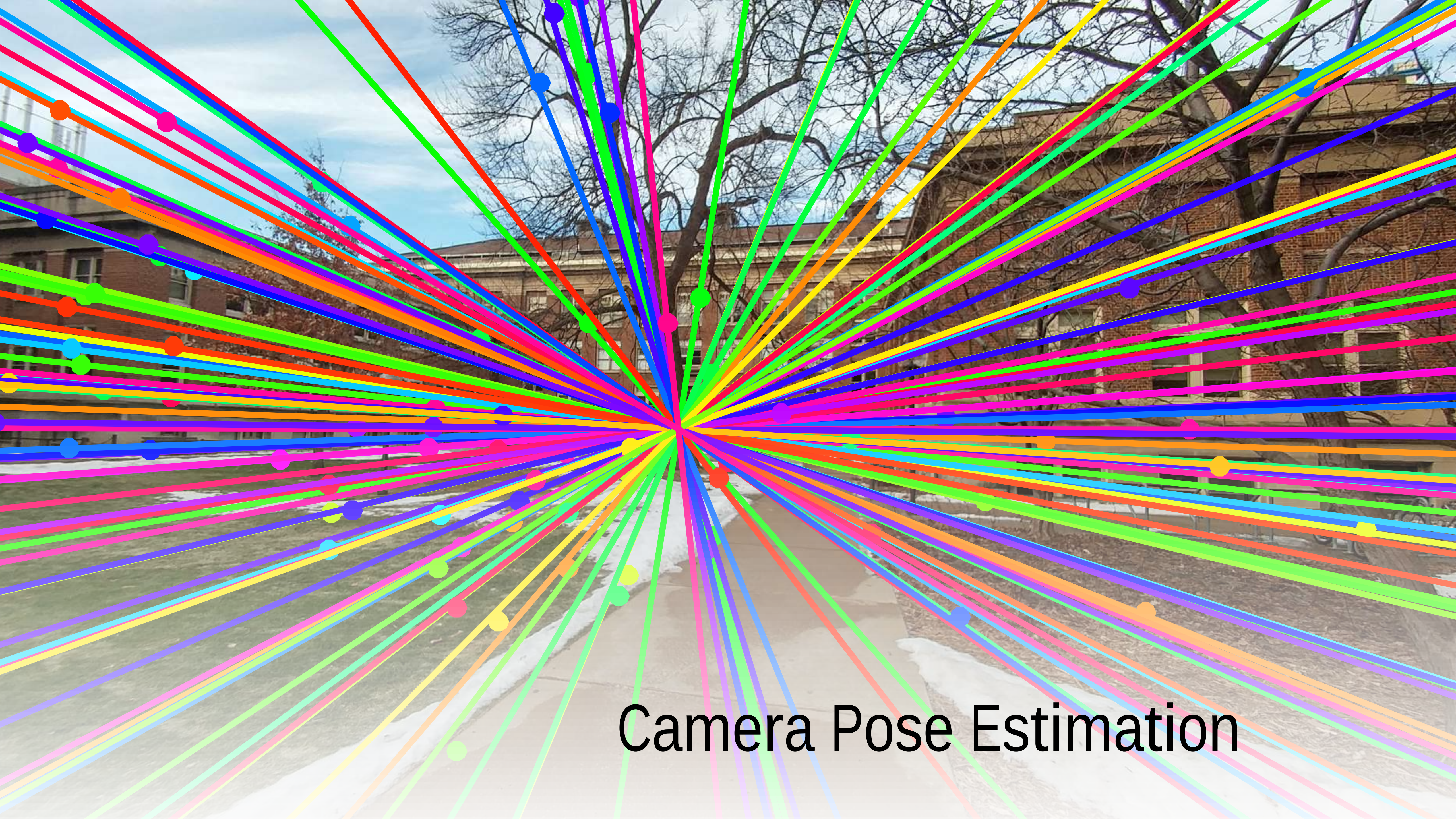
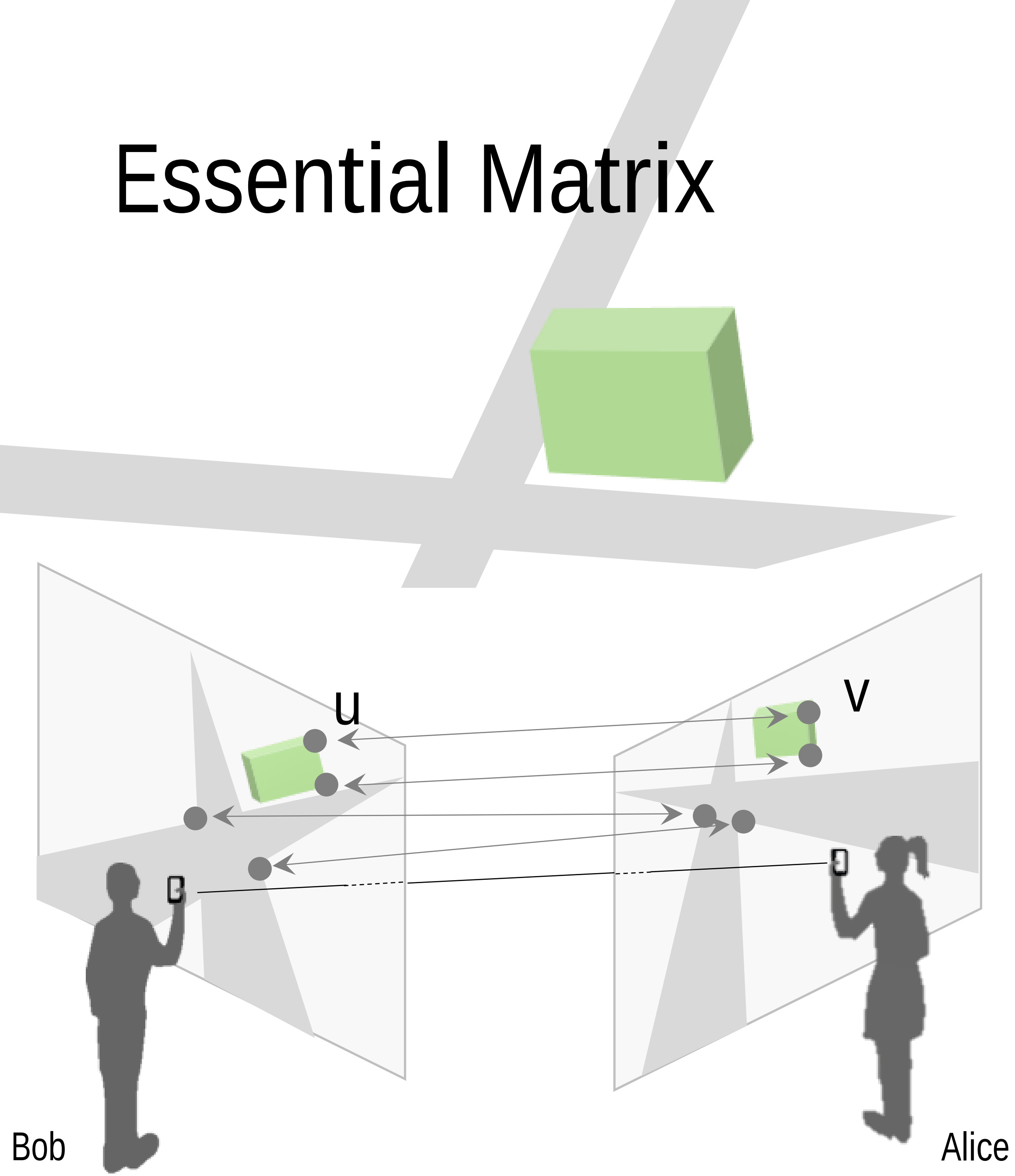


Camera Pose Estimation

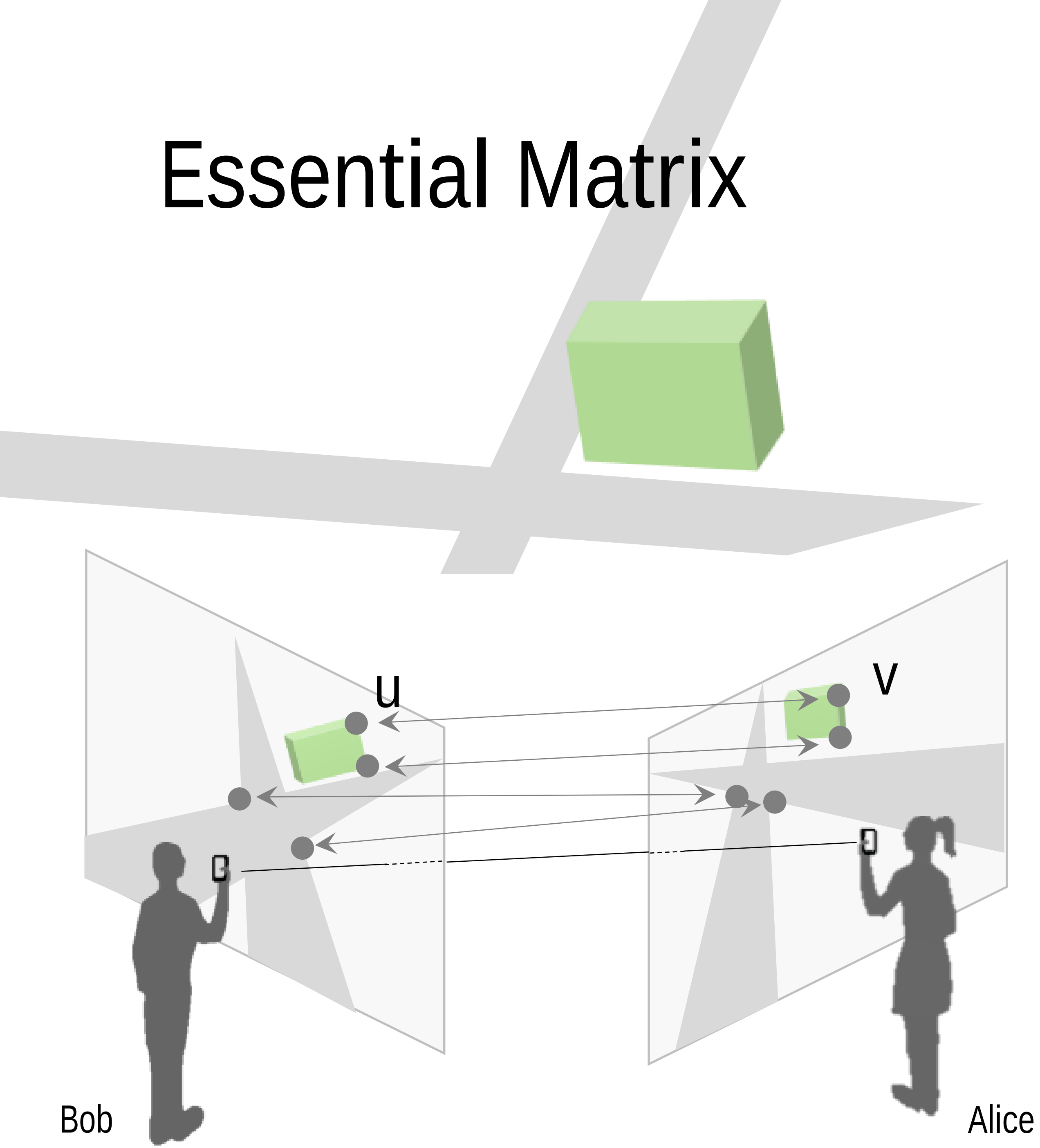
Essential Matrix



$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

Essential Matrix



Essential Matrix:

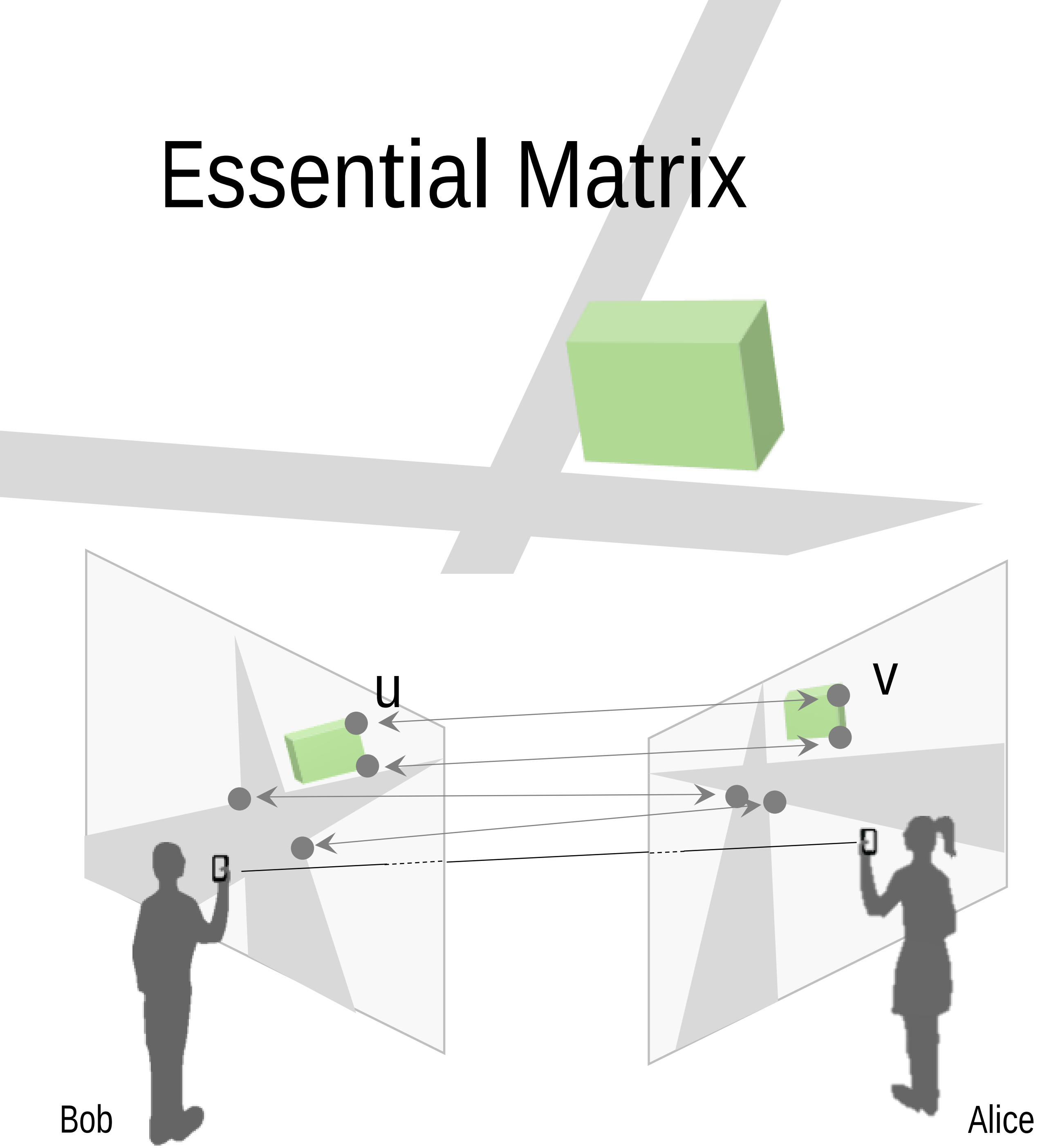
$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

$$\rightarrow E = \underline{K^{T} F K} \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Essential Matrix



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

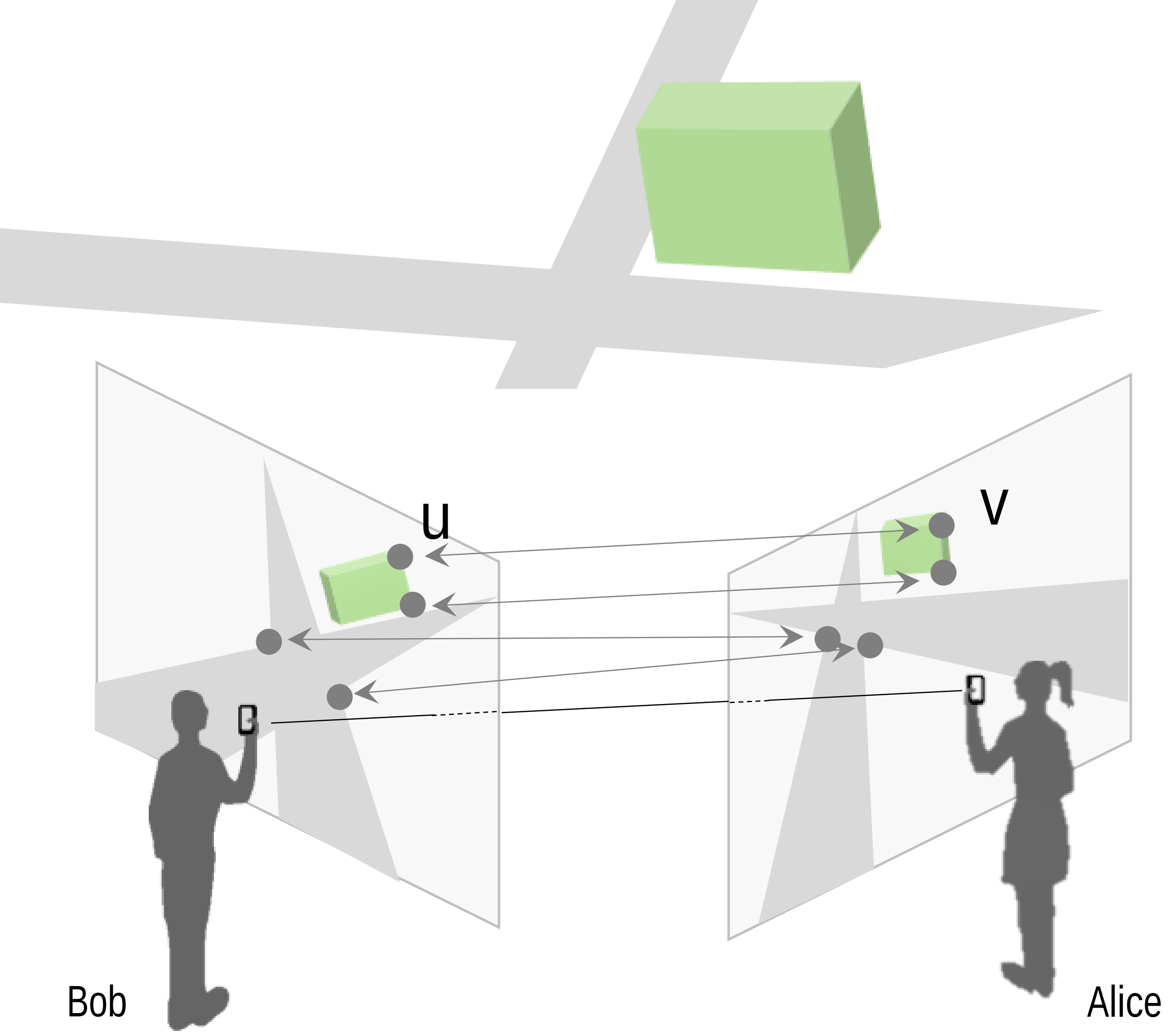
$$\longrightarrow E = \underline{K^{T}FK} \qquad \text{where } E = [t]_{\times} R$$

Calibrated fundamental matrix

Property of essential matrix:

$$E = UDV^{T} = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}V^{T}$$

Essential Matrix



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

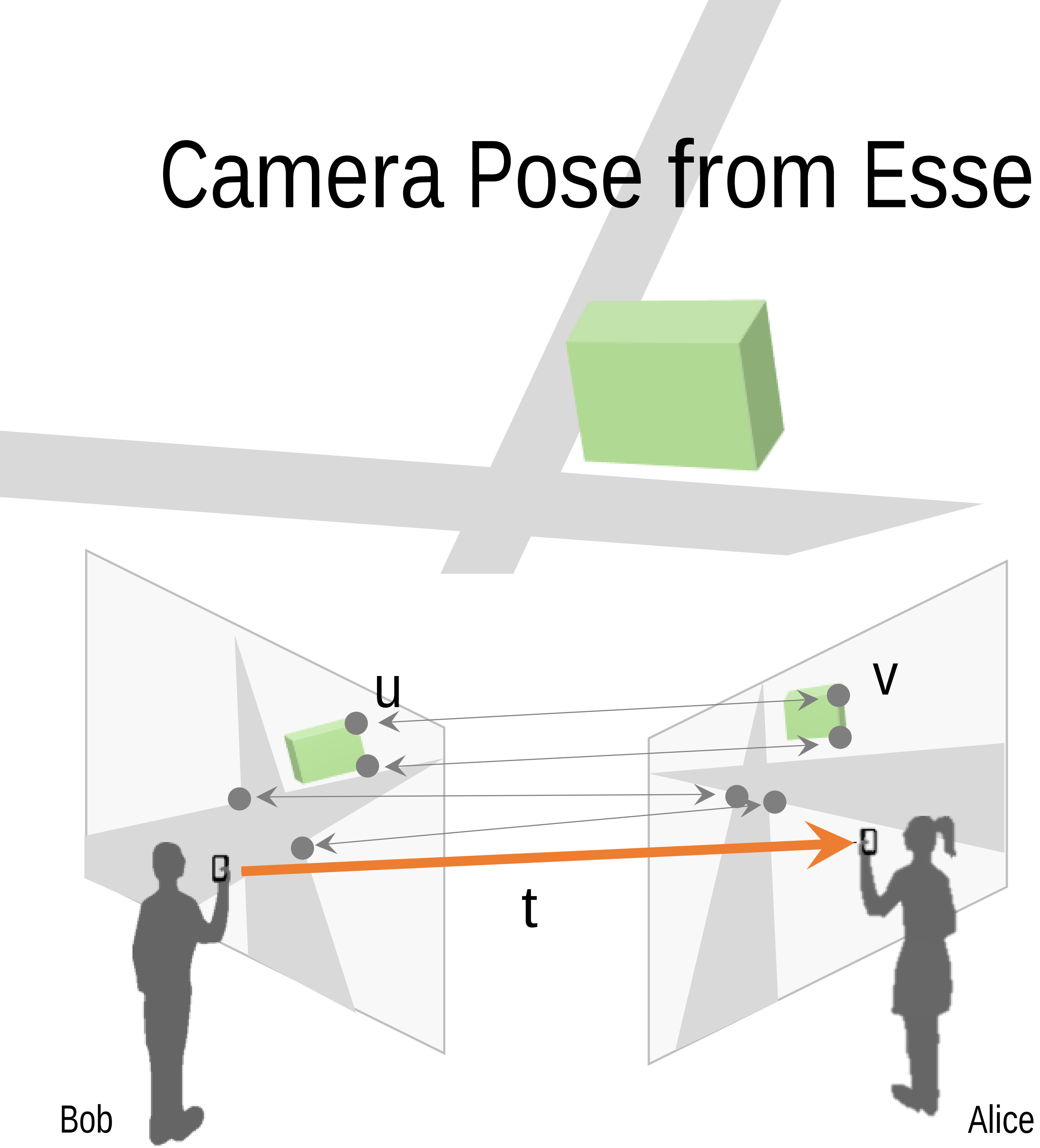
$$\rightarrow E = K^{T} F K \qquad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Property of essential matrix:

$$E = UDV^{T} = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^{T}$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

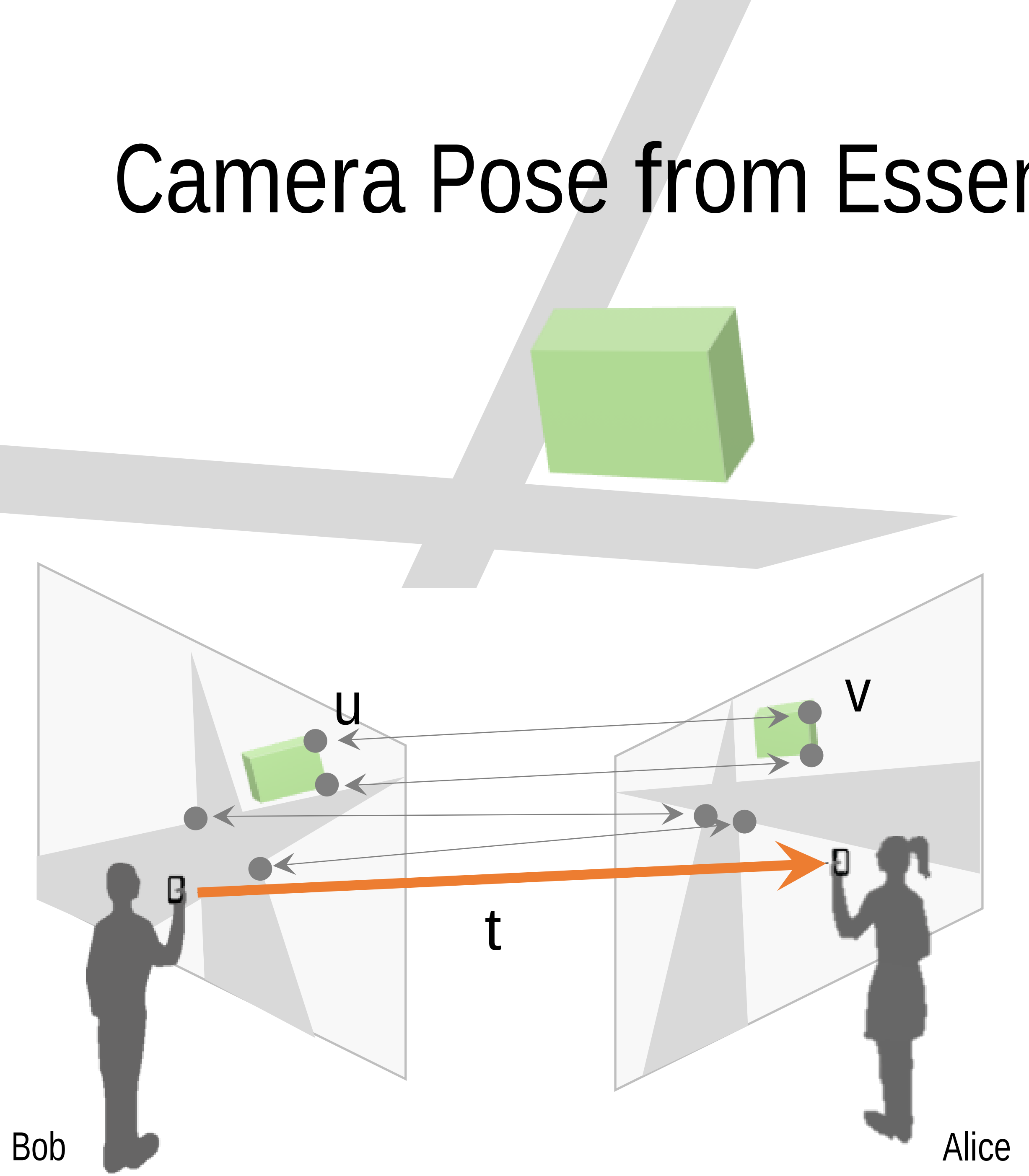
$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

$$\rightarrow E = K^{T}FK \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

$$t =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} RK^{-1} = K^{-T}EK^{-1}$$

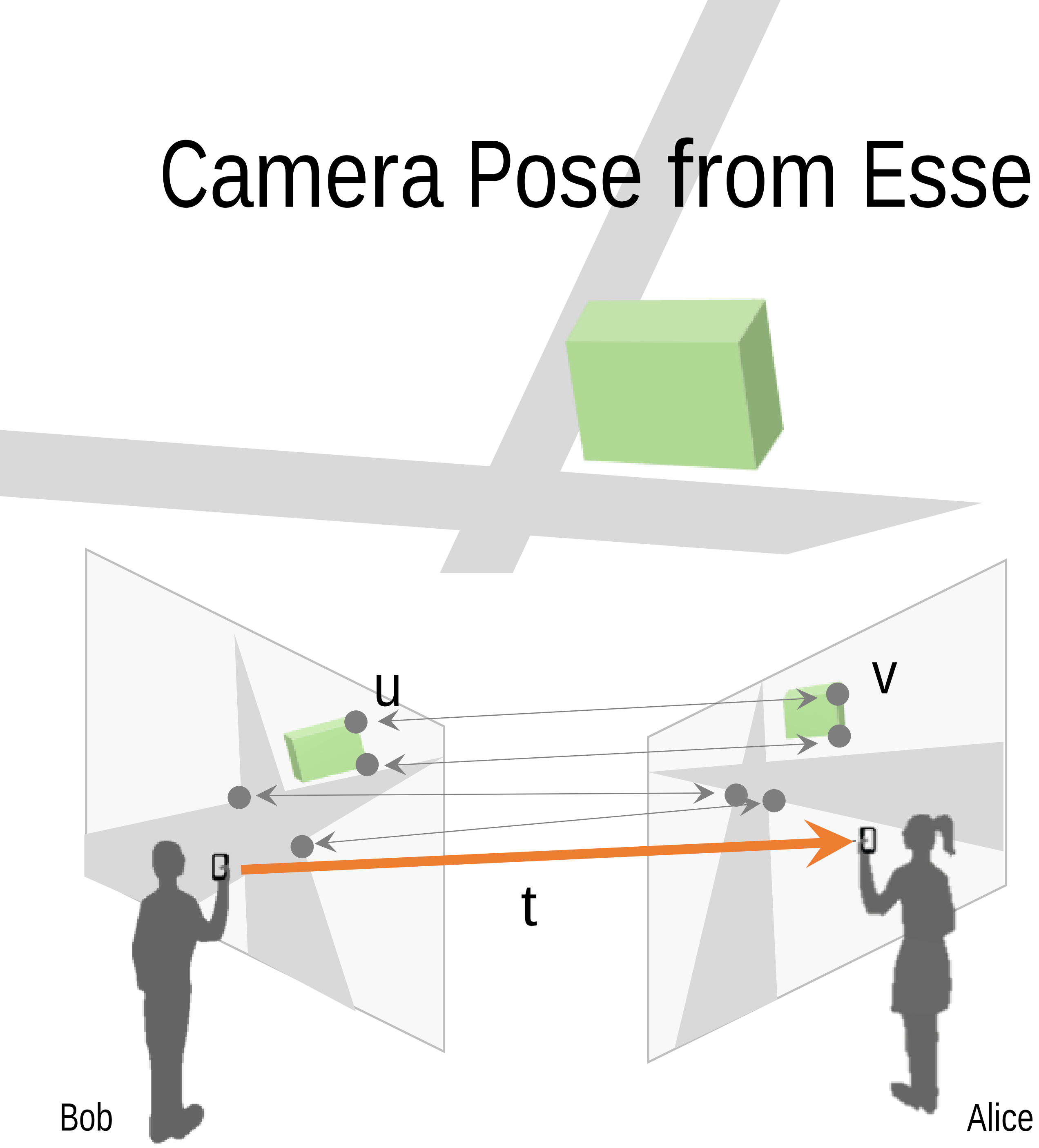
$$\rightarrow E = \underline{K^{T}FK}$$ where $E = [t]_{\times} R$

Calibrated fundamental matrix

Left null space of E is translation vector, t:

$$t =$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

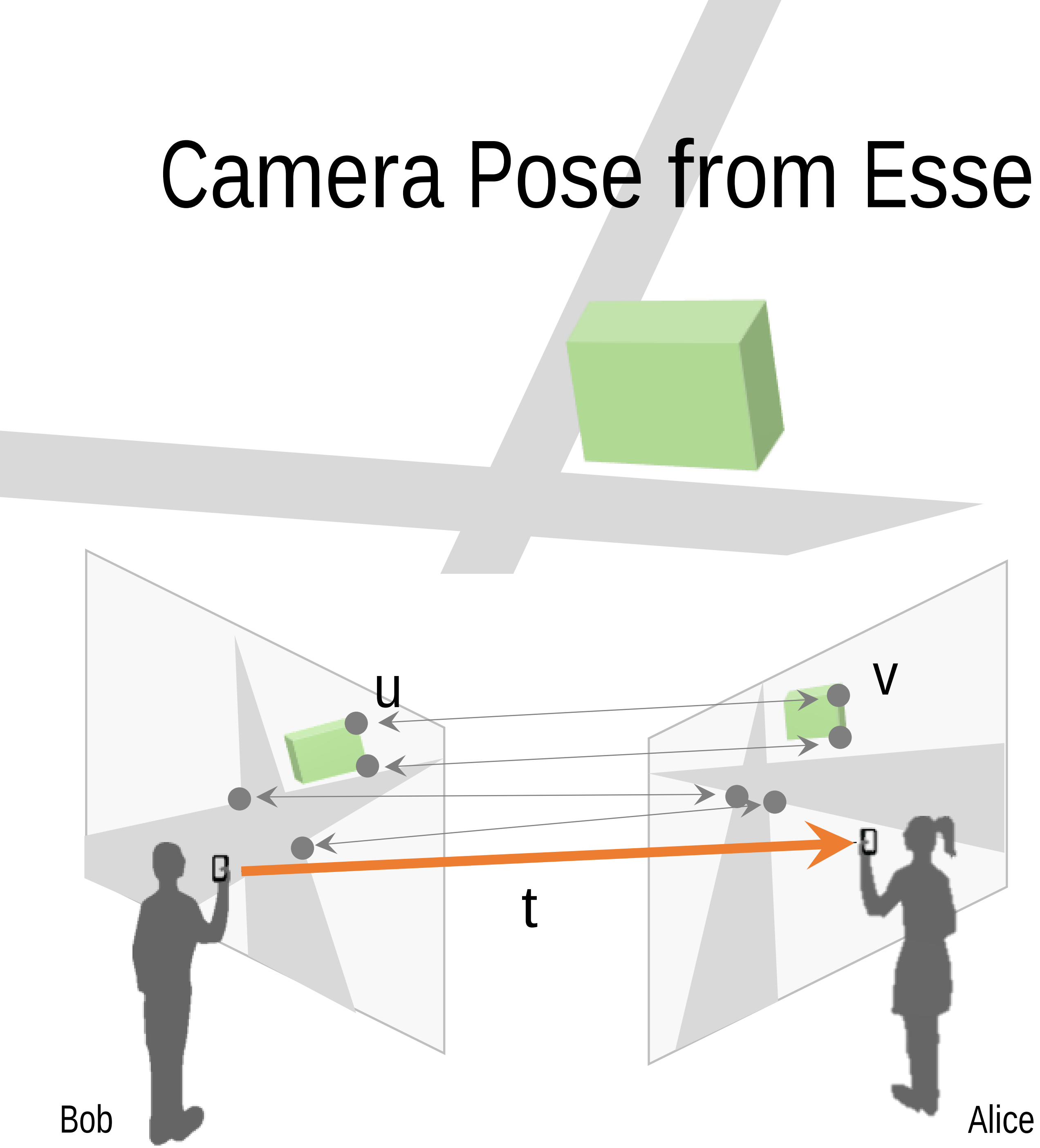
$$\rightarrow E = K^{T} F K \quad \text{where} \quad E = [t]_{\times} R$$

Calibrated fundamental matrix

Left null space of E is translation vector, t:

$$t = \pm \text{null}(E^{T}) = \pm \text{null}(([t]_{\times} R)^{T})$$

Camera Pose from Essential Matrix (Translation)



Essential Matrix:

$$F = F(R, t)$$

$$= K^{-T}[t]_{\times} R K^{-1} = K^{-T} E K^{-1}$$

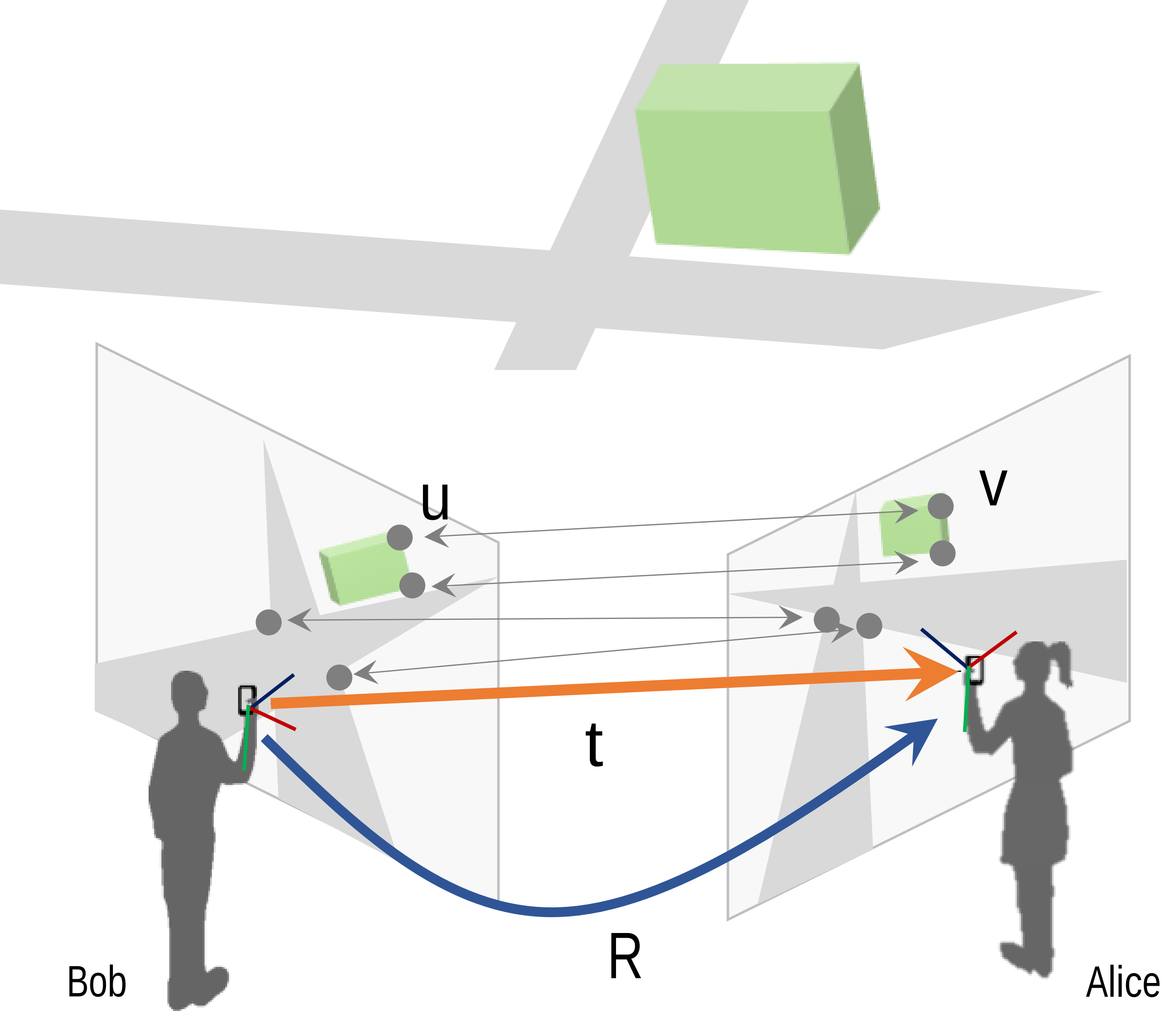
$$\rightarrow E = \underbrace{K^{T} F K}_{\text{Calibrated fundamental matrix}} \quad \text{where} \quad E = [t]_{\times} R$$

Left null space of E is translation vector, t:

$$t = \pm \text{null}(E^{T}) = \pm \text{null}\left(\left([t]_{\times} R\right)^{T}\right)$$

$$\because t^{T}[t]_{\times} R = -\left([t]_{\times} t\right)^{T} R = -\underbrace{\left(t \times t\right)^{T}}_{\text{Self-cross product}} R = 0$$

Essential Matrix Decomposition

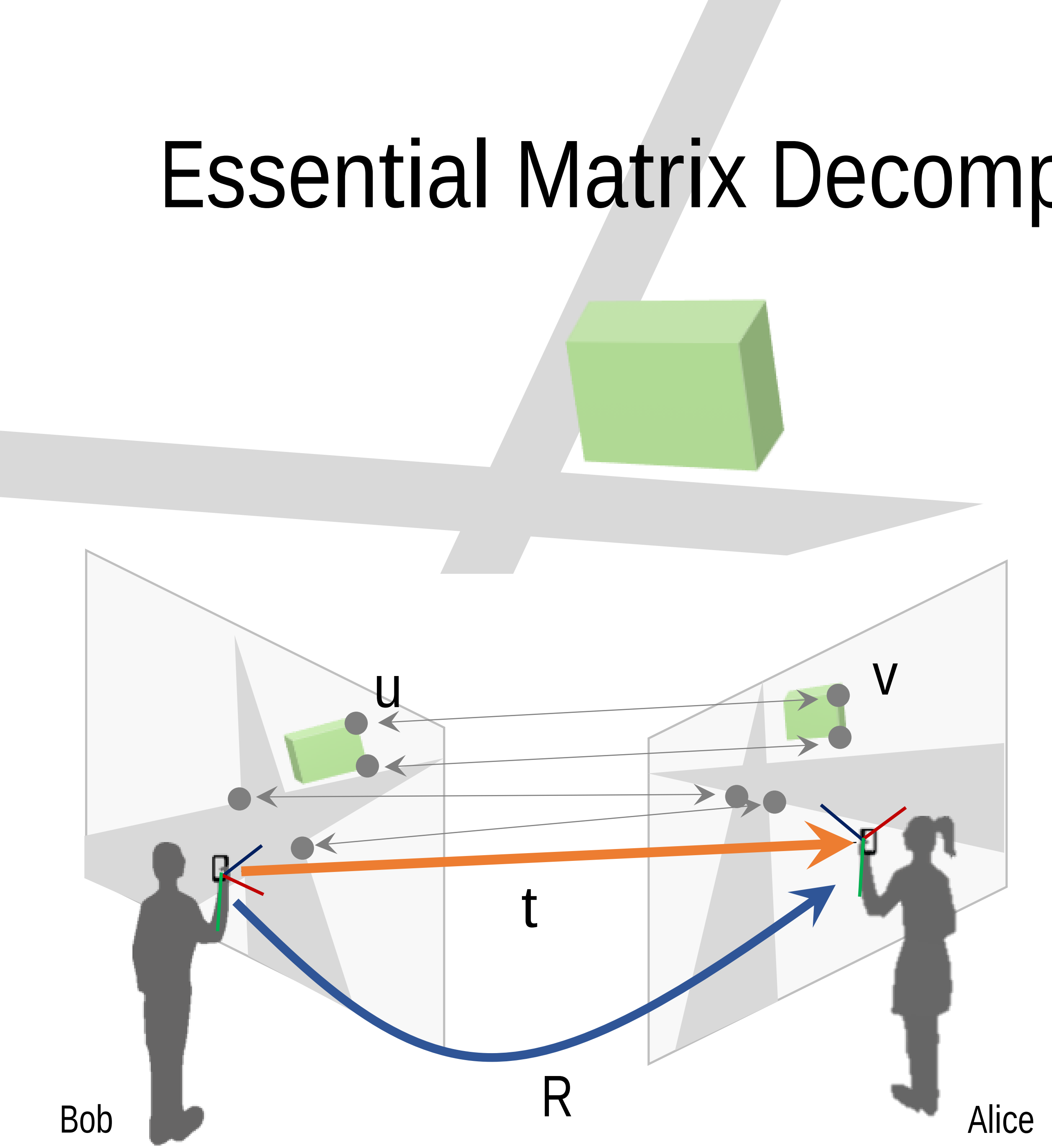


Left null space of E is translation vector, t:

$$t = \text{null}(E^{T}) = \text{null}\left(\left([t]_{\times} R\right)^{T}\right)$$

Can I invert $[t]_{\times}$?

Essential Matrix Decomposition



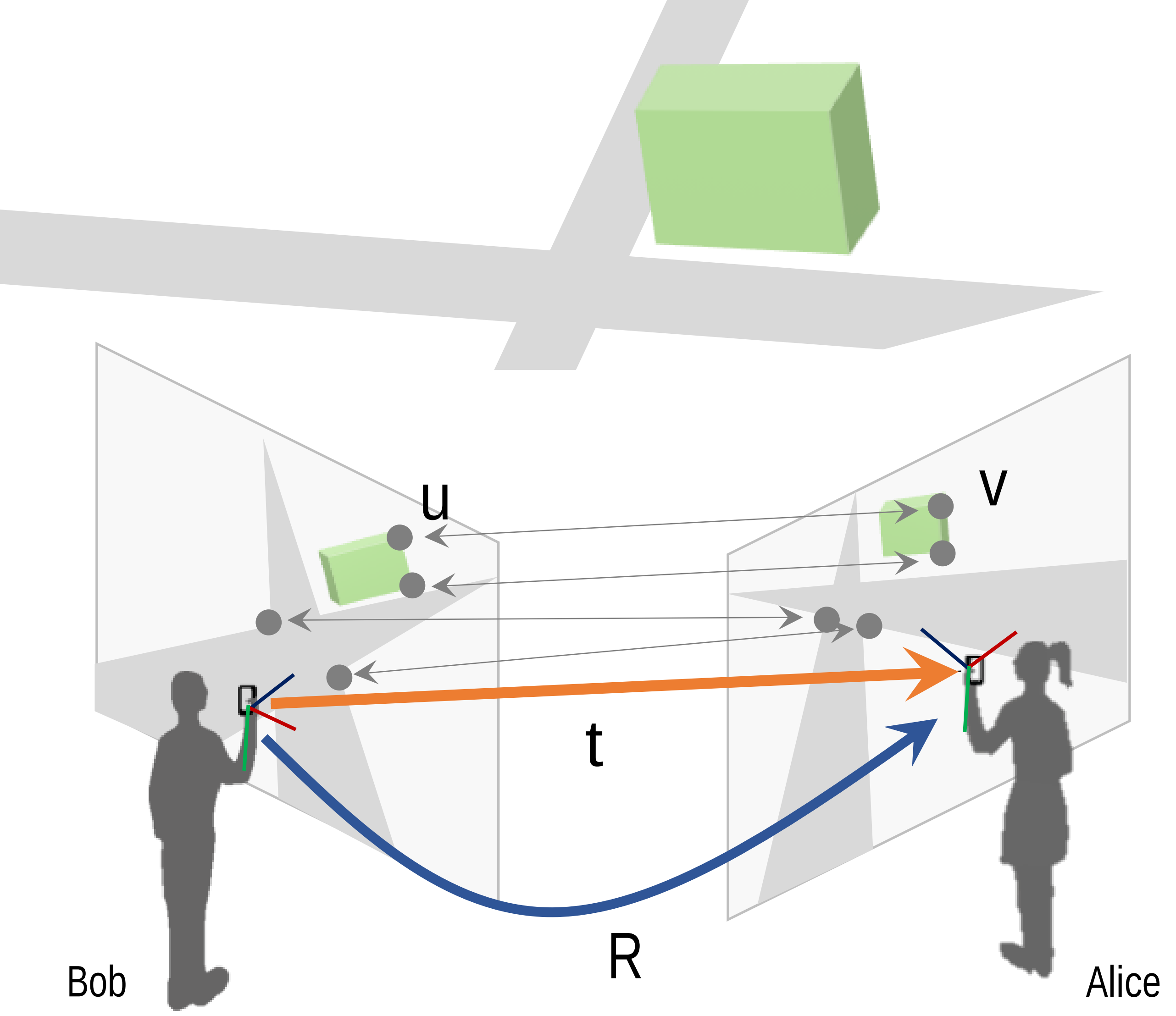
Left null space of E is translation vector, t:

$$t = \mathrm{null}(E^T) = \mathrm{null}\left(\left([t]_\times R\right)^T\right)$$

$$\longrightarrow \quad t = u_3 \qquad \text{where} \quad U = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$$

$$E = UDV^T = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

Essential Matrix Decomposition



Left null space of E is translation vector, t:

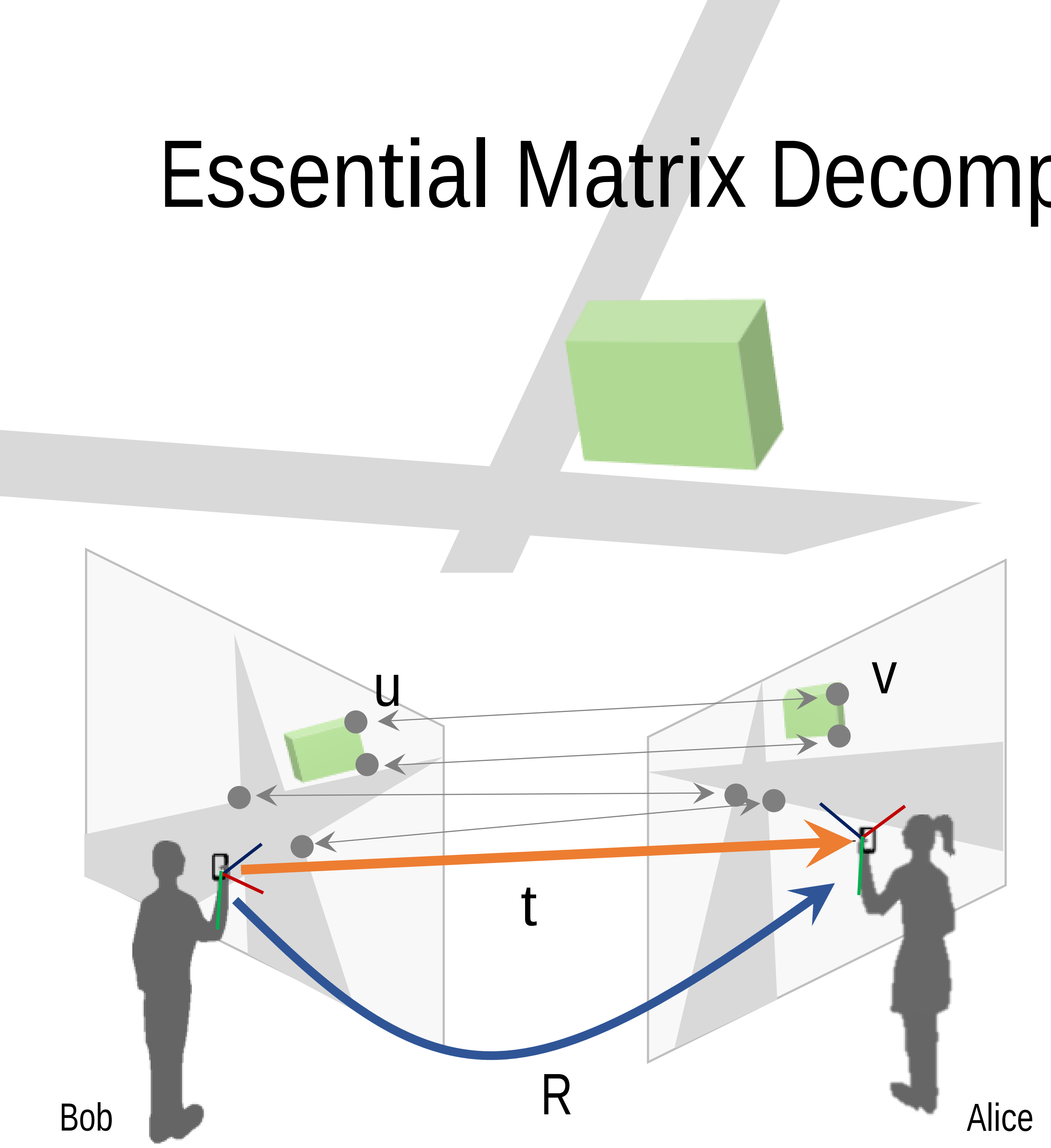
$$t = \mathrm{null}(E^T) = \mathrm{null}\left(\left([t]_\times R\right)^T\right)$$

$\longrightarrow$ $t = u_3$ where $U = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$

$$E = UDV^T = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}V^T$$

$\longrightarrow$ $t = u_1 \times u_2$ (orthogonal matrix, U)

Essential Matrix Decomposition



Left null space of E is translation vector, t:

$$t = \text{null}(E^T) = \text{null}\left(\left([t]_\times R\right)^T\right)$$

$$\longrightarrow \quad t = u_3 \qquad \text{where} \quad U = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$$

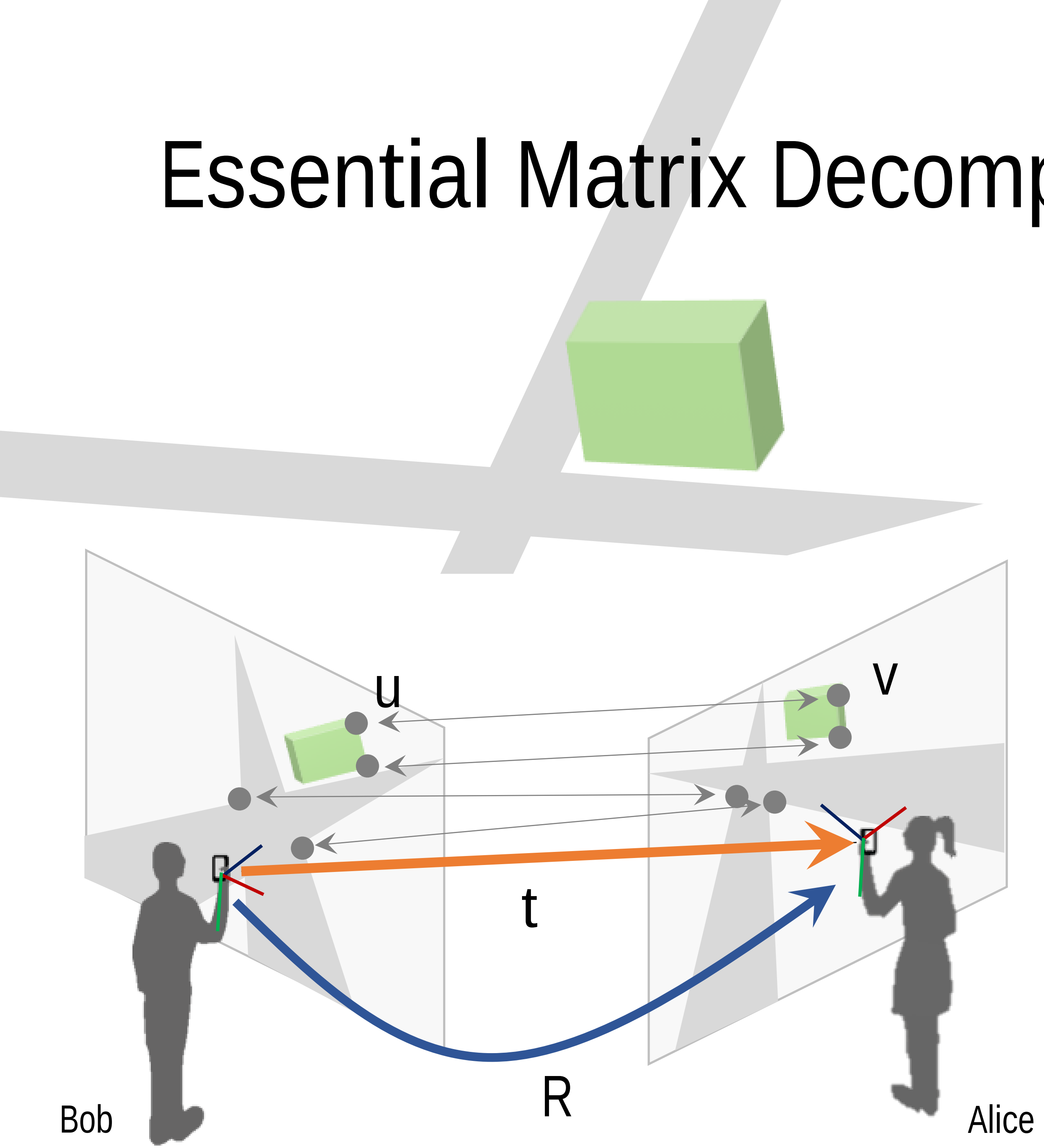
$$E = UDV^T = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}V^T$$

$$\longrightarrow \quad t = u_1 \times u_2 \quad \text{(orthogonal matrix, U)}$$

$$[t]_\times = [u_1 \times u_2]_\times = u_2 u_1^T - u_1 u_2^T$$

:

Essential Matrix Decomposition



Left null space of E is translation vector, t:

$$t = \mathrm{null}(E^T) = \mathrm{null}\left(\left([t]_\times R\right)^T\right)$$

$\longrightarrow$ $t = u_3$ where $U = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$

$$E = UDV^T = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}V^T$$

$\longrightarrow$ $t = u_1 \times u_2$ (orthogonal matrix, U)

$$[t]_\times = [u_1 \times u_2]_\times = u_2 u_1^T - u_1 u_2^T$$

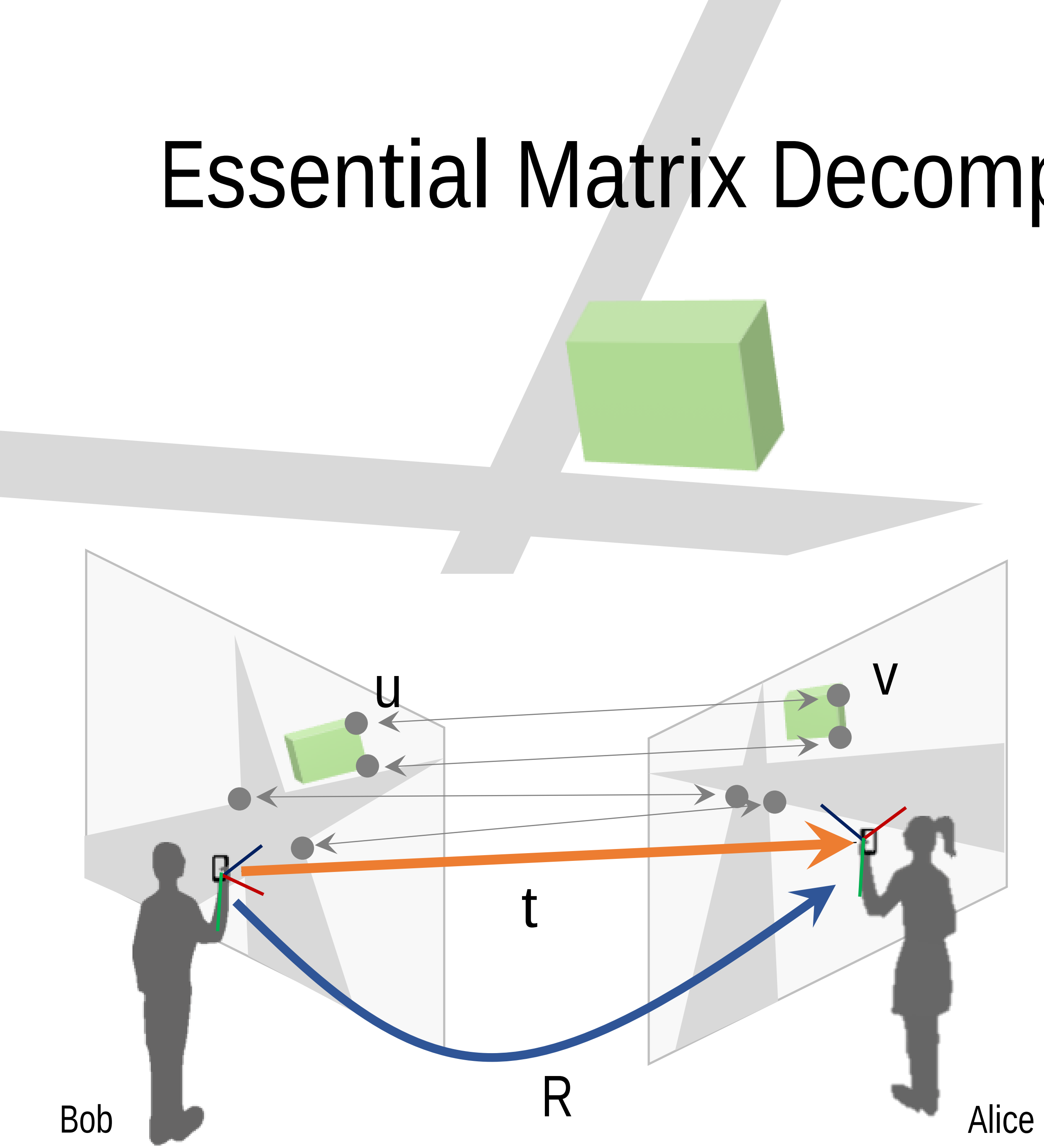
$$= U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}U^T$$

Prove!

Essential Matrix Decomposition

$$E = [t]_{\times} R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R$$

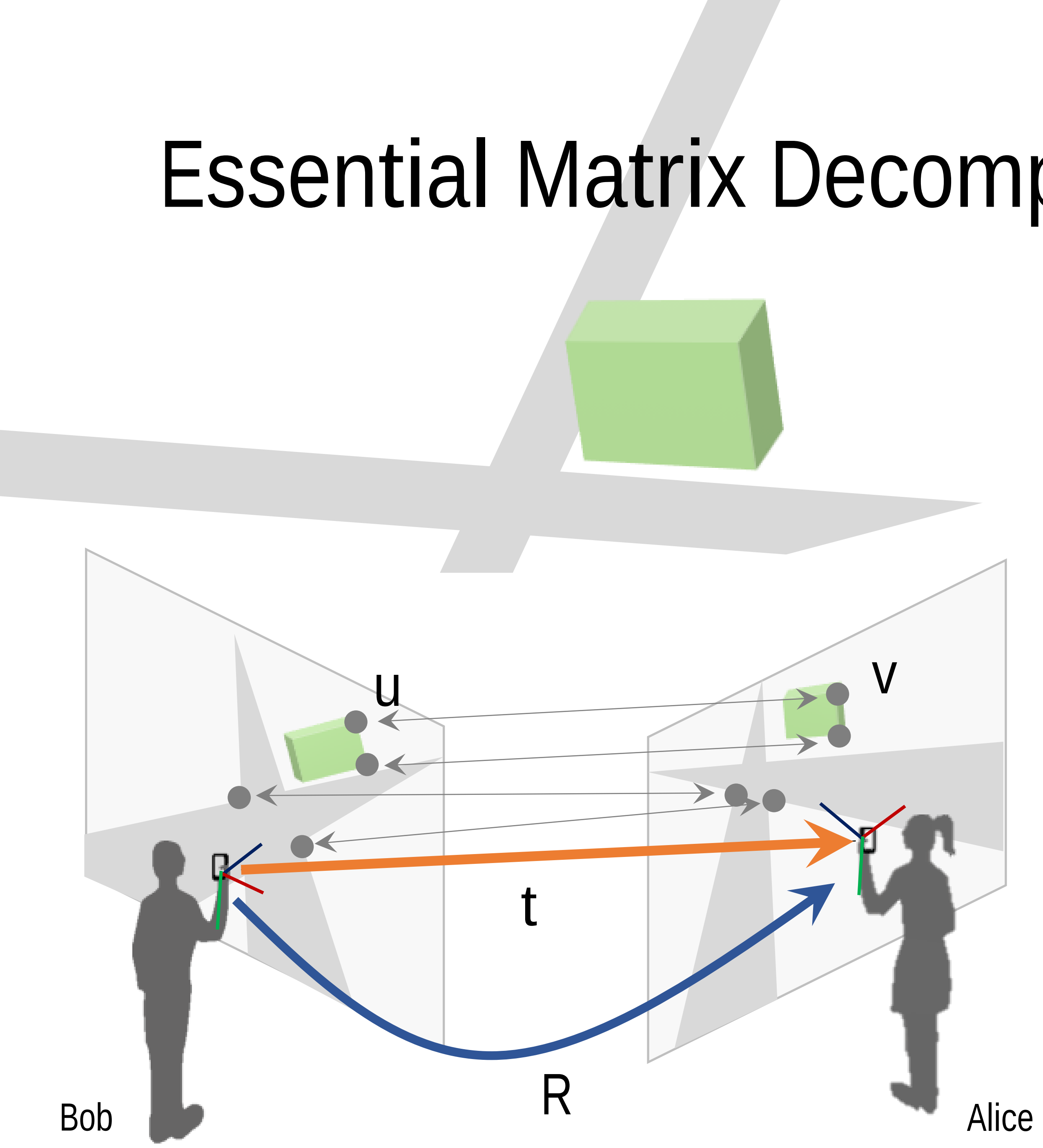
where $R \in SO(3)$



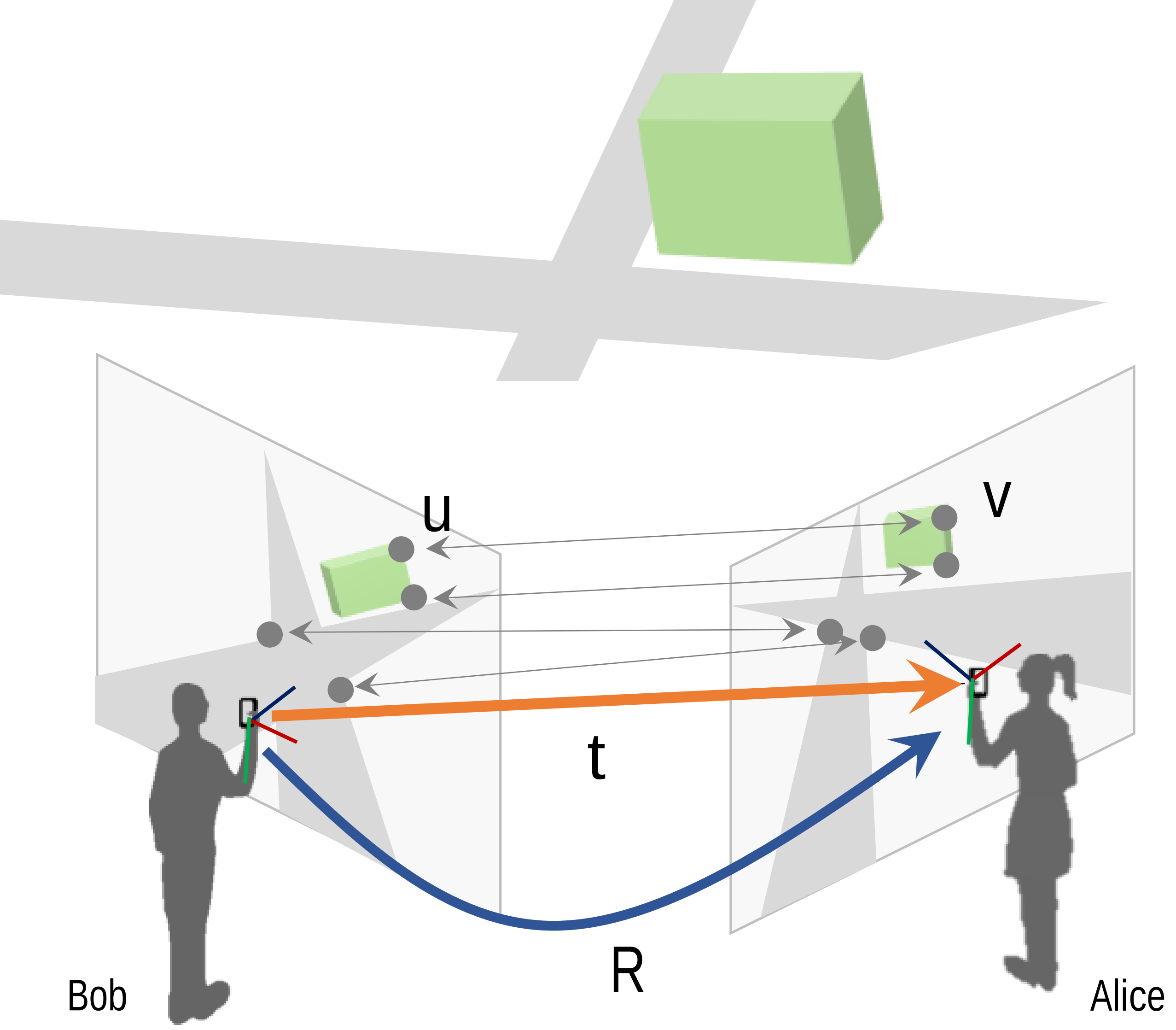
Essential Matrix Decomposition

$$E = [t]_\times R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

where $R \in SO(3)$



Essential Matrix Decomposition



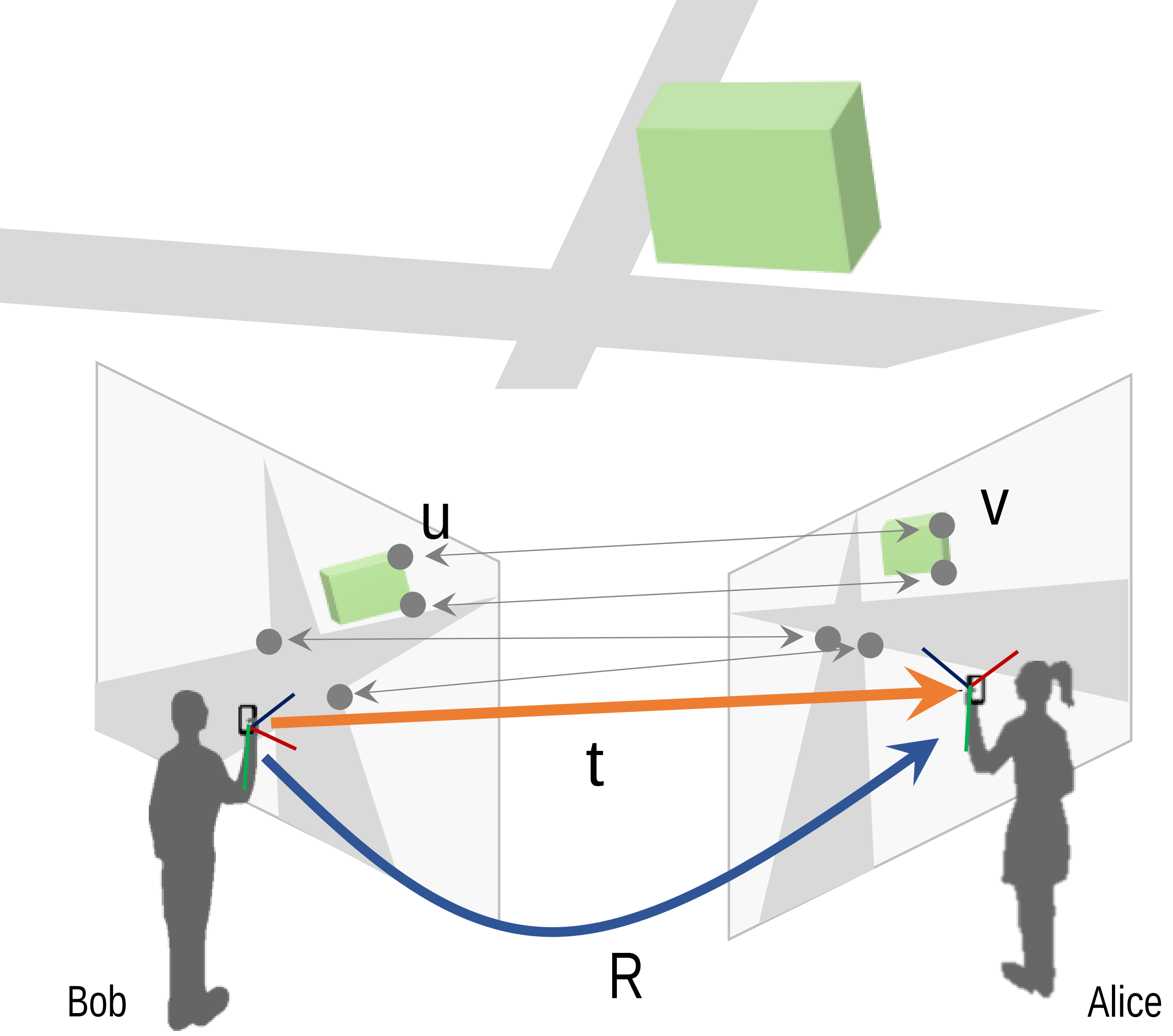
$$E = [t]_\times R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

where $R \in SO(3)$

Define $R = UWV^T$

$$E = [t]_\times R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T UWV^T = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} WV^T$$

Essential Matrix Decomposition



$$E = [t]_{\times} R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^{T} R = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^{T}$$

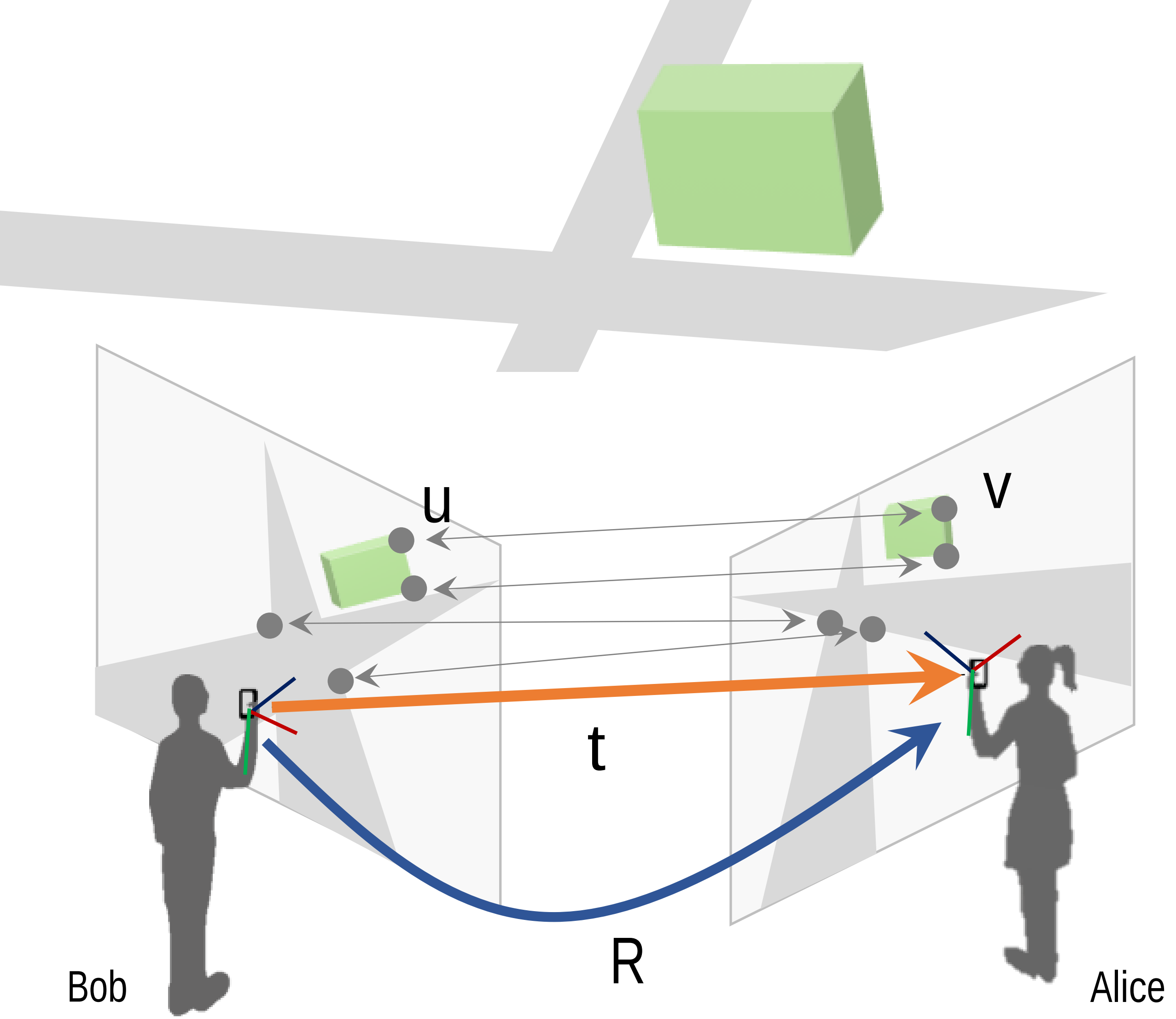
where $R \in SO(3)$

Define $R = \underline{U}WV^{T}$

$$E = [t]_{\times} R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^{T} UWV^{T} = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} WV^{T}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W$$

Essential Matrix Decomposition



$$E = [t]_{\times} R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

where $R \in SO(3)$

Define $R = UWV^T$

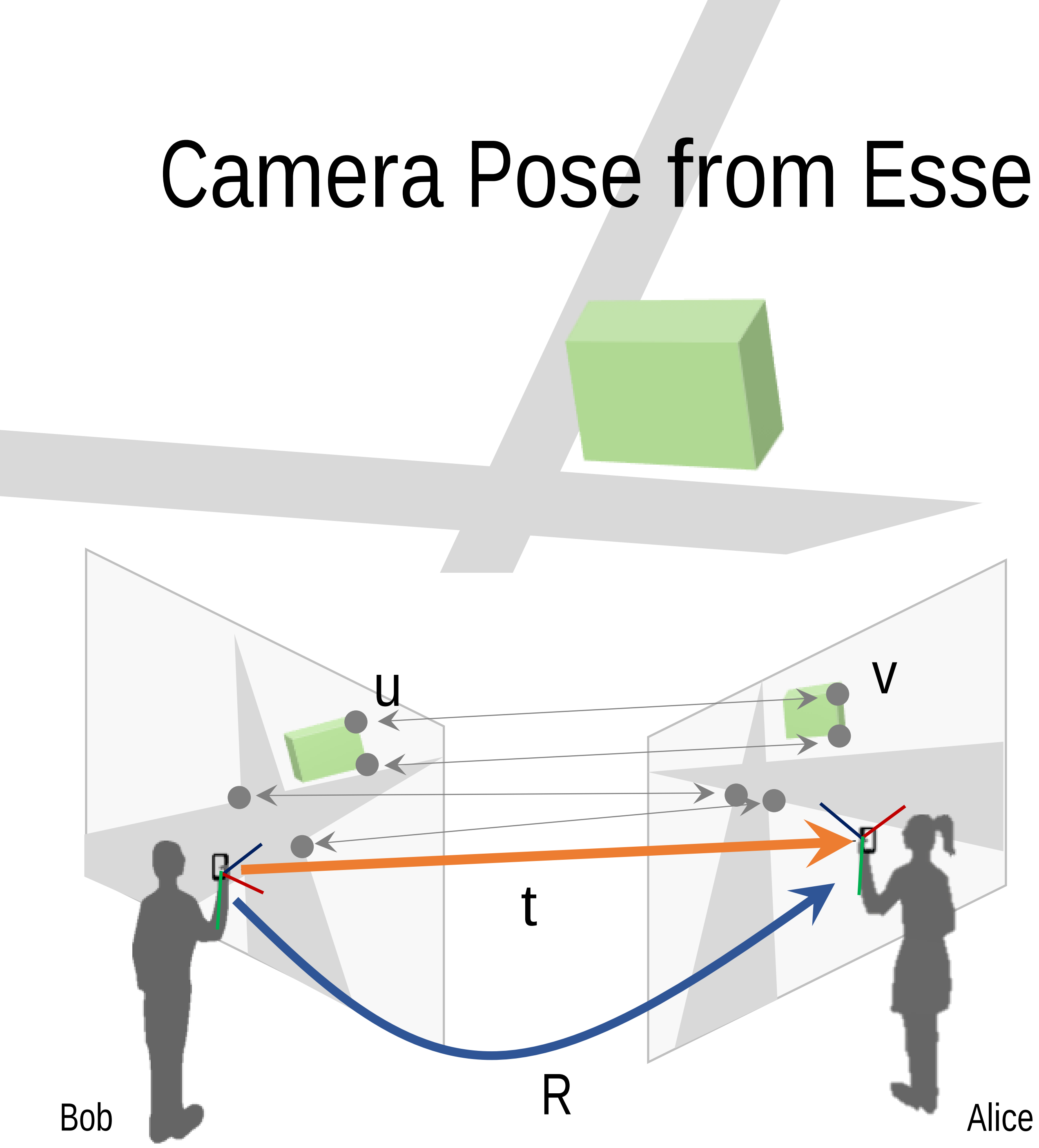
$$E = [t]_{\times} R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T UWV^T = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} WV^T$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} W \qquad W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ or } \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Camera Pose from Essential Matrix (Rotation)

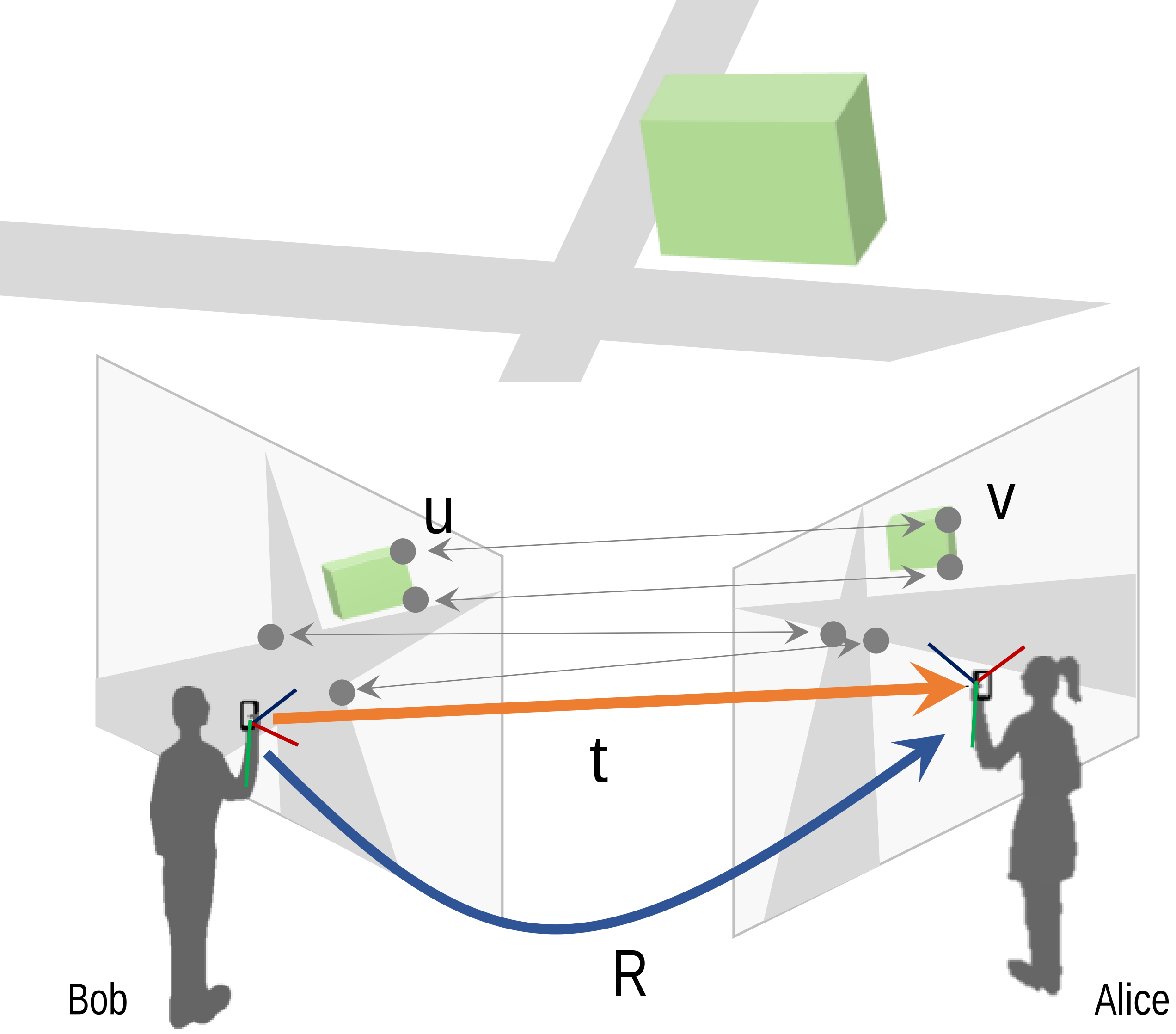
$$E = [t]_\times R = U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R = U \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

where $R \in SO(3)$



$$R = U \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^T, \text{ or } U \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^T$$

Where Am I?



$$E = [t]_\times R = U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} U^T R = U\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix} V^T$$

$$t = \pm u_3$$

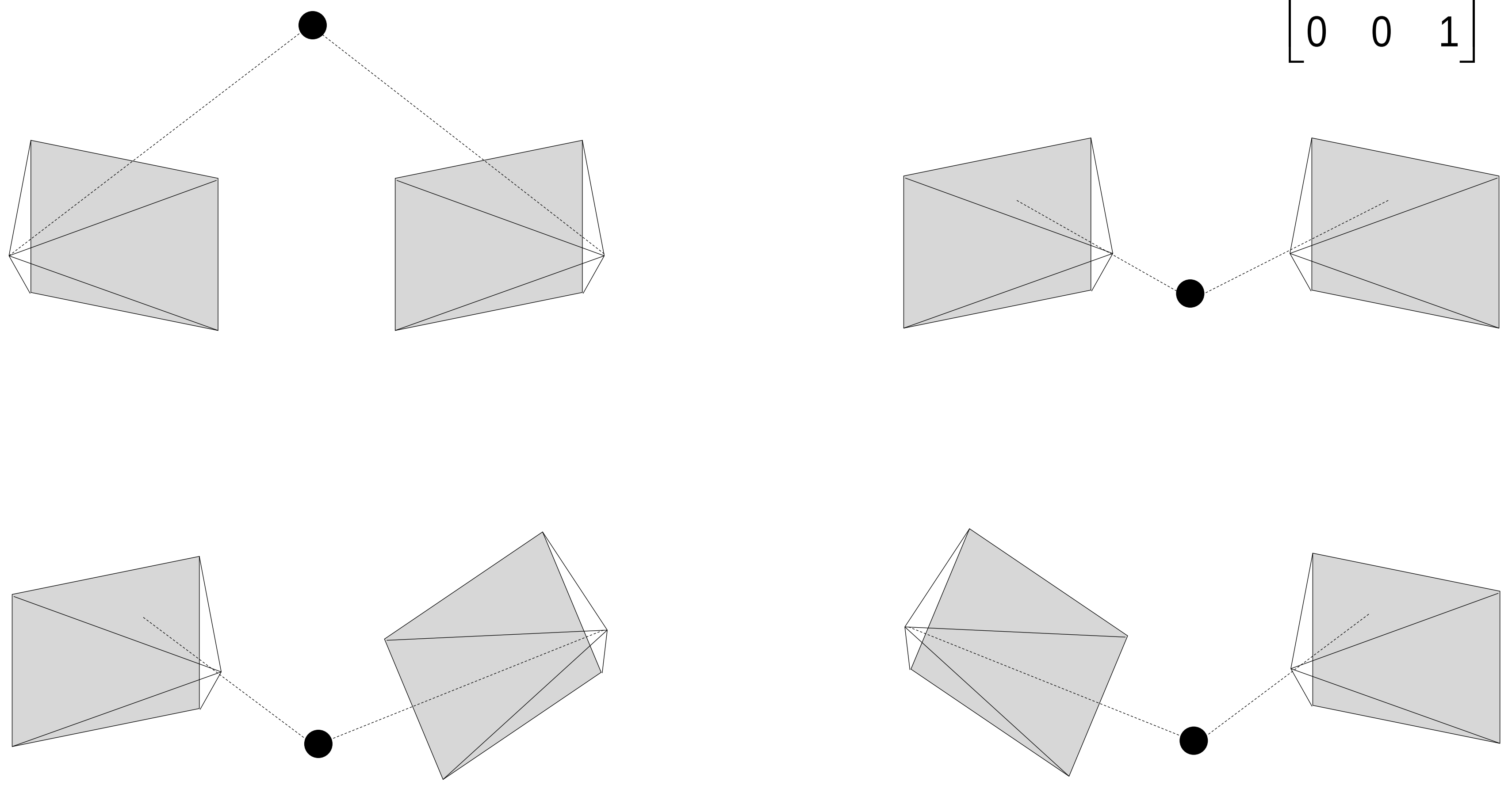
$$R = U\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^T, \text{ or } U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} V^T$$

$\longrightarrow$ Four configurations

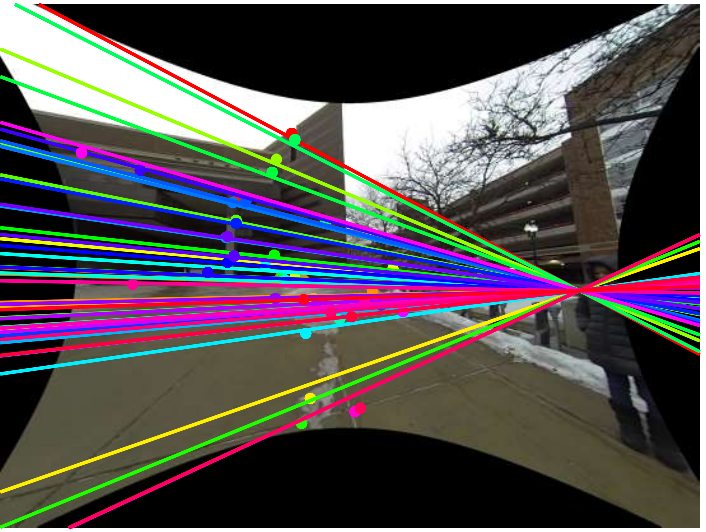
Four Configurations

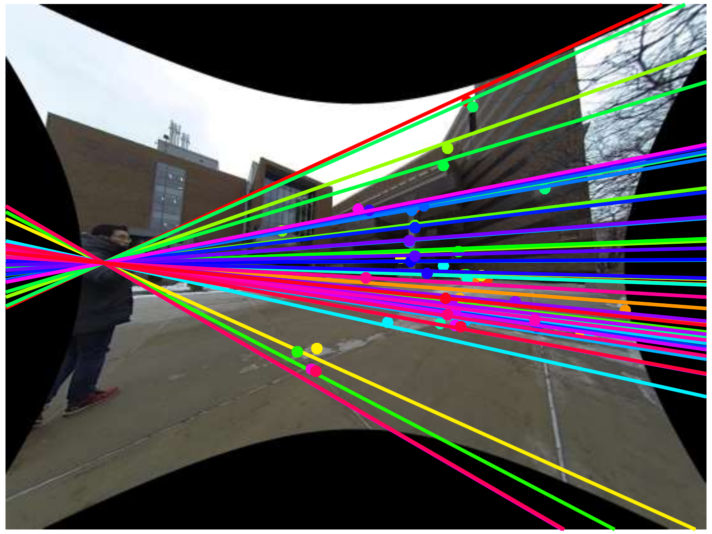
$$\mathbf{t} = \pm \mathbf{u}_3$$

$$R = U\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^T, \text{ or } U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^T$$



Camera Pose Estimation





$$E = K^T F K$$

```
function E = ComputeEssentialMatrix(F, K)

E = K' * F * K;

[u d v] = svd(E);

d(1,1) = 1;
d(2,2) = 1;
d(3,3) = 0;

E = u * d * v';
```

SVD cleanup

D =

1.0468	0	0
0	0.9975	0
0	0	0.0000

Before cleanup

D =

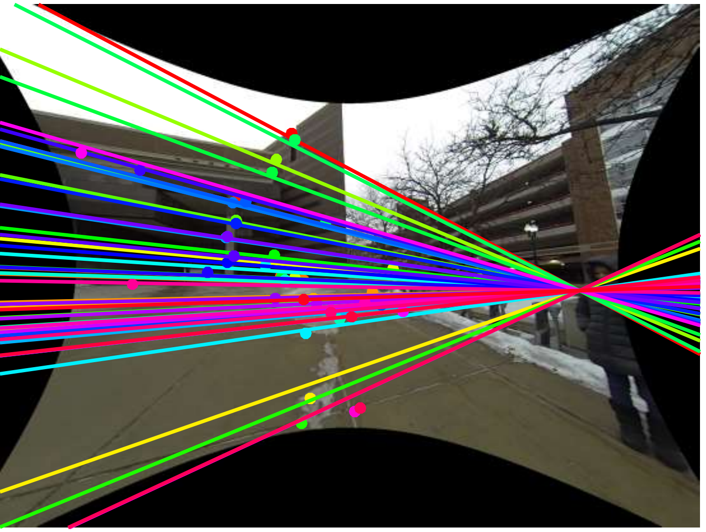
1.0000	0	0
0	1.0000	0
0	0	0.0000

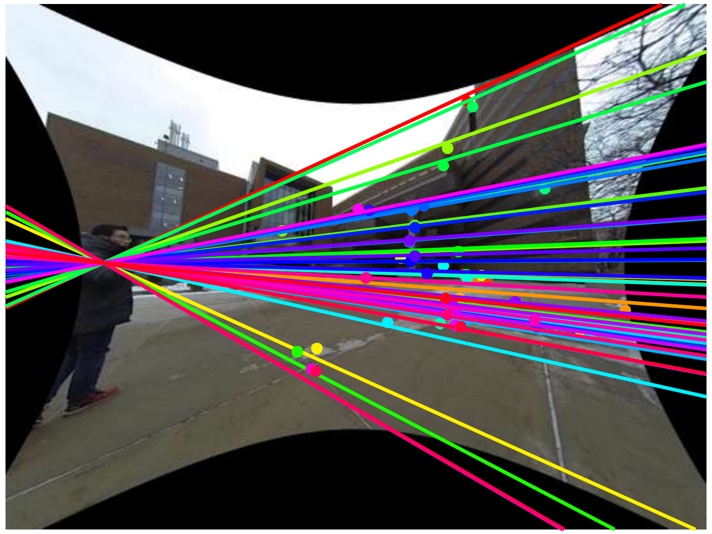
After cleanup

Camera Pose Estimation

$$t = \pm u_3$$

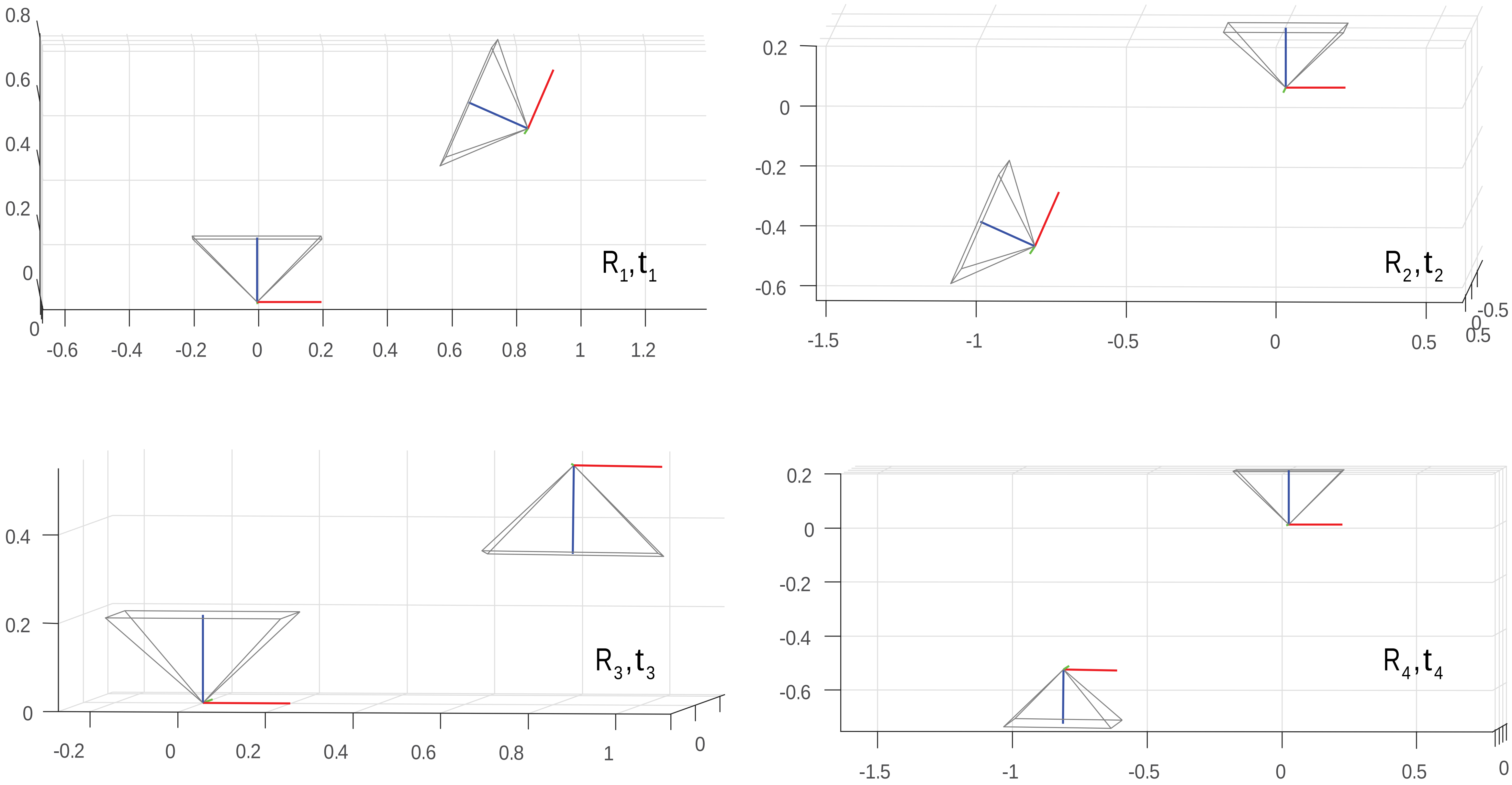
$$R = U\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^T \text{, or } U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^T$$





```
function [R1 t1, R2, t2, R3, t3, R4, t4] = ...
CameraPoseFromEssentialMatrix(E)

[U D V] = svd(E);

W = [0 -1 0;
     1 0 0;
     0 0 1];

t1 = U(:,3);
R1 = U * W * V';
if det(R1) < 0
  t1 = -t1;  R1 = -R1;
end

...
```

$\det(R) = 1$


0.8
0.6
0.4
0.2
0
0
-0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 1.2
R_1,t_1
0.2
0
-0.2
-0.4
-0.6
-1.5 -1 -0.5 0 0.5
-0.5
0
0.5
R_2,t_2
0.4
0.2
0
-0.2 0 0.2 0.4 0.6 0.8 1
0
R_3,t_3
0.2
0
-0.2
-0.4
-0.6
-1.5 -1 -0.5 0 0.5
0
R_4,t_4

Camera Image Projection