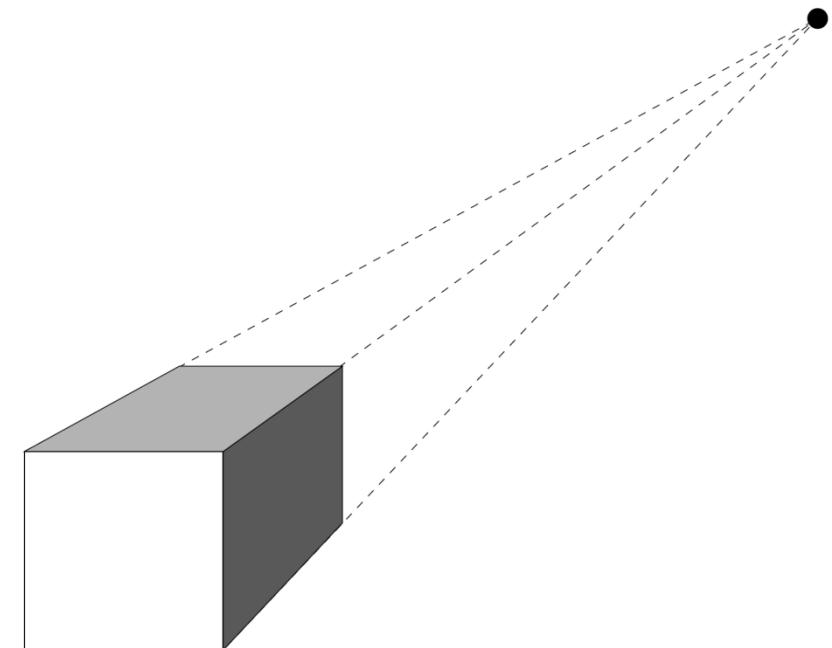
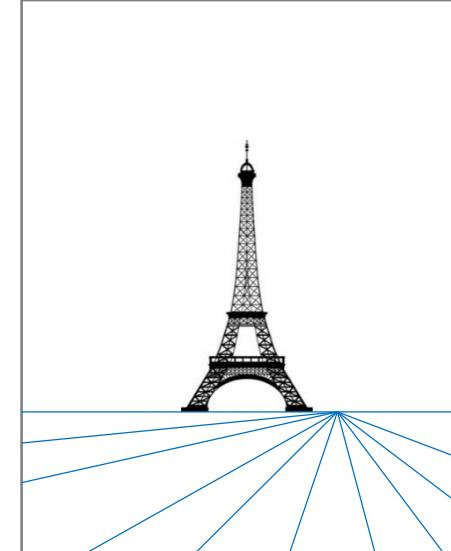
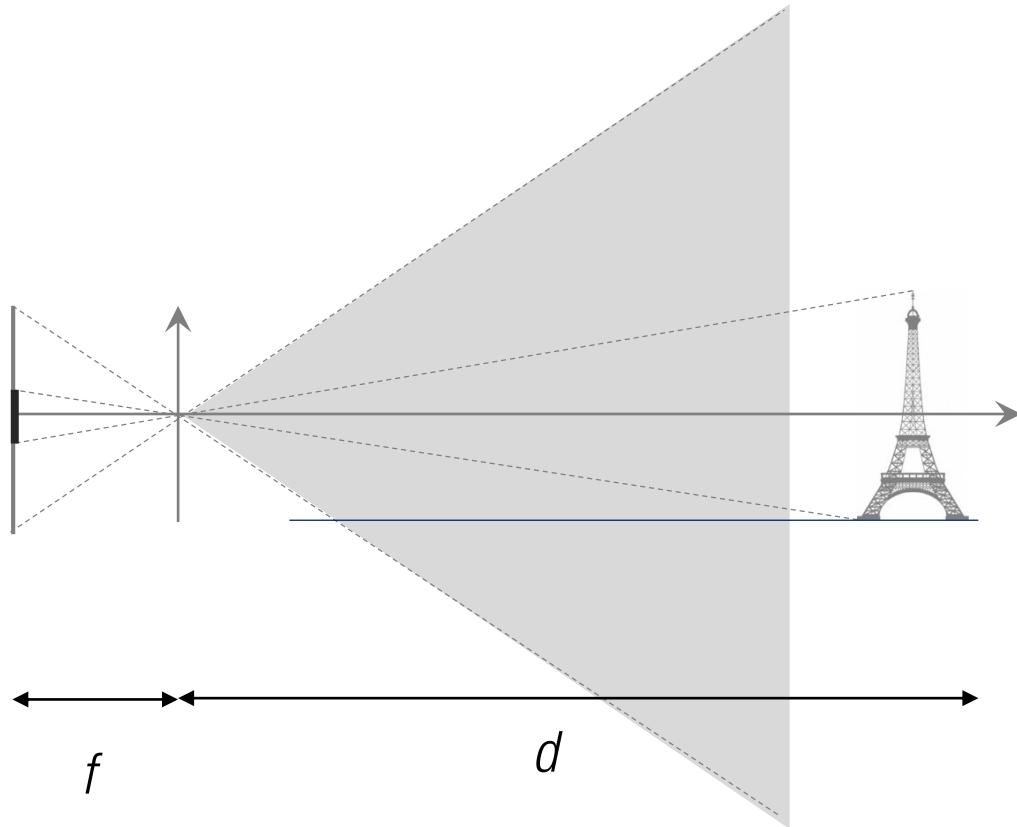
Where Was I?



Christian Kleiman
Photo Sphere - Jun 2014

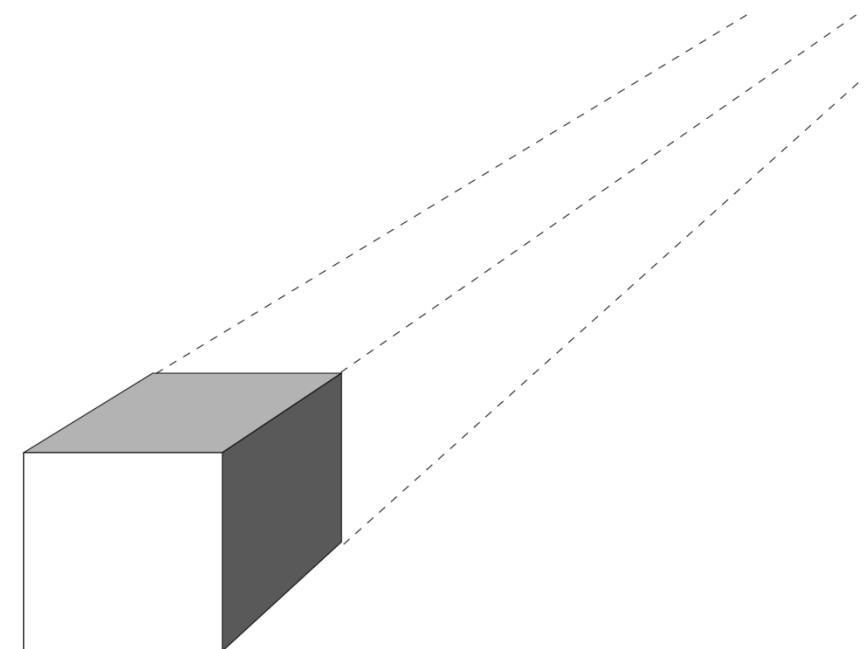
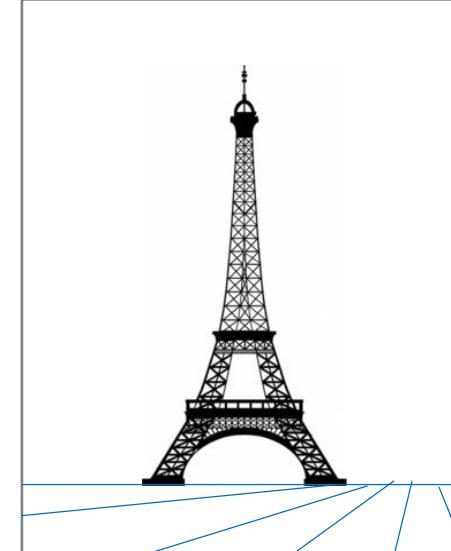
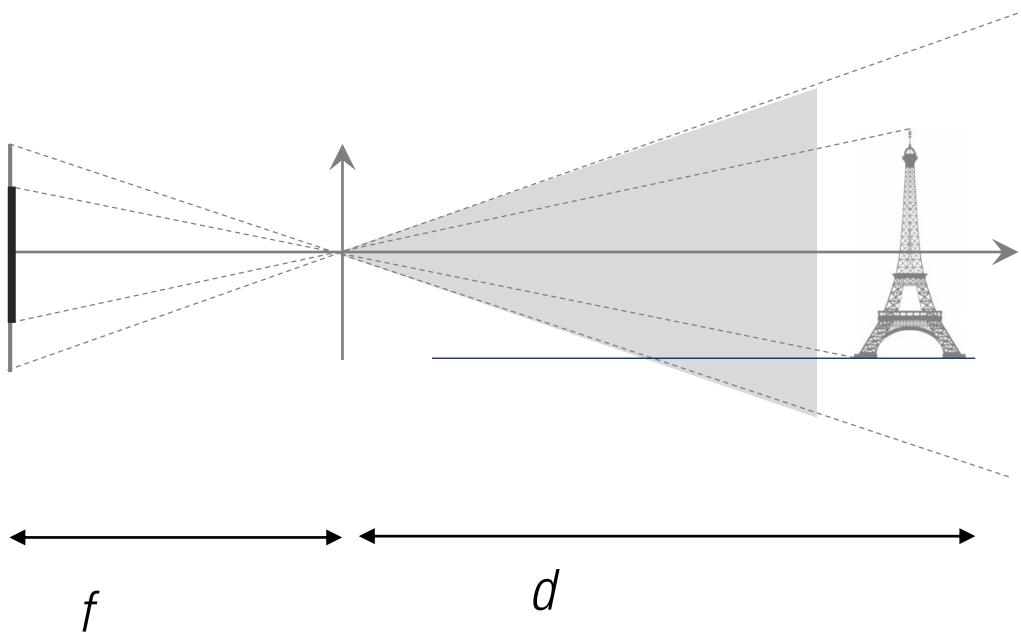


Focal Length



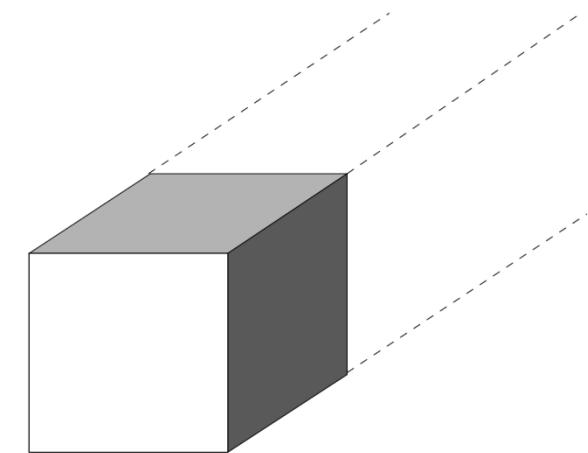
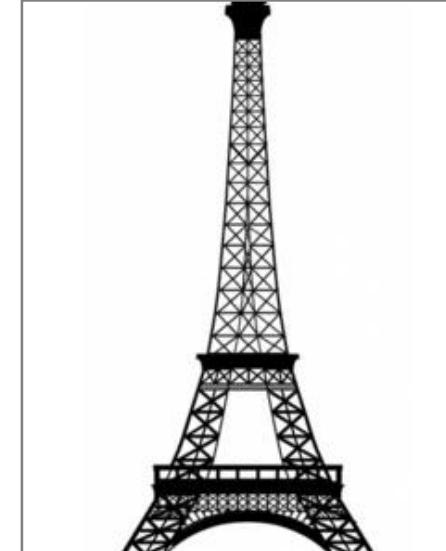
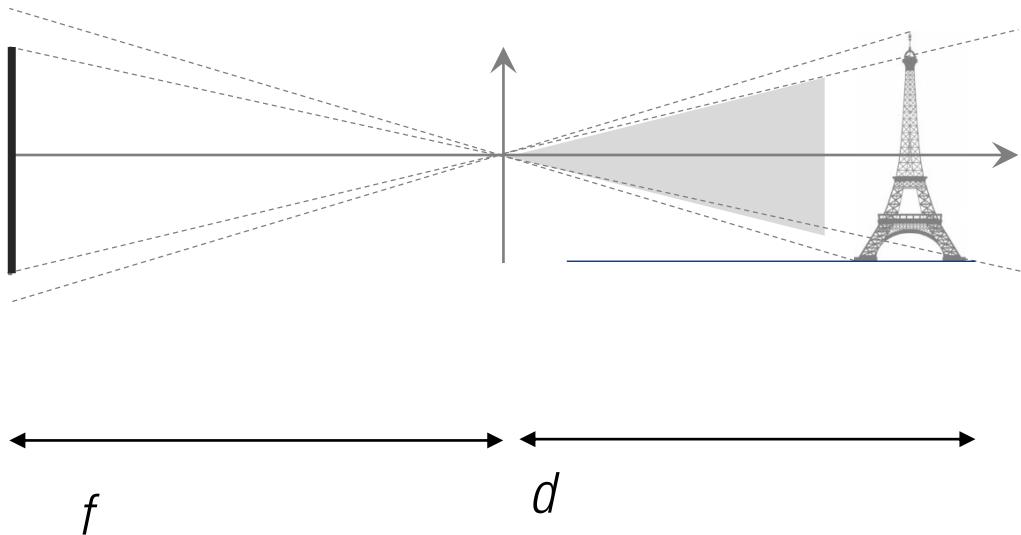
Strong perspective

Focal Length



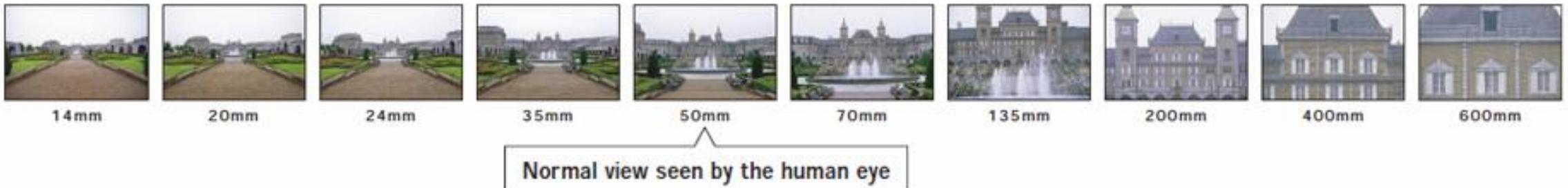
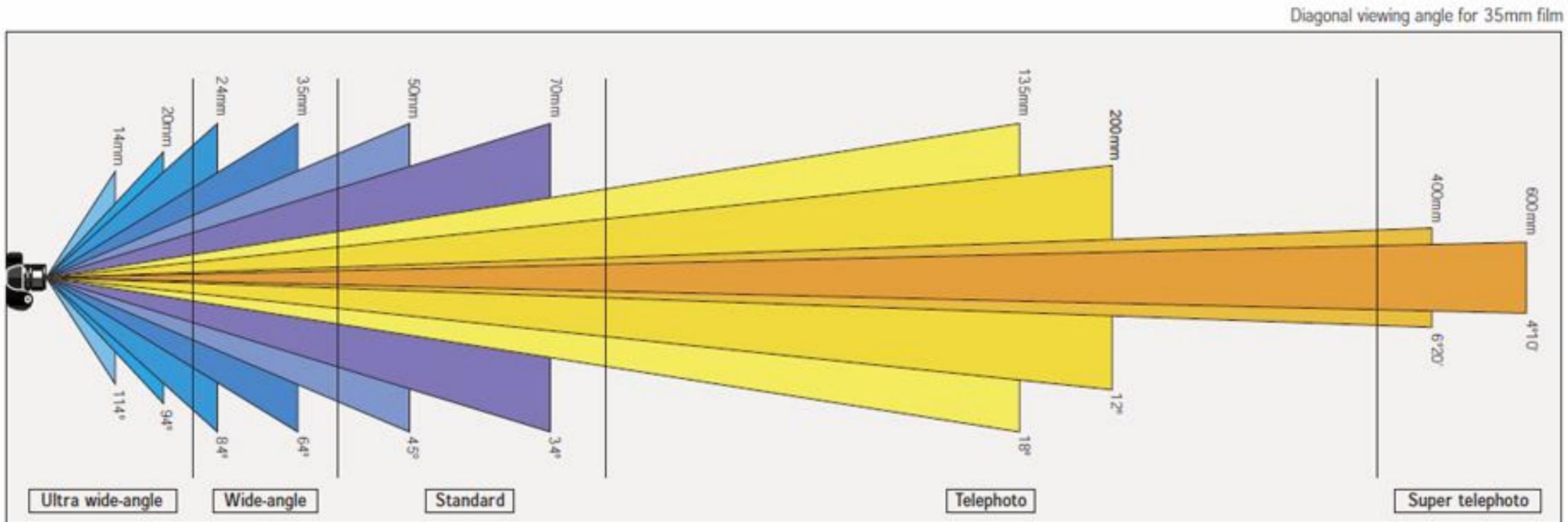
Weak perspective

Focal Length

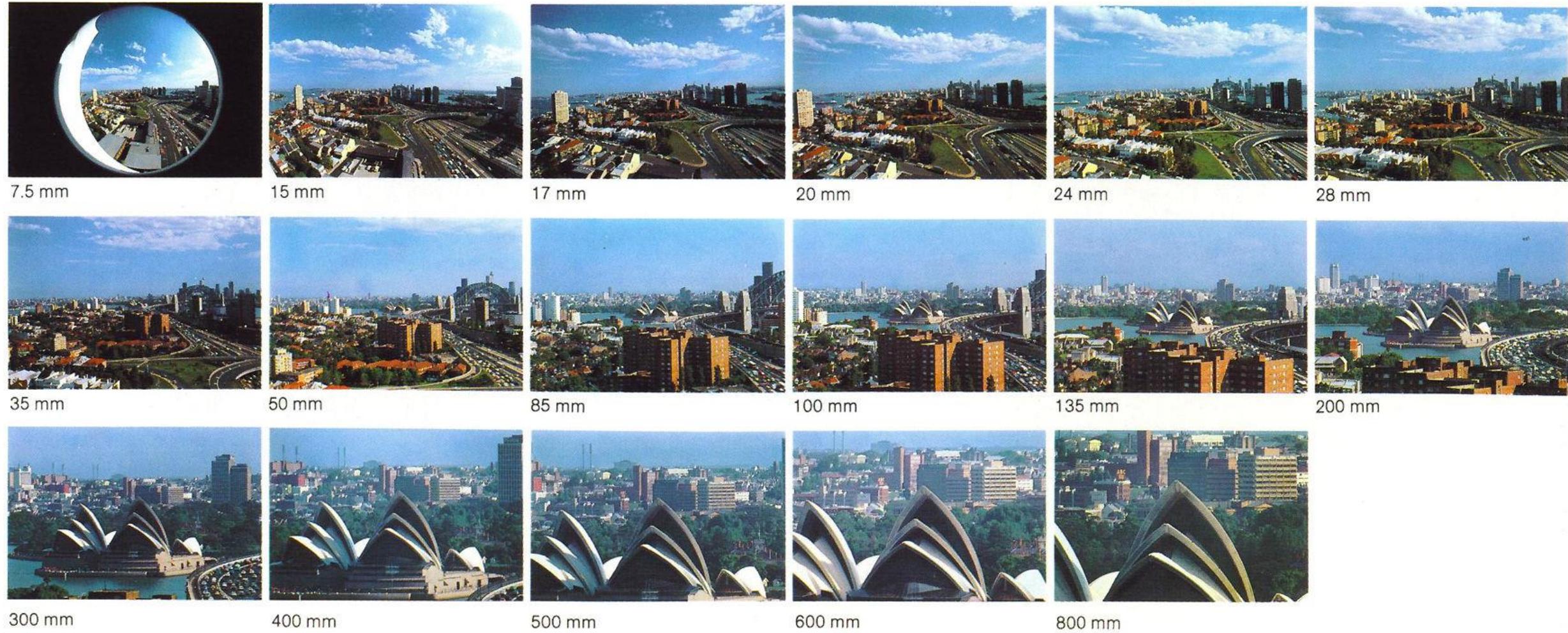


Orthographic

Focal Length



Angle of View





Dolly Zoom (Vertigo Effect)



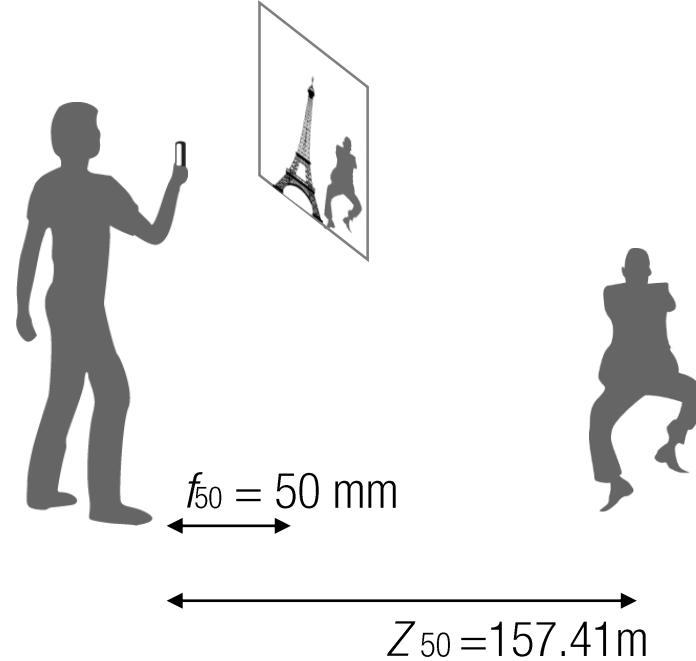
VERTIGO (1958)

Dolly Zoom (Vertigo Effect)

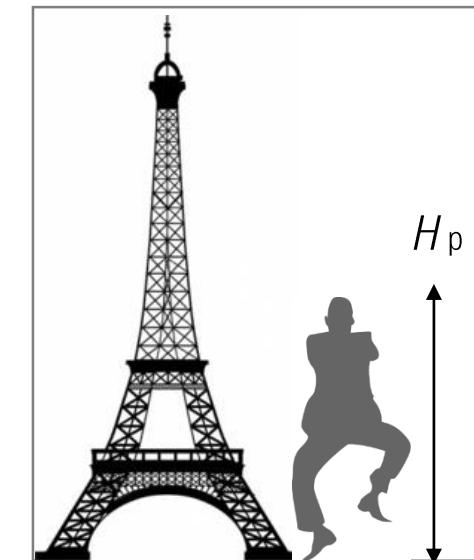
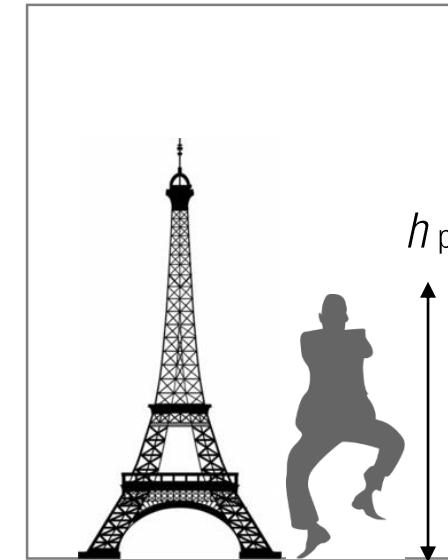
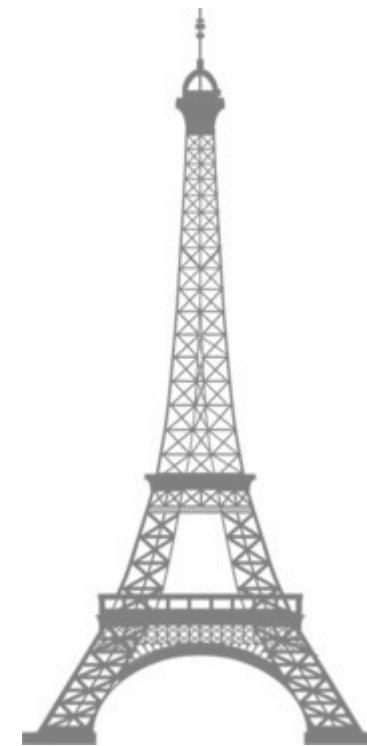


JAWS (1975)

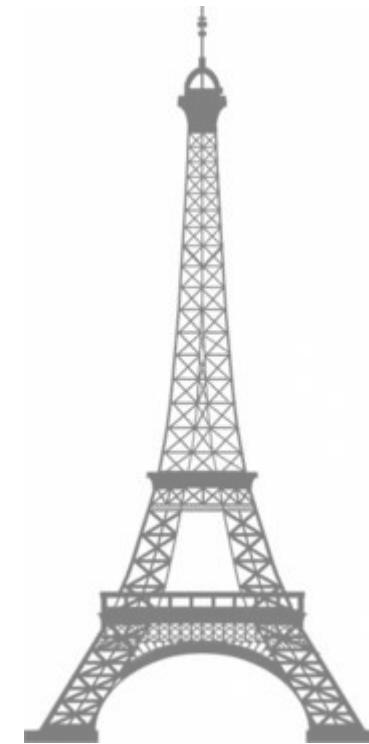
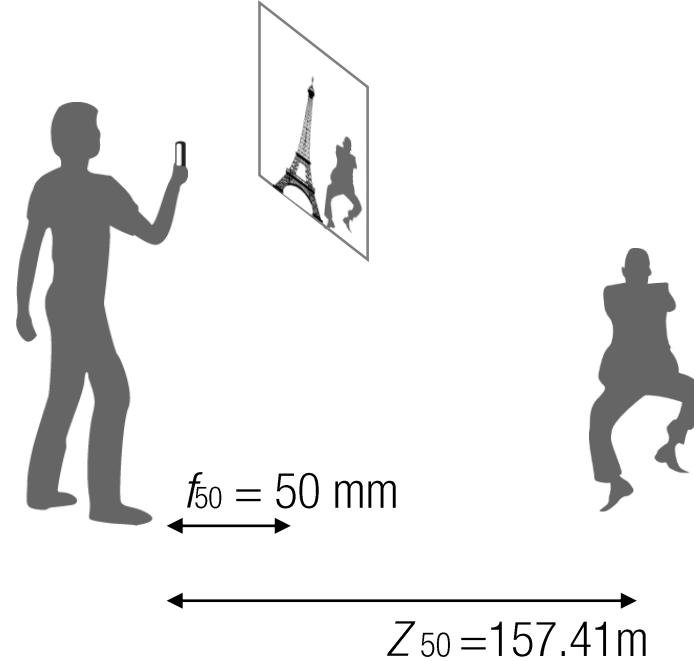
Dolly Zoom



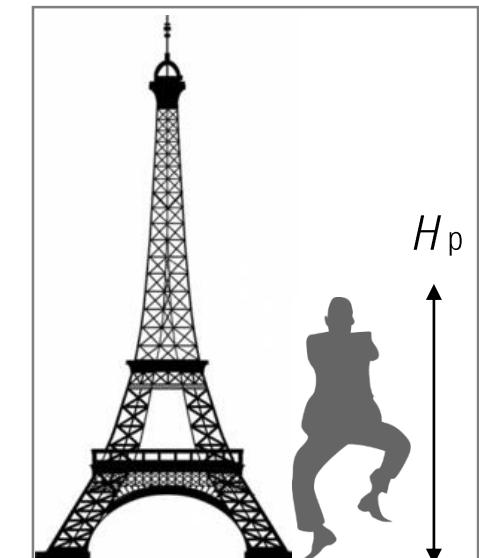
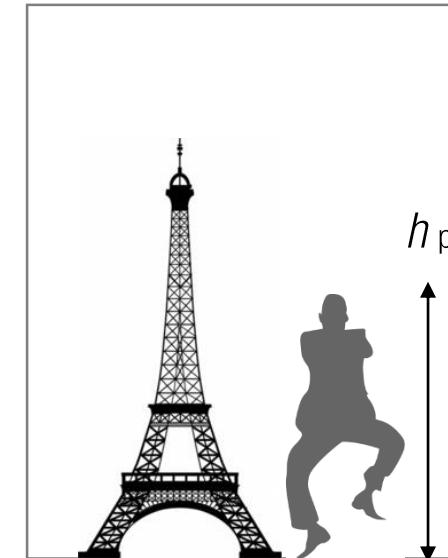
Given focal length ($f_m=100\text{mm}$),
what Z_{100} to make the height of the person remain the same as $f_m=50\text{mm}$?



Dolly Zoom

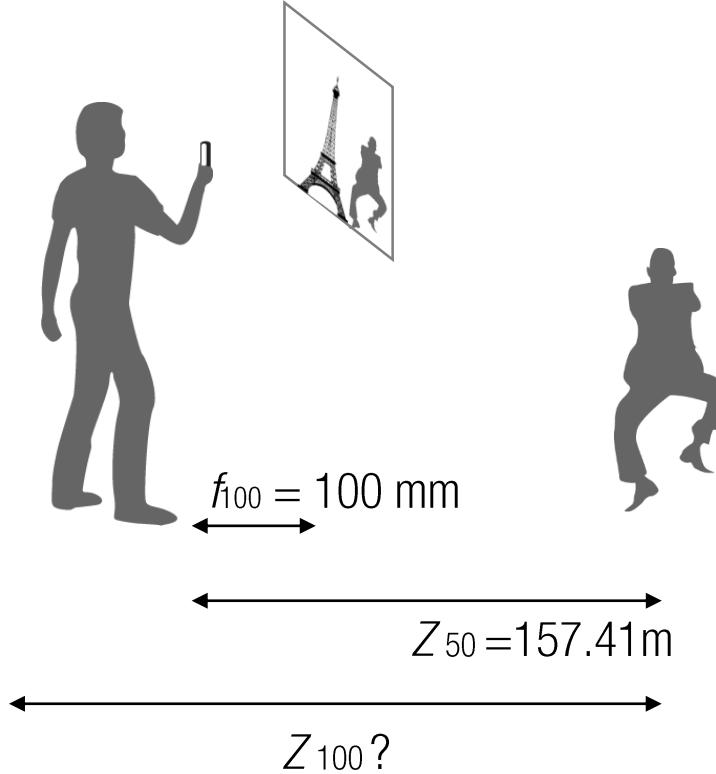


Given focal length ($f_m=100\text{mm}$),
what Z_{100} to make the height of the person remain the same as $f_m=50\text{mm}$?

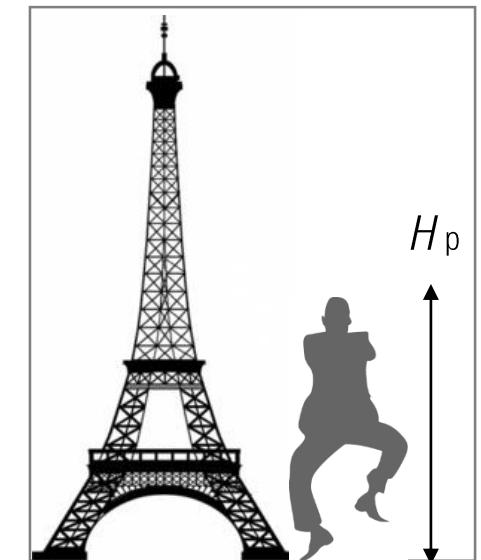
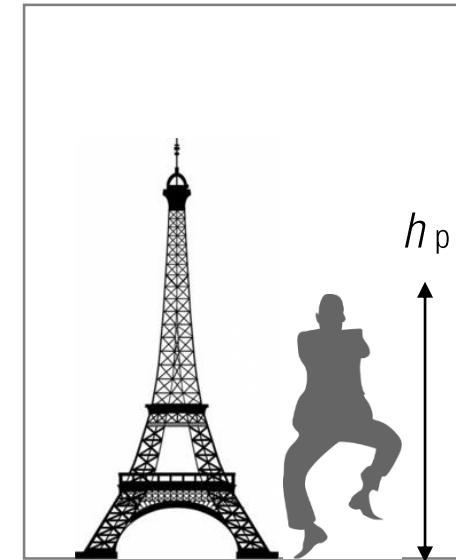
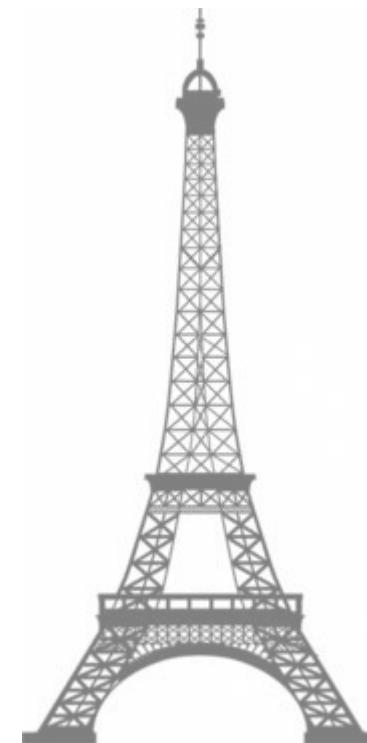


$$h_{50} = f_{50} \frac{Y}{Z_{50}} \quad h_{100} = f_{100} \frac{Y}{Z_{100}} \quad \text{s.t.} \quad h_{100} = h_{50}$$

Dolly Zoom



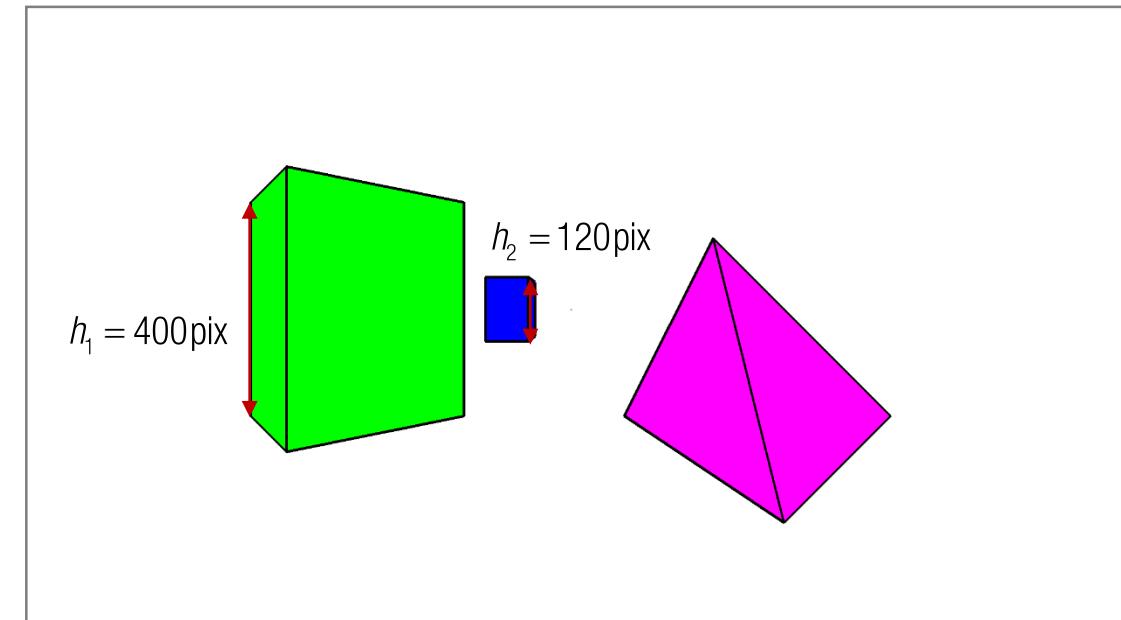
Given focal length ($f_m=100\text{mm}$),
what Z_{100} to make the height of the person remain the same as $f_m=50\text{mm}$?



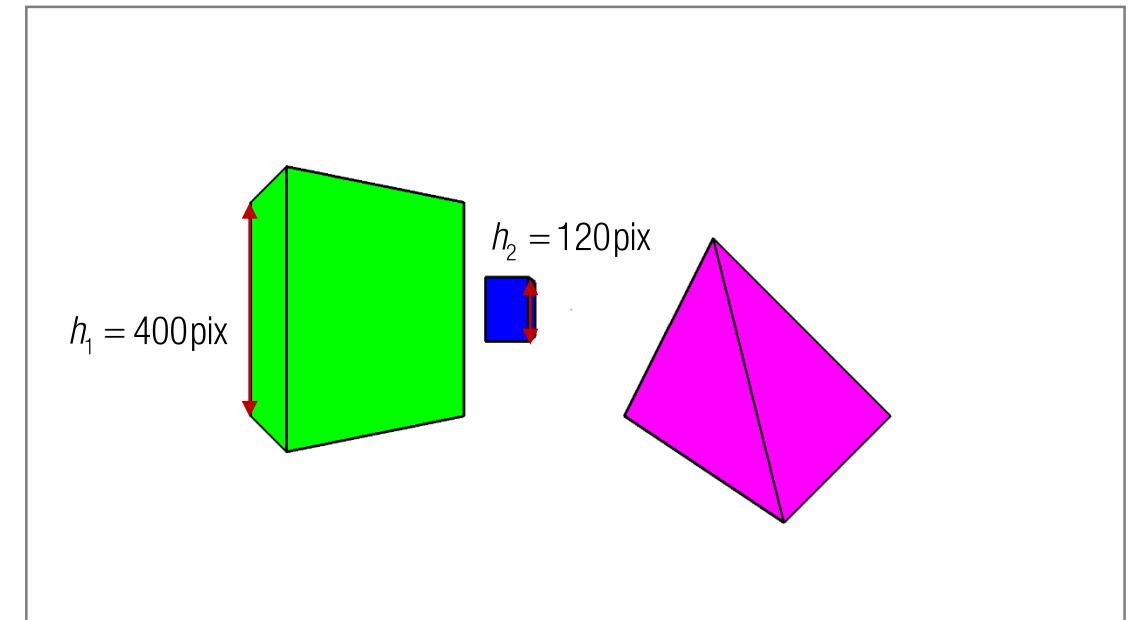
$$h_{50} = f_{50} \frac{Y}{Z_{50}} \quad h_{100} = f_{100} \frac{Y}{Z_{100}} \quad \text{s.t.} \quad h_{100} = h_{50}$$

$$Z_{100} = \frac{f_{100}}{f_{50}} Z_{50} \quad Z_{100} = \frac{100}{50} 157.41 = 314.8 \text{m}$$

Where am I with Dolly Zoom?

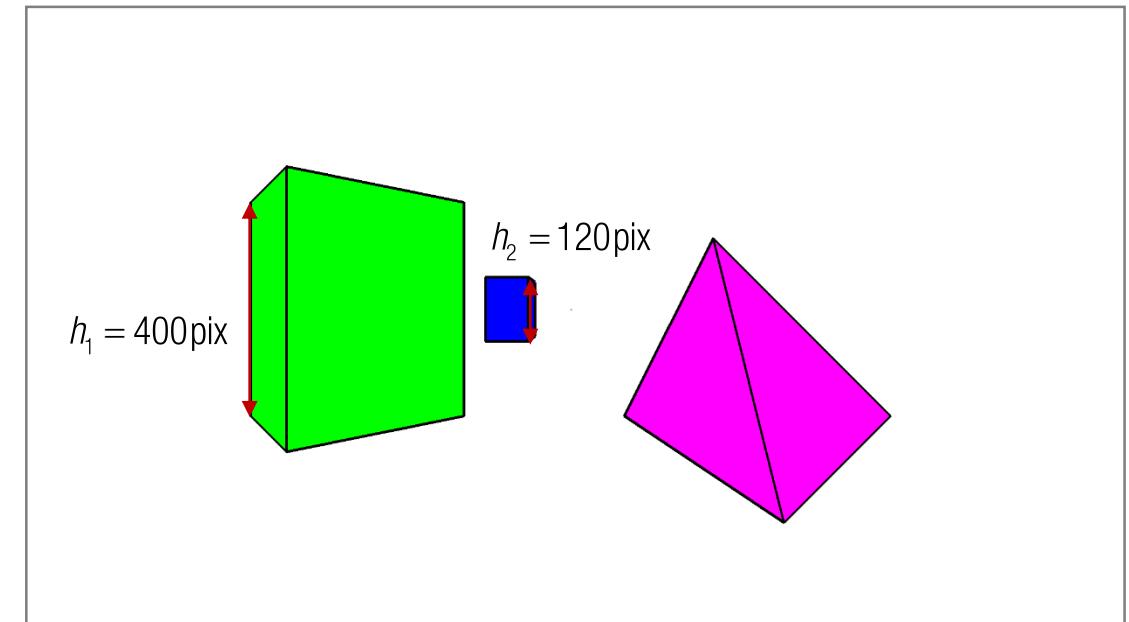
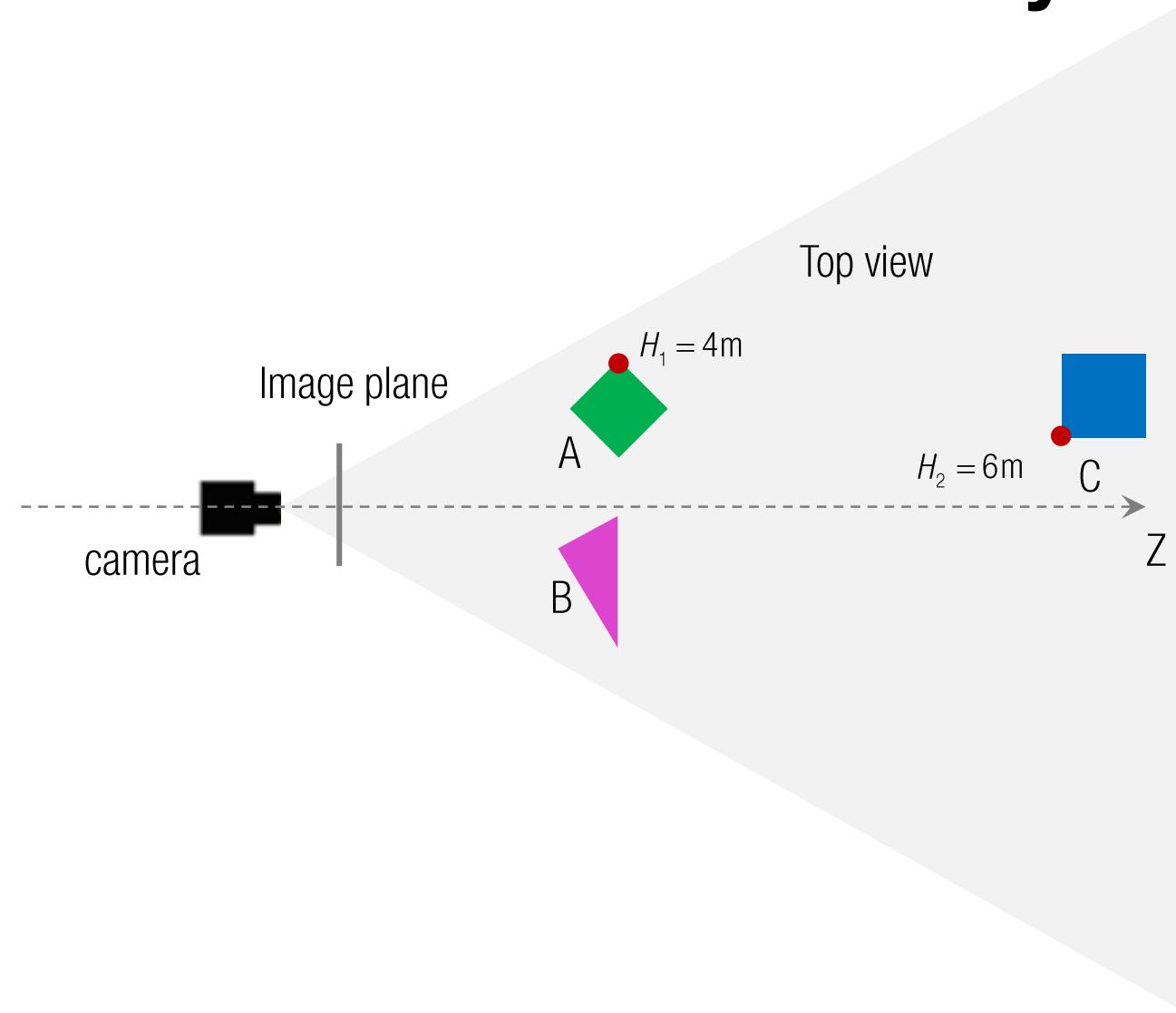


Where am I with Dolly Zoom?



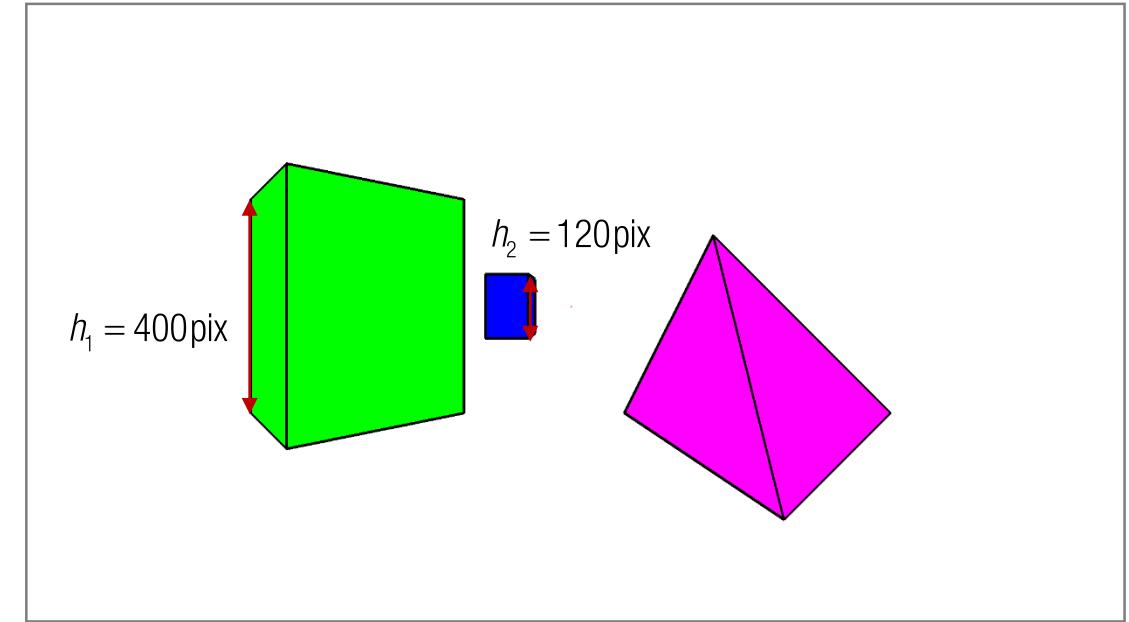
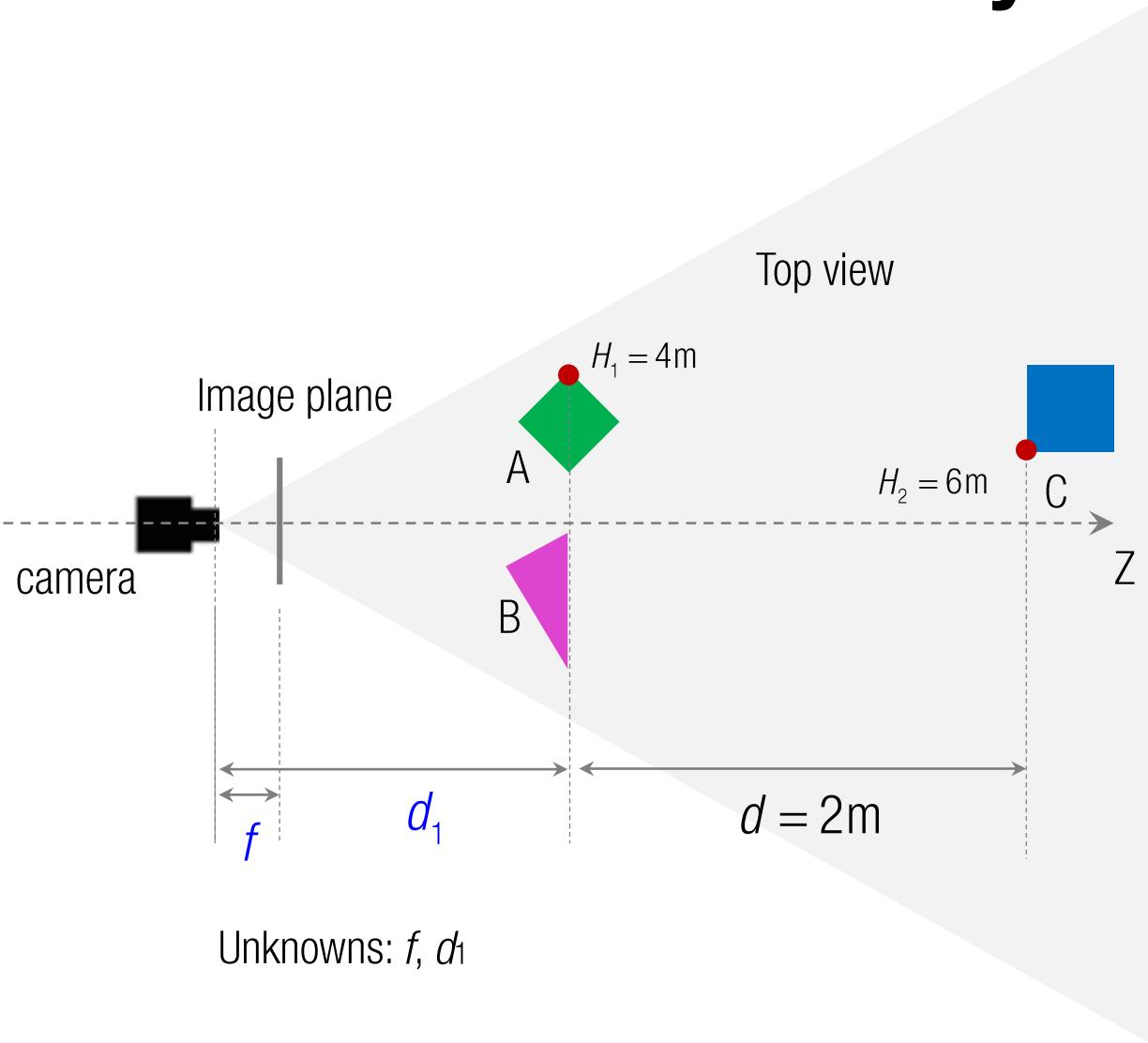
How far I need to step back with zoom factor x2?
How will h_2 change?

Where am I with Dolly Zoom?



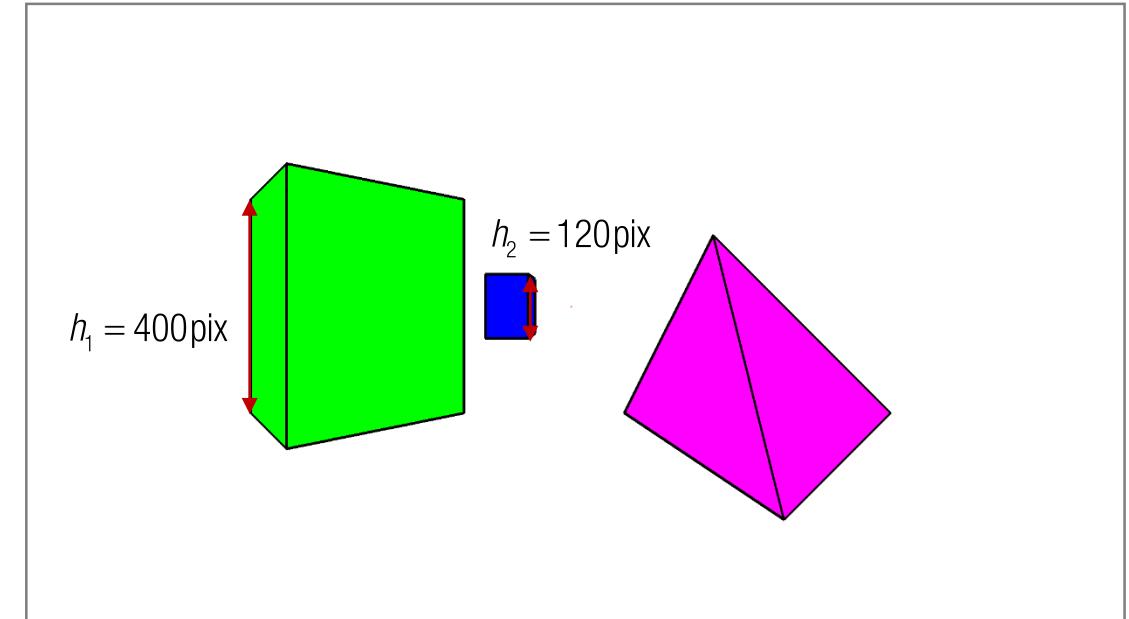
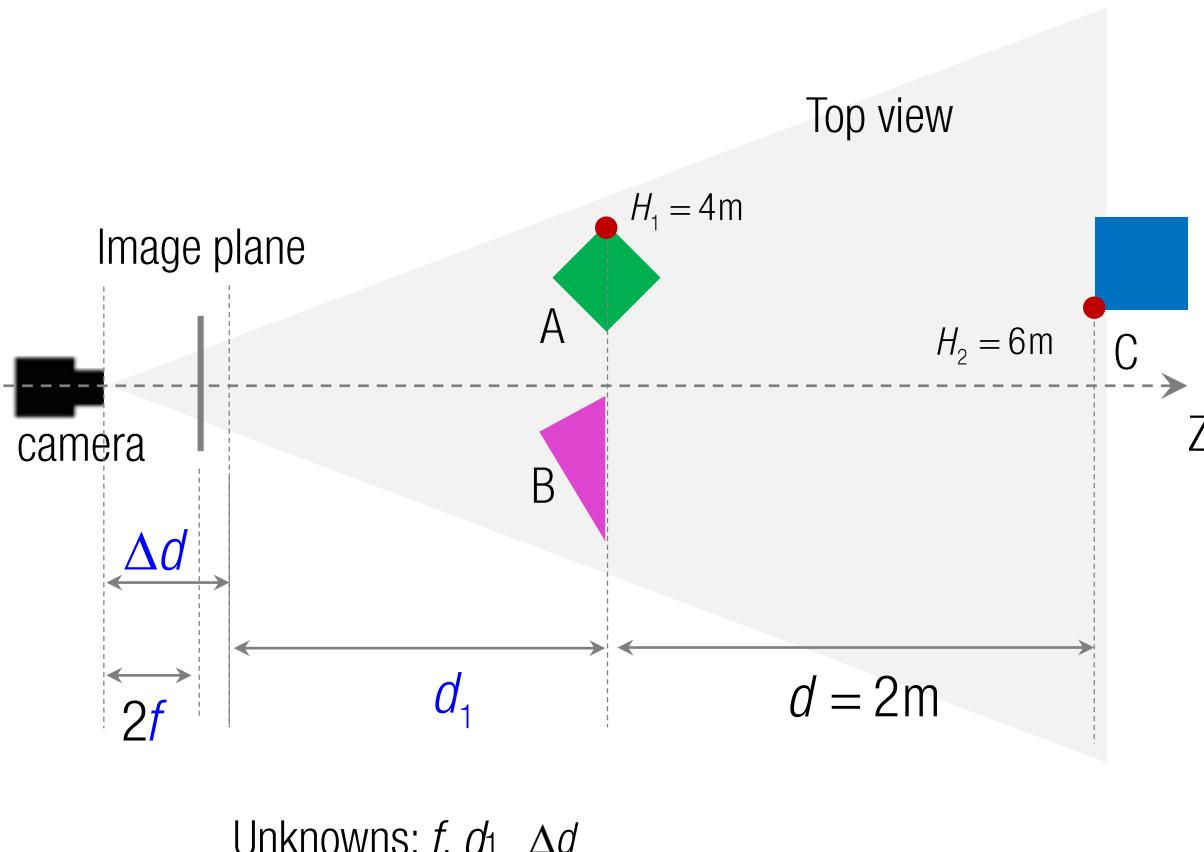
How far I need to step back with zoom factor x2?
How will h_2 change?

Where am I with Dolly Zoom?



How far I need to step back with zoom factor x2?
How will h_2 change?

Where am I with Dolly Zoom?

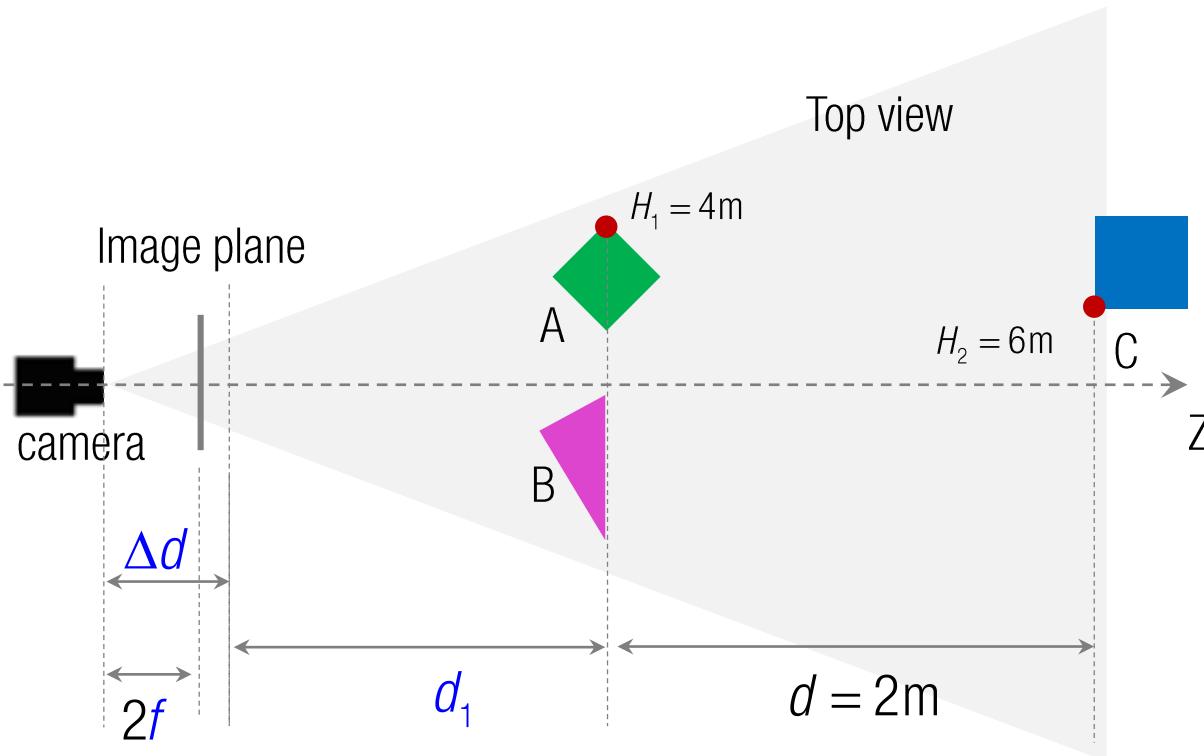


How far I need to step back with zoom factor x2?
How will h_2 change?

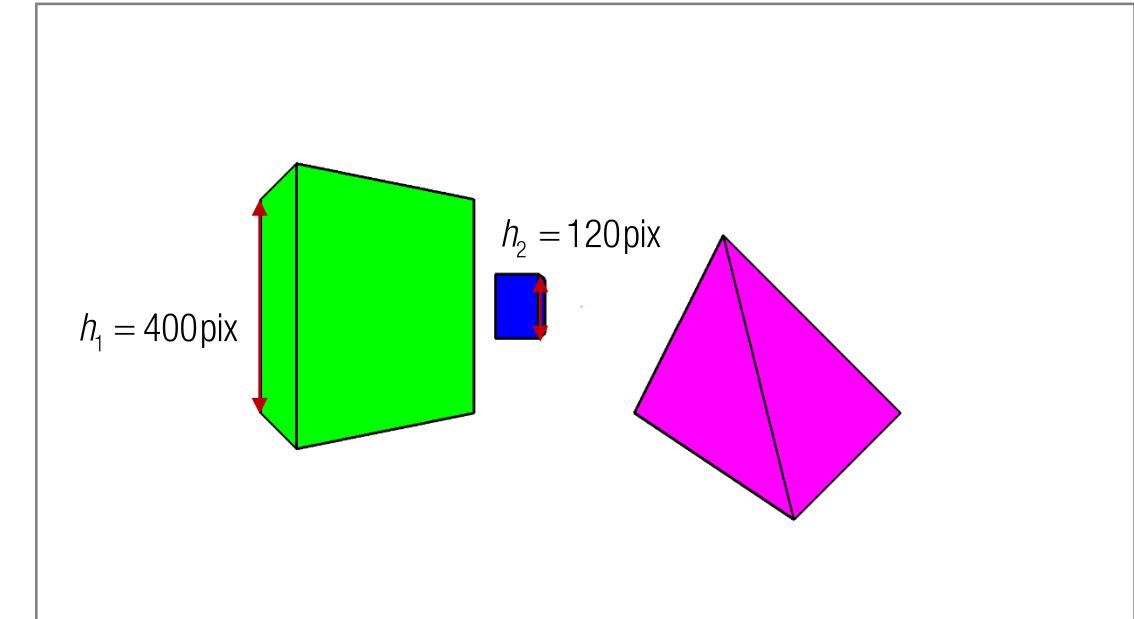
Where am I with Dolly Zoom?

Equations:

$$h_1 = f \frac{H_1}{d_1}$$



Unknowns: $f, d_1, \Delta d$



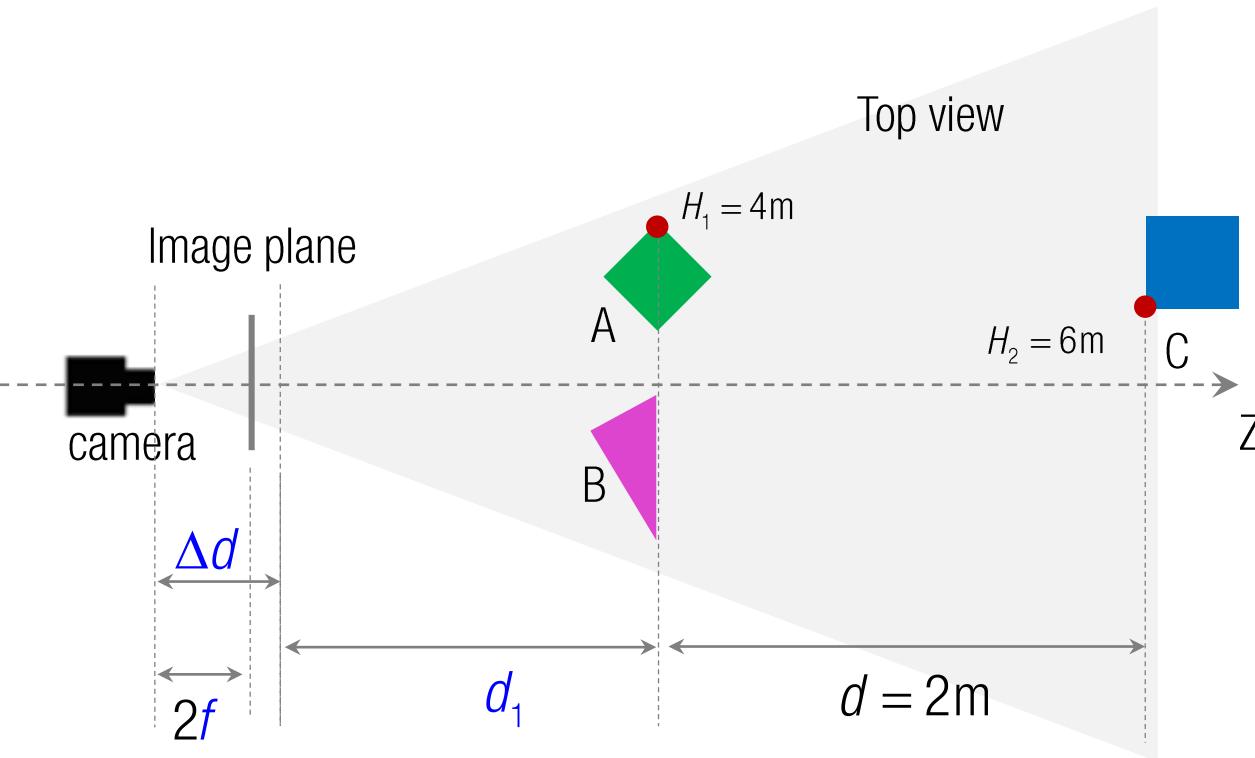
How far I need to step back with zoom factor x2?
How will h_2 change?

Where am I with Dolly Zoom?

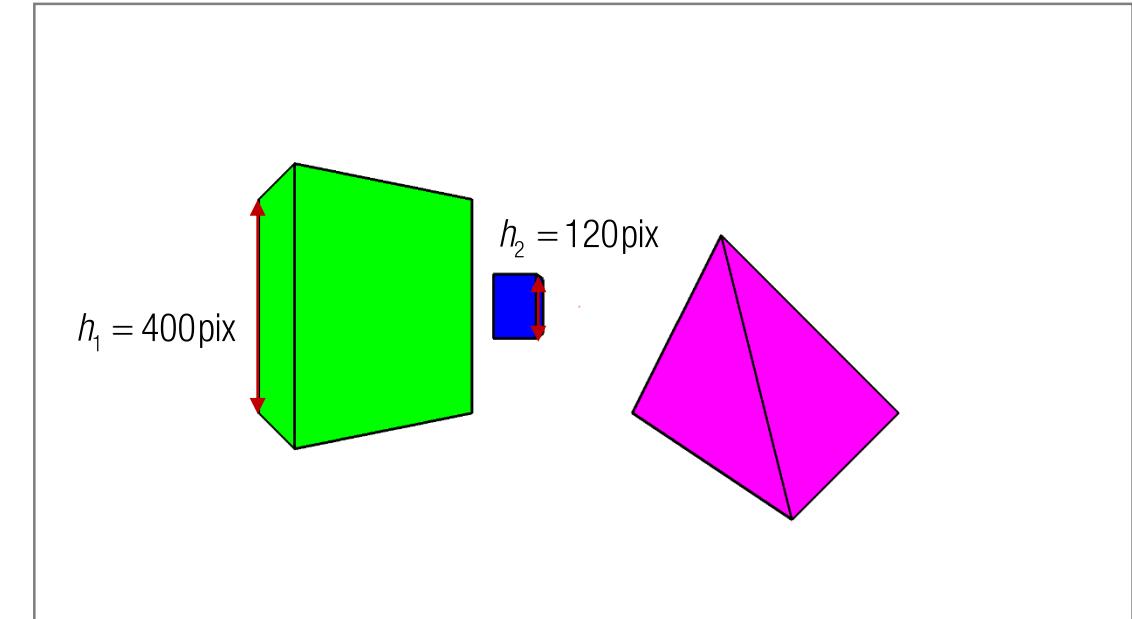
Equations:

$$h_1 = f \frac{H_1}{d_1}$$

$$h_1 = 2f \frac{H_1}{\Delta d + d_1}$$



Unknowns: $f, d_1, \Delta d$

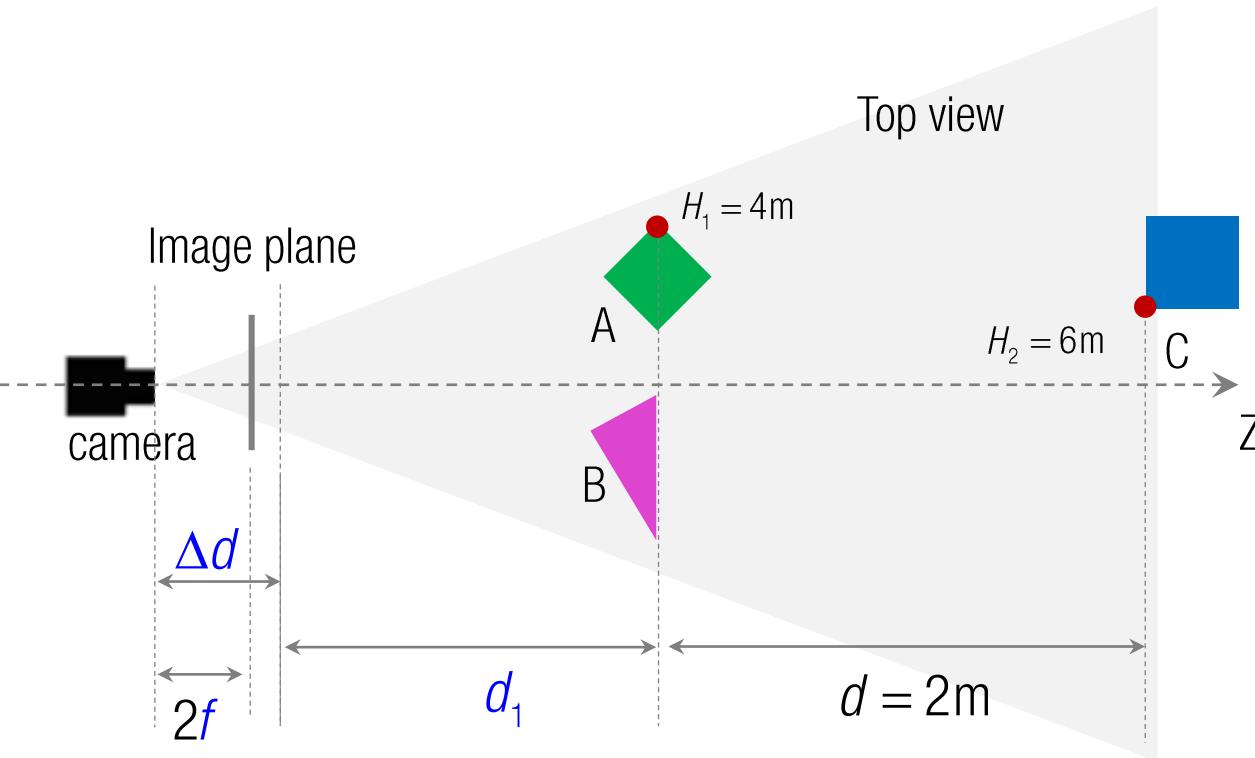


How far I need to step back with zoom factor x2?
How will h_2 change?

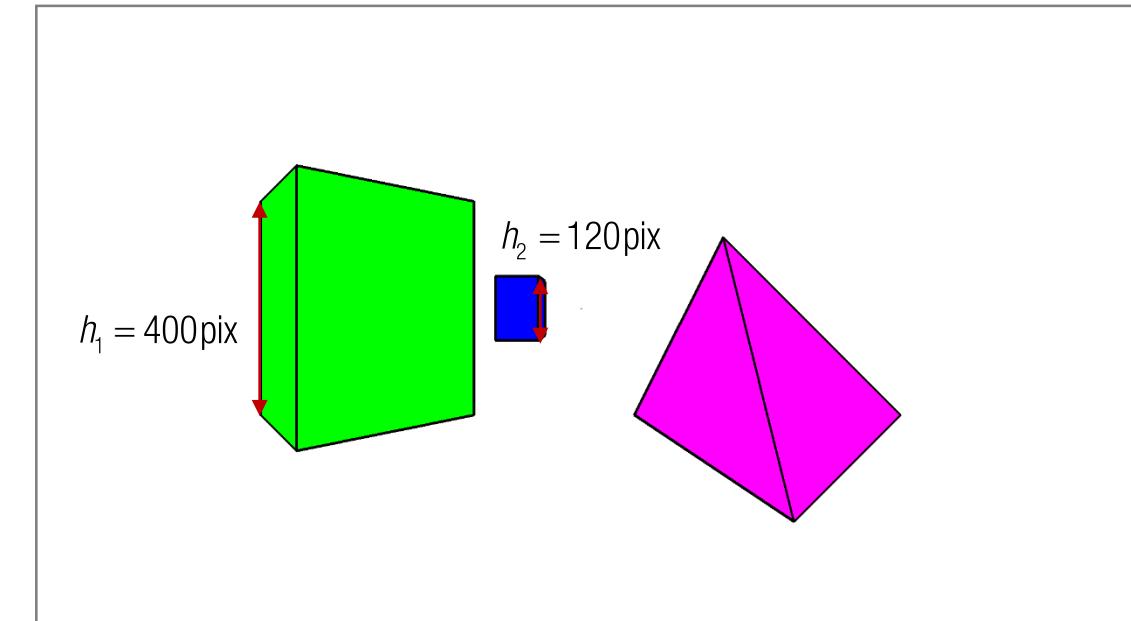
Where am I with Dolly Zoom?

Equations:

$$h_1 = f \frac{H_1}{d_1} \quad h_1 = 2f \frac{H_1}{\Delta d + d_1} \longrightarrow \Delta d = d_1$$



Unknowns: $f, d_1, \Delta d$



How far I need to step back with zoom factor x2?
How will h_2 change?

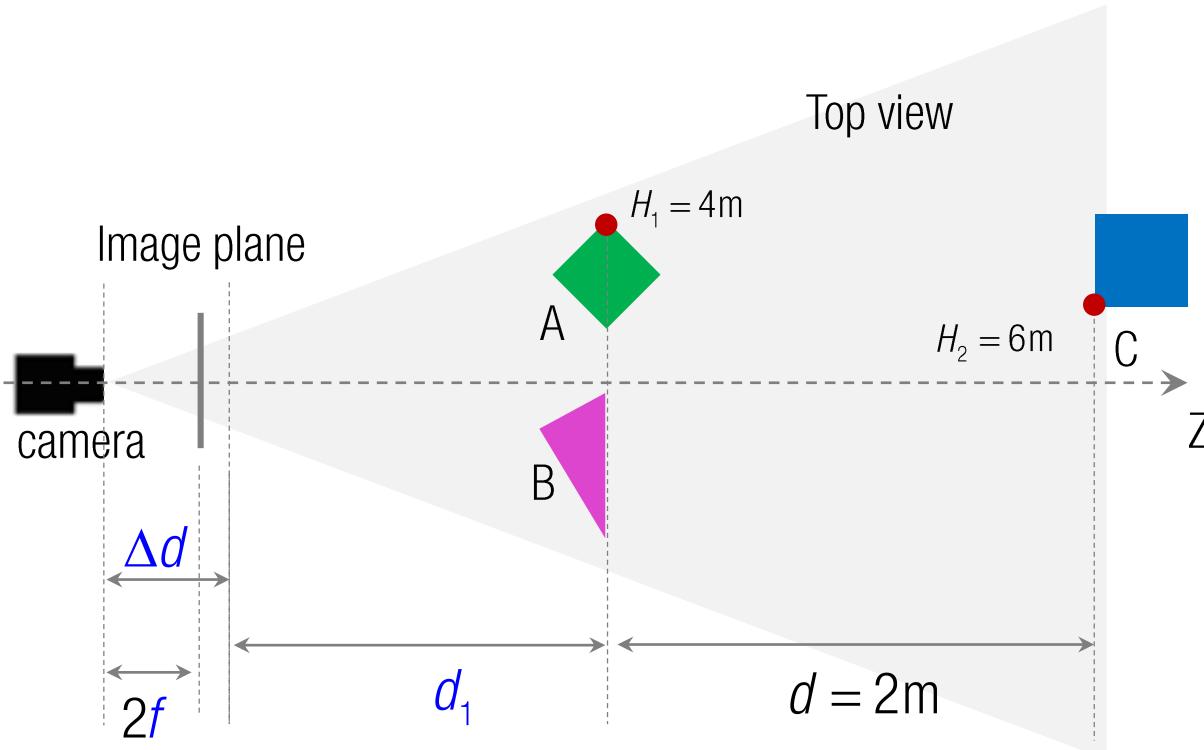
Where am I with Dolly Zoom?

Equations:

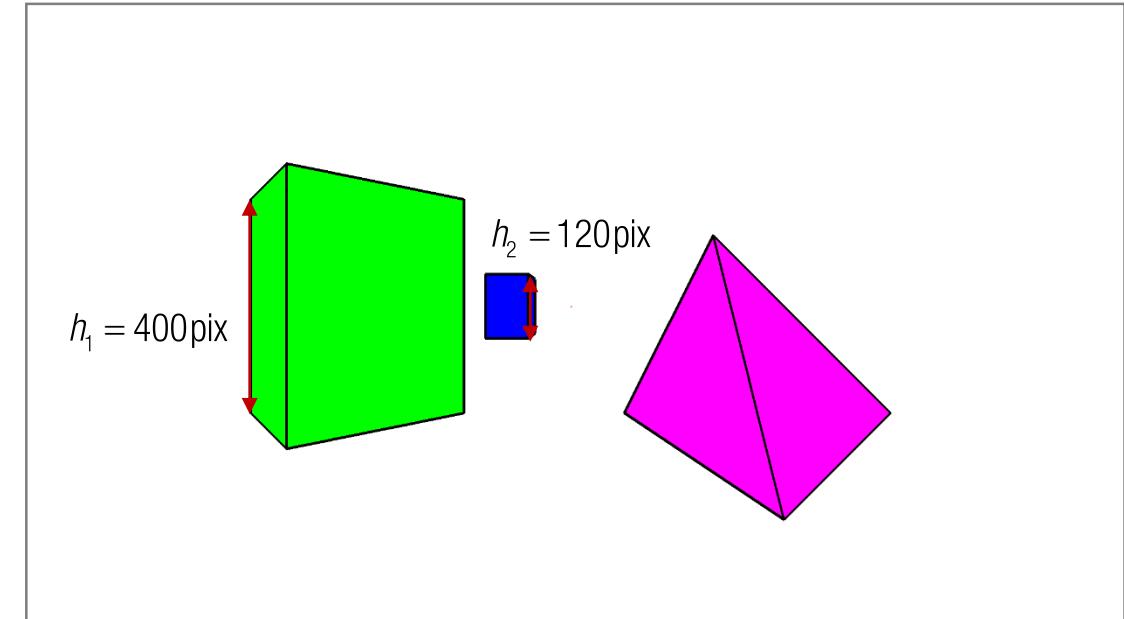
$$h_1 = f \frac{H_1}{d_1}$$

$$h_1 = 2f \frac{H_1}{\Delta d + d_1} \longrightarrow \Delta d = d_1$$

$$h_2 = f \frac{H_2}{d_1 + d}$$

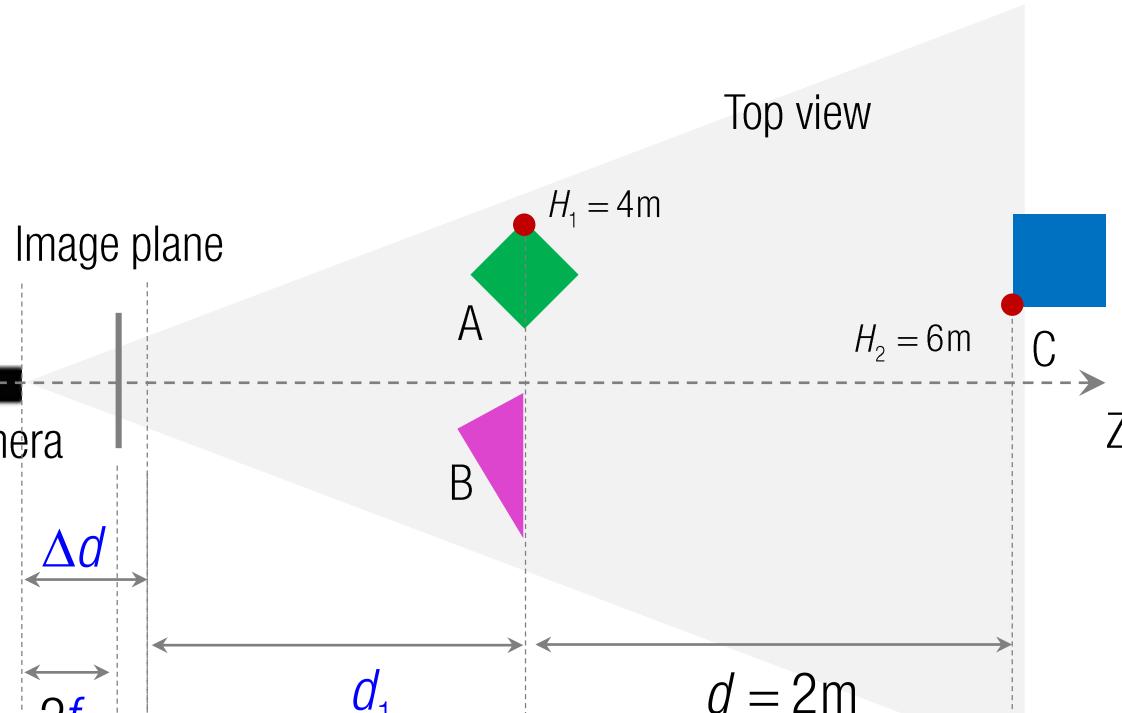


Unknowns: $f, d_1, \Delta d$



How far I need to step back with zoom factor x2?
How will h_2 change?

Where am I with Dolly Zoom?



Unknowns: $f, d_1, \Delta d$

Equations:

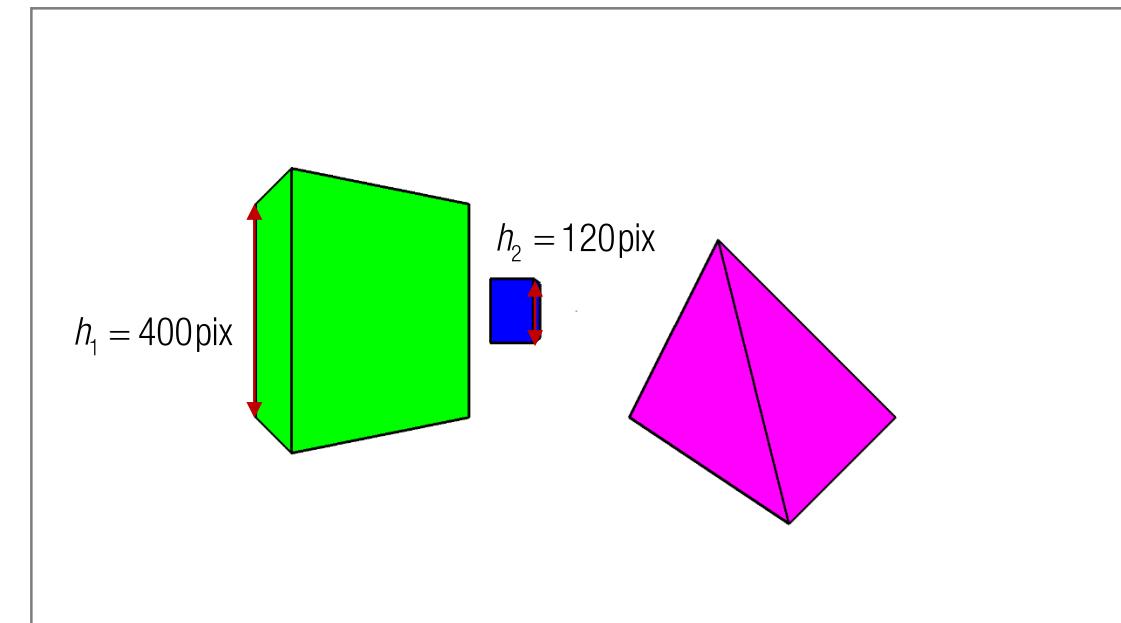
$$h_1 = f \frac{H_1}{d_1}$$

$$h_2 = f \frac{H_2}{d_1 + d}$$

$$d_1 = \frac{1}{1 - \frac{h_2 H_1}{h_1 H_2}} d = 2.5\text{m}$$

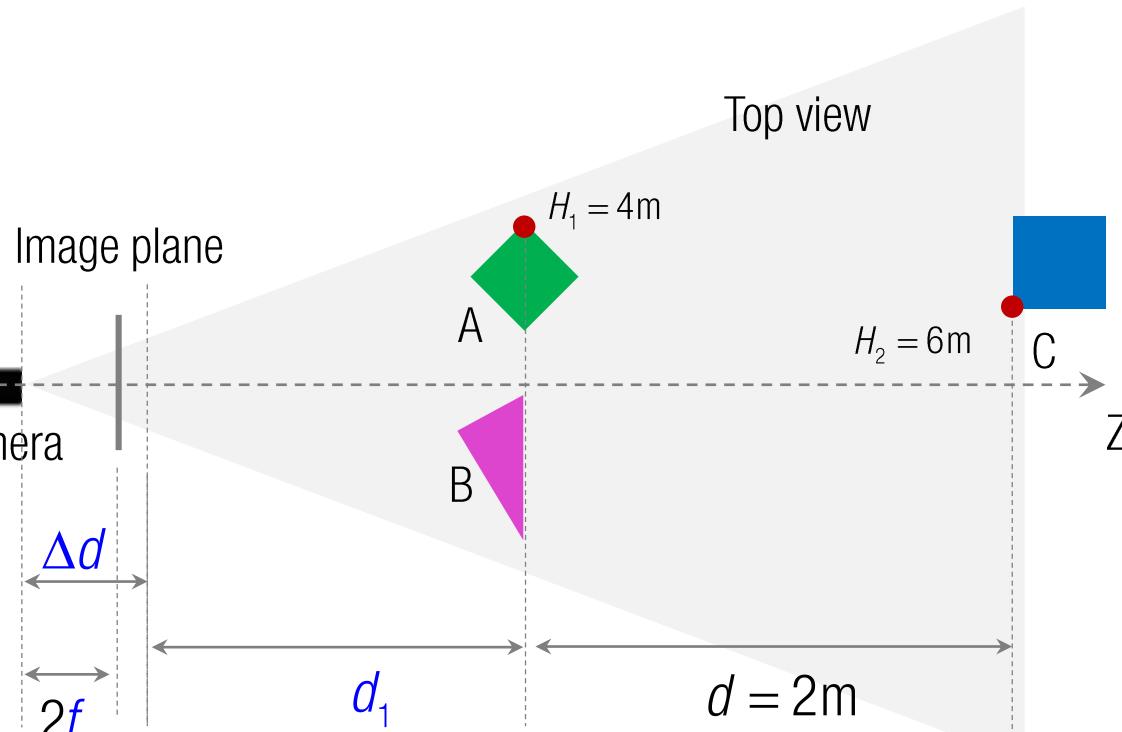
$$h_1 = 2f \frac{H_1}{\Delta d + d_1} \rightarrow \Delta d = d_1$$

$$\Delta d = 2.5\text{m}$$



$$h_2 = 120\text{pix} \rightarrow h'_2 = 200\text{pix}$$

Where am I with Dolly Zoom?



Unknowns: $f, d_1, \Delta d$

Equations:

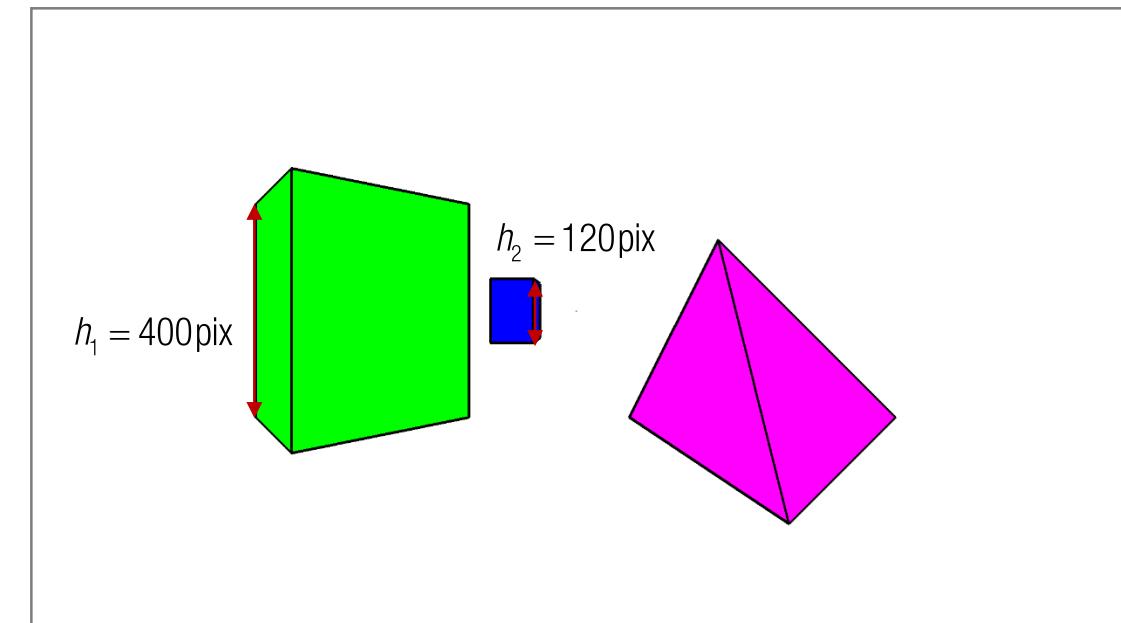
$$h_1 = f \frac{H_1}{d_1}$$

$$h_2 = f \frac{H_2}{d_1 + d}$$

$$d_1 = \frac{1}{1 - \frac{h_2 H_1}{h_1 H_2}} d = 2.5\text{m}$$

$$h_1 = 2f \frac{H_1}{\Delta d + d_1} \rightarrow \Delta d = d_1$$

$$\Delta d = 2.5\text{m} \quad f = 250\text{pix}$$



$$h_2 = 120\text{pix} \rightarrow h_2' = 200\text{pix}$$

Where am I with Dolly Zoom?

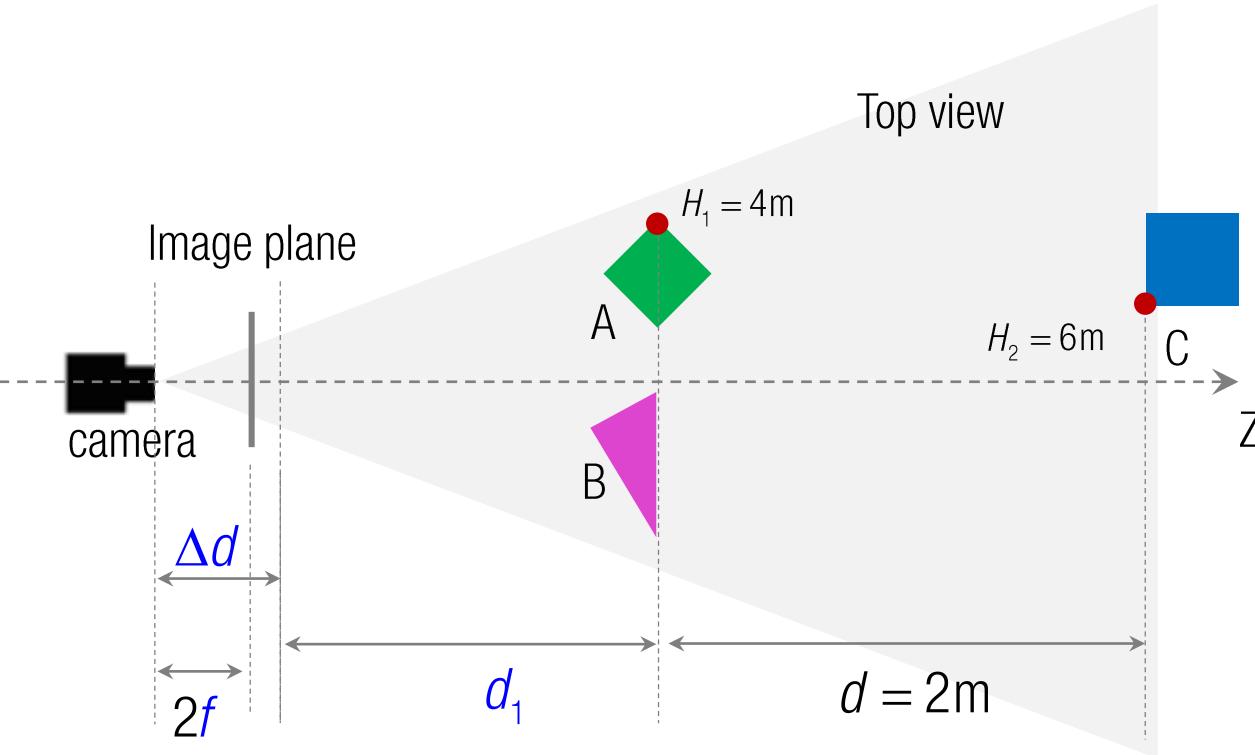
Equations:

$$h_1 = f \frac{H_1}{d_1}$$

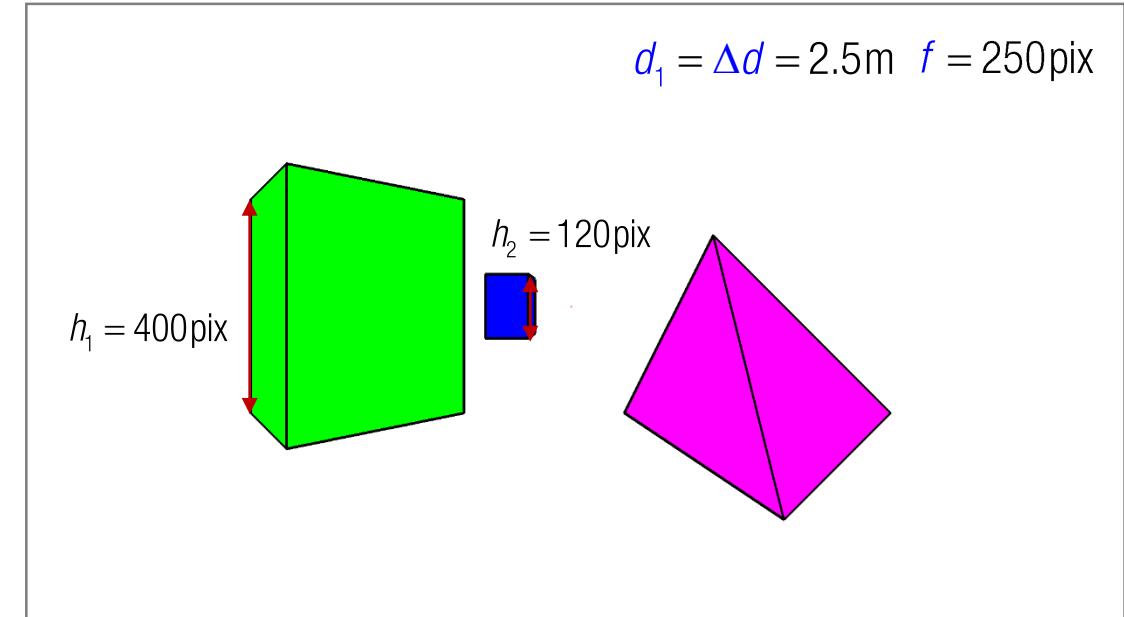
$$h_1 = 2f \frac{H_1}{\Delta d + d_1}$$

$$h_2 = f \frac{H_2}{d_1 + d}$$

$$h'_2 = 2f \frac{H_2}{\Delta d + d_1 + d}$$



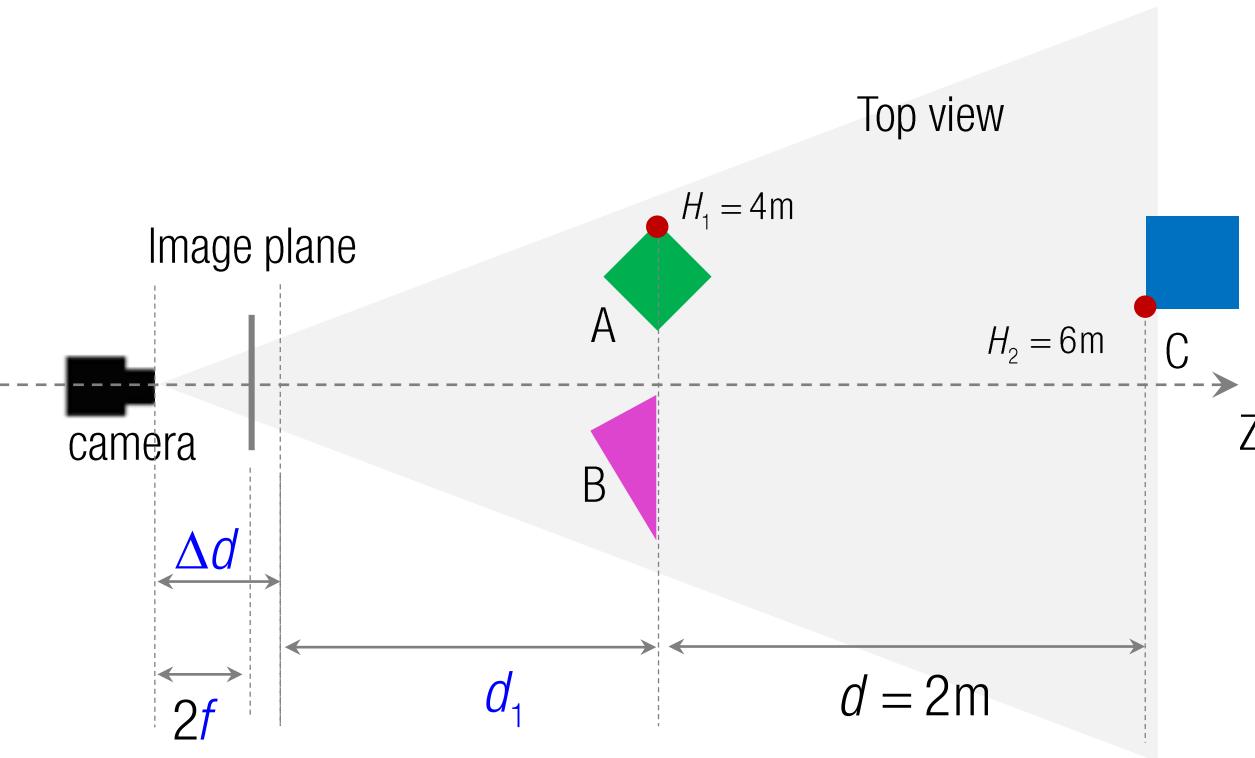
Unknowns: $f, d_1, \Delta d$



How far I need to step back with zoom factor x2?

How will h_2 change?

Where am I with Dolly Zoom?



Unknowns: $f, d_1, \Delta d$

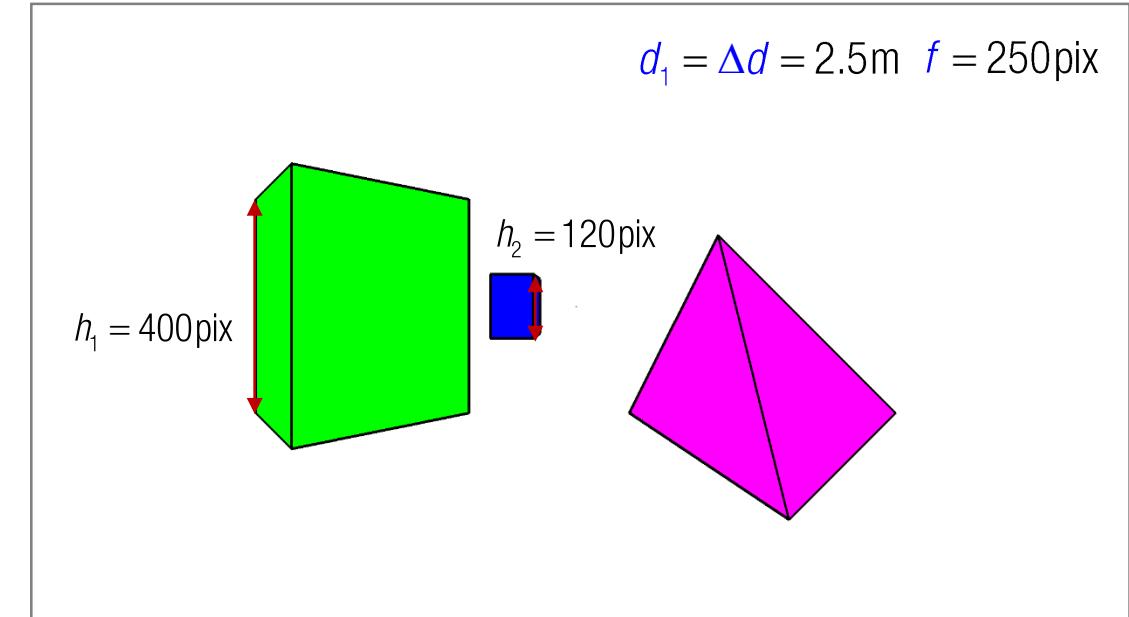
Equations:

$$h_1 = f \frac{H_1}{d_1}$$

$$h_1 = 2f \frac{H_1}{\Delta d + d_1}$$

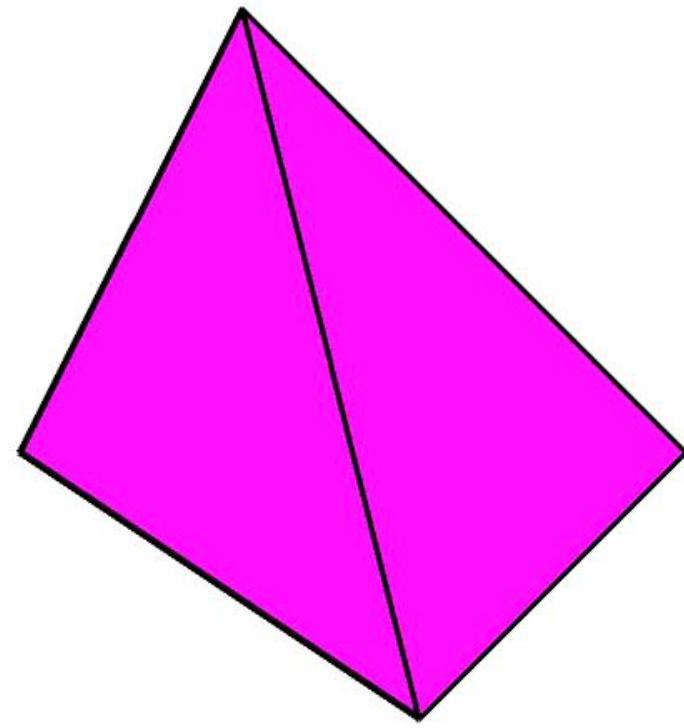
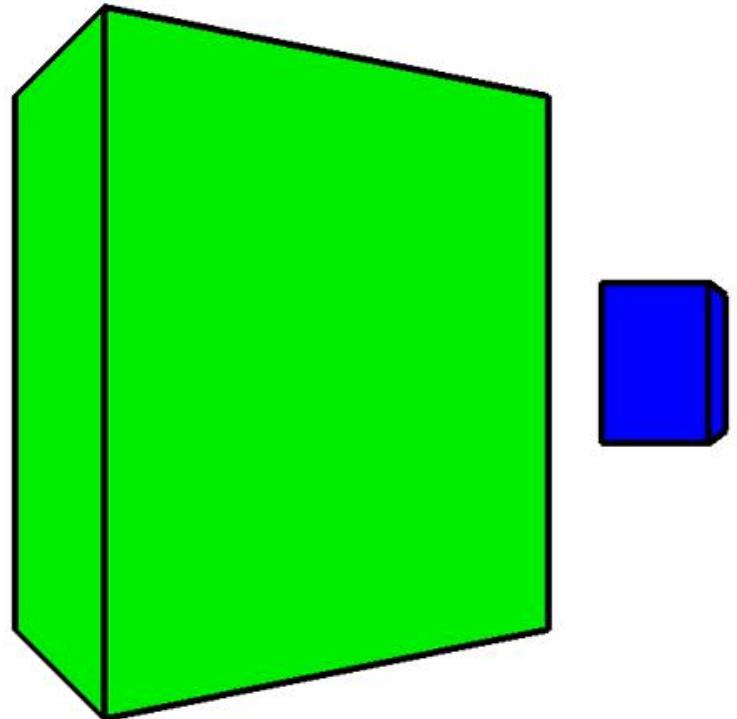
$$h_2 = f \frac{H_2}{d_1 + d}$$

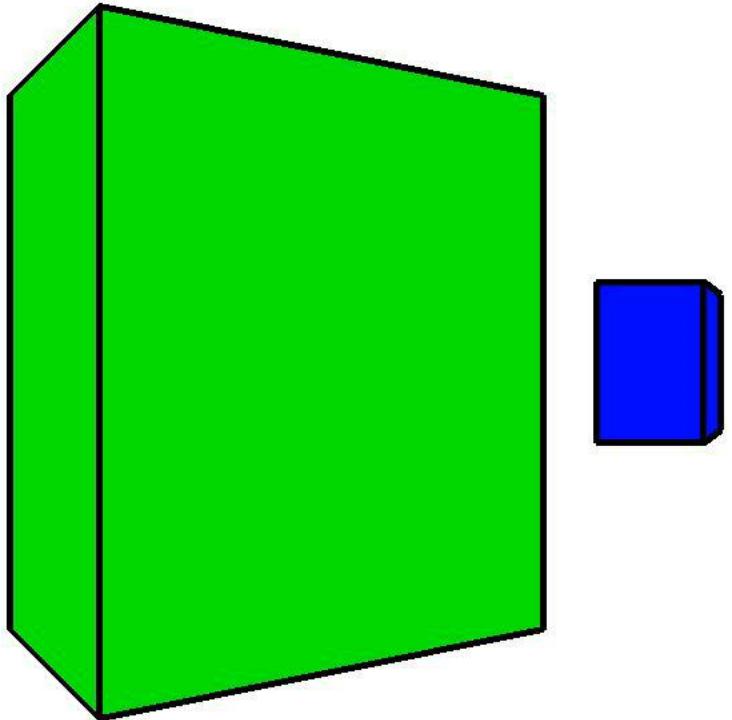
$$h'_2 = 2f \frac{H_2}{\Delta d + d_1 + d} = 429\text{pix}$$



How far I need to step back with zoom factor x2?

How will h_2 change?





```
%% load data and set parameters
points = load('points.mat');

d_ref = 4;
f_ref = 400;
pos = 0 : -0.1 : -9.9;

H1 = points.points_A(1,2) - points.points_A(2,2);
H2 = points.points_C(1,2) - points.points_C(2,2);

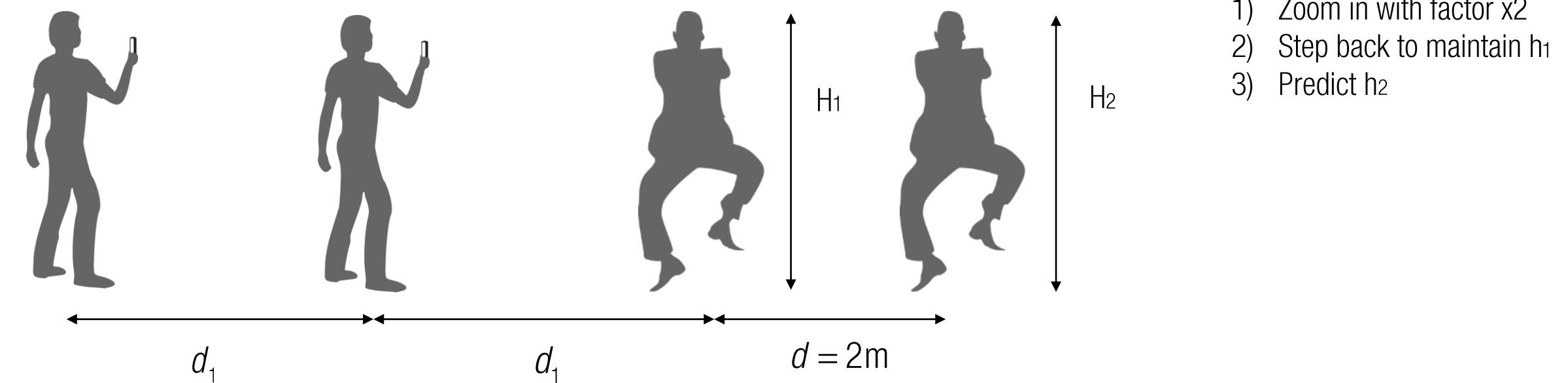
%% Dolly Zoom: keep one object's height constant

f = compute_focal_length(d_ref, f_ref, pos);

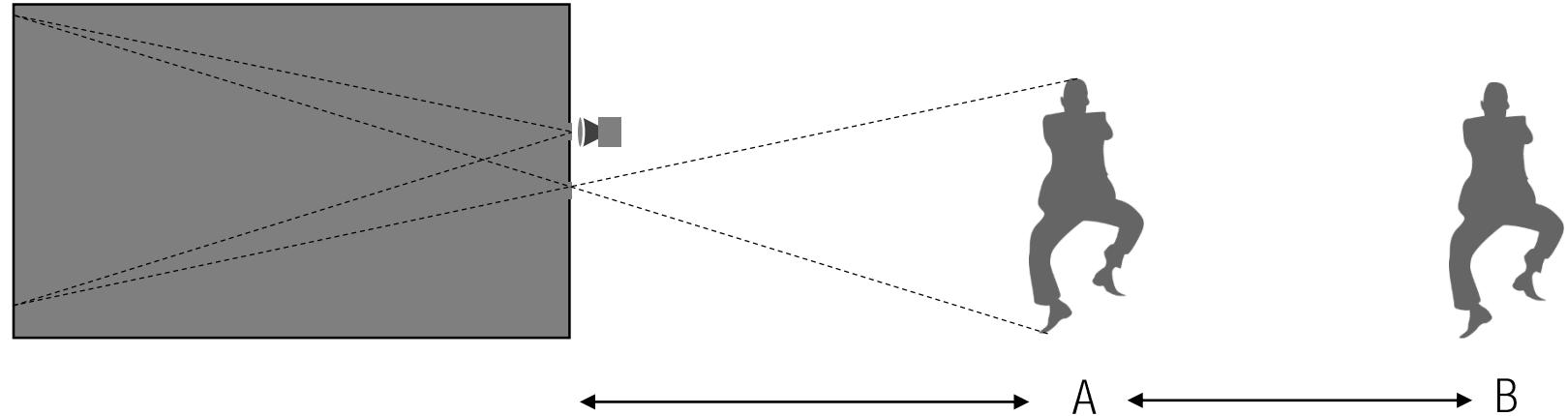
for i = 1 : length(f)
    figure(1), hold on, axis equal;
    xlim([0,1920]), ylim([0,1080]);
    project_objects(f(i), pos(i), points, 1);
    pause(0.1);
end;

function [ f ] = compute_focal_length( d_ref, f_ref, pos )
    f = (d_ref-pos) / d_ref * f_ref;
```

Where am I with Dolly Zoom?



Camera Obscura HW #1



- 1) Where was I with respect to A?
- 2) How far B from A?