

# Lens Distortion



Lens configuration (internal parameter)

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = L \left( \begin{bmatrix} \mathbf{K} & \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} \right)$$

Spatial relationship between sensor and pinhole (internal parameter)

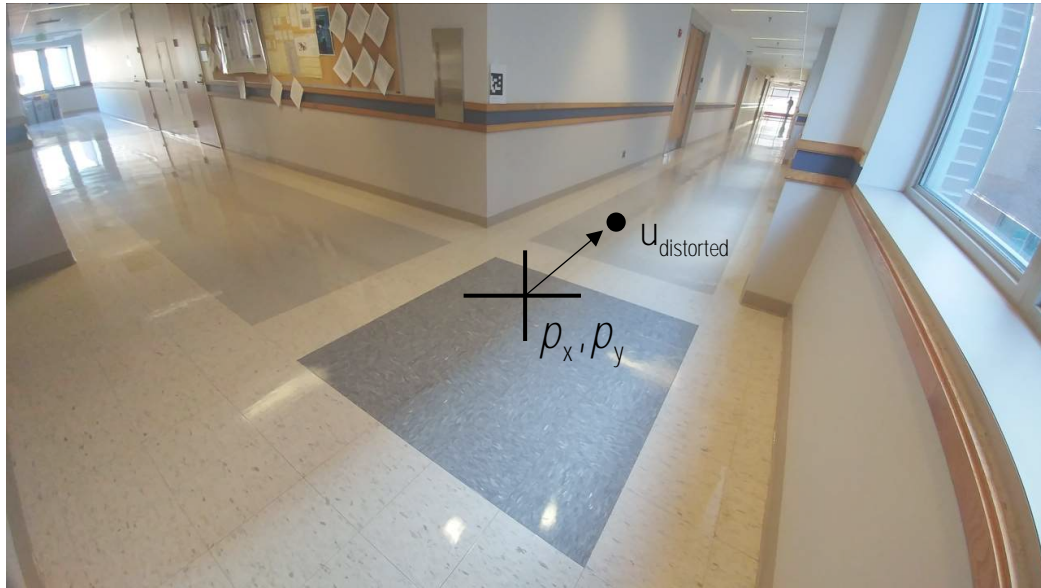
Camera body configuration (extrinsic parameter)



Lens Radial Distortion

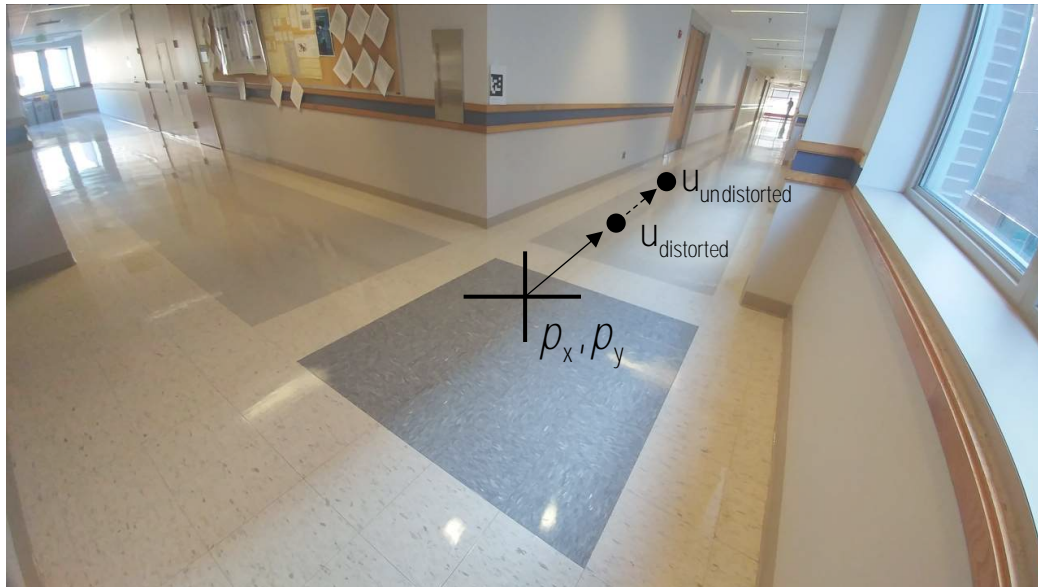
# Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



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$$\bar{u}_{\text{distorted}} = L(\boldsymbol{\rho})\bar{u}_{\text{undistorted}}$$

$$\text{where } \boldsymbol{\rho} = \|\bar{u}_{\text{undistorted}}\|$$

$$L(\boldsymbol{\rho}) = 1 + k_1\rho^2 + k_2\rho^4 + \dots$$

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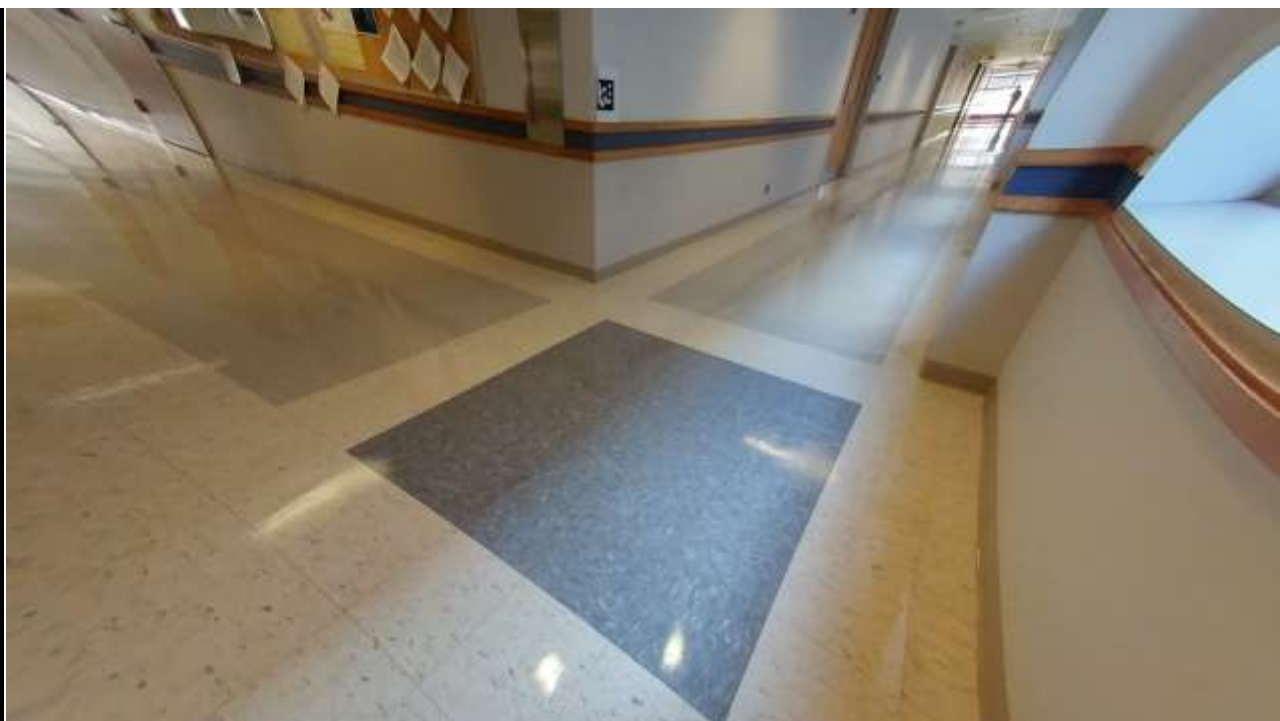
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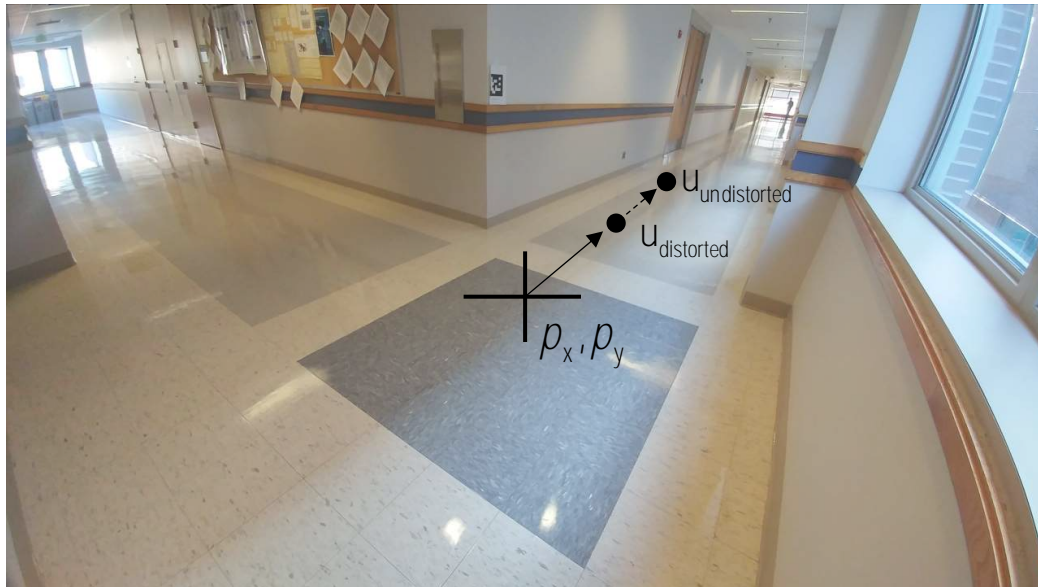
$$k_1 > 0$$



$$k_1 < 0$$

# Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



$$\bar{u}_{\text{distorted}} = L(\boldsymbol{\rho})\bar{u}_{\text{undistorted}}$$

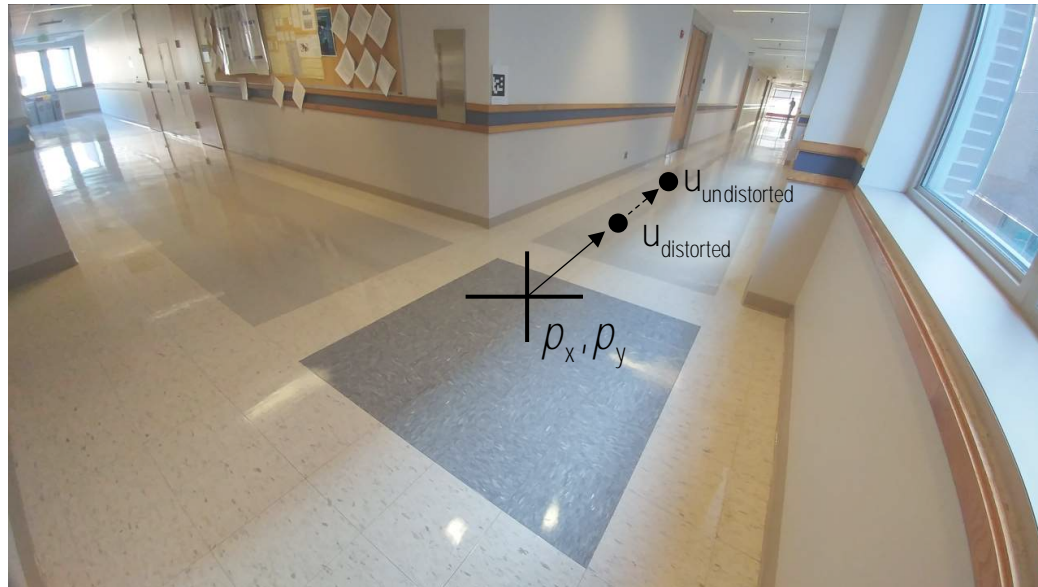
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Assumption: Lens distortion is a function of distance from the principal point.



Normalized point:

$$\bar{u}_{\text{distorted}} = K^{-1}u_{\text{distorted}}, \quad \bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$$

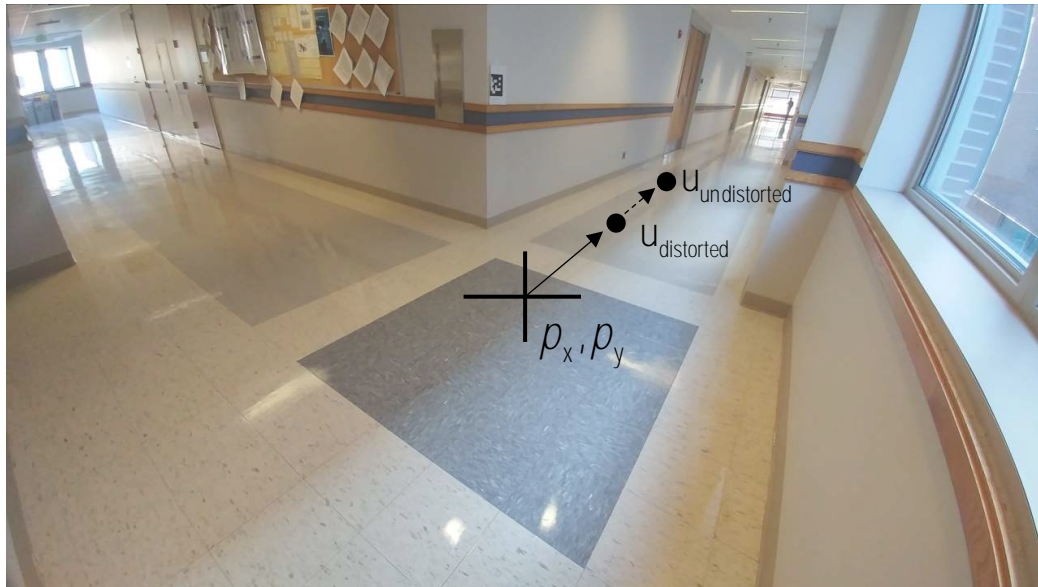
$$\bar{u}_{\text{distorted}} = L(\boldsymbol{\rho})\bar{u}_{\text{undistorted}}$$

where  $\boldsymbol{\rho} = \|\bar{u}_{\text{undistorted}}\|$

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$$L(\boldsymbol{\rho}) = 1 + k_1\rho^2 + k_2\rho^4 + \dots$$

$$f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

$$\rho_x = \frac{W_{\text{img}}}{2} = \frac{3840}{2} = 1920 \text{pix} \quad \rho_y = \frac{H_{\text{img}}}{2} = \frac{2160}{2} = 1080 \text{pix}$$

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Distorted image



```
im = imread('image.jpg');  
f = 1224;  
k = -0.08;  
px = size(im,2)/2;  
py = size(im,1)/2;
```

$$f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

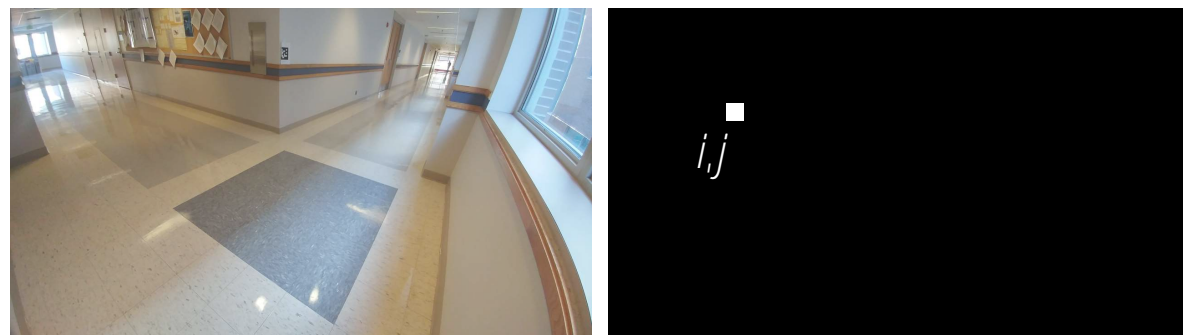
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# Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image



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k = -0.08;  
px = size(im,2)/2;  
py = size(im,1)/2;
```

```
im_new = zeros(size(im)); % create a new image
```

```
for i = 1 : size(im,1)  
    for j = 1 : size(im,2)
```

$$f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

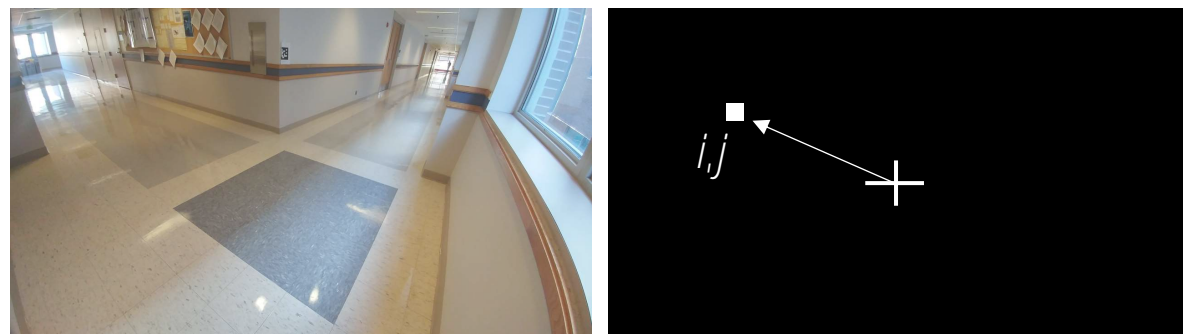
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```

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im_new = zeros(size(im)); % create a new image
```

```
for i = 1 : size(im,1)  
    for j = 1 : size(im,2)  
        dx = ([j;i]-[px;py])/f;  
        r = norm(dx);
```

$$\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$$
$$\rho = \|\bar{u}_{\text{undistorted}}\|$$

$$f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

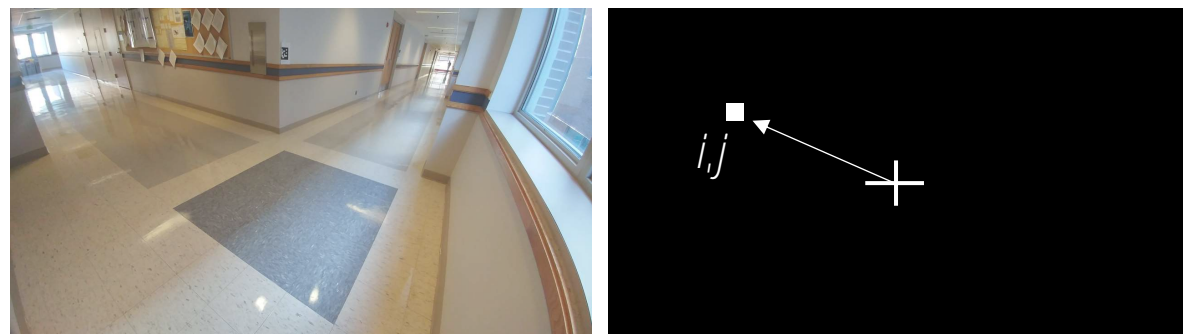
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for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        dx = ([j;i]-[px;py])/f;
        r = norm(dx);
        l = 1 + k*r*r;
        x = f*l*dx+[cx;cy];
```

$$\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$$

$$\rho = \|\bar{u}_{\text{undistorted}}\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \dots$$

$$K\bar{u}_{\text{distorted}} = u_{\text{distorted}}$$

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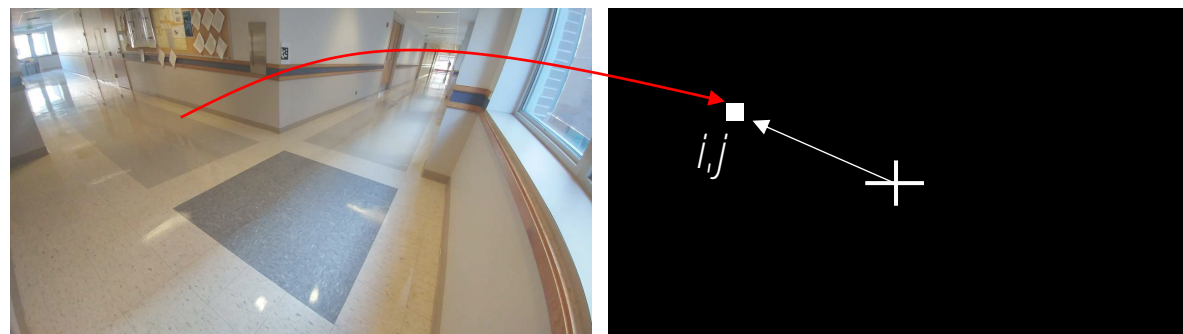
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```
for i = 1 : size(im,1)
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```
    for j = 1 : size(im,2)
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```
        dx = ([j;i]-[px;py])/f;
```

```
        r = norm(dx);
```

```
        l = 1 + k*r*r;
```

```
        x = f*l*dx+[cx;cy];
```

$$\bar{u}_{undistorted} = K^{-1}u_{undistorted}$$

$$\rho = \|\bar{u}_{undistorted}\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \dots$$

$$K\bar{u}_{distorted} = u_{distorted}$$

```
        if floor(x(1))<=0 || floor(x(1))>size(im,2) || floor(x(2))<=0 || floor(x(2))>size(im,1)
            continue;
        end
```

```
        im_new(i,j,:) = im(floor(x(2)), floor(x(1)),:);
```

```
    end
end
```

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Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{ pix}$$

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```
for i = 1 : size(im,1)
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        r = norm(dx);
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        l = 1 + k*r*r;
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        x = f*l*dx+[cx;cy];
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        if floor(x(1))<=0 || floor(x(1))>size(im,2) || floor(x(2))<=0 || floor(x(2))>size(im,1)
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        end
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        im_new(i,j,:) = im(floor(x(2)), floor(x(1)),:);
```

```
    end
end
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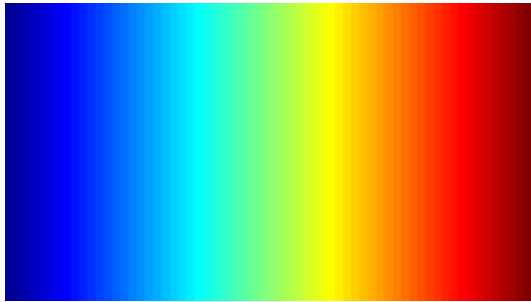


# Radial Distortion Model (MATLAB Efficient)

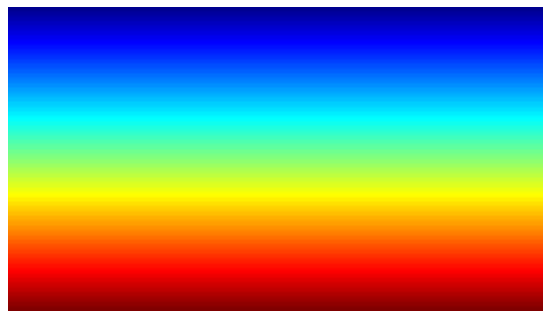
Assumption: Lens distortion is a function of distance from the principal point.

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));  
h = size(X, 1); w = size(X,2);
```

← XY coordinate



X



Y

$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

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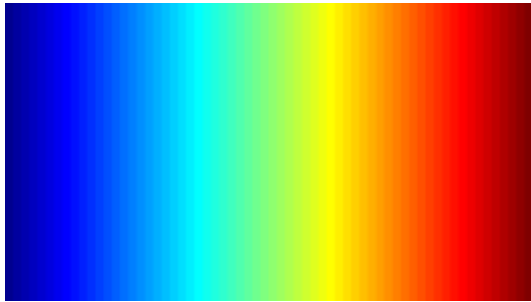
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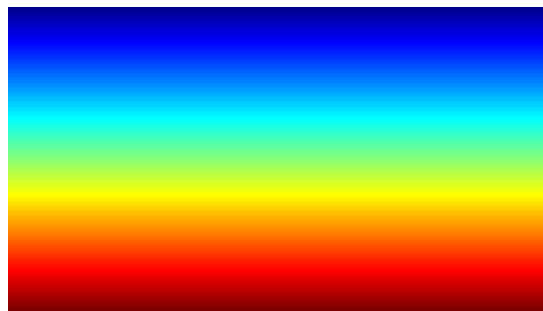
← XY coordinate

```
X_n = (X-px)/f;  
Y_n = (Y-py)/f;
```

←  $\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$



X



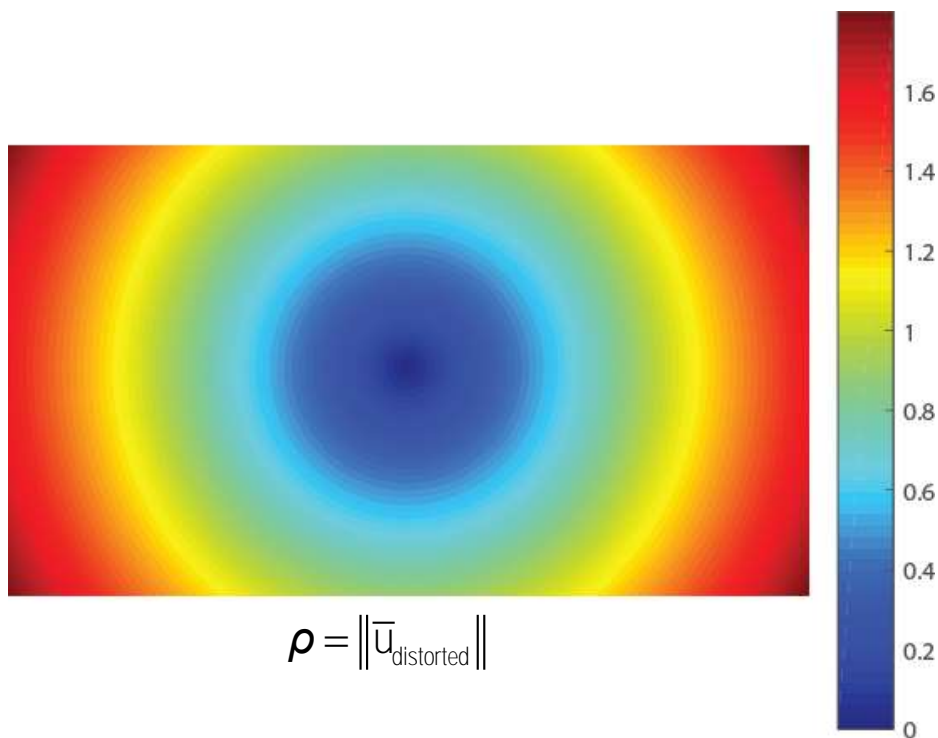
Y

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[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));  
h = size(X, 1); w = size(X,2);
```

```
X_n = (X-px)/f;  
Y_n = (Y-py)/f;
```

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

← XY coordinate

←  $\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$

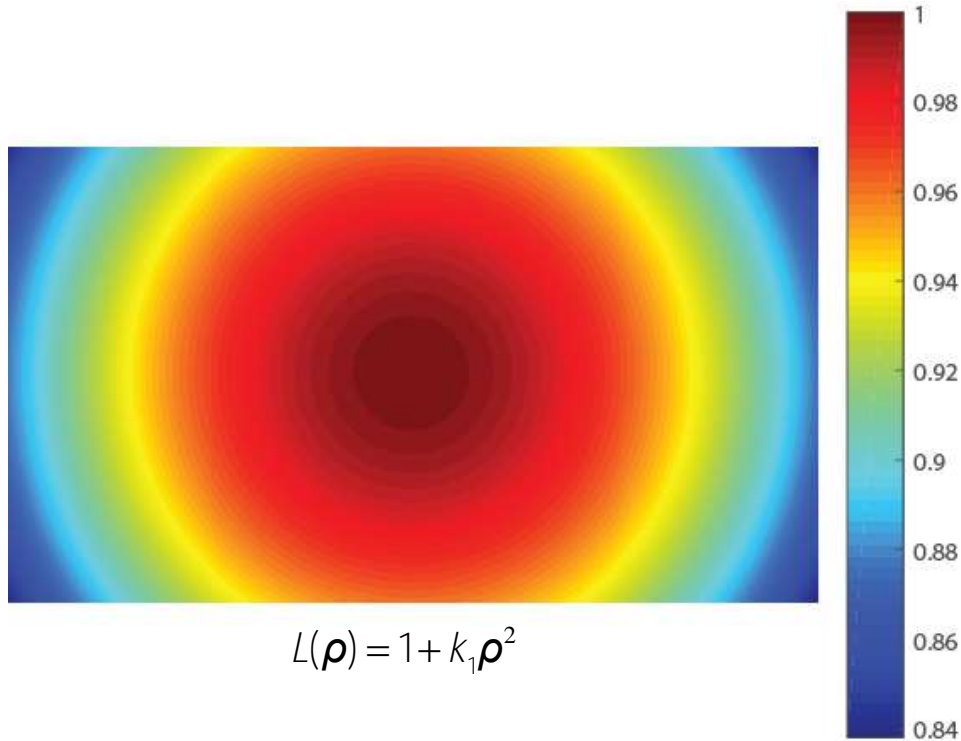
←  $\rho = \|\bar{u}_{\text{undistorted}}\|$

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# Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



$$L(\rho) = 1 + k_1 \rho^2$$

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

```
L = 1 + k * r_u.^2;
```

← XY coordinate

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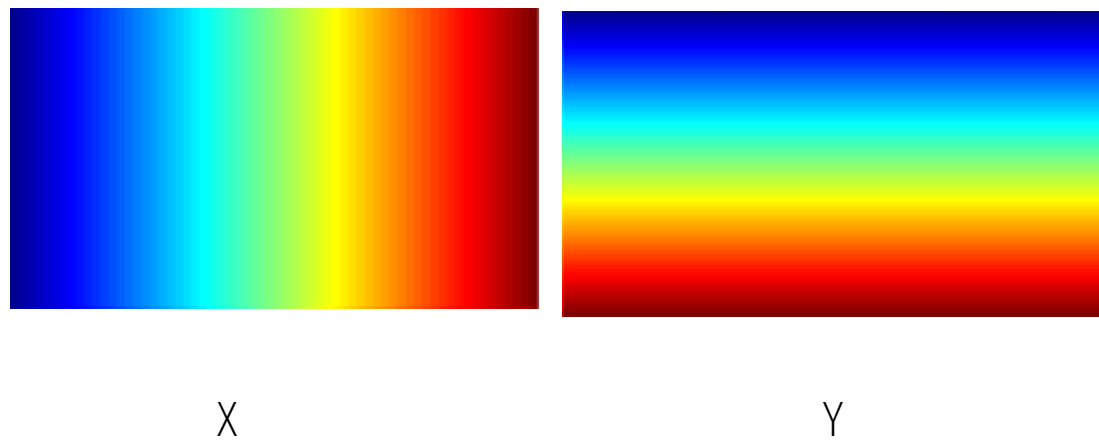
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← XY coordinate

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X_n = (X-px)/f;
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```

←  $\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

←  $\rho = \|\bar{u}_{\text{undistorted}}\|$

```
L = 1 + k * r_u.^2;
```

←  $L(\rho) = 1 + k_1\rho^2$

```
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
```

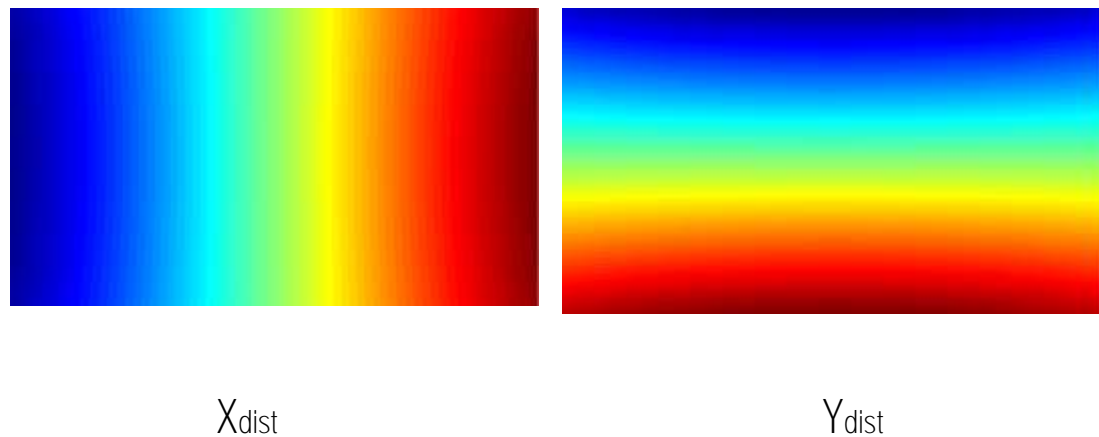
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X_n = (X-px)/f;
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r_u = sqrt(X_n.^2+Y_n.^2);
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←  $\rho = \|\bar{u}_{\text{undistorted}}\|$

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L = 1 + k * r_u.^2;
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X_dist_n = X_n.* L;
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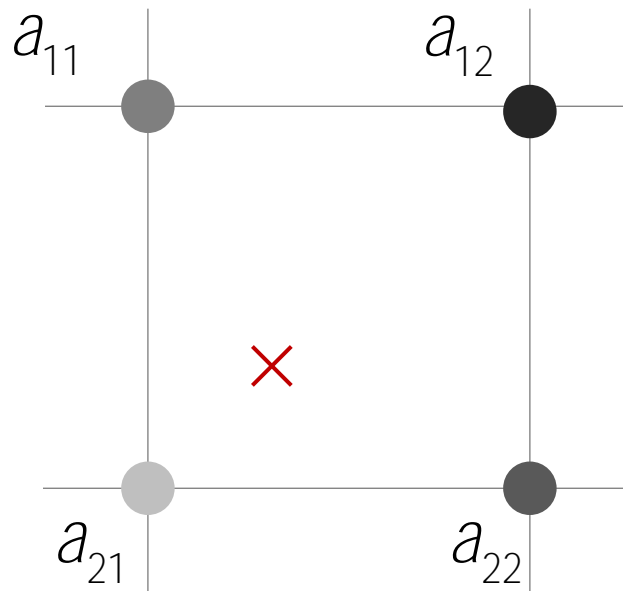
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←  $\bar{u}_{undistorted} = K^{-1}u_{undistorted}$

```
r_u = sqrt(X_n.^2+Y_n.^2);
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L = 1 + k * r_u.^2;
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X_dist_n = X_n.* L;
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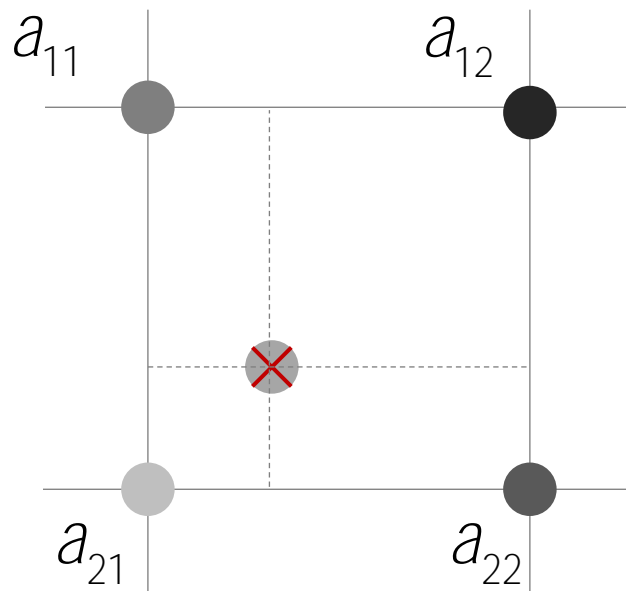
←  $K\bar{u}_{distorted} = u_{distorted}$

```
imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), X_dist(:), Y_dist(:)), [h, w]);
```

Bilinear interpolation

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```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

← XY coordinate

```
X_n = (X-px)/f;
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←  $\bar{u}_{undistorted} = K^{-1}u_{undistorted}$

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r_u = sqrt(X_n.^2+Y_n.^2);
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imUndistortion(:, :, 2) = reshape(interp2(im(:, :, 2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:, :, 3) = reshape(interp2(im(:, :, 3), X_dist(:), Y_dist(:)), [h, w]);
```

Bilinear interpolation

UndistortImageRadial.m

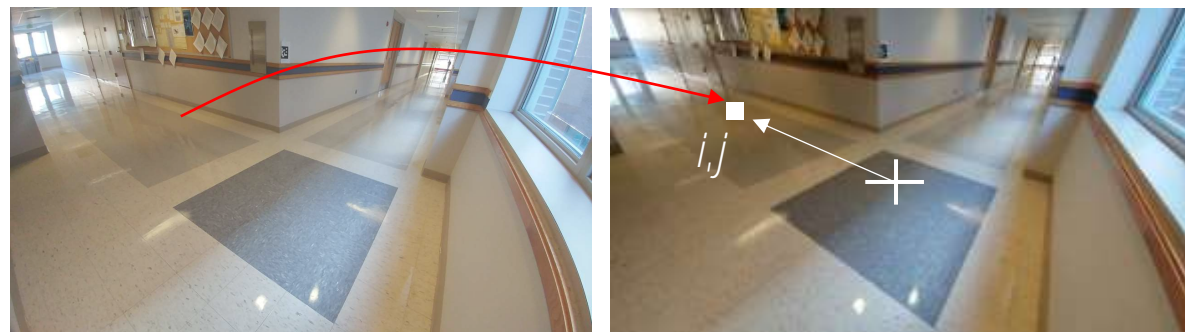


# Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image



```
[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));  
h = size(X, 1); w = size(X,2);  
X = X(:);  
Y = Y(:);
```

```
pt = [X'; Y'];  
pt = bsxfun(@minus, pt, [px;py]);  
pt = bsxfun(@rdivide, pt, [f;f]);  
r_u = sqrt(sum(pt.^2, 1));  
pt = bsxfun(@times, pt, 1 + k * r_u.^2);  
pt = bsxfun(@times, pt, [f;f]);  
pt = bsxfun(@plus, pt, [px;py]);
```

$$\bar{u}_{\text{undistorted}} = K^{-1}u_{\text{undistorted}}$$

$$\rho = \|\bar{u}_{\text{distorted}}\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \dots$$

$$K\bar{u}_{\text{distorted}} = u_{\text{distorted}}$$

```
imUndistortion(:, :, 1) = reshape(interp2(im(:, :, 1), pt(1, :), pt(2, :)), [h, w]);  
imUndistortion(:, :, 2) = reshape(interp2(im(:, :, 2), pt(1, :), pt(2, :)), [h, w]);  
imUndistortion(:, :, 3) = reshape(interp2(im(:, :, 3), pt(1, :), pt(2, :)), [h, w]);
```

Bilinear interpolation

UndistortImageRadial.m

# Practice with Your Cellphone Camera

Code download: <http://www-users.cs.umn.edu/~hspark/CSci5980/code/>

UndistortImageRadial.m



```
im = imread('1227161240_HDR.jpg');
im = double(im);
k = -0.05;
f = 1224;
px = size(im,2)/2;
py = size(im,1)/2;
```

```
[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
r_u = sqrt(X_n.^2+Y_n.^2);
L = 1 + k * r_u.^2;
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
X_dist = X_dist_n*f + px;
Y_dist = Y_dist_n*f + py;
```

```
imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), X_dist(:), Y_dist(:)), [h, w]);
```





Lens Radial Distortion Correction