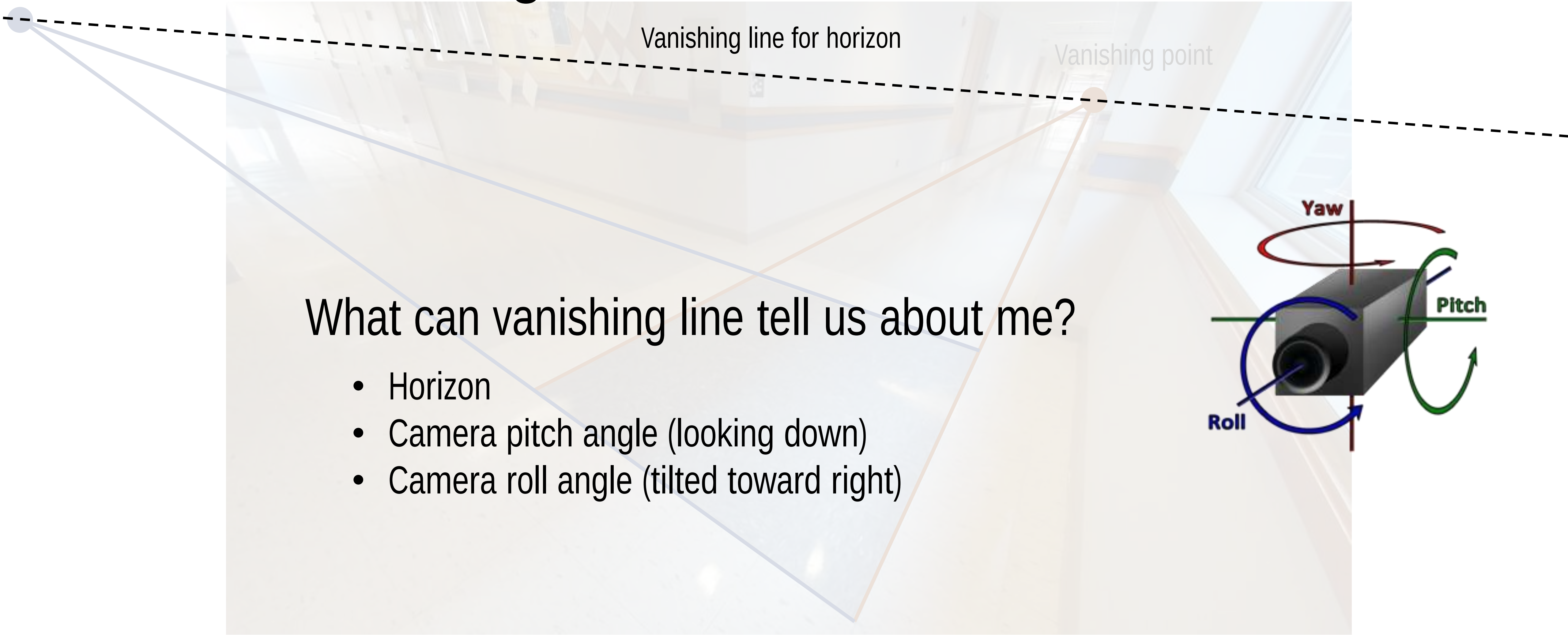
Where am I?
Perspective-n-Point

Recall: Vanishing Line

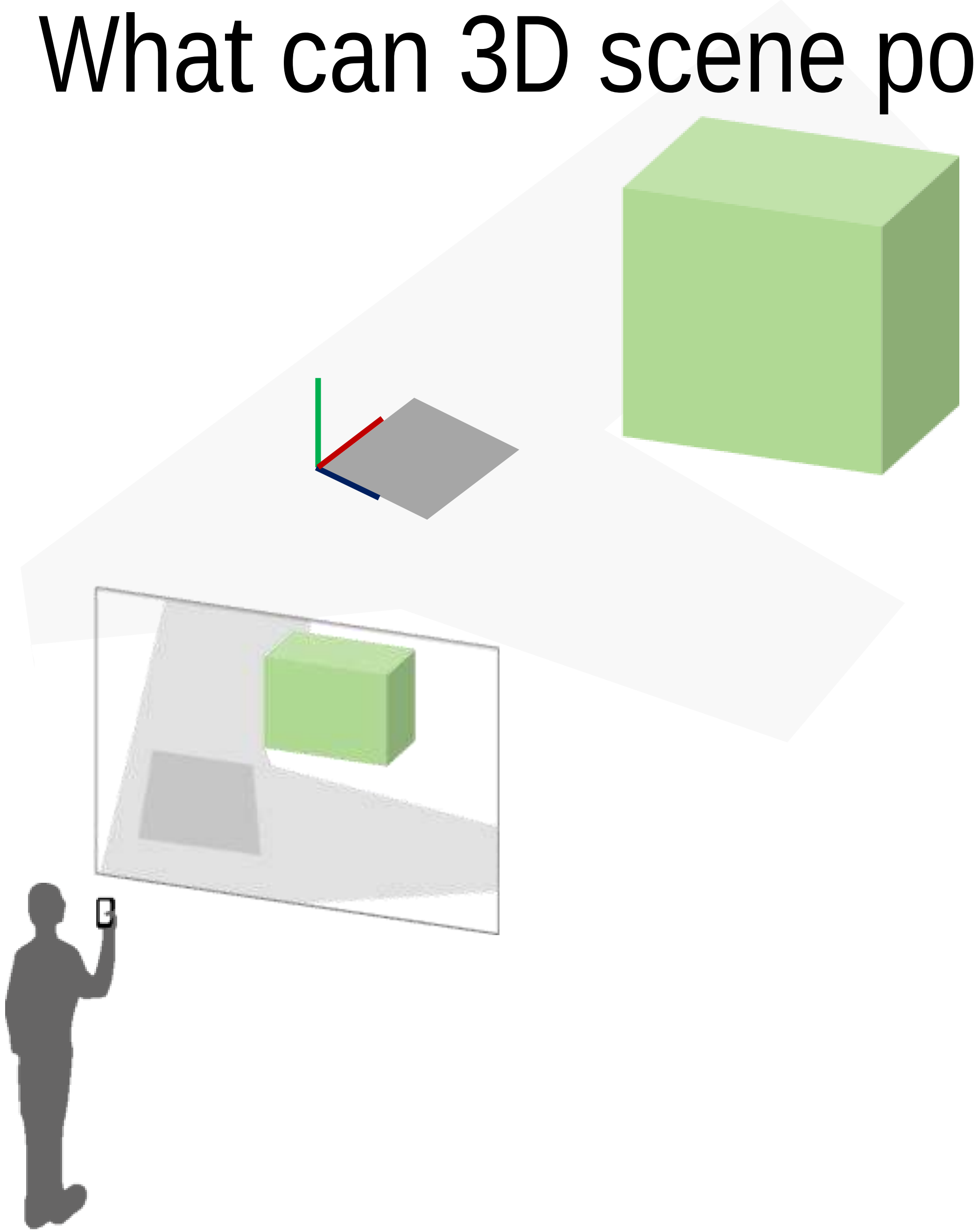


What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)

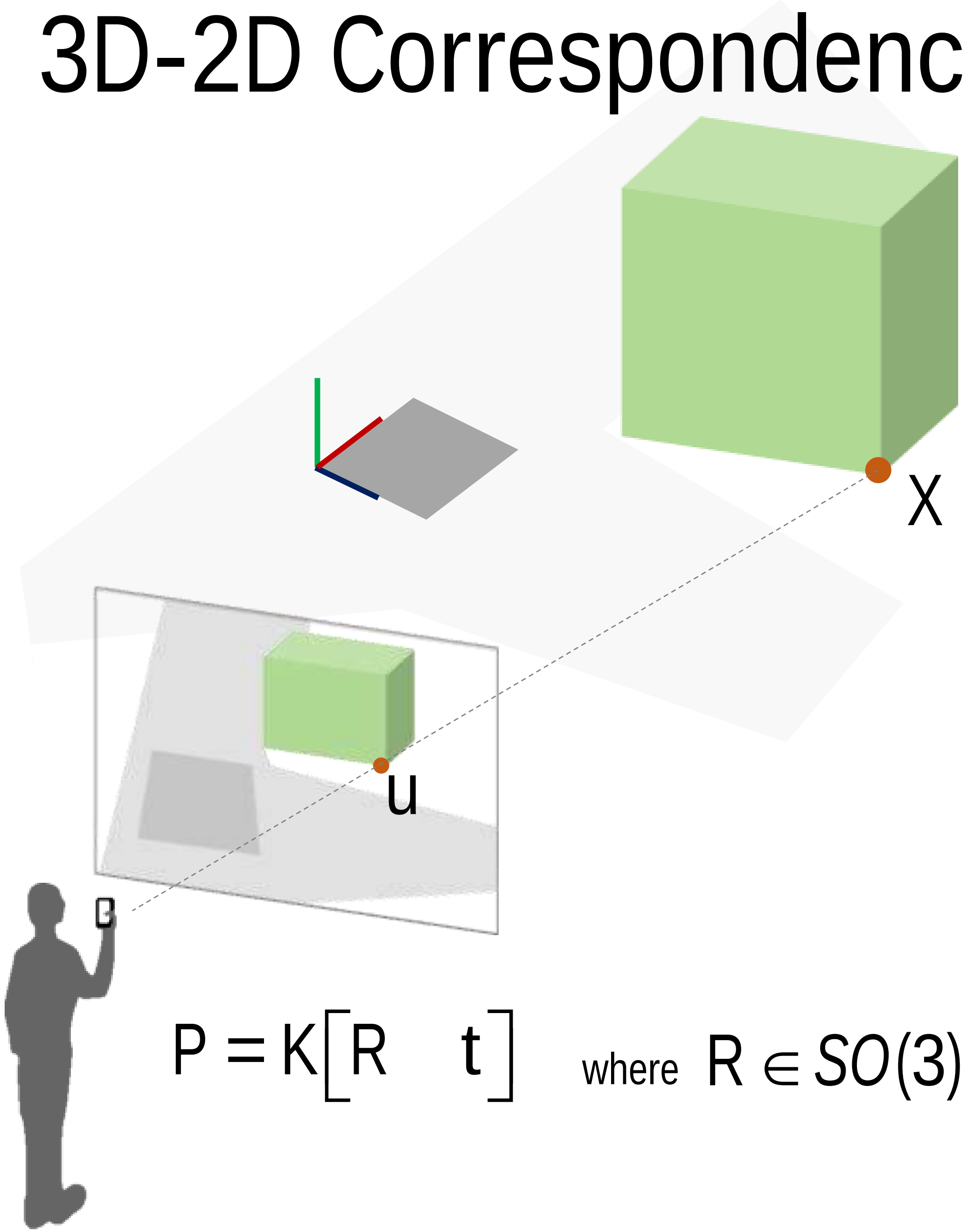
Keller Hall

What can 3D scene points tell us about?



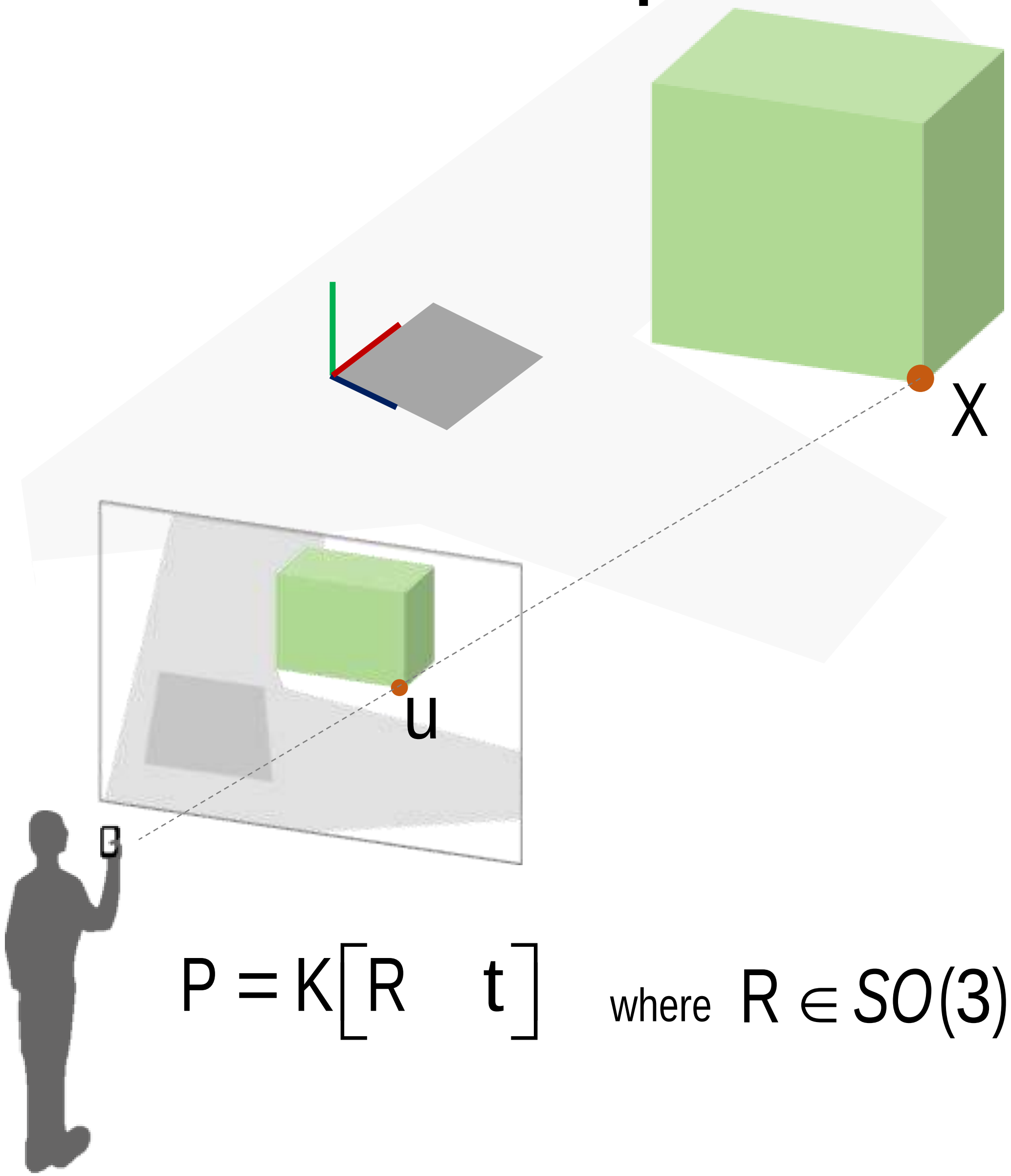
http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D Correspondence



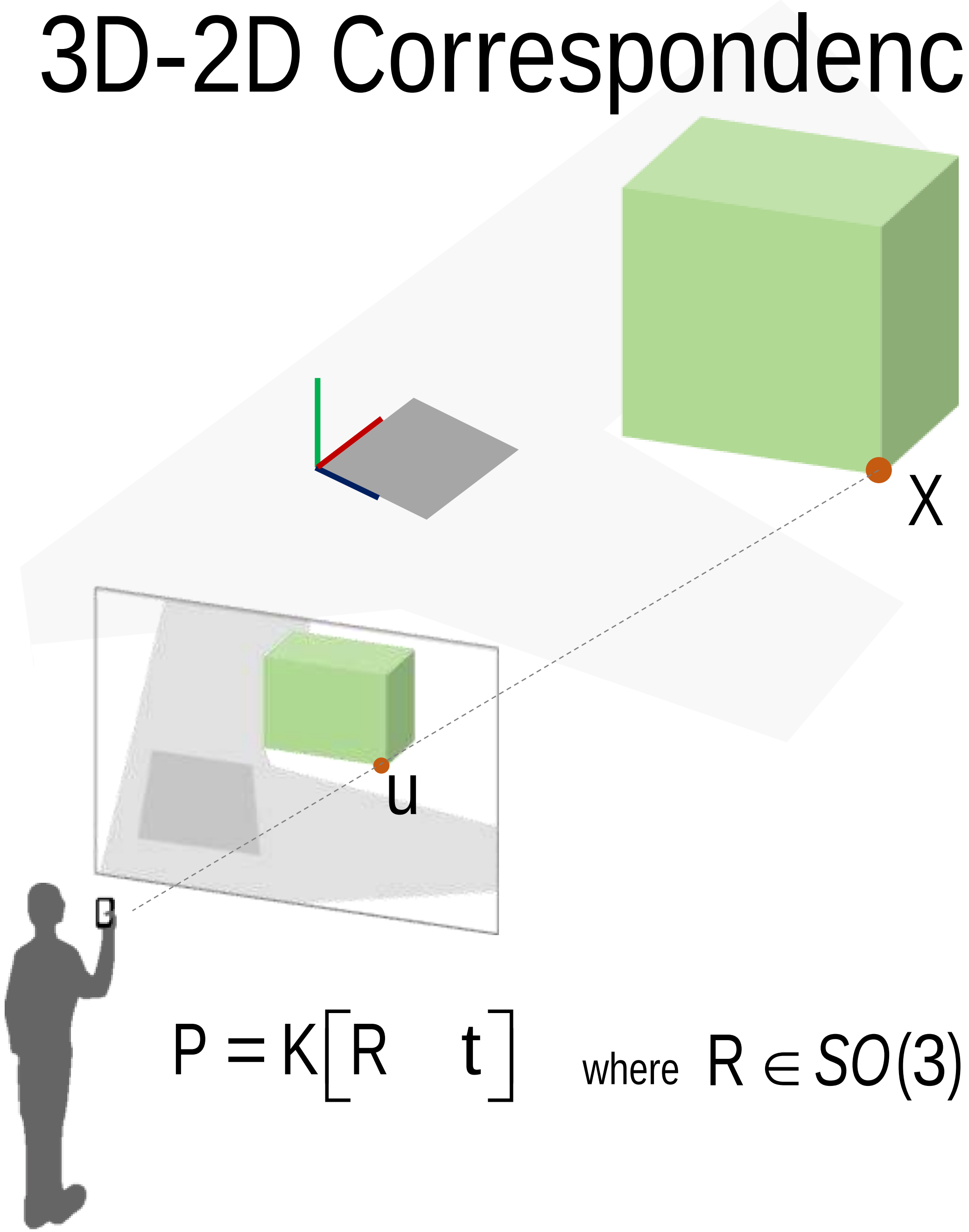
$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

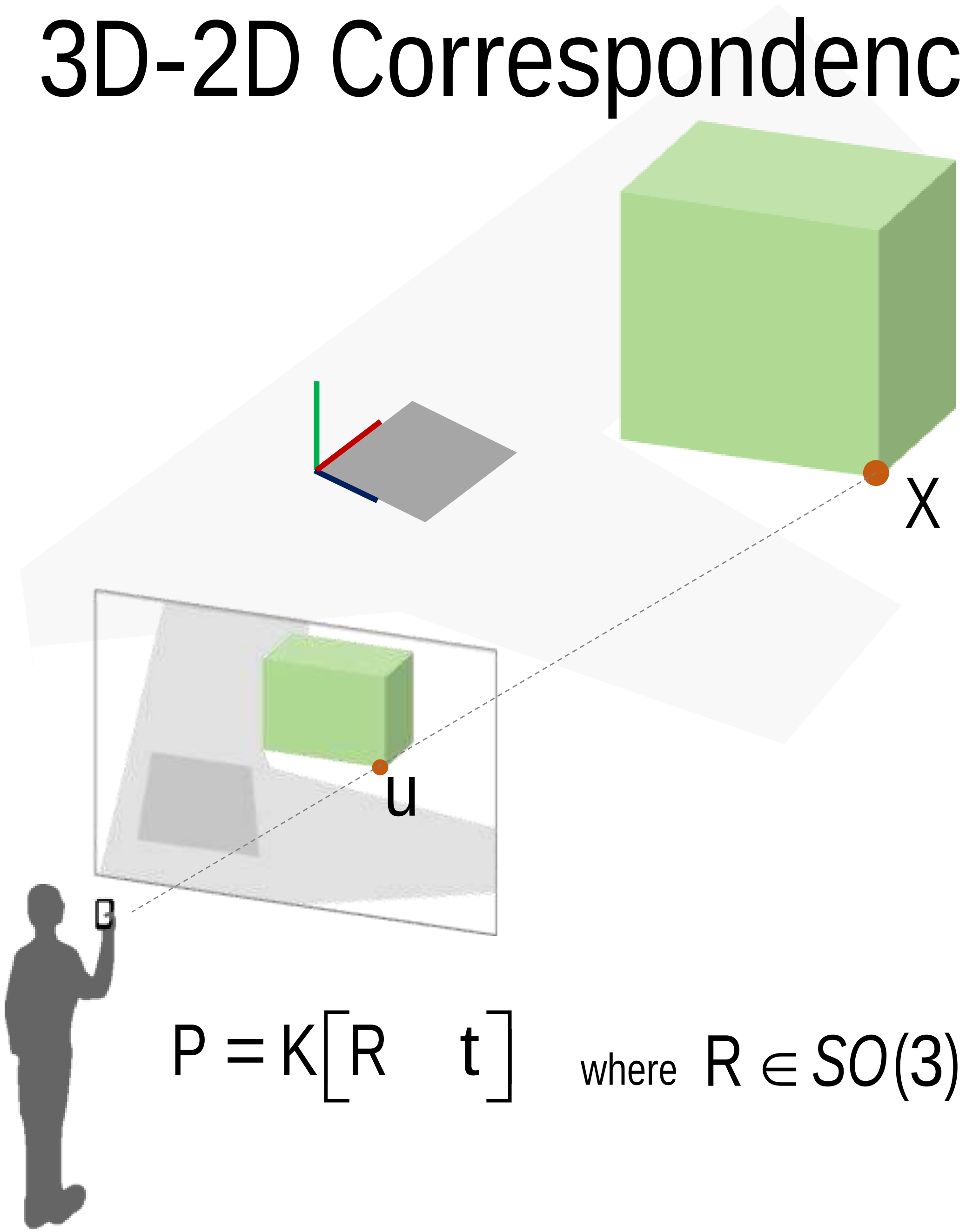
$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where} \quad R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

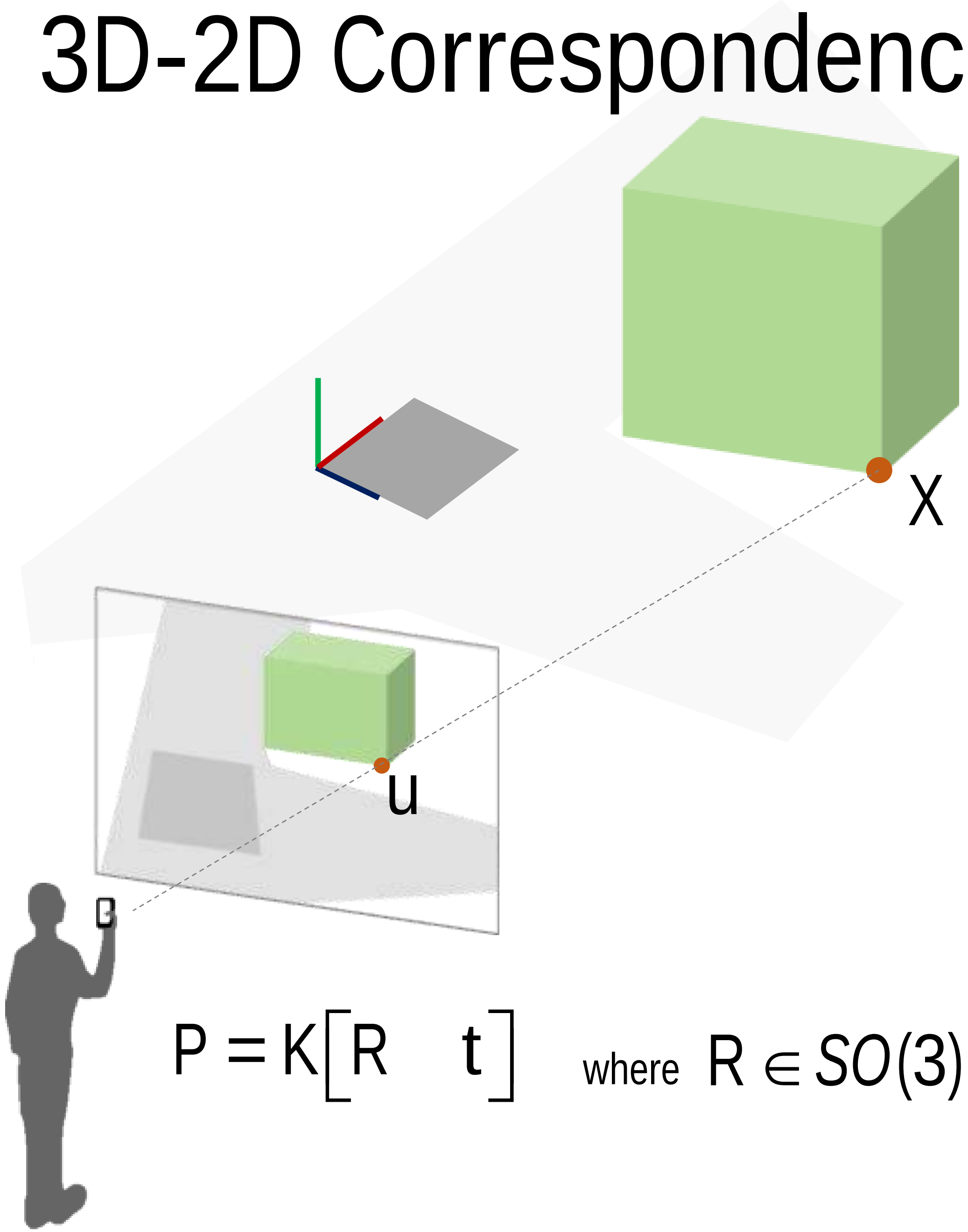
Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)

# of equations per correspondence: 2

3D-2D Correspondence

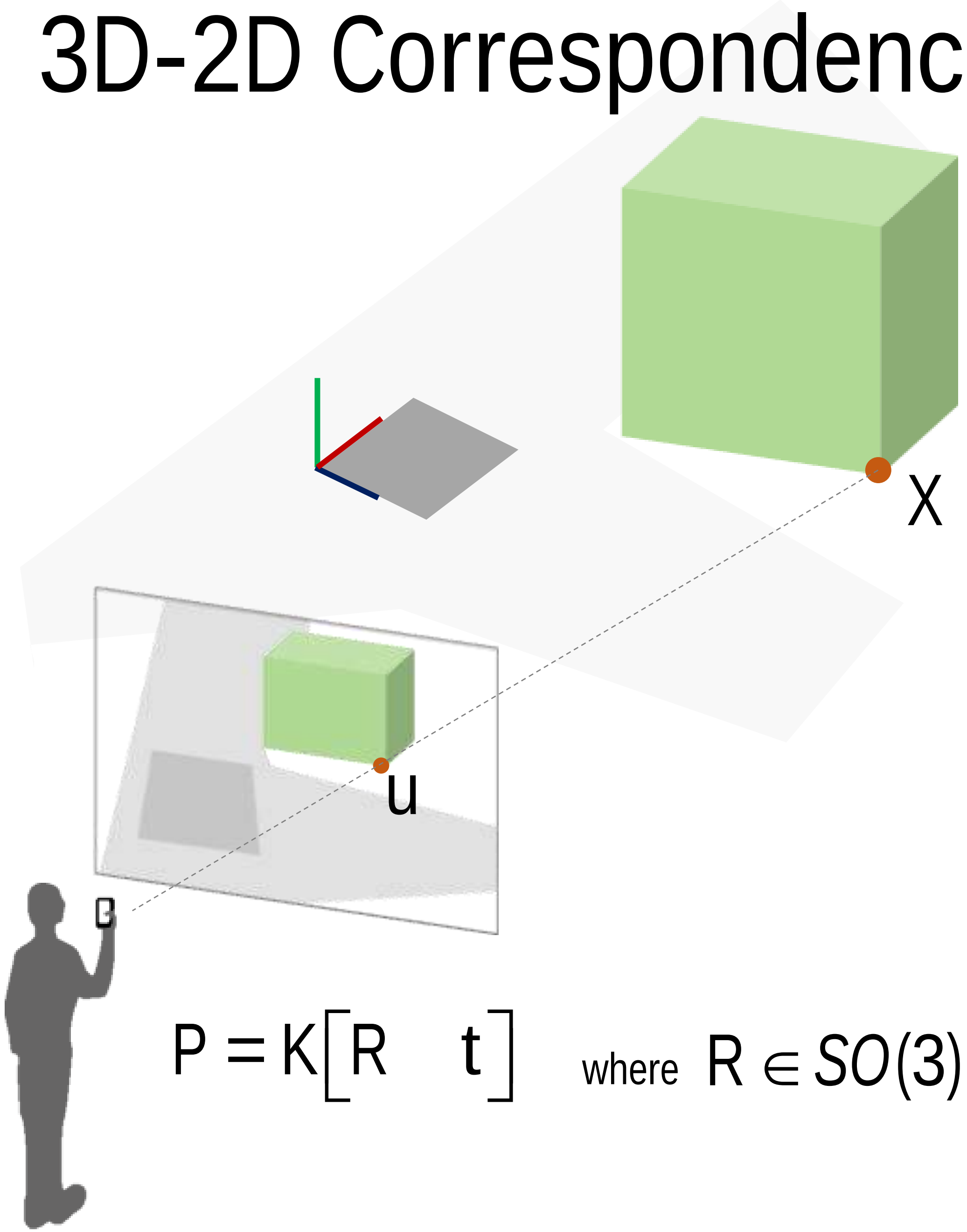


$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

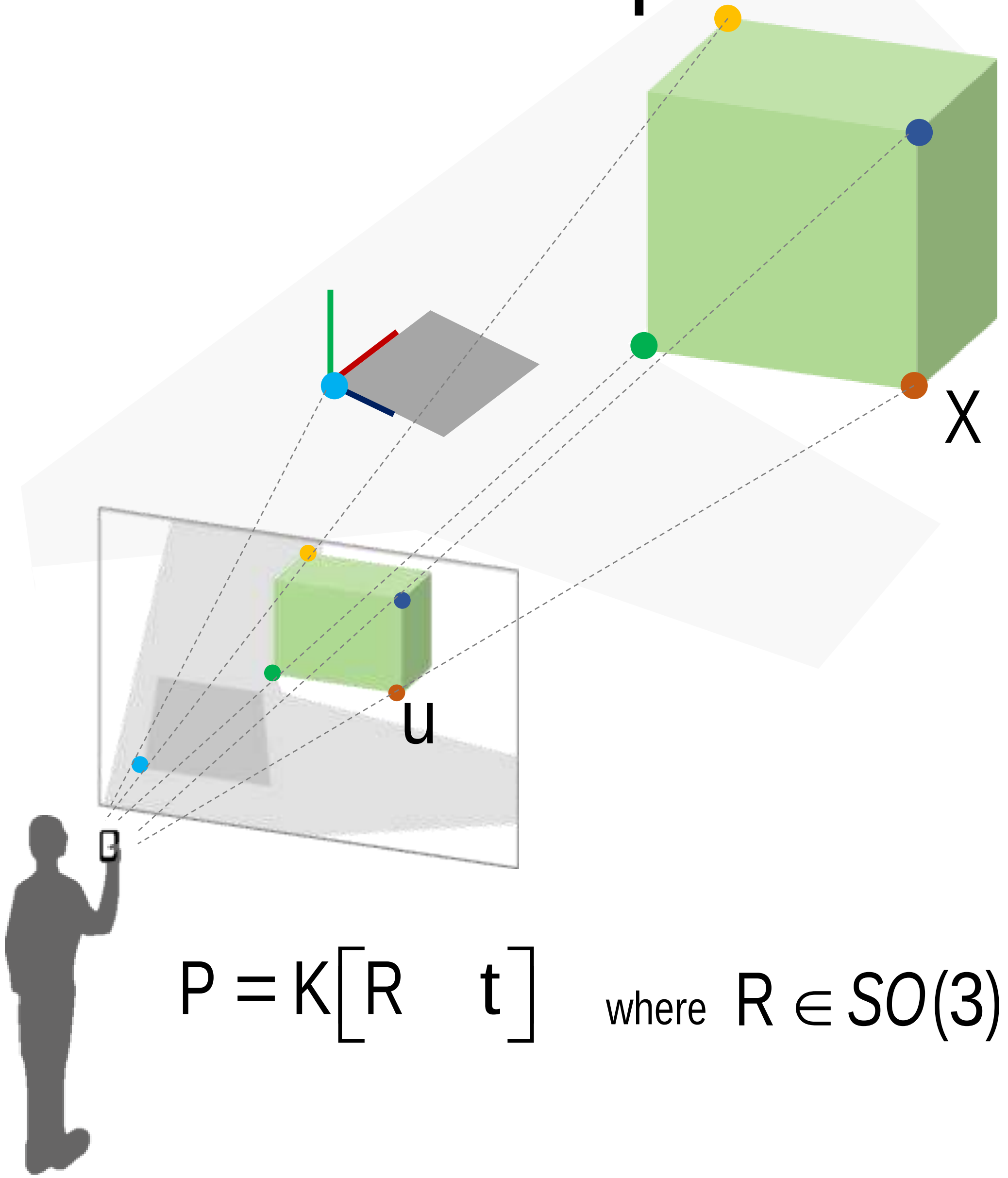
3D-2D correspondence: $u \leftrightarrow X$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\begin{bmatrix} X & Y & Z & 1 & & & & & -u^xX & -u^xY & -u^xZ & -u^x \\ & & & & X & Y & Z & 1 & -u^yX & -u^yY & -u^yZ & -u^y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2x12

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

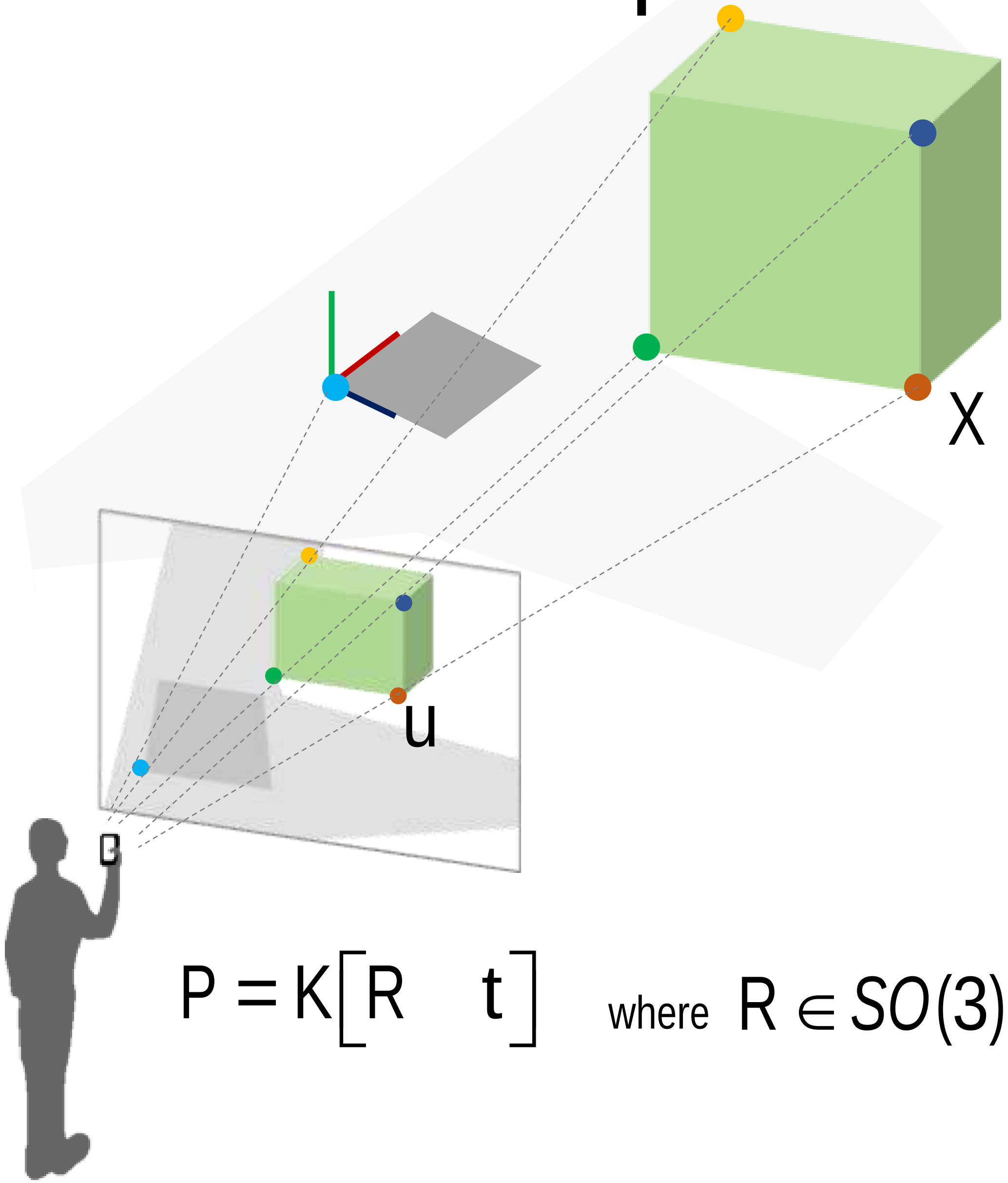
3D-2D correspondence: $u \leftrightarrow X$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^x X & -u_1^x Y & -u_1^x Z & -u_1^x \\
 & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^y X & -u_1^y Y & -u_1^y Z & -u_1^y \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
X_m & Y_m & Z_m & 1 & & & & & -u_m^x X & -u_m^x Y & -u_m^x Z & -u_m^x \\
 & & & & X_m & Y_m & Z_m & 1 & -u_m^y X & -u_m^y Y & -u_m^y Z & -u_m^y
\end{bmatrix}
\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$2m$x12

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

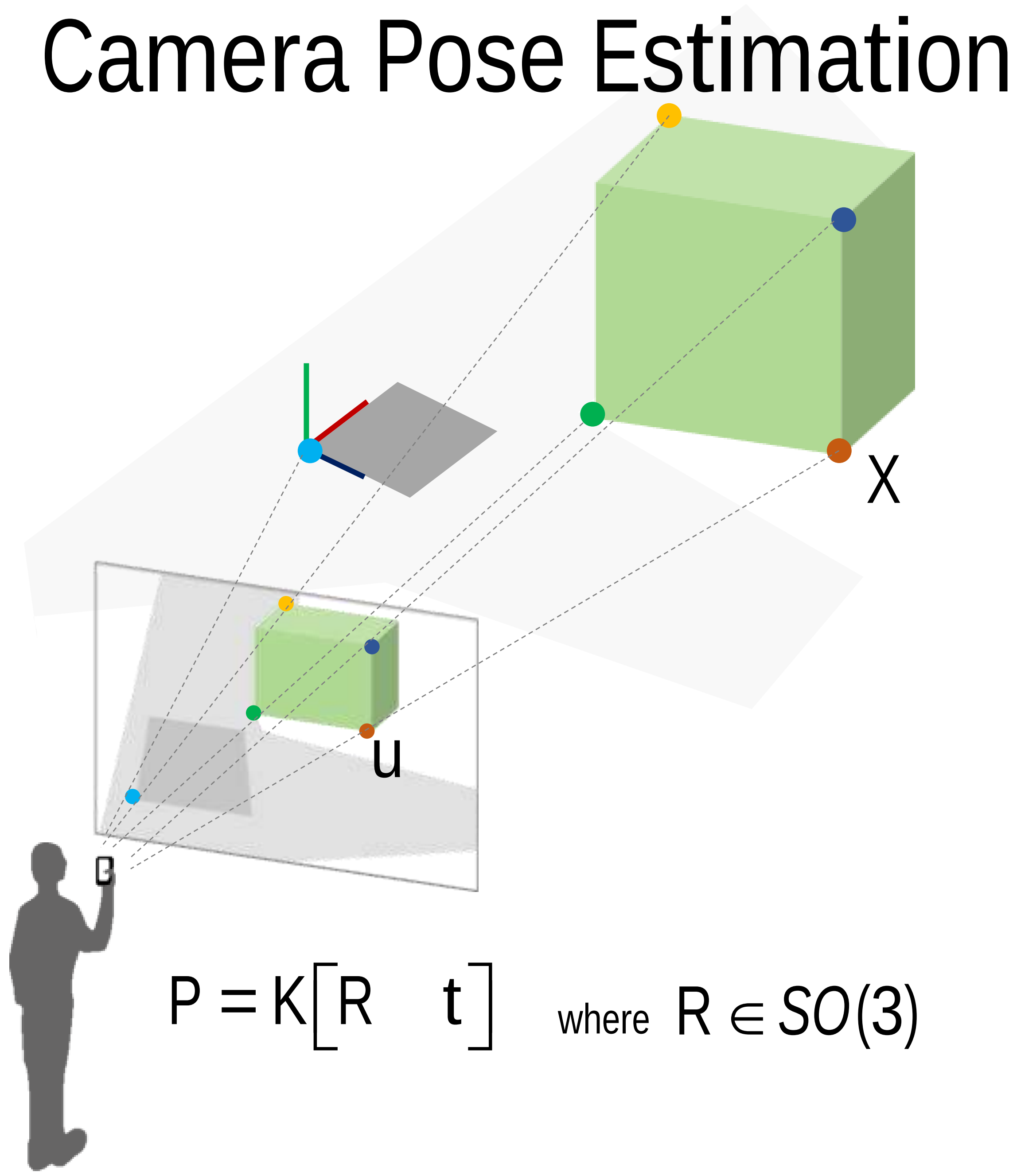
3D-2D correspondence: $u \leftrightarrow X$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^xX & -u_1^xY & -u_1^xZ & -u_1^x \\ & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^yX & -u_1^yY & -u_1^yZ & -u_1^y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ X_m & Y_m & Z_m & 1 & & & & & -u_m^xX & -u_m^xY & -u_m^xZ & -u_m^x \\ & & & & X_m & Y_m & Z_m & 1 & -u_m^yX & -u_m^yY & -u_m^yZ & -u_m^y \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{0}$$

$2m$x12

Camera Pose Estimation

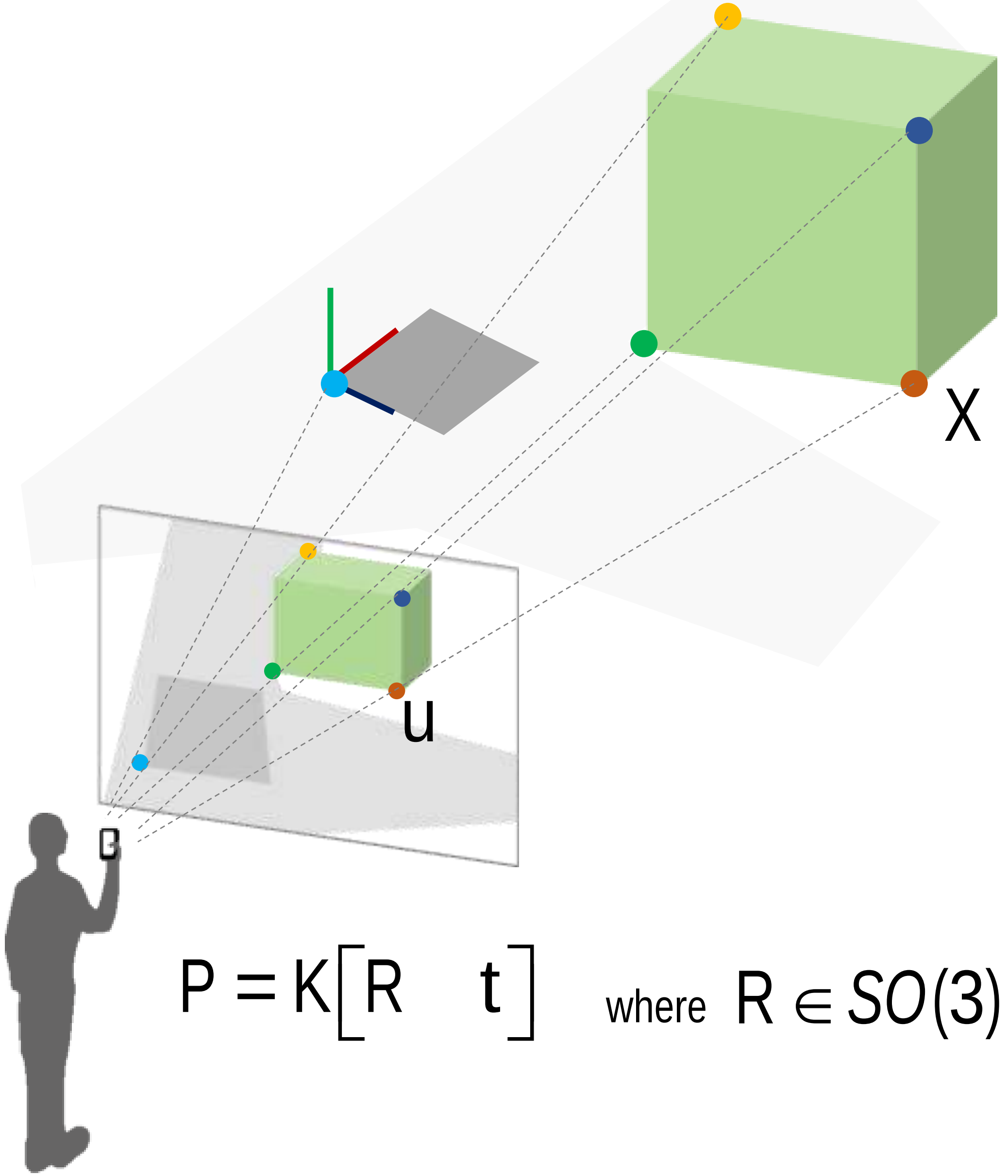


$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

If K is given,

$$K\begin{bmatrix} R & t \end{bmatrix} = \gamma\begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

Camera Pose Estimation



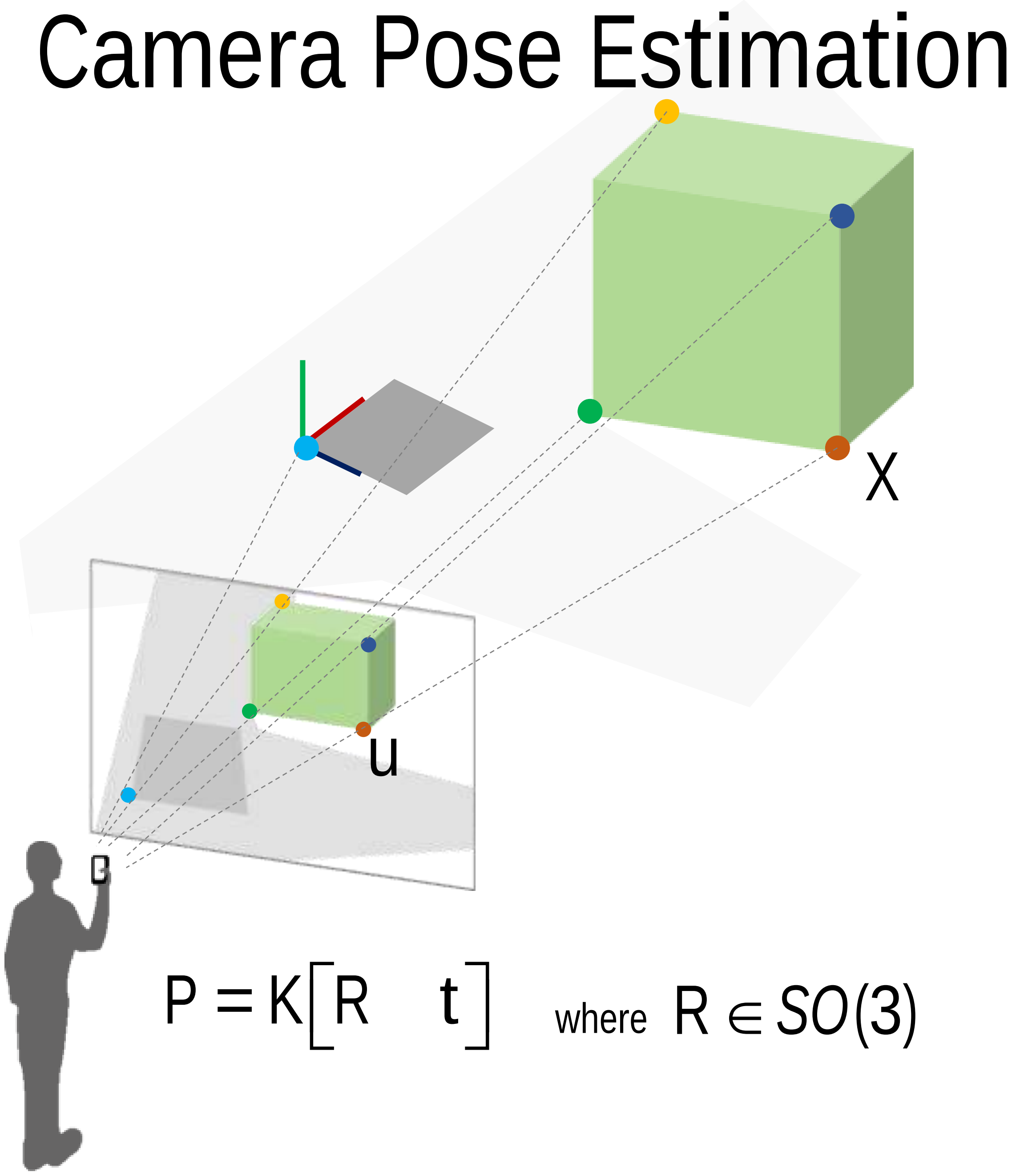
$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

If K is given,

$$K\begin{bmatrix} R & t \end{bmatrix} = \gamma \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

$$\longrightarrow \gamma R = K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

Camera Pose Estimation



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where} \quad R \in SO(3)$$

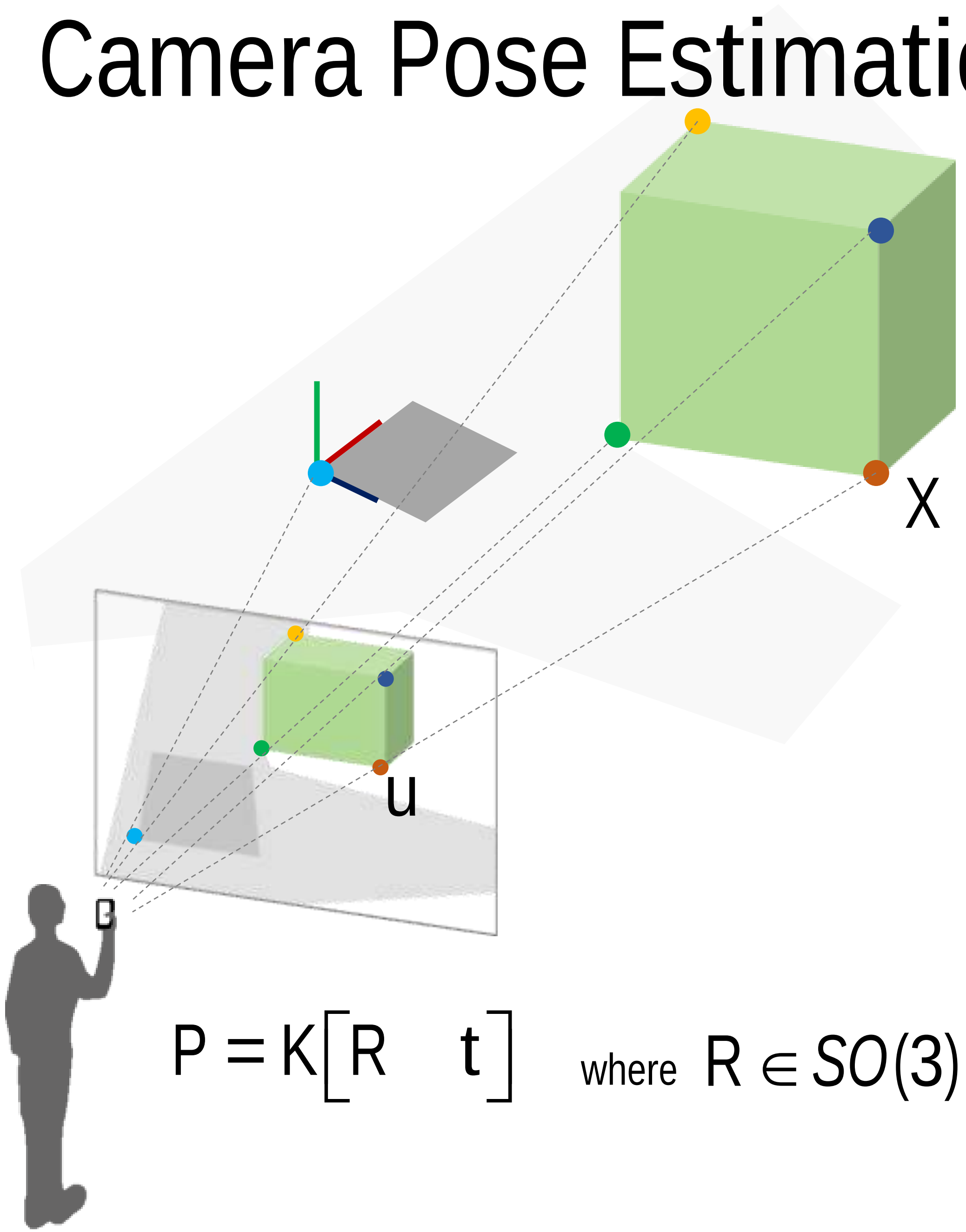
If K is given,

$$K\begin{bmatrix} R & t \end{bmatrix} = \gamma \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

$$\longrightarrow \gamma R = K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

$$K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} = U\begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix}V^T$$

Camera Pose Estimation



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

If K is given,

$$K\begin{bmatrix} R & t \end{bmatrix} = \gamma \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

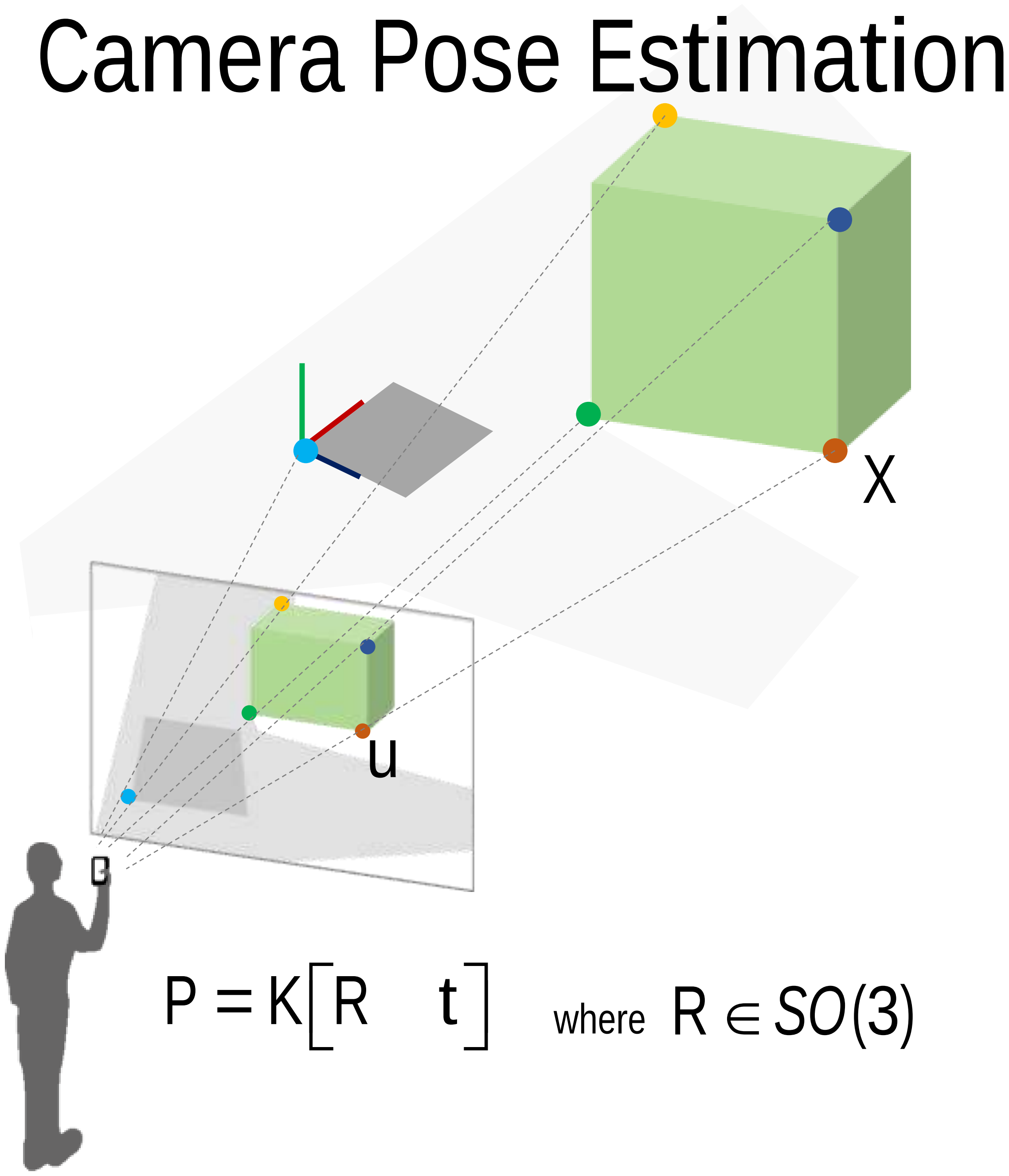
$$\longrightarrow \gamma R = K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

$$K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} = U \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} V^T$$

$$\longrightarrow \gamma \approx d_{11}$$

$R = UV^T$: SVD cleanup

Camera Pose Estimation



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

If K is given,

$$K\begin{bmatrix} R & t \end{bmatrix} = \gamma \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

$$\longrightarrow \gamma R = K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}$$

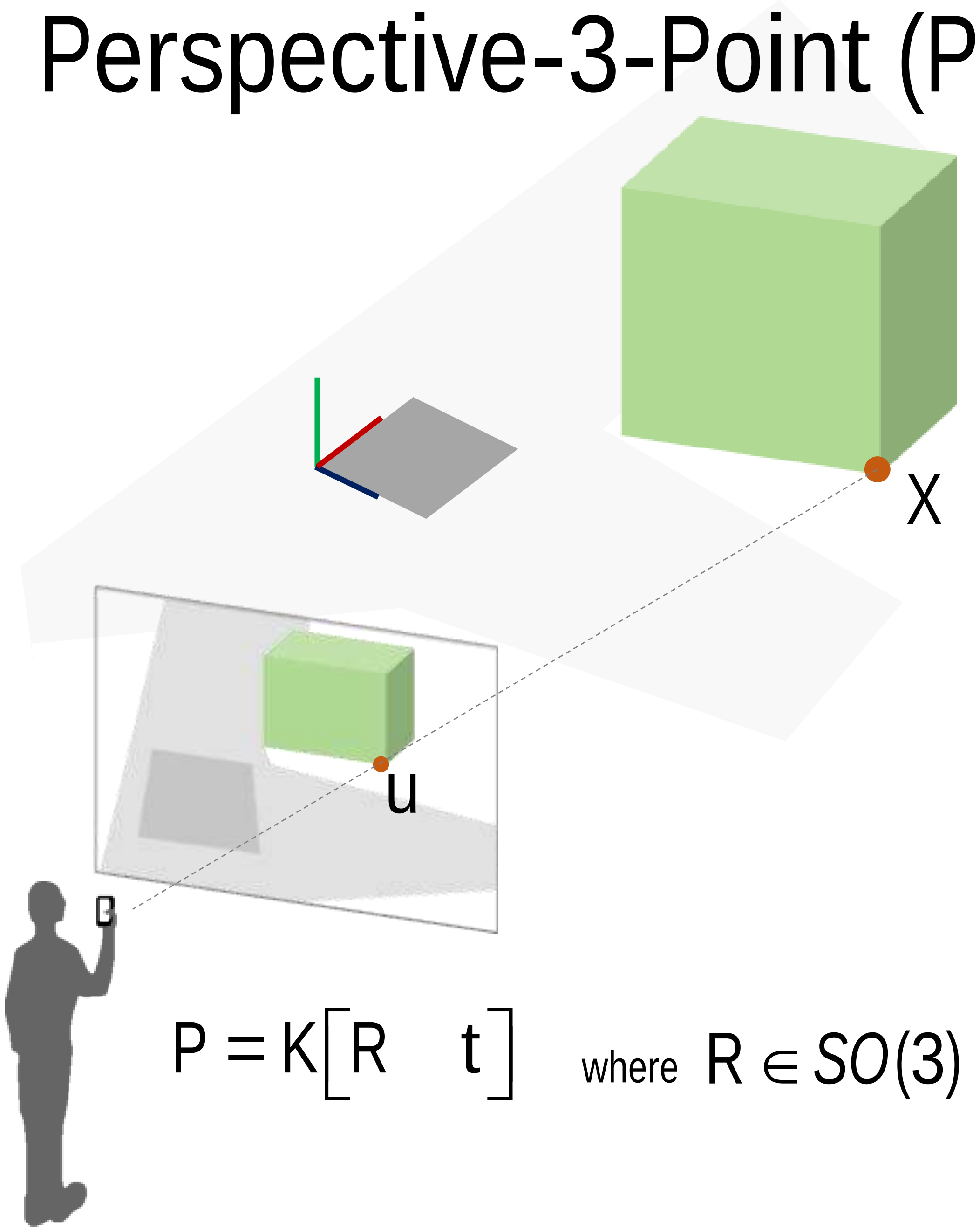
$$K^{-1}\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} = U \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} V^T$$

$$\longrightarrow \gamma \approx d_{11}$$

$R = UV^T$: SVD cleanup

$$\longrightarrow t = \frac{K^{-1}p_4}{d_{11}}$$: Translation and scale recovery

Perspective-3-Point (P3P)



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where} \quad R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

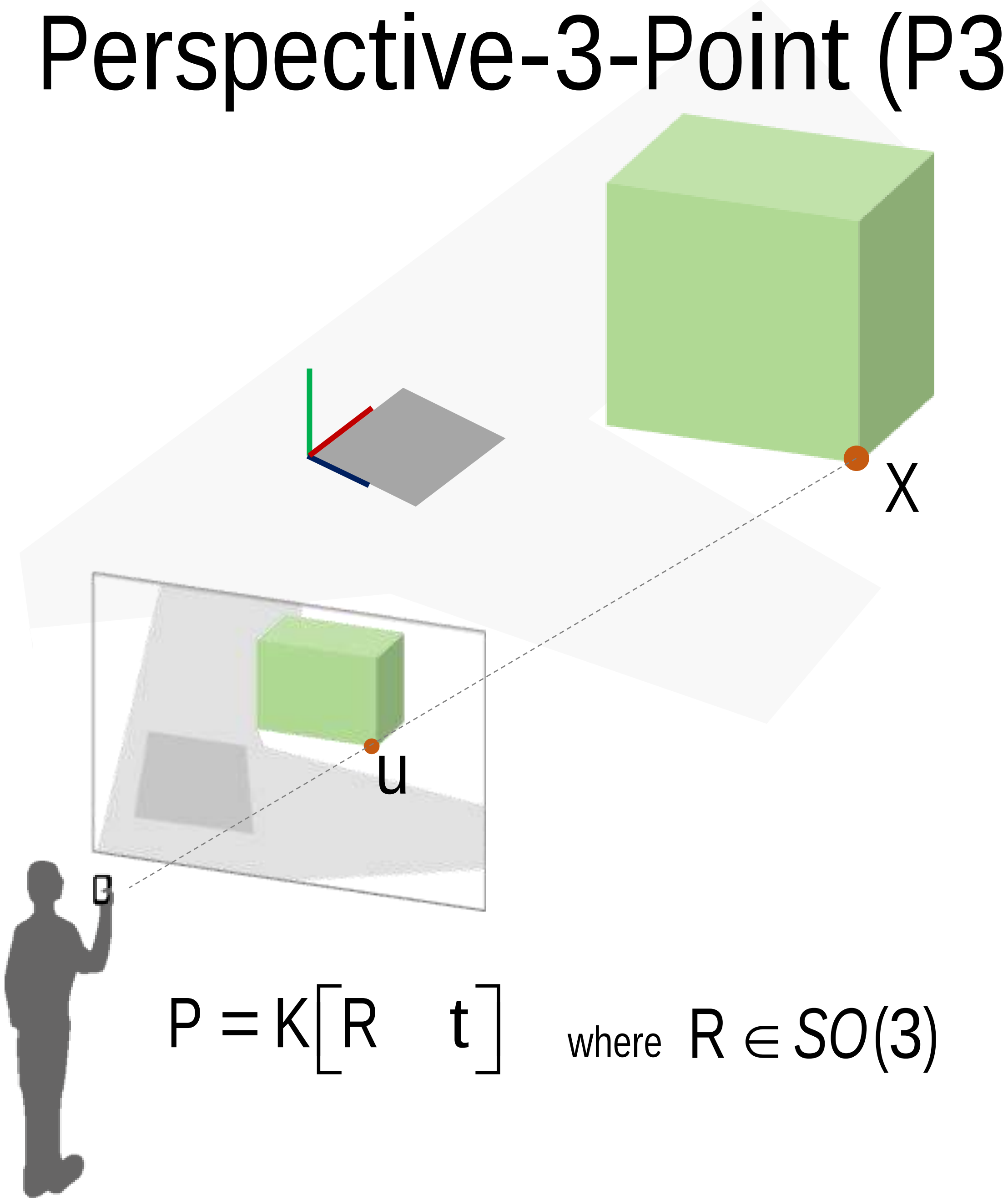
$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)

6 dof when K is known.

# of equations per correspondence: 2

Perspective-3-Point (P3P)



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

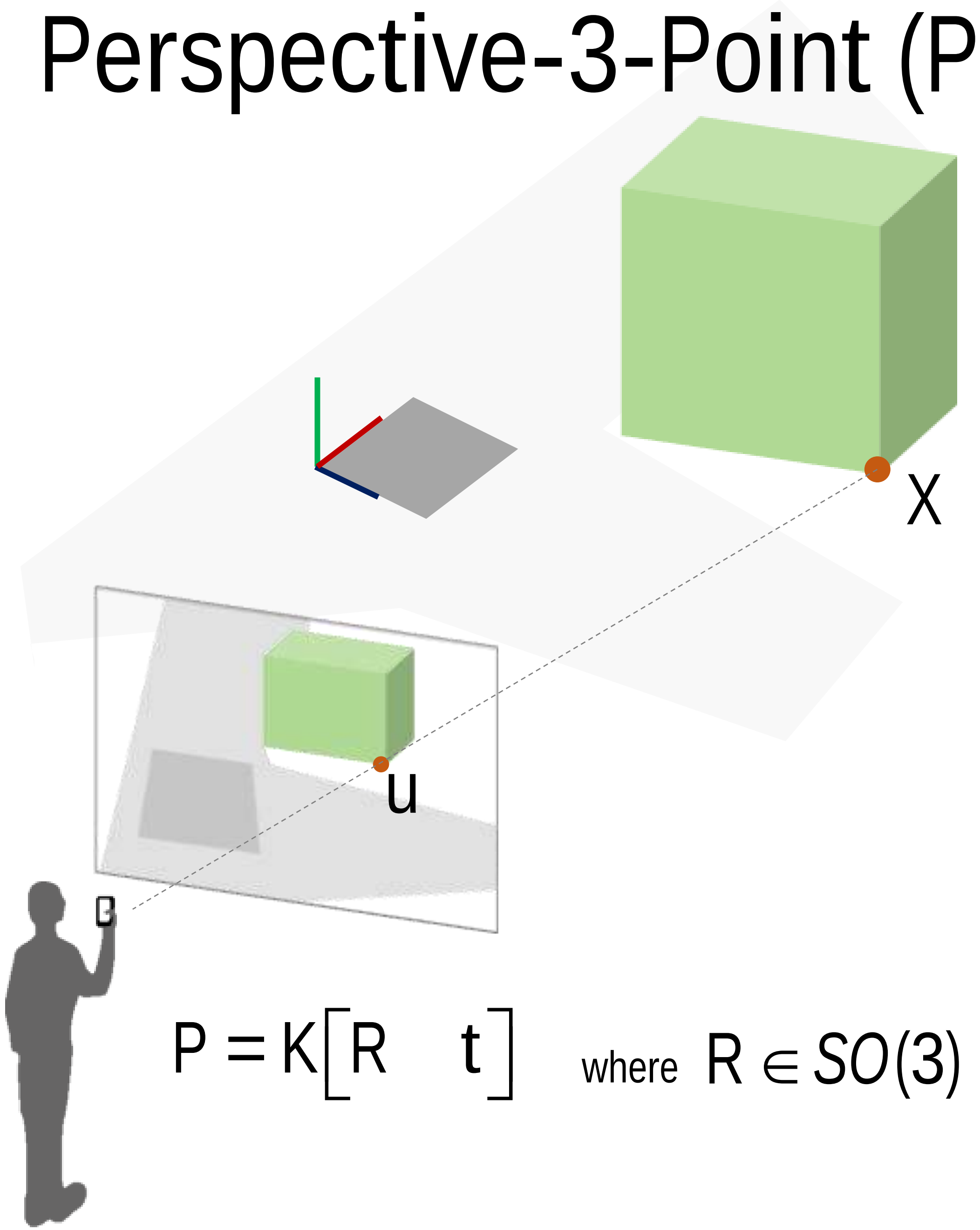
$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)
6 dof when K is known.
# of equations per correspondence: 2

Perspective-3-Point (P3P)



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D correspondence: $u \leftrightarrow X$

$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

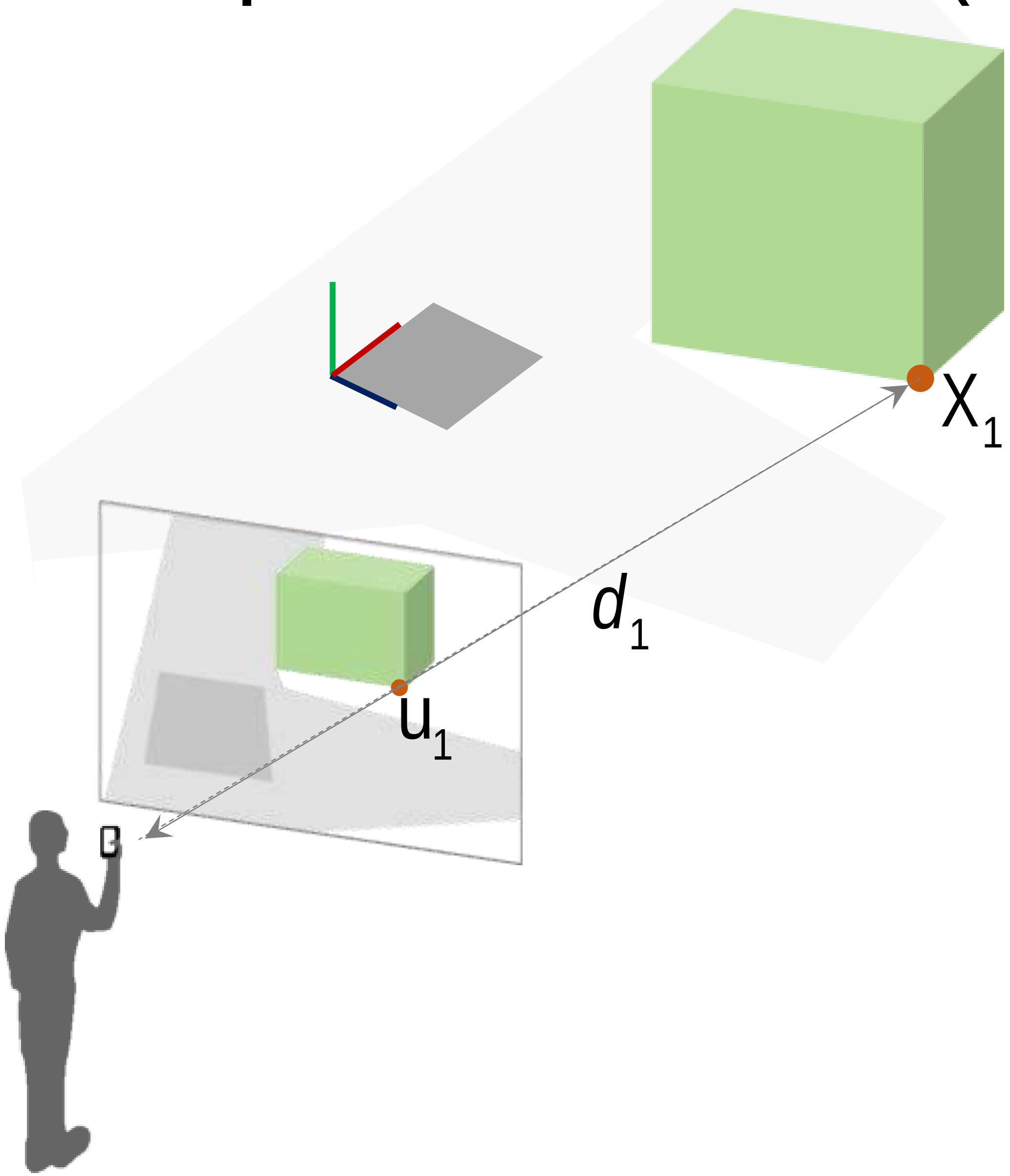
# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)

6 dof when K is known.

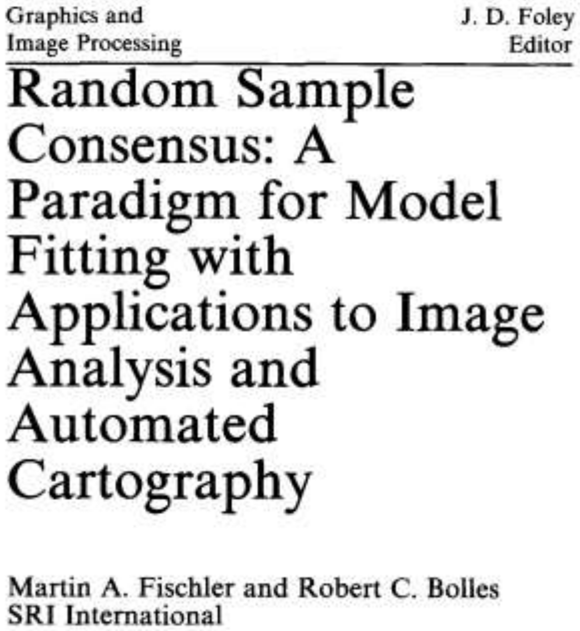
# of equations per correspondence: 2

3 correspondences should be enough.

Perspective-3-Point (P3P)



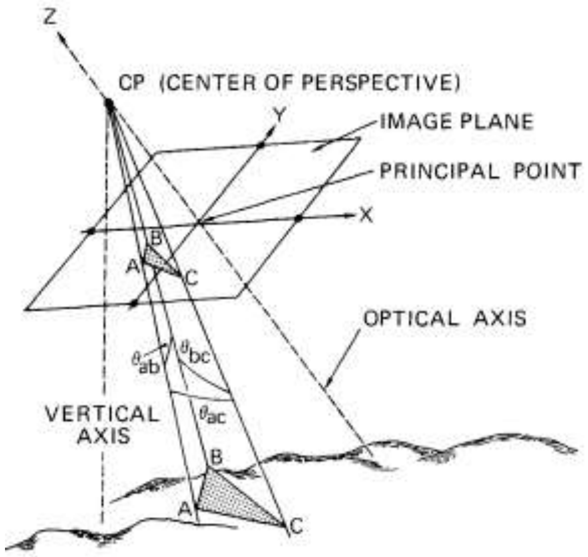
RANSAC with PnP



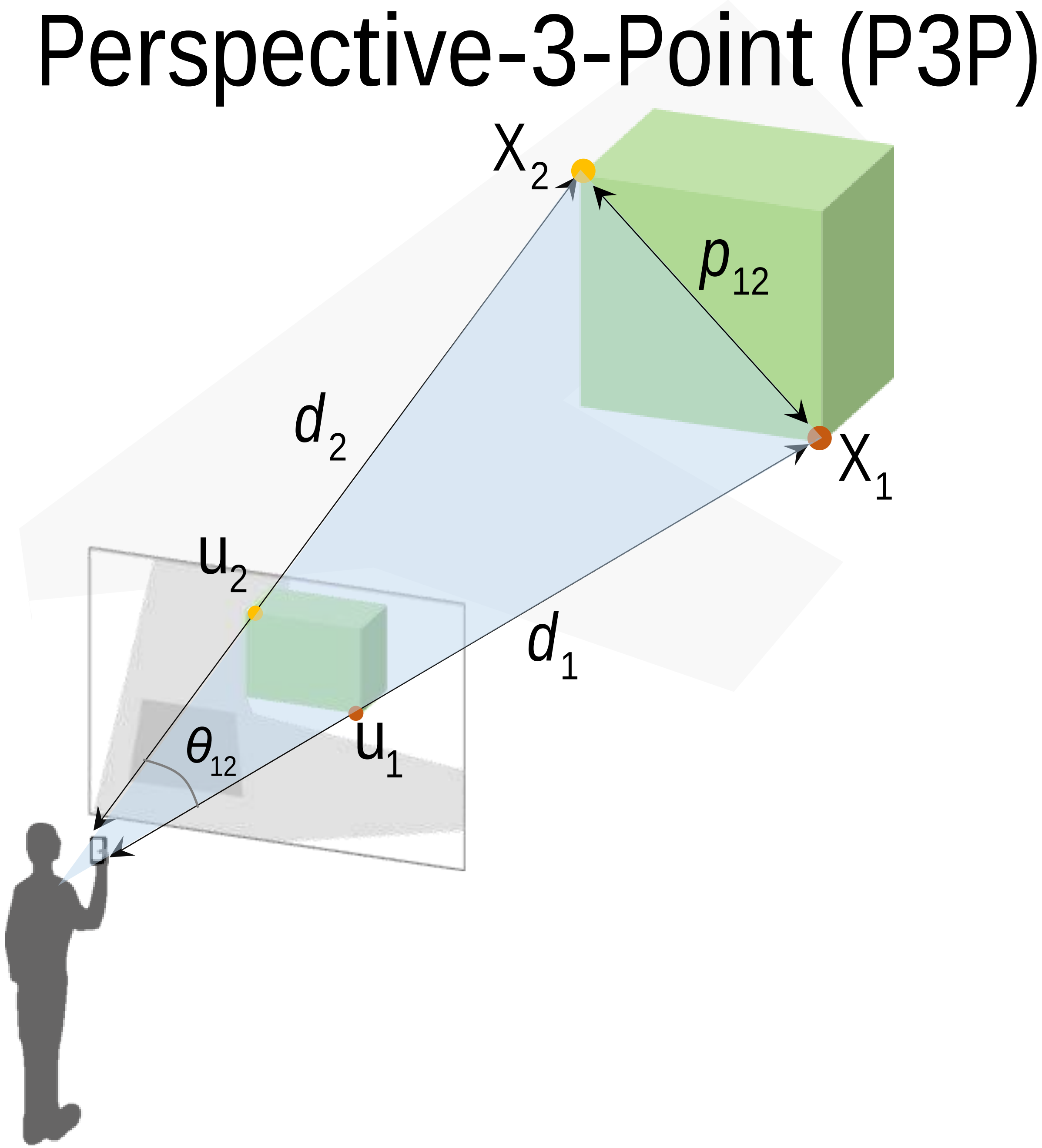
Graphics and Image Processing — J. D. Foley, Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

Martin A. Fischler and Robert C. Bolles
SRI International



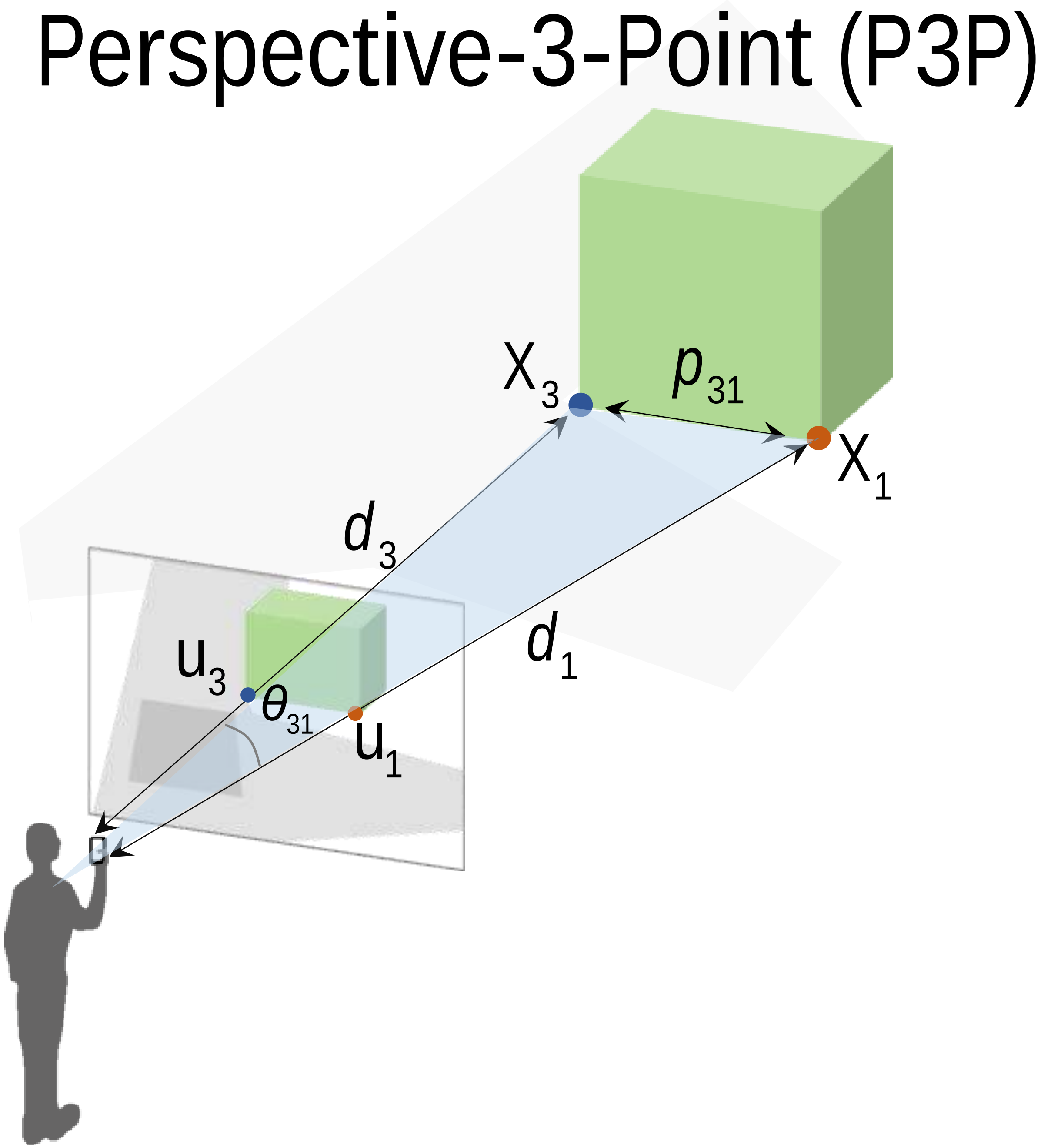
Perspective-3-Point (P3P)



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} = p_{12}^2$$

Perspective-3-Point (P3P)

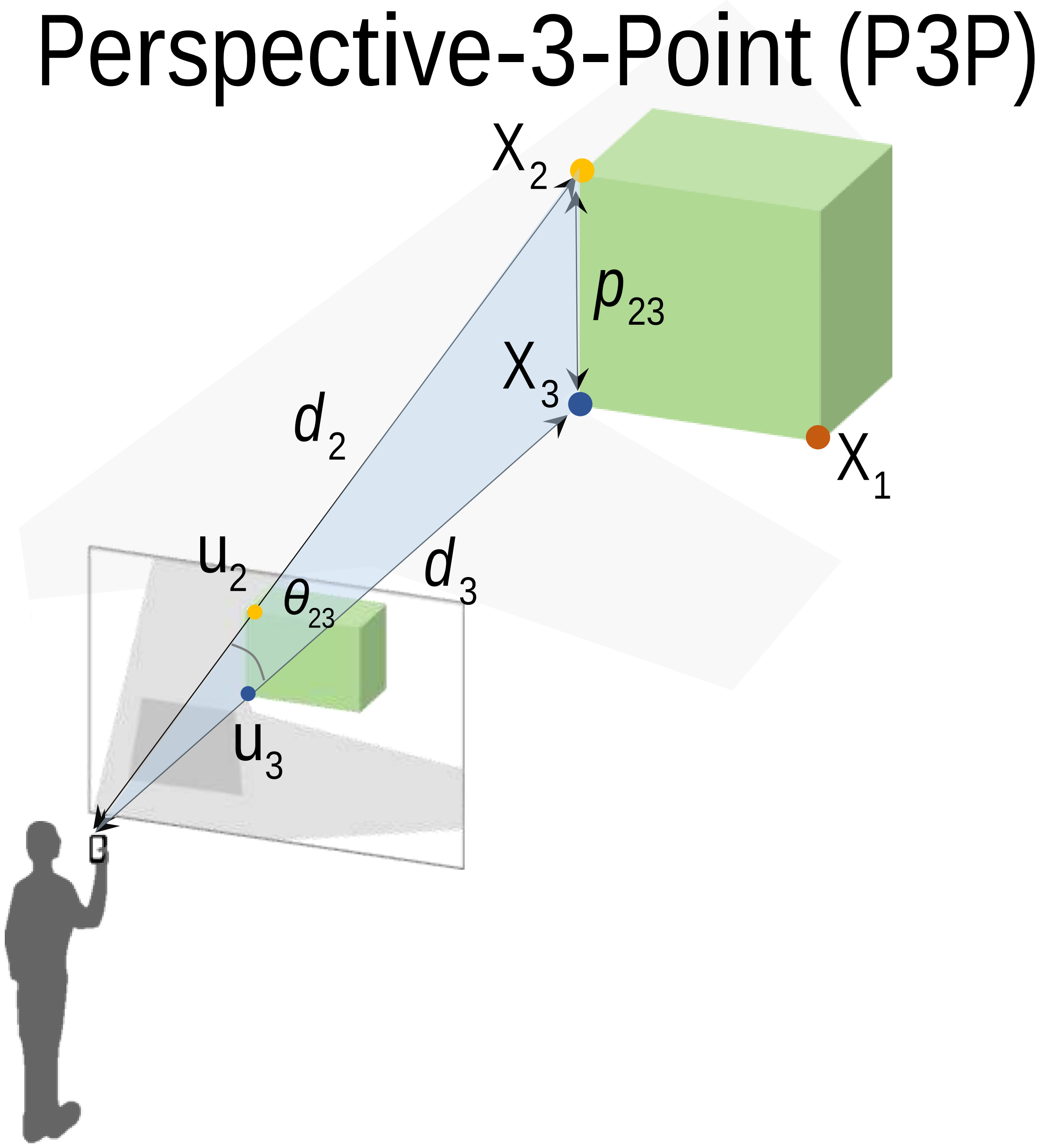


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$

$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$

Perspective-3-Point (P3P)



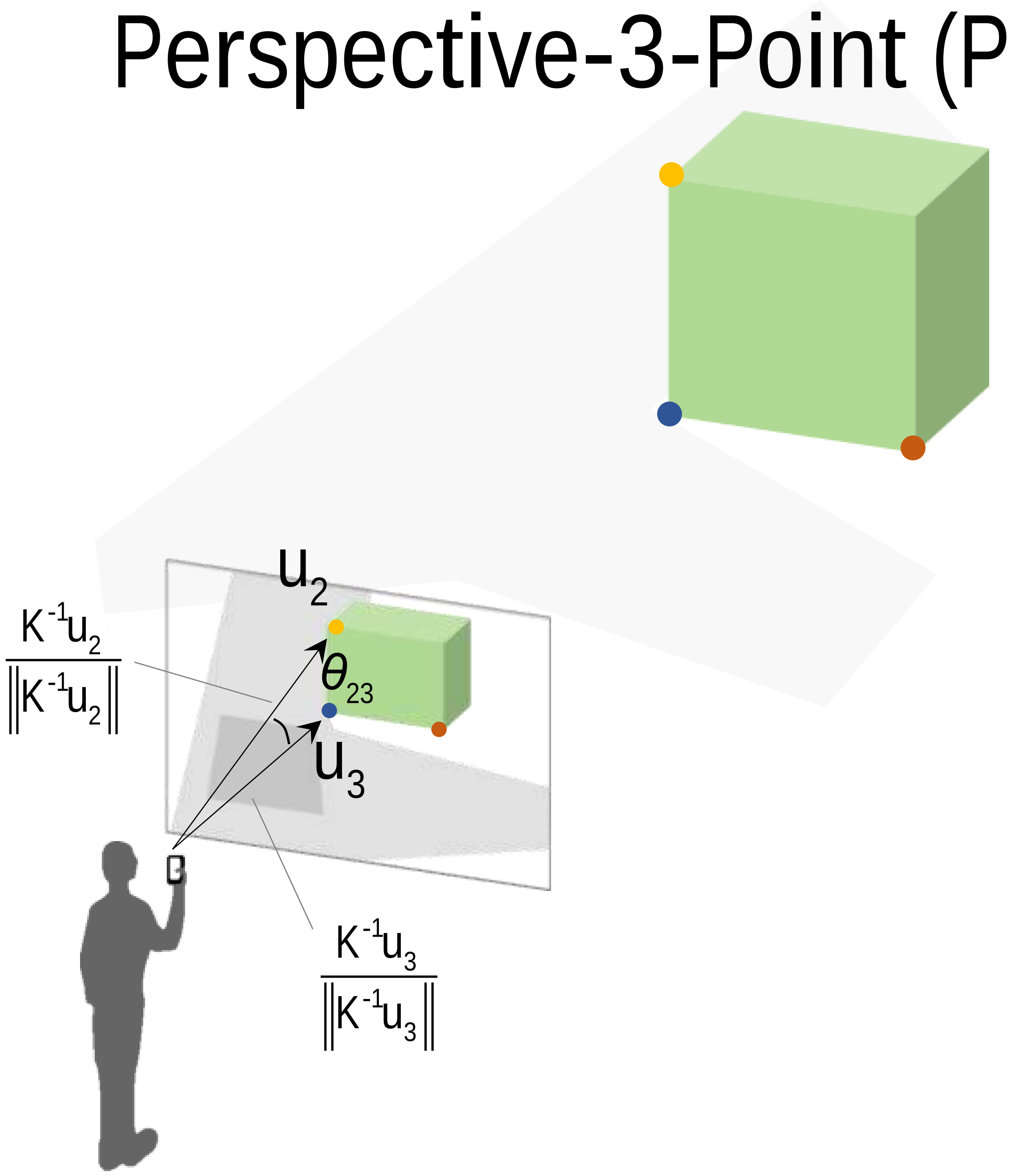
2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3 d_1 \cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2 d_3 \cos\theta_{23} = p_{23}^2$$

3 equations

Unknowns: d_1, d_2, d_3

Perspective-3-Point (P3P)



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3 d_1 \cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2 d_3 \cos\theta_{23} = p_{23}^2$$

3 equations

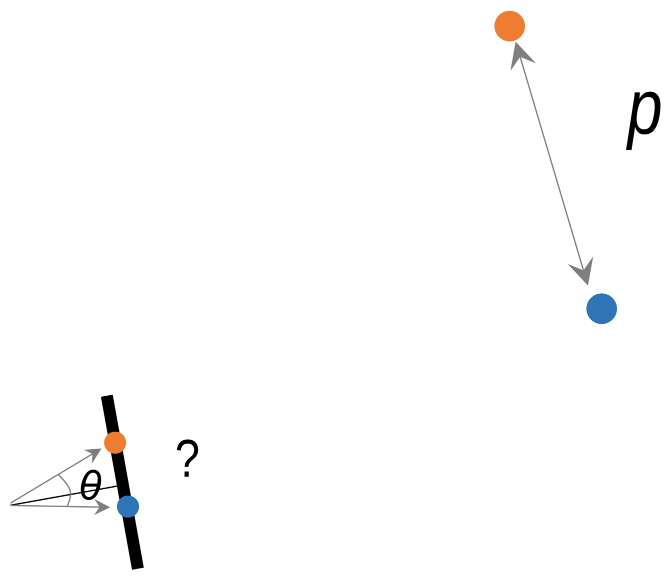
Unknowns: d_1, d_2, d_3

Note:

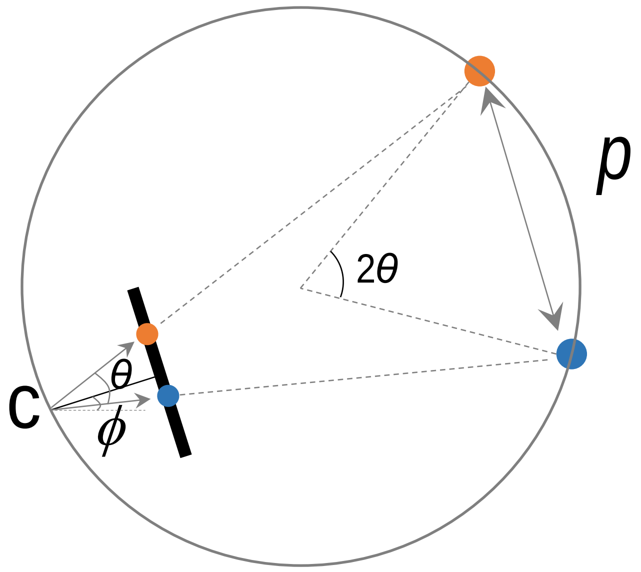
$$\cos\theta_{12} = \frac{(K^{-1}u_1)^T (K^{-1}u_2)}{\|K^{-1}u_1\|\|K^{-1}u_2\|} \qquad \cos\theta_{23} = \frac{(K^{-1}u_2)^T (K^{-1}u_3)}{\|K^{-1}u_2\|\|K^{-1}u_3\|}$$

$$\cos\theta_{31} = \frac{(K^{-1}u_1)^T (K^{-1}u_3)}{\|K^{-1}u_1\|\|K^{-1}u_3\|}$$

Geometric Interpretation: 1D Camera

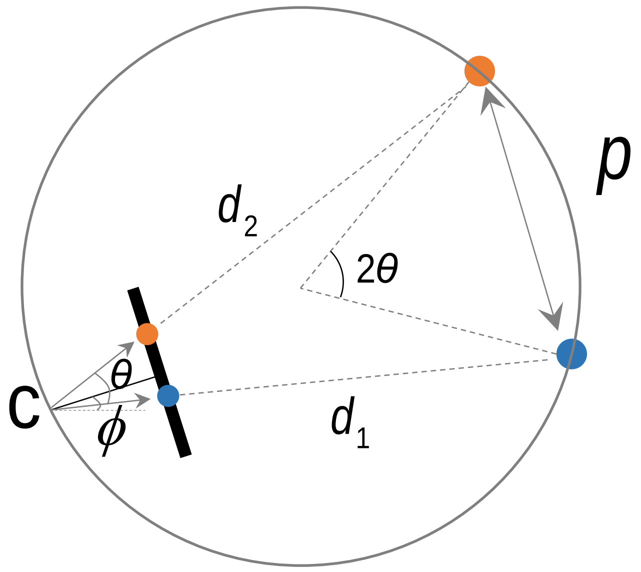


Geometric Interpretation: 1D Camera



Property of inscribed angle

Geometric Interpretation: 1D Camera

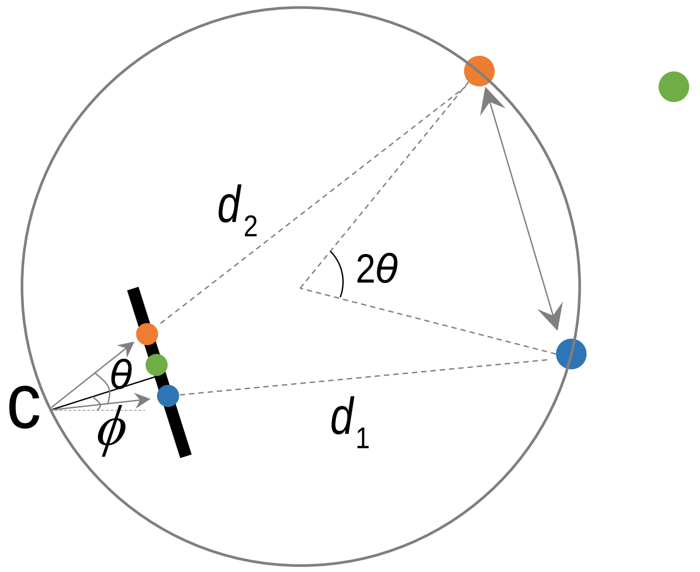


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta = p^2$$

Infinite number of solutions

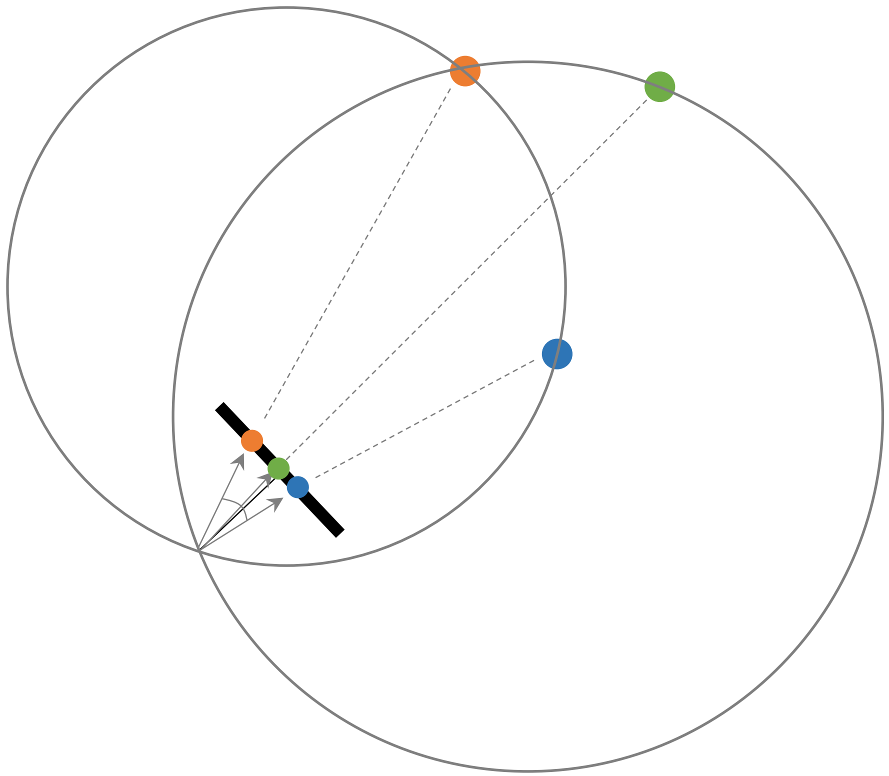
Geometric Interpretation: 1D Camera



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta = p^2$$

Geometric Interpretation: 1D Camera

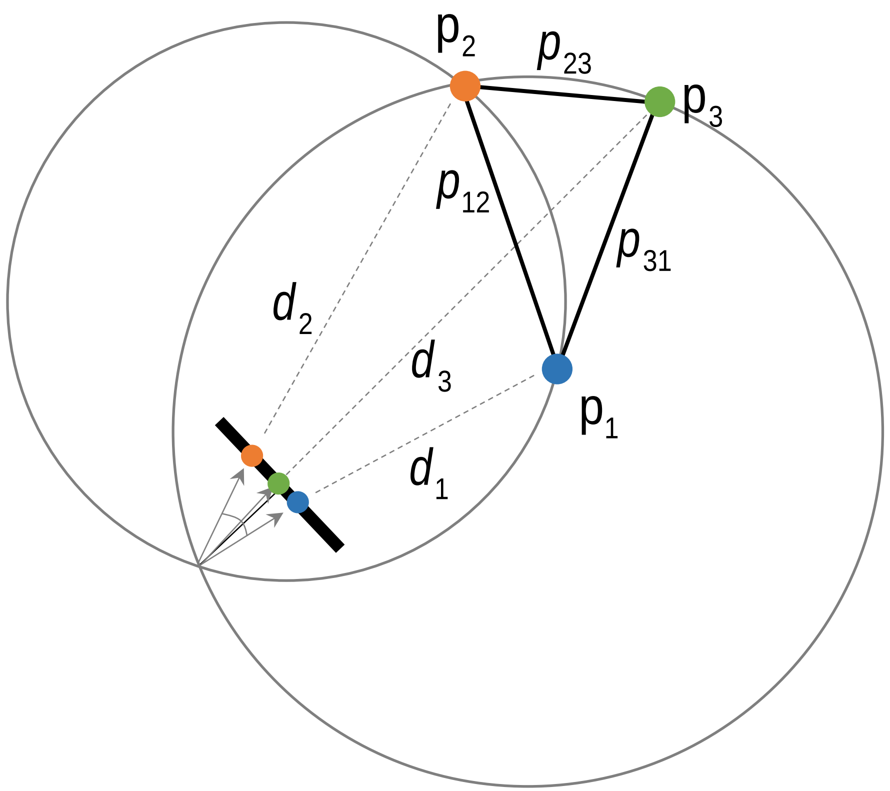


Finite number of solutions

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta = p^2$$

Geometric Interpretation: 1D Camera

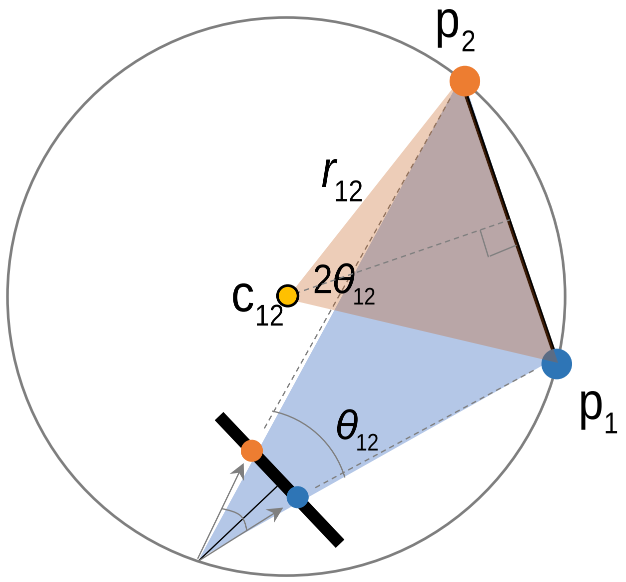


Finite number of solutions

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3 d_1 \cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2 d_3 \cos\theta_{23} = p_{23}^2$$

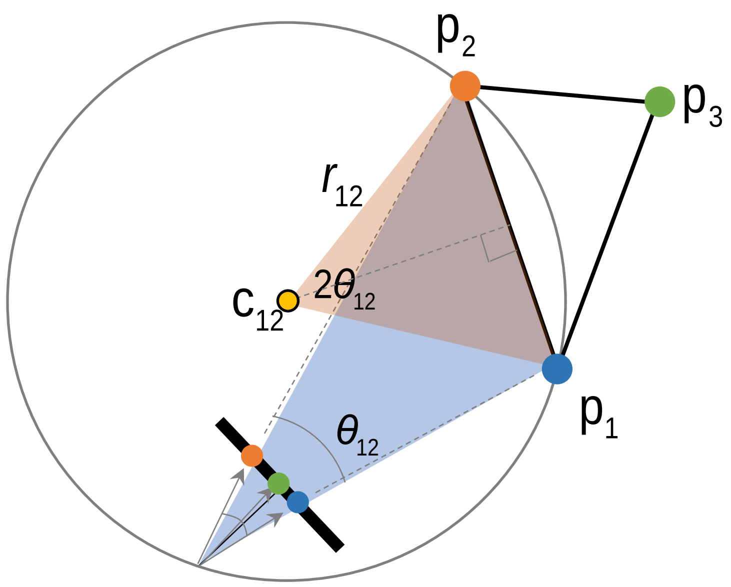
Geometric Interpretation: 1D Camera



$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

where $$r_{12} = \frac{\| p_2 - p_1 \|}{2 \sin\theta_{12}}$$

Geometric Interpretation: 1D Camera

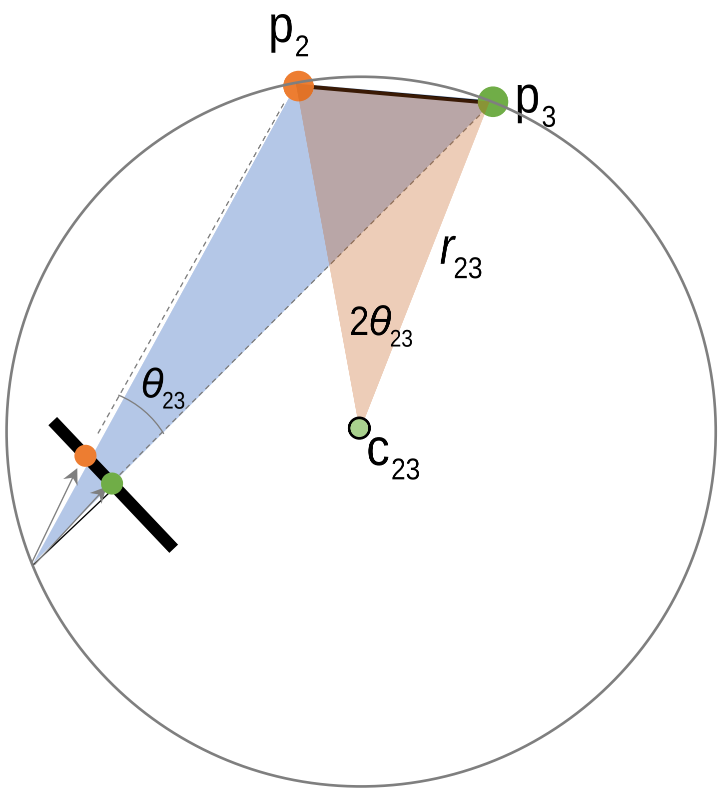


$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12}$$

$$u_{12} \perp p_2 - p_1$$

where $r_{12} = \frac{\lVert p_2 - p_1 \rVert}{2\sin\theta_{12}}$

Geometric Interpretation: 1D Camera



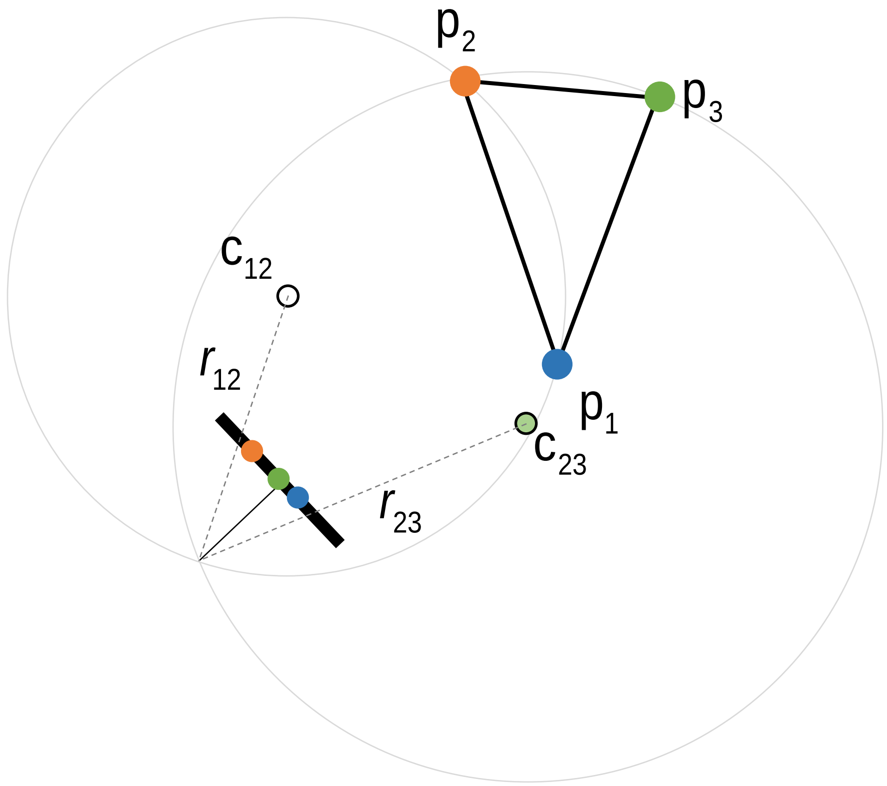
$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12}\cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

where $r_{12} = \dfrac{\|p_2 - p_1\|}{2\sin\theta_{12}}$

$$c_{23} = \frac{p_3 + p_2}{2} \pm r_{23}\cos\theta_{23} u_{23} \qquad u_{23} \perp p_3 - p_2$$

where $r_{23} = \dfrac{\|p_3 - p_2\|}{2\sin\theta_{23}}$

Geometric Interpretation: 1D Camera



$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

$$\text{where } r_{12} = \frac{\|p_2 - p_1\|}{2\sin\theta_{12}}$$

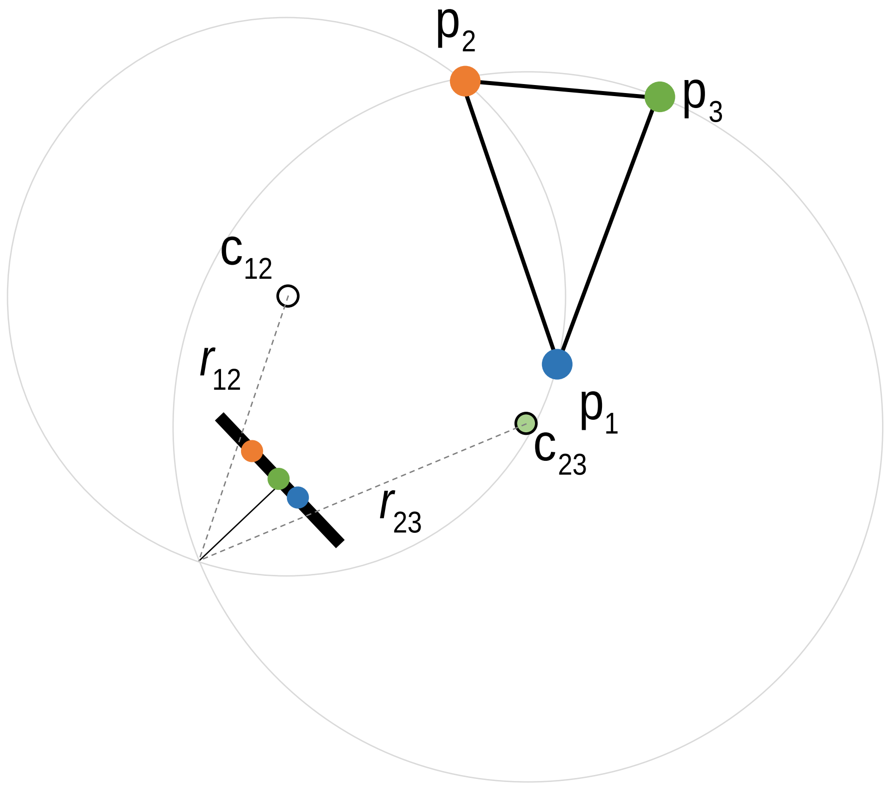
$$c_{23} = \frac{p_3 + p_2}{2} \pm r_{23} \cos\theta_{23} u_{23} \qquad u_{23} \perp p_3 - p_2$$

$$\text{where } r_{23} = \frac{\|p_3 - p_2\|}{2\sin\theta_{23}}$$

$$\|x - c_{12}\|^2 = r_{12}^2 \qquad \|x - c_{23}\|^2 = r_{23}^2$$

HW: Drive x and orientation.

Geometric Interpretation: Family of Solutions



$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

$$\text{where } r_{12} = \frac{\|p_2 - p_1\|}{2\sin\theta_{12}}$$

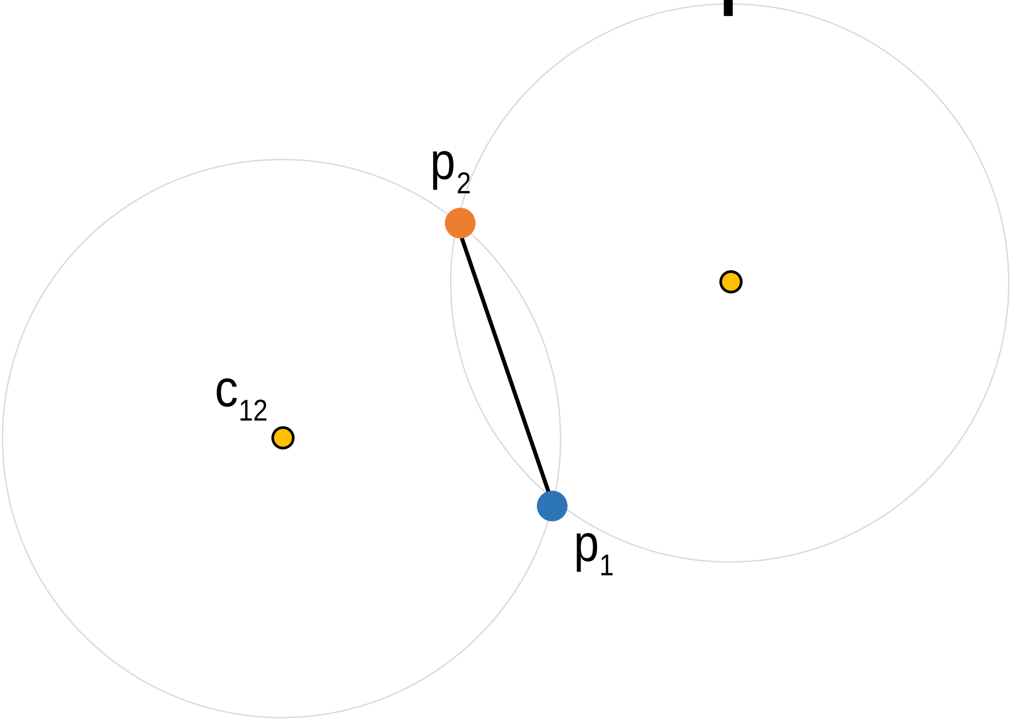
$$c_{23} = \frac{p_3 + p_2}{2} \pm r_{23} \cos\theta_{23} u_{23} \qquad u_{23} \perp p_3 - p_2$$

$$\text{where } r_{23} = \frac{\|p_3 - p_2\|}{2\sin\theta_{23}}$$

$$\|x - c_{12}\|^2 = r_{12}^2 \qquad \|x - c_{23}\|^2 = r_{23}^2$$

HW: Drive x and orientation.

Geometric Interpretation: Family of Solutions

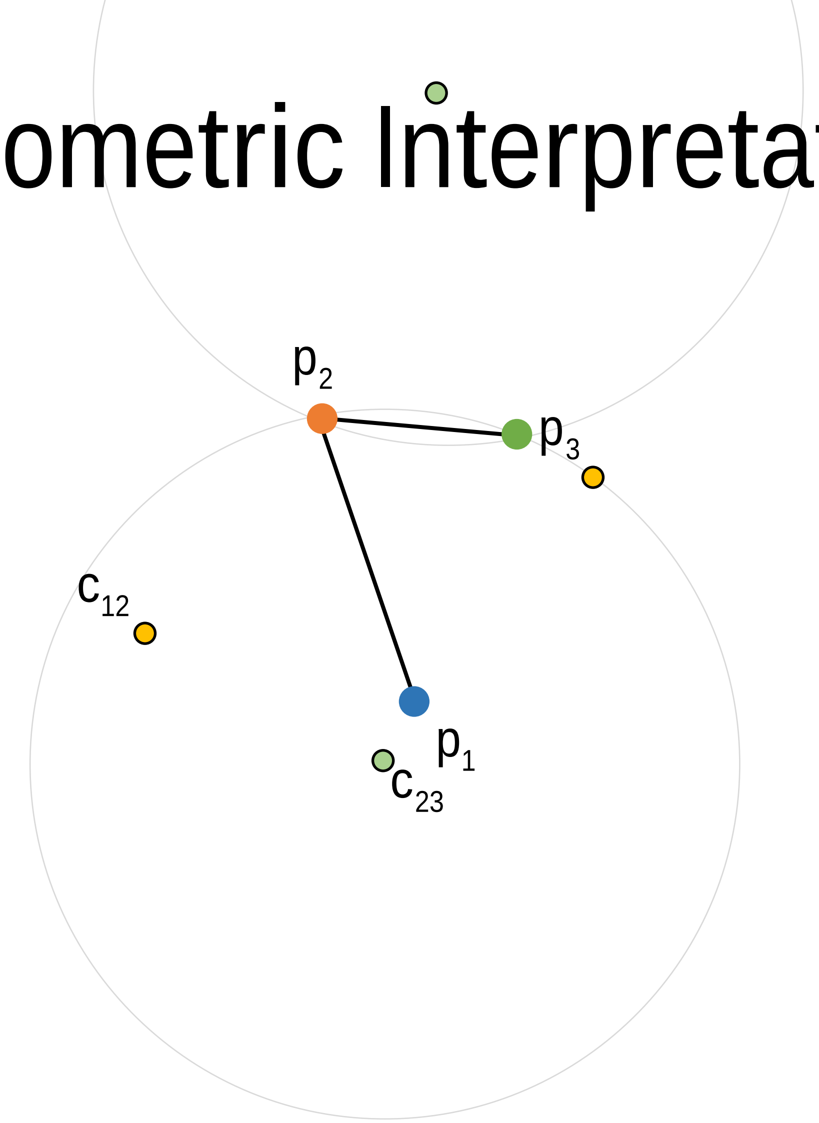


$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12}$$

$$u_{12} \perp p_2 - p_1$$

where $r_{12} = \dfrac{\|p_2 - p_1\|}{2\sin\theta_{12}}$

Geometric Interpretation: Family of Solutions



4 combinations

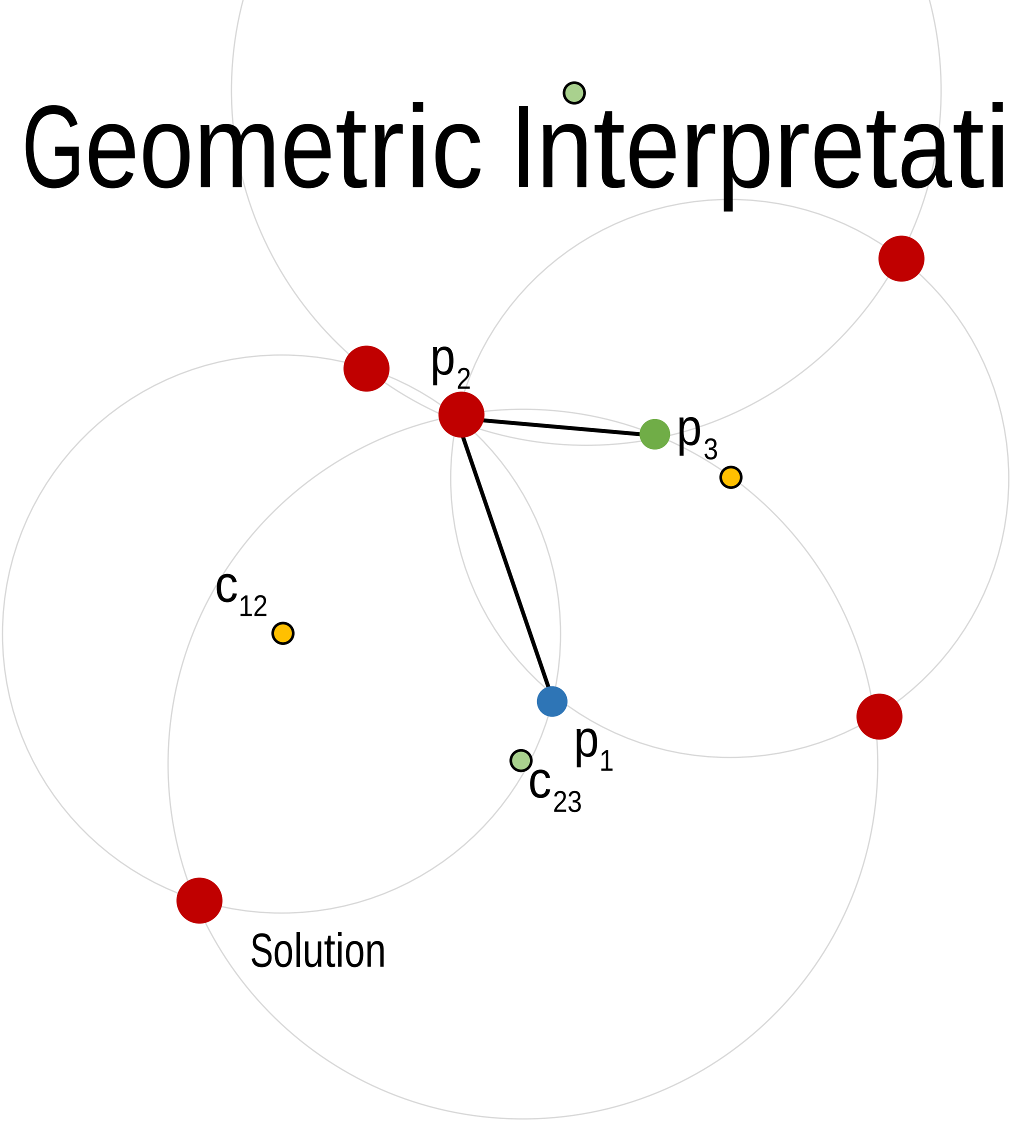
$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

$$\text{where } \; r_{12} = \frac{\|p_2 - p_1\|}{2\sin\theta_{12}}$$

$$c_{23} = \frac{p_3 + p_2}{2} \pm r_{23} \cos\theta_{23} u_{23} \qquad u_{23} \perp p_3 - p_2$$

$$\text{where } \; r_{23} = \frac{\|p_3 - p_2\|}{2\sin\theta_{23}}$$

Geometric Interpretation: Family of Solutions



$$c_{12} = \frac{p_1 + p_2}{2} \pm r_{12} \cos\theta_{12} u_{12} \qquad u_{12} \perp p_2 - p_1$$

$$\text{where} \quad r_{12} = \frac{\|p_2 - p_1\|}{2\sin\theta_{12}}$$

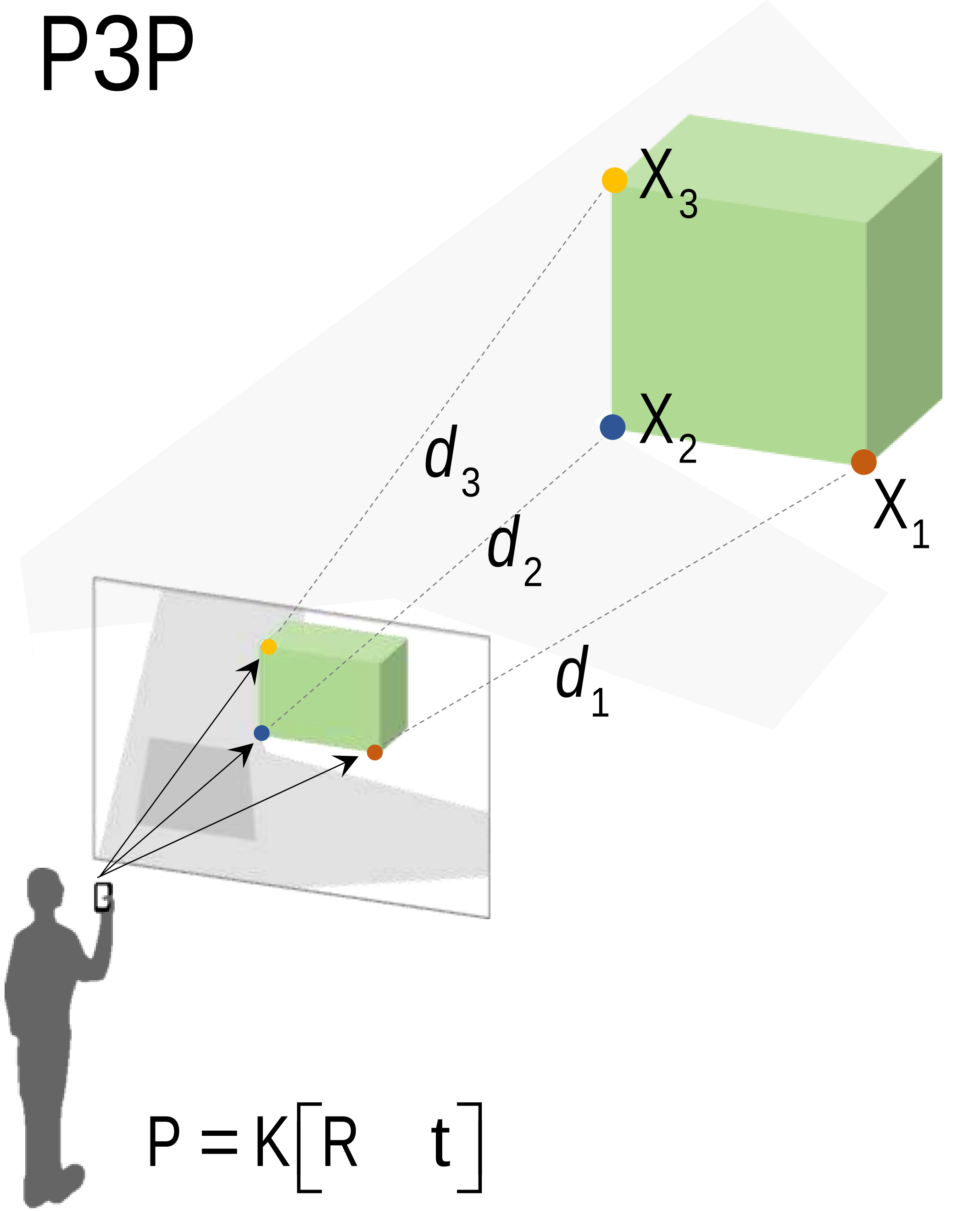
$$c_{23} = \frac{p_3 + p_2}{2} \pm r_{23} \cos\theta_{23} u_{23} \qquad u_{23} \perp p_3 - p_2$$

$$\text{where} \quad r_{23} = \frac{\|p_3 - p_2\|}{2\sin\theta_{23}}$$

4 combinations of circle centers

→ 4 solutions except for p_2 (p_2 is counted four times.).

P3P



$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2 d_1 d_2 \cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2 d_3 d_1 \cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2 d_1 d_2 \cos\theta_{23} = p_{23}^2$$

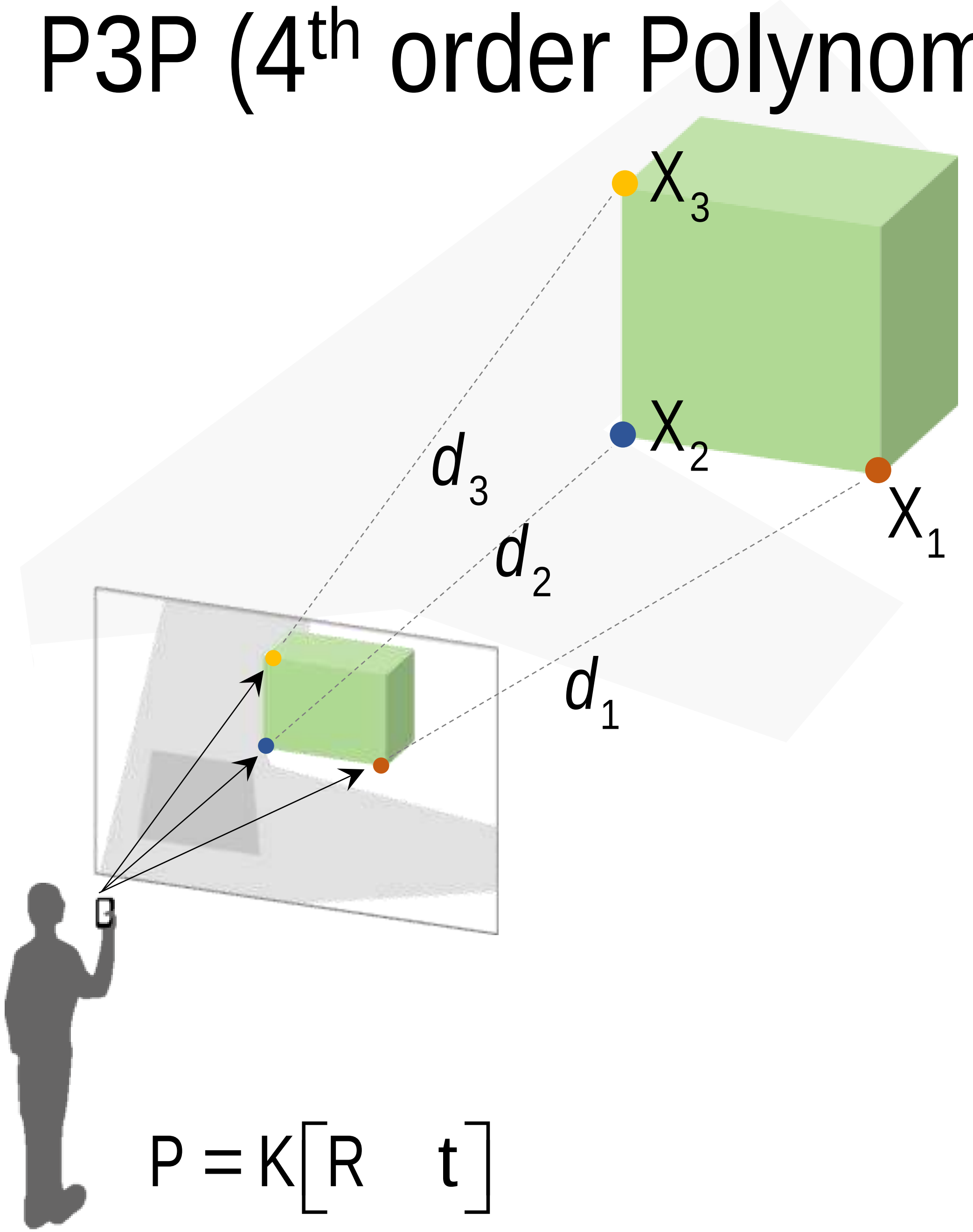
3 equations

The number of possible solutions: 8 = 2x2x2

$d_1 > 0$ $d_2 > 0$ $d_3 > 0$: 4 = 2x2x2/2

→ requires additional fourth point to verify the solution.

P3P (4th order Polynomial)



$P = K\begin{bmatrix} R & t \end{bmatrix}$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

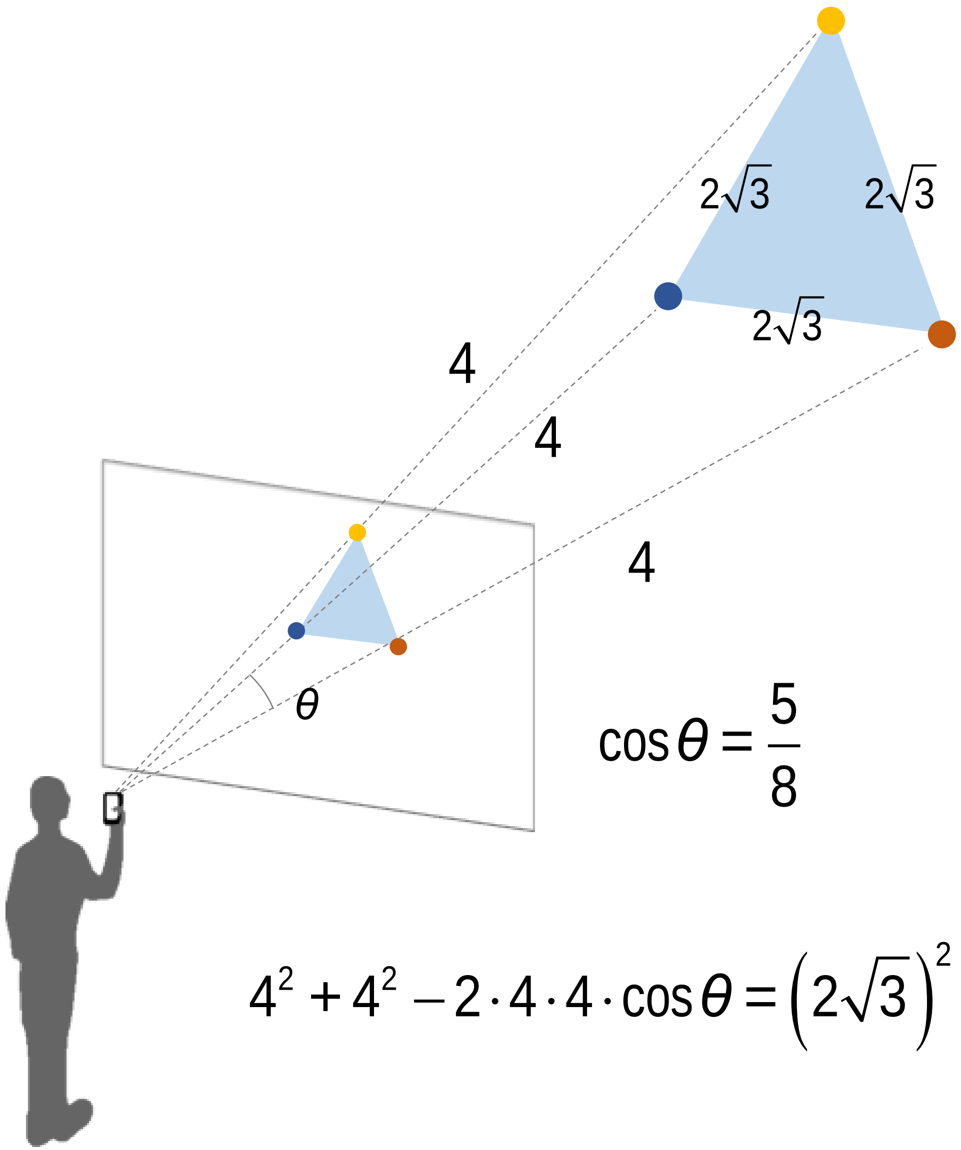
3 equations

4th order polynomial:

$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

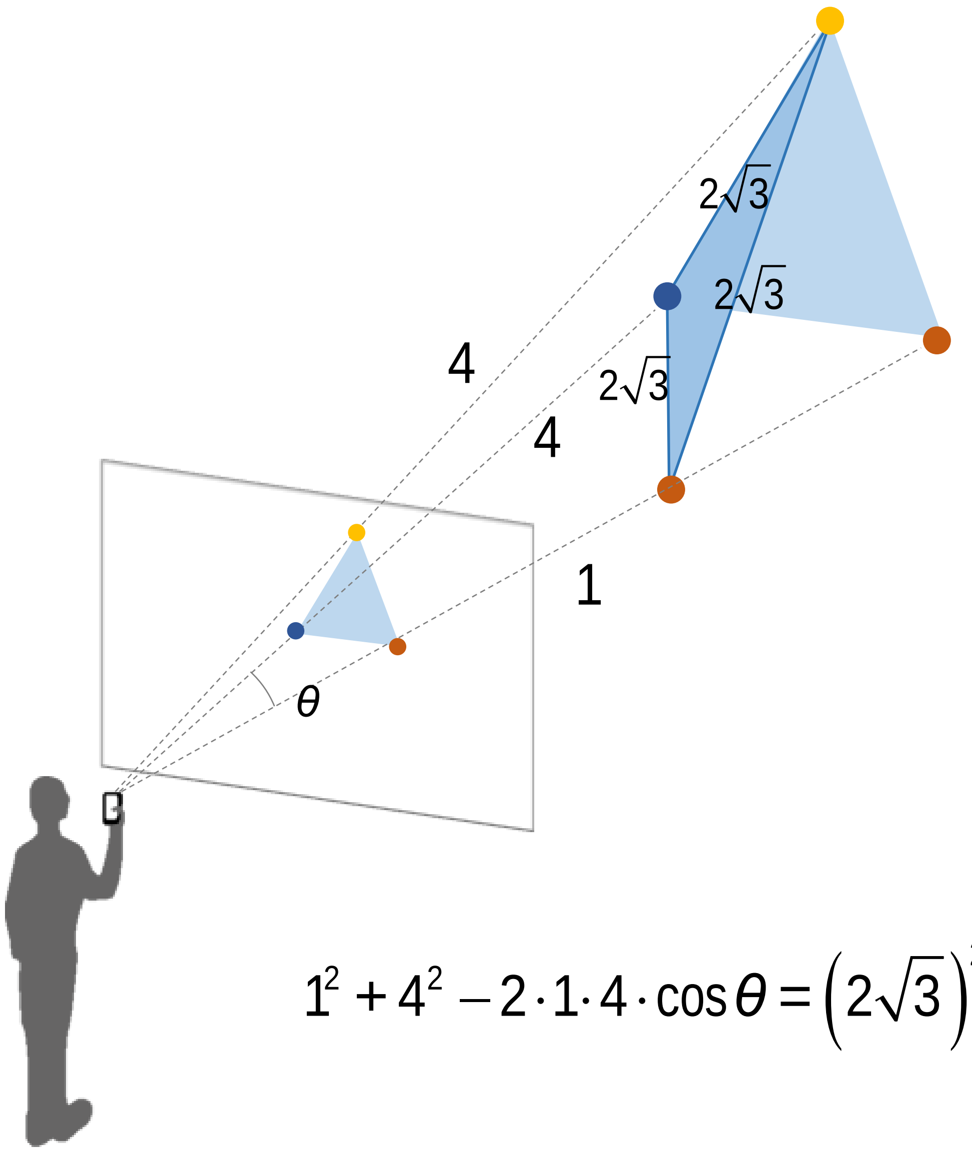
Closed form solutions exist.

Four Solution Example



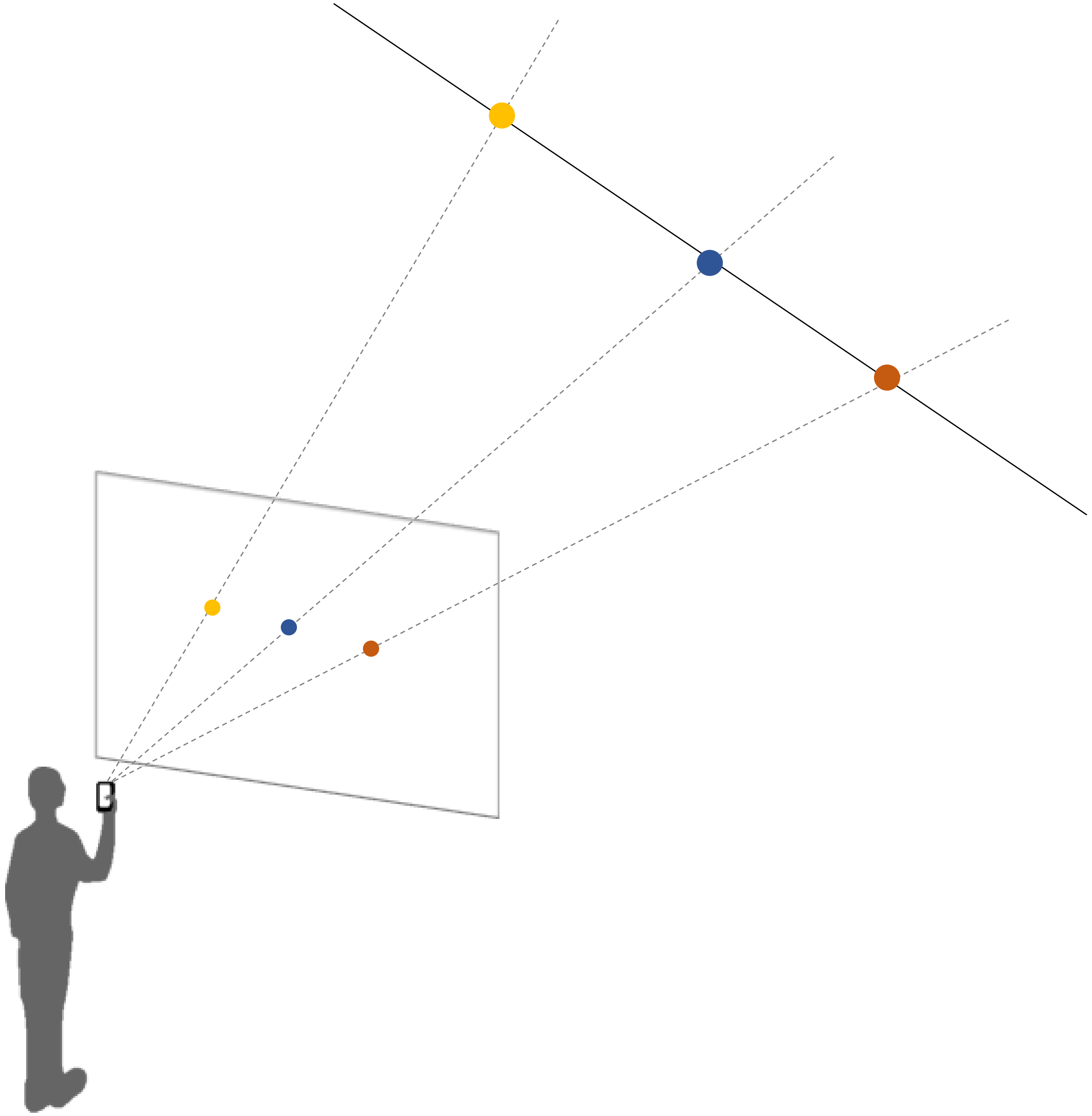
$$\cos\theta = \frac{5}{8}$$

$$4^2 + 4^2 - 2\cdot 4\cdot 4\cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

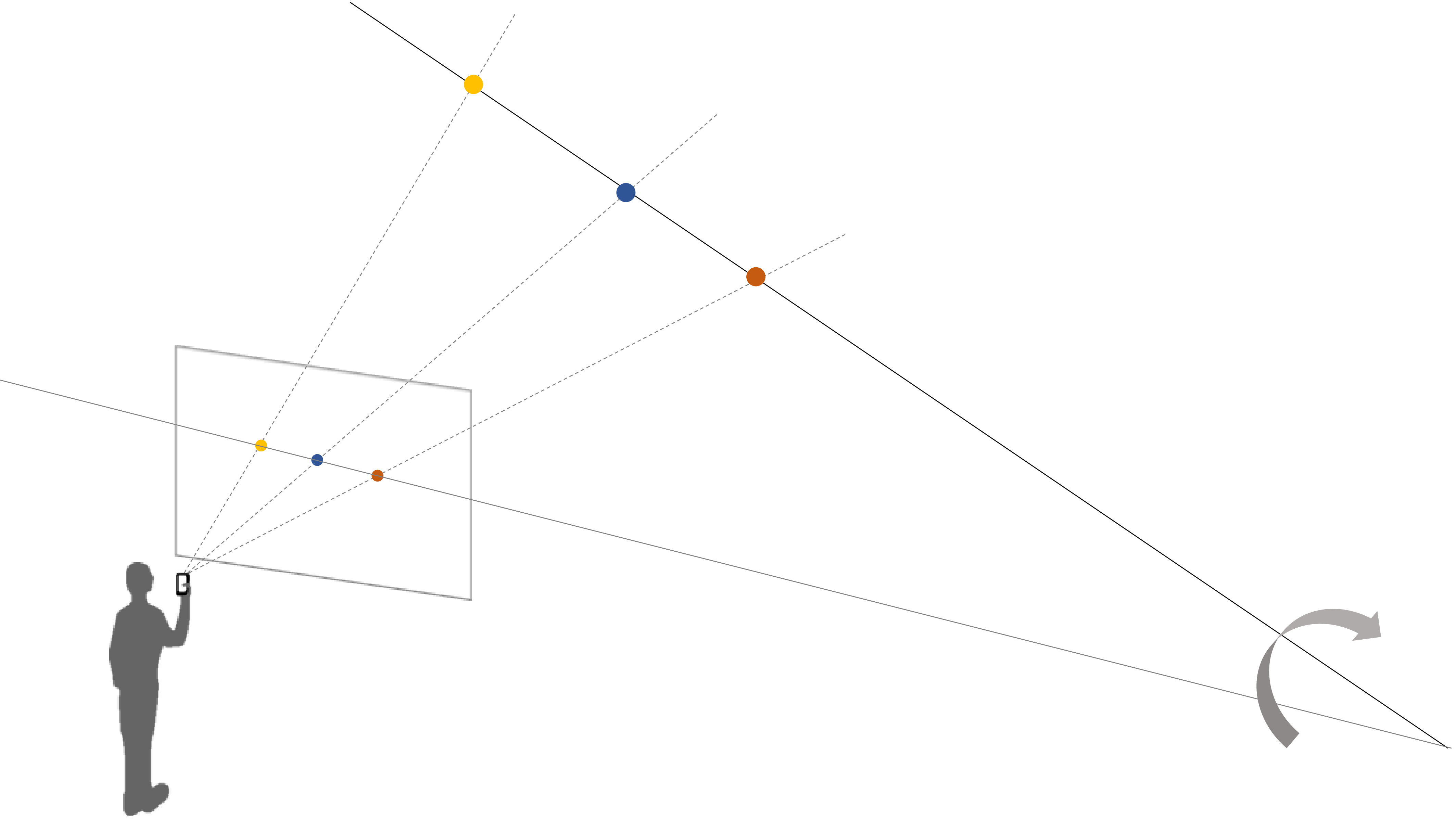


$$1^2 + 4^2 - 2\cdot 1\cdot 4\cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

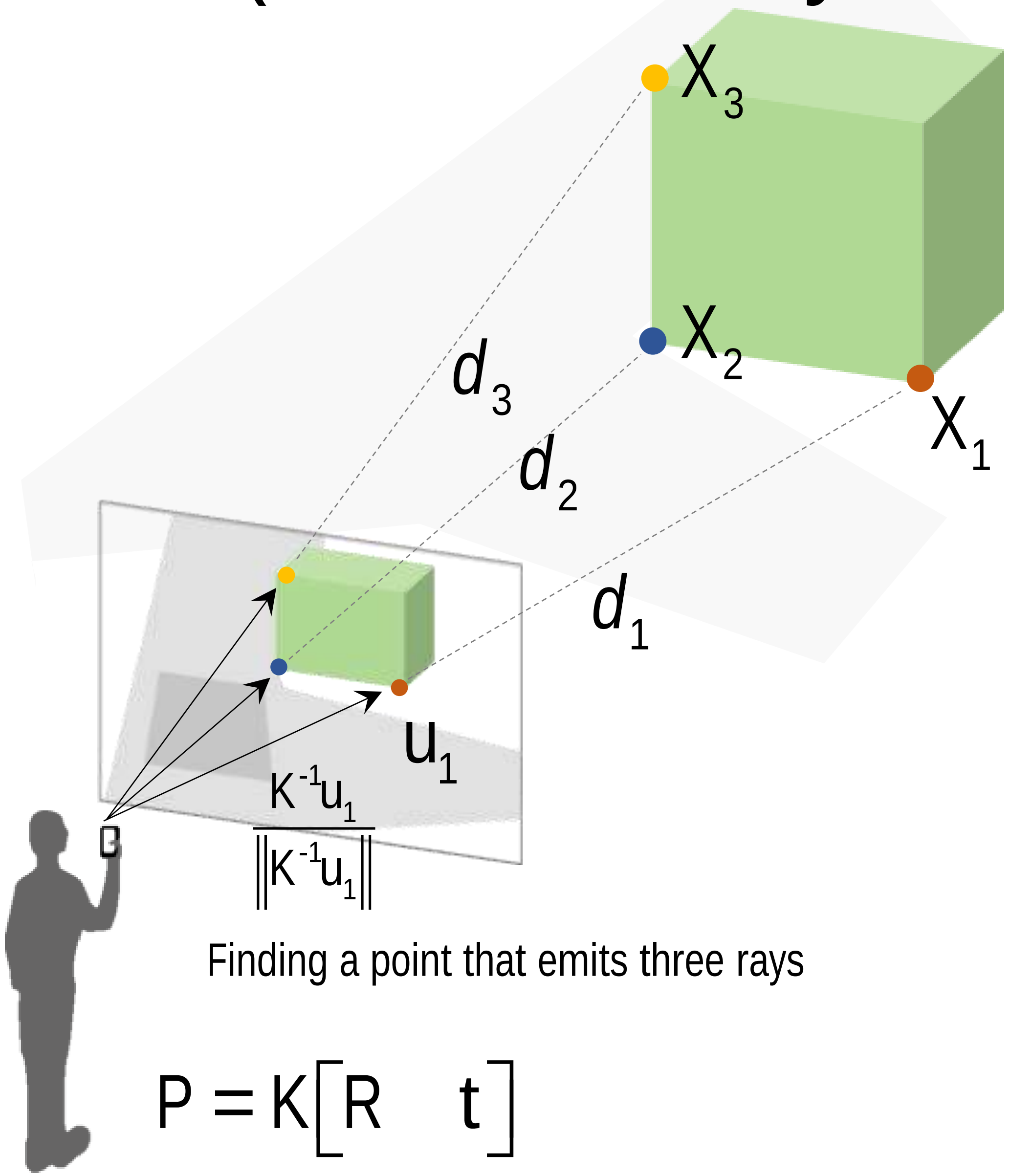
Ambiguity



Ambiguity (Colinear points)



P3P (4^{th} order Polynomial)



Finding a point that emits three rays

$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

2^{nd} Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

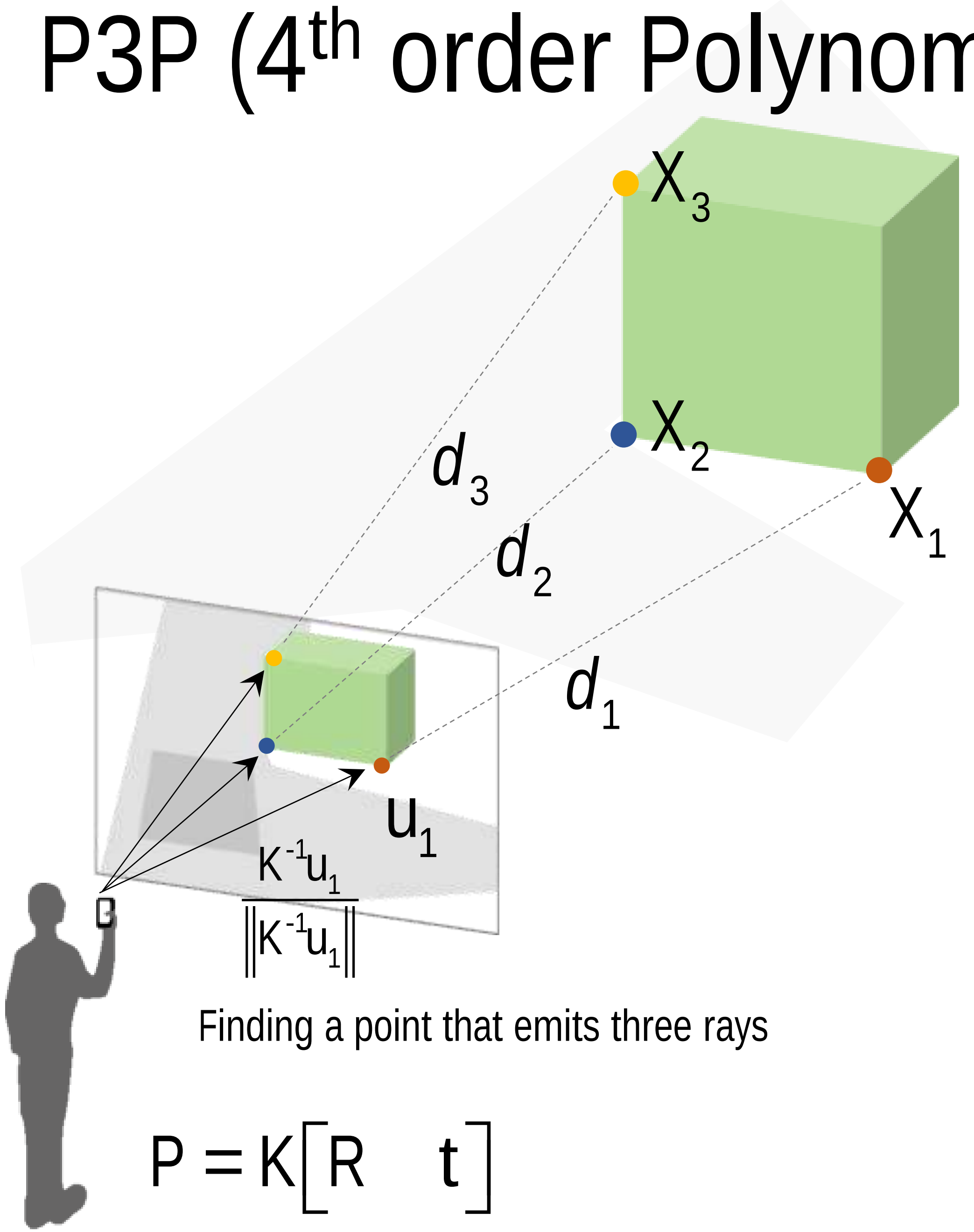
4^{th} order polynomial:

→ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

→ Compute t using X_1, X_2, X_3, d_1, d_2, and d_3.

P3P (4th order Polynomial)



Finding a point that emits three rays

$$P = K\begin{bmatrix} R & t \end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

4th order polynomial:

→ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

→ Compute t using X_1, X_2, X_3, d_1, d_2, and d_3.

→ $\begin{bmatrix} \tilde{X}_1 & \tilde{X}_2 & \tilde{X}_3 \end{bmatrix} = R\begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}$

Rotation matrix computation

where $\tilde{X}_1 = d_1 \frac{K^{-1}u_1}{\|K^{-1}u_1\|}$ $\tilde{X}_2 = d_2 \frac{K^{-1}u_2}{\|K^{-1}u_2\|}$ $\tilde{X}_3 = d_3 \frac{K^{-1}u_3}{\|K^{-1}u_3\|}$