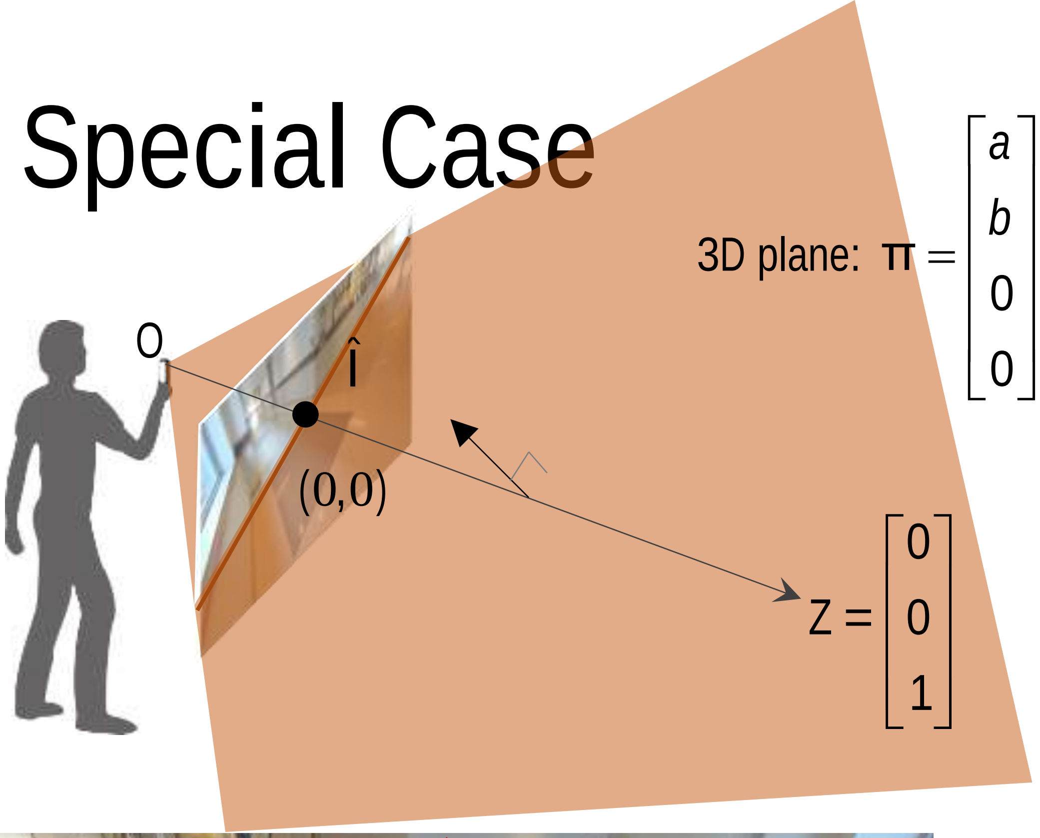
Point and Line at Infinity

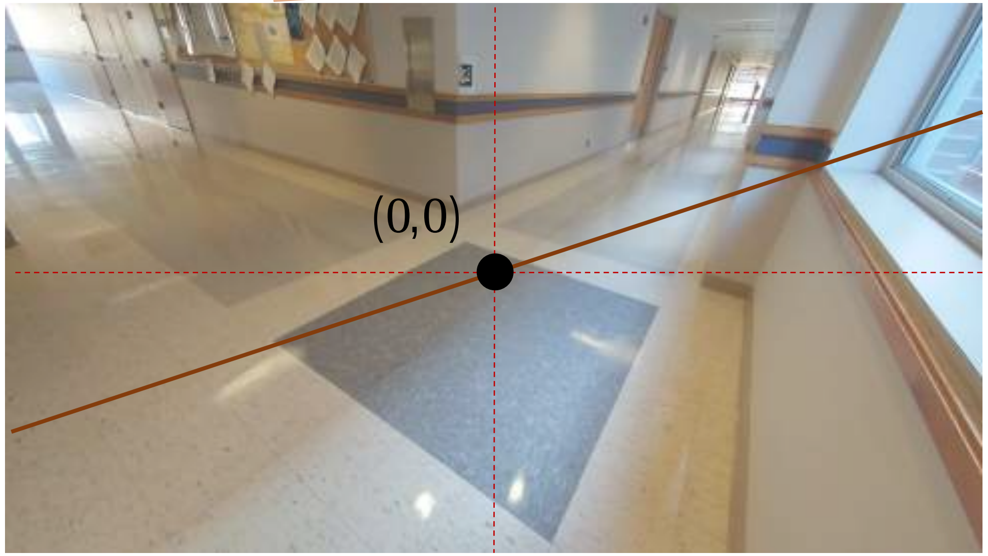
Special Case

Lines passe through the center of image

$$\hat{\mathbf{l}} = \mathrm{K}^{\mathrm{T}}\mathbf{l} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

Special Case





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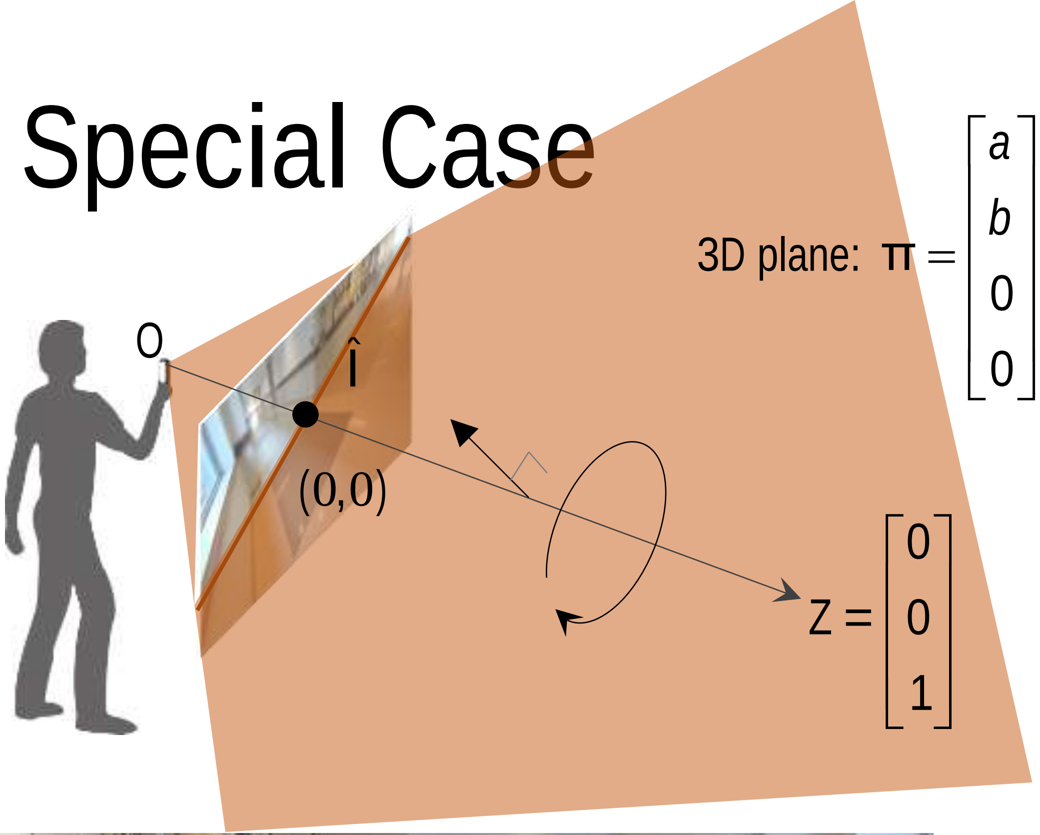

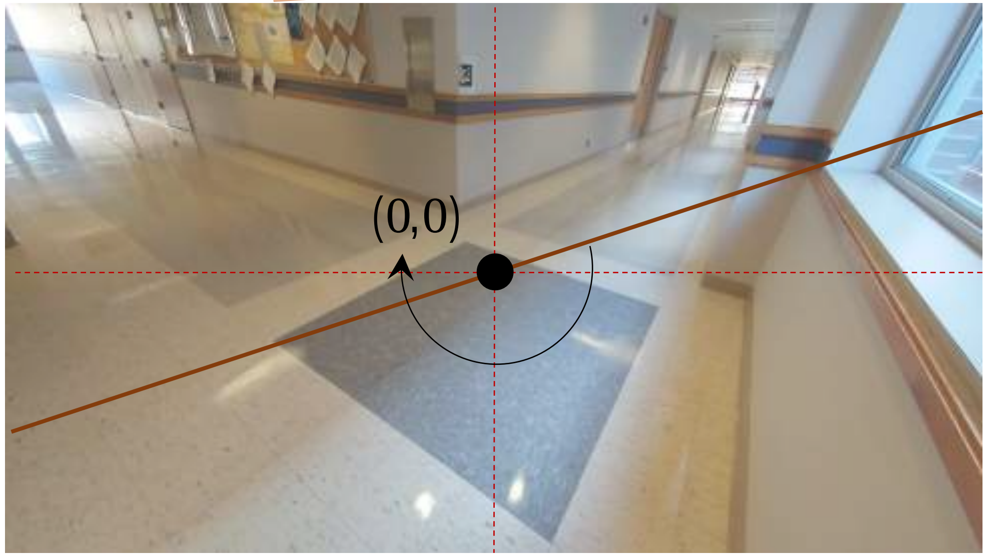

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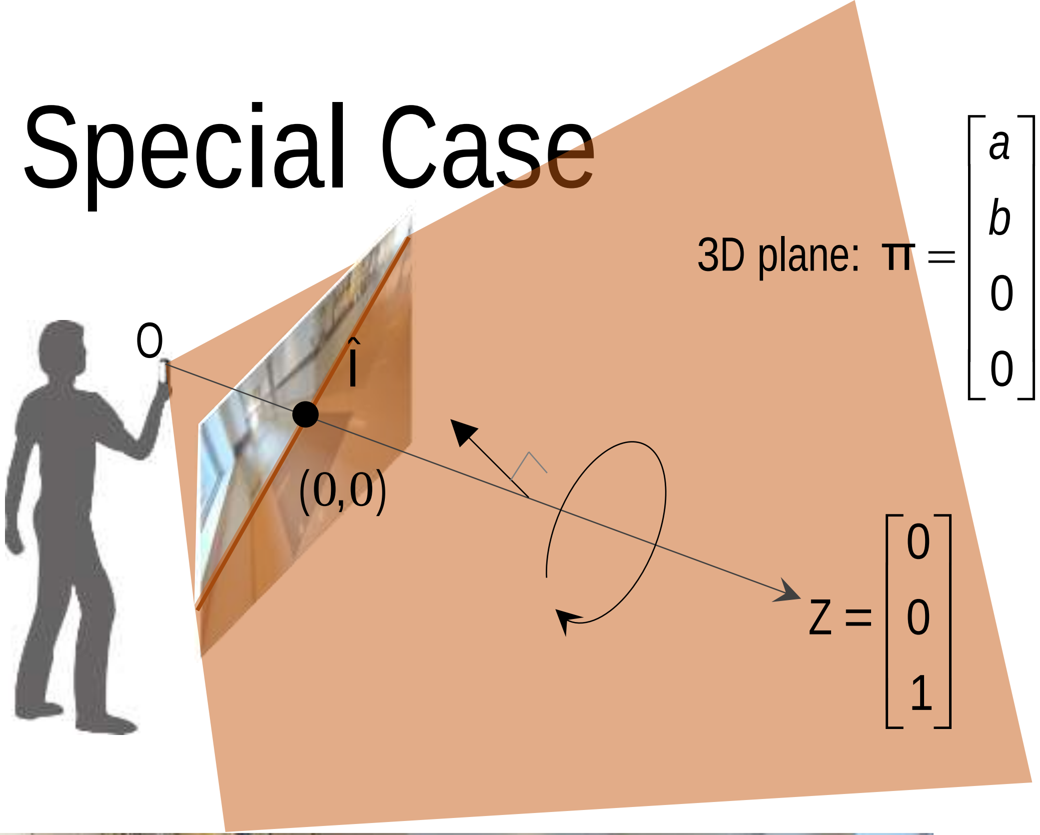
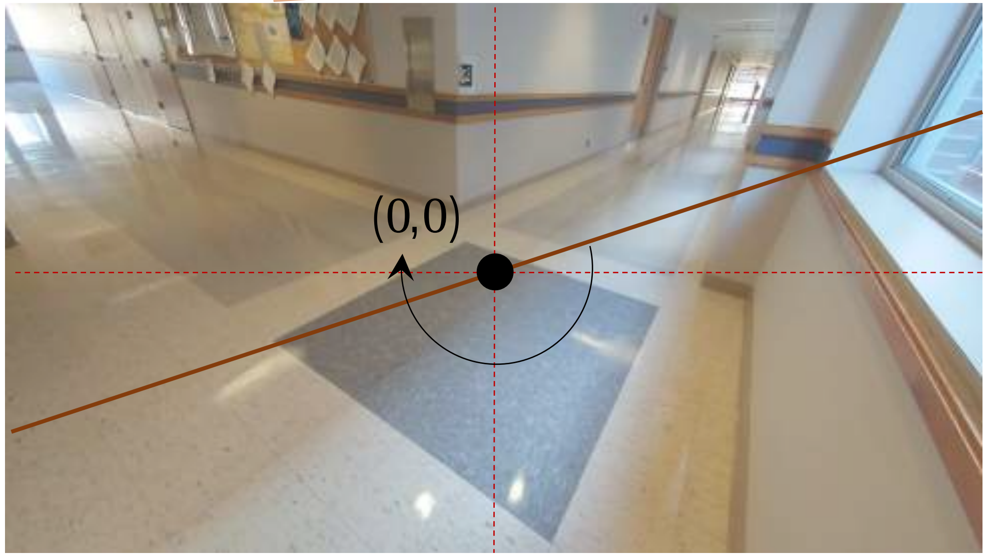


When the last element of line, $\hat{l}$ is zero:

- the line passes through the center of image.
- the surface normal of the 3D plane is perpendicular to Z axis
- the 3D plane is a plane rotating around Z axis

$$\begin{bmatrix} a & b & 0 \end{bmatrix} Z = 0$$



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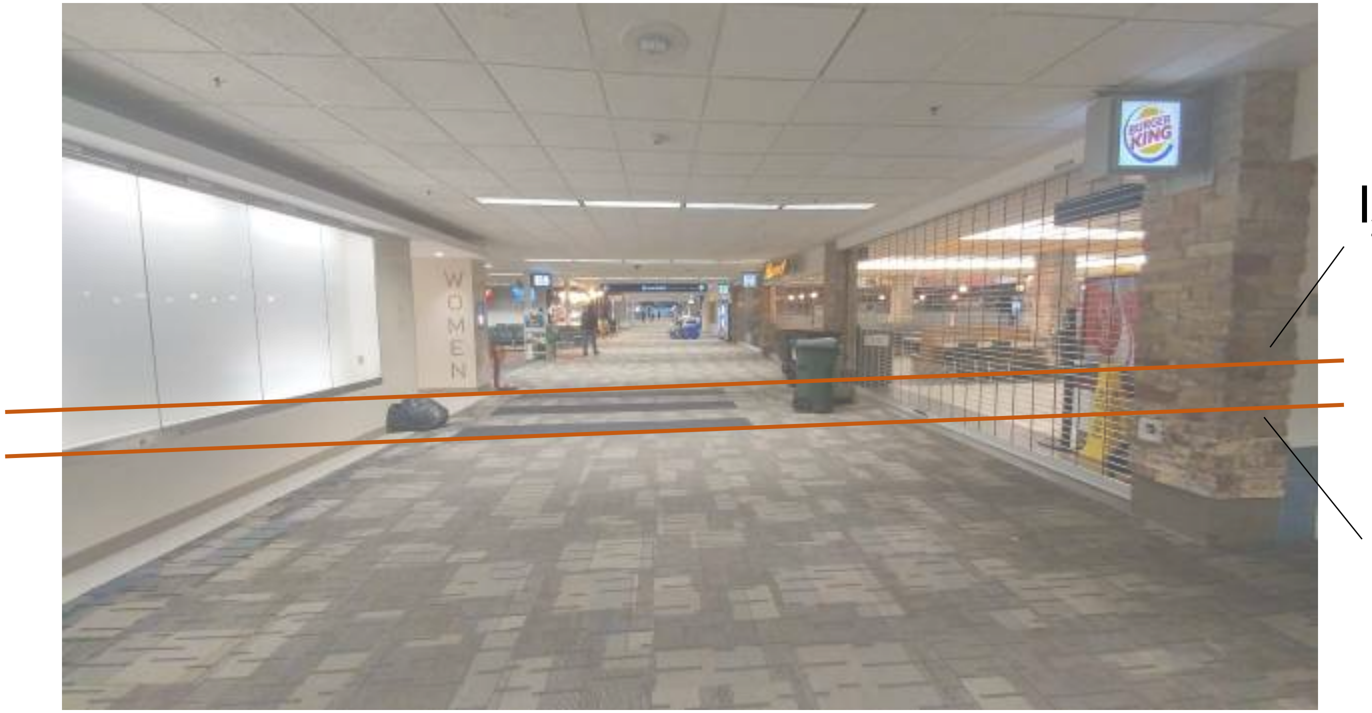
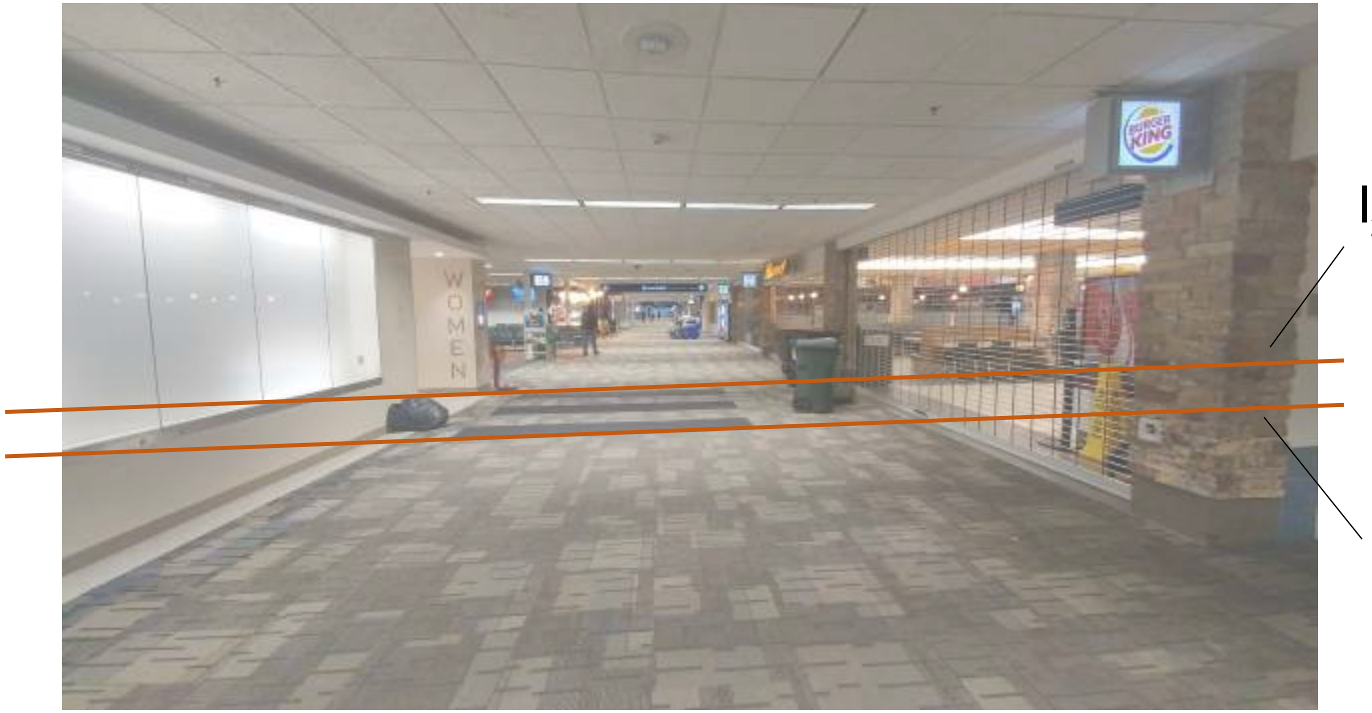
Special Case: 2D Point at Infinity



$$\mathbf{l}_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix}$$

$$\mathbf{l}_1 = \begin{bmatrix} a \\ b \\ c_1 \end{bmatrix}$$

Special Case: 2D Point at Infinity



$$\mathbf{l}_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix}$$

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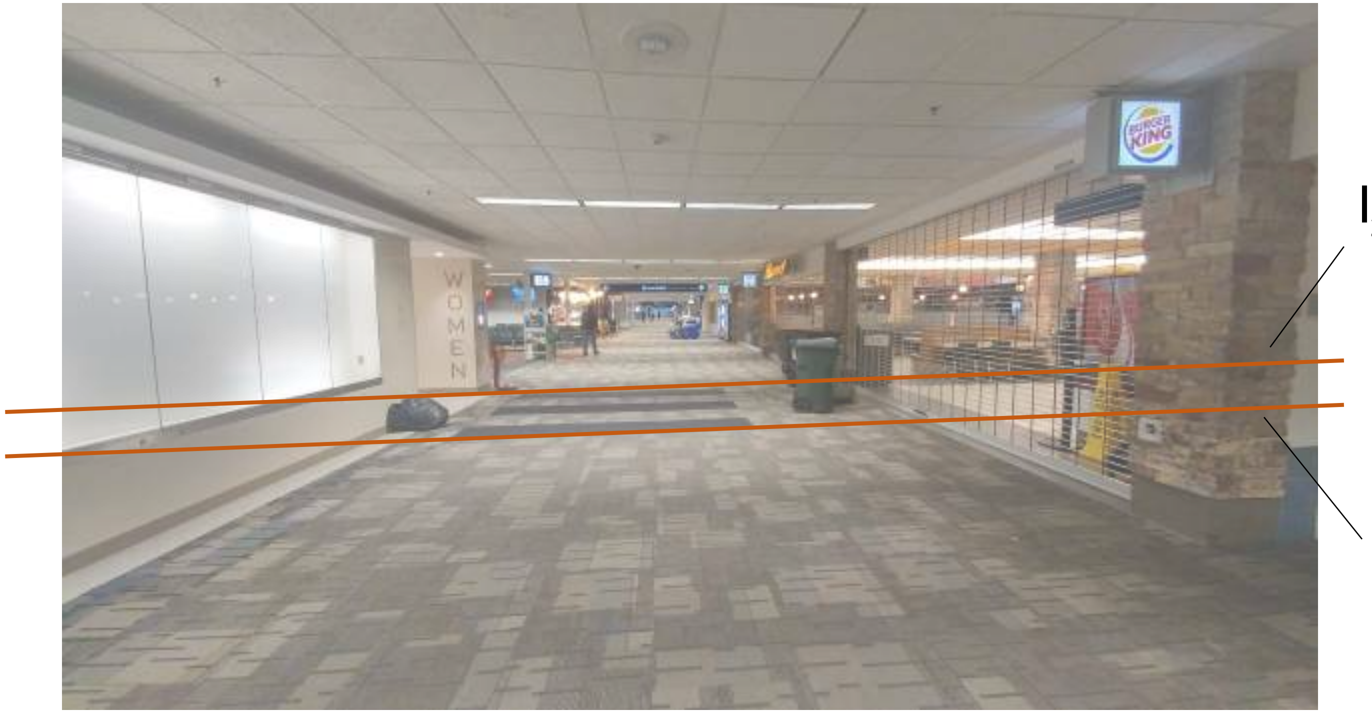
The intersection of two parallel lines in an image:

$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 = (c_2 - c_1) \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$$

Special Case: 2D Point at Infinity



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This point does not correspond to a finite point in the image because:

$$\mathrm{x} = \begin{bmatrix} -b/0 \\ a/0 \\ 0/0 \end{bmatrix}$$

Special Case: 2D Point at Infinity

Top view

The intersection of two parallel lines in an image:

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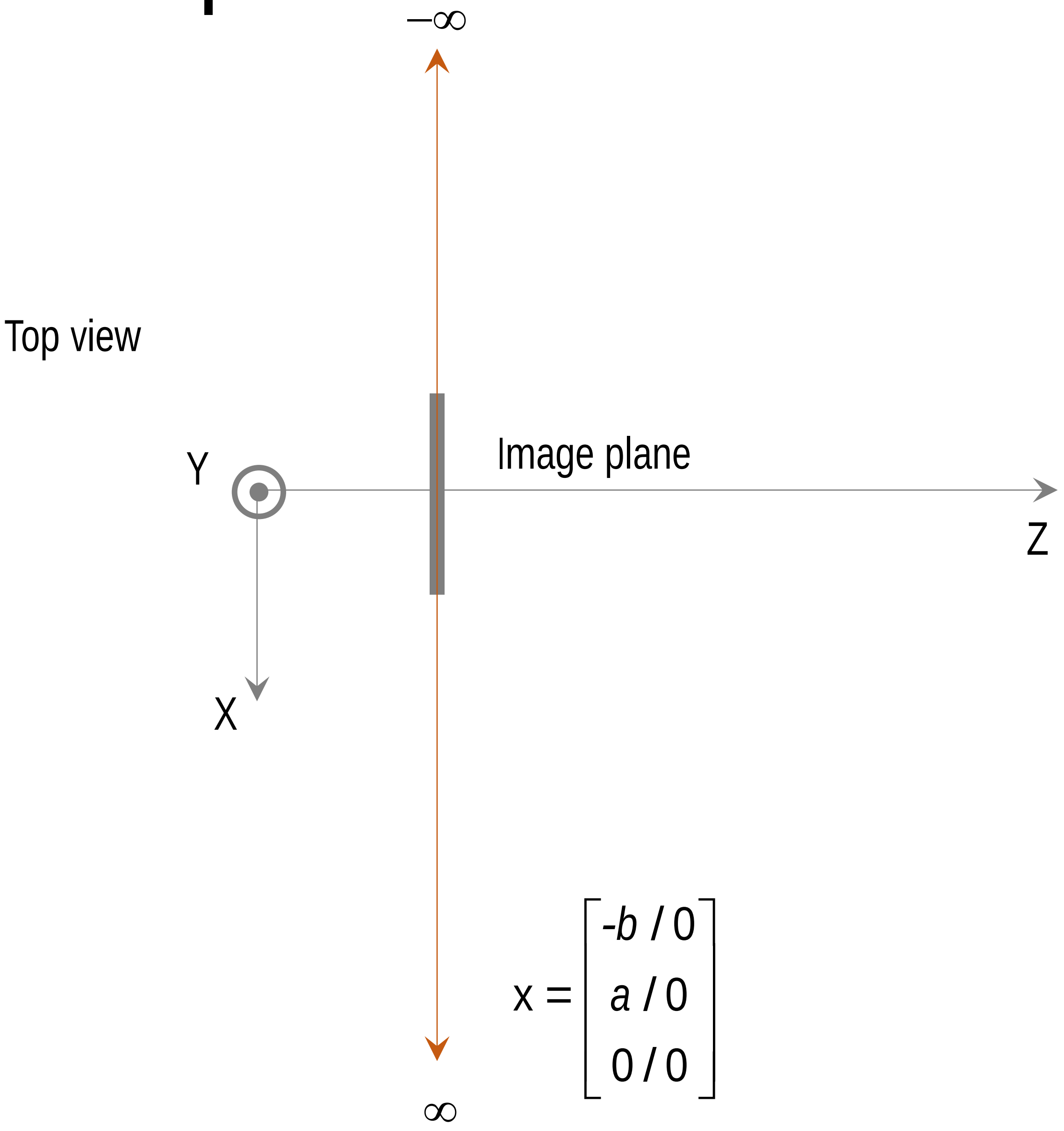
Special Case: 2D Point at Infinity



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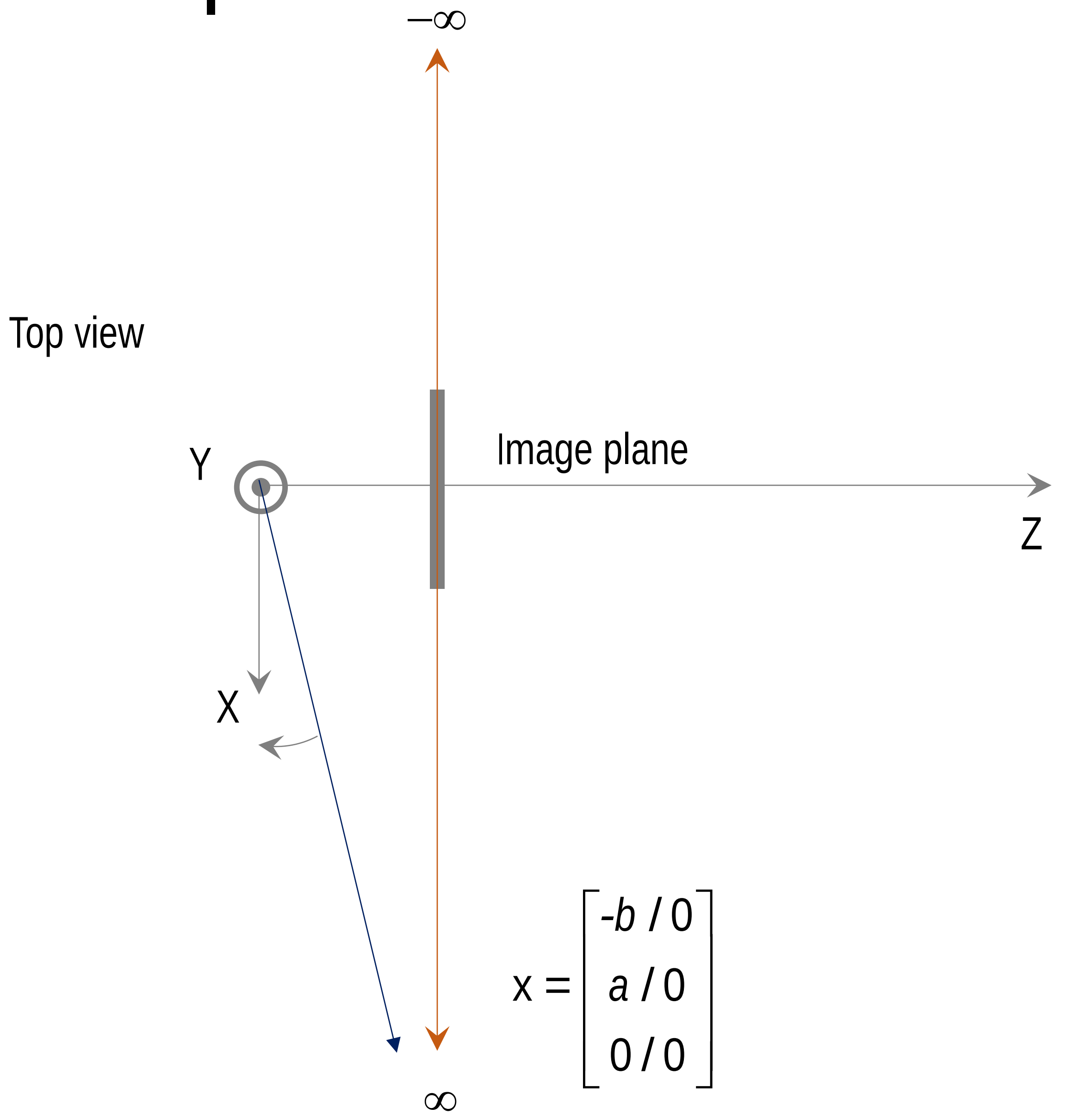

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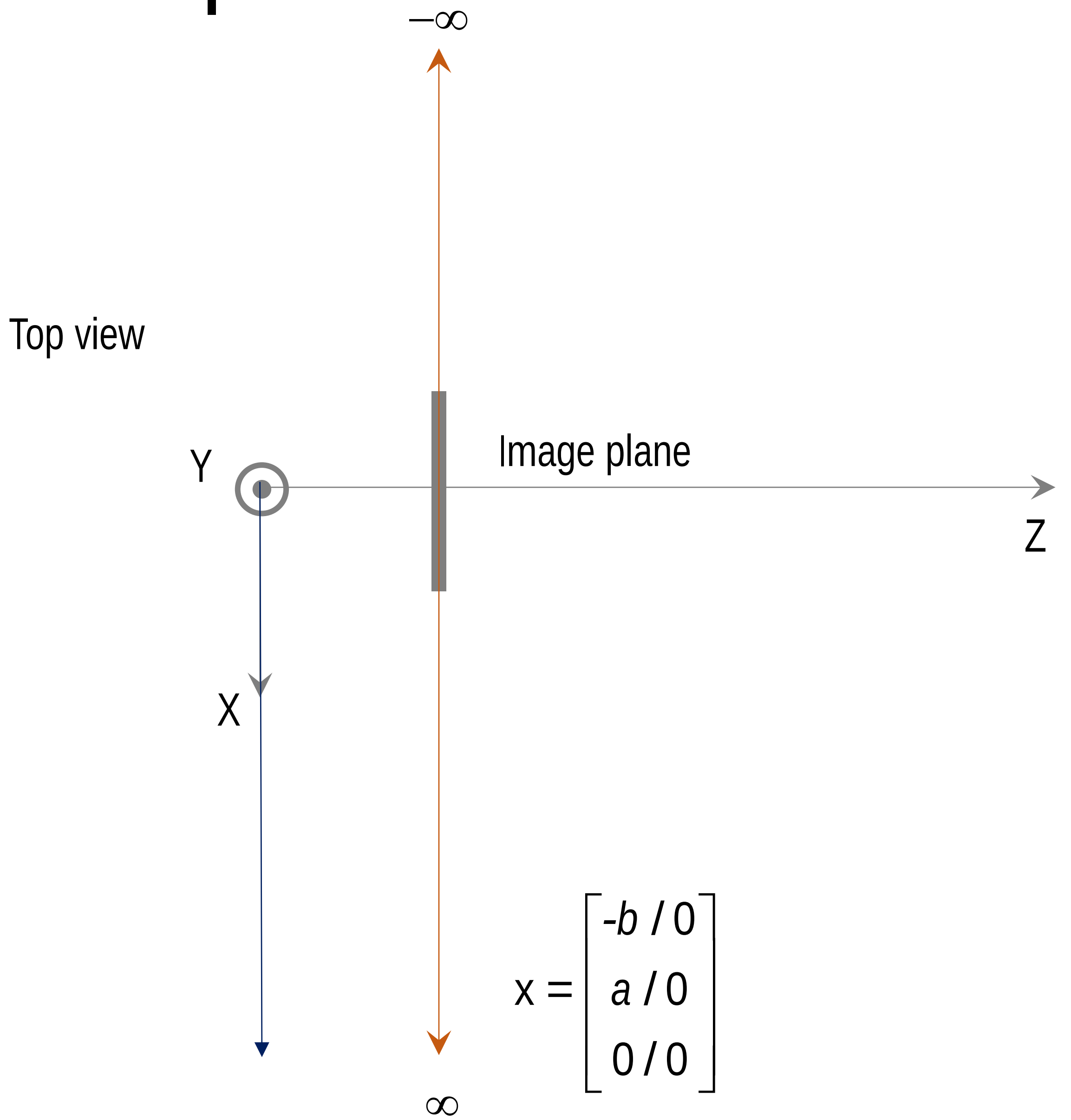

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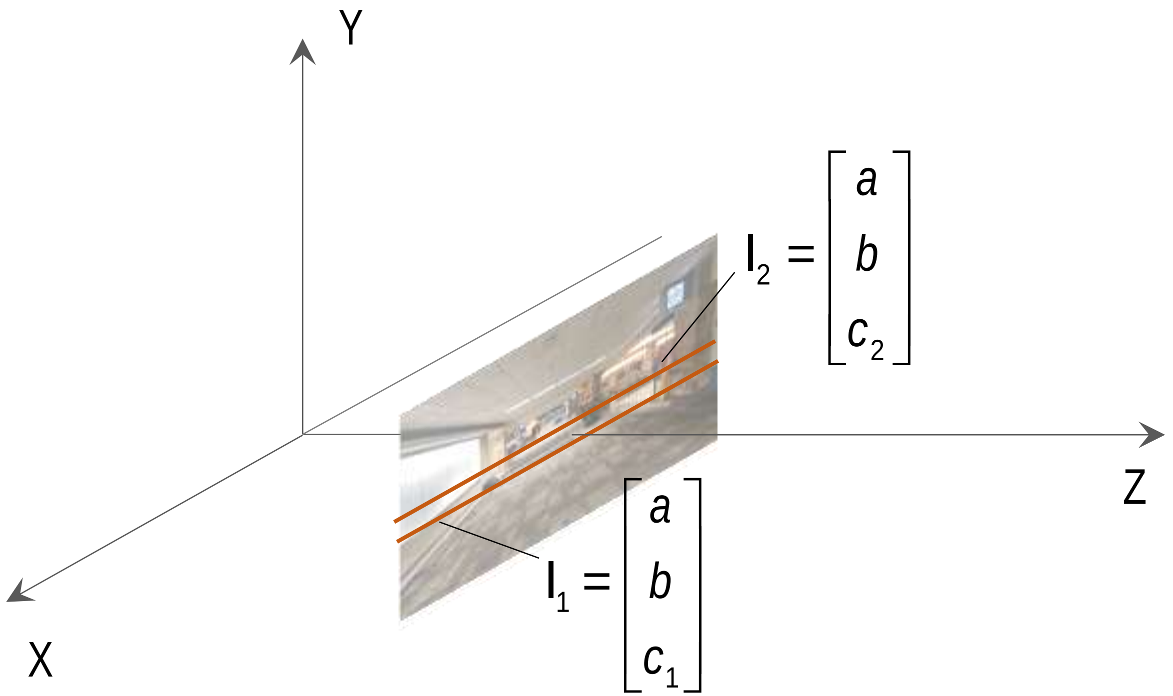
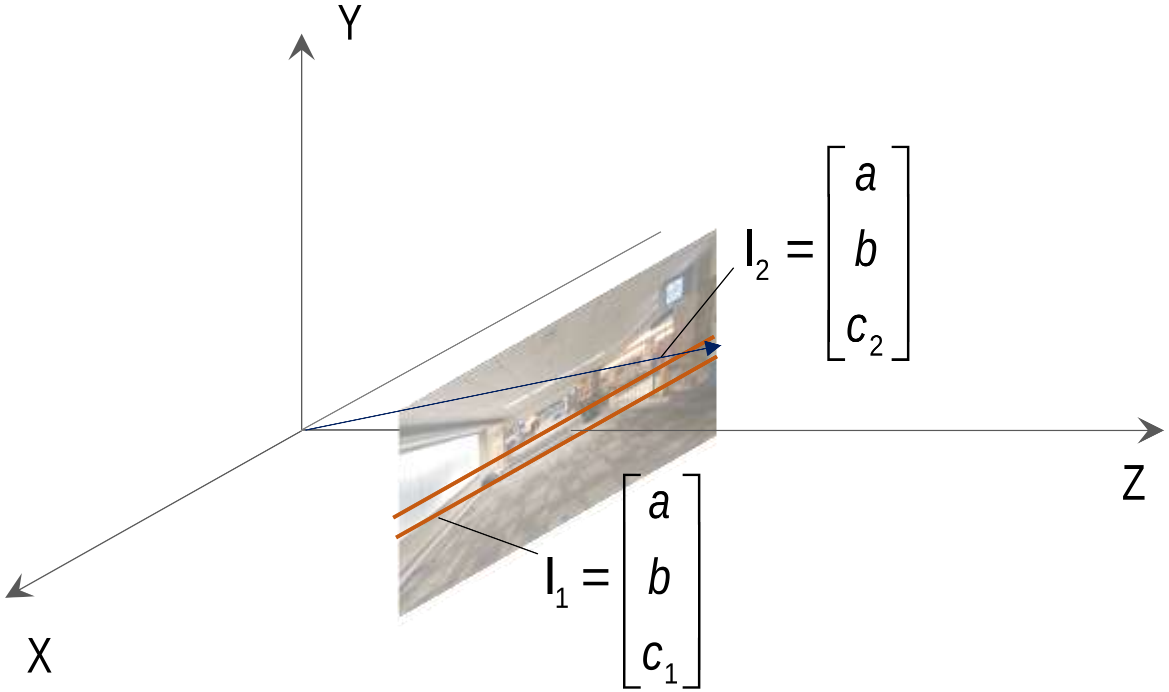
Point and Line at Infinity



Parallel lines intersect at the point at infinity:

Point at infinity: $x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

Point and Line at Infinity



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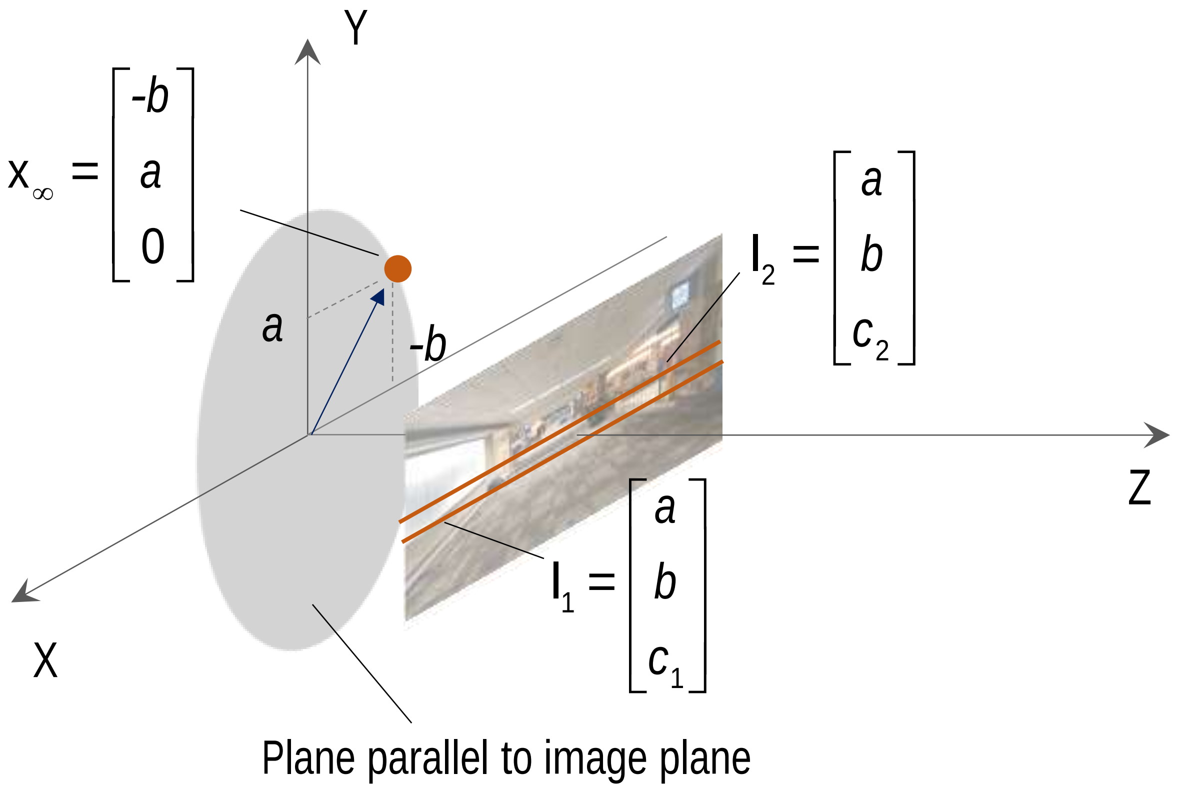
Point and Line at Infinity

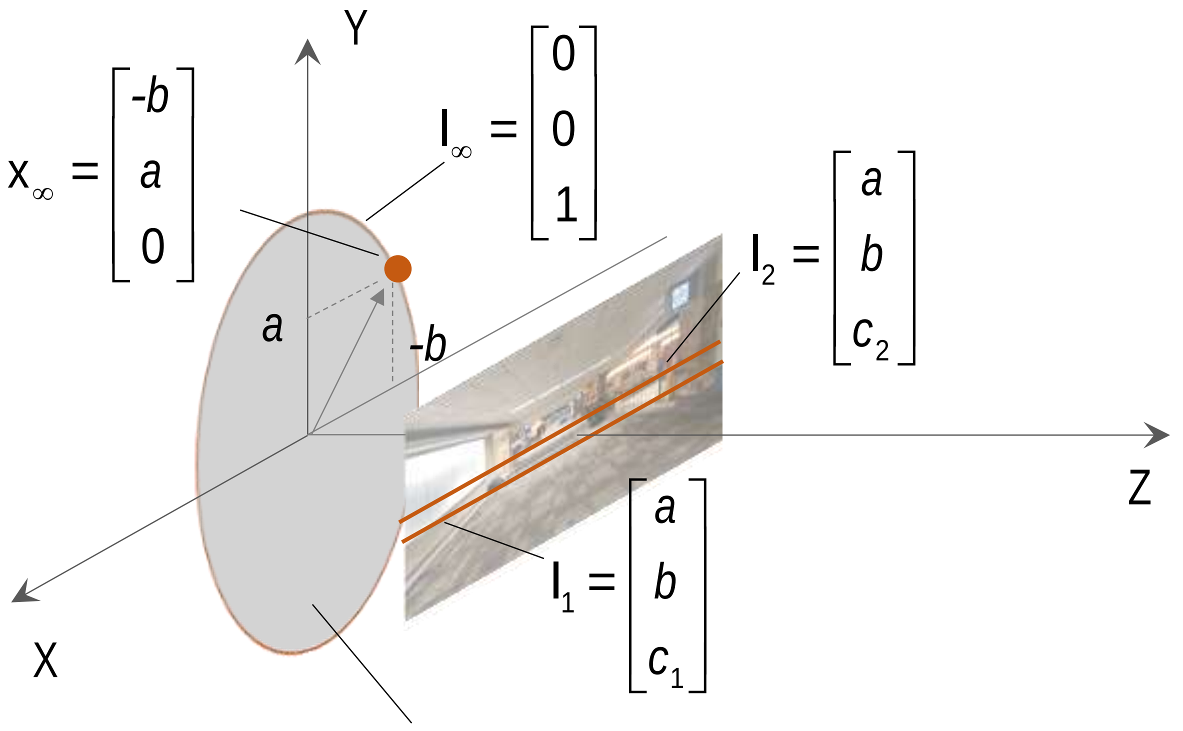


Parallel lines intersect at the point at infinity:

Point at infinity: $x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

A point at infinity corresponds to a direction.

Point and Line at Infinity



Plane parallel to image plane

Parallel lines intersect at the point at infinity:

Point at infinity: $x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

A point at infinity corresponds to a direction.

All points at infinity lie in the line at infinity:

$$l_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = Z \quad \because x_{ideal}^T l_\infty = 0$$

where the surface normal is parallel to the Z axis.