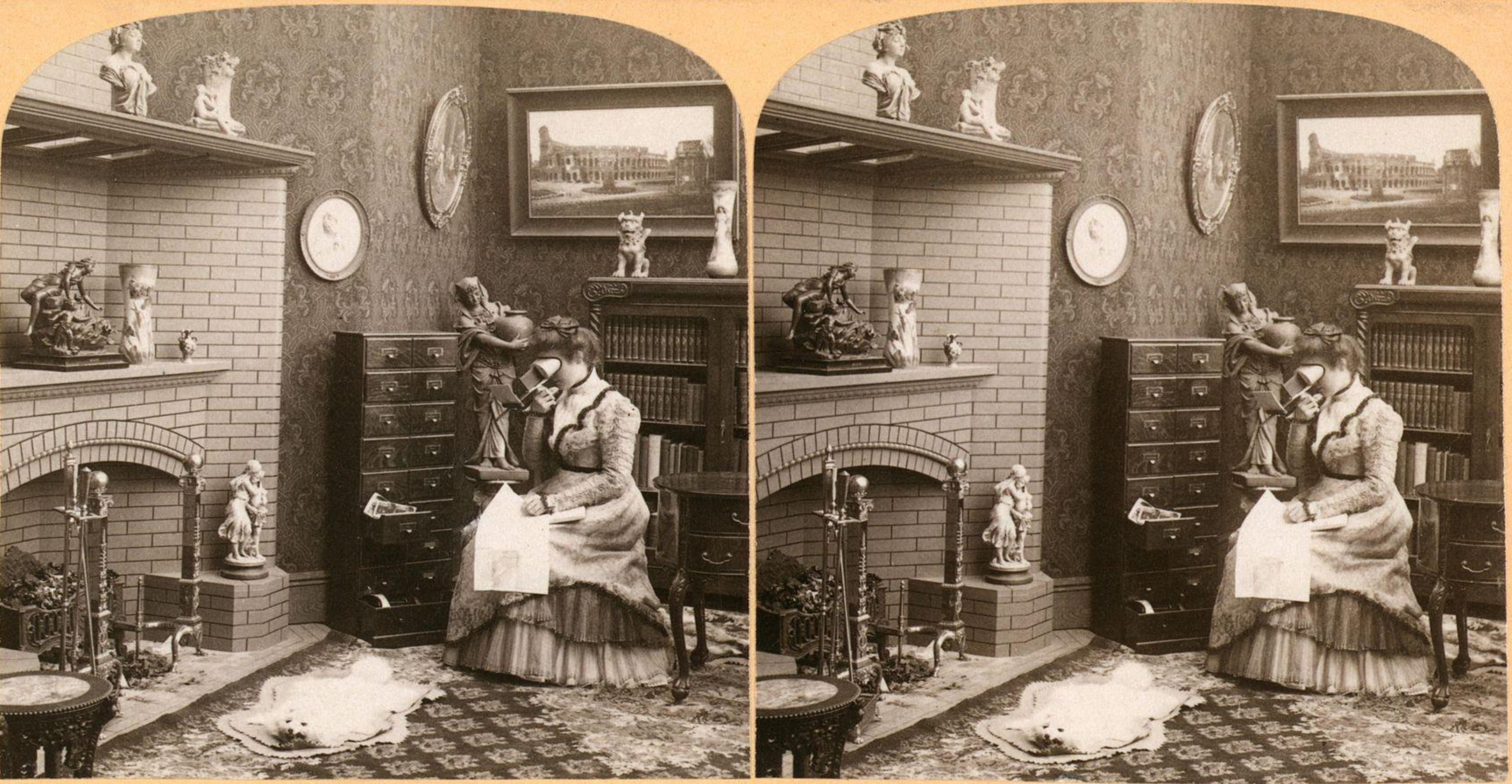

Two View (Epipolar) Geometry



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

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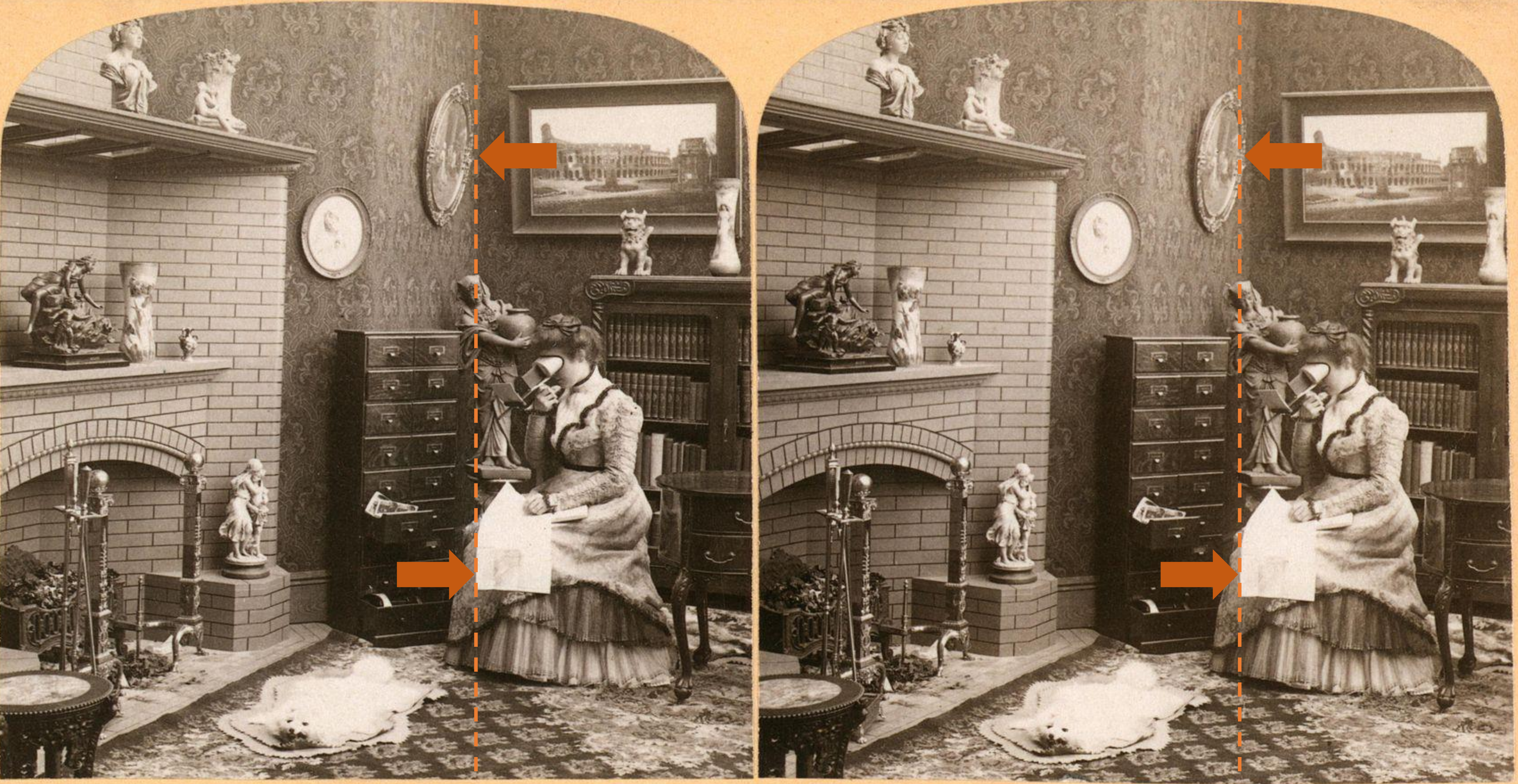
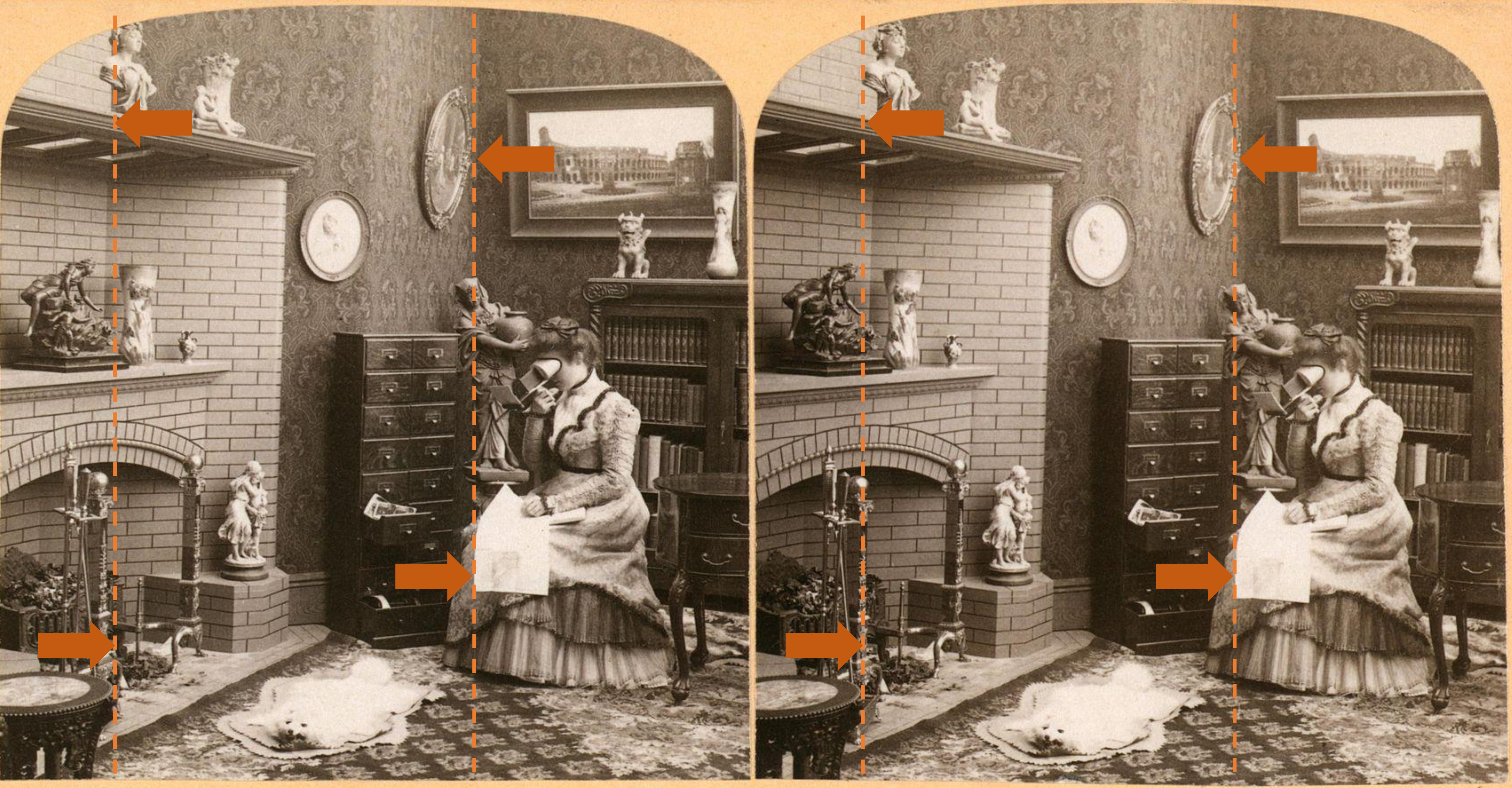


The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

Circa 1900

Stereo: Holmes Stereoscope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mode


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VITA
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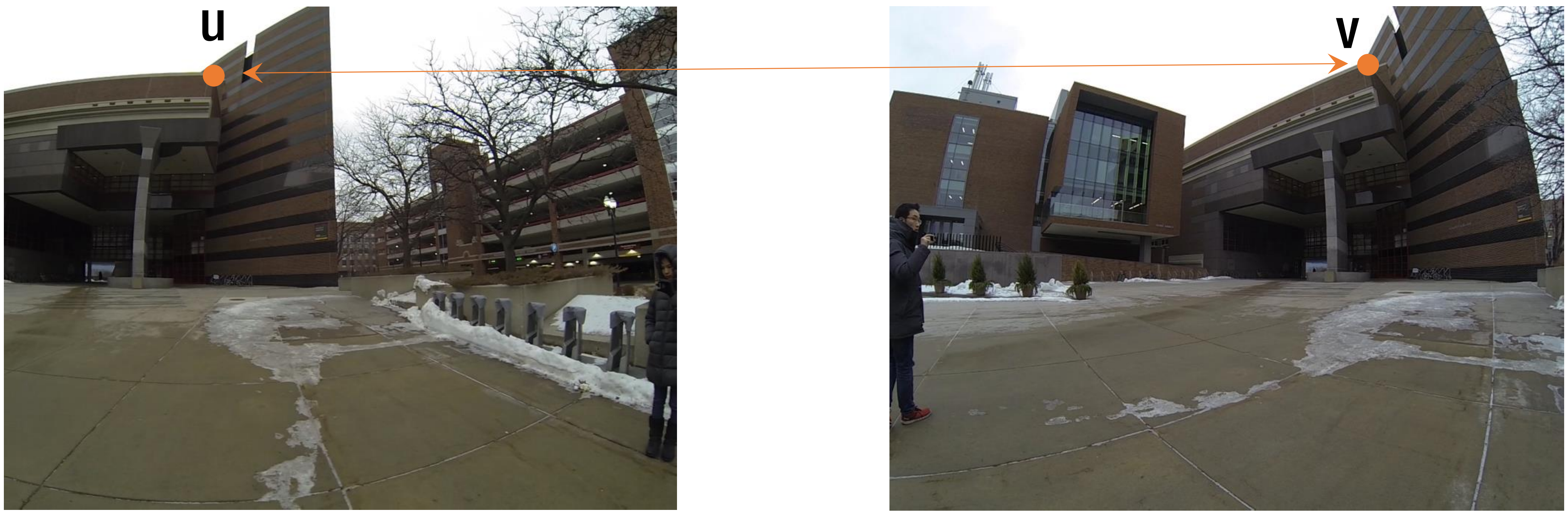
Left image (Bob)

Right image (Alice)

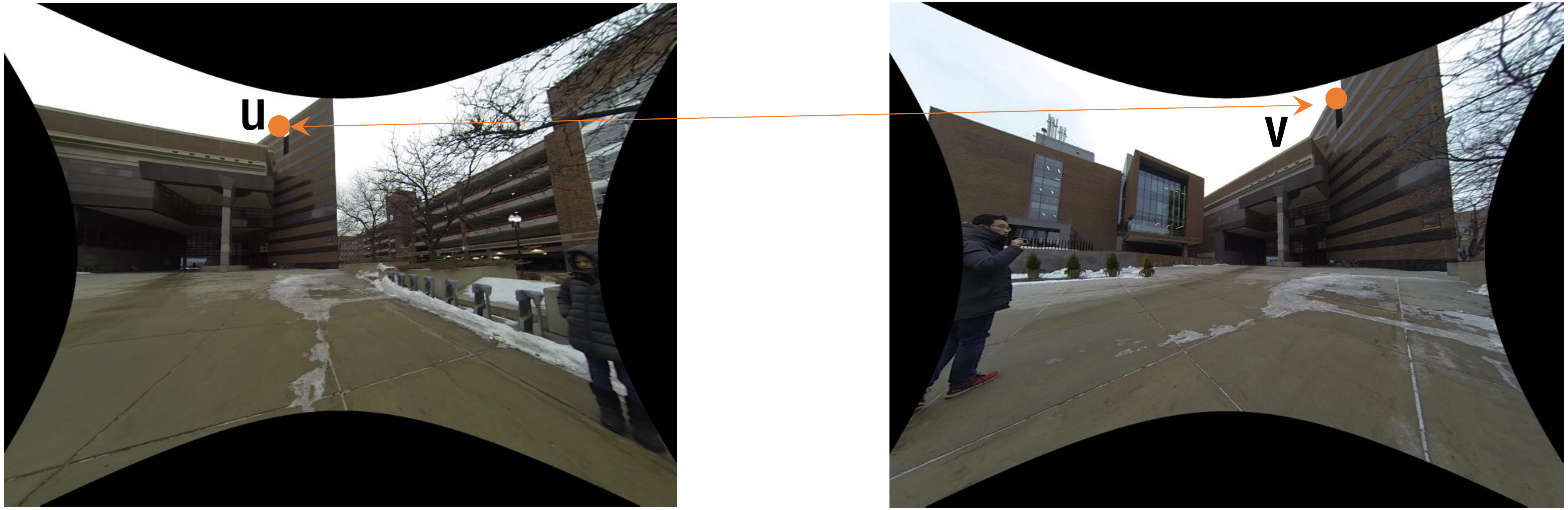
2D Correspondence



Left image (Bob)

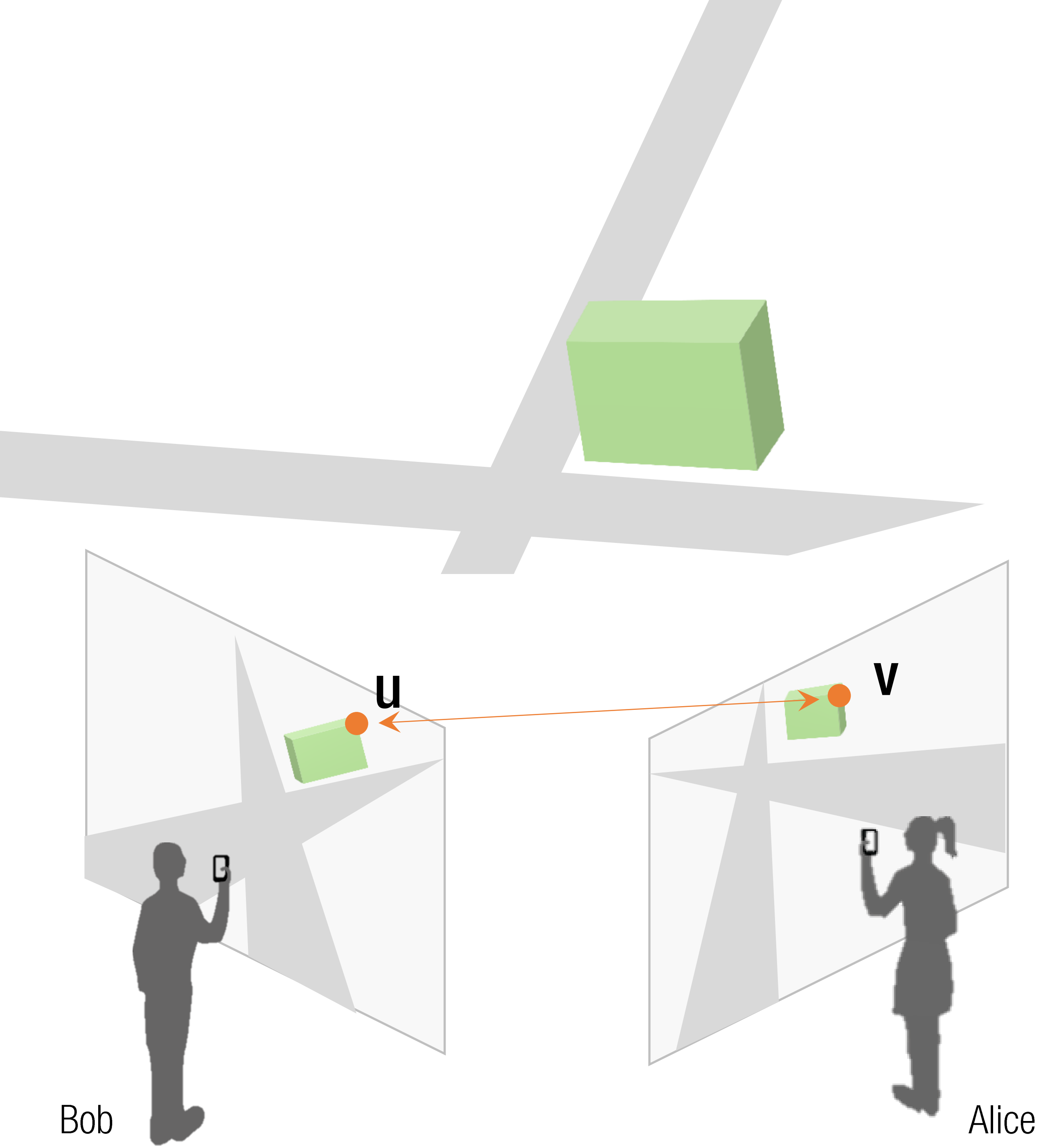
Right image (Alice)

2D Correspondence



Left image (Bob)

Right image (Alice)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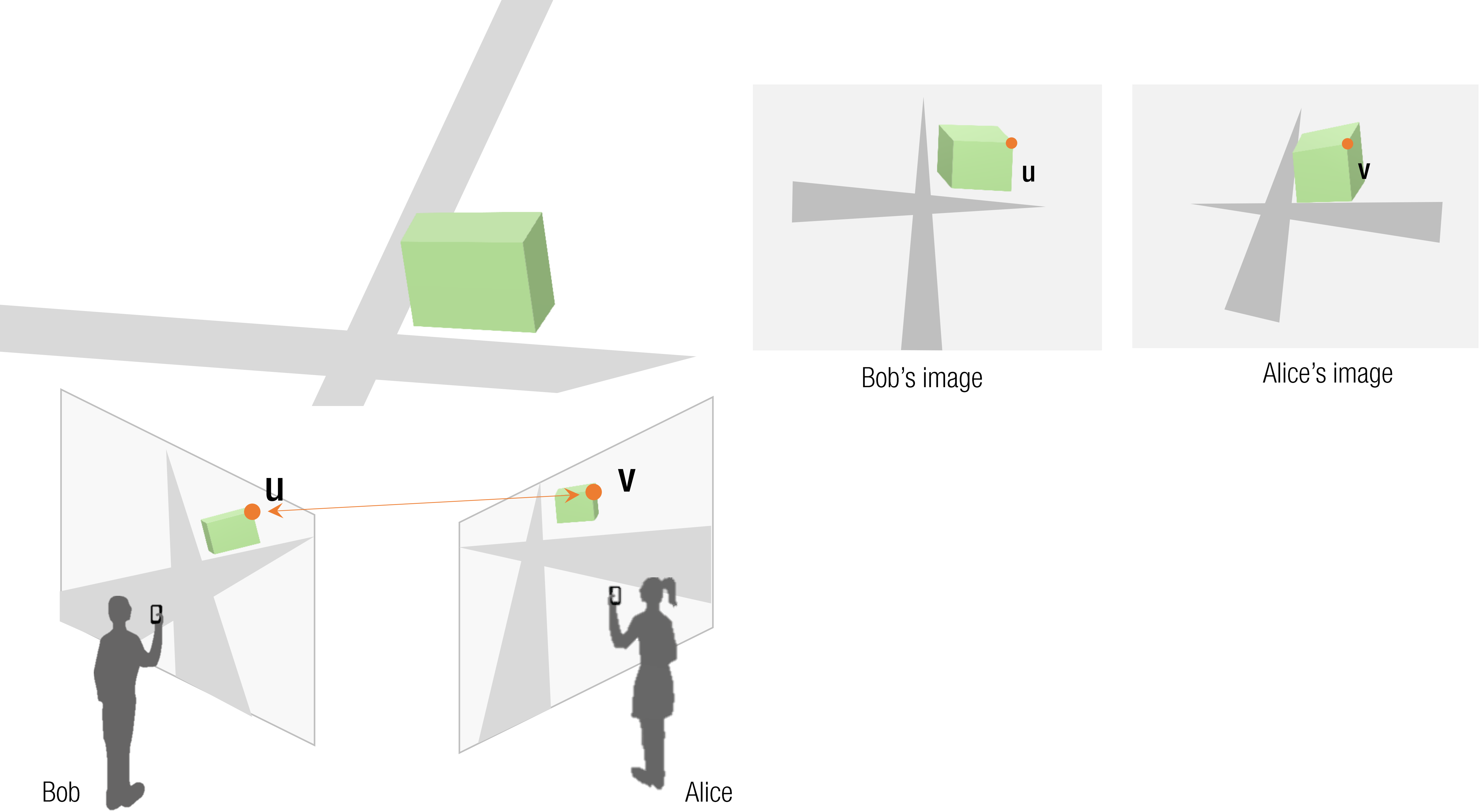
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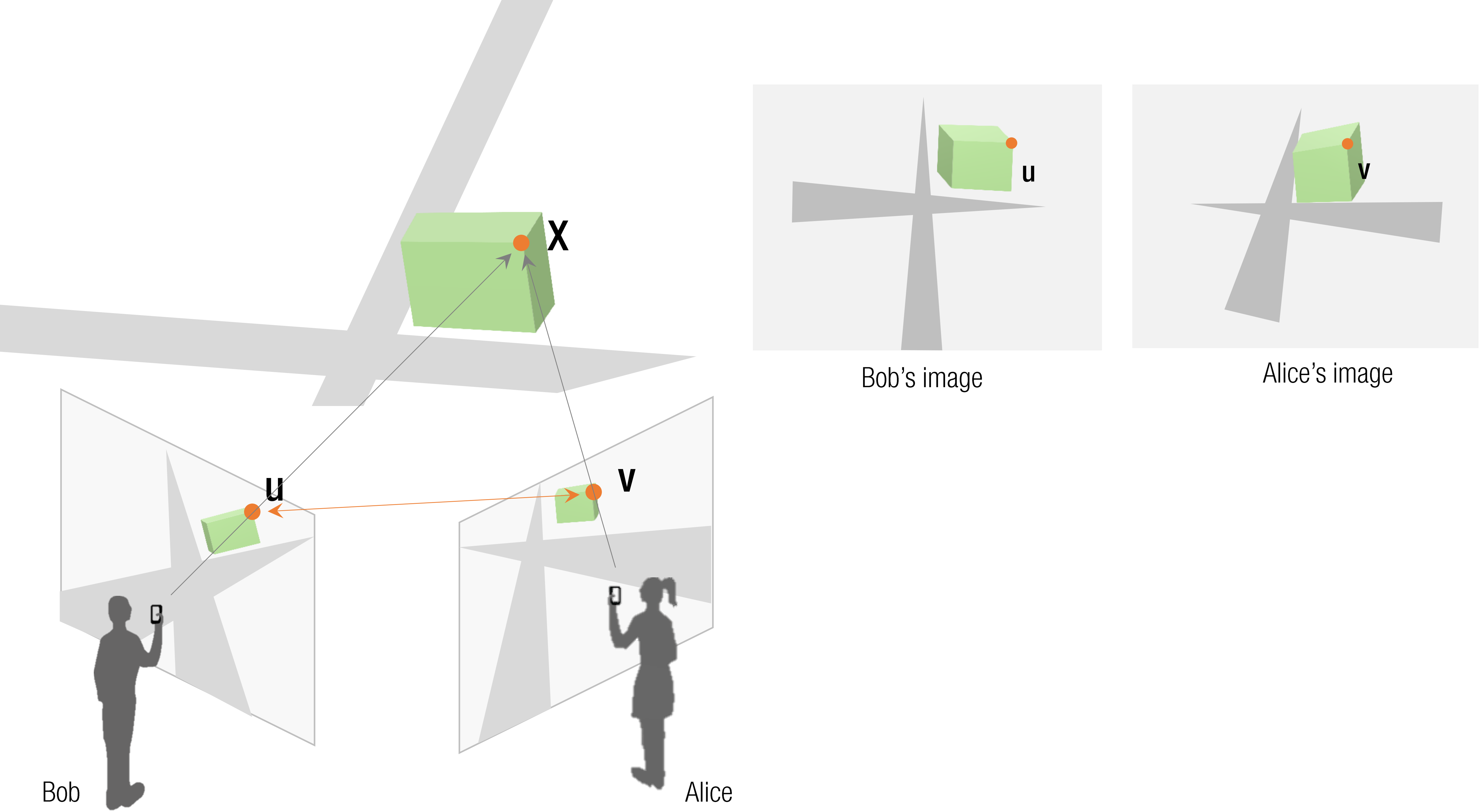
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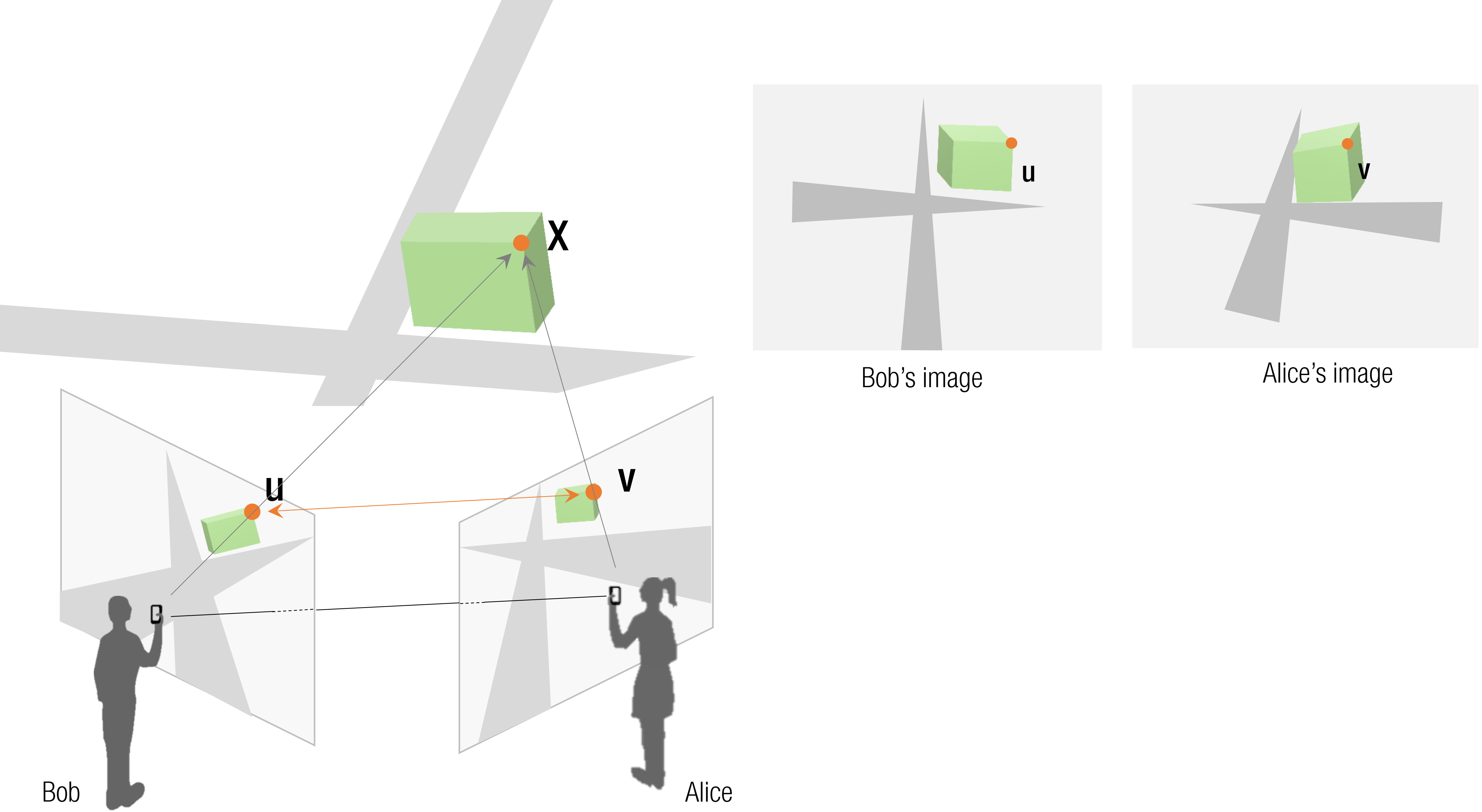
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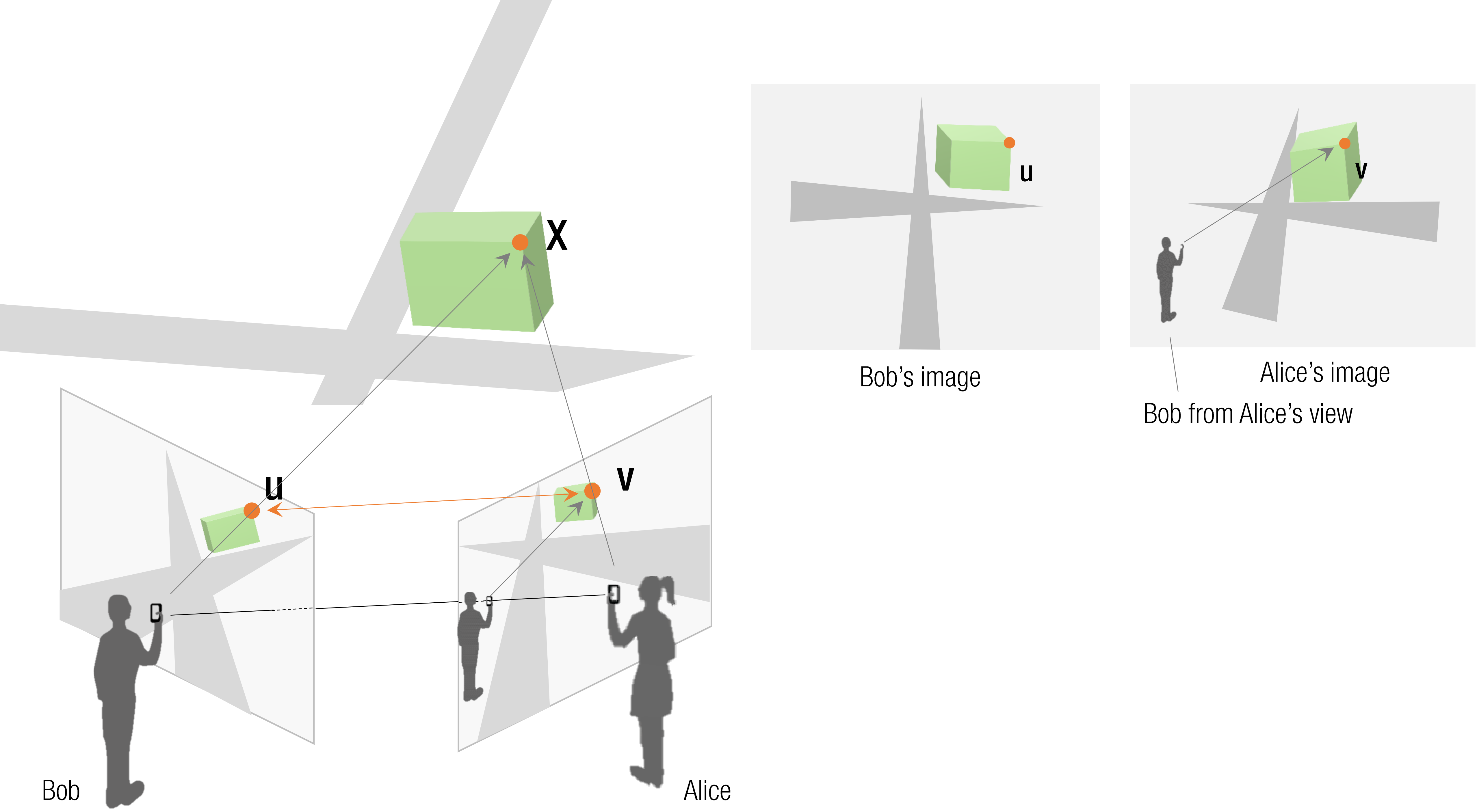
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Bob's image
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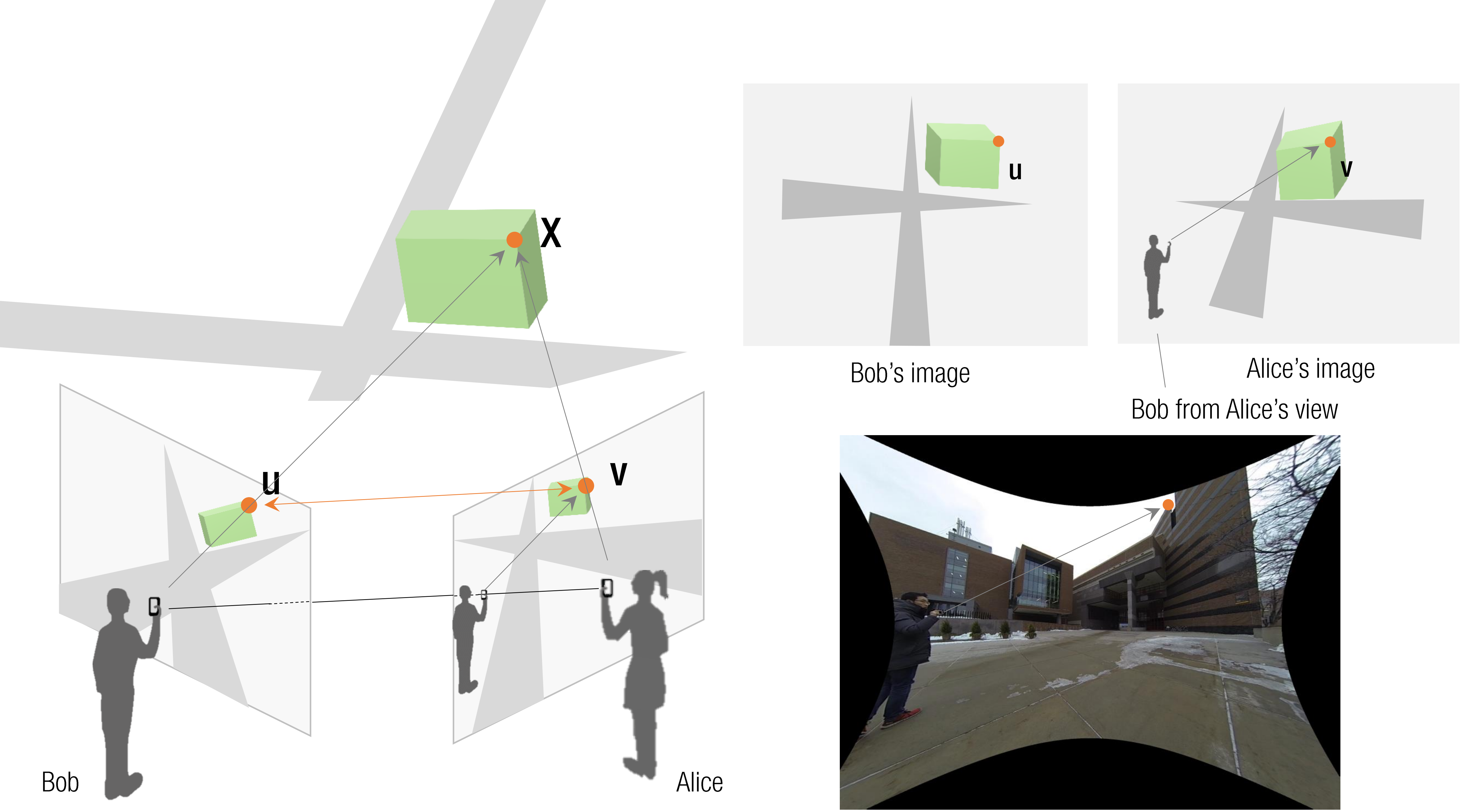
Alice's image
Bob from Alice's view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Bob's image
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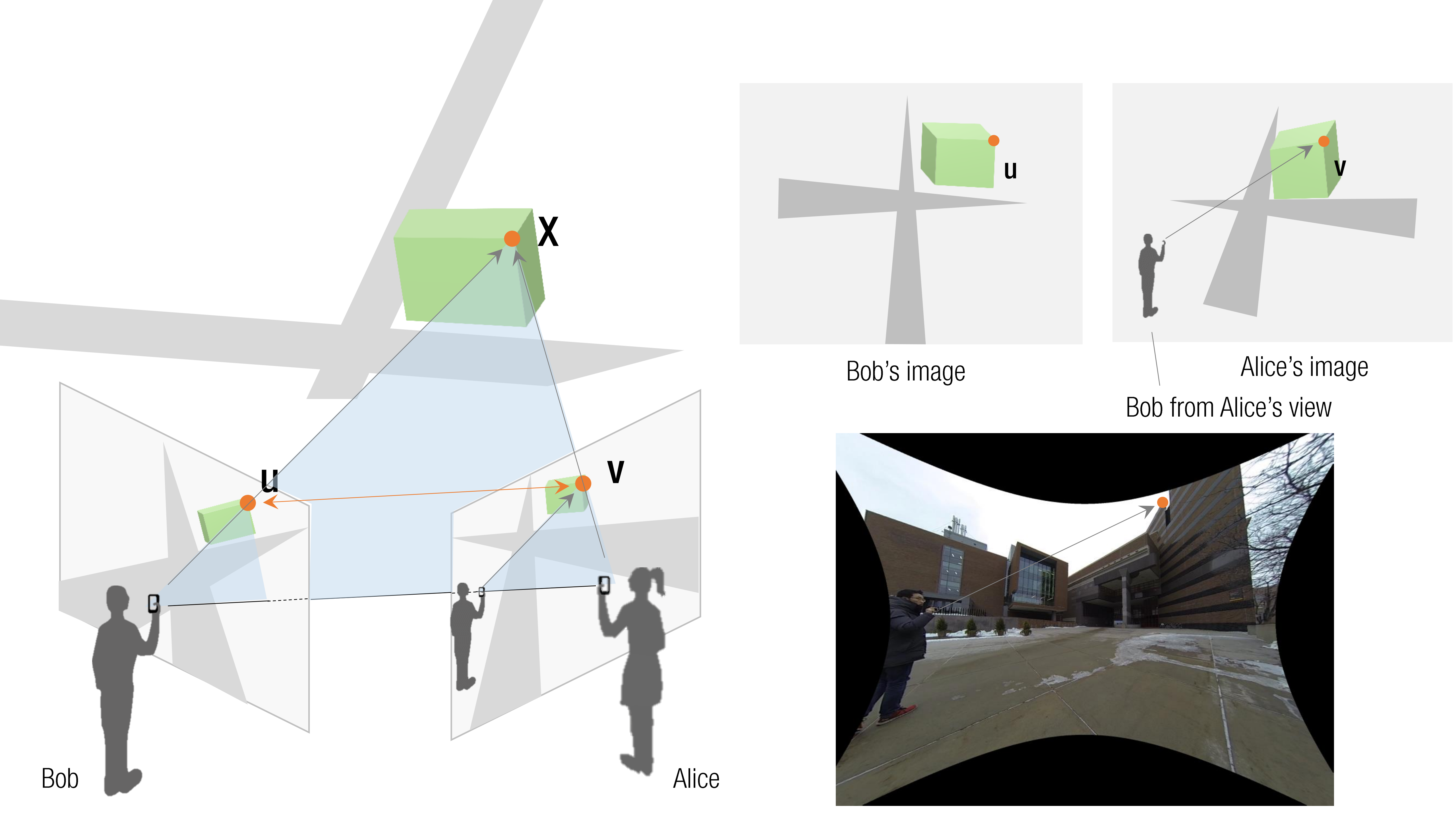
Bob from Alice's view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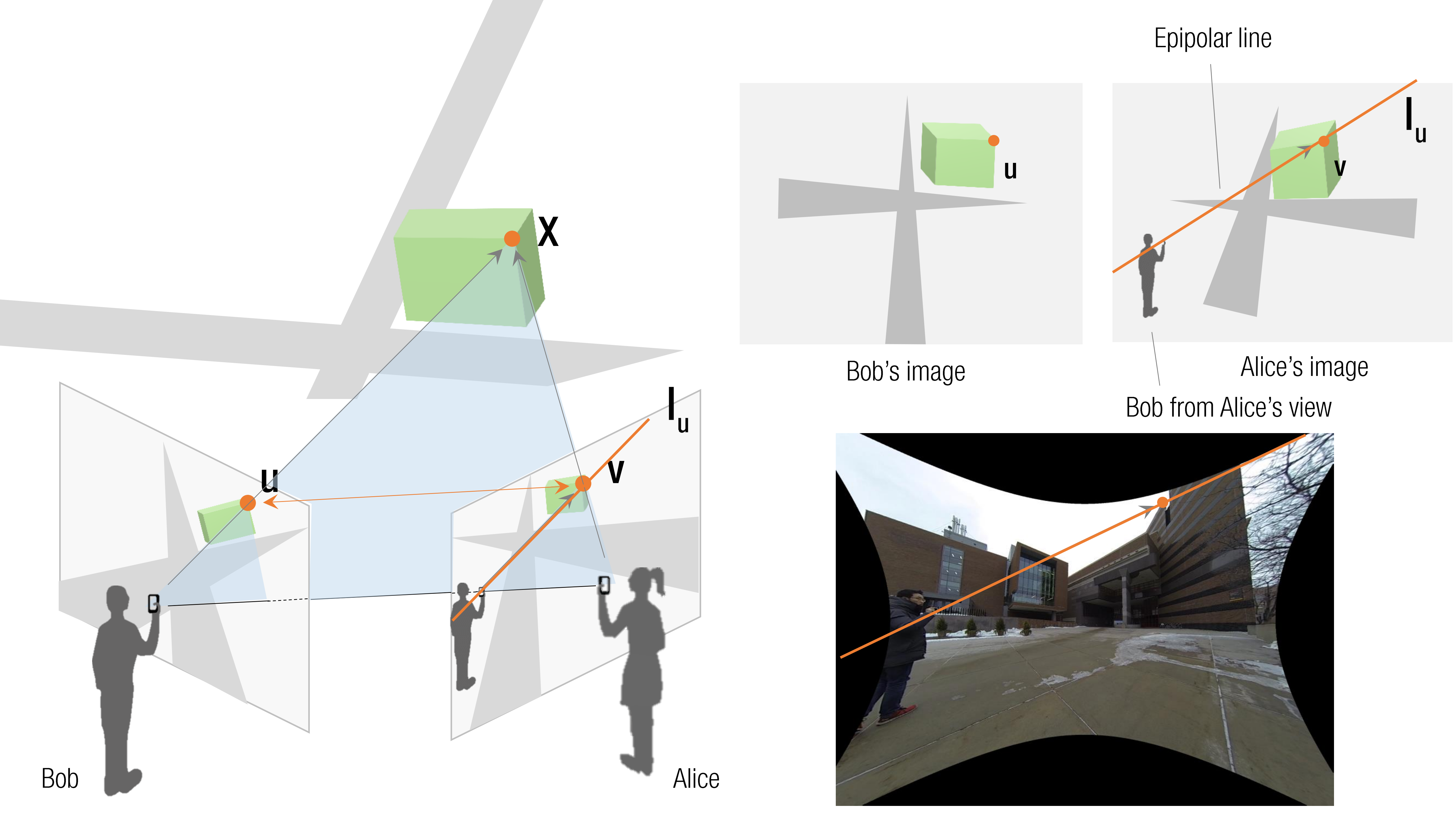
Alice's image
Bob from Alice's view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Epipolar line
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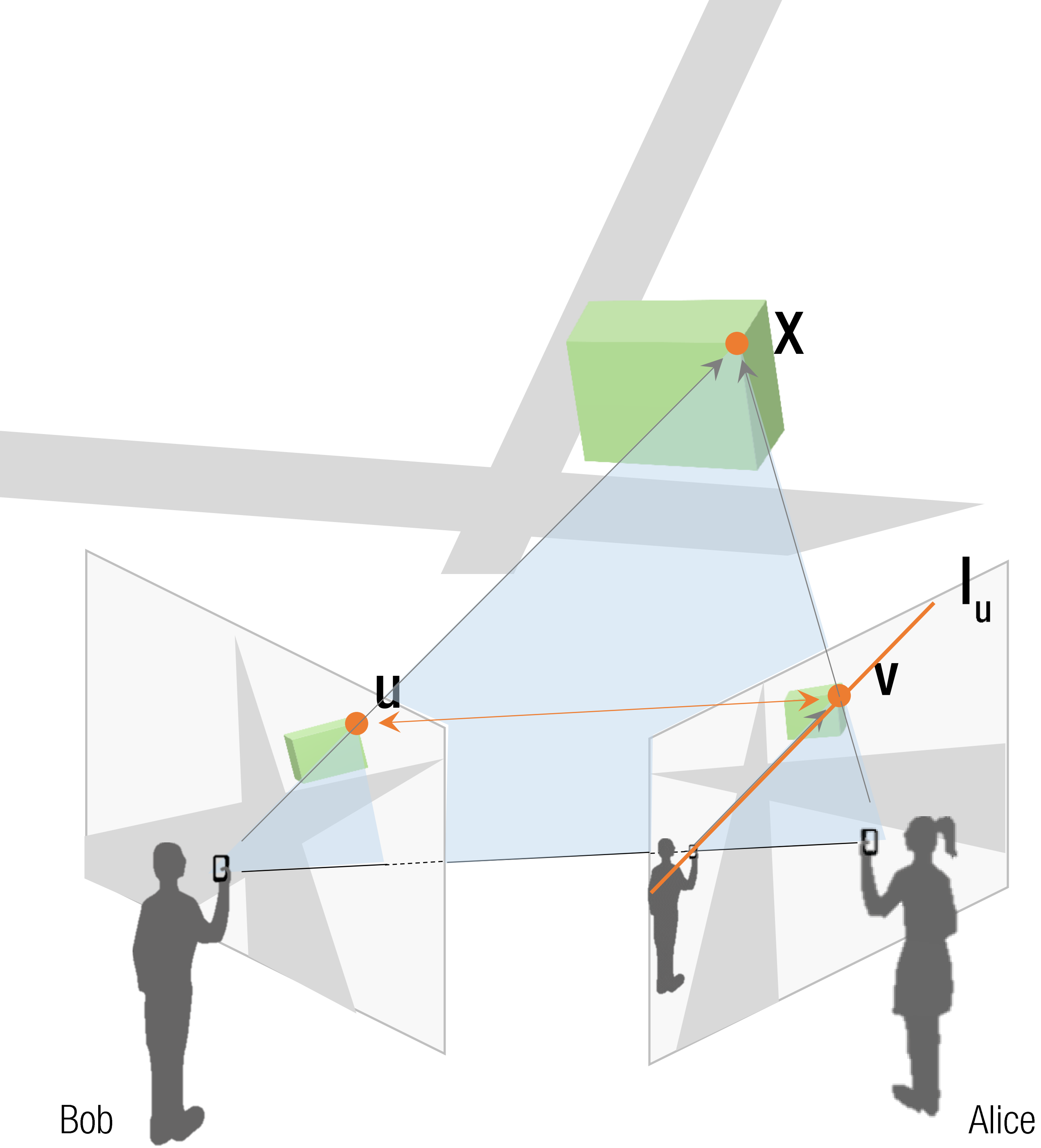
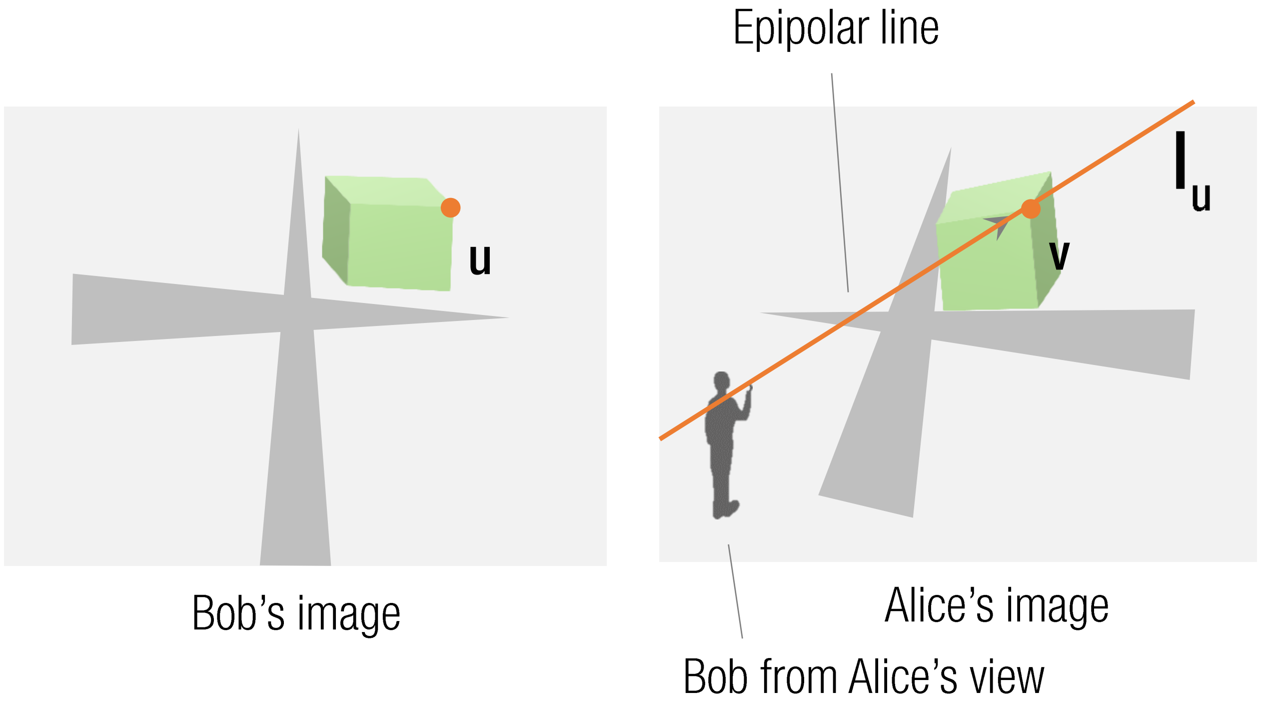
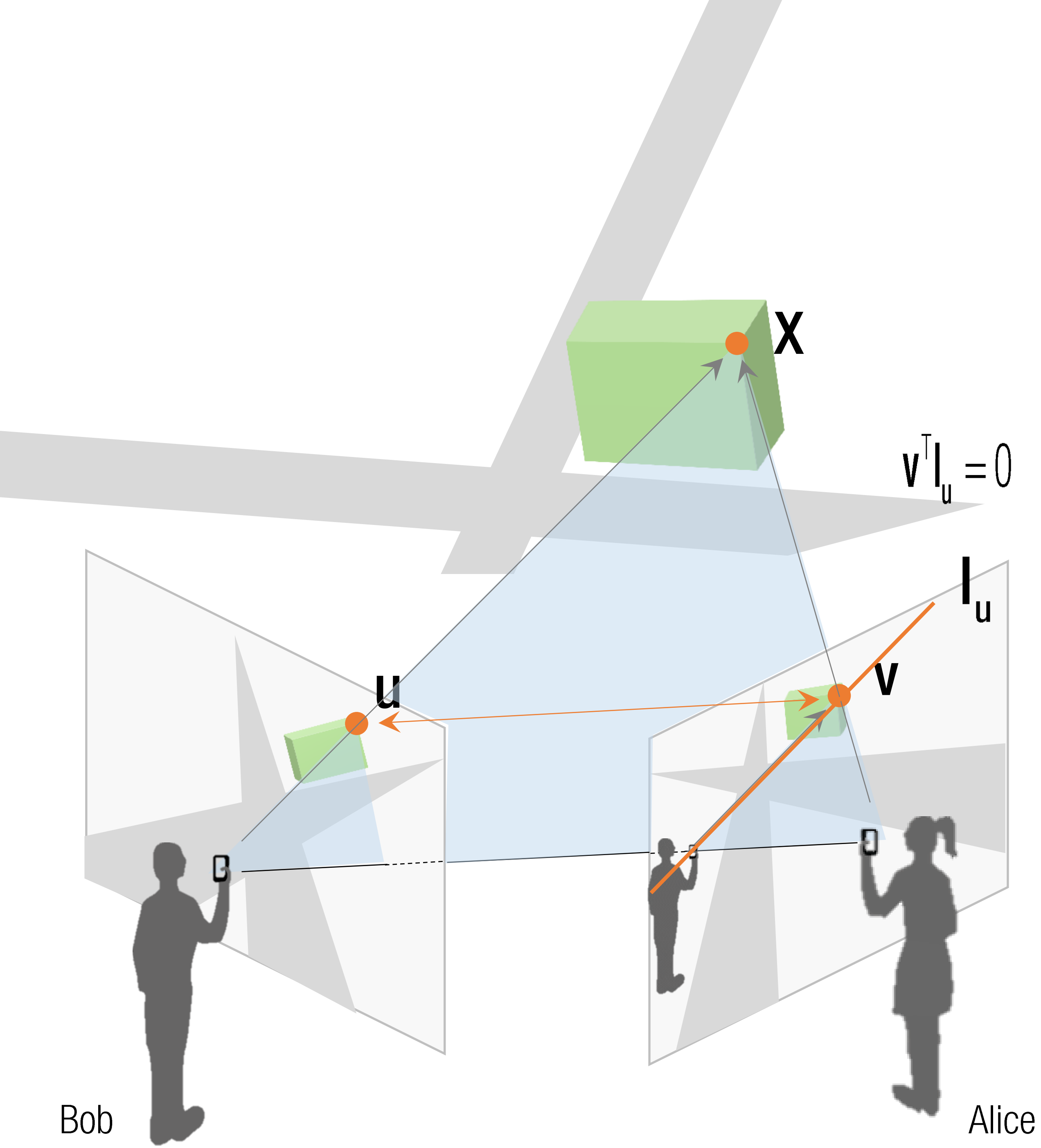
Bob's image
Alice's image
Bob from Alice's view

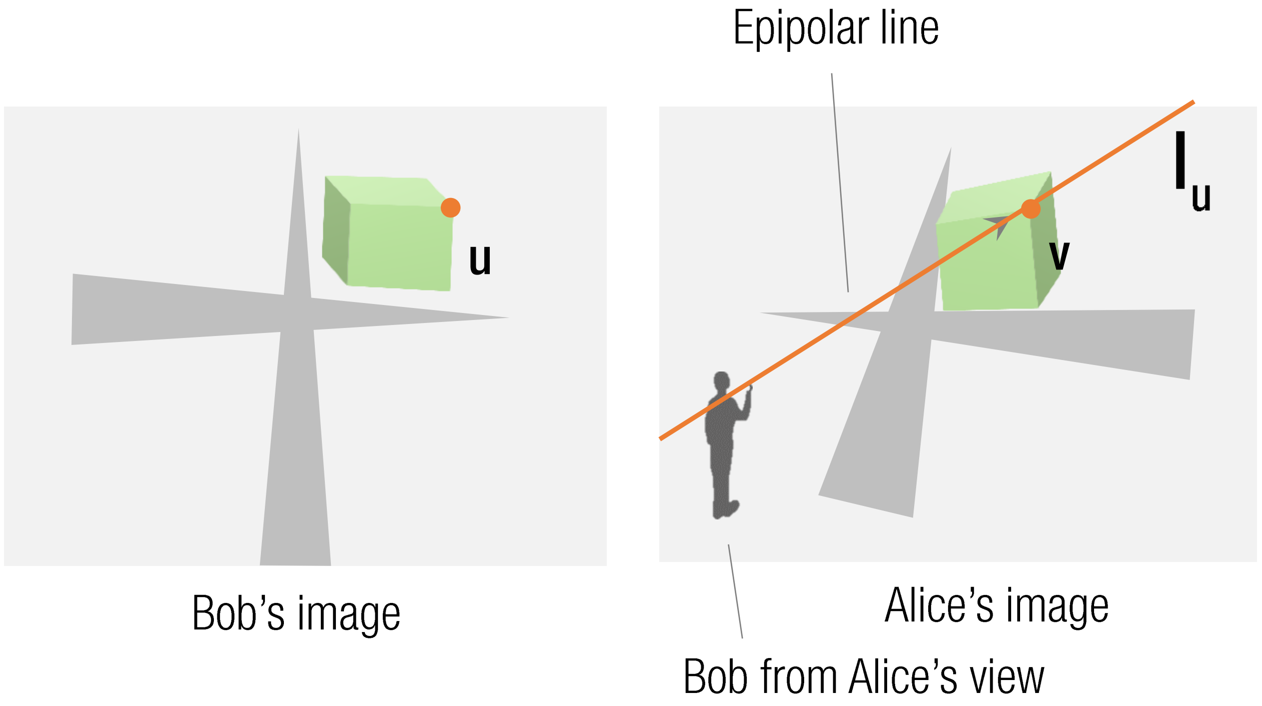




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

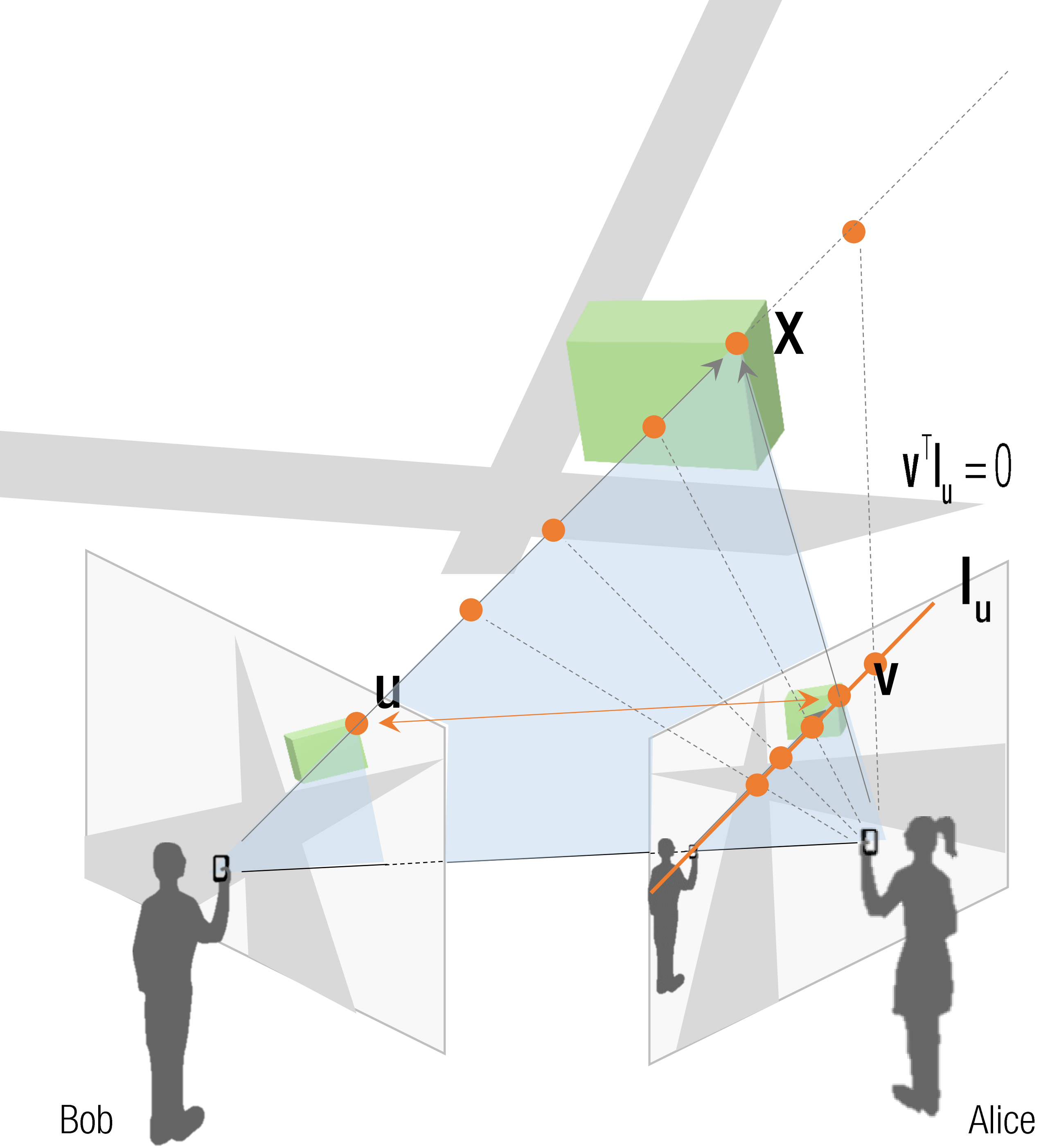


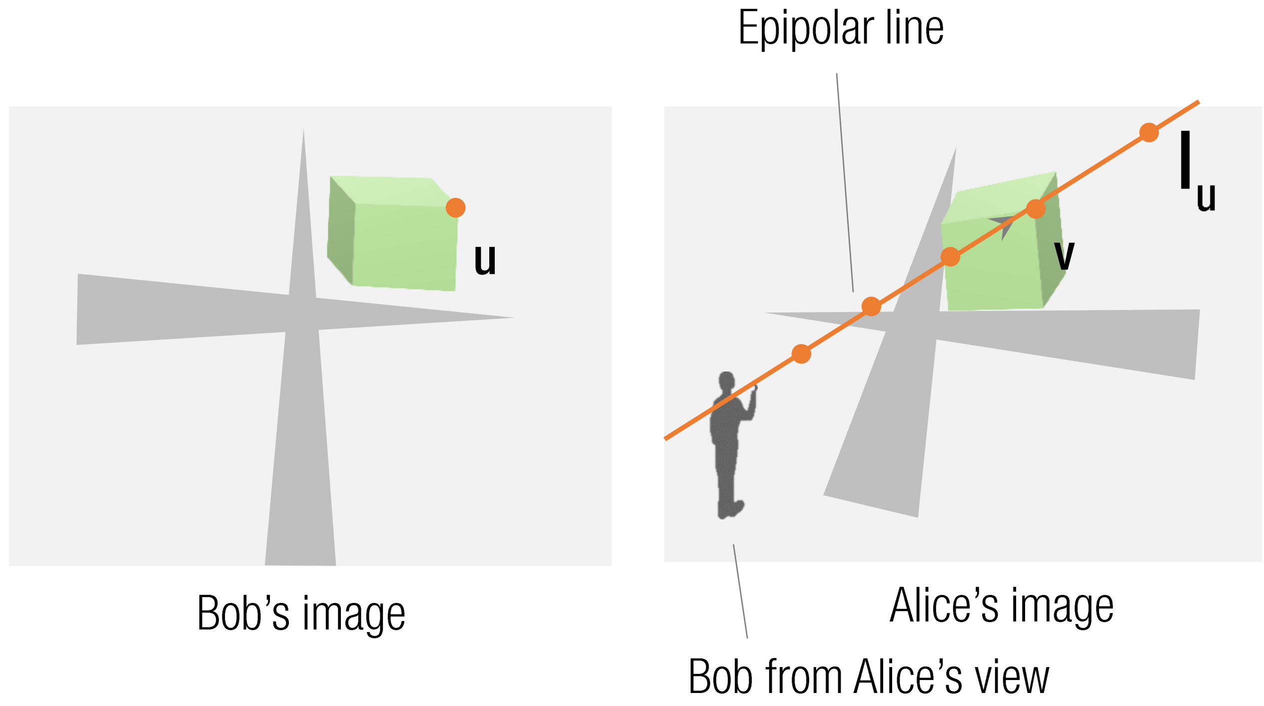


Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$





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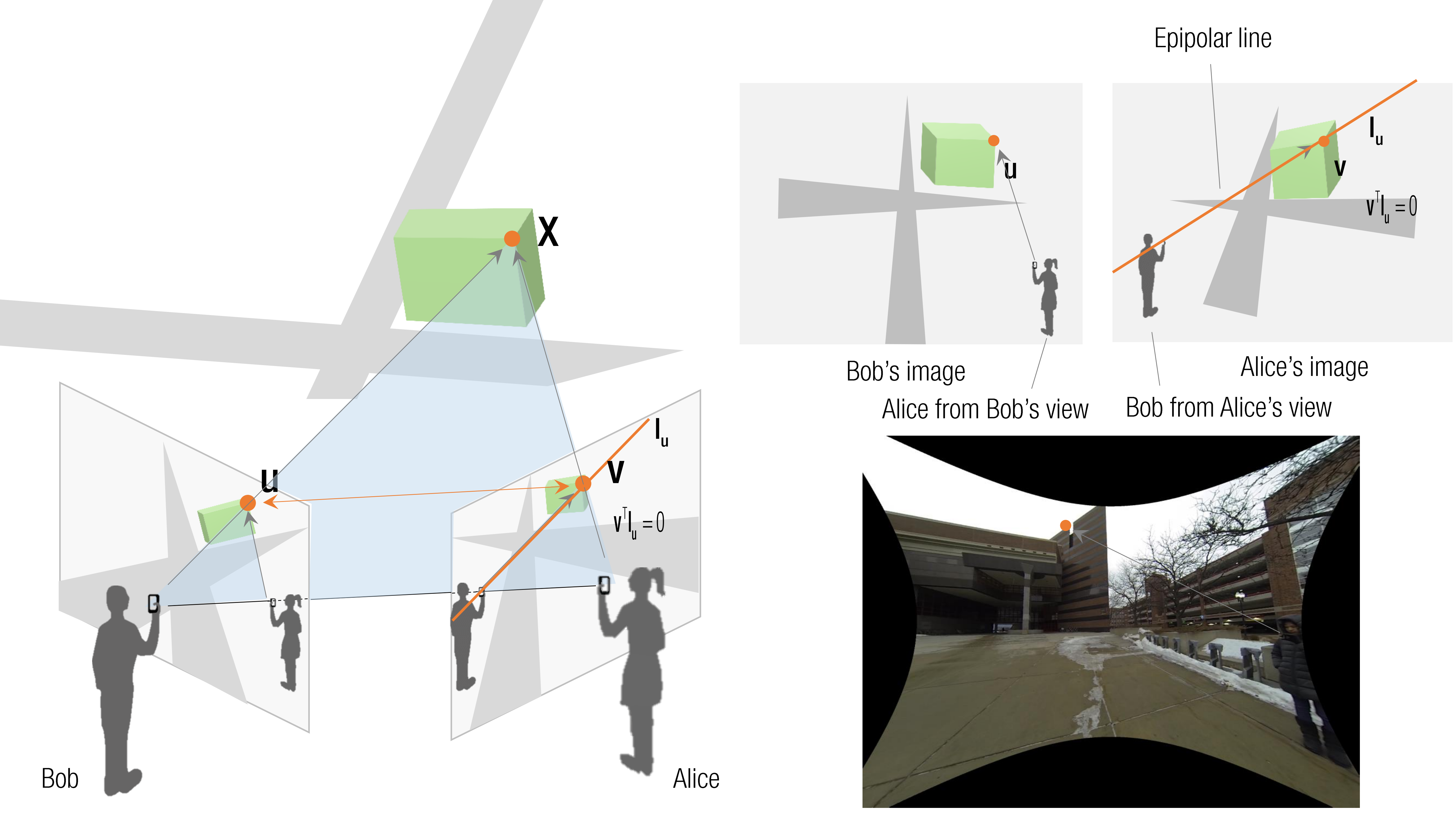
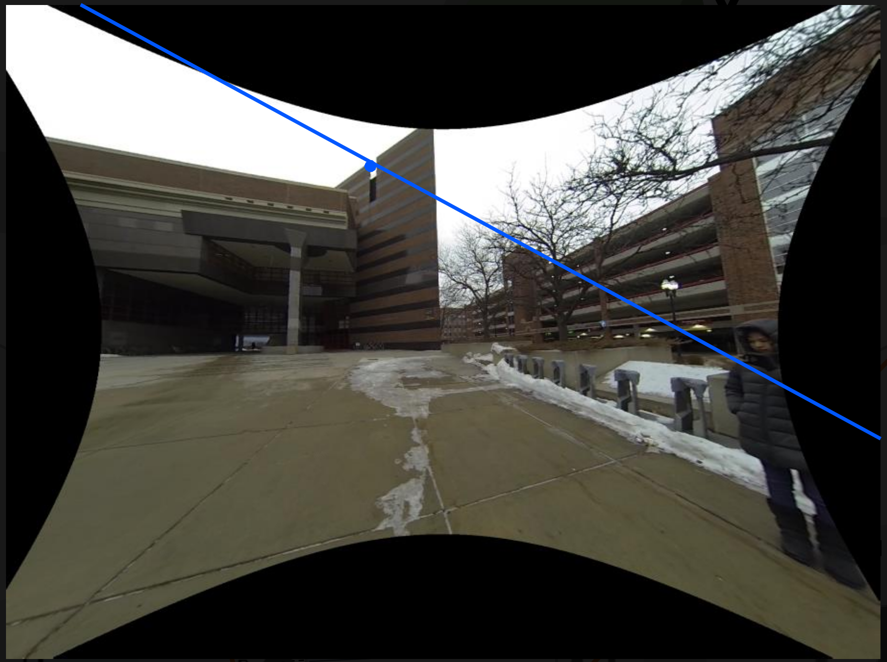
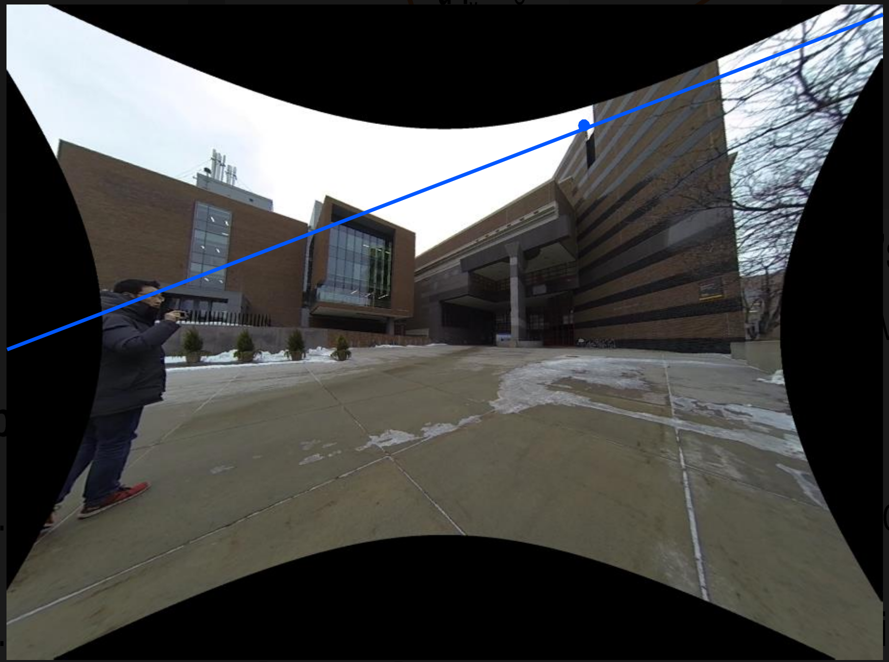
3. Any point along the epipolar line can be a candidate of correspondences.


X
u
v
l_u
$v^T l_u = 0$
Bob
Alice
Epipolar line
l_u
u
v
$v^T l_u = 0$
Bob's image
Alice's image
Alice from Bob's view
Bob from Alice's view

Epipolar line





s image

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Ep

1. ... olar line in

2. ... in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$

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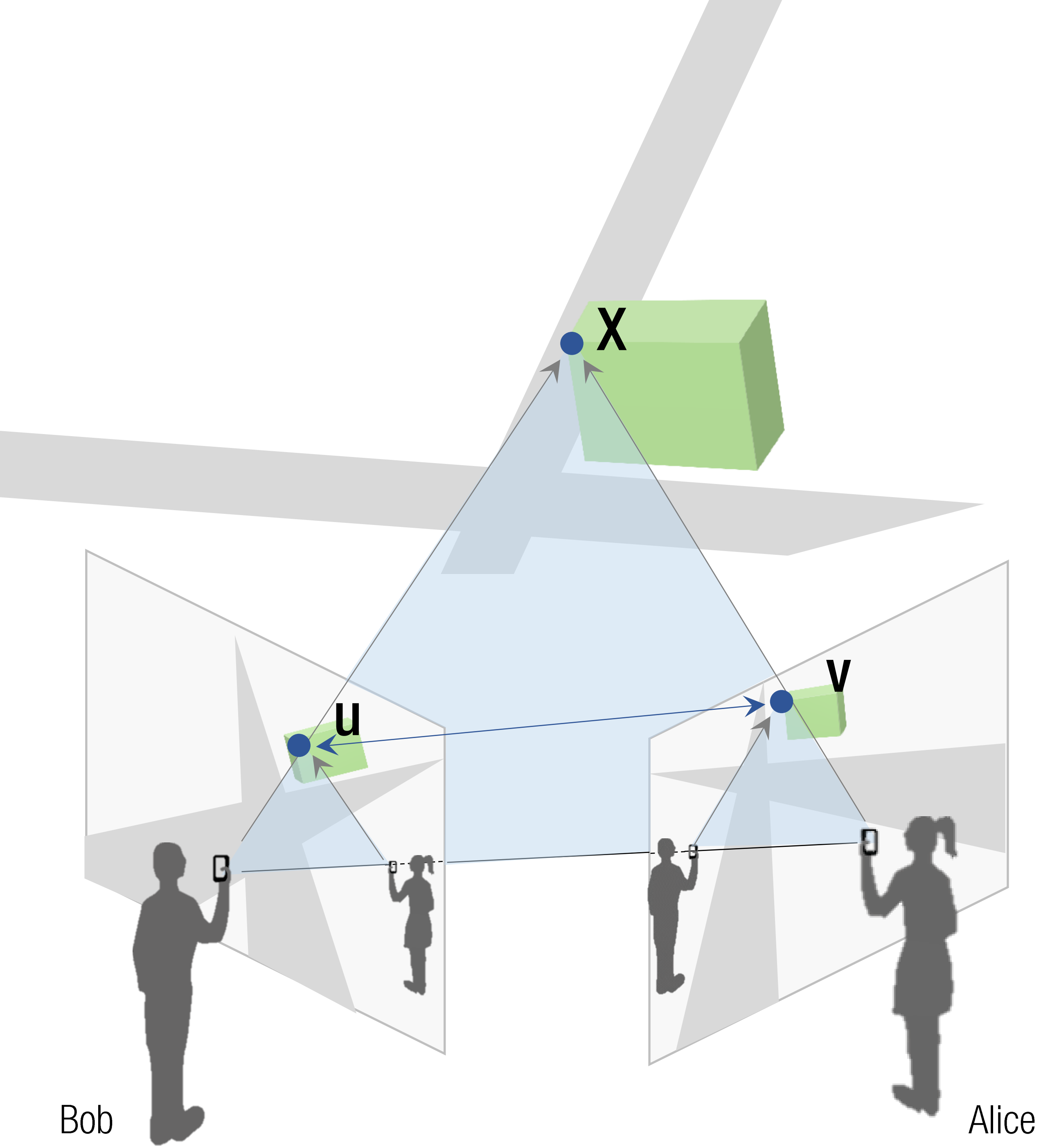
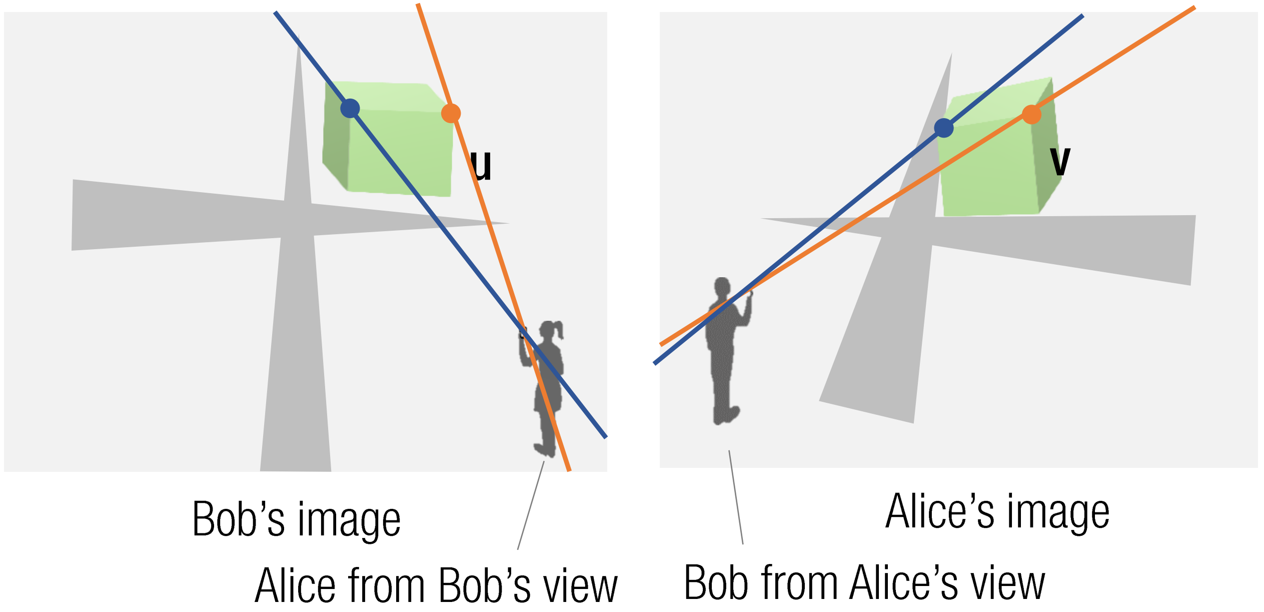
Bob Alice





Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
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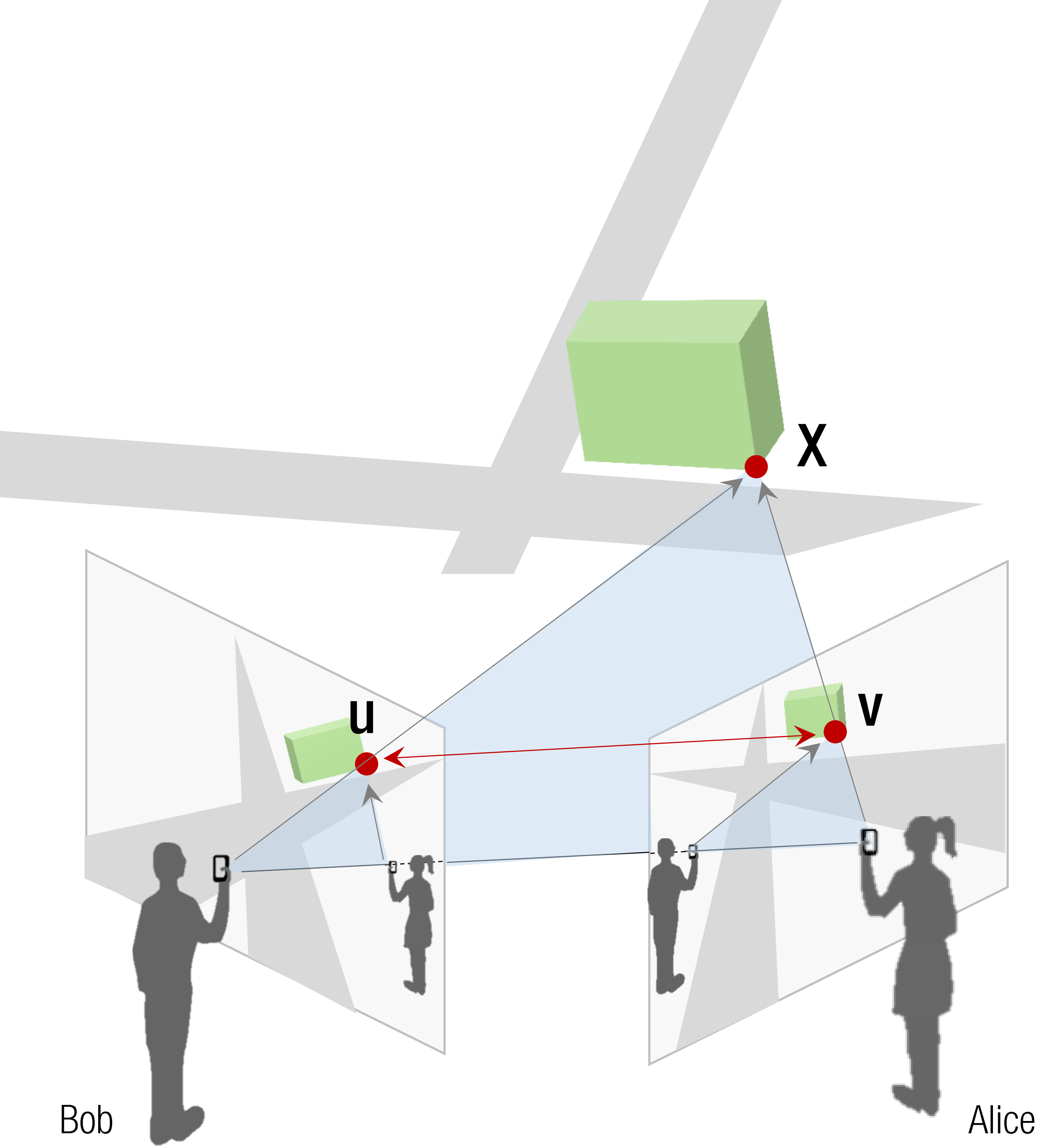

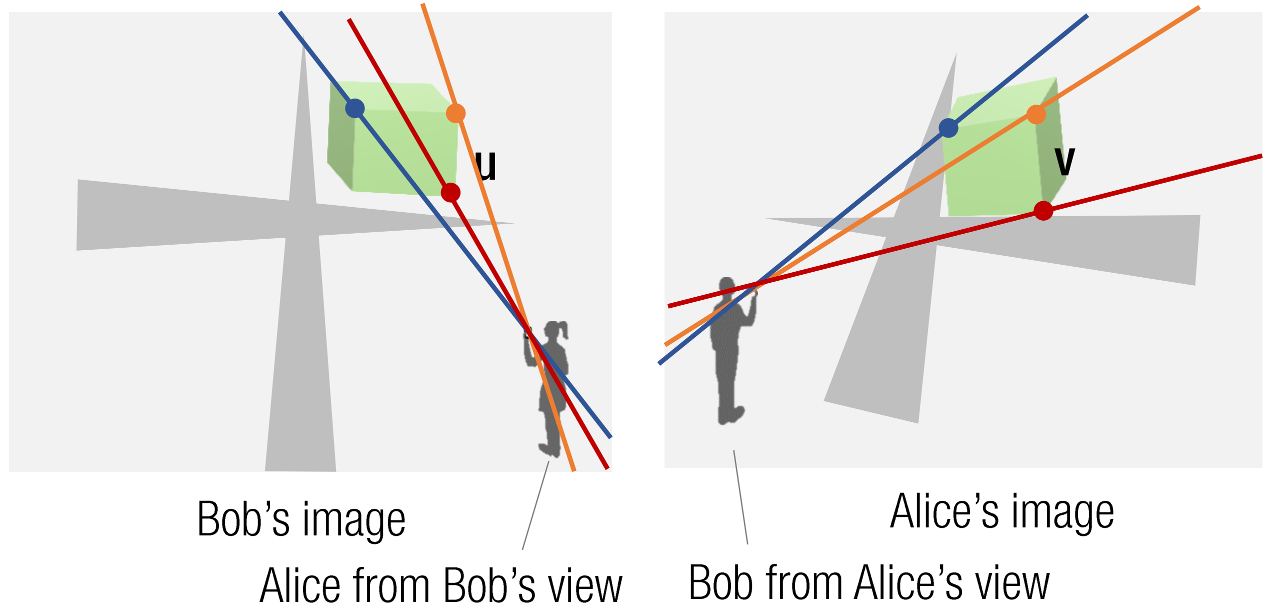

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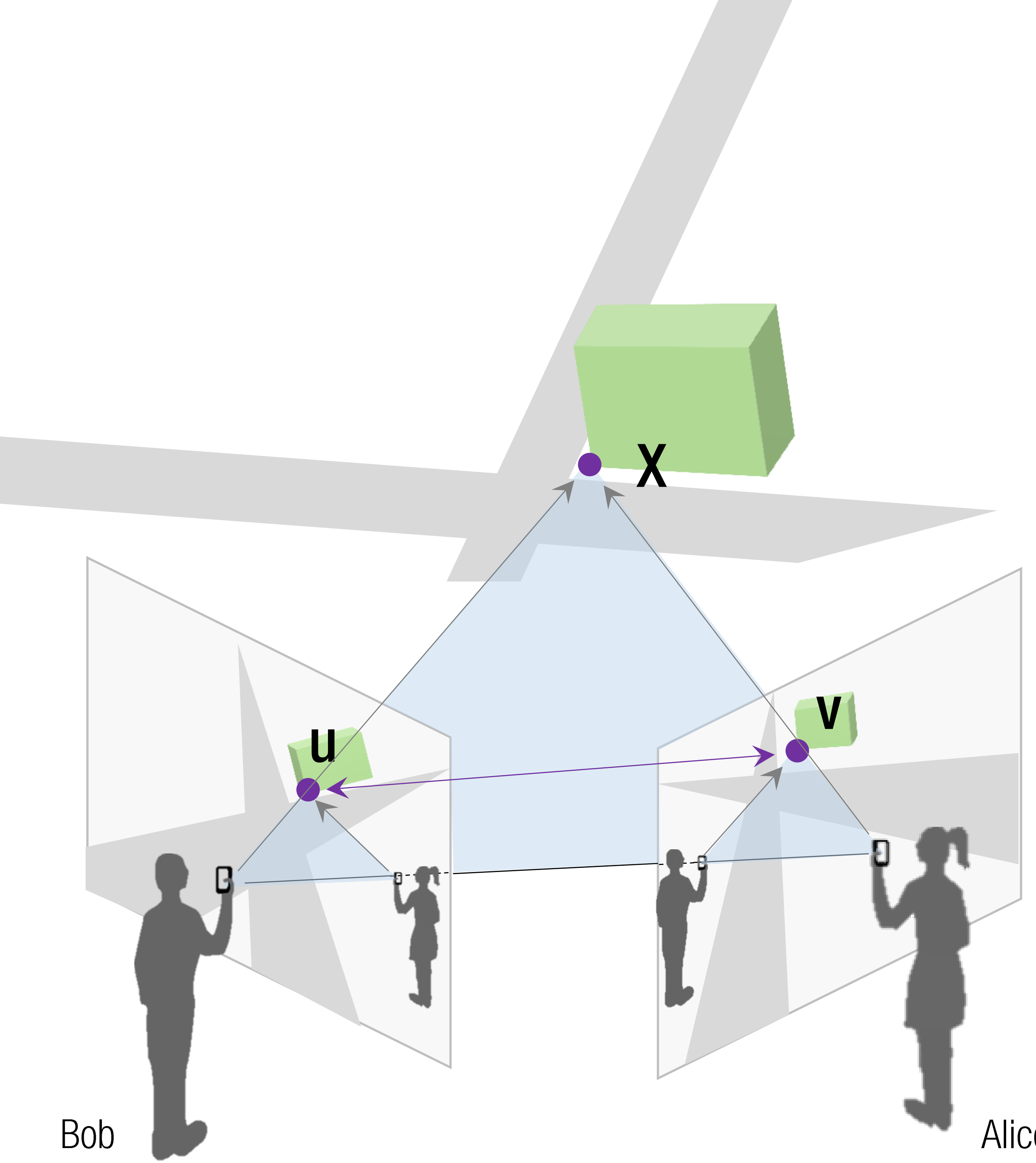
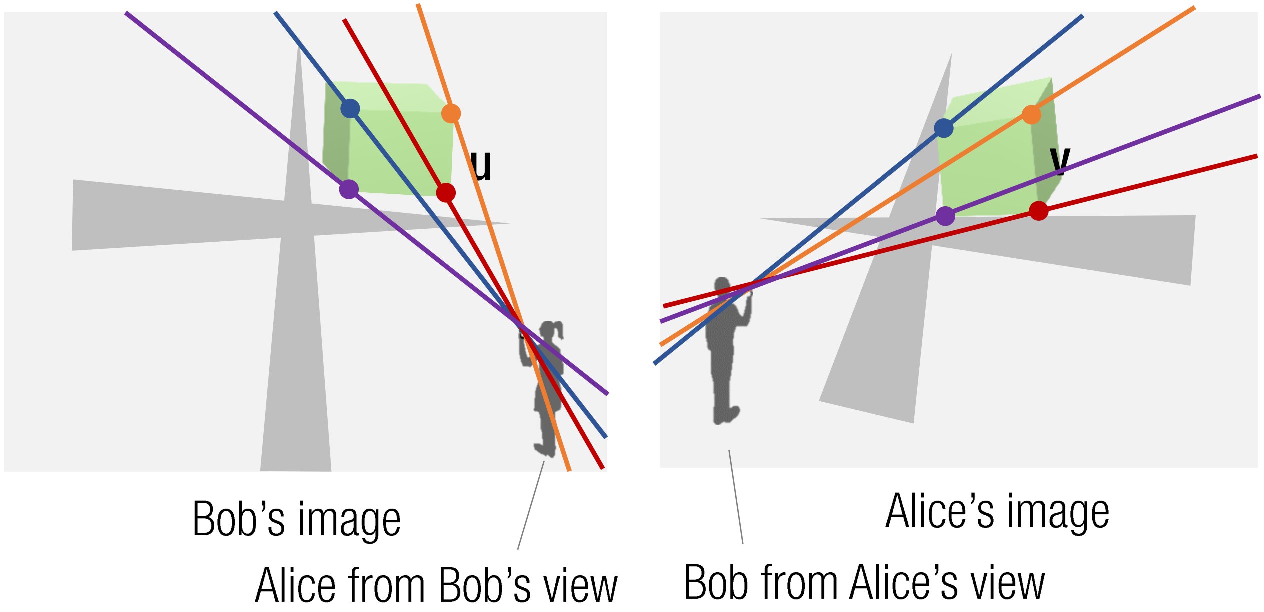


Bob

Alice



Epipolar constraint between two images:

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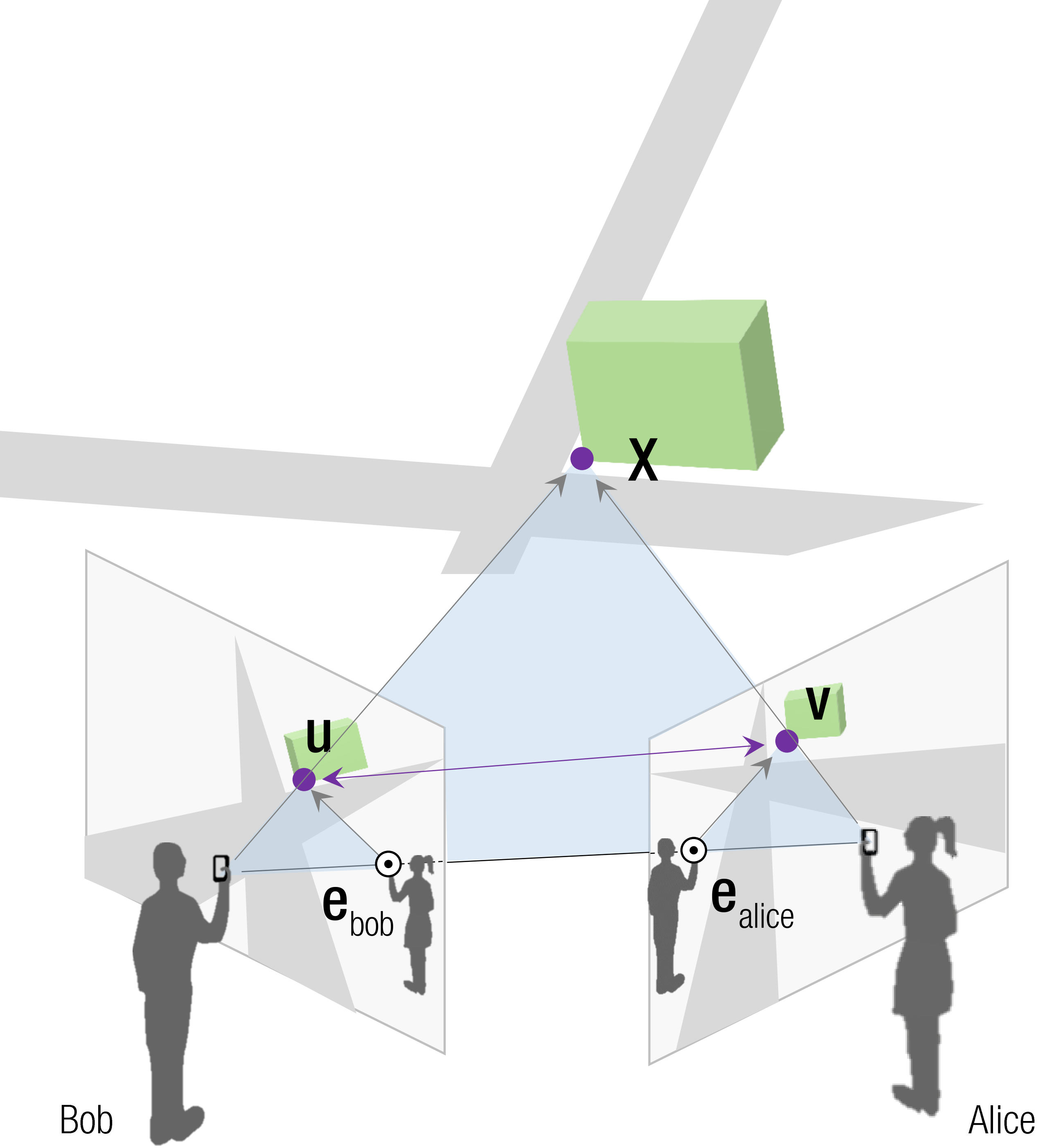
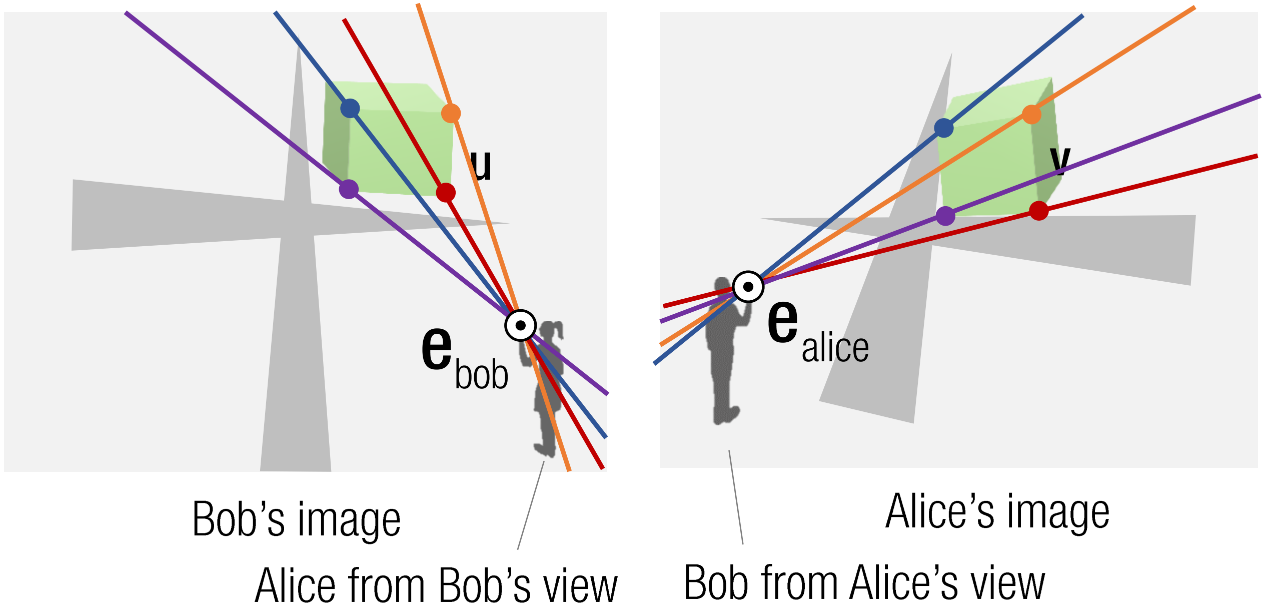
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T \mathbf{l}_u = 0$ $\mathbf{u}^T \mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole.





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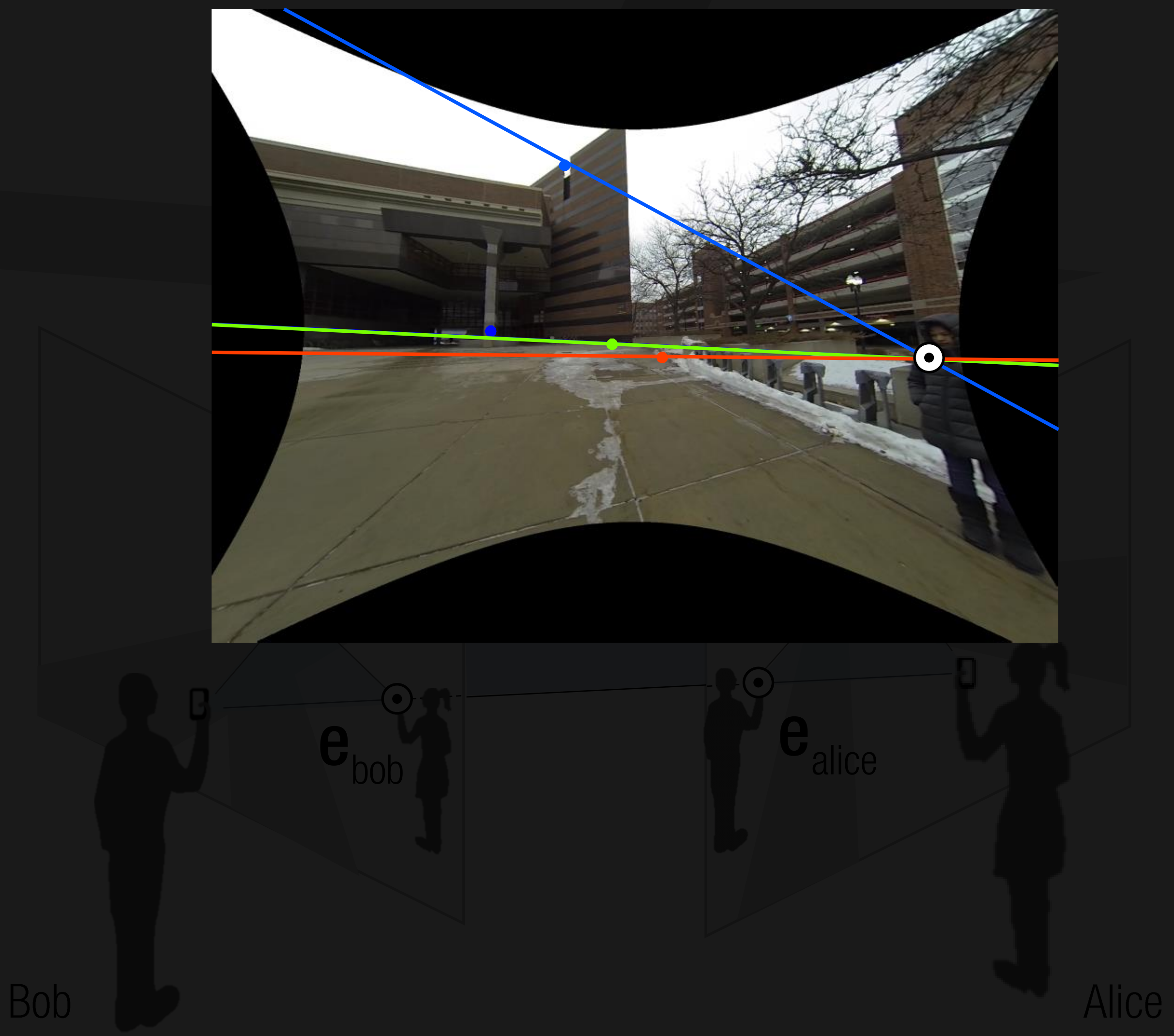

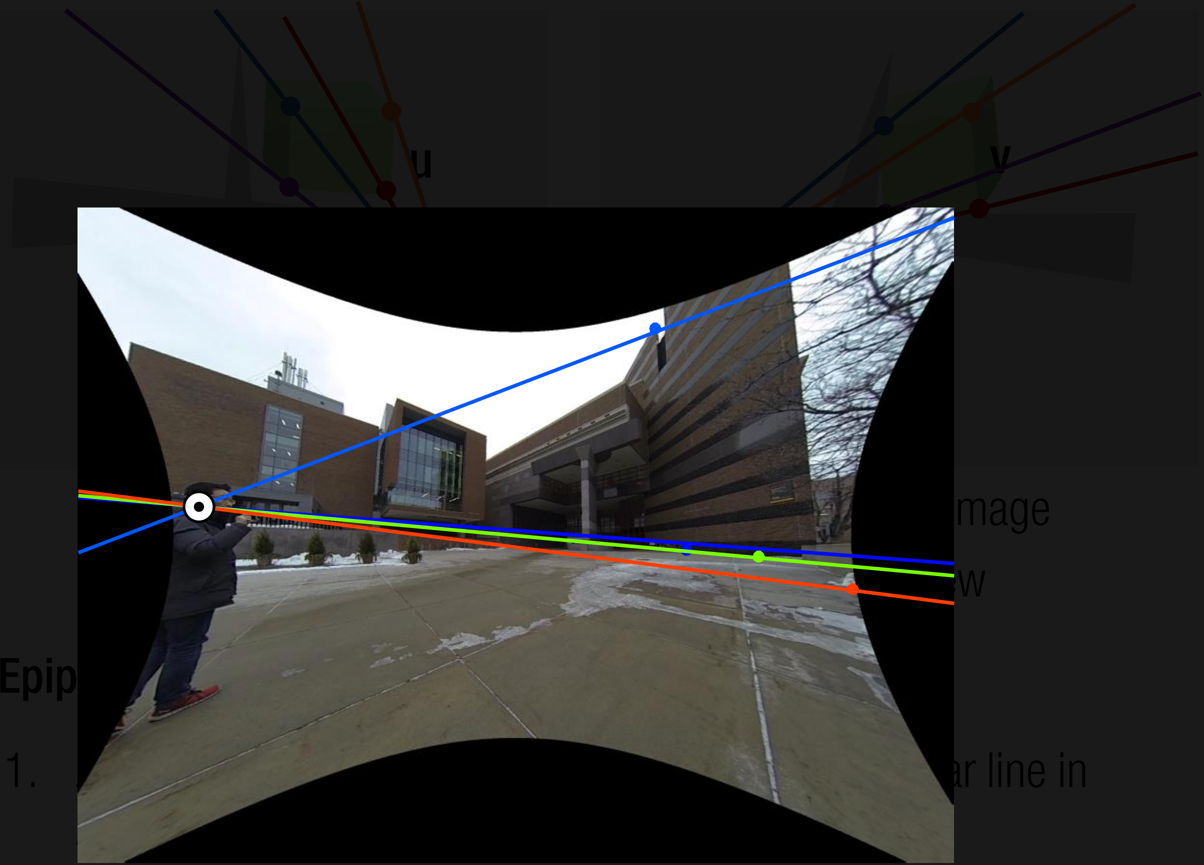

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Epip

1. ar line in

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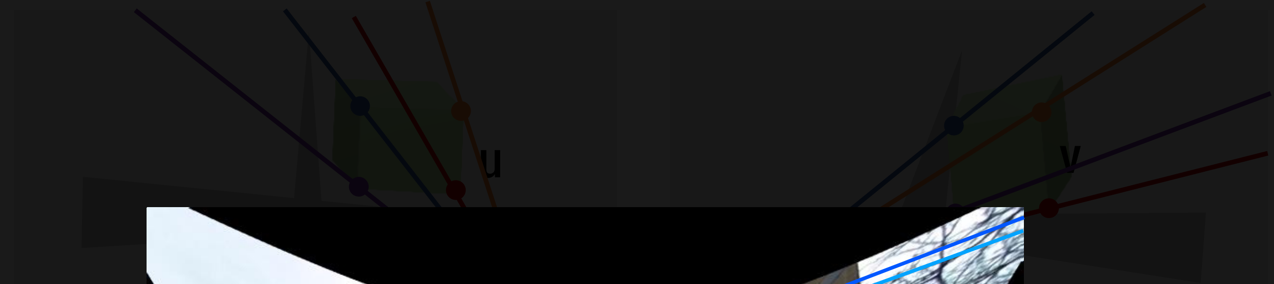

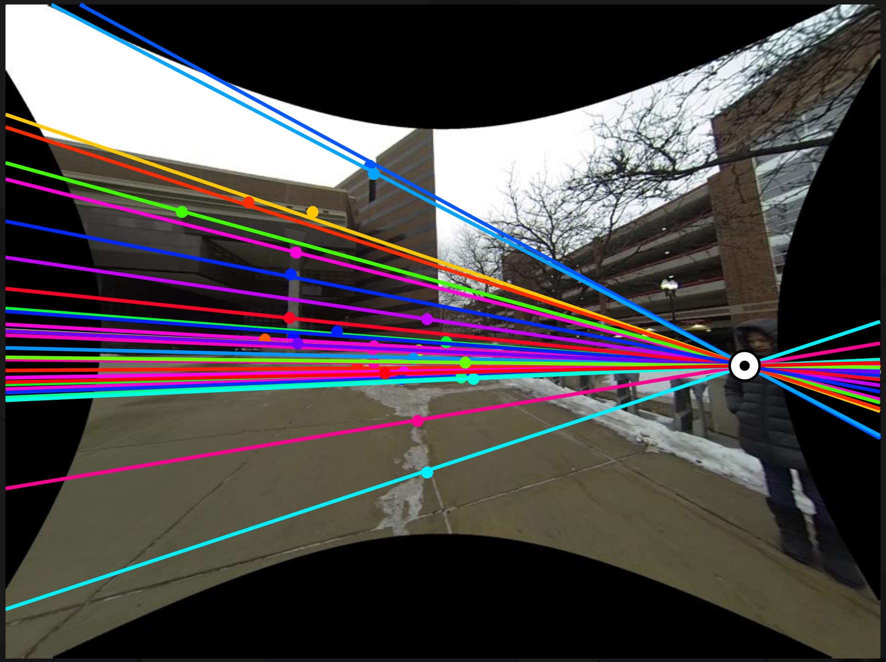

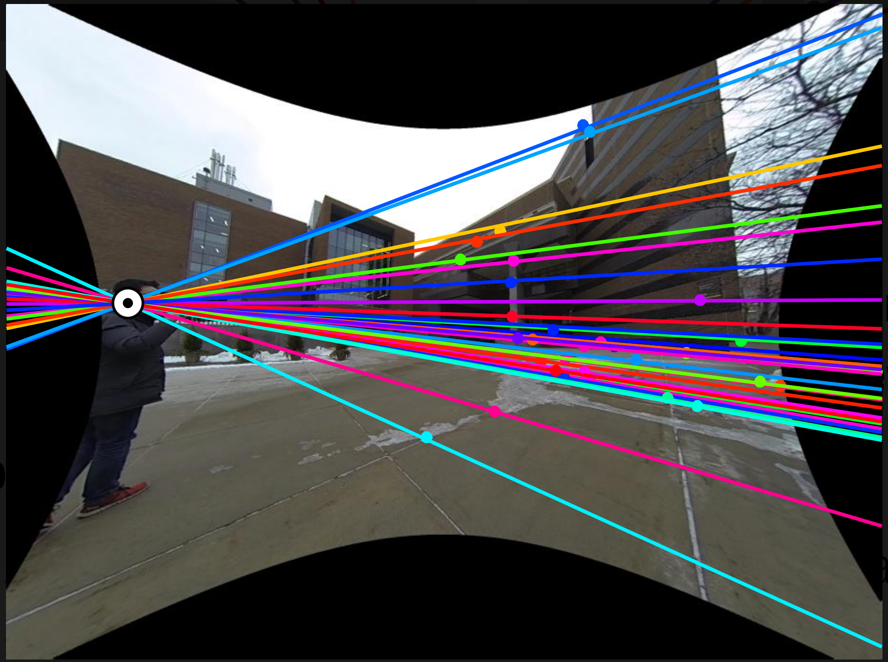

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Epip

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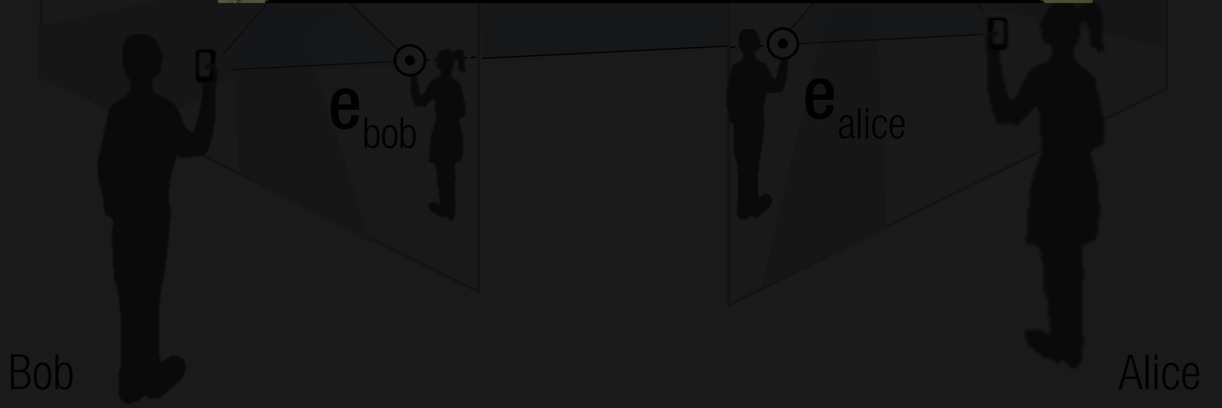
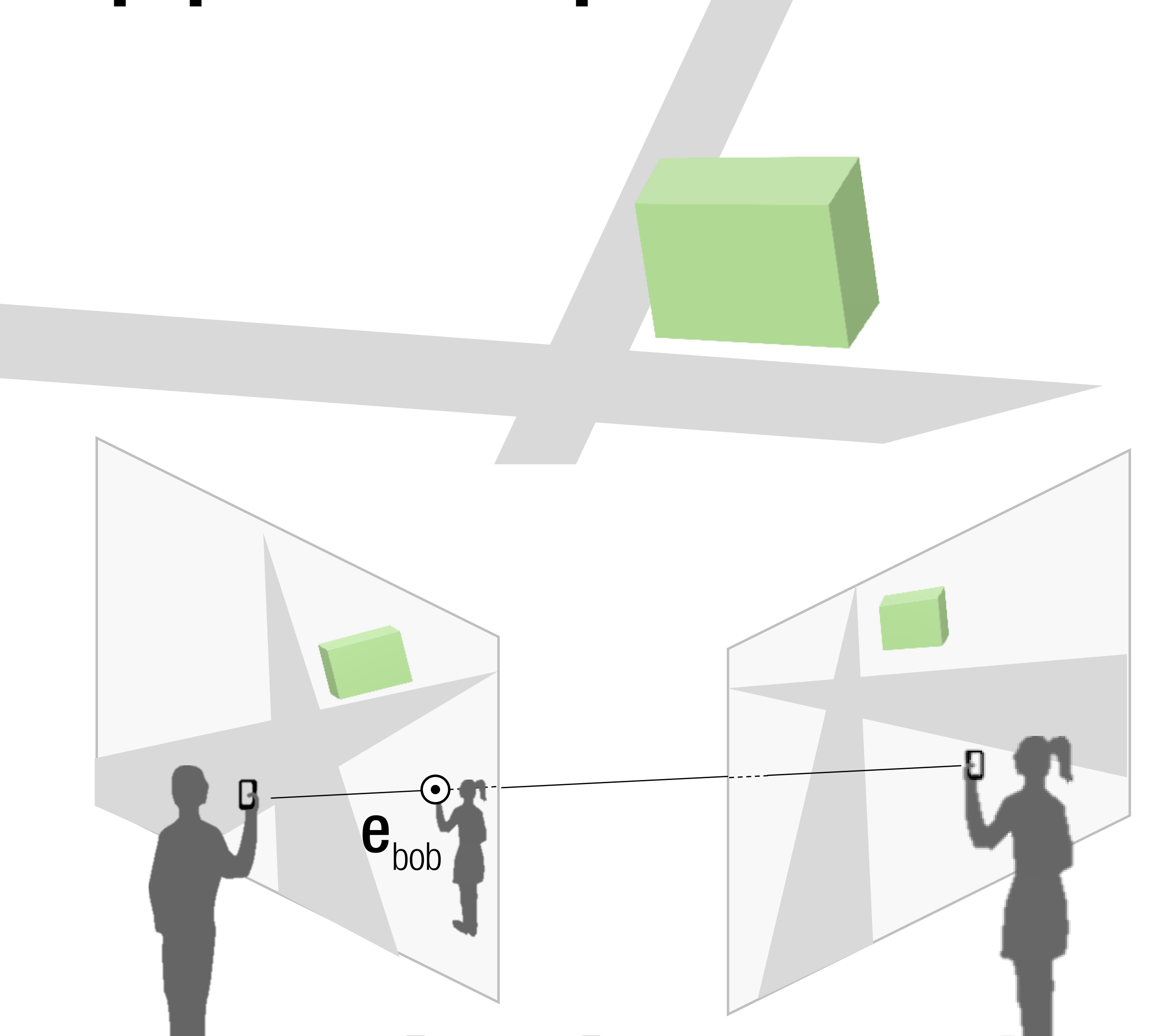
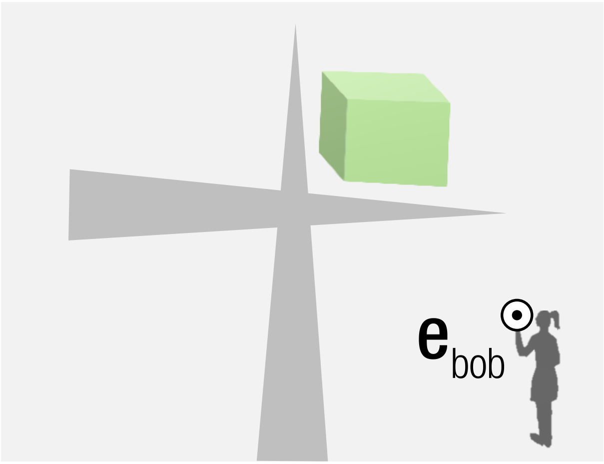


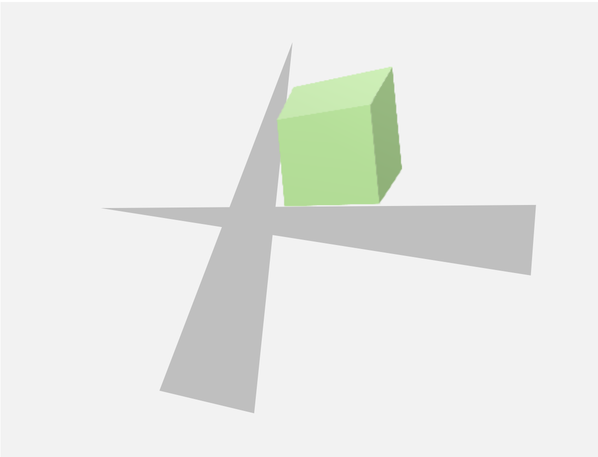
Epipole Computation



$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times1} \end{bmatrix} \qquad \mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$



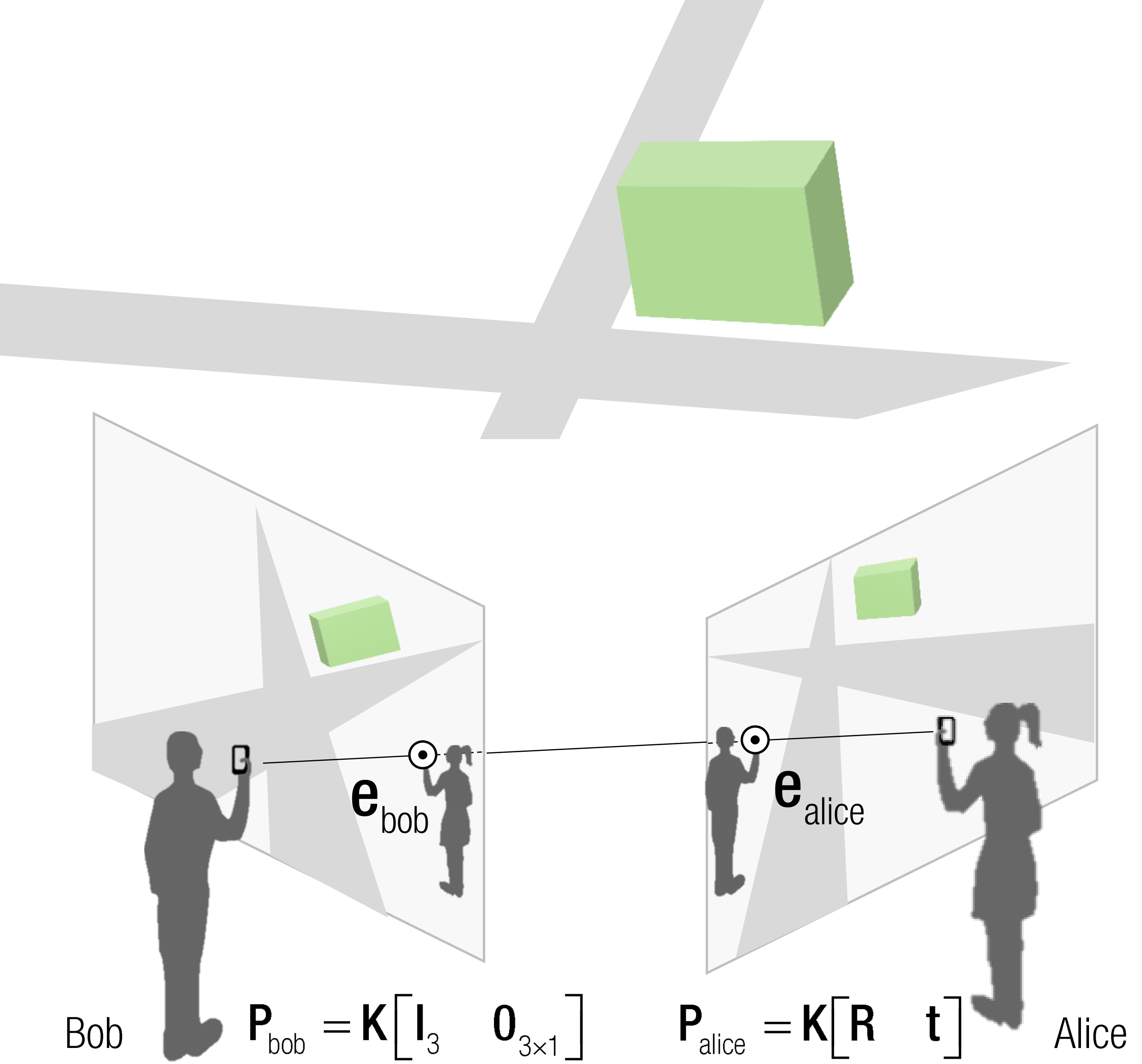
Bob's image

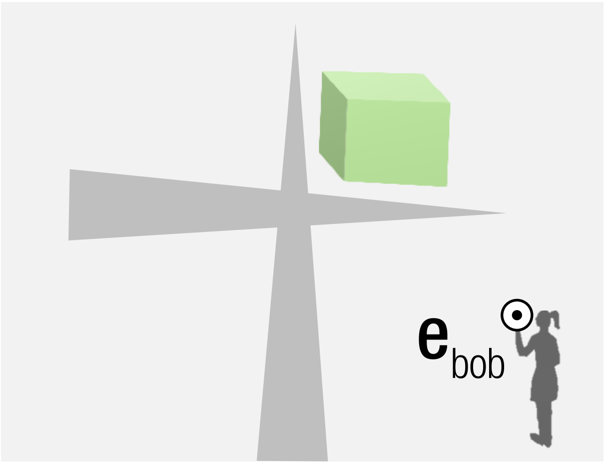


Alice's image

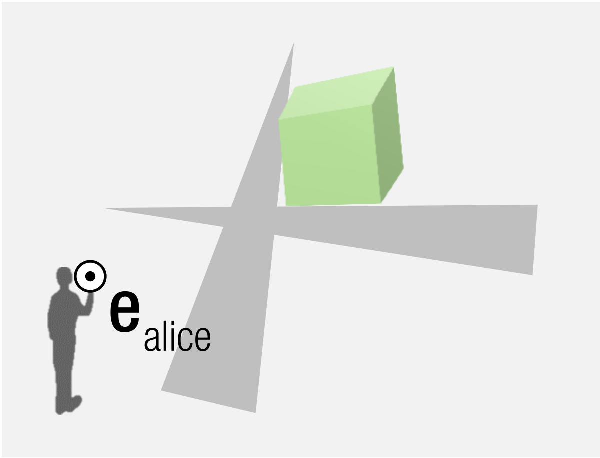
$$\lambda \mathbf{e}_{bob} = \underbrace{\mathbf{K}\begin{bmatrix} \mathbf{I}_3 & \mathbf{0} \end{bmatrix}}_{\text{Bob's camera projection matrix}} \begin{bmatrix} -\mathbf{R}^{\mathsf{T}}\mathbf{t} \\ 1 \end{bmatrix} = -\mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

Epipole Computation





Bob's image



Alice's image

$$\lambda \mathbf{e}_{bob} = \underbrace{K\begin{bmatrix} I_3 & 0 \end{bmatrix}}_{\text{Bob's camera projection matrix}} \begin{bmatrix} -R^T t \\ 1 \end{bmatrix} = -KR^T t$$

Bob's camera projection matrix

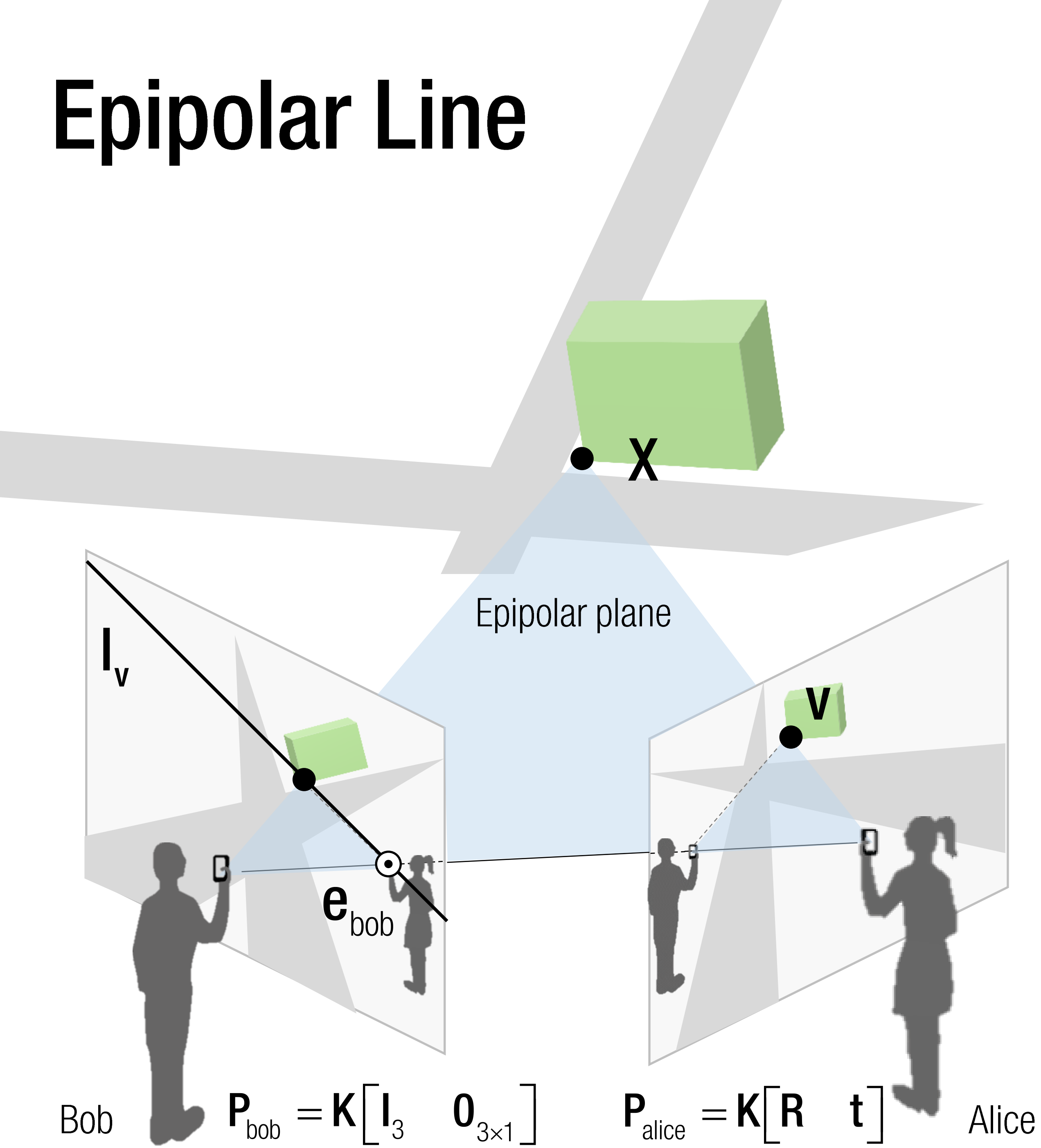
$$\lambda \mathbf{e}_{alice} = \underbrace{K\begin{bmatrix} R & t \end{bmatrix}}_{\text{Alice's camera projection matrix}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = Kt$$

Alice's camera projection matrix

Epipolar Line

$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\top}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} =$$



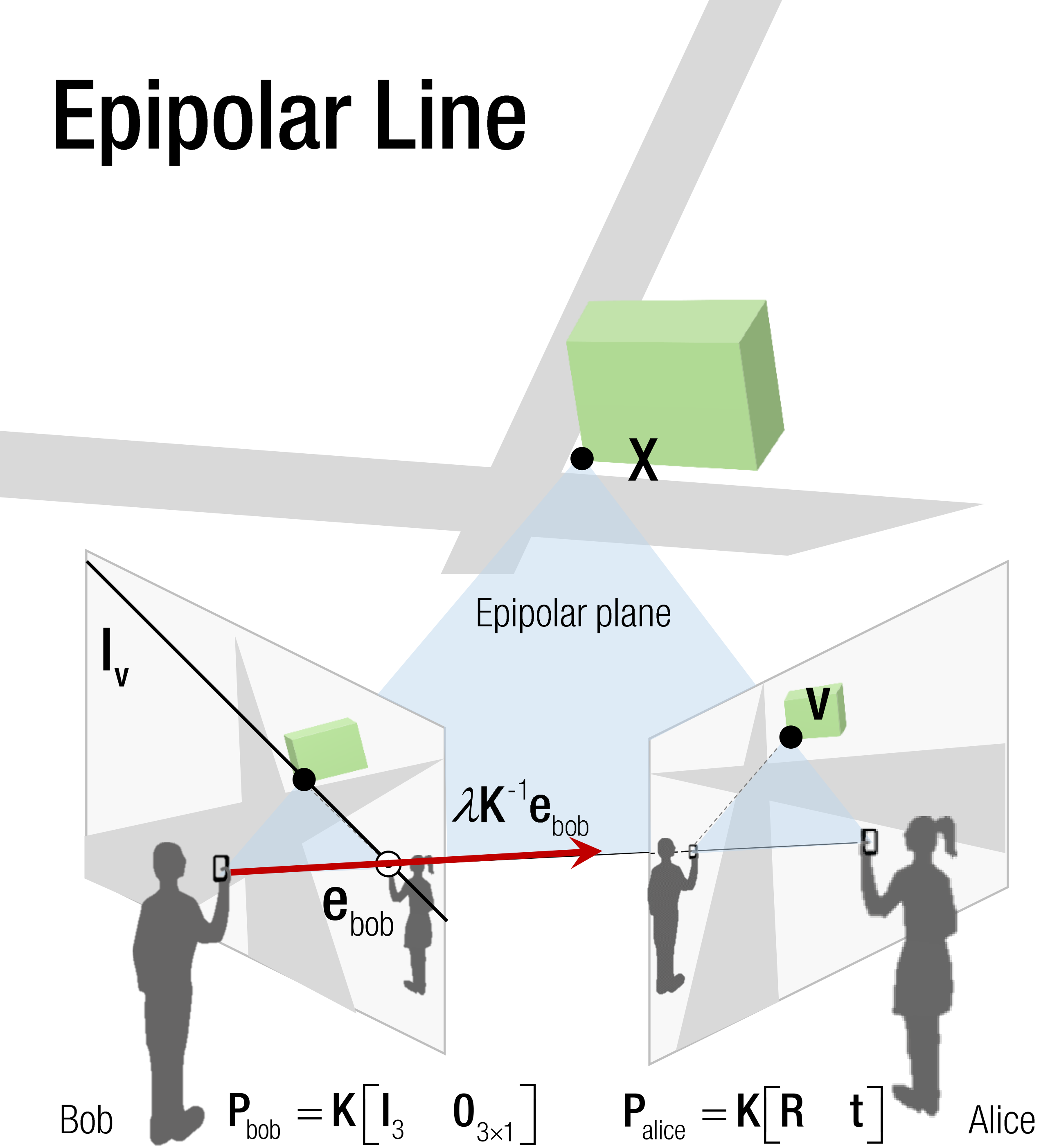
$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix} \qquad \mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

Epipolar Line

$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} =$$

$$\rightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^{\mathsf{T}}\mathbf{t}$$

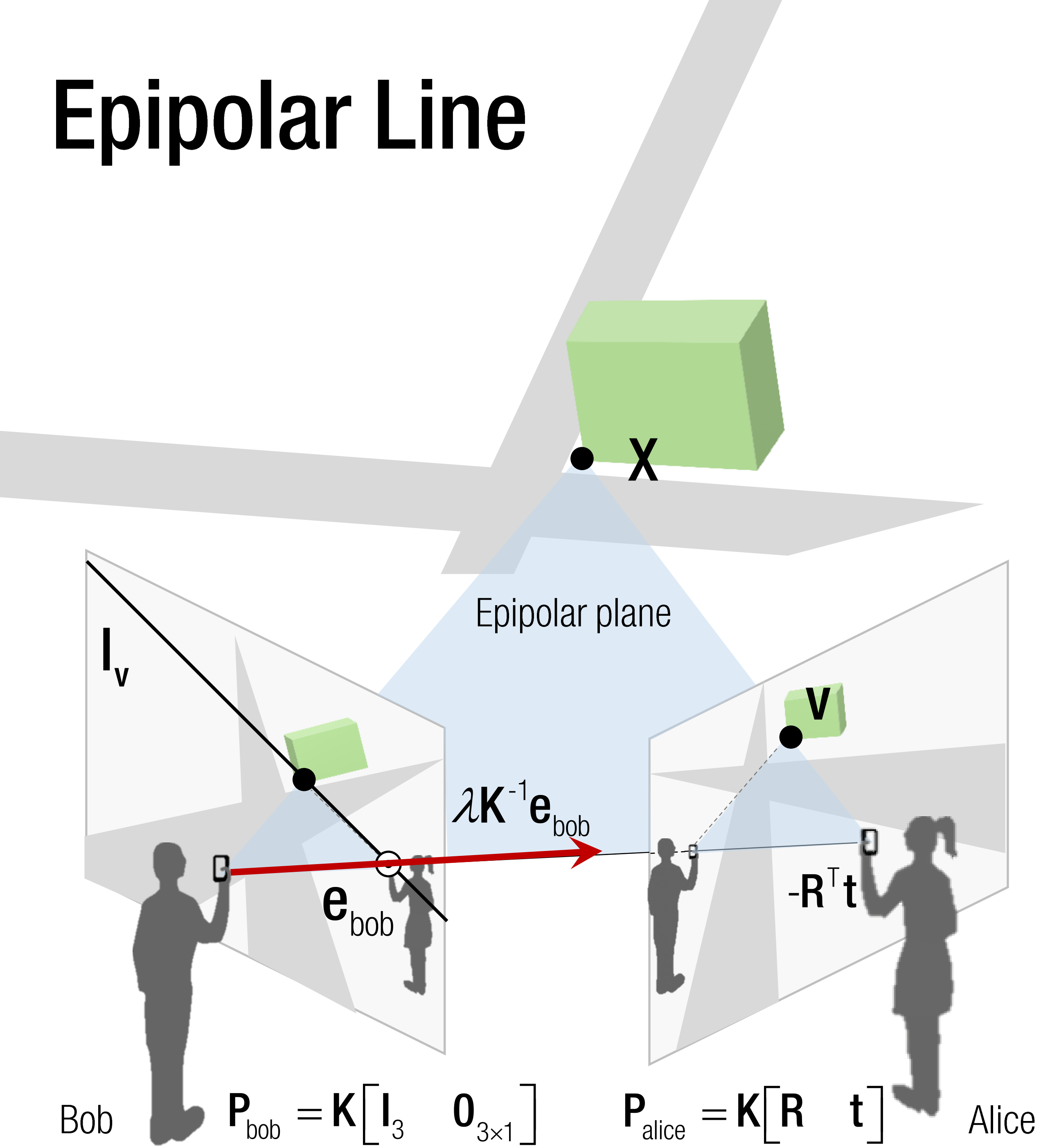


Epipolar Line

$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{v} =$$

$$\rightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^{\mathsf{T}}\mathbf{t}$$



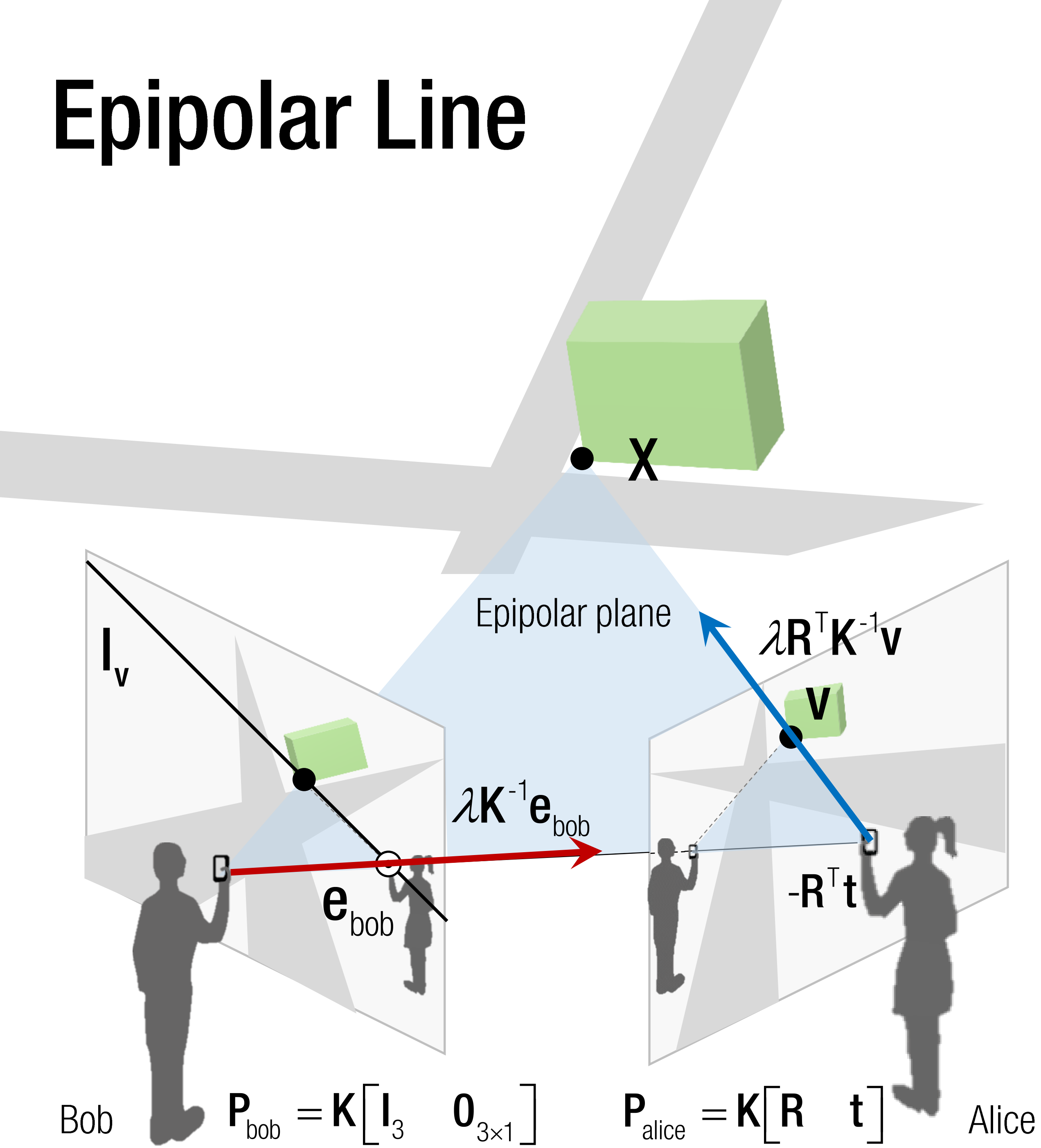
Epipolar Line

$$\lambda e_{bob} = KR^{T}t \qquad \lambda e_{alice} = Kt$$

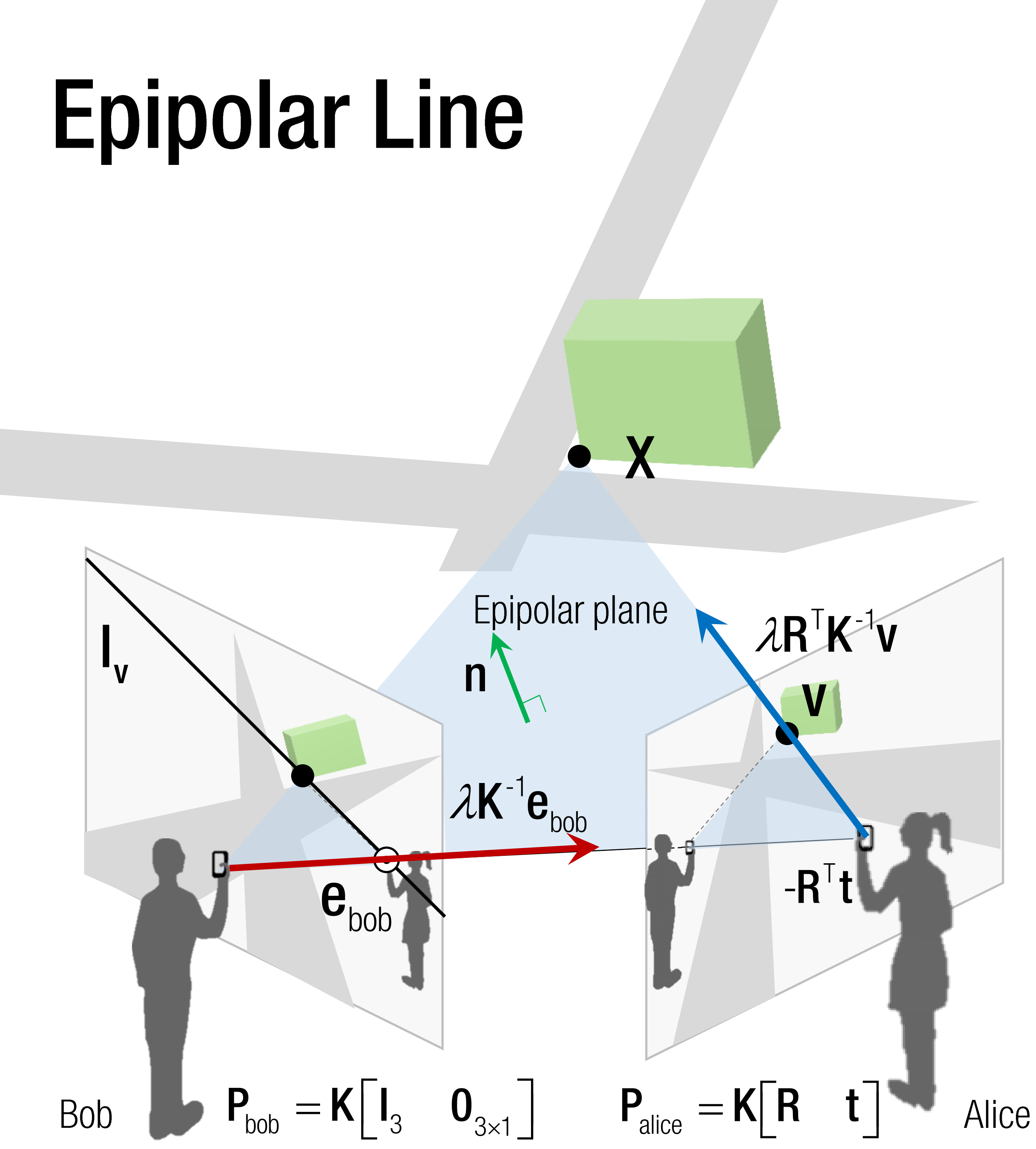
$$l_v =$$

→ $\lambda K^{-1} e_{bob} = R^{T}t$

→ $\underbrace{\lambda_1 R^{T} K^{-1} v}_{\text{Direction}} \; - \underbrace{R^{T} t}_{\text{Alice's camer location}}$



Epipolar Line



$\lambda e_{bob} = KR^T t$ $\quad\quad \lambda e_{alice} = Kt$

$l_v =$

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$

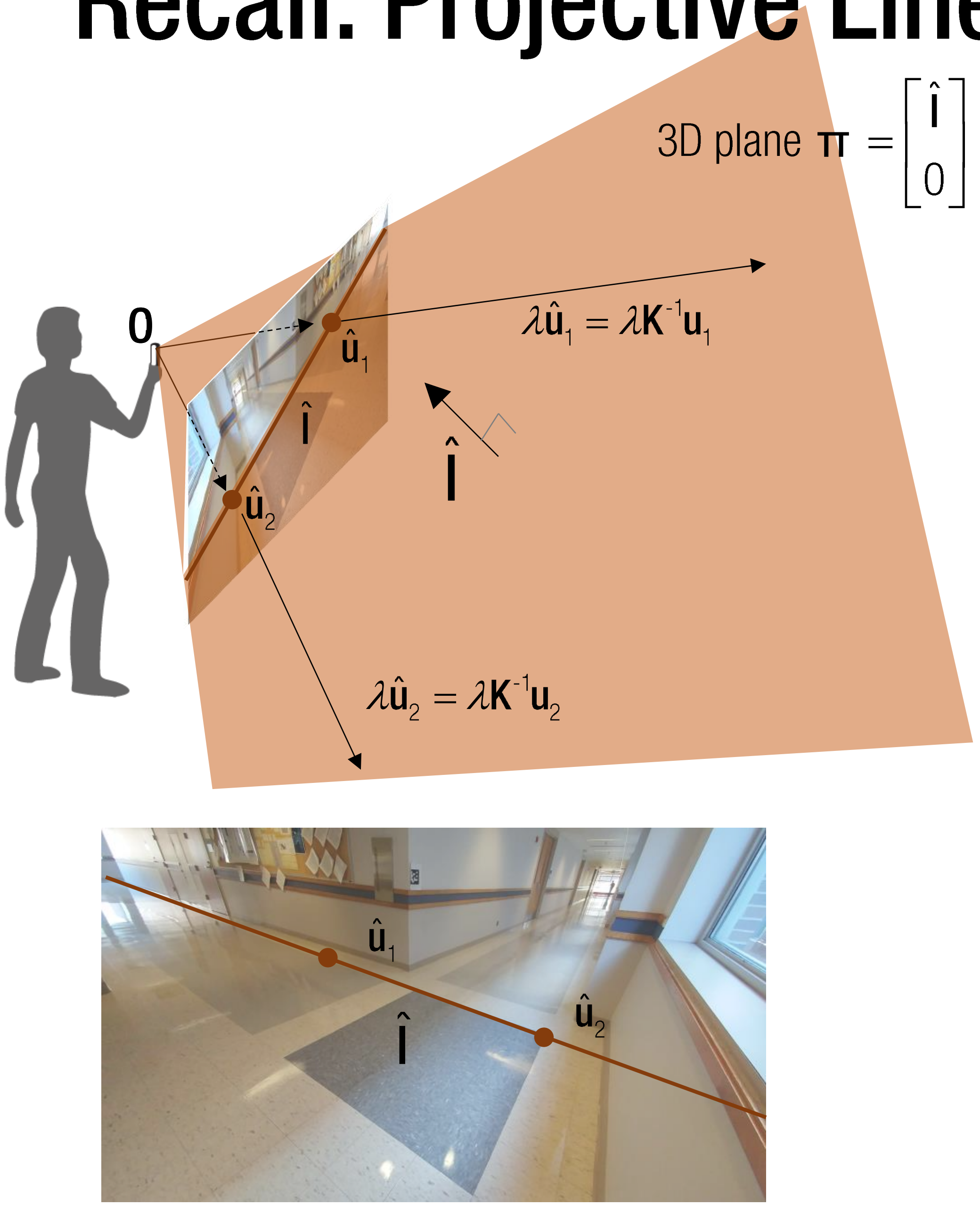
→ $n = R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right)$

$= R^T t \times R^T K^{-1} v$

$= R^T \left(t \times K^{-1} v\right)$: only works for rotation

$= R^T [t]_\times K^{-1} v$

Recall: Projective Line vs. Plane



Normalized coordinate:

$$\hat{\mathbf{u}}_1 = \mathbf{K}^{-1}\mathbf{u}_1 \qquad \hat{\mathbf{u}}_2 = \mathbf{K}^{-1}\mathbf{u}_2$$

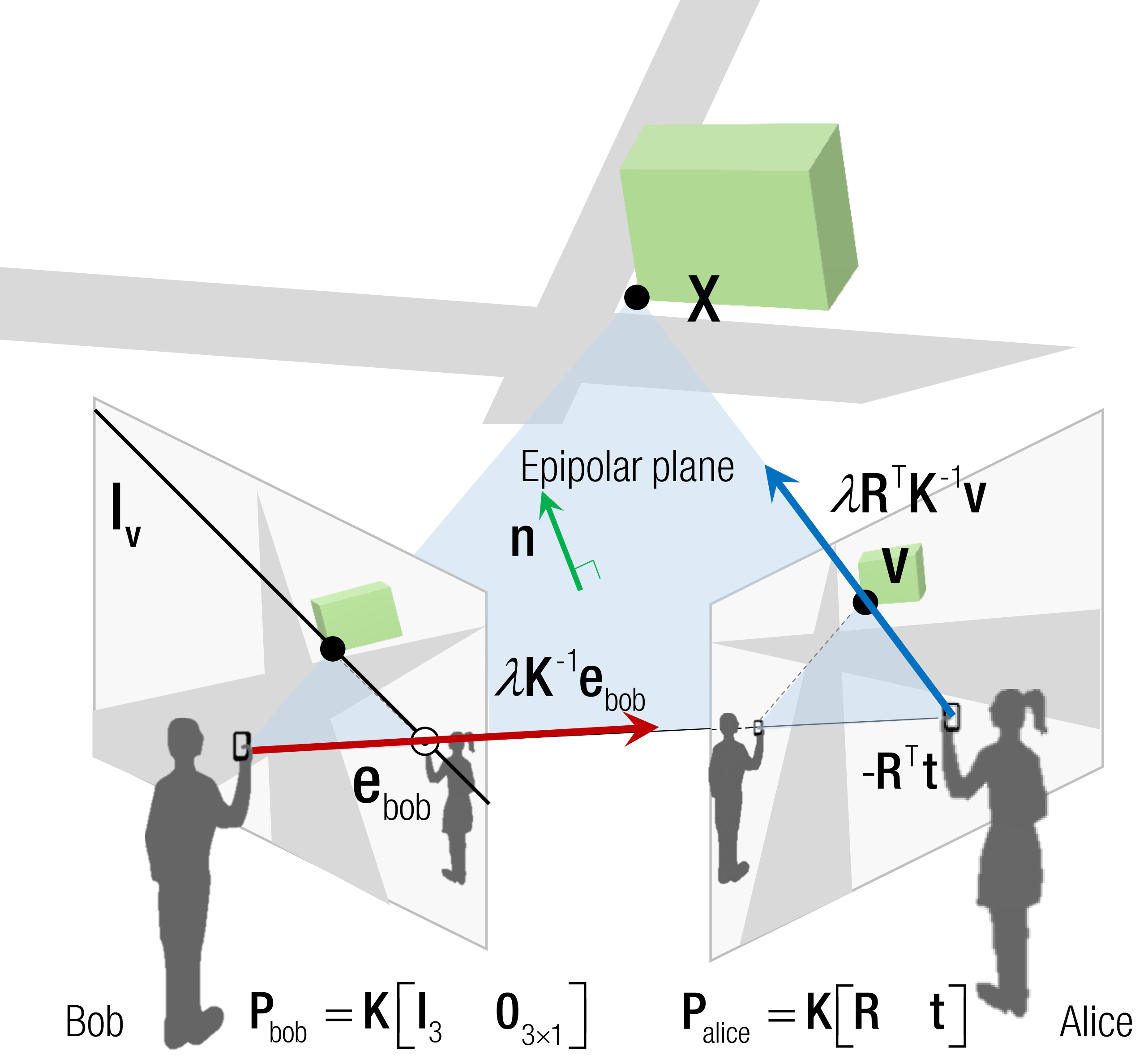
$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{u}}_1 \times \hat{\mathbf{u}}_2$$

where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathsf{T}}\mathbf{l} = \mathbf{K}^{\mathsf{T}}\mathbf{l}$ due to duality

Plane normal: $\left(\lambda_1\hat{\mathbf{u}}_1\right) \times \left(\lambda_2\hat{\mathbf{u}}_2\right) = \lambda\hat{\mathbf{l}}$

$$\therefore \pi = \begin{bmatrix} \hat{\mathbf{l}} \\ 0 \end{bmatrix}$$

Epipolar Line



$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

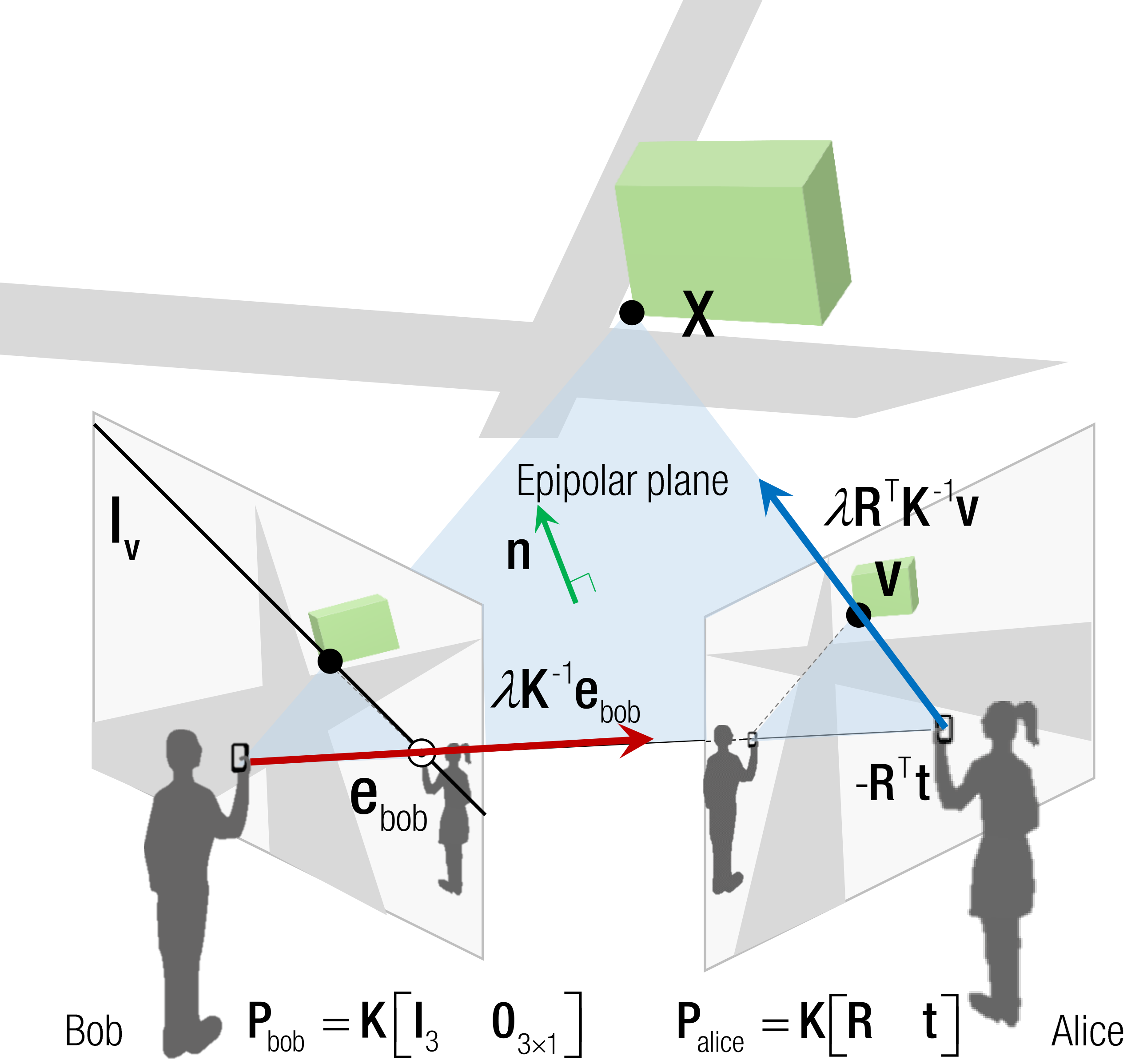
$$l_v =$$

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$

→
$$\begin{aligned} n &= R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right) \\ &= R^T t \times R^T K^{-1} v \\ &= R^T \left(t \times K^{-1} v\right) \quad \text{: only works for rotation} \\ &= R^T [t]_\times K^{-1} v \end{aligned}$$

Epipolar Line



$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$

$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$: Epipolar line

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$

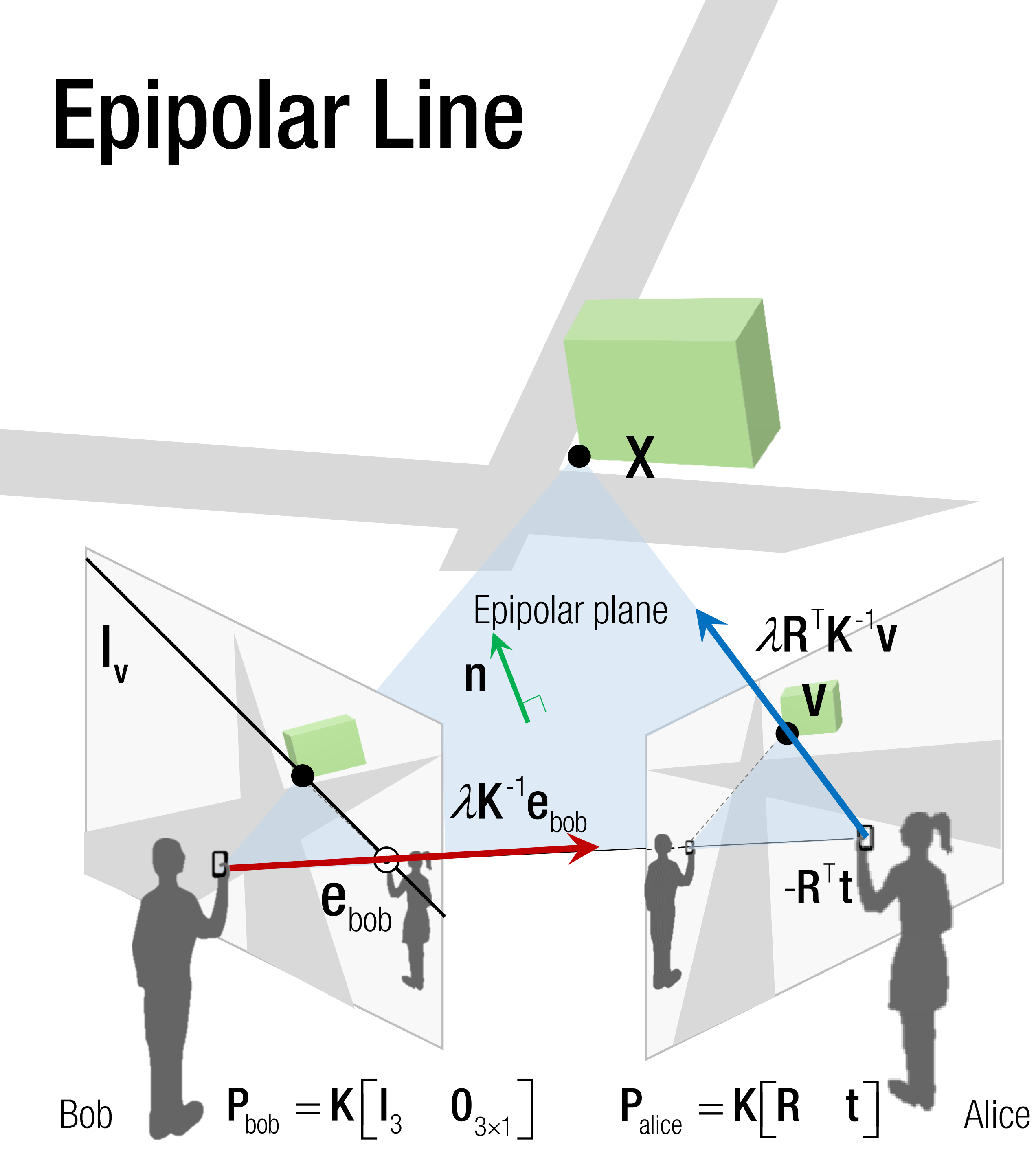
→ $n = R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right)$

$= R^T t \times R^T K^{-1} v$

$= R^T \left(t \times K^{-1} v\right)$: only works for rotation

$= R^T [t]_\times K^{-1} v$

Epipolar Line



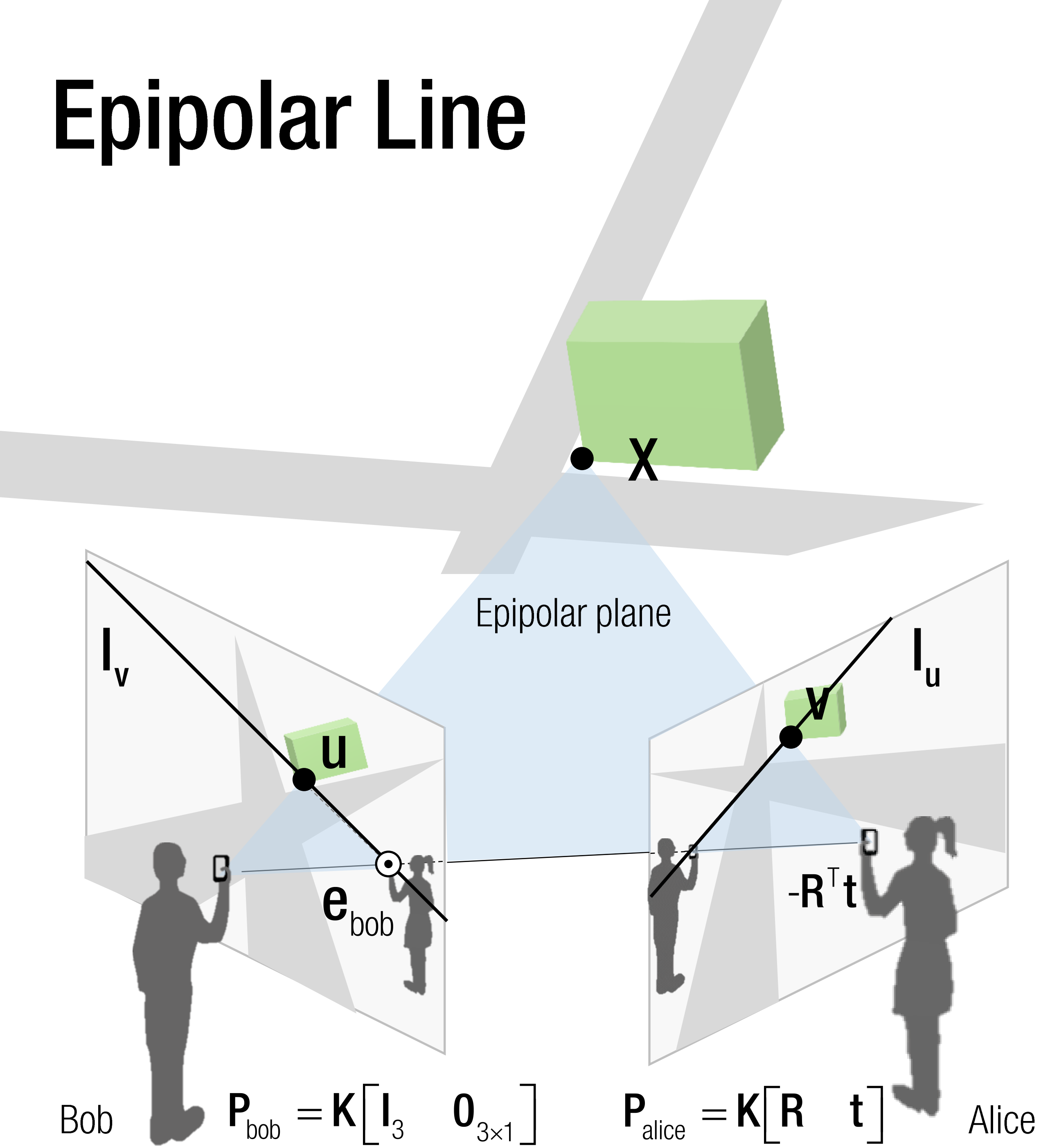
$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

Epipolar constraint:

$$u^T l_v = u^T K^{-T} R^T [t]_\times K^{-1} v = 0$$

Epipolar Line



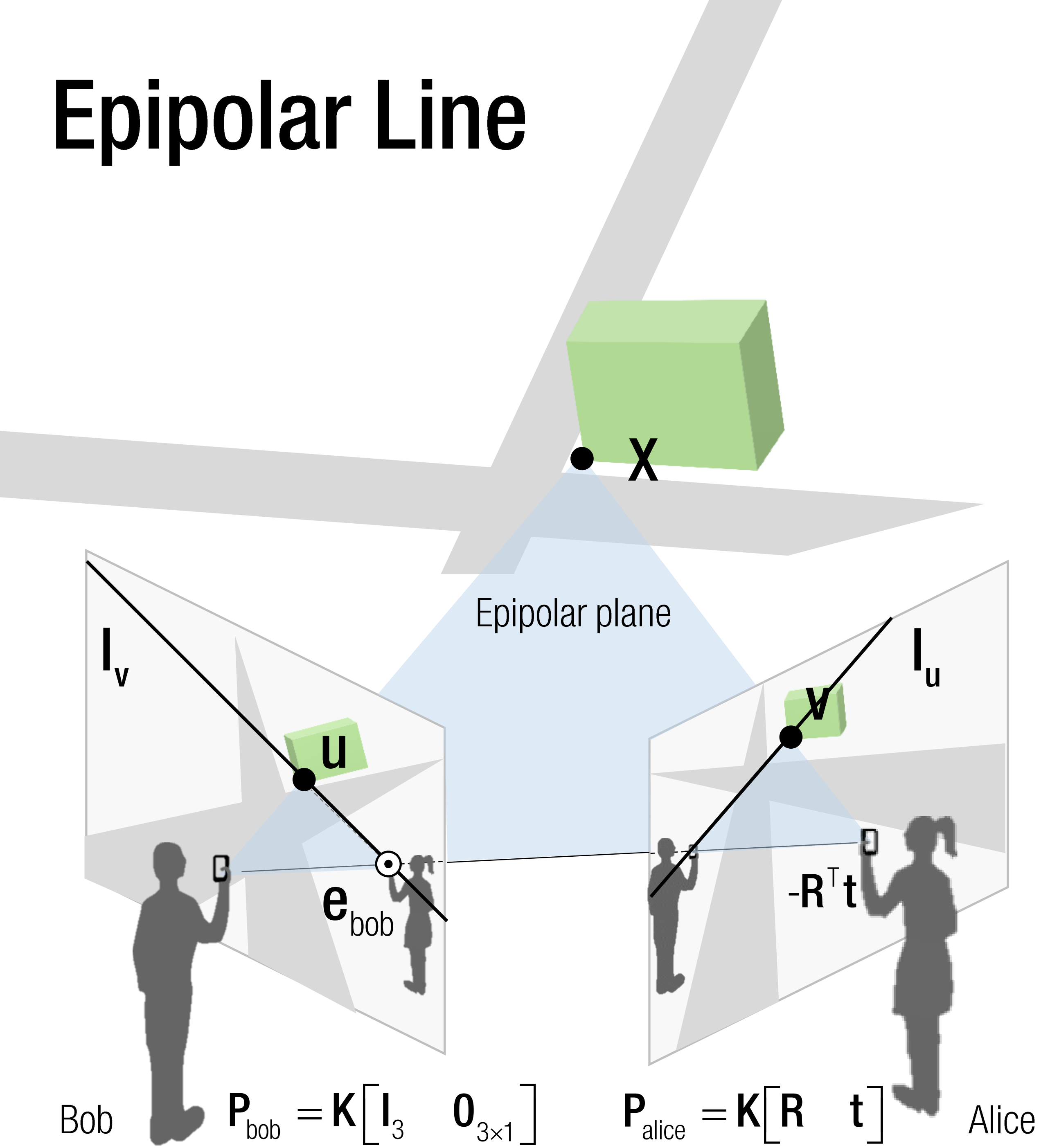
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^\mathsf{T}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_\mathbf{v} = \mathbf{K}^{-\mathsf{T}}\mathbf{n} = \mathbf{K}^{-\mathsf{T}}\mathbf{R}^\mathsf{T}\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v}$$: Epipolar line

Epipolar constraint:

$$\mathbf{u}^\mathsf{T}\mathbf{l}_\mathbf{v} = \mathbf{u}^\mathsf{T}\mathbf{K}^{-\mathsf{T}}\mathbf{R}^\mathsf{T}\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v} = 0$$

Epipolar Line



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$

$$\lambda e_{bob} = KR^{T}t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T}n = K^{-T}R^{T}[t]_{\times}K^{-1}v$$: Epipolar line

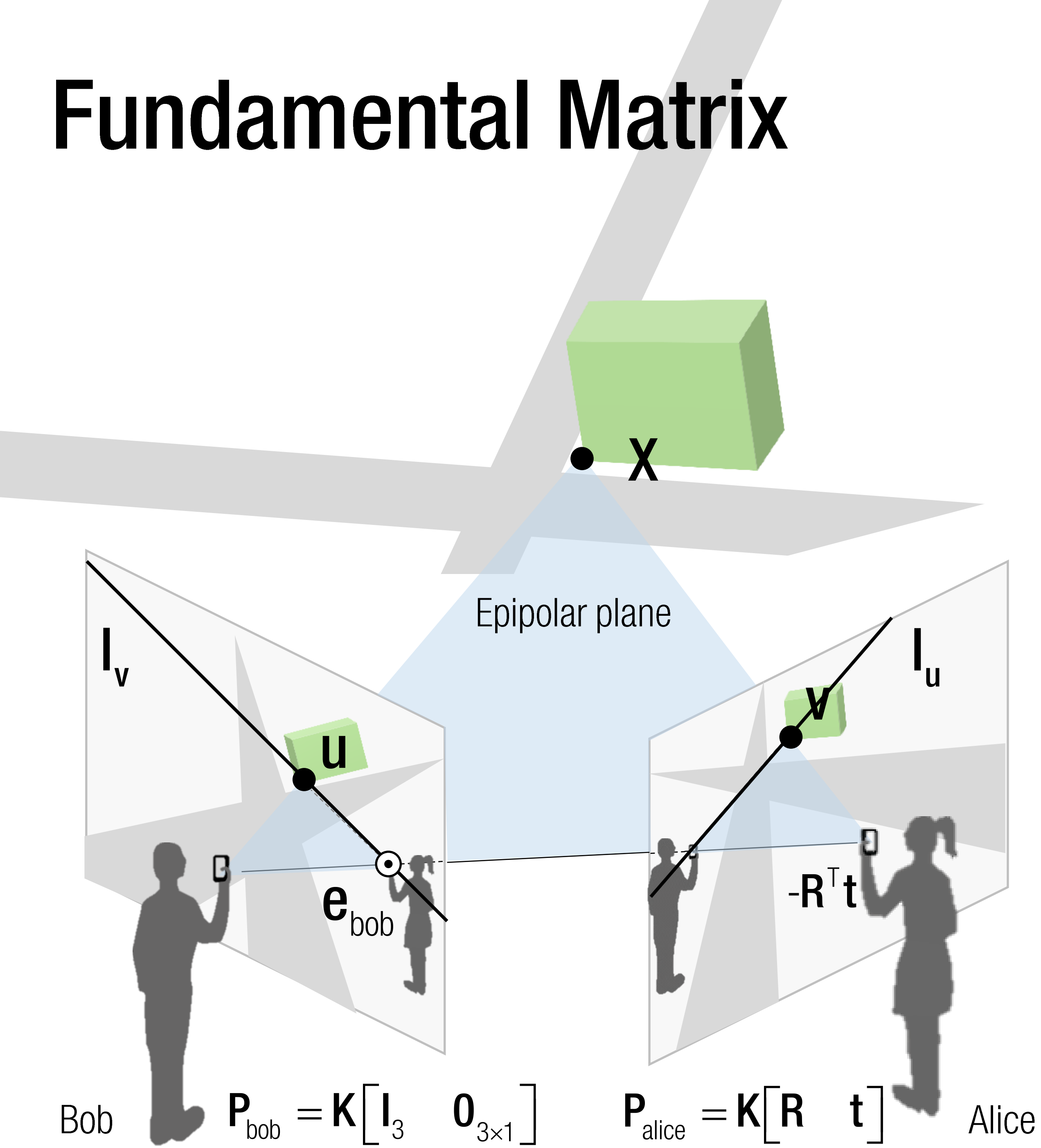
Epipolar constraint:

$$u^{T}l_v = \underbrace{u^{T}K^{-T}R^{T}[t]_{\times}K^{-1}}_{l_u^{T}}v = 0$$

$$l_u = -K^{-T}[t]_{\times}RK^{-1}u \qquad \because [t]_{\times}^{T} = -[t]_{\times}$$

Skew symetric matrix

Fundamental Matrix



$\lambda e_{bob} = KR^T t$ $\lambda e_{alice} = Kt$

$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

Epipolar constraint:

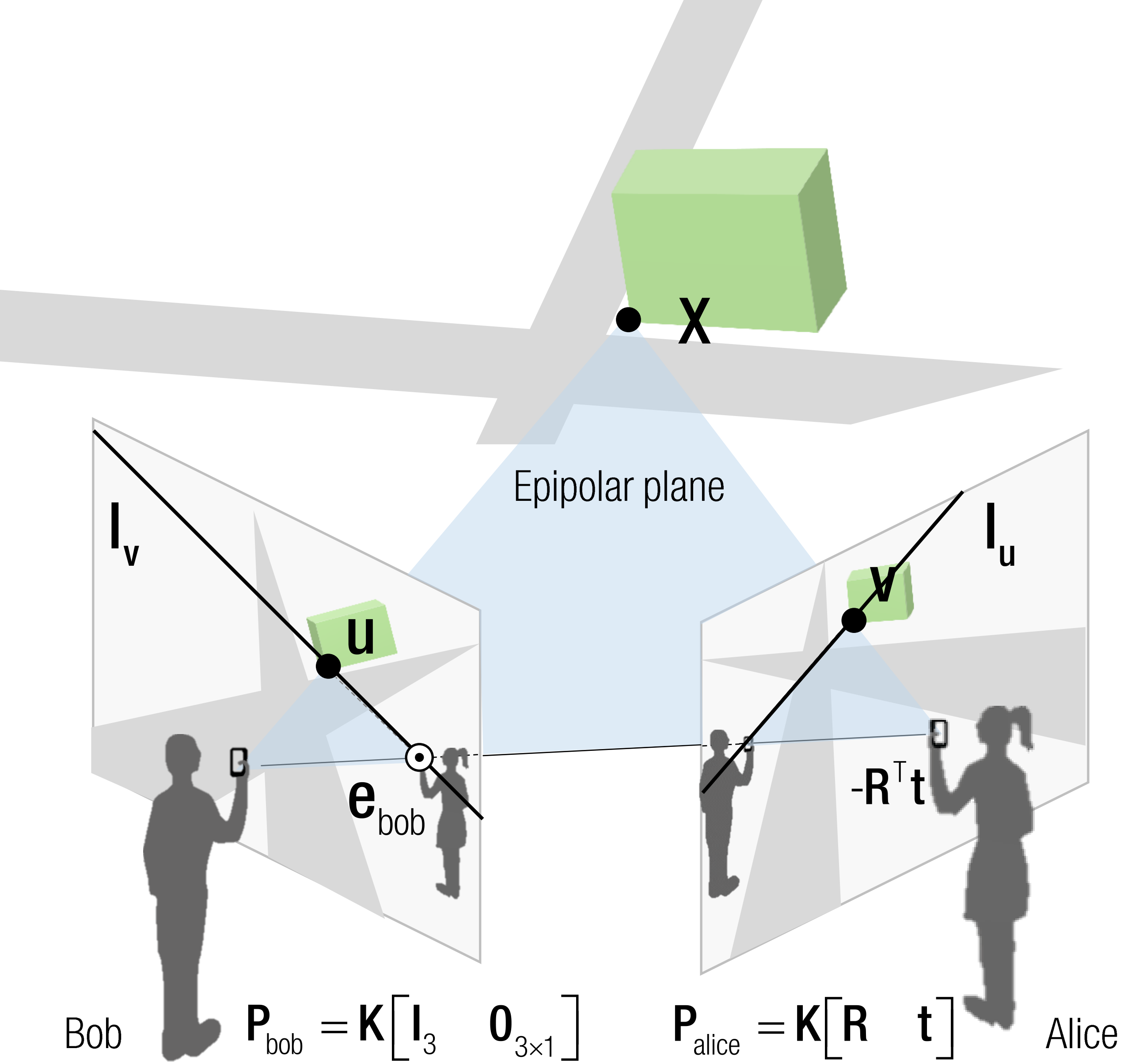
$$u^T l_v = \underbrace{u^T K^{-T} R^T [t]_\times K^{-1}}_{l_u^T} v = 0$$

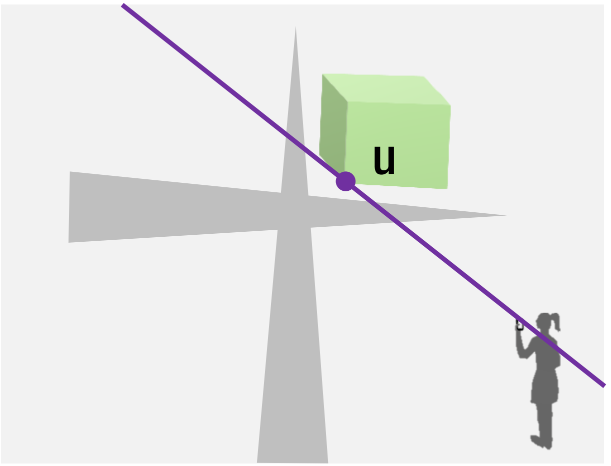
$$l_u = \underbrace{-K^{-T} [t]_\times R K^{-1}}_{\text{Common for all points}} u$$

$\because [t]_\times^T = -[t]_\times$

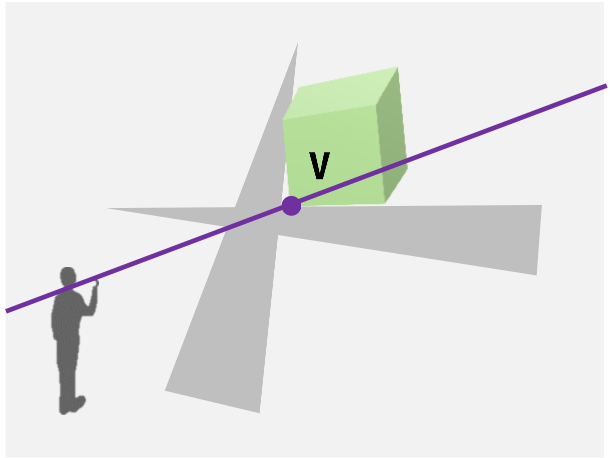
Skew symetric matrix

Fundamental Matrix





Bob's image

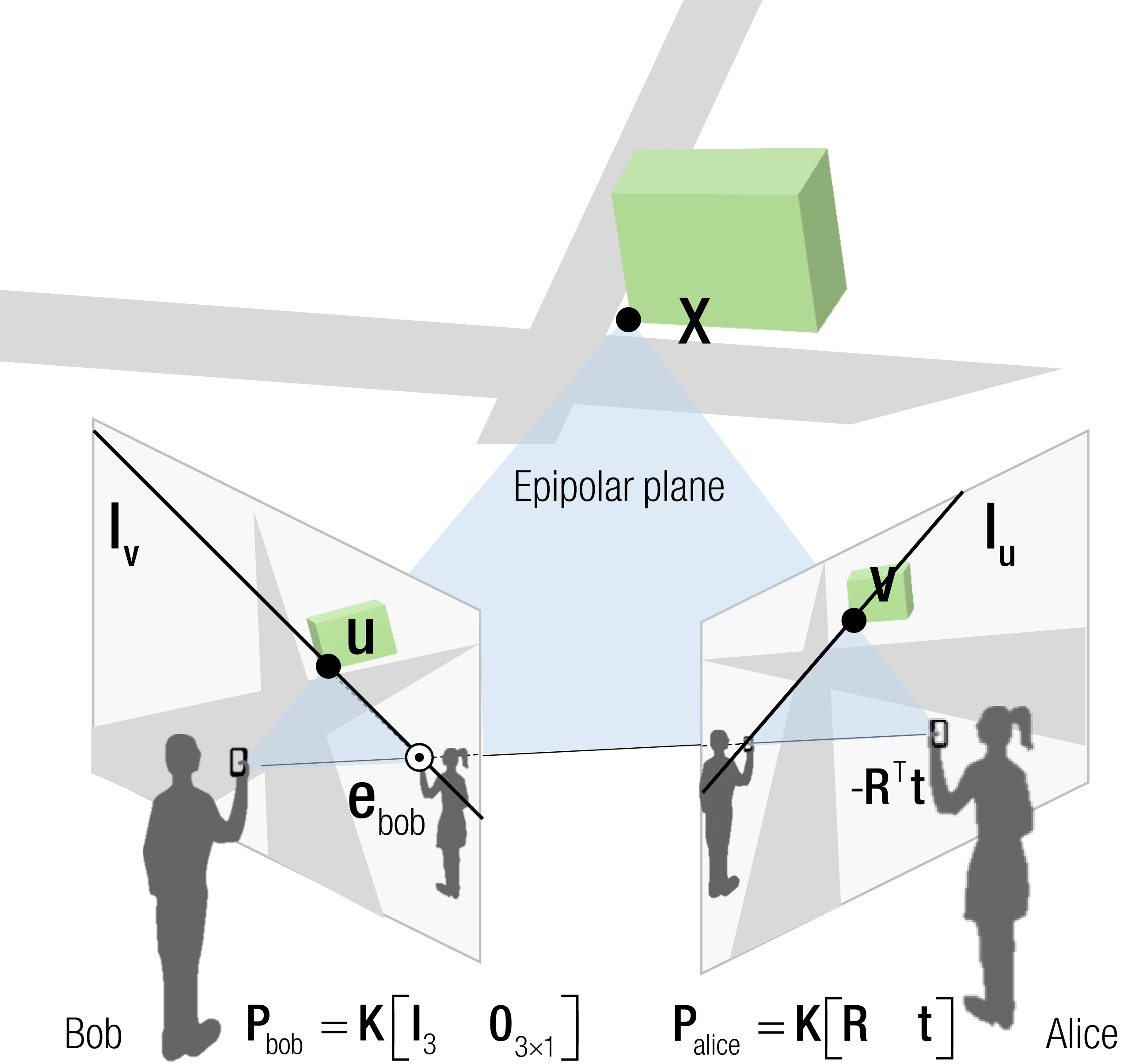


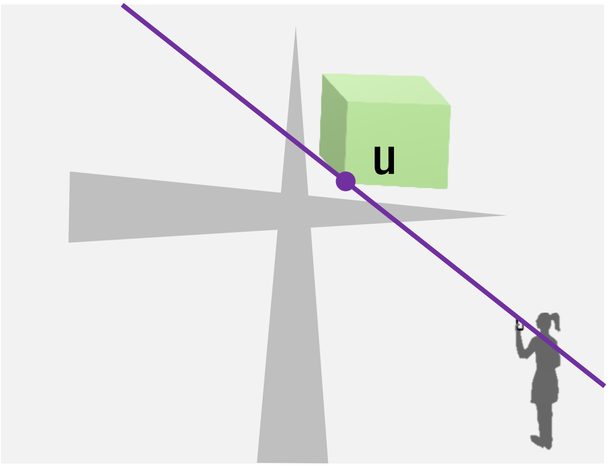
Alice's image

$$v^T l_u = v^T \underline{K^{-T}[t]_\times R K^{-1}} u = 0$$

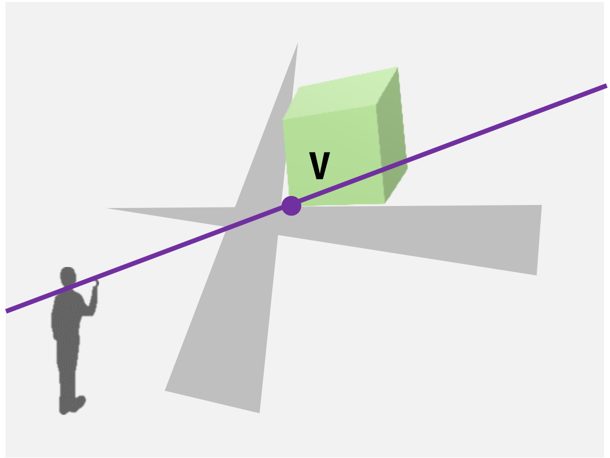
Common for all points

Fundamental Matrix





Bob's image



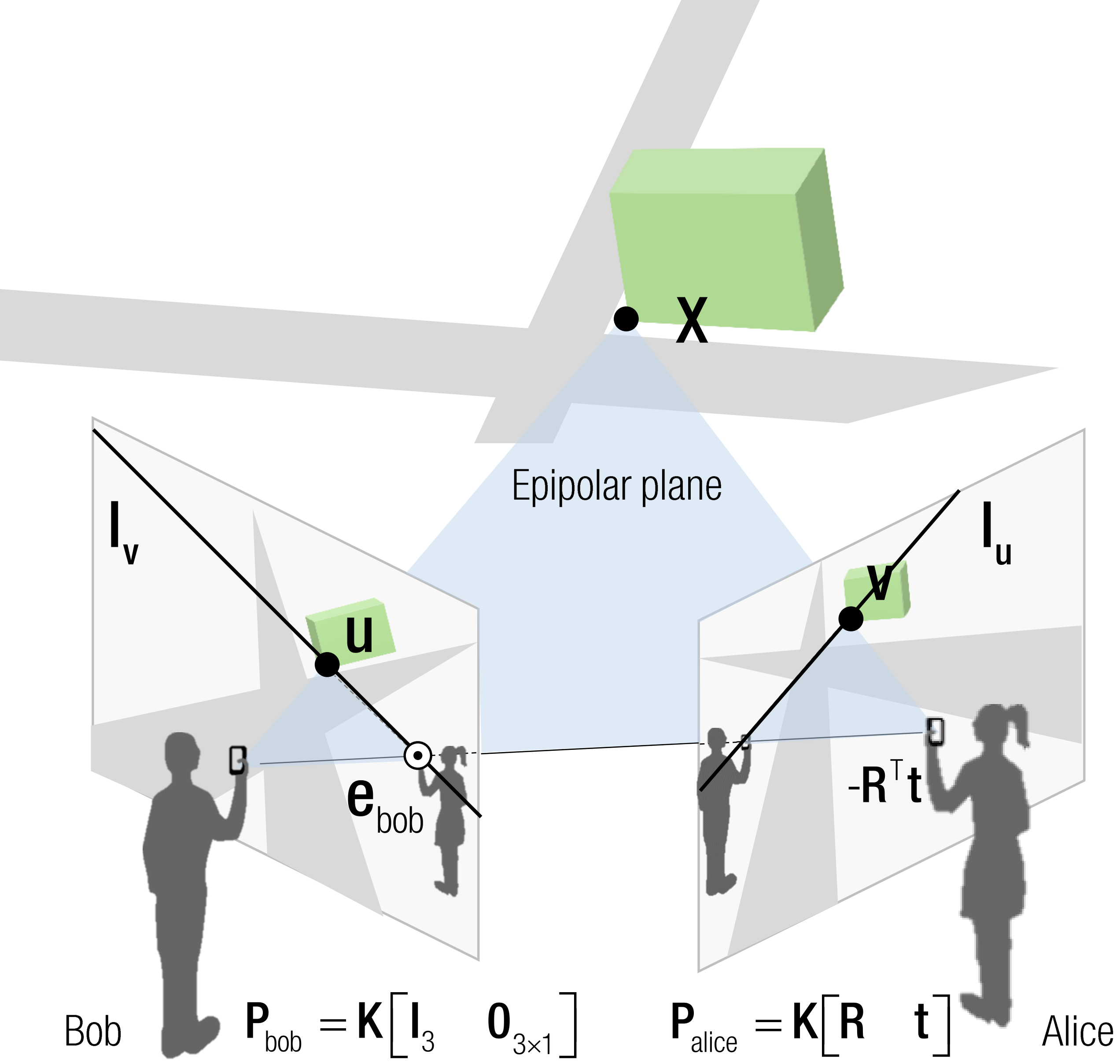
Alice's image

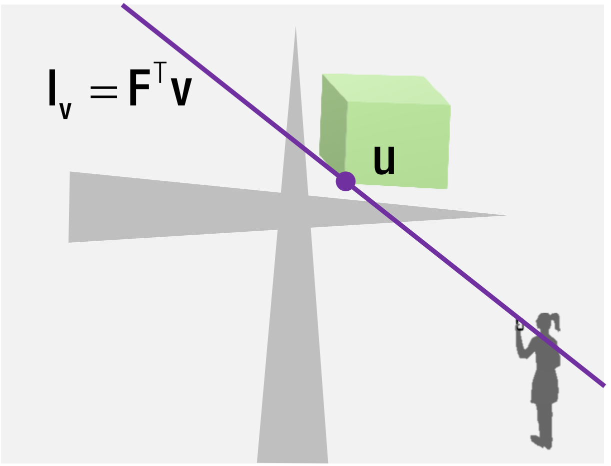
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = 0$$

$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

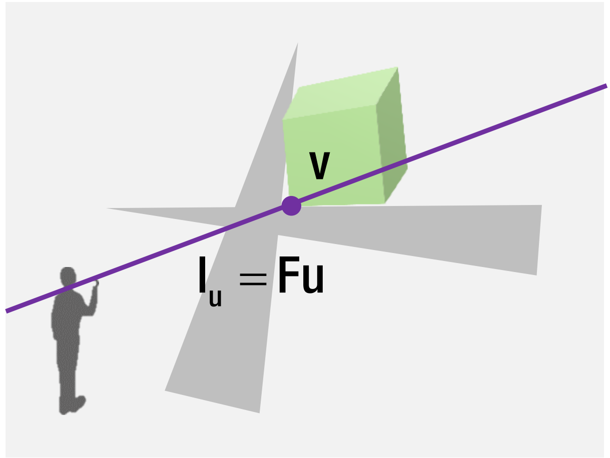
where $\underbrace{\mathbf{F} = \mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1}}_{\text{Fundamental matrix}}$

Fundamental Matrix





Bob's image



Alice's image

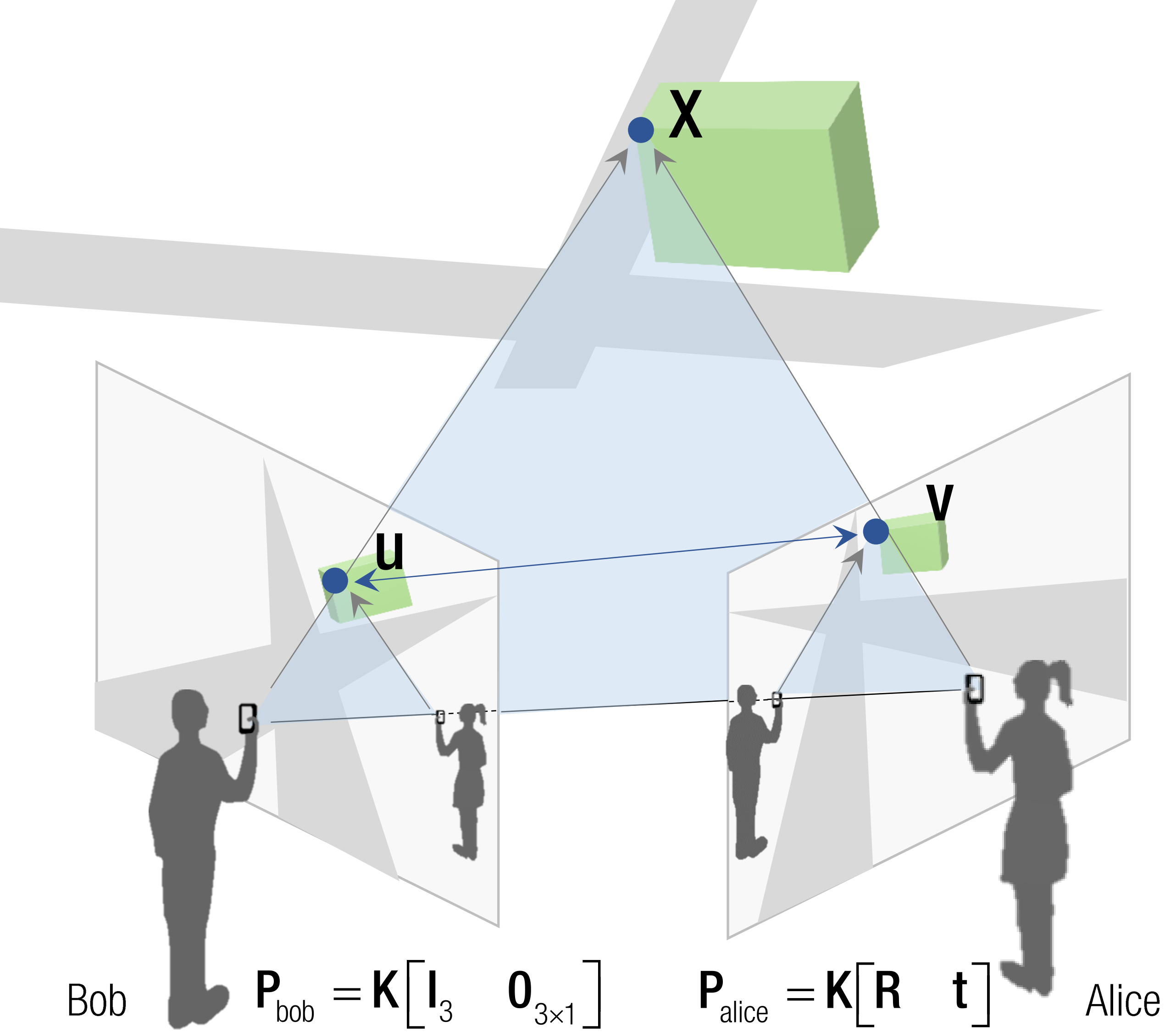
$$v^T l_u = v^T \underline{K^{-T}[t]_\times RK^{-1}} u = 0$$

Common for all points

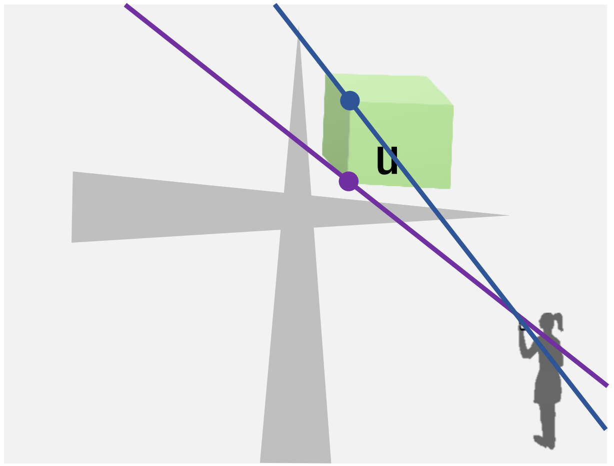
$$= v^T F u = 0$$

$$= v^T (Fu) = u^T (F^T v) = 0$$

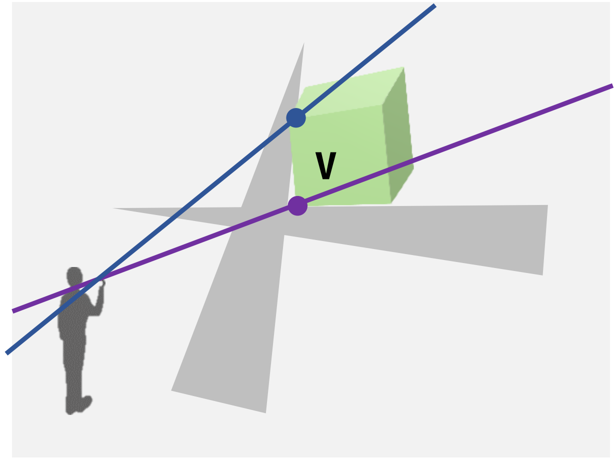
Fundamental Matrix



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



Alice's image

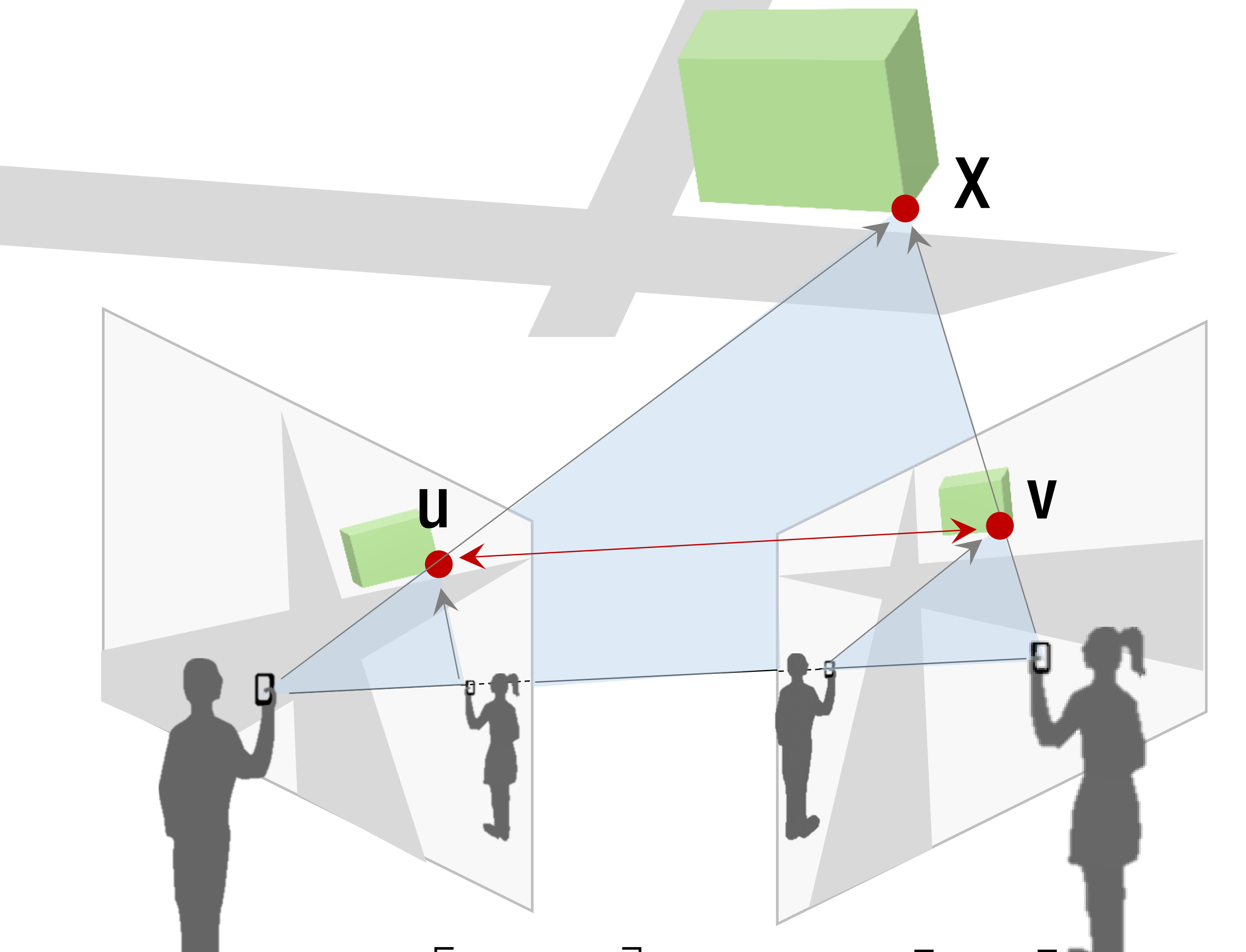
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underline{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1}} \mathbf{u} = 0$$

Common for all points

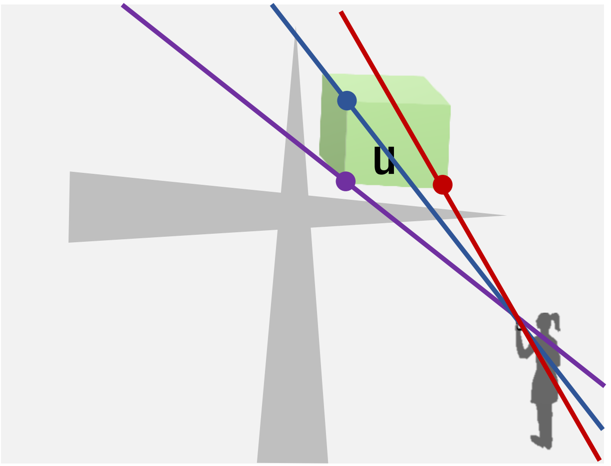
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

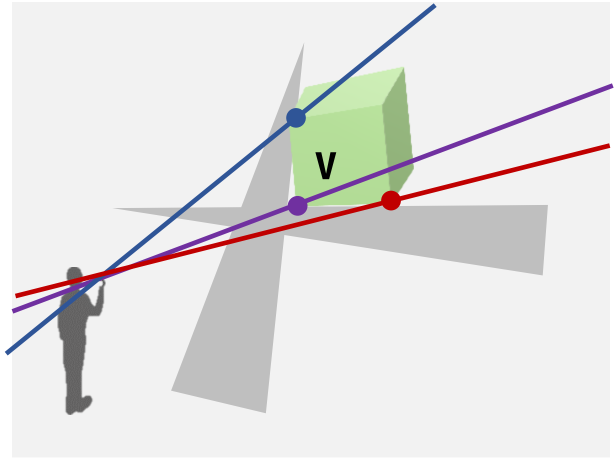
Fundamental Matrix



Bob $P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$ Alice



Bob's image



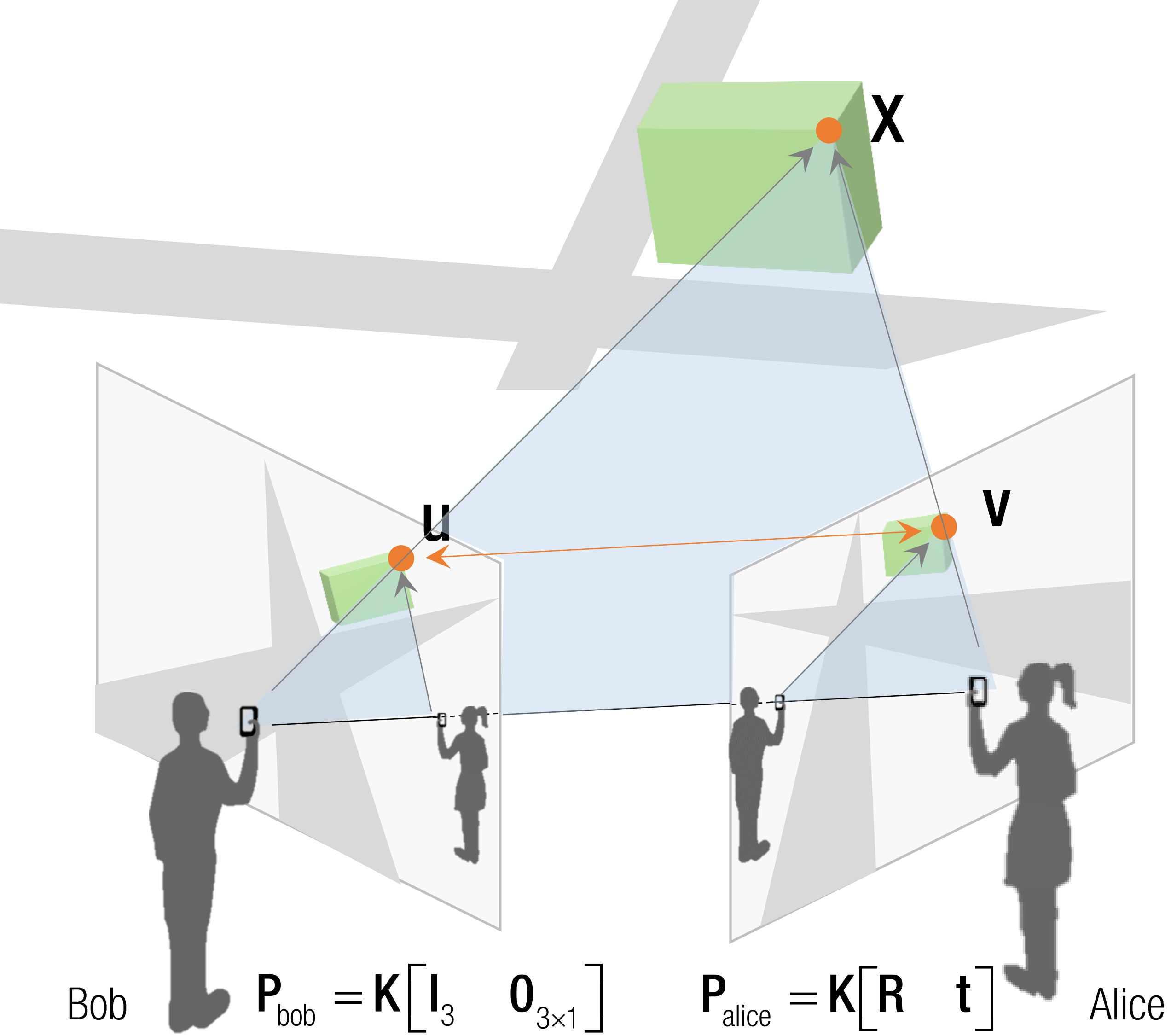
Alice's image

$$v^T l_u = v^T \underbrace{K^{-T}[t]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

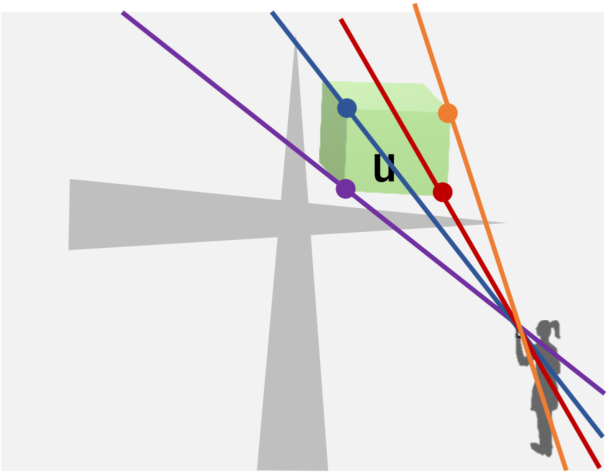
$$= v^T F u = 0$$

$$= v^T (Fu) = u^T (F^T v) = 0$$

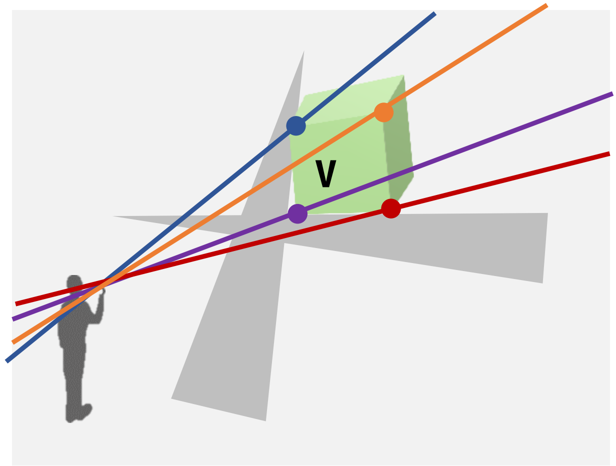
Fundamental Matrix



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



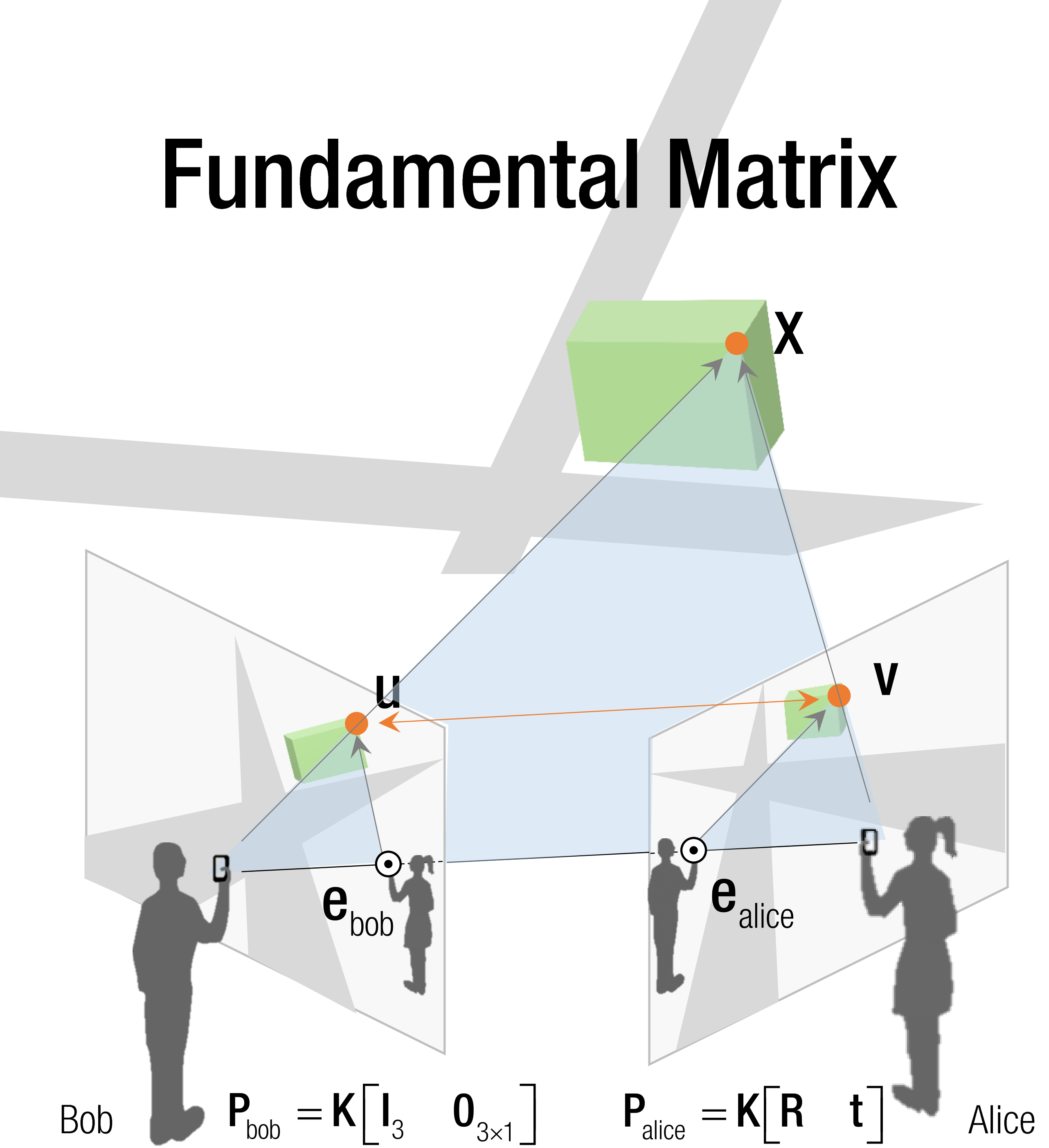
Alice's image

$$v^T l_u = \underbrace{v^T K^{-T} \left[t\right]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

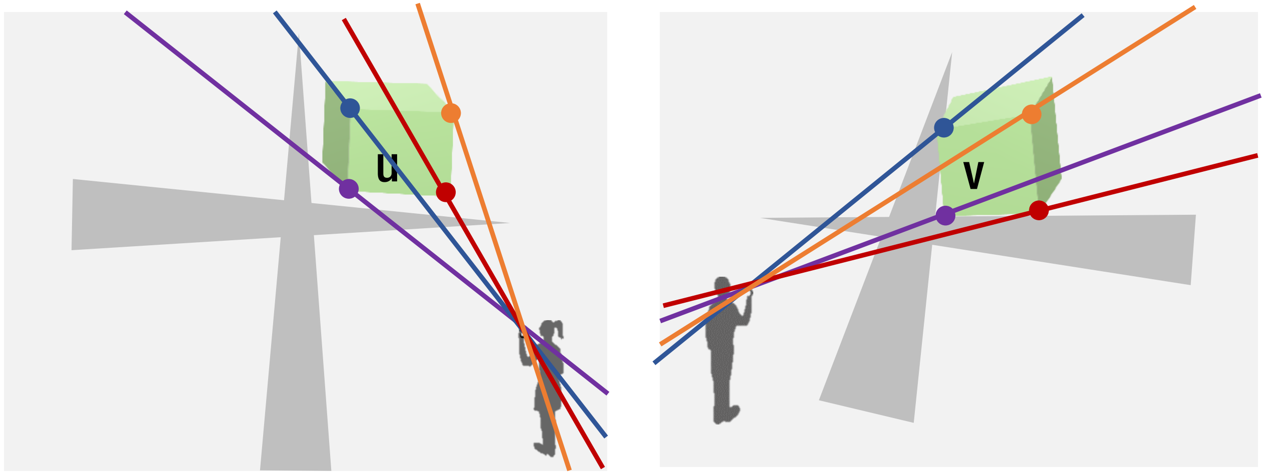
$$= v^T F u = 0$$

$$= v^T (Fu) = u^T \left(F^T v\right) = 0$$

Fundamental Matrix



$P_{bob} = K[I_3 \quad 0_{3\times1}]$ $P_{alice} = K[R \quad t]$



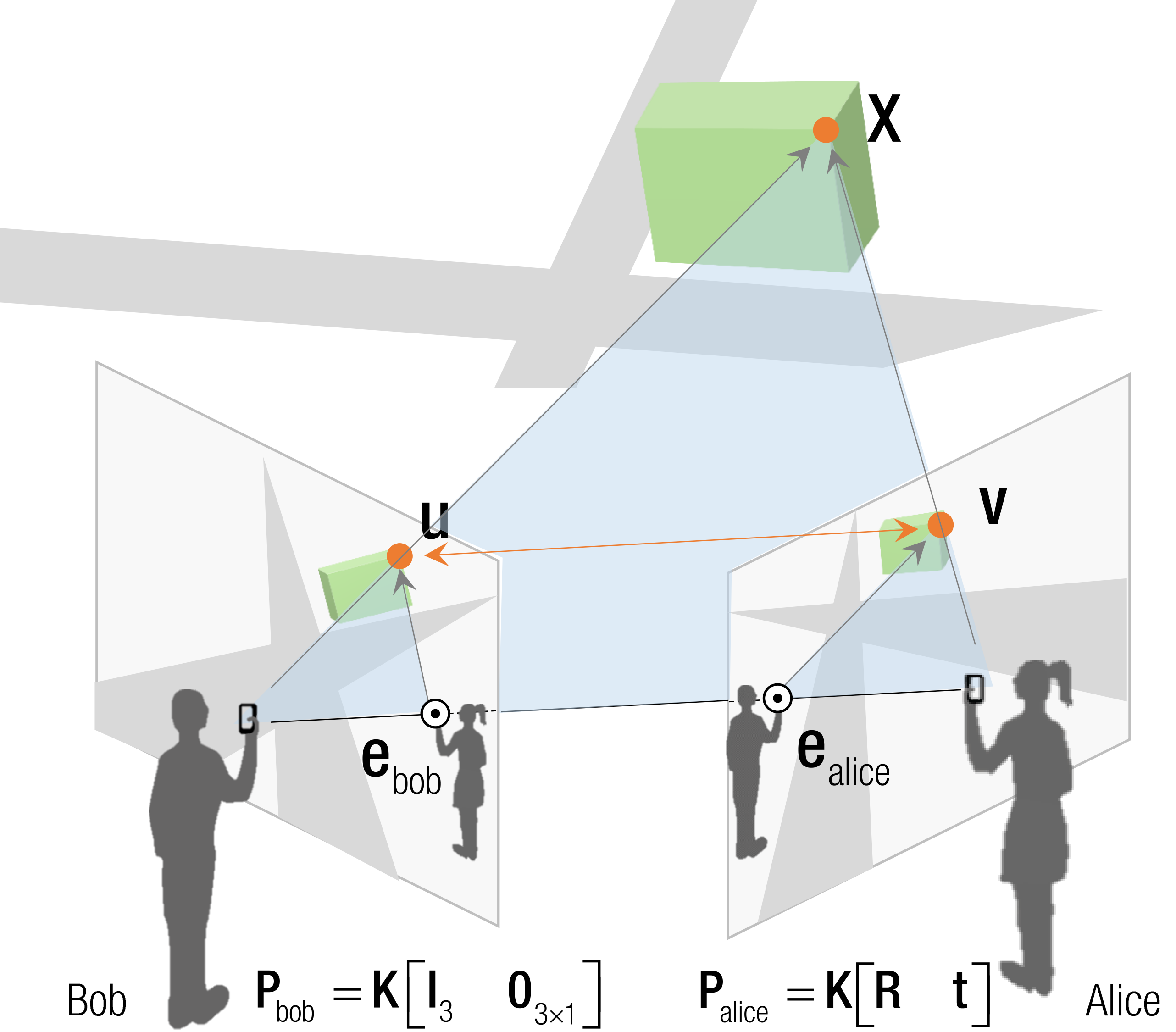
Bob's image

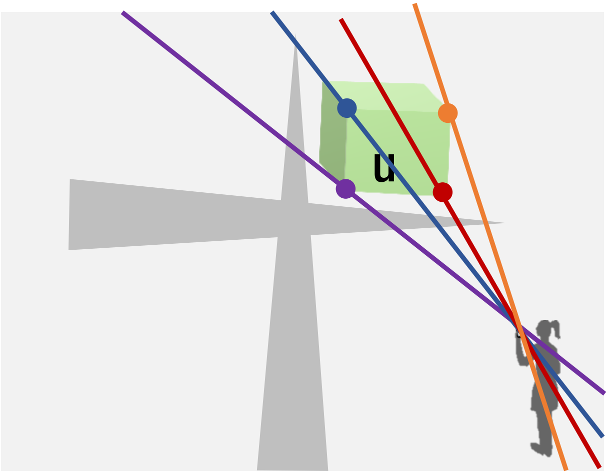
Alice's image

Properties of Fundamental Matrix

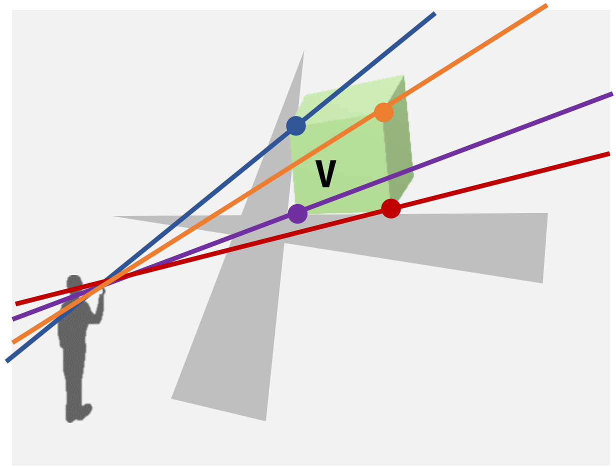
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.

Fundamental Matrix





Bob's image

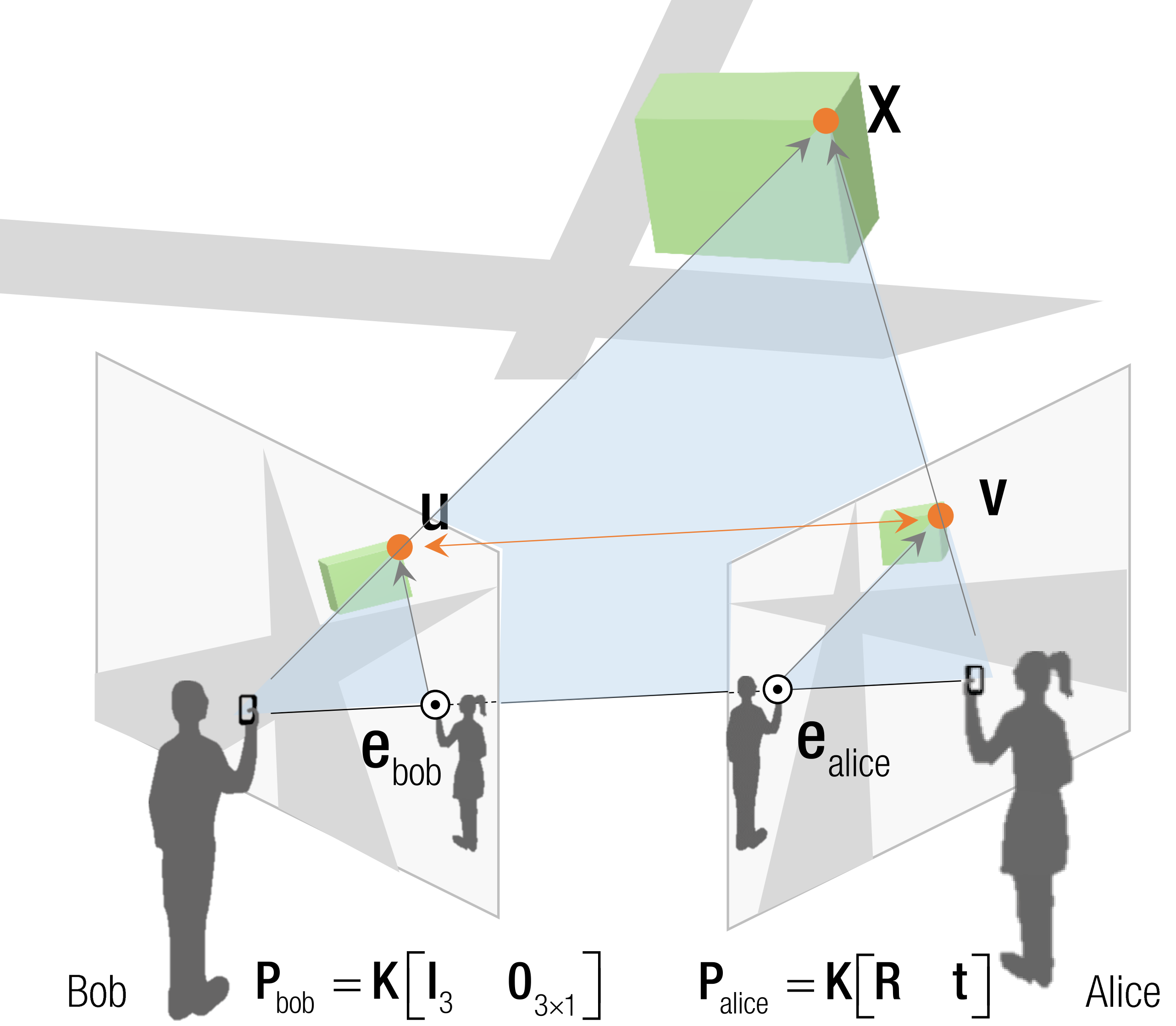


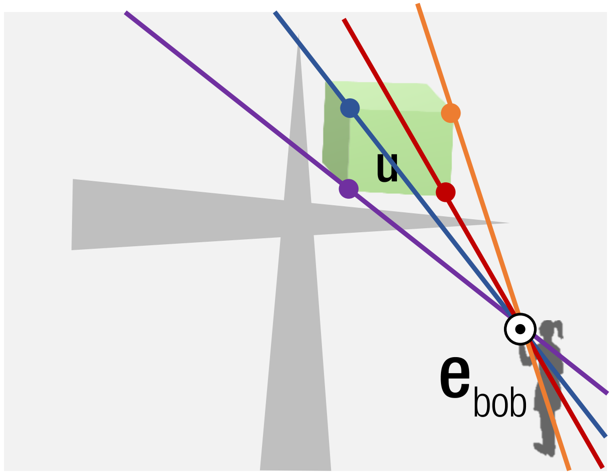
Alice's image

Properties of Fundamental Matrix

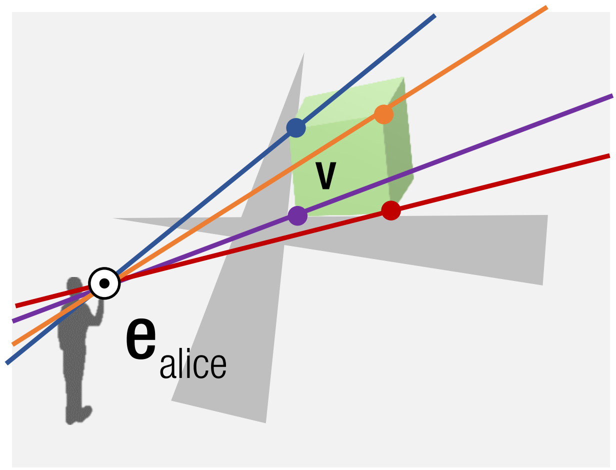
- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u}$ $\quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$

Fundamental Matrix





Bob's image



Alice's image

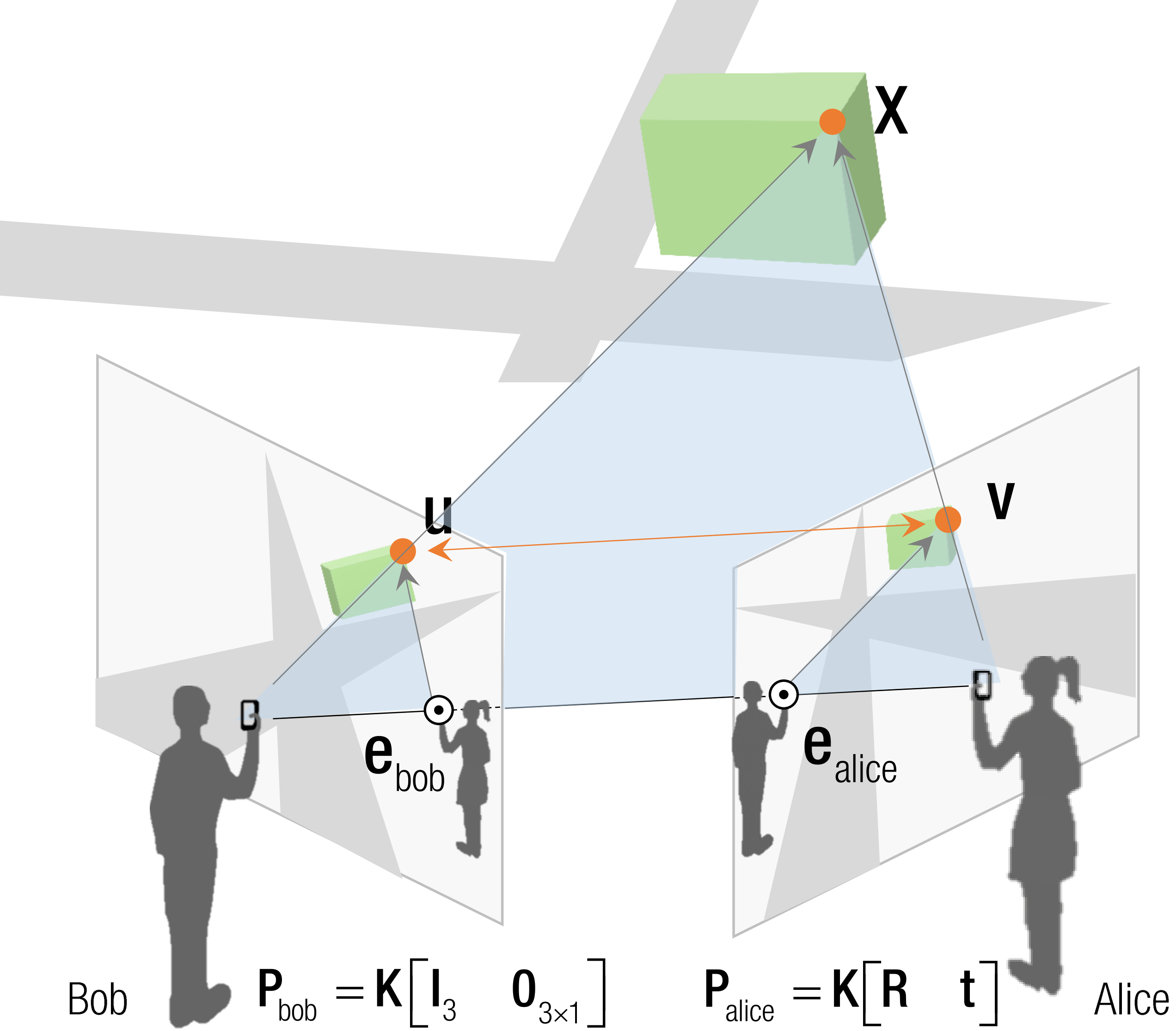
Properties of Fundamental Matrix

- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{\text{bob}}$, $\mathbf{P}_{\text{alice}}$, then $\mathbf{F}^{\mathsf{T}}$ is for $\mathbf{P}_{\text{alice}}$, $\mathbf{P}_{\text{bob}}$.
- Epipolar line: $\mathbf{l}_{\mathbf{u}} = \mathbf{F}\mathbf{u} \quad \mathbf{l}_{\mathbf{v}} = \mathbf{F}^{\mathsf{T}}\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{\text{bob}} = \mathbf{0} \quad \mathbf{F}^{\mathsf{T}}\mathbf{e}_{\text{alice}} = \mathbf{0}$

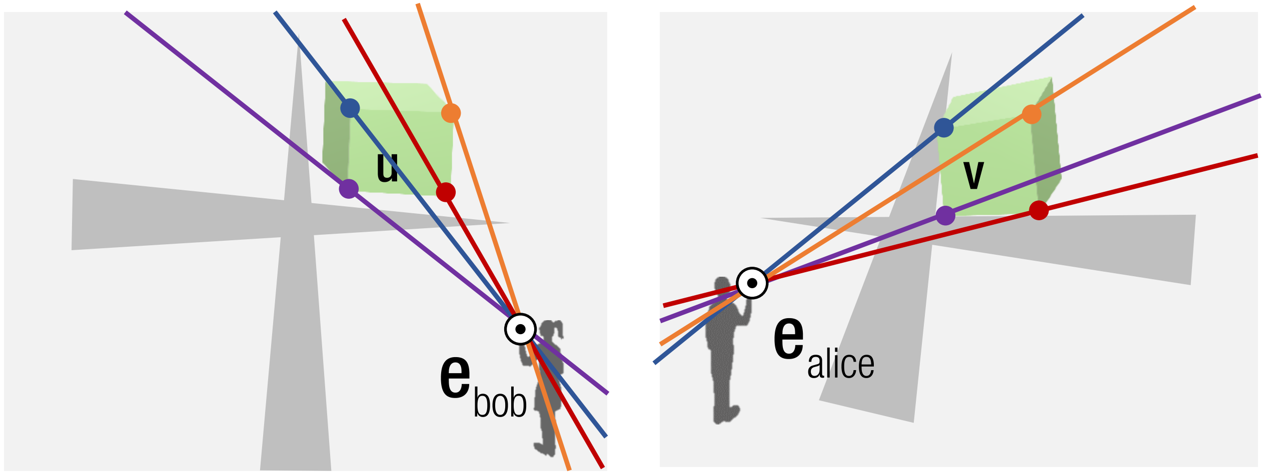
$$\because \mathbf{v}_i^{\mathsf{T}}\mathbf{F}\mathbf{e}_{\text{bob}} = 0, \quad \mathbf{u}_i^{\mathsf{T}}\mathbf{F}^{\mathsf{T}}\mathbf{e}_{\text{alice}} = 0, \quad \forall i$$

$$\longrightarrow \mathbf{e}_{\text{bob}} = \text{null}(\mathbf{F}), \quad \mathbf{e}_{\text{alice}} = \text{null}(\mathbf{F}^{\mathsf{T}})$$

Fundamental Matrix



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $\quad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$



Bob's image Alice's image

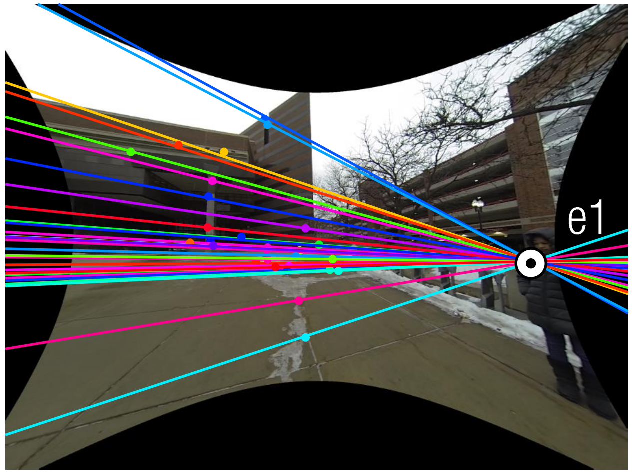
Properties of Fundamental Matrix

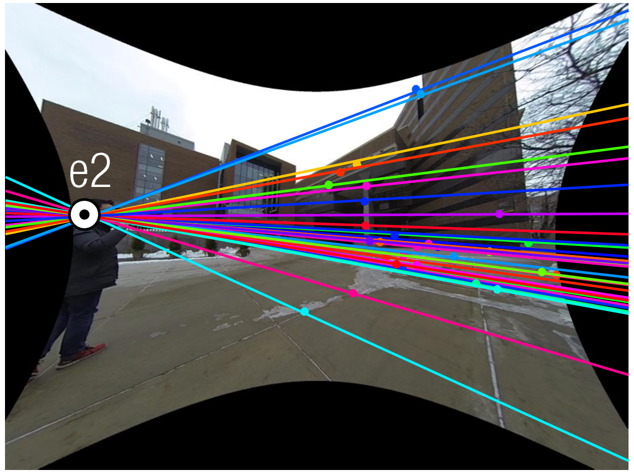
- Transpose: if **F** is for **P**bob, **P**alice, then **F**^{T} is for **P**alice, **P**bob.
- Epipolar line: $\mathbf{l_u} = \mathbf{Fu} \quad \mathbf{l_v} = \mathbf{F^T v}$
- Epipole: $\mathbf{Fe_{bob}} = \mathbf{0} \quad \mathbf{F^T e_{alice}} = \mathbf{0}$
- rank(**F**)=2: degree of freedom 9 (3x3 matrix)-1 (scale)-1 (rank)=7

$$\mathbf{F} = \mathbf{K}^{-T} \underbrace{[\mathbf{t}]_\times \mathbf{R}}_{\text{rank 2 matrix}} \mathbf{K}^{-1}$$

Fundamental Matrix

$$F = K^{-T}[t]_{\times} R K^{-1}$$





K =

568.9961	0	643.2106
0	568.9884	477.9828
0	0	1.0000

R =

0.4344	0.0271	0.9003
-0.0139	0.9996	-0.0234
-0.9006	-0.0024	0.4346

t =

-1.8360
-0.1582
1.1219

Download:
+ EpipoleVisualization.m
+ keller1.png
+ keller2.png
+epipolar.mat

Fill out missing parts:
+ Compute fundamental matrix
+ Compute epipole 1
+ Compute epipole 2'

Output:
+ Visualization of epipolar lines

Camera Motion

Camera Motion

Image 2

Image 1

Forward motion

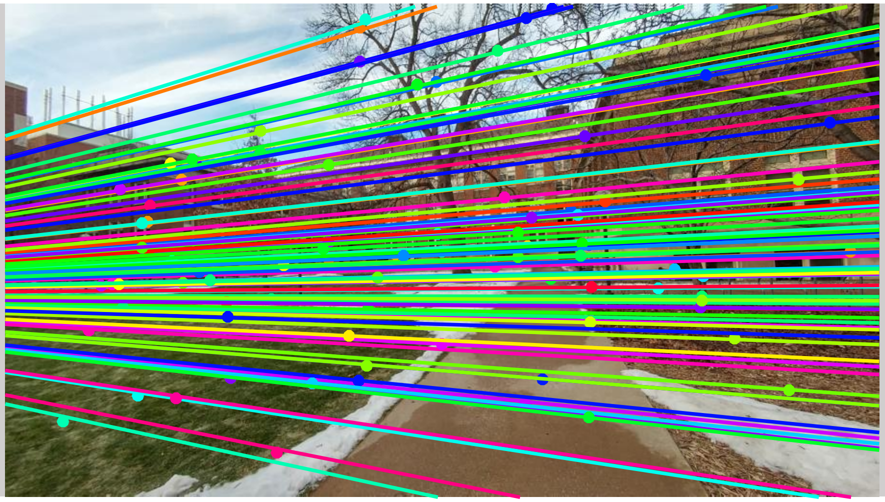


Image 2

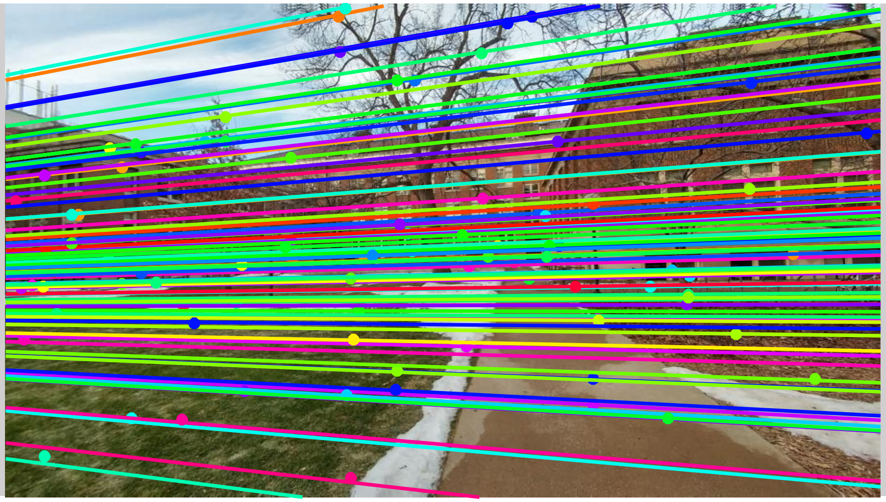


Image 1

Lateral motion