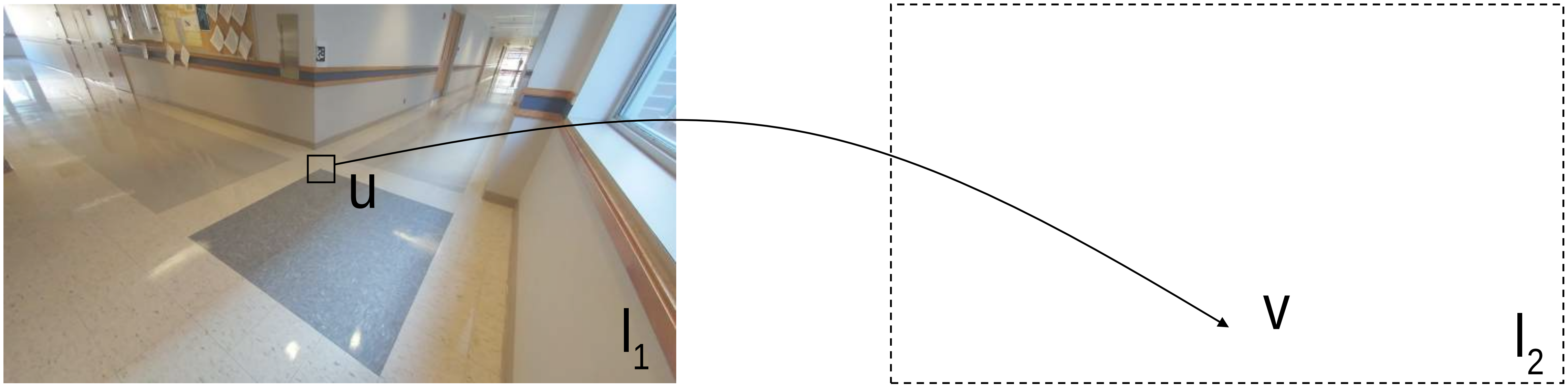
Image Transform

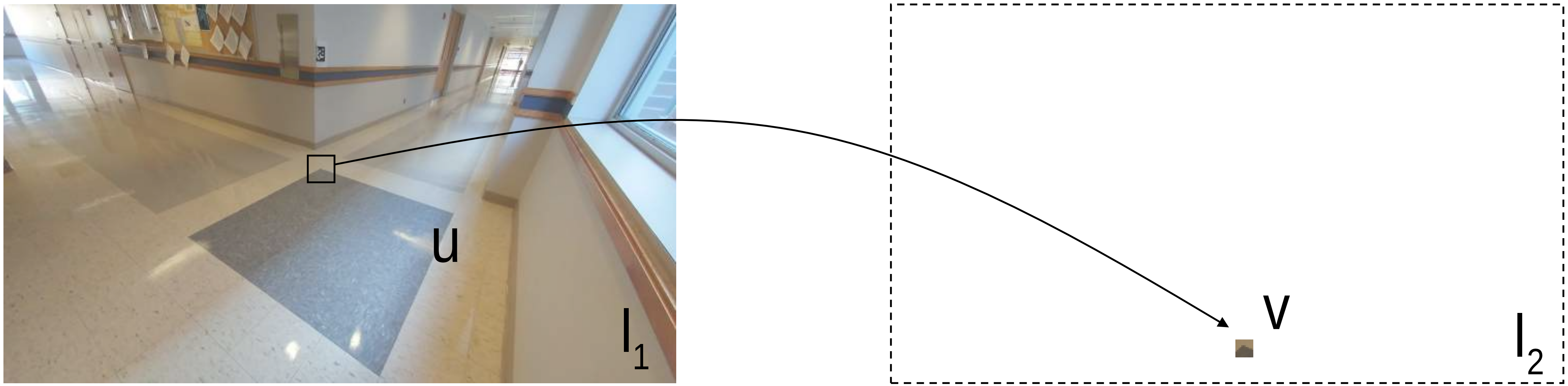
Panorama Image (Keller+Lind Hall)

Image Warping (Coordinate Transform)



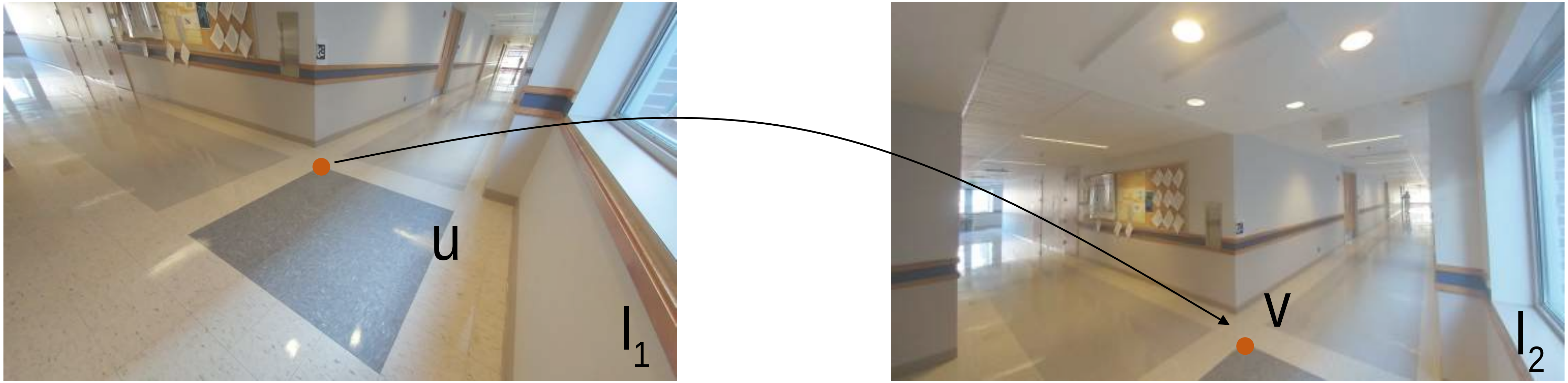
$$I_2(v) = I_1(u)$$

Image Warping (Coordinate Transform)



$$I_2(v) = I_1(u)$$: Pixel transport

Image Warping (Coordinate Transform)



$$I_2(v) = I_1(u)$$: Pixel transport

Cf) Image Filtering (Pixel Transform)

$$I_2 = g(I_1)$$: Pixel transform

Uniform Scaling

$$I_2(v) = I_1(u)$$

Uniform Scaling

$$I_2(v) = I_1(u)$$

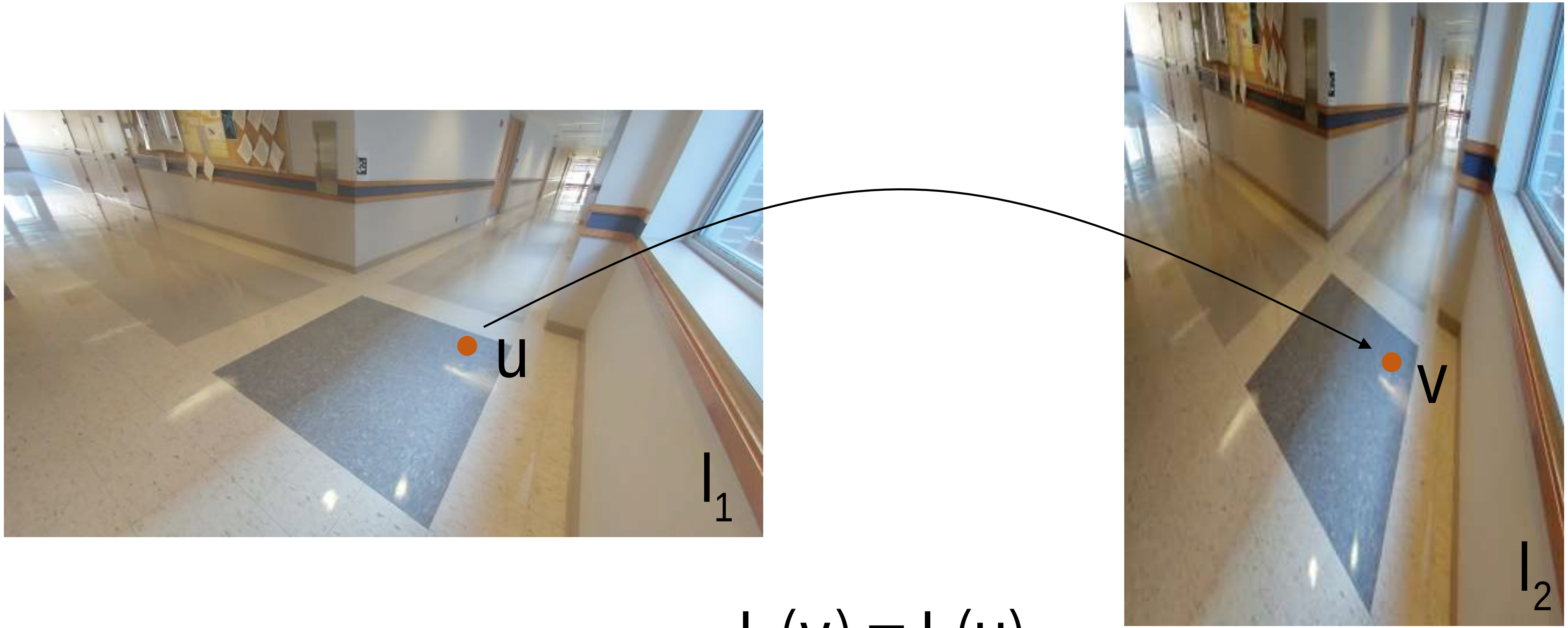
$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Uniform Scaling

$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & & \\ & s_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \qquad s_x = s_y$$

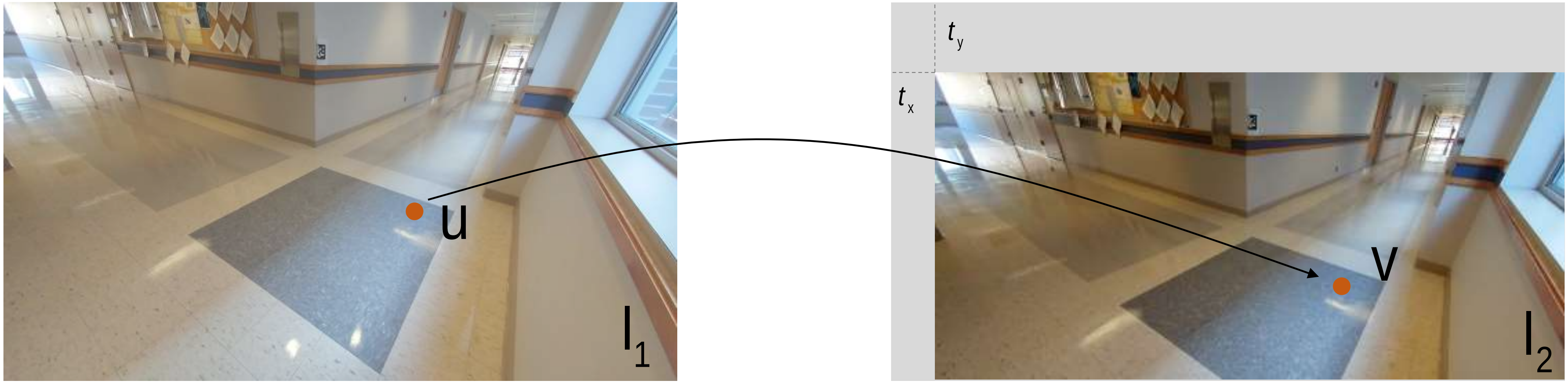
Aspect Ratio Change



$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & & \\ & s_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \qquad s_x \neq s_y$$

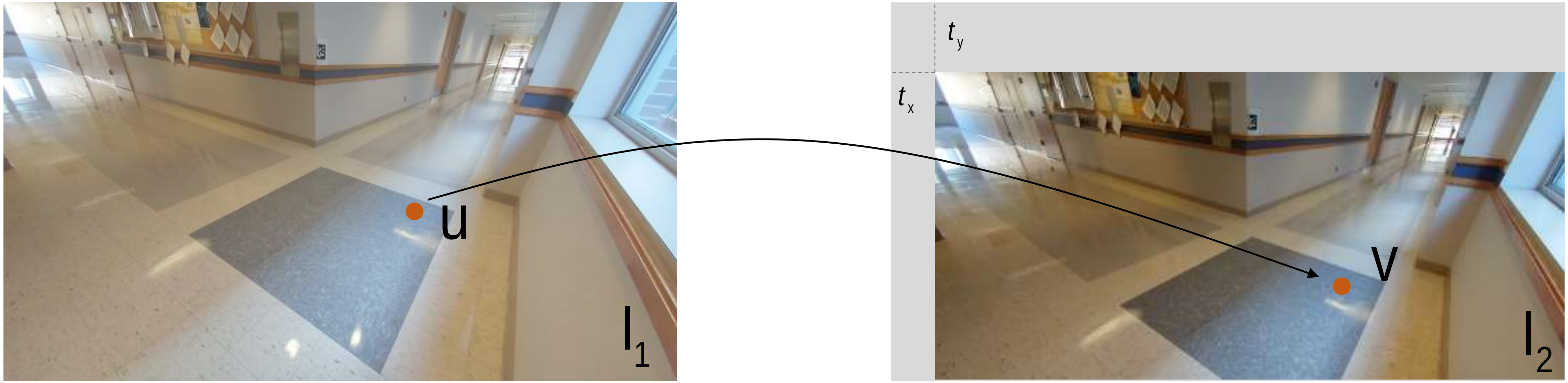
Translation



$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

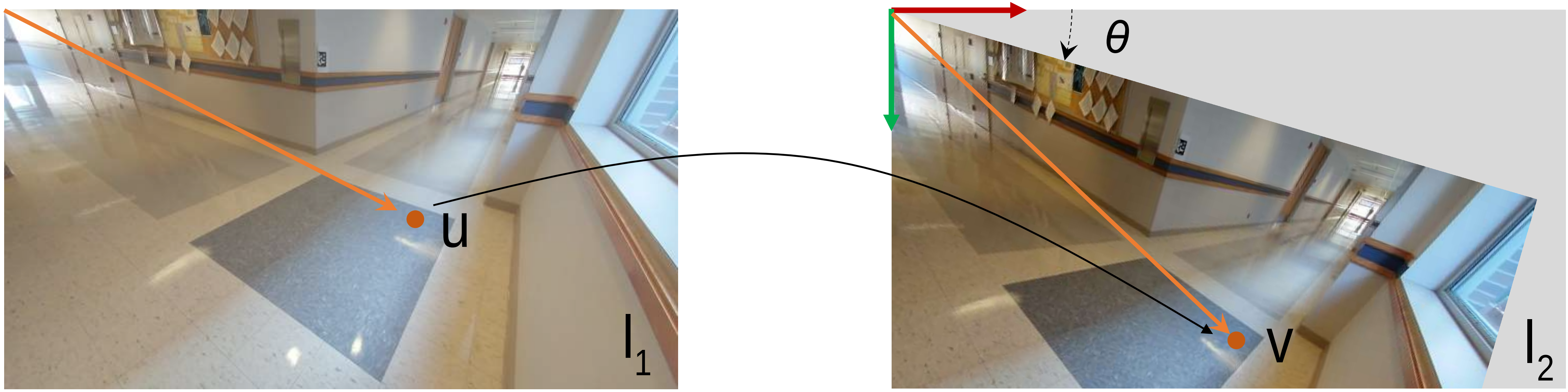
Translation



$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & t_x \\ & 1 & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

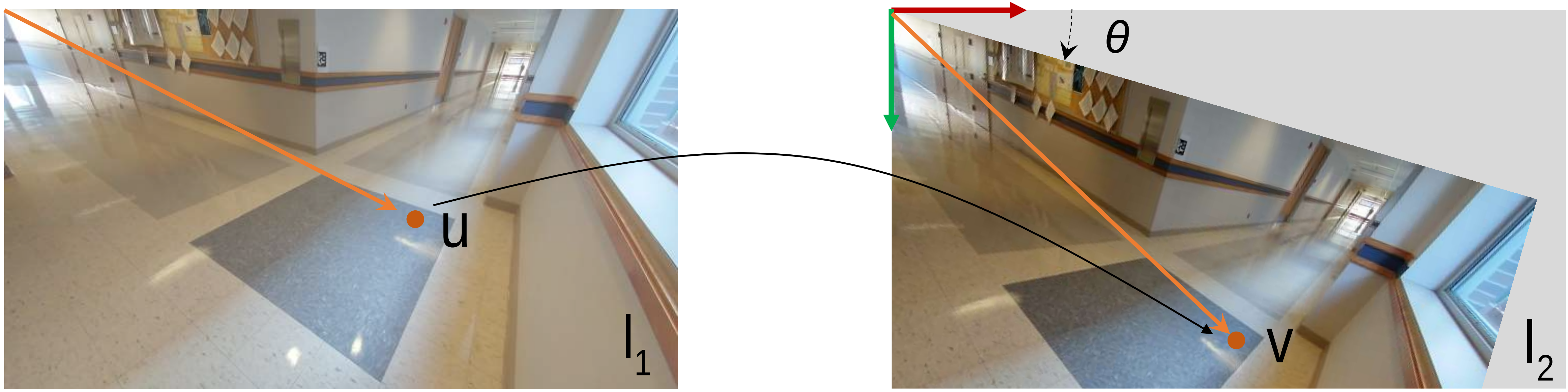
Rotation



$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

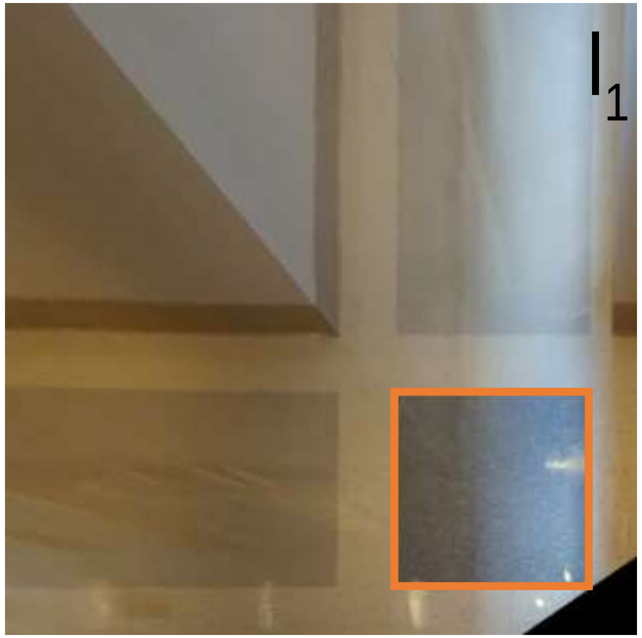
Rotation

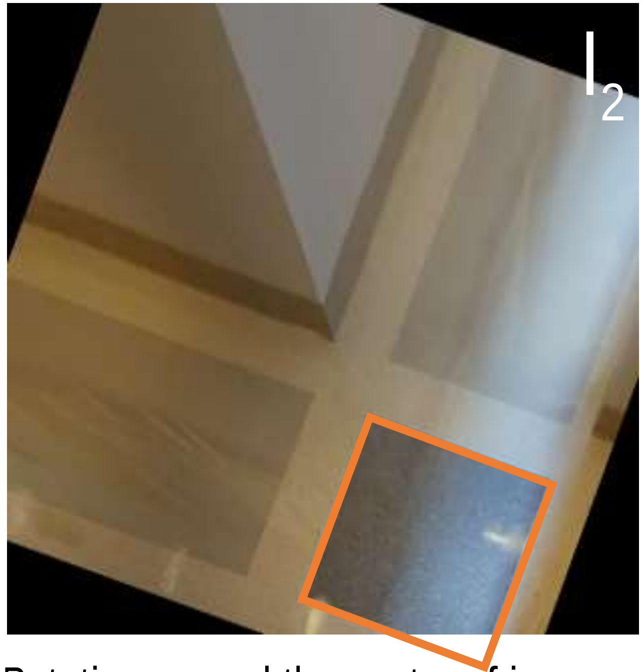


$$I_2(v) = I_1(u)$$

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean Transform SE(3)

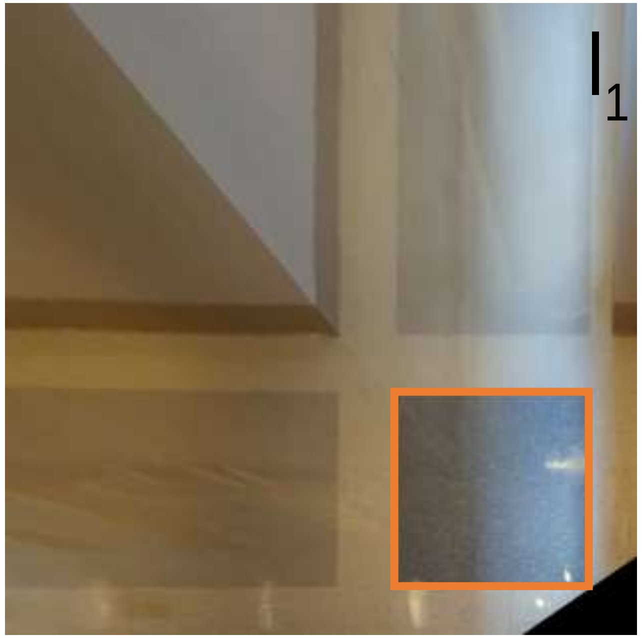


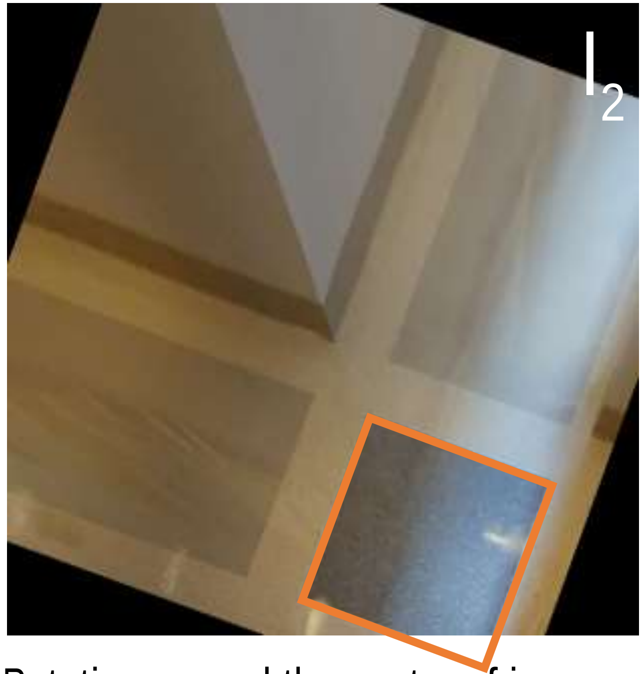


Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean Transform SE(3)

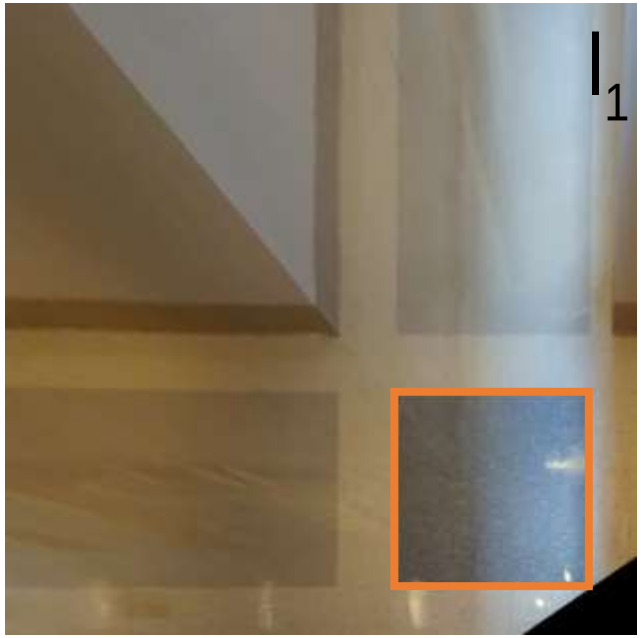


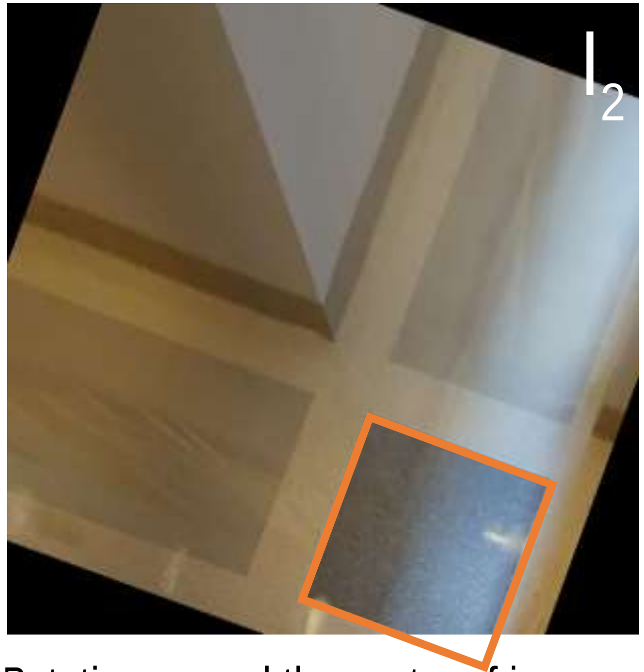


Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean Transform SE(3)





Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

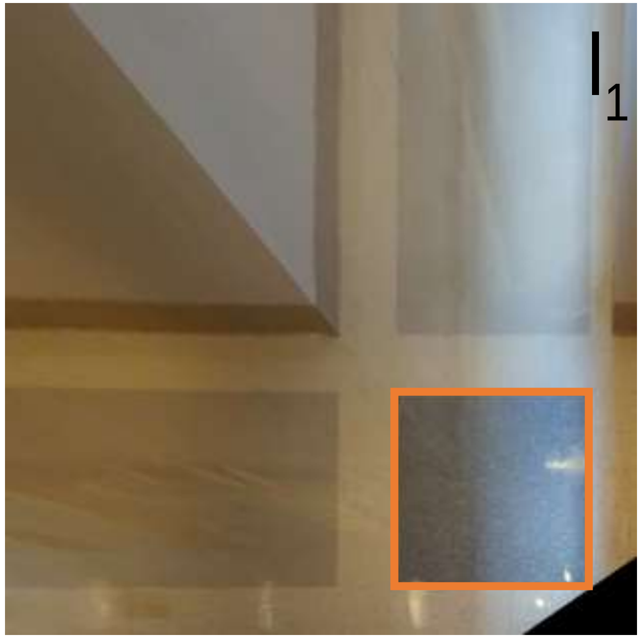
Invariant properties

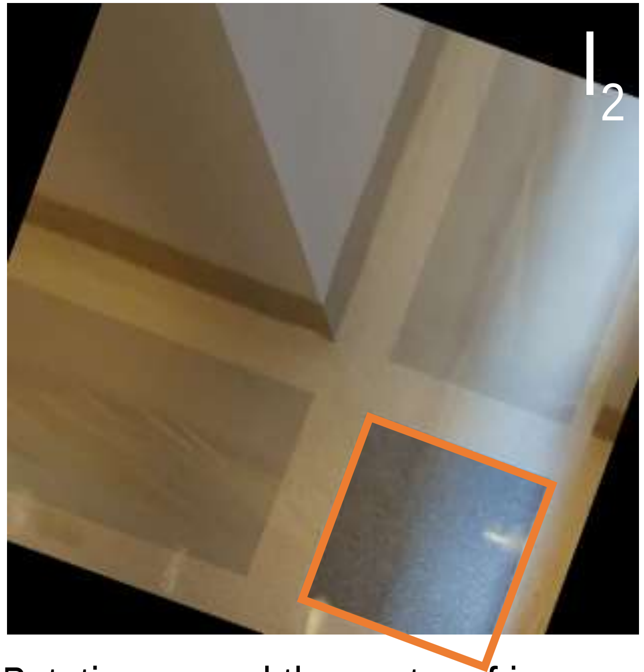
- Length
- Angle
- Area

Degree of freedom

3 (2 translation+1 rotation)

Euclidean Transform SE(3)





Rotation around the center of image

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} \mathrm{v} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{R} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{u} \\ 1 \end{bmatrix}$$

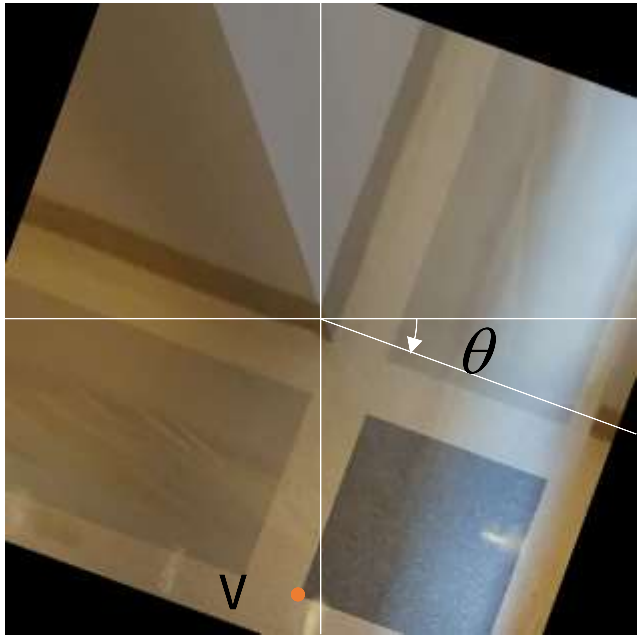
Invariant properties

- Length
- Angle
- Area

Degree of freedom

3 (2 translation+1 rotation)

Euclidean Transform SE(3)

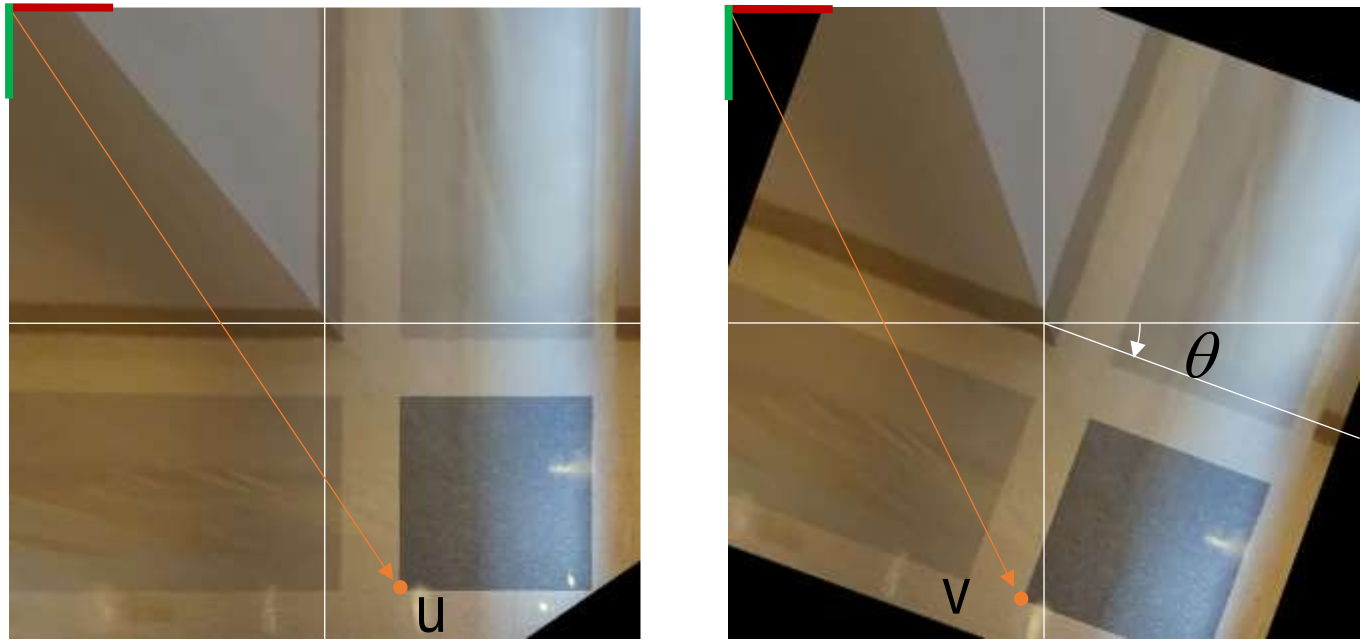


Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

t ?

Euclidean Transform SE(3)

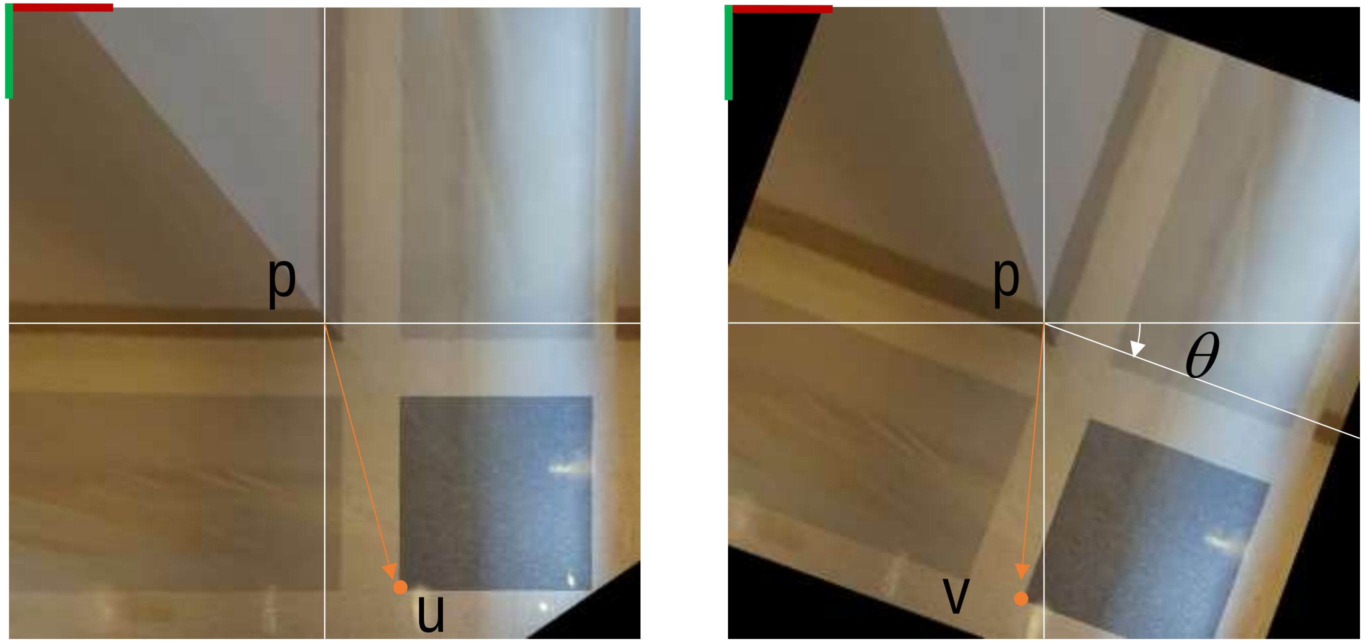


Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

t ?

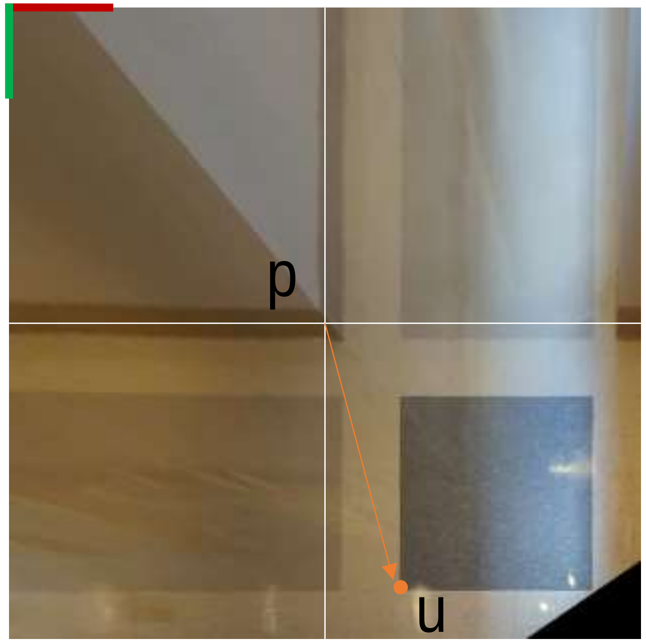
Euclidean Transform SE(3)

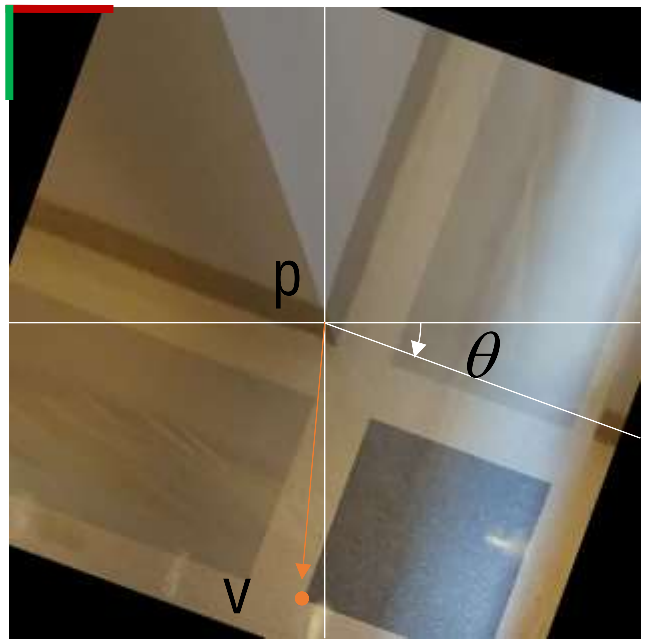


Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

Euclidean Transform SE(3)





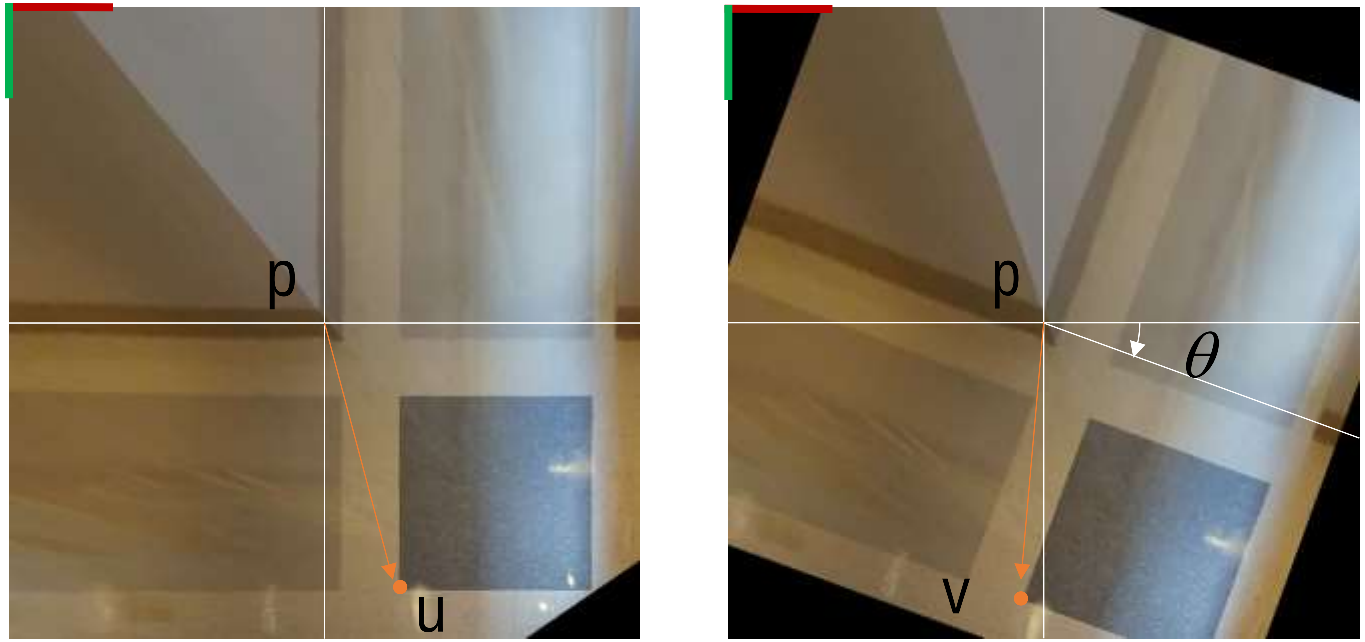
Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

$$\bar{u} = u - p \qquad \bar{v} = v - p$$

$$\longrightarrow \bar{v} = R\bar{u}$$

Euclidean Transform SE(3)



Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

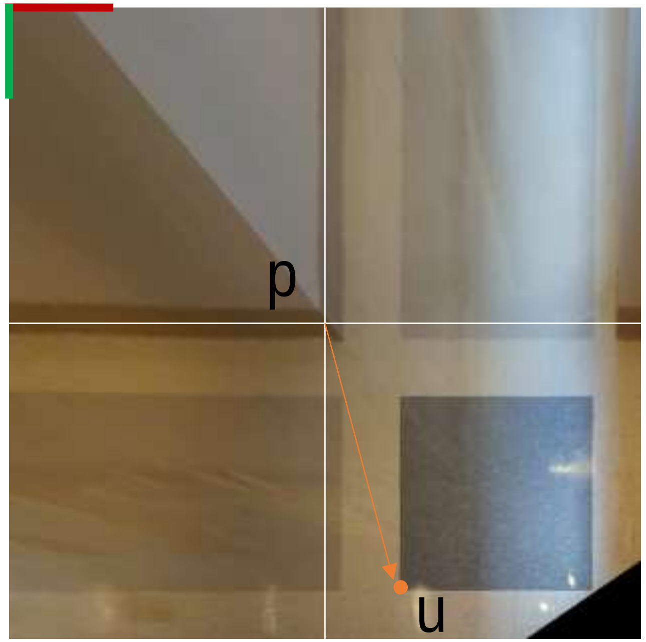
$\bar{u} = u - p \qquad \bar{v} = v - p$

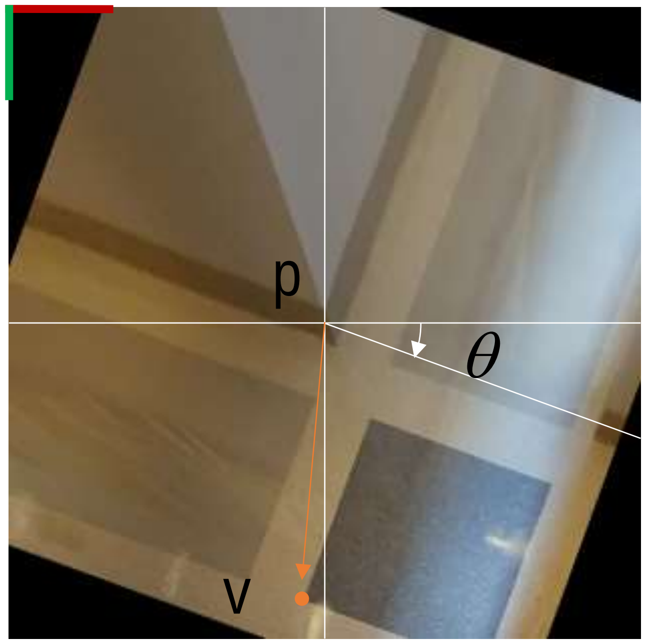
$\longrightarrow \bar{v} = R\bar{u}$

$\longrightarrow v - p = R(u - p)$

$\longrightarrow v = Ru - Rp + p$

Euclidean Transform SE(3)





Rotate about the image center

$$\begin{bmatrix} v \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix} \longrightarrow v = Ru + t$$

$$\bar{u} = u - p \qquad \bar{v} = v - p$$

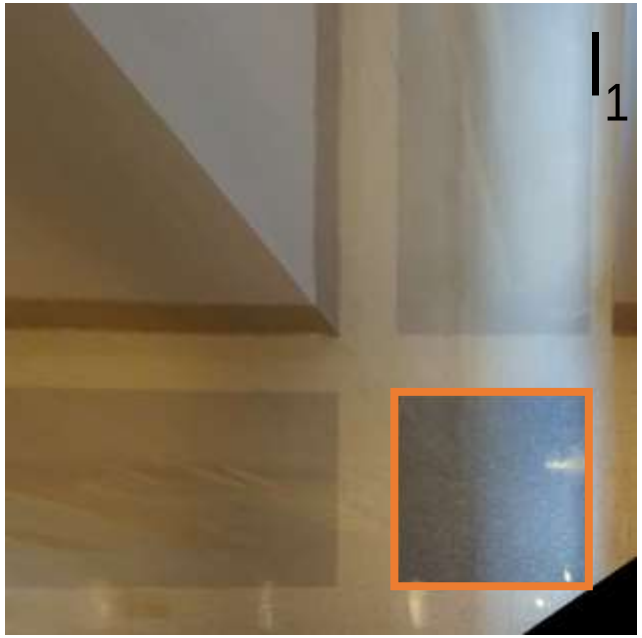
$$\longrightarrow \bar{v} = R\bar{u}$$

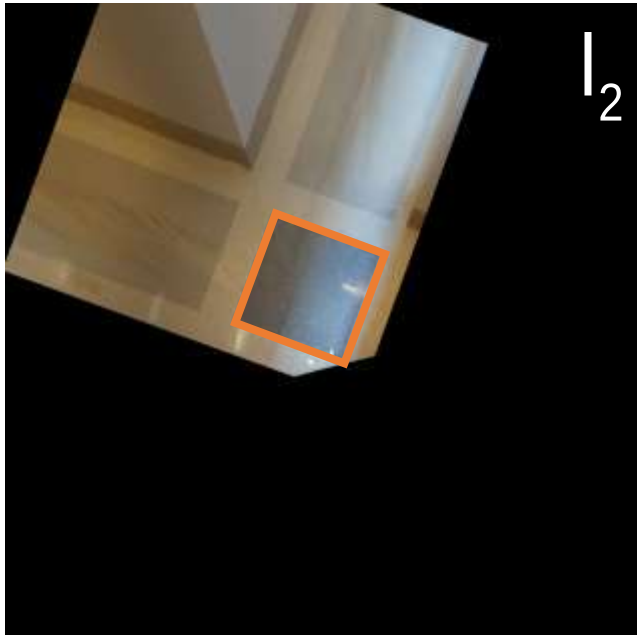
$$\longrightarrow v - p = R(u - p)$$

$$\longrightarrow v = Ru - Rp + p$$

$$\longrightarrow t = -Rp + p$$

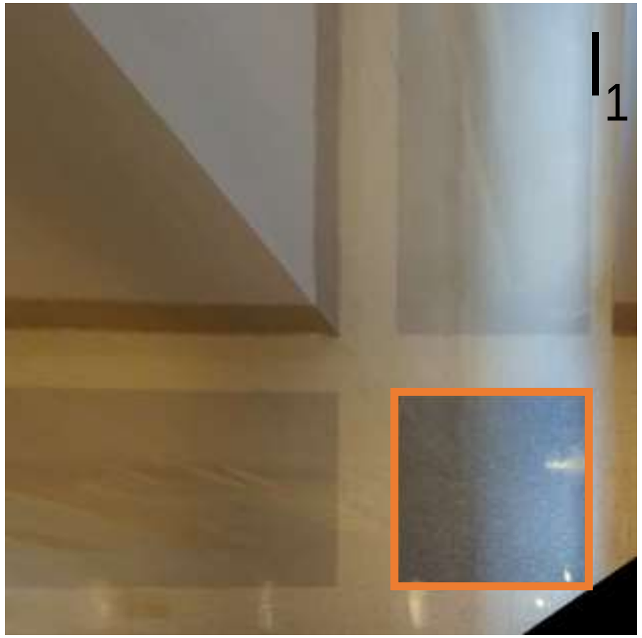
Similarity Transform

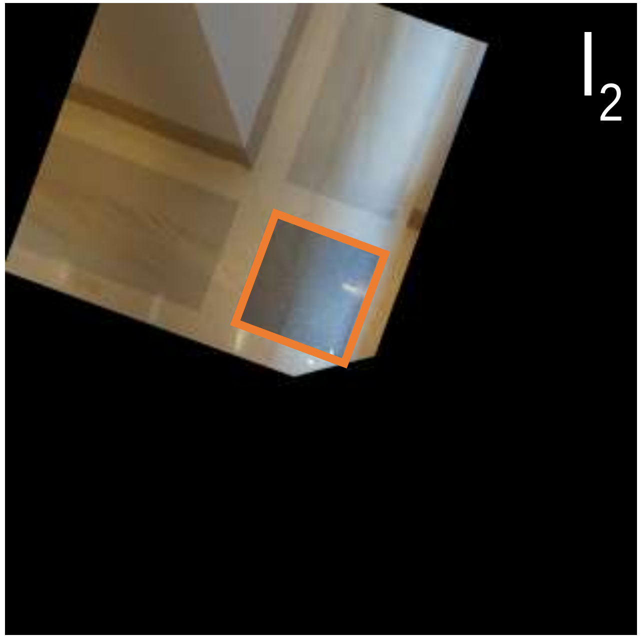




$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \quad ? \quad \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

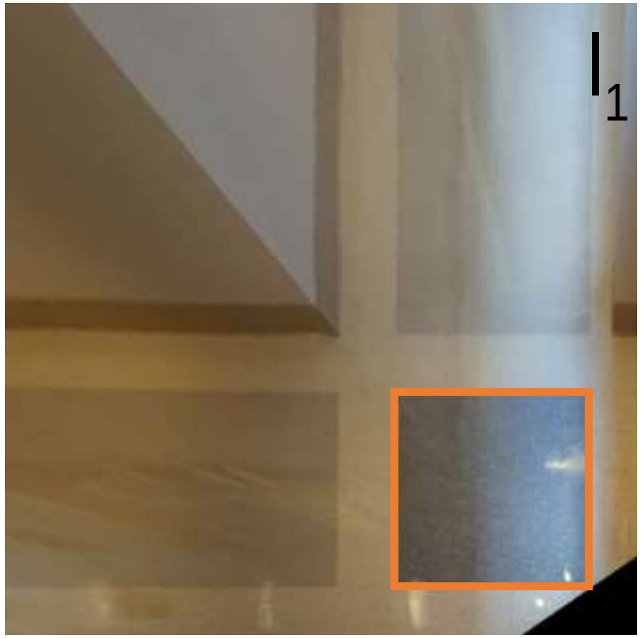
Similarity Transform

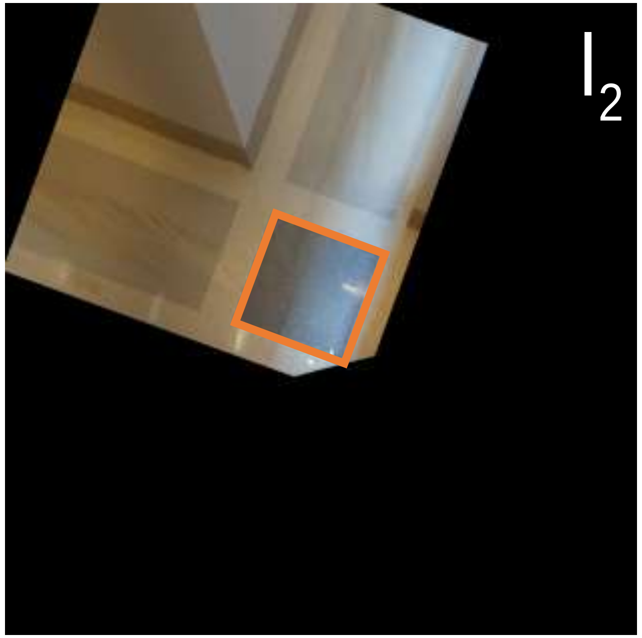




$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & & \\ & \alpha & \\ & & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Similarity Transform





Invariant properties

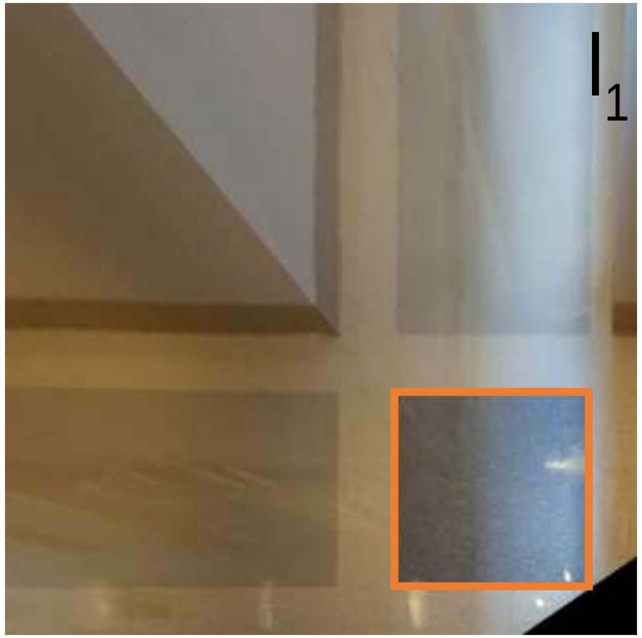
- Length ratio
- Angle

Degree of freedom

4 (2 translation+1 rotation+1 scale)

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & & \\ & \alpha & \\ & & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \mathrm{R} & \mathrm{t} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

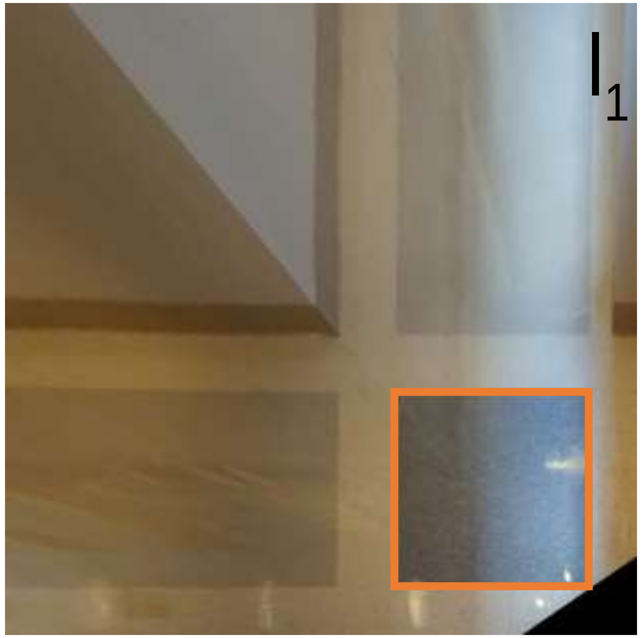
Affine Transform



$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

Affine Transform

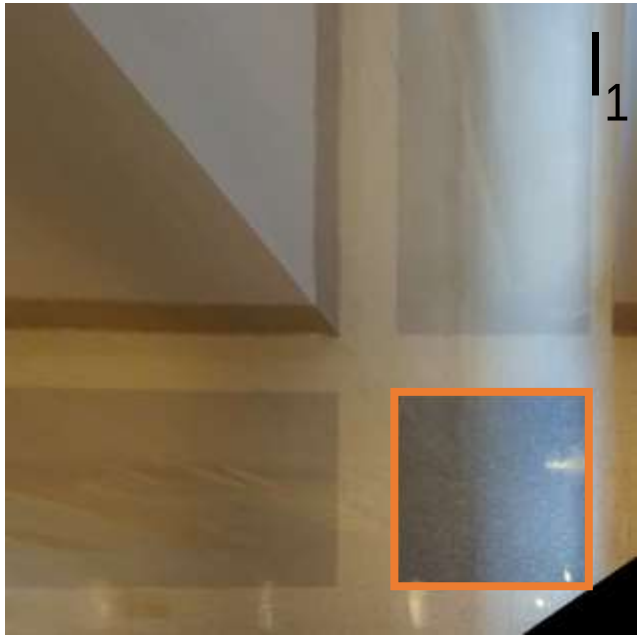


$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Affine Transform



Invariant properties

- Parallelism
- Ratio of area
- Ratio of length

Degree of freedom

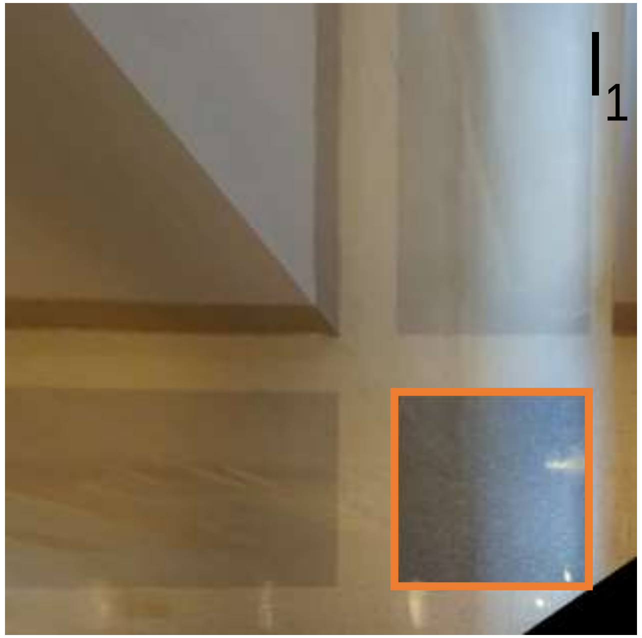
6

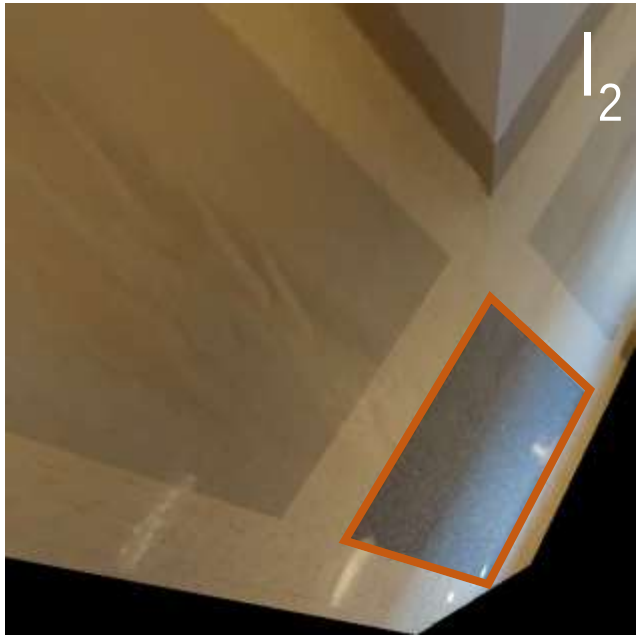
$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Euclidean transform

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

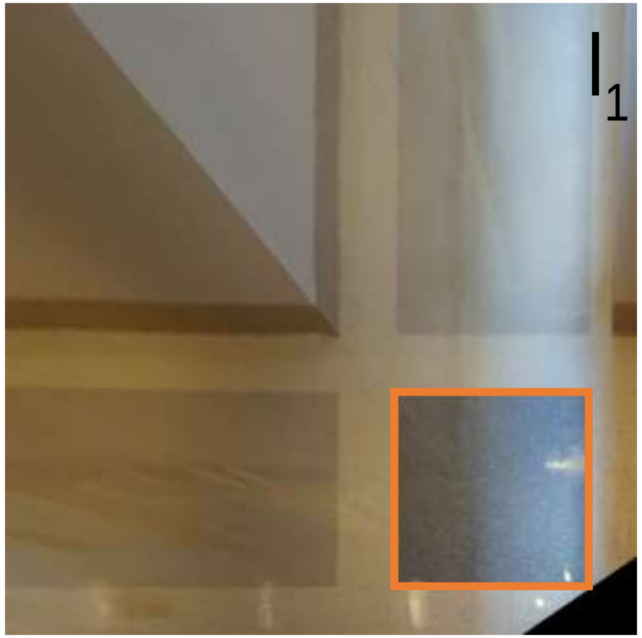
Perspective Transform (Homography)

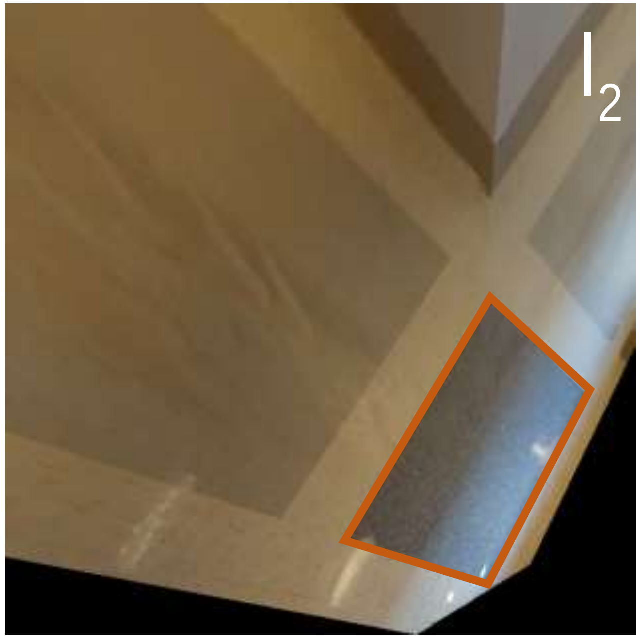




$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Perspective Transform (Homography)

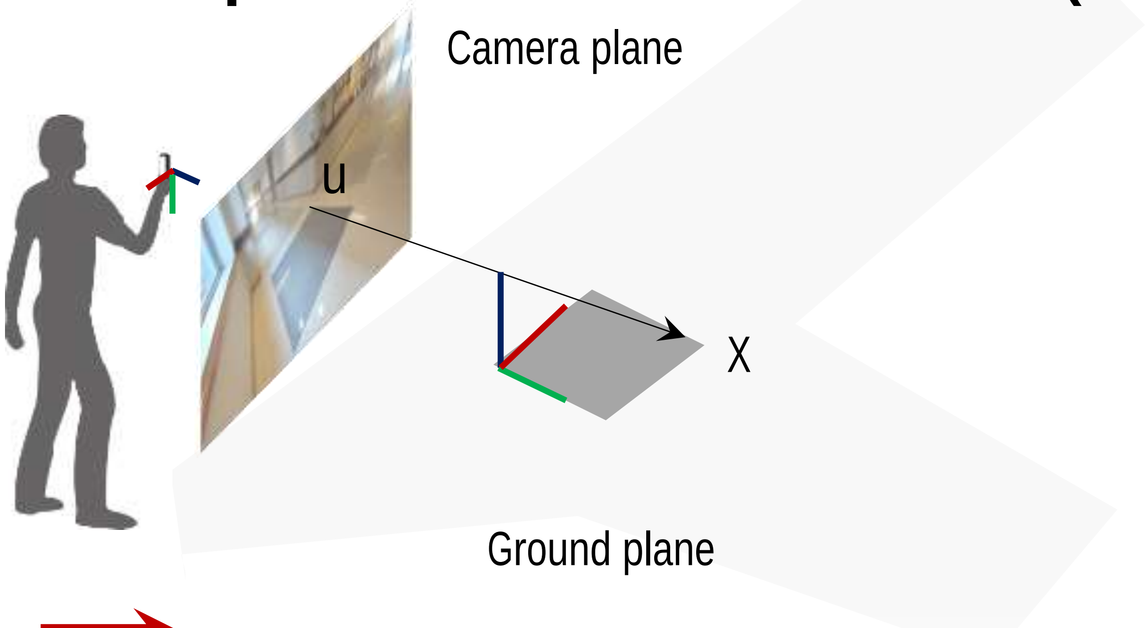




$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

: General form of plane to plane linear mapping

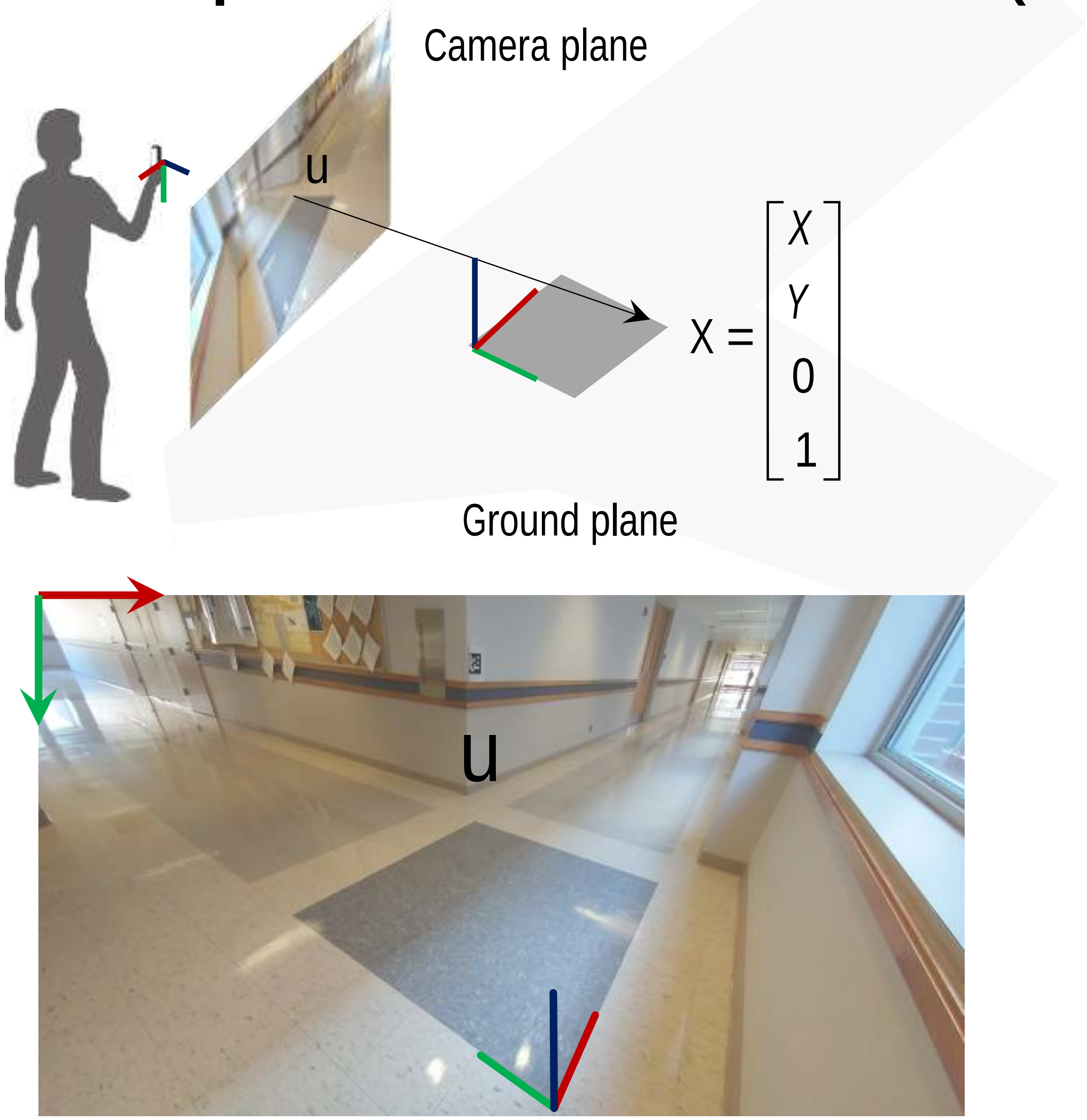
Perspective Transform (Homography)



$$\lambda \underline{u} = K \begin{bmatrix} R & t \end{bmatrix} \underline{X}$$

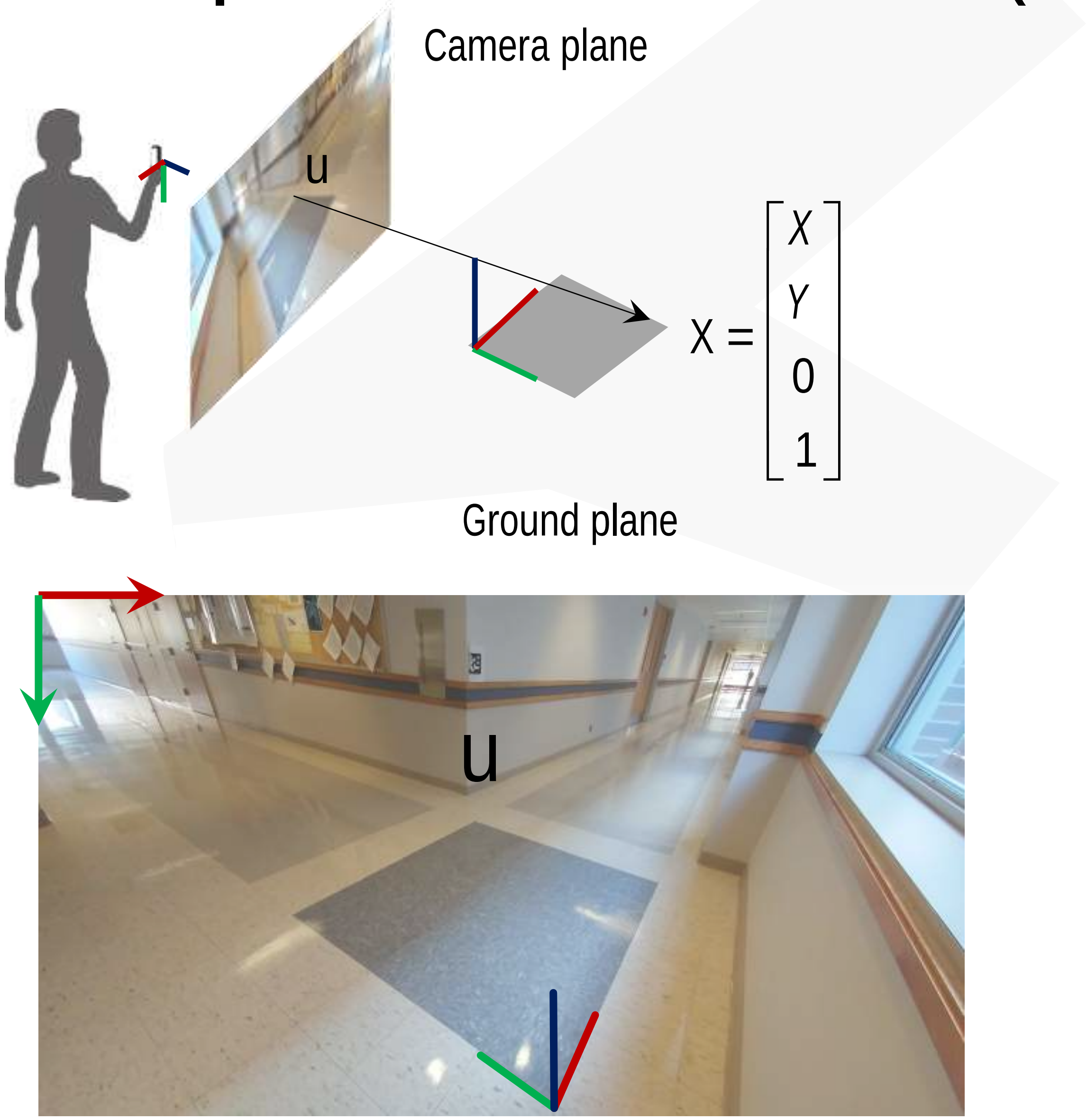
Camera plane　　Ground plane

Perspective Transform (Homography)



$$\underbrace{\lambda u}_{\text{Camera plane}} = K\begin{bmatrix} R & t \end{bmatrix}\underbrace{X}_{\text{Ground plane}}$$

Perspective Transform (Homography)

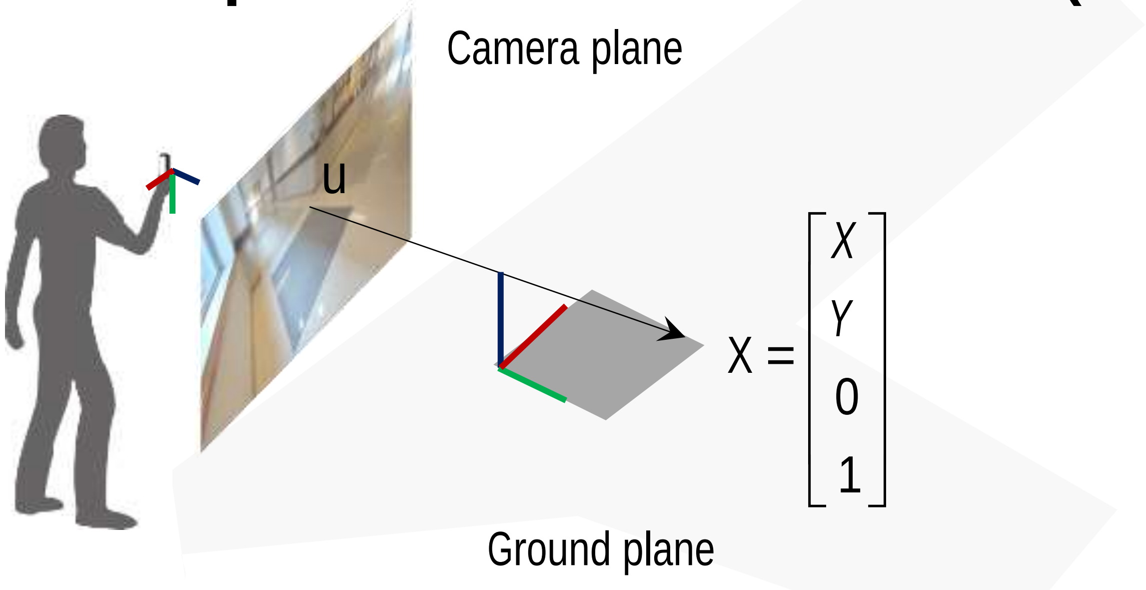


$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

Camera plane (λu) Ground plane (X)

$$\longrightarrow \quad \lambda u = K\begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Perspective Transform (Homography)



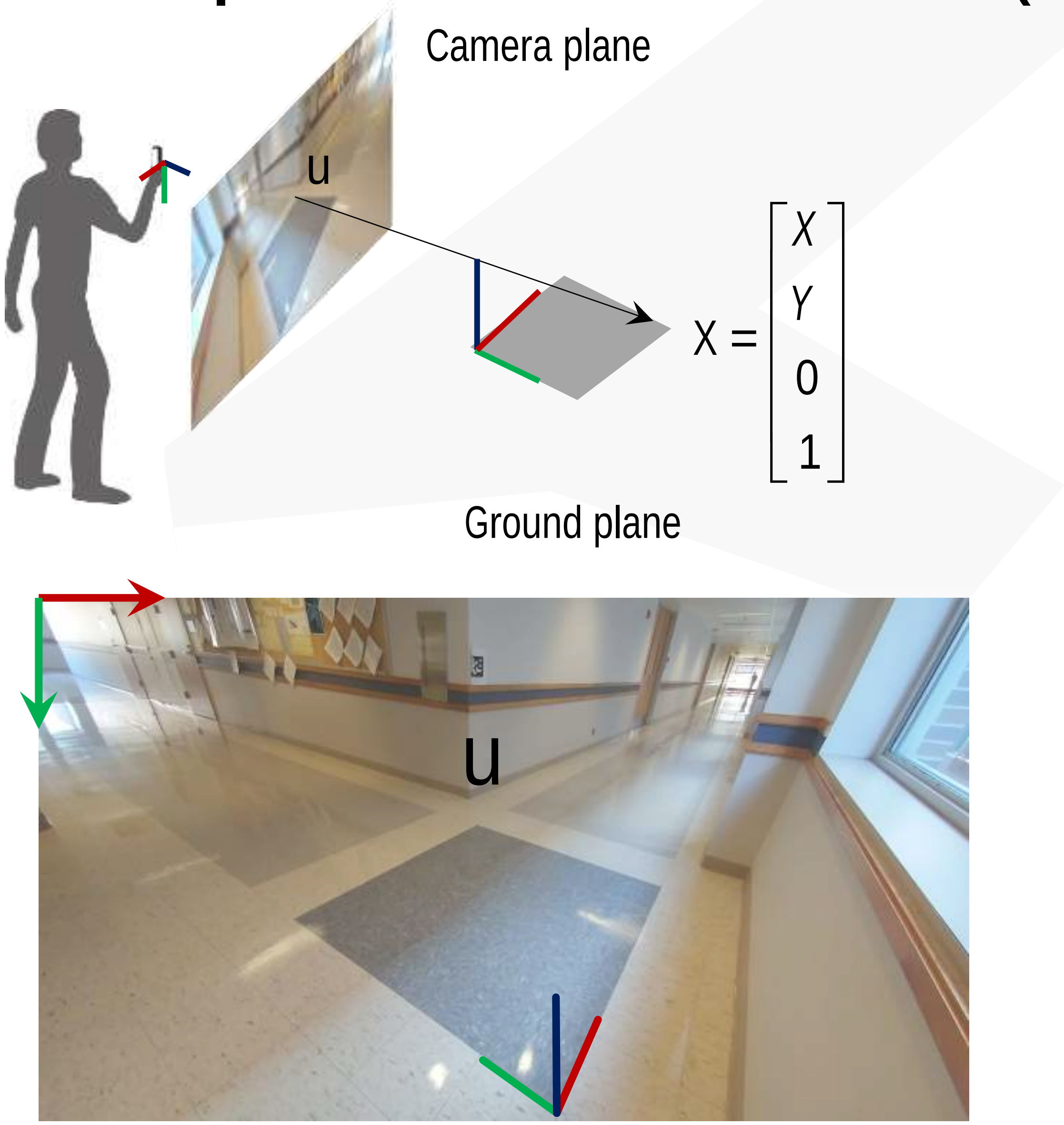
$$\lambda \mathbf{u} = K\begin{bmatrix} R & \mathbf{t} \end{bmatrix} X$$

Camera plane — Ground plane

$$\longrightarrow \lambda \mathbf{u} = K\begin{bmatrix} r_1 & r_2 & r_3 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \mathbf{u} = K\begin{bmatrix} r_1 & r_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Perspective Transform (Homography)



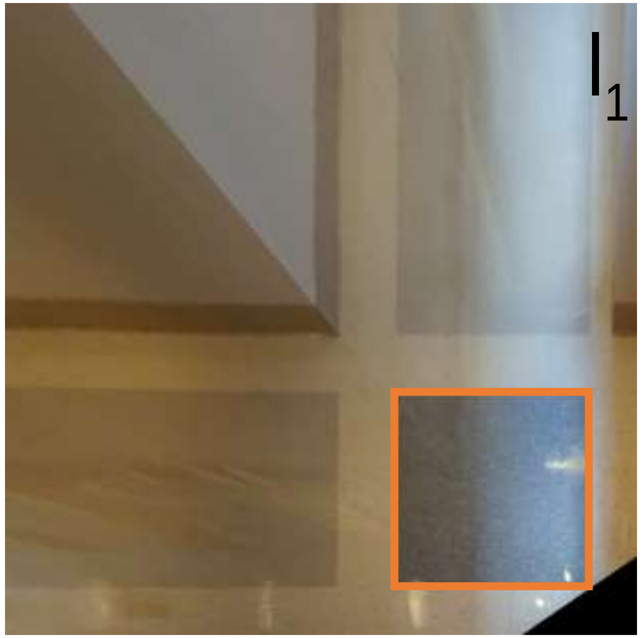
$$\underbrace{\lambda u}_{\text{Camera plane}} = K\begin{bmatrix} R & t \end{bmatrix}\underbrace{X}_{\text{Ground plane}}$$

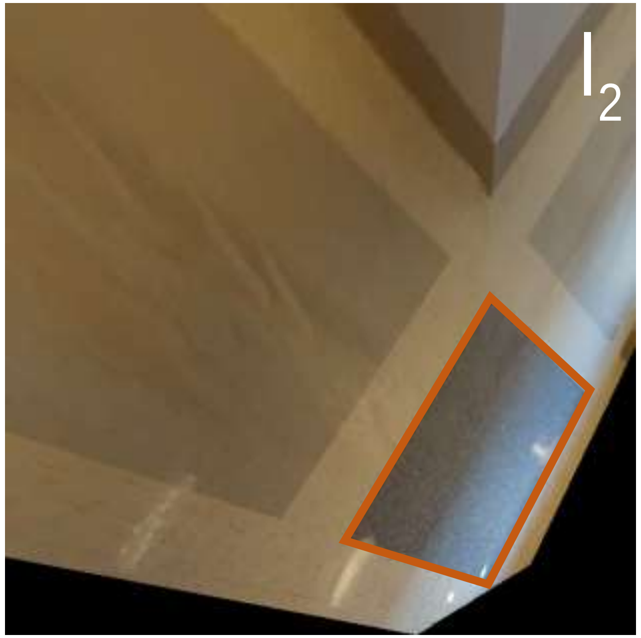
$$\longrightarrow \lambda u = K\begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda u = K\begin{bmatrix} r_1 & r_2 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda\begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = H\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Perspective Transform (Homography)

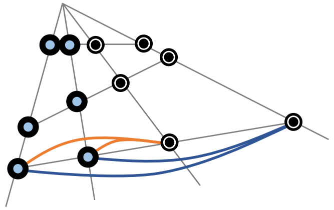




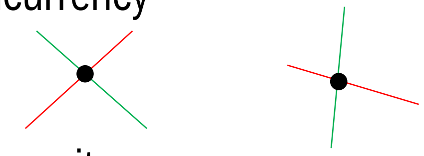
$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

Invariant properties

- Cross ratio



- Concurrency

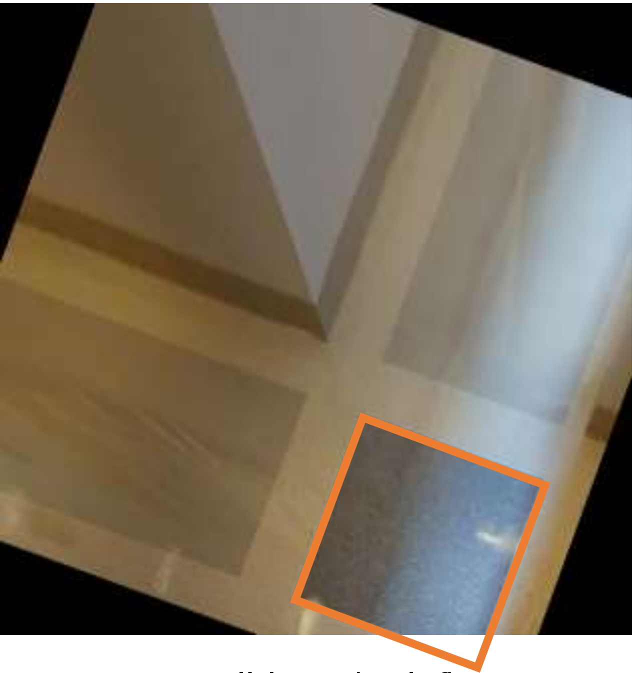


- Colinearity

Degree of freedom

8 (9 variables – 1 scale)

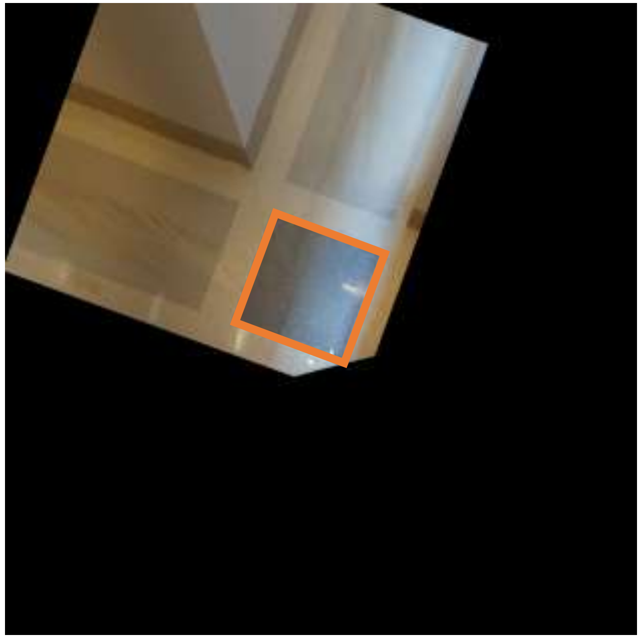
Hierarchy of Transformations



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Similarity (4 dof)

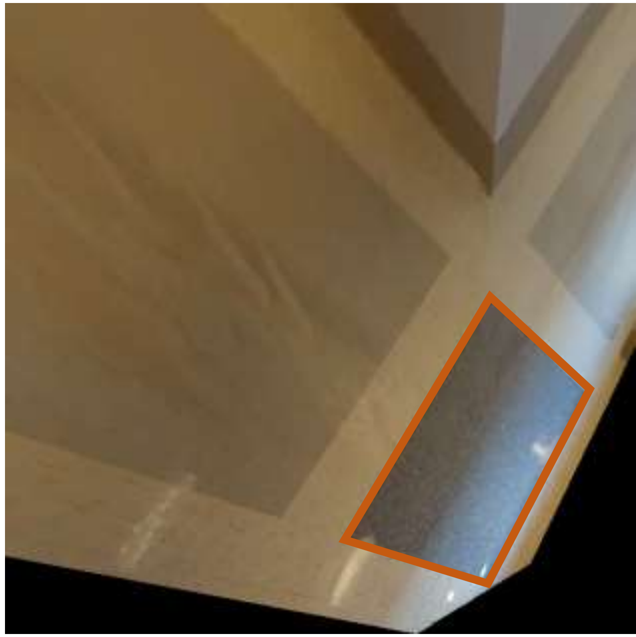
- Length ratio
- Angle

$$\begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

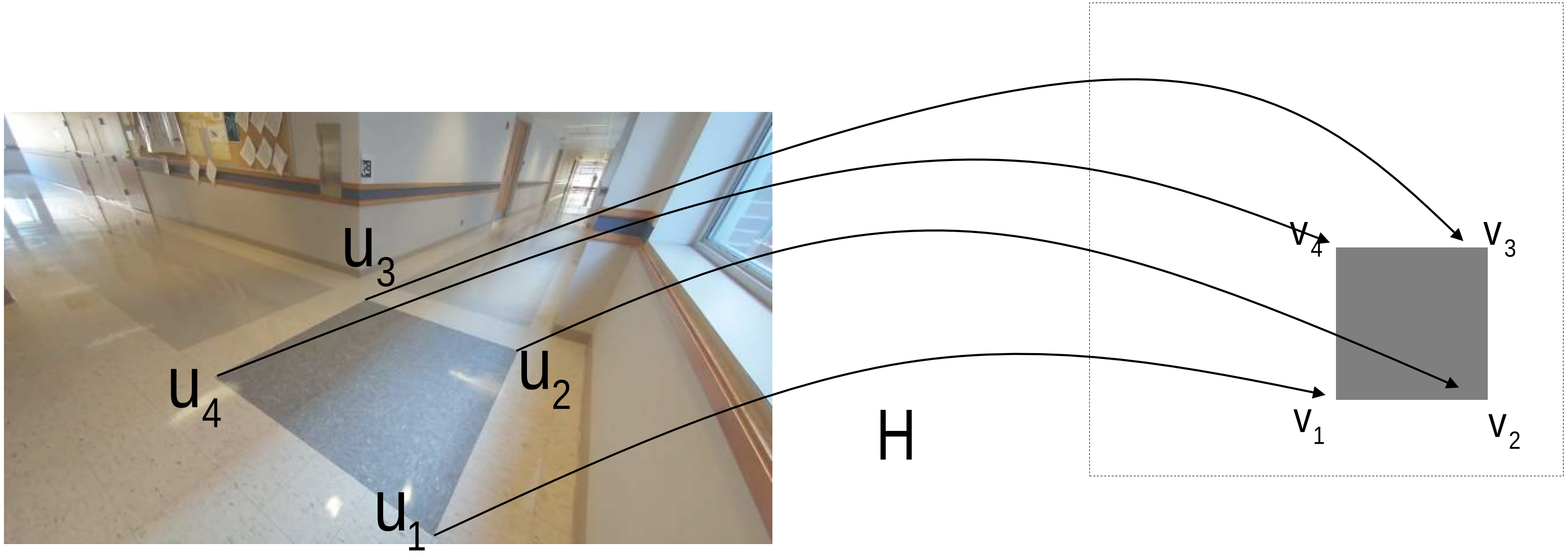


Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

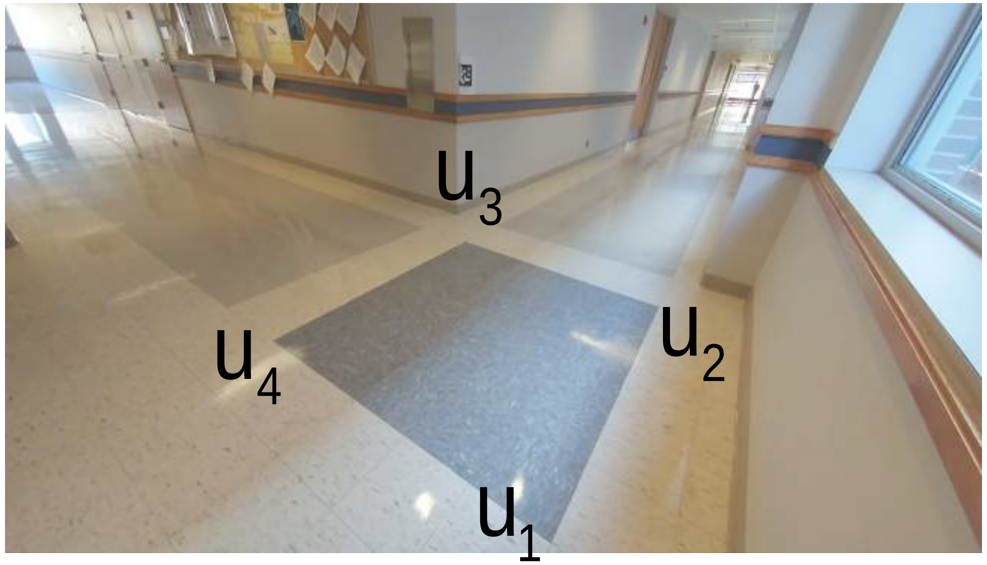
$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

Fun with Homography



The image can be rectified as if it is seen from top view.

Fun with Homography



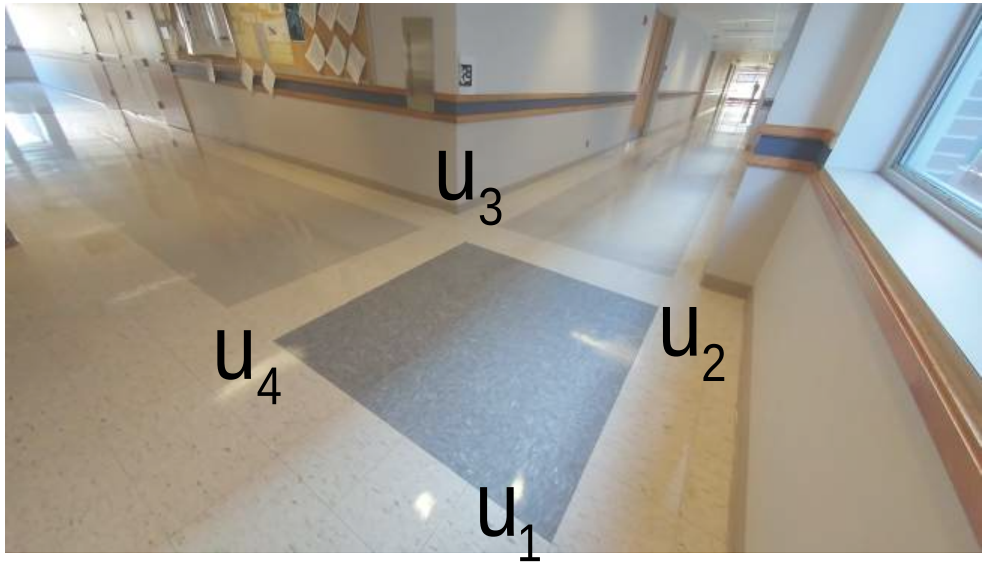
RectificationViaHomography.m

```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, H);
```

Fun with Homography



RectificationViaHomography.m

```
u = [u1'; u2'; u3'; u4'];
v = [v1'; v2'; v3'; v4'];

% Need at least non-colinear four points
H = ComputeHomography(v, u);

im_warped = ImageWarping(im, H);
```

ImageWarping.m

```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
u_z = H(3,1)*v_x + H(3,2)*v_y + H(3,3);

u_x = u_x./u_z;
u_y = u_y./u_z;
```

$$\lambda \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} = H \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

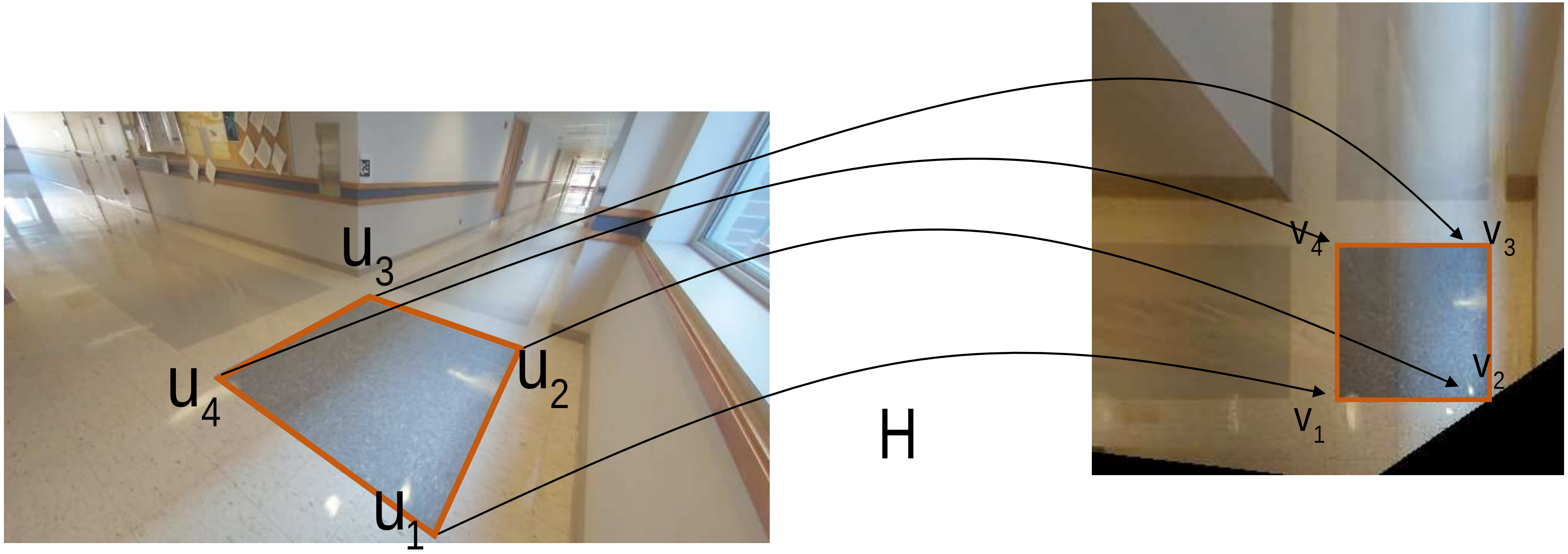
```
im_warped(:,:,1) = reshape(interp2(im(:,:,1), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,2) = reshape(interp2(im(:,:,2), u_x(:), u_y(:)), [h, w]);
im_warped(:,:,3) = reshape(interp2(im(:,:,3), u_x(:), u_y(:)), [h, w]);

im_warped = uint8(im_warped);
```

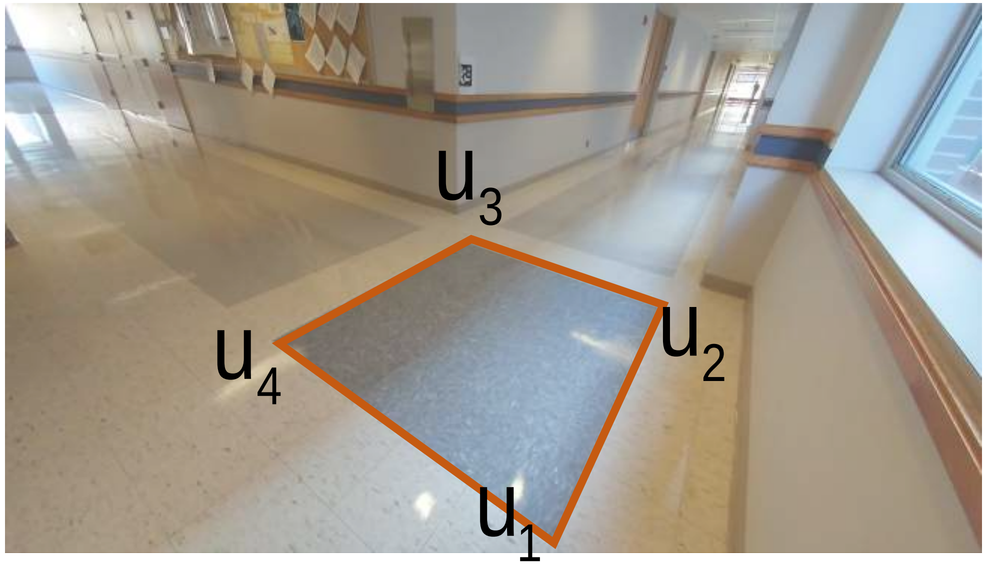
Cf) ImageWarpingEuclidean.m

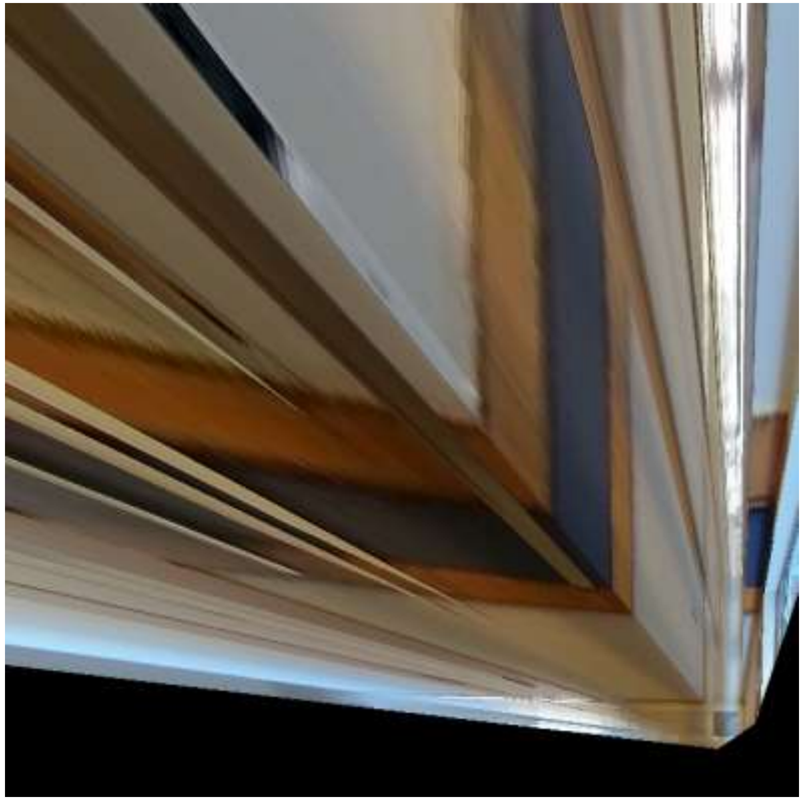
```
u_x = H(1,1)*v_x + H(1,2)*v_y + H(1,3);
u_y = H(2,1)*v_x + H(2,2)*v_y + H(2,3);
```

Fun with Homography



Fun with Homography





Fun with Homography

Fun with Homography

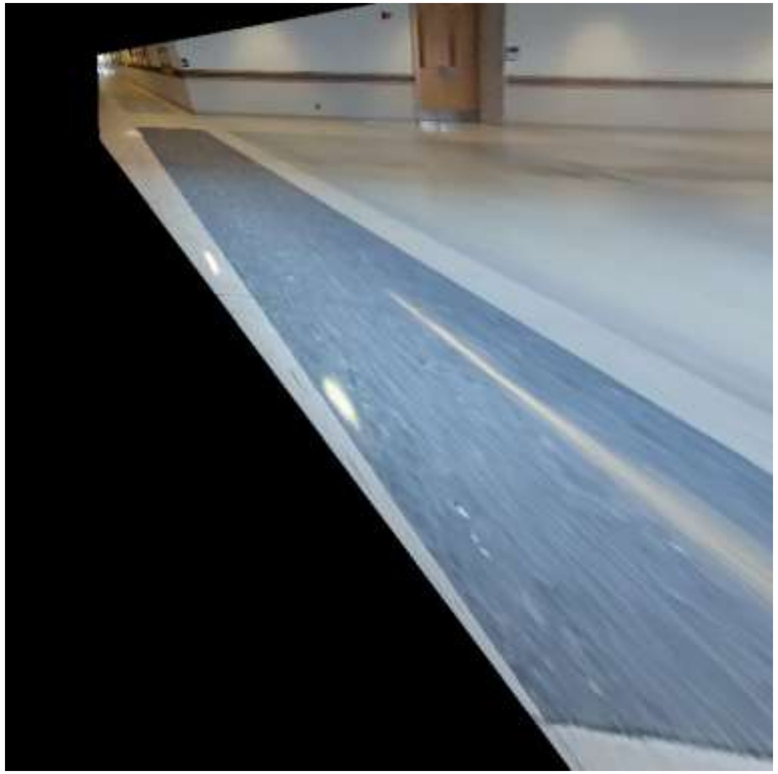


Image Warping

Image Inpainting

Virtual Advertisement

Image Transform via Plane

Keller entrance left

Keller entrance right

Image Transform via 3D Plane

$$\lambda \mathrm{u} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \mathrm{X}$$

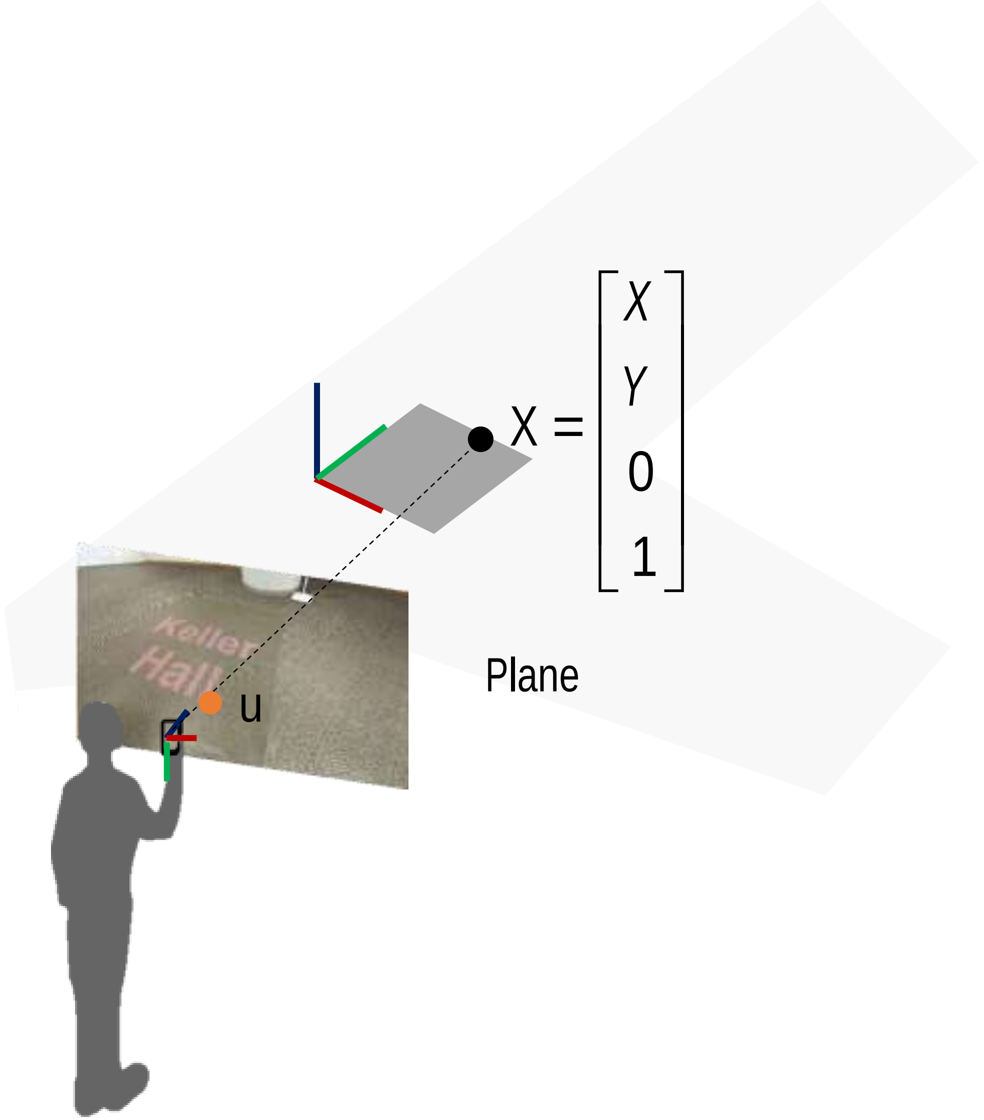


Image Transform via 3D Plane

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

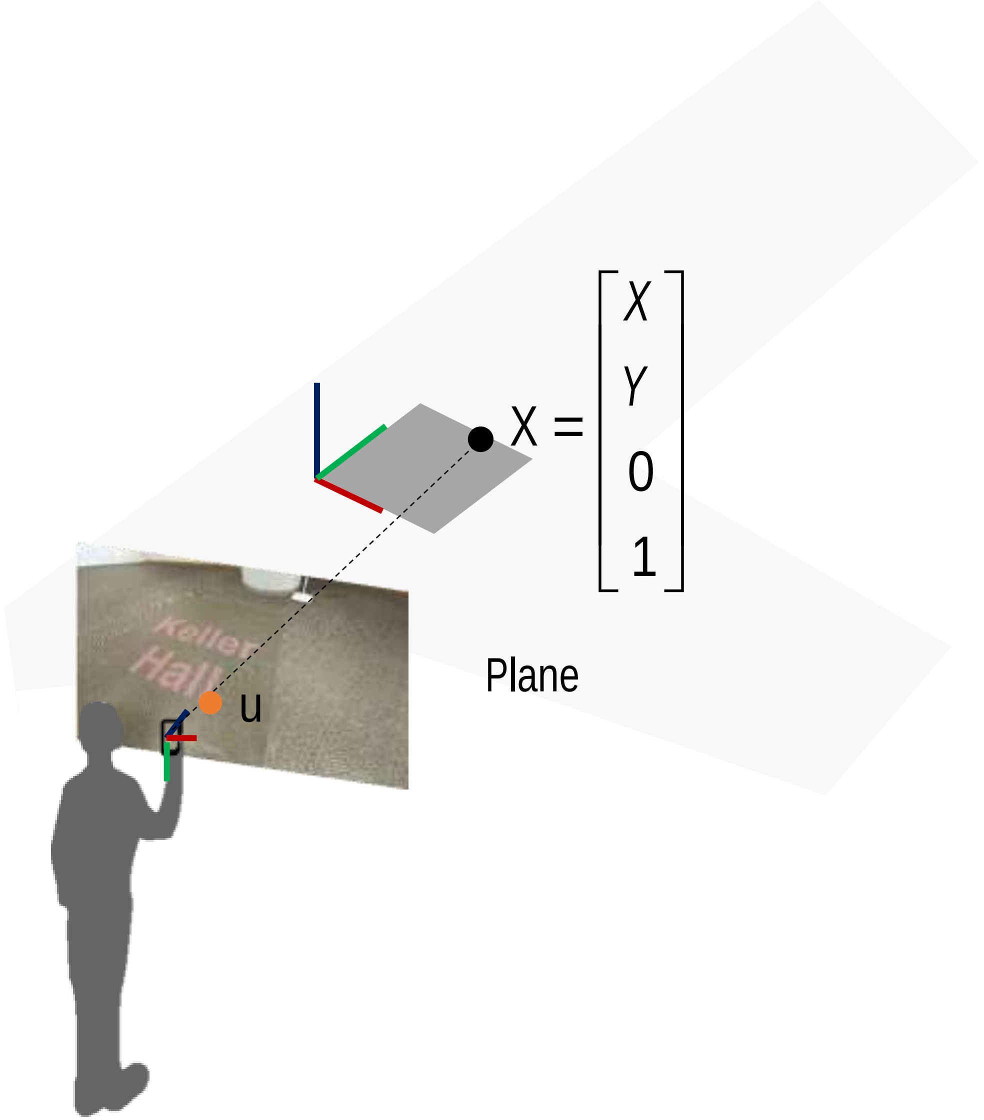
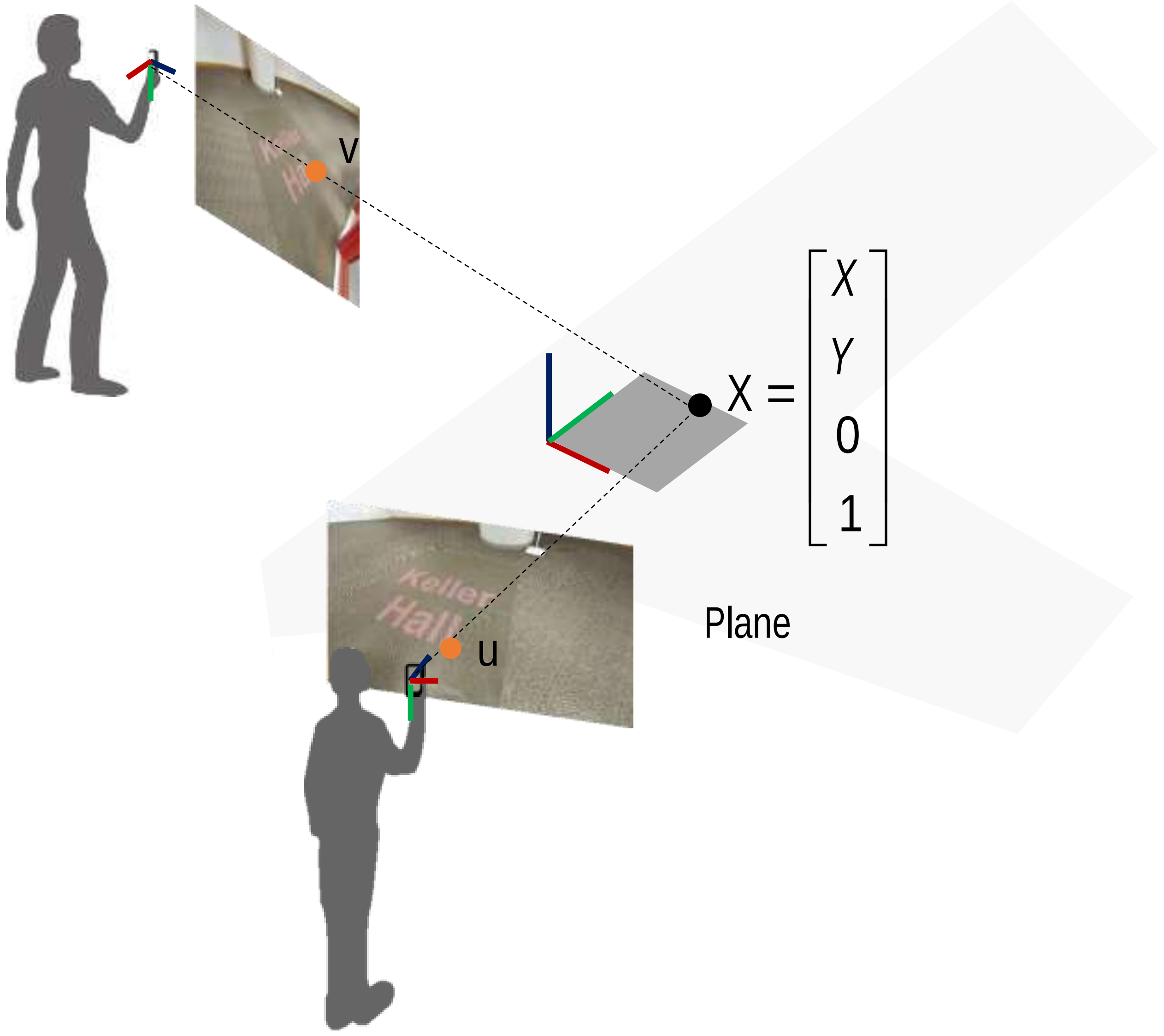


Image Transform via 3D Plane



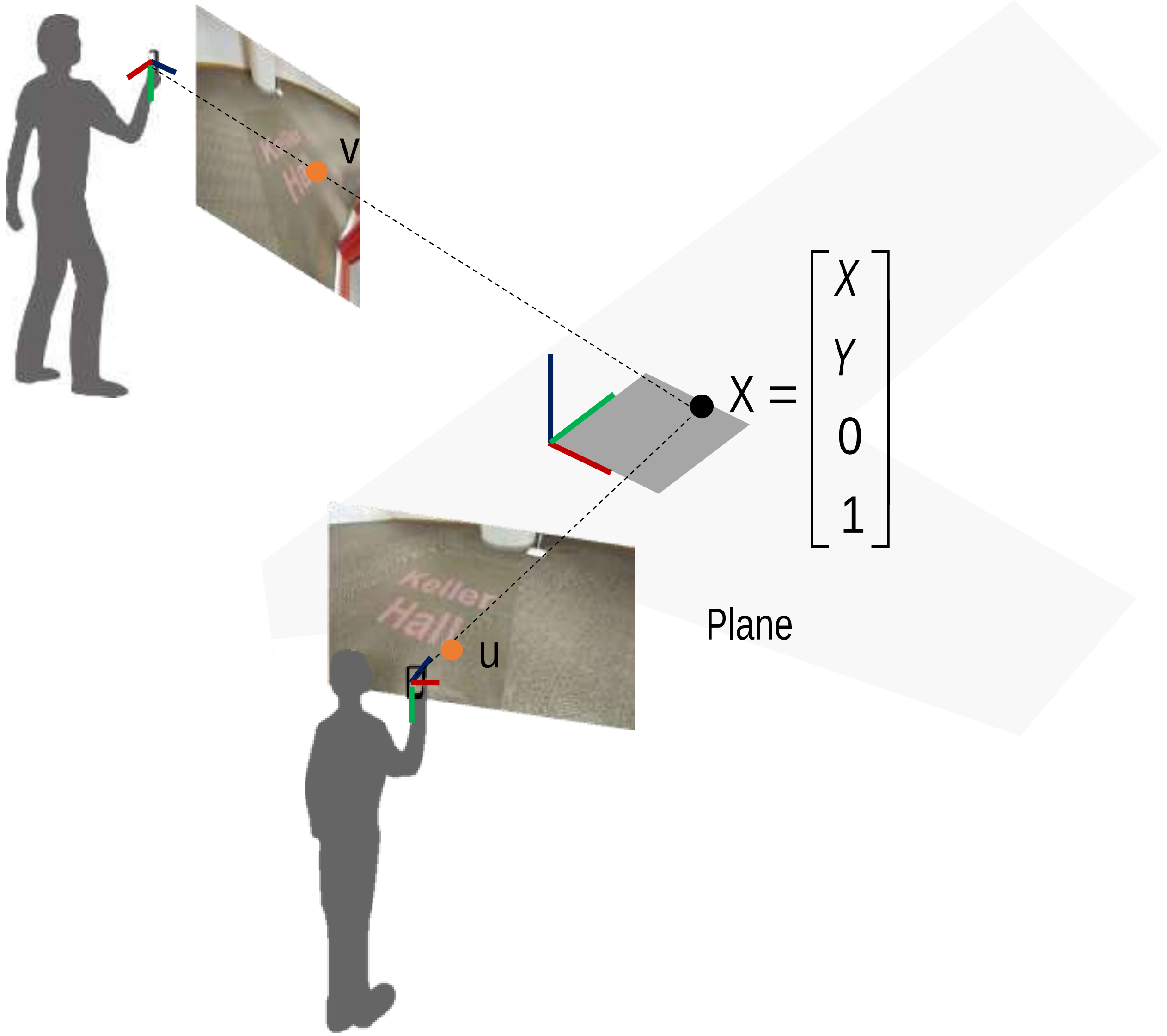
$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \mathrm{u} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix}\mathrm{X}$$

$$\longrightarrow \quad \lambda \mathrm{u} = \mathrm{K}\begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu \mathrm{v} = \mathrm{K}'\begin{bmatrix} \mathrm{r}_1' & \mathrm{r}_2' & \mathrm{t}' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Image Transform via 3D Plane



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

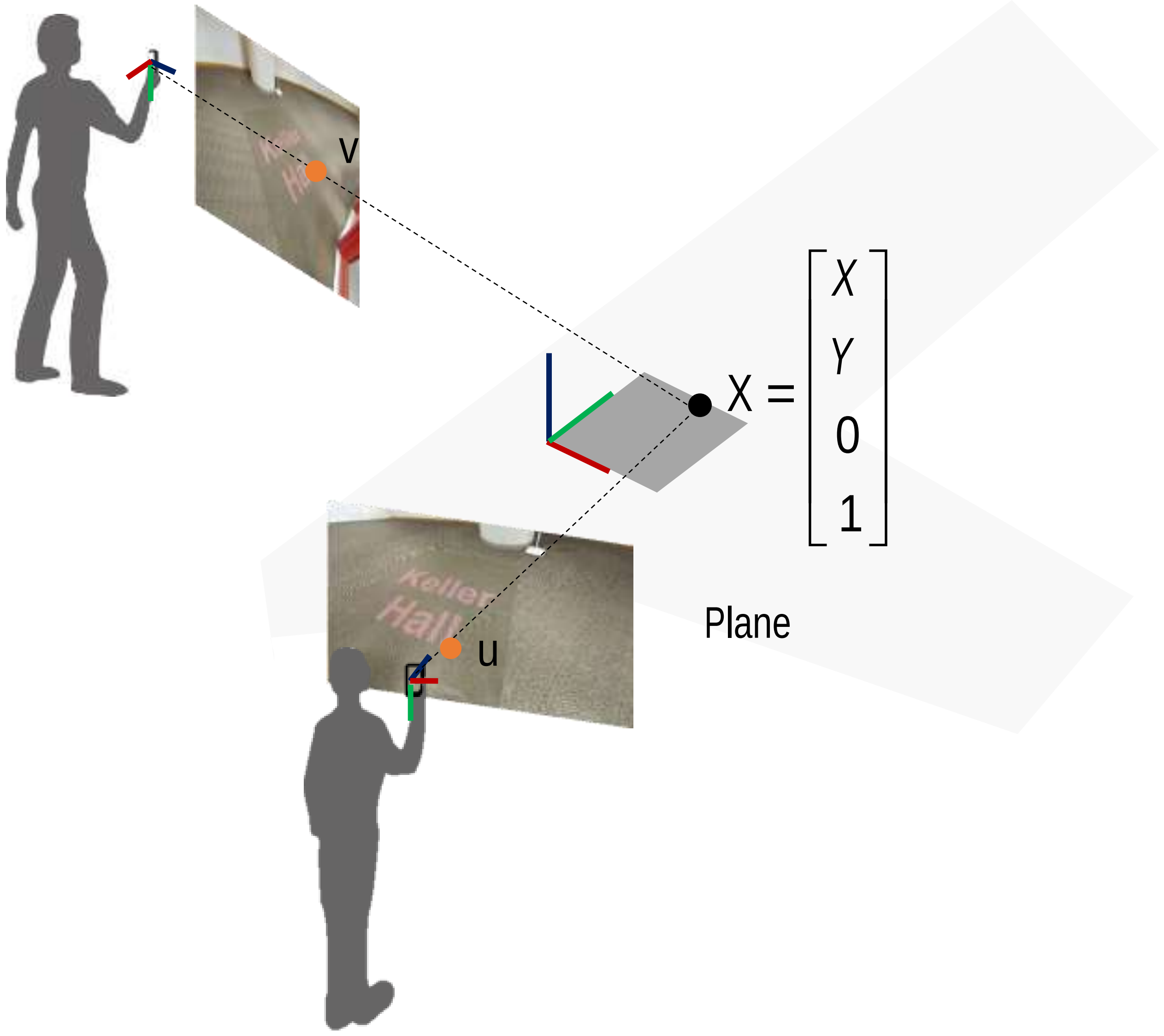
$$\lambda u = K\begin{bmatrix} R & t \end{bmatrix} X$$

$$\longrightarrow \quad \lambda u = K\begin{bmatrix} r_1 & r_2 & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu v = K'\begin{bmatrix} r_1' & r_2' & t' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

How are two image coordinates (u,v) related?

Image Transform via 3D Plane



$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\mathbf{X}$$

$$\longrightarrow \quad \lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mu \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

How are two image coordinates (u,v) related?

$$\longrightarrow \quad \lambda \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}^{-1}\mathbf{K}^{-1}\mathbf{u} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mu \begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}^{-1}\mathbf{K}'^{-1}\mathbf{v}$$

$$\alpha \mathbf{v} = \mathbf{K}'\begin{bmatrix} \mathbf{r}_1' & \mathbf{r}_2' & \mathbf{t}' \end{bmatrix}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}^{-1}\mathbf{K}^{-1}\mathbf{u}$$

$$\alpha \mathbf{v} = \mathbf{H}\mathbf{u}$$

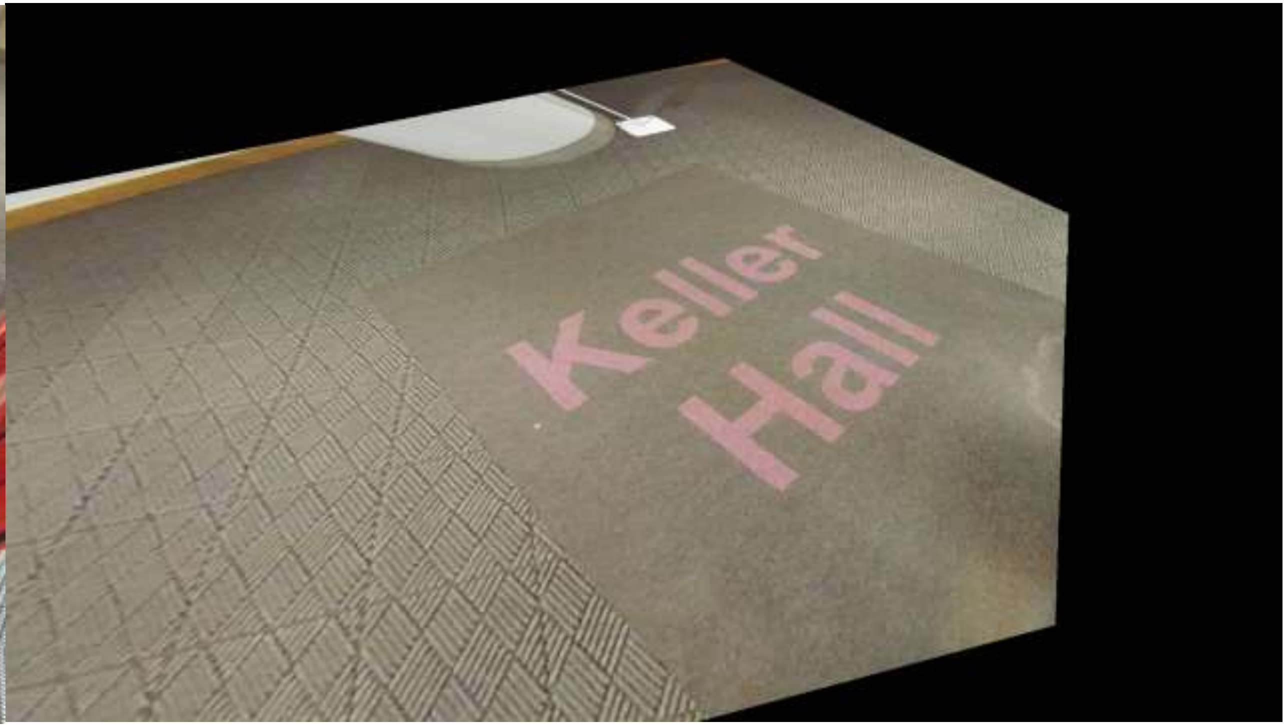
Image Transform via 3D Plane

Keller entrance left

Keller entrance right

Image Transform via 3D Plane

Keller entrance left



Right image to left

Image Transform via 3D Plane

Image Transform via 3D Plane

Left image to right

Right image to left

Image Transform via 3D Plane

360 Panorama

https://www.youtube.com/watch?v=H6SsB3JYqQg

Image Transform by Pure 3D Rotation

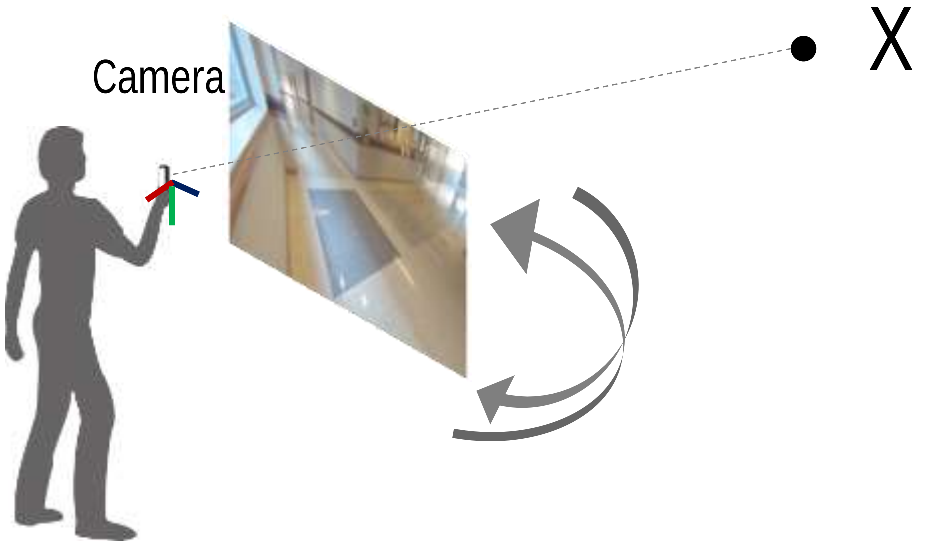
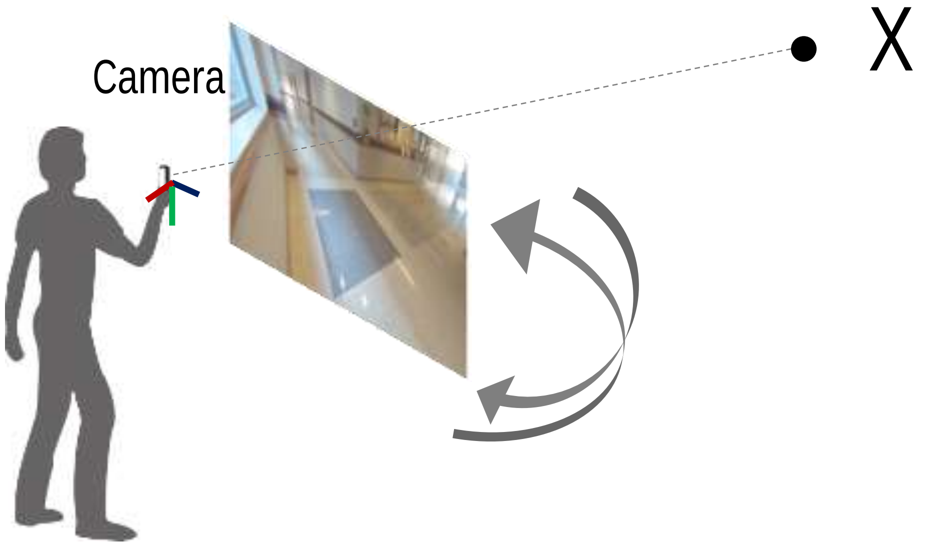
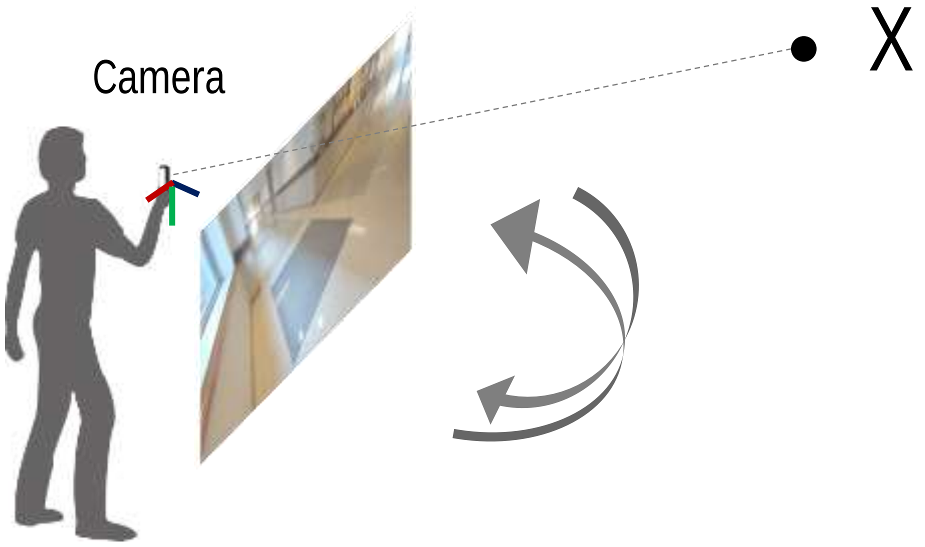


Image Transform by Pure 3D Rotation



$$\lambda_1 u = KX$$

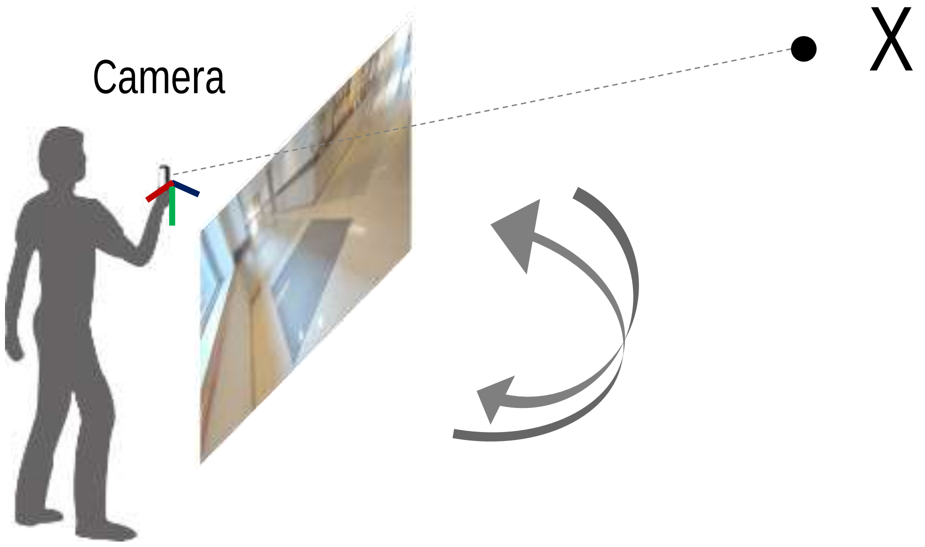
Image Transform by Pure 3D Rotation



$$\lambda_1 u = KX$$

$$\lambda_2 v = KRX$$

Image Transform by Pure 3D Rotation

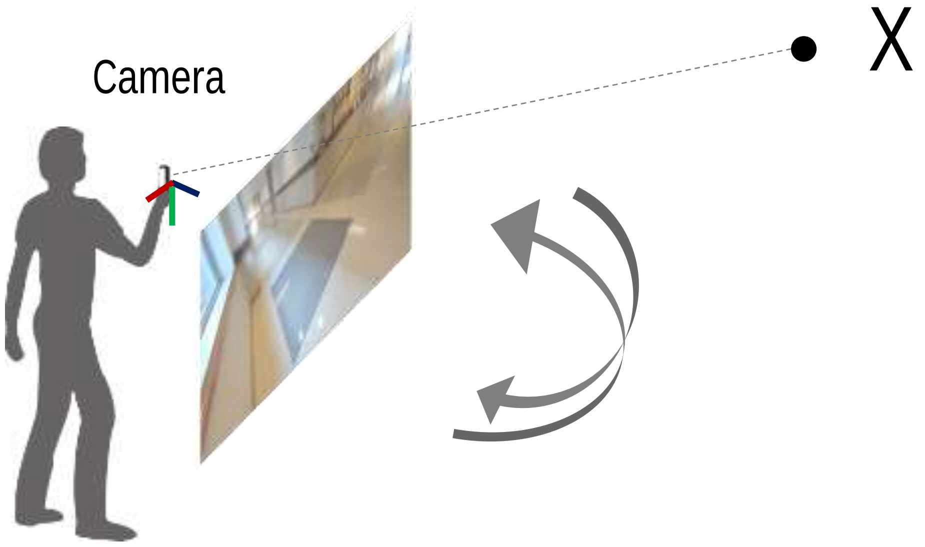


$$\lambda_1 u = KX$$

$$\lambda_2 v = KRX$$

$$\longrightarrow X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v$$

Image Transform by Pure 3D Rotation



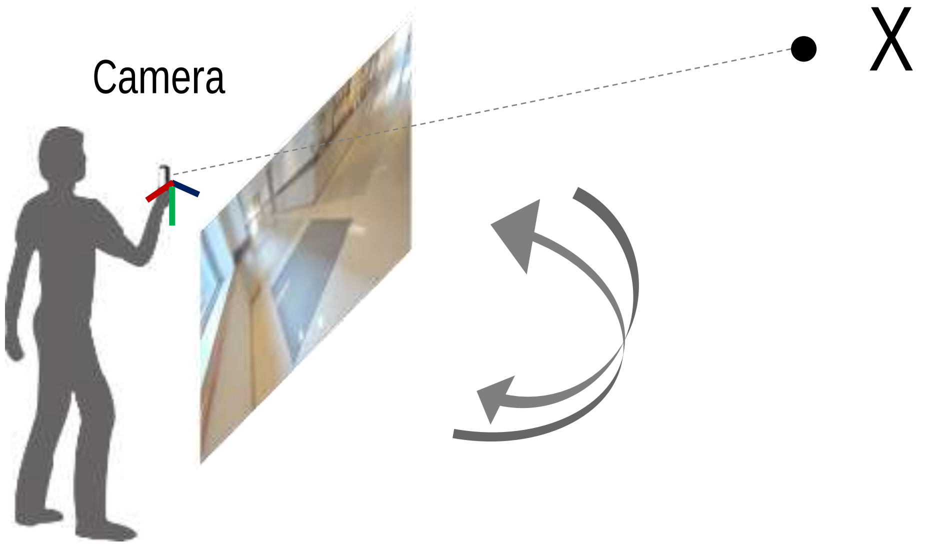
$$\lambda_1 u = KX$$

$$\lambda_2 v = KRX$$

$$\longrightarrow X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v$$

$$\longrightarrow \lambda v = KRK^{-1} u$$

Image Transform by Pure 3D Rotation



$$\lambda_1 u = KX$$

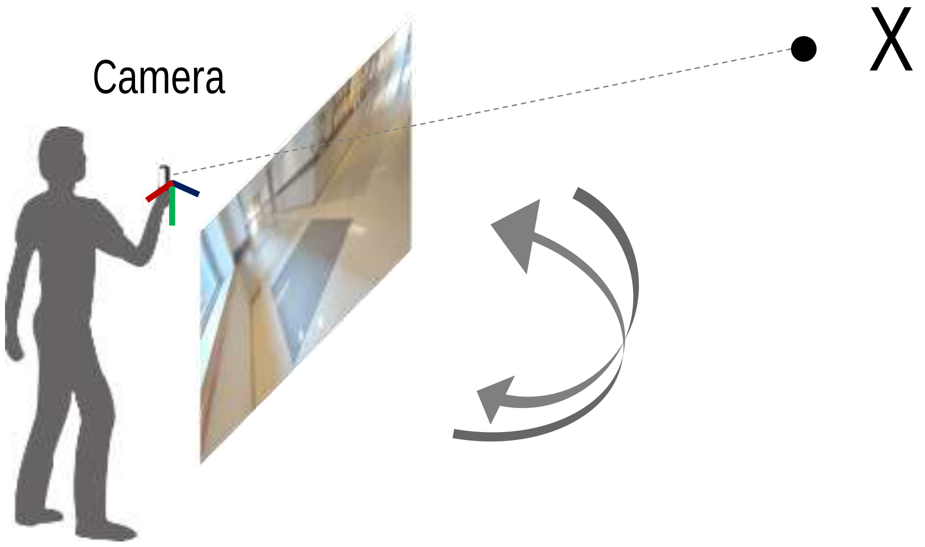
$$\lambda_2 v = KRX$$

$$\longrightarrow X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v$$

$$\longrightarrow \lambda v = KRK^{-1} u$$

$$\longrightarrow H = KRK^{-1}$$

Image Transform by Pure 3D Rotation



$$\lambda_1 u = KX$$

$$\lambda_2 v = KRX$$

$$\longrightarrow X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v$$

$$\longrightarrow \lambda v = KRK^{-1} u$$

$$\longrightarrow H = KRK^{-1}$$

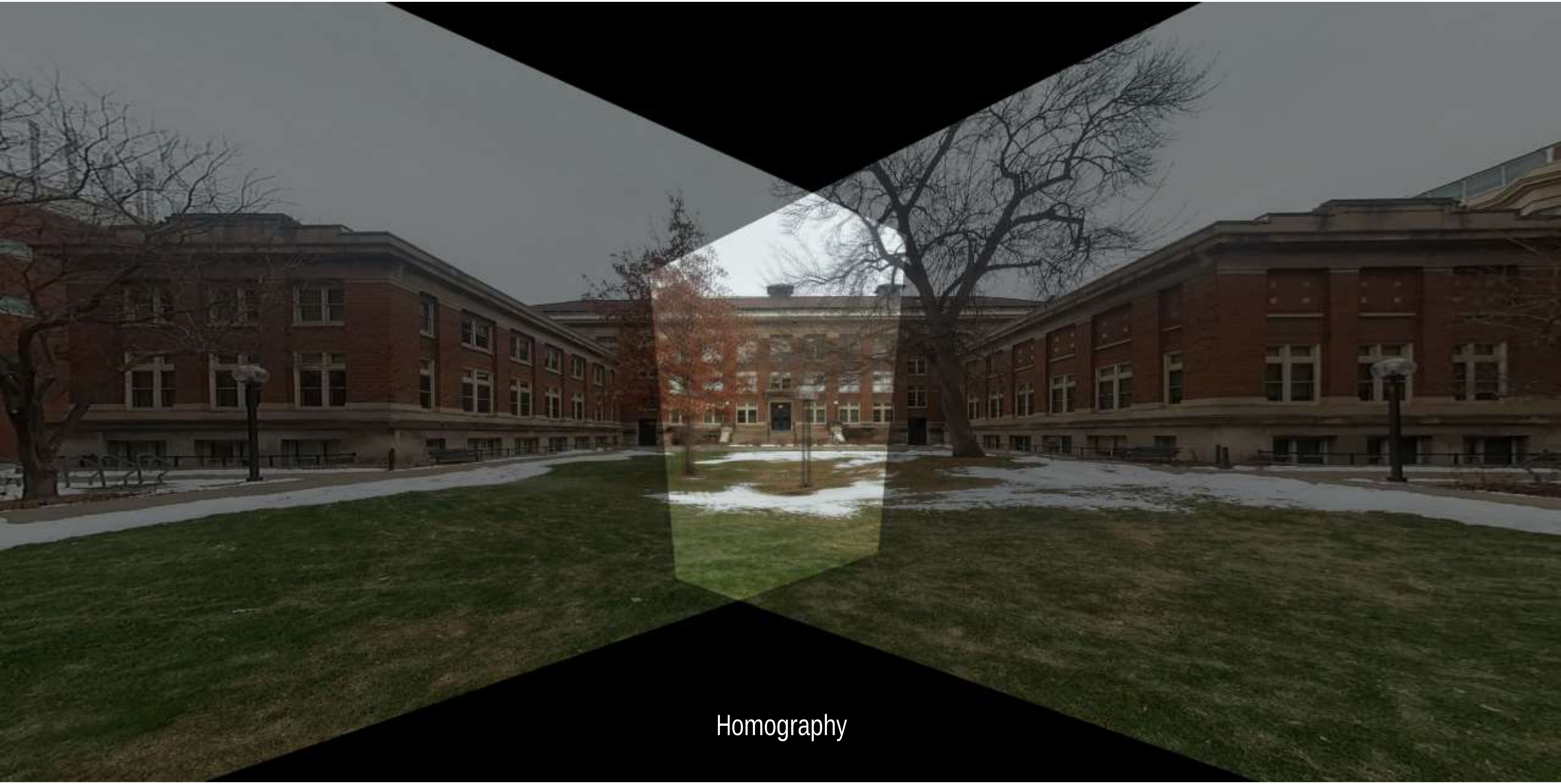
$$\longrightarrow R = K^{-1} H K$$

$$\lambda u = Hv$$

Lind Hall Left

Lind Hall Right

Euclidean Transform (Translation)


Homography

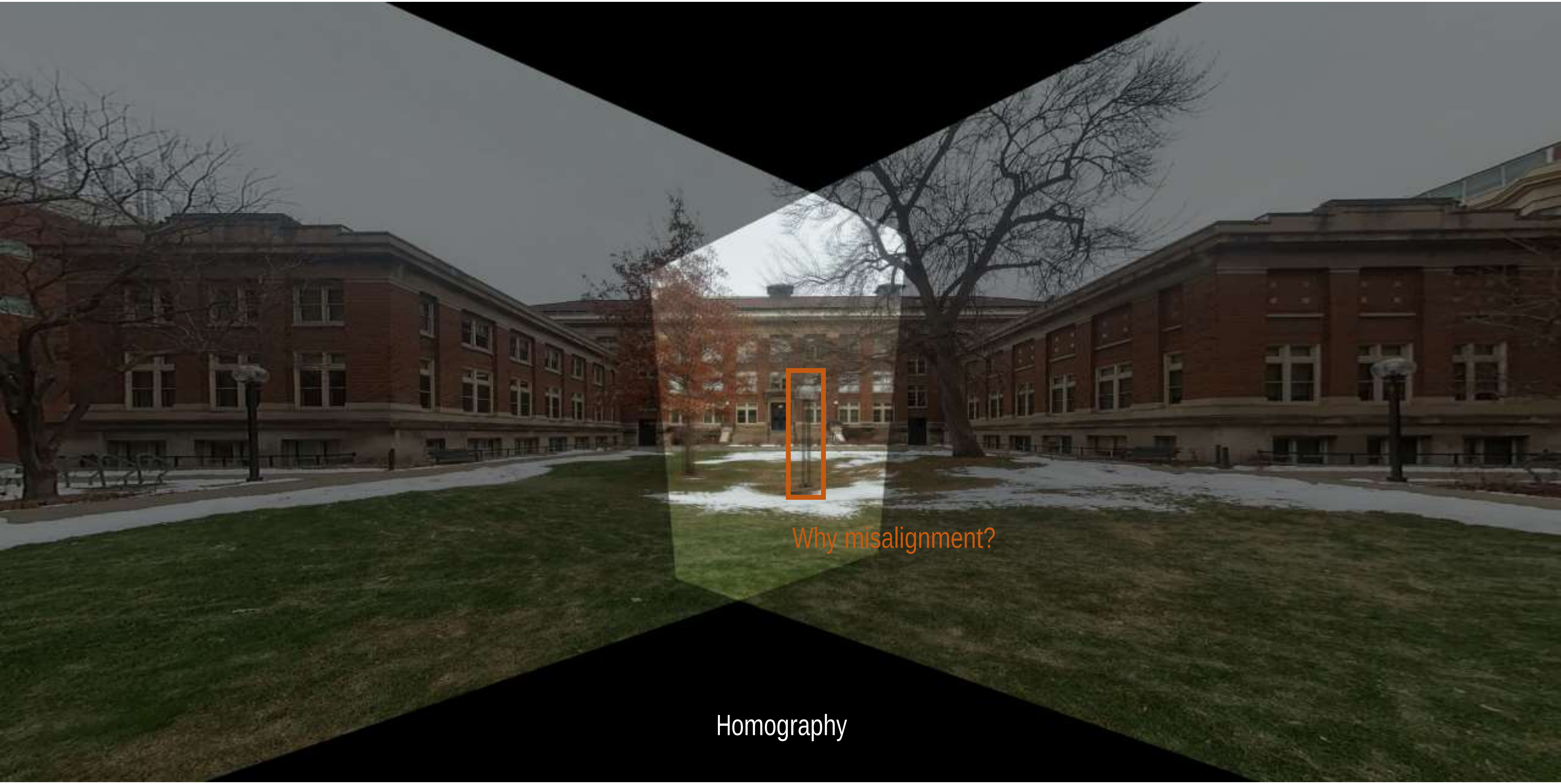
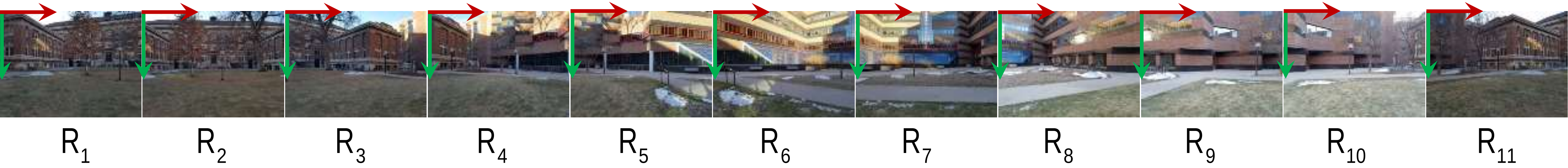

Why misalignment?
Homography

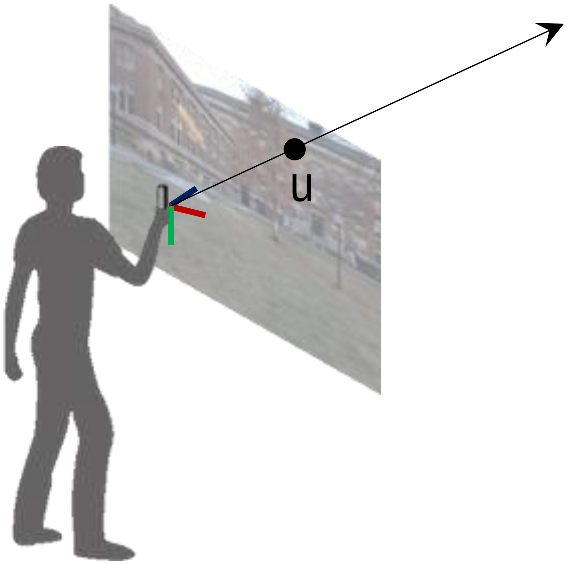
Image Panorama (Cylindrical Projection)

Image Panorama (Cylindrical Projection)

Image Panorama (Cylindrical Projection)

Image Panorama (Cylindrical Projection)

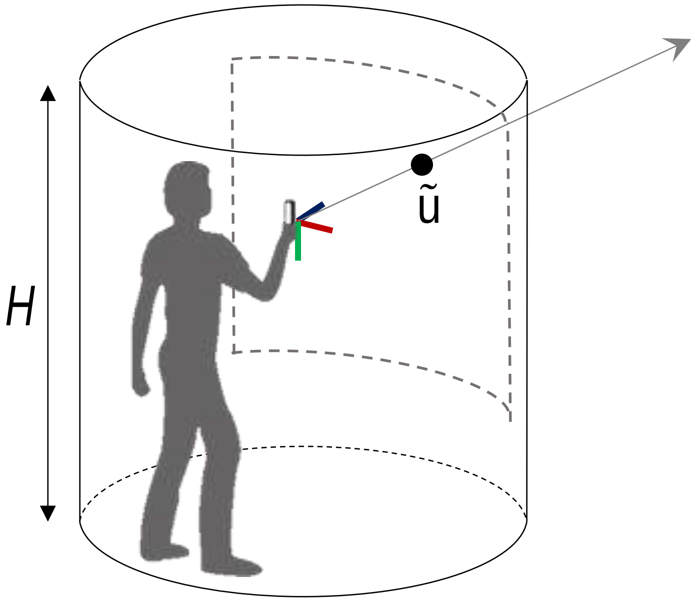




$$X = \lambda K^{-1} R^{T} u$$

$$R = K^{-1} H K$$

Image Panorama (Cylindrical Projection)

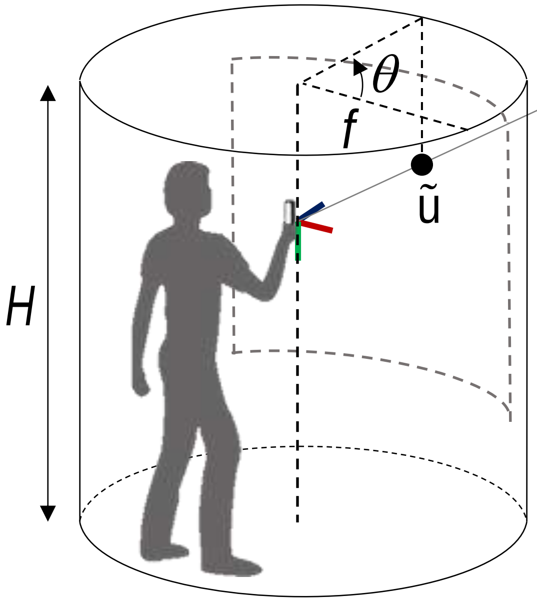


$X = \lambda K^{-1} R^{T} u$

$R = K^{-1} H K$

$\tilde{u} =$

Image Panorama (Cylindrical Projection)

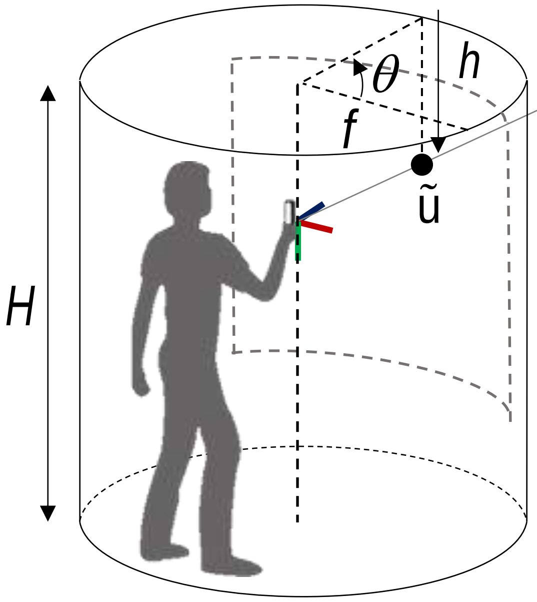


$$X = \lambda K^{-1} R^{T} u$$

$$R = K^{-1} H K$$

$$\tilde{u} = \begin{bmatrix} f\cos\theta \\ \\ f\sin\theta \end{bmatrix}$$

Image Panorama (Cylindrical Projection)

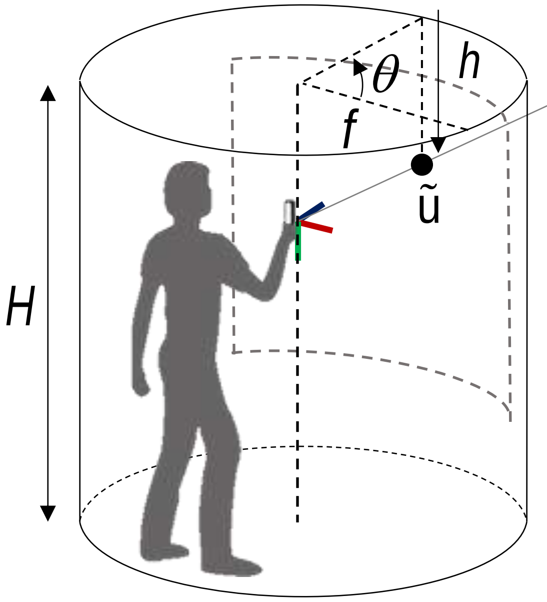


$$X = \lambda K^{-1} R^{T} u$$

$$R = K^{-1} H K$$

$$\tilde{u} = \begin{bmatrix} f\cos\theta \\ h - \frac{H}{2} \\ f\sin\theta \end{bmatrix}$$

Image Panorama (Cylindrical Projection)



$$X = \lambda K^{-1} R^{T} u$$

$$R = K^{-1} H K$$

$$\tilde{u} = \begin{bmatrix} f\cos\theta \\ h - \frac{H}{2} \\ f\sin\theta \end{bmatrix} = \lambda K^{-1} R^{T} u$$

Image Panorama (Cylindrical Projection)