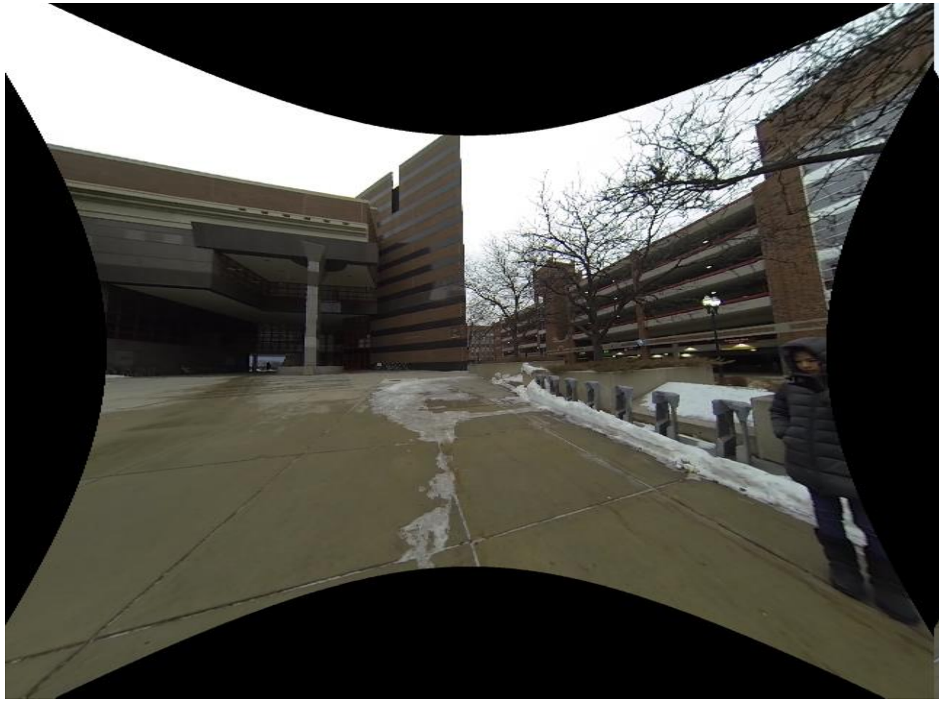
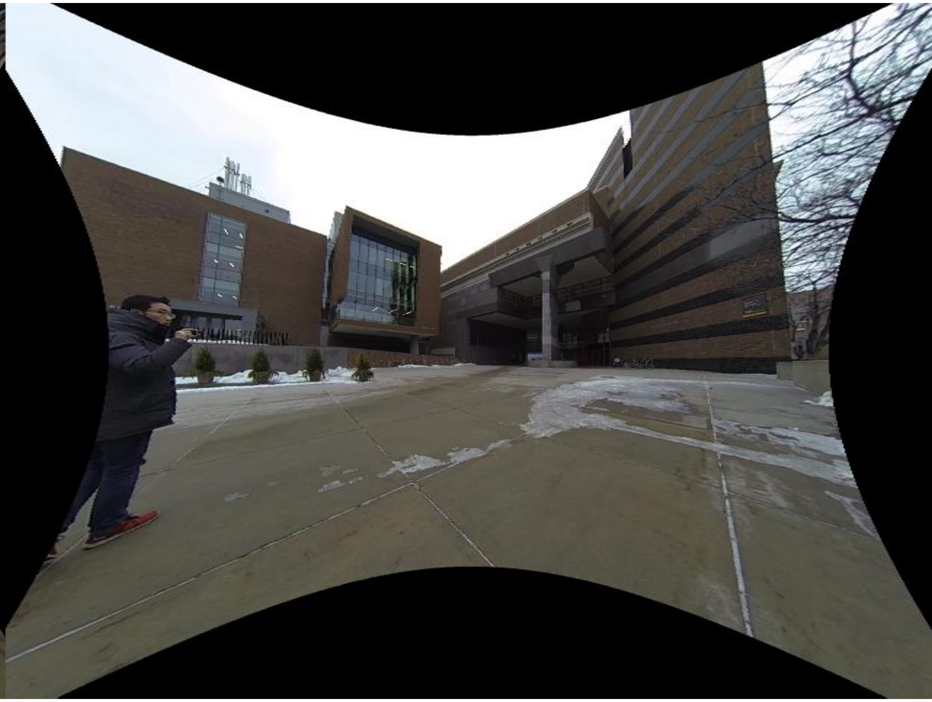
Two View (Epipolar) Geometry

2D Correspondences

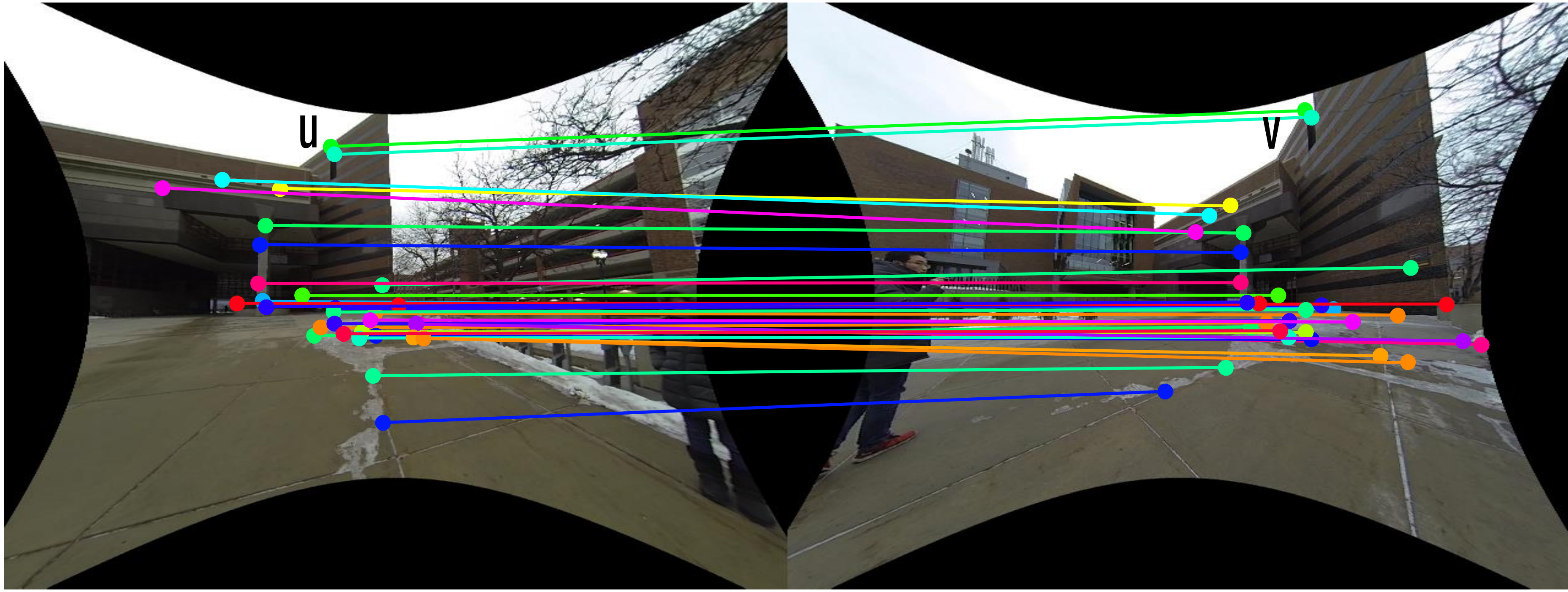


Bob's image



Alice's image

2D Correspondences

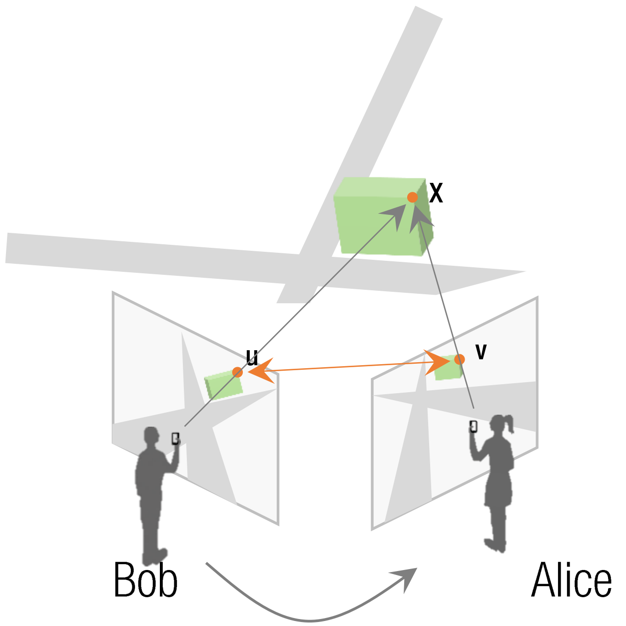


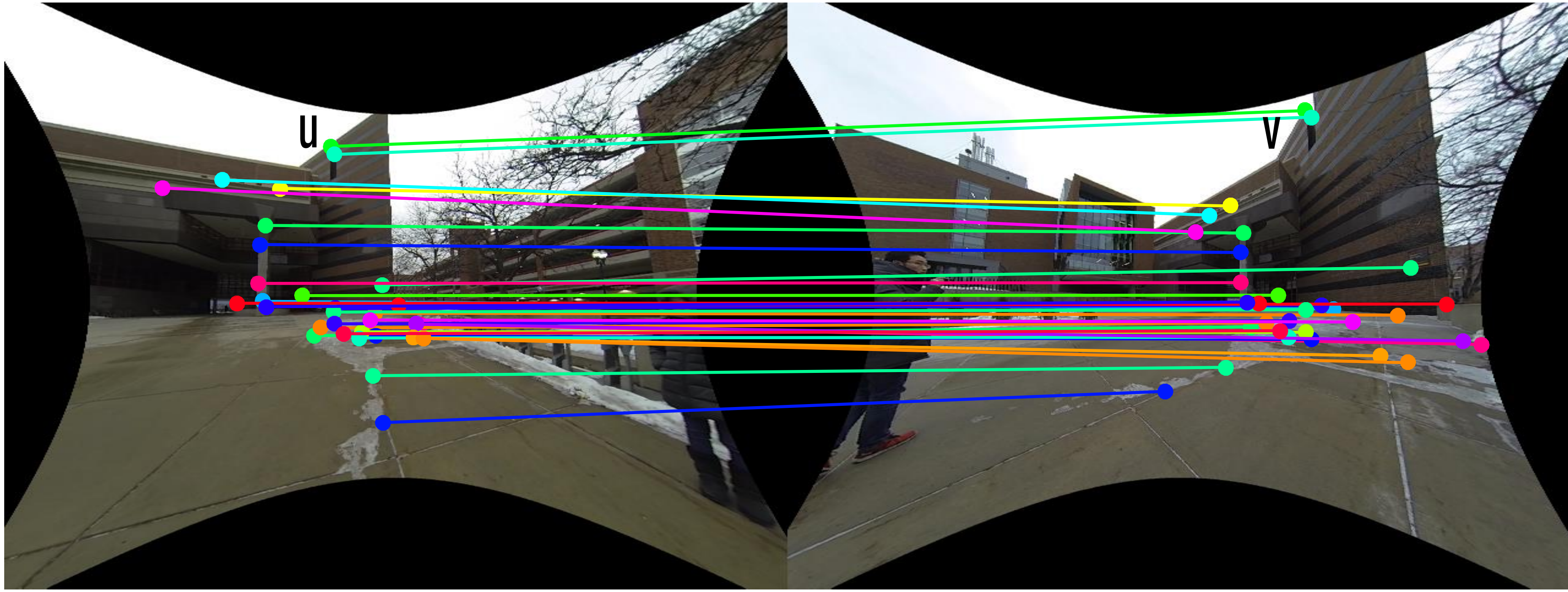
Bob's image

Alice's image

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

2D Correspondences





Bob's image

Alice's image

$$\mathbf{F} = \mathbf{F}(\mathbf{R}, \mathbf{t})$$

$$= \mathbf{K}^{-T}[\mathbf{t}]_{\times}\mathbf{R}\mathbf{K}^{-1}$$

$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

How to compute fundamental matrix?

8 Point Algorithm (Longuet-Higgins, Nature 1981)

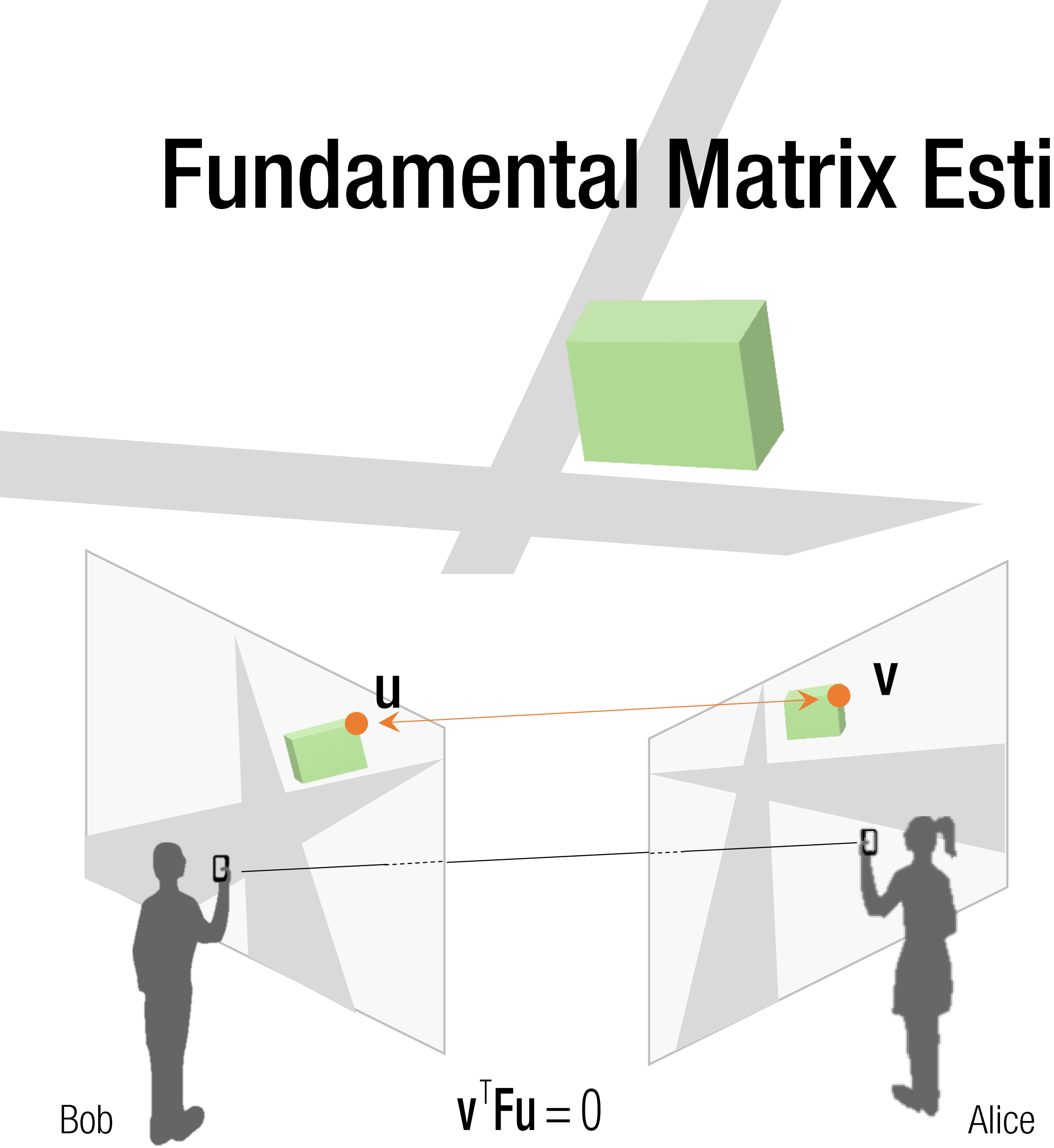
A computer algorithm for reconstructing a scene from two projections

H. C. Longuet-Higgins

Laboratory of Experimental Psychology, University of Sussex,
Brighton BN1 9QG, UK

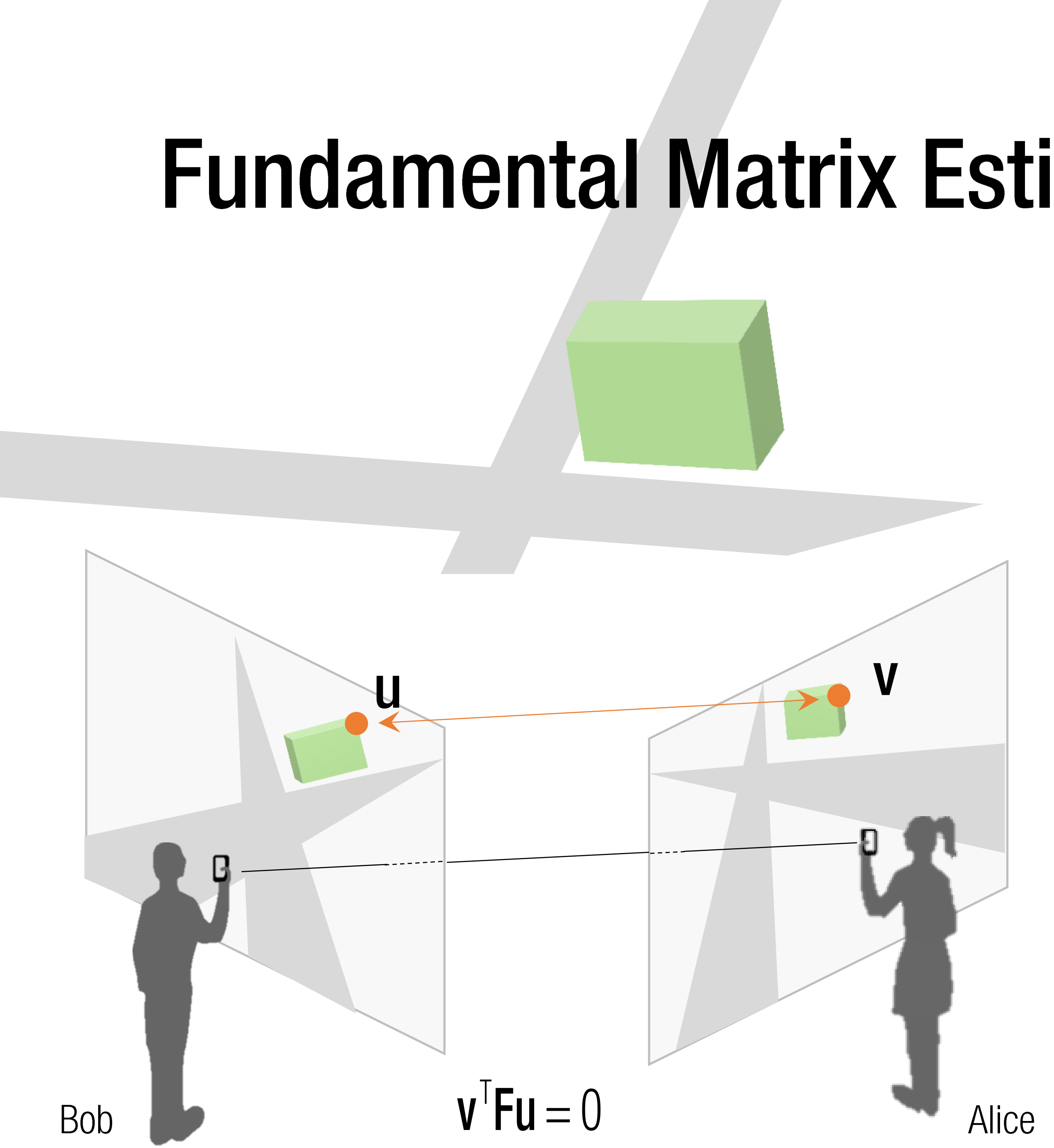
A simple algorithm for computing the three-dimensional ture of a scene from a correlated pair of perspective proj is described here, when the spatial relationship between t projections is unknown. This problem is relevant not o photographic surveying[1] but also to binocular vision[2], wh non-visual information available to the observer abo

Fundamental Matrix Estimation



$v^T F u = 0$

Fundamental Matrix Estimation

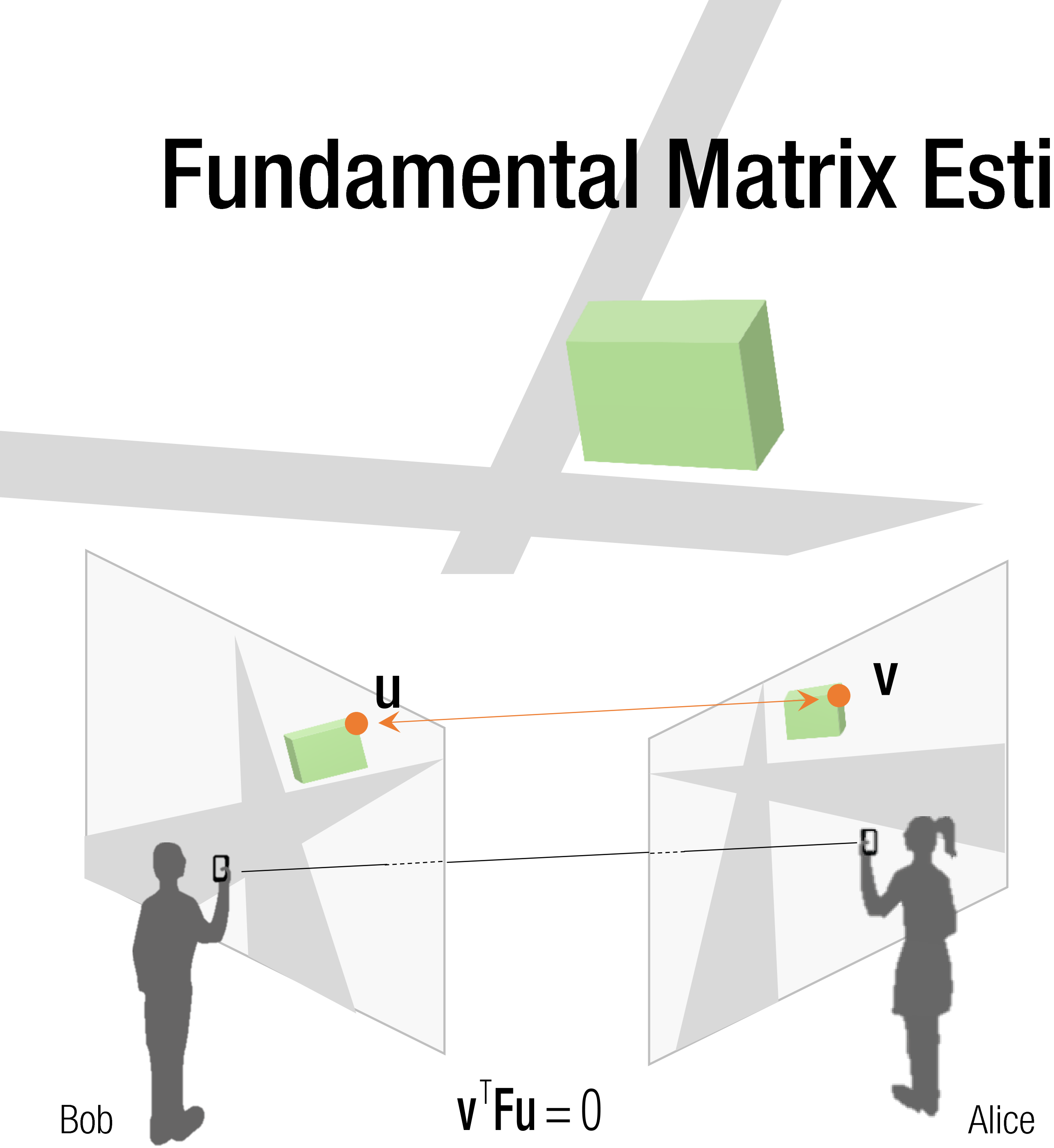


$$\mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:

Fundamental Matrix Estimation



$$\mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

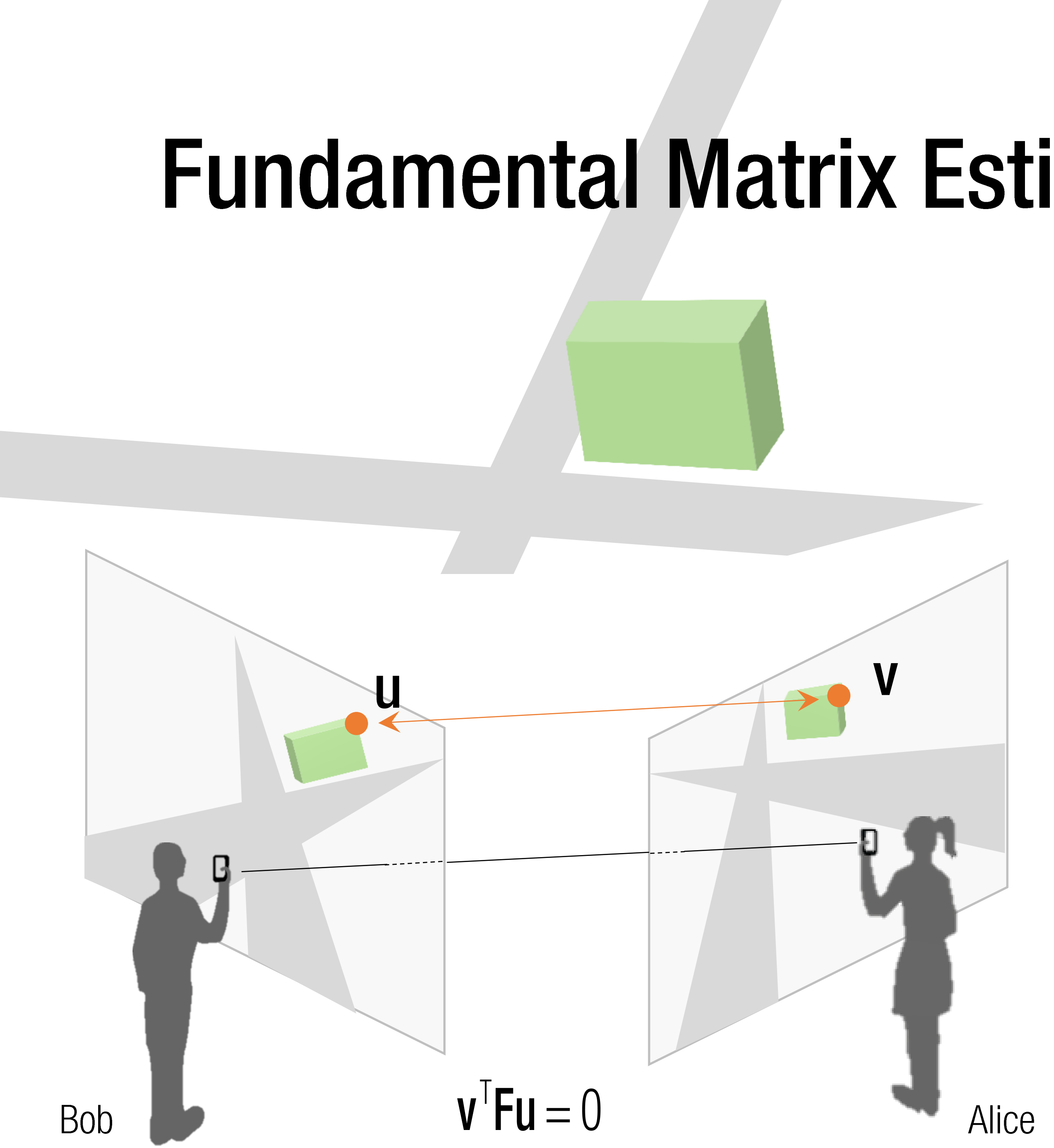
$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:
7 = 9 (3x3 matrix) – 1(scale) – 1 (rank 2)

We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:

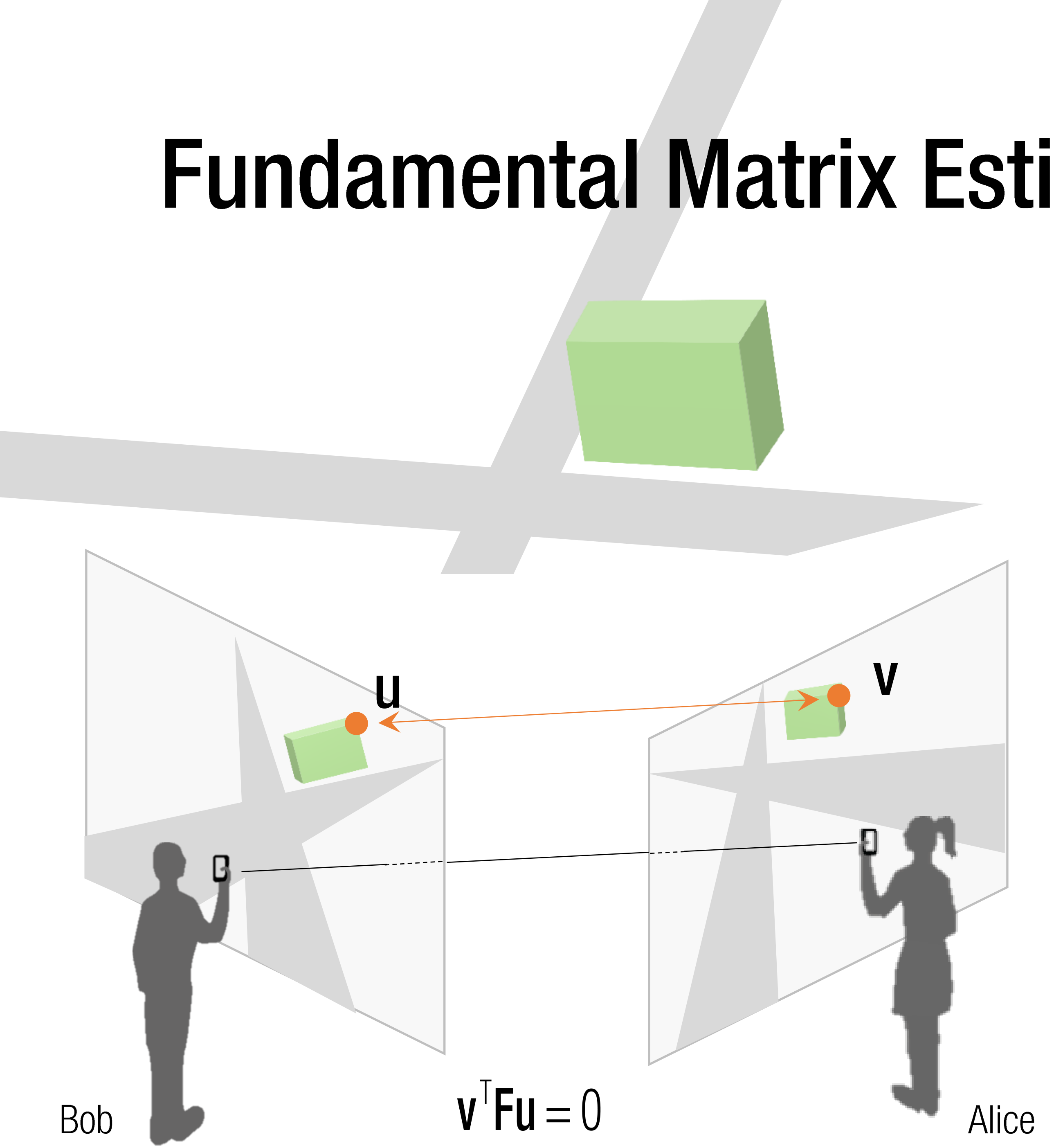
Fundamental Matrix Estimation

$$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$



$$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = 0$$

Fundamental Matrix Estimation

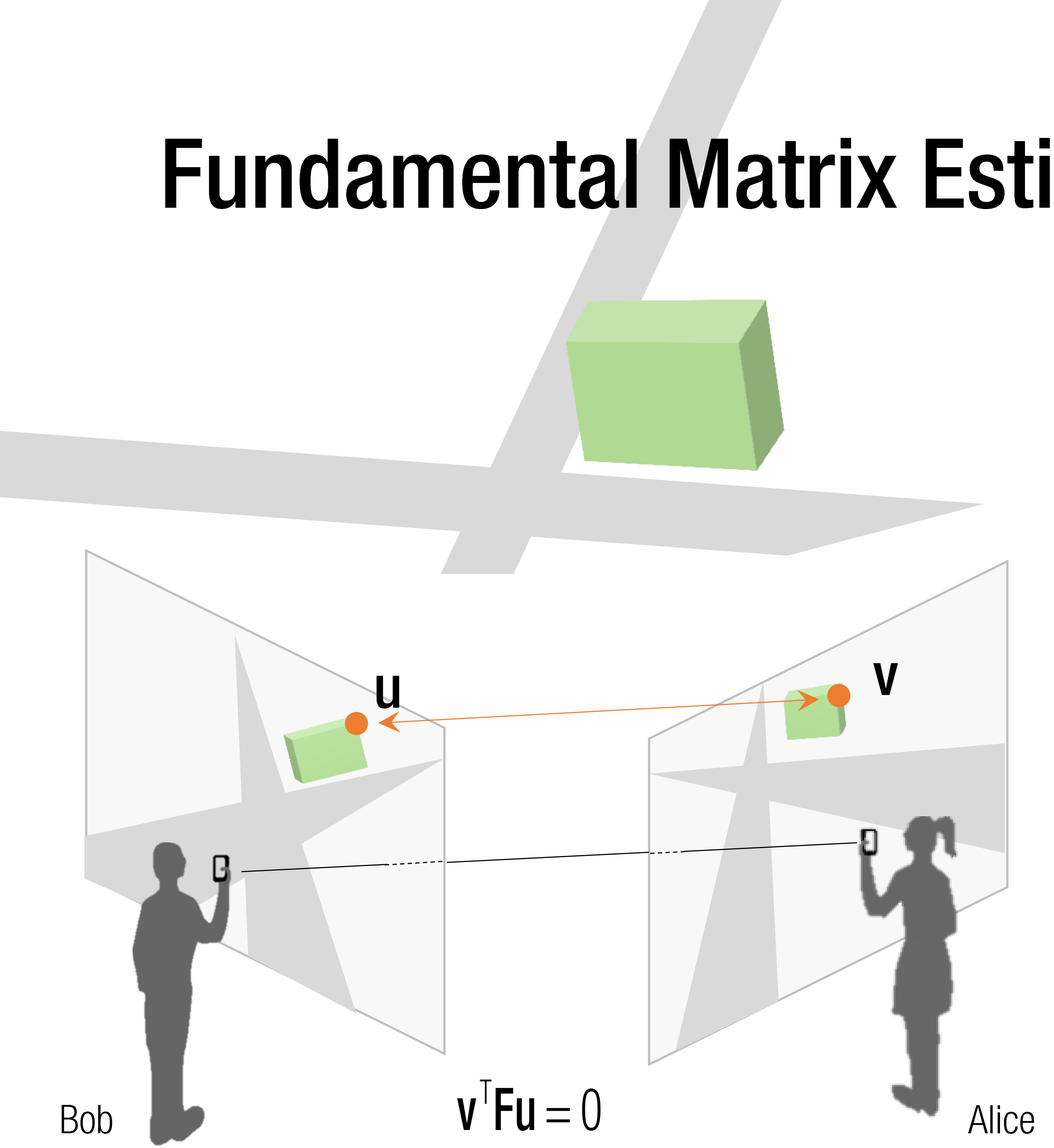


$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

Fundamental Matrix Estimation



$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

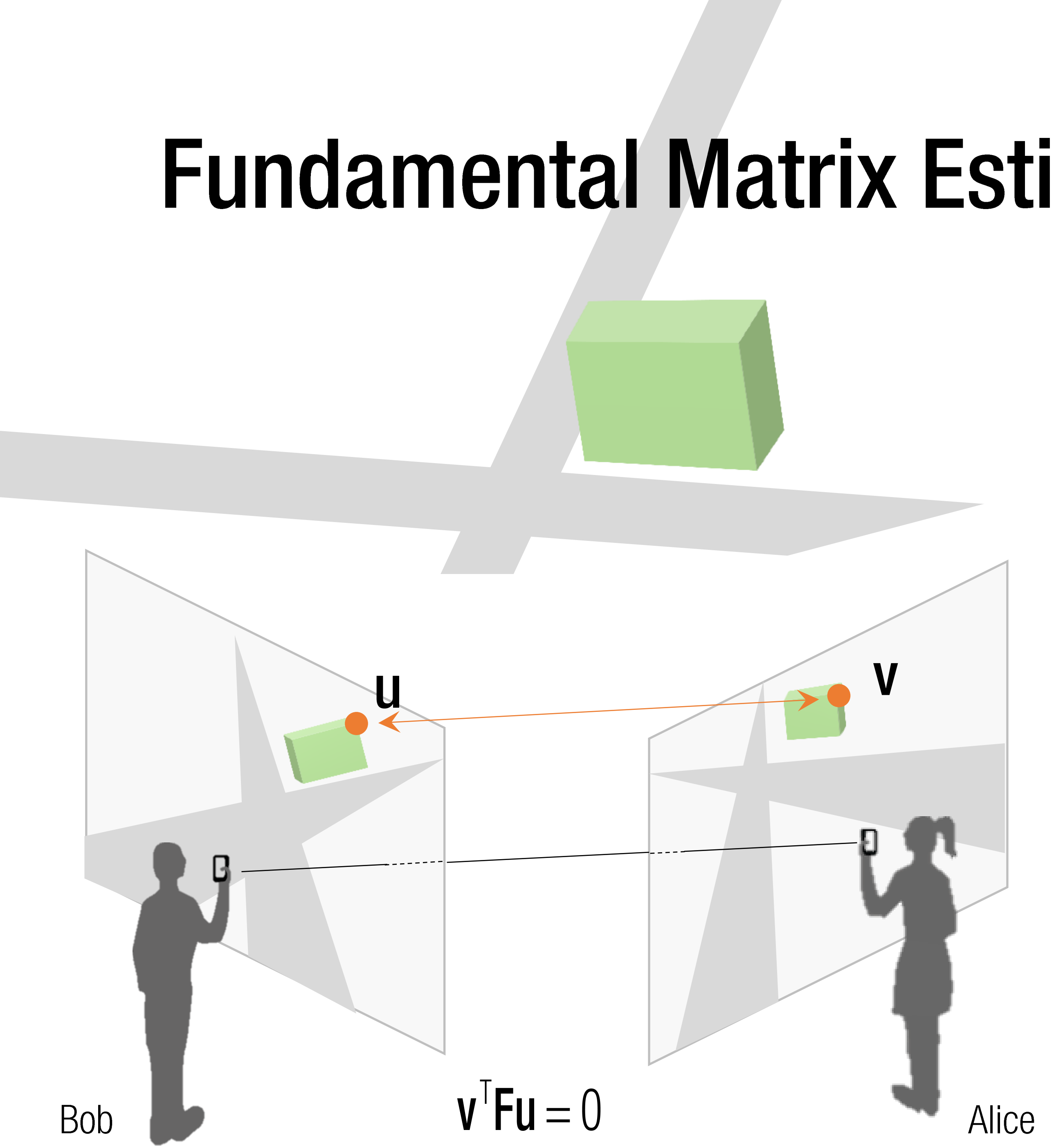
$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

Linear in **F**.

Fundamental Matrix Estimation



$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

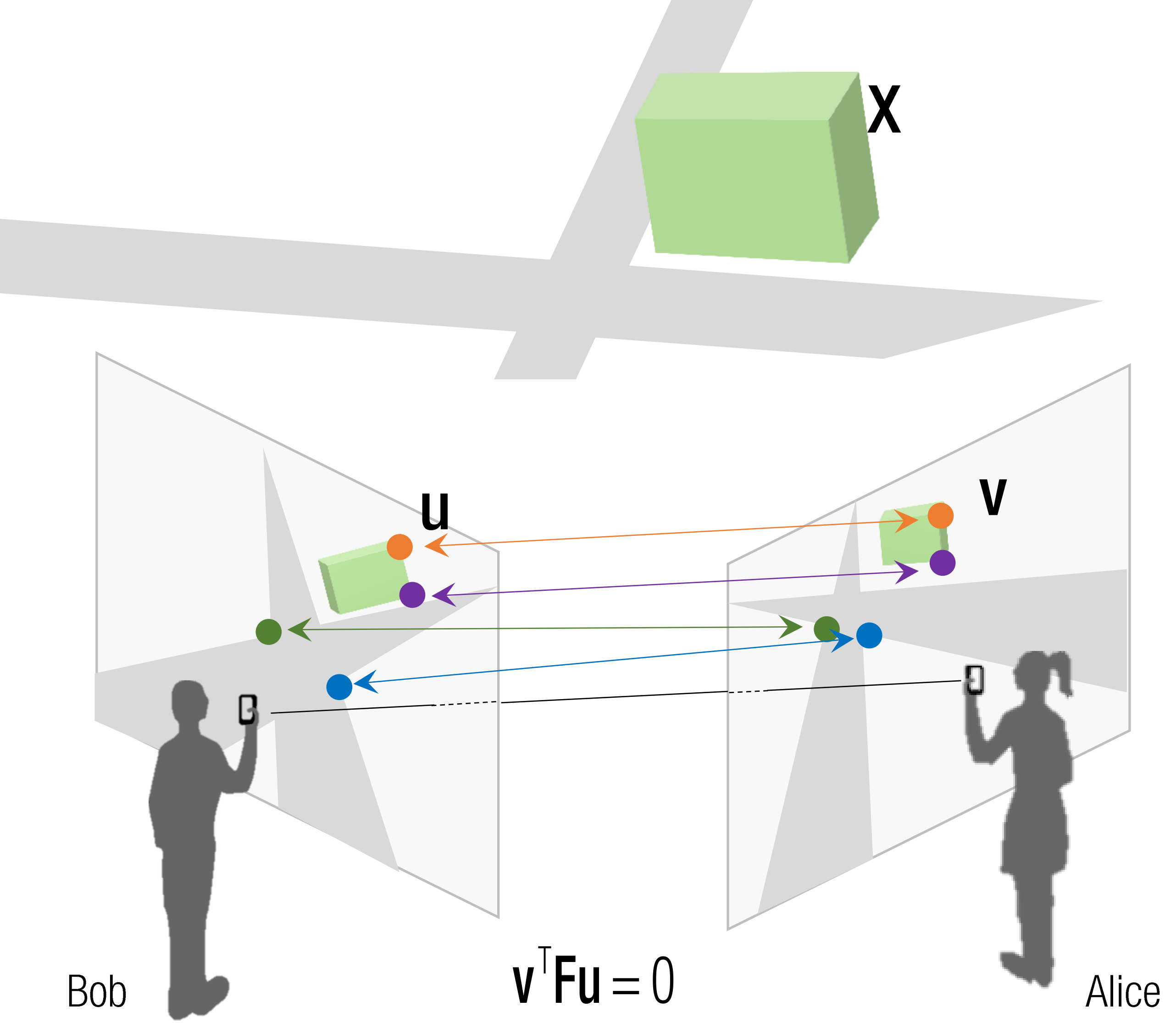
Linear in **F**.

$$\longrightarrow \begin{bmatrix} u^x v^x & u^y v^x & v^x & u^x v^y & u^y v^y & v^y & u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

\# of unknowns: 9

\# of equations per correspondence: 1

Fundamental Matrix Estimation



$$\mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

$$\mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

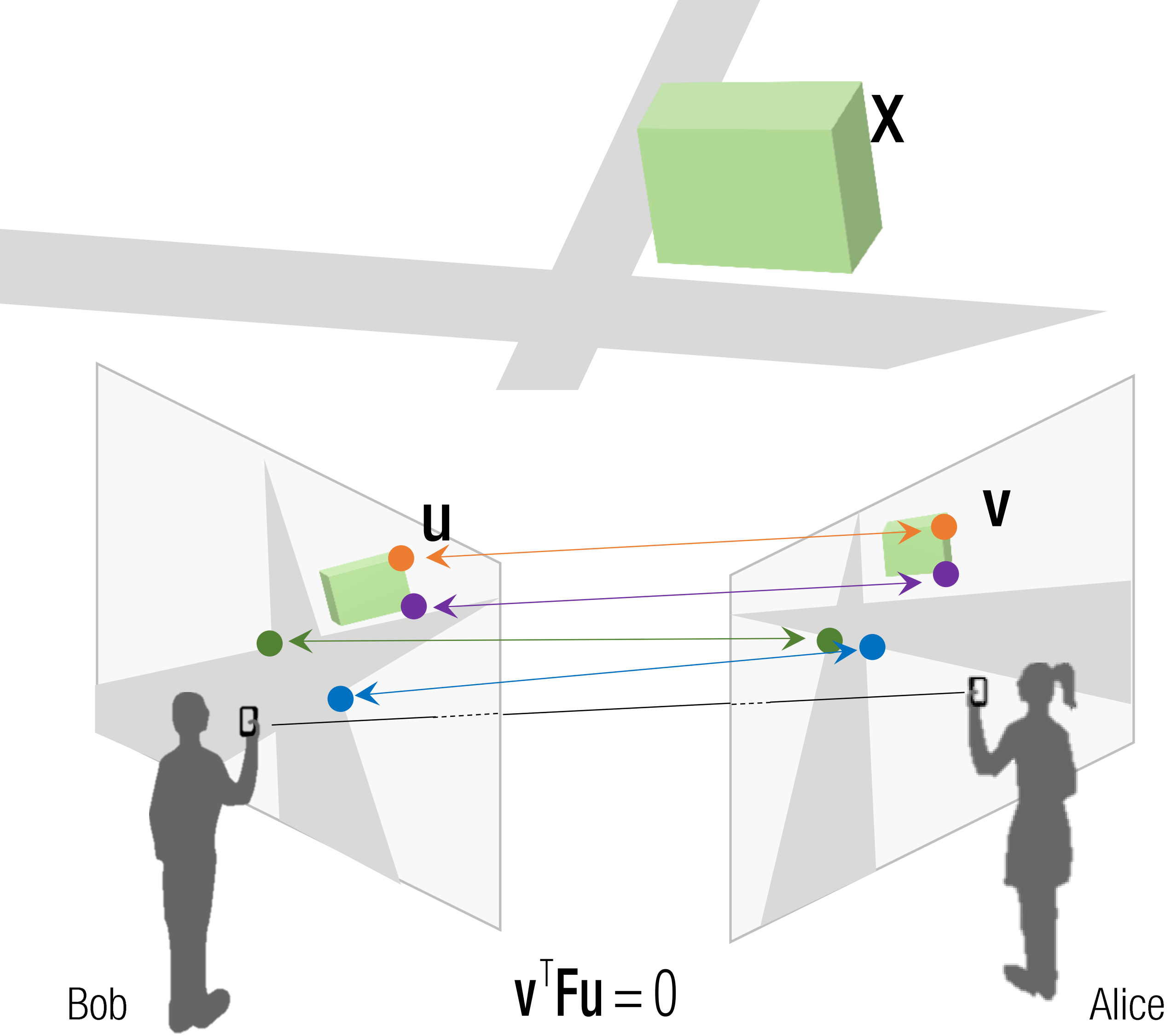
$$= 0$$

Linear in **F**.

$$\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}_{m\times 1}$$

What is minimum m?

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

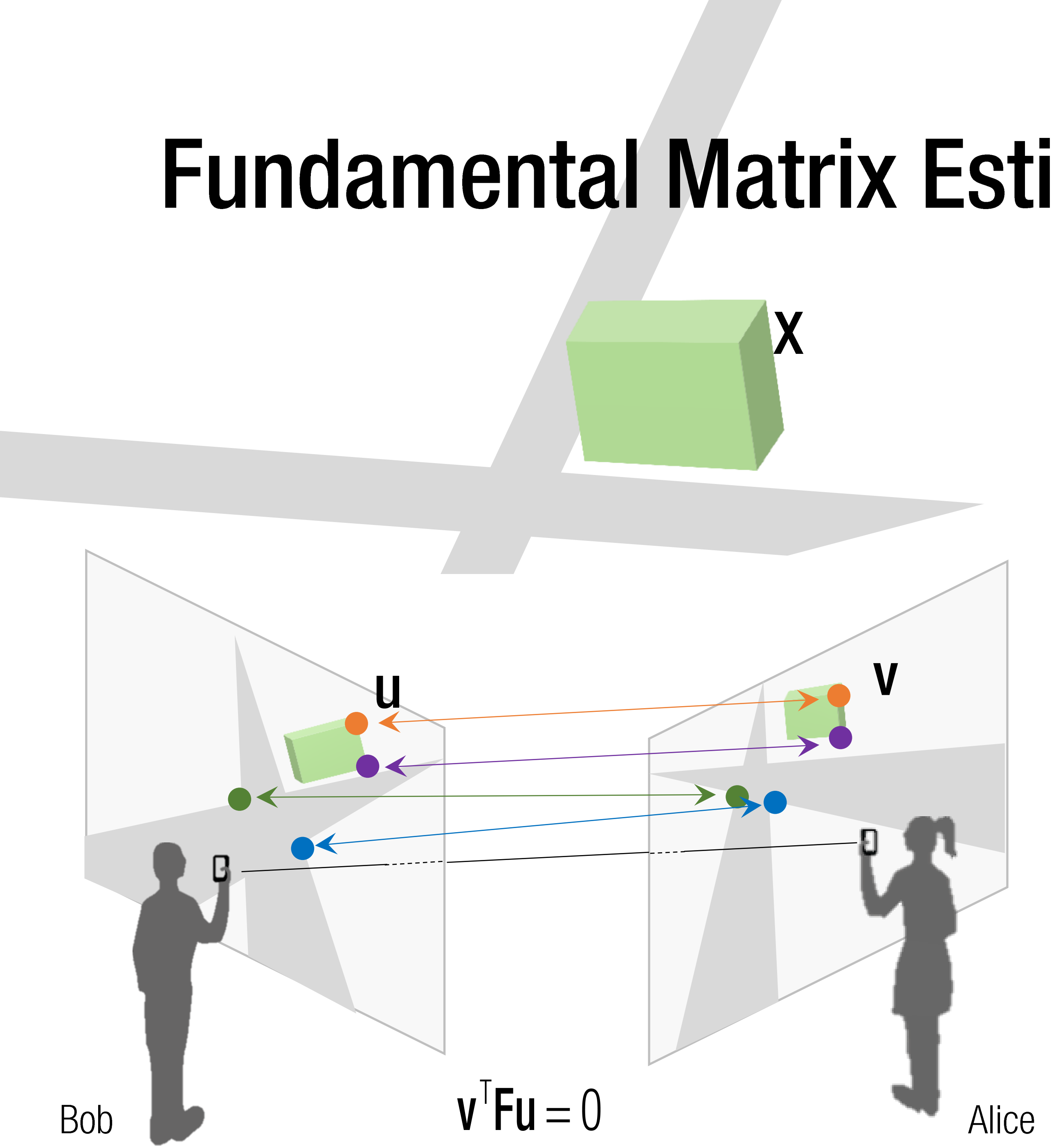
$$= 0$$

Linear in **F**.

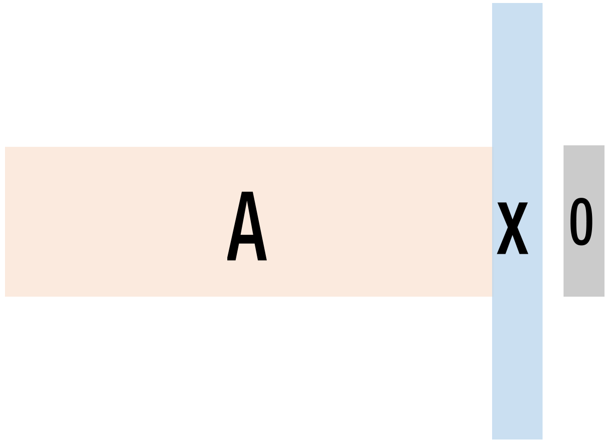
$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{X}} = \mathbf{0}_{m \times 1}$$

What is minimum m?

Fundamental Matrix Estimation

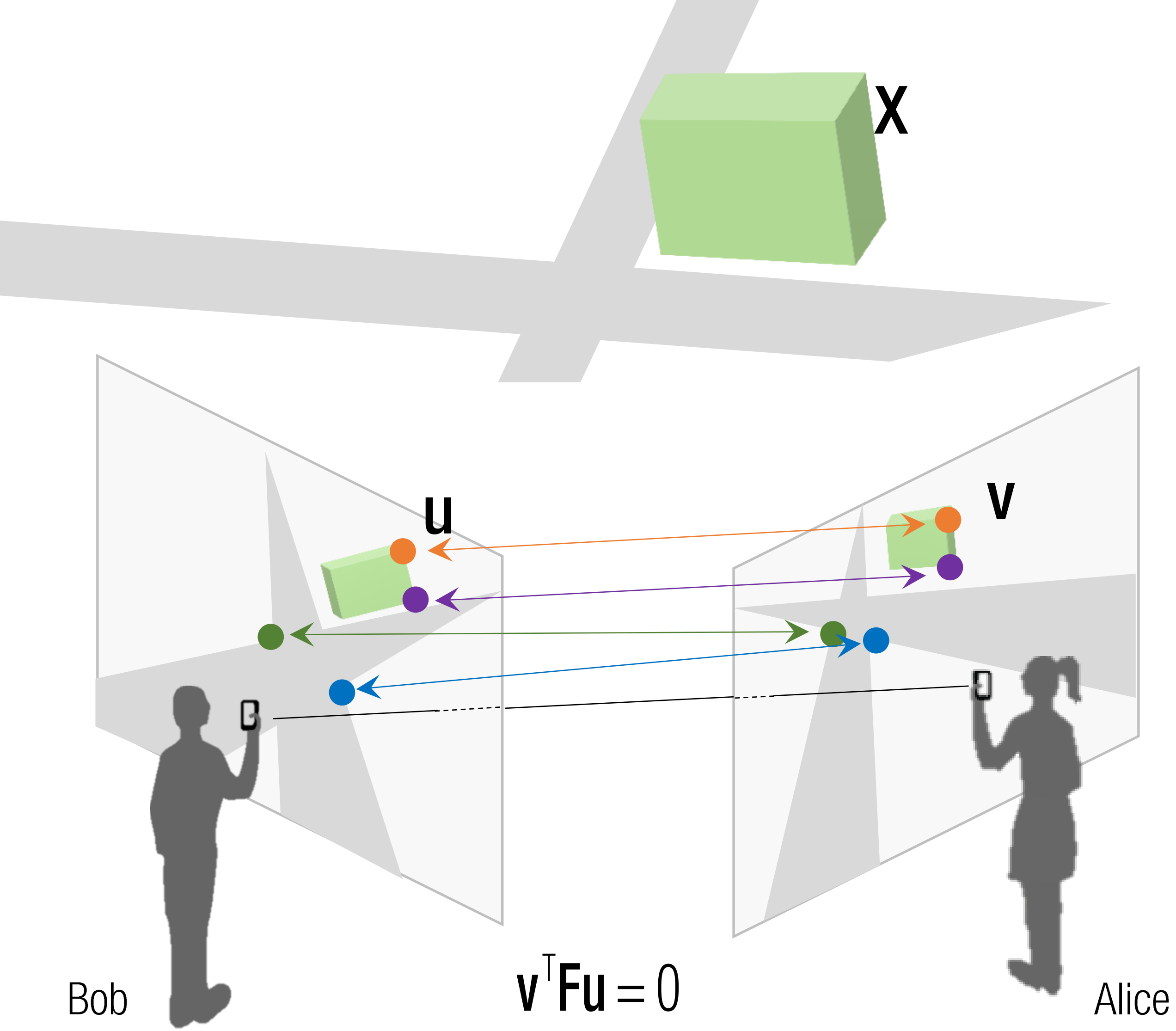


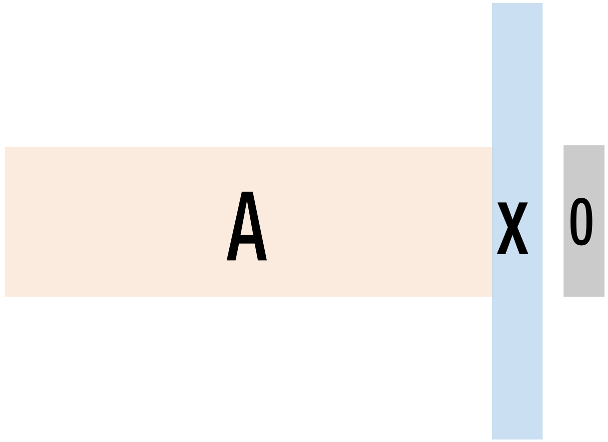
$$v^T F u = 0$$



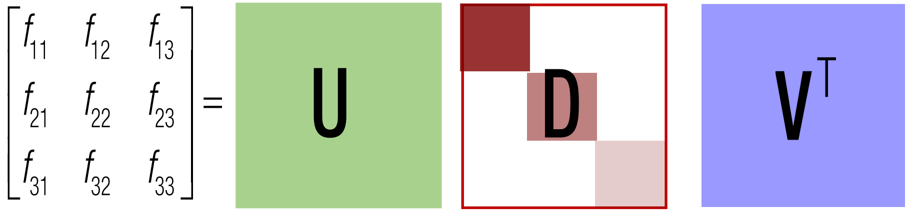
The solution is not necessarily satisfy rank 2 constraint.

Fundamental Matrix Estimation

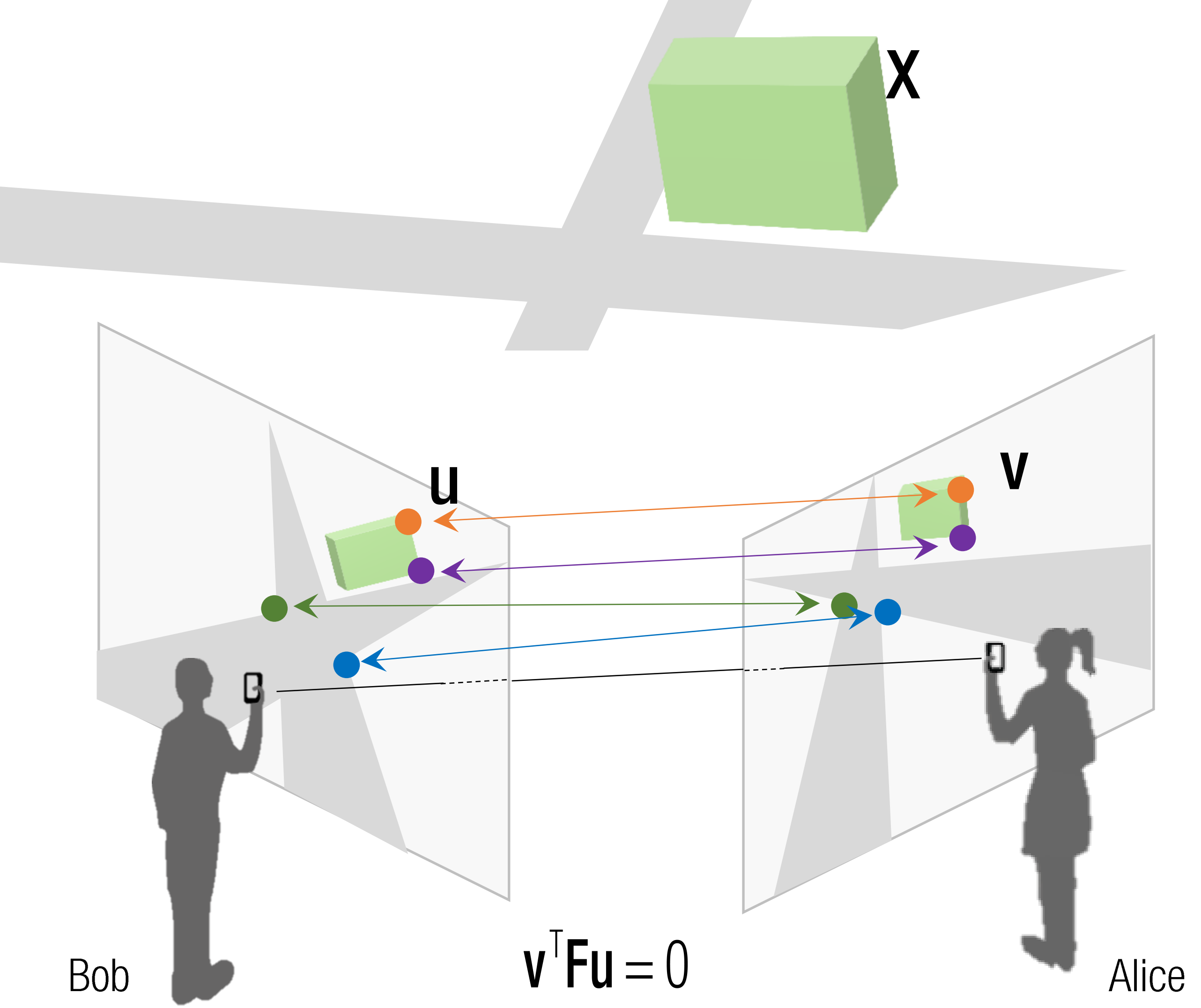


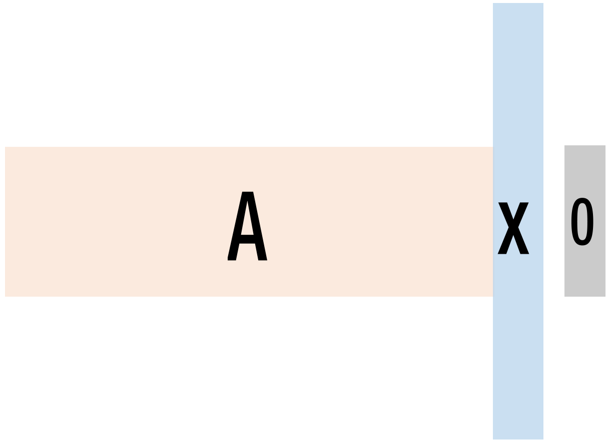


The solution is not necessarily satisfy rank 2 constraint.

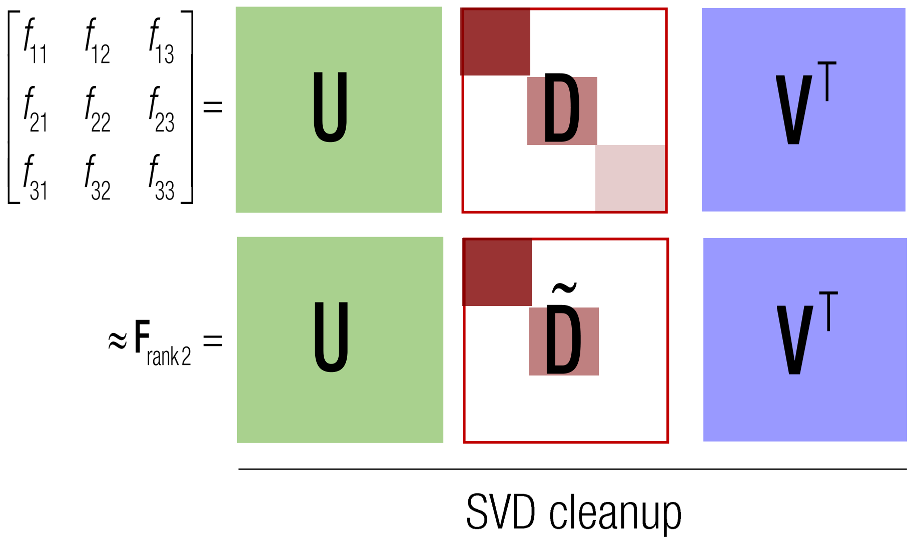


Fundamental Matrix Estimation

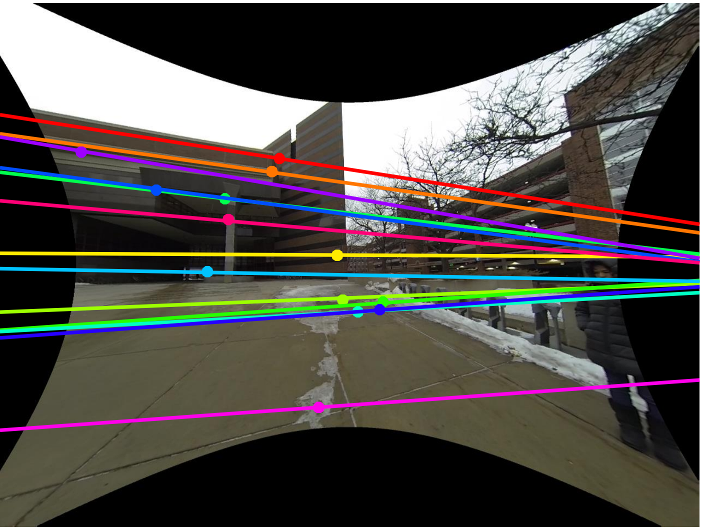


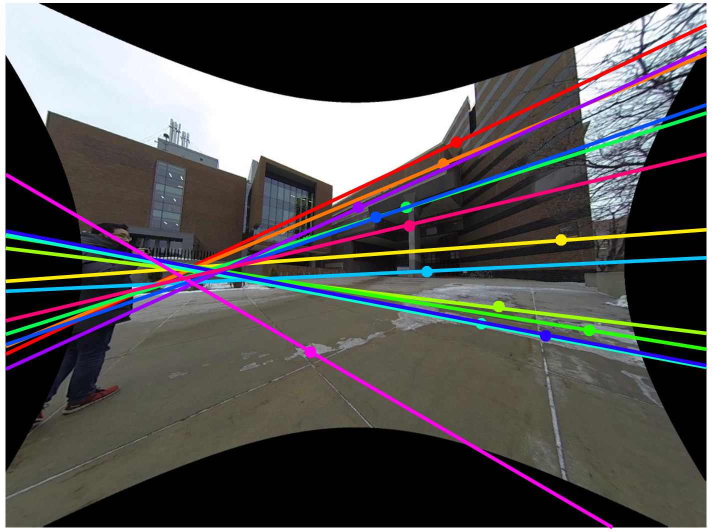


The solution is not necessarily satisfy rank 2 constraint.



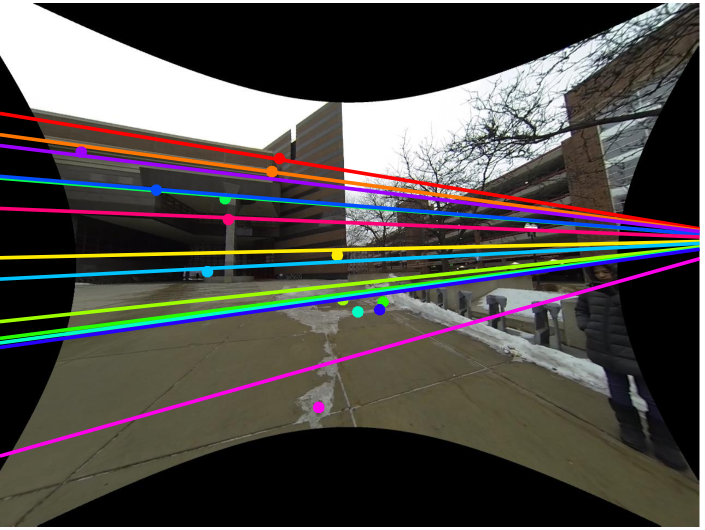
Fundamental Matrix Estimation

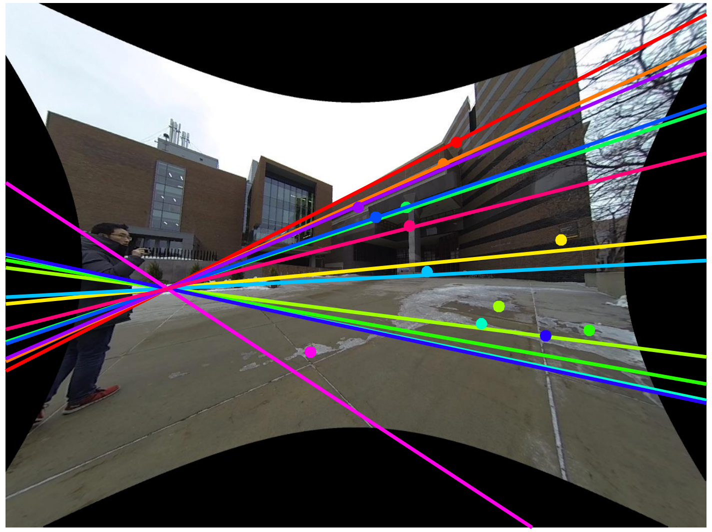




```
function [F F1] = ComputeFundamentalMatrix(x1, x2)
A = [];
for i = 1 : size(x1,1)
   A =[A; x1(i,1)*x2(i,1) x1(i,2)*x2(i,1) x2(i,1) x1(i,1)*x2(i,2) ...
      x1(i,2)*x2(i,2) x2(i,2) x1(i,1) x1(i,2) 1];
end

f = SolveHomogeneousEq(A);

F = [f(1:3)'; f(4:6)'; f(7:9)'];
```

Fundamental Matrix Estimation

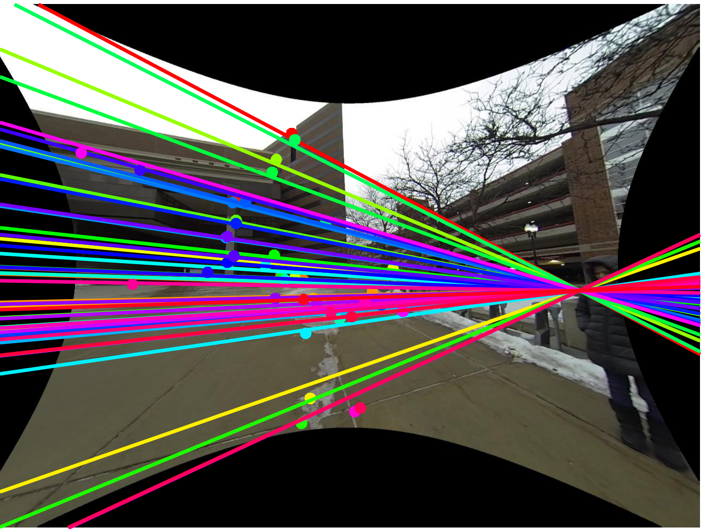


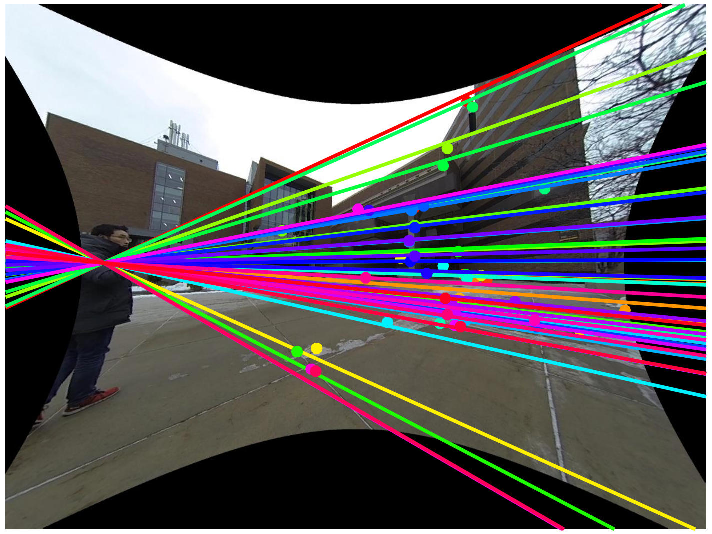


```
function [F F1] = ComputeFundamentalMatrix(x1, x2)
A = [];
for i = 1 : size(x1,1)
    A =[A; x1(i,1)*x2(i,1) x1(i,2)*x2(i,1) x2(i,1) x1(i,1)*x2(i,2) ...
        x1(i,2)*x2(i,2) x2(i,2) x1(i,1) x1(i,2) 1];
end

f = SolveHomogeneousEq(A);

F = [f(1:3)'; f(4:6)'; f(7:9)'];

[u d v] = svd(F);
F1 = F;
d(3,3) = 0;
F = u*d*v';
```

SVD cleanup

Fundamental Matrix Estimation

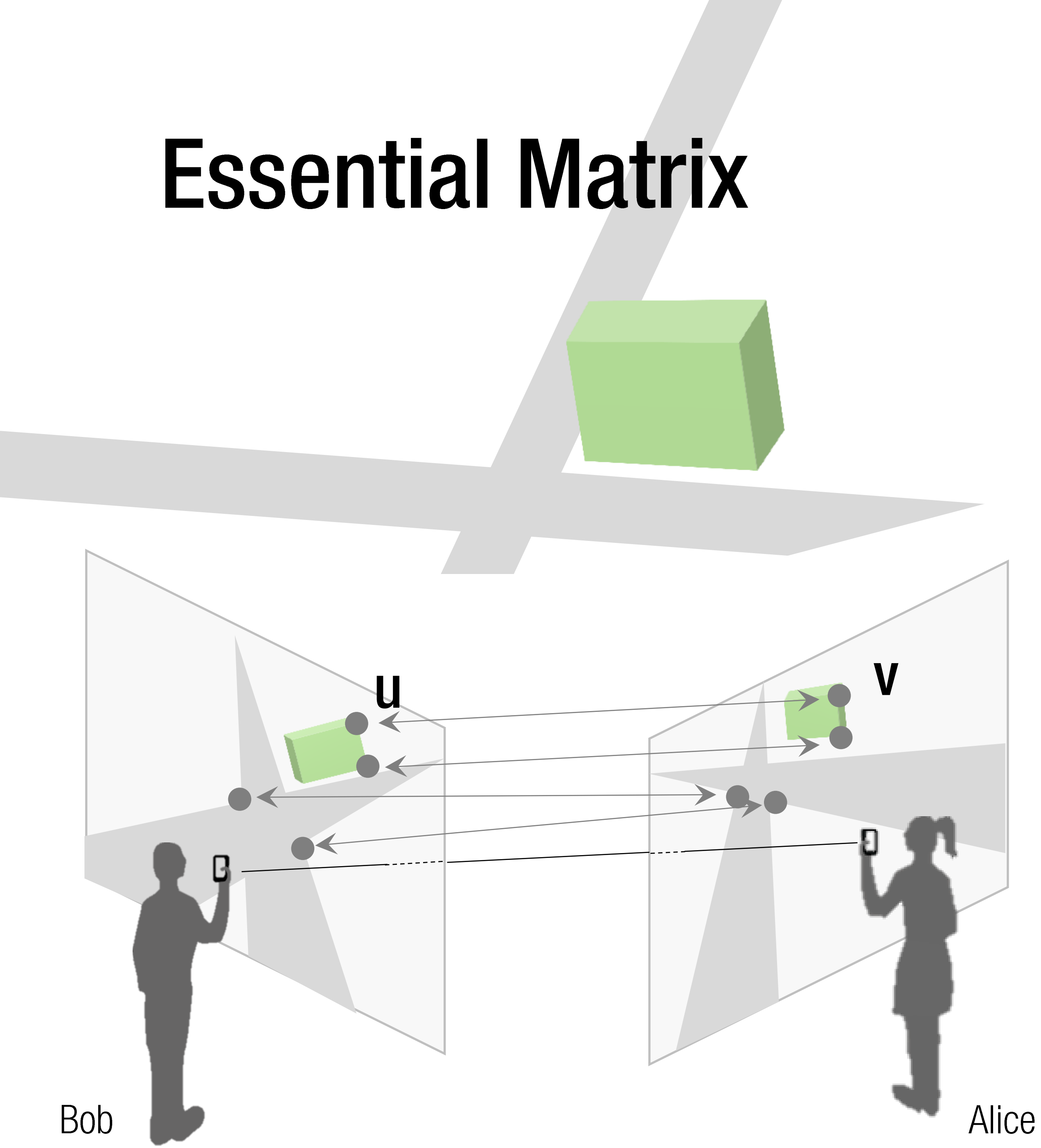




```
function [F F1] = ComputeFundamentalMatrix(x1, x2)
A = [];
for i = 1 : size(x1,1)
   A =[A; x1(i,1)*x2(i,1) x1(i,2)*x2(i,1) x2(i,1) x1(i,1)*x2(i,2) ...
      x1(i,2)*x2(i,2) x2(i,2) x1(i,1) x1(i,2) 1];
end

f = SolveHomogeneousEq(A);

F = [f(1:3)'; f(4:6)'; f(7:9)'];

[u d v] = svd(F);
F1 = F;
d(3,3) = 0;
F = u*d*v';
```

SVD cleanup

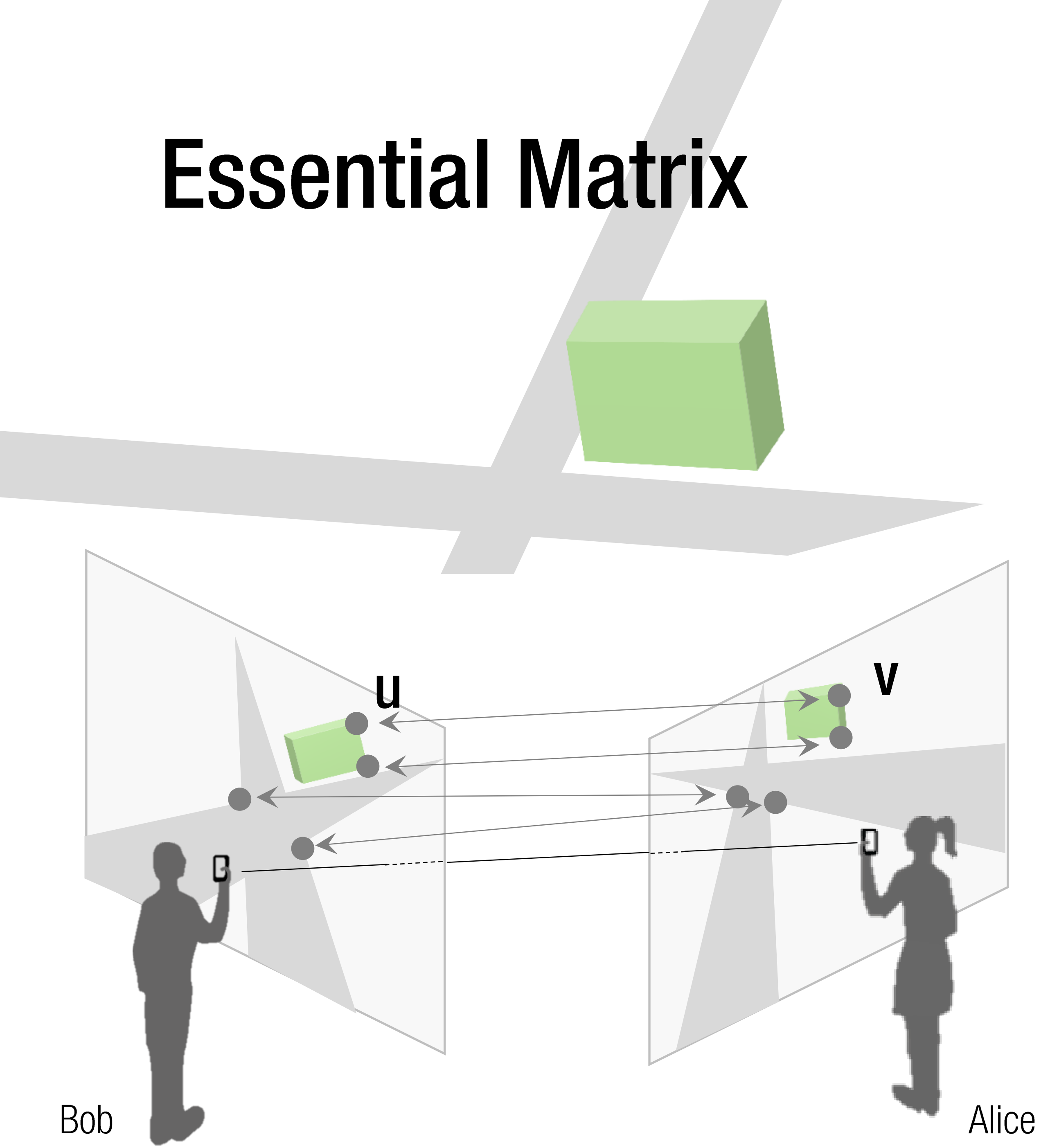
Essential Matrix



$$F = F(R, t)$$

$$= K^{-T} \left[t \right]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

Essential Matrix



Essential Matrix:

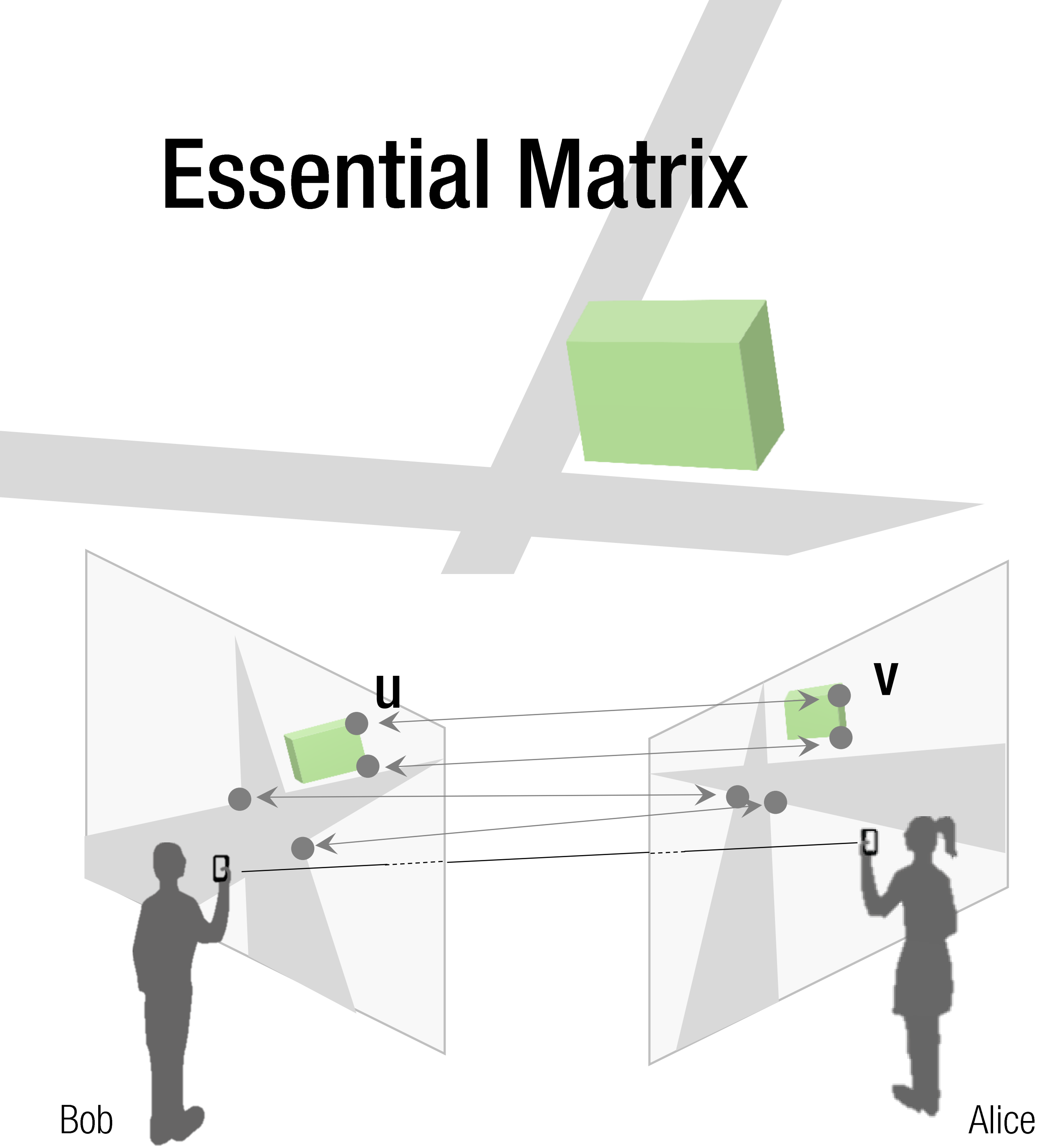
$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

$$\rightarrow E = \underline{K^{T} F K} \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Essential Matrix



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

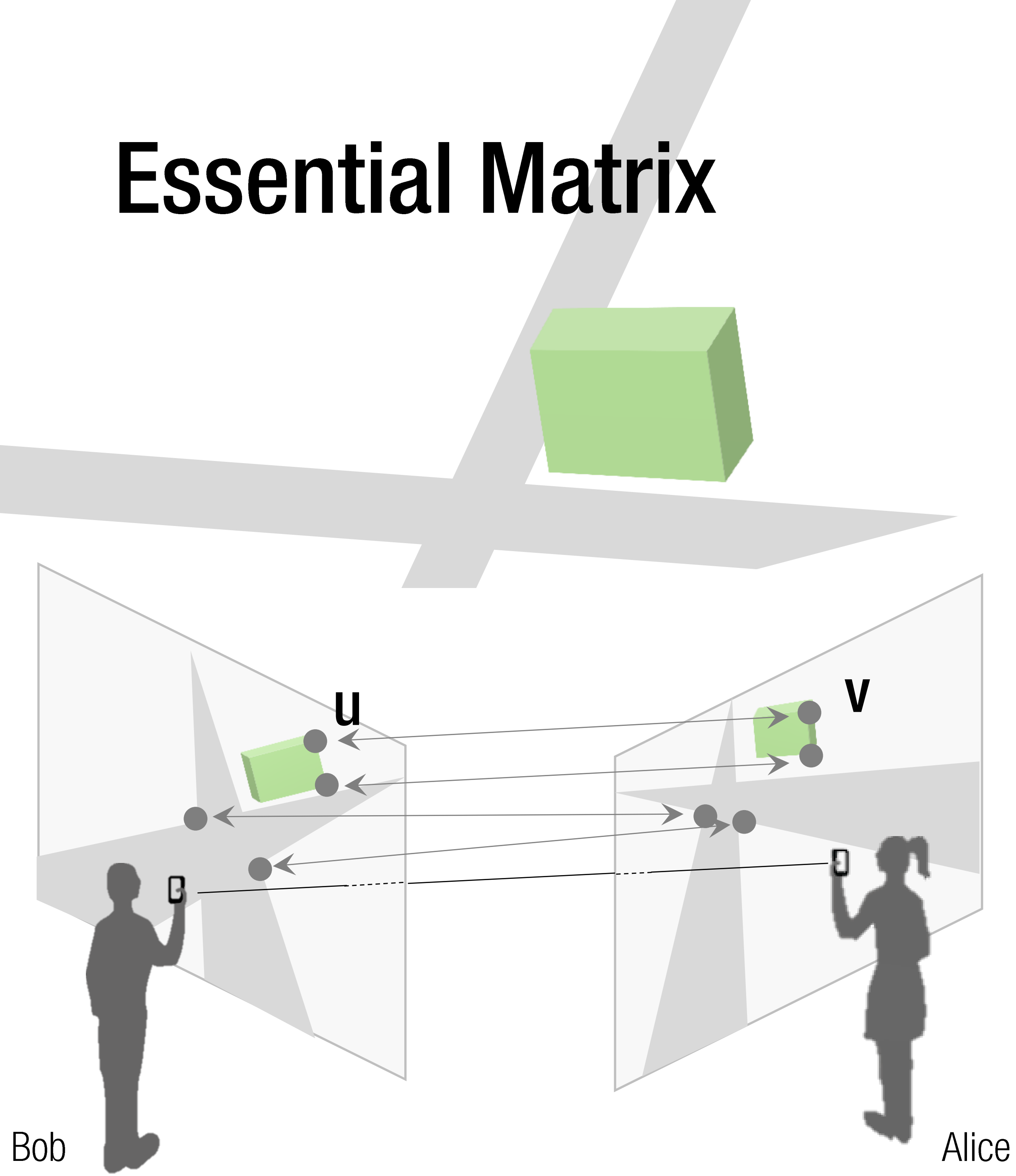
$$\longrightarrow E = \underline{K^{T} F K} \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Property of essential matrix:

$$E = UDV^{T} = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^{T}$$

Essential Matrix



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

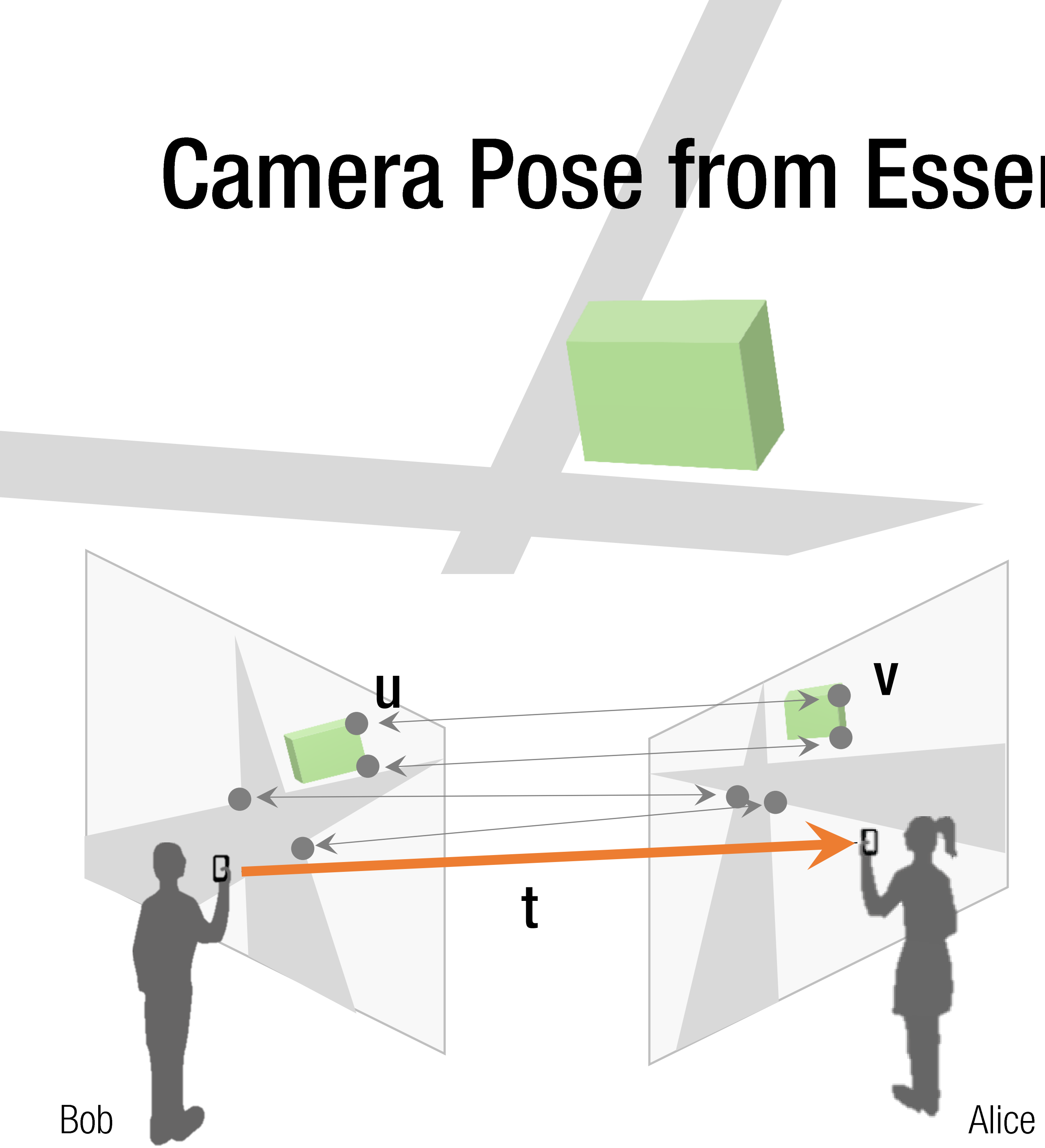
$$\rightarrow E = \underline{K^{T} F K} \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Property of essential matrix:

$$E = UDV^{T} = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^{T}$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

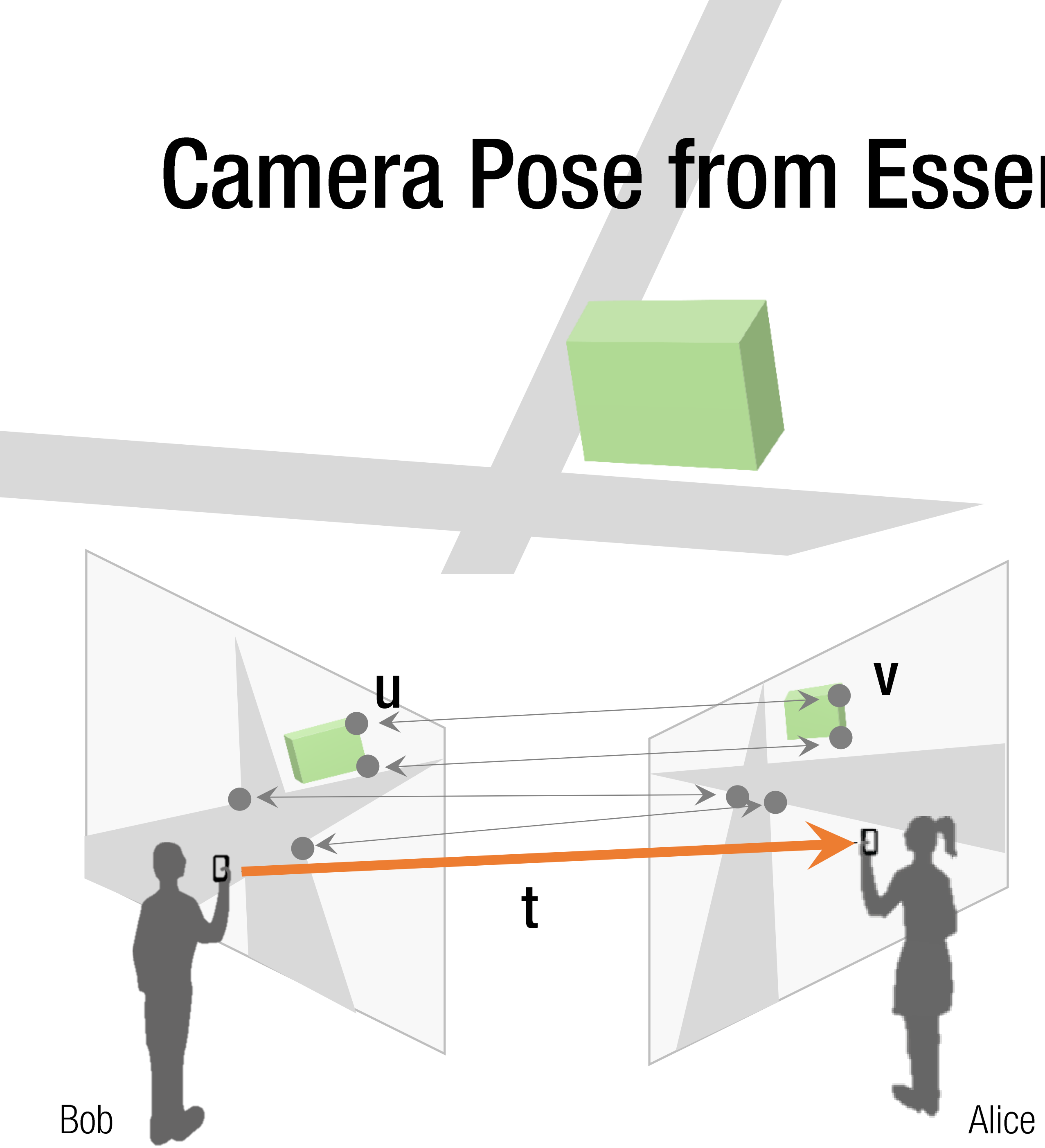
$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

$$\rightarrow E = \underline{K^{T}FK} \qquad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

$$t =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

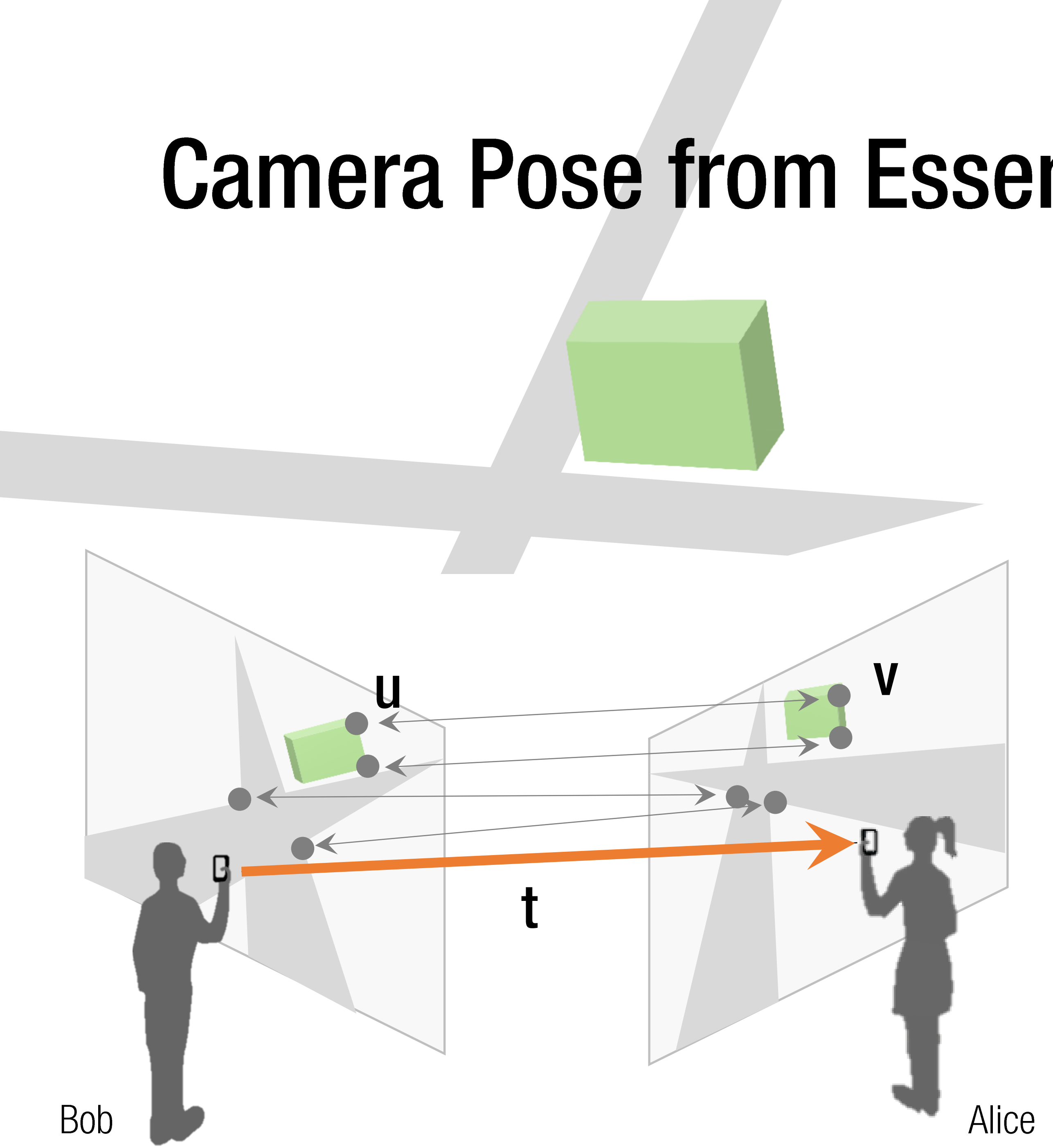
$$\longrightarrow E = \underline{K^{T}FK} \qquad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Left null space of E is translation vector, **t**:

$$t =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$\mathbf{F} = \mathbf{F}(\mathbf{R}, \mathbf{t})$$

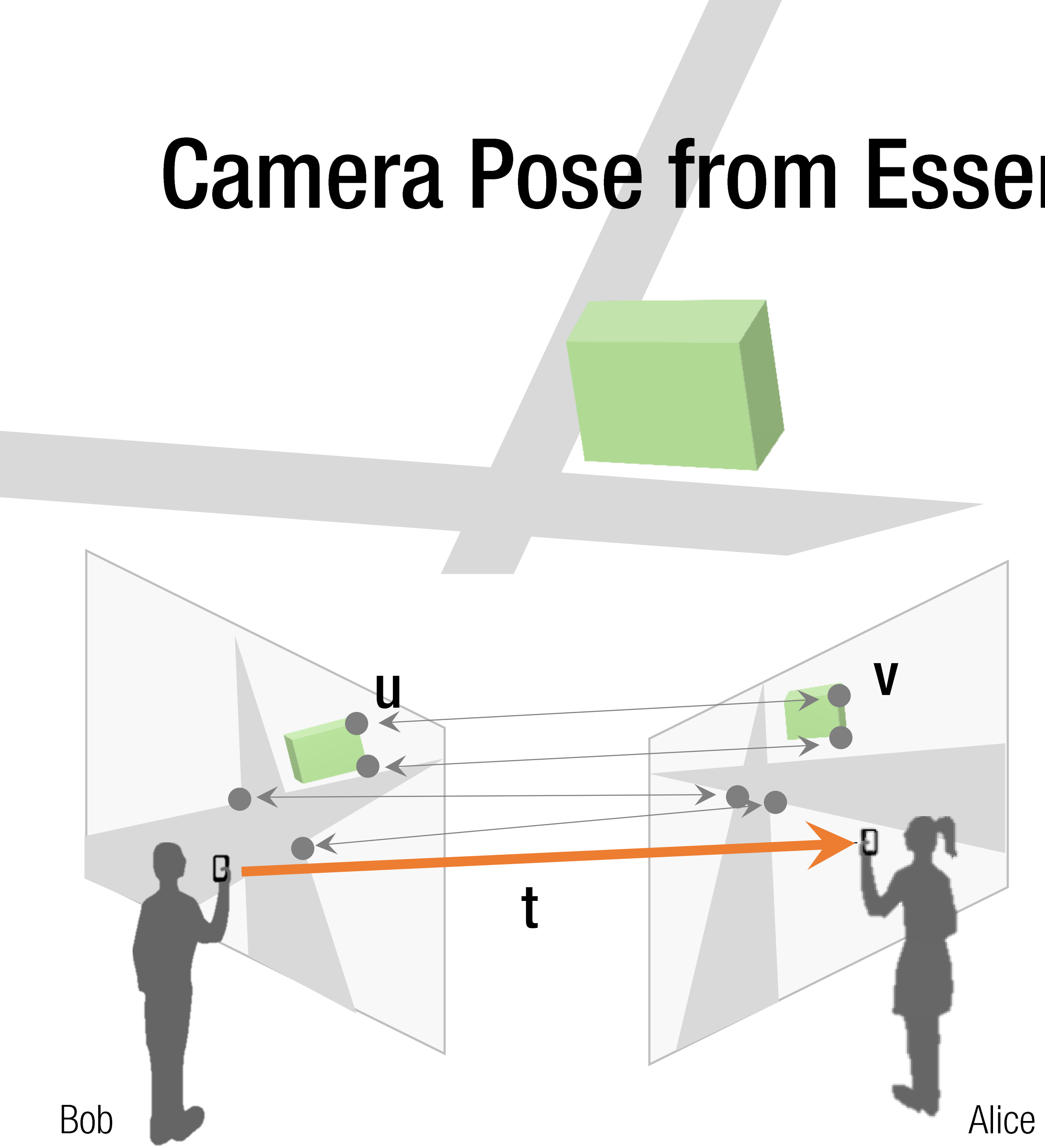
$$= \mathbf{K}^{-T}[\mathbf{t}]_{\times}\mathbf{R}\mathbf{K}^{-1} = \mathbf{K}^{-T}\mathbf{E}\mathbf{K}^{-1}$$

$$\rightarrow \mathbf{E} = \underbrace{\mathbf{K}^{T}\mathbf{F}\mathbf{K}}_{\text{Calibrated fundamental matrix}} \quad \text{where} \quad \mathbf{E} = [\mathbf{t}]_{\times}\mathbf{R}$$

Left null space of E is translation vector, **t**:

$$\mathbf{t} = \pm\text{null}(\mathbf{E}^{T}) = \pm\text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{T}\right)$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

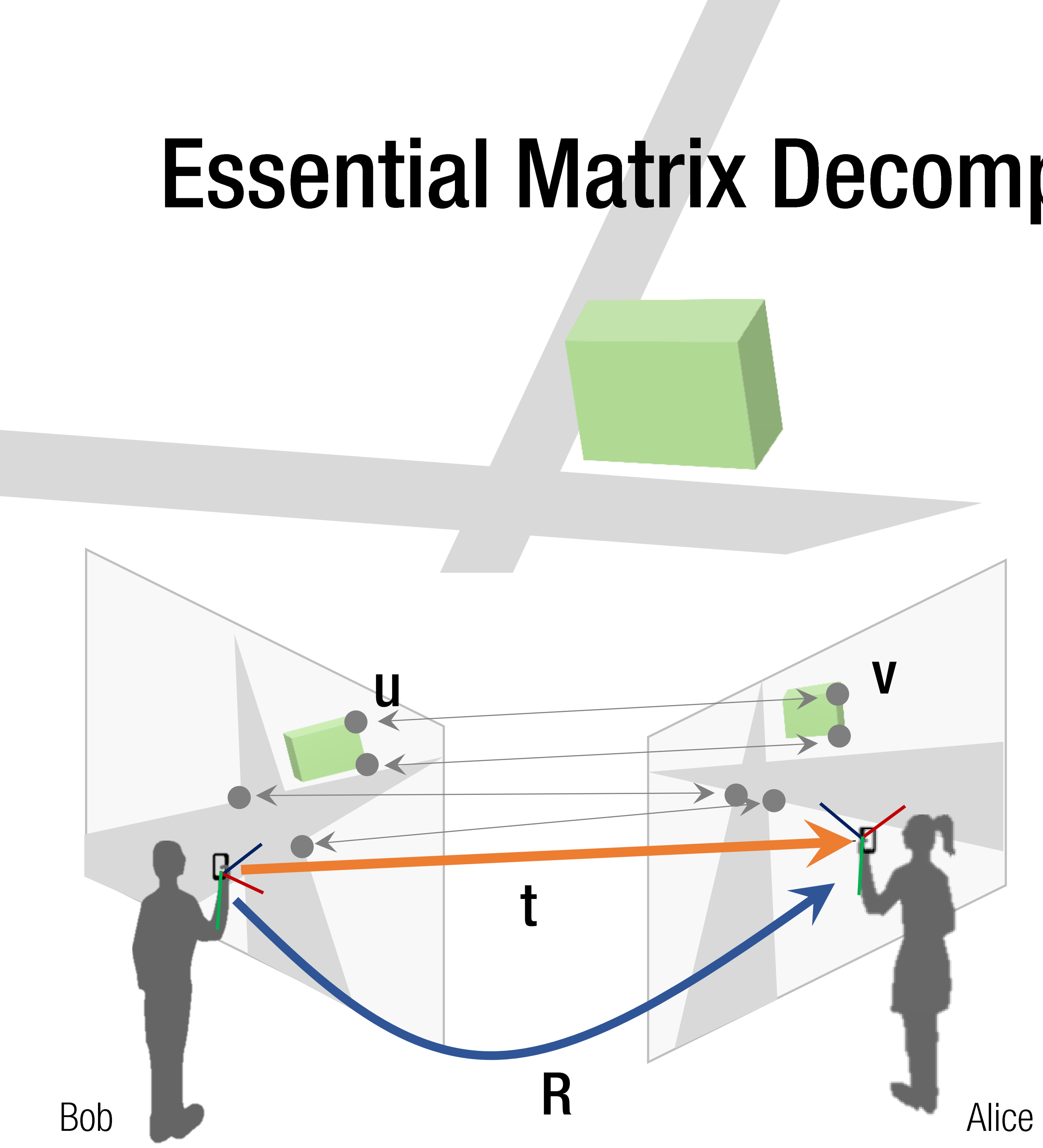
$$\longrightarrow E = \underbrace{K^{T} F K}_{\text{Calibrated fundamental matrix}} \qquad \text{where} \quad E = [t]_{\times} R$$

Left null space of E is translation vector, **t**:

$$t = \pm \text{null}(E^{T}) = \pm \text{null}\left(\left([t]_{\times} R\right)^{T}\right)$$

$$\because t^{T}[t]_{\times} R = -\left([t]_{\times} t\right)^{T} R = -\underbrace{(t \times t)^{T}}_{\text{Self-cross product}} R = 0$$

Essential Matrix Decomposition

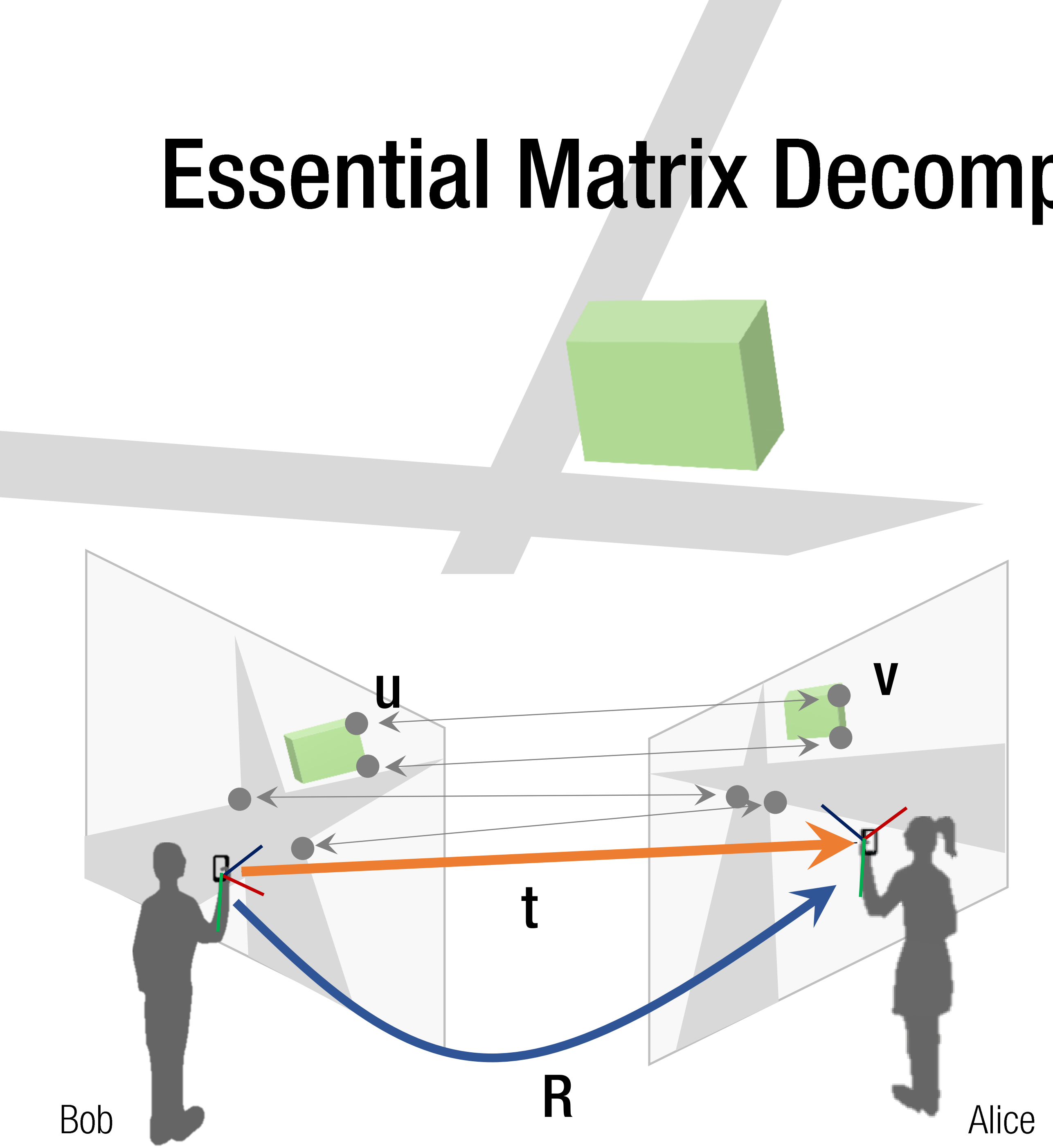


Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^{\mathsf{T}}) = \text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

Can I invert $[\mathbf{t}]_{\times}$?

Essential Matrix Decomposition



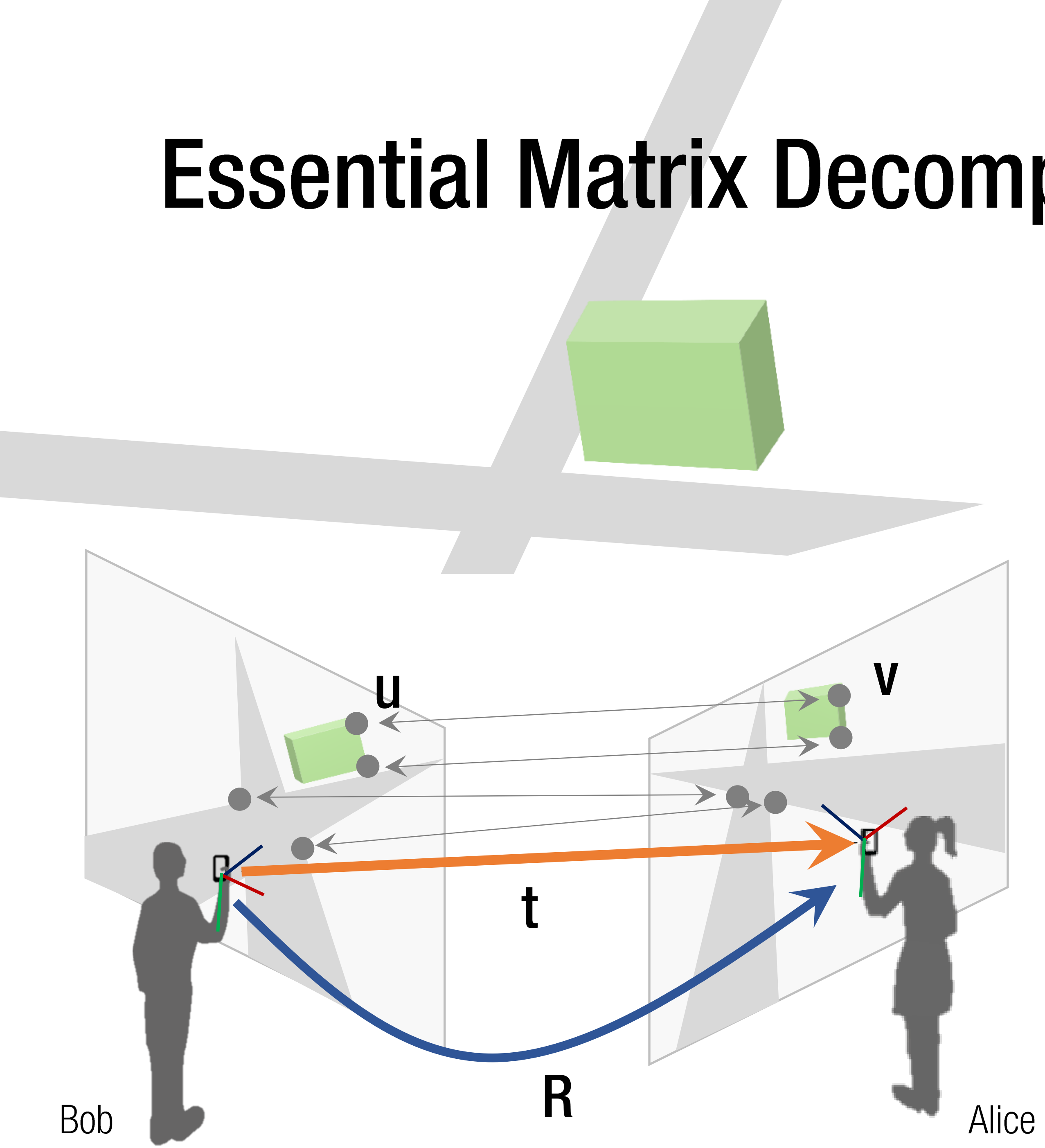
Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^{\mathsf{T}}) = \text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

$$\longrightarrow \quad \mathbf{t} = \mathbf{u}_3 \qquad \text{where} \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$$

$$\mathbf{E} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathsf{T}} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

Essential Matrix Decomposition



Left null space of E is translation vector, **t**:

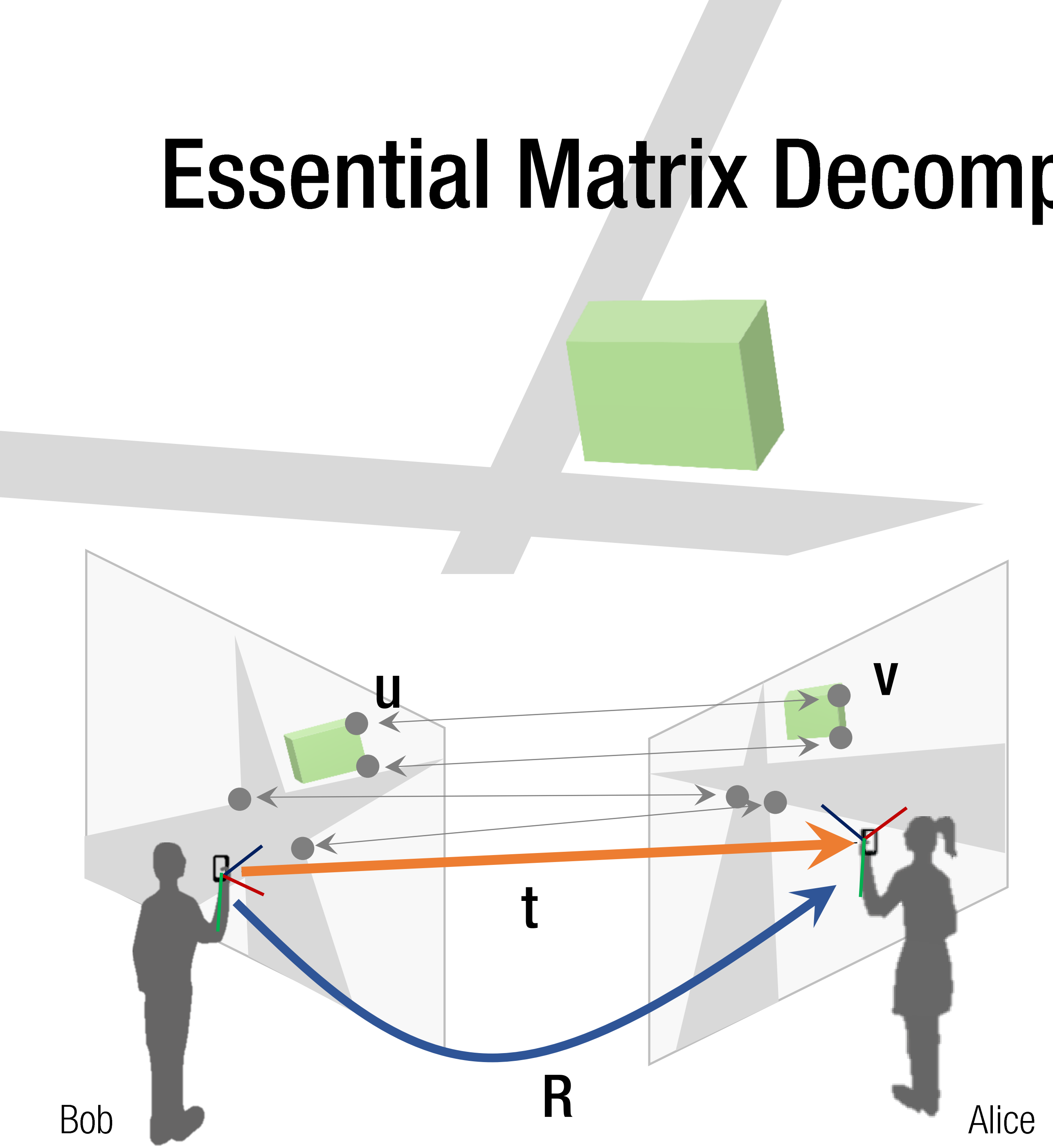
$$\mathbf{t} = \text{null}(\mathbf{E}^{\mathsf{T}}) = \text{null}\left(\left([\mathbf{t}]_{\times}\mathbf{R}\right)^{\mathsf{T}}\right)$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_3$ where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$

$$\mathbf{E} = \mathbf{UDV}^{\mathsf{T}} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

Essential Matrix Decomposition



Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^\top) = \text{null}\left(\left([\mathbf{t}]_\times \mathbf{R}\right)^\top\right)$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_3$ where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$

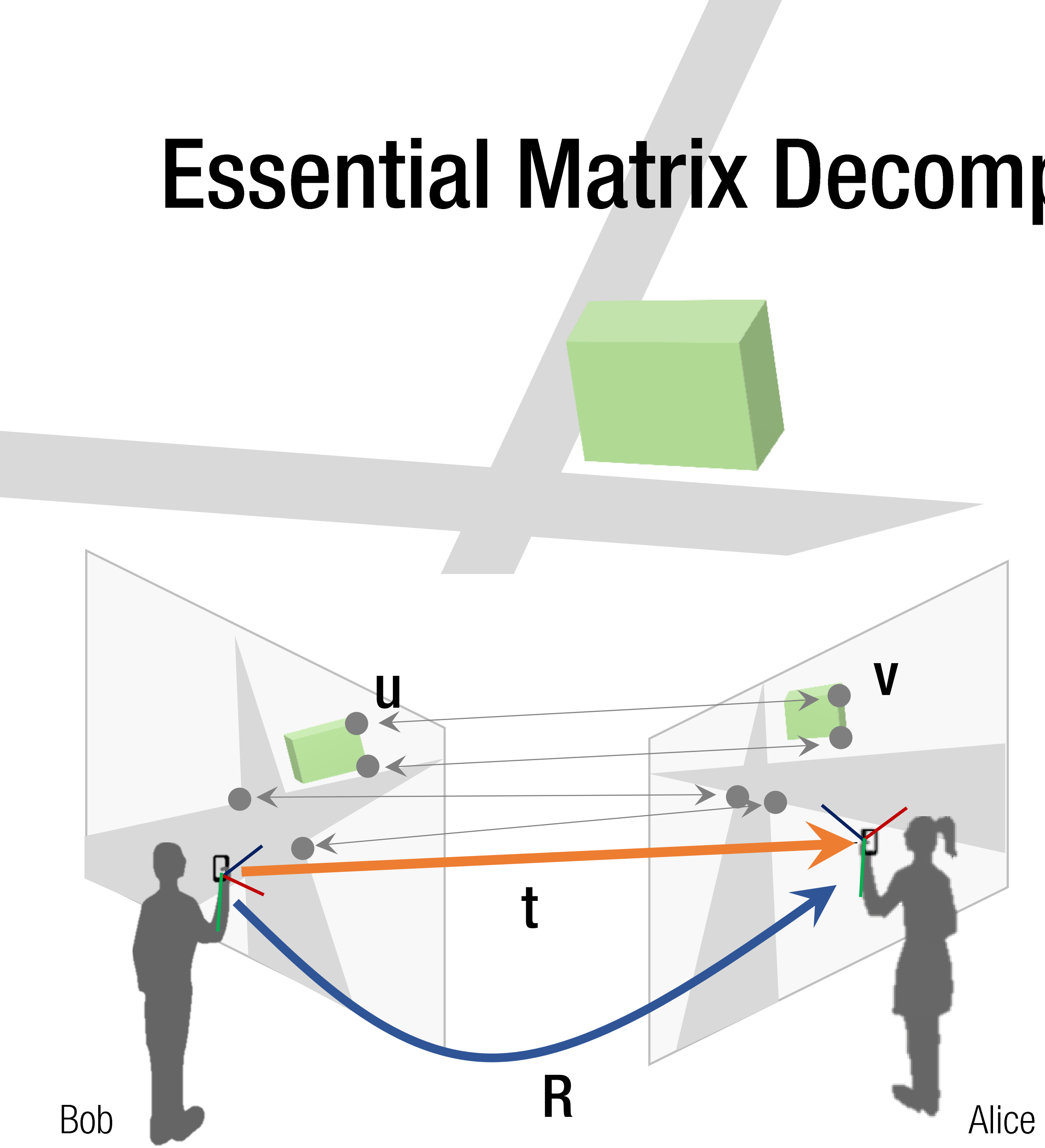
$$\mathbf{E} = \mathbf{U}\mathbf{D}\mathbf{V}^\top = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\top$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

$$[\mathbf{t}]_\times = [\mathbf{u}_1 \times \mathbf{u}_2]_\times = \mathbf{u}_2\mathbf{u}_1^\top - \mathbf{u}_1\mathbf{u}_2^\top$$

:

Essential Matrix Decomposition



Left null space of E is translation vector, **t**:

$$\mathbf{t} = \text{null}(\mathbf{E}^T) = \text{null}\left(\left([\mathbf{t}]_\times \mathbf{R}\right)^T\right)$$

$\longrightarrow$ $\mathbf{t} = \mathbf{u}_3$ where $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$

$$\mathbf{E} = \mathbf{UDV}^T = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^T$$

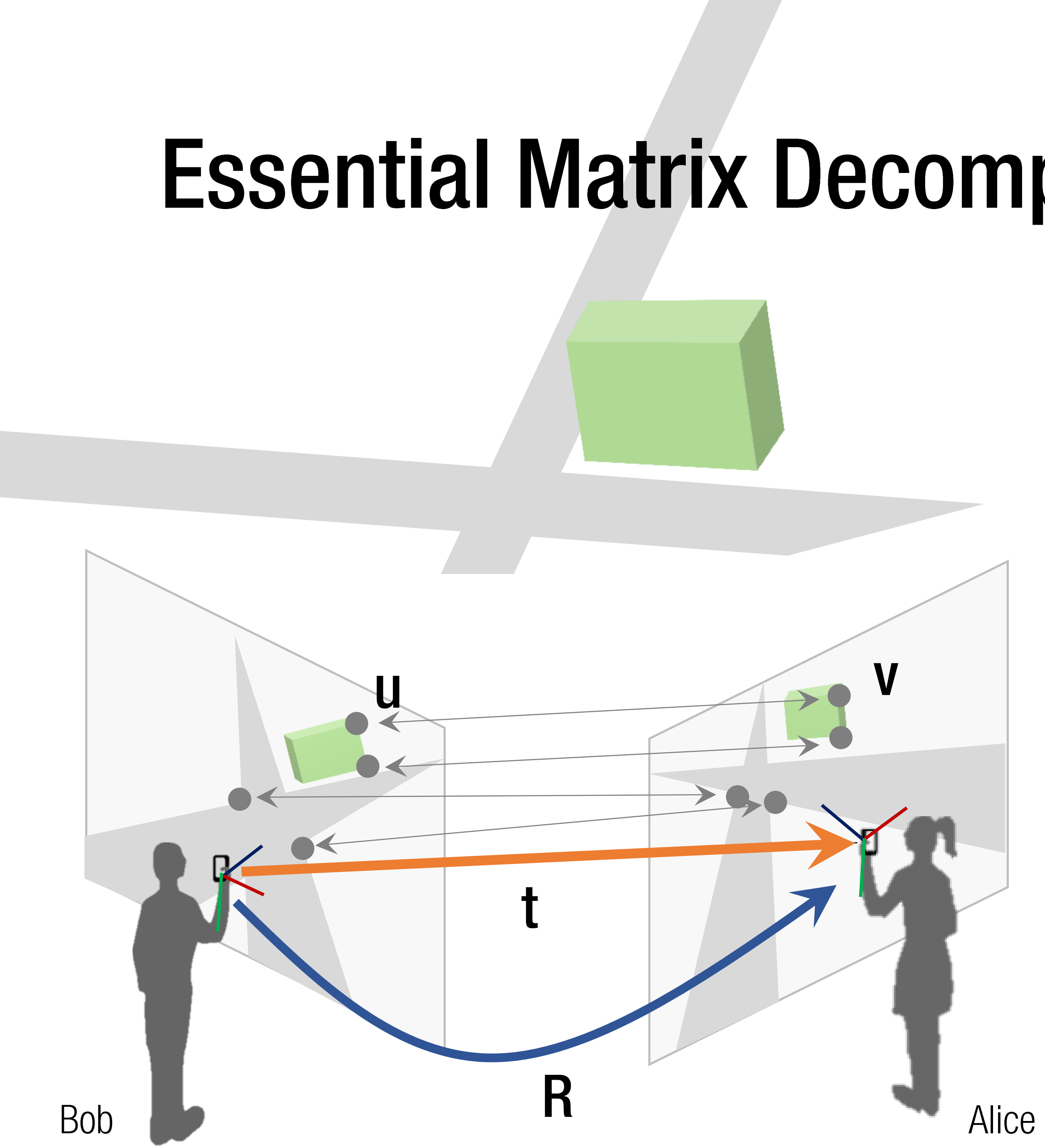
$\longrightarrow$ $\mathbf{t} = \mathbf{u}_1 \times \mathbf{u}_2$ (orthogonal matrix, **U**)

$$[\mathbf{t}]_\times = [\mathbf{u}_1 \times \mathbf{u}_2]_\times = \mathbf{u}_2\mathbf{u}_1^T - \mathbf{u}_1\mathbf{u}_2^T$$

$$= \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^T$$

Prove!

Essential Matrix Decomposition



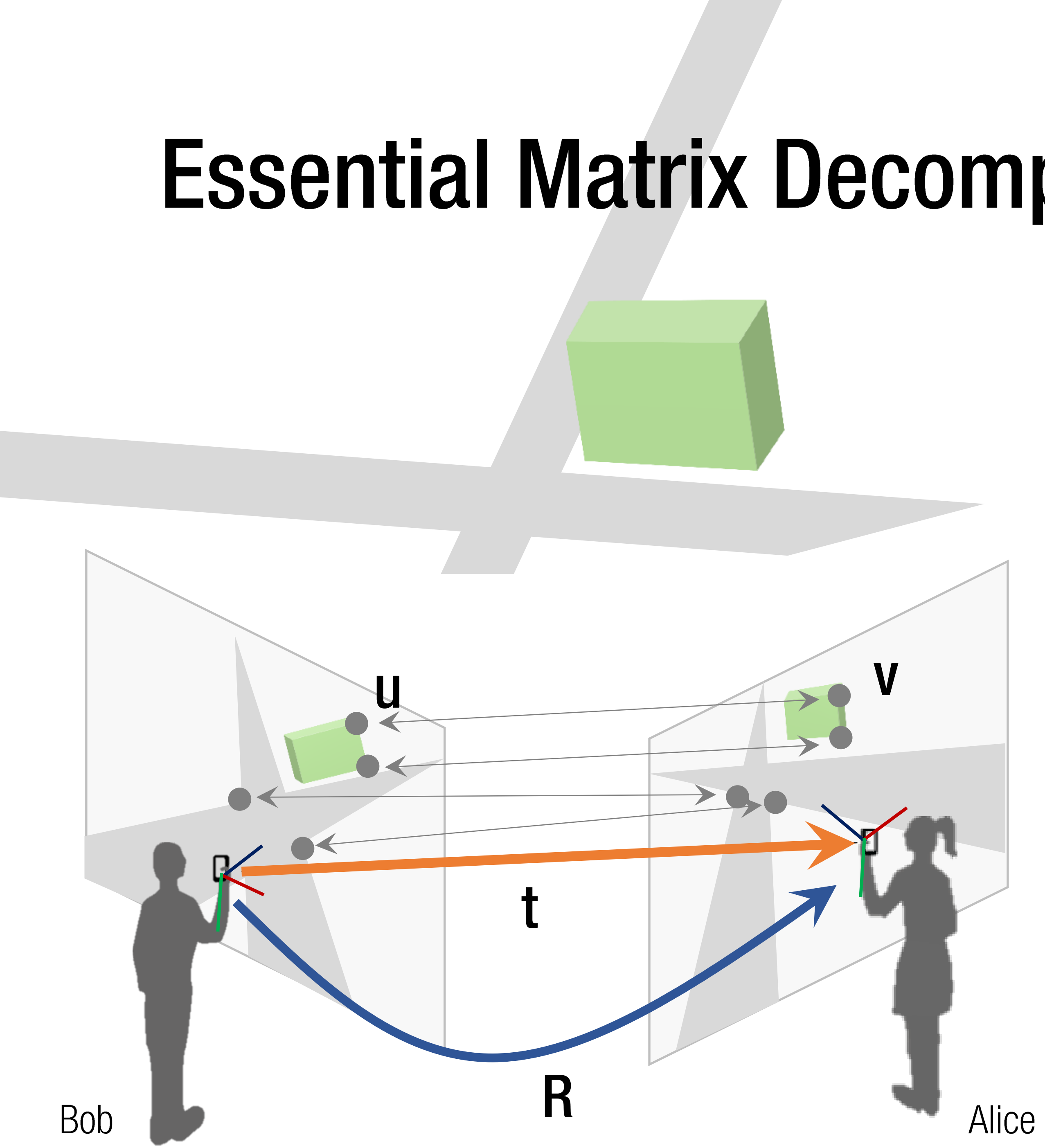
$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^T\mathbf{R}$$

where $\mathbf{R} \in SO(3)$

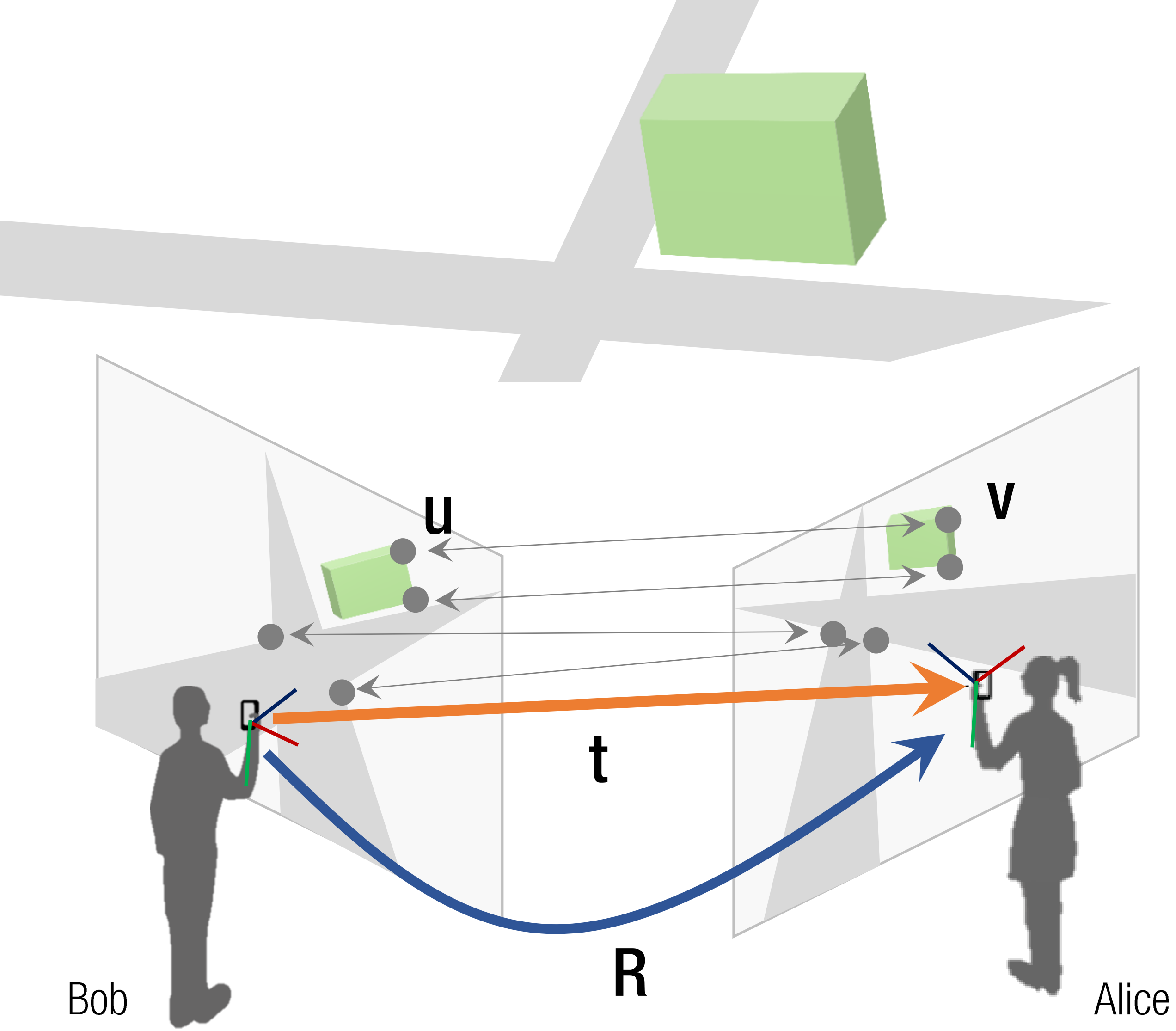
Essential Matrix Decomposition

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{U}^\mathsf{T} \mathbf{R} = \mathbf{U} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} \mathbf{V}^\mathsf{T}$$

where $\mathbf{R} \in SO(3)$



Essential Matrix Decomposition



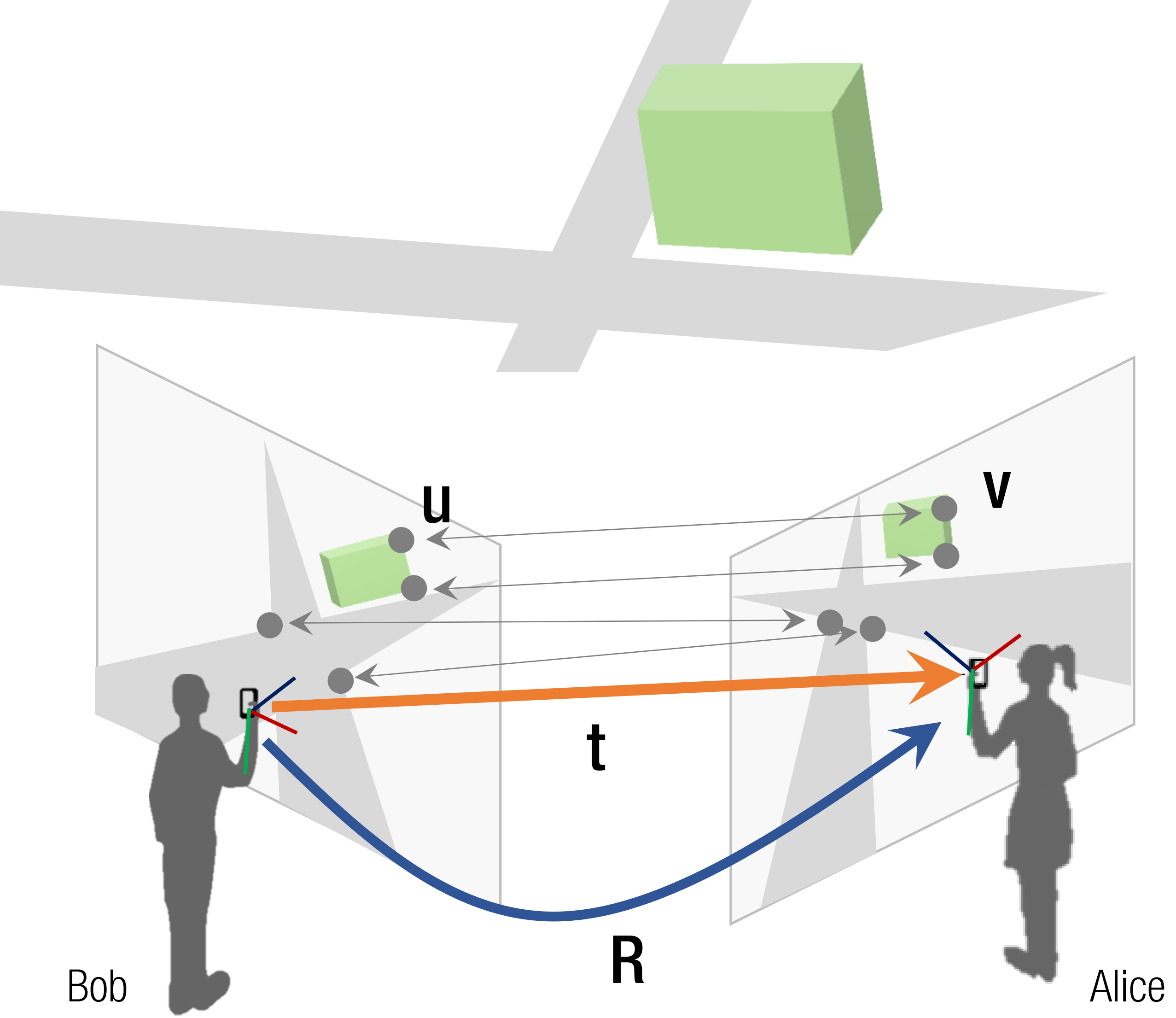
$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\top \mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\top$$

where $\mathbf{R} \in SO(3)$

Define $\mathbf{R} = \mathbf{U}\underline{\mathbf{W}}\mathbf{V}^\top$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\top \mathbf{U}\mathbf{W}\mathbf{V}^\top = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W}\mathbf{V}^\top$$

Essential Matrix Decomposition



$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

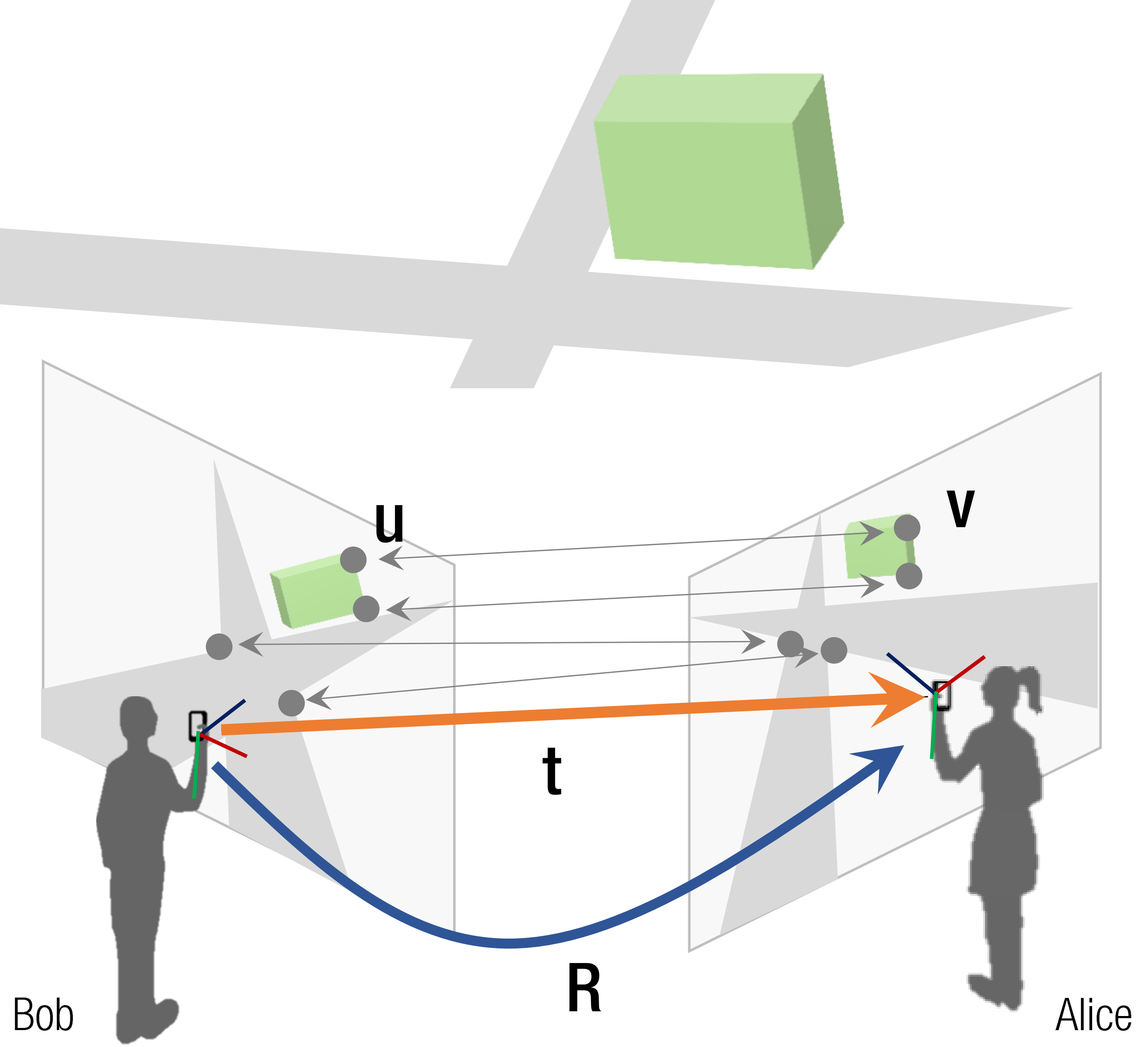
where $\mathbf{R} \in SO(3)$

Define $\mathbf{R} = \mathbf{UWV}^\mathsf{T}$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{UWV}^\mathsf{T} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{WV}^\mathsf{T}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W}$$

Essential Matrix Decomposition



$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

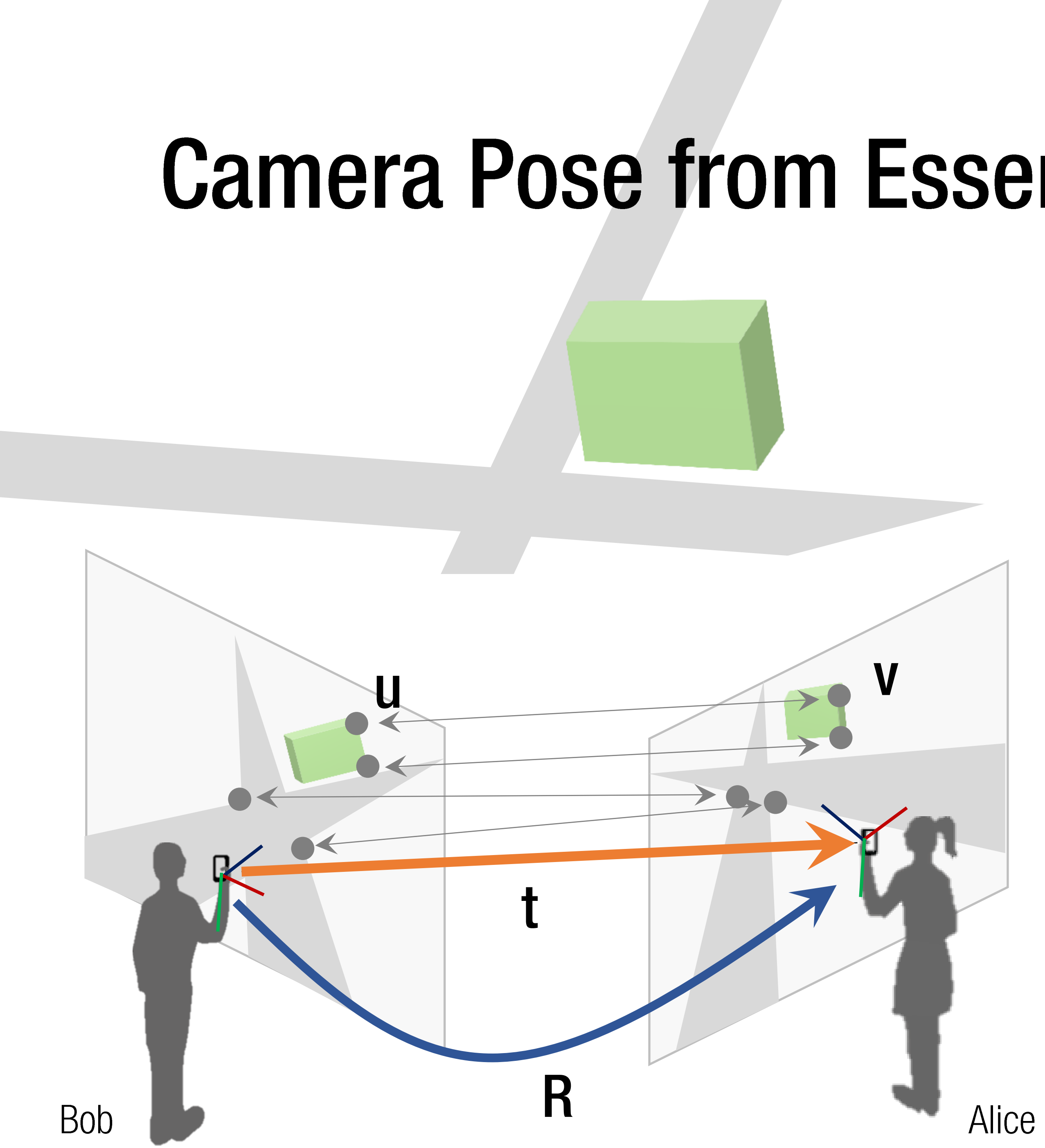
where $\mathbf{R} \in SO(3)$

Define $\mathbf{R} = \mathbf{U}\underline{\mathbf{W}}\mathbf{V}^\mathsf{T}$

$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{U}\mathbf{W}\mathbf{V}^\mathsf{T} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W}\mathbf{V}^\mathsf{T}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{W} \qquad \mathbf{W} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Camera Pose from Essential Matrix (Rotation)

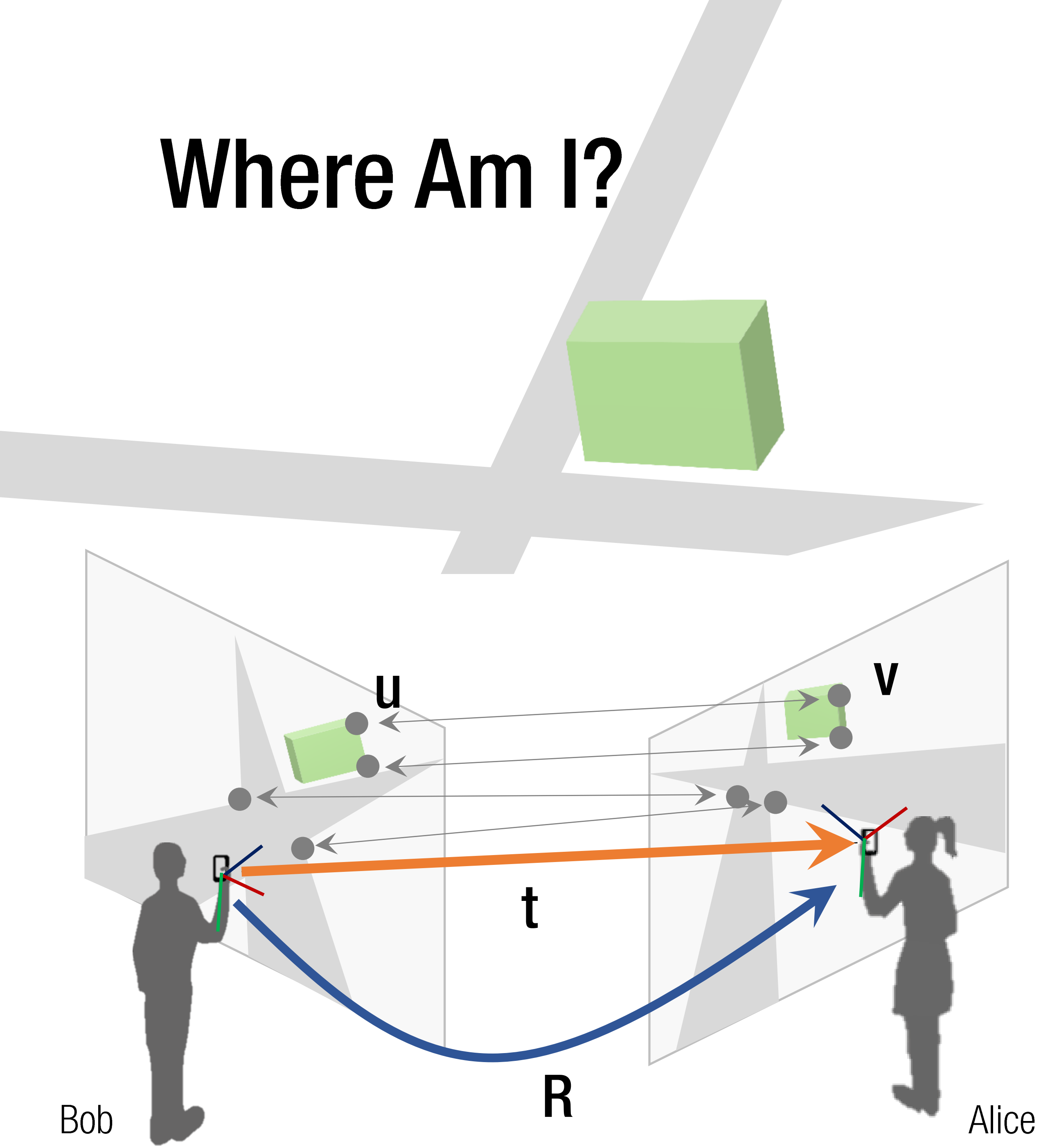


$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

where $\mathbf{R} \in SO(3)$

$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

Where Am I?



$$\mathbf{E} = [\mathbf{t}]_\times \mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{U}^\mathsf{T}\mathbf{R} = \mathbf{U}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

$$\mathbf{t} = \pm \mathbf{u}_3$$

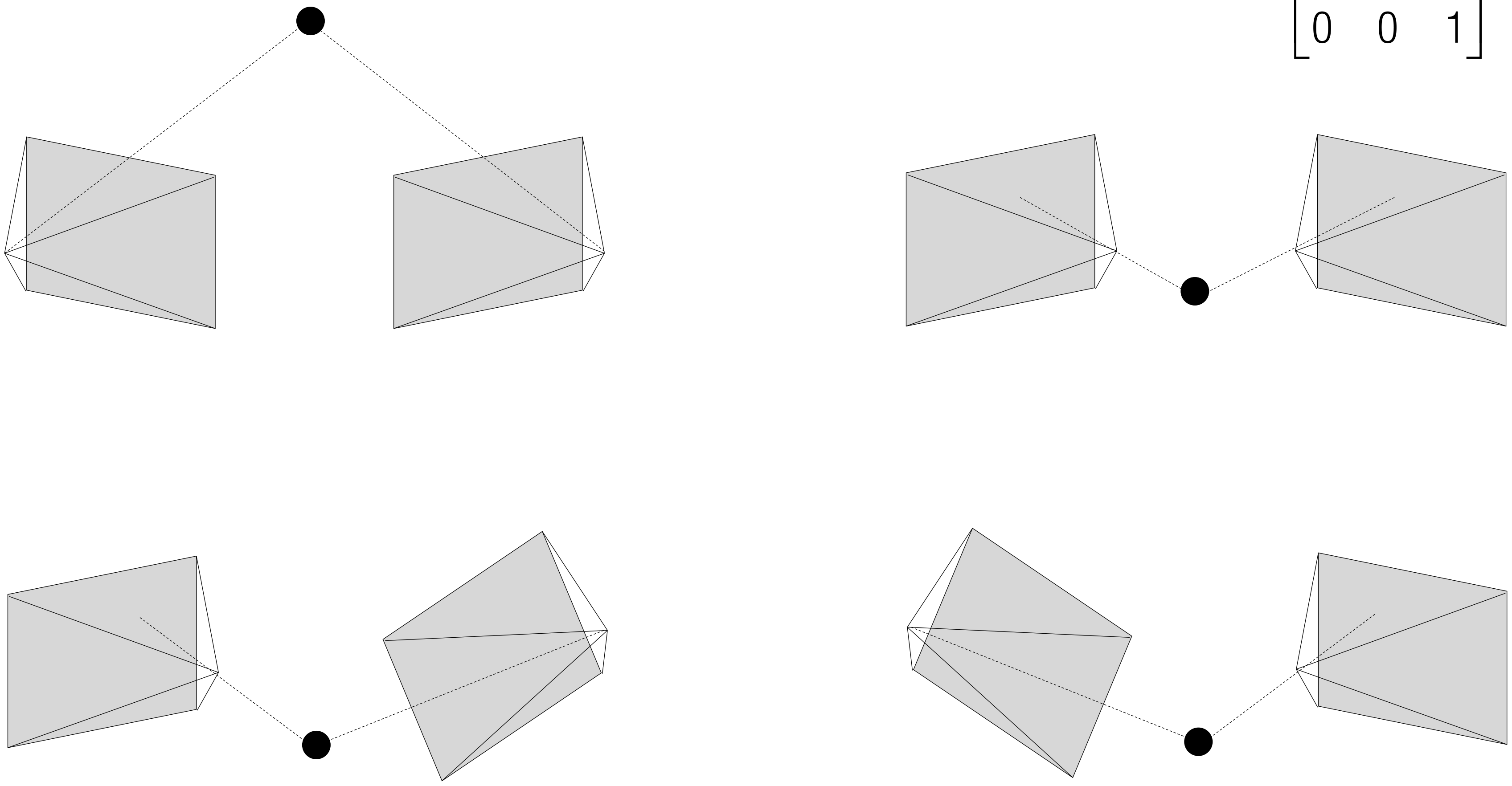
$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^\mathsf{T}$$

→ Four configurations

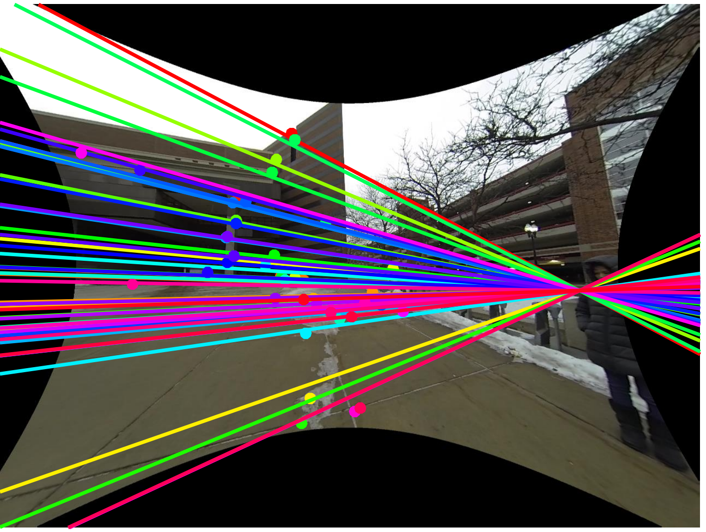
Four Configurations

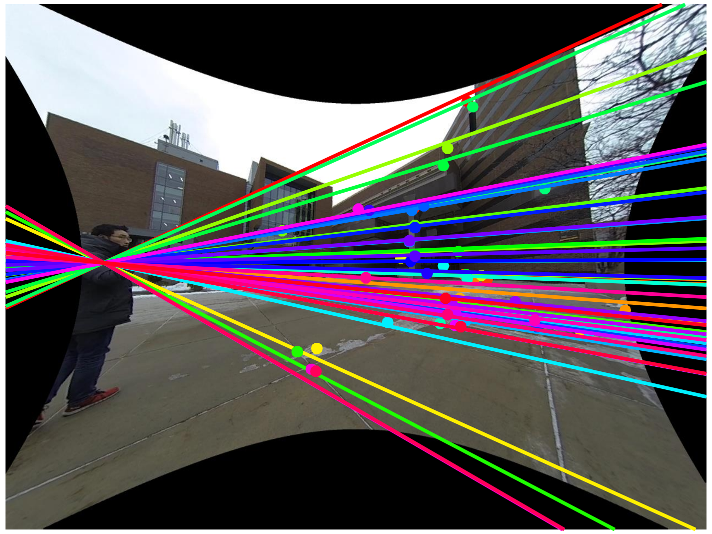
$$\mathbf{t} = \pm \mathbf{u}_3$$

$$\mathbf{R} = \mathbf{U}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathrm{T}}, \text{ or } \mathbf{U}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{V}^{\mathrm{T}}$$



Camera Pose Estimation





$$E = K^T F K$$

```
function E = ComputeEssentialMatrix(F, K)

E = K' * F * K;

[u d v] = svd(E);

d(1,1) = 1;
d(2,2) = 1;
d(3,3) = 0;

E = u * d * v';
```

SVD cleanup

D =

1.0468	0	0
0	0.9975	0
0	0	0.0000

Before cleanup

D =

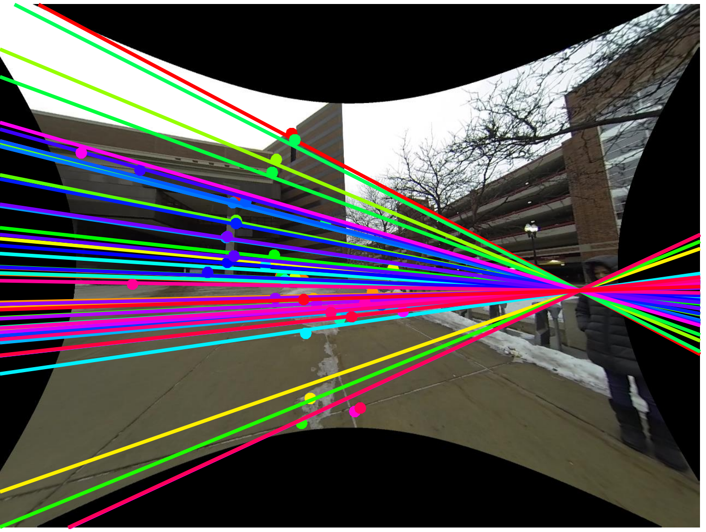
1.0000	0	0
0	1.0000	0
0	0	0.0000

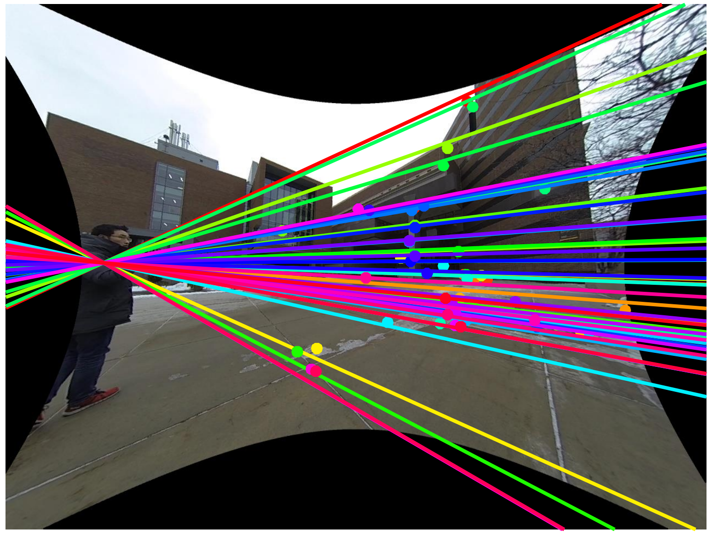
After cleanup

Camera Pose Estimation

$$\mathbf{t} = \pm \mathbf{u}_3$$

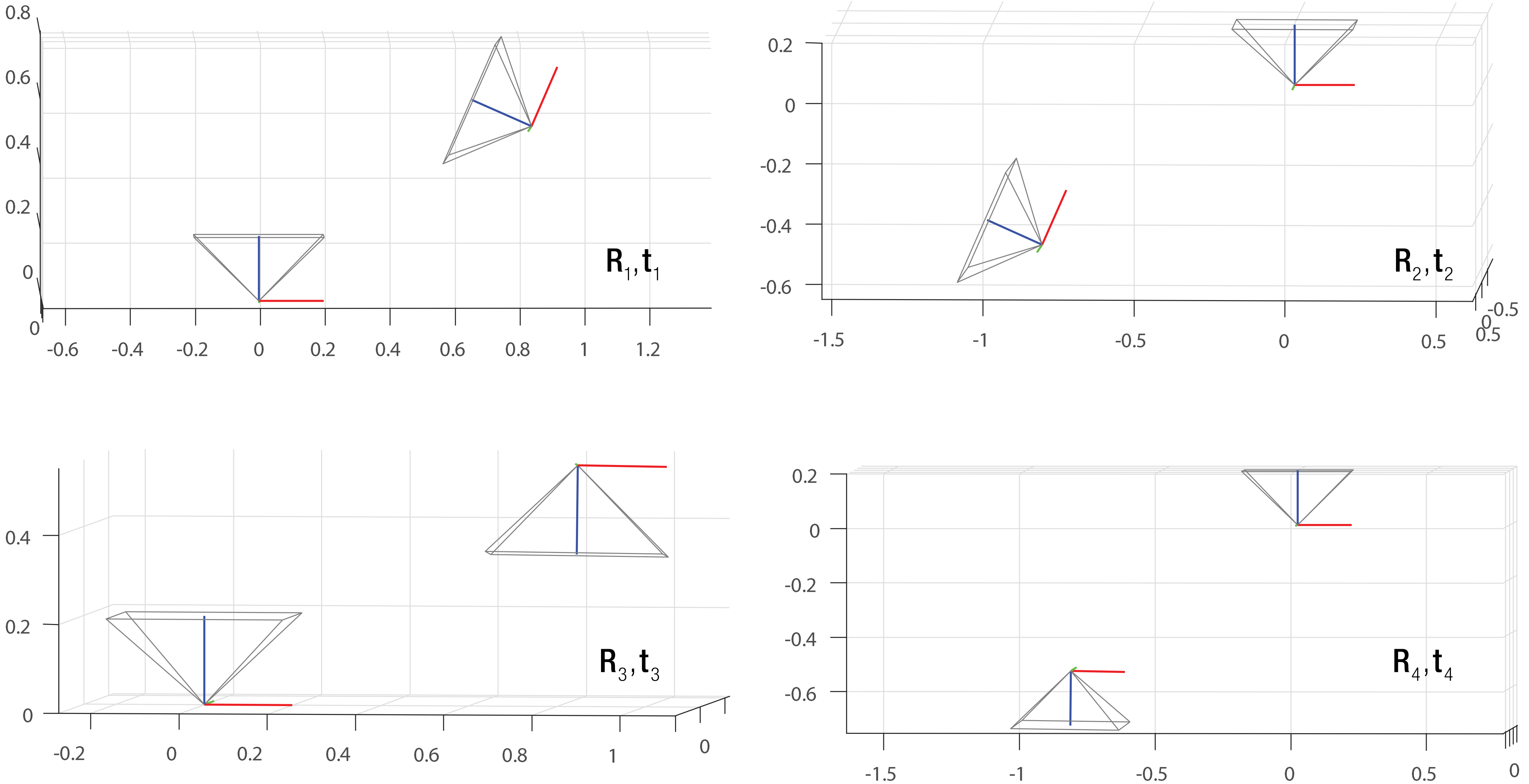
$$\mathbf{R} = \mathbf{U} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}^{\mathrm{T}}, \text{ or } \mathbf{U} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}^{\mathrm{T}}$$



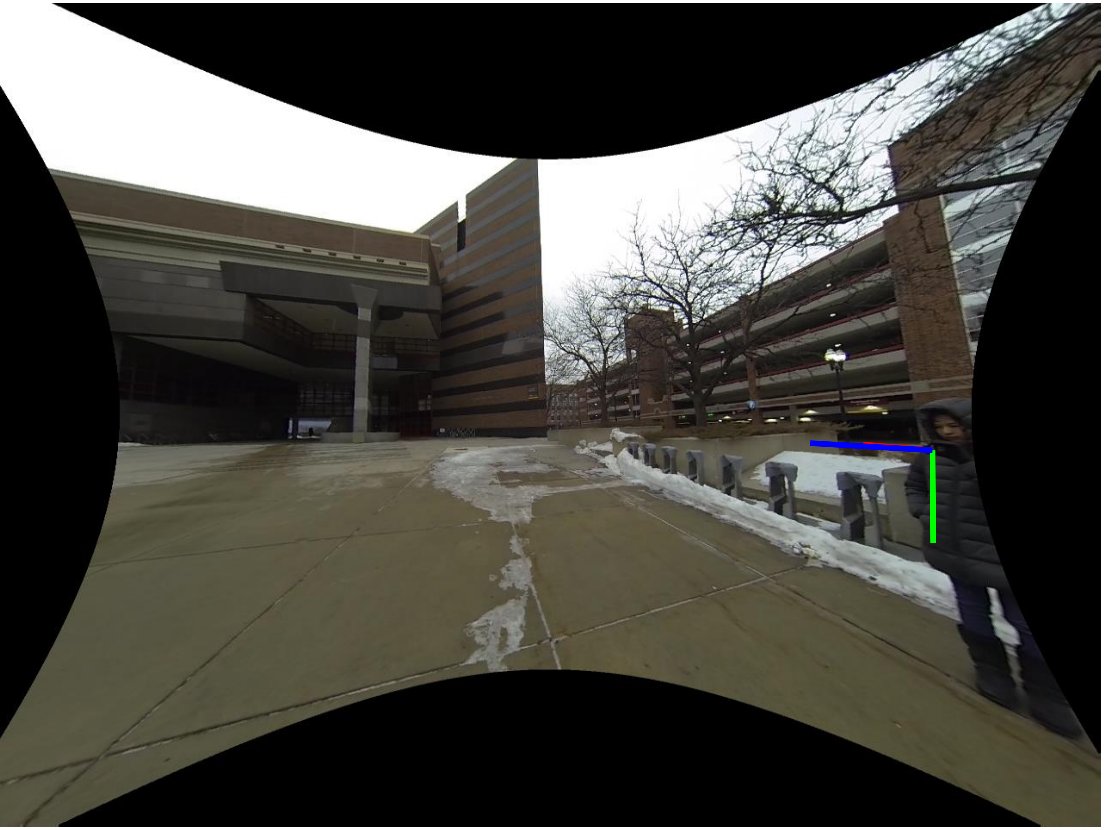


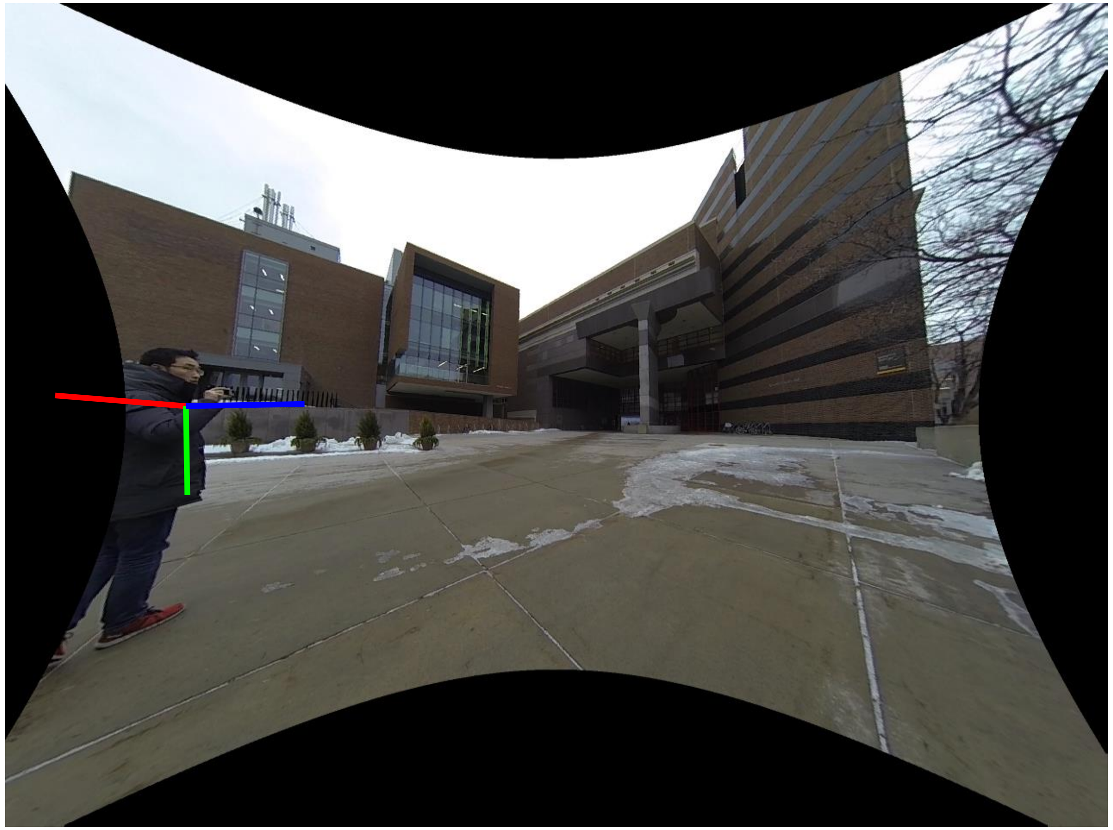
```
function [R1 t1, R2, t2, R3, t3, R4, t4] = ...
CameraPoseFromEssentialMatrix(E)

[U D V] = svd(E);

W = [0 -1 0;
    1 0 0;
    0 0 1];

t1 = U(:,3);
R1 = U * W * V';
if det(R1) < 0
  t1 = -t1;  R1 = -R1;
end

...
```

$$\det(\mathbf{R}) = 1$$


0.8
0.6
0.4
0.2
0
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
R_1, t_1
0.2
0
-0.2
-0.4
-0.6
-1.5
-1
-0.5
0
0.5
-0.5
0
0.5
R_2, t_2
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
0
R_3, t_3
0.2
0
-0.2
-0.4
-0.6
-1.5
-1
-0.5
0
0.5
0
R_4, t_4

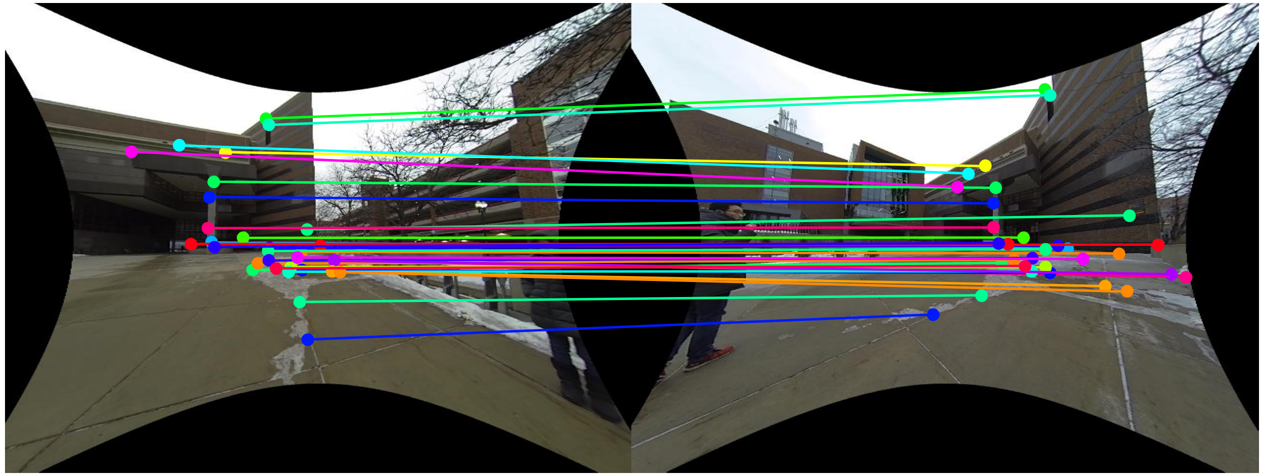
Camera Image Projection





Feature Matching

How Many Correspondences?



$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \underbrace{0_{m\times 1}}_{\mathbf{0}}$$

What is minimum m?

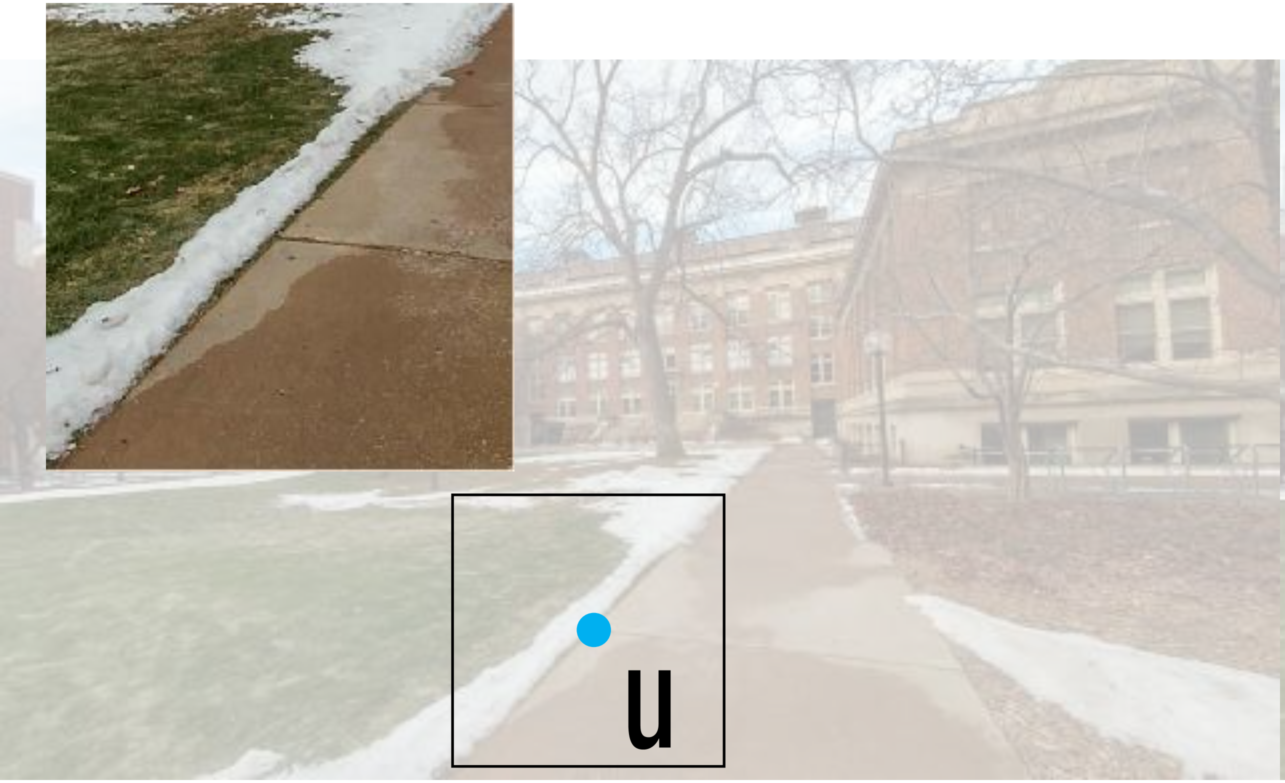

u

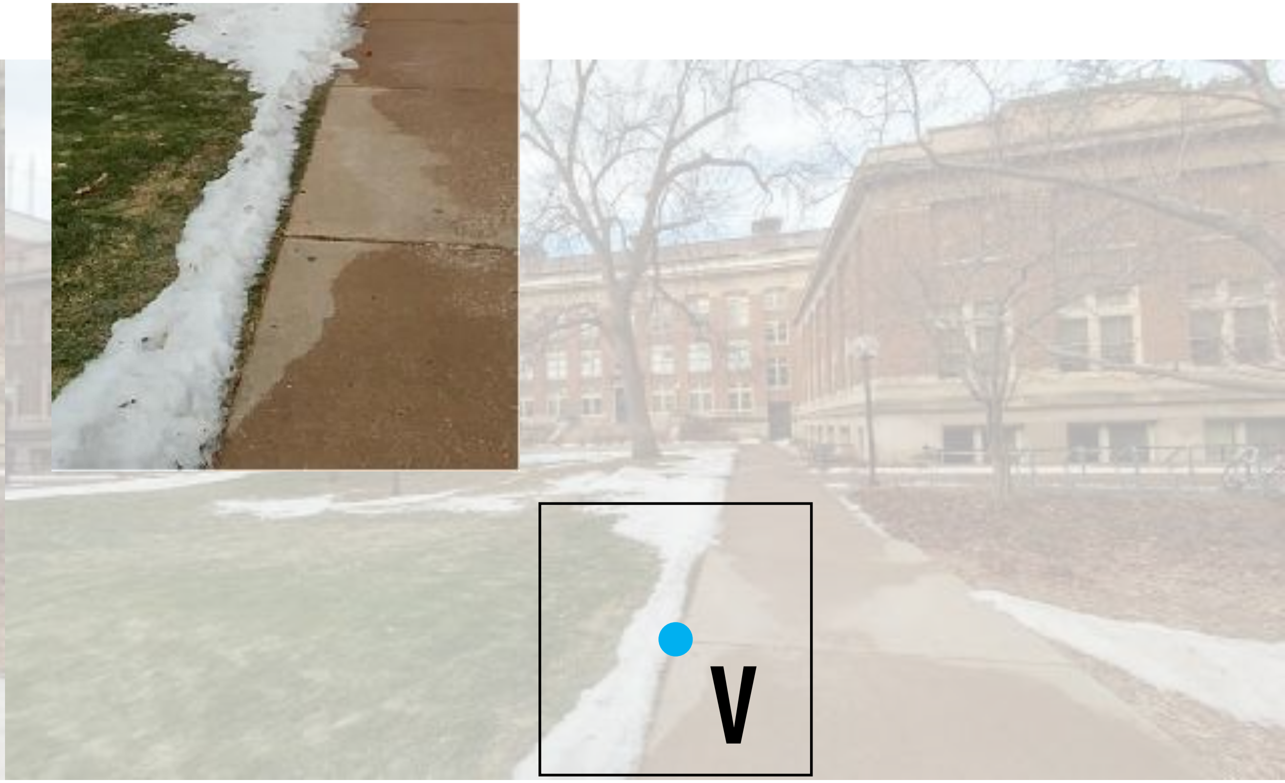

v

Local Patch

Local Patch (Scale)

Local Patch (Scale)





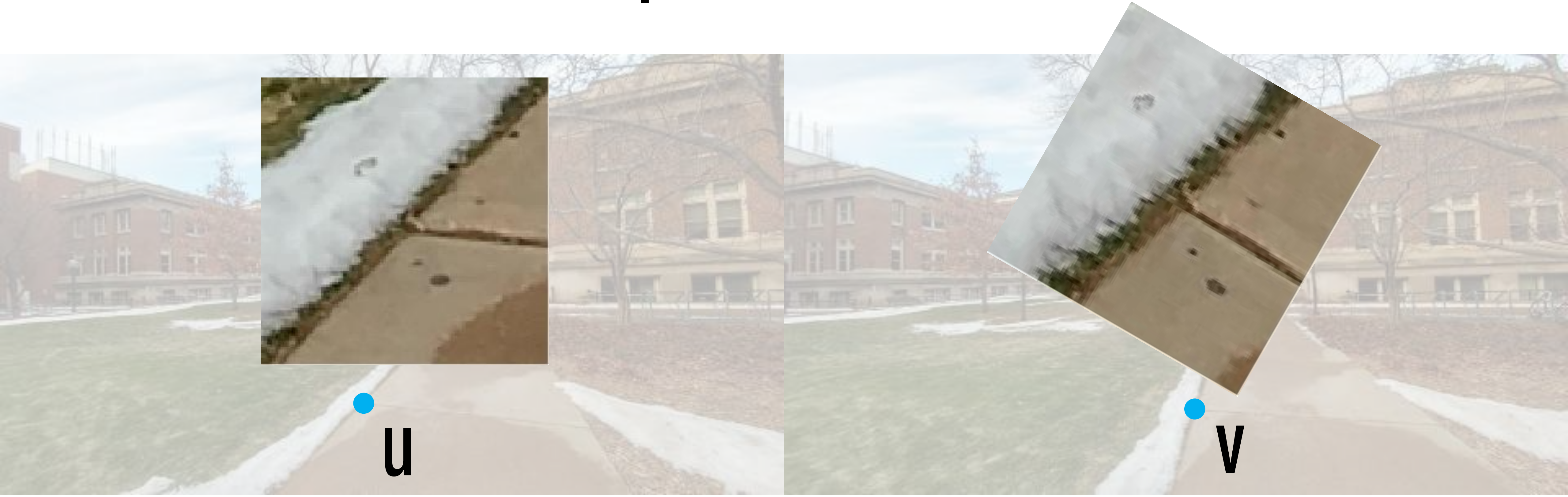
Local Patch (Scale)

Local Visual Descriptor

Desired properties:

- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.

Local Visual Descriptor



Desired properties:

- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.
- Orientation aware

Local Scale Invariant Feature Transform (SIFT)

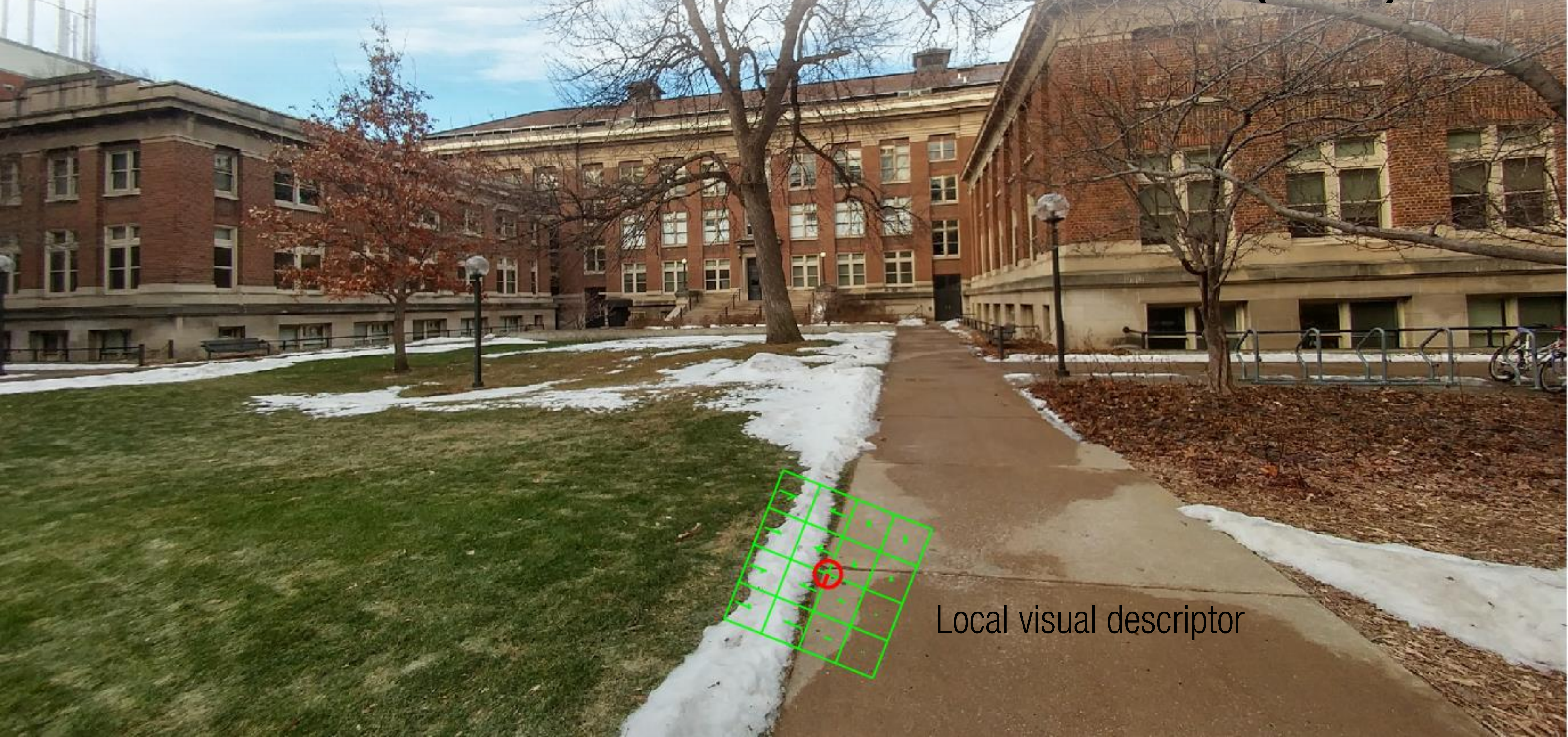
Desired properties:

- Repeatability: the same point is repeatedly detected.
- Discriminativity: the point is unique.
- Orientation aware

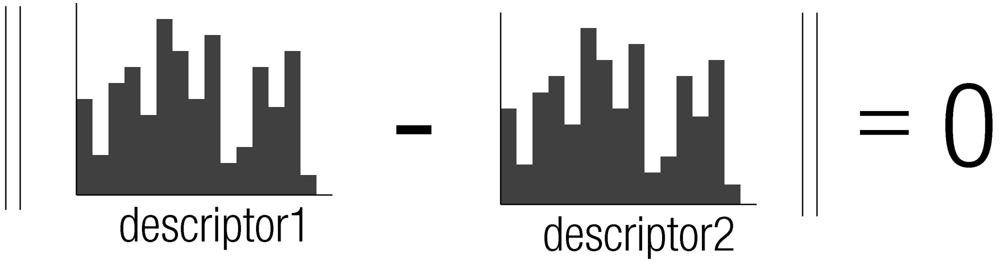
More detail from CSCI 5561

Local Scale Invariant Feature Transform (SIFT)

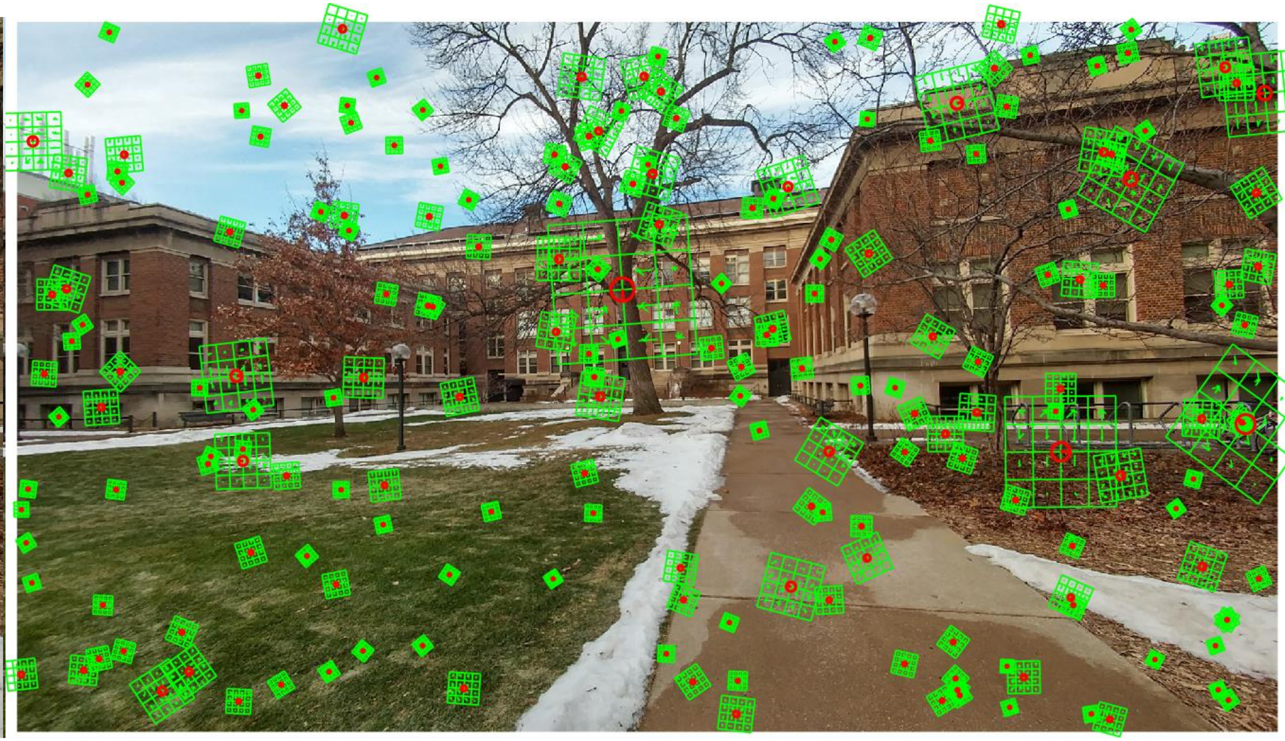
Local Scale Invariant Feature Transform (SIFT)



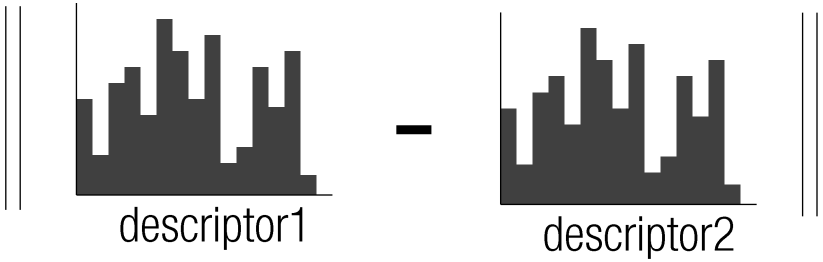
Local Scale Invariant Feature Transform (SIFT)



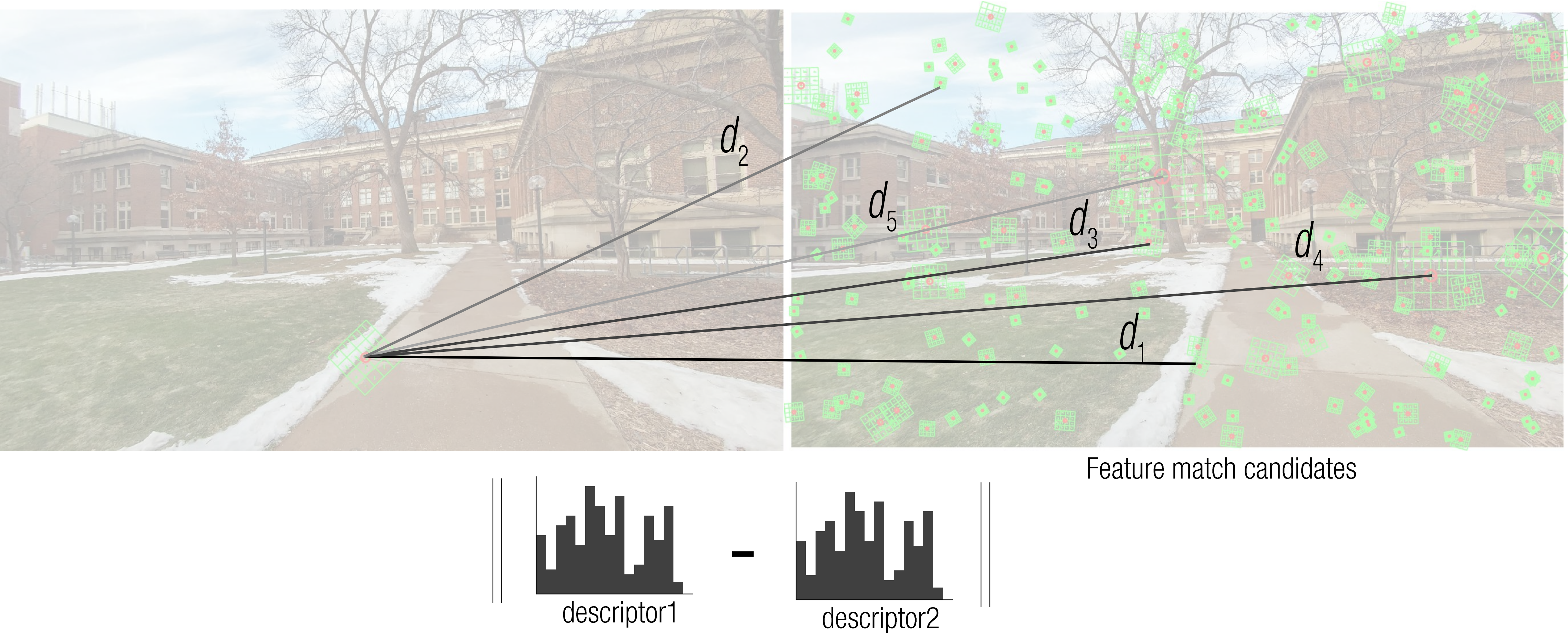
Local Scale Invariant Feature Transform (SIFT)



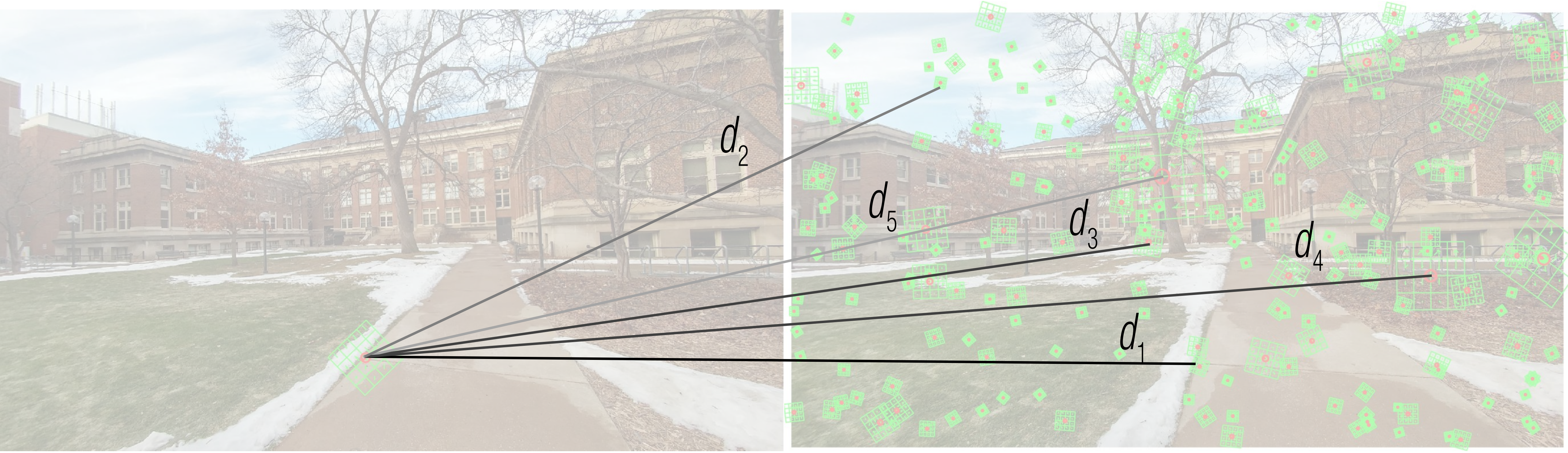
Feature match candidates



Nearest Neighbor Search



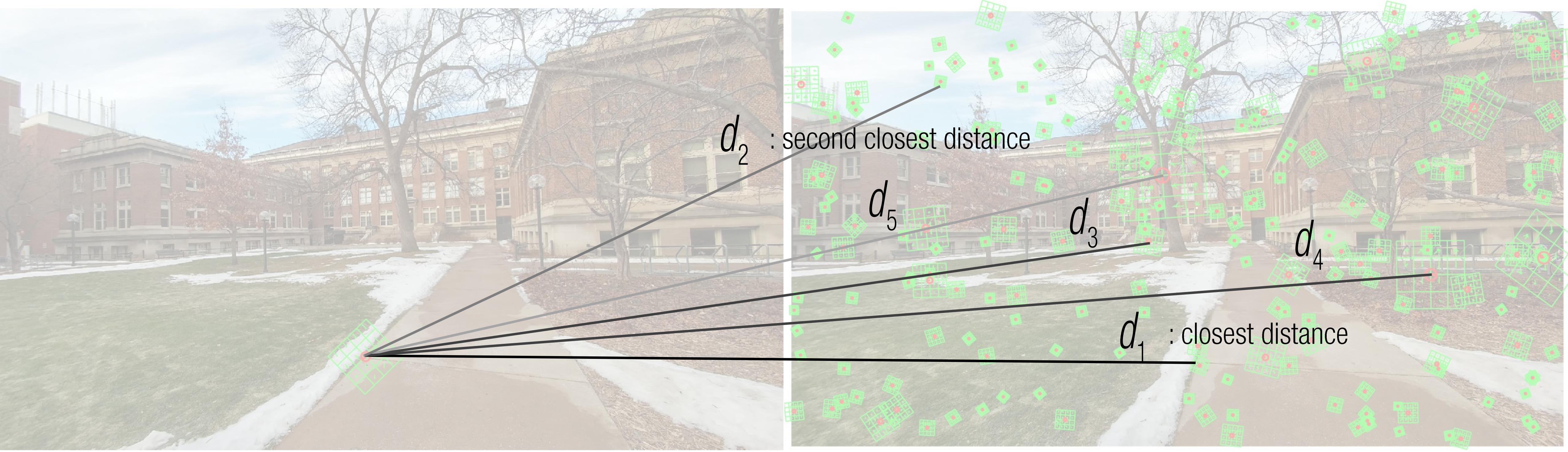
Nearest Neighbor Search



Feature match candidates

Discriminativity: how is the feature point unique?

Nearest Neighbor Search w/ Ratio Test

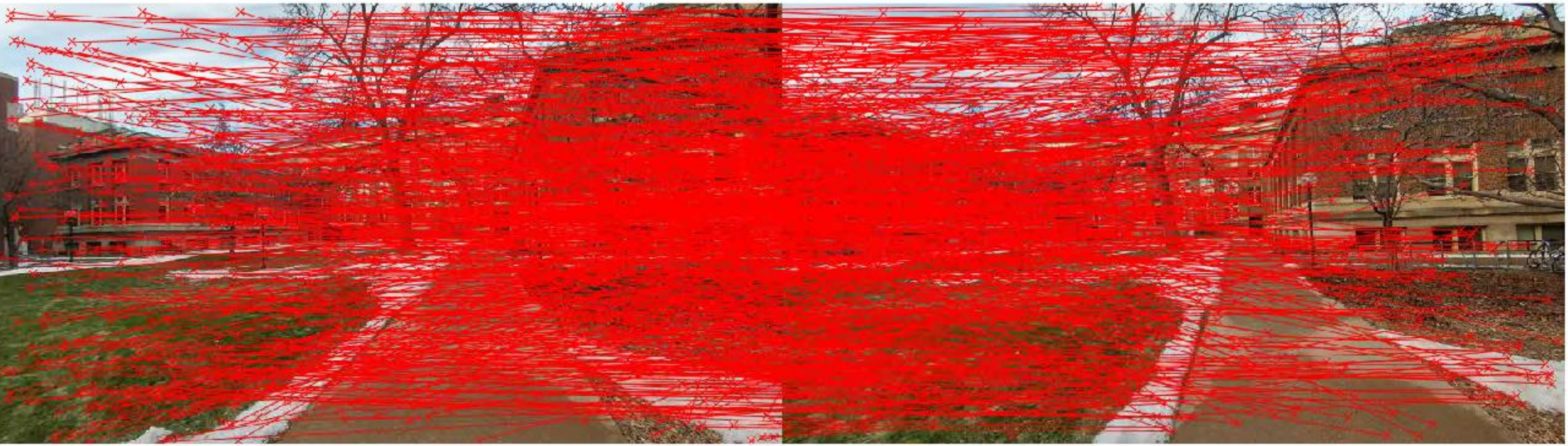


Feature match candidates

Discriminativity: how is the feature point unique?

$$\frac{d_1}{d_2} < 0.7$$

Nearest Neighbor Search w/o Ratio Test

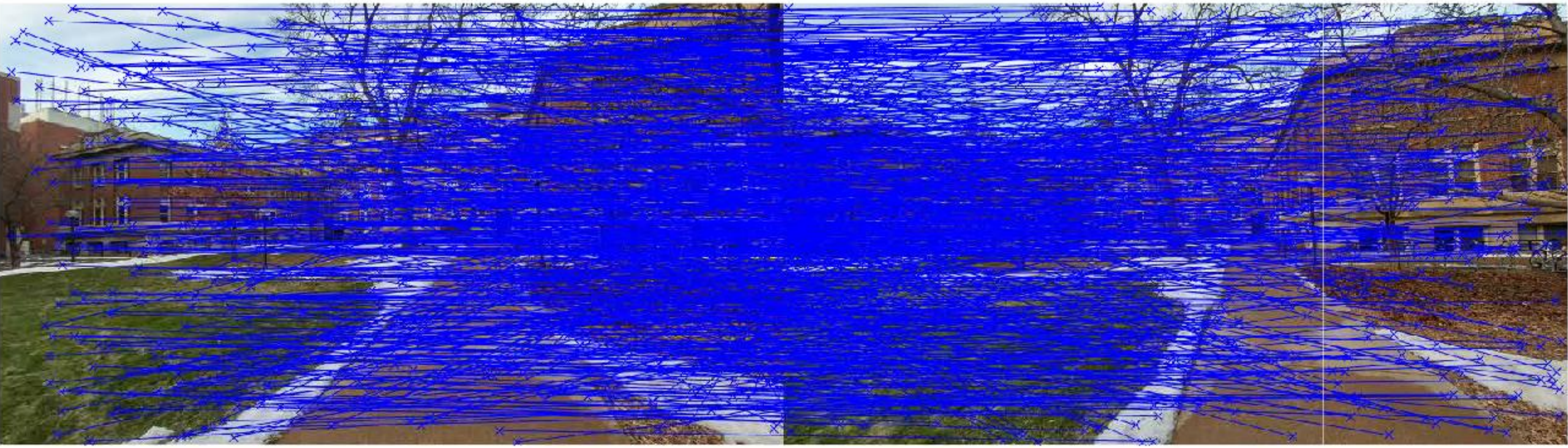


Left image→right image

Nearest Neighbor Search w/ Ratio Test

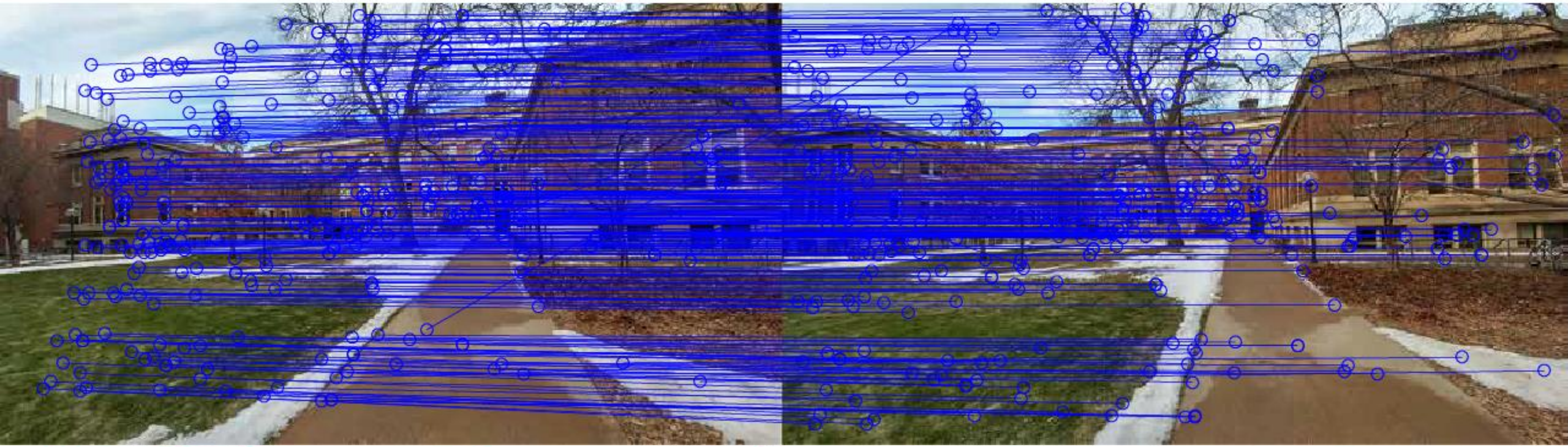
Left image→right image

Nearest Neighbor Search w/o Ratio Test



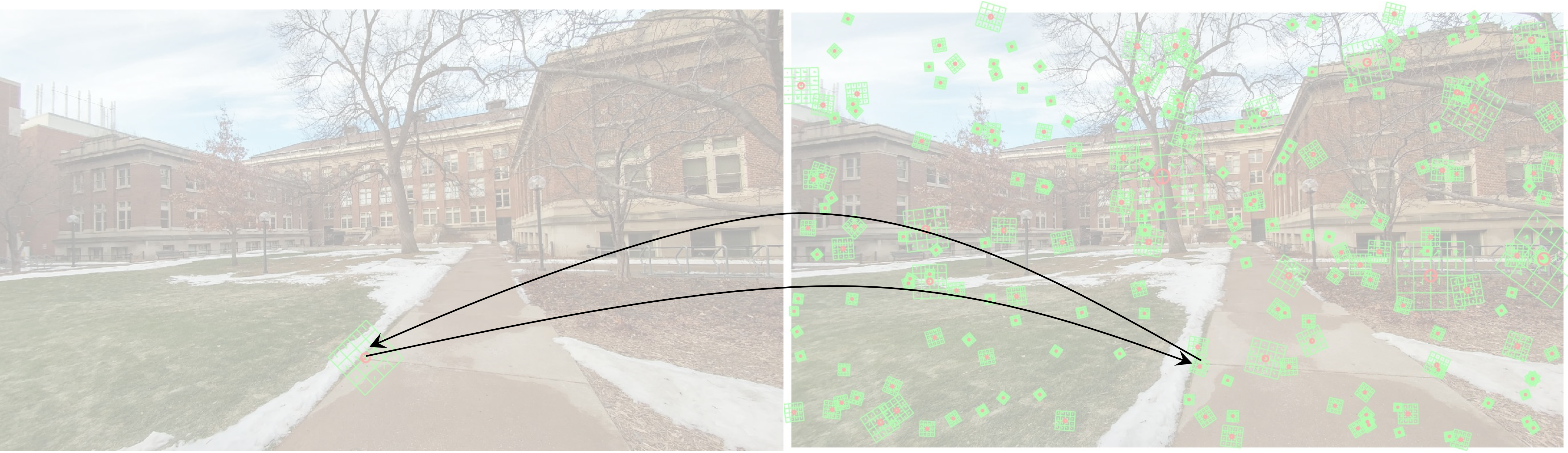
Left image←right image

Nearest Neighbor Search w/ Ratio Test



Left image←right image

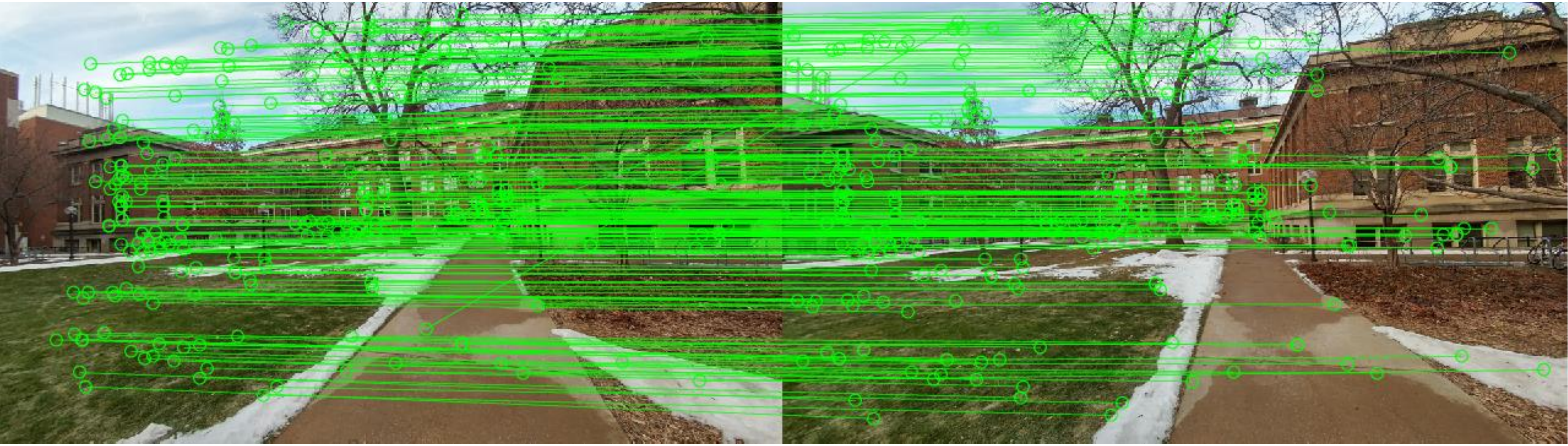
Bi-directional Consistency Check



Feature match candidates

Consistency: would a feature match correspond to each other?

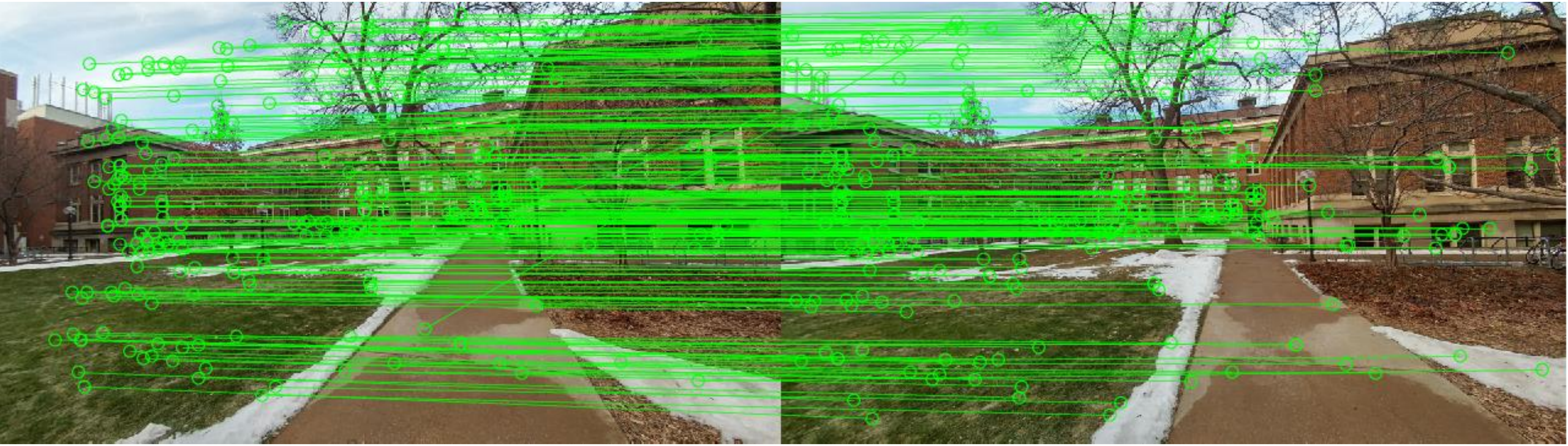
Bi-directional Consistency Check



RANSAC: Random Sample Consensus

Fundamental Matrix Computation: Linear Least Squares

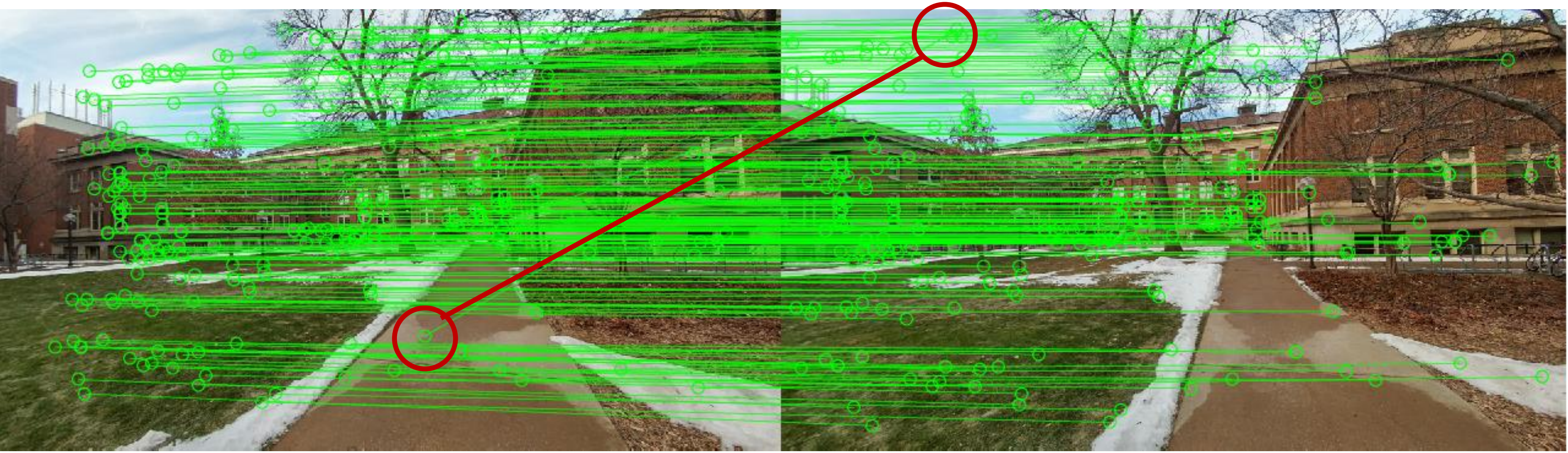
$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$



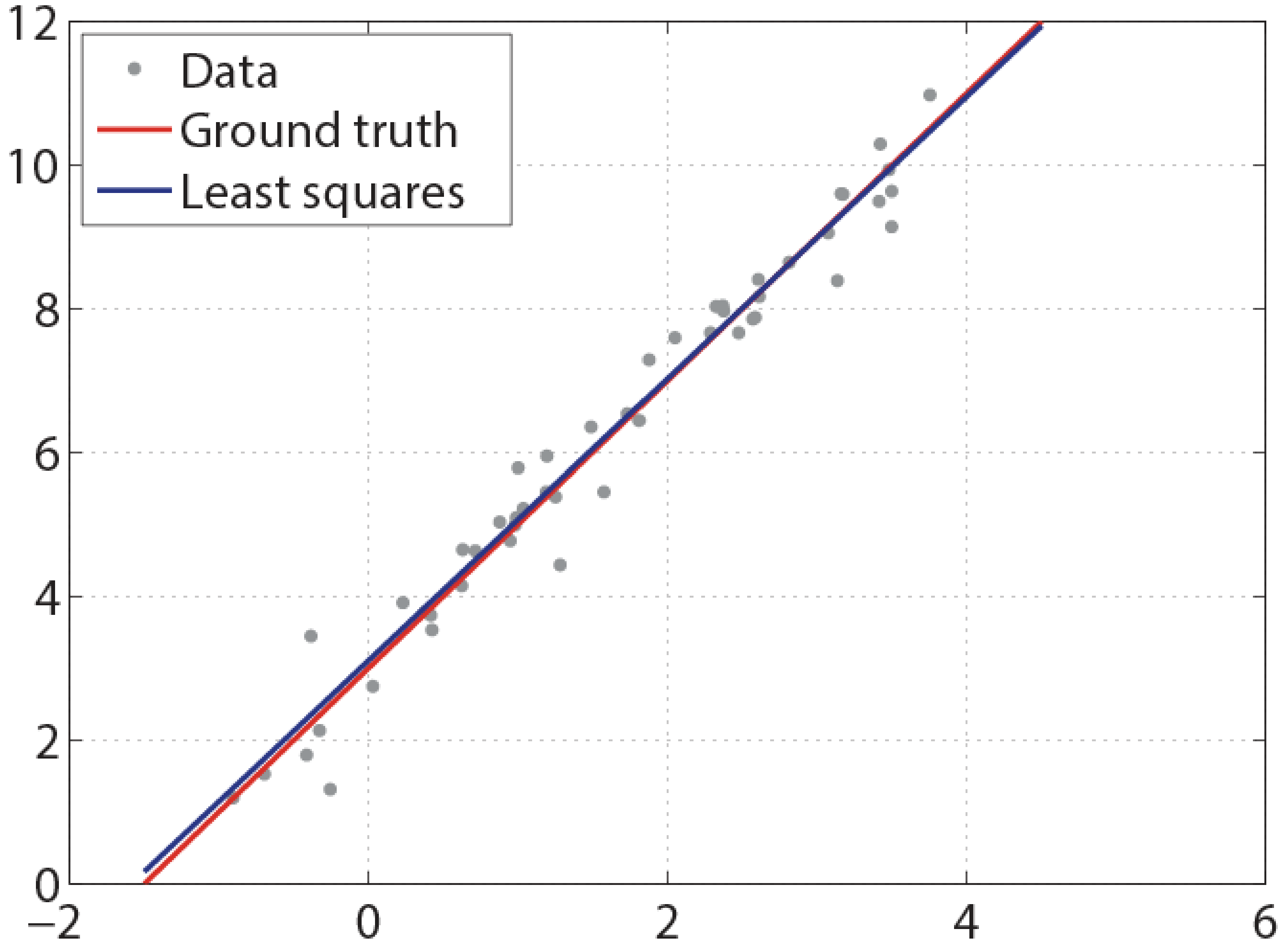
Fundamental Matrix Computation: Linear Least Squares

$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \underbrace{0_{m \times 1}}_{\mathbf{0}}$$

Outlier?



Recall: Line Fitting (Ax=b)

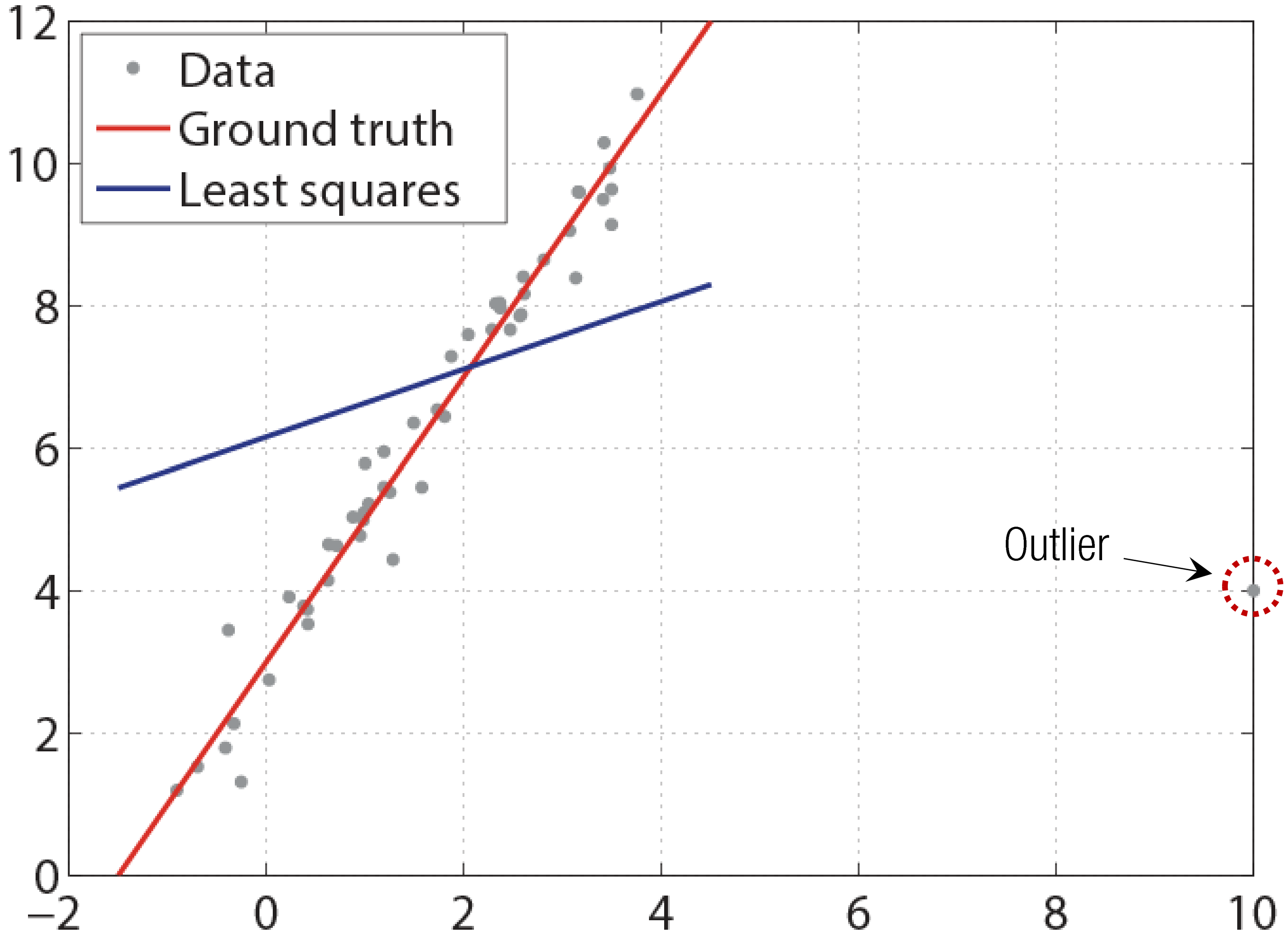


$$\underbrace{\begin{bmatrix} u_1 & 1 \\ u_2 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} m \\ d \end{bmatrix}}_{\mathbf{x}} \approx \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}}_{\mathbf{b}}$$

$$\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{A}^T \mathbf{b}$$

$$\mathbf{x} = \left[\mathbf{A}^T \mathbf{A} \right]^{-1} \mathbf{A}^T \mathbf{b}$$

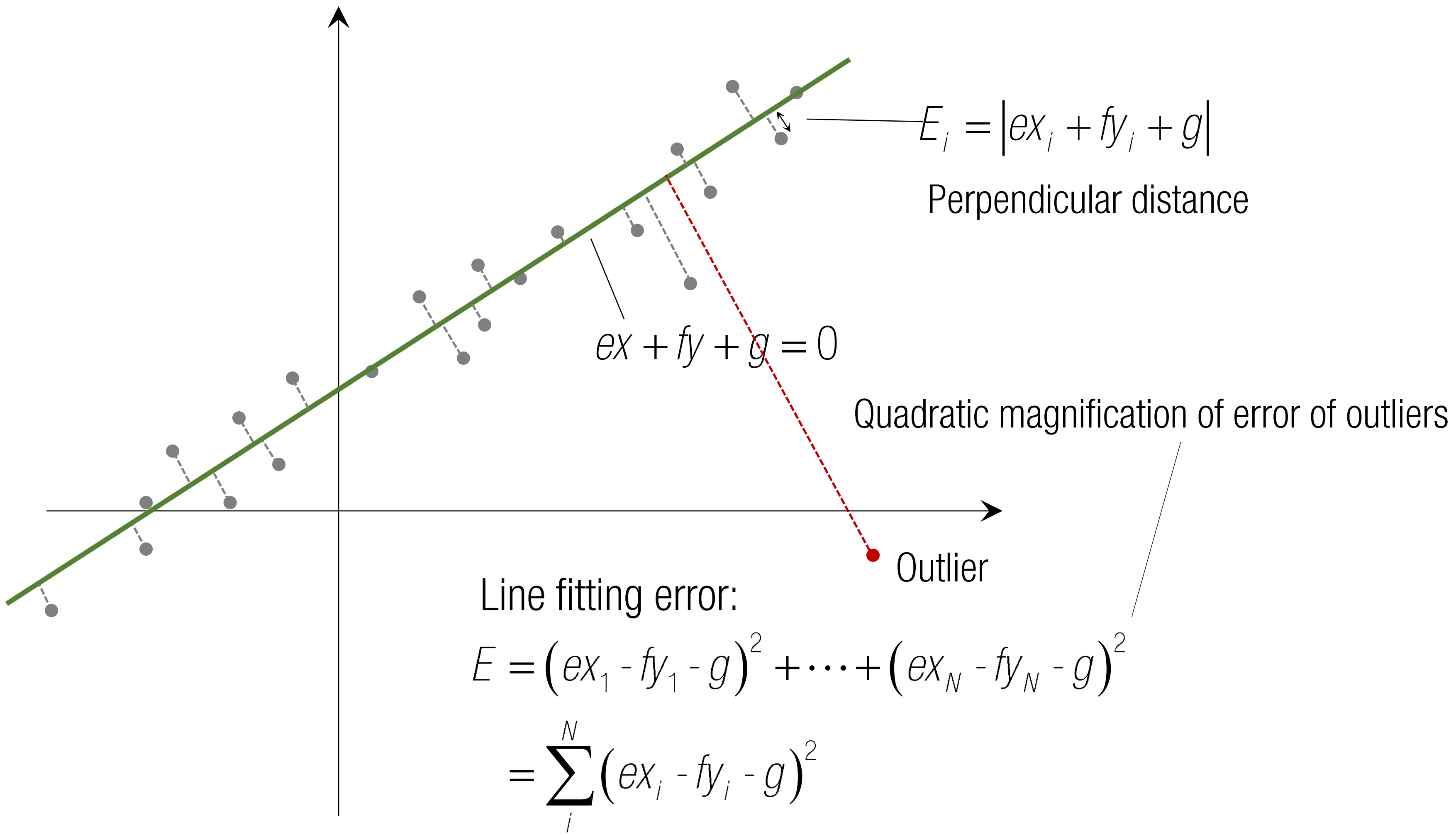
Outlier



$$\underbrace{\begin{bmatrix} u_1 & 1 \\ u_2 & 1 \\ \vdots & \vdots \\ u_n & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} m \\ d \end{bmatrix}}_{x} \approx \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}}_{b}$$

$$A^T A x = A^T b$$

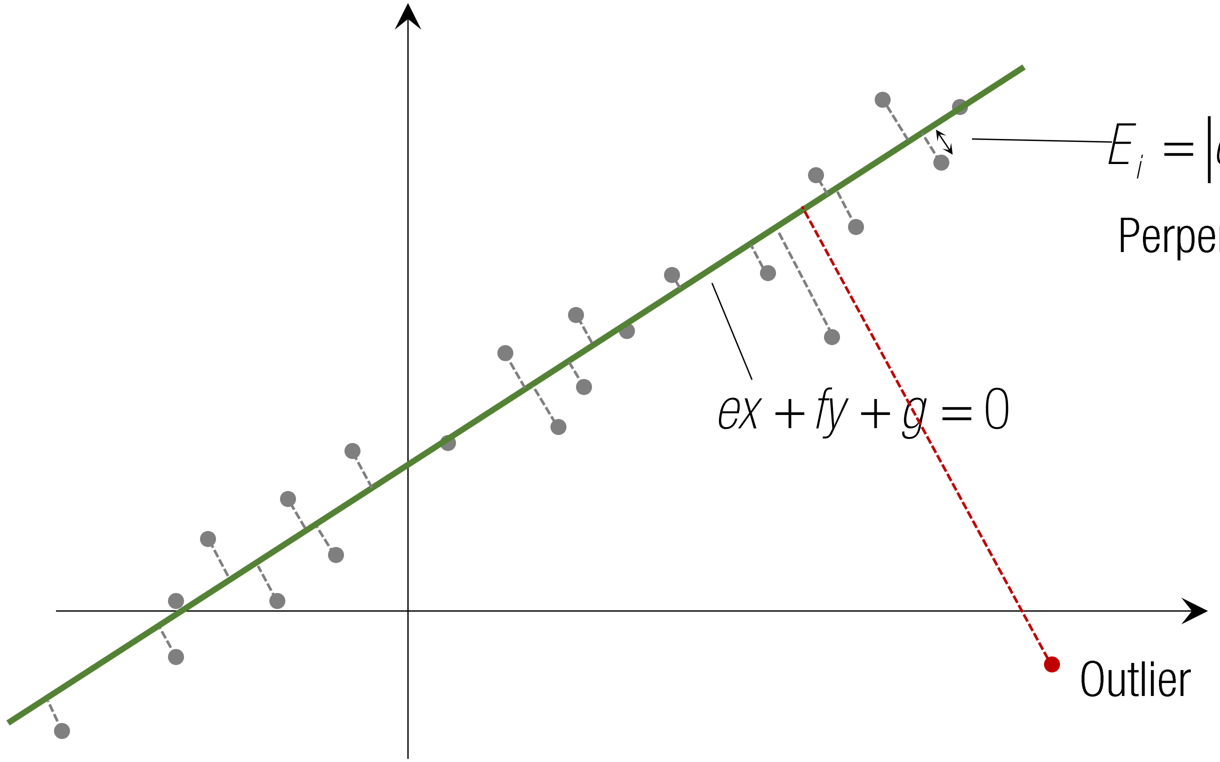
$$x = \left[A^T A \right]^{-1} A^T b$$



Line fitting error:

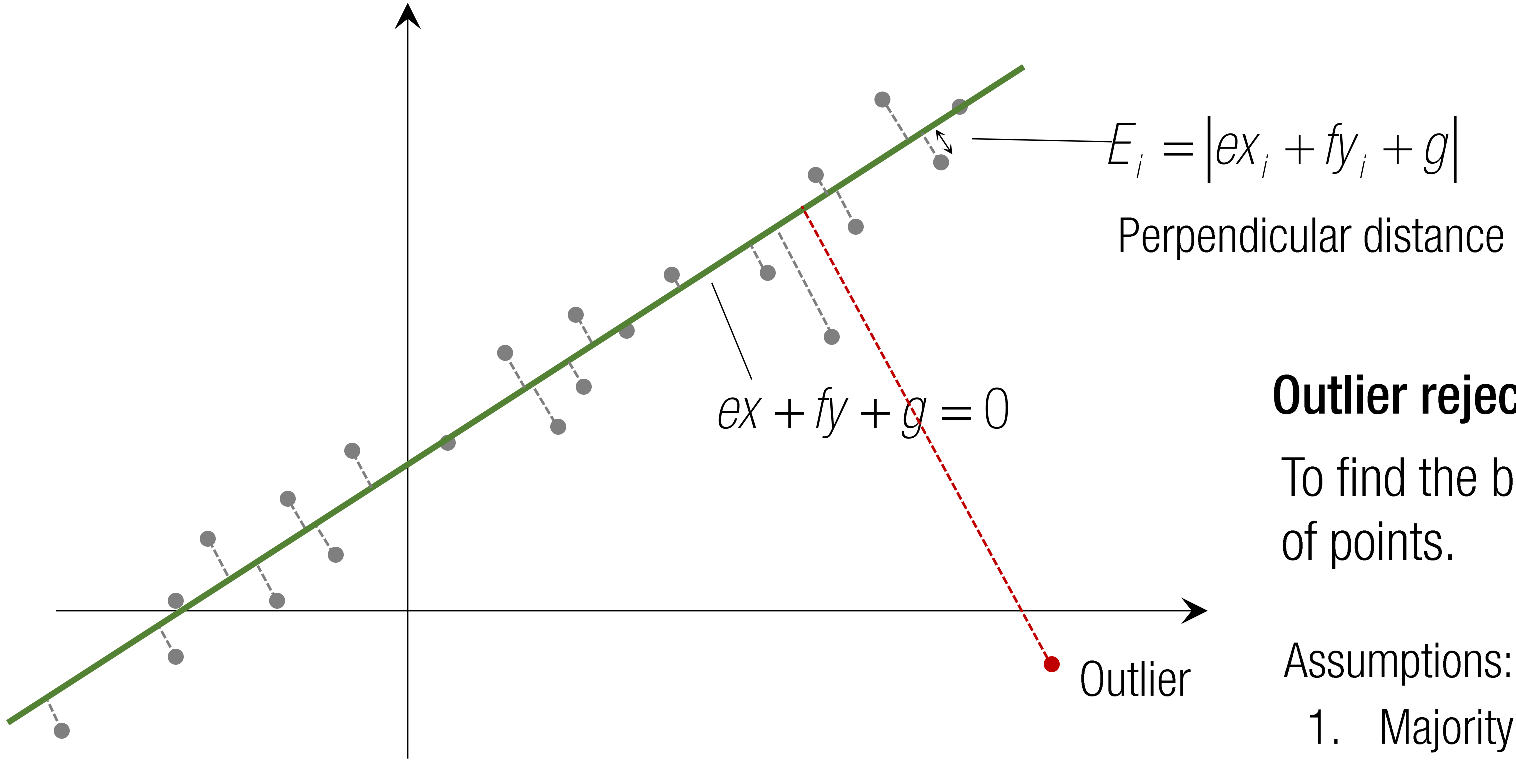
$$E = (ex_1 - fy_1 - g)^2 + \cdots + (ex_N - fy_N - g)^2$$

$$= \sum_{i}^{N} (ex_i - fy_i - g)^2$$



Outlier rejection strategy:

To find the best line that explanes the maximum number of points.



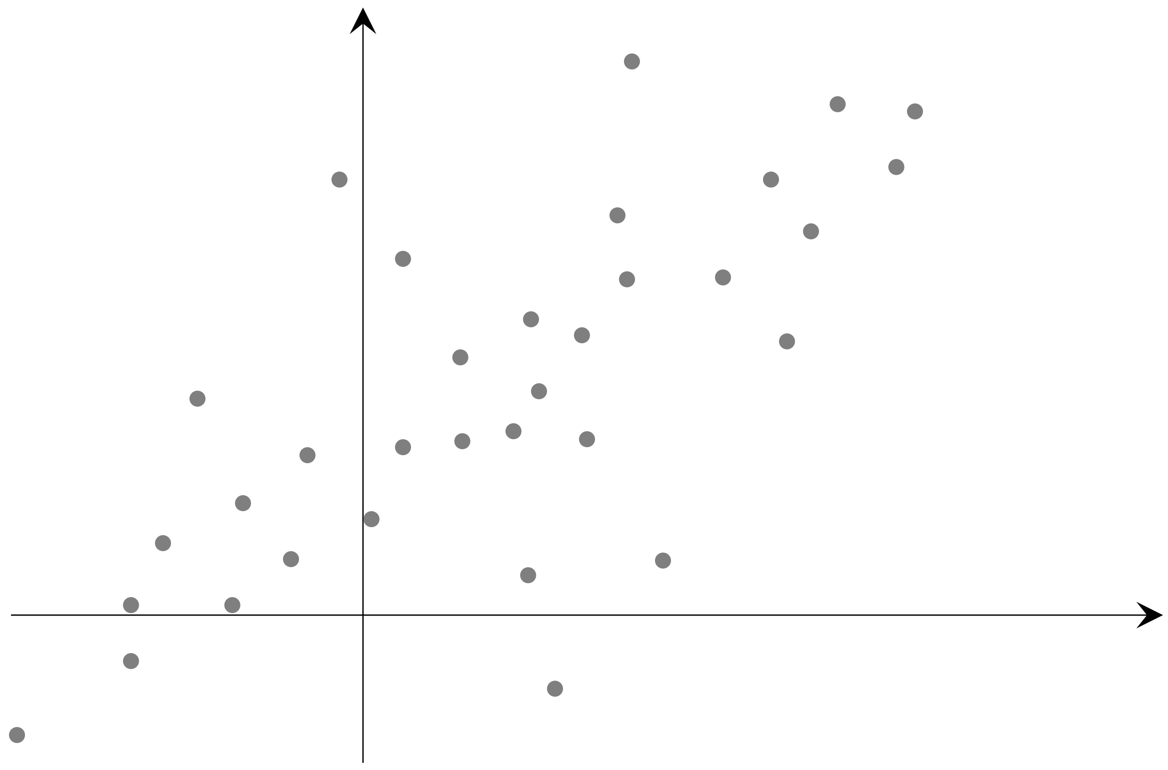
Outlier rejection strategy:

To find the best line that explanes the maximum number of points.

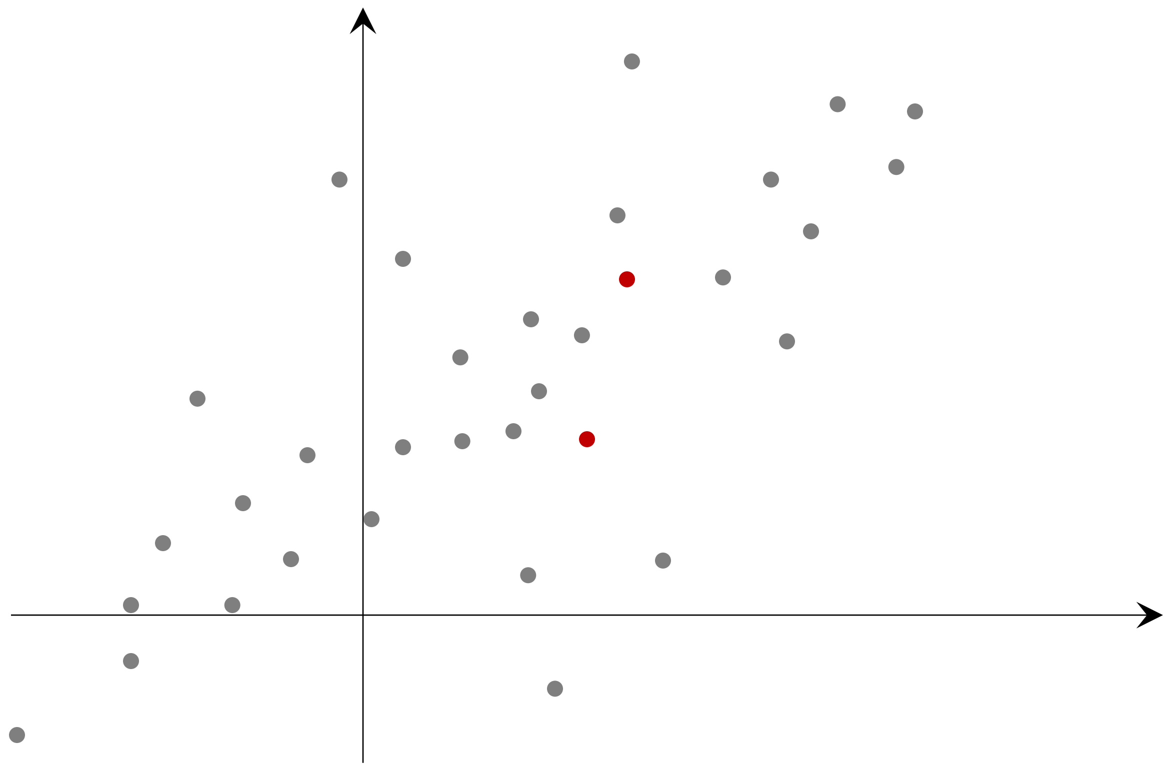
Assumptions:

1. Majority of good samples agree with the underlying model (good apples are same and simple.).

2. Bad samples does not consistently agree with a single model (all bad apples are different and complicated.).

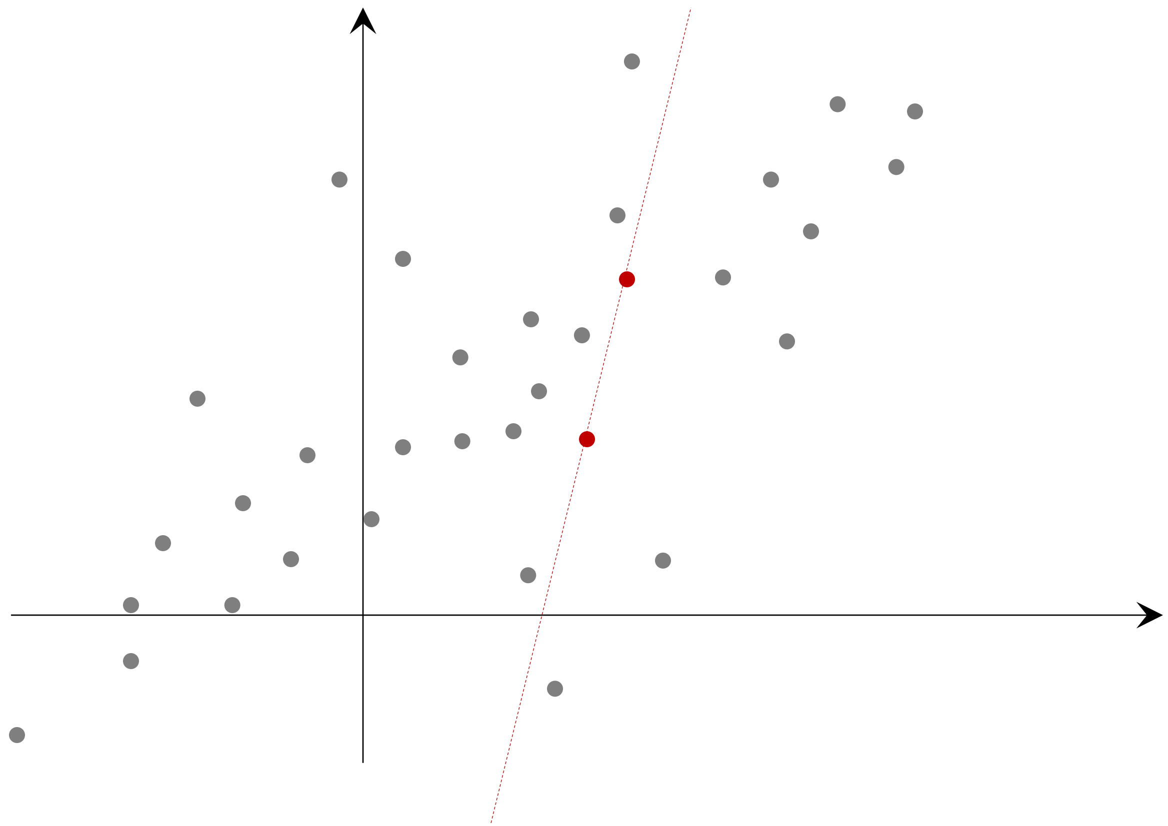


RANSAC: Random Sample Consensus



1. Random sampling

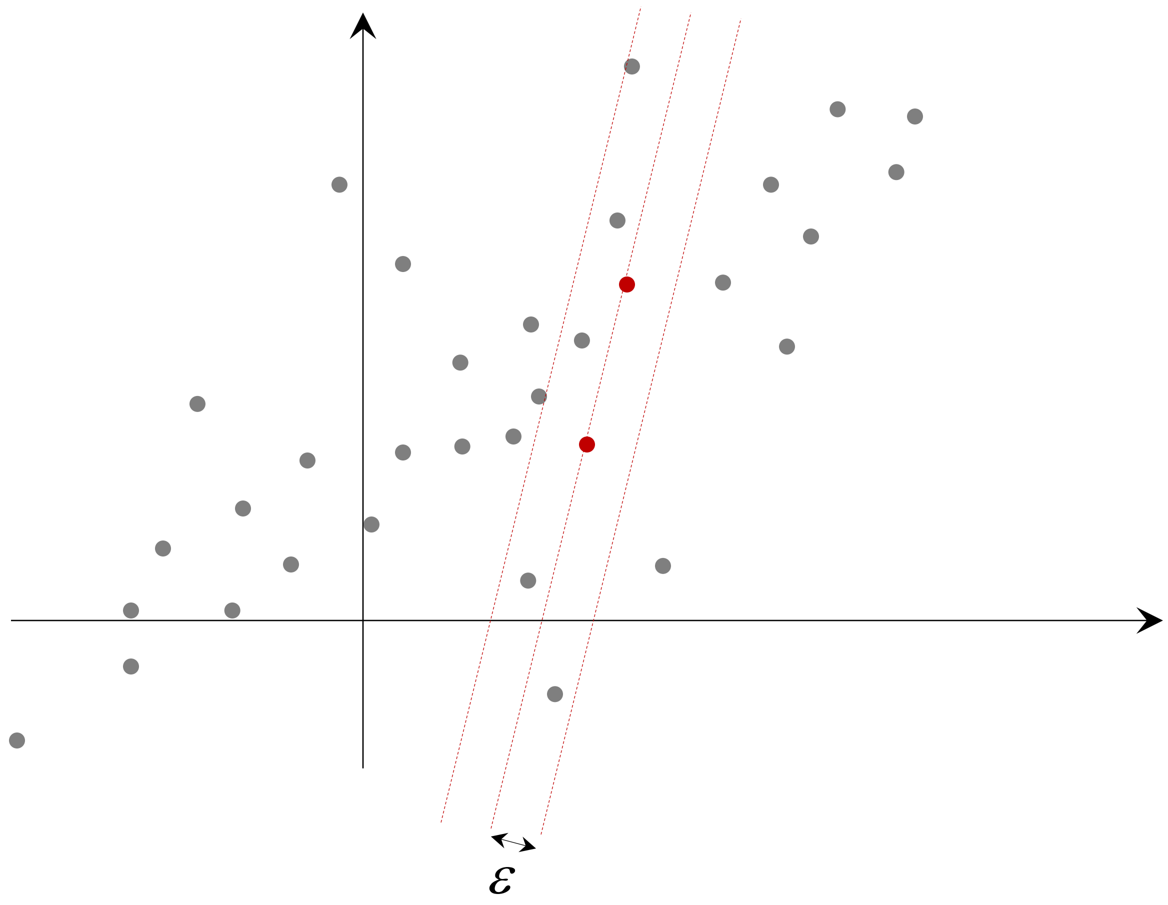
RANSAC: Random Sample Consensus



1. Random sampling

2. Model building

RANSAC: Random Sample Consensus

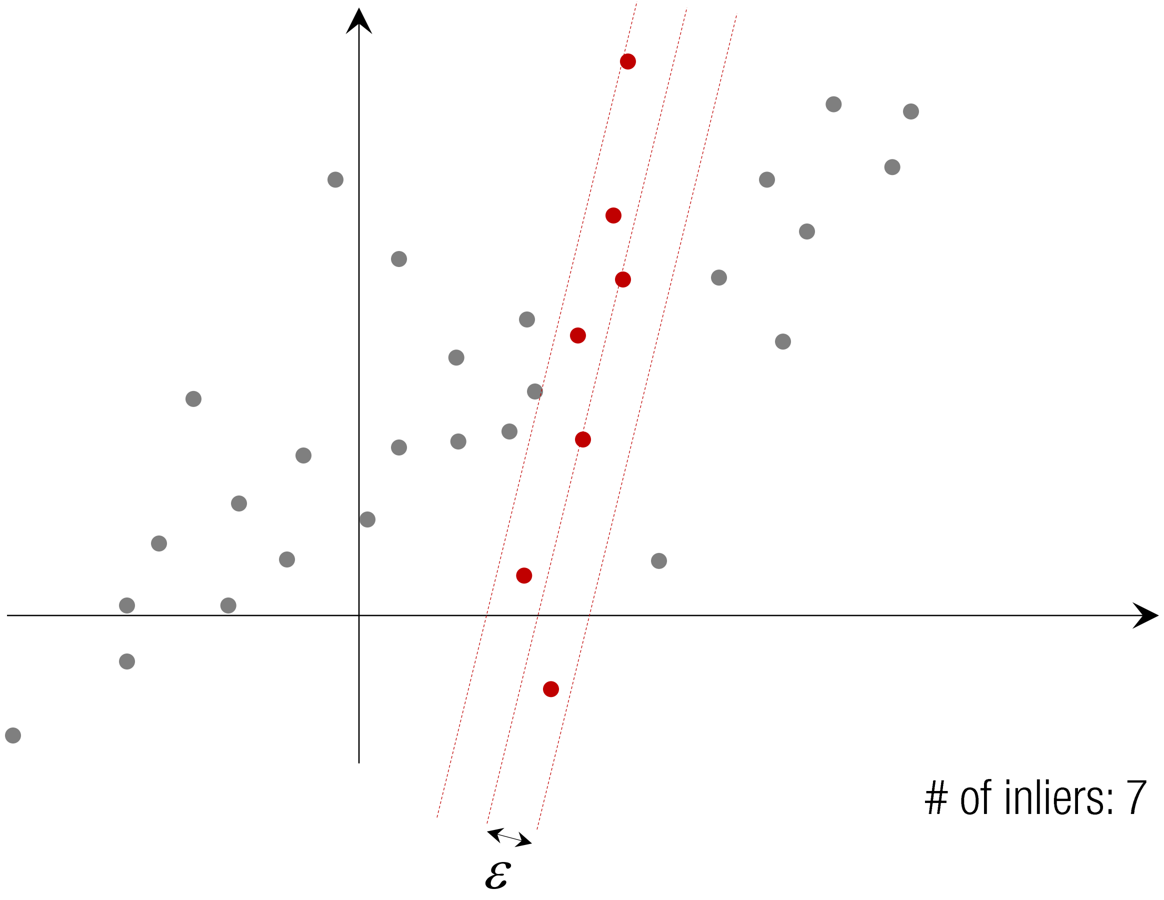


1. Random sampling

2. Model building

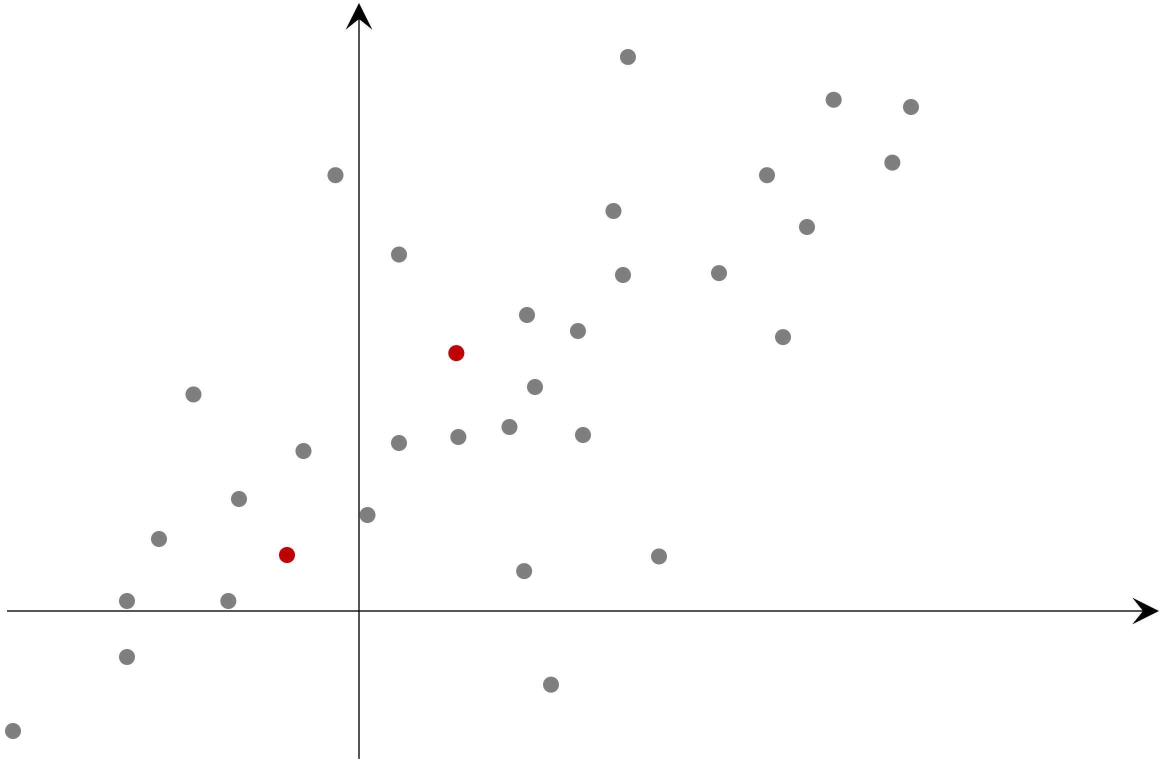
3. Thresholding

RANSAC: Random Sample Consensus



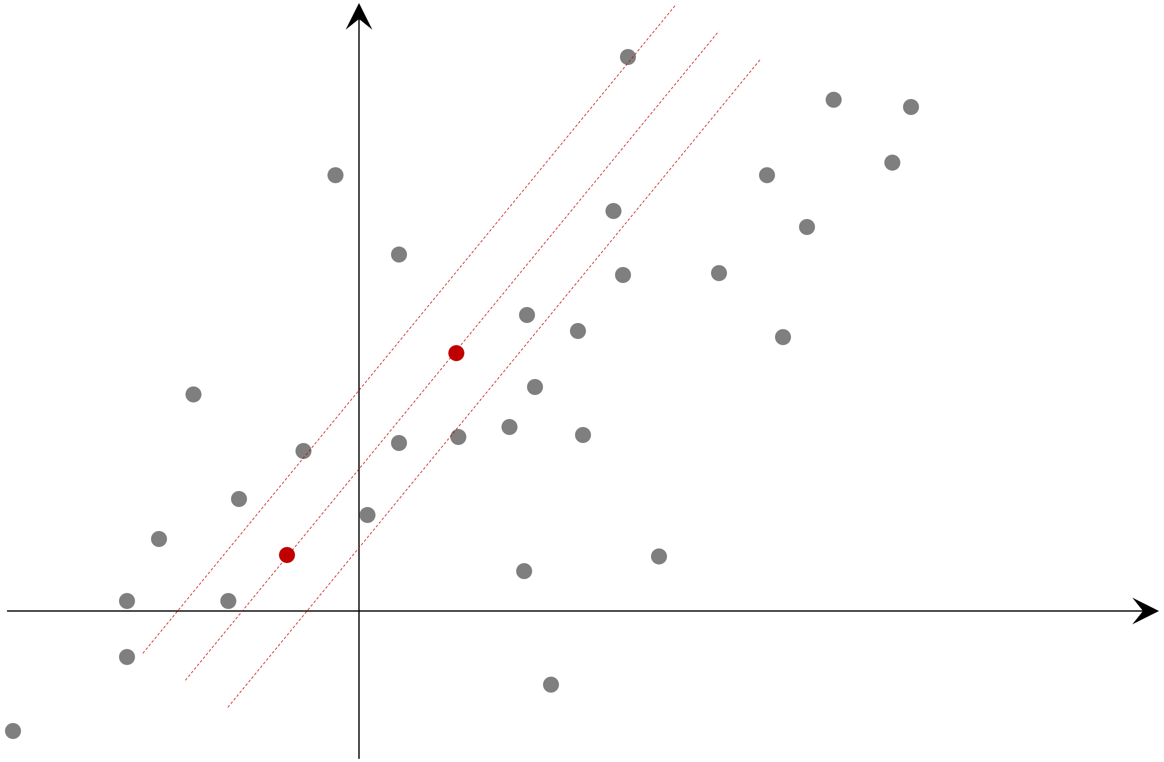
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



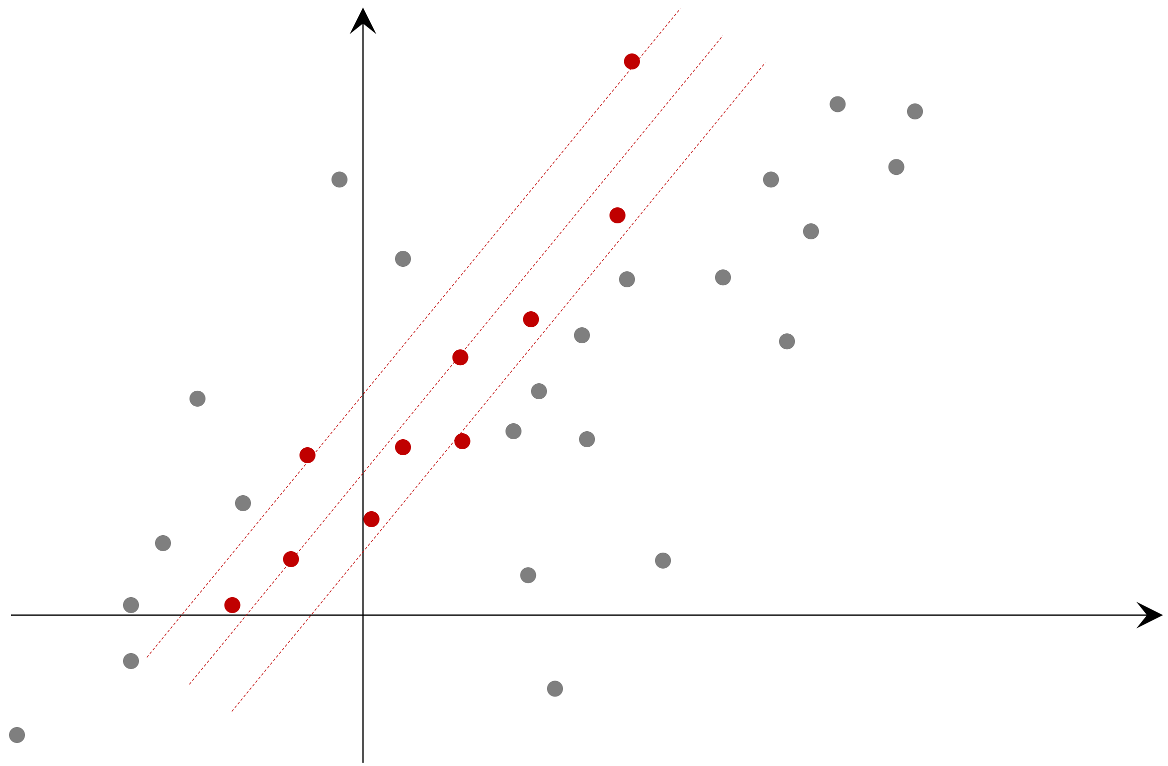
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



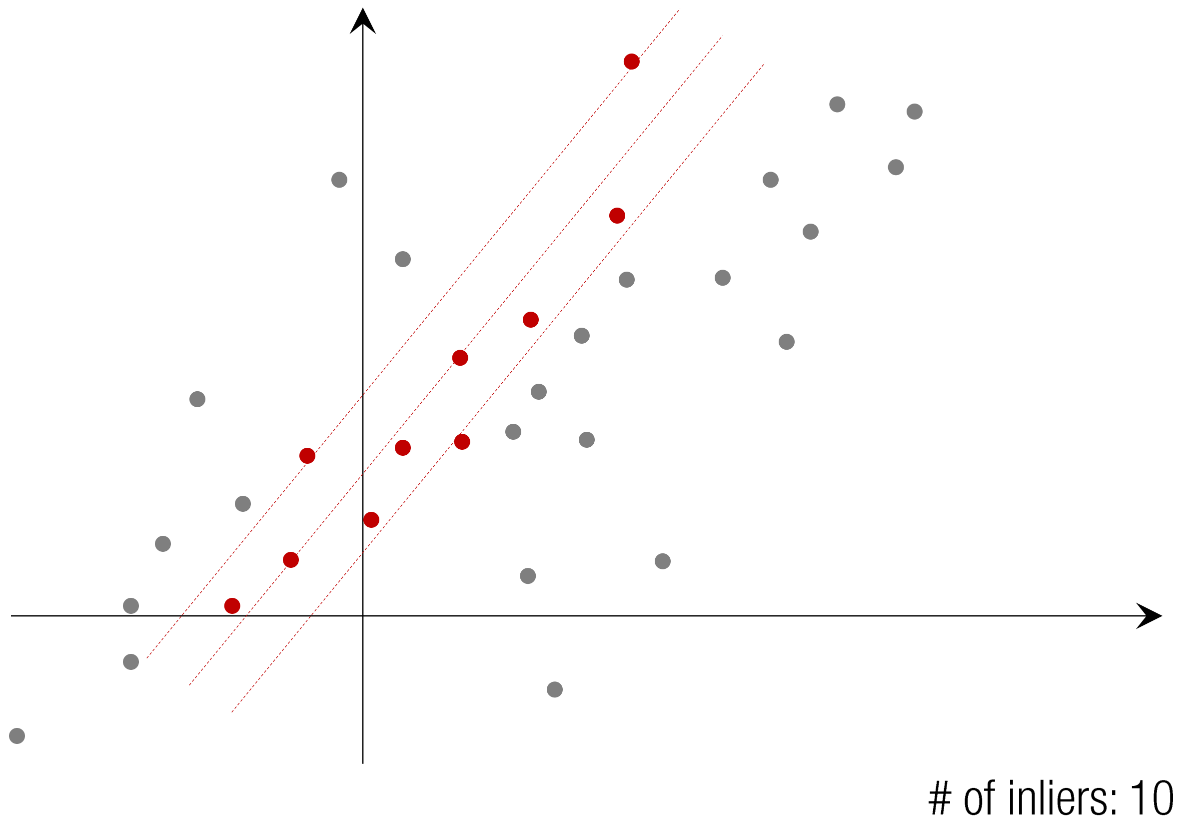
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



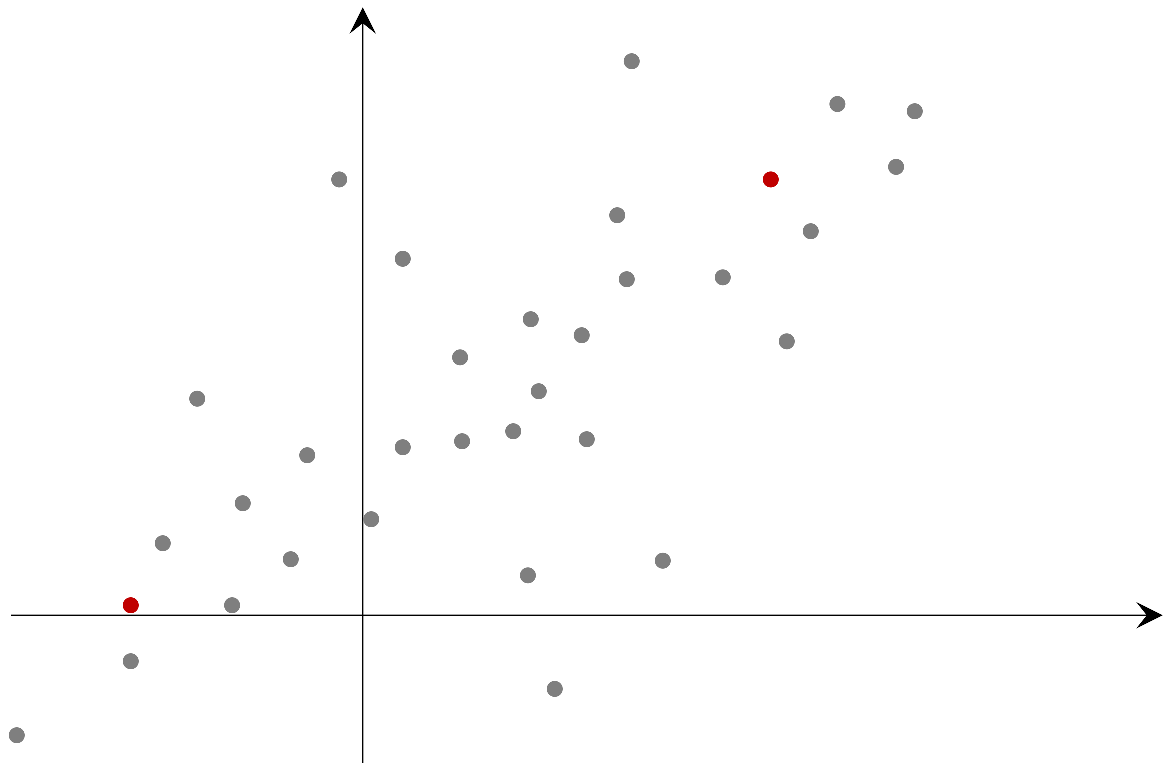
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



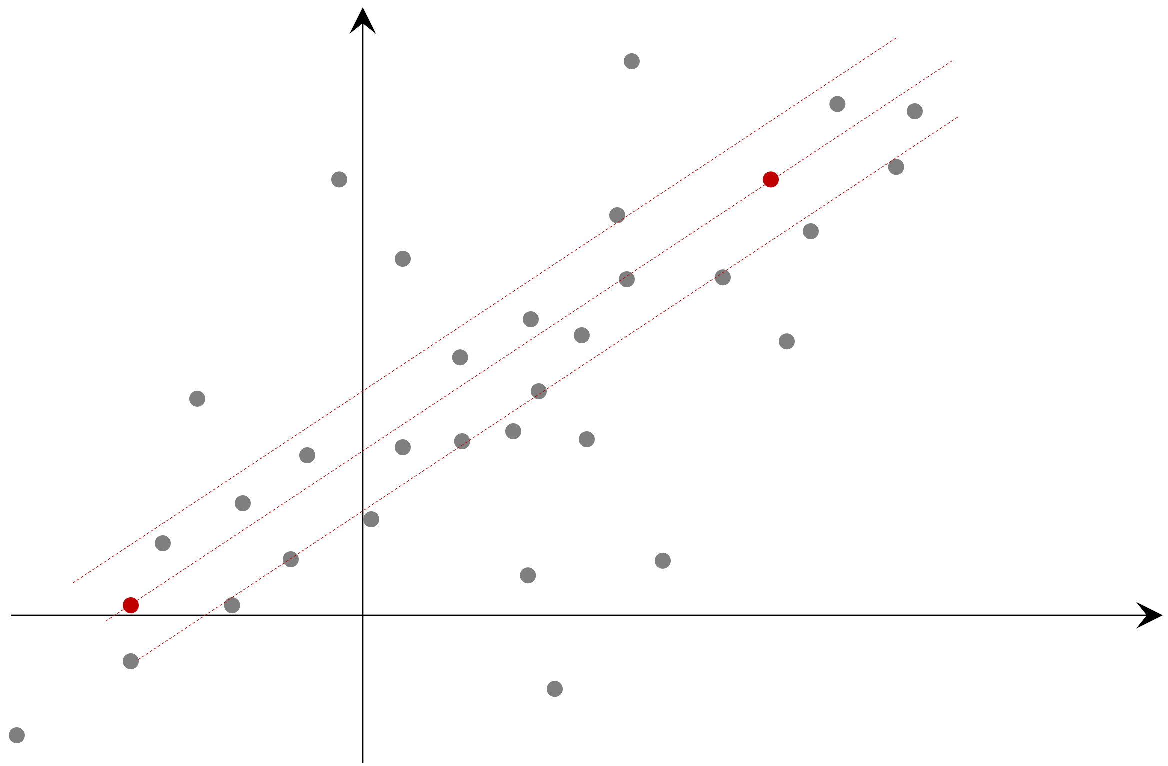
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



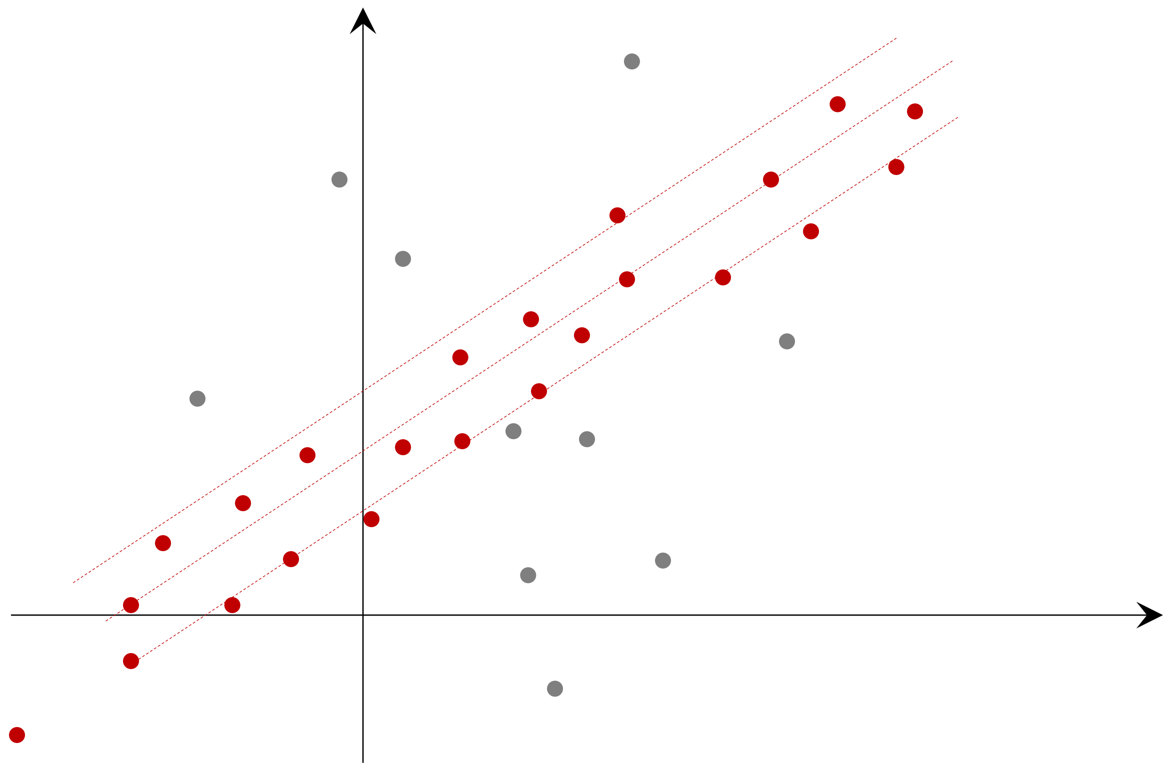
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



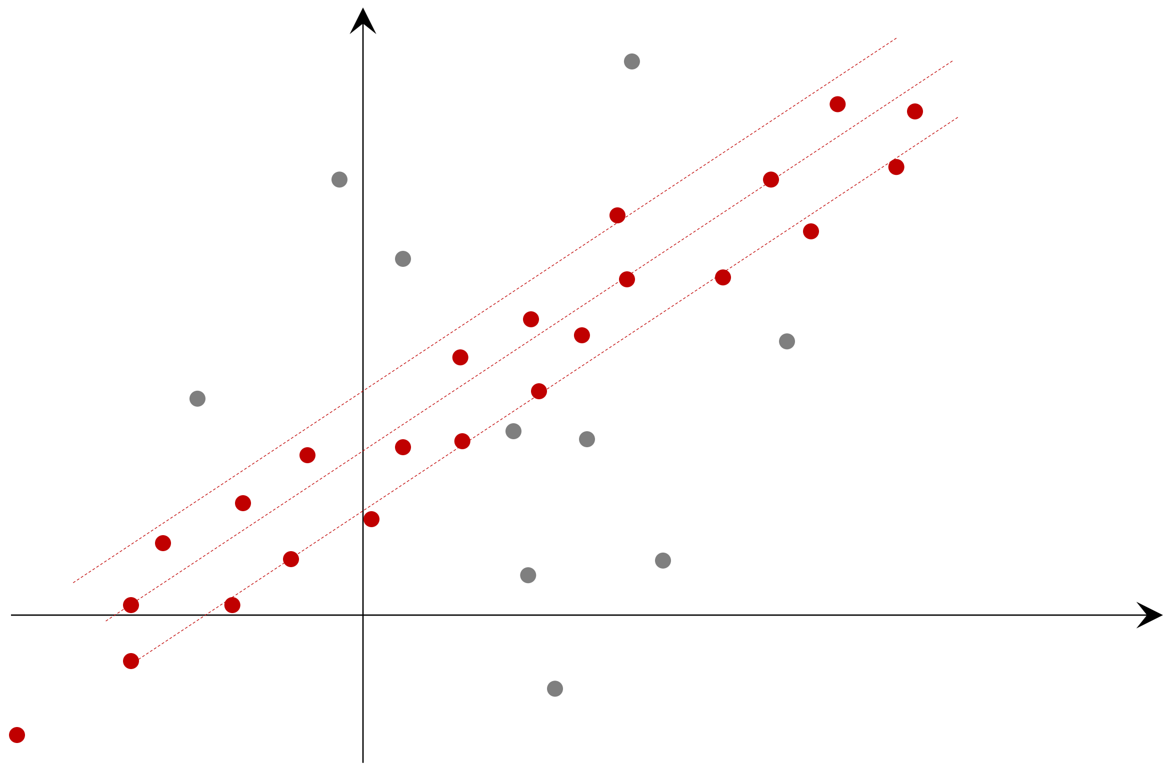
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus

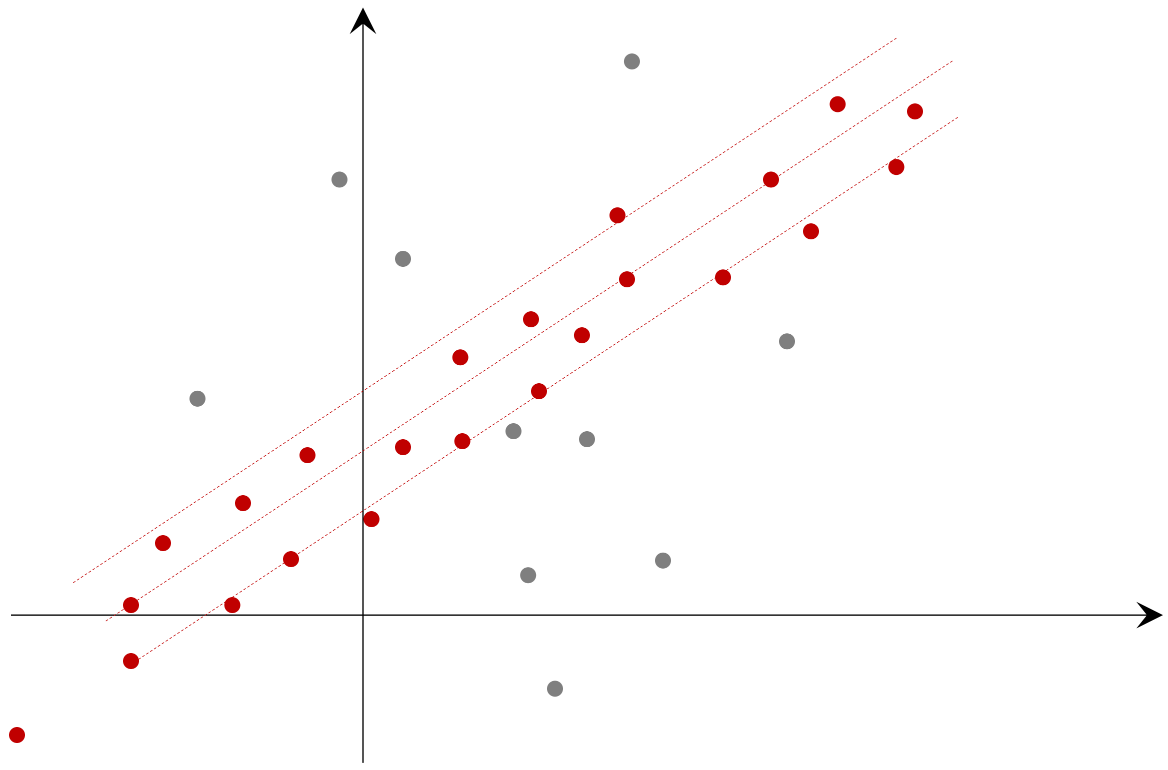


1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

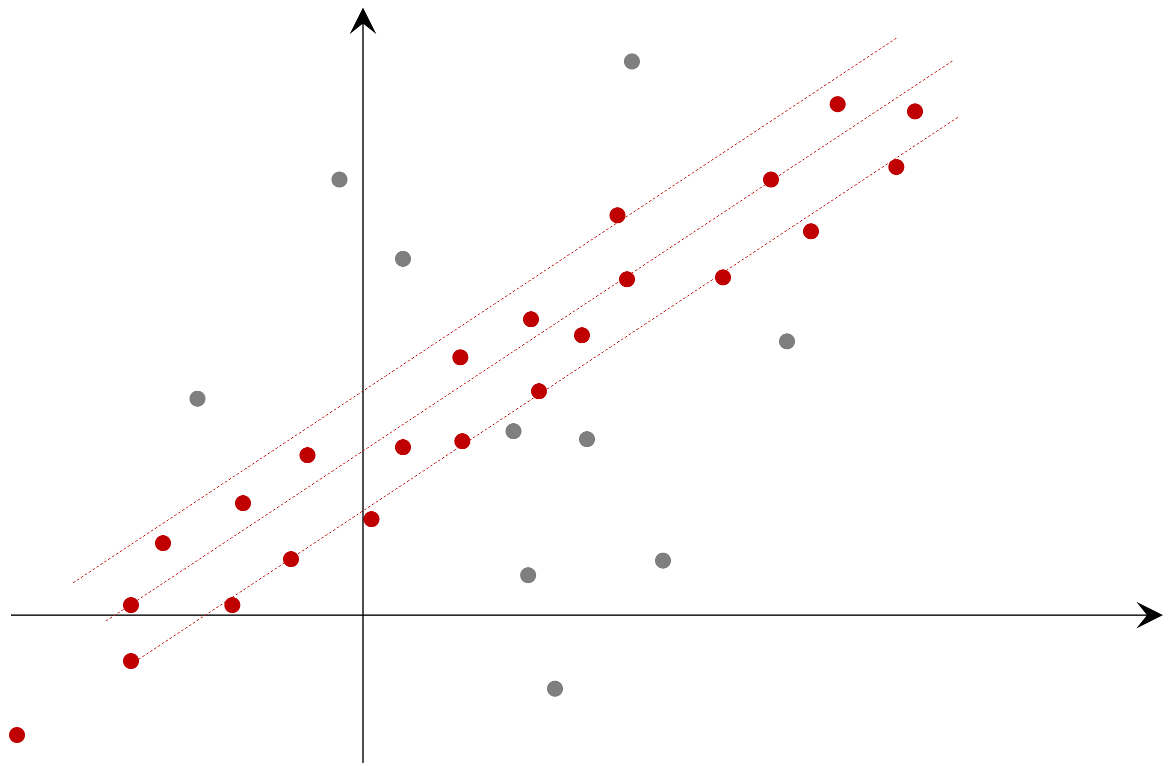
# of inliers: 23

Maximum number of inliers

RANSAC: Random Sample Consensus



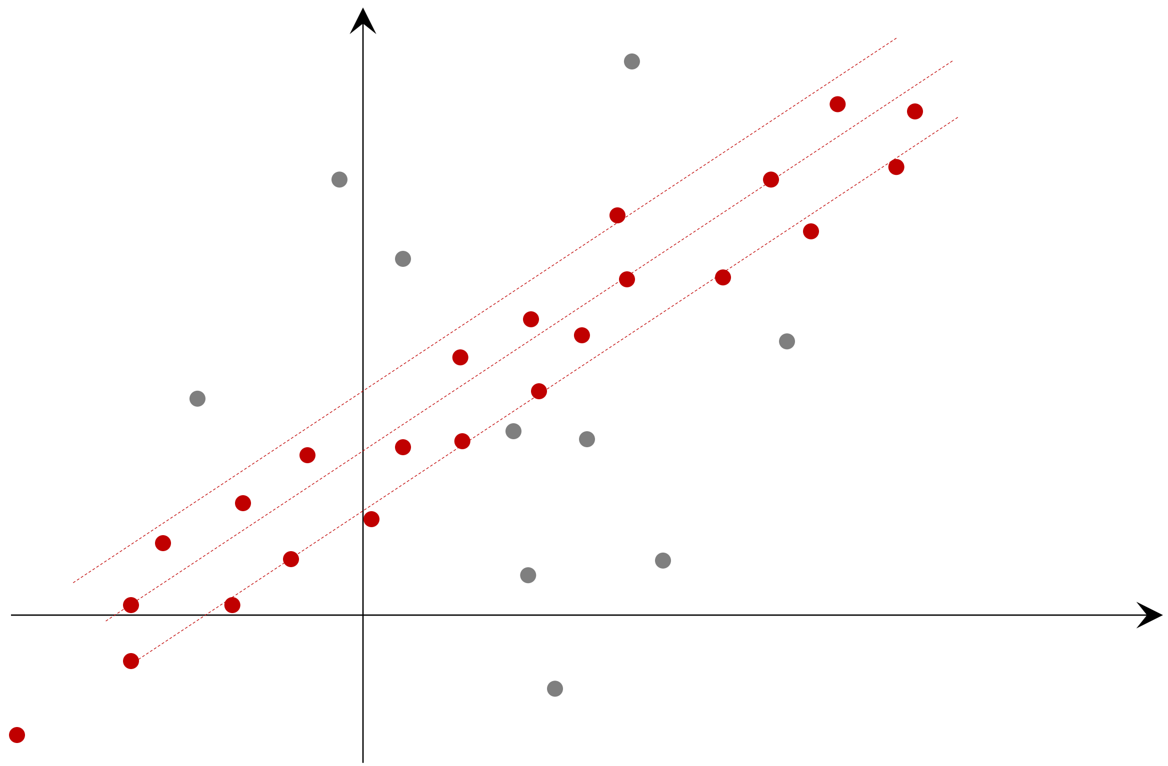
Required number of iterations with *p* success rate:



Required number of iterations with *p* success rate:

Probability of choosing an inlier:

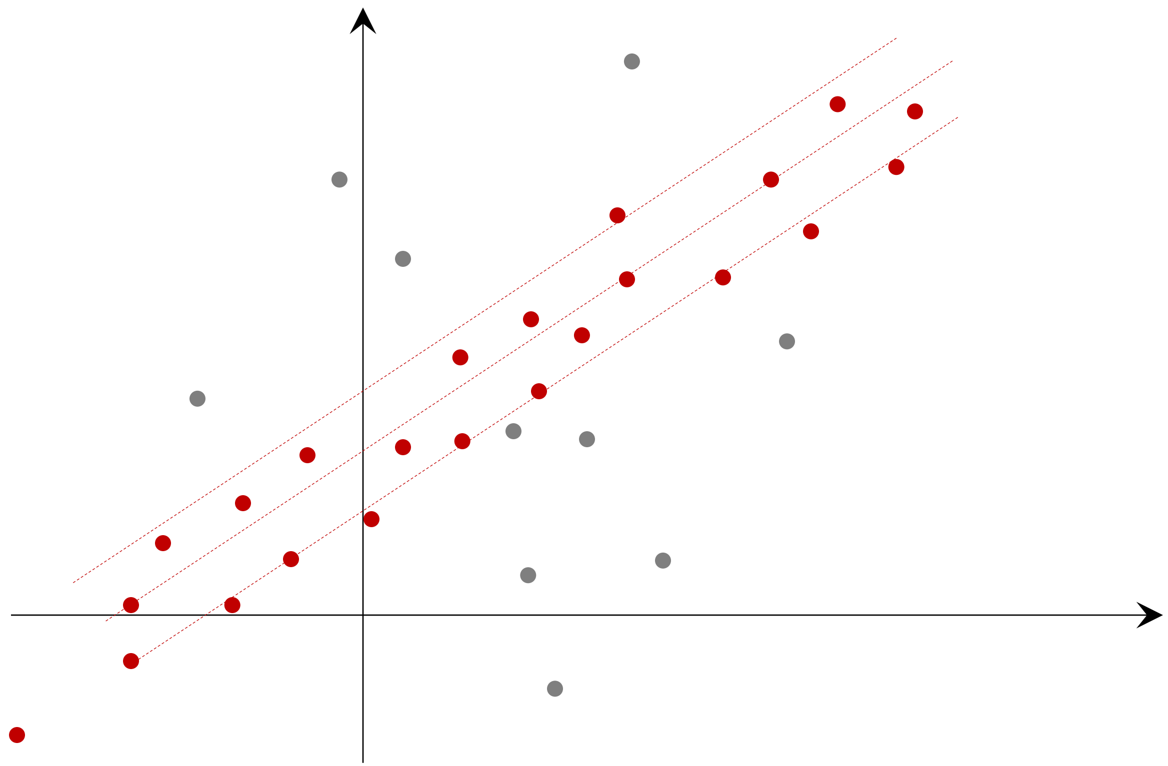
$$w = \frac{\text{\# of inliers}}{\text{\# of samples}}$$



Required number of iterations with *p* success rate:

Probability of choosing an inlier: $w = \dfrac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

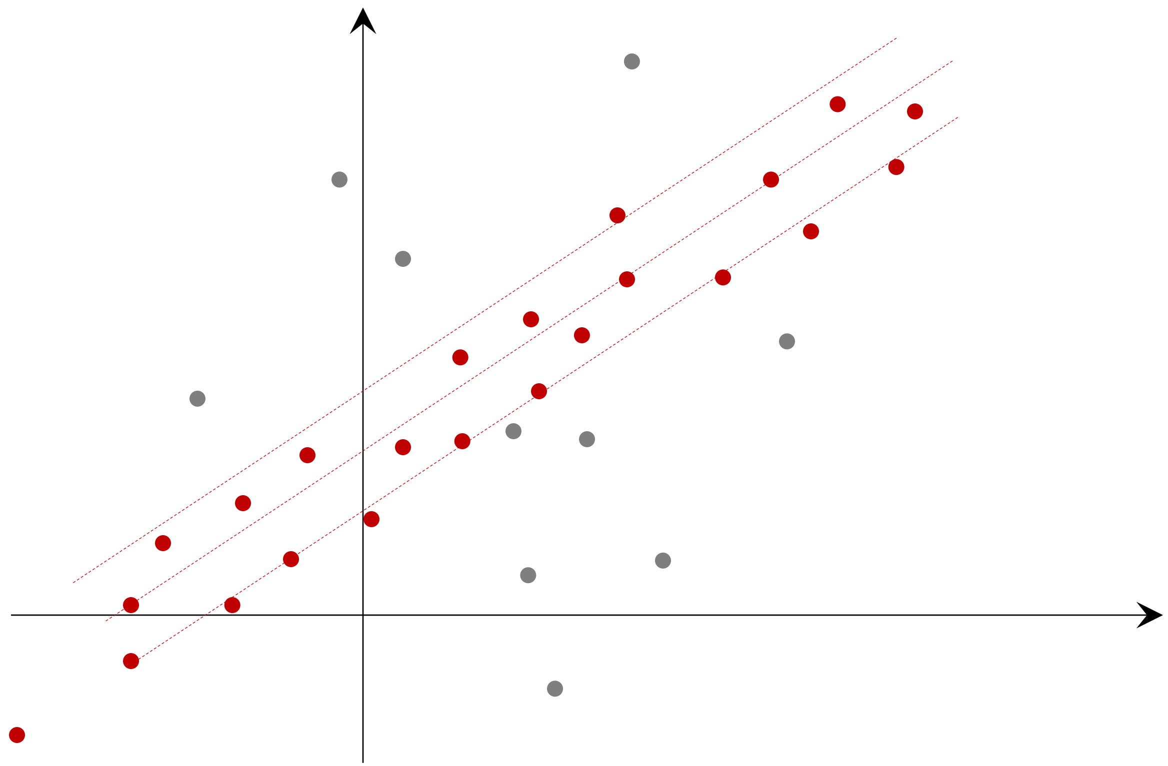


Required number of iterations with p success rate:

Probability of choosing an inlier: $$w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during k iterations: $\left(1 - w^n\right)^k$



Required number of iterations with *p* success rate:

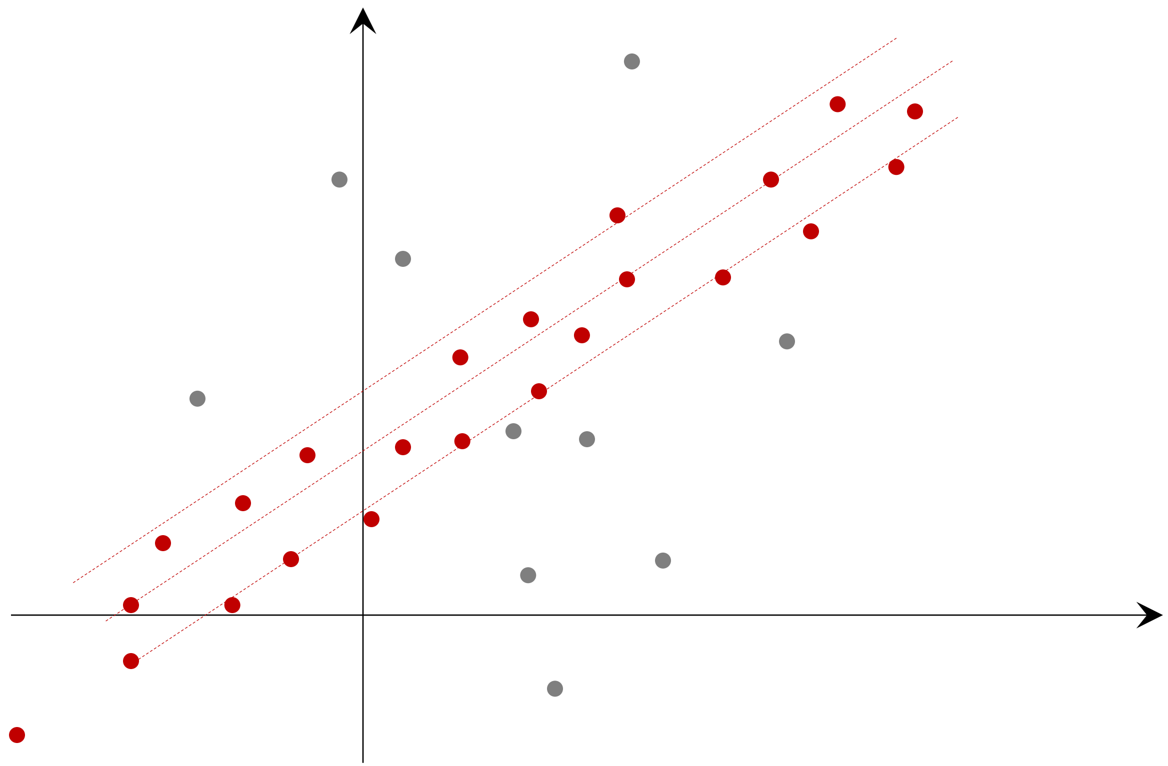
Probability of choosing an inlier: $w = \dfrac{\#\text{ of inliers}}{\#\text{ of samples}}$

Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during *k* iterations: $\left(1-w^n\right)^k$

$\left(1-w^n\right)^k = 1-p$ where *p* is desired RANSAC success rate.

$$k = \frac{\log\left(1-p\right)}{\log\left(1-w^n\right)}$$



Required number of iterations with *p* success rate:

$$k = \frac{\log(1-p)}{\log(1-w^n)} \quad \text{where} \quad w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$$

Probability of choosing an inlier: $w = \frac{\#\text{ of inliers}}{\#\text{ of samples}}$

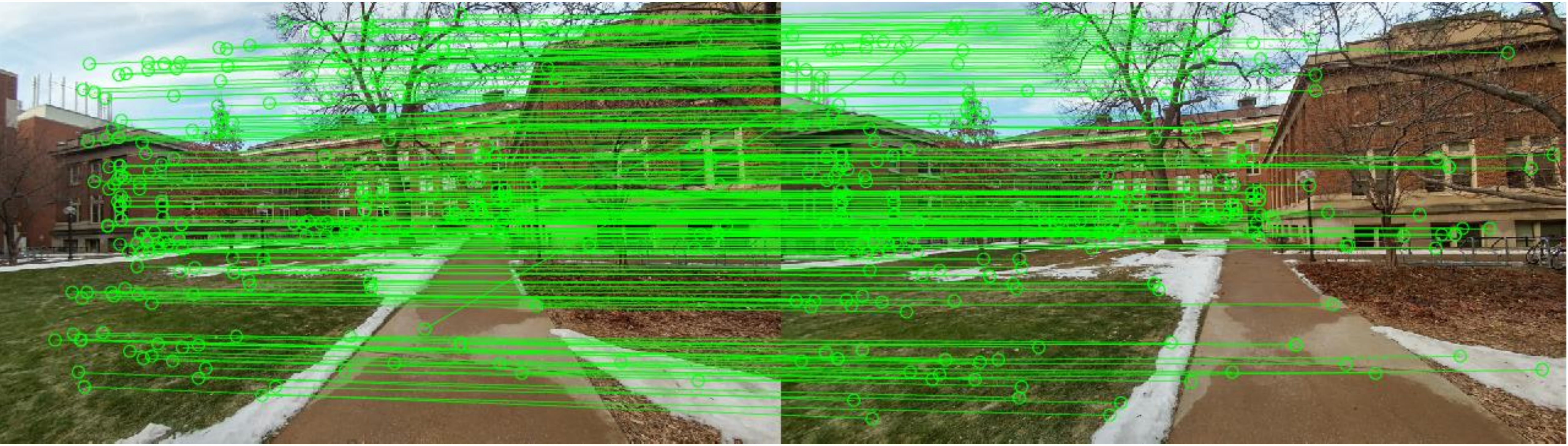
Probability of building a correct model: w^n where n is the number of samples to build a model.

Probability of not building a correct model during *k* iterations: $(1-w^n)^k$

$(1-w^n)^k = 1-p$ where *p* is desired RANSAC success rate.

$$k = \frac{\log(1-p)}{\log(1-w^n)}$$

Fundamental Matrix Computation via RANSAC



Fundamental Matrix Computation via RANSAC

$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ \vdots \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

Fundamental Matrix Computation via RANSAC

Epipolar line: $\mathbf{l_u} = \mathbf{F}\mathbf{u}$

Distance: $$d = \frac{\left|au_x + bu_y + c\right|}{\sqrt{a^2 + b^2}} = \frac{\left|\mathbf{F}\mathbf{u}\right|}{\sqrt{(\mathbf{F}_{1,:}\mathbf{u})^2 + (\mathbf{F}_{2,:}\mathbf{u})^2}}$$

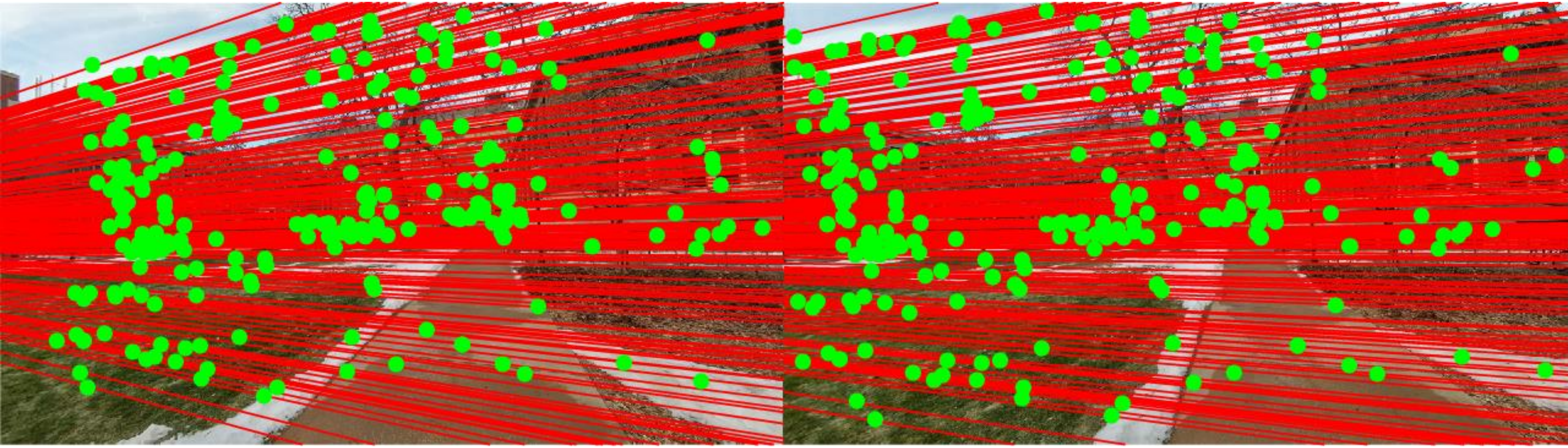
Fundamental Matrix Computation via RANSAC

# of inliers: 65 out of 260

Fundamental Matrix Computation via RANSAC

# of inliers: 65 out of 260

Fundamental Matrix Computation via RANSAC



# of inliers: 186 out of 260

Four Camera Pose Config. From Essential Matrix