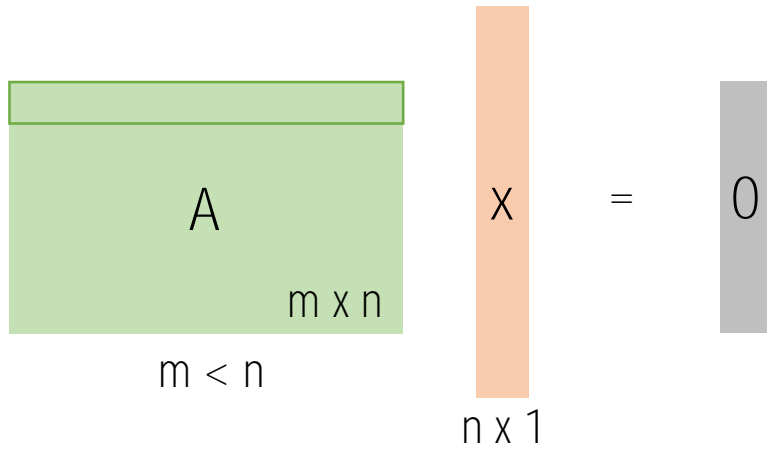


# Nullspace

# eqs < # unknowns



The diagram illustrates the matrix equation  $Ax = 0$ . On the left, a green rectangle represents matrix  $A$ , with dimensions  $m \times n$  and the condition  $m < n$  written below it. To its right is a vertical orange bar representing vector  $x$ , with dimensions  $n \times 1$  written below it. An equals sign follows, and to its right is a vertical gray bar representing the zero vector  $0$ .

$$A \quad x = 0$$

$m \times n$        $n \times 1$

$m < n$

# Nullspace

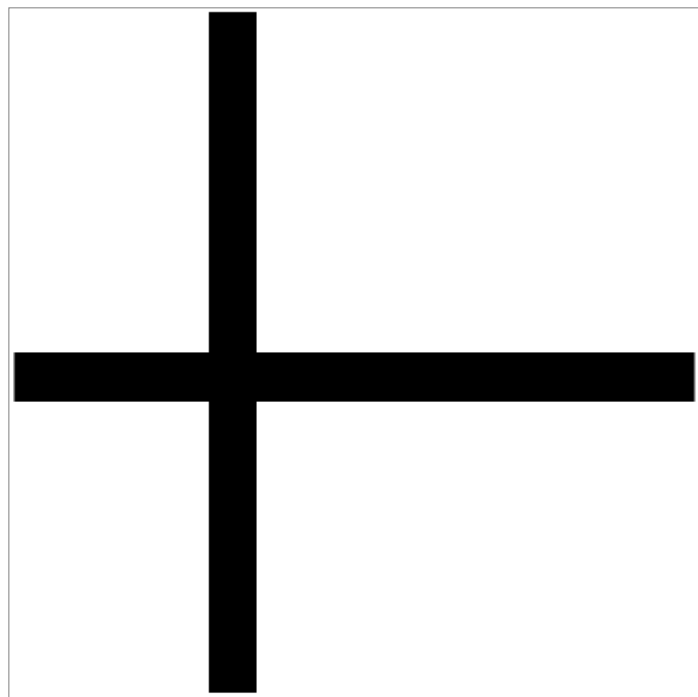
# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .

A diagram illustrating the equation  $Ax=0$ . On the left, a green rectangle represents matrix  $A$  with dimensions  $m \times n$  and  $m < n$ . To its right is an orange vertical bar representing vector  $x$  with dimensions  $n \times 1$ . An equals sign follows, and to its right is a gray vertical bar representing the zero vector  $0$ .

→  $x = \text{null}(A)$

# More SVD

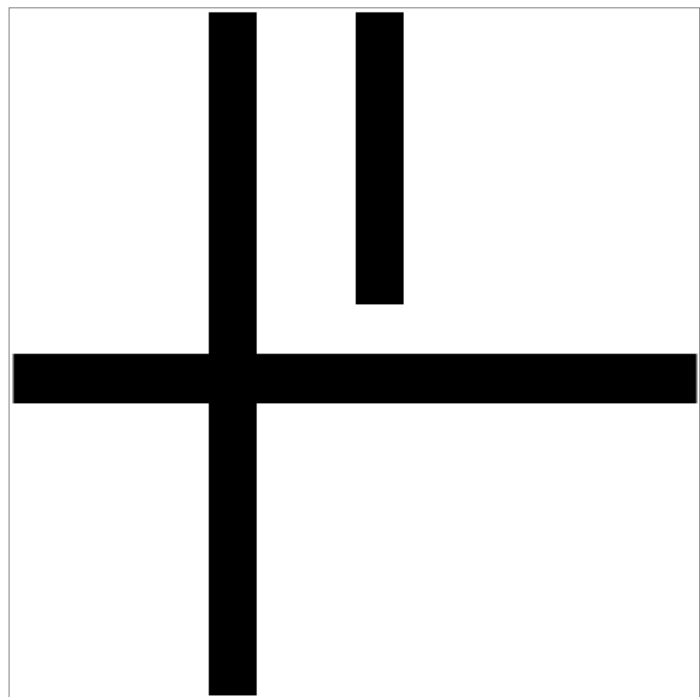


14x14

=



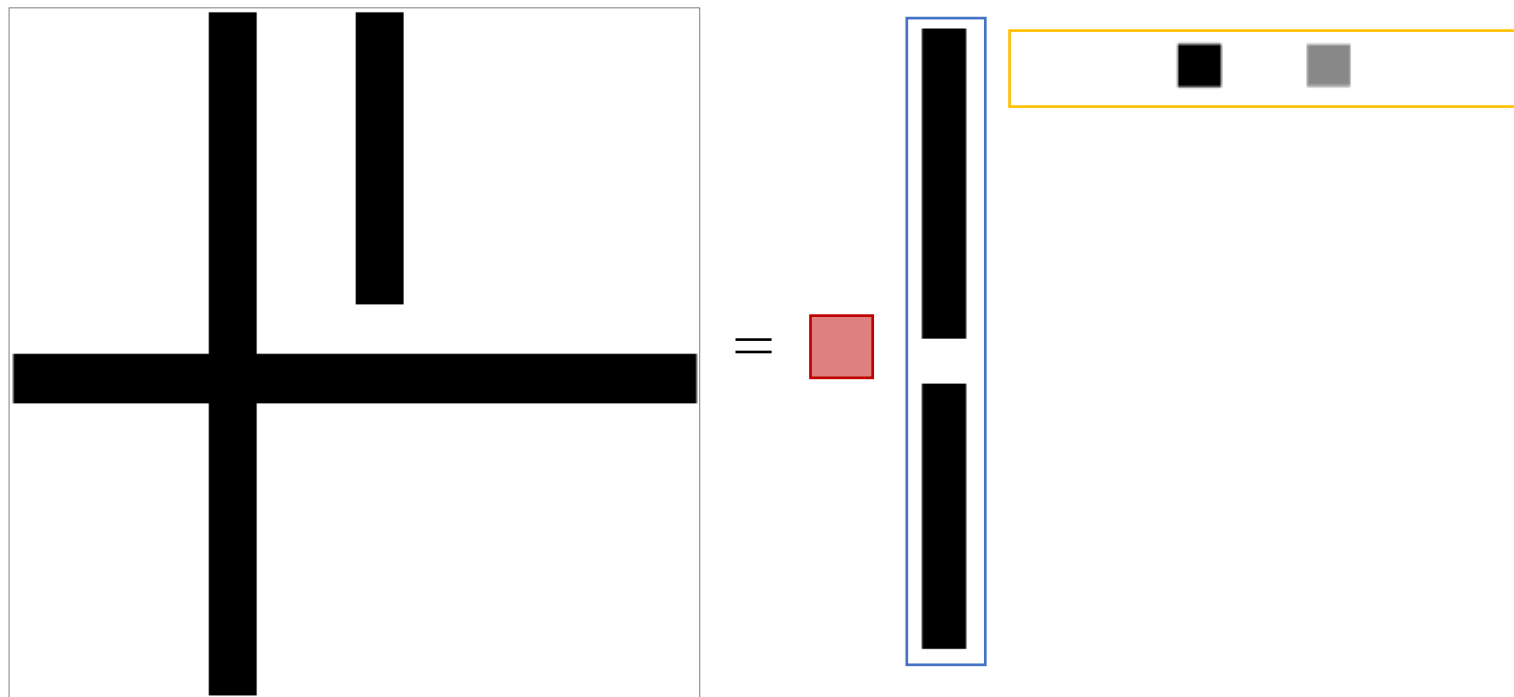
# More SVD



=

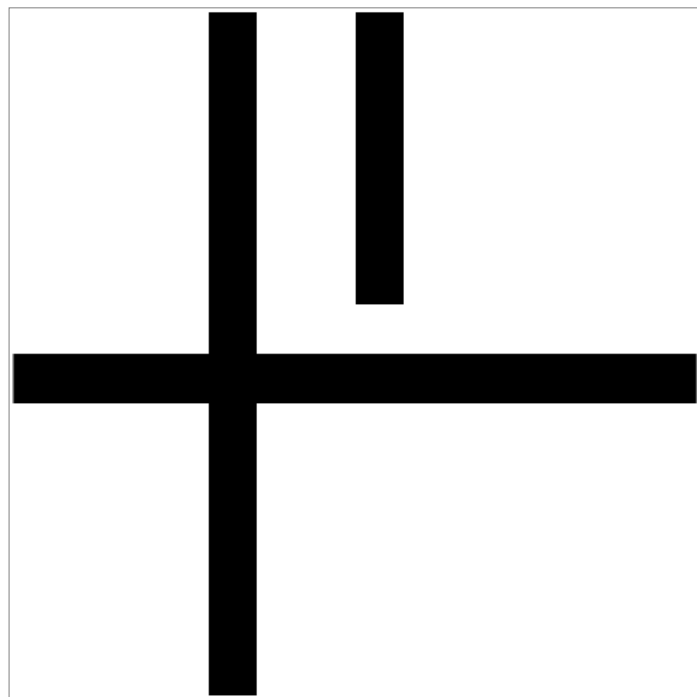
14x14

# More SVD



14x14

# More SVD



14x14

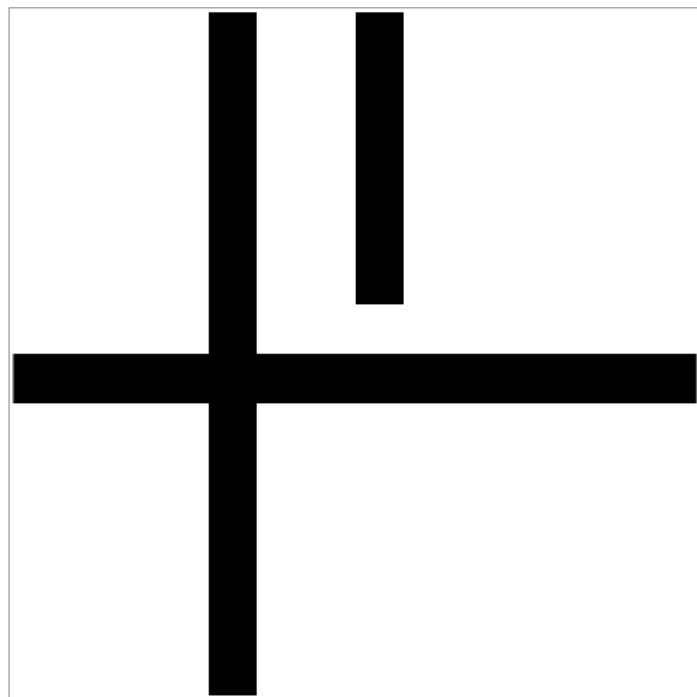
=



+

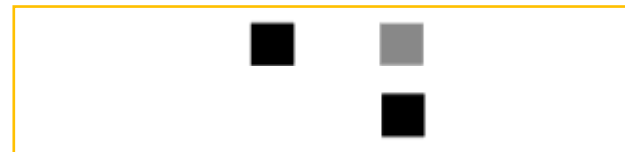
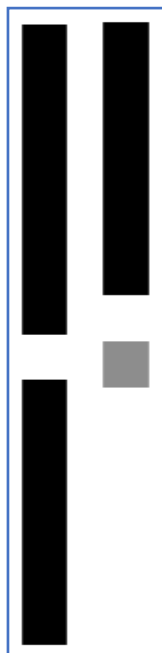


# More SVD



14x14

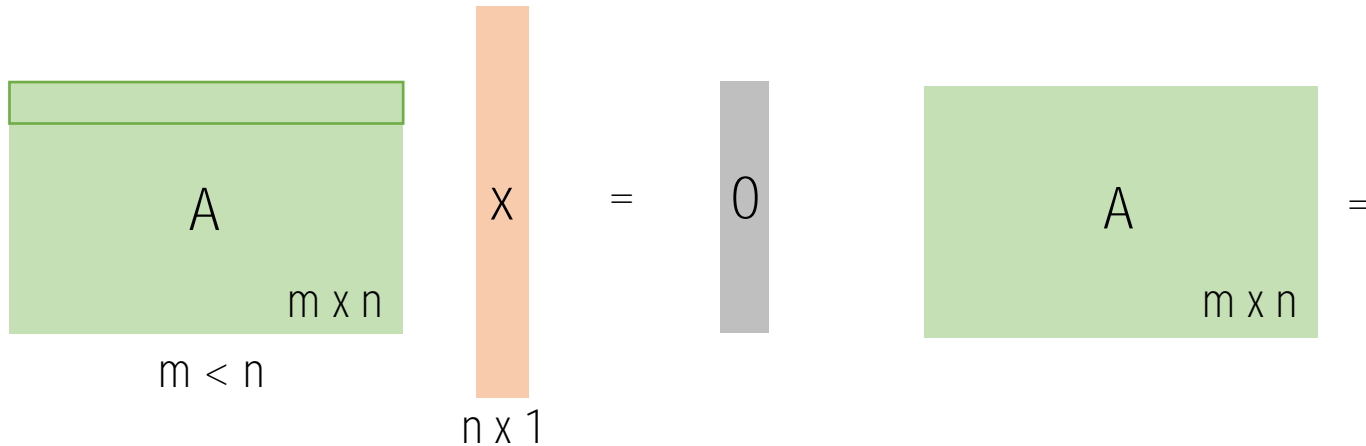
=



# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .



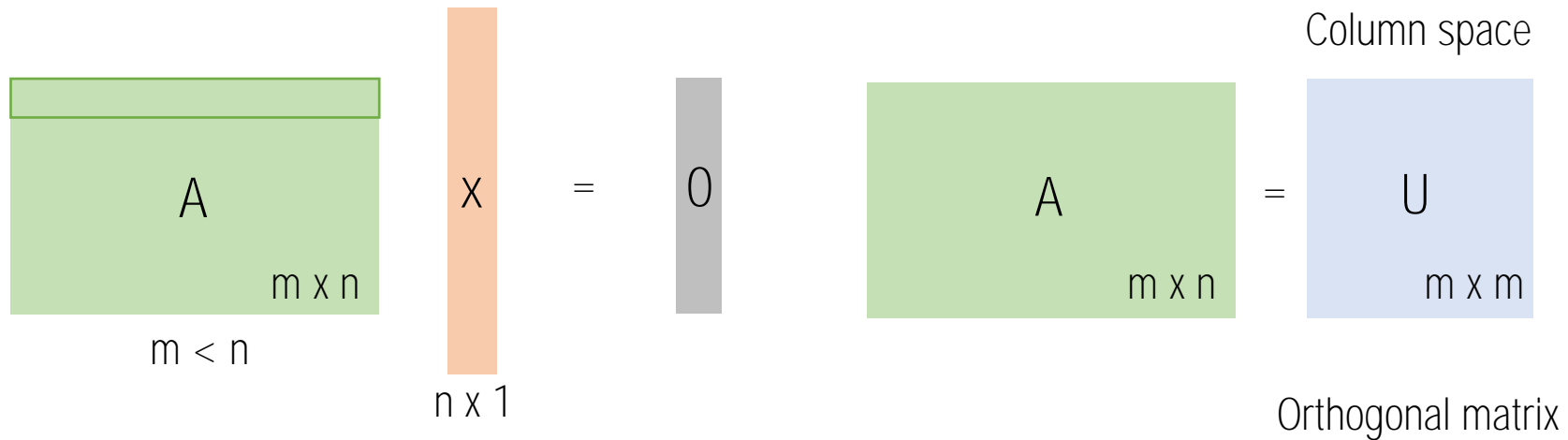
→  $x = \text{null}(A)$



# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .

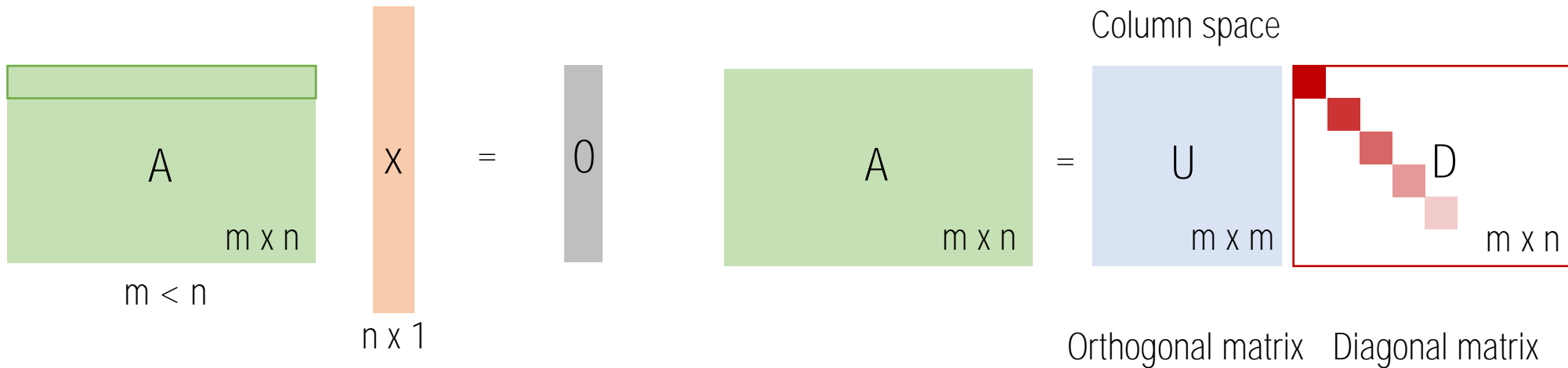


→  $x = \text{null}(A)$

# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .

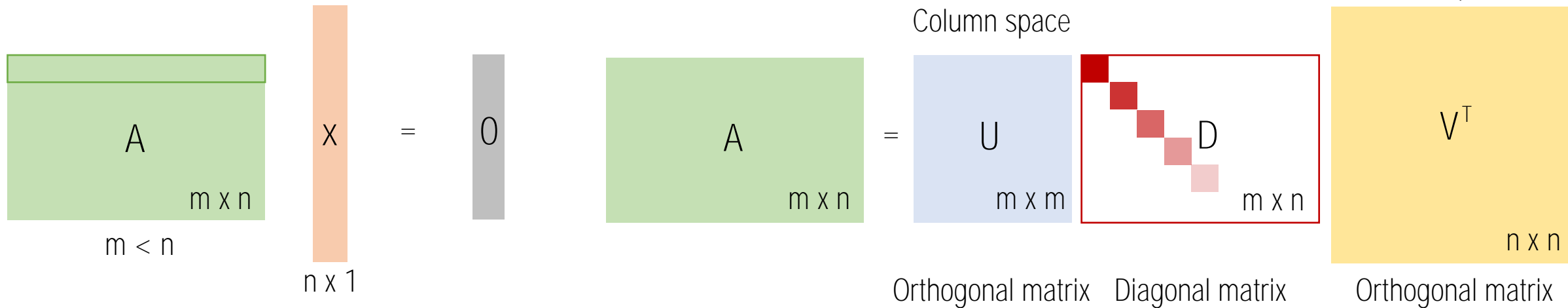


→  $x = \text{null}(A)$

# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .

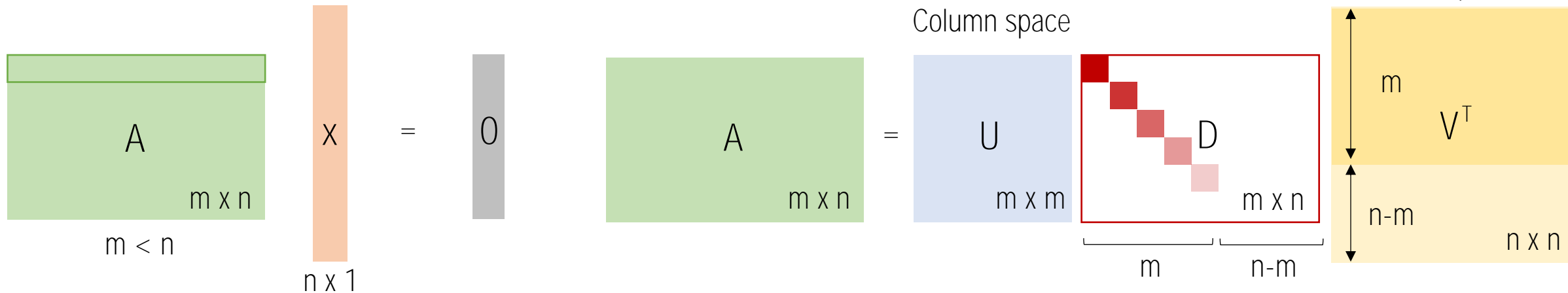


→  $x = \text{null}(A)$

# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .

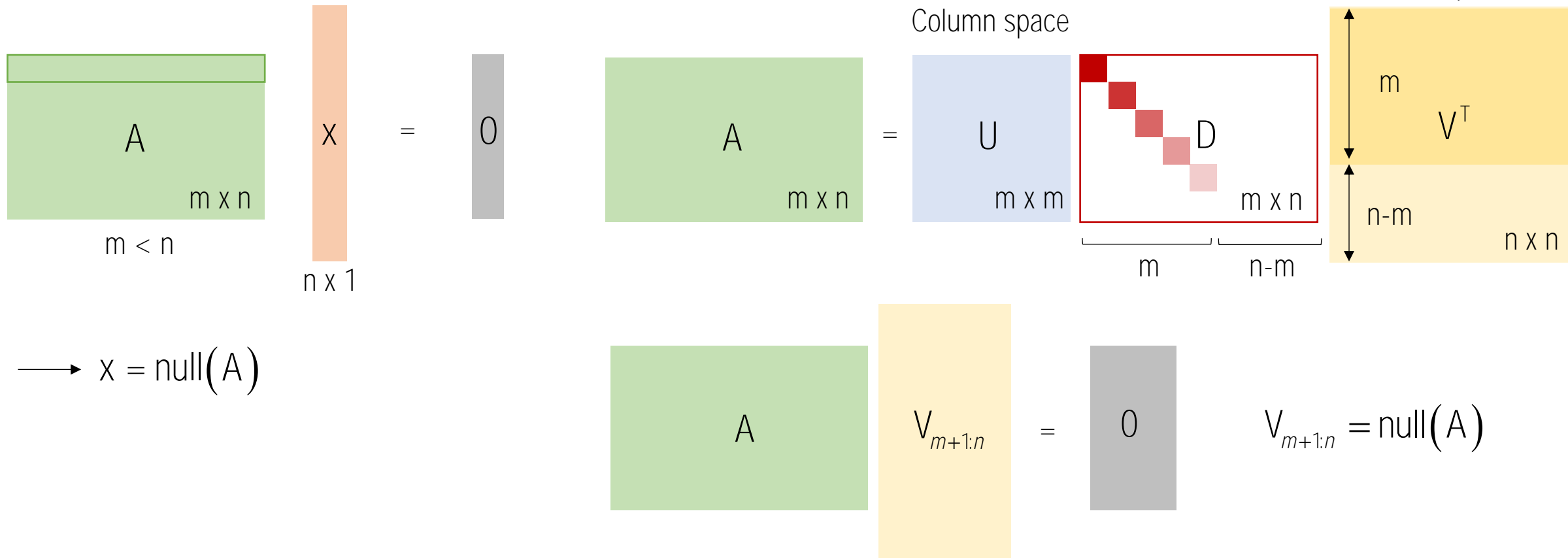


→  $x = \text{null}(A)$

# Singular Value Decomposition (SVD)

# eqs < # unknowns

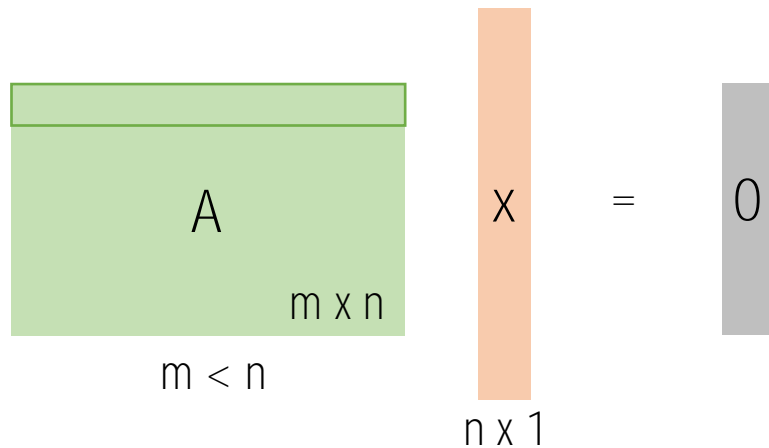
There exist at least  $n-m$  vectors that makes  $Ax=0$ .



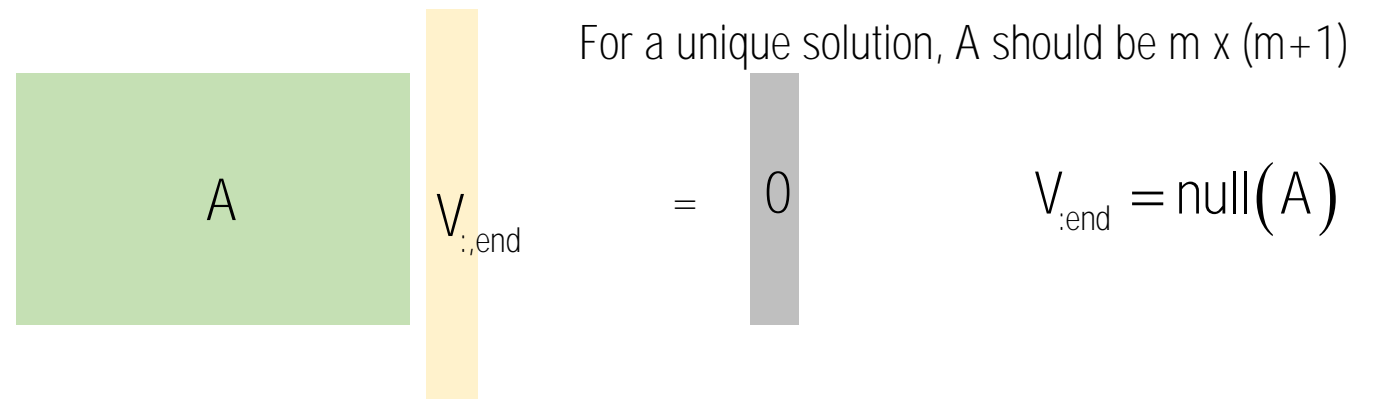
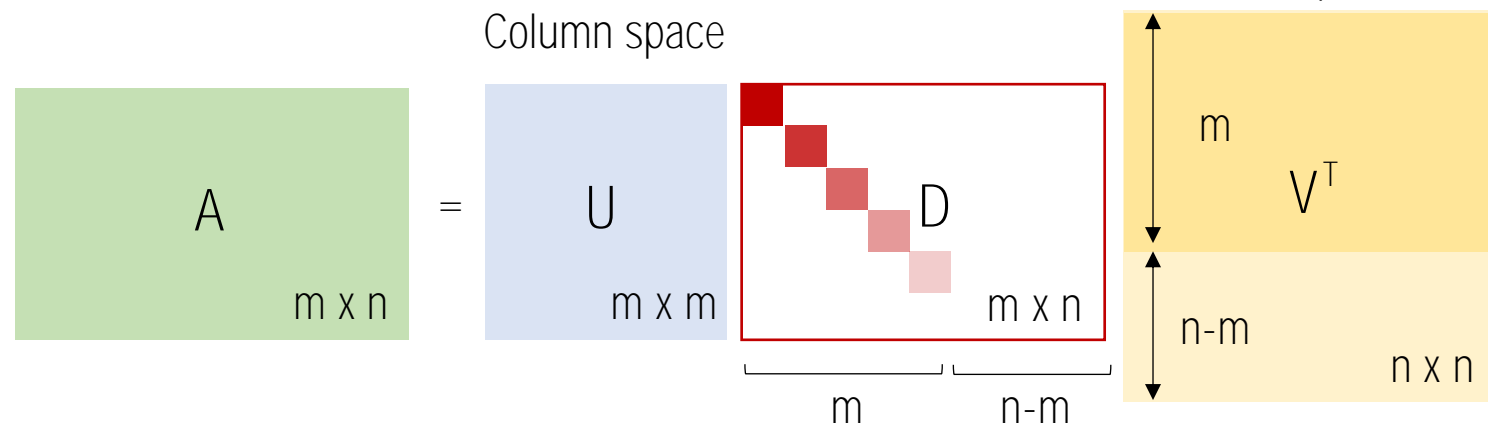
# Singular Value Decomposition (SVD)

# eqs < # unknowns

There exist at least  $n-m$  vectors that makes  $Ax=0$ .



→  $x = \text{null}(A)$



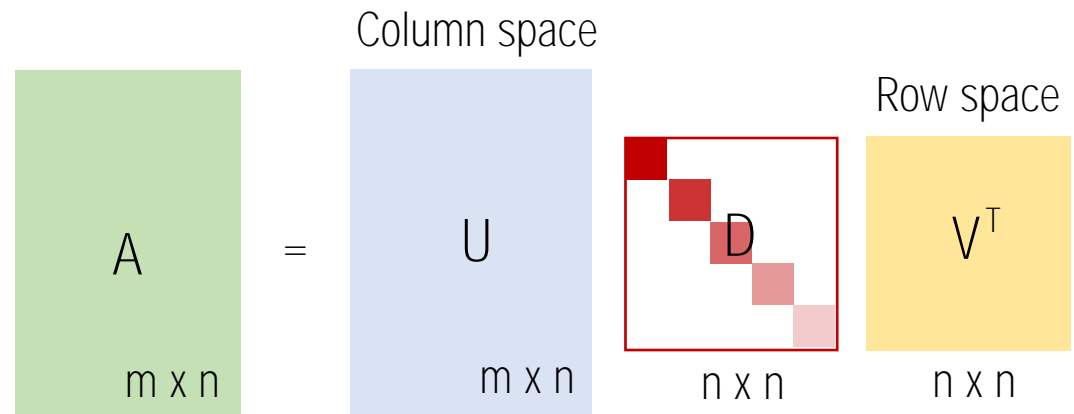
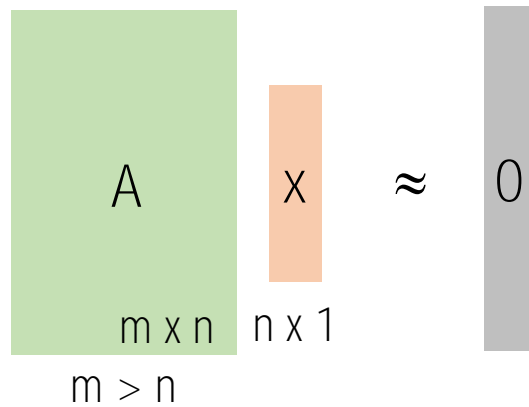
For a unique solution,  $A$  should be  $m \times (m+1)$

$V_{:,end} = \text{null}(A)$

# Singular Value Decomposition (SVD)

# eqs > # unknowns

There exist no nullspace of A.

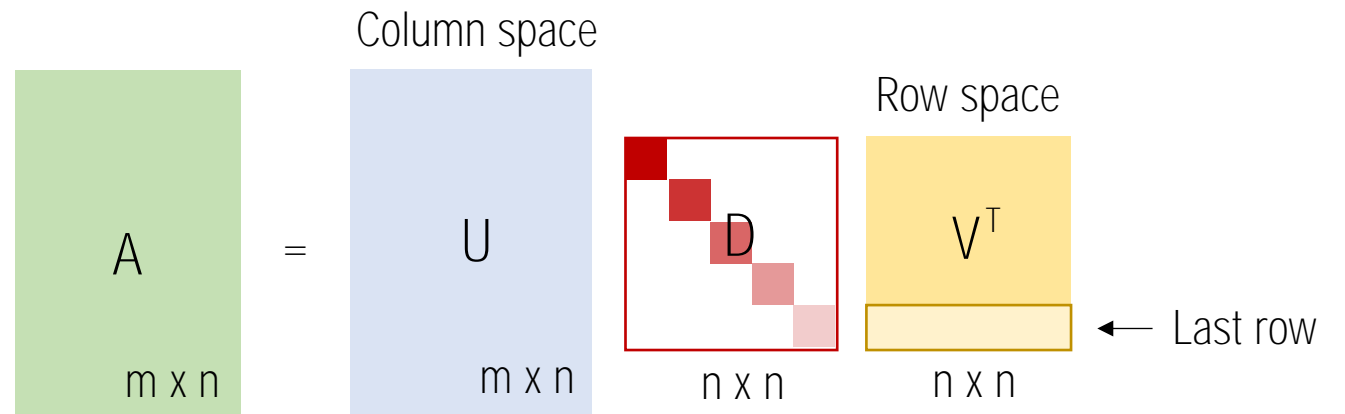
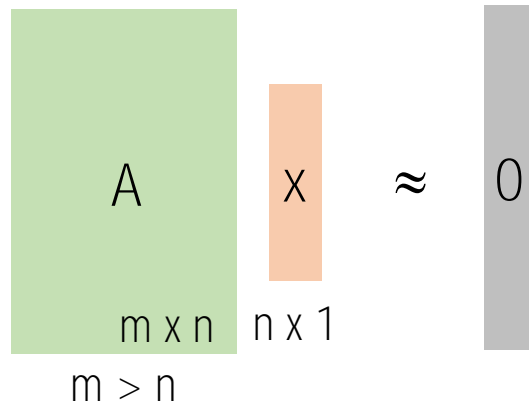


$$\underset{x}{\text{minimize}} \|Ax\|^2 \text{ subject to } \|x\| = 1$$

# Singular Value Decomposition (SVD)

# eqs > # unknowns

There exist no nullspace of A.

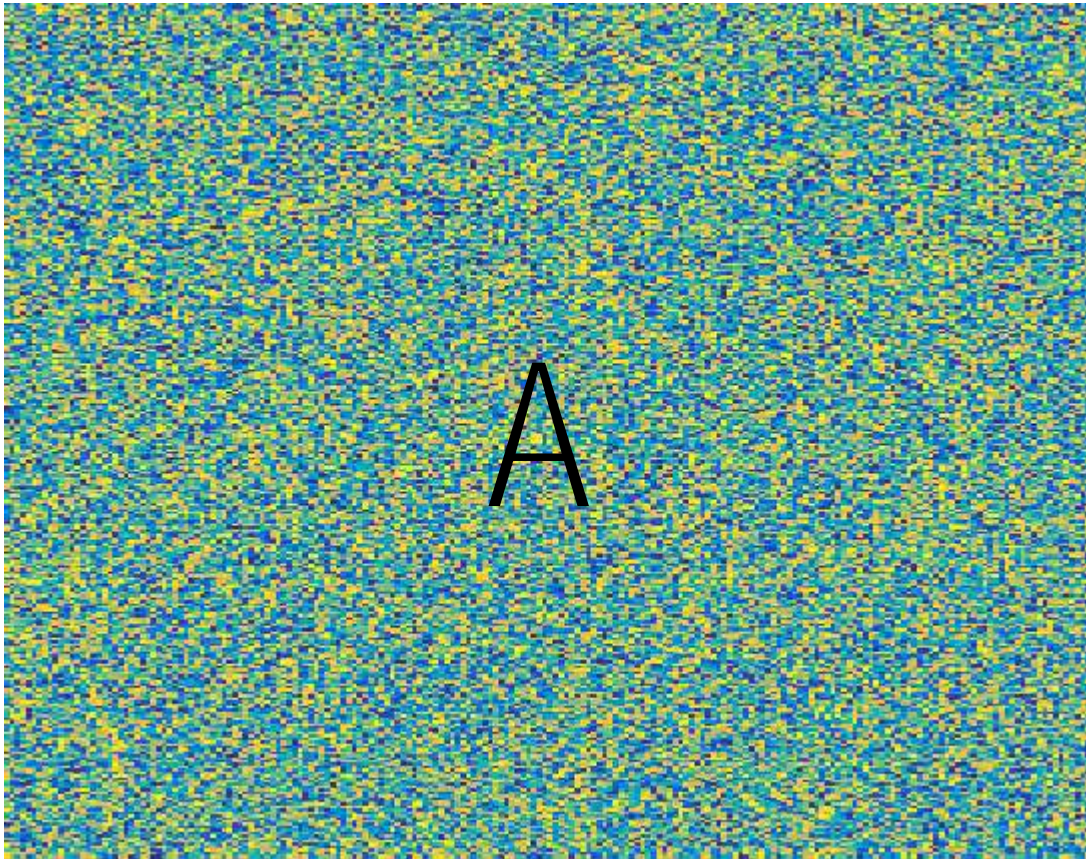


$$\underset{x}{\text{minimize}} \|Ax\|^2 \text{ subject to } \|x\| = 1$$

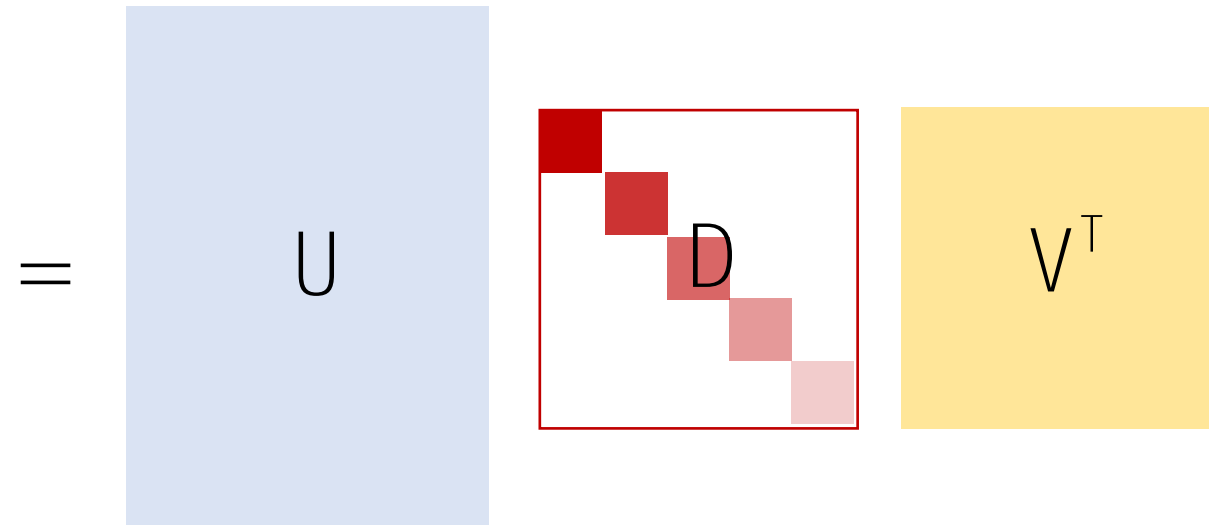
Approximated nullspace of A:  $V_{:,end}$



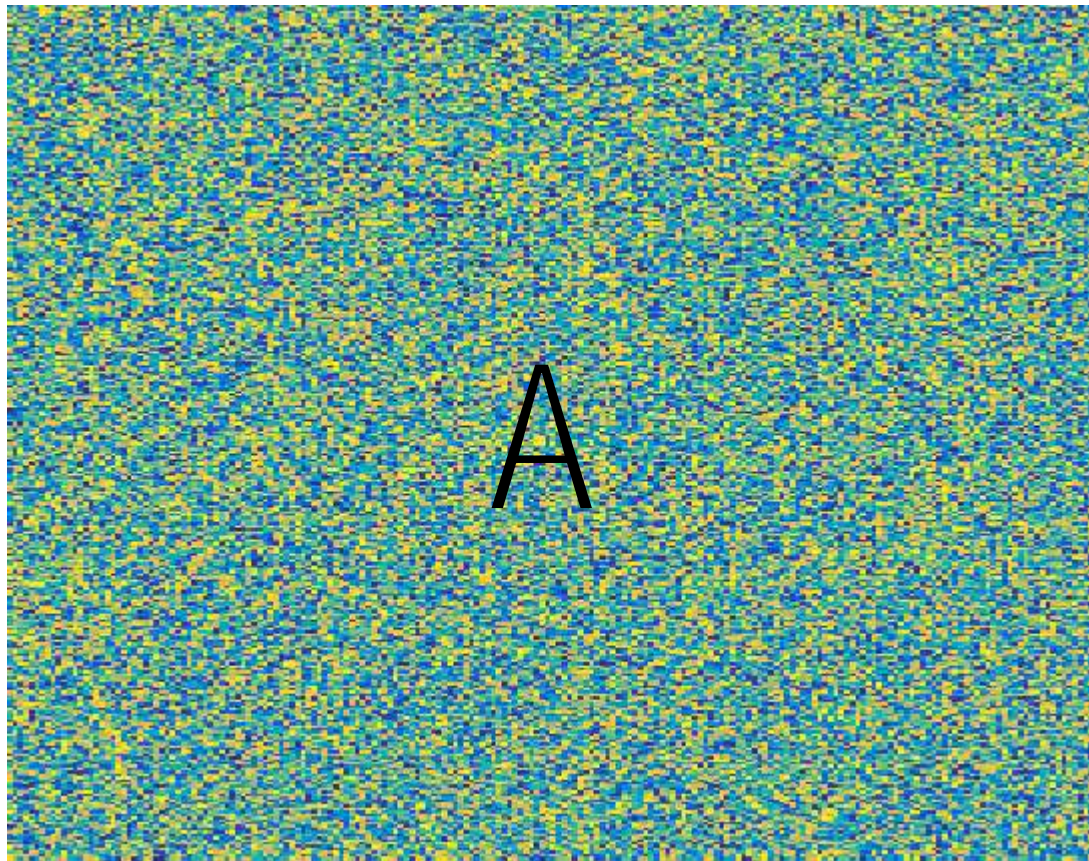
# Random Matrix SVD



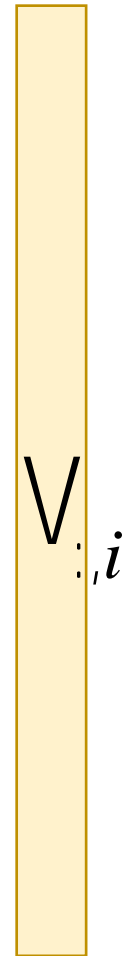
Random matrix



# Residual (Nullspace Approximation)

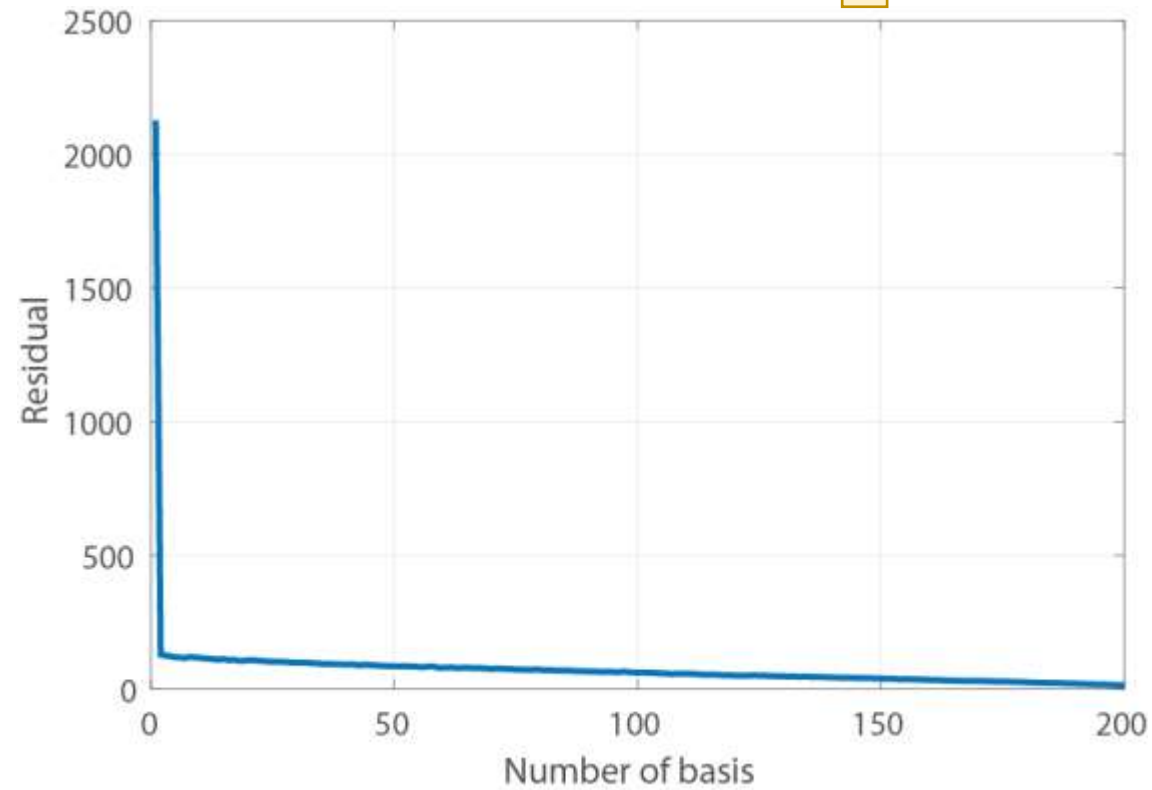


Random matrix

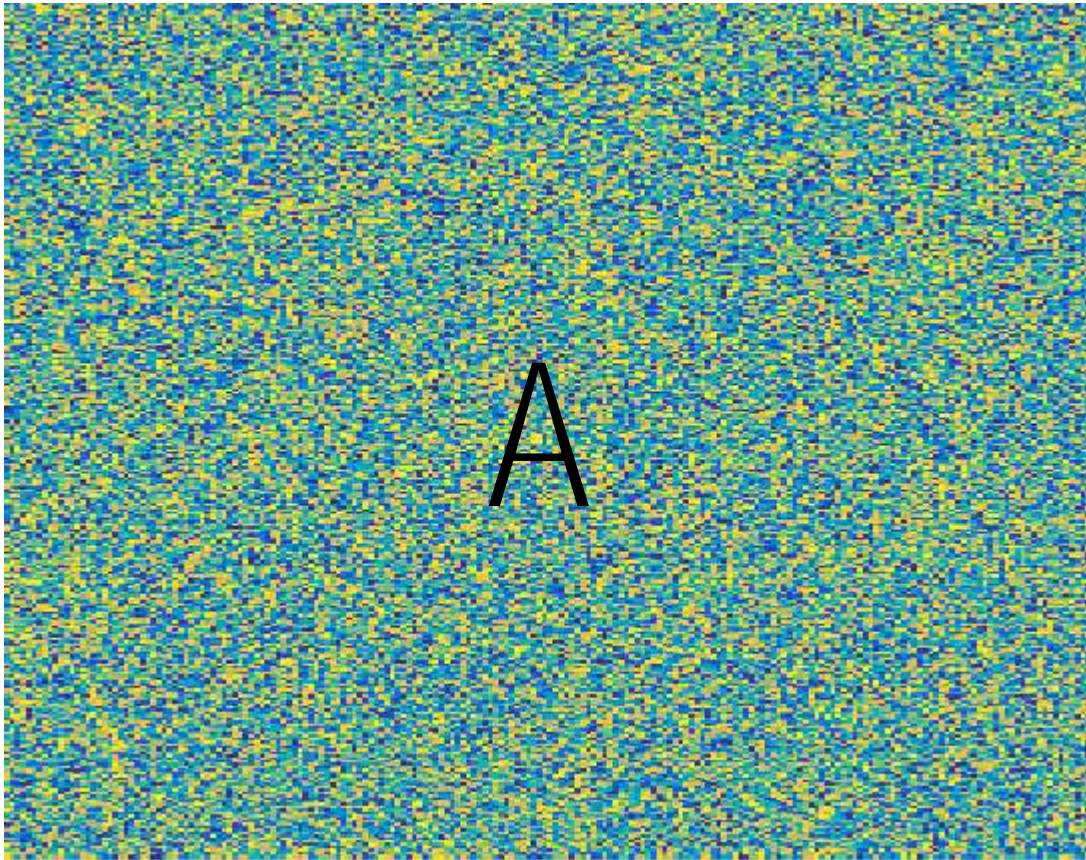


Approximated nullspace of A:

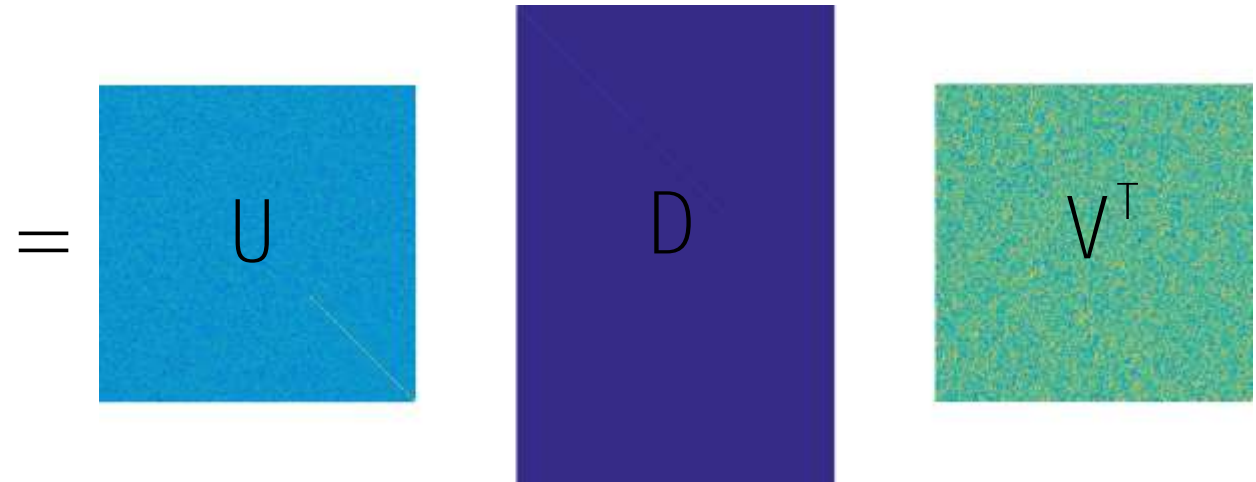
$V_{:,end}$



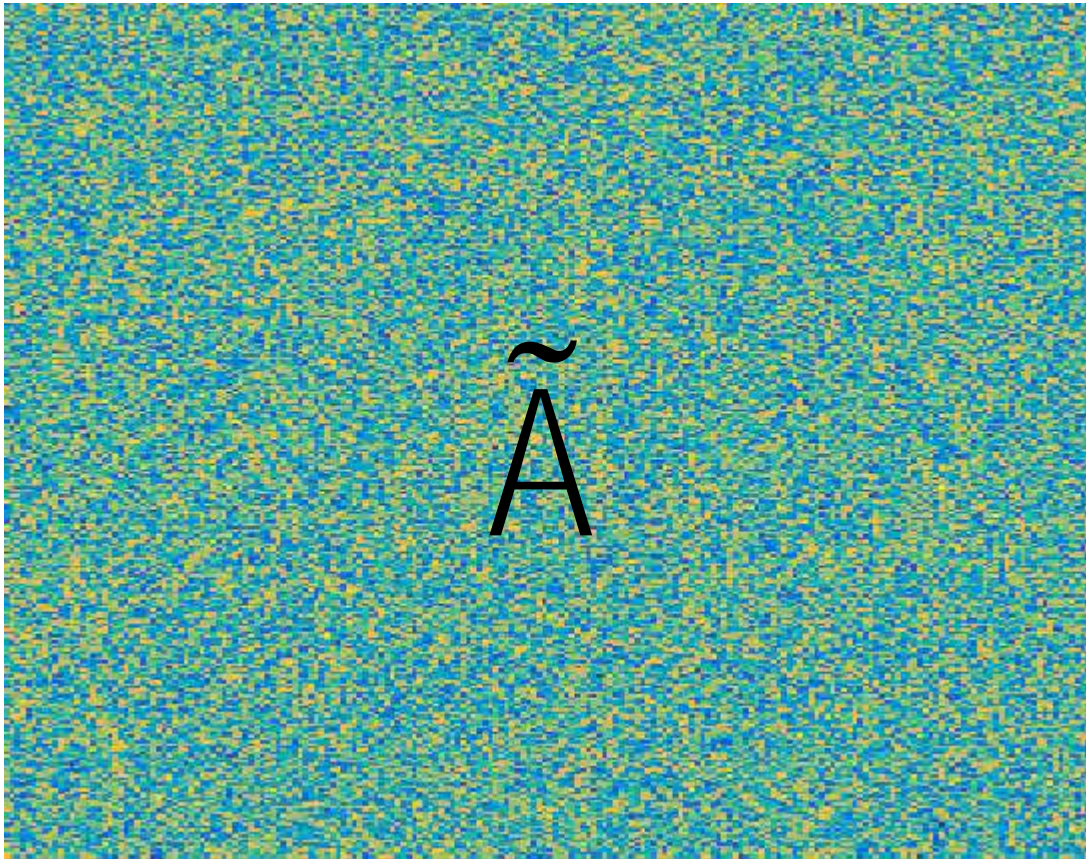
# SVD Matrix Approximation



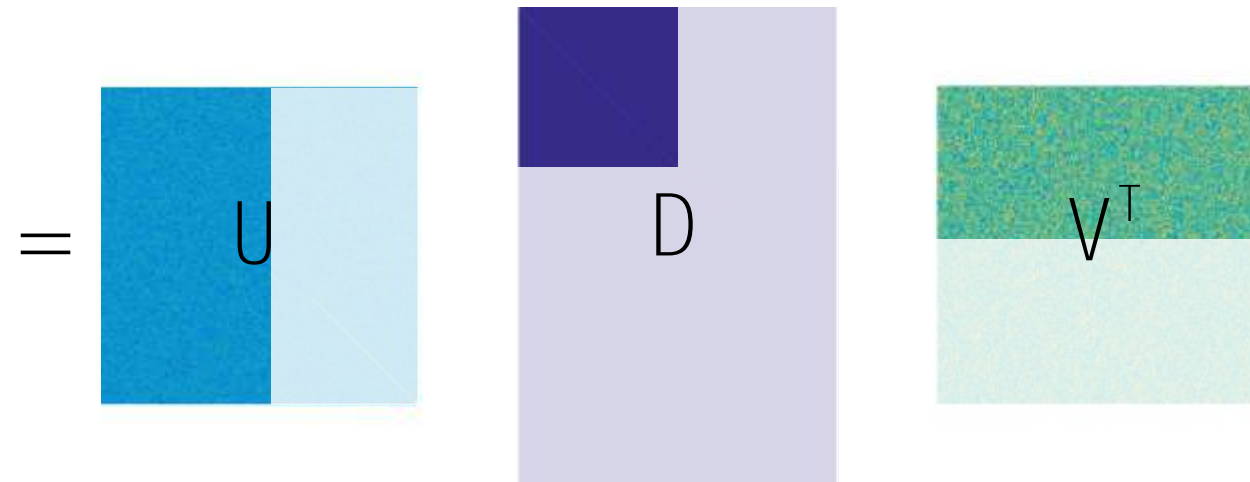
Random matrix



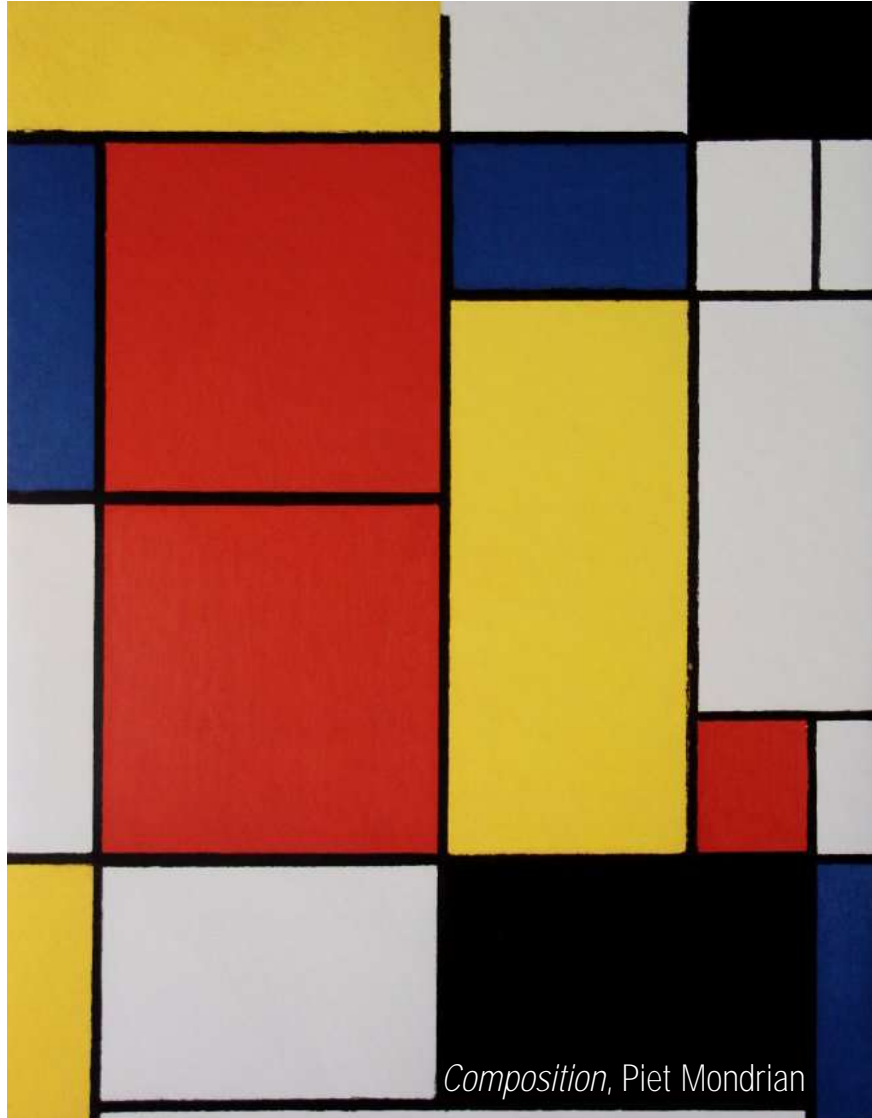
# SVD Matrix Approximation



Reconstructed matrix



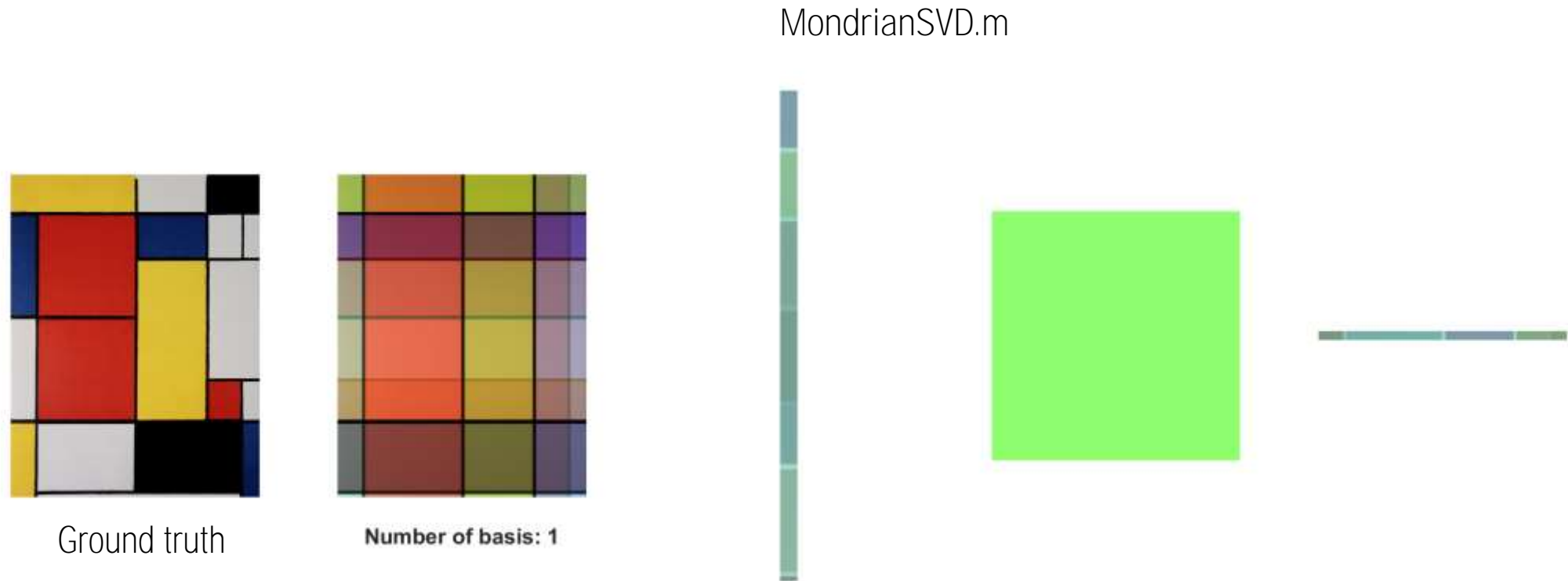
# Mondrian Painting SVD



$$= \begin{matrix} \text{U} \\ m \times n \end{matrix} \begin{matrix} \text{D} \\ n \times n \end{matrix} \begin{matrix} \text{V}^T \\ n \times n \end{matrix}$$

The diagram illustrates the Singular Value Decomposition (SVD) of the painting. It shows three matrices: a light blue matrix labeled 'U' with dimensions  $m \times n$ , a red matrix labeled 'D' with dimensions  $n \times n$  containing a diagonal of red squares, and a yellow matrix labeled 'V<sup>T</sup>' with dimensions  $n \times n$ .

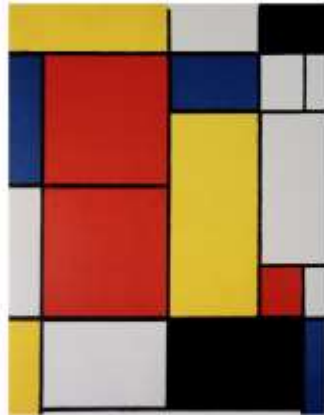
# Mondrian Painting SVD Approximation



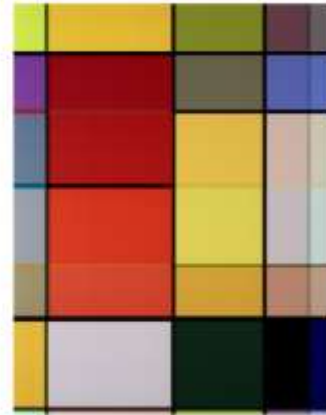
$$A = U D V^T$$

# Mondrian Painting SVD Approximation

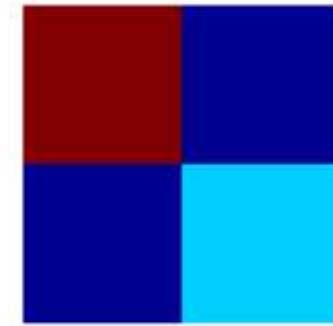
MondrianSVD.m



Ground truth



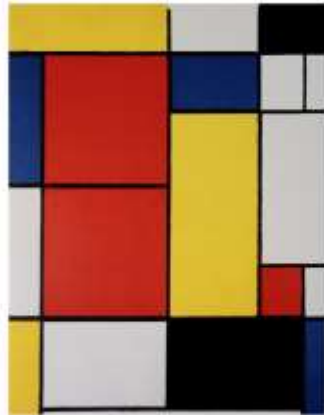
Number of basis: 2



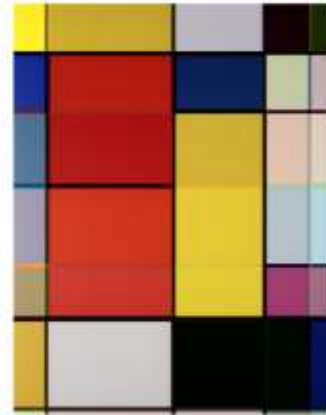
$$A = U D V^T$$

# Mondrian Painting SVD Approximation

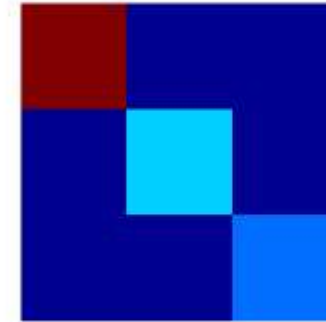
MondrianSVD.m



Ground truth



Number of basis: 3

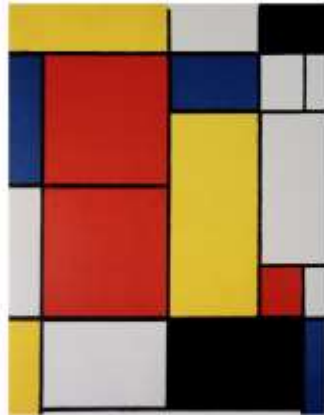


$$A = U D V^T$$



# Mondrian Painting SVD Approximation

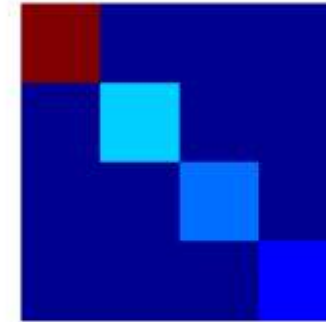
MondrianSVD.m



Ground truth



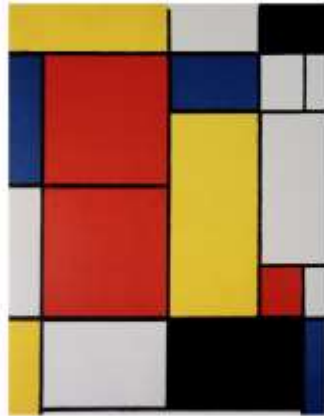
Number of basis: 4



$$A = U D V^T$$

# Mondrian Painting SVD Approximation

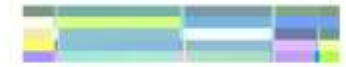
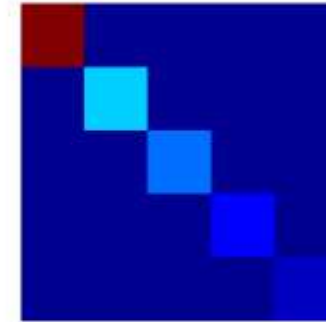
MondrianSVD.m



Ground truth



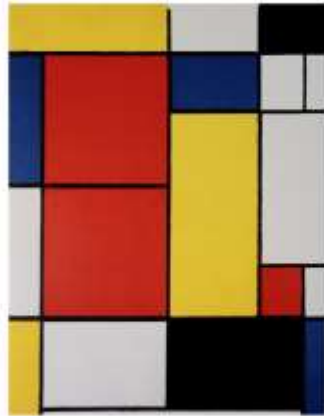
Number of basis: 5



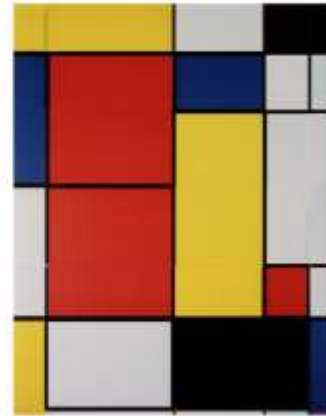
$$A = U D V^T$$

# Mondrian Painting SVD Approximation

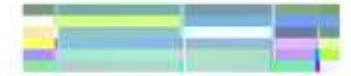
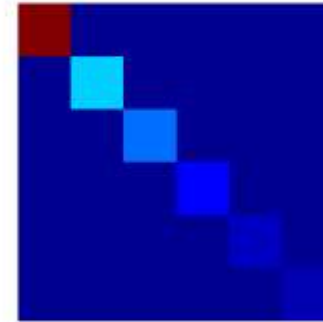
MondrianSVD.m



Ground truth



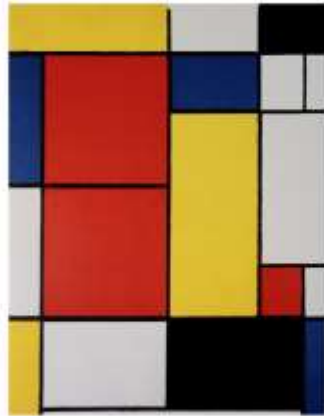
Number of basis: 6



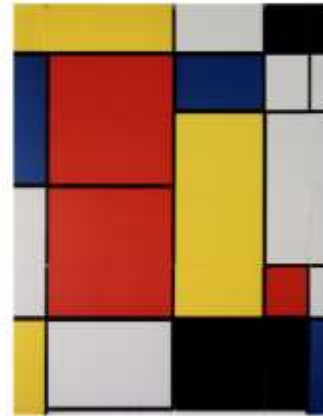
$$A = U D V^T$$

# Mondrian Painting SVD Approximation

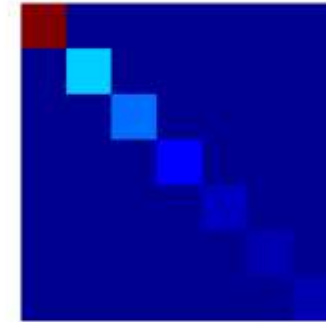
MondrianSVD.m



Ground truth

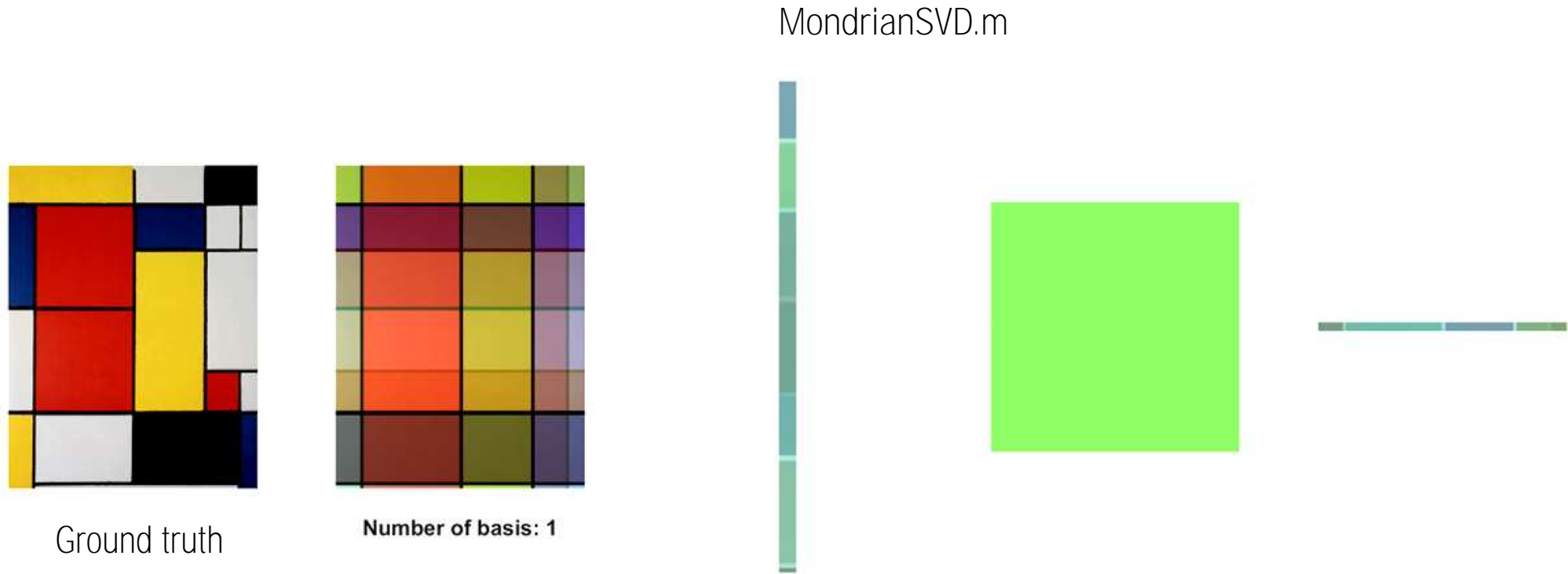


Number of basis: 7



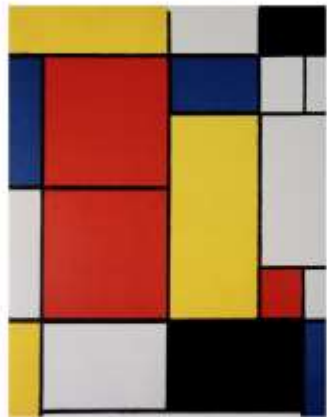
$$A = U D V^T$$

# Mondrian Painting SVD Approximation



$$A = U D V^T$$

# Reconstruction Error



Ground truth



Number of basis: 7

A

