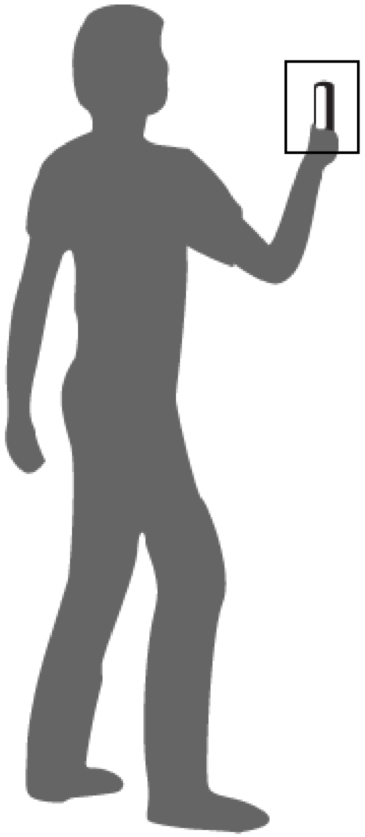
Camera Calibration

Real-time Facial Reenactment

Live capture using a commodity webcam

Thies et al. "Face2Face: Realtime Face Capture and Reenactment of RGB Videos"

Camera Intrinsic Parameter

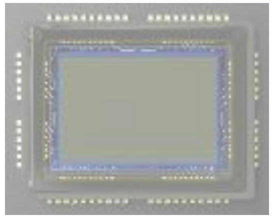


Pixel space

Metric space

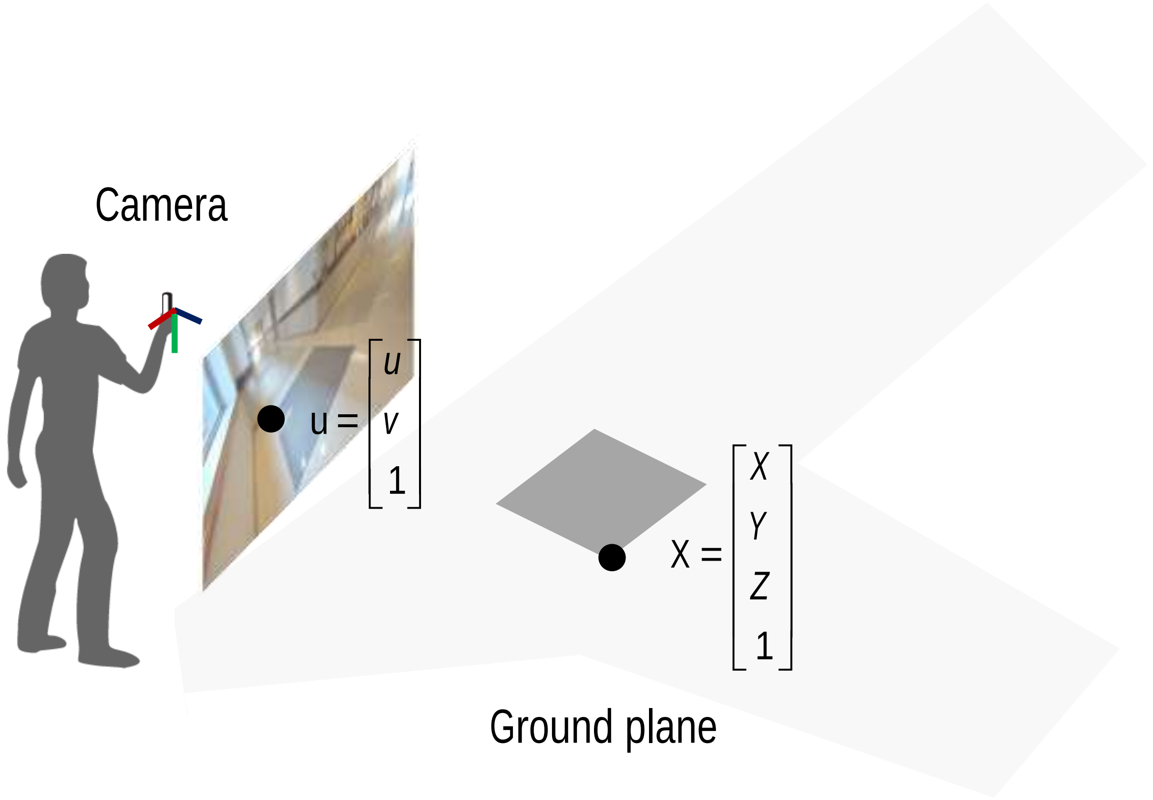
$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix}}_{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

+



Camera intrinsic parameter
: metric space to pixel space

Camera Calibration in Pixel Space

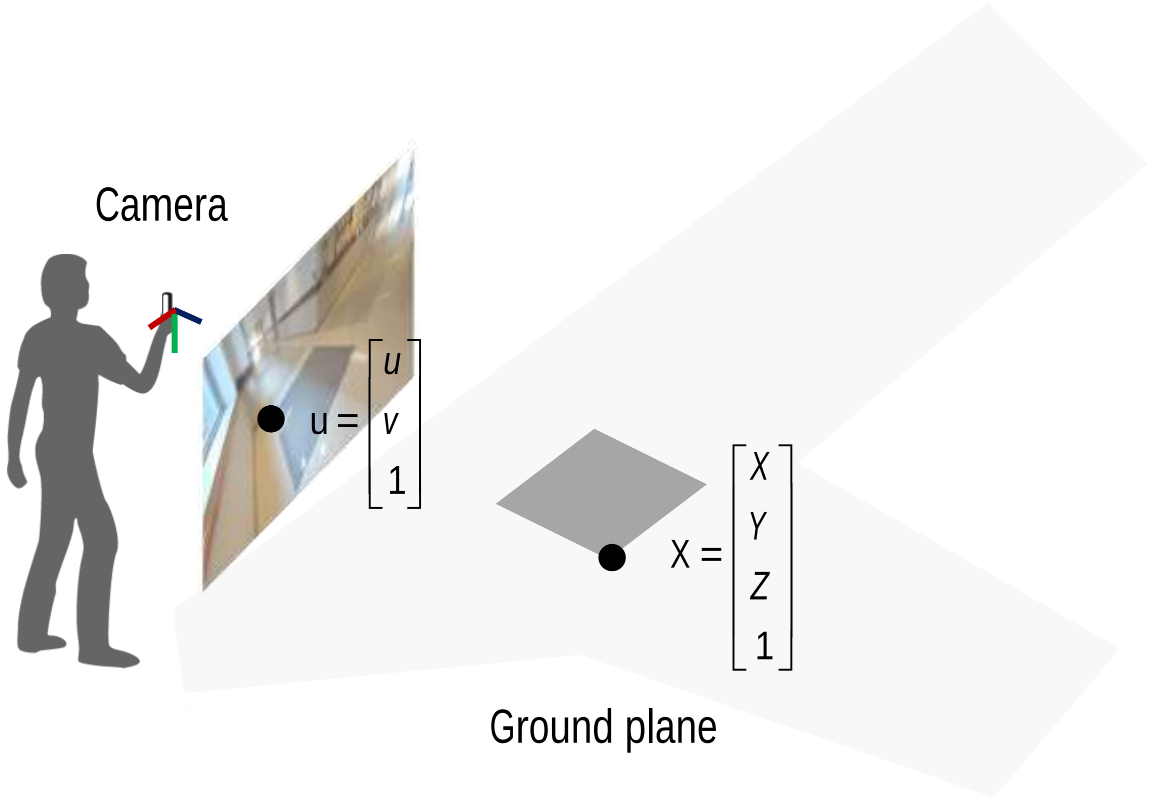


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns:

# of equations:

Camera Calibration in Pixel Space

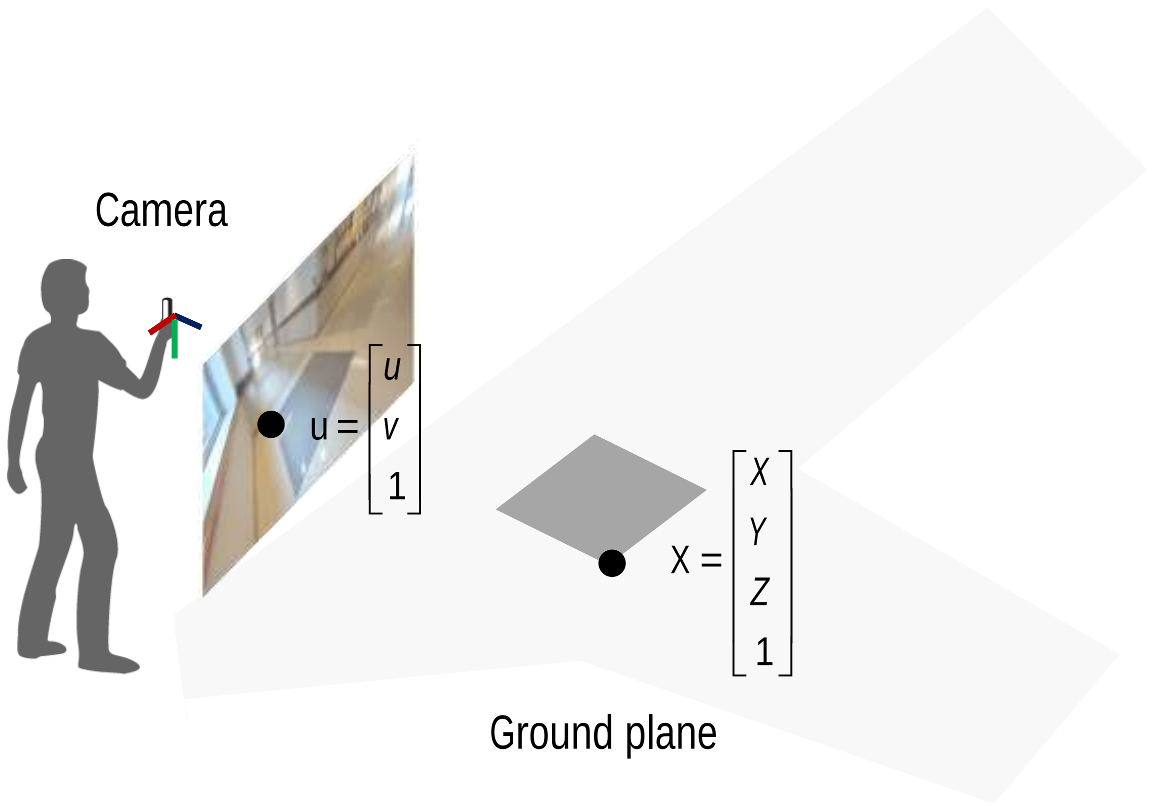


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

\# of unknowns: 3 (K) + 6 (R and t) + 3 (X)

\# of equations:

Camera Calibration in Pixel Space

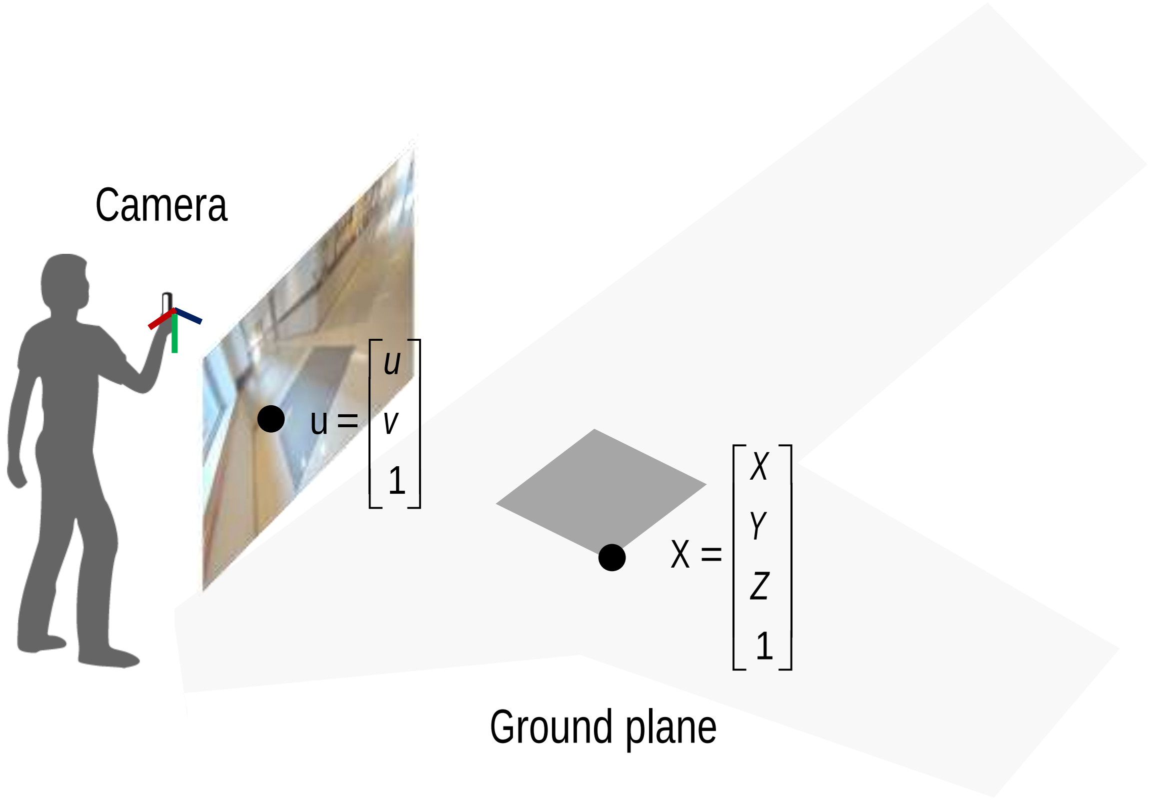


$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (K) + 6 (R and t) + 3 (X)

# of equations: 2

Camera Calibration in Pixel Space



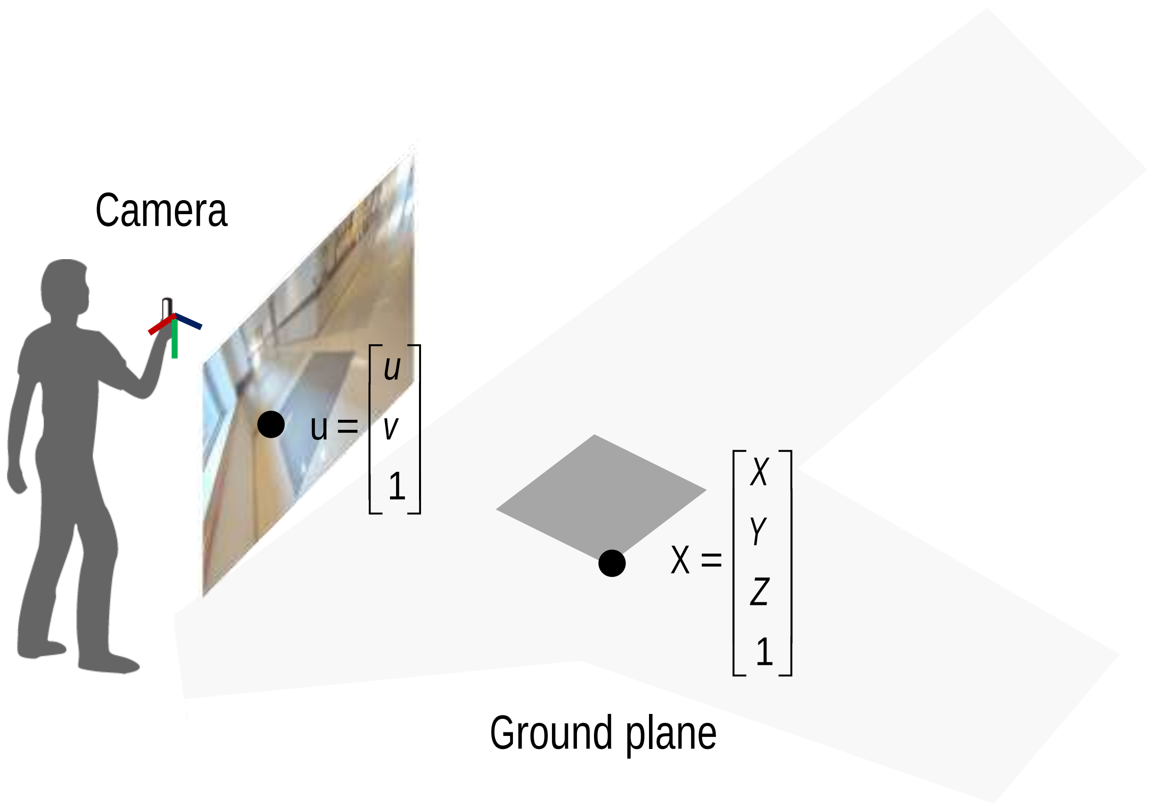
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

\# of unknowns: 3 (K) + 6F (R and t) + 3P (X)

\# of equations: 2P

where F is # of images and P is # of points.

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

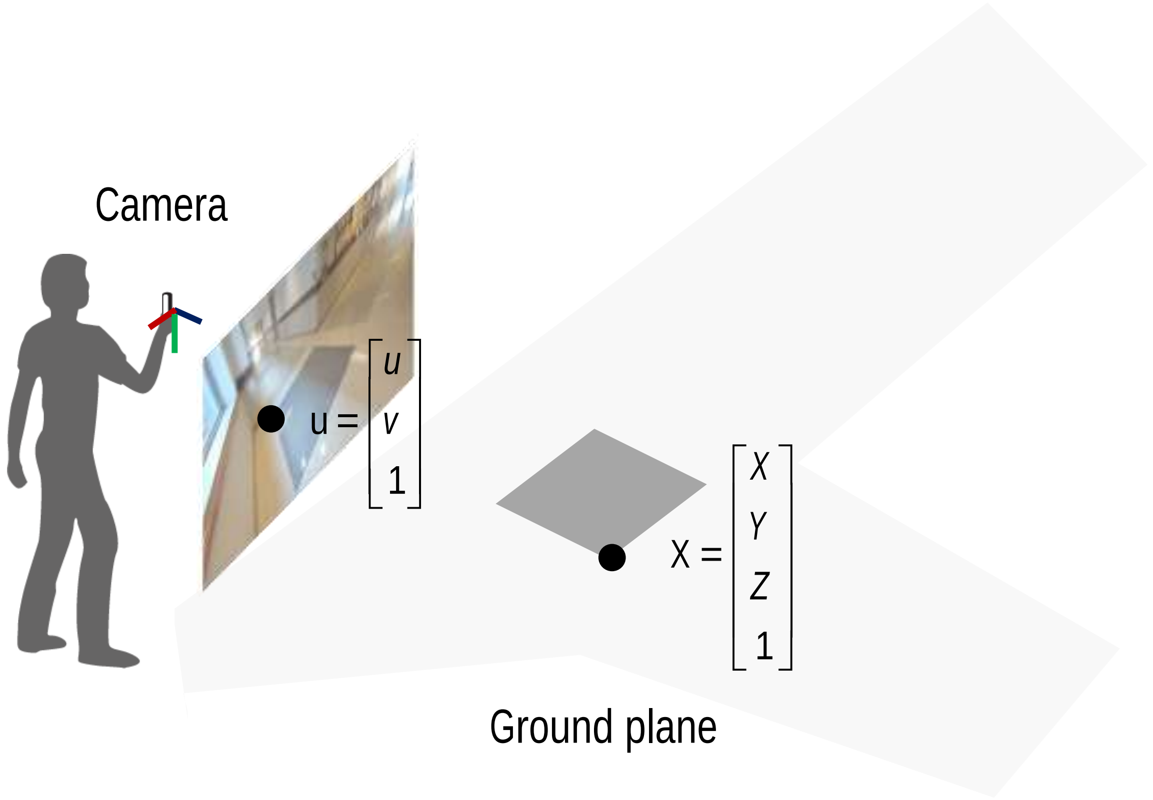
\# of unknowns: 3 (K) + 6F (R and t) + 3P (X)

\# of equations: 2P

where F is # of images and P is # of points.

\# of unknowns > # of equations

Camera Calibration in Pixel Space



$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

\# of unknowns: 3 (K) + 6F (R and t) + 3P (X)

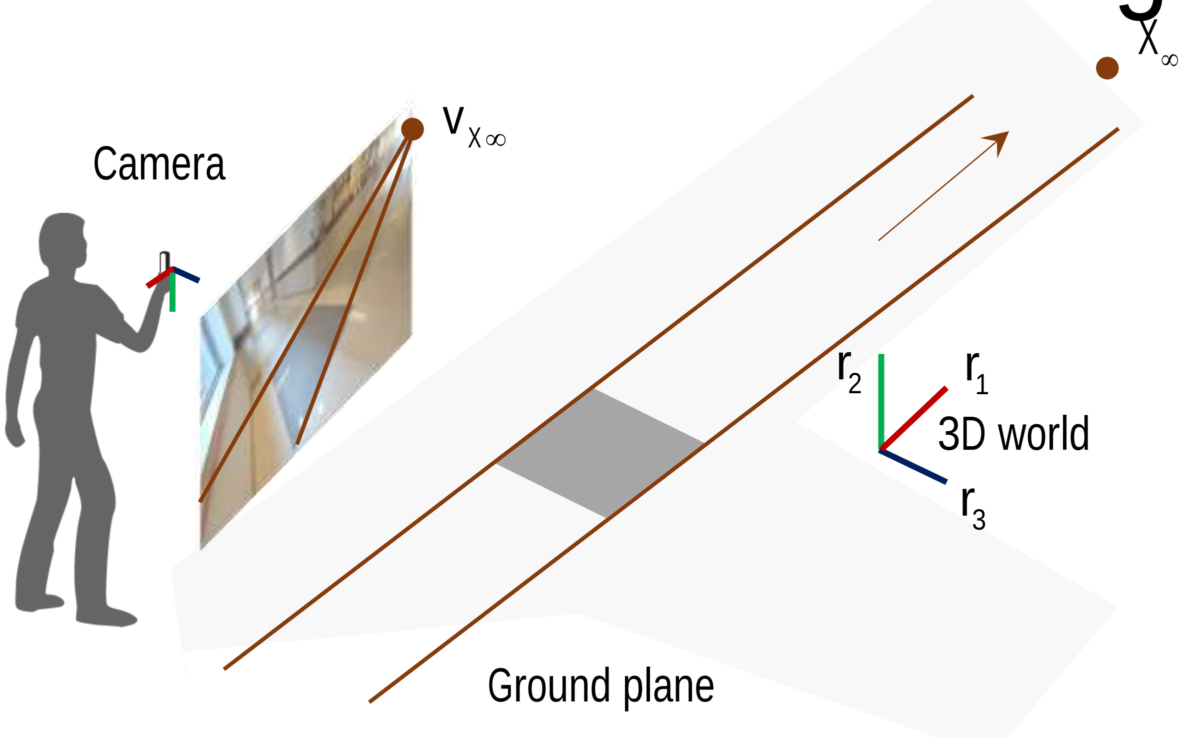
\# of equations: 2P

where F is # of images and P is # of points.

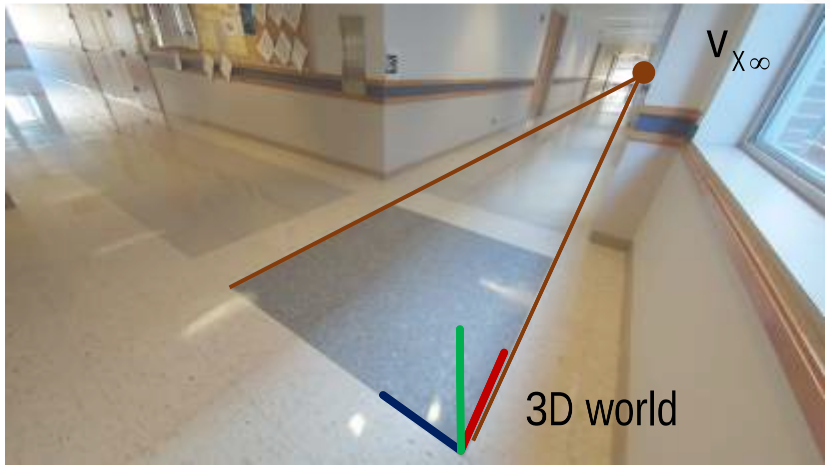
\# of unknowns > # of equations

What do we know about the scene?

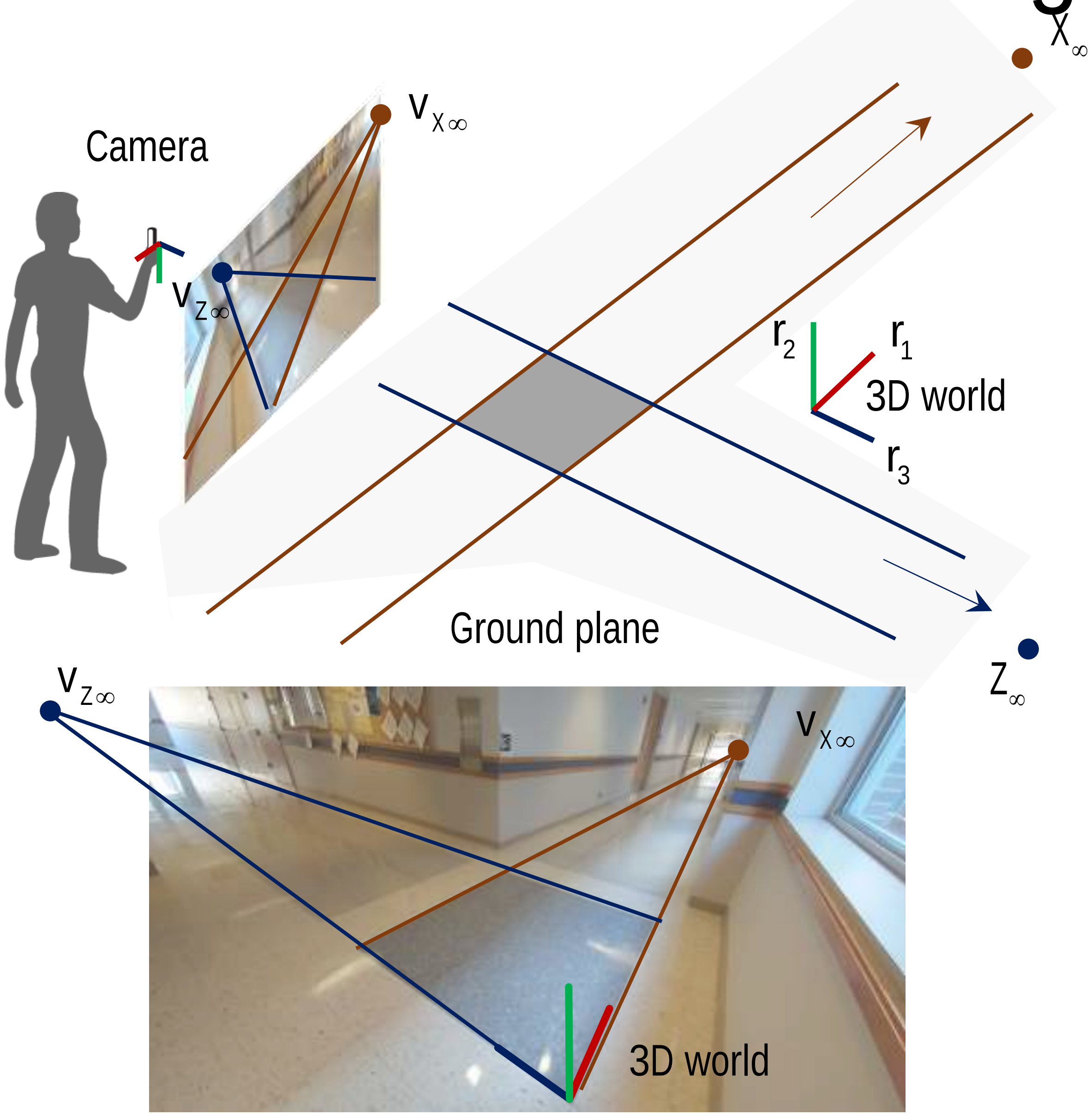
Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R X_{\infty}$$

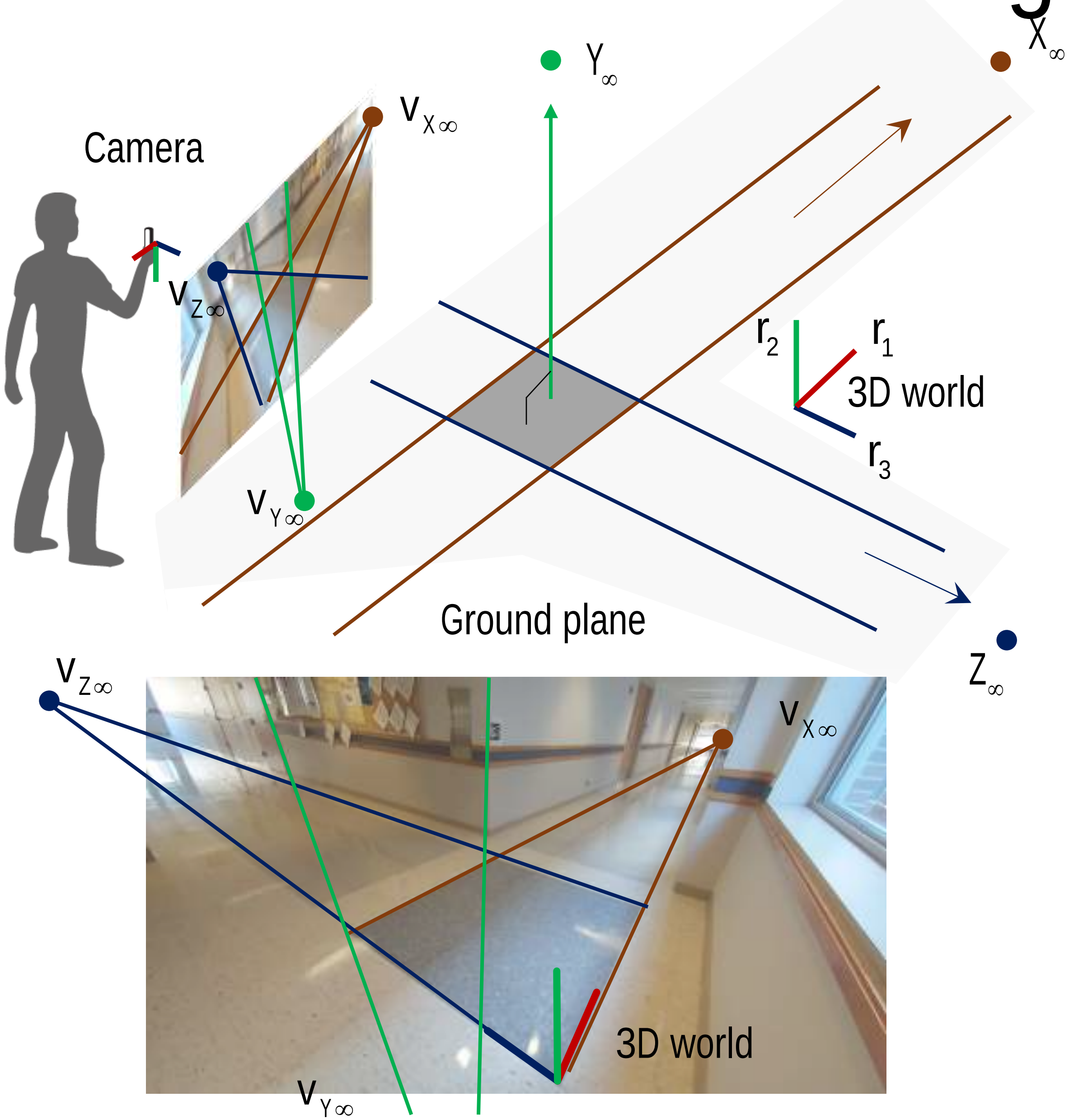


Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R X_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R Z_\infty$$

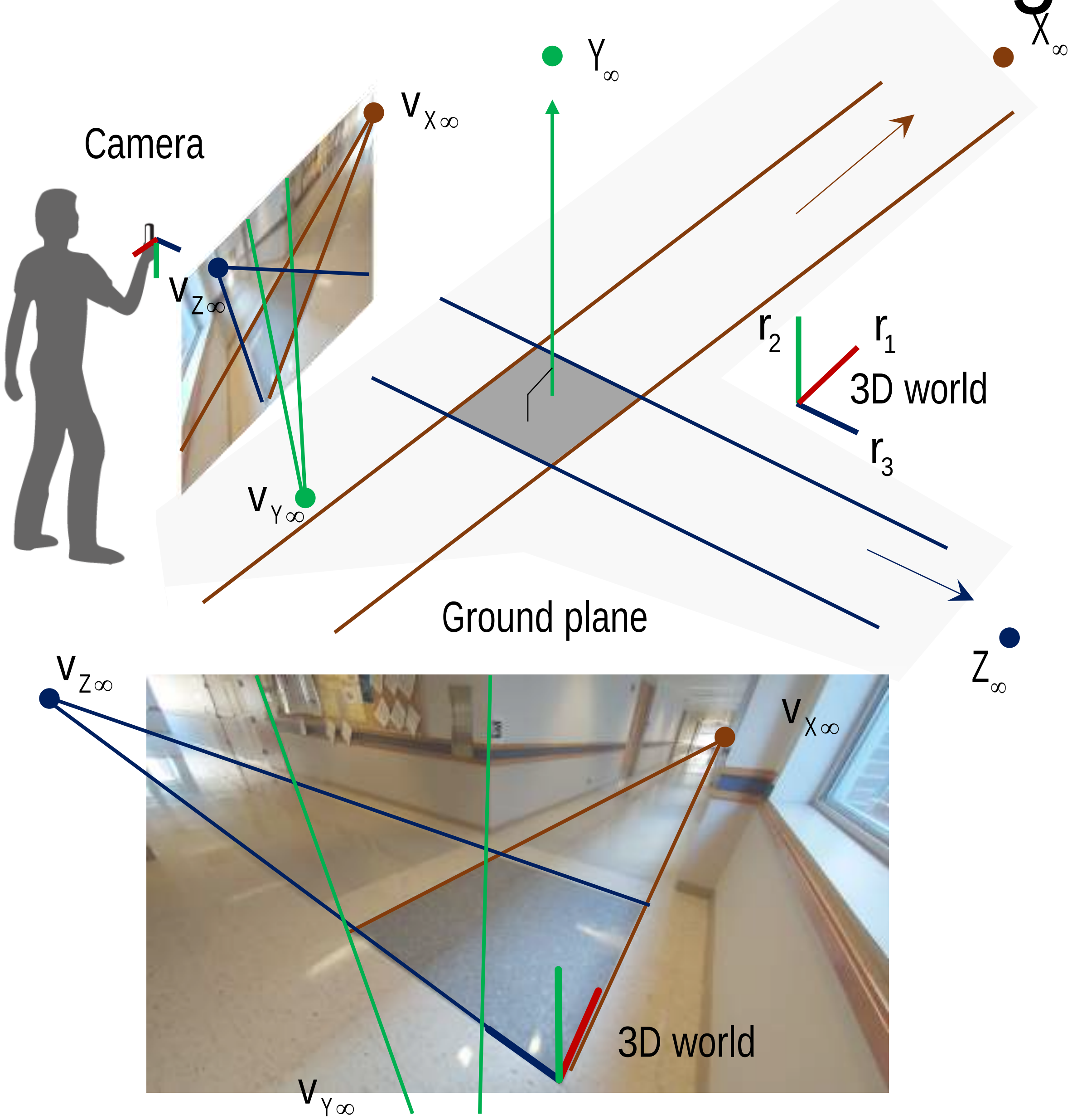
Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R X_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R Z_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} R Y_\infty$$

Note that the camera extrinsic is still unknown (R and t).

Camera Calibration using Vanishing Points



$$\lambda \mathrm{v}_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathrm{R} X_\infty \quad \lambda \mathrm{v}_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathrm{R} Z_\infty \quad \lambda \mathrm{v}_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathrm{R} Y_\infty$$

Note that the camera extrinsic is still unknown (R and t).

Known property of points at infinity:

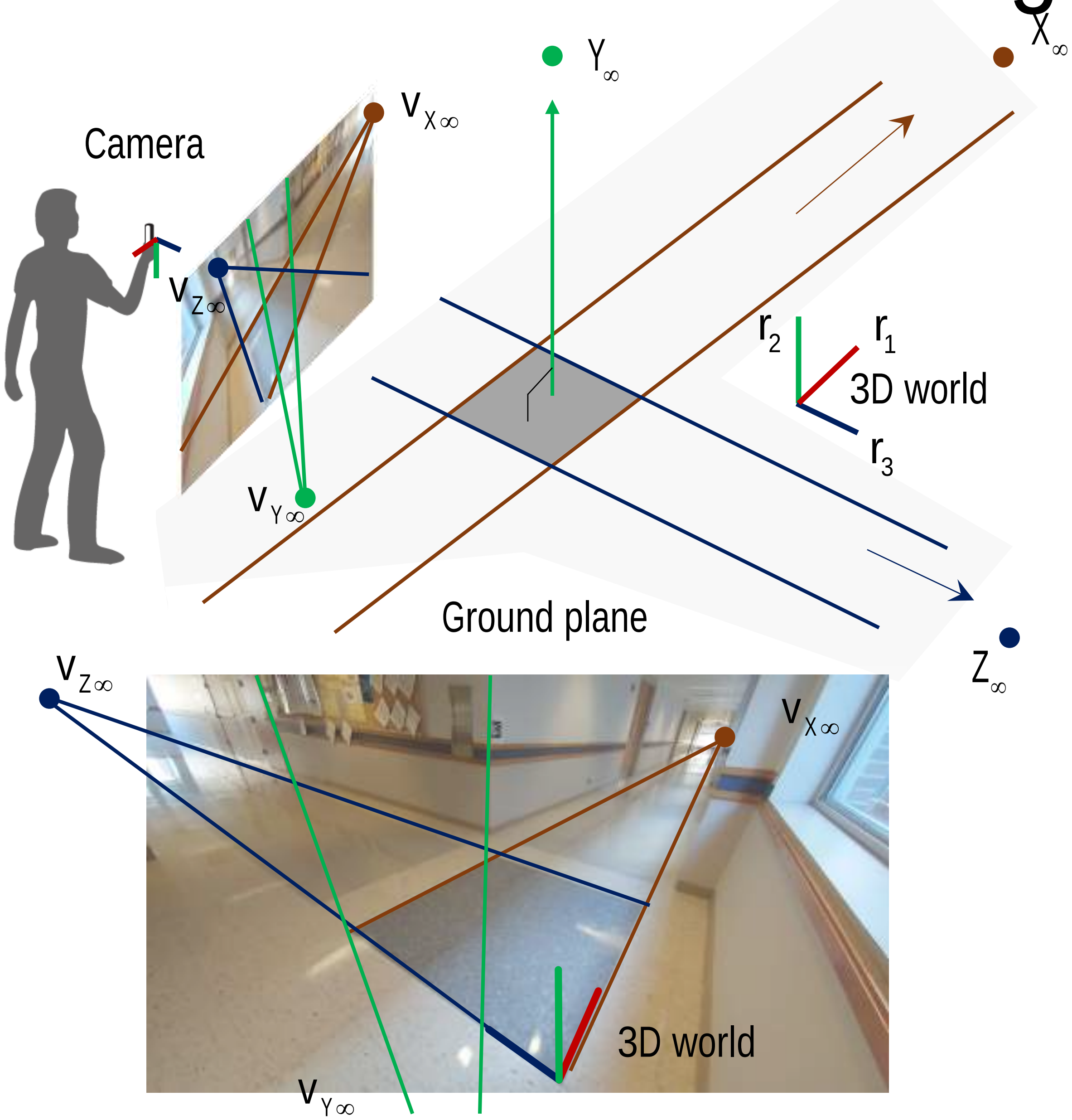
$$(X_\infty)^{\mathsf{T}}(Y_\infty) = 0$$

$$(Y_\infty)^{\mathsf{T}}(Z_\infty) = 0$$

$$(Z_\infty)^{\mathsf{T}}(X_\infty) = 0$$

These axes are perpendicular to each other.

Camera Calibration using Vanishing Points



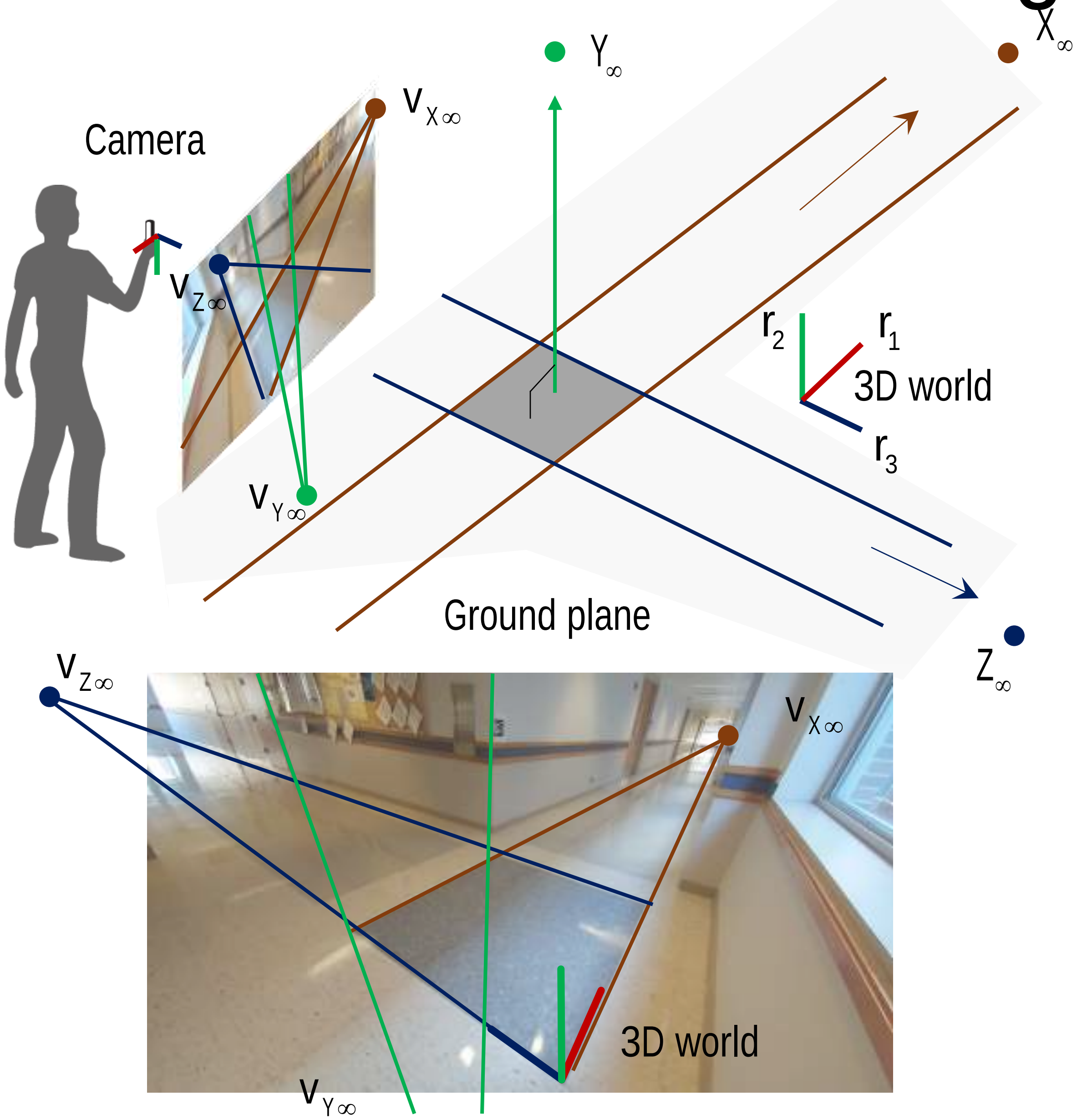
$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RX_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RZ_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RY_\infty$$

Note that the camera extrinsic is still unknown (R and t).

Known property of points at infinity:

$$\begin{aligned} (X_\infty)^T(Y_\infty) &= 0 \\ (Y_\infty)^T(Z_\infty) &= 0 \\ (Z_\infty)^T(X_\infty) &= 0 \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} (RX_\infty)^T(RY_\infty) &= 0 \\ (RY_\infty)^T(RZ_\infty) &= 0 \\ (RZ_\infty)^T(RX_\infty) &= 0 \end{aligned}$$

Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RX_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RZ_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RY_\infty$$

$$\lambda K^{-1} v_{X\infty} = RX_\infty \qquad \lambda K^{-1} v_{Y\infty} = RY_\infty \qquad \lambda K^{-1} v_{Z\infty} = RZ_\infty$$

Note that the camera extrinsic is still unknown (R and t).

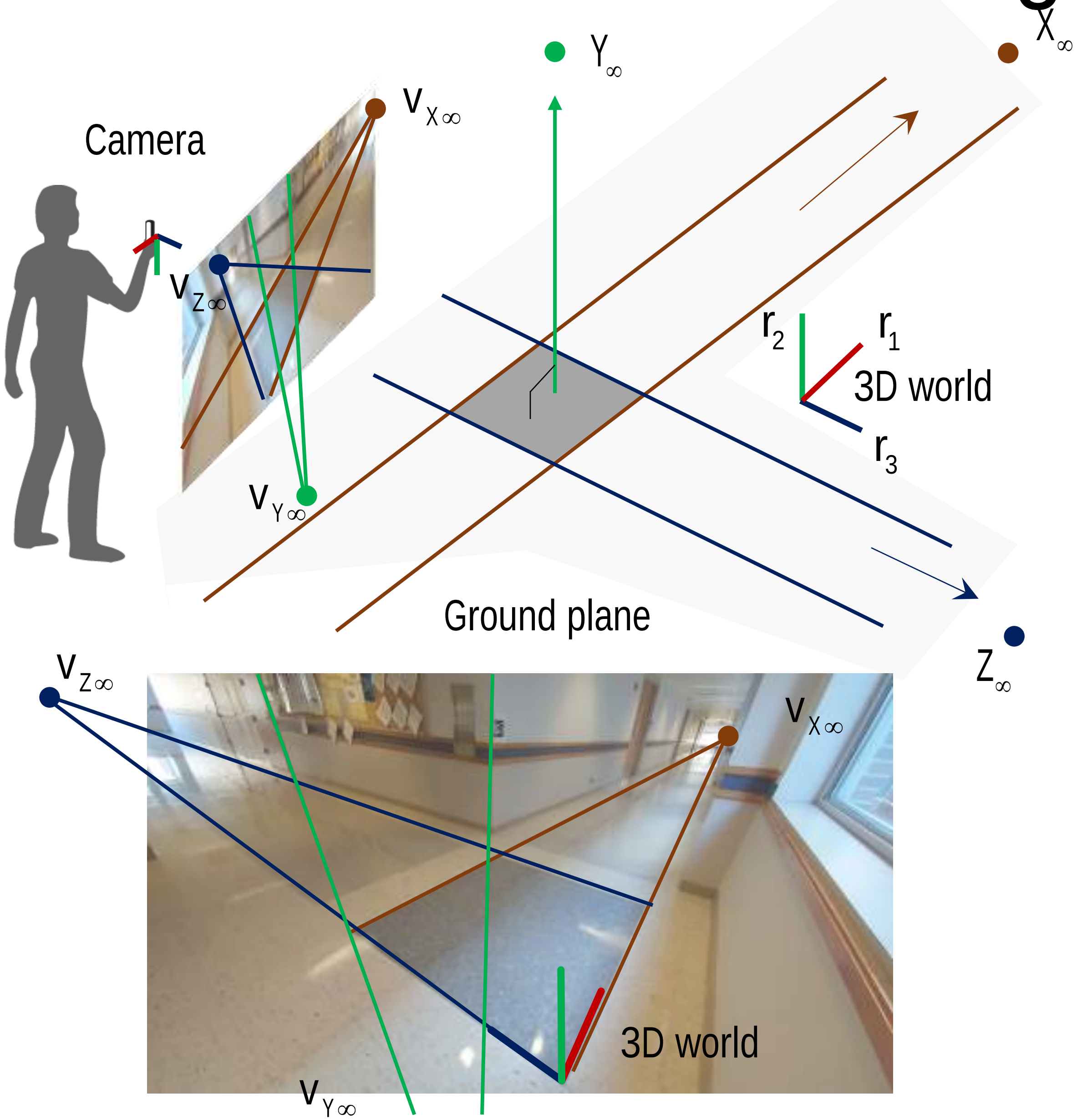
Known property of points at infinity:

$$(X_\infty)^T (Y_\infty) = 0 \qquad (RX_\infty)^T (RY_\infty) = 0$$

$$(Y_\infty)^T (Z_\infty) = 0 \quad \longleftrightarrow \quad (RY_\infty)^T (RZ_\infty) = 0$$

$$(Z_\infty)^T (X_\infty) = 0 \qquad (RZ_\infty)^T (RX_\infty) = 0$$

Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RX_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RZ_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RY_\infty$$

$$\lambda K^{-1} v_{X\infty} = RX_\infty \qquad \lambda K^{-1} v_{Y\infty} = RY_\infty \qquad \lambda K^{-1} v_{Z\infty} = RZ_\infty$$

Note that the camera extrinsic is still unknown (R and t).

Known property of points at infinity:

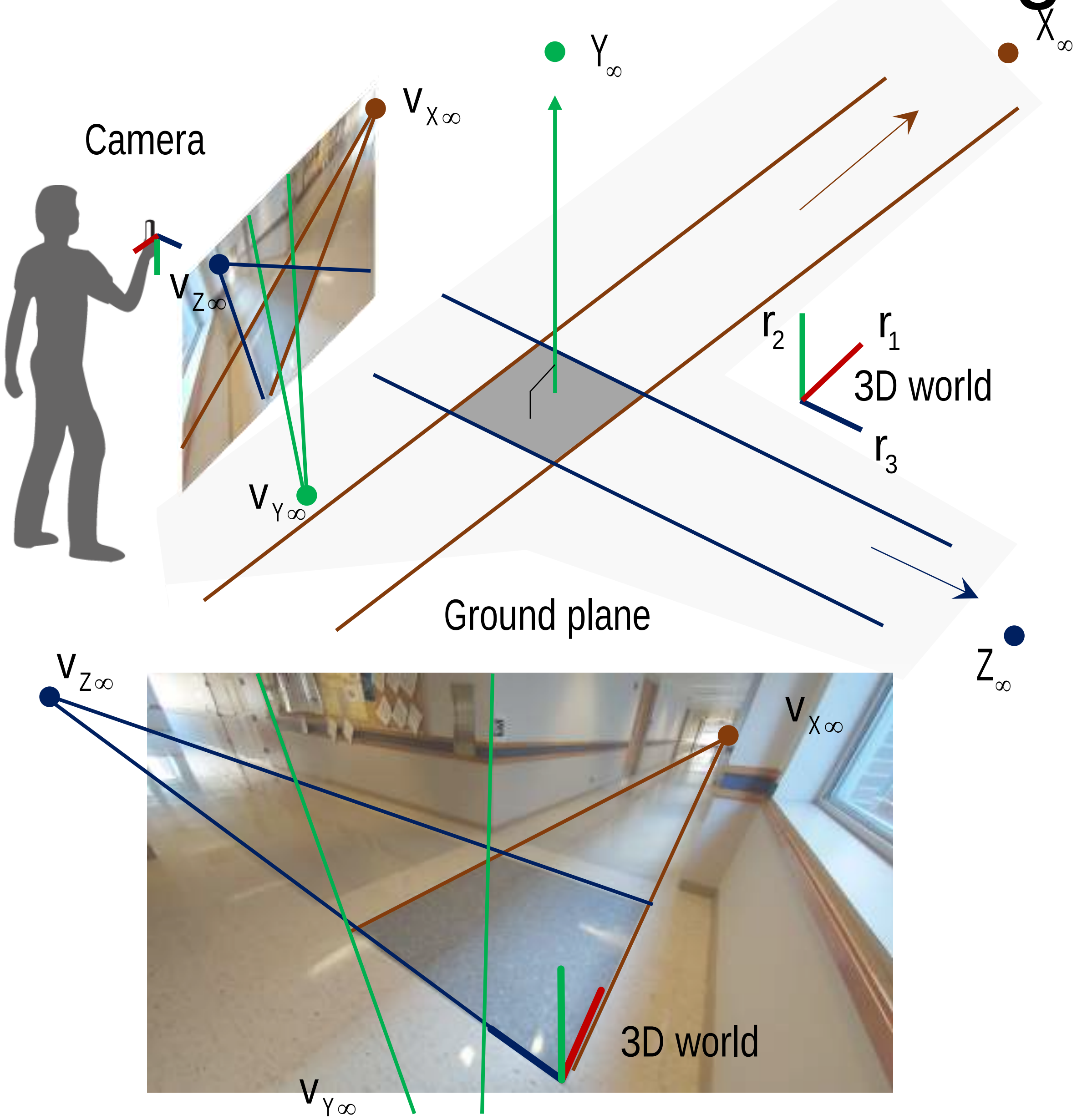
$$(X_\infty)^T (Y_\infty) = 0 \qquad (RX_\infty)^T (RY_\infty) = 0$$

$$(Y_\infty)^T (Z_\infty) = 0 \quad \longleftrightarrow \quad (RY_\infty)^T (RZ_\infty) = 0$$

$$(Z_\infty)^T (X_\infty) = 0 \qquad (RZ_\infty)^T (RX_\infty) = 0$$

$$\left(K^{-1} v_{X\infty}\right)^T \left(K^{-1} v_{Y\infty}\right) = \left(K^{-1} v_{Y\infty}\right)^T \left(K^{-1} v_{Z\infty}\right) = \left(K^{-1} v_{Z\infty}\right)^T \left(K^{-1} v_{X\infty}\right) = 0$$

Camera Calibration using Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RX_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RZ_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RY_\infty$$

$$\lambda K^{-1} v_{X\infty} = RX_\infty \qquad \lambda K^{-1} v_{Y\infty} = RY_\infty \qquad \lambda K^{-1} v_{Z\infty} = RZ_\infty$$

Note that the camera extrinsic is still unknown (R and t).

Known property of points at infinity:

$$(X_\infty)^T (Y_\infty) = 0 \qquad (RX_\infty)^T (RY_\infty) = 0$$

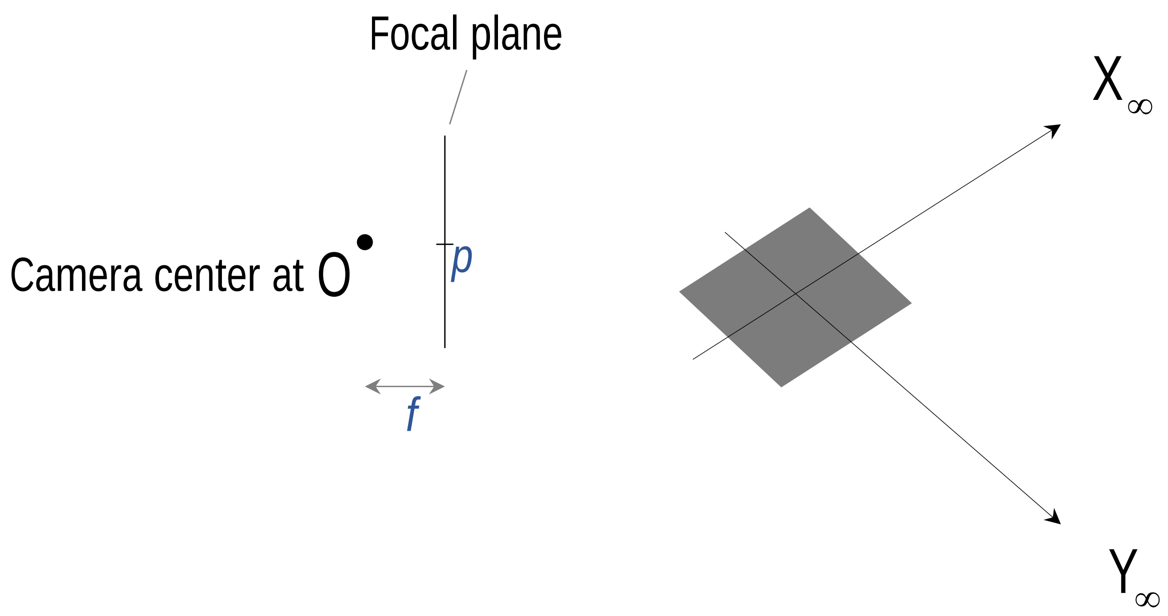
$$(Y_\infty)^T (Z_\infty) = 0 \quad \longleftrightarrow \quad (RY_\infty)^T (RZ_\infty) = 0$$

$$(Z_\infty)^T (X_\infty) = 0 \qquad (RZ_\infty)^T (RX_\infty) = 0$$

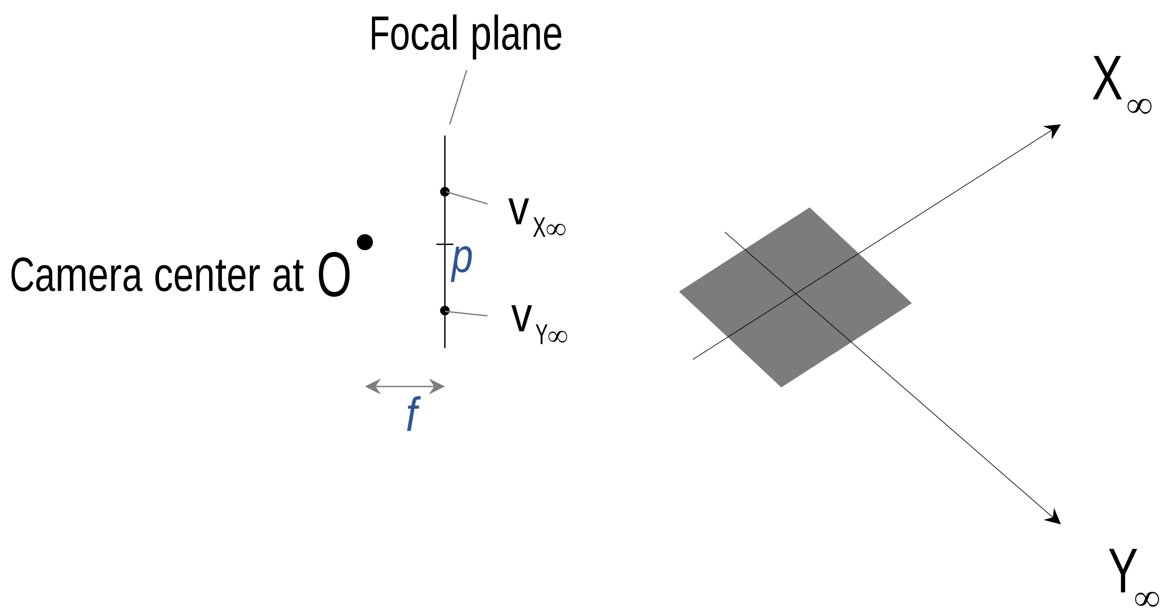
$$(K^{-1} v_{X\infty})^T (K^{-1} v_{Y\infty}) = (K^{-1} v_{Y\infty})^T (K^{-1} v_{Z\infty}) = (K^{-1} v_{Z\infty})^T (K^{-1} v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

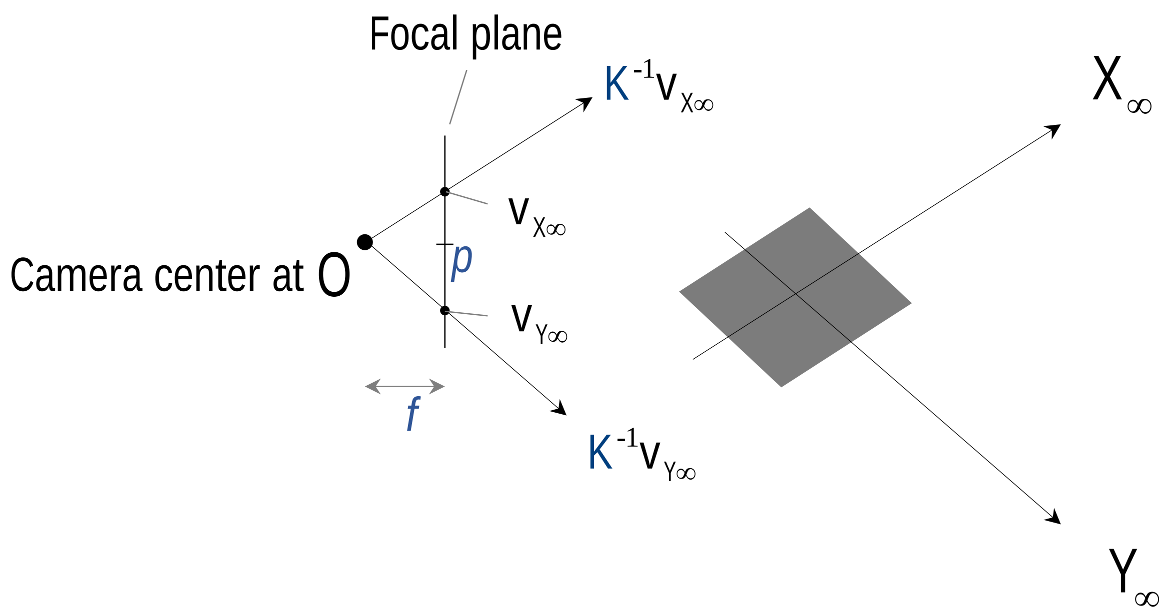
Geometric Interpretation with 1D Camera



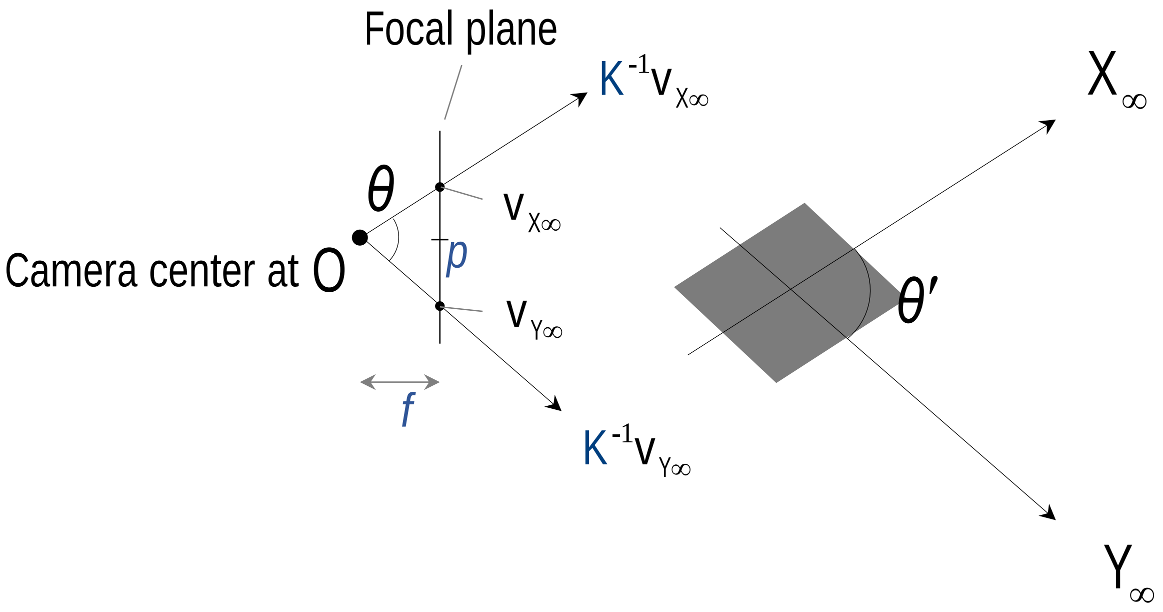
Geometric Interpretation with 1D Camera



Geometric Interpretation with 1D Camera

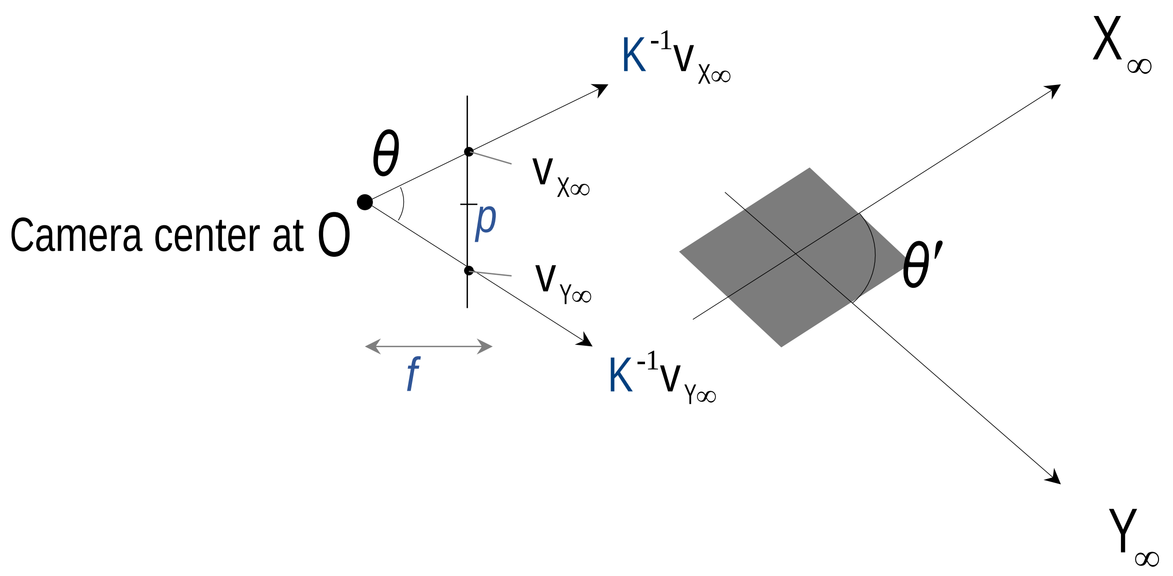


Geometric Interpretation with 1D Camera



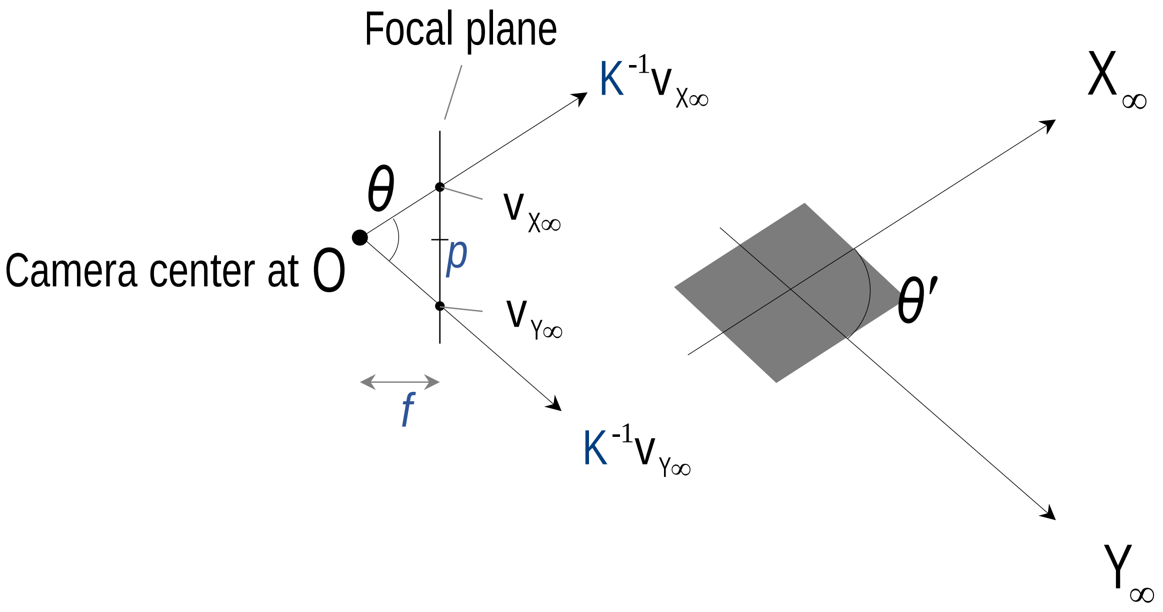
$\theta = \theta'$

Geometric Interpretation with 1D Camera



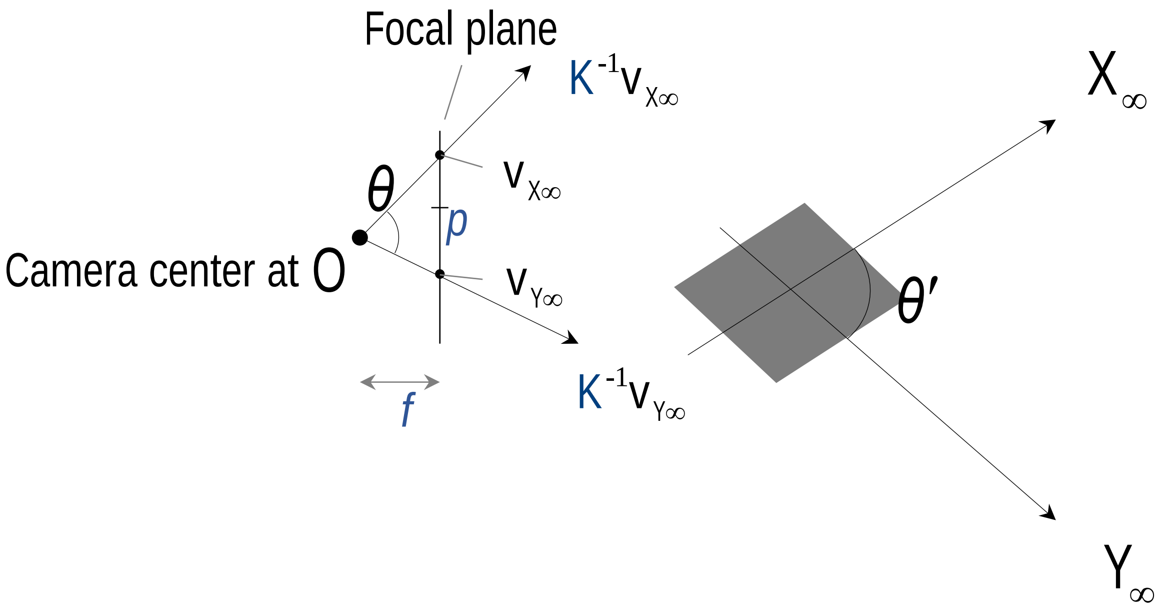
$\theta \neq \theta'$

Geometric Interpretation with 1D Camera



$\theta = \theta'$

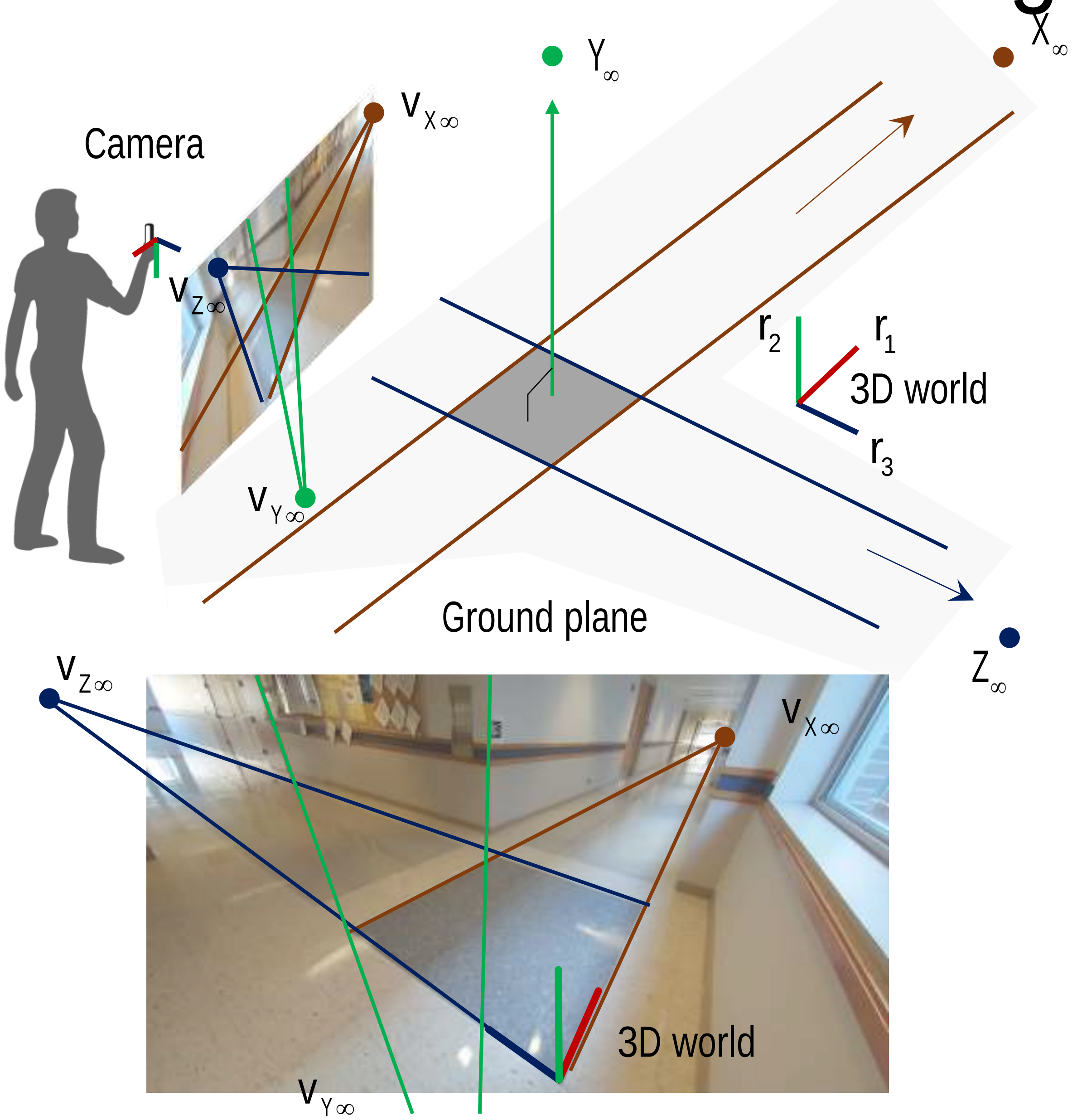
Geometric Interpretation with 1D Camera



$\theta \neq \theta'$

Given two vanishing points, the focal length and principal point are uniquely defined.
For the 2D camera case, another vanishing point is needed to uniquely define f, p_x, and p_y.

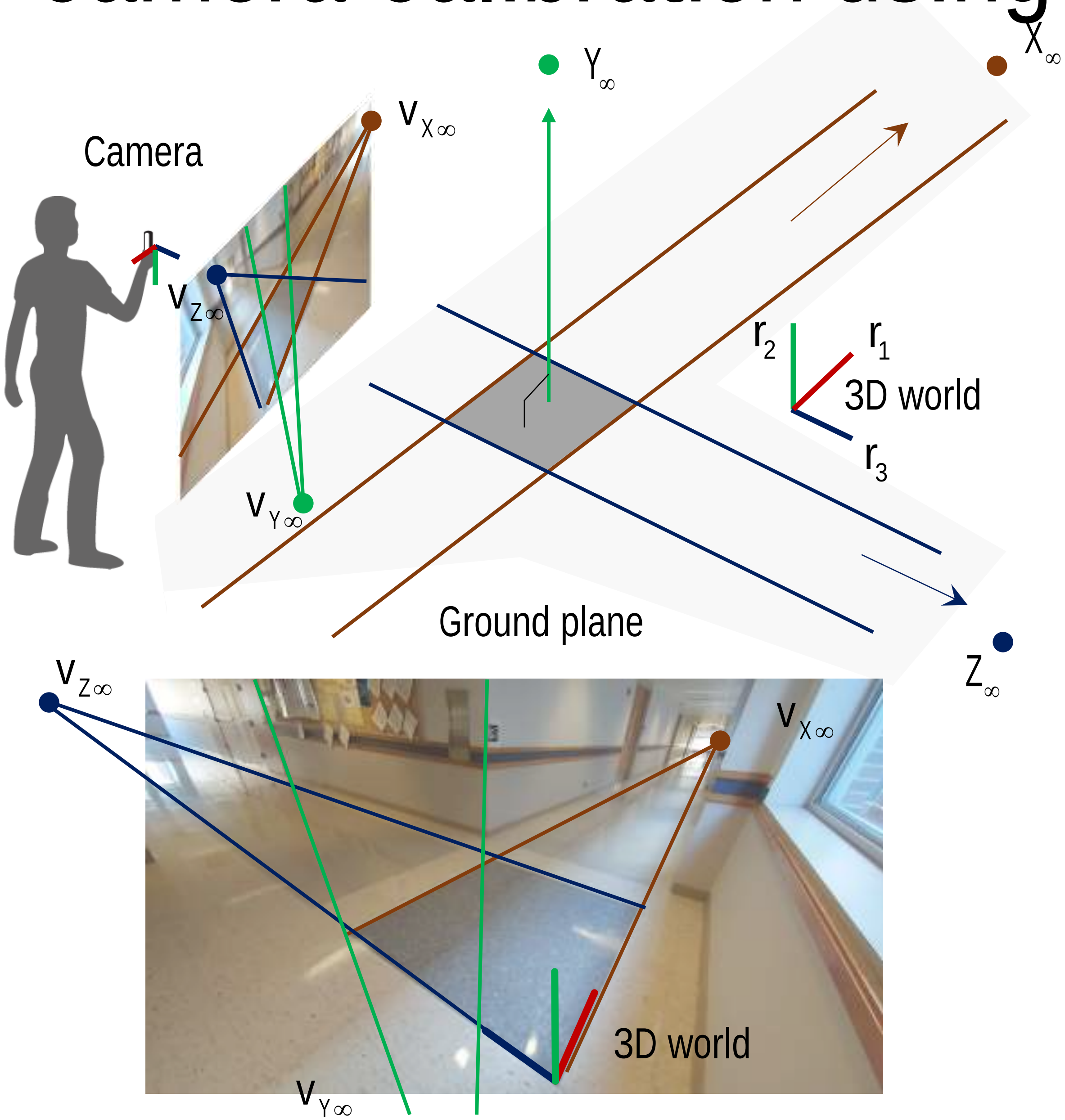
Camera Calibration using Vanishing Points



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

Camera Calibration using Vanishing Points

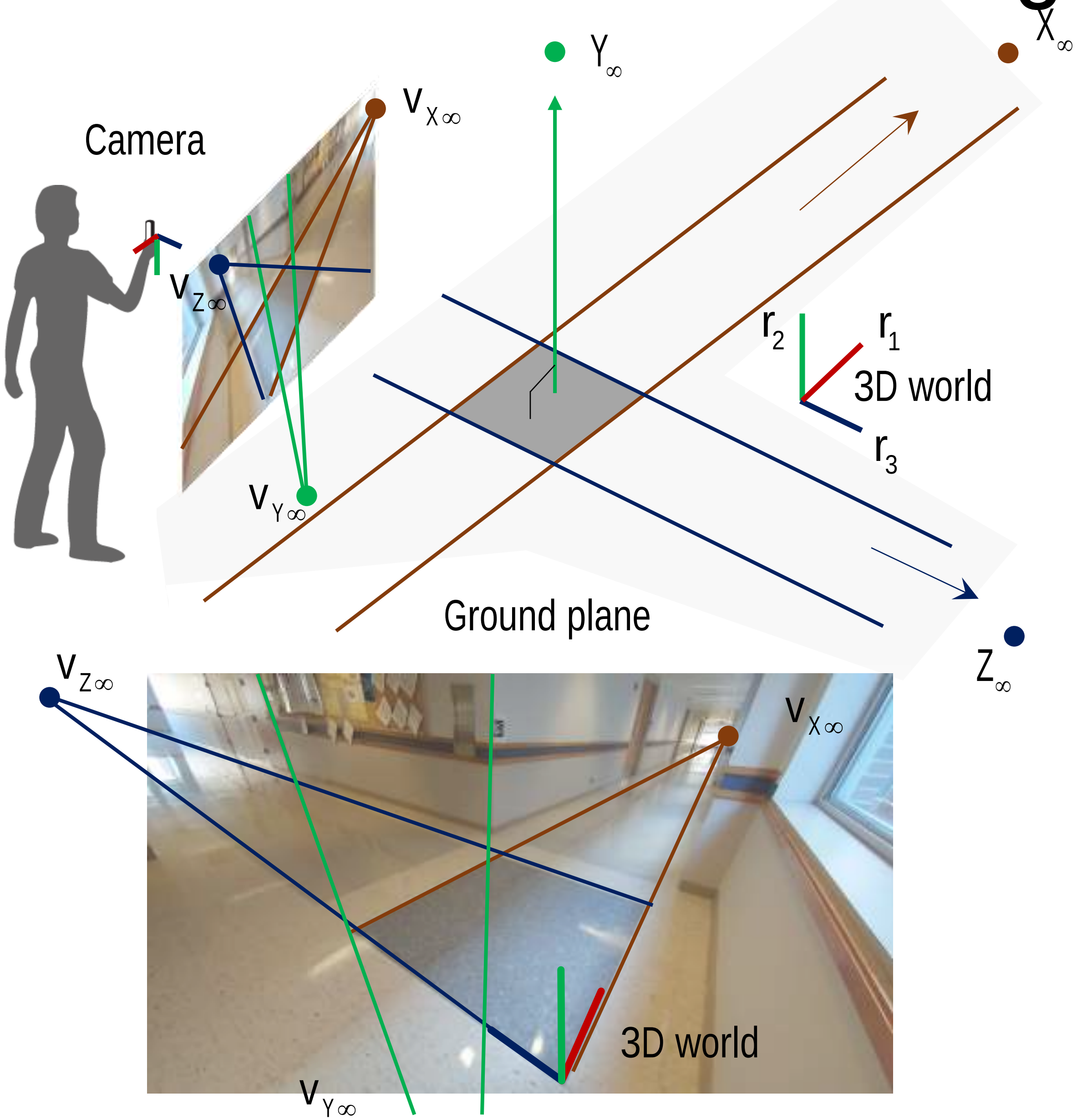


$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right)=v_i^TK^{-T}K^{-1}v_j \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

Camera Calibration using Vanishing Points



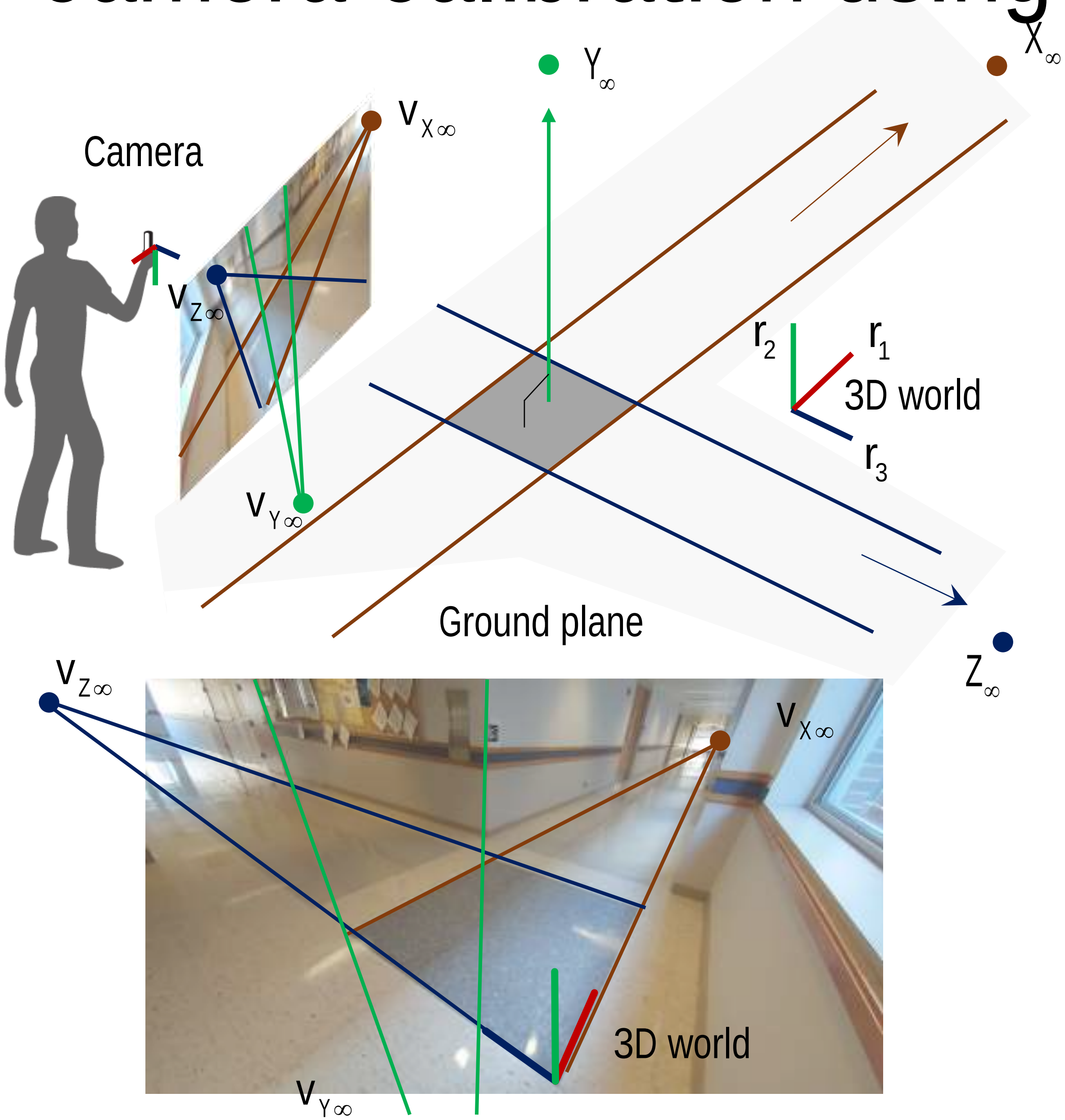
$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right) = \left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right) = \left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right) = 0$$

: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right) = v_i^T K^{-T}K^{-1}v_j \qquad i, j = X,Y,Z, \text{ and } i \neq j$$

$$K^{-T}K^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

Camera Calibration using Vanishing Points



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

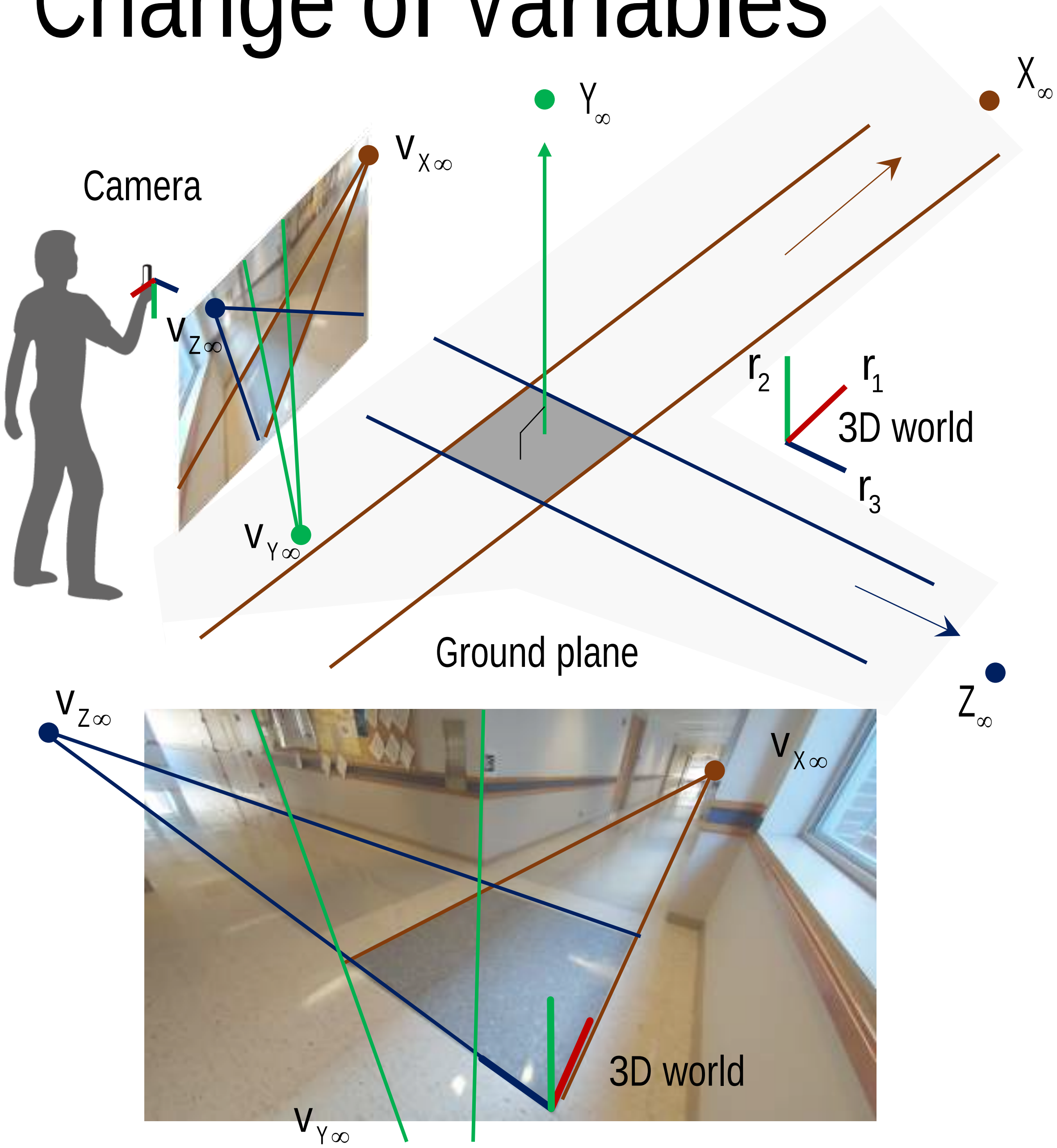
: 3 unknowns and 3 equations

$$\longrightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right)=v_i^T K^{-T}K^{-1}v_j \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$K^{-T}K^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1\end{bmatrix}$$

Change of Variables



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

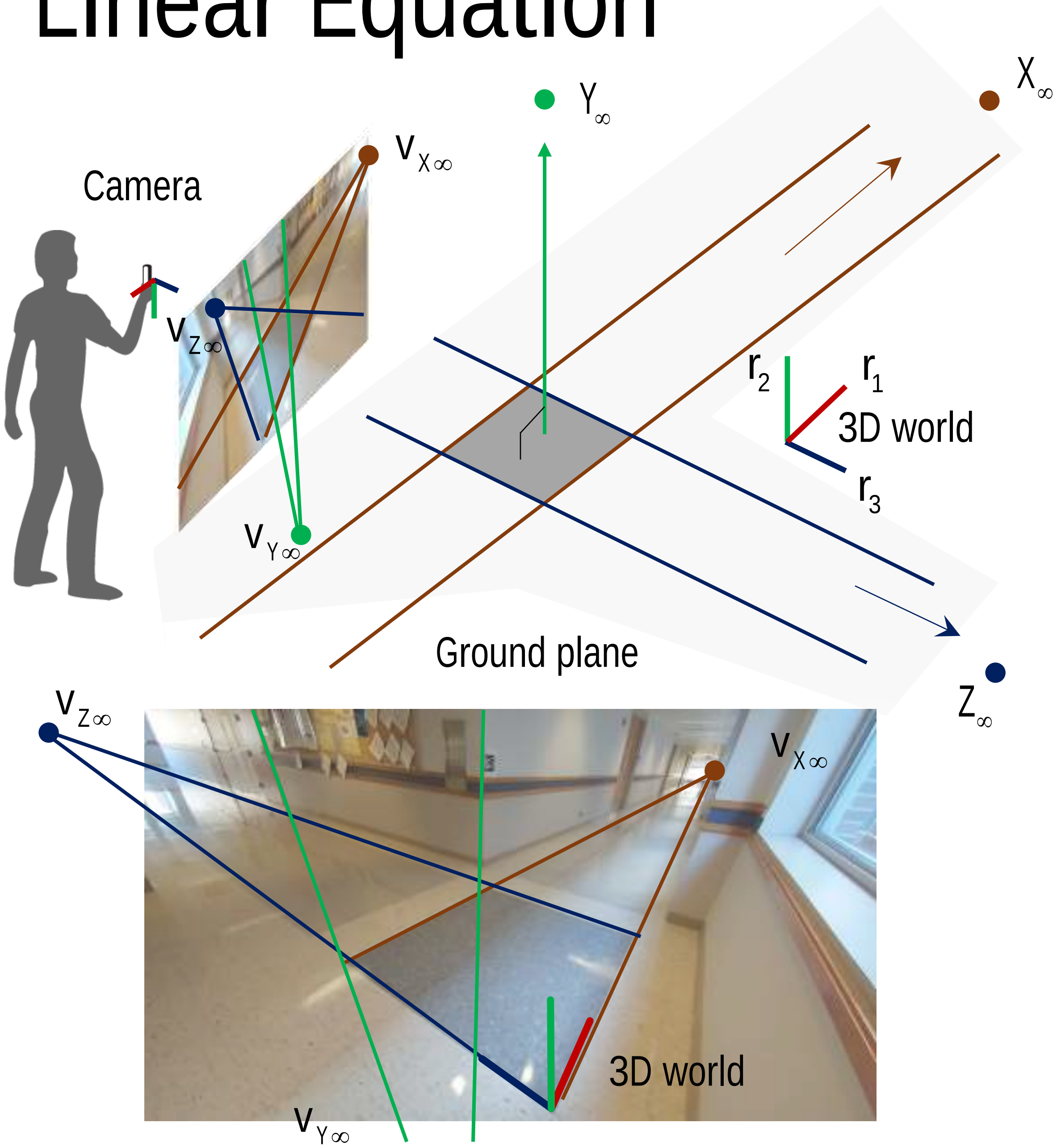
$$\rightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right)=v_i^T K^{-T}K^{-1}v_j \qquad i,j=X,Y,Z,\ \text{and}\ i\neq j$$

$$K^{-T}K^{-1}=\begin{bmatrix}1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1\end{bmatrix}\begin{bmatrix}1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{f^2} & & -\frac{p_x}{f^2} \\ & \frac{1}{f^2} & -\frac{p_y}{f^2} \\ -\frac{p_x}{f^2} & -\frac{p_y}{f^2} & \frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1\end{bmatrix}=\begin{bmatrix}b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4\end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

Linear Equation



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right) = \left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right) = \left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right) = 0$$

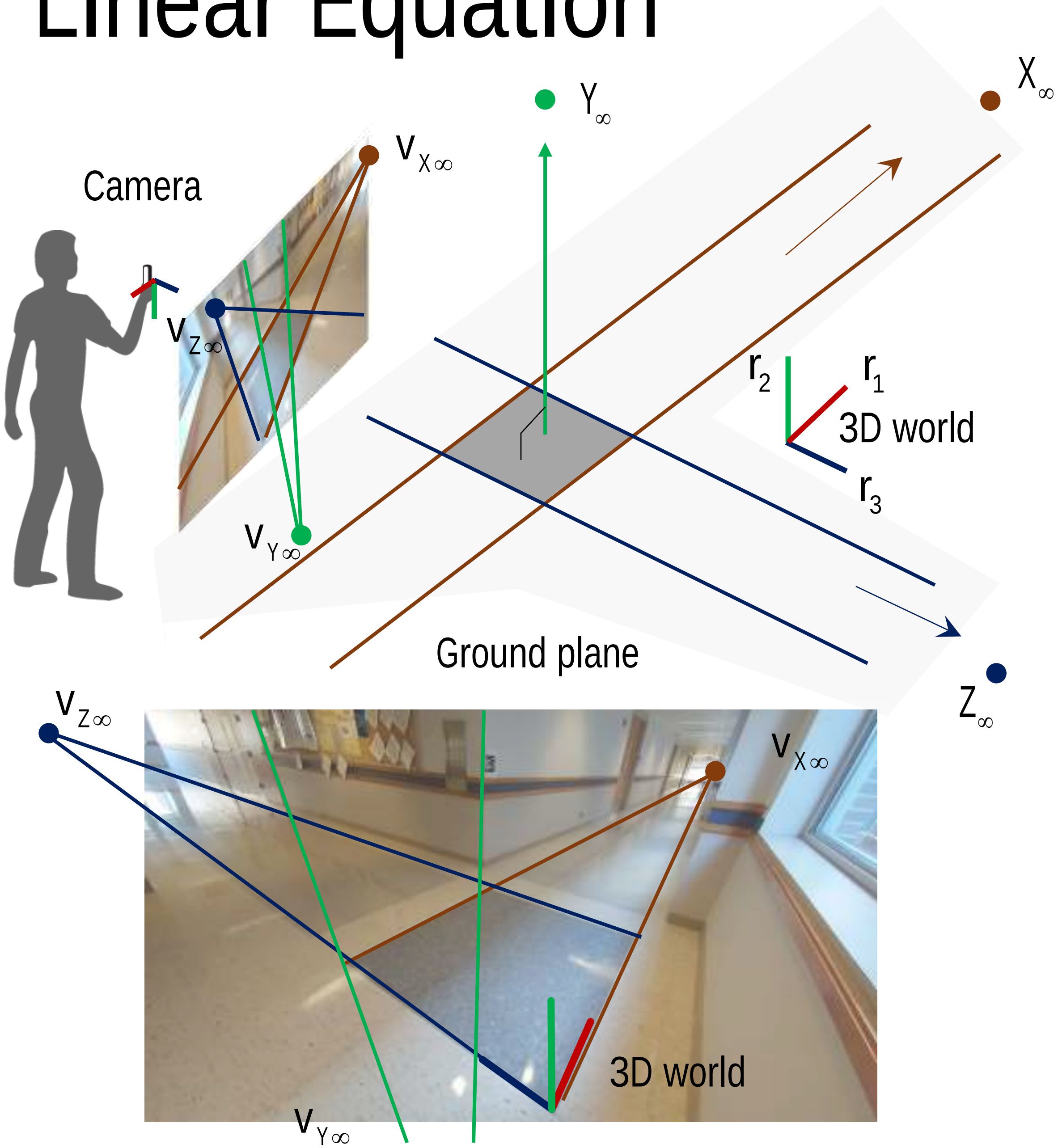
: 3 unknowns and 3 equations

→ $\left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right) = v_i^T K^{-T} K^{-1} v_j$ $i, j = X, Y, Z,$ and $i \neq j$

→ $v_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} v_j$:

Linear in b

Linear Equation



$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right) = \left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right) = \left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right) = 0$$

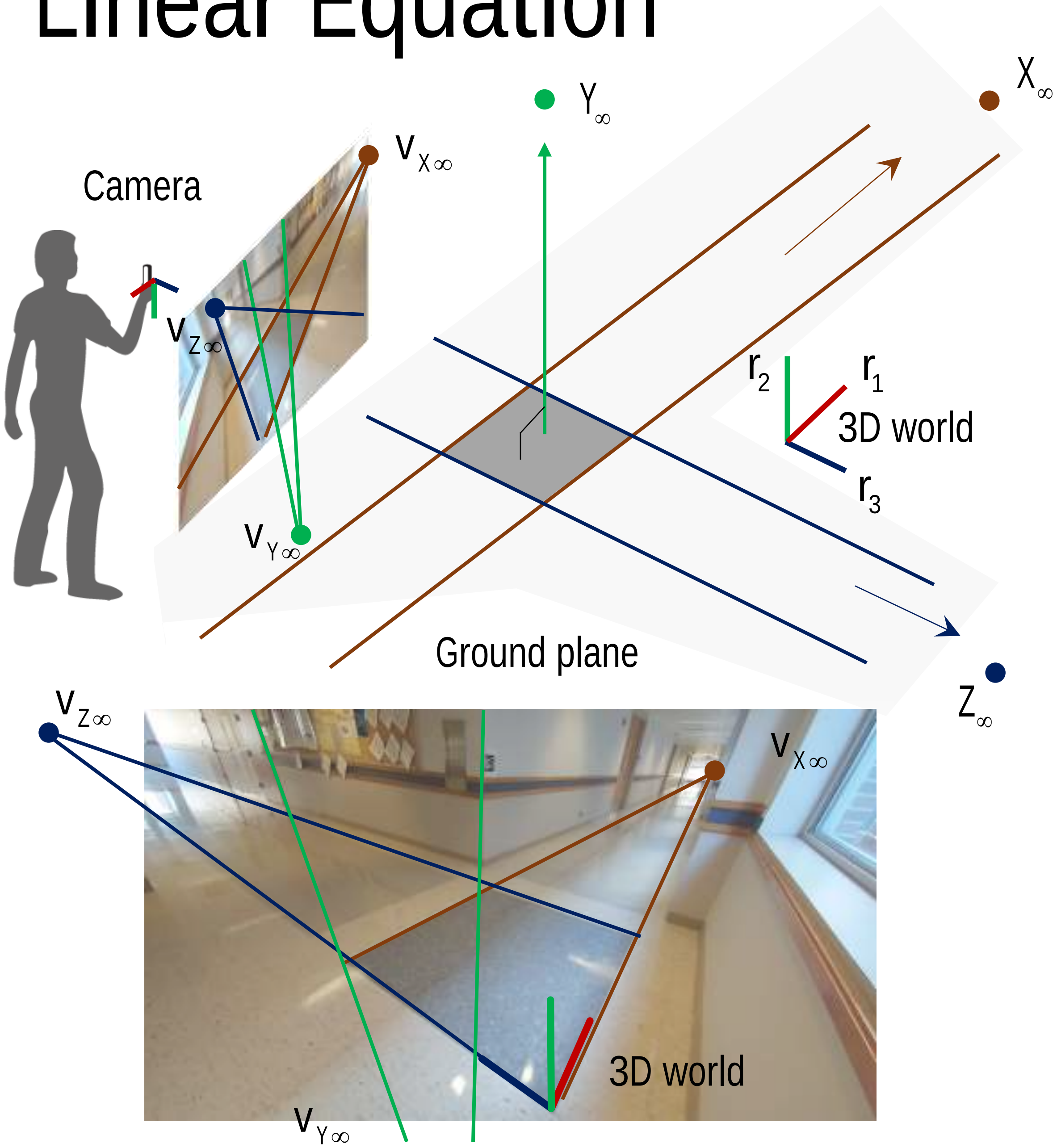
: 3 unknowns and 3 equations

$$\rightarrow \quad \left(K^{-1}v_i\right)^T\left(K^{-1}v_j\right) = v_i^T K^{-T} K^{-1} v_j \qquad i, j = X, Y, Z, \text{ and } i \neq j$$

$$\rightarrow \quad v_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} v_j = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

Linear in b

Linear Equation



$$(K^{-1}v_{X\infty})^T(K^{-1}v_{Y\infty}) = (K^{-1}v_{Y\infty})^T(K^{-1}v_{Z\infty}) = (K^{-1}v_{Z\infty})^T(K^{-1}v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

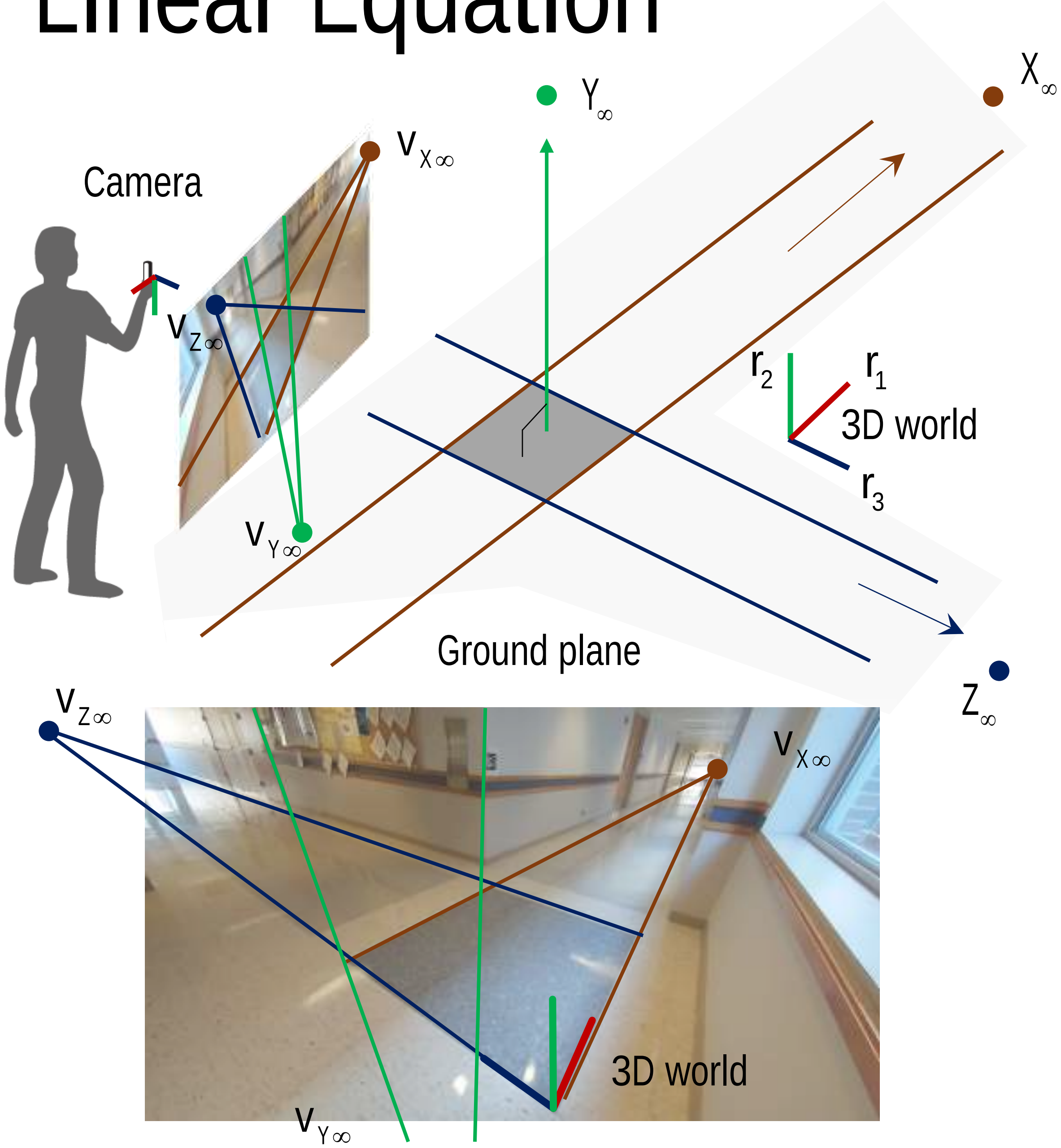
→ $(K^{-1}v_i)^T(K^{-1}v_j) = v_i^T K^{-T}K^{-1}v_j \qquad i, j = X, Y, Z, \text{ and } i \neq j$

→ $$v_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} v_j = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

Linear in b

→ $$\begin{bmatrix} u_iu_j + v_iv_j & u_i + u_j & v_i + v_j & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

Linear Equation



$$(K^{-1}v_{X\infty})^T(K^{-1}v_{Y\infty}) = (K^{-1}v_{Y\infty})^T(K^{-1}v_{Z\infty}) = (K^{-1}v_{Z\infty})^T(K^{-1}v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

$\rightarrow \quad (K^{-1}v_i)^T(K^{-1}v_j) = v_i^T K^{-T} K^{-1} v_j \qquad i, j = X, Y, Z, \text{ and } i \neq j$

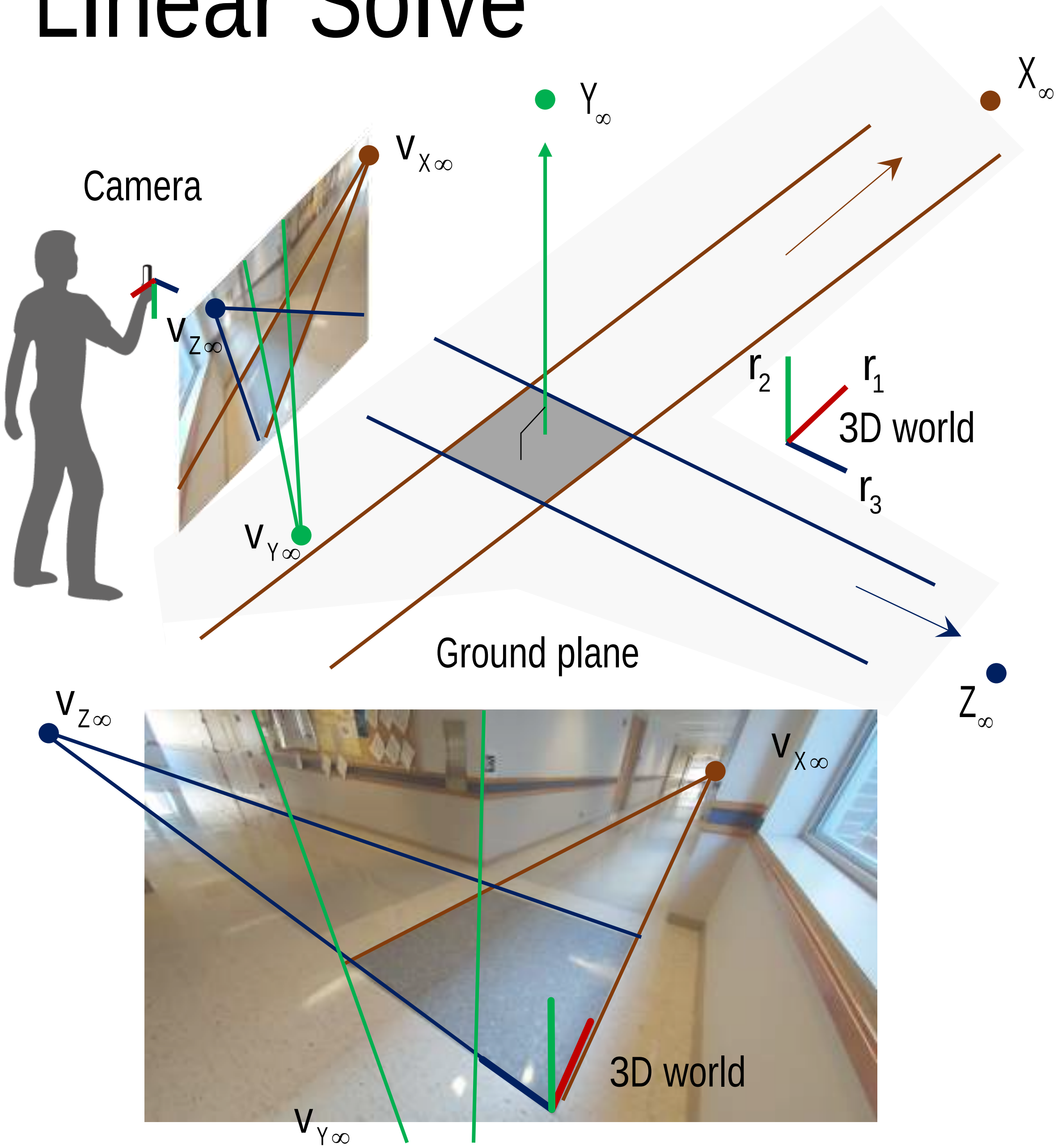
$$\rightarrow \quad v_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} v_j = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

Linear in b

$$\rightarrow \quad \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

3x4

Linear Solve



$$(K^{-1}v_{X\infty})^T(K^{-1}v_{Y\infty}) = (K^{-1}v_{Y\infty})^T(K^{-1}v_{Z\infty}) = (K^{-1}v_{Z\infty})^T(K^{-1}v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

→ $(K^{-1}v_i)^T(K^{-1}v_j) = v_i^T K^{-T} K^{-1} v_j$ $\quad i, j = X, Y, Z,$ and $i \neq j$

→ $$v_i^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} v_j = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} u_j \\ v_j \\ 1 \end{bmatrix} = 0$$

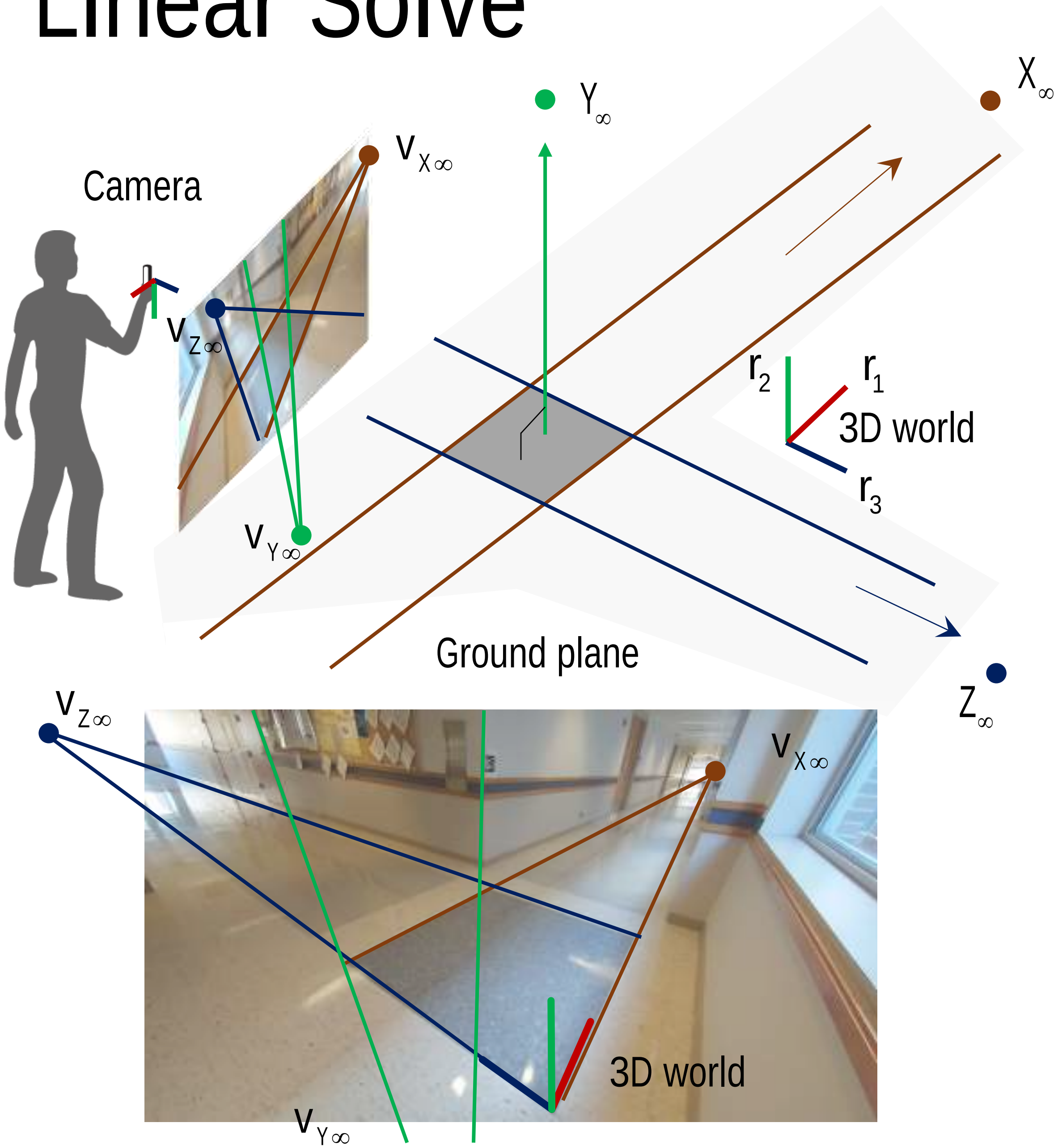
Linear in b

→ $$\begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

A x = 0

3x4

Linear Solve



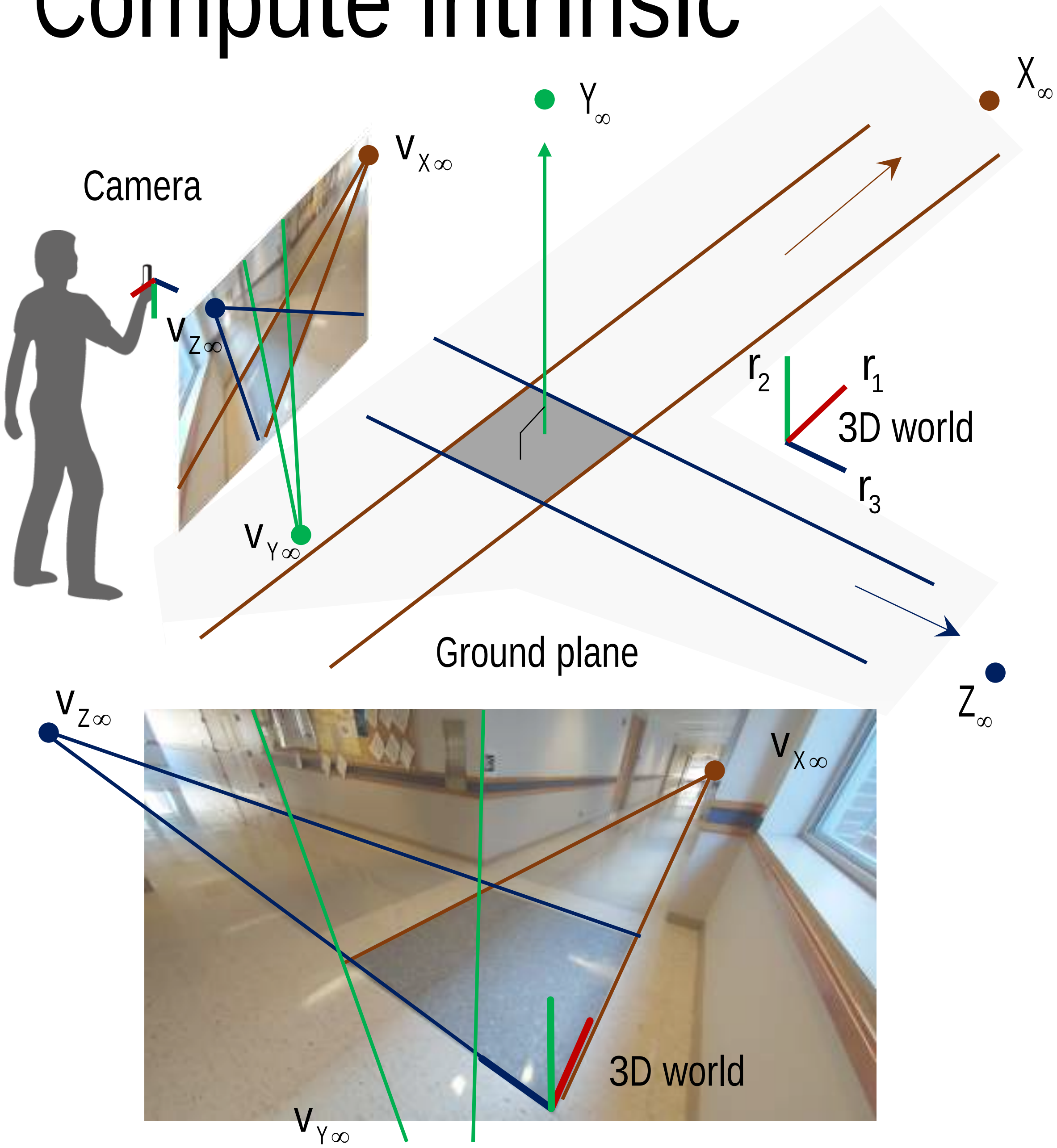
$$\left(K^{-1}v_{X\infty}\right)^T\left(K^{-1}v_{Y\infty}\right)=\left(K^{-1}v_{Y\infty}\right)^T\left(K^{-1}v_{Z\infty}\right)=\left(K^{-1}v_{Z\infty}\right)^T\left(K^{-1}v_{X\infty}\right)=0$$

: 3 unknowns and 3 equations

$$\rightarrow \quad x=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1=\frac{1}{f^2},\quad b_2=-\frac{p_x}{f^2},\quad b_3=-\frac{p_y}{f^2},\quad b_4=\frac{p_x^2}{f^2}+\frac{p_y^2}{f^2}+1$

Compute Intrinsic



$$(K^{-1}v_{X\infty})^T(K^{-1}v_{Y\infty}) = (K^{-1}v_{Y\infty})^T(K^{-1}v_{Z\infty}) = (K^{-1}v_{Z\infty})^T(K^{-1}v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

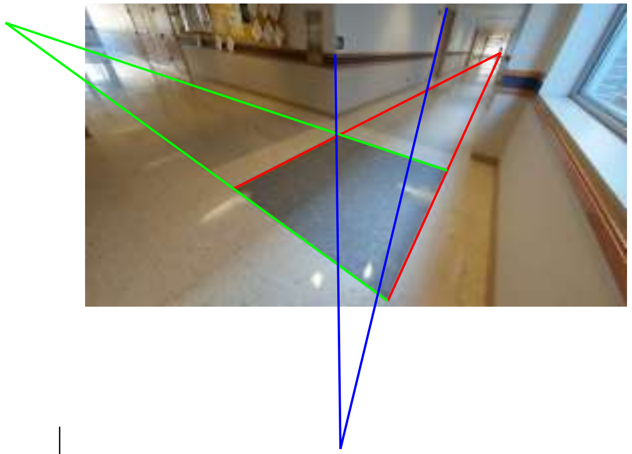
$$\rightarrow \quad x = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

$$\rightarrow \quad p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
z21 = [2564; 31; 1];z22 = [2439; 551; 1];

m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```



Vanishing points

CameraCalibration.m

Camera Calibration

$$A = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];
```

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

l31 = GetLineFromTwoPoints(z11,z12);
l32 = GetLineFromTwoPoints(z21,z22);

x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
z = GetPointFromTwoLines(l31,l32);
```

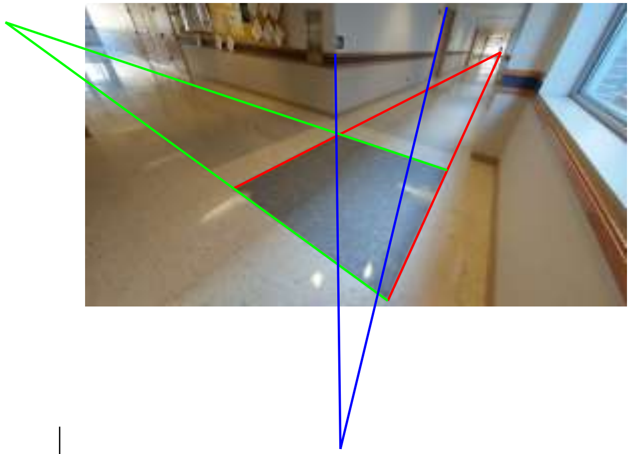

Vanishing points

CameraCalibration.m

Camera Calibration

$$A = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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x = GetPointFromTwoLines(l11,l12);
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```

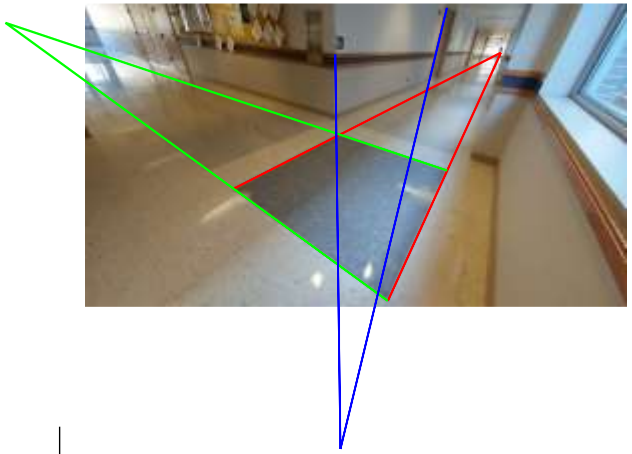

Vanishing points

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

CameraCalibration.m

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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x = GetPointFromTwoLines(l11,l12);
y = GetPointFromTwoLines(l21,l22);
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```

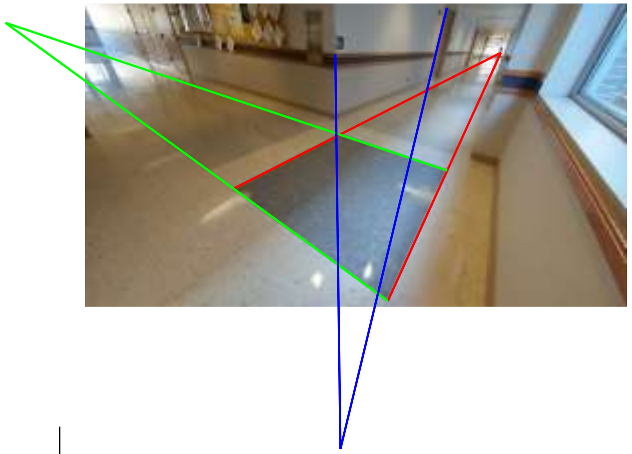

Vanishing points

CameraCalibration.m

$$A = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

```
px = -x(2)/x(1);
py = -x(3)/x(1);
f = sqrt(x(4)/x(1)-px^2-py^2);

K = [f 0 px;
    0 f py;
    0 0 1]
```

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Camera Calibration

```
function CameraCalibration

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

z11 = [1772; 364; 1];z12 = [1778; 823; 1];
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l21 = GetLineFromTwoPoints(m21,m22);
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```
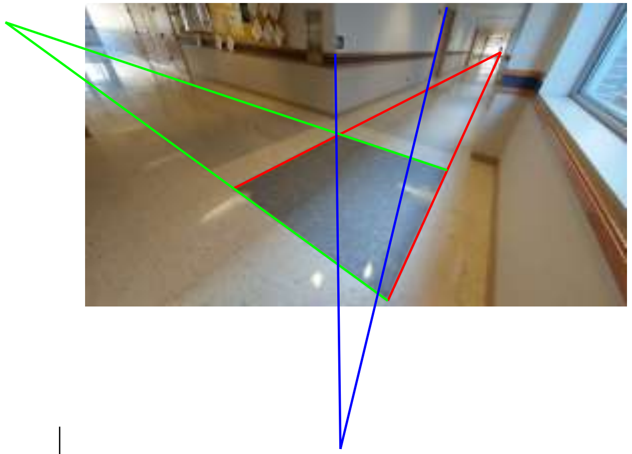



Vanishing points

CameraCalibration.m

$$A = \begin{bmatrix} u_1u_2 + v_1v_2 & u_1 + u_2 & v_1 + v_2 & 1 \\ u_3u_2 + v_3v_2 & u_3 + u_2 & v_3 + v_2 & 1 \\ u_1u_3 + v_1v_3 & u_1 + u_3 & v_1 + v_3 & 1 \end{bmatrix}$$

```
A = [x(1)*y(1)+x(2)*y(2) x(1)+y(1) x(2)+y(2) 1;
    z(1)*y(1)+z(2)*y(2) z(1)+y(1) z(2)+y(2) 1;
    x(1)*z(1)+x(2)*z(2) x(1)+z(1) x(2)+z(2) 1];

[u d v] = svd(A);
x = v(:,end);
```

Linear solve using SVD

```
px = -x(2)/x(1);
py = -x(3)/x(1);
f = sqrt(x(4)/x(1)-px^2-py^2);

K = [f 0 px;
    0 f py;
    0 0 1]
```

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

```
K =

   1317.2        0   1931.8
        0   1317.2   1146.1
        0        0        1
```

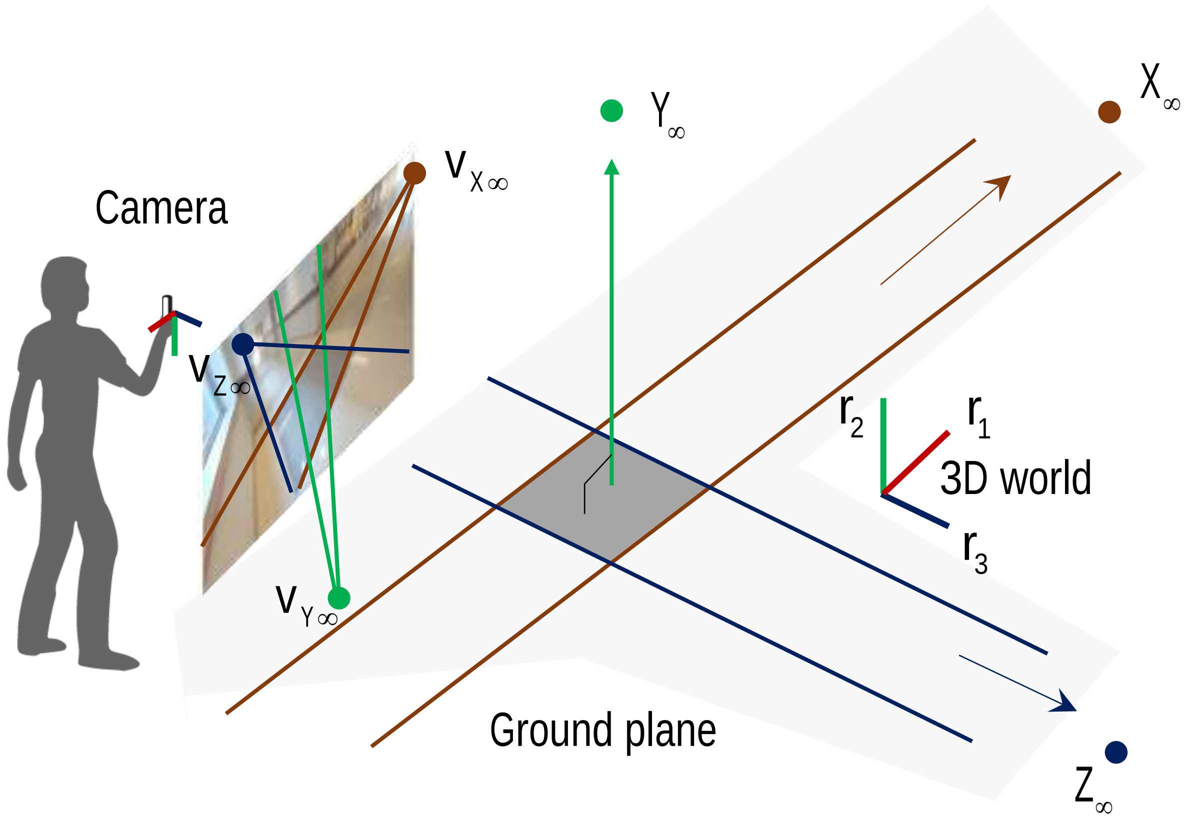
f = 1224

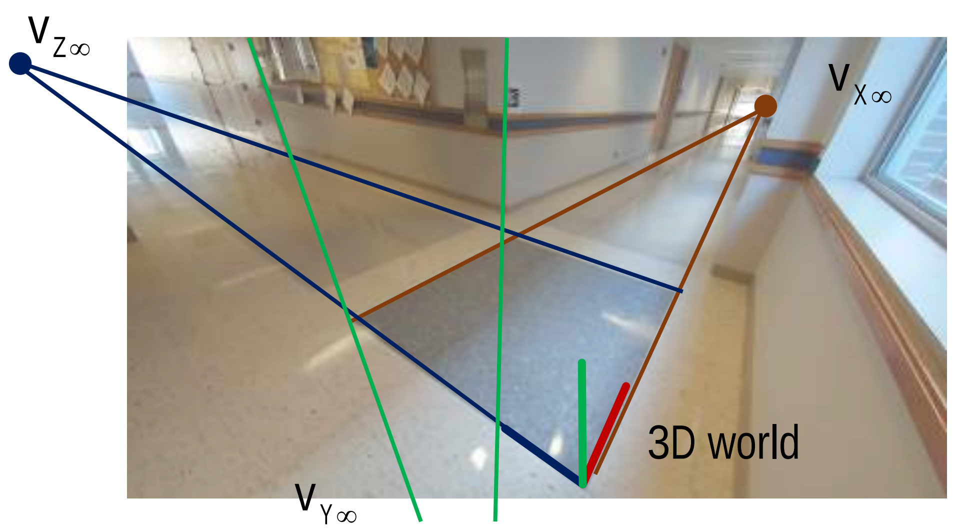
p_x = size(im,2)/2 = 1920

p_y = size(im,1)/2 = 1080

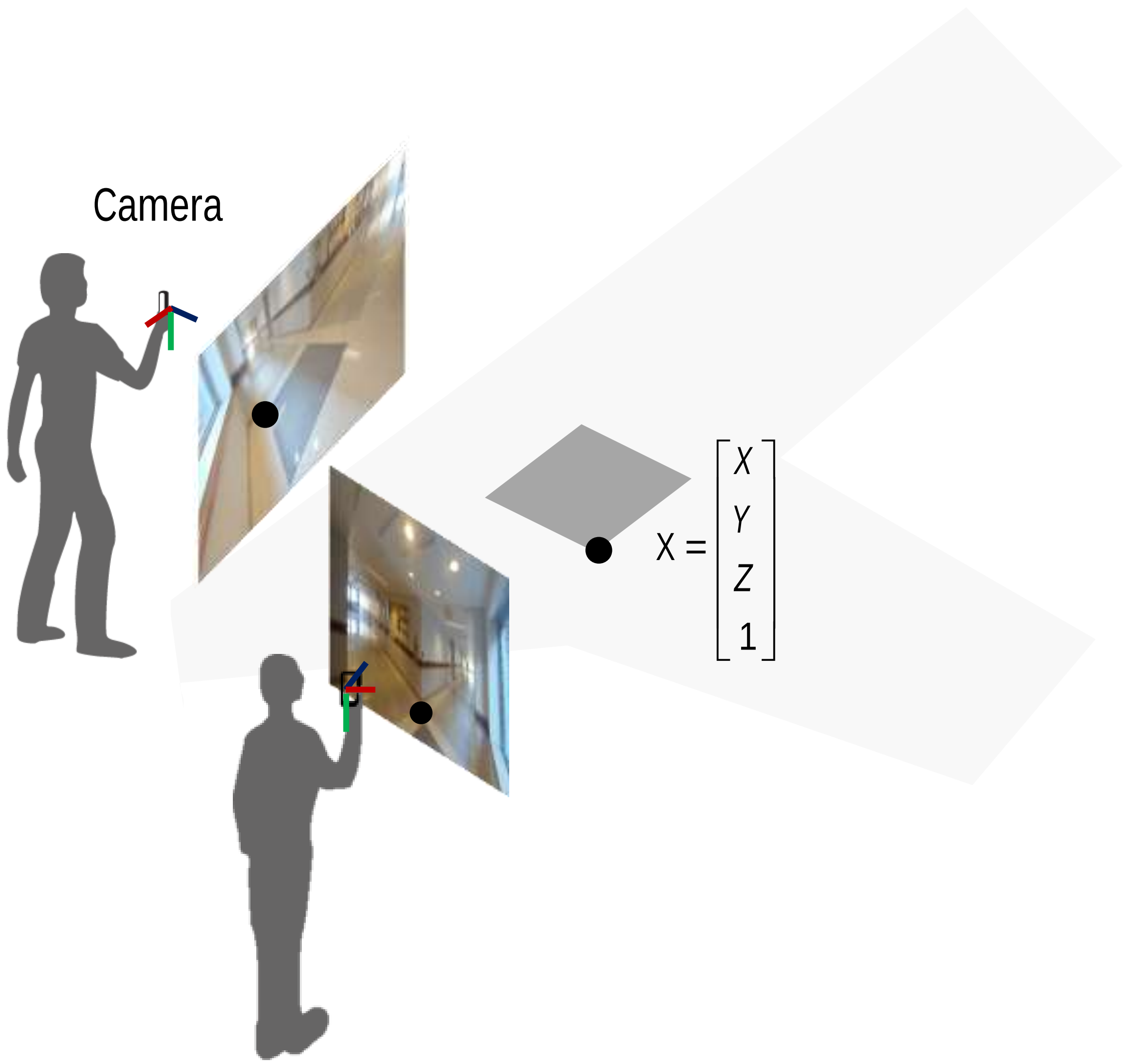
Previous manual estimate

Camera Calibration via Vanishing Points

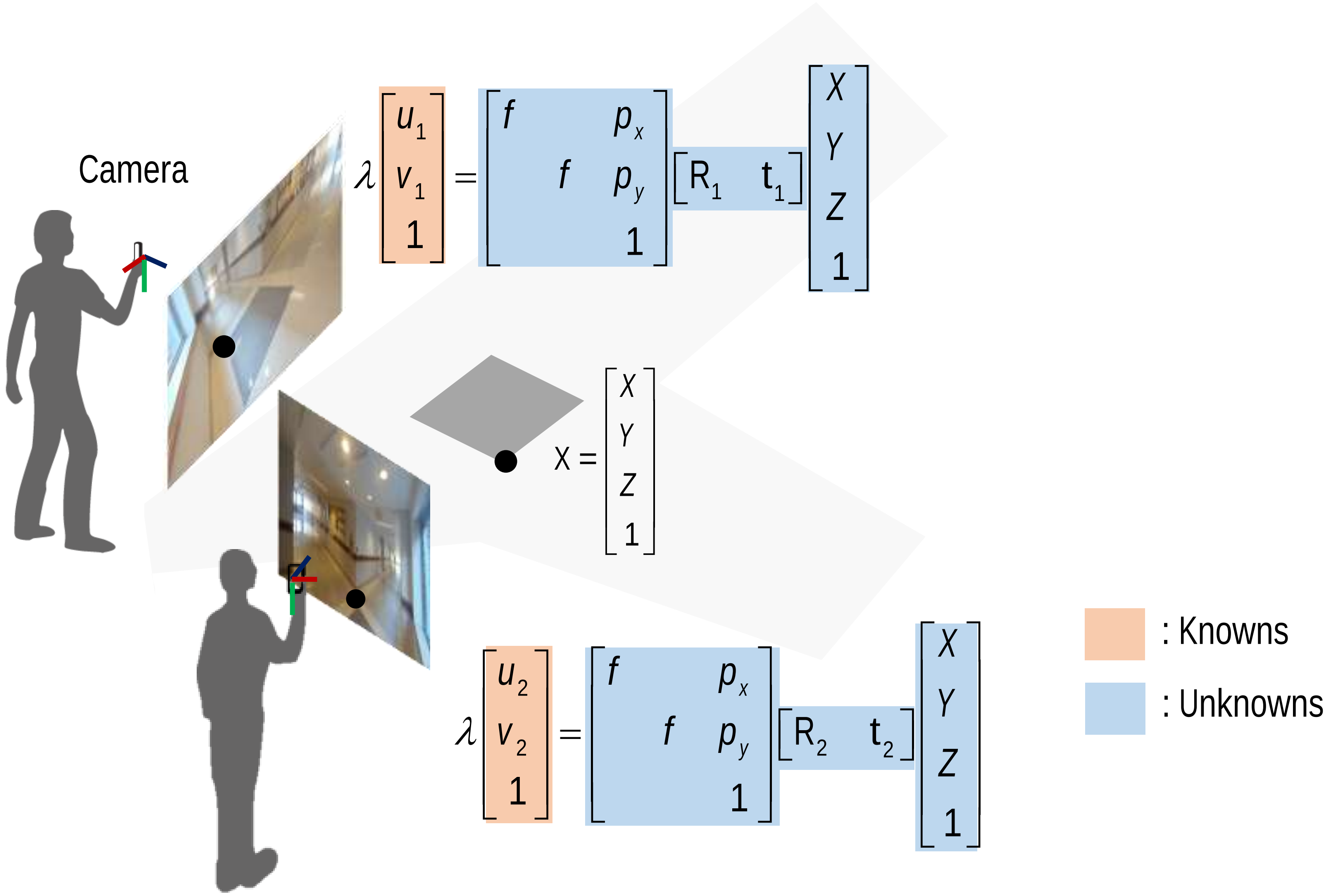




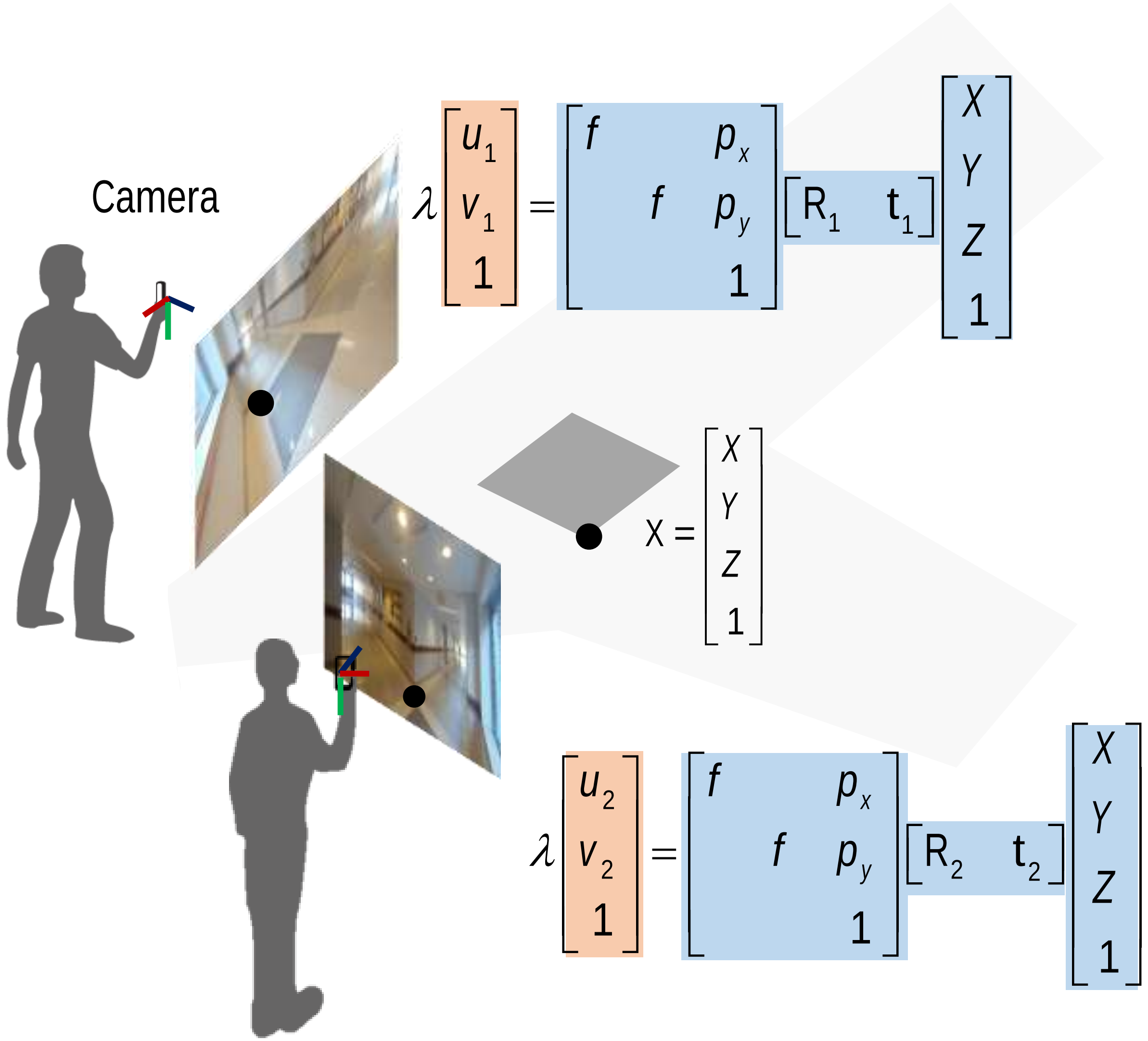
Multiview Camera Calibration



Multiview Camera Calibration



Multiview Camera Calibration



# of unknowns:

n: the number of images

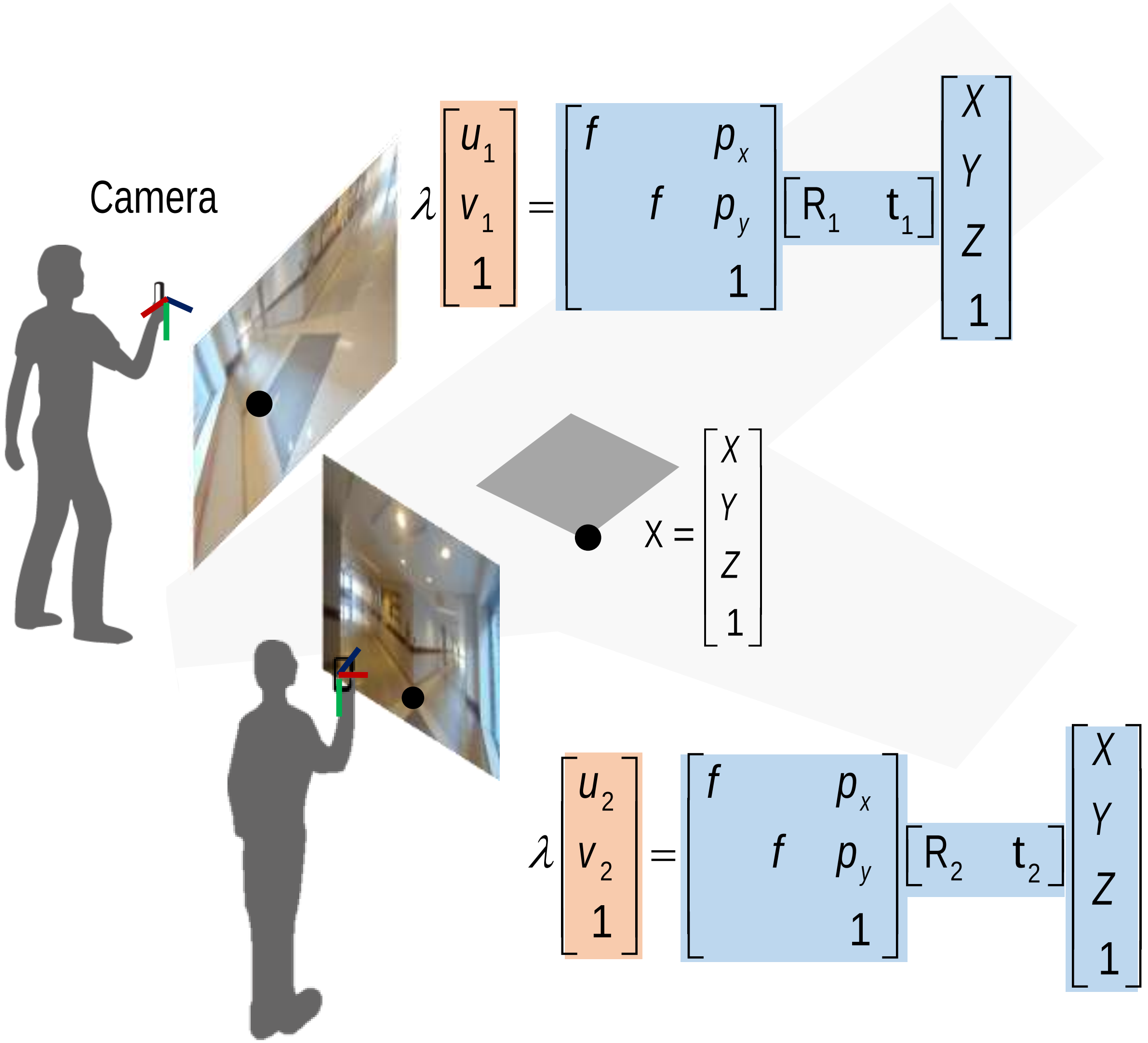
p: the number of points

# of equations:

: Knowns

: Unknowns

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_2 & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (K) + 6*n* (R and t) + 3*p* (X)

n: the number of images

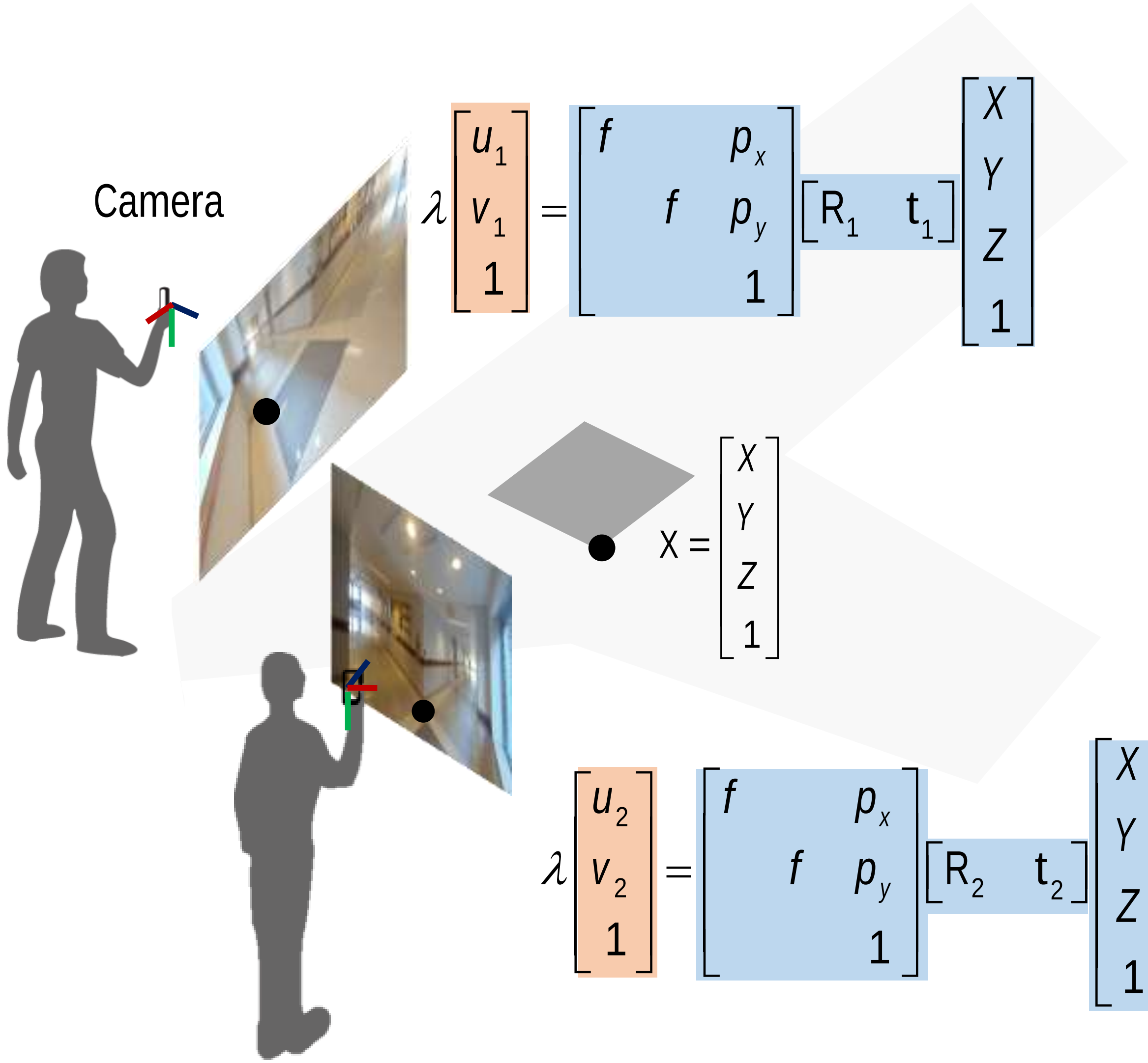
p: the number of points

# of equations:

: Knowns

: Unknowns

Multiview Camera Calibration



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_2 & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

\# of unknowns: 3 (K) + 6*n* (R and t) + 3*p* (X)

n: the number of images

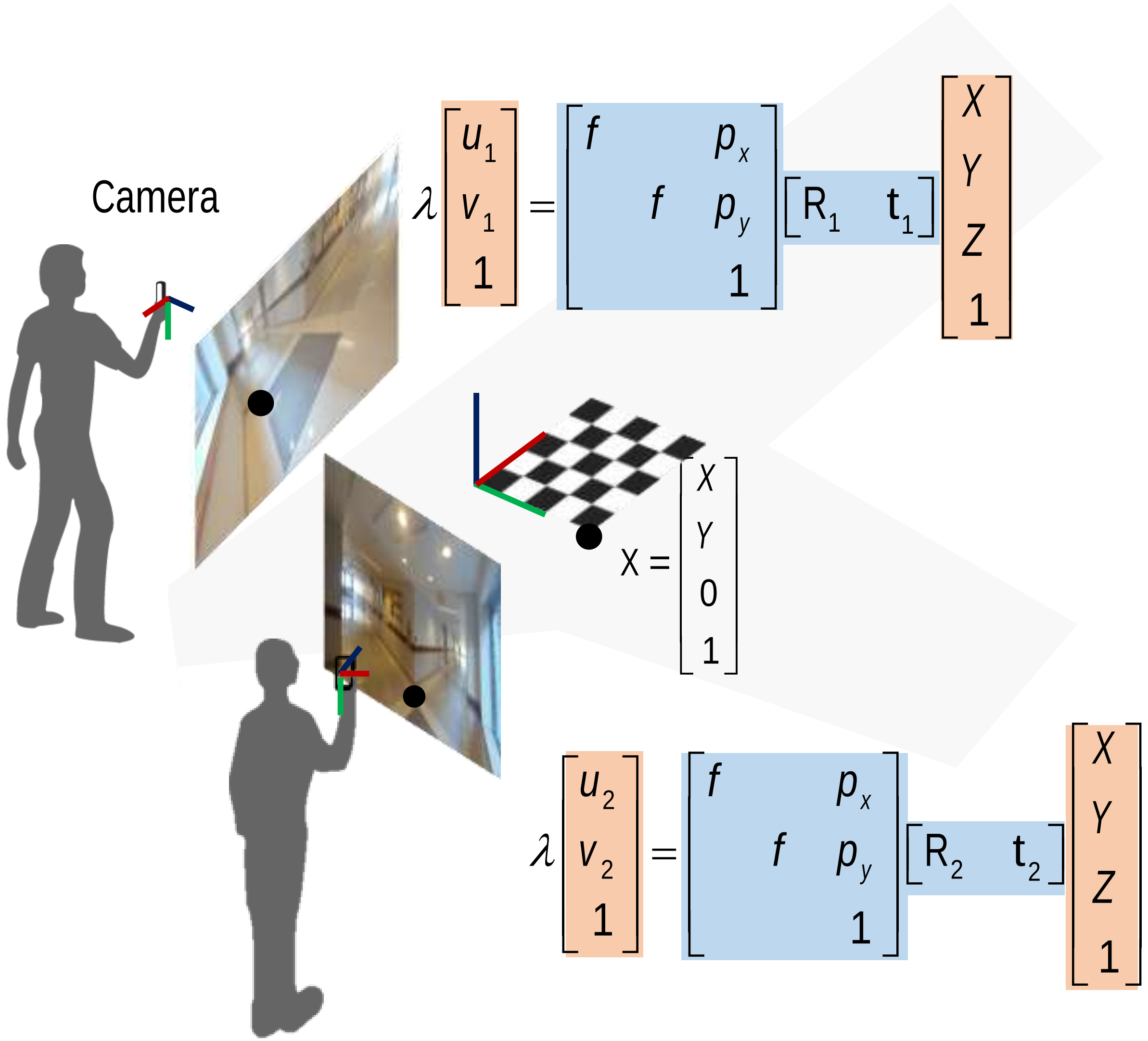
p: the number of points

\# of equations: 2*np*

: Knowns

: Unknowns

Insight: Known Common 3D Points



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_2 & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (K) + 6*n* (R and t)

n: the number of images

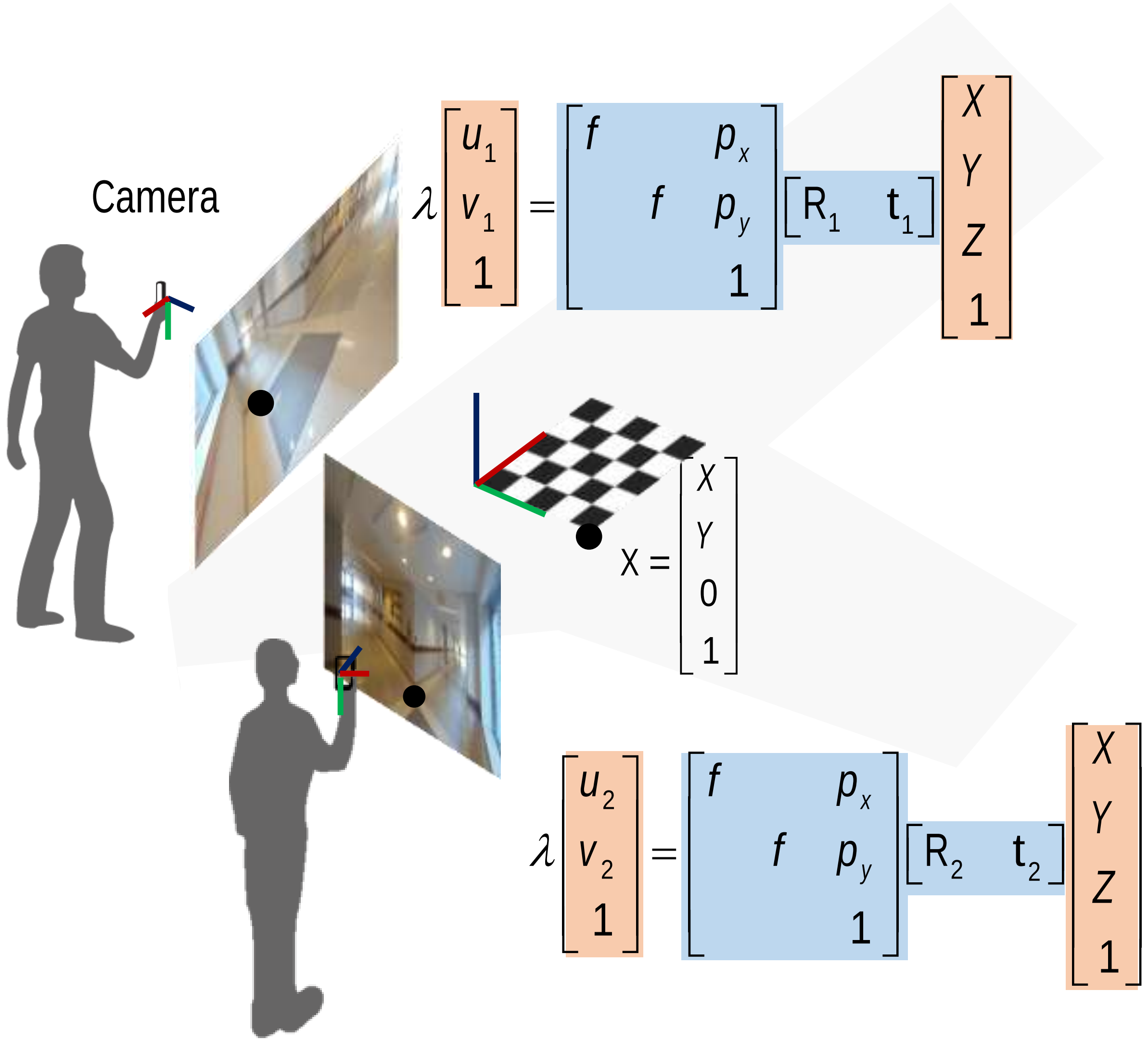
p: the number of points

# of equations: 2*np*

: Knowns

: Unknowns

Insight: Known Common 3D Points



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_2 & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (K) + 6*n* (R and t)

n: the number of images

p: the number of points

# of equations: 2*np*

We can solve for K, R, t if $3 + 6n < 2\,nm$

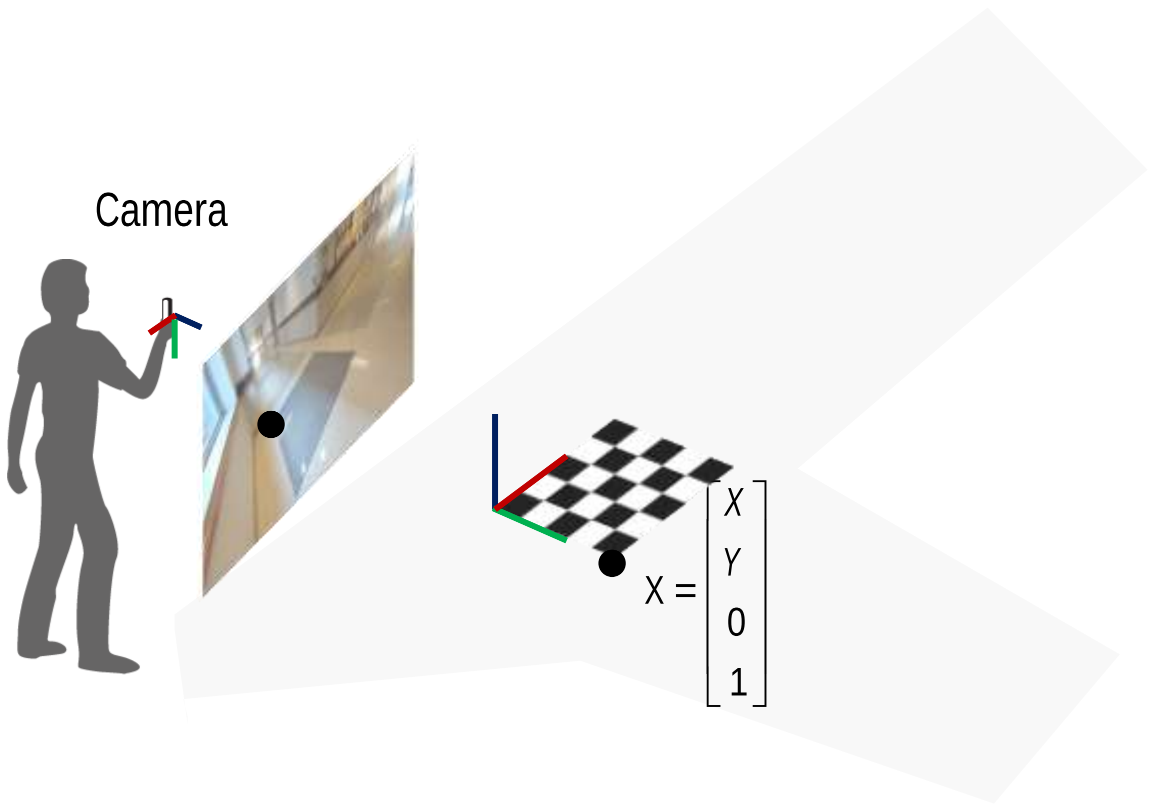
: Knowns

: Unknowns

Homography

Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_1 & r_2 & r_3 & t \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

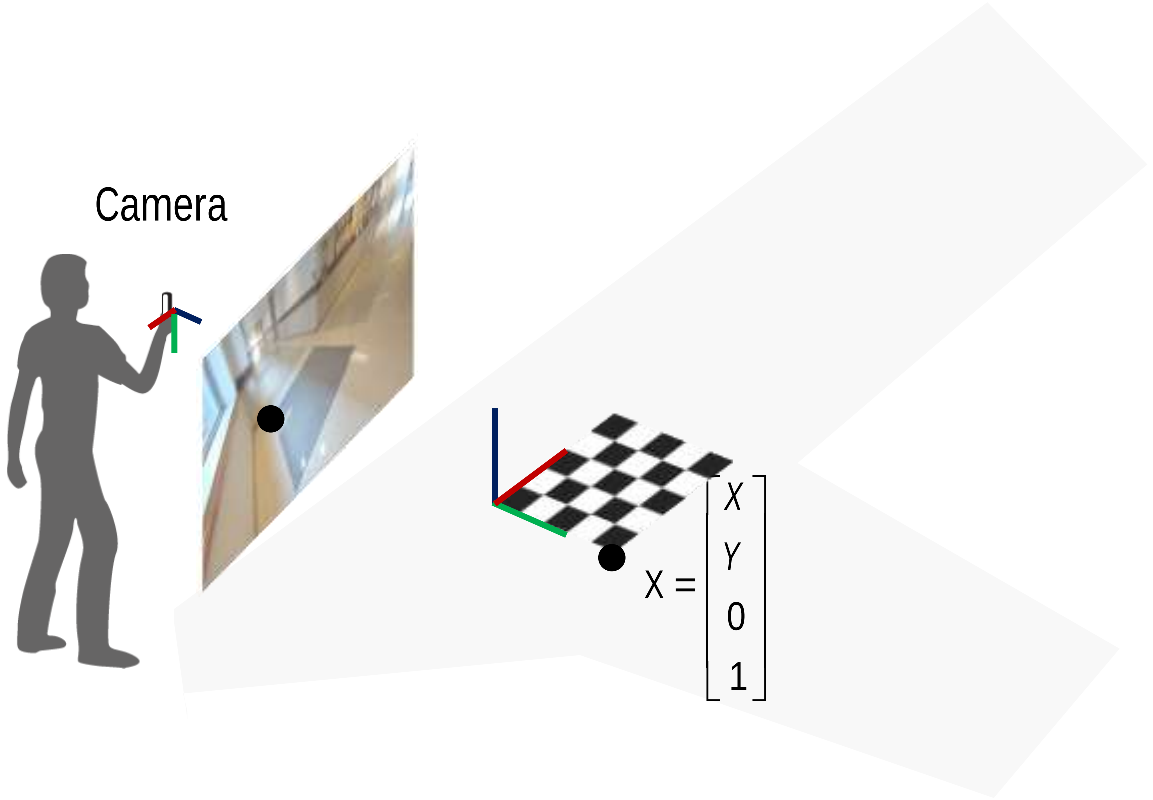


Homography

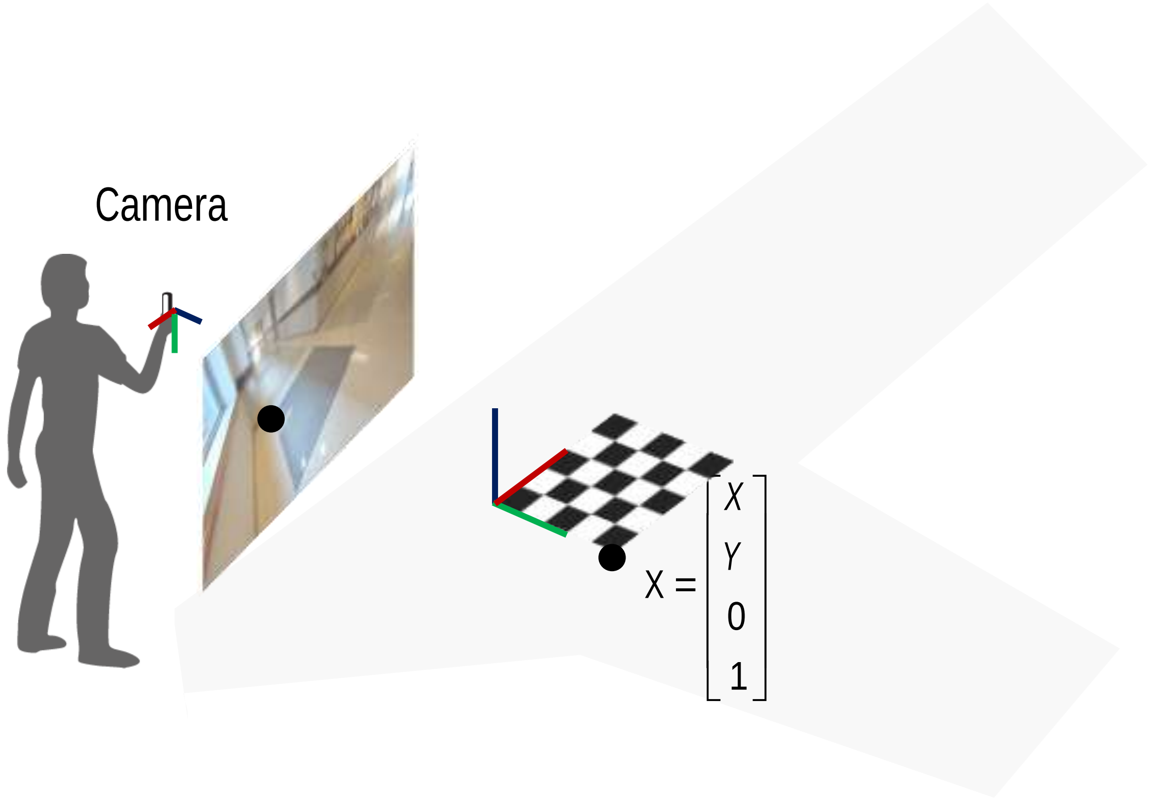
Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$



Homography

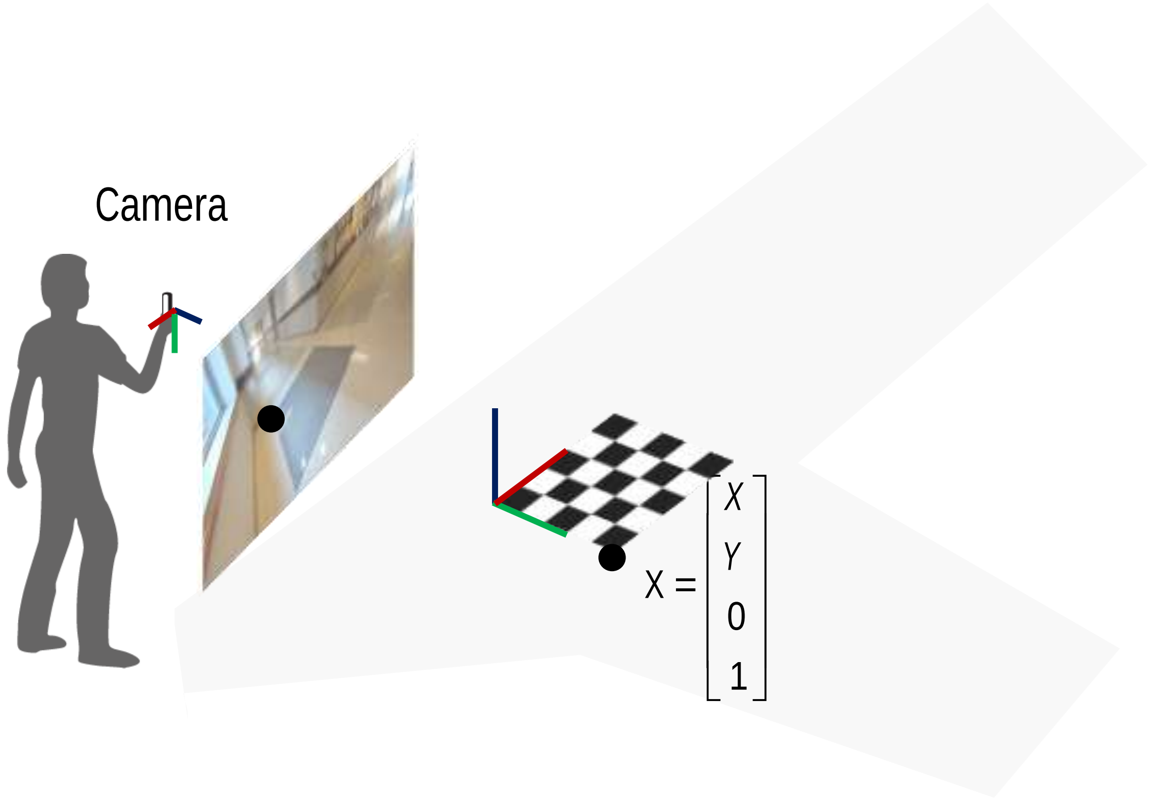


Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Points in 2D plane are mapped to an image with homography:

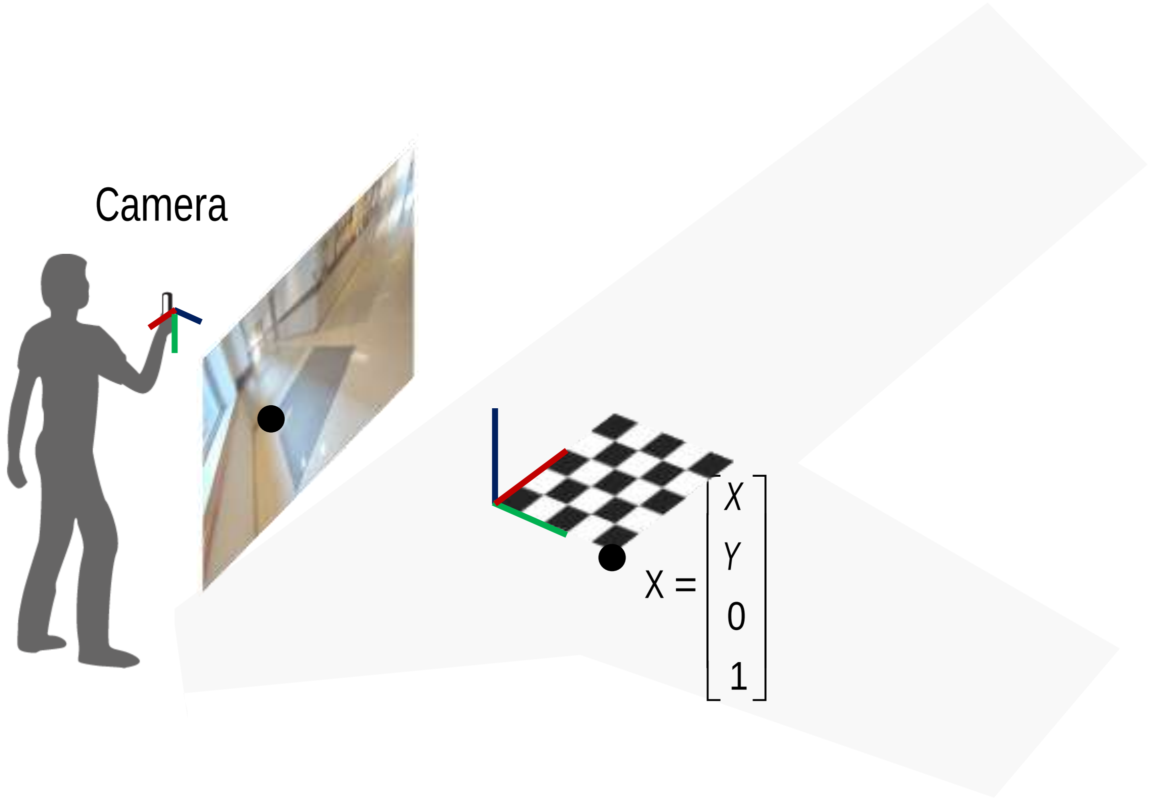
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

1. Compute homography

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Homography

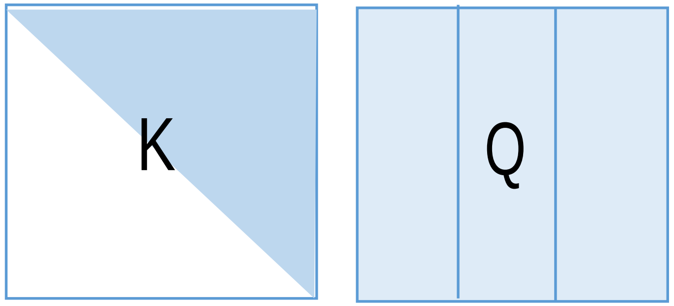


Points in 2D plane are mapped to an image with homography:

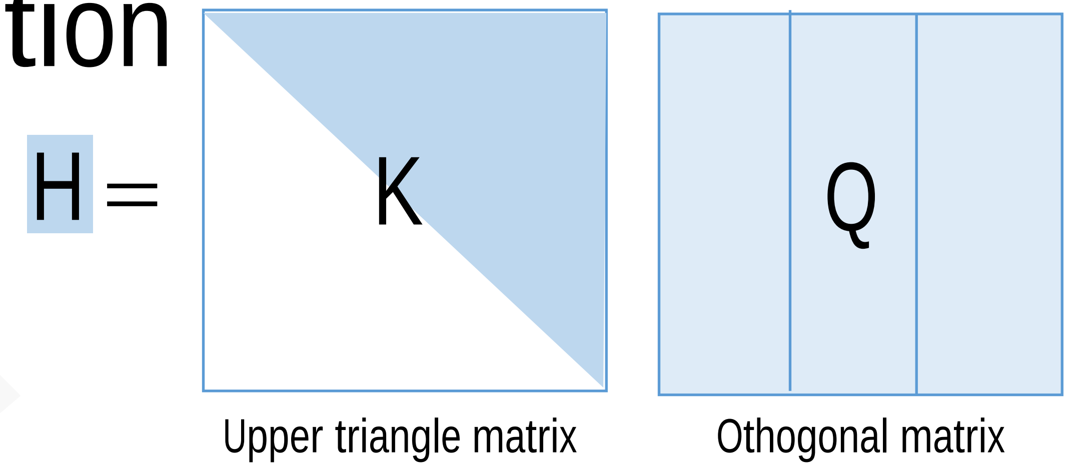
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

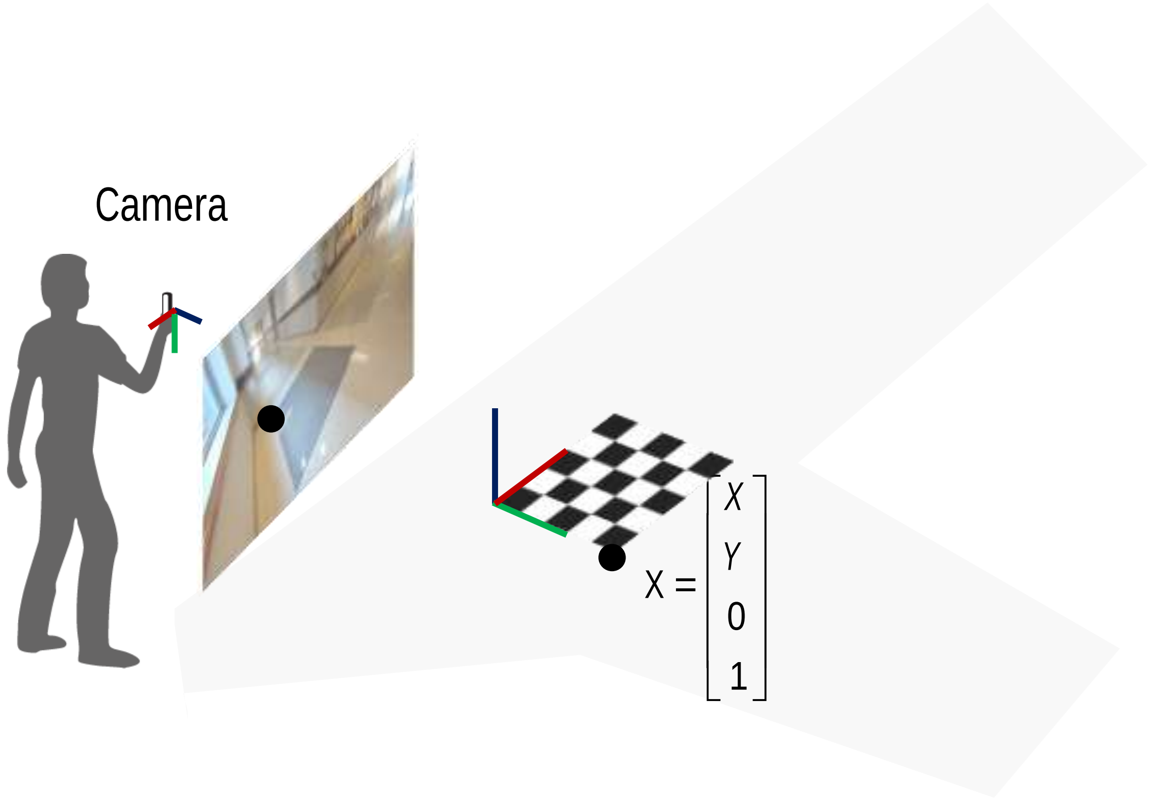
$$= \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

H =

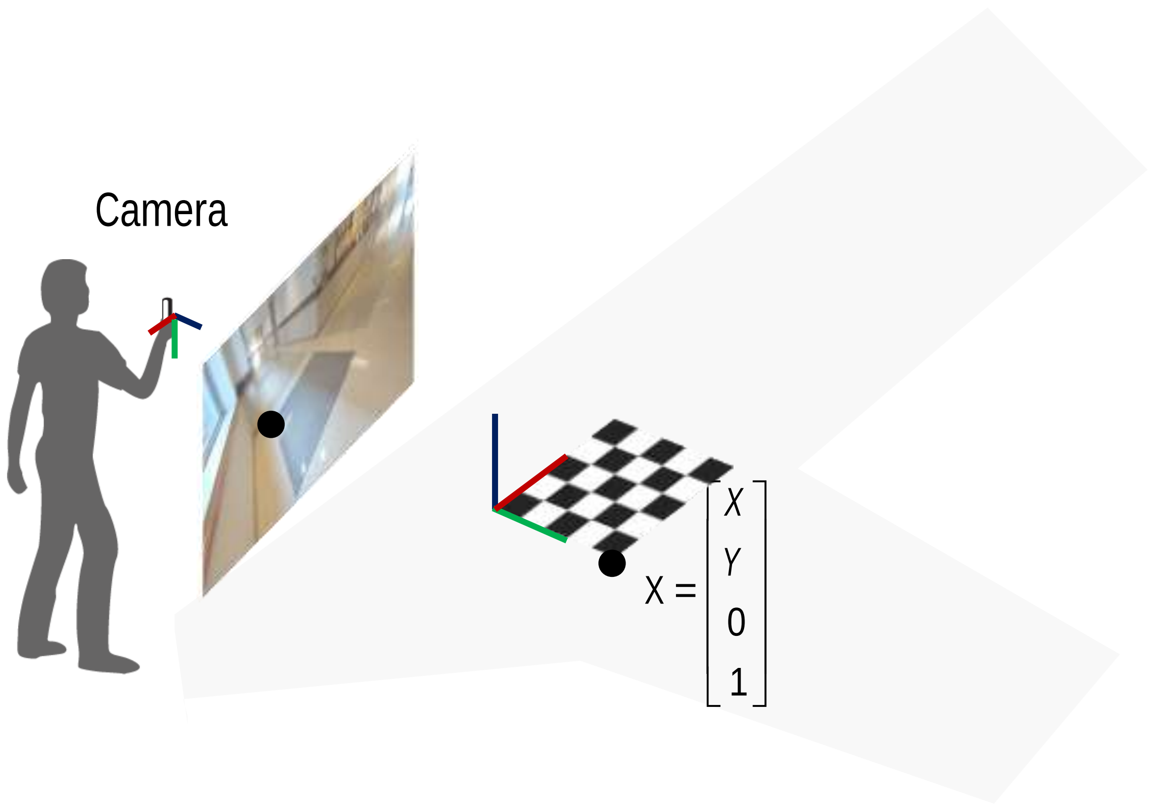


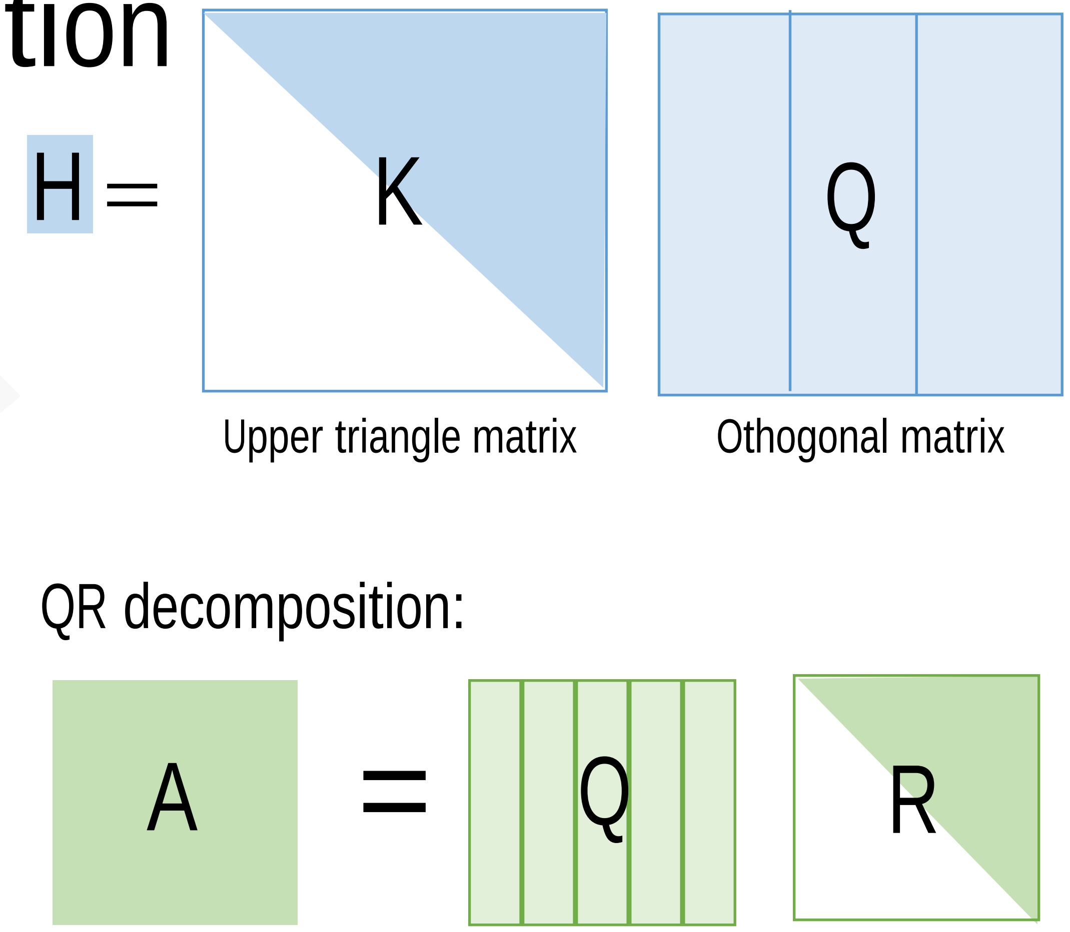
Method1: RQ Decomposition



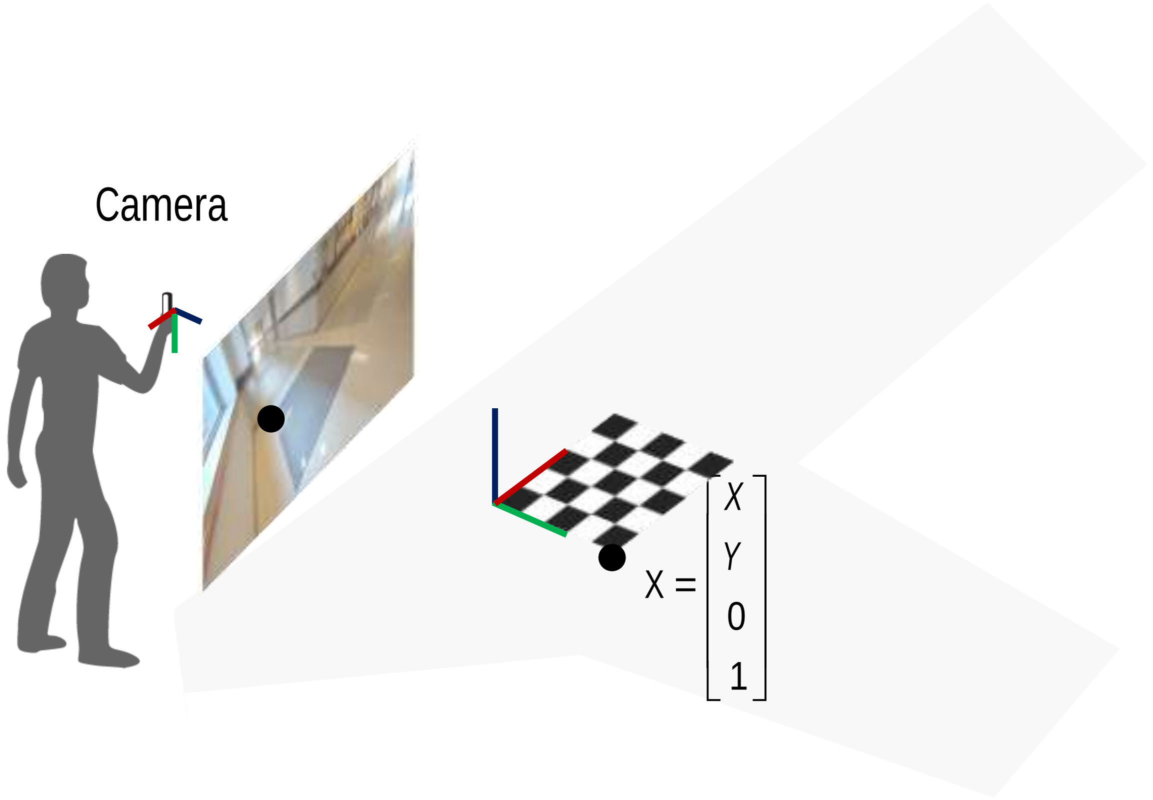


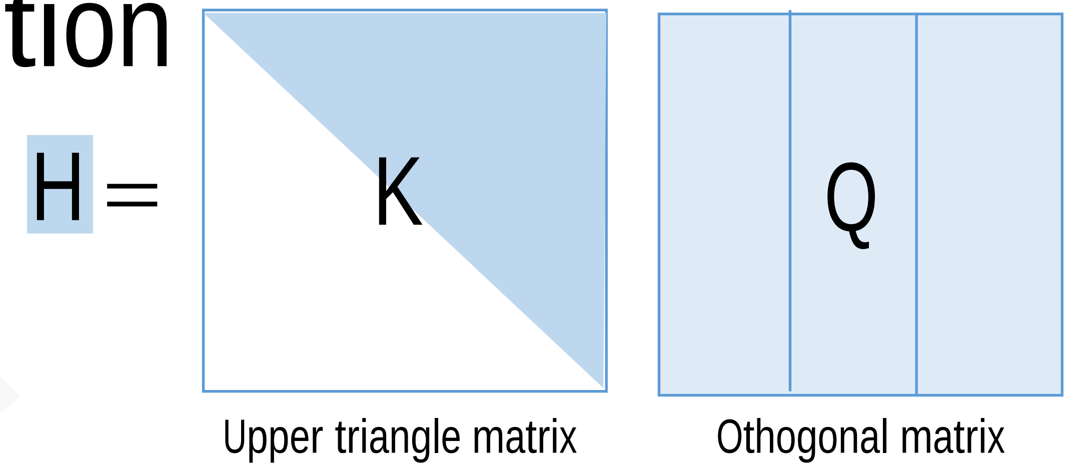
Method1: RQ Decomposition



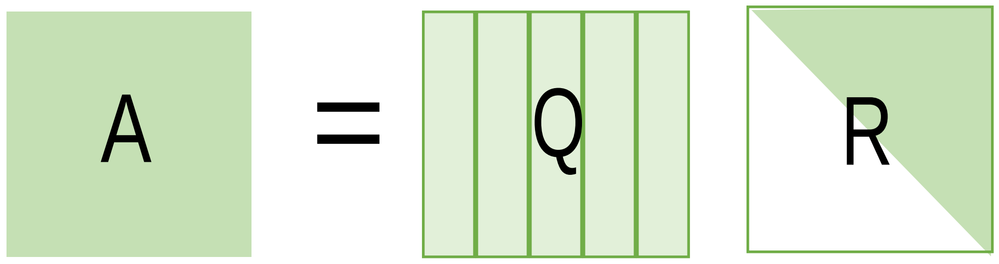


Method1: RQ Decomposition





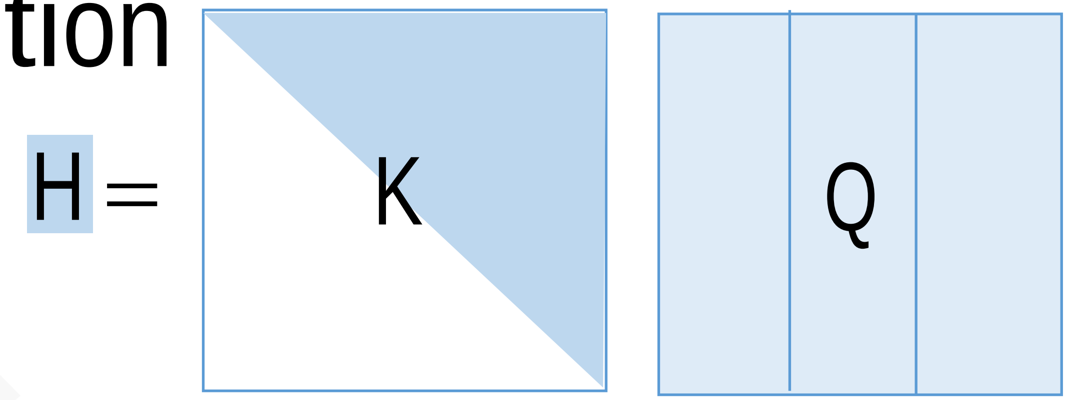
QR decomposition:

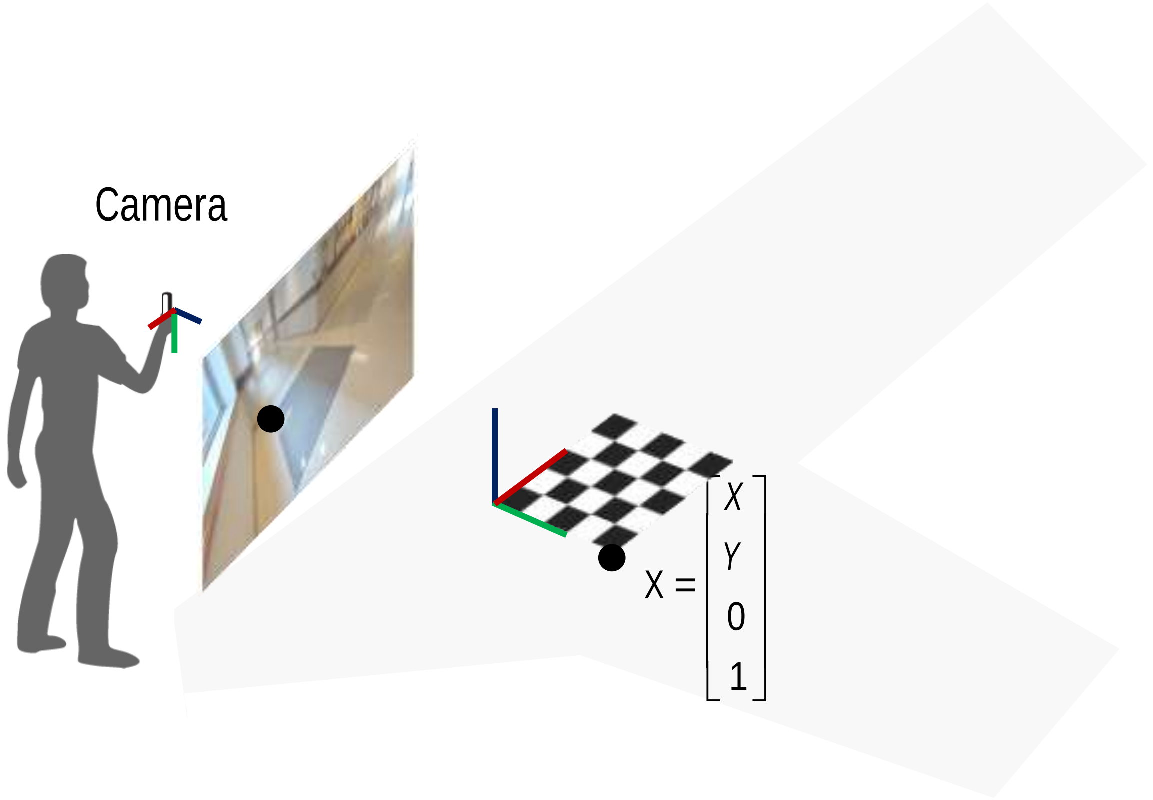


MATLAB

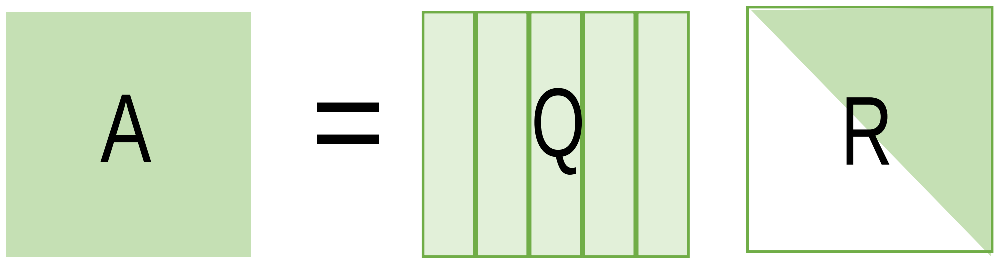
[Q R] = qr(A)

Method1: RQ Decomposition





QR decomposition:

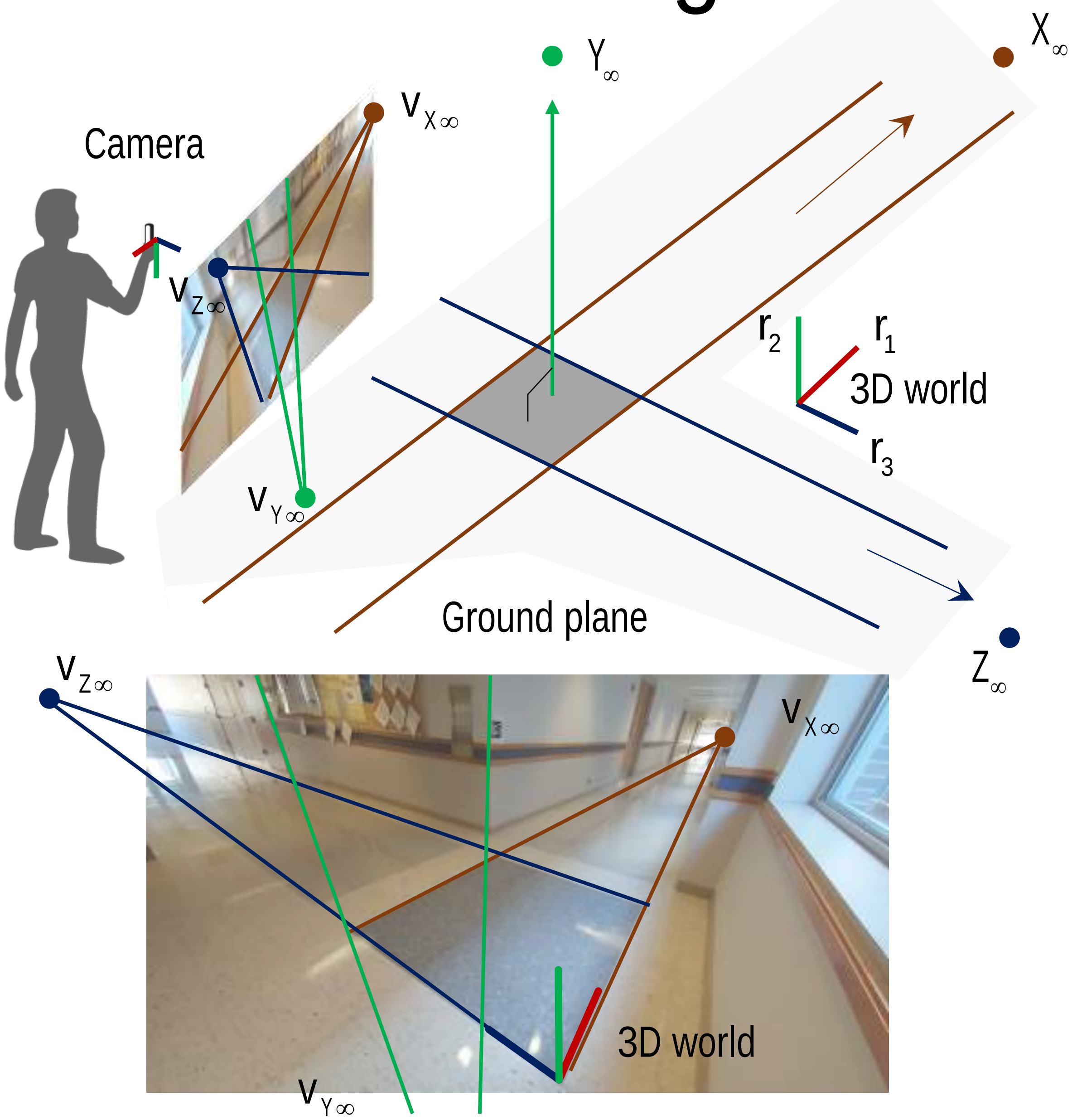


MATLAB

[Q R] = qr(A)

HW: How to convert QR to RQ?

Recall: Vanishing Points



$$\lambda v_{X\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RX_\infty \quad \lambda v_{Z\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RZ_\infty \quad \lambda v_{Y\infty} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} RY_\infty$$

$$\lambda K^{-1} v_{X\infty} = RX_\infty \qquad \lambda K^{-1} v_{Y\infty} = RY_\infty \qquad \lambda K^{-1} v_{Z\infty} = RZ_\infty$$

Note that the camera extrinsic is still unknown (R and t).

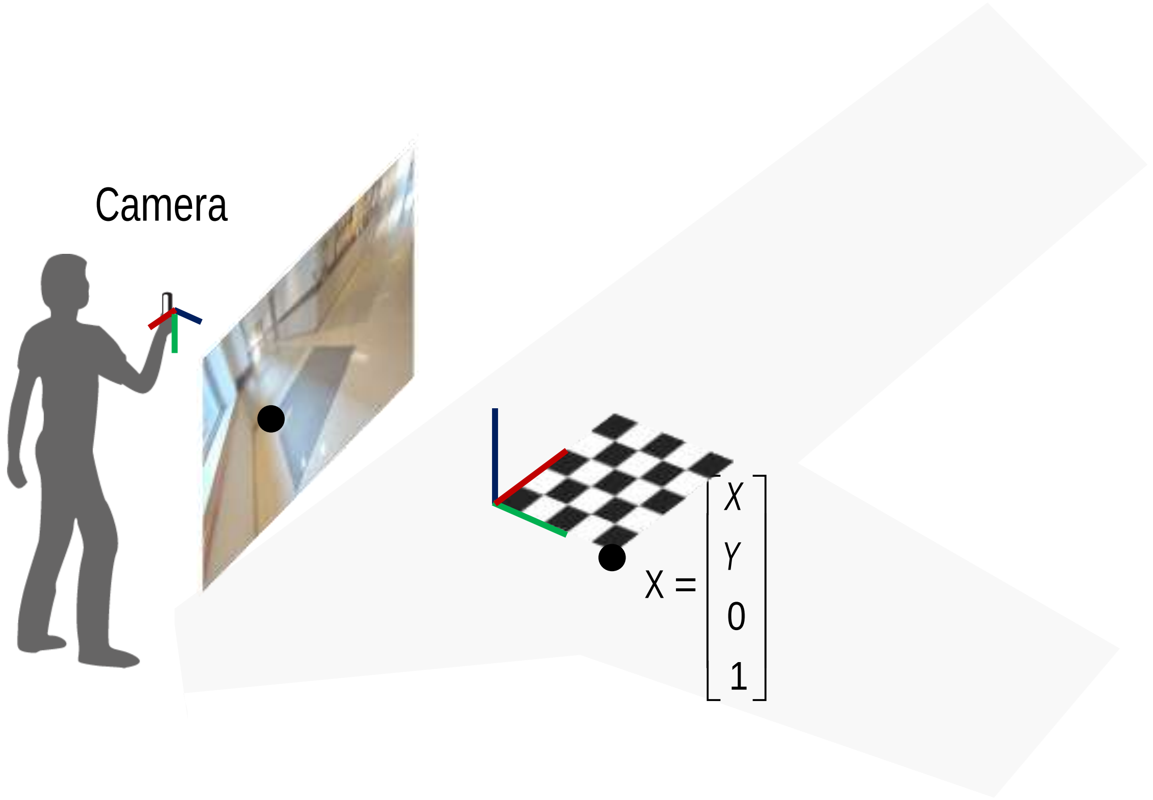
Known property of points at infinity:

$$(X_\infty)^T (Y_\infty) = 0 \qquad (RX_\infty)^T (RY_\infty) = 0$$
$$(Y_\infty)^T (Z_\infty) = 0 \quad \longleftrightarrow \quad (RY_\infty)^T (RZ_\infty) = 0$$
$$(Z_\infty)^T (X_\infty) = 0 \qquad (RZ_\infty)^T (RX_\infty) = 0$$

$$(K^{-1} v_{X\infty})^T (K^{-1} v_{Y\infty}) = (K^{-1} v_{Y\infty})^T (K^{-1} v_{Z\infty}) = (K^{-1} v_{Z\infty})^T (K^{-1} v_{X\infty}) = 0$$

: 3 unknowns and 3 equations

Method2: Rotation



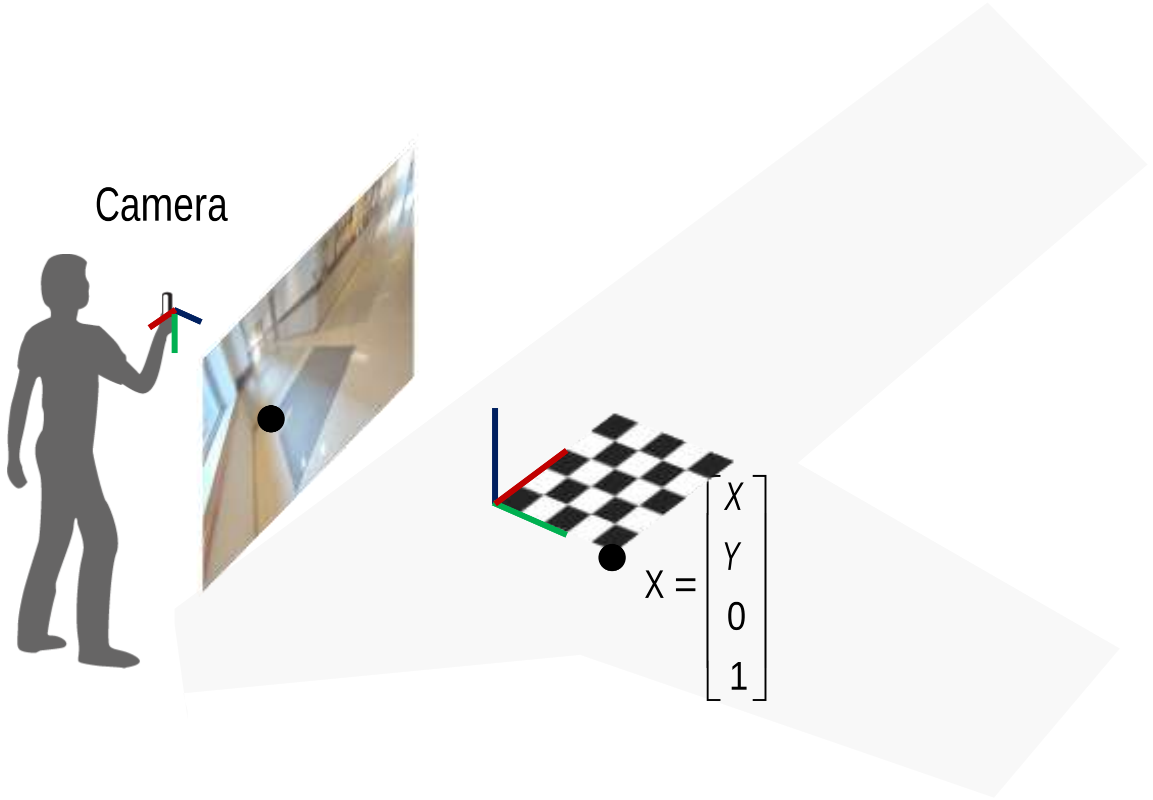
: Knowns

: Unknowns

Homography factorization:

$$\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

Method2: Rotation



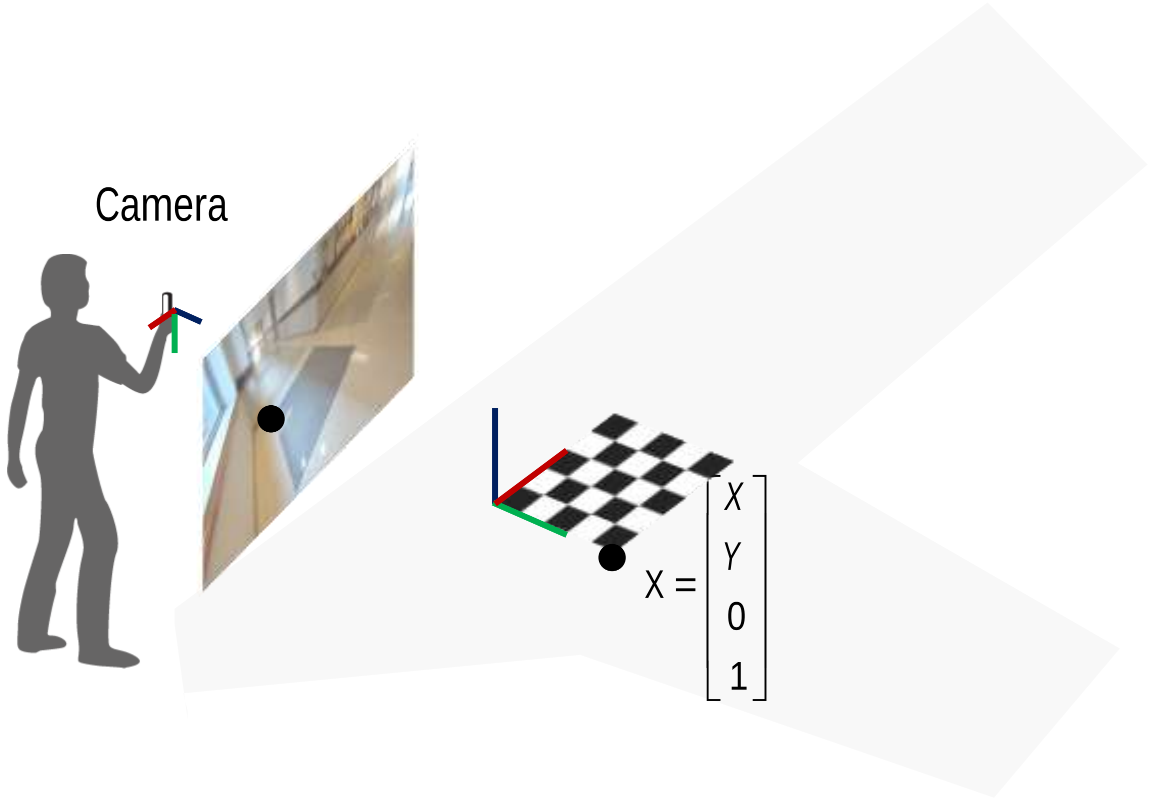
Homography factorization:

: Knowns

: Unknowns

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

Method2: Rotation



: Knowns

: Unknowns

Homography factorization:

$$\begin{pmatrix}\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\end{pmatrix}^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

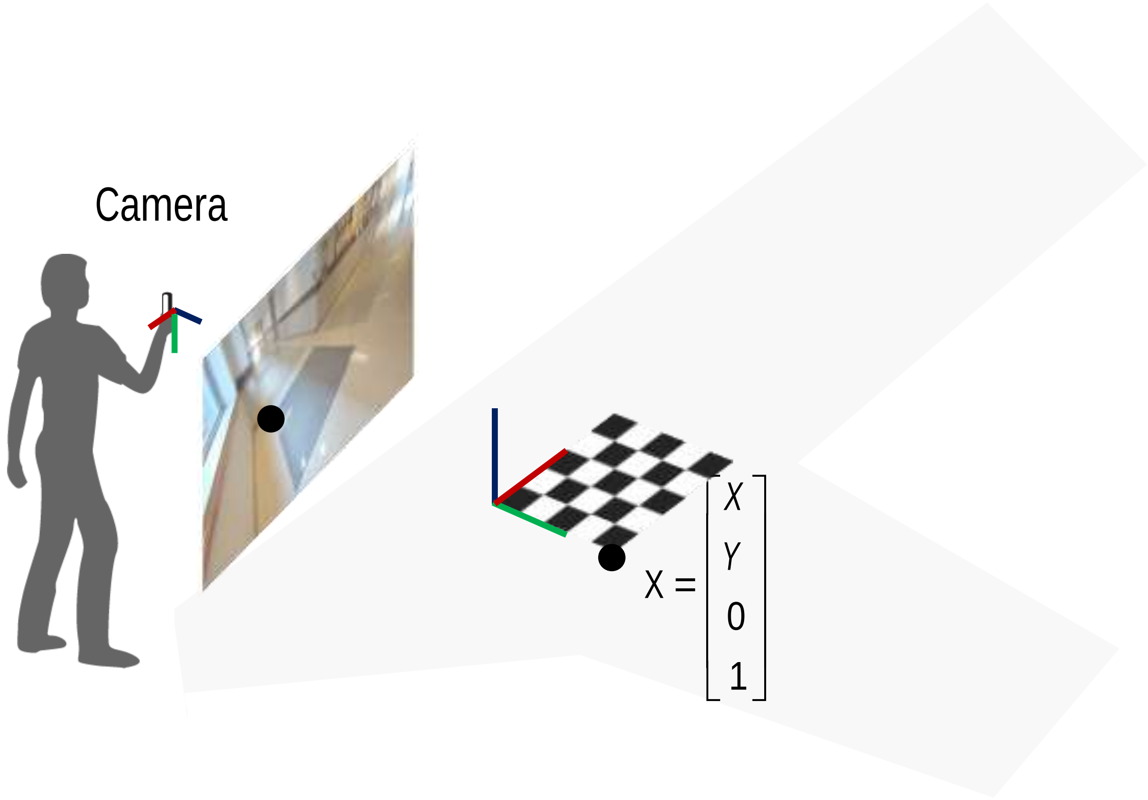
$$r_1 = K^{-1} h_1 \qquad r_2 = K^{-1} h_2 \qquad r_3 = K^{-1} h_3$$

Method2: Rotation

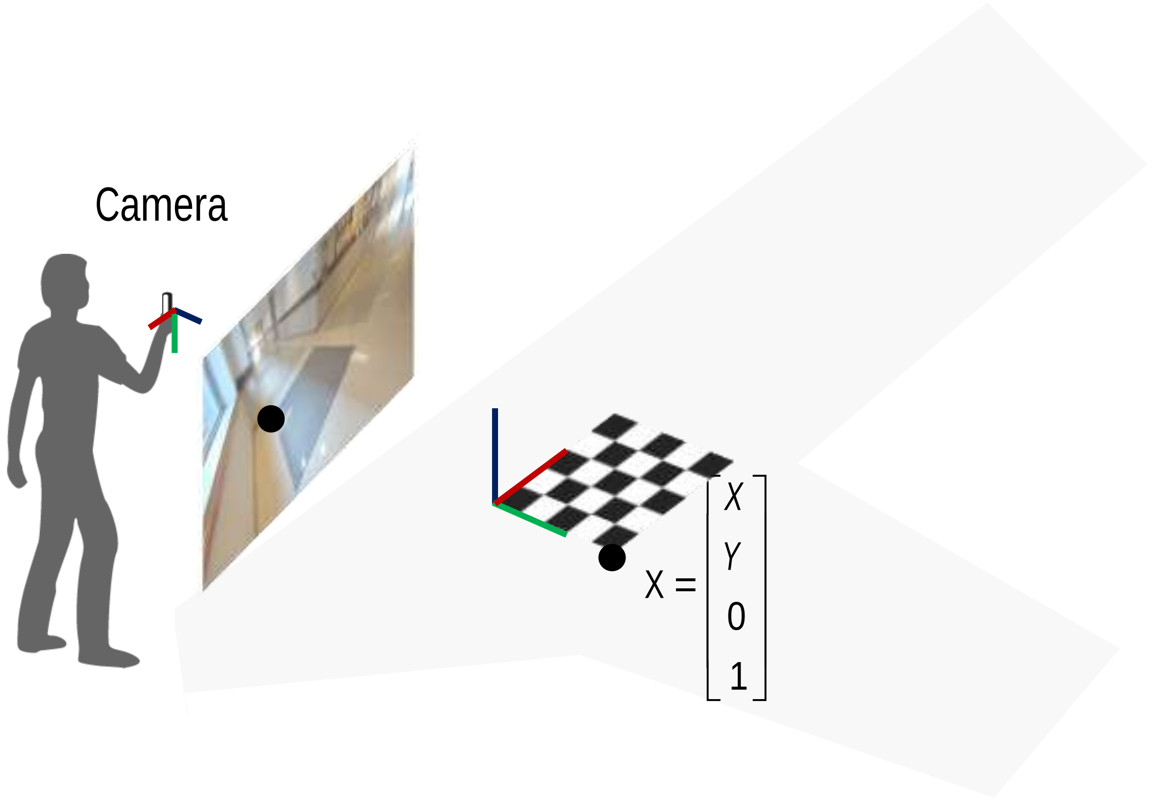
$$r_1 = K^{-1}h_1 \qquad r_2 = K^{-1}h_2 \qquad r_3 = K^{-1}h_3$$

Orthogonality of rotation matrix property:

$$r_1^T r_2 = 0 \qquad \|r_1\| = 1 \qquad \|r_2\| = 1$$



Method2: Rotation



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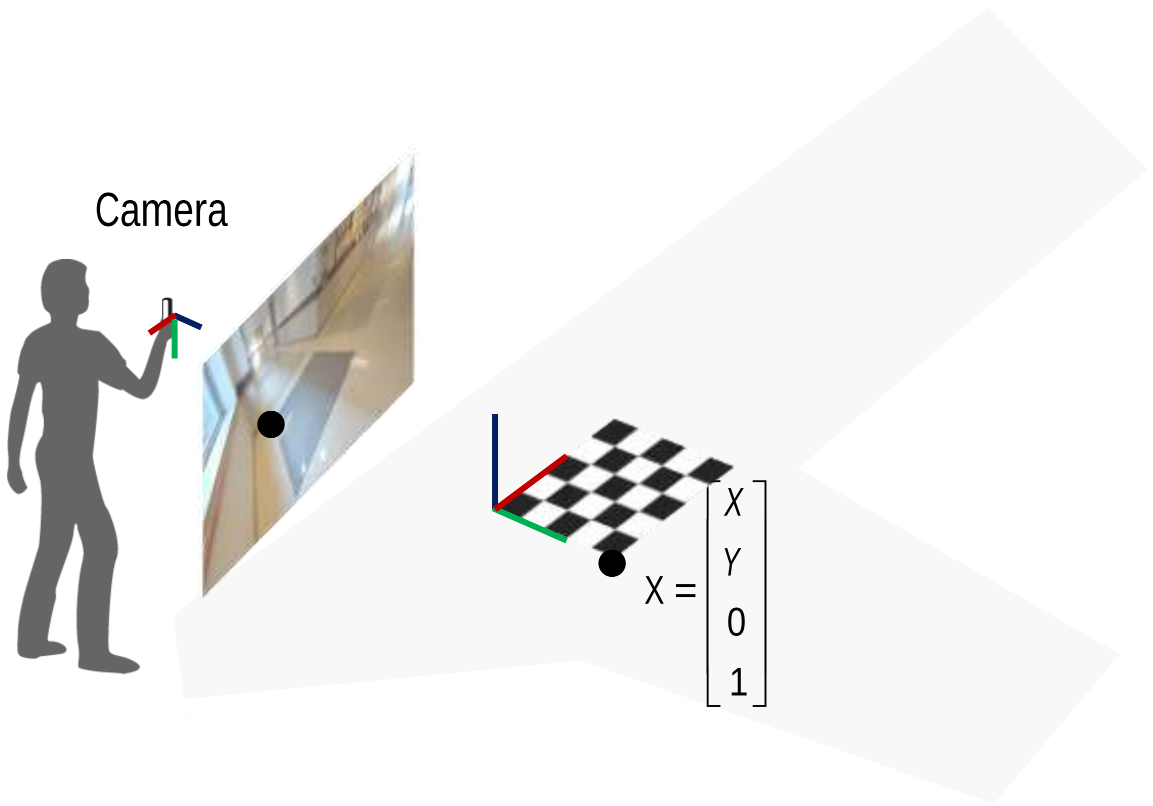
$$r_1^T r_2 = 0 \qquad \|r_1\| = 1 \qquad \|r_2\| = 1$$

$$\longrightarrow \left(K^{-1}h_1\right)^T \left(K^{-1}h_2\right) = h_1^T K^{-T} K^{-1} h_2 = 0$$

Method2: Rotation



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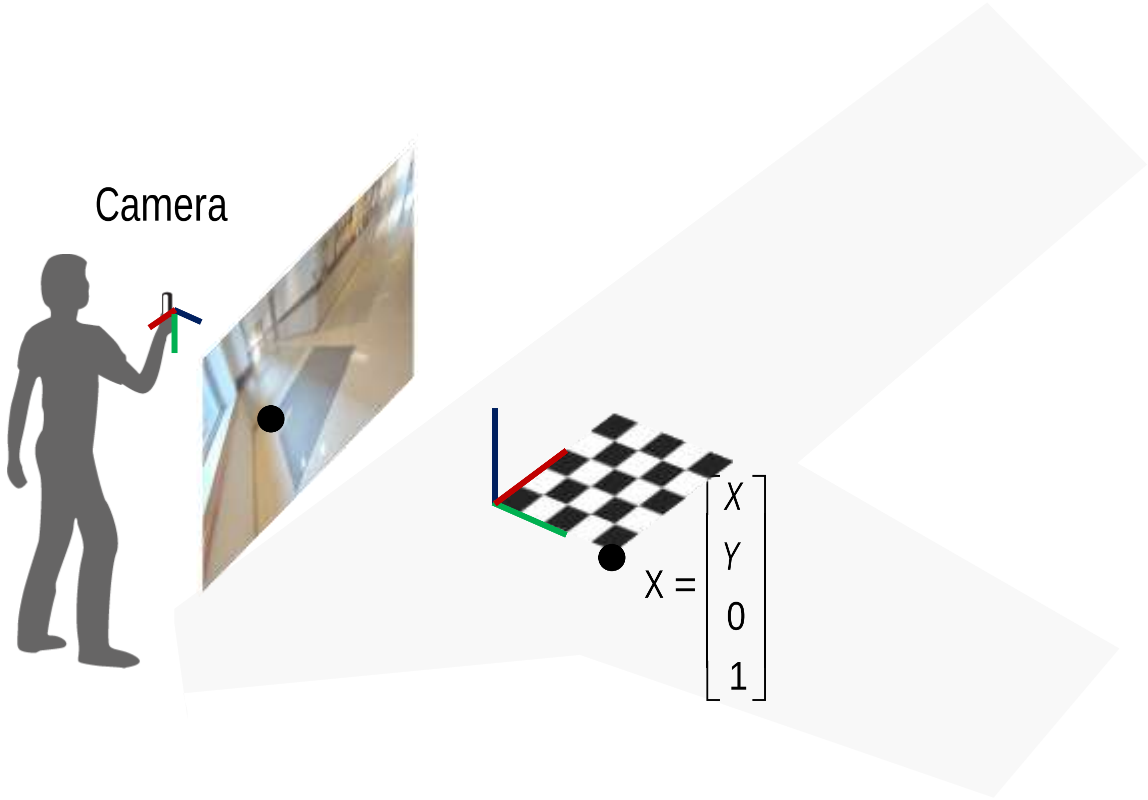
$$\|K^{-1}h_1\| = \|K^{-1}h_2\| \quad \text{or,} \quad h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$$

Method2: Rotation



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

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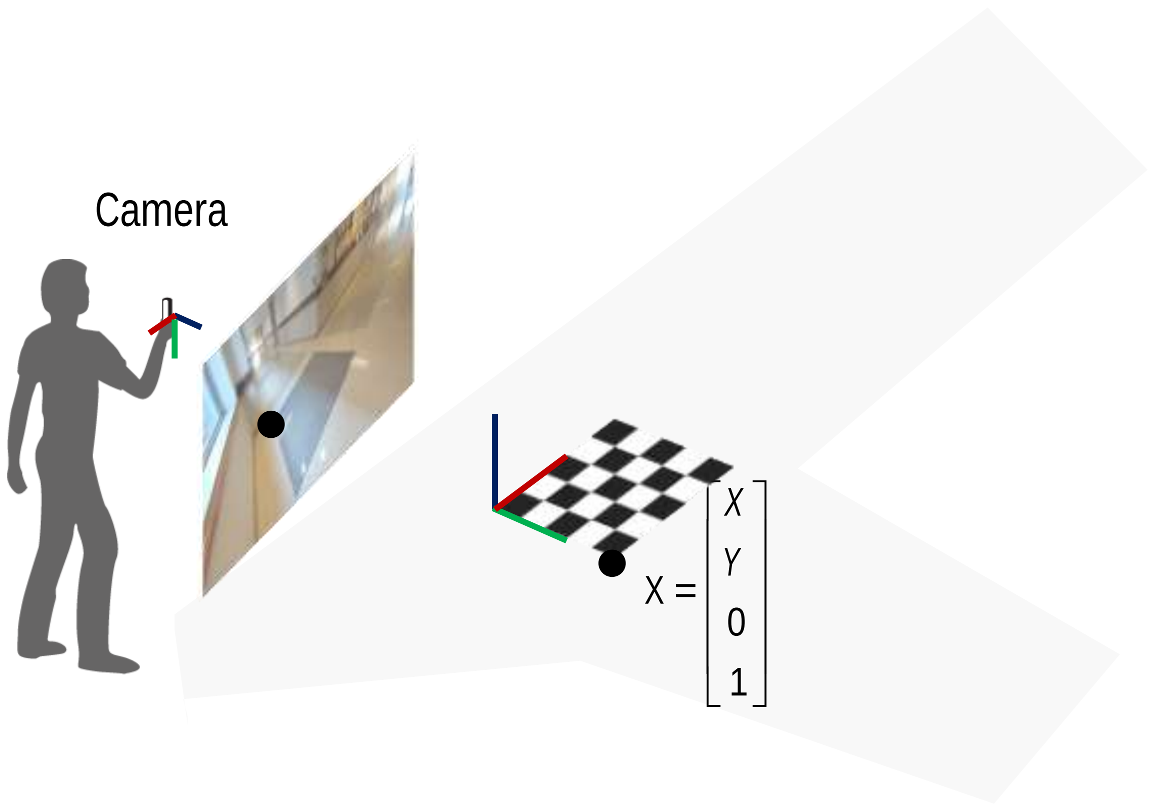
$$K^{-T}K^{-1} =$$

Method2: Rotation



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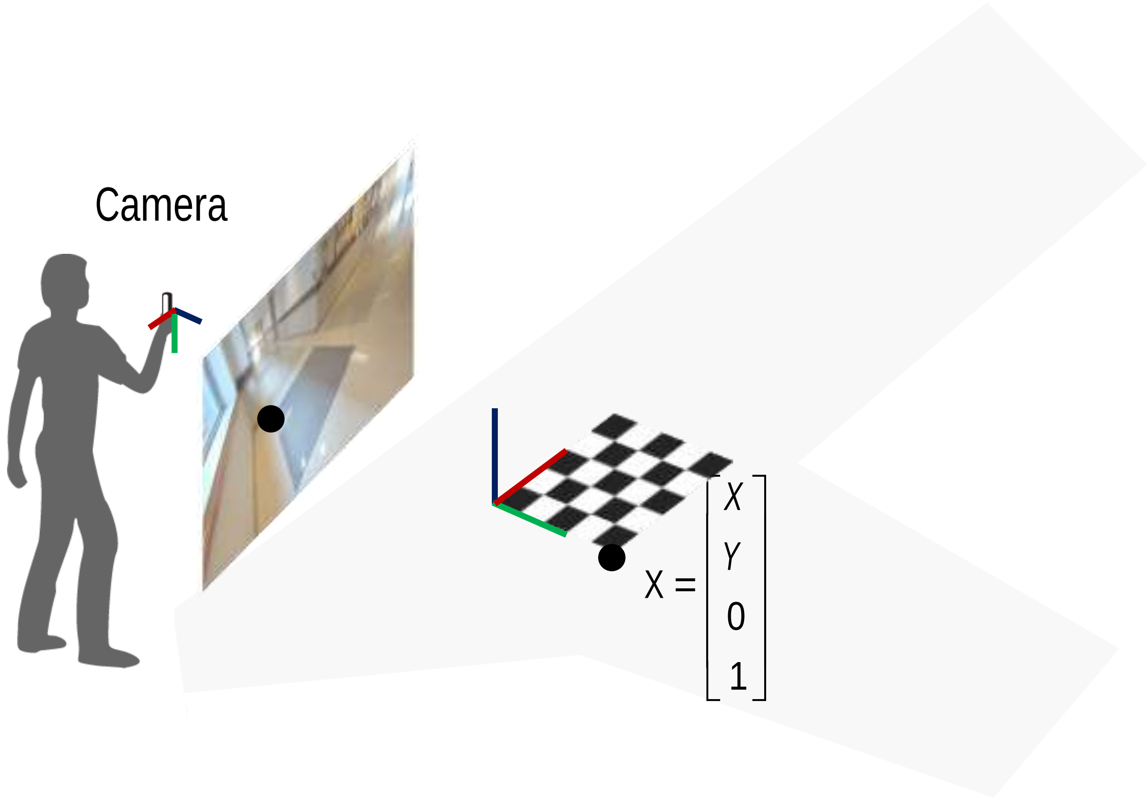
$$K^{-T}K^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix}$$

Method2: Rotation



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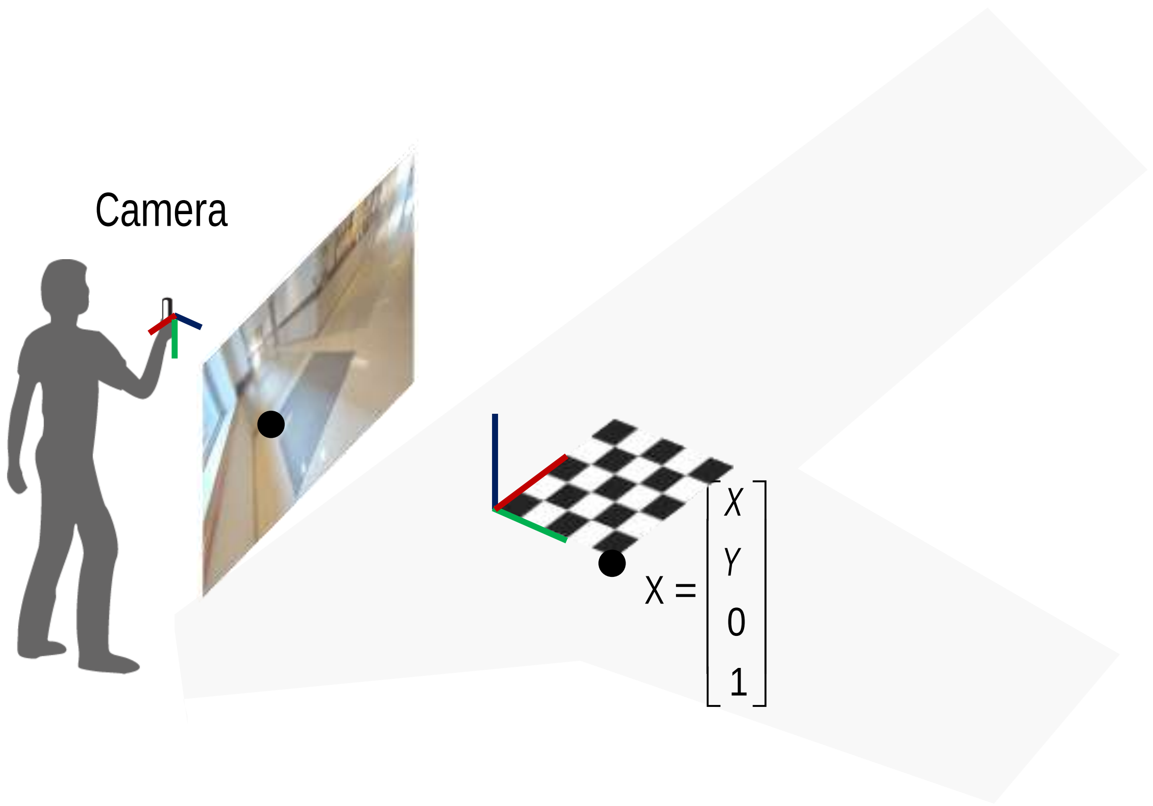
$$K^{-T}K^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix}\begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

Method2: Rotation



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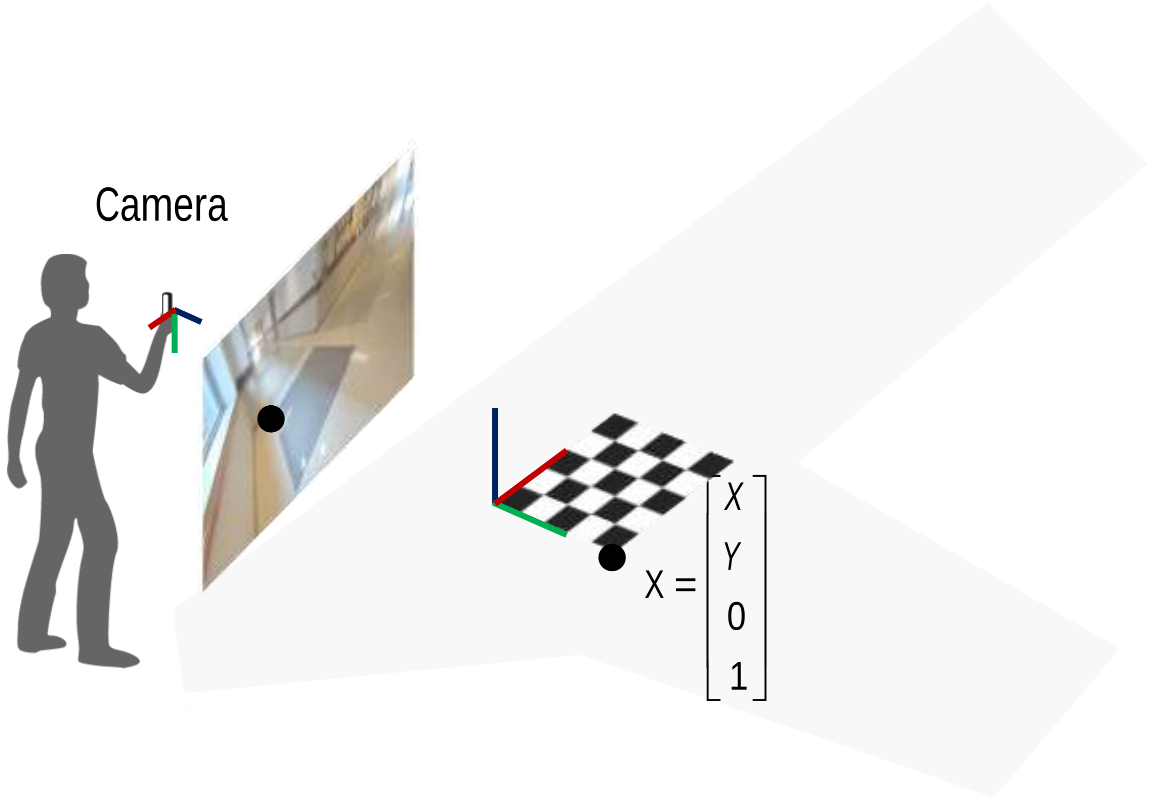
$$K^{-T}K^{-1} = \begin{bmatrix} 1/f & & \\ & 1/f & \\ -p_x/f & -p_y/f & 1 \end{bmatrix} \begin{bmatrix} 1/f & & -p_x/f \\ & 1/f & -p_y/f \\ & & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix}}_{B}$$

where $b_1 = \frac{1}{f^2}, \quad b_2 = -\frac{p_x}{f^2}, \quad b_3 = -\frac{p_y}{f^2}, \quad b_4 = \frac{p_x^2}{f^2} + \frac{p_y^2}{f^2} + 1$

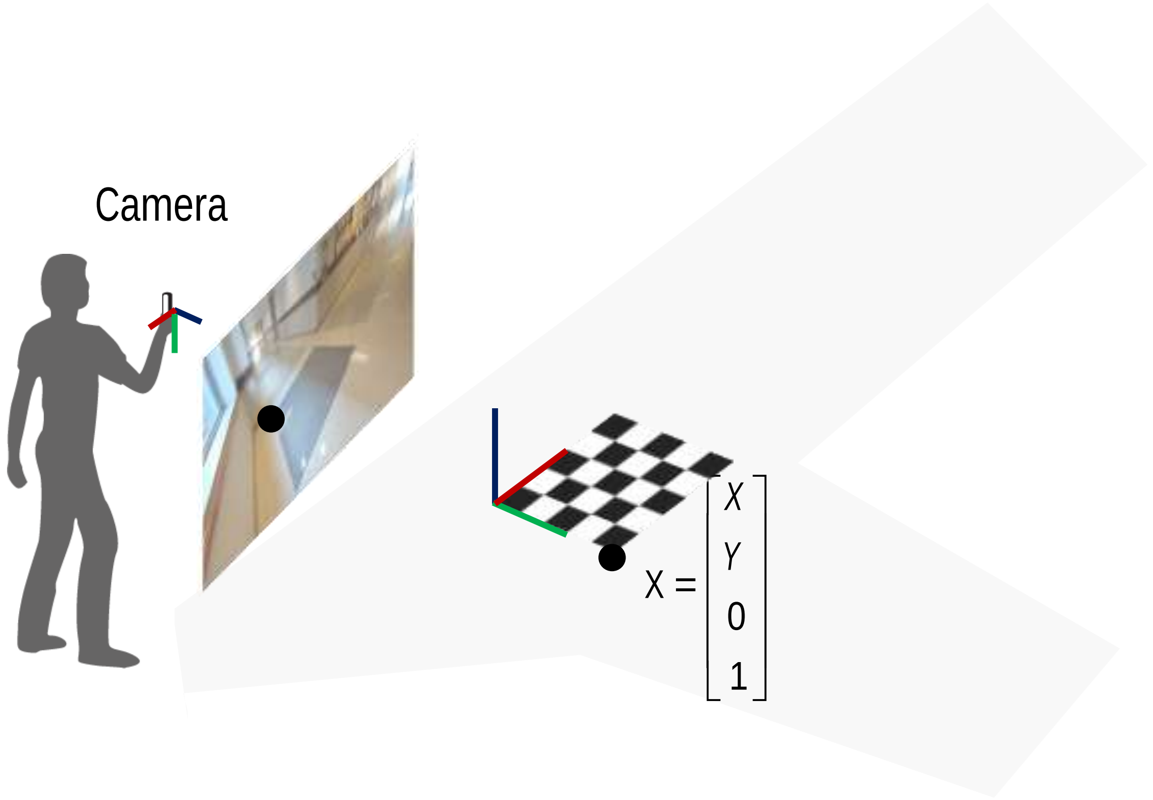
Linear in B: $\quad h_1^T B h_2 = 0 \qquad h_1^T B h_1 = h_2^T B h_2$

Method2: Rotation

$$\mathbf{h}_1^{\mathsf{T}}\mathbf{B}\mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$



Method2: Rotation



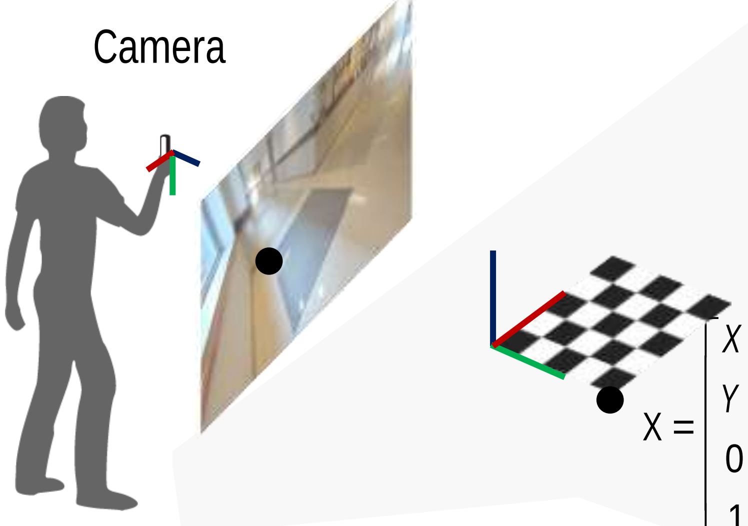
$$\mathrm{h}_1^T \mathrm{B} \mathrm{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathrm{h}_1^T \mathrm{B} \mathrm{h}_1 = \mathrm{h}_2^T \mathrm{B} \mathrm{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

Method2: Rotation

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

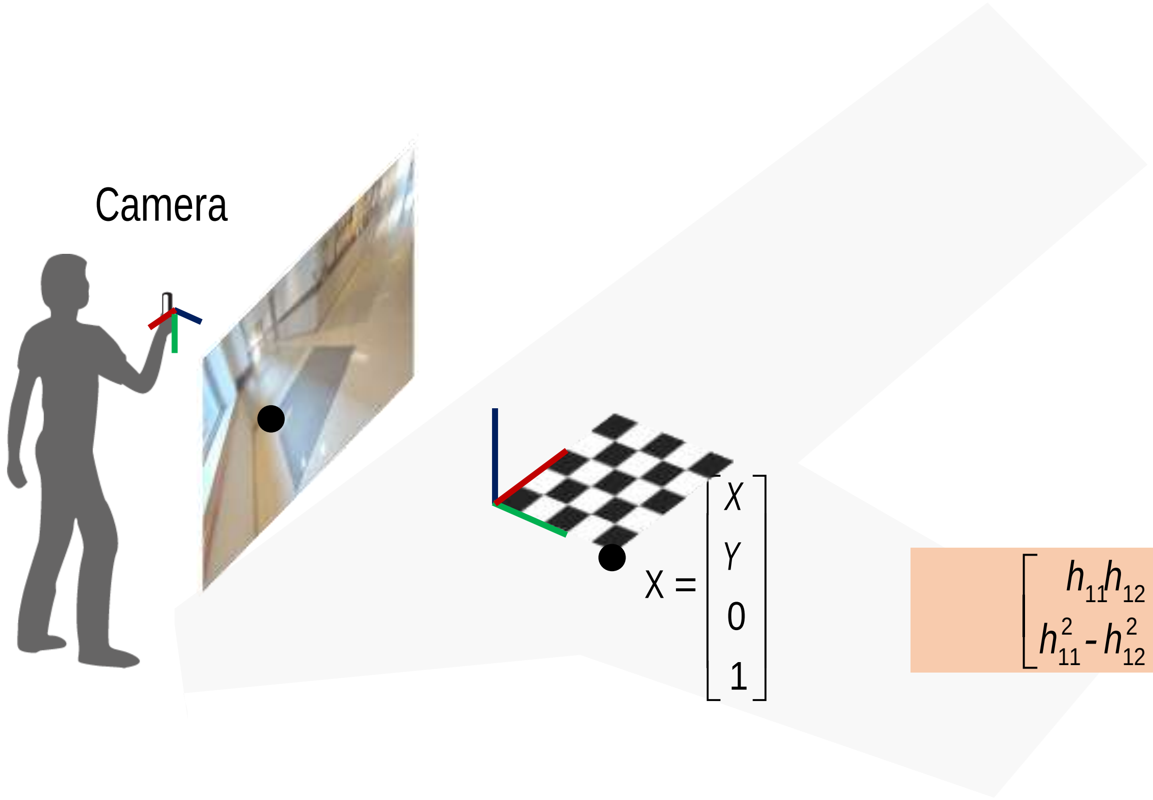
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

2x4

Method2: Rotation

$$h_1^T B h_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$h_1^T B h_1 = h_2^T B h_2$$

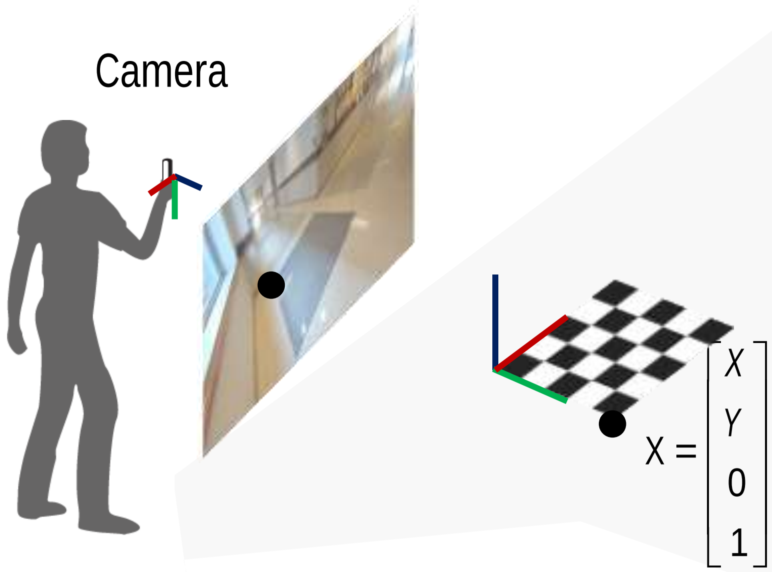
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = 0$$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$h_1^T B h_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$h_1^T B h_1 = h_2^T B h_2$$

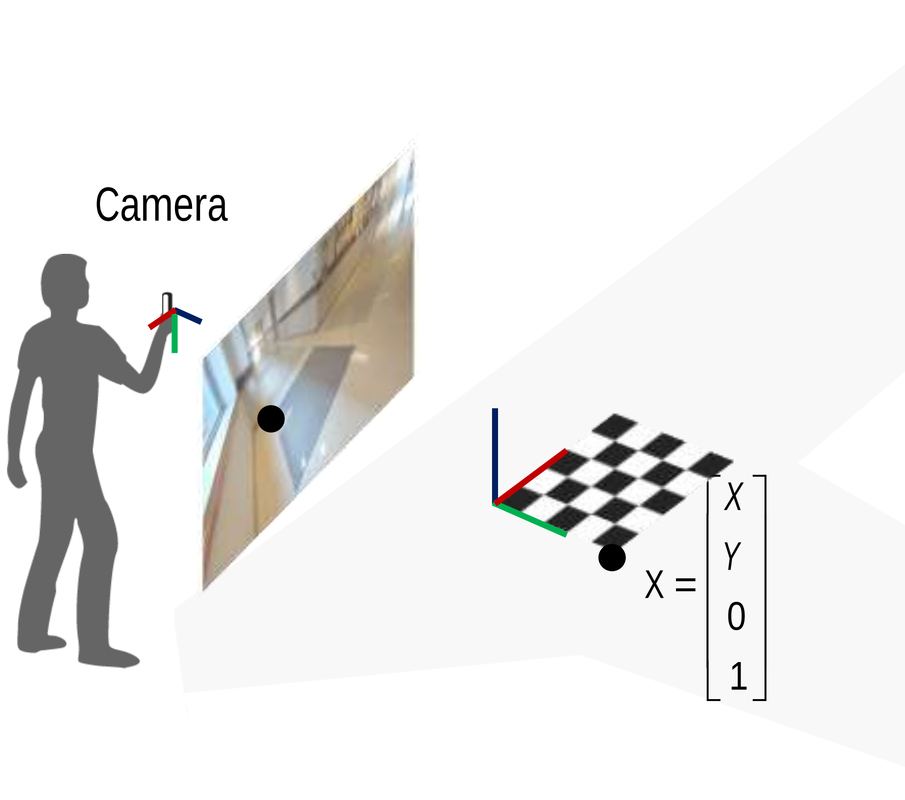
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{x} = 0$$

2x4

Each image produces 2 equations and therefore, x can be computed with minimum 2 images.

Method2: Rotation



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

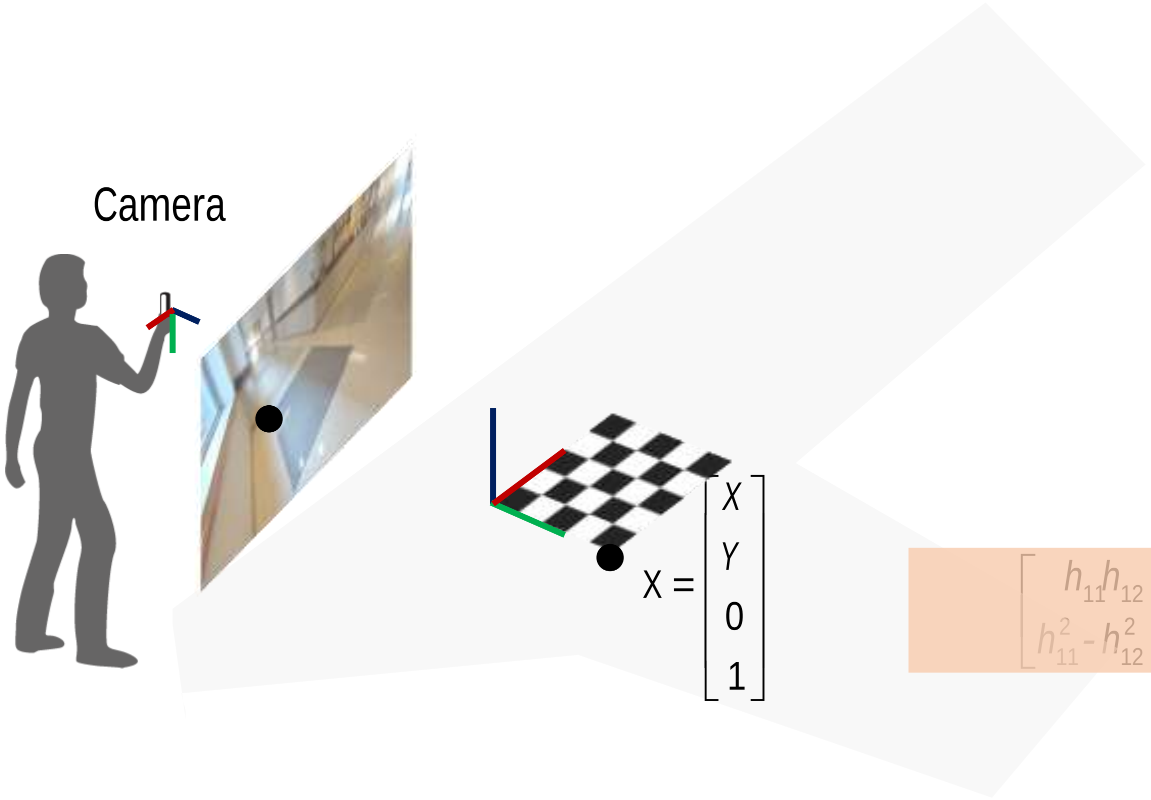
$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12}+h_{21}h_{22} & h_{11}h_{32}+h_{31}h_{12} & h_{21}h_{32}+h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2-h_{12}^2+h_{21}^2-h_{22}^2 & 2(h_{11}h_{31}-h_{12}h_{32}) & 2(h_{21}h_{31}-h_{22}h_{32}) & h_{31}^2-h_{32}^2 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\mathbf{X}} = $$

2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_2 = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = 0$$

$$\mathbf{h}_1^T \mathbf{B} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{B} \mathbf{h}_2$$

$$\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}^T \begin{bmatrix} b_1 & & b_2 \\ & b_1 & b_3 \\ b_2 & b_3 & b_4 \end{bmatrix} \begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} h_{11}h_{12} + h_{21}h_{22} & h_{11}h_{32} + h_{31}h_{12} & h_{21}h_{32} + h_{31}h_{22} & h_{32}h_{31} \\ h_{11}^2 - h_{12}^2 + h_{21}^2 - h_{22}^2 & 2(h_{11}h_{31} - h_{12}h_{32}) & 2(h_{21}h_{31} - h_{22}h_{32}) & h_{31}^2 - h_{32}^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{X} = \mathbf{0}$$

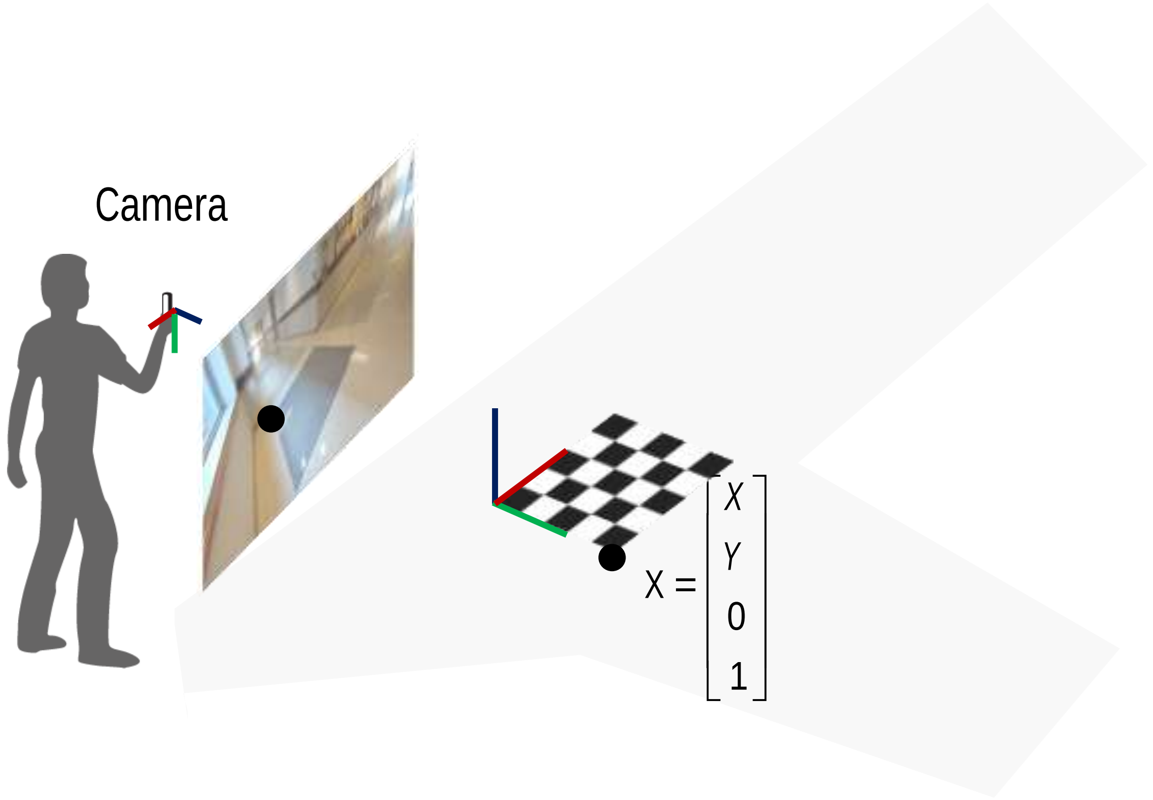
2x4

$$p_x = -\frac{b_2}{b_1}, \quad p_y = -\frac{b_3}{b_1}, \quad f = \sqrt{\frac{b_4}{b_1} - (p_x^2 + p_y^2)}$$

Method2: Rotation

: Knowns

: Unknowns



Homography factorization:

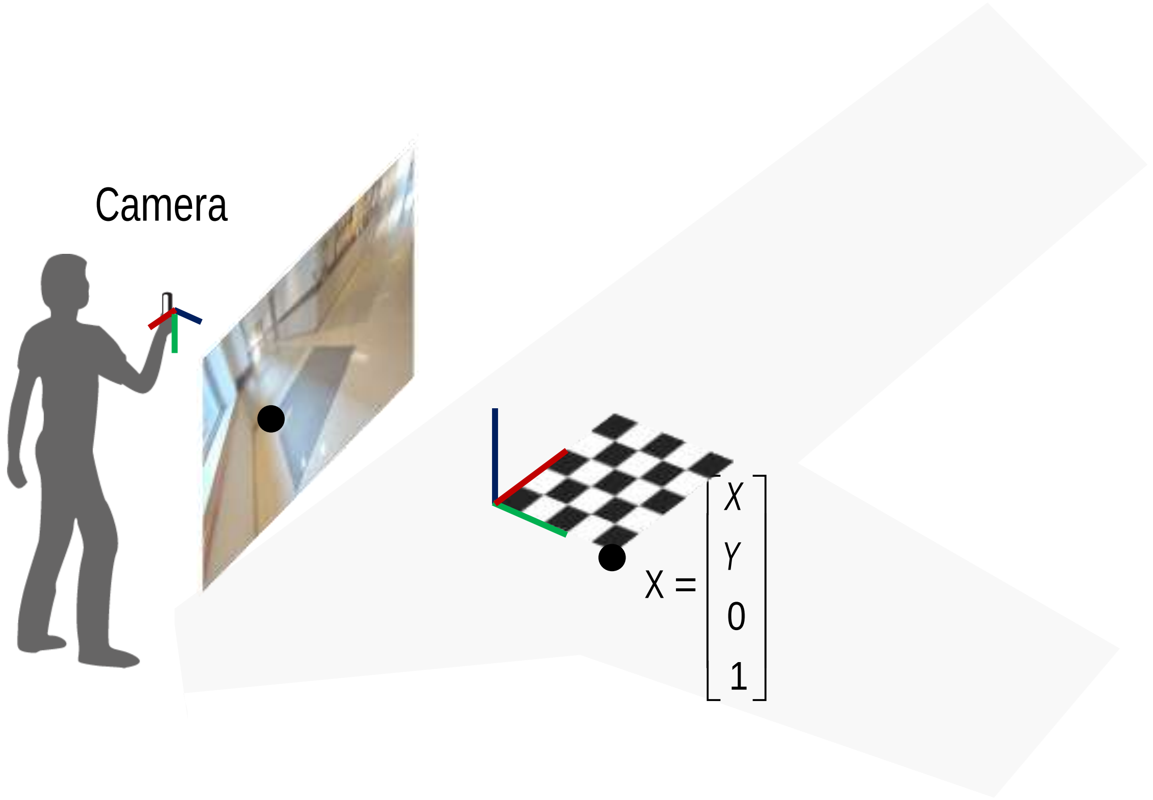
$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

$$r_1 = K^{-1} h_1 \qquad r_2 = K^{-1} h_2 \qquad r_3 = K^{-1} h_3$$

Method2: Rotation

: Knowns

: Unknowns



Homography factorization:

$$\left(\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

$$r_1 = K^{-1}h_1 \qquad r_2 = K^{-1}h_2 \qquad r_3 = K^{-1}h_3$$

$$r_1 = \frac{K^{-1}h_1}{\left\|K^{-1}h_1\right\|}, \quad r_2 = \frac{K^{-1}h_2}{\left\|K^{-1}h_1\right\|}, \quad t = \frac{K^{-1}h_3}{\left\|K^{-1}h_1\right\|}, \quad r_3 = r_1 \times r_2$$

Divided by constant factor