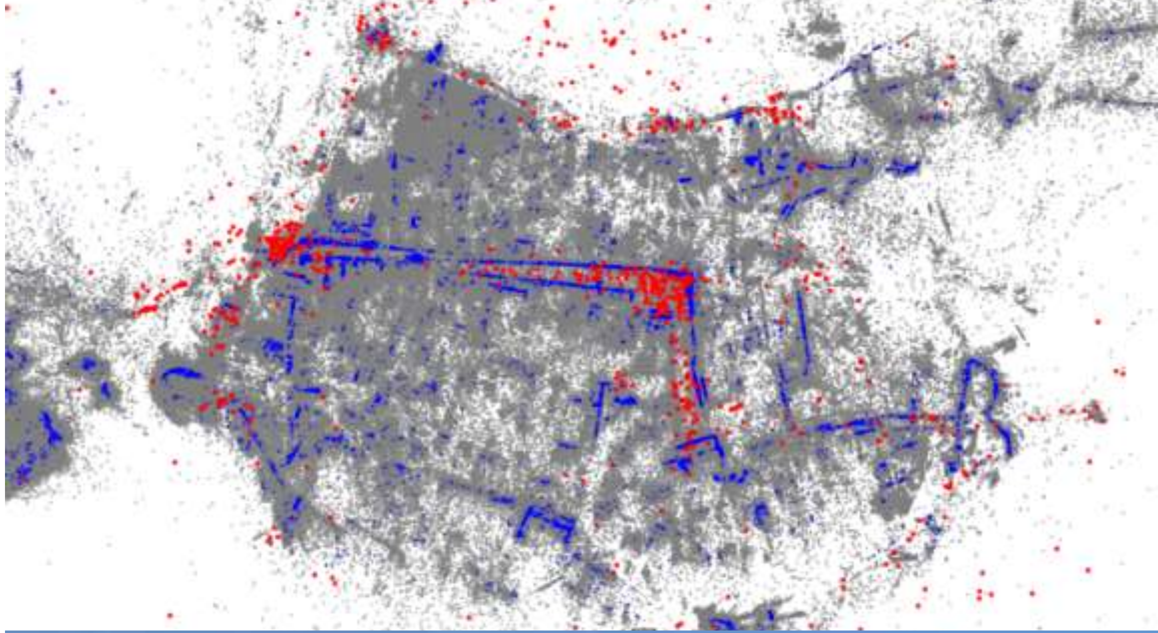
Structure from Motion: A Factorization Approach



Announcement

- HW #4 and 5 are graded.

- HW #6 is out
  - Two deadlines: May 3 (Problem 2,3,4,5) and May 10 (Problem 6 with full reconstruction)
  - 5980 students: < 5 image reconstruction
  - 8980 students: <10 image reconstruction

- Course Evaluation Today

Longuet-Higgins: Epipolar Geometry

A computer algorithm for reconstructing a scene from two projections

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A simple algorithm for computing the three-dimensiona ture of a scene from a correlated pair of perspective proj is described here, when the spatial relationship between t projections is unknown. This problem is relevant not photographic surveying[1] but also to binocular vision[2], wh non-visual information available to the observer abo

Carlos Tomasi

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Shape and Motion from Image Streams under Orthography: a Factorization Method

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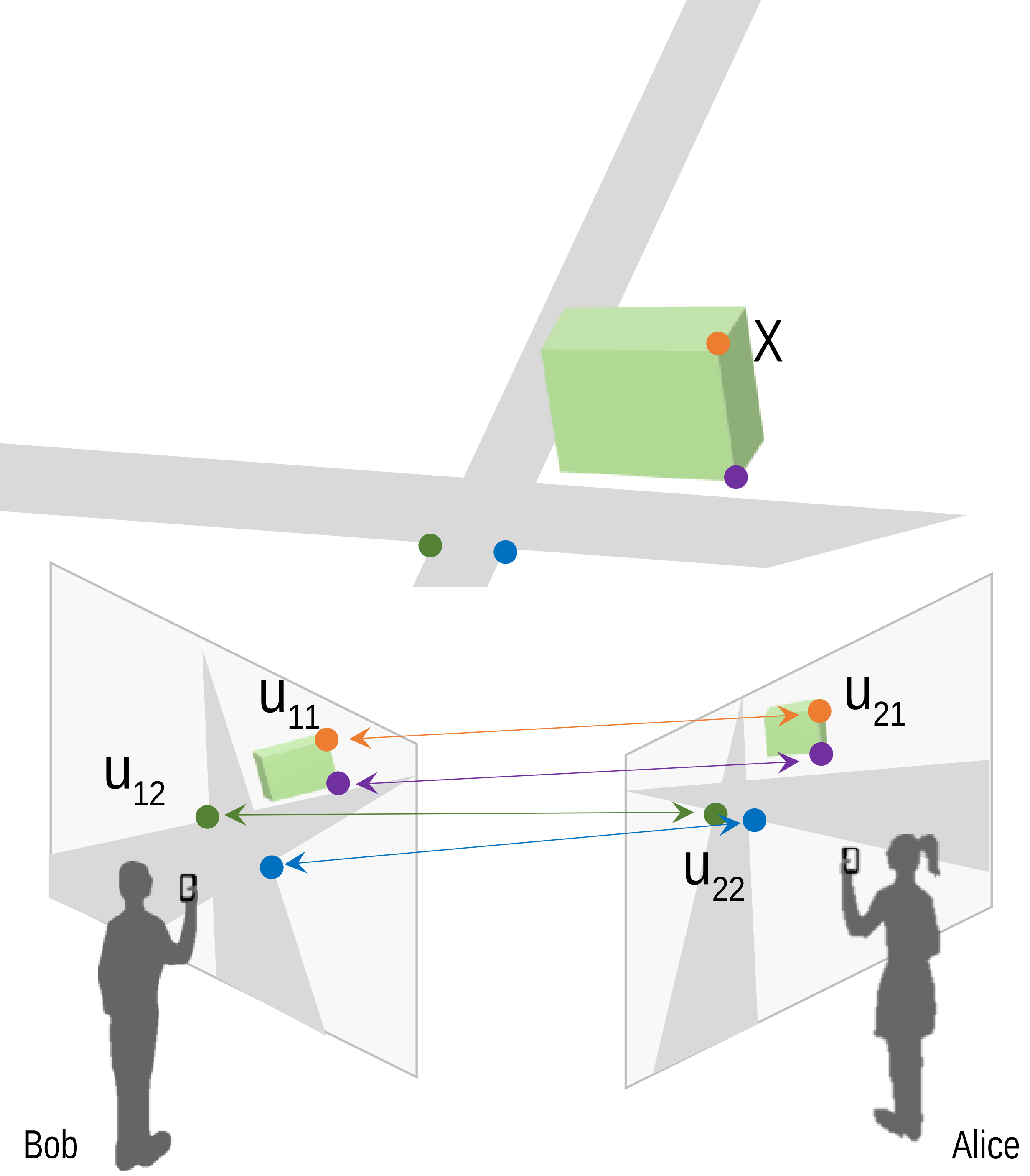
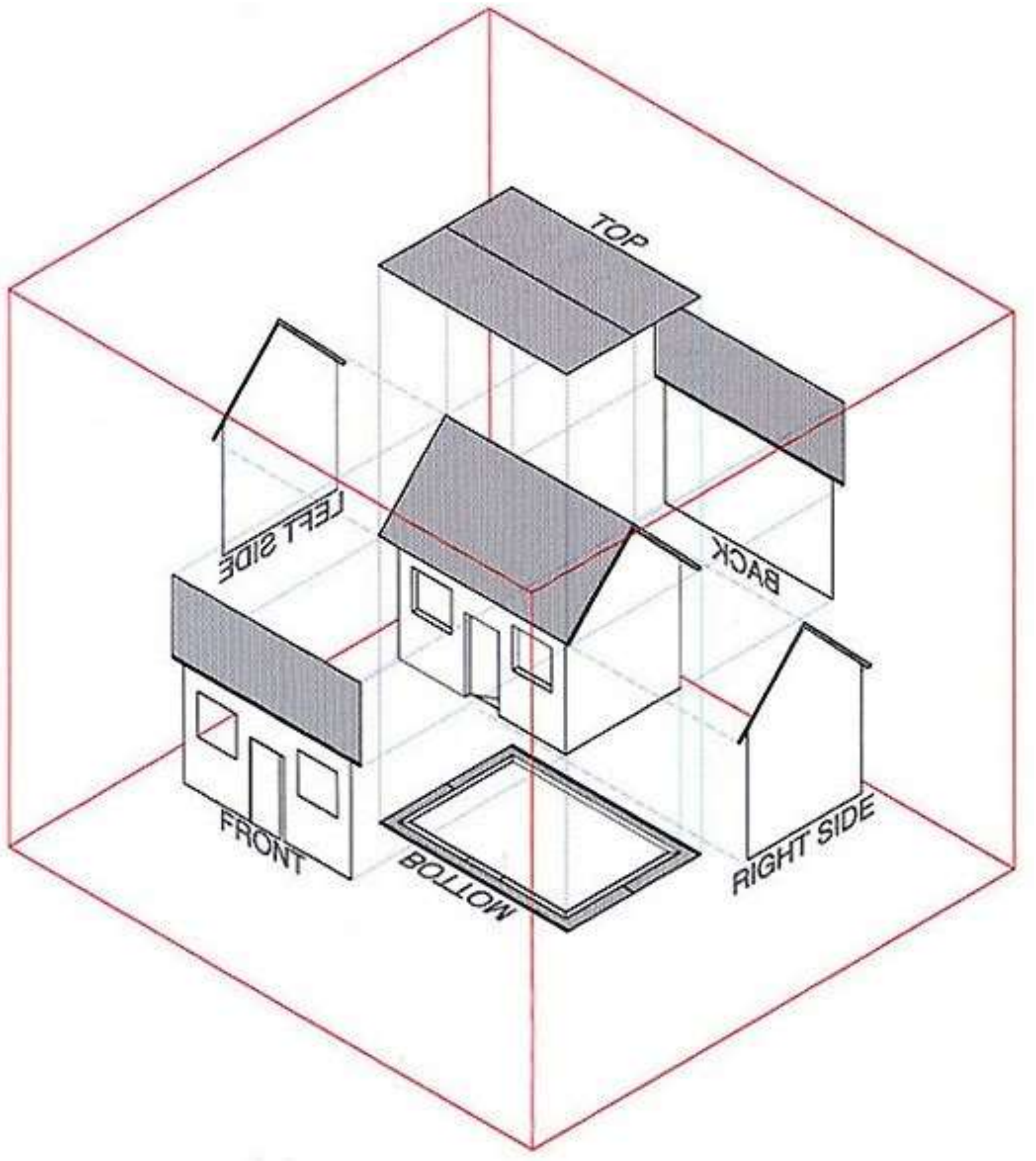
Tomasi-Kanade Factorization



$$\lambda_{11}u_{11} = R_1X_1 + t_1 \qquad \lambda_{21}u_{11} = R_2X_2 + t_2$$

$$\lambda_{12}u_{12} = R_1X_2 + t_1 \qquad \lambda_{22}u_{22} = R_2X_2 + t_2$$

Orthographic Camera



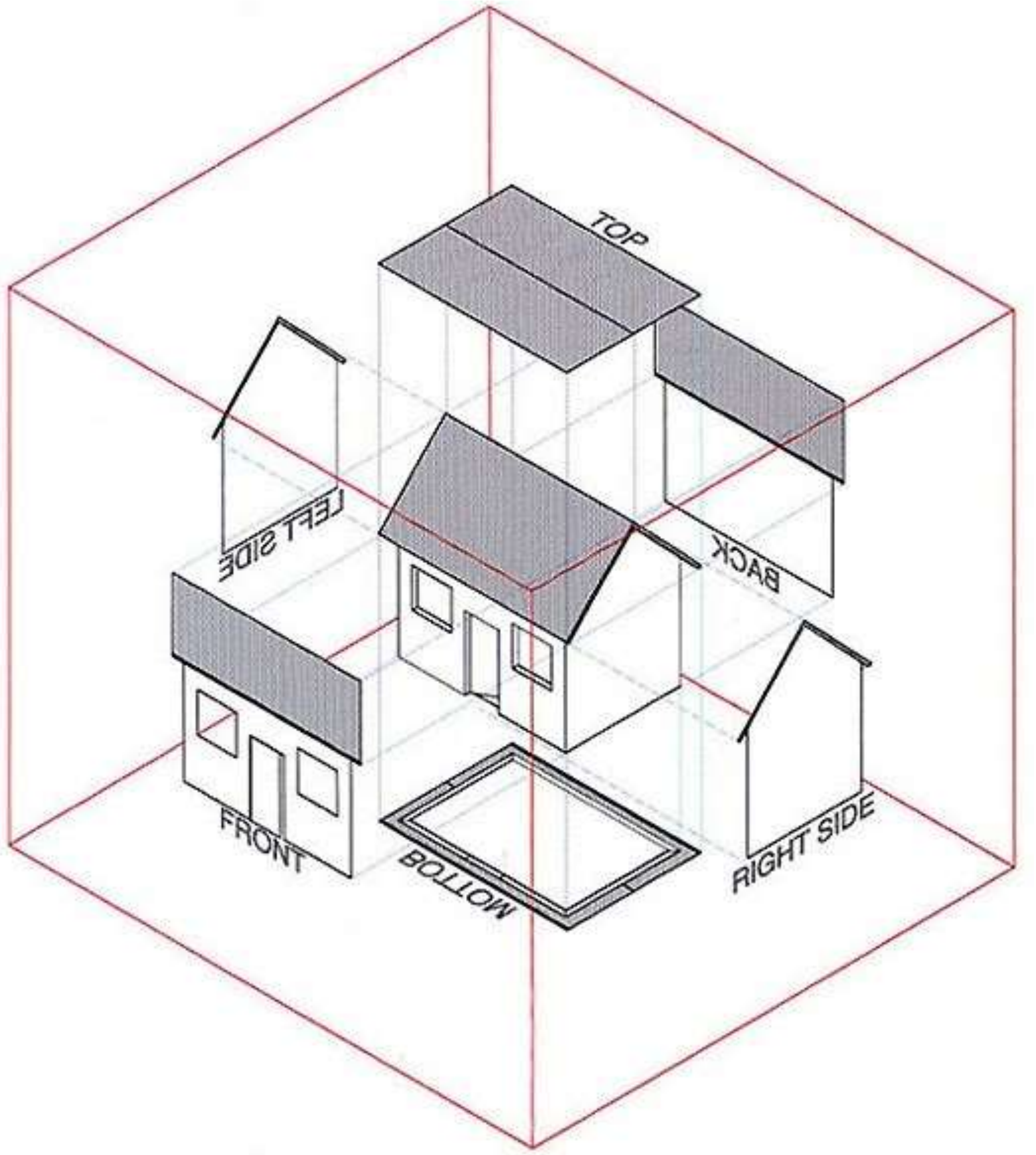
Affine camera:

$$P_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

Orthographic Camera



Affine camera:

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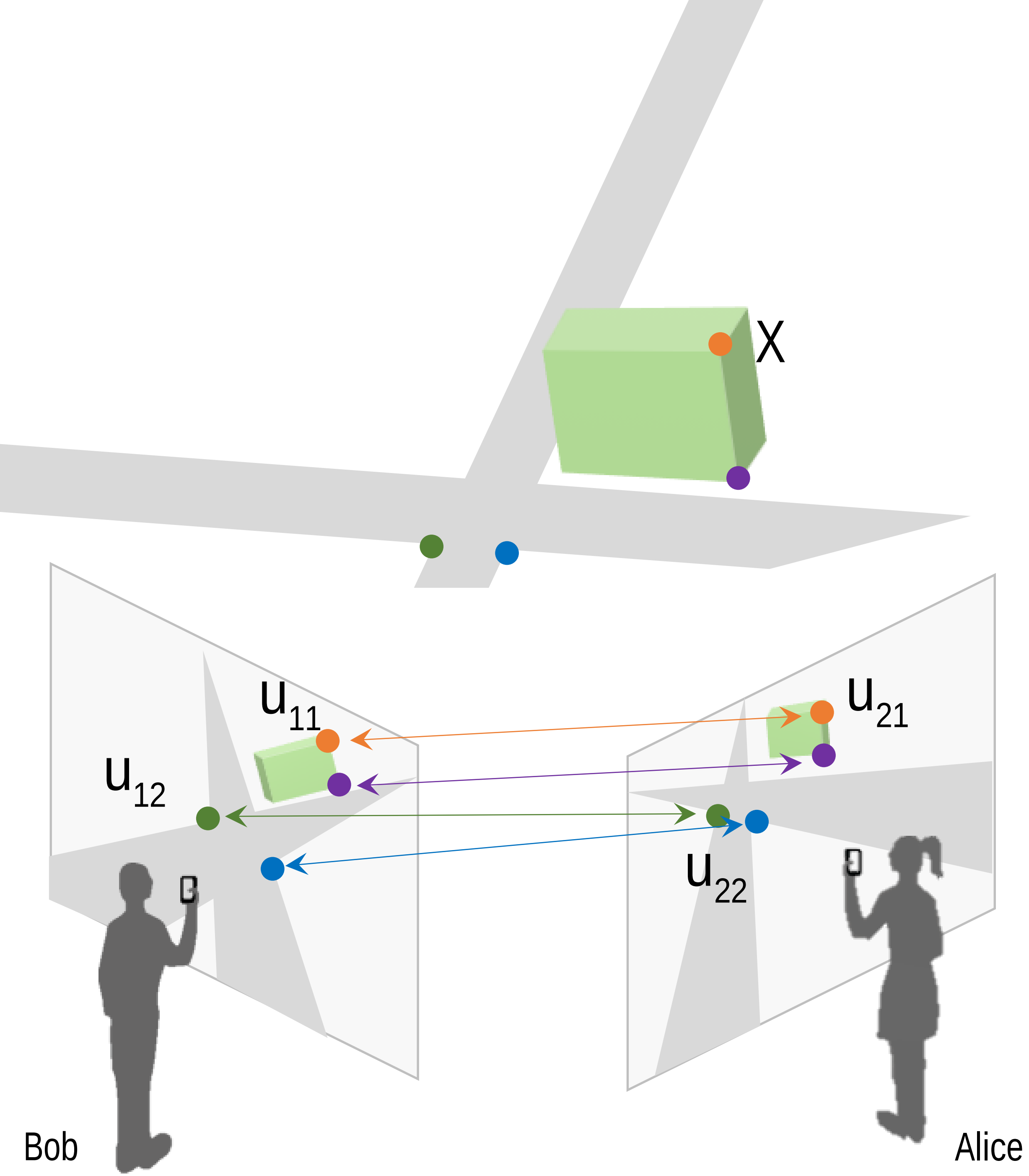
Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

$$P_O = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix}$$



$$\lambda_{11}u_{11} = R_1X_1 + t_1 \qquad \lambda_{21}u_{11} = R_2X_2 + t_2$$

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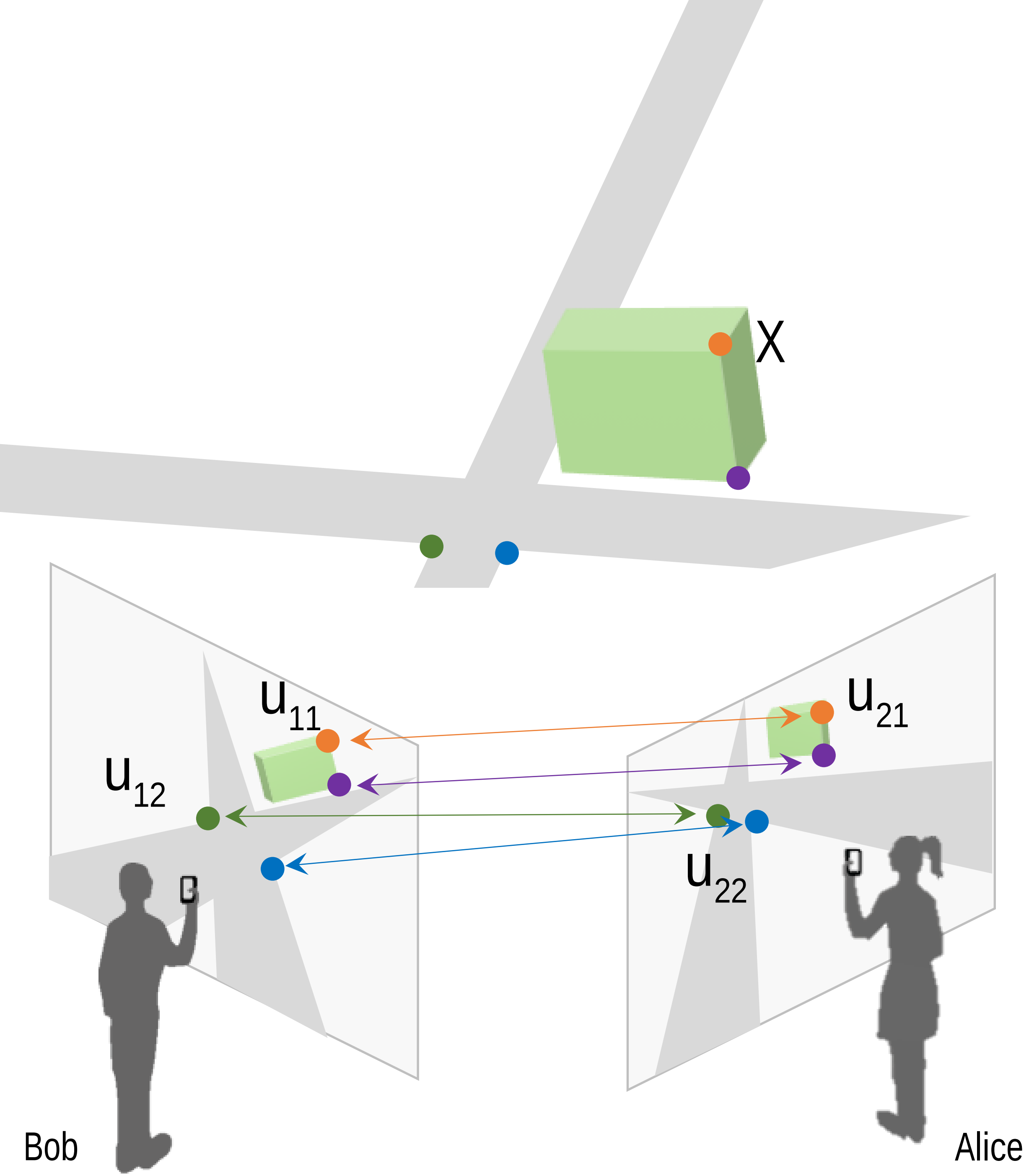
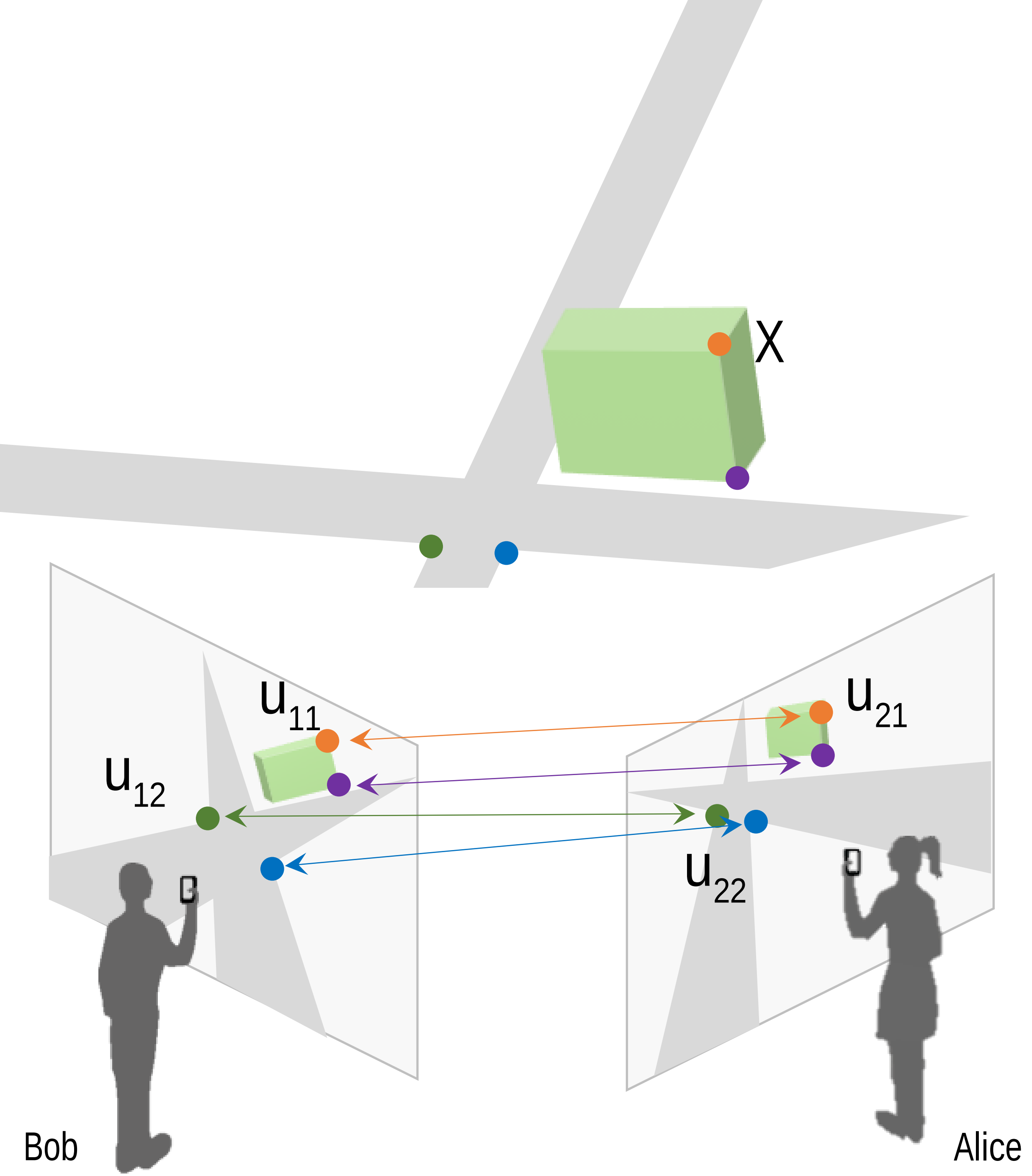


Orthographic projection:

$$u_{11} = \hat{R}_1 X_1 + \hat{t}_1 \qquad u_{11} = \hat{R}_2 X_2 + \hat{t}_2$$

$$u_{12} = \hat{R}_1 X_2 + \hat{t}_1 \qquad u_{22} = \hat{R}_2 X_2 + \hat{t}_2$$

where $\hat{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{bmatrix} \quad \hat{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$



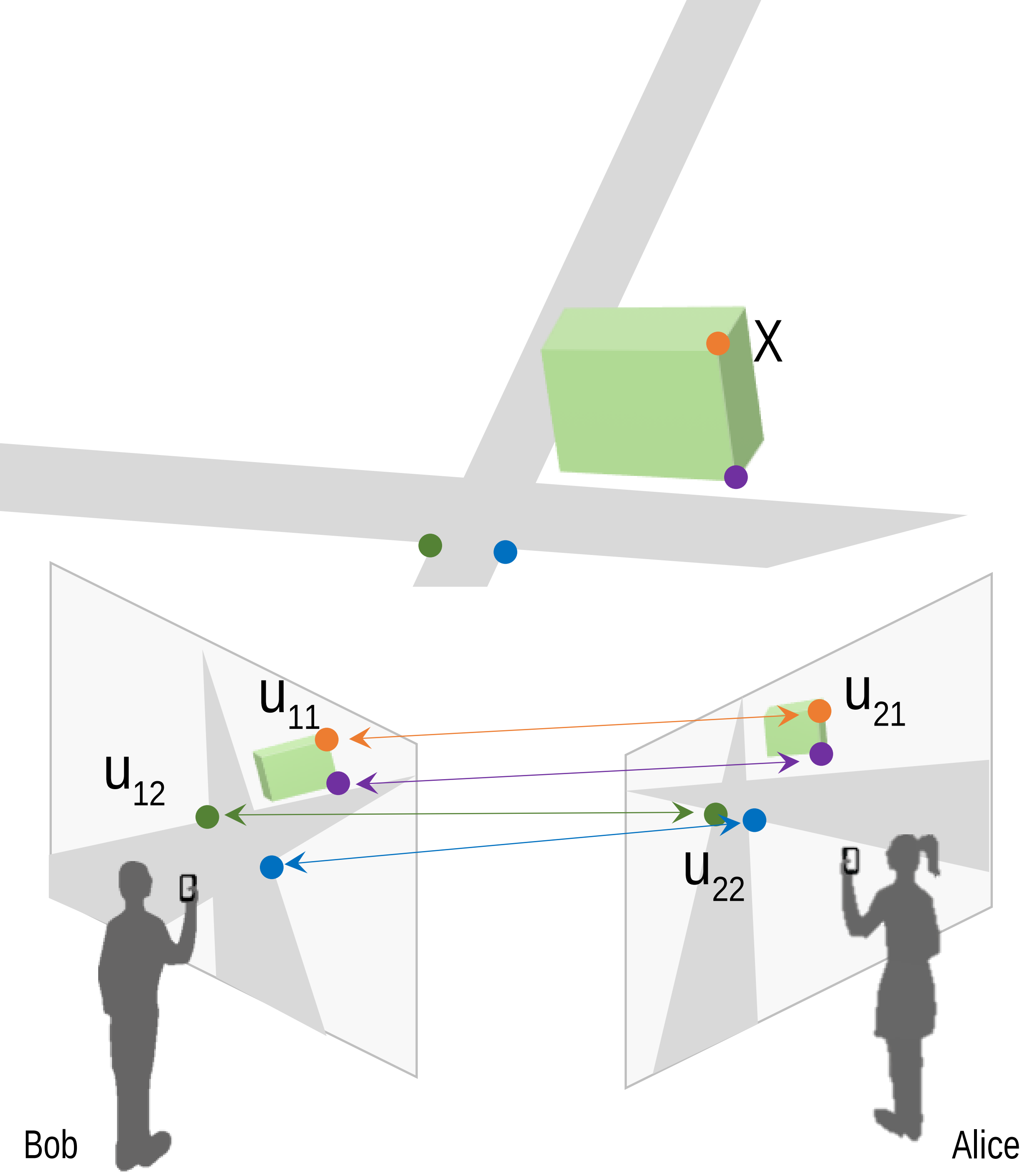
Orthographic projection:

$$\tilde{u}_{11} = \hat{R}_1 \tilde{X}_1 \qquad \tilde{u}_{11} = \hat{R}_2 \tilde{X}_2$$

$$\tilde{u}_{12} = \hat{R}_1 \tilde{X}_2 \qquad \tilde{u}_{22} = \hat{R}_2 \tilde{X}_2$$

where $\hat{R} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \end{bmatrix} \quad \hat{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

$$\tilde{X}_j = X_j - \frac{1}{P}\sum_{j}^{P} X_j \longrightarrow \tilde{u}_{ij} = u_{ij} - \frac{1}{P}\sum_{j}^{P} u_{ij}$$

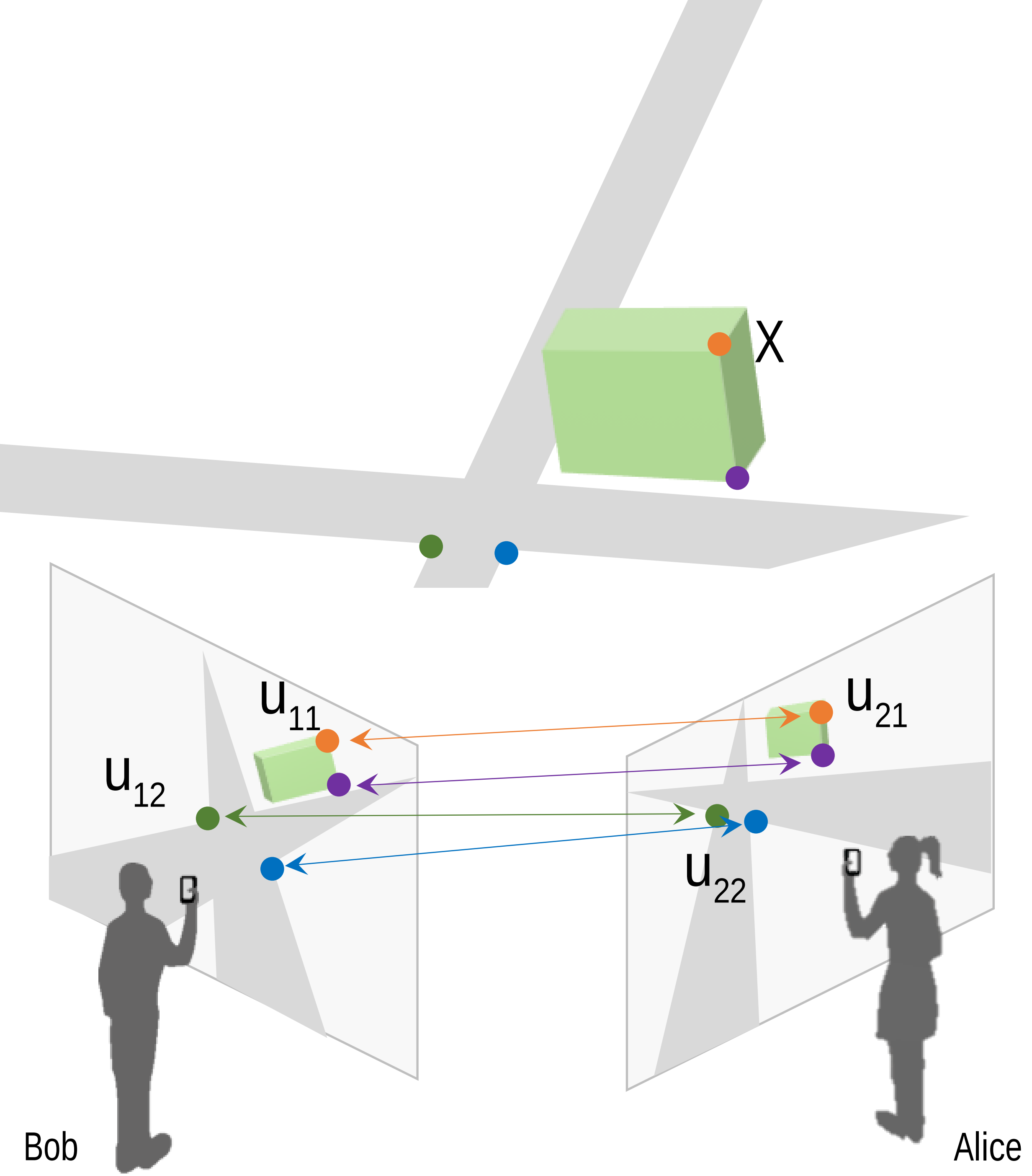


Orthographic projection:

$$\tilde{u}_{11} = \hat{R}_1 \tilde{X}_1 \qquad \tilde{u}_{11} = \hat{R}_2 \tilde{X}_2$$

$$\tilde{u}_{12} = \hat{R}_1 \tilde{X}_2 \qquad \tilde{u}_{22} = \hat{R}_2 \tilde{X}_2$$

$$\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \ddots & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{R}_1 \\ \vdots \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & \cdots & \tilde{X}_P \end{bmatrix}$$



Orthographic projection:

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$2F \times P$ $2F \times 3$ $3 \times P$

Knowns   Unknowns

$$\mathbf{W} = \begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \mathbf{M}\,\mathbf{S} = \begin{bmatrix} \hat{R}_1 \\ \vdots \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & \cdots & \tilde{X}_P \end{bmatrix}$$

W	M	S
$2F$xP	$2F$x3	3xP
Measurement matrix	Motion mtx	Shape mtx

$$\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & W & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{R}_1 \\ M \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & S & \tilde{X}_P \end{bmatrix}$$

$2F \times P$	$2F \times 3$	$3 \times P$
Measurement matrix	Motion mtx	Shape mtx

rank( ) = 3

$$\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & W & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{R}_1 \\ M \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & S & \tilde{X}_P \end{bmatrix}$$

$2F \times P$	$2F \times 3$	$3 \times P$
Measurement matrix	Motion mtx	Shape mtx

rank( ) = 3

$$W = UDV^T \xrightarrow{?} \begin{array}{l} M = UD^{1/2} \\ S = D^{1/2}V^T \end{array}$$

$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & W & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{2F\times P} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ M \\ \hat{R}_F \end{bmatrix}}_{2F\times 3} \underbrace{\begin{bmatrix} \tilde{X}_1 & S & \tilde{X}_P \end{bmatrix}}_{3\times P}$$

Measurement matrix | Motion mtx | Shape mtx

rank( ) = 3

$$W = UDV^T \xrightarrow{?} \begin{aligned} M &= UD^{1/2} \\ S &= D^{1/2}V^T \end{aligned}$$

No. There exists infinite M and S combinations.

$$M = UD^{1/2}Q$$

$$S = Q^{-1}D^{1/2}V^T$$

Any nonsingular Q can produce solutions.

Is there any constraint?

$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & W & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{2F\times P} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ M \\ \hat{R}_F \end{bmatrix}}_{2F\times 3} \underbrace{\begin{bmatrix} \tilde{X}_1 & S & \tilde{X}_P \end{bmatrix}}_{3\times P}$$

Measurement matrix | Motion mtx | Shape mtx

$\text{rank}(\ \ ) = 3$

$$W = UDV^T$$

$$M = UD^{1/2}Q$$

$$S = Q^{-1}D^{1/2}V^T$$

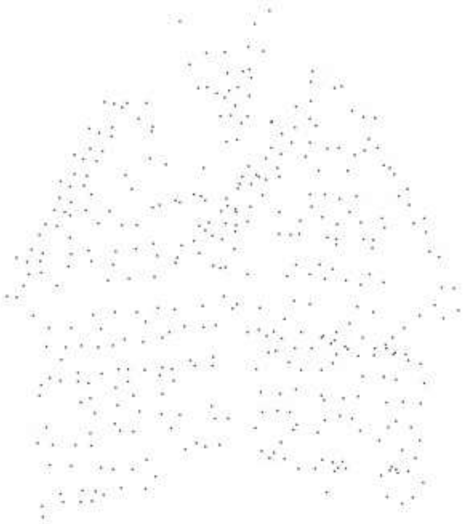
Orthogonal constraint:

$$M = UD^{1/2}Q = \begin{bmatrix} r_{1x}^T \\ r_{1y}^T \\ \vdots \\ r_{Fx}^T \\ r_{Fy}^T \end{bmatrix}$$

$$\longrightarrow r_{1x}^T r_{1x} = 1, \quad r_{1y}^T r_{1y} = 1, \quad r_{1x}^T r_{1y} = 0$$

# of unknowns: 9 (Q: 3x3)

# of equations: 3*F*



$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{W} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ \vdots \\ \hat{R}_F \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} \tilde{X}_1 & \cdots & \tilde{X}_P \end{bmatrix}}_{S}$$