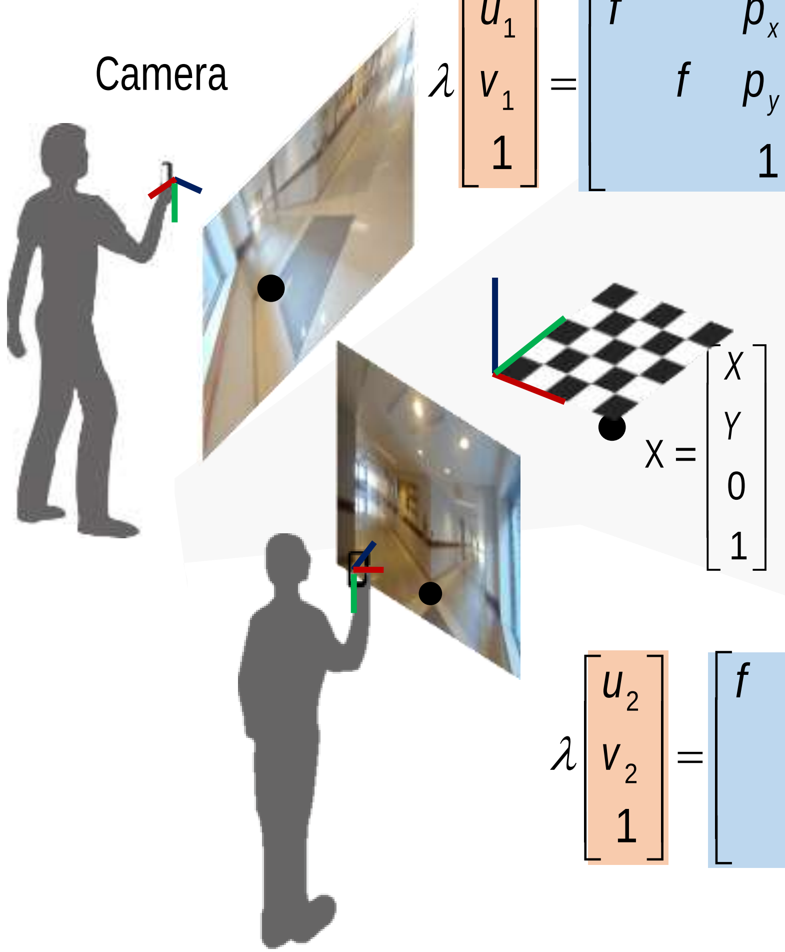
Where am I via Homography?

Recall: Camera Calibration from Multiple Imags



$$\lambda \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_1 & t_1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} R_2 & t_2 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# of unknowns: 3 (K) + 6*n* (R and t)

n: the number of images

# of equations: 2*nm* (X)

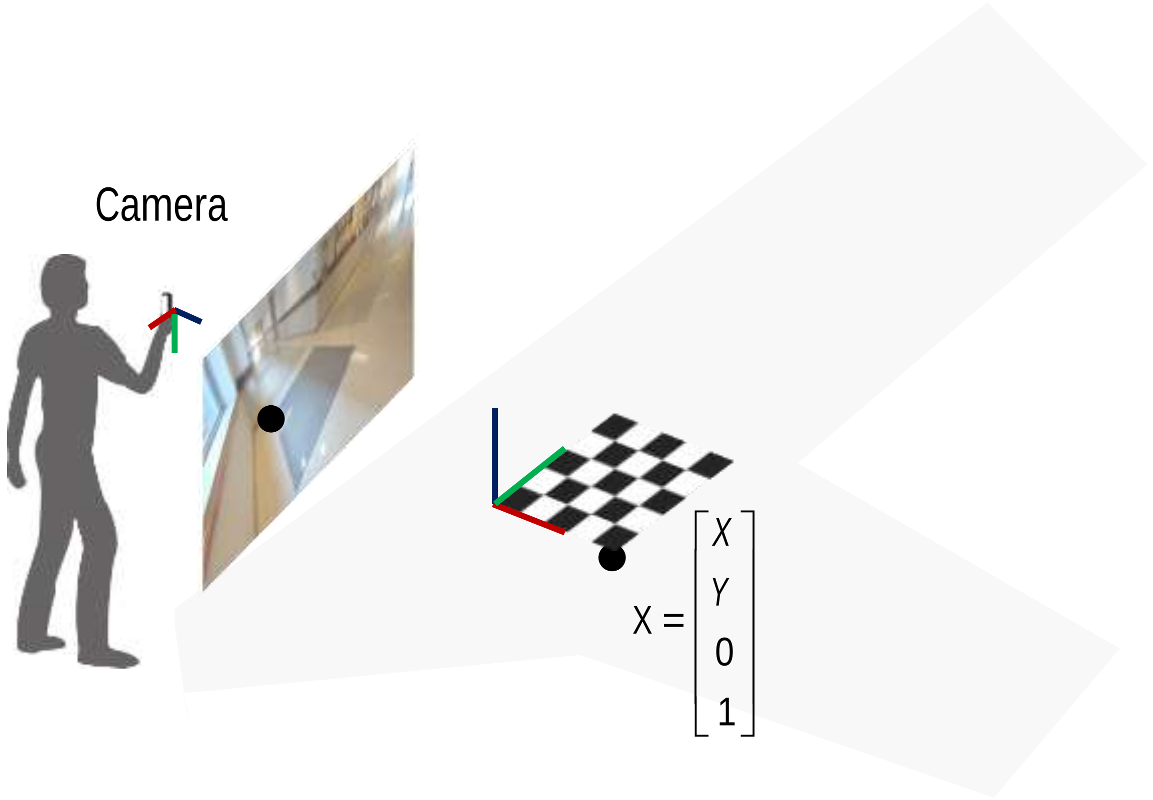
m: the number of known 3D points

We can solve for K, R, t if $3 + 6n < 2\,nm$

: Knowns

: Unknowns

Homography Mapping



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

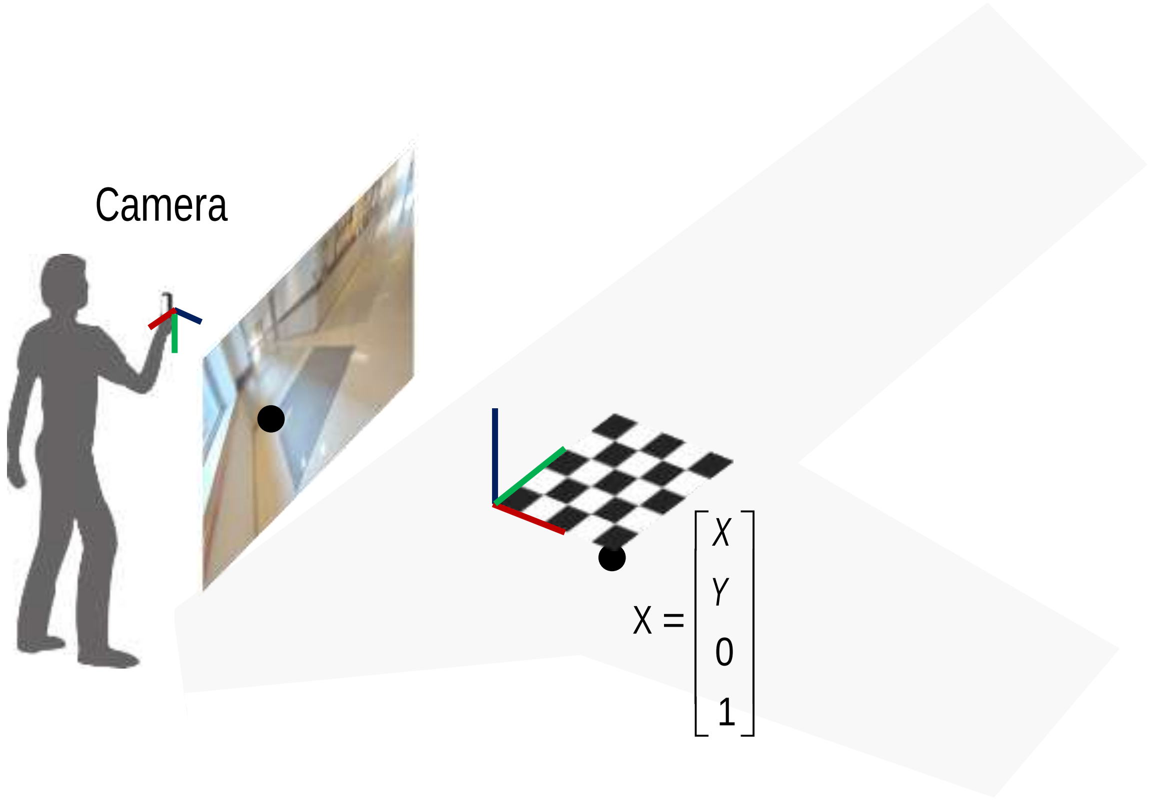
Points in 2D plane are mapped to an image with homography:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



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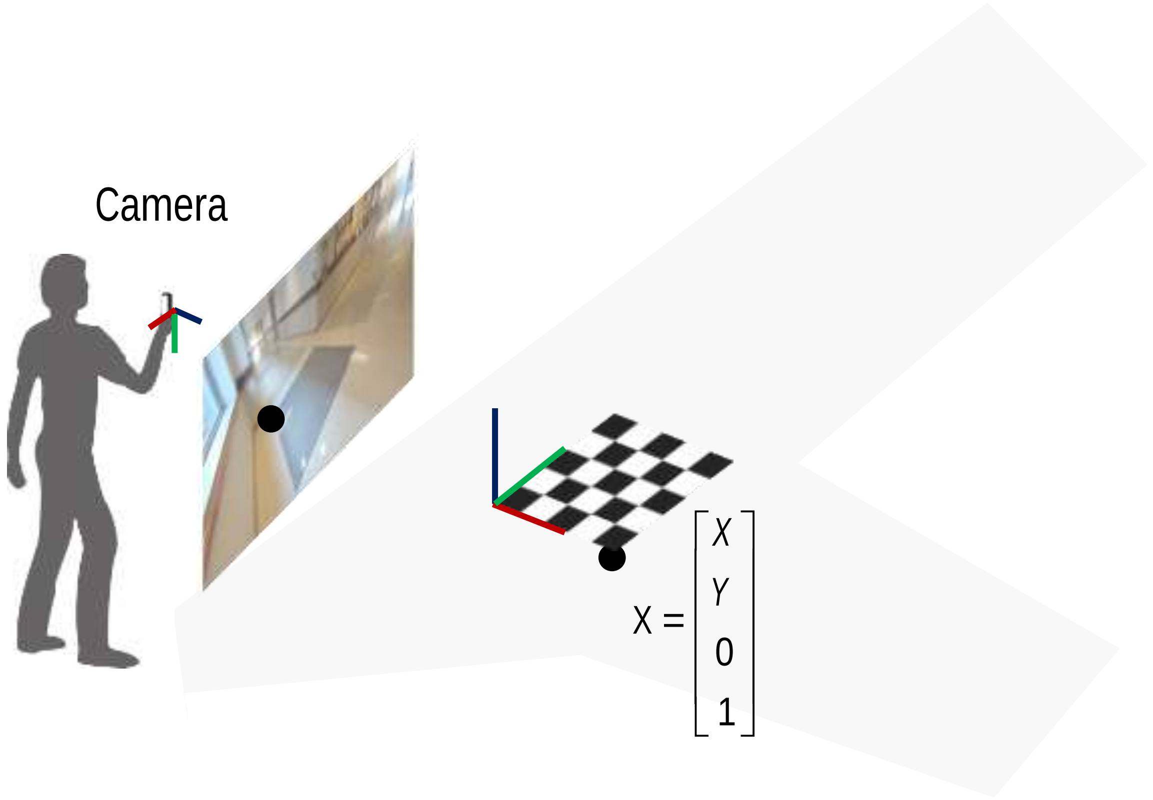
$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \underbrace{\begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix}}_{\mathrm{Q}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \underbrace{\mathrm{H}}_{3\mathrm{x}3} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



Points in 2D plane are mapped to an image with homography:

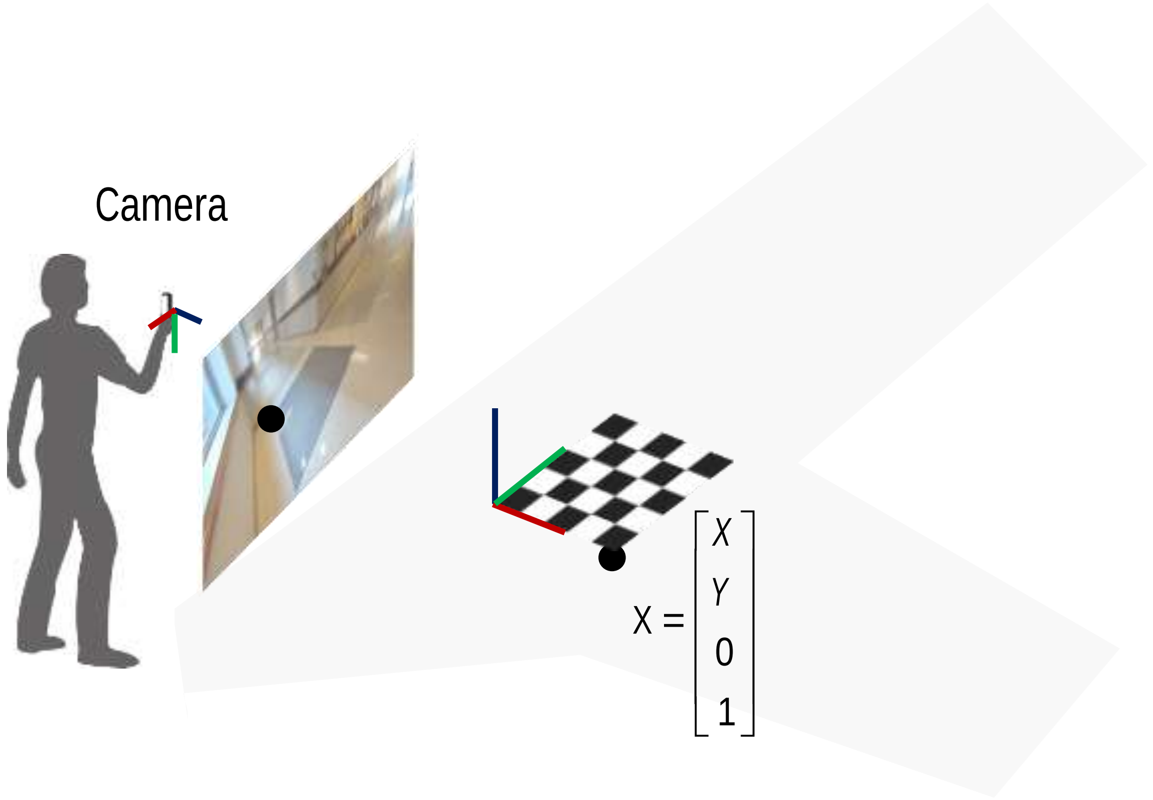
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathrm{K}} \underbrace{\begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix}}_{\mathrm{Q}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \underset{3\mathrm{x}3}{\underline{\mathrm{H}}} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

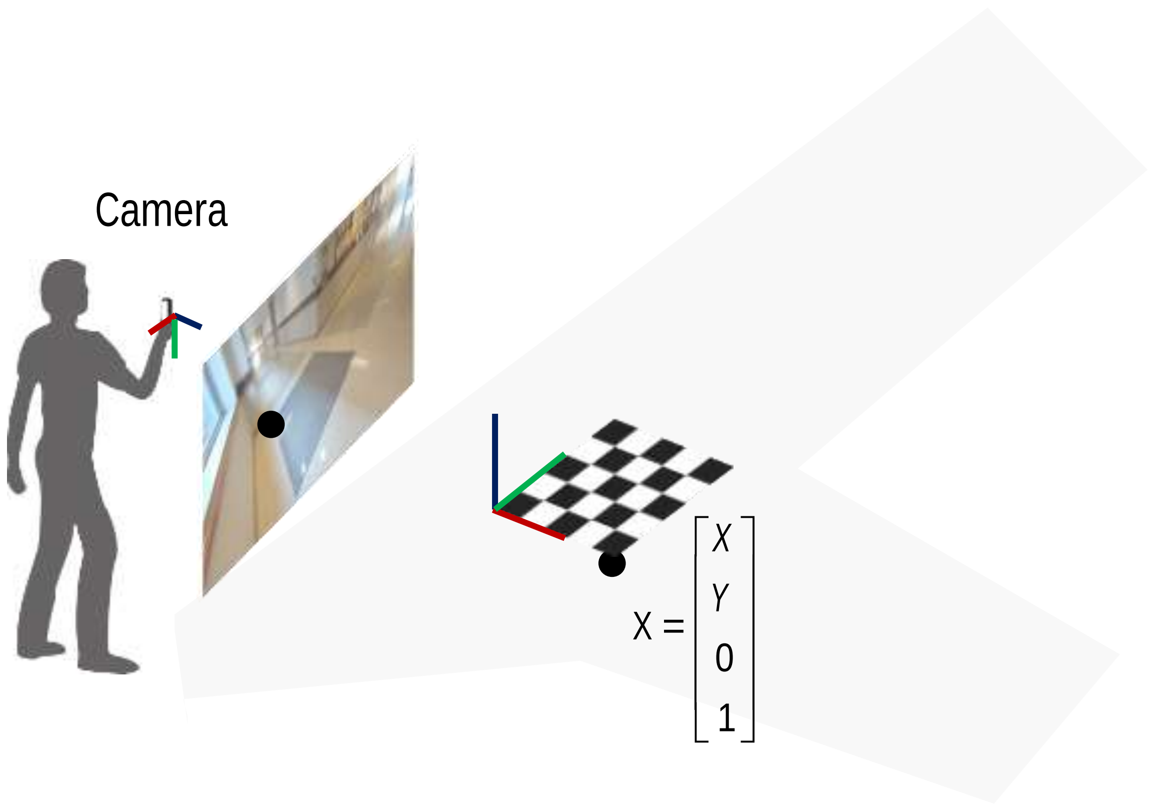
Points in 2D plane are mapped to an image with homography:

$$K^{-1}H = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

: Knowns

: Unknowns

Homography Mapping



$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

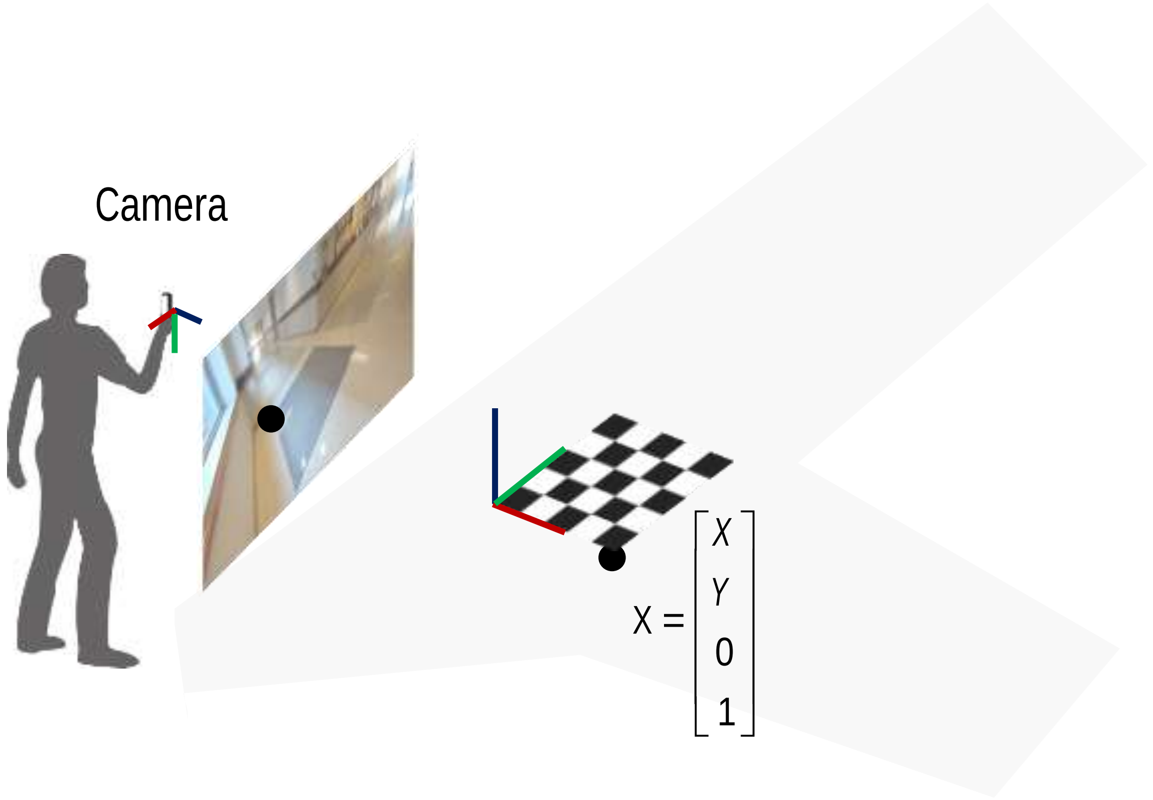
Points in 2D plane are mapped to an image with homography:

$$K^{-1}H = K^{-1}\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

$$\rightarrow r_1 = \frac{K^{-1}h_1}{\left\| K^{-1}h_1 \right\|}, \quad r_2 = \frac{K^{-1}h_2}{\left\| K^{-1}h_1 \right\|}, \quad t = \frac{K^{-1}h_3}{\left\| K^{-1}h_1 \right\|}$$

Common denominator

Homography Mapping



Points in 2D plane are mapped to an image with homography:

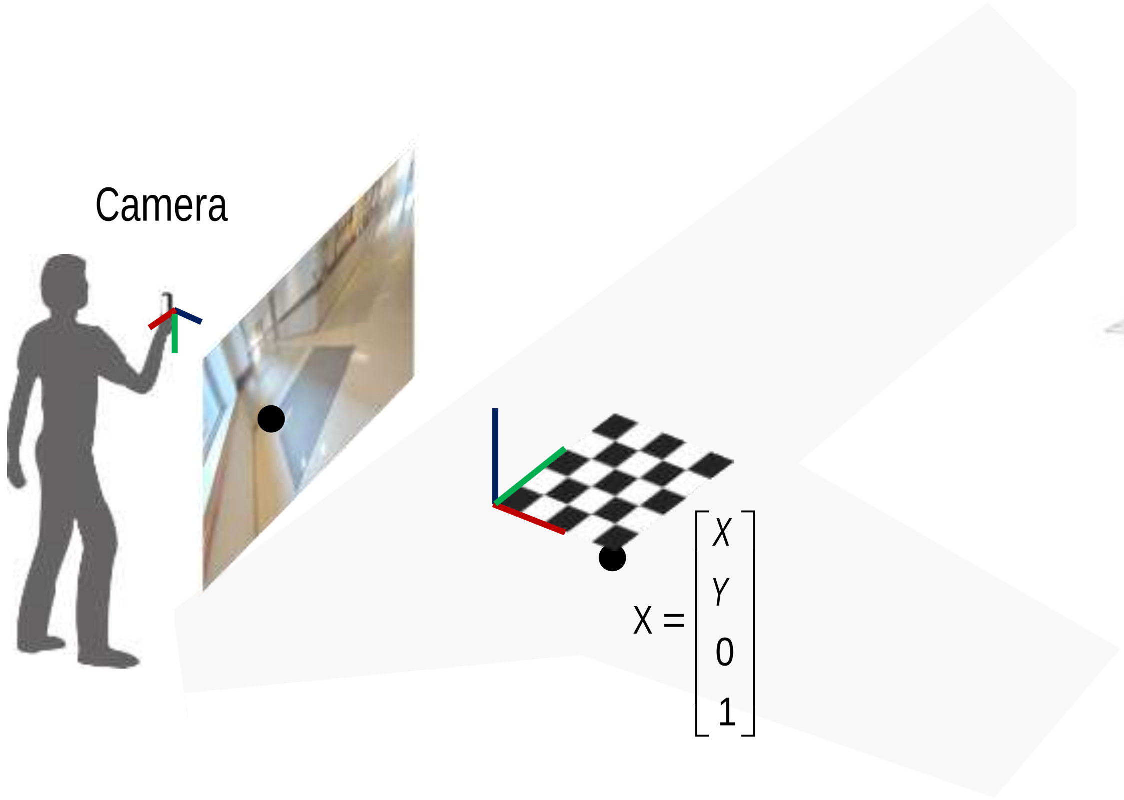
$$K^{-1}H = K^{-1}\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}$$

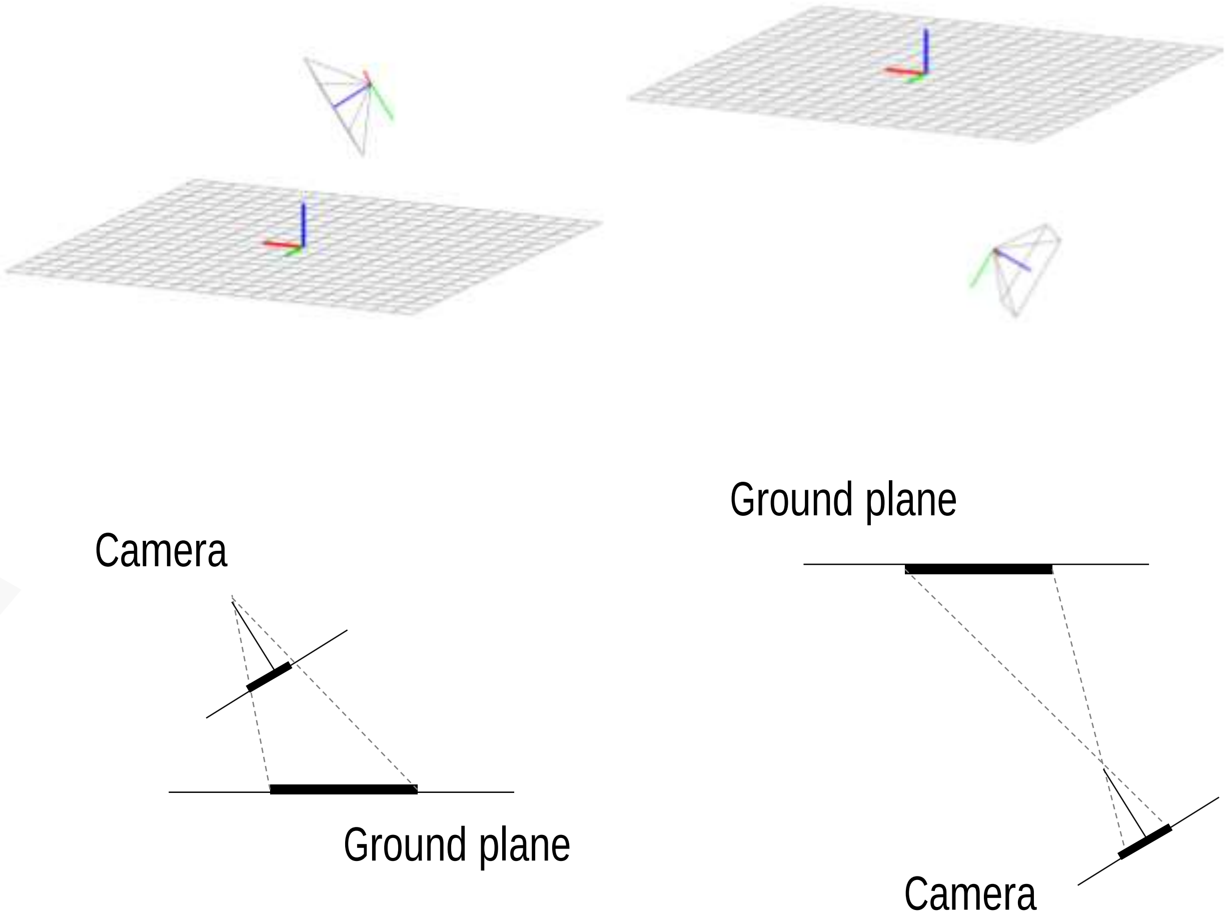
$$\rightarrow r_1 = \frac{K^{-1}h_1}{\left\|K^{-1}h_1\right\|},\quad r_2 = \frac{K^{-1}h_2}{\left\|K^{-1}h_1\right\|},\quad t = \frac{K^{-1}h_3}{\left\|K^{-1}h_1\right\|}$$

Common denominator

$$\rightarrow r_3 = r_1 \times r_2$$

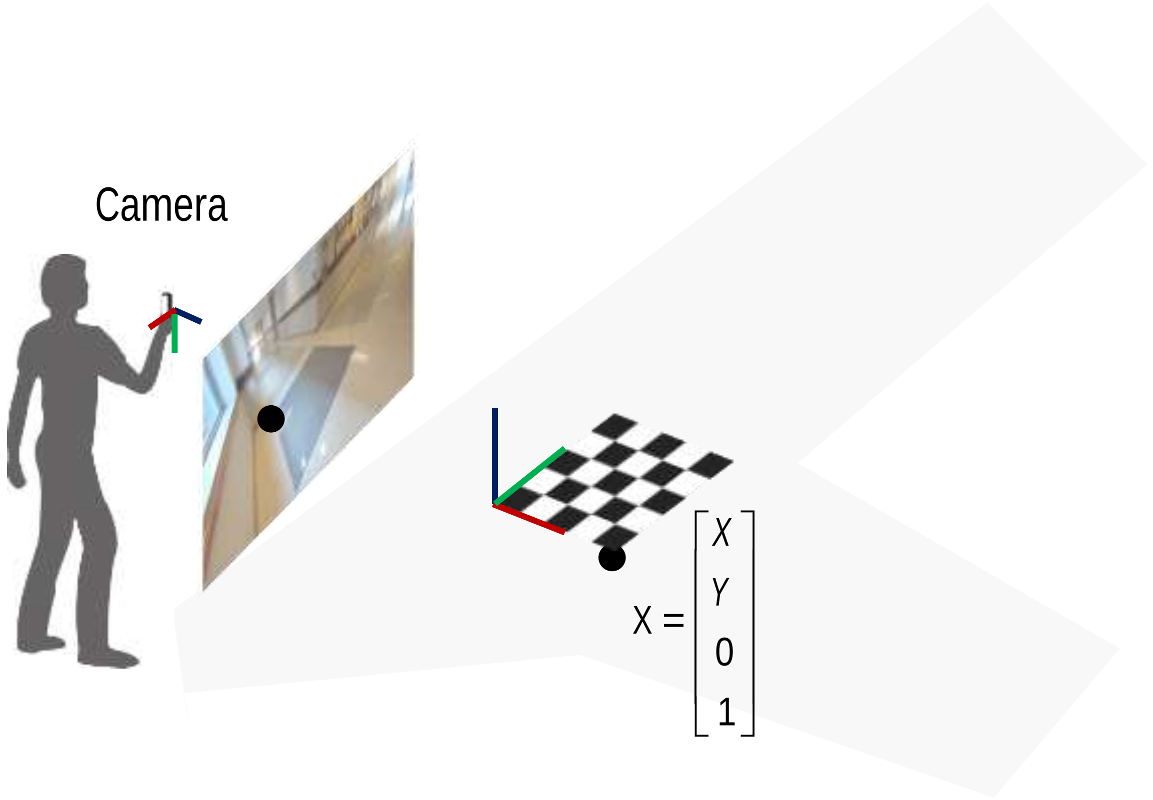
Geometric Ambiguity





Two configurations

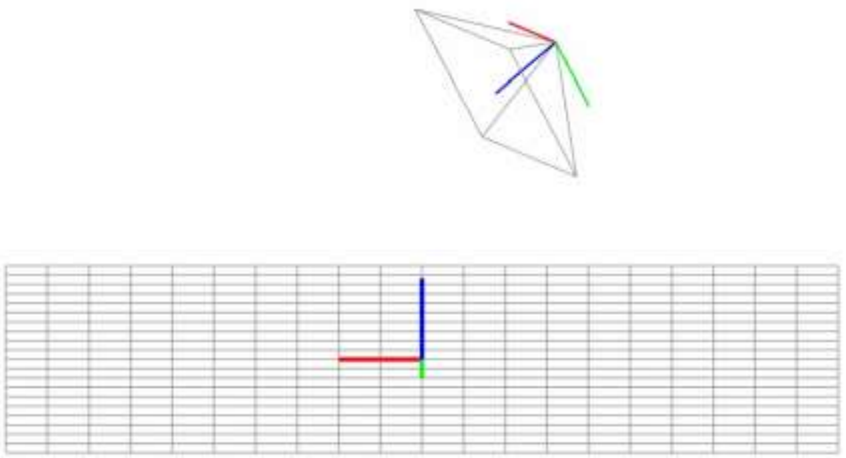
Homography Mapping



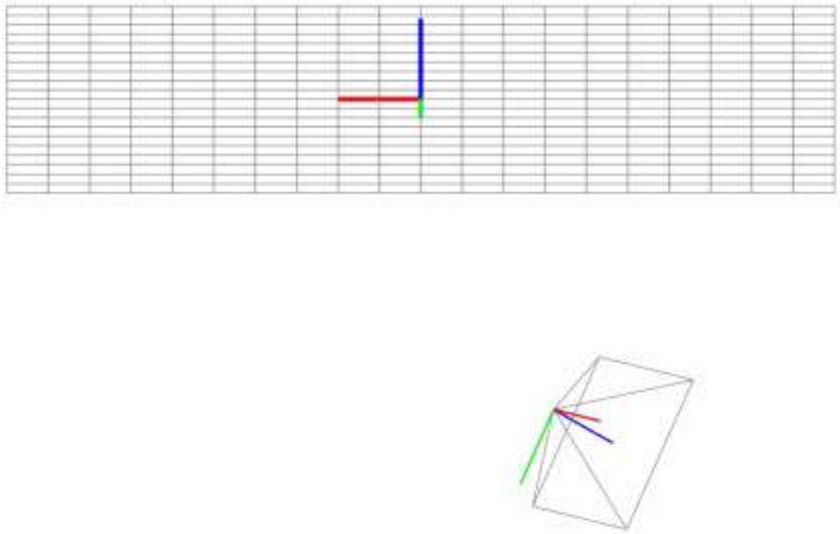
ComputeCameraFromHomography.m

```
function ComputeCameraFromHomography

f = 1300;
K = [f 0 size(im,2)/2;
    0 f size(im,1)/2;
    0 0 1];

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];

u = [m11(1:2)';m12(1:2)';m13(1:2)'; m14(1:2)'];
X = [0 0;1 0;1 1;0 1];
X = [X ones(4,1)]; % homogeneous coordinate

H = ComputeHomography(u, X)

denom = norm(inv(K)*H(:,1));
r1 = inv(K)*H(:,1)/denom;
r2 = inv(K)*H(:,2)/denom;
t = inv(K)*H(:,3)/denom;

r3 = Vec2Skew(r1)*r2;
```

Homography Mapping

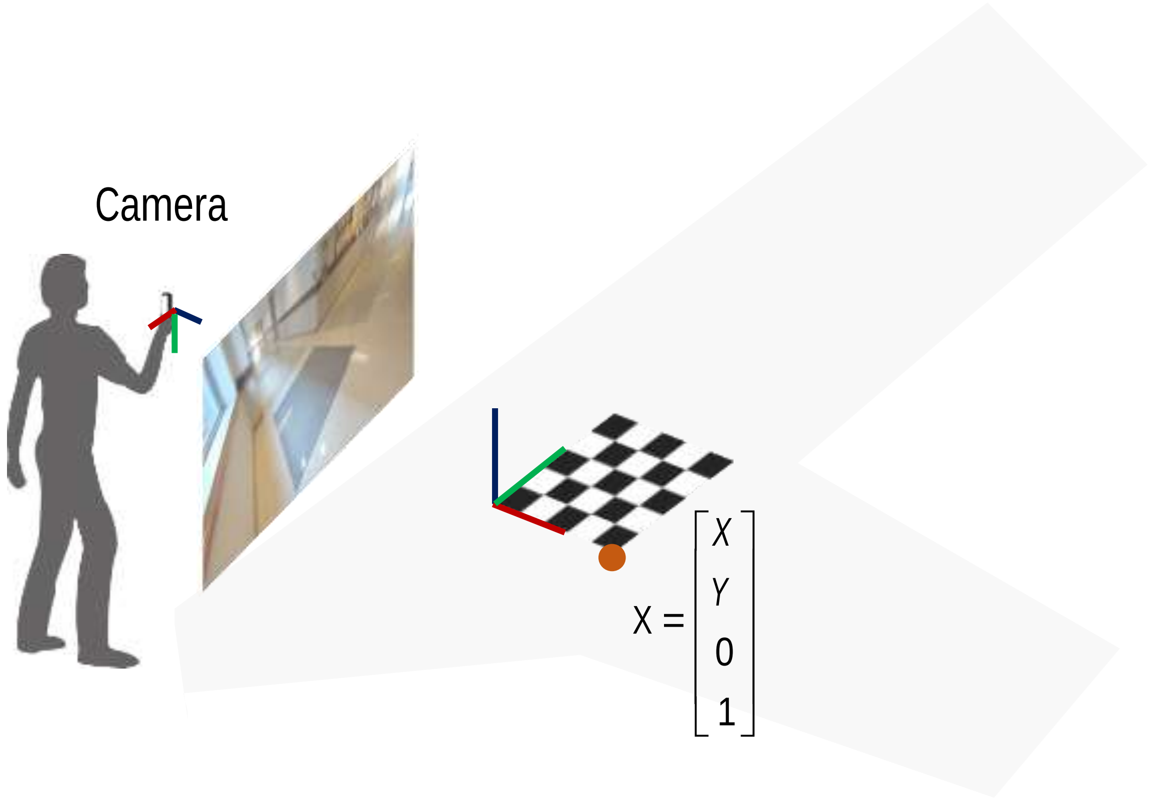


H



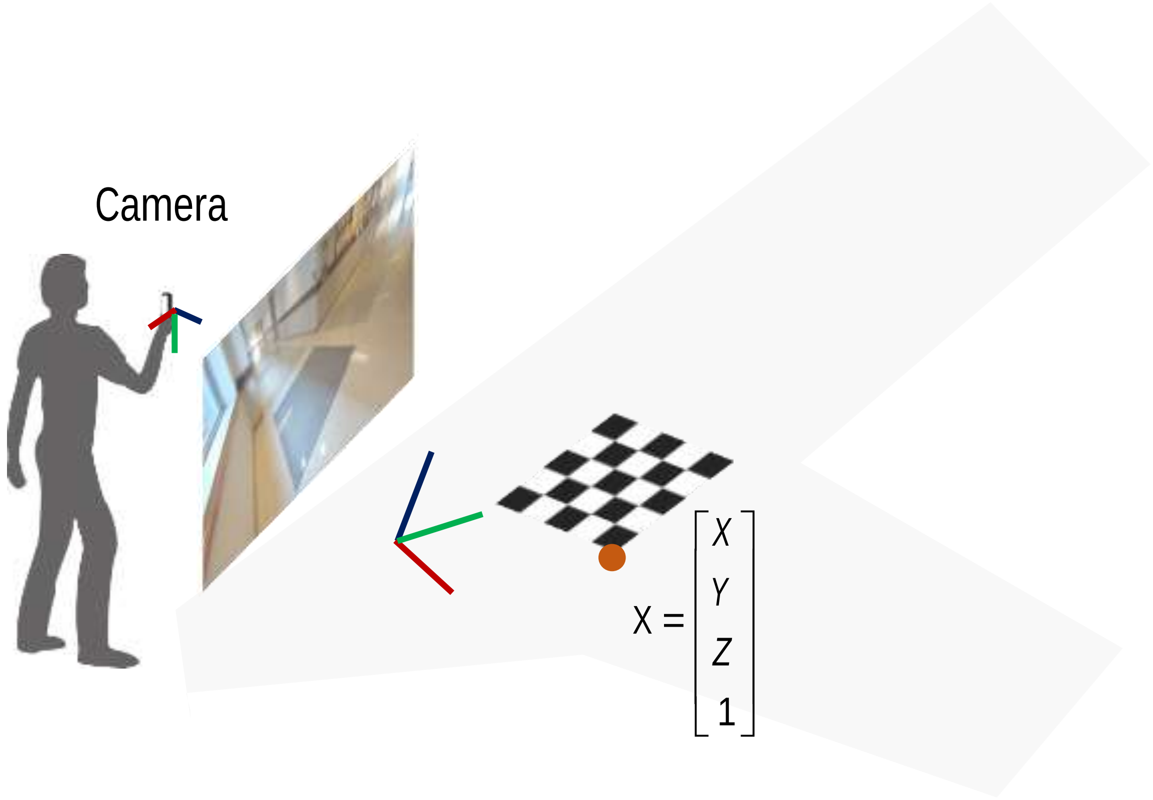
-H

Plane Representation

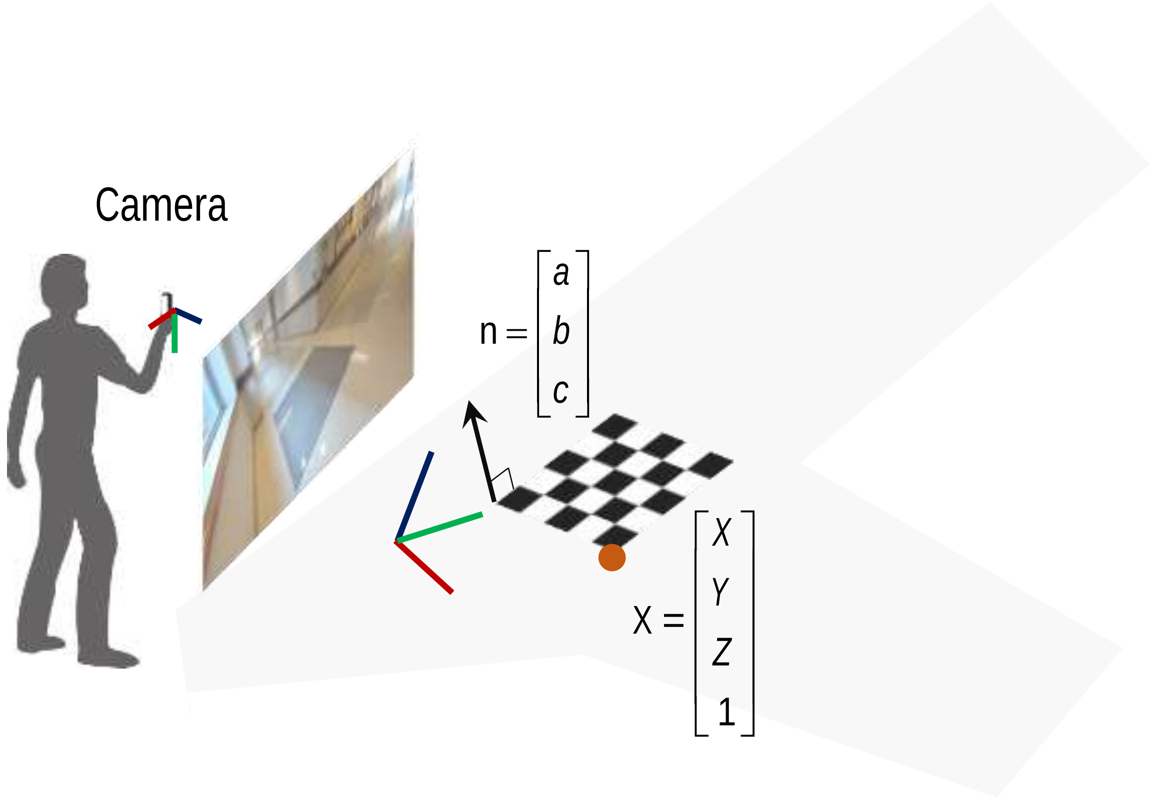


$$X = \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

Plane Representation



Plane Representation



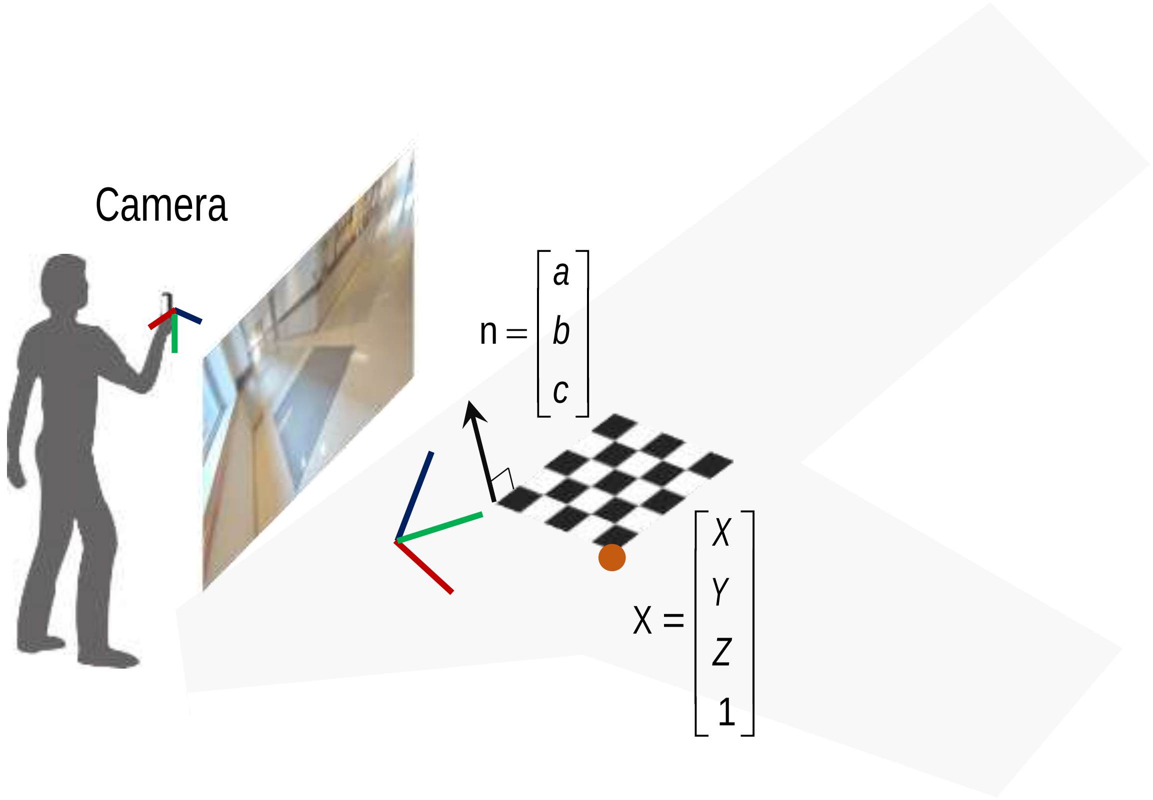
Plane equation:

$$aX + bY + cZ + d = 0$$

Surface normal:

$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

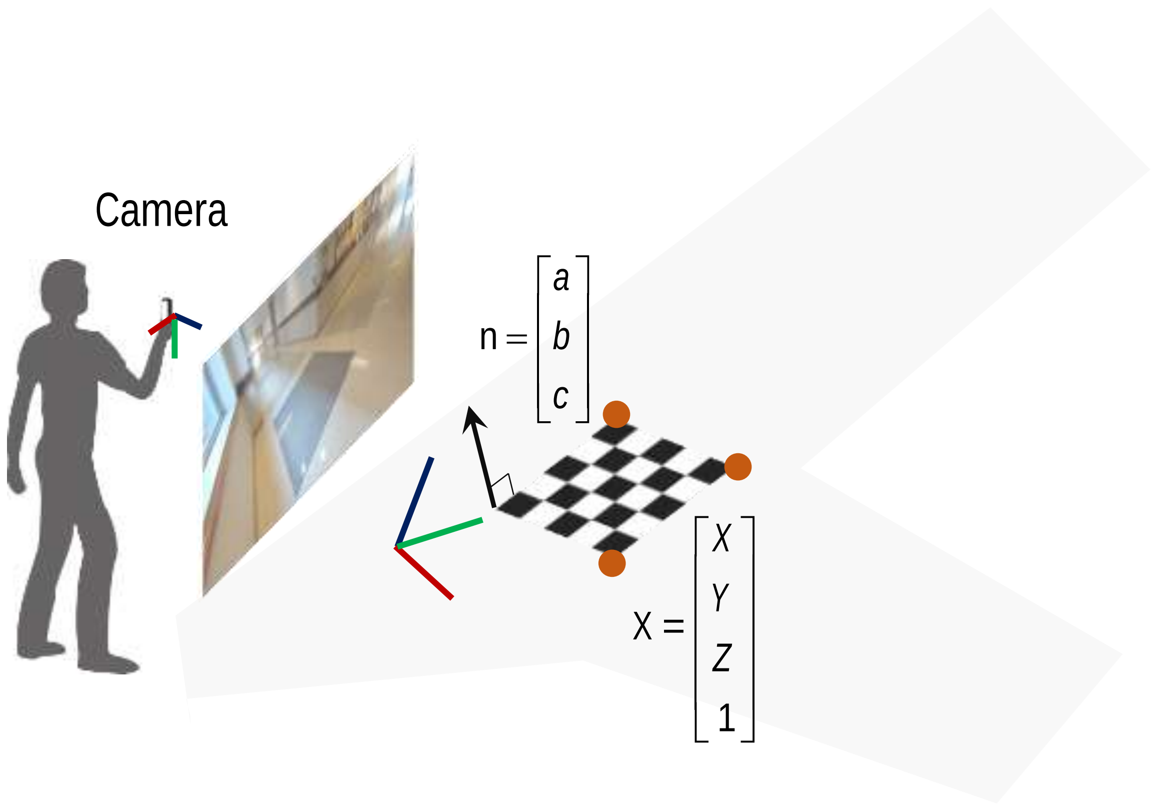
Surface normal:

$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$aX + bY + cZ + d = 0$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

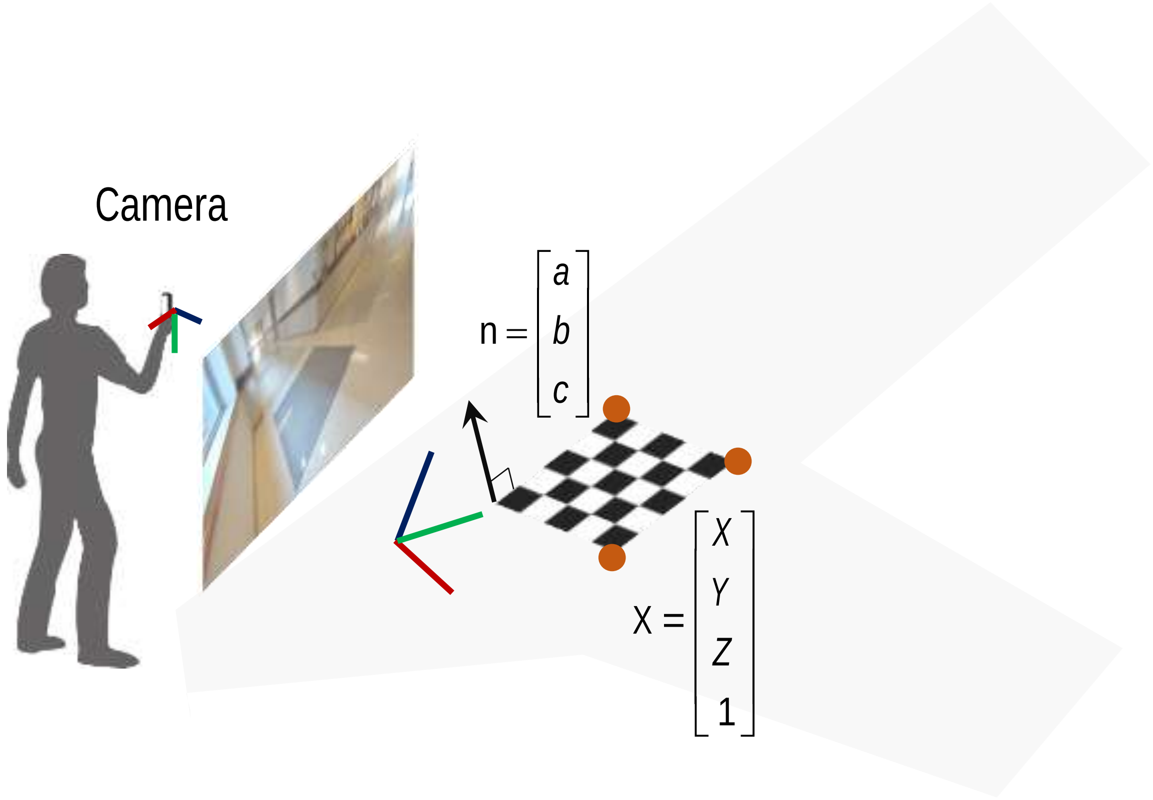
Surface normal:

$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$aX_1 + bY_1 + cZ_1 + d = 0$$
$$aX_2 + bY_2 + cZ_2 + d = 0$$
$$aX_3 + bY_3 + cZ_3 + d = 0$$

Plane Representation



Plane equation:

$$aX + bY + cZ + d = 0$$

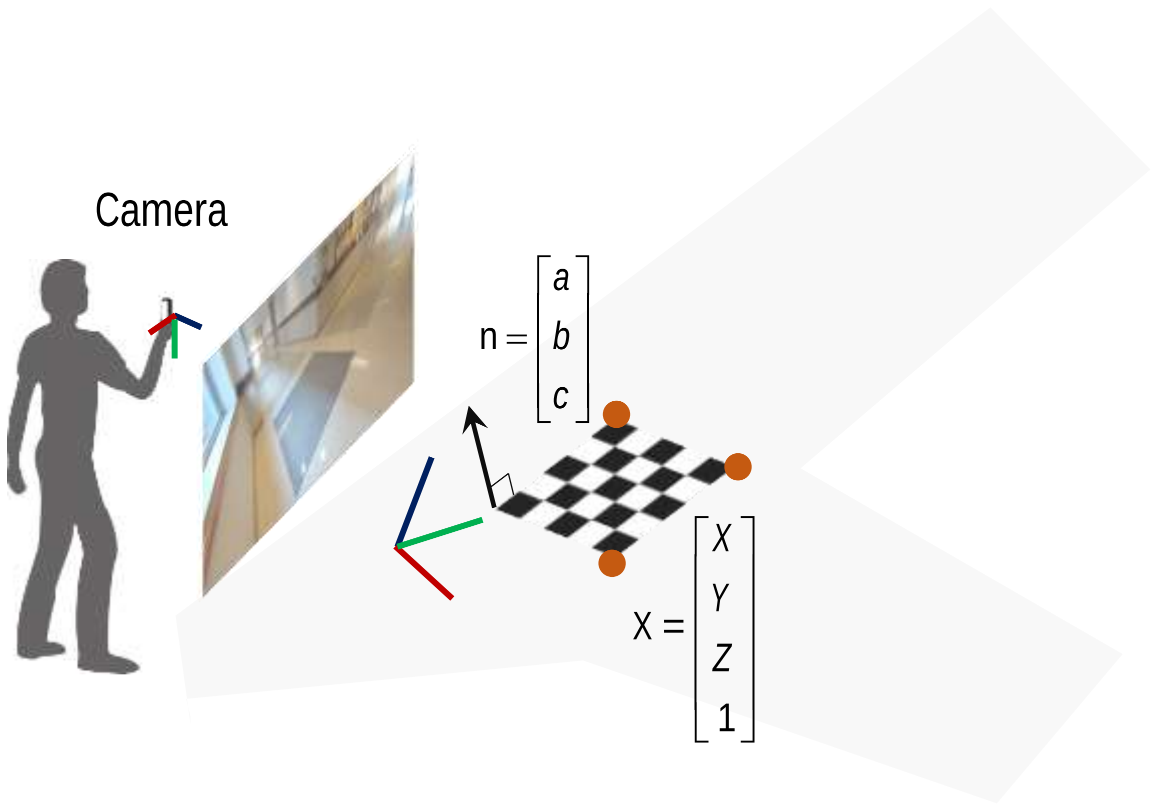
Surface normal:

$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Plane Representation



$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$X = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Plane equation:

$$aX + bY + cZ + d = 0$$

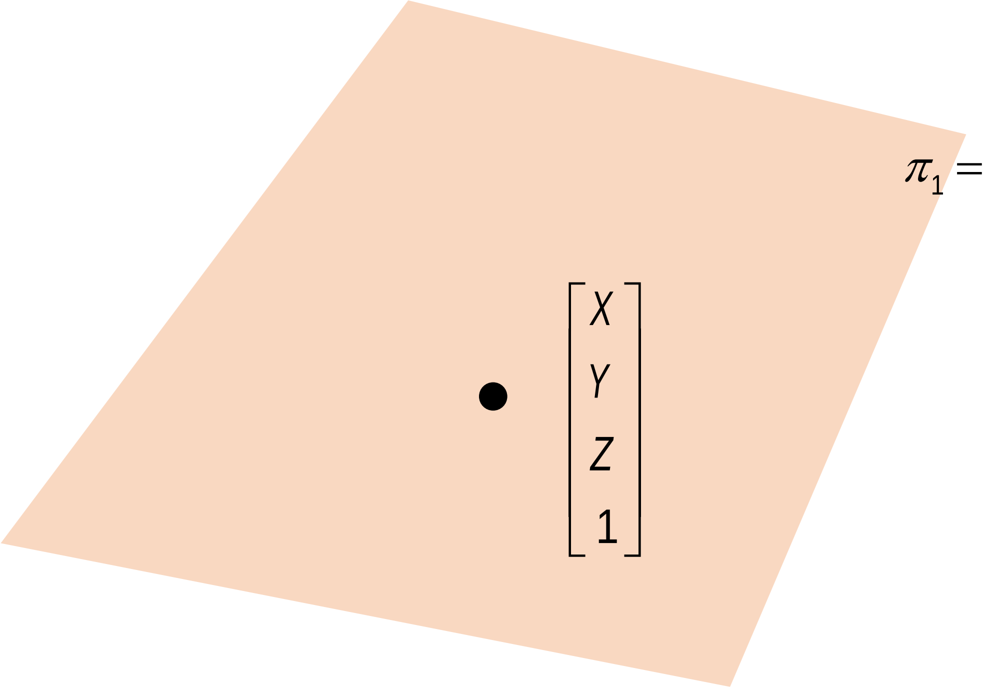
Surface normal:

$$n = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

How many points to define a plane?

$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{0}$$

Plane-Point Duality



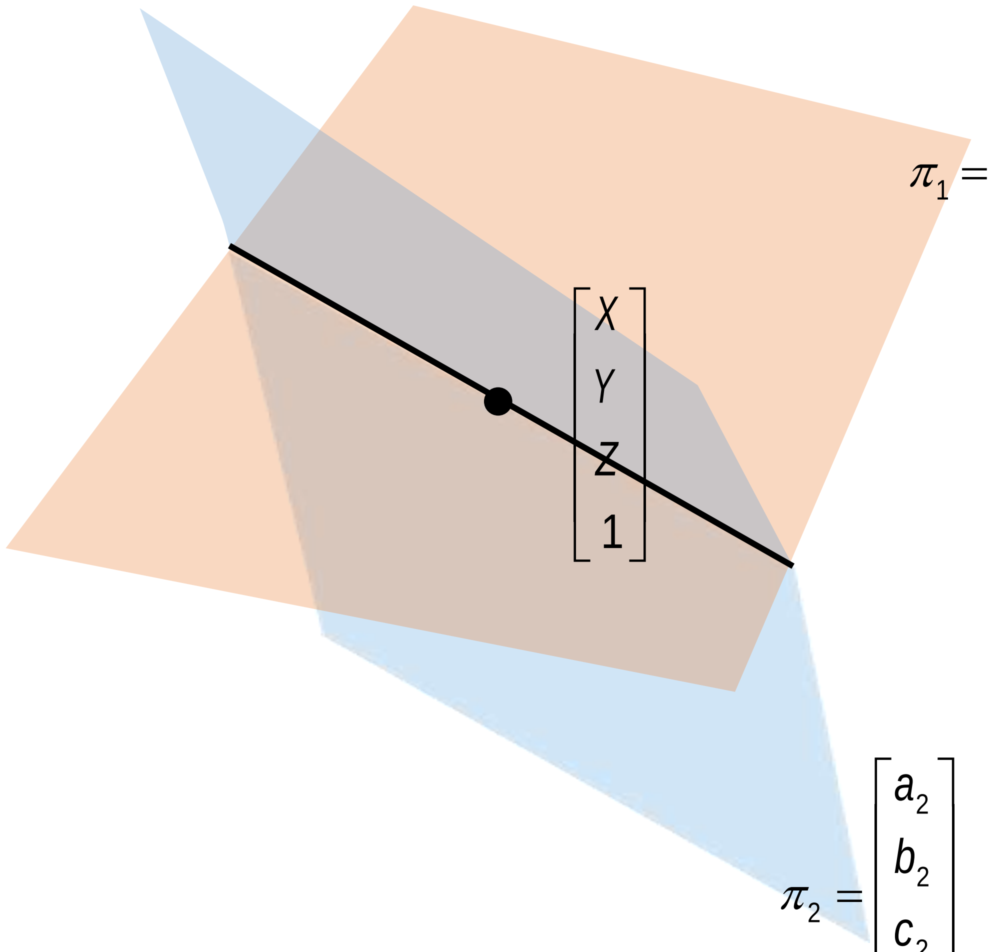
$$\pi_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Plane equation:

$$a_1 X + b_1 Y + c_1 Z + d_1 = 0$$

Plane-Point Duality



$$\pi_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

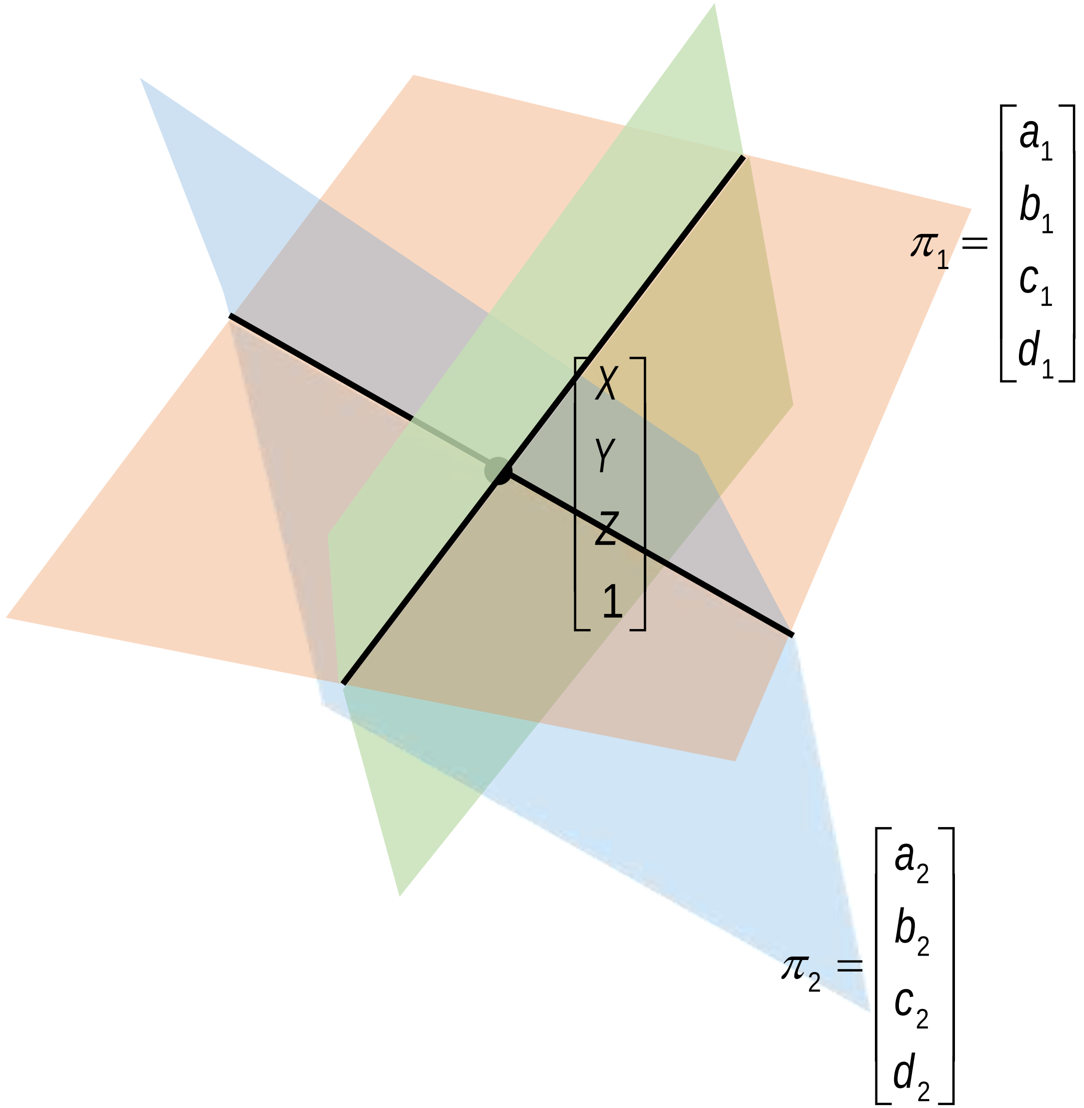
$$\pi_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix}$$

Plane equation:

$$a_1X + b_1Y + c_1Z + d_1 = 0$$

$$a_2X + b_2Y + c_2Z + d_2 = 0$$

Plane-Point Duality



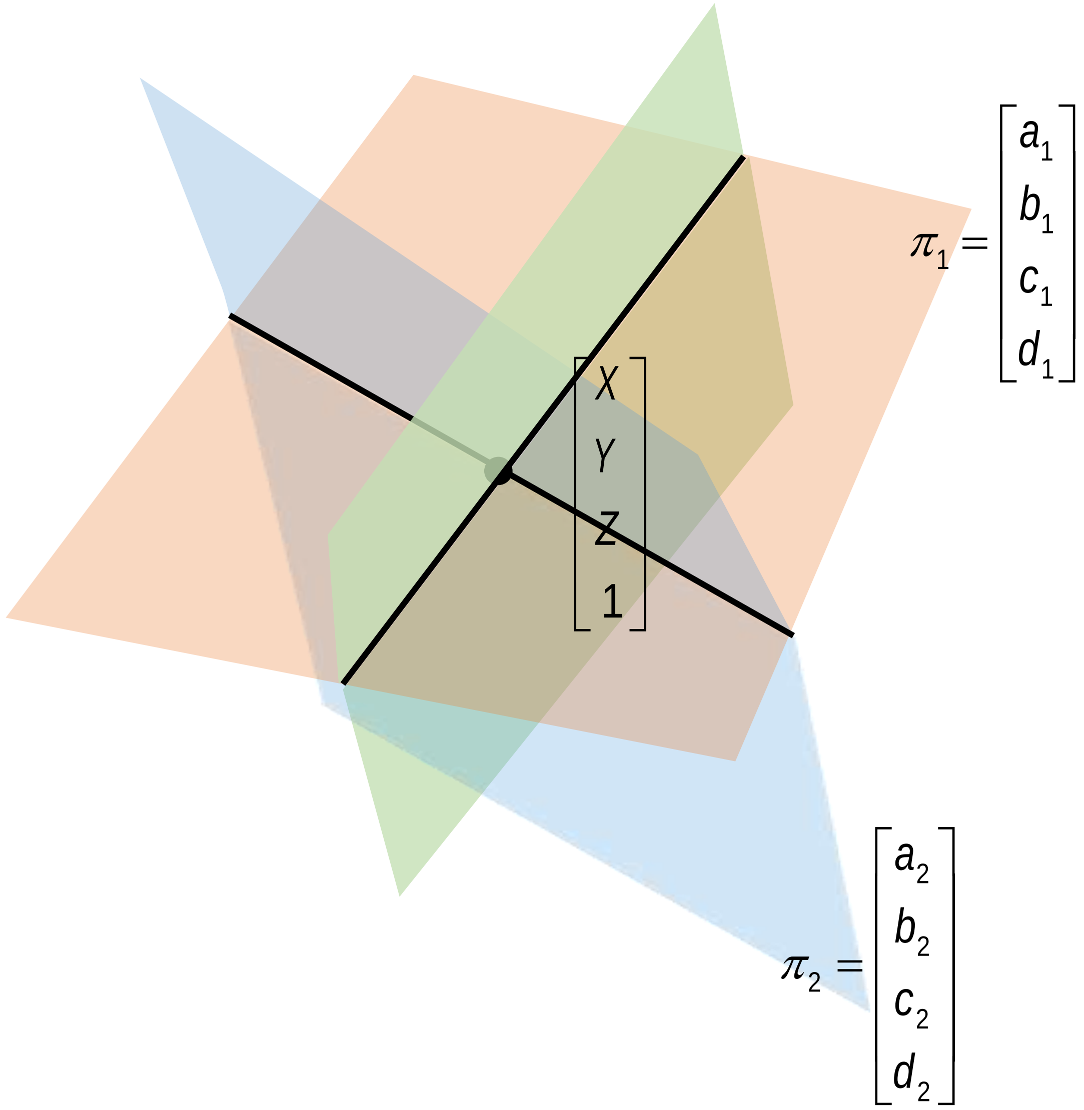
Plane equation:

$$a_1 X + b_1 Y + c_1 Z + d_1 = 0$$

$$a_2 X + b_2 Y + c_2 Z + d_2 = 0$$

$$a_3 X + b_3 Y + c_3 Z + d_3 = 0$$

Plane-Point Duality



Plane equation:

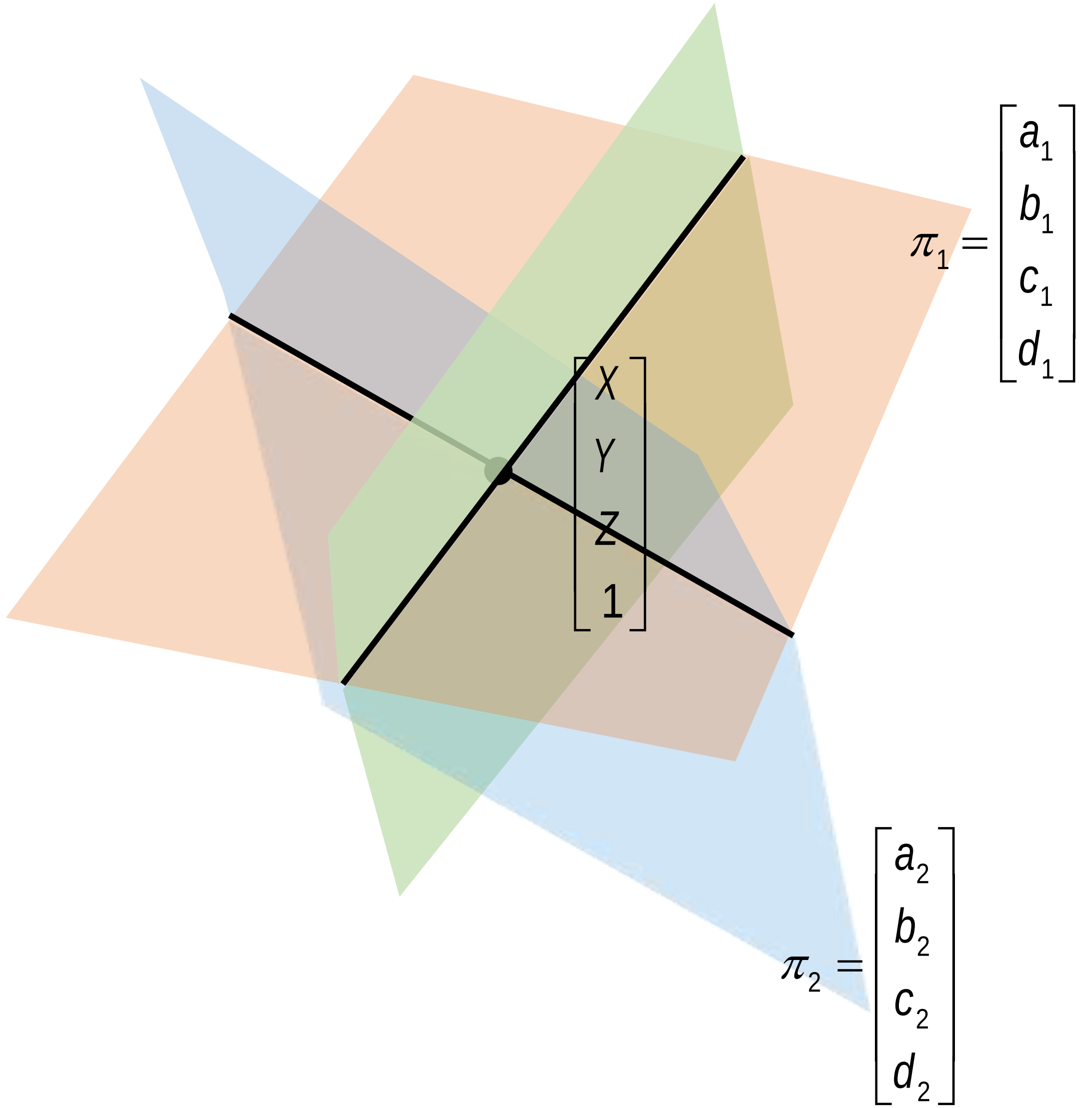
$$a_1X + b_1Y + c_1Z + d_1 = 0$$

$$a_2X + b_2Y + c_2Z + d_2 = 0$$

$$a_3X + b_3Y + c_3Z + d_3 = 0$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Plane-Point Duality



Plane equation:

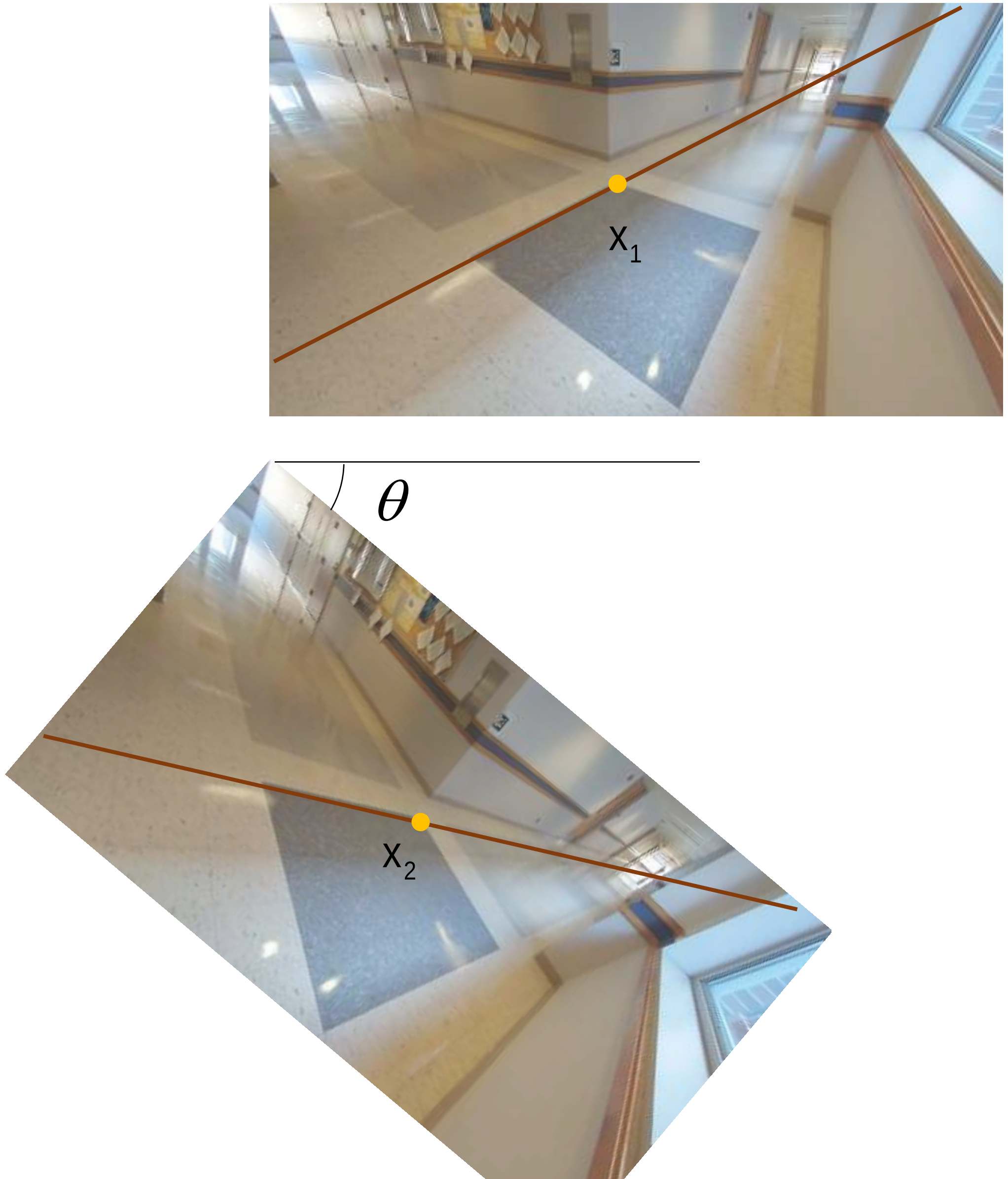
$$a_1X + b_1Y + c_1Z + d_1 = 0$$

$$a_2X + b_2Y + c_2Z + d_2 = 0$$

$$a_3X + b_3Y + c_3Z + d_3 = 0$$

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}}_{\mathrm{A}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\mathrm{X}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathrm{0}}$$

Recall: 2D Point and Line Duality



The 2D line joining two points:

$$l = x_1 \times x_2$$

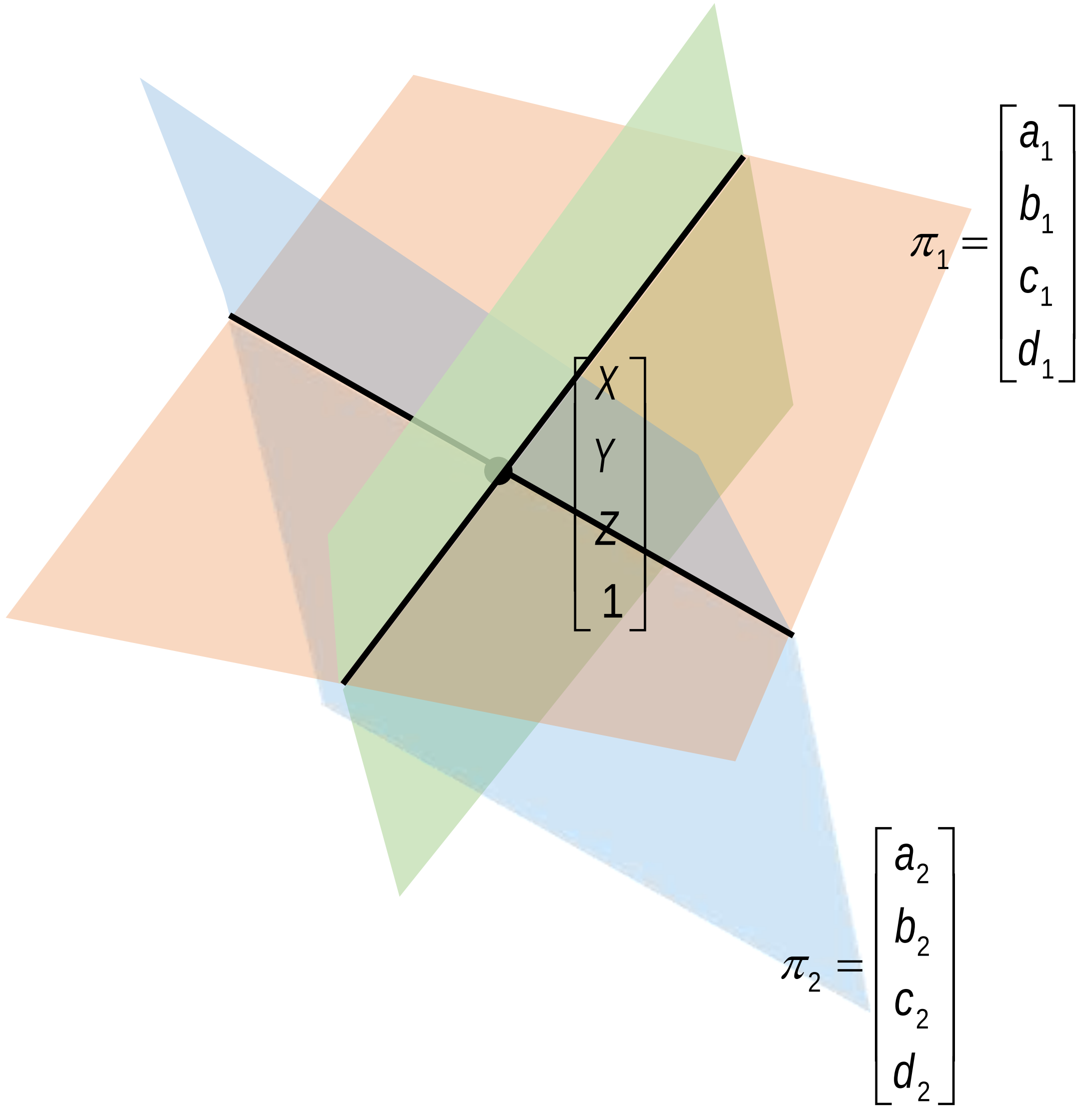
The intersection between two lines:

$$x = l_1 \times l_2$$

Given any formula, we can switch the meaning of point and line to get another formula.

$$x_2 = Tx_1 \leftrightarrow l_2 = T^{-T} l_1 \qquad \text{T: Transformation}$$

Plane-Point Duality

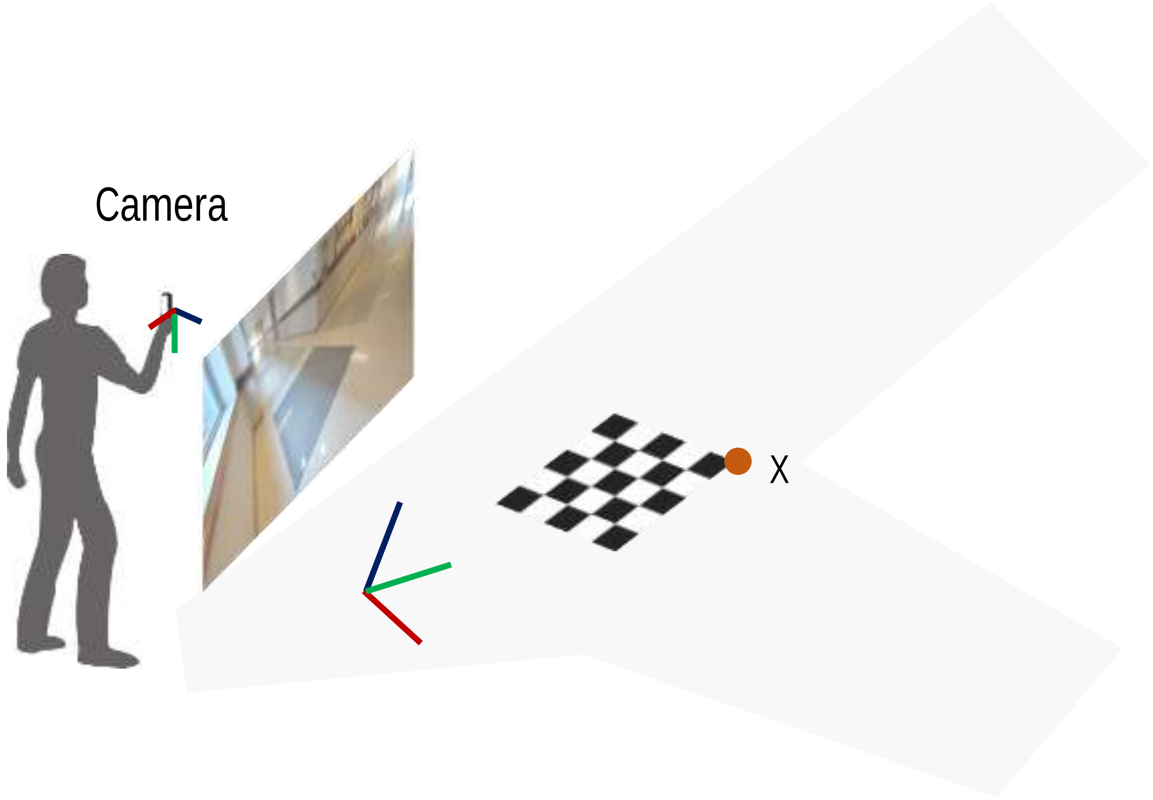


$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Given any formula, we can switch the meaning of point and plane to get another formula.

Plane Representation



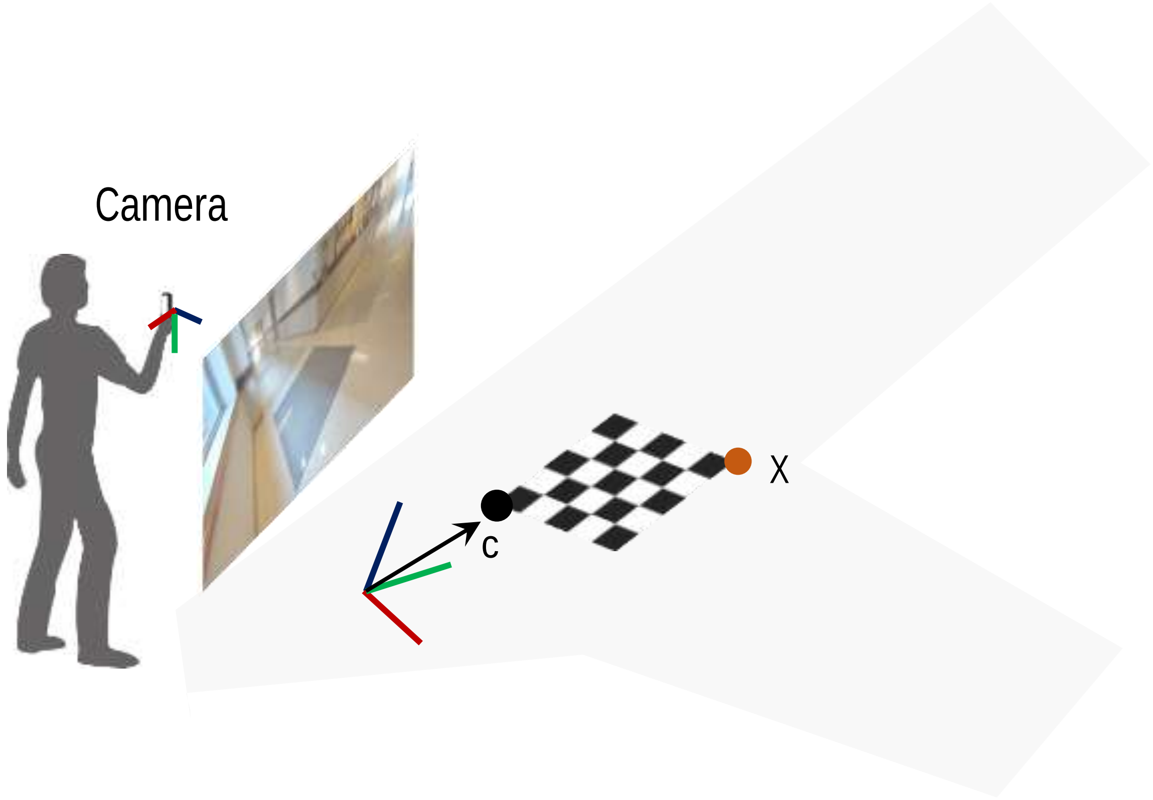
How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3DOF

Plane Representation



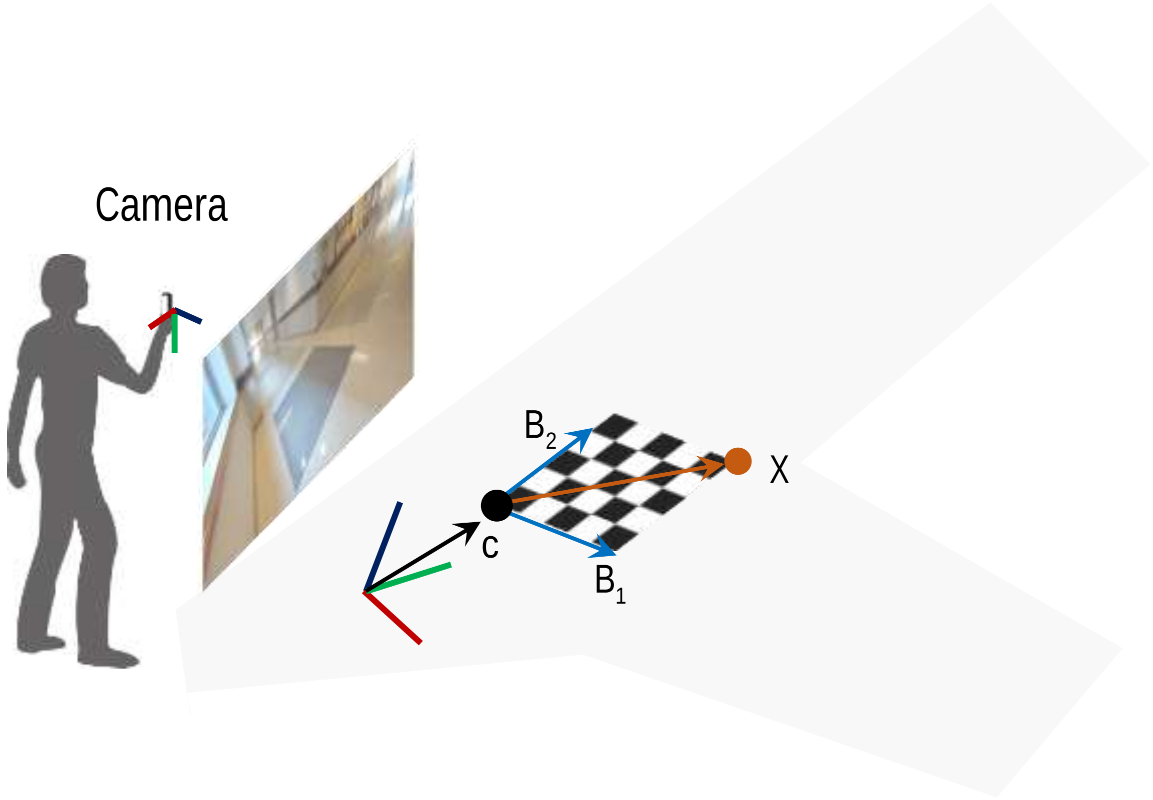
How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{c} +$$

3DOF

Plane Representation



How to parametrize a point in the plane?

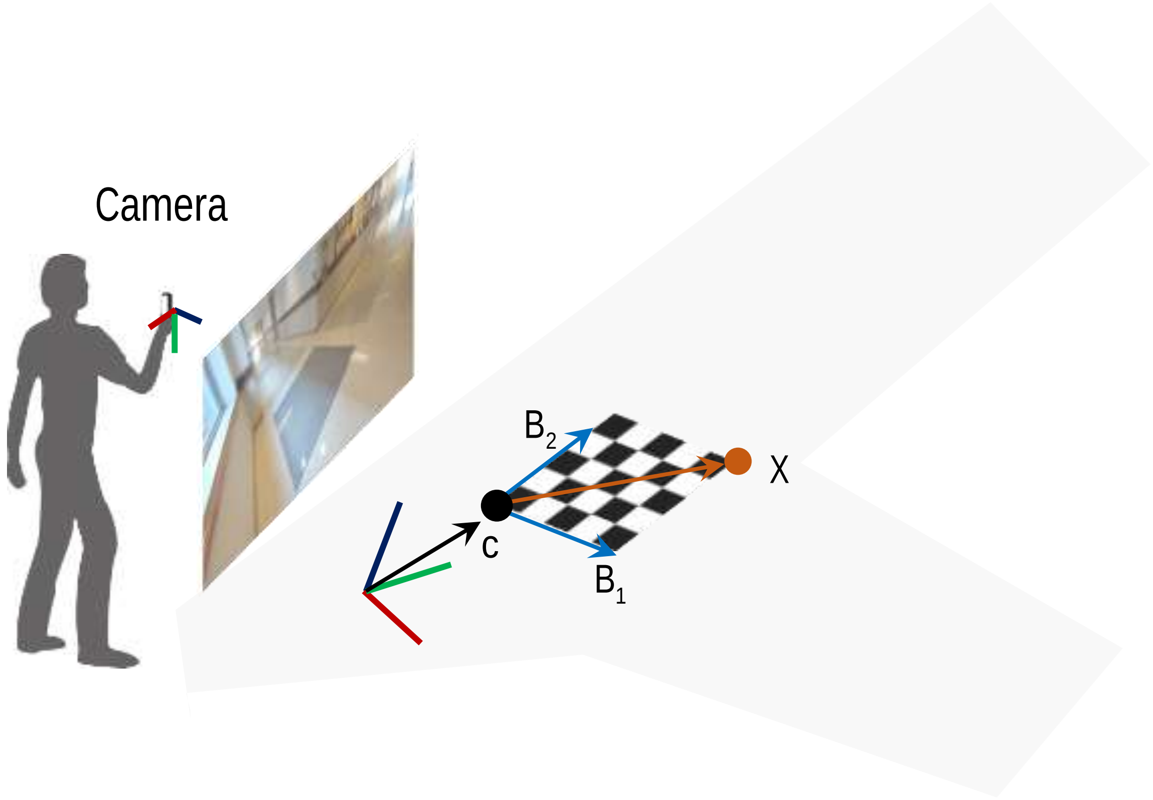
$$aX + bY + cZ + d = 0$$

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{c} + \mu_1 \mathrm{B}_1 + \mu_2 \mathrm{B}_2$$

3DOF

Basis

Plane Representation



How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

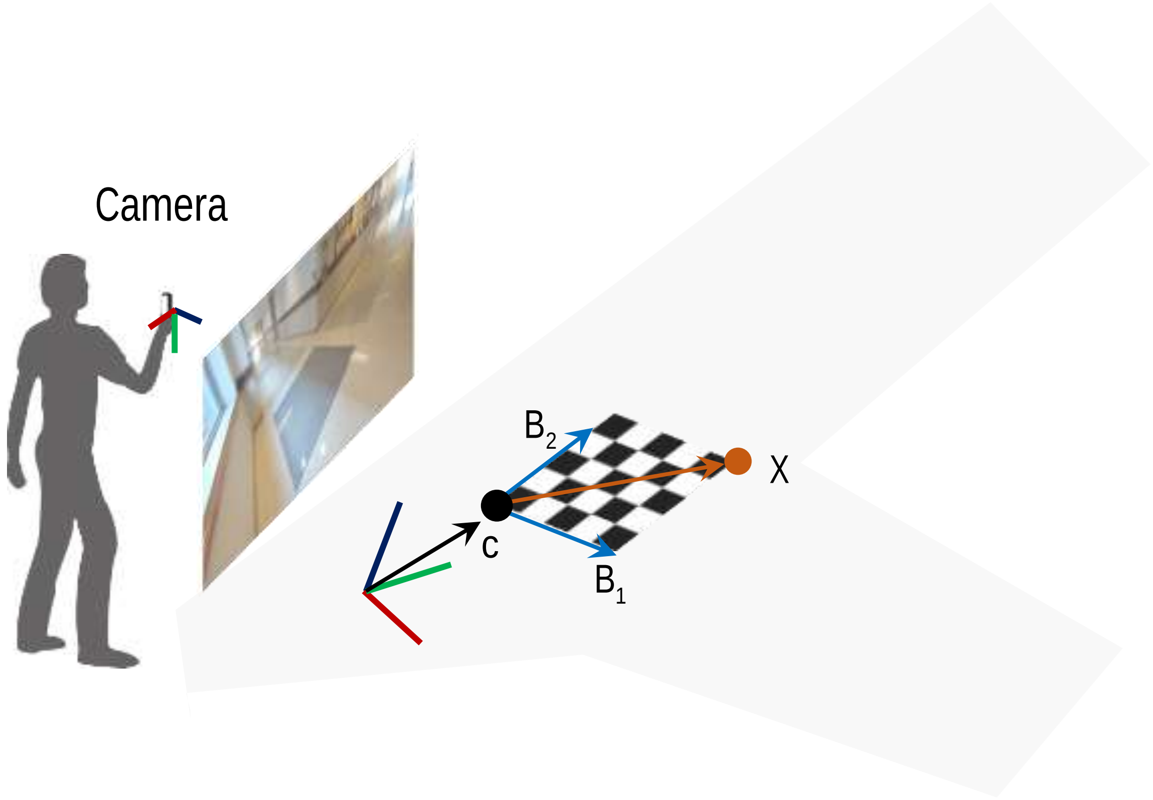
$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{c} + \mu_1 \mathrm{B}_1 + \mu_2 \mathrm{B}_2 = \begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \mathrm{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Basis

3DOF

2DOF

Plane Representation



How to parametrize a point in the plane?

$$aX + bY + cZ + d = 0$$

$$\mathrm{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathrm{c} + \mu_1 \mathrm{B}_1 + \mu_2 \mathrm{B}_2 = \begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \mathrm{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Basis

3DOF

2DOF

Plane projection:

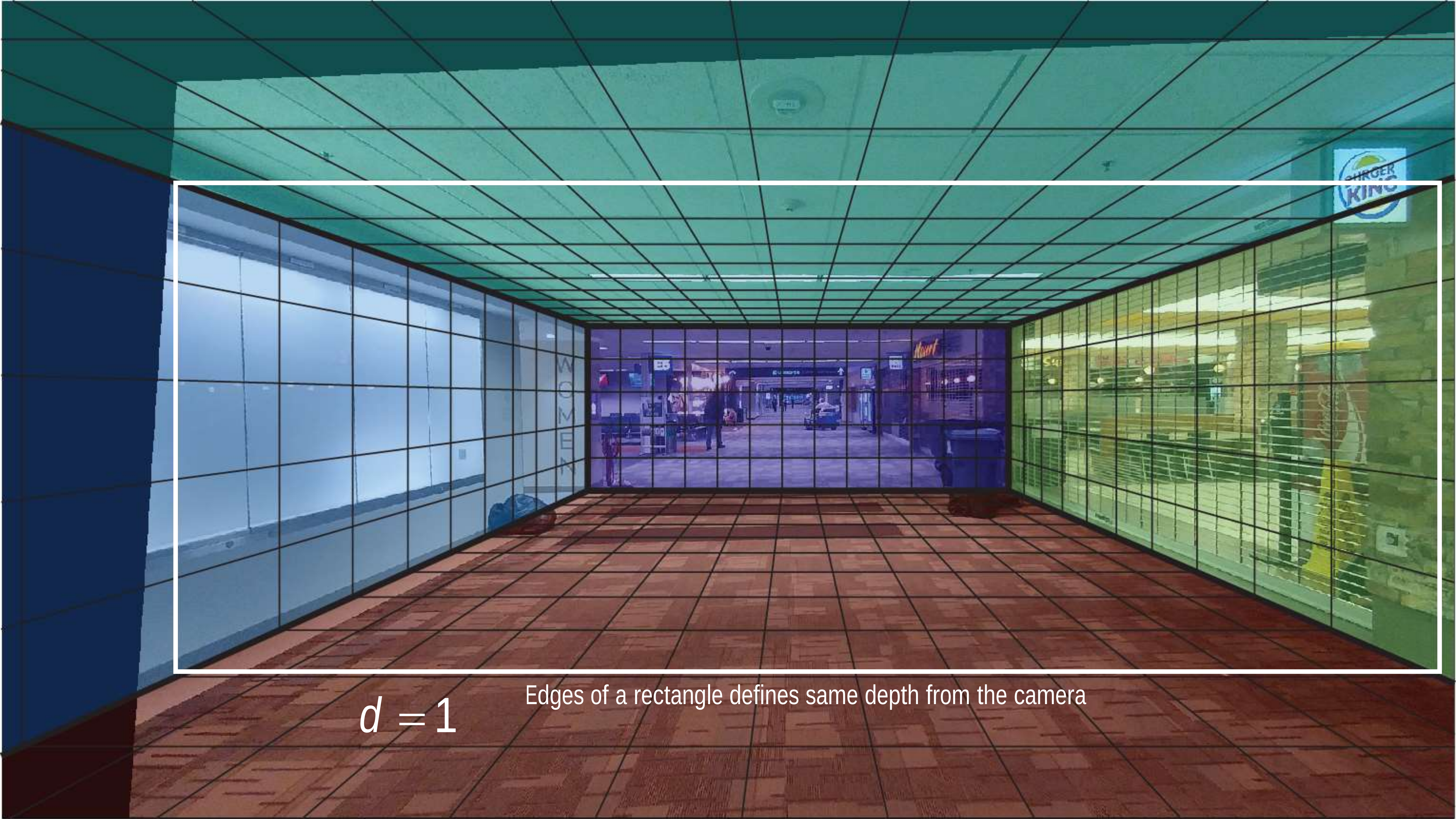
$$\lambda \mathrm{u} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathrm{K}\begin{bmatrix} \mathrm{R} & \mathrm{t} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \mathrm{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} \\ 1 \end{bmatrix}$$

$$= \mathrm{K}\begin{bmatrix} \mathrm{R}\mathrm{B}_1 & \mathrm{R}\mathrm{B}_2 & \mathrm{R}\mathrm{c} + \mathrm{t} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

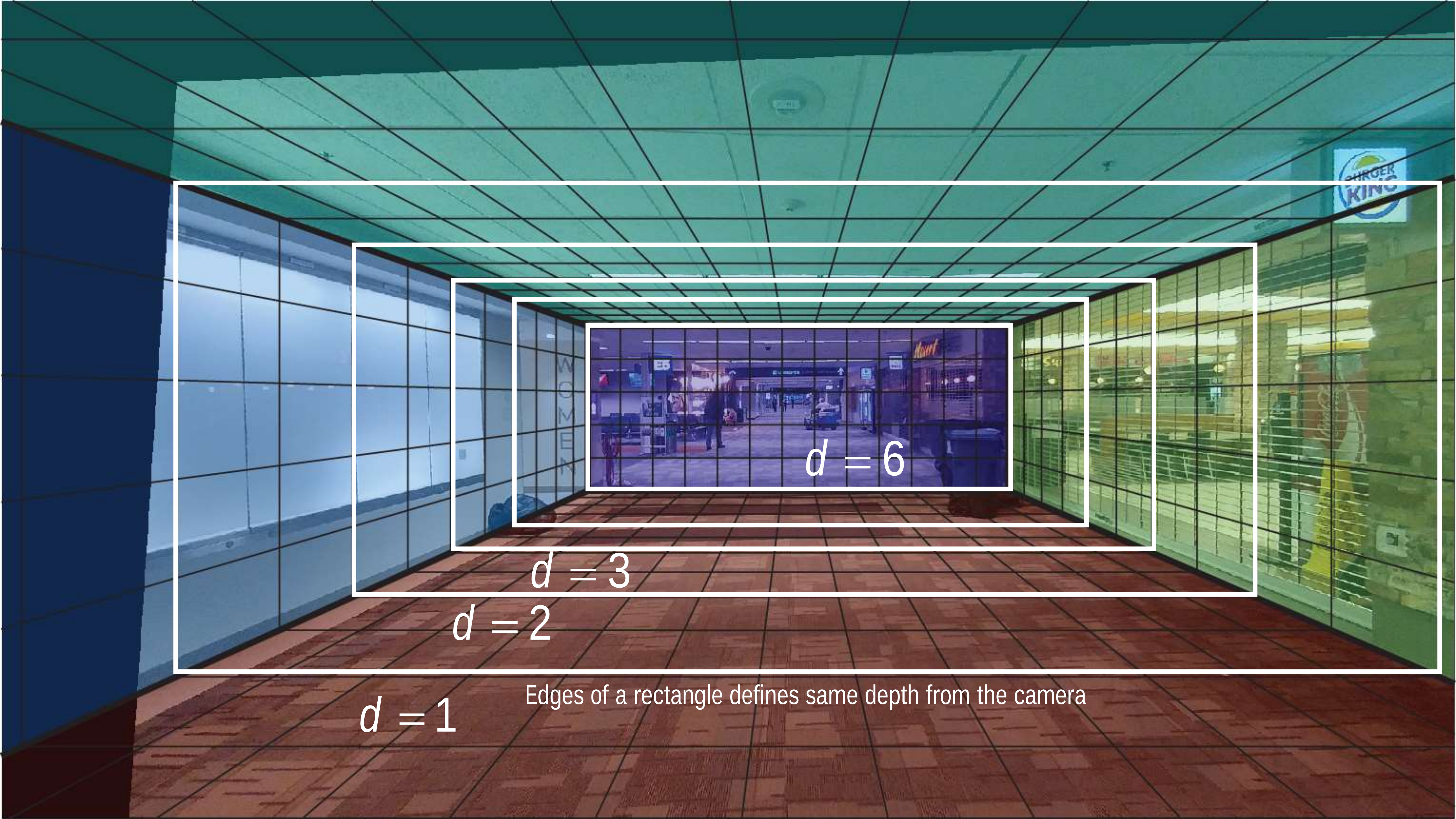

WOMEN
E5
BURGER KING

HW #3 Tour into your photo


WOMEN
BURGER KING

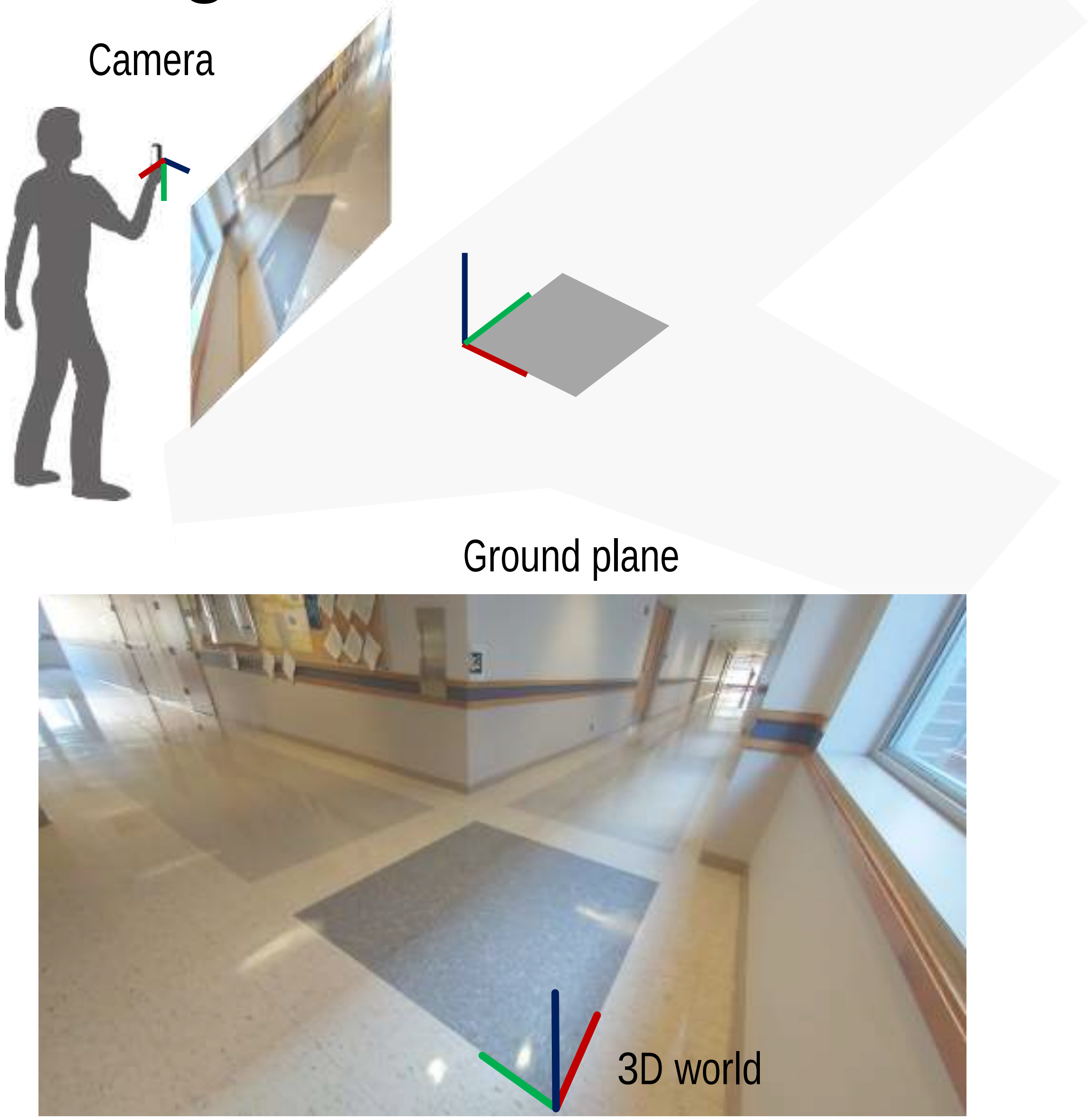


Edges of a rectangle defines same depth from the camera



$d = 6$
$d = 3$
$d = 2$
$d = 1$
Edges of a rectangle defines same depth from the camera

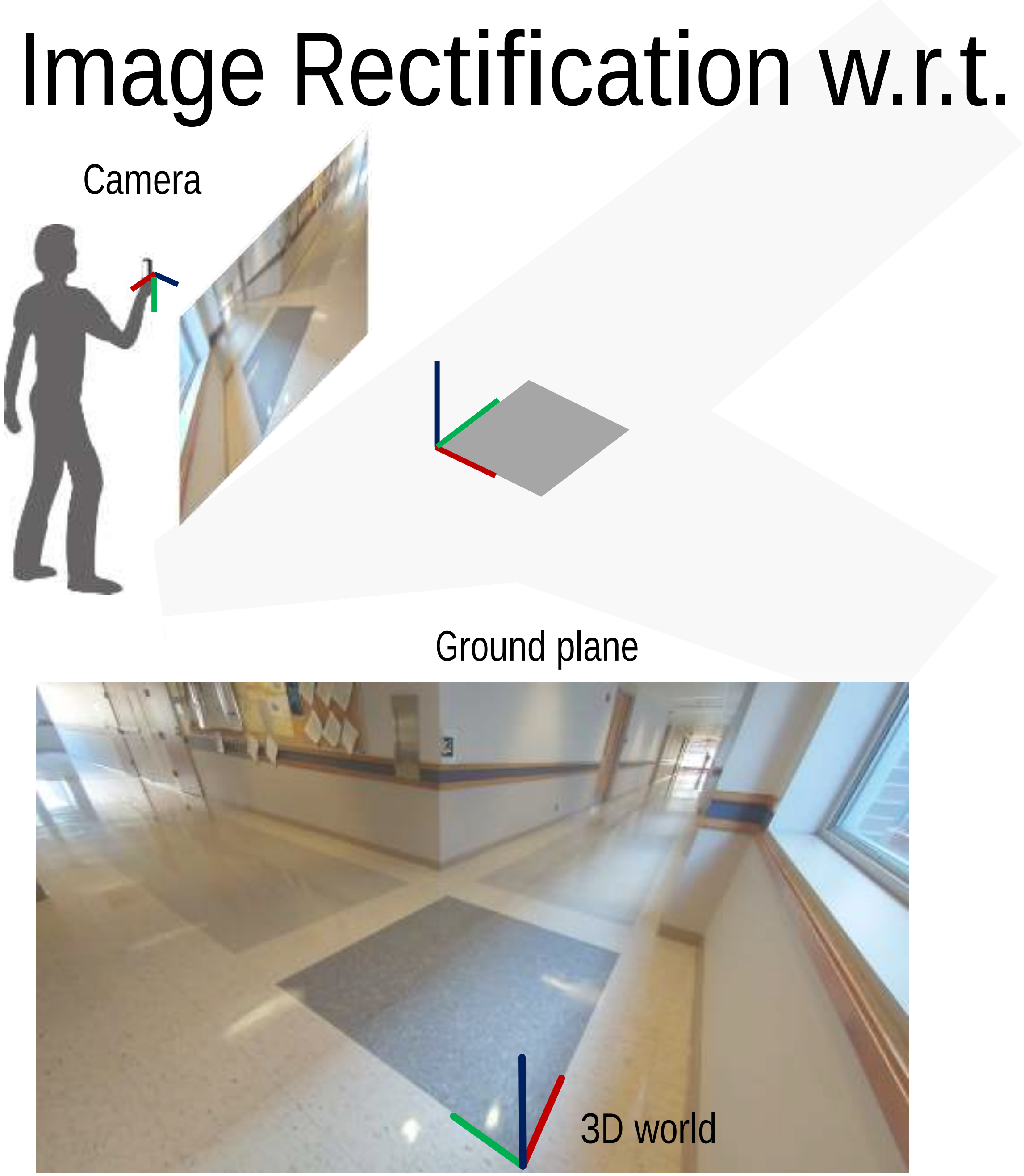
Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

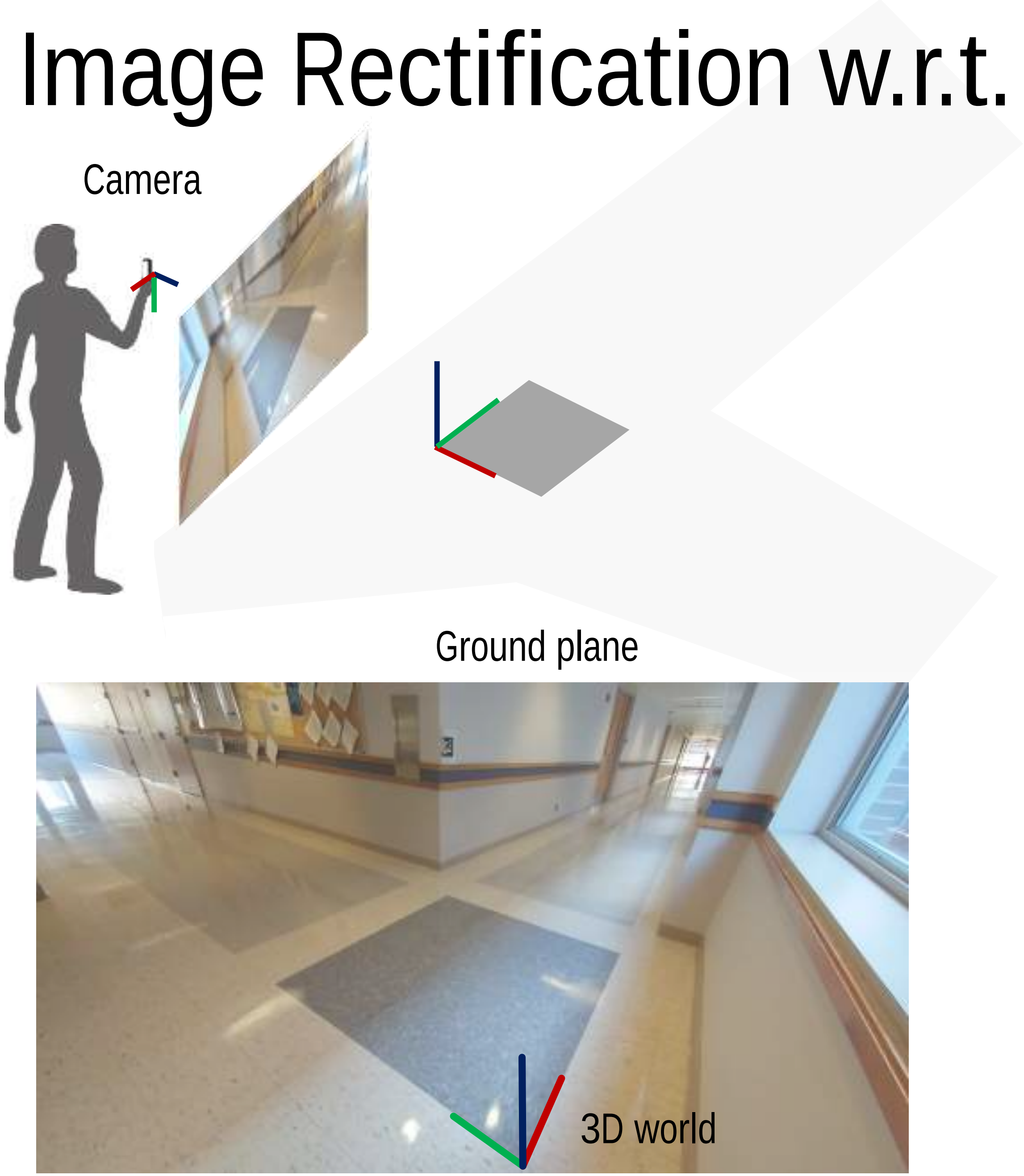
→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathrm{r}_3 = \mathrm{r}_1 \times \mathrm{r}_2$$

Camera pose from homography

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

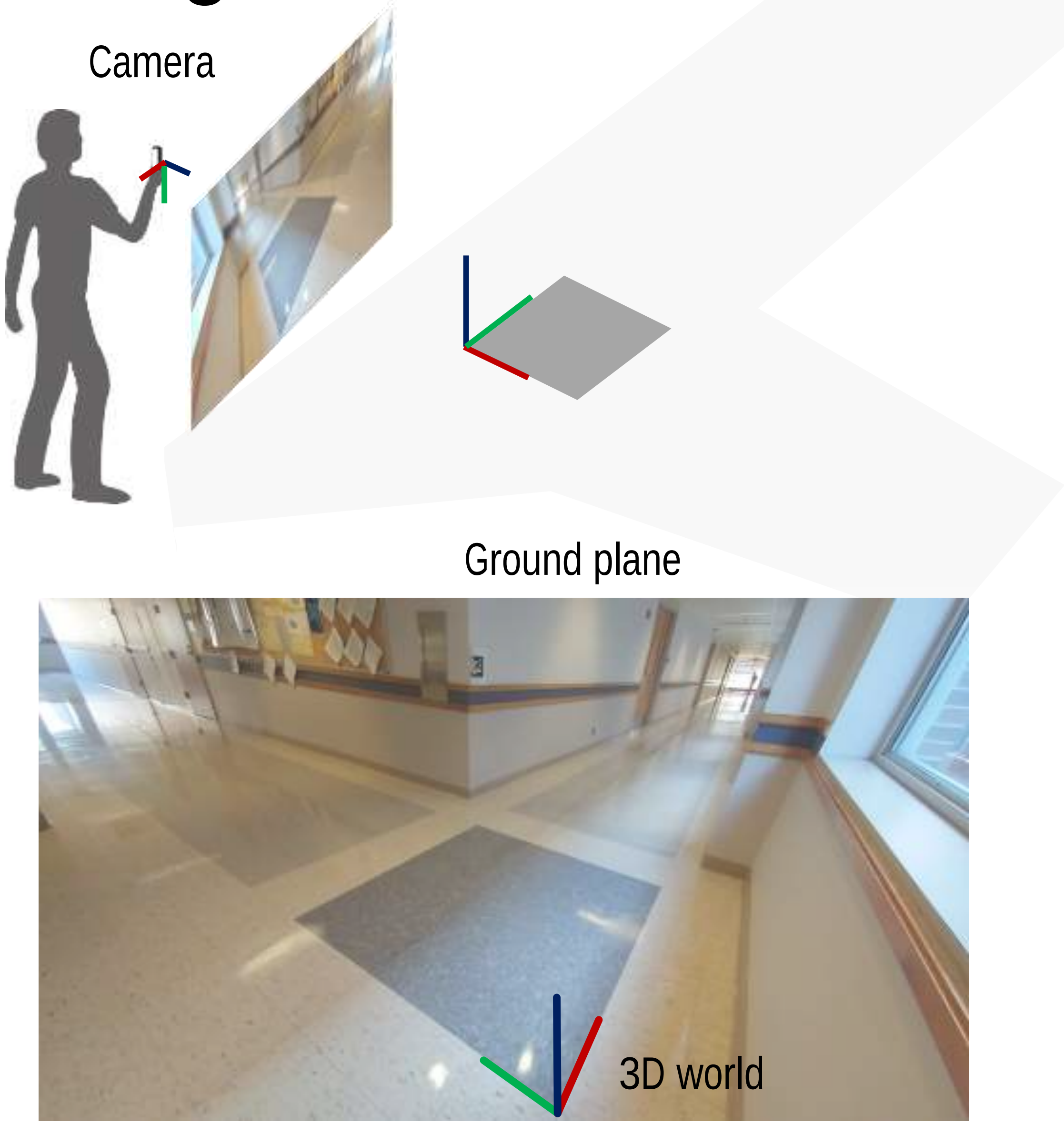
$$\mathrm{r}_3 = \mathrm{r}_1 \times \mathrm{r}_2$$

Camera pose from homography

$$\mathrm{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Image rotation

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathrm{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

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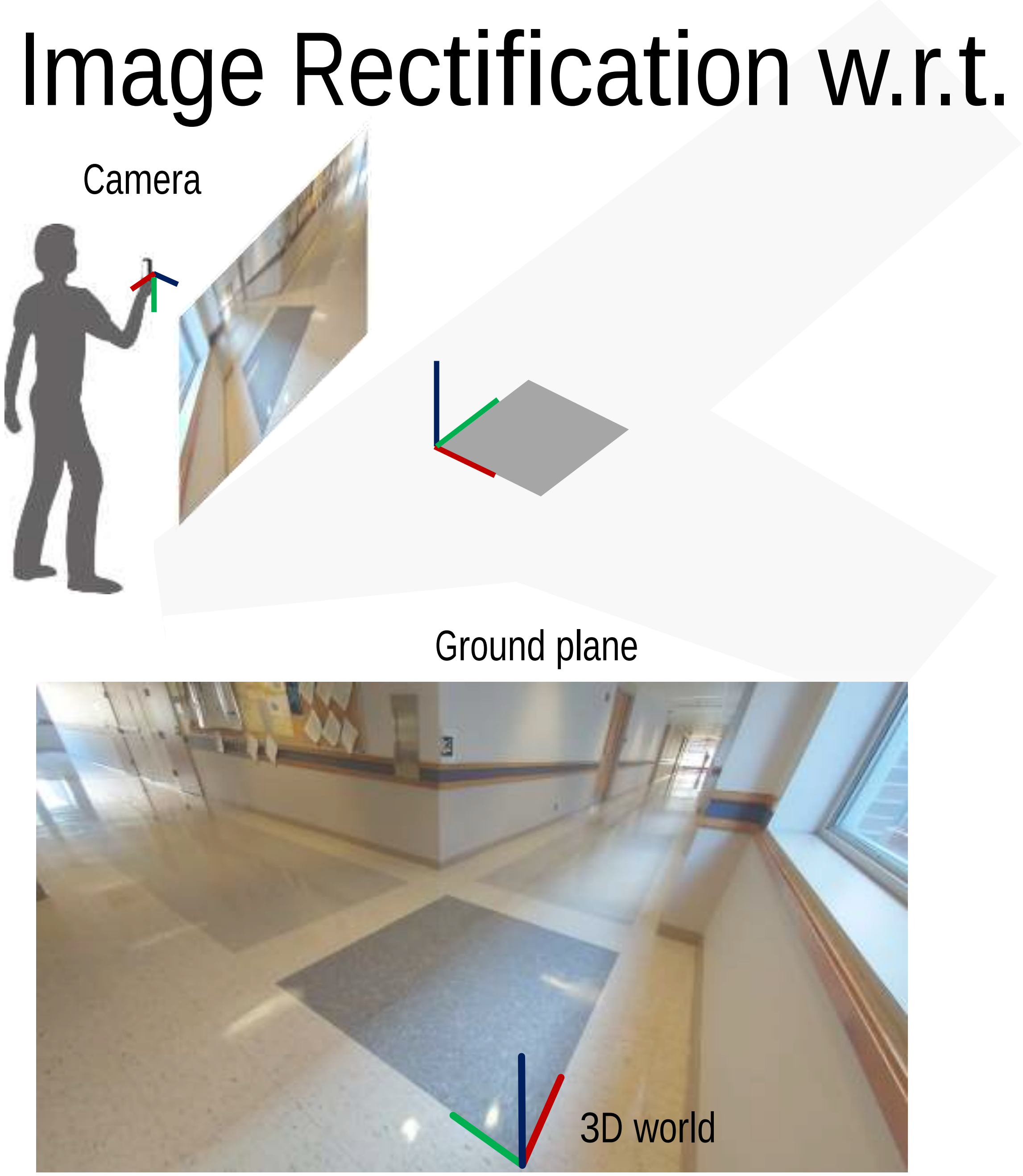
Camera pose from homography

$$\mathrm{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathrm{R}} = \begin{bmatrix} & \tilde{\mathrm{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathrm{r}}_z & \end{bmatrix}$$

Image rotation   Rectified rotation

Image Rectification w.r.t. Ground Plane



$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{R} = \begin{bmatrix} & \tilde{r}_x & \\ 0 & 0 & -1 \\ & \tilde{r}_z & \end{bmatrix}$$

Image rotation

Rectified rotation

$$\tilde{r}_x = \frac{r_x - \left(r_x \cdot \tilde{r}_y\right)\tilde{r}_y}{\left\| r_x - \left(r_x \cdot \tilde{r}_y\right)\tilde{r}_y \right\|}$$

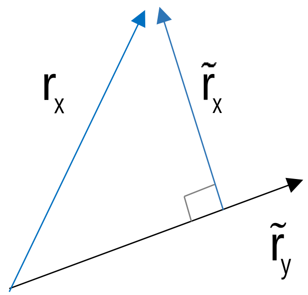
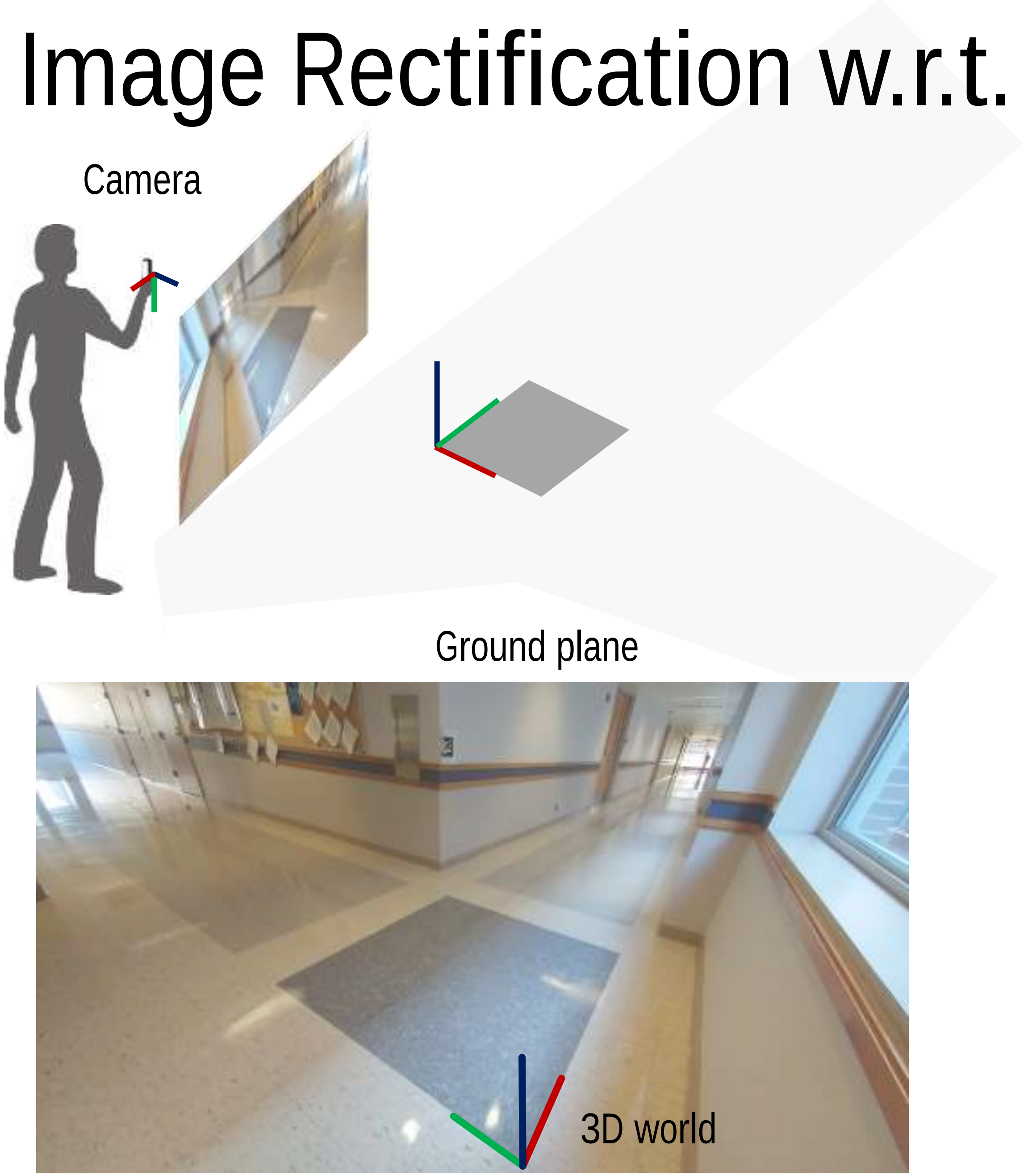


Image Rectification w.r.t. Ground Plane



$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{R} = \begin{bmatrix} & \tilde{r}_x & \\ 0 & 0 & -1 \\ & \tilde{r}_z & \end{bmatrix}$$

Image rotation

Rectified rotation

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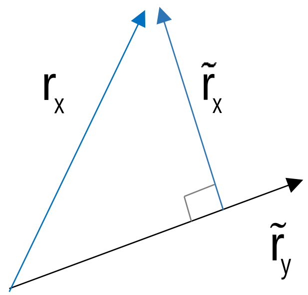
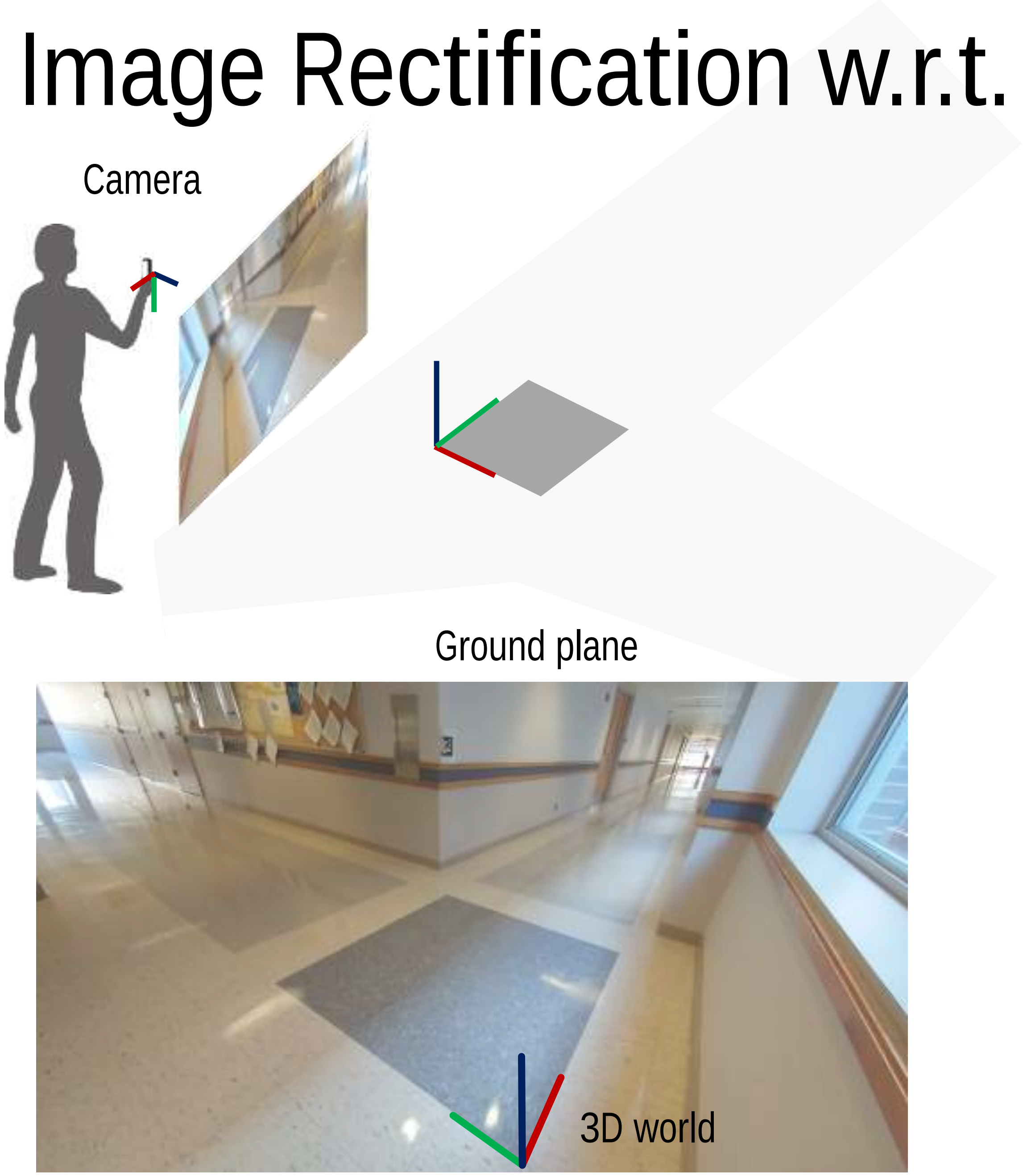


$$\tilde{r}_z = \tilde{r}_x \times \tilde{r}_y$$

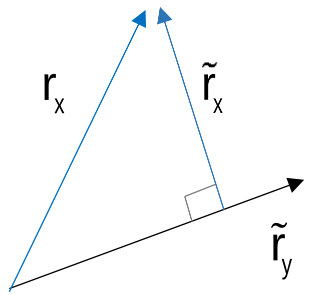
Image Rectification w.r.t. Ground Plane



$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{R} = \begin{bmatrix} & \tilde{r}_x & \\ 0 & 0 & -1 \\ & \tilde{r}_z & \end{bmatrix}$$

Image rotation

Rectified rotation



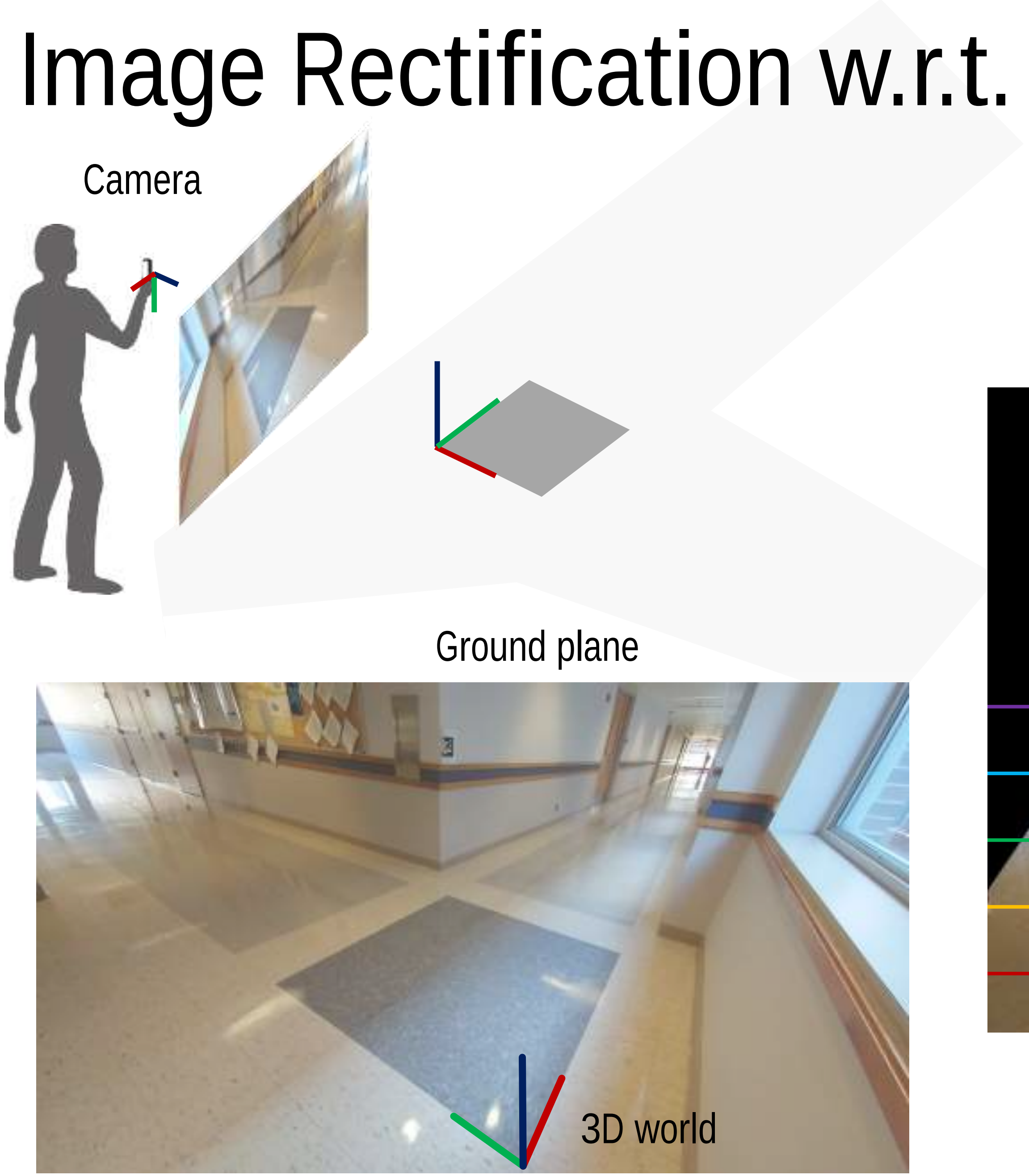
$$\tilde{r}_x = \frac{r_x - \left(r_x \cdot \tilde{r}_y\right)\tilde{r}_y}{\left\| r_x - \left(r_x \cdot \tilde{r}_y\right)\tilde{r}_y \right\|}$$

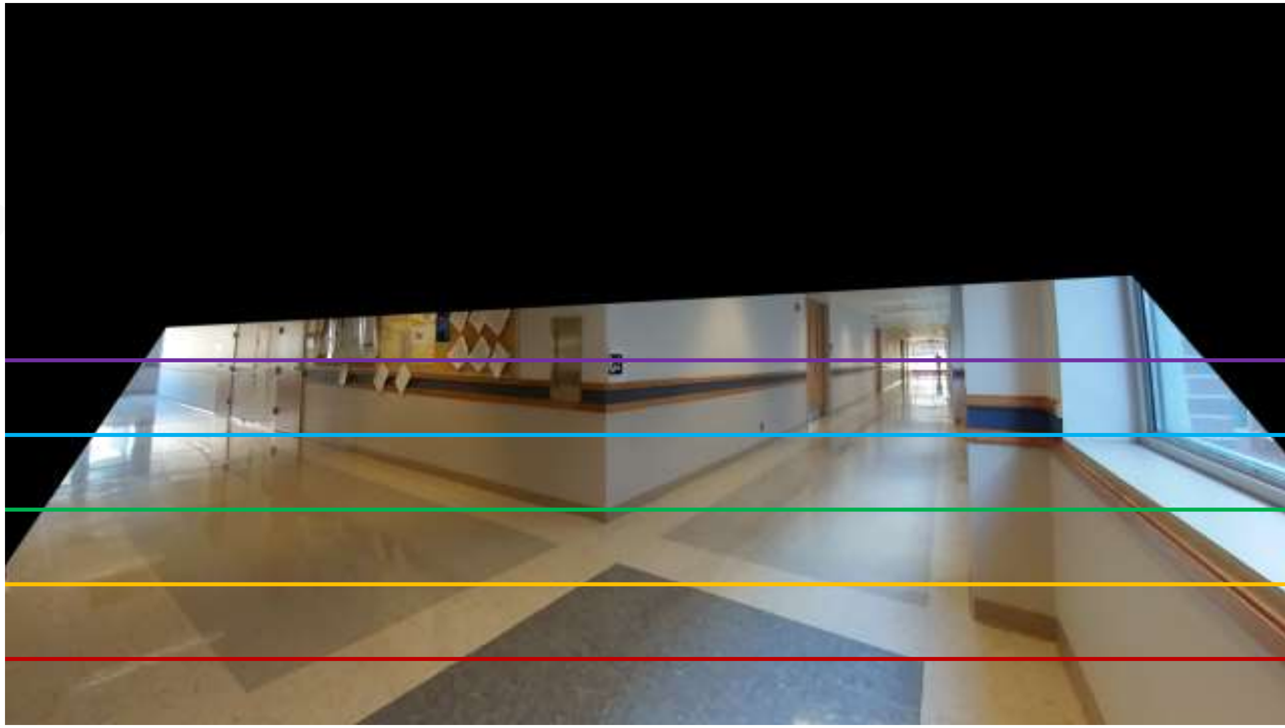
$$\tilde{r}_z = \tilde{r}_x \times \tilde{r}_y$$

$$\lambda \tilde{R}^T K^{-1} \tilde{u} = R^T K^{-1} u \longrightarrow \lambda \tilde{u} = K \tilde{R} R^T K^{-1} u$$

Image Rectification w.r.t. Ground Plane

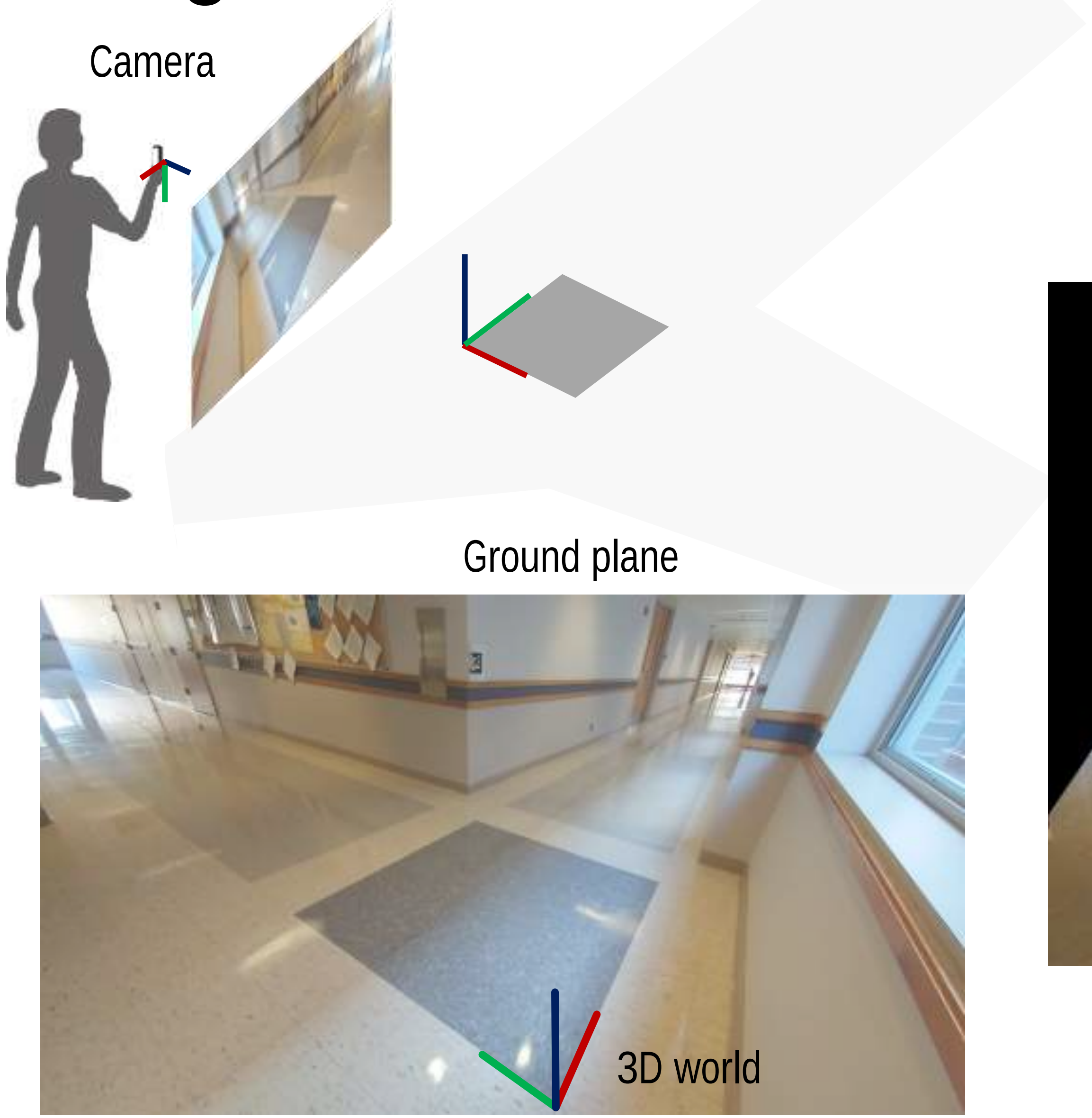
Image Rectification w.r.t. Ground Plane





Same depth

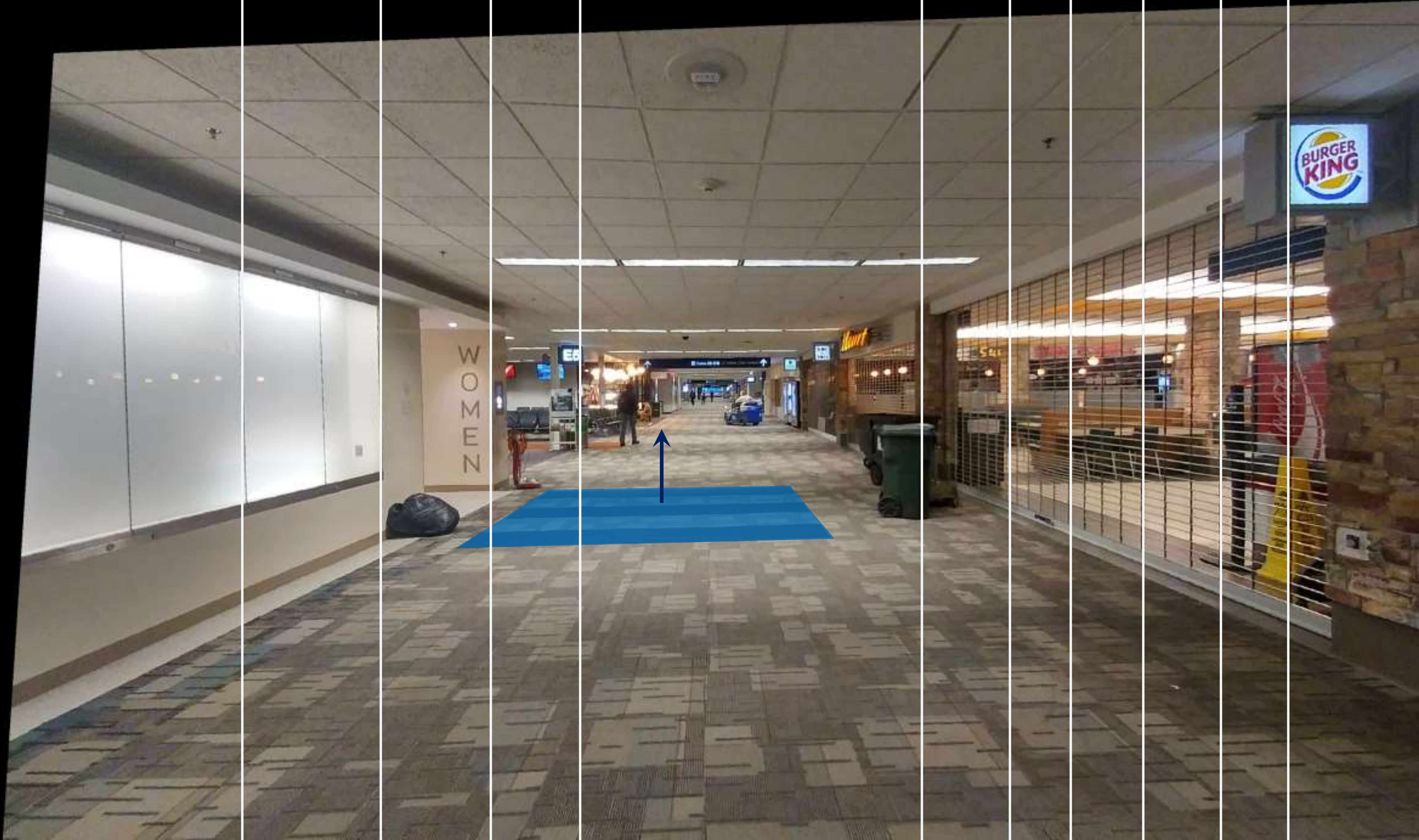
Image Rectification w.r.t. Ground Plane



BURGER KING
WOMEN
E5

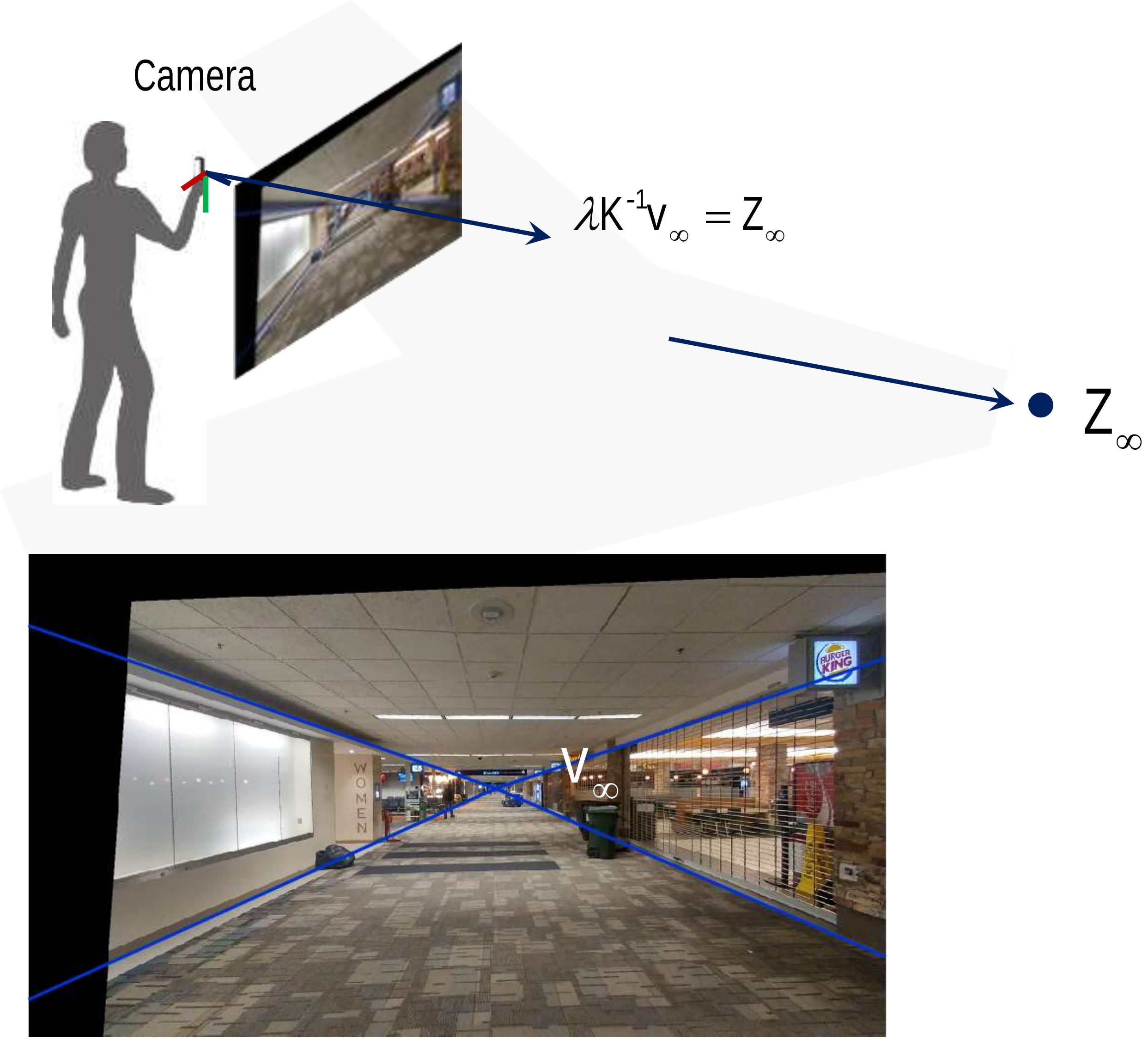
WOMEN
E5
BURGER KING


BURGER KING
WOMEN
E5


WOMEN
E6
BURGER KING


Vanishing point
WOMEN
BURGER KING

Vanishing Point

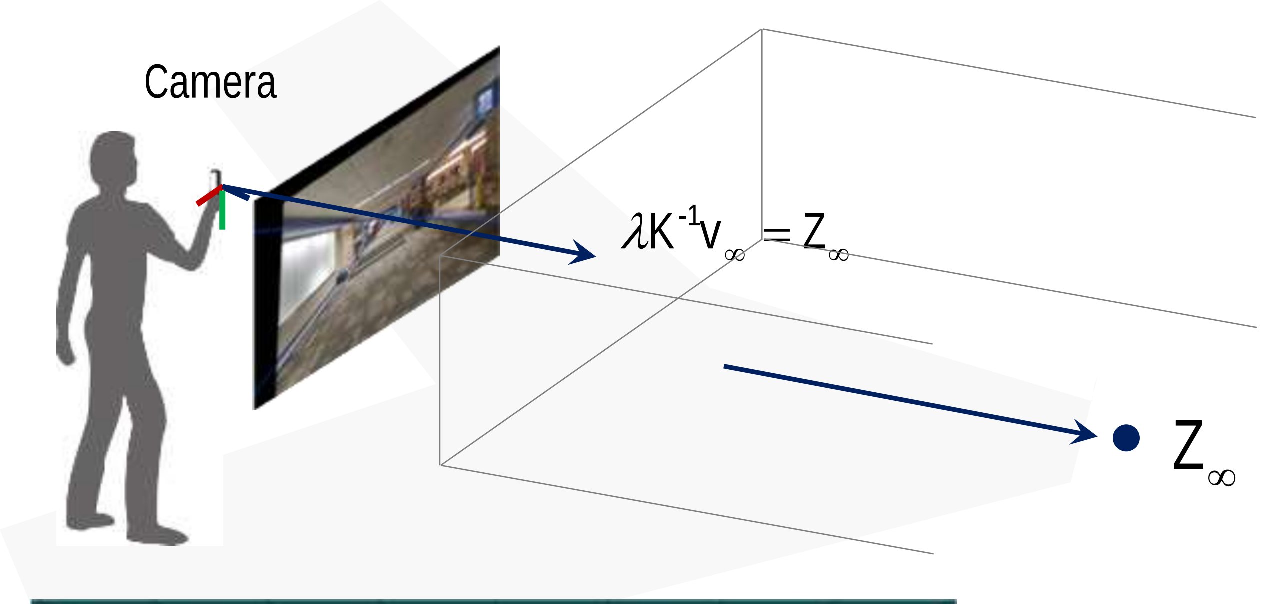


Vanishing point projection:

$$\lambda v_{\infty} = K Z_{\infty}$$

$$\longrightarrow \quad \lambda K^{-1} v_{\infty} = Z_{\infty}$$

Box Representation

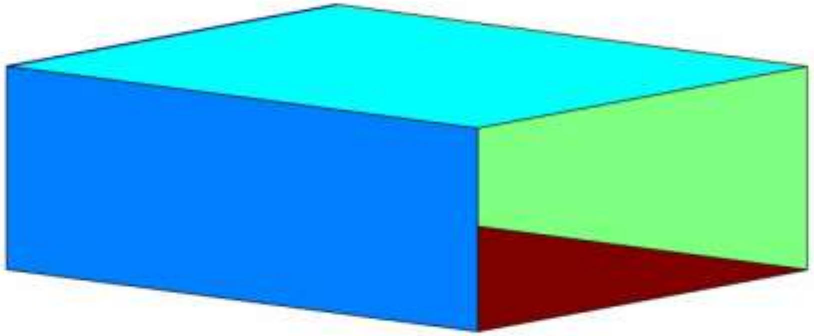


Vanishing point projection:

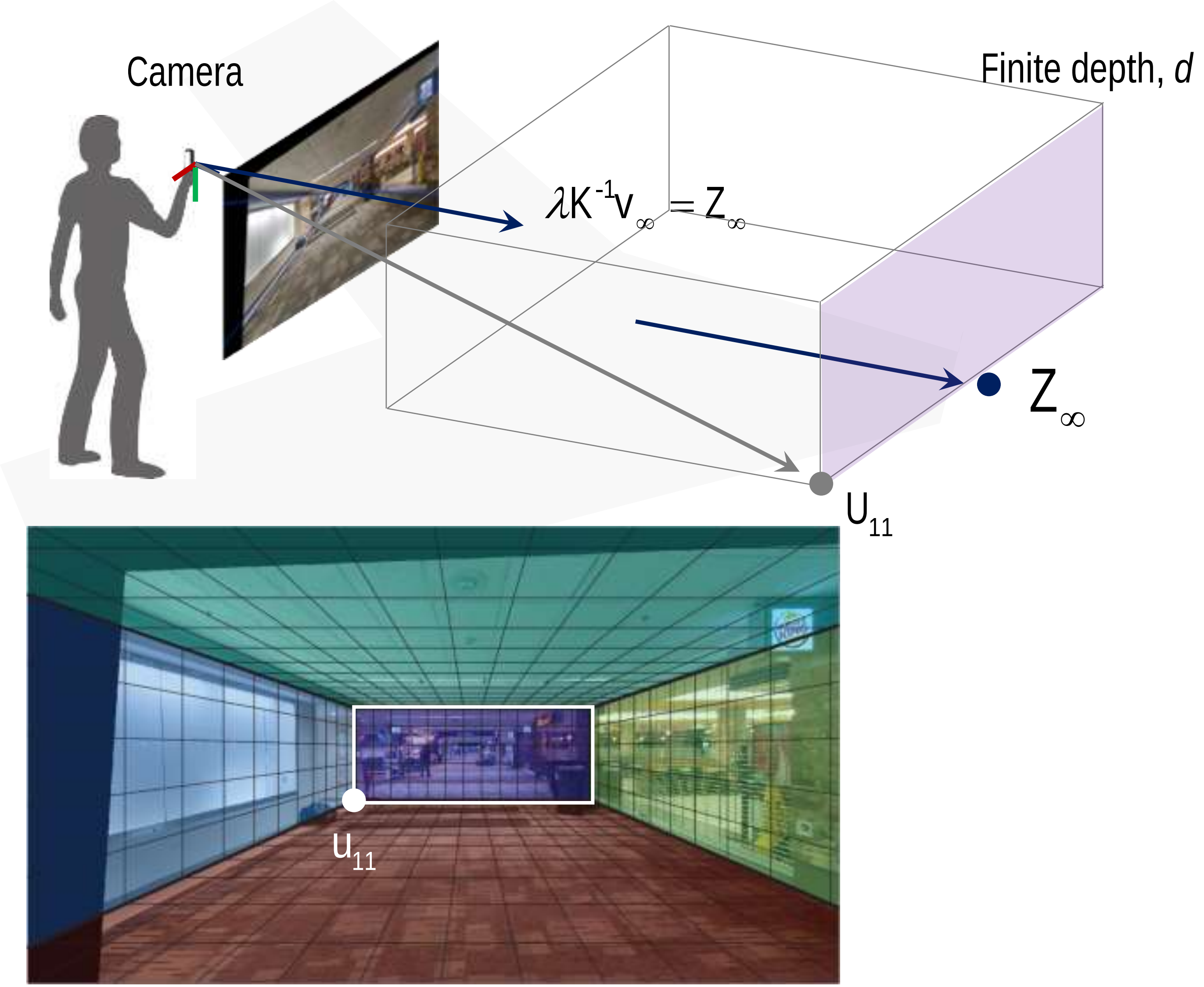
$$\lambda v_\infty = K Z_\infty$$

$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

Define the direction of the box



Box Representation



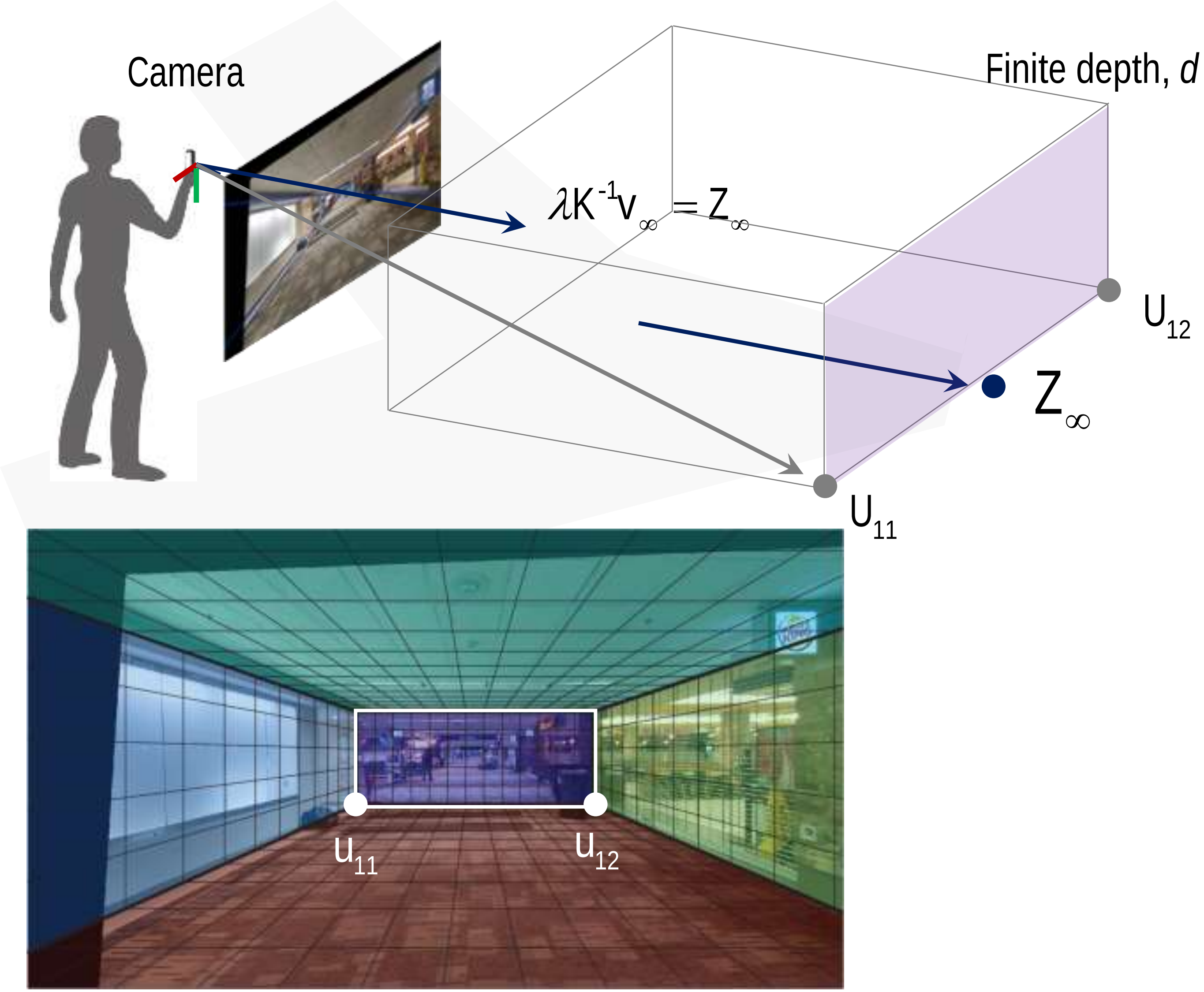
Vanishing point projection:

$$\lambda v_\infty = K Z_\infty$$

$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

$$U_{11} = d K^{-1} u_{11}$$

Box Representation



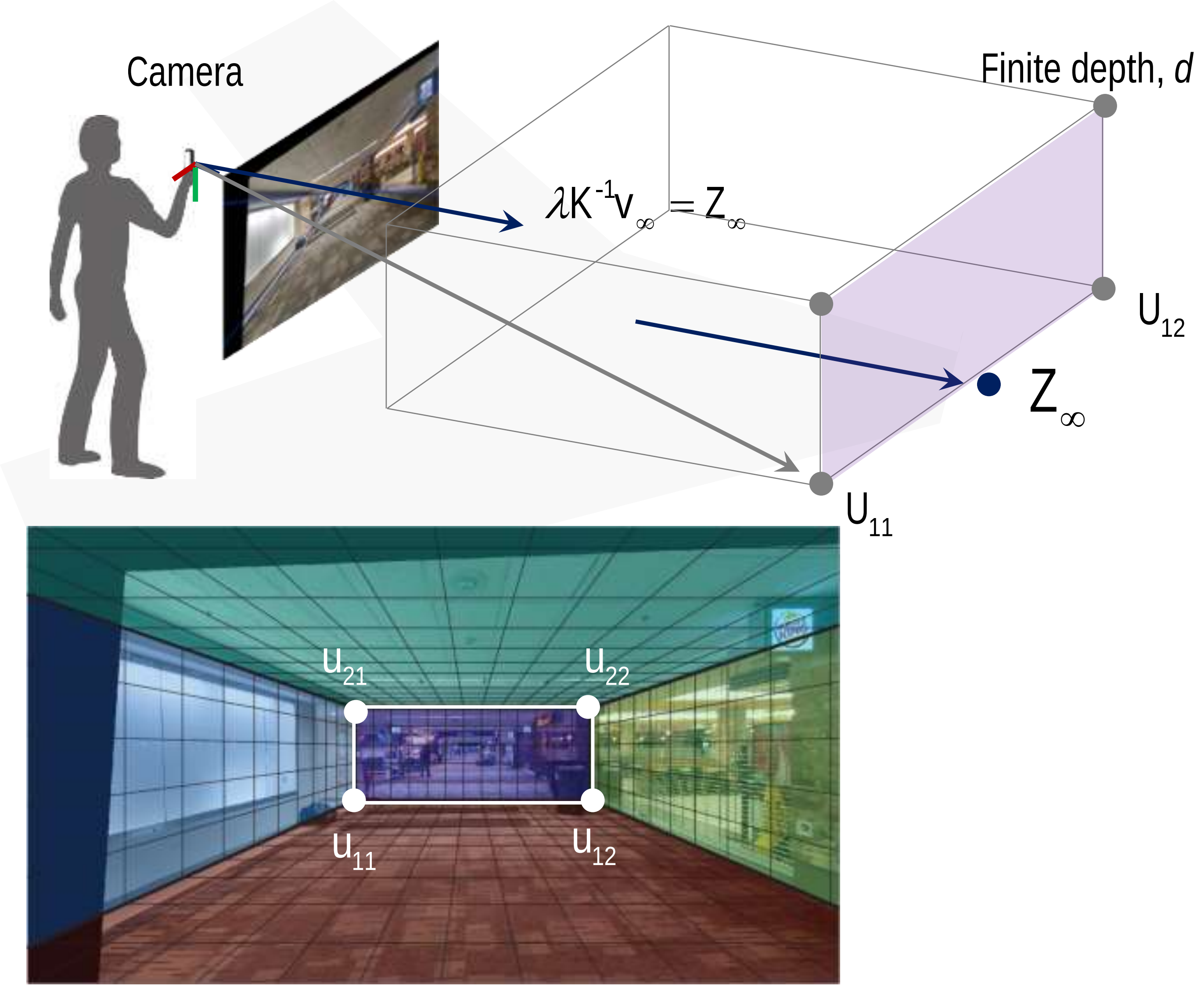
Vanishing point projection:

$$\lambda v_\infty = K Z_\infty$$

$$\longrightarrow \lambda K^{-1} v_\infty = Z_\infty$$

$U_{11} = dK^{-1}u_{11}$ $\quad$ $U_{12} = dK^{-1}u_{12}$ $\quad$: Same x coord.

Box Representation



Vanishing point projection:

$$\lambda v_\infty = K Z_\infty$$

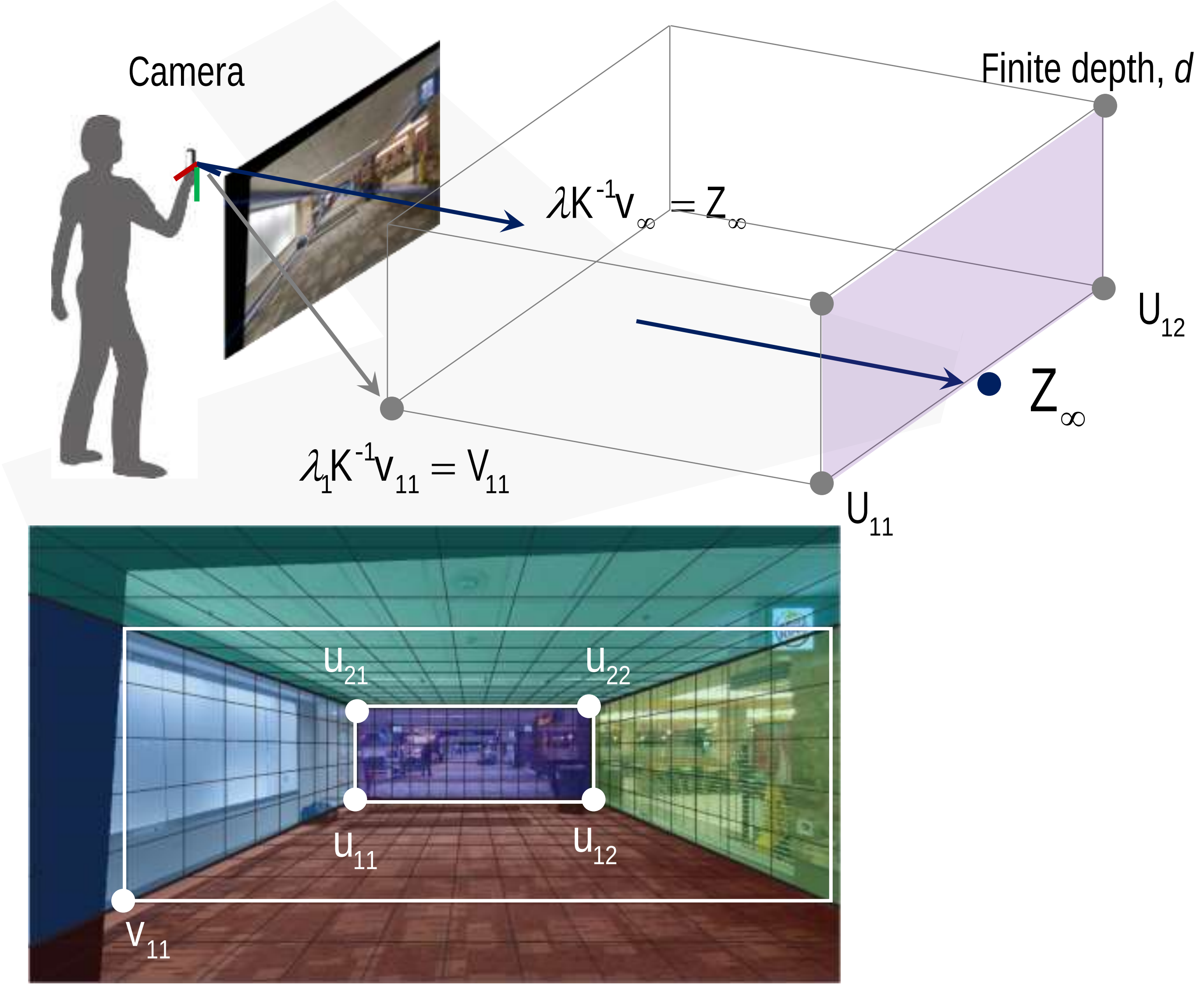
$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

$U_{11} = dK^{-1}\boxed{u_{11}}$ $\quad$ $U_{12} = dK^{-1}u_{12}$ $\quad$: Same x coord.

$U_{21} = dK^{-1}\boxed{u_{21}}$ $\quad$ $U_{22} = dK^{-1}u_{22}$

Same y coord.

Box Representation



Vanishing point projection:

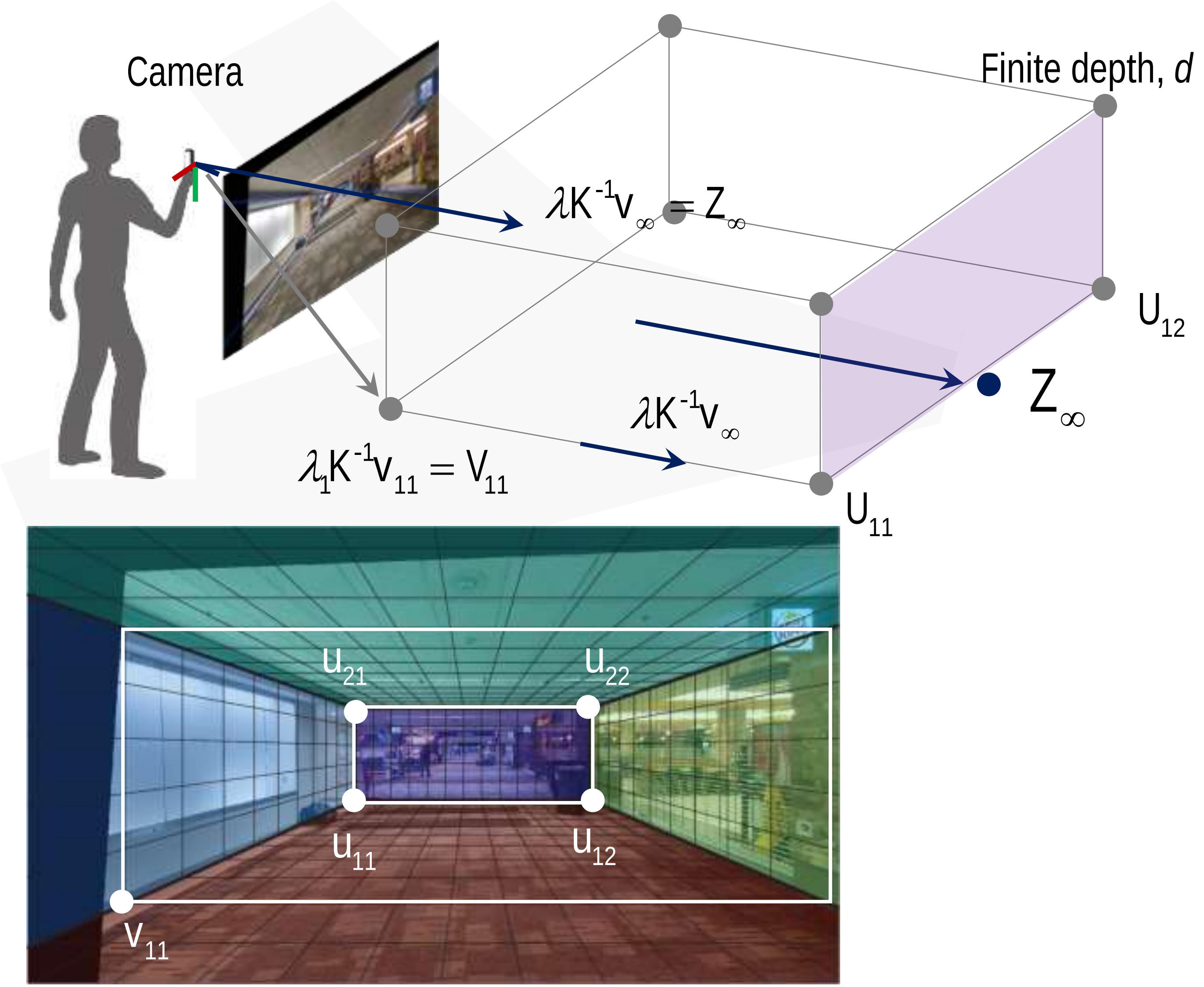
$$\lambda v_\infty = K Z_\infty$$

$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

Depth of frontal surface?

$$\underline{\lambda_1} K^{-1} v_{11} = V_{11}$$

Box Representation



Vanishing point projection:

$$\lambda v_\infty = K Z_\infty$$

$$\longrightarrow \quad \lambda K^{-1} v_\infty = Z_\infty$$

Depth of frontal surface?

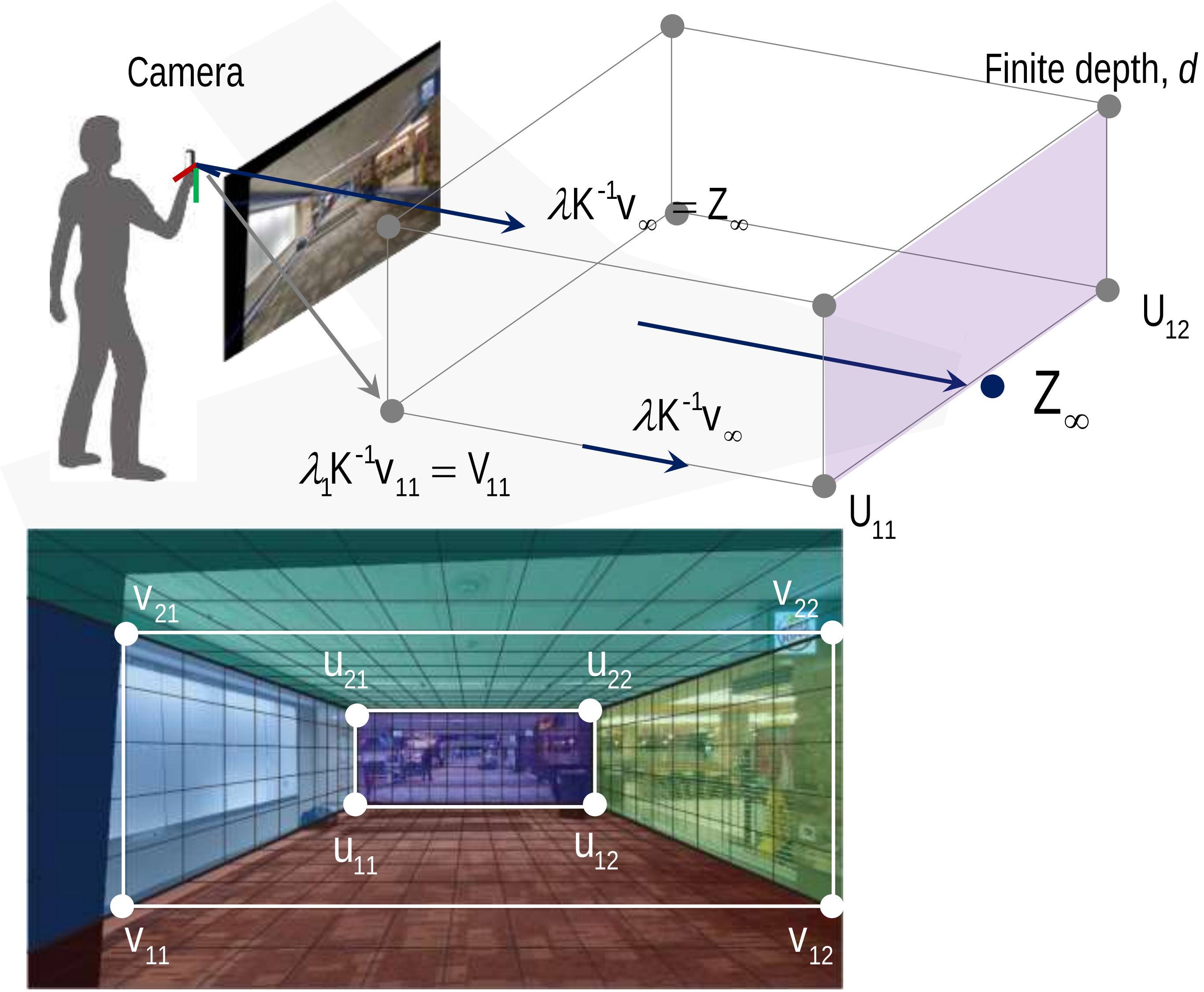
$$\underline{\lambda_1} K^{-1} v_{11} = V_{11}$$

Line between U_{11} and V_{11} is parallel to the vanishing point direction.

$$\lambda_1 K^{-1} v_{11} + \lambda K^{-1} v_\infty = U_{11} = d K^{-1} u_{11}$$

HW: express λ_1 using d.

Box Representation



Vanishing point projection:

$$\lambda v_\infty = KZ_\infty$$

$$\longrightarrow \quad \lambda K^{-1}v_\infty = Z_\infty$$

Depth of frontal surface?

$$\underline{\lambda_1} K^{-1}v_{11} = V_{11}$$

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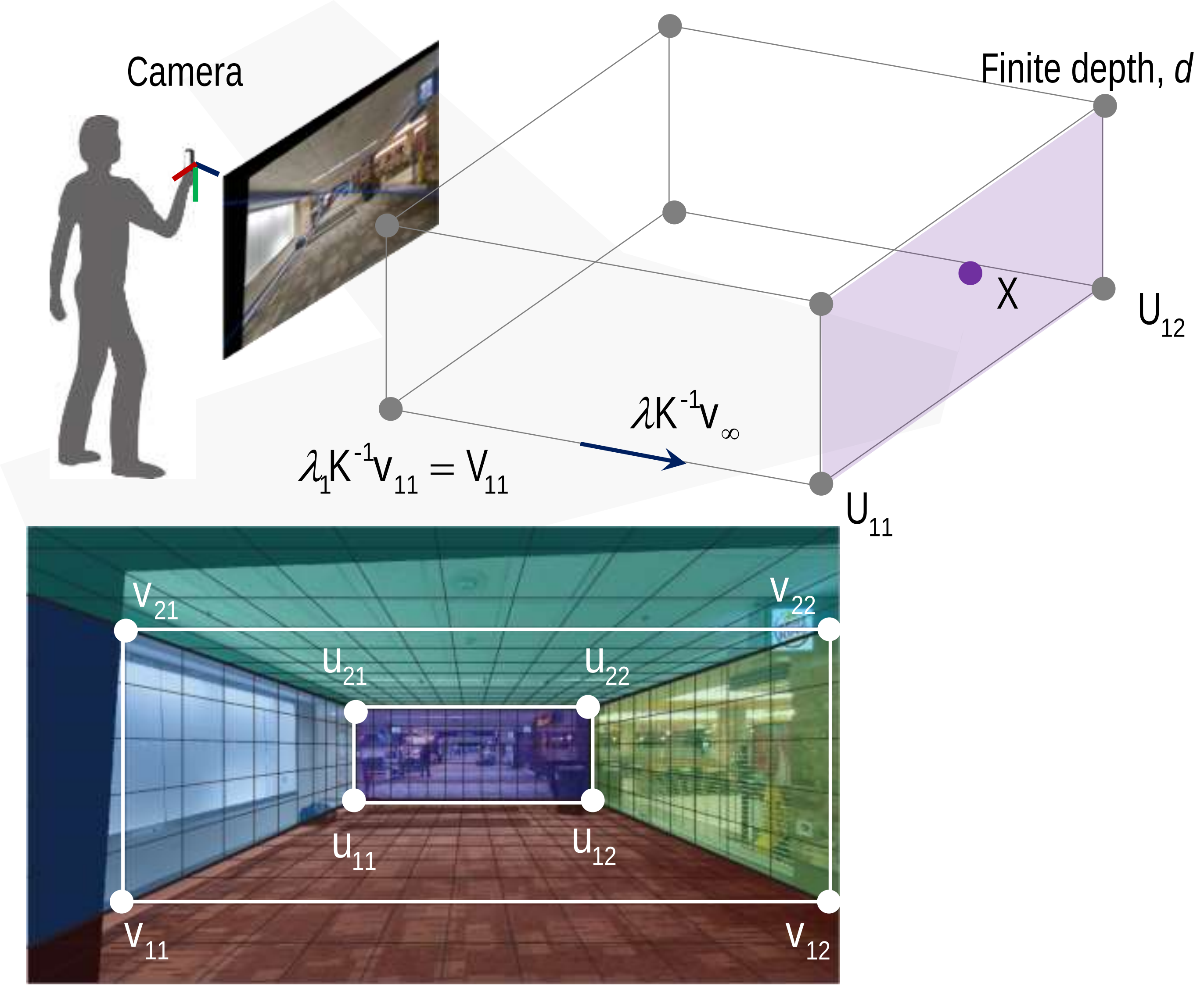
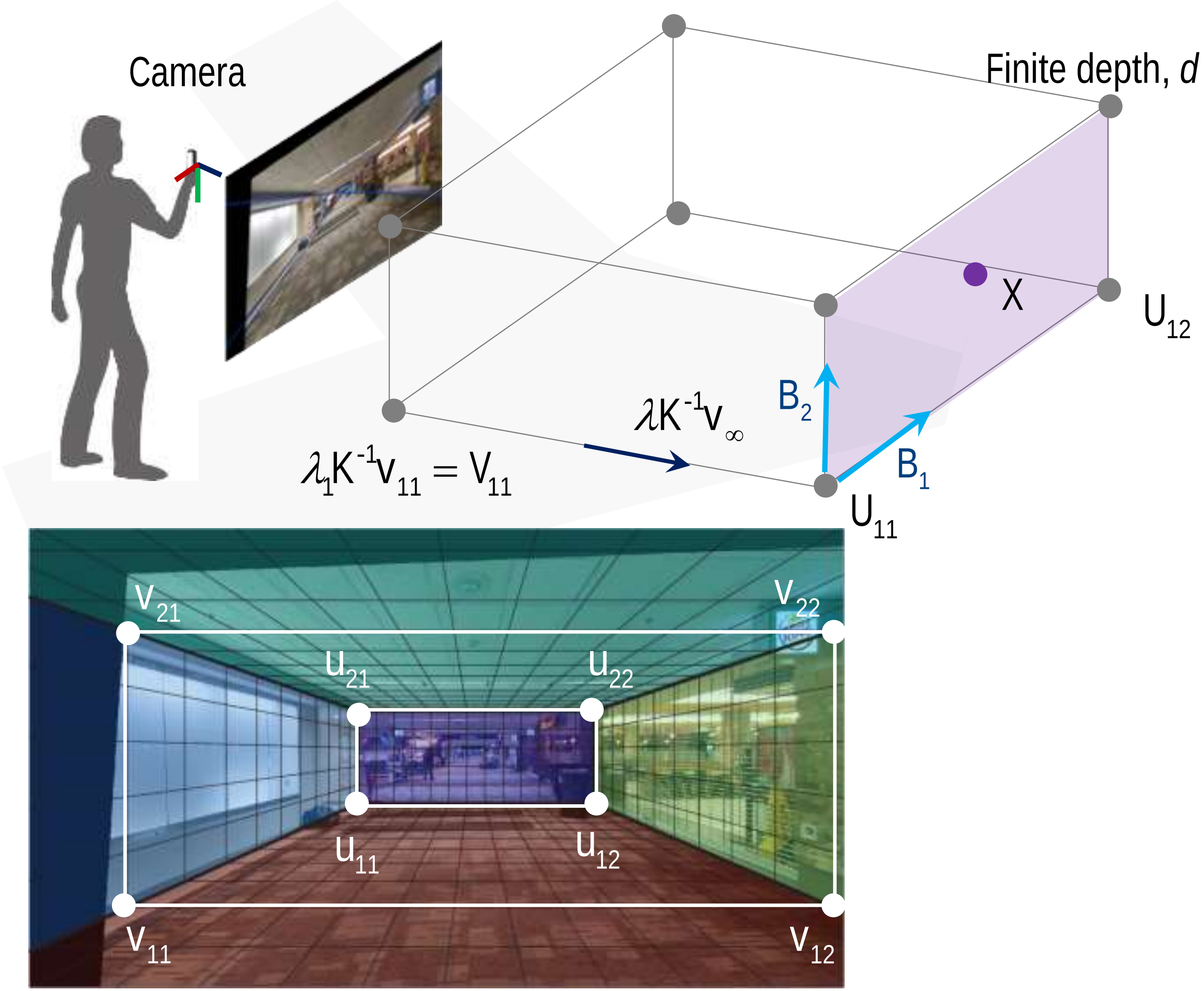


Point in a plane:

$$X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Box Representation



Point in a plane:

$$X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = U_{11} + \mu_1 B_1 + \mu_2 B_2$$

$$= \begin{bmatrix} B_1 & B_2 & c \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

2DOF

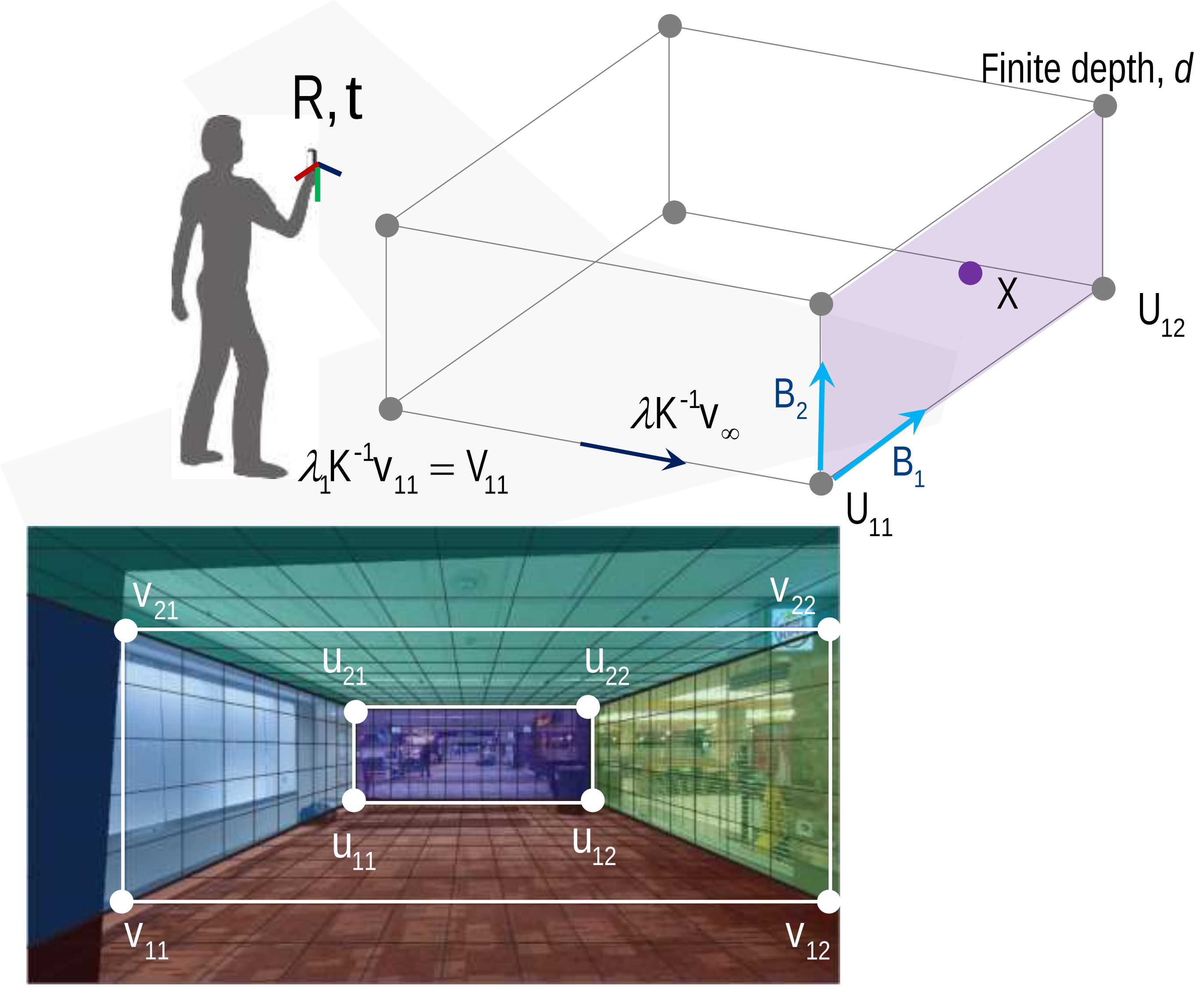
Homography mapping from 3D plane to image:

$$\lambda u = K \begin{bmatrix} B_1 & B_2 & c \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = H \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Box Representation

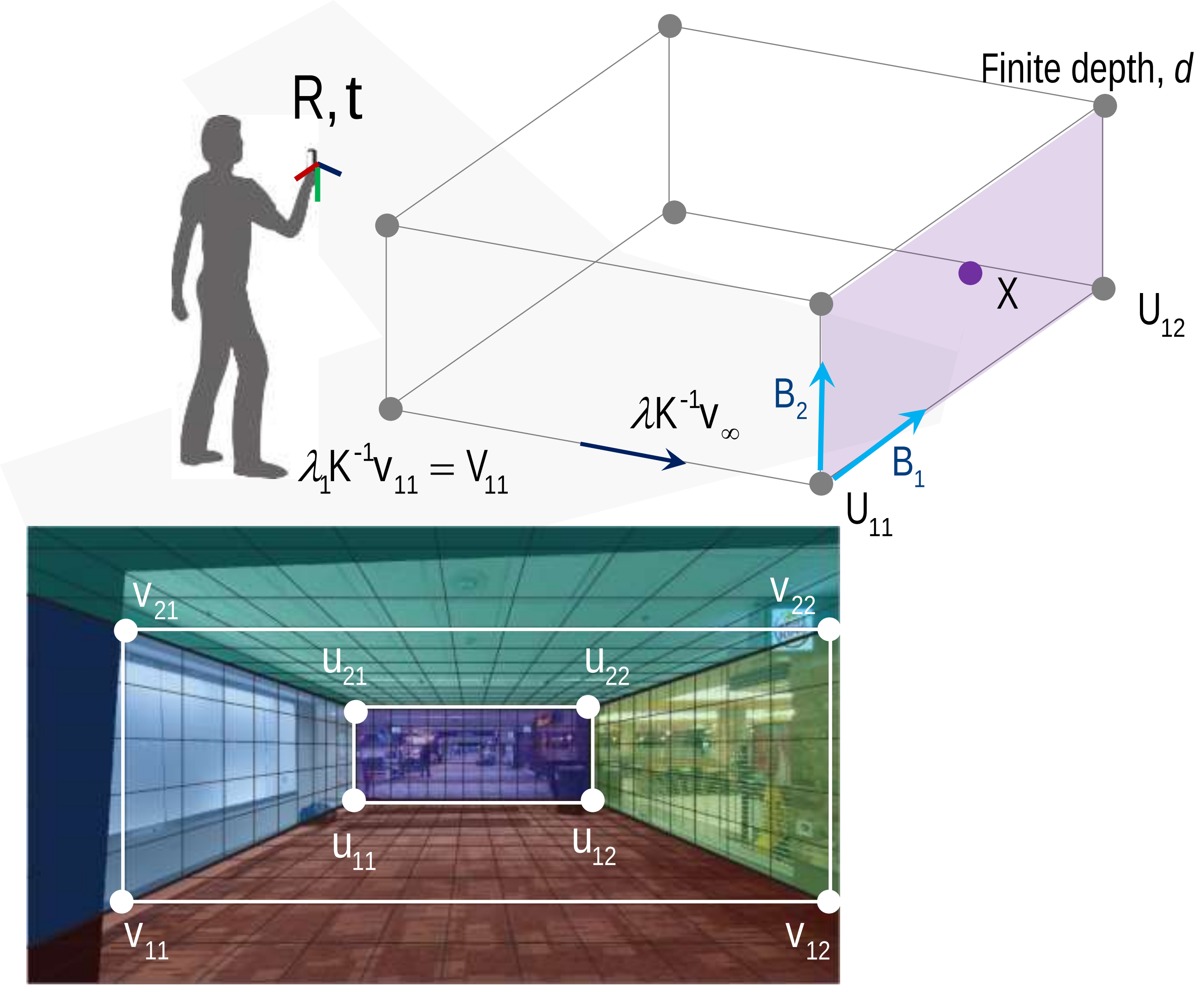
Homography mapping from 3D plane to image:

$$\boxed{\lambda \mathrm{u}} = \mathrm{K}\begin{bmatrix} \mathrm{B}_1 & \mathrm{B}_2 & \mathrm{c} \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \boxed{\mathrm{H}}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$



Texture Mapping

Homography



Homography mapping from 3D plane to image:

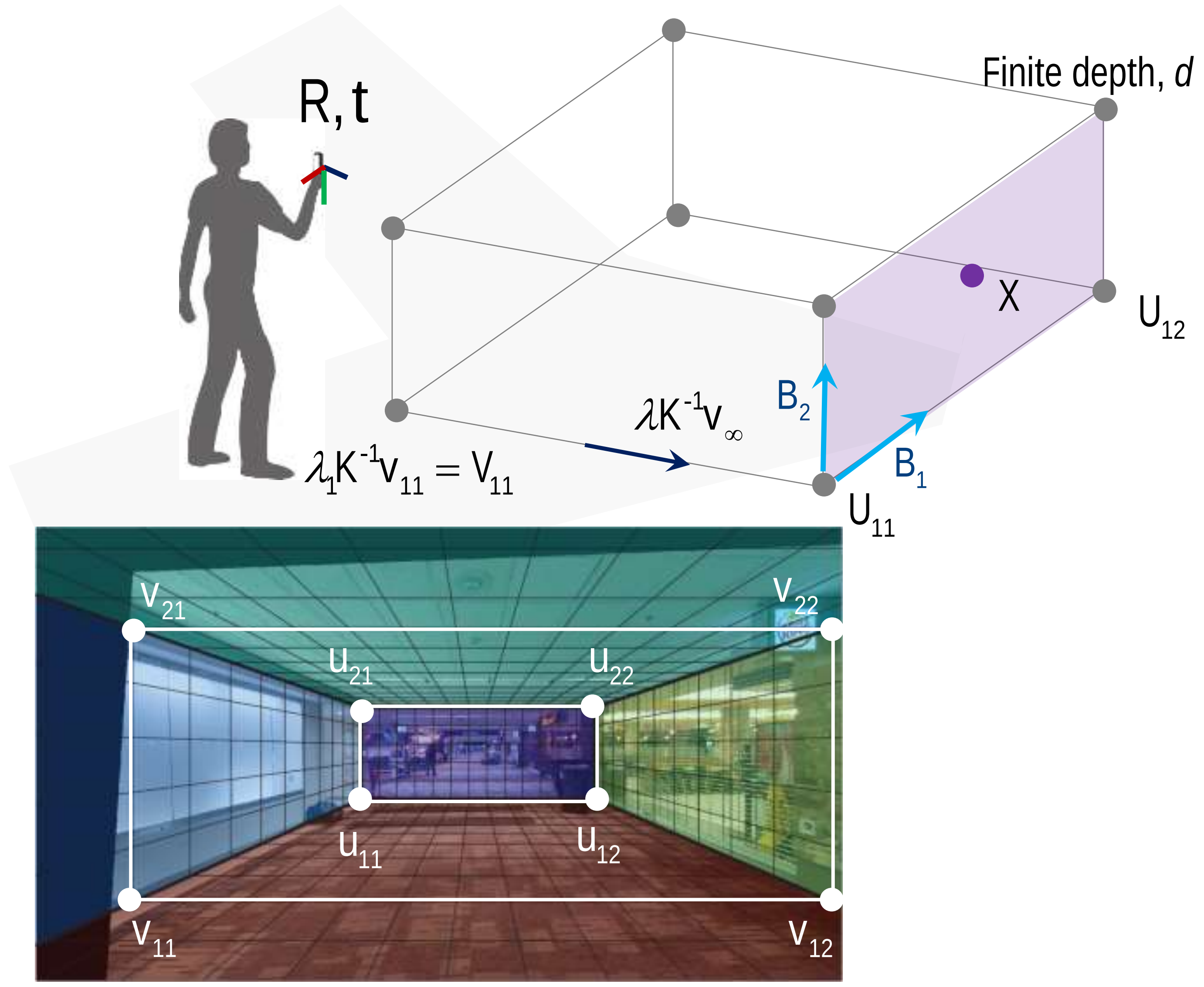
$$\lambda u = K\begin{bmatrix} B_1 & B_2 & c \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = H\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{u} = K\begin{bmatrix} R & t \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = K\begin{bmatrix} R & t \end{bmatrix}\begin{bmatrix} \begin{bmatrix} B_1 & B_2 & c \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} \\ 1 \end{bmatrix}$$

$$= K\begin{bmatrix} RB_1 & RB_2 & Rc + t \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \tilde{H}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Homography



Homography mapping from 3D plane to image:

$$\lambda u = K\begin{bmatrix} B_1 & B_2 & c \end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = H\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{u} = K\begin{bmatrix} R & t \end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} = K\begin{bmatrix} R & t \end{bmatrix}\begin{bmatrix}\begin{bmatrix} B_1 & B_2 & c \end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}\\ 1\end{bmatrix}$$

$$= K\begin{bmatrix} RB_1 & RB_2 & Rc + t \end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \tilde{H}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

$$\rightarrow \quad \lambda \tilde{H}^{-1}\tilde{u} = \begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \lambda_1 H^{-1} u \quad \rightarrow \quad \lambda \tilde{u} = \tilde{H} H^{-1} u$$

HW #3 Tour into your photo