Projective Line

THE MAKING OF

CHEMICAL BROTHERS 'WIDE OPEN'

Announcement

- HW #1 is team assignment up to 2 members.
- HW #1 write-up is due Feb 2nd
- HW #1 presentation (5 min) is Feb 9th
- Submission through Moodle
- One write-up and one presentation per team
- Code online (http://www-users.cs.umn.edu/~hspark/CSci5980/code/)
- Paper selection due by Feb 14th (please consult with me before selection).
- Office hour: Tue/Thr 2-4p @ Keller Hall 5-225E

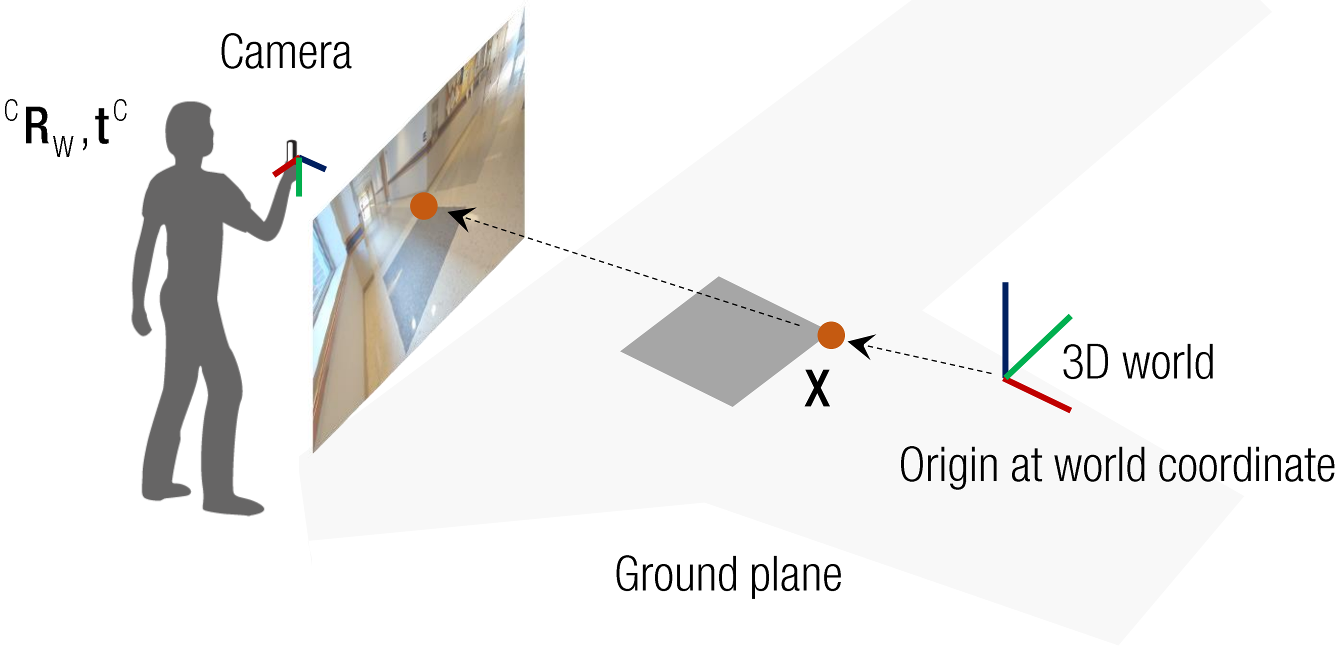
Lens configuration (internal parameter)

$$\begin{bmatrix}\mathbf{x}\\ 1\end{bmatrix} = \mathit{L}\left(\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix}\right)$$

Spatial relationship between sensor and pinhole (internal parameter)

Camera body configuration (extrinsic parameter)

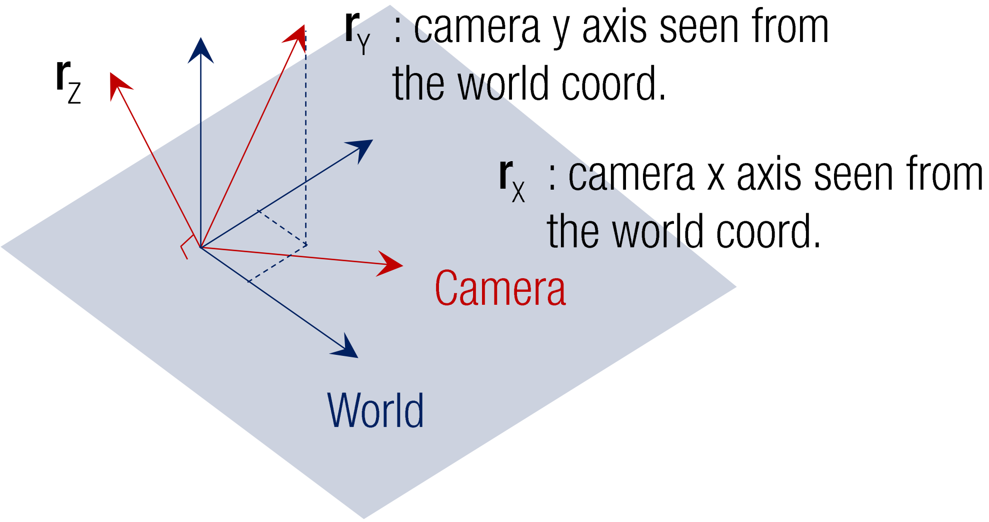
Camera Projection Matrix



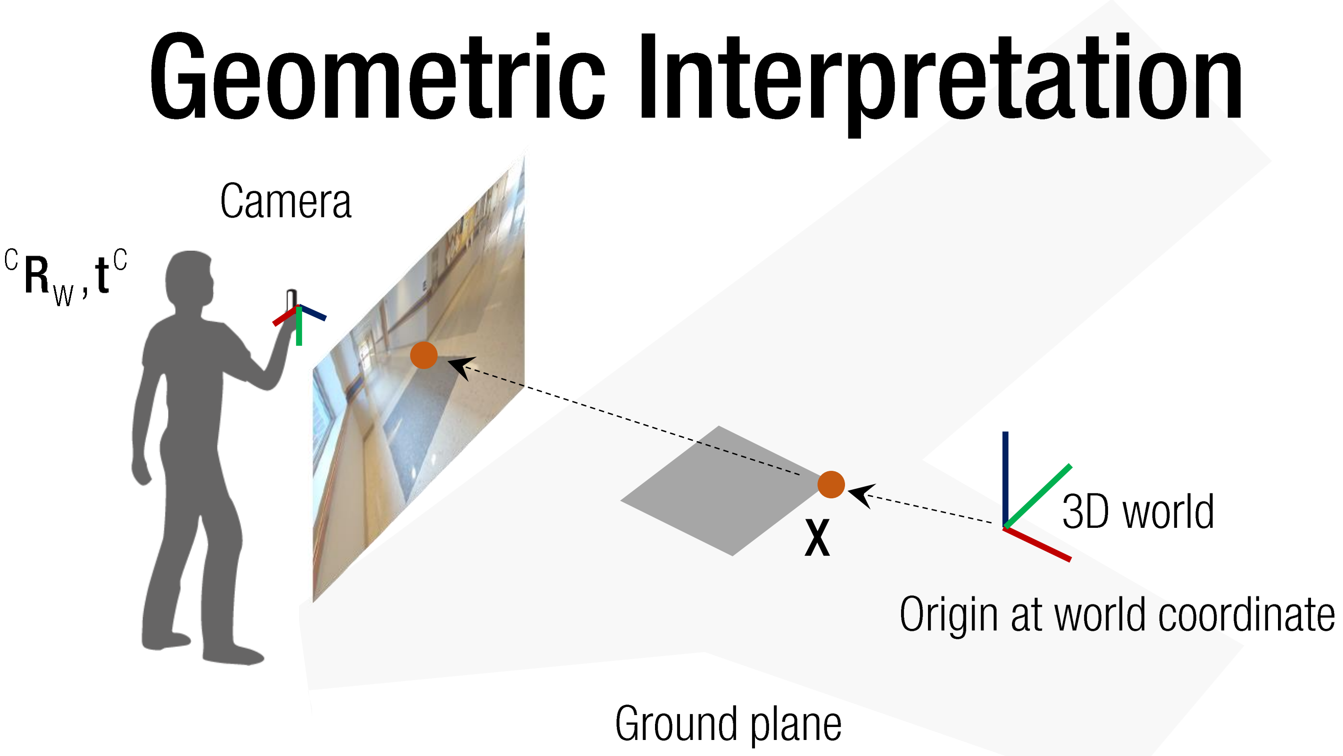
Camera projection of world point:

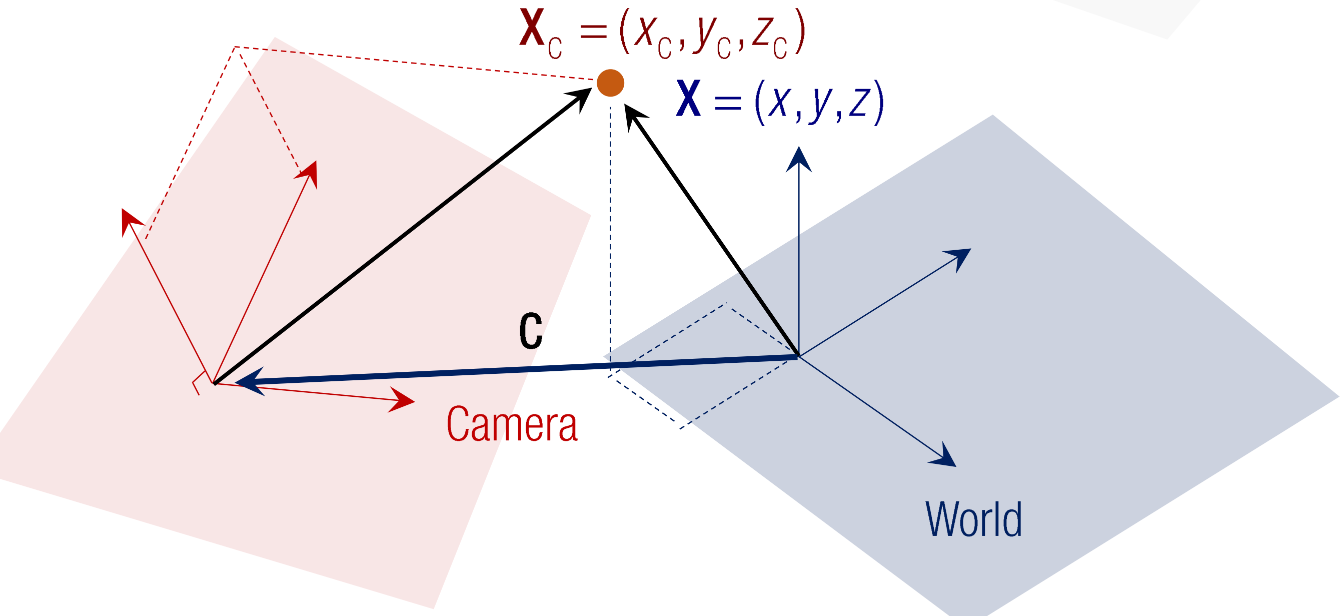
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ Z_C \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \Bigg[\underbrace{\begin{matrix} r_{x1} & \mathbf{r}_X & r_{x3} \\ r_{y1} & \mathbf{r}_Y & r_{y3} \\ r_{z1} & \mathbf{r}_Z & r_{z3} \end{matrix}}_{{}^{C}\mathbf{R}_W} \quad \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{C}\mathbf{t}} \Bigg] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



Geometric Interpretation





Coordinate transformation from world to camera:

$$\mathbf{X}_C = {}^{C}\mathbf{R}_W \mathbf{X} + {}^{C}\mathbf{t} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where ${}^{C}\mathbf{t}$ is translation from world to camera seen from camera.

Rotate and then, translate.

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cf) **Translate and then, rotate.**

$$\mathbf{X}_C = {}^{C}\mathbf{R}_W (\mathbf{X} - \mathbf{C}) = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} 1 & & & -C_x \\ & 1 & & -C_y \\ & & 1 & -C_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where $\mathbf{C}$ is translation from world to camera seen from world.

Image Projection

$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix}$$

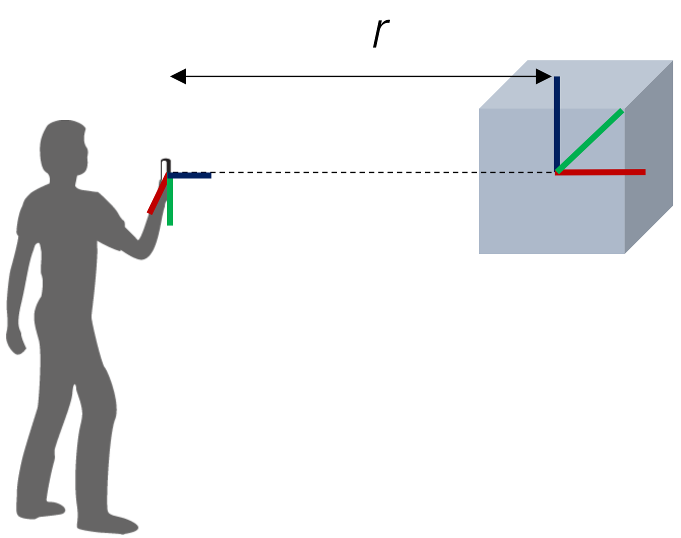


Image Projection

$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

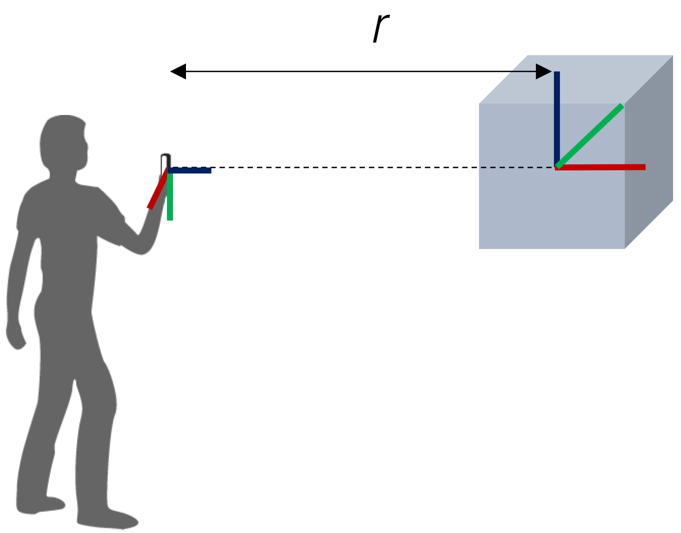


Image Projection

$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

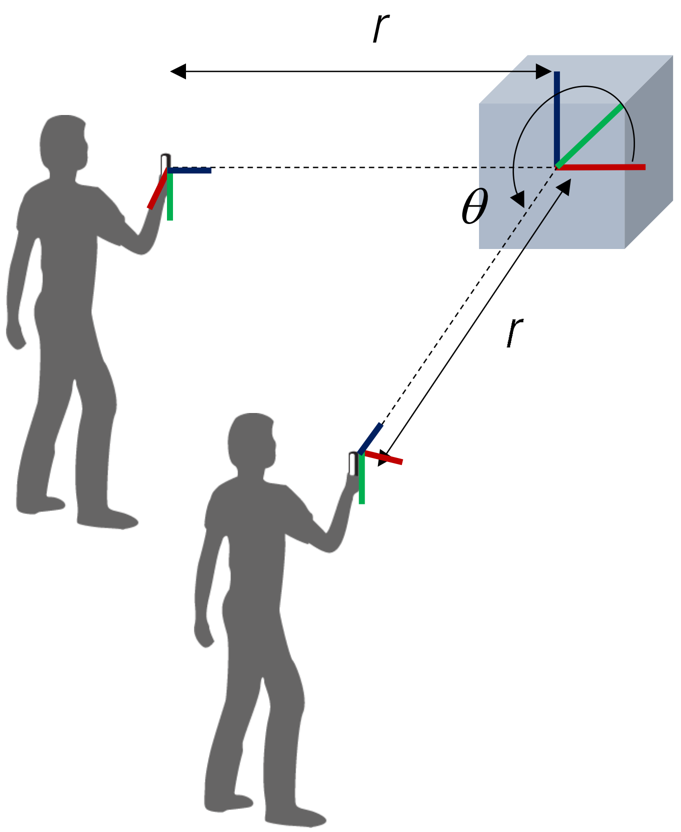


Image Projection

$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix}$$

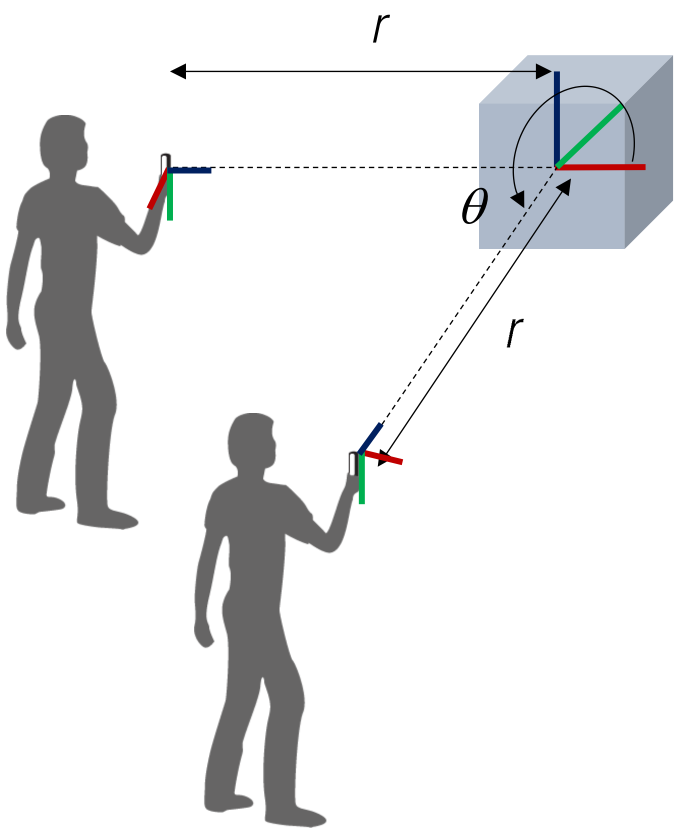


Image Projection

$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \\ -\cos\theta & -\sin\theta & 0 \end{bmatrix}$$

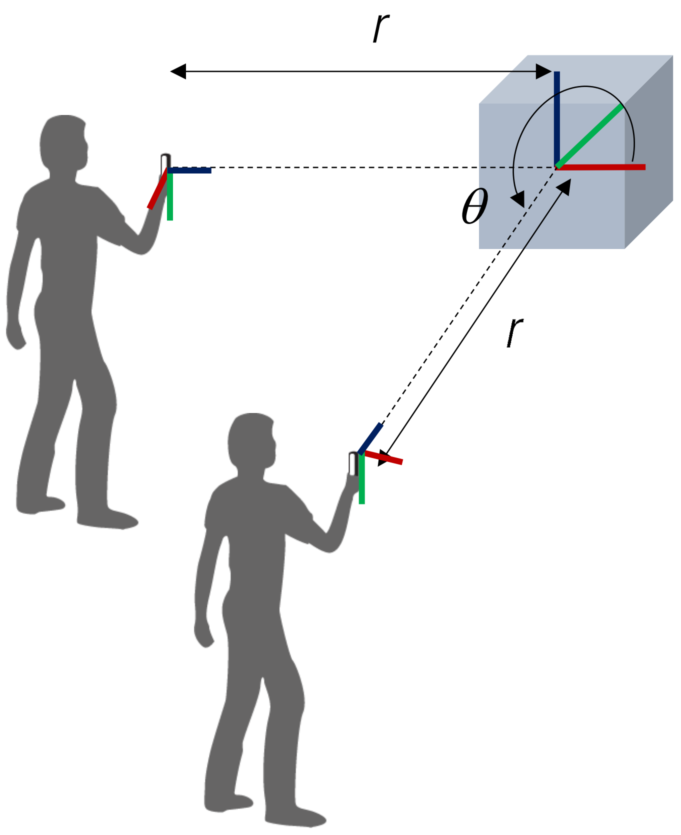
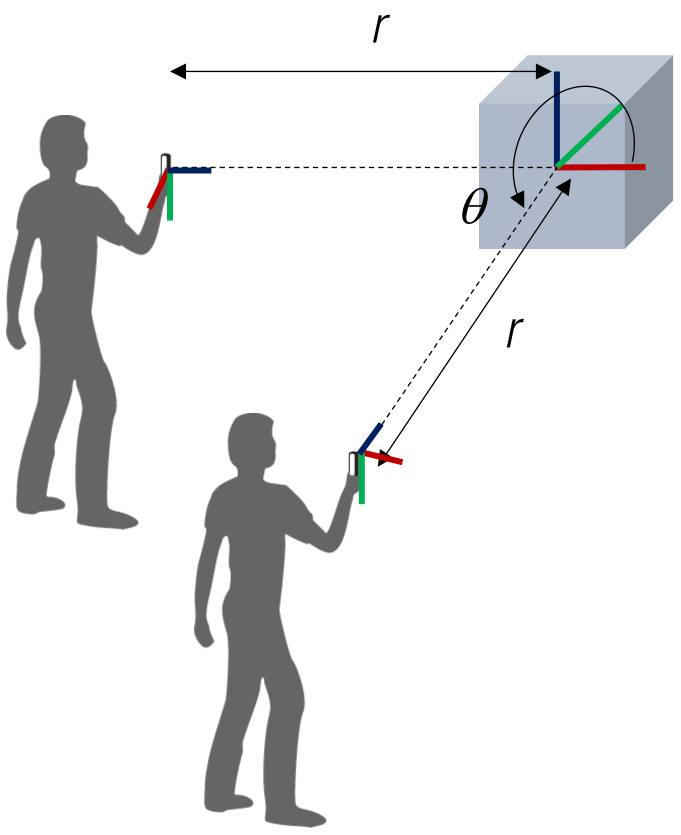


Image Projection



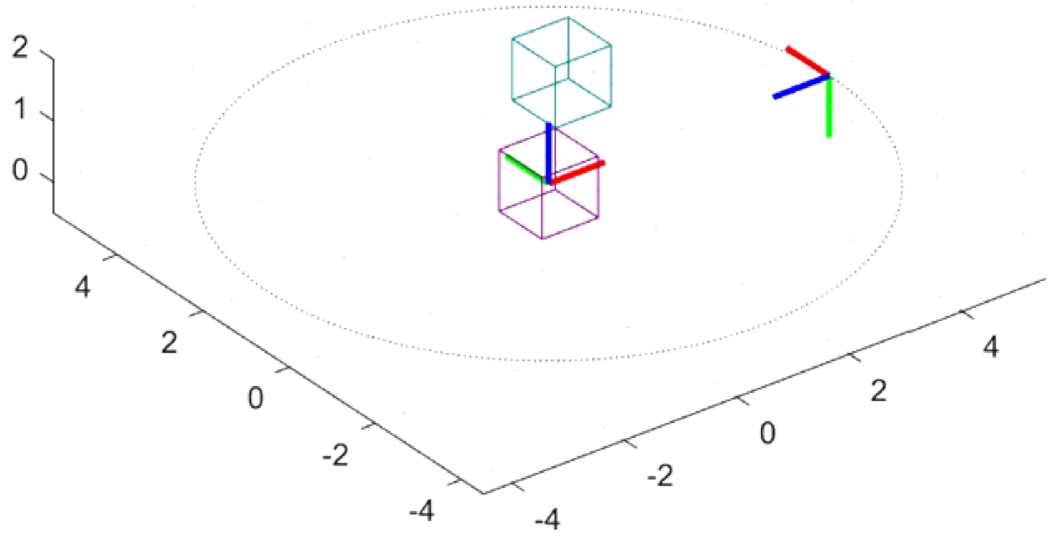
$$\mathbf{C} = \begin{bmatrix} -r \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

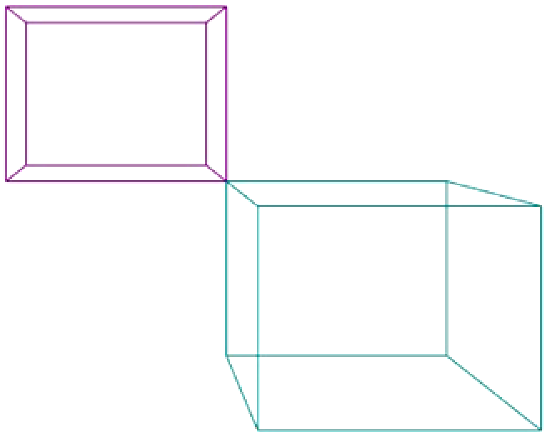
$$\mathbf{C} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \\ -\cos\theta & -\sin\theta & 0 \end{bmatrix}$$

RotateCamera.m

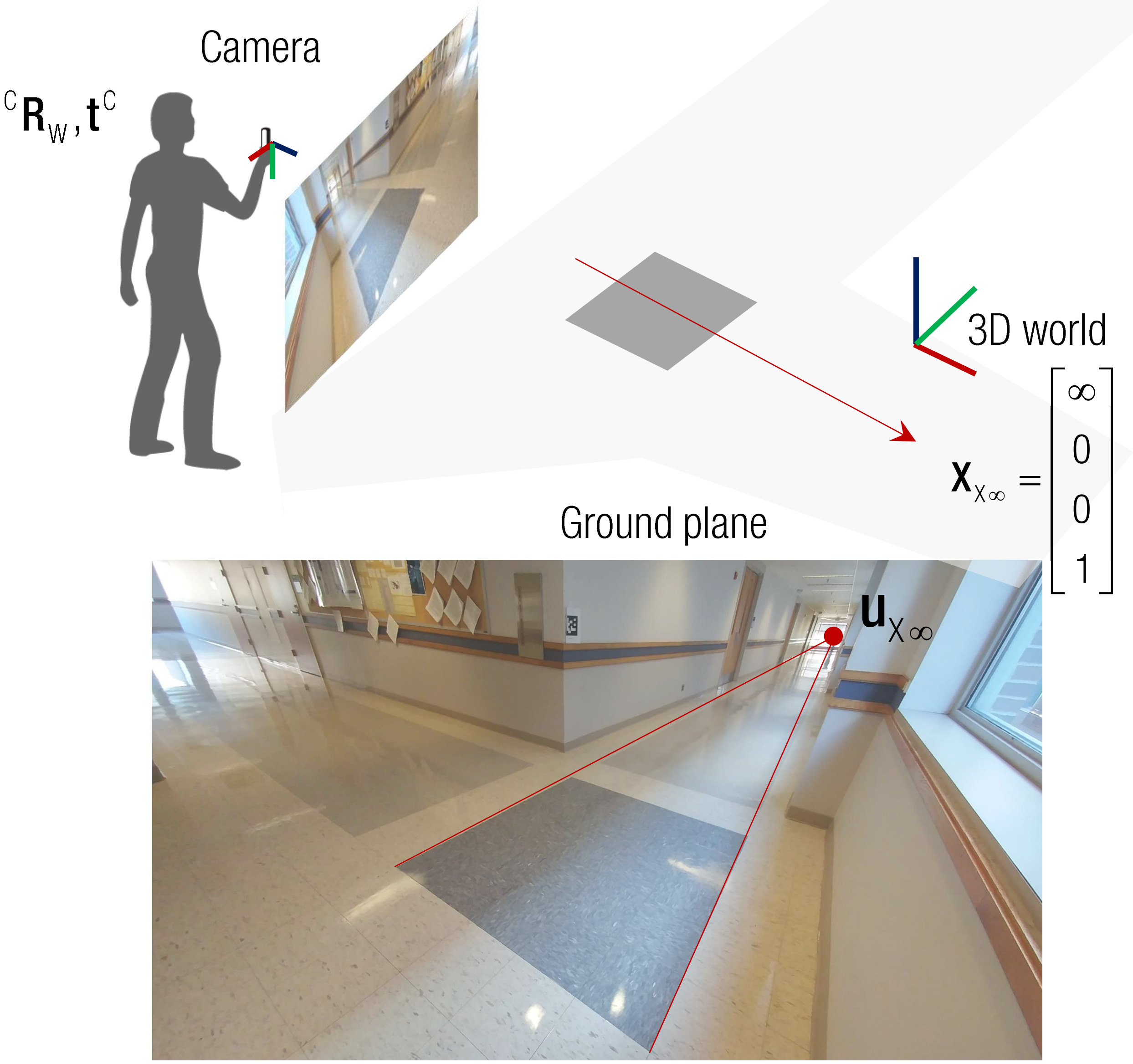
```
for i = 1 : length(theta)
   camera_offset = [radius*cos(theta(i)); radius*sin(theta(i)); 0];
   camera_center = camera_offset + center_of_mass';

   rz = [-cos(theta(i)); -sin(theta(i)); 0];
   ry = [0 0 -1]';
   rx = [-sin(theta(i)); cos(theta(i)); 0];
   R = [rx'; ry'; rz'];
   C = camera_center;
   P = K * R * [ eye(3) -C];

   proj = [];
   for j = 1 : size(sqaure_point,1)
      u = P * [sqaure_point(j,:)';1];
      proj(j,:) = u'/u(3);
   end
end
```

Image Projection





Geometric Interpretation



Camera projection of world point:

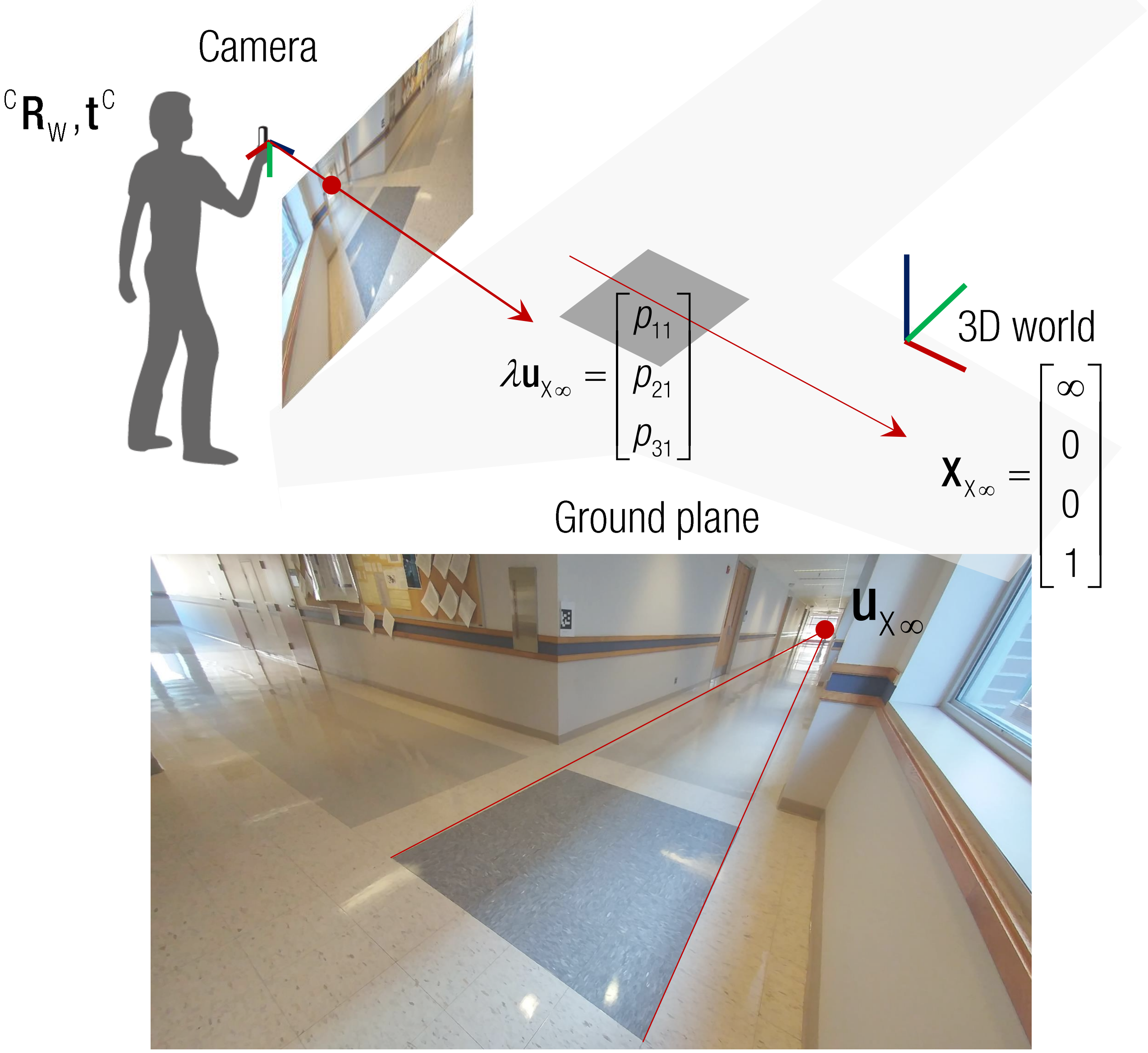
$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^{C}\mathbf{R}_{W}} & \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{C}\mathbf{t}} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \boxed{p_{11}} & p_{12} & p_{13} & p_{14} \\ \boxed{p_{21}} & p_{22} & p_{23} & p_{24} \\ \boxed{p_{31}} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad u = \lim_{X \to \infty} \frac{p_{11} X + p_{14}}{p_{31} X + p_{34}} = \frac{p_{11}}{p_{31}}$$

$$v = \lim_{X \to \infty} \frac{p_{21} X + p_{24}}{p_{31} X + p_{34}} = \frac{p_{21}}{p_{31}}$$

Geometric Interpretation



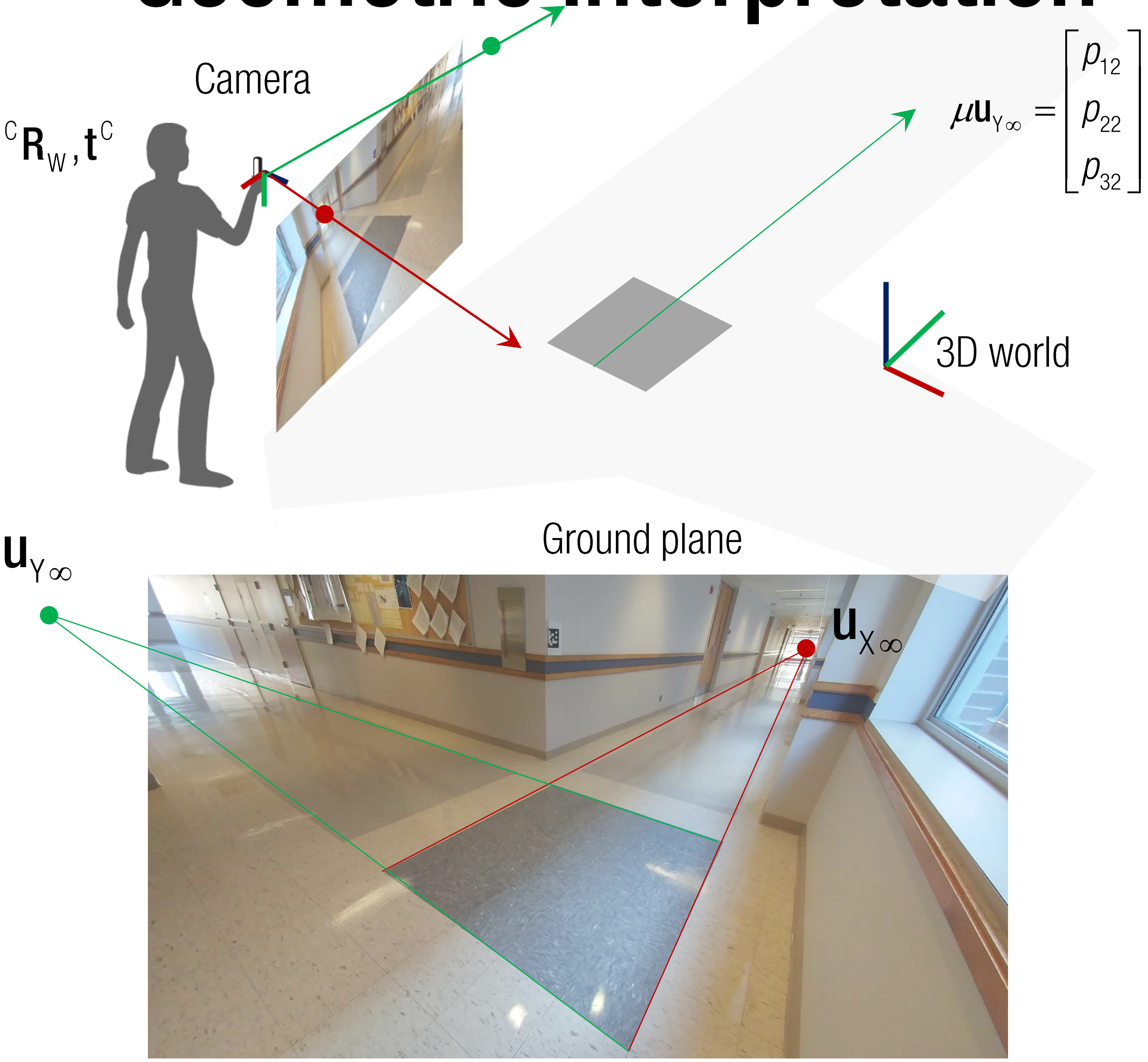
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \left[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{^{C}\mathbf{R}_{W}} \; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{^{C}\mathbf{t}} \right] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} \infty \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X \to \infty} \frac{p_{11}X + p_{14}}{p_{31}X + p_{34}} = \frac{p_{11}}{p_{31}} \\ v &= \lim_{X \to \infty} \frac{p_{21}X + p_{24}}{p_{31}X + p_{34}} = \frac{p_{21}}{p_{31}} \end{aligned} \longrightarrow \lambda \mathbf{u}_{X\infty} = \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix}$$

Geometric Interpretation



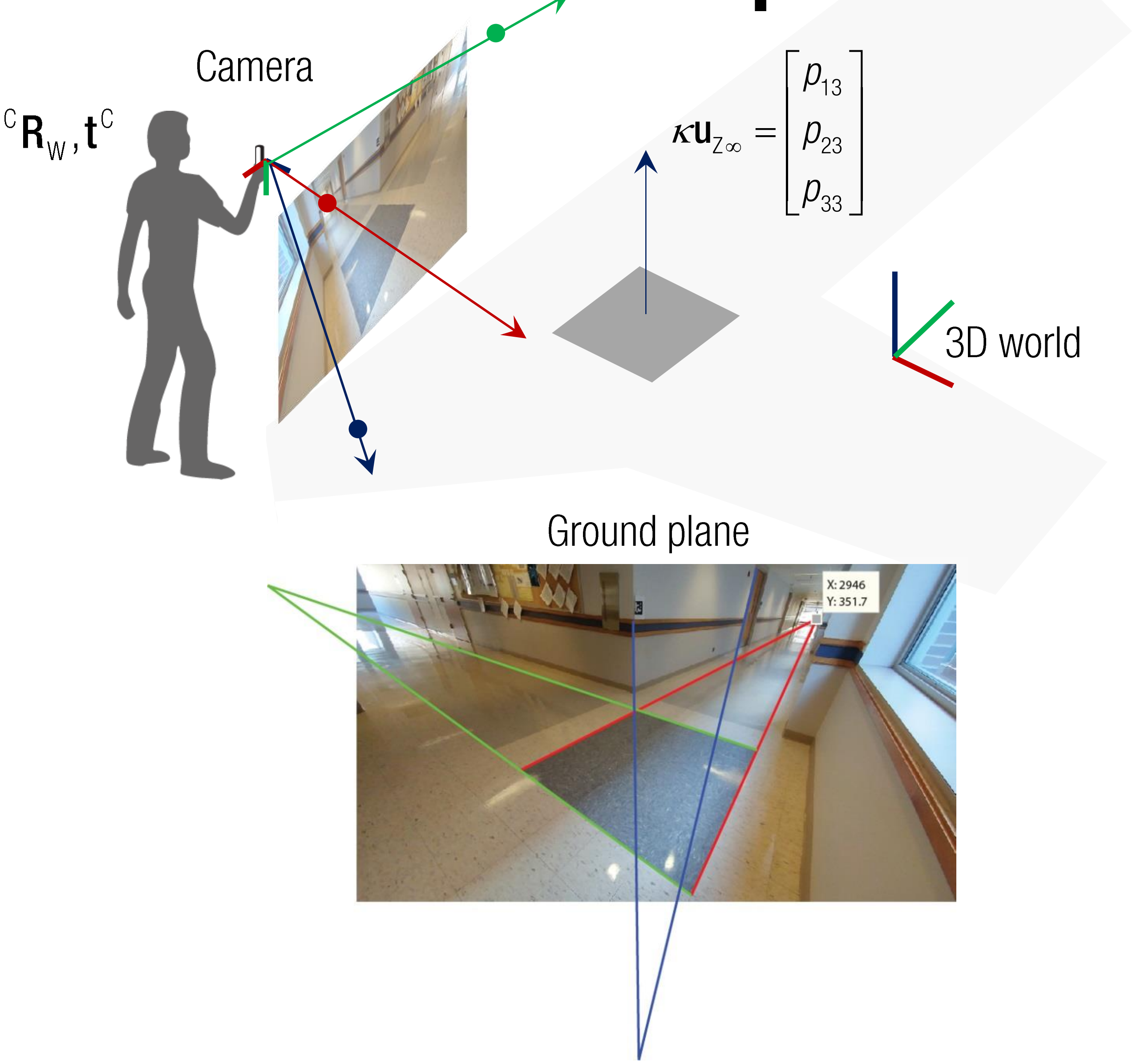
Camera projection of world point:

$$\lambda\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x\\ & f & p_y\\ & & 1\end{bmatrix}}_{\mathbf{K}} \begin{bmatrix} \underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3}\\ r_{y1} & r_{y2} & r_{y3}\\ r_{z1} & r_{z2} & r_{z3}\end{matrix}}_{^{C}\mathbf{R}_{W}} & \underbrace{\begin{matrix} t_x\\ t_y\\ t_z\end{matrix}}_{^{C}\mathbf{t}}\end{bmatrix} \begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} = \mathbf{P}\begin{bmatrix}\mathbf{X}\\ 1\end{bmatrix}$$

$$\longrightarrow \lambda\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14}\\ p_{21} & p_{22} & p_{23} & p_{24}\\ p_{31} & p_{32} & p_{33} & p_{34}\end{bmatrix}\begin{bmatrix}0\\ \infty\\ 0\\ 1\end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X\to\infty}\frac{p_{12}Y + p_{14}}{p_{32}Y + p_{34}} = \frac{p_{12}}{p_{32}}\\ v &= \lim_{X\to\infty}\frac{p_{22}Y + p_{24}}{p_{32}Y + p_{34}} = \frac{p_{22}}{p_{32}}\end{aligned} \longrightarrow \mu\mathbf{u}_{Y\infty} = \mu\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}p_{12}\\ p_{22}\\ p_{32}\end{bmatrix}$$

Geometric Interpretation



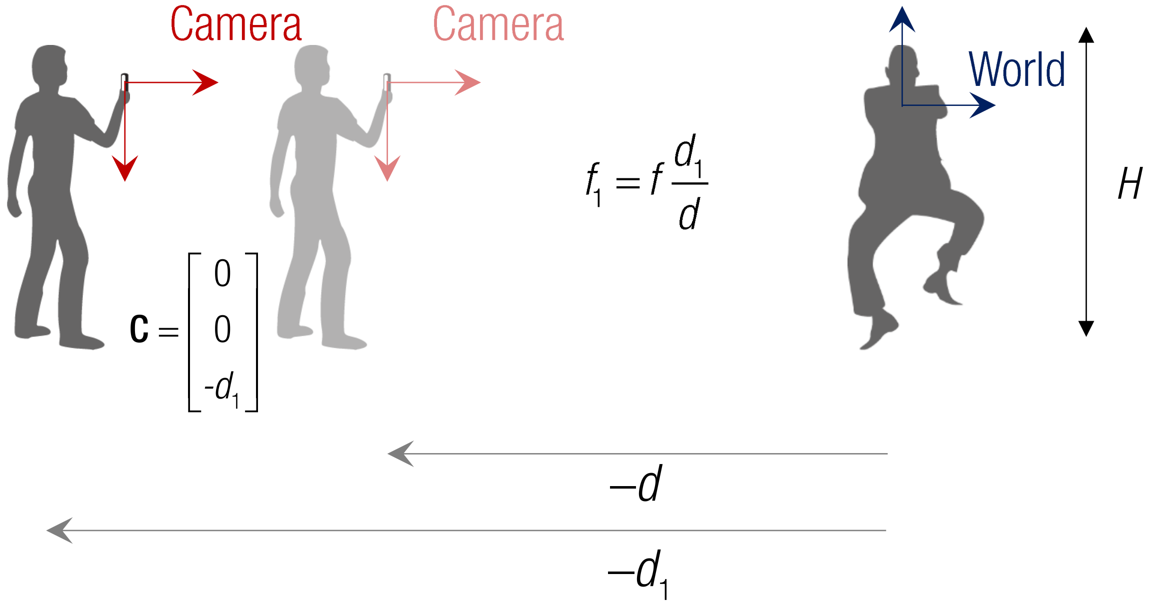
Camera projection of world point:

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}}_{\mathbf{K}} \Bigg[\underbrace{\begin{matrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{matrix}}_{{}^{C}\mathbf{R}_{W}} \; \underbrace{\begin{matrix} t_x \\ t_y \\ t_z \end{matrix}}_{{}^{C}\mathbf{t}} \Bigg] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\longrightarrow \lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \infty \\ 1 \end{bmatrix}$$

$$\longrightarrow \begin{aligned} u &= \lim_{X\to\infty} \frac{p_{13}Z + p_{14}}{p_{33}Z + p_{34}} = \frac{p_{13}}{p_{33}} \\ v &= \lim_{X\to\infty} \frac{p_{23}Z + p_{24}}{p_{33}Z + p_{34}} = \frac{p_{23}}{p_{33}} \end{aligned} \longrightarrow \kappa \mathbf{u}_{Z\infty} = \kappa \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} p_{13} \\ p_{23} \\ p_{33} \end{bmatrix}$$

Affine Camera



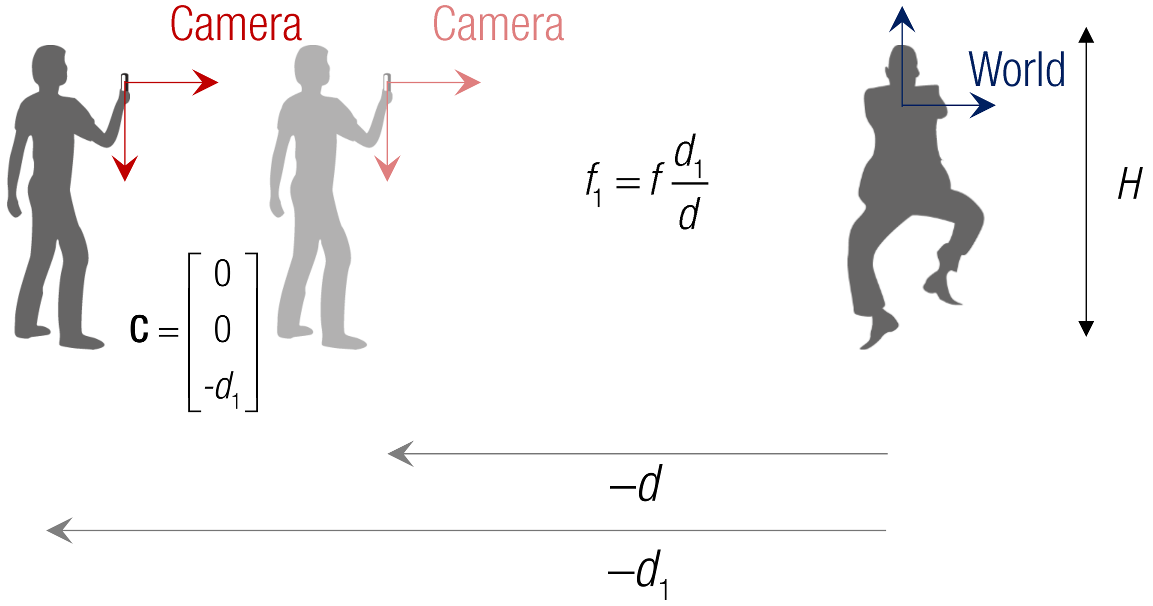
Affine camera:

$$\mathbf{P} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ \boxed{0 \quad 0 \quad 0} & & & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ \boxed{0 \quad 0 \quad 0 \quad 1} & & & \end{bmatrix}$$

Weak perspectiveness

Strong perspectiveness

Affine Camera



Affine camera:

$$\mathbf{P}_{\mathrm{A}} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

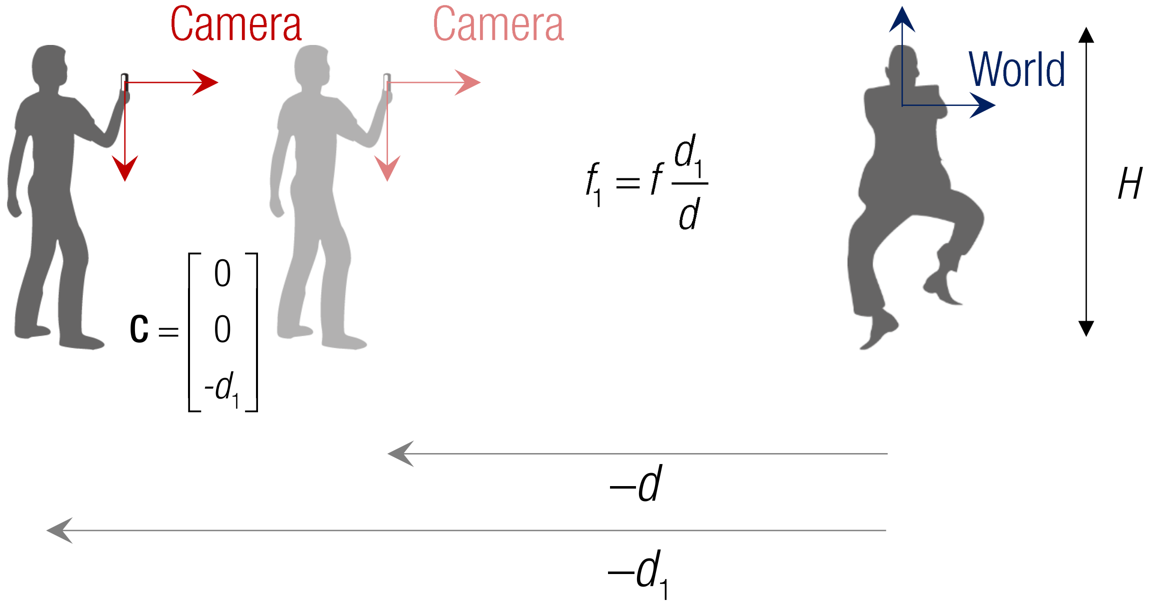
No scaler

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{P}_{\mathrm{A}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Weak perspectiveness

Strong perspectiveness

Affine Camera



$$\mathbf{C} = \begin{bmatrix} 0 \\ 0 \\ -d_1 \end{bmatrix}$$

$$f_1 = f\frac{d_1}{d}$$

Weak perspectiveness

Strong perspectiveness

Affine camera:

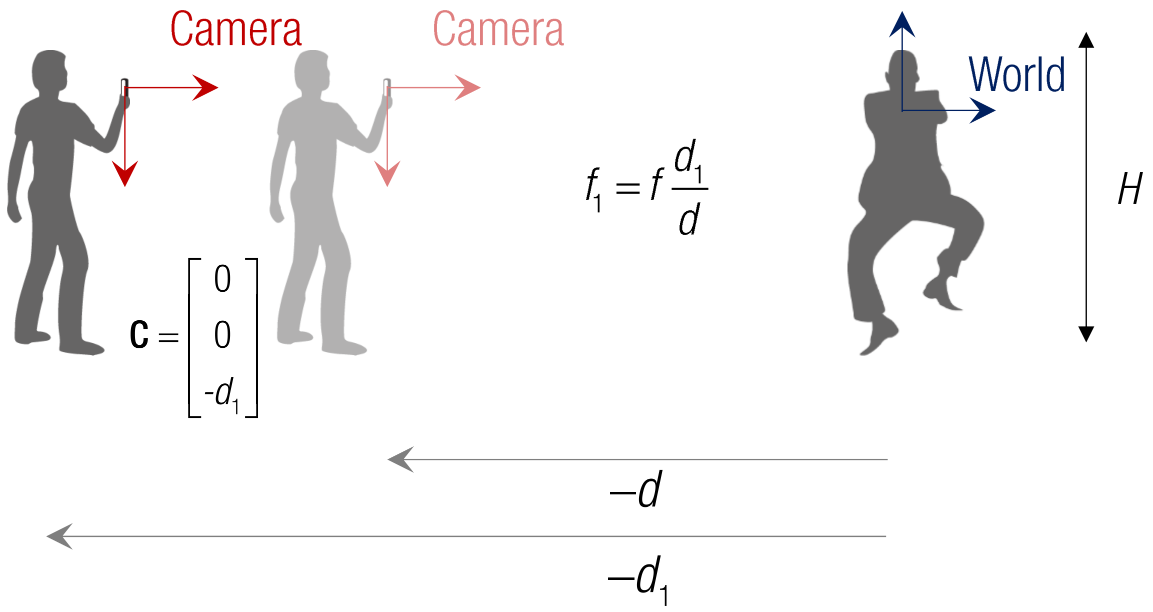
$$\mathbf{P}_{\mathrm{A}} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No scaler

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{P}_{\mathrm{A}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Affine Camera



Weak perspectiveness

Strong perspectiveness

Affine camera:

$$\mathbf{P}_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d\begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

No scaler

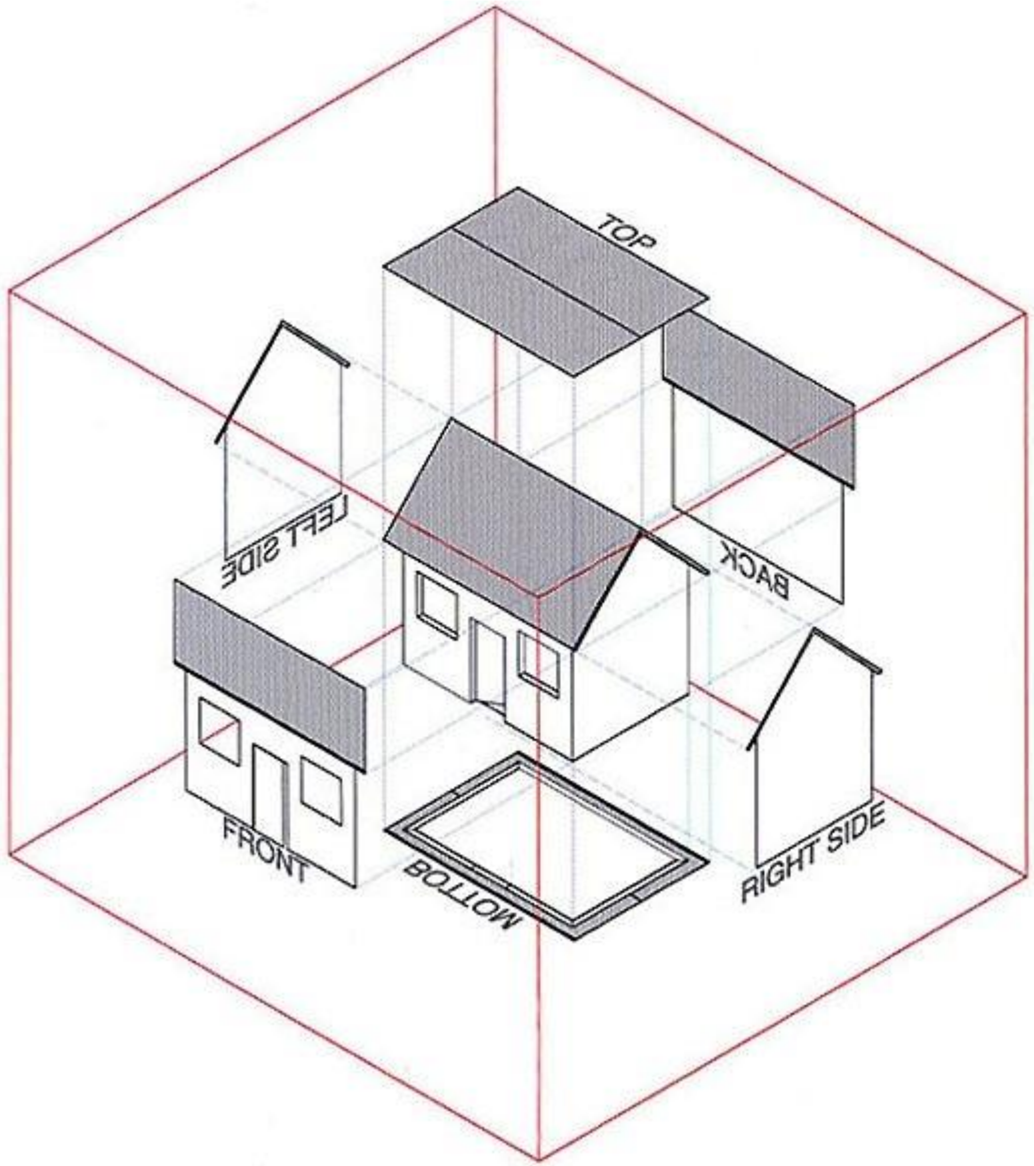
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{P}_A\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = d\begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{23} \end{bmatrix}\begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

No denominator

Orthographic Camera



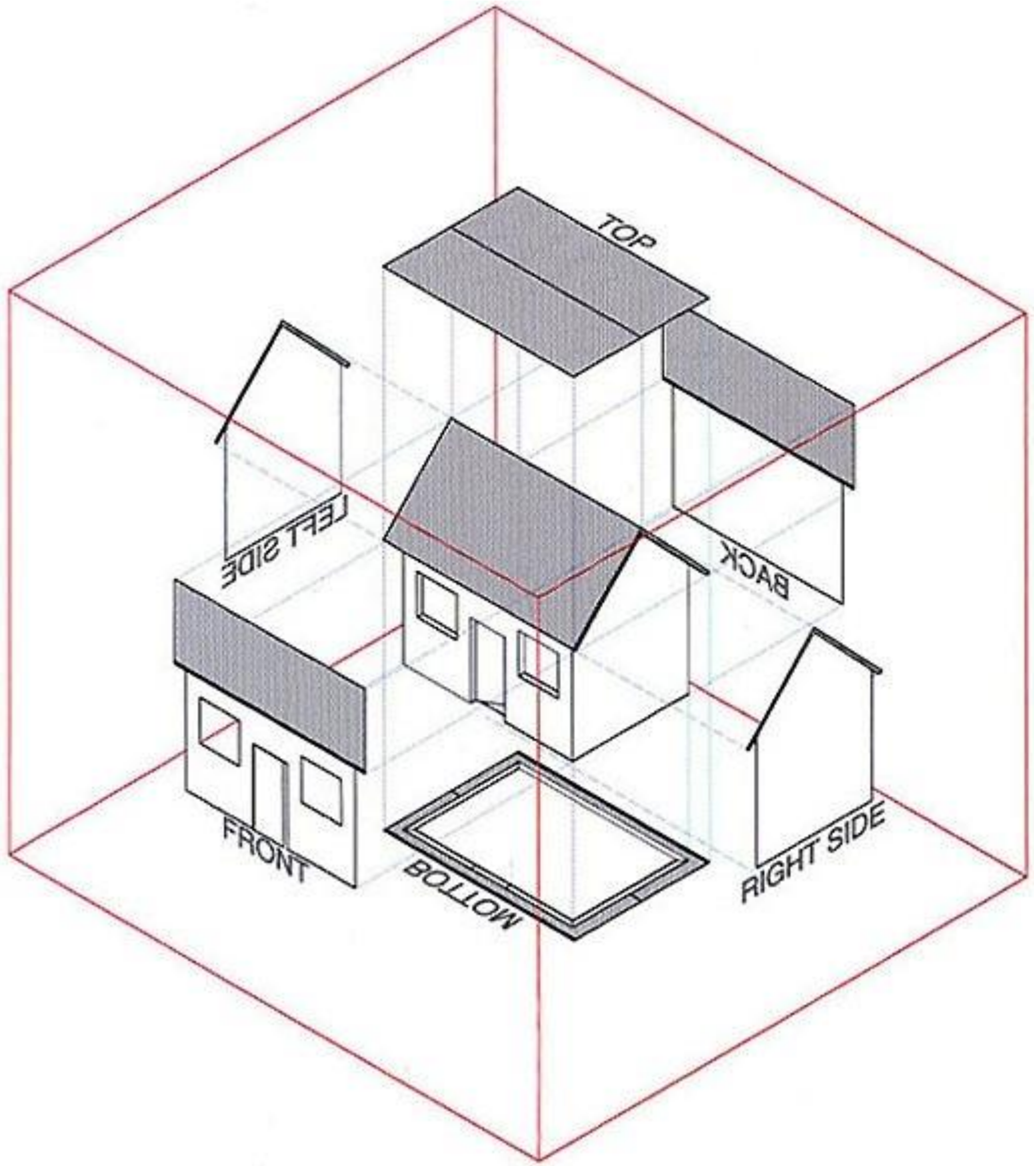
Affine camera:

$$\mathbf{P}_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

Orthographic Camera



Affine camera:

$$\mathbf{P}_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & 0 & 0 \\ r_{y1} & r_{y2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

$$\mathbf{P}_0 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Lens configuration (internal parameter)

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = {\color{red}L}\left({\color{blue}\mathbf{K}} \begin{bmatrix} {\color{magenta}\mathbf{R}} & {\color{magenta}\mathbf{t}} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} \right)$$

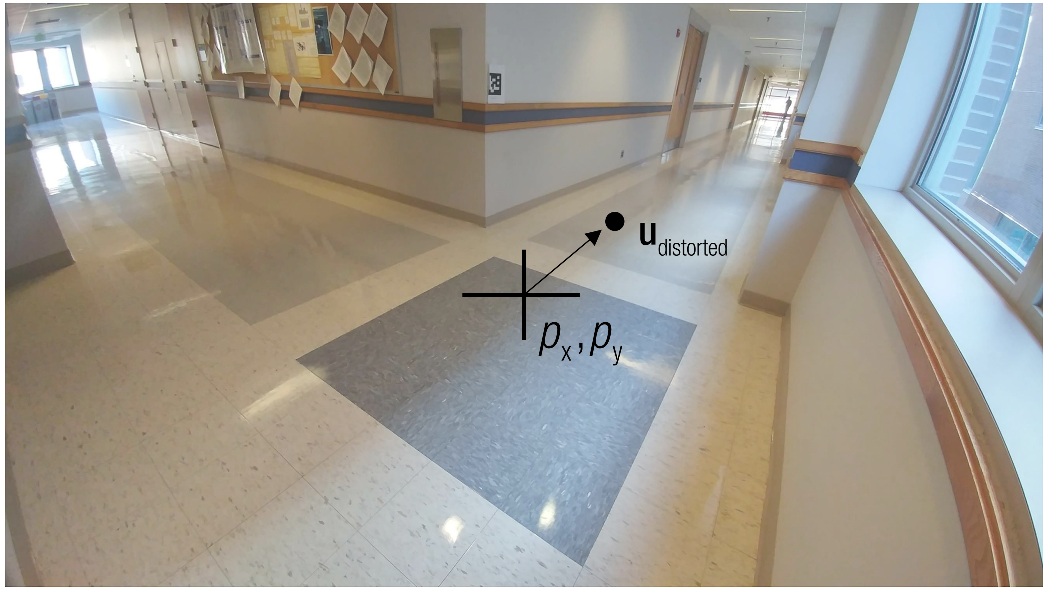
Spatial relationship between sensor and pinhole (internal parameter)

Camera body configuration (extrinsic parameter)


Lens Radial Distortion

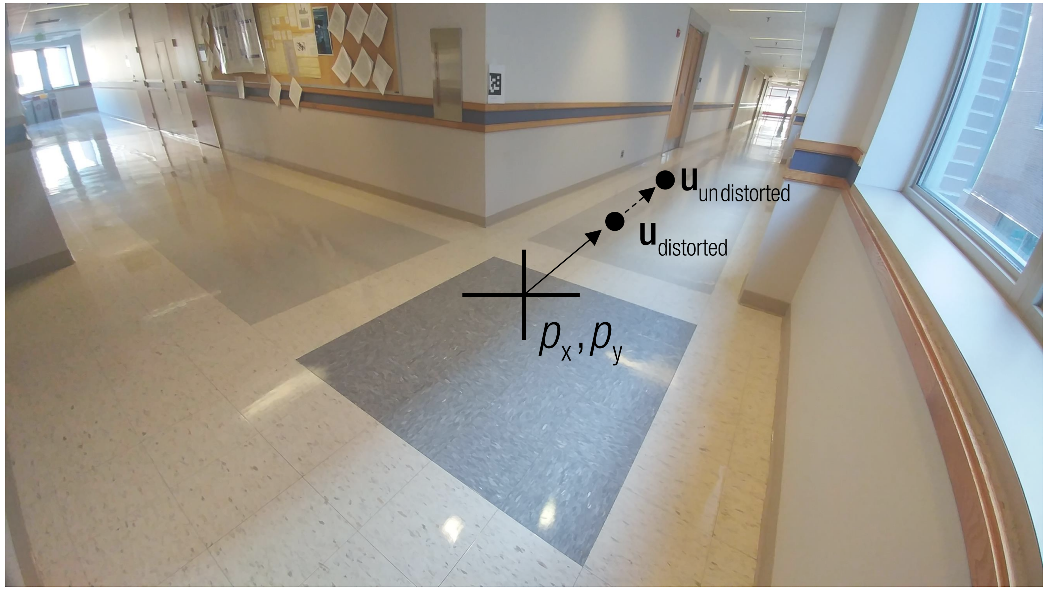
Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

where $\quad \rho = \left\|\bar{\mathbf{u}}_{\text{undistorted}}\right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

$k_1 > 0$

$k_1 < 0$

Radial Distortion Model

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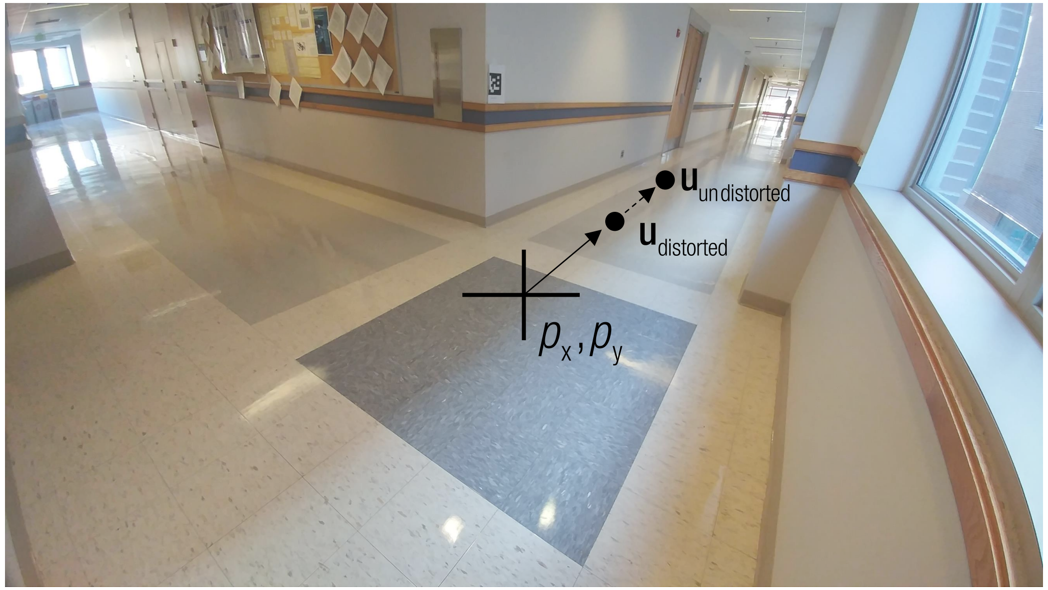


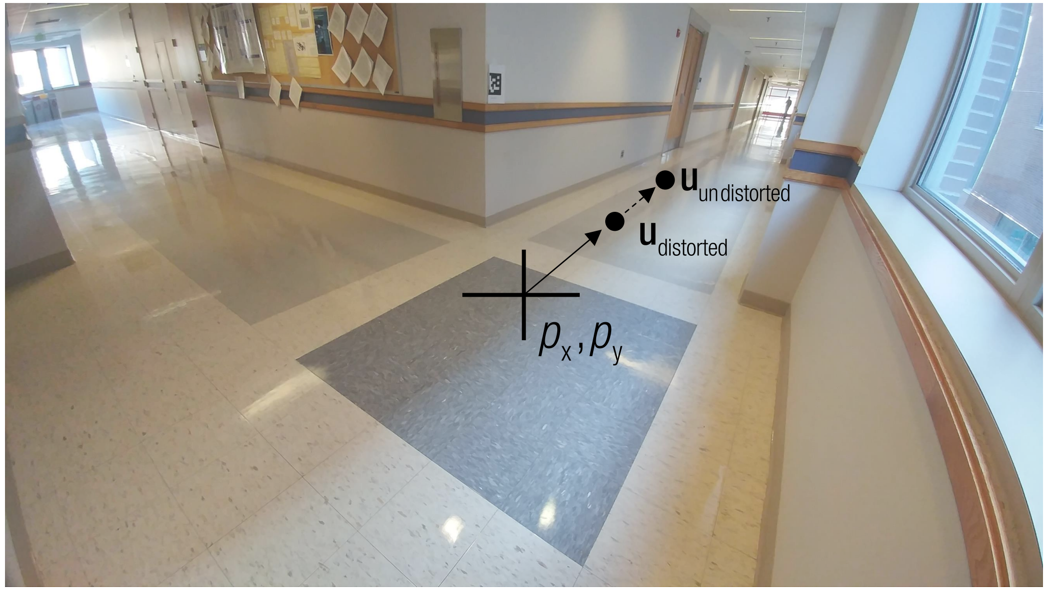
$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

where $\rho = \left\| \bar{\mathbf{u}}_{\text{undistorted}} \right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

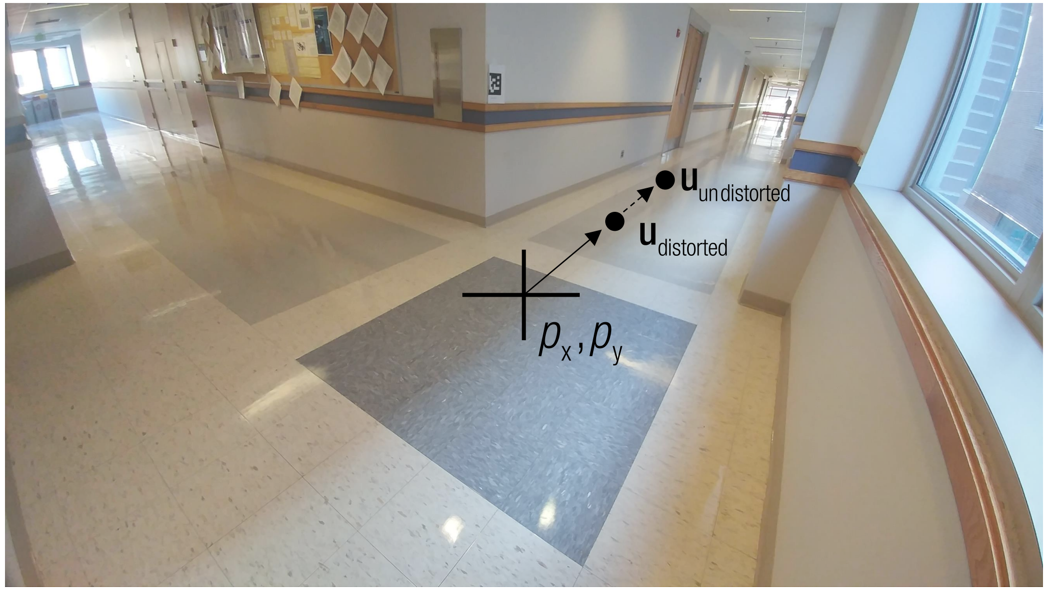
$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

where $\rho = \left\| \bar{\mathbf{u}}_{\text{undistorted}} \right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.



$$f = f_m \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \text{pix}$$

$$p_x = \frac{W_{\text{img}}}{2} = \frac{3840}{2} = 1920 \text{pix} \qquad p_y = \frac{H_{\text{img}}}{2} = \frac{2160}{2} = 1080 \text{pix}$$

Normalized point:

$$\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{distorted}}, \quad \bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\bar{\mathbf{u}}_{\text{distorted}} = L(\rho)\bar{\mathbf{u}}_{\text{undistorted}}$$

where $\rho = \left\| \bar{\mathbf{u}}_{\text{undistorted}} \right\|$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;
```

Distorted image

$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224 \mathrm{pix}$$

$$p_{\mathrm{x}} = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920 \mathrm{pix} \qquad p_{\mathrm{y}} = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080 \mathrm{pix}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image


```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;

im_new = zeros(size(im));   % create a new image

for i = 1 : size(im,1)
   for j = 1 : size(im,2)
```

$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\,\mathrm{pix}$$

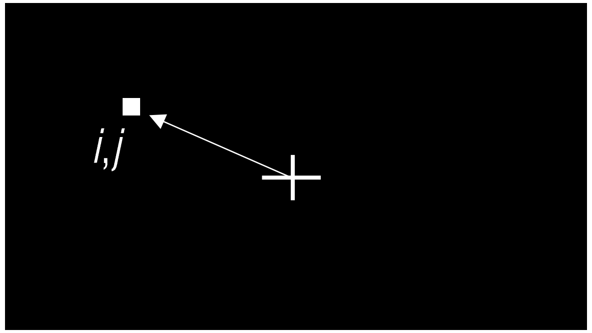
$$p_{\mathrm{x}} = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920\,\mathrm{pix} \qquad p_{\mathrm{y}} = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080\,\mathrm{pix}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image




```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;

im_new = zeros(size(im));   % create a new image

for i = 1 : size(im,1)
   for j = 1 : size(im,2)
      dx = ([j;i]-[px;py])/f;
      r = norm(dx);
```

$$\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\rho = \left\|\bar{\mathbf{u}}_{\text{undistorted}}\right\|$$

$$f = f_{\text{m}}\frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153\frac{3840}{0.0048} = 1224\,\text{pix}$$

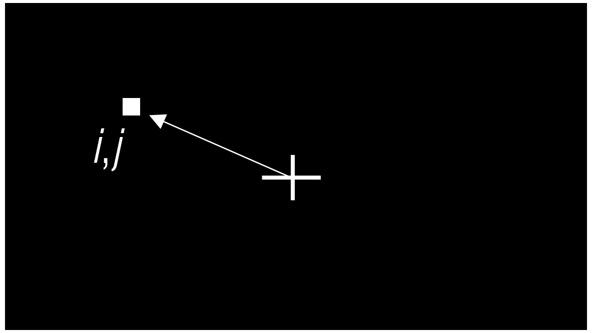
$$p_{\text{x}} = \frac{W_{\text{img}}}{2} = \frac{3840}{2} = 1920\,\text{pix} \qquad p_{\text{y}} = \frac{H_{\text{img}}}{2} = \frac{2160}{2} = 1080\,\text{pix}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image

Undistorted image




```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;

im_new = zeros(size(im));   % create a new image

for i = 1 : size(im,1)
   for j = 1 : size(im,2)
      dx = ([j;i]-[px;py])/f;
      r = norm(dx);
      l = 1 + k*r*r;
      x = f*l*dx+[cx;cy];
```

$$\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\rho = \left\|\bar{\mathbf{u}}_{\text{undistorted}}\right\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

$$\mathbf{K}\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{u}_{\text{distorted}}$$

$$f = f_{\text{m}}\frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153\frac{3840}{0.0048} = 1224\,\text{pix}$$

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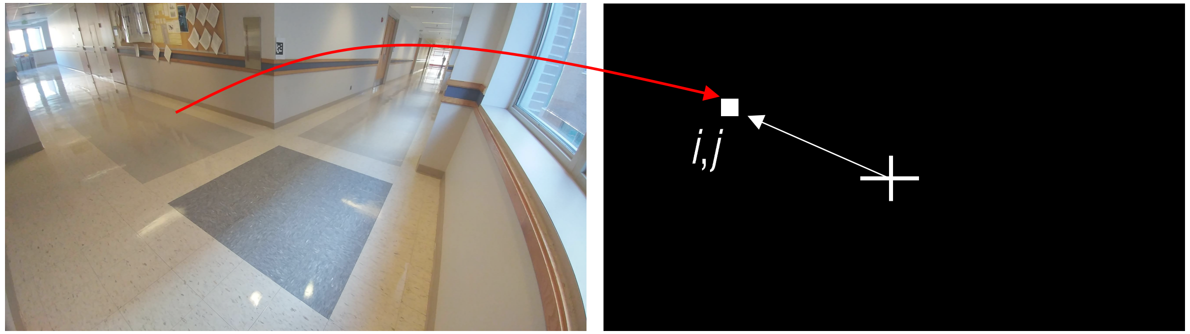
Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image Undistorted image



$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\,\mathrm{pix}$$

$$p_x = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920\,\mathrm{pix} \qquad p_y = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080\,\mathrm{pix}$$

```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;

im_new = zeros(size(im));   % create a new image

for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        dx = ([j;i]-[px;py])/f;
        r = norm(dx);
        l = 1 + k*r*r;
        x = f*l*dx+[cx;cy];

        if floor(x(1))<=0 || floor(x(1))>size(im,2) || floor(x(2))<=0 || floor(x(2))>size(im,1)
            continue;
        end

        im_new(i,j,:) = im(floor(x(2)), floor(x(1)),:);
    end
end
```

$$\bar{\mathbf{u}}_{\mathrm{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\mathrm{undistorted}}$$

$$\rho = \left\| \bar{\mathbf{u}}_{\mathrm{undistorted}} \right\|$$

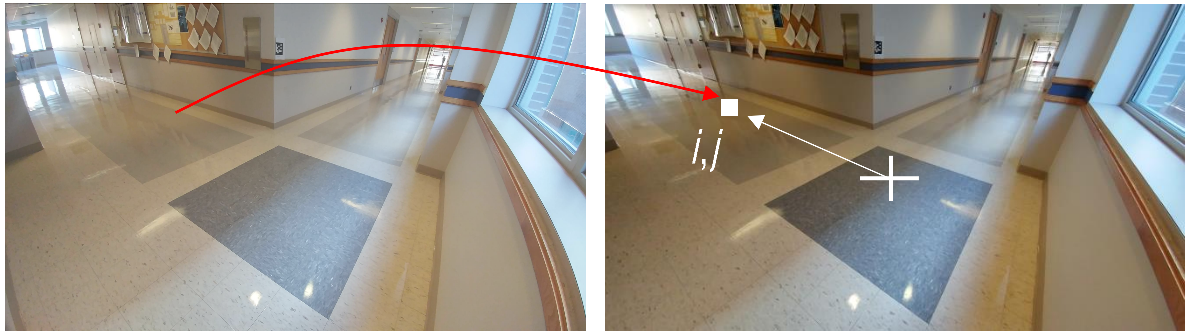
$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

$$\mathbf{K}\bar{\mathbf{u}}_{\mathrm{distorted}} = \mathbf{u}_{\mathrm{distorted}}$$

Radial Distortion Model

Assumption: Lens distortion is a function of distance from the principal point.

Distorted image　　　　Undistorted image



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153\frac{3840}{0.0048} = 1224\text{pix}$$

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```
im = imread('image.jpg');
f = 1224;
k = -0.08;
px = size(im,2)/2;
py = size(im,1)/2;

im_new = zeros(size(im));   % create a new image

for i = 1 : size(im,1)
   for j = 1 : size(im,2)
      dx = ([j;i]-[px;py])/f;
      r = norm(dx);
      l = 1 + k*r*r;
      x = f*l*dx+[cx;cy];

      if floor(x(1))<=0 || floor(x(1))>size(im,2) || floor(x(2))<=0 || floor(x(2))>size(im,1)
         continue;
      end

      im_new(i,j,:) = im(floor(x(2)), floor(x(1)),:);
   end
end
```

$$\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\rho = \left\|\bar{\mathbf{u}}_{\text{undistorted}}\right\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

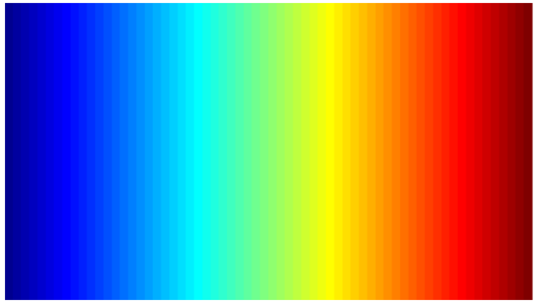
$$\mathbf{K}\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{u}_{\text{distorted}}$$

Radial Distortion Model (MATLAB Efficient)

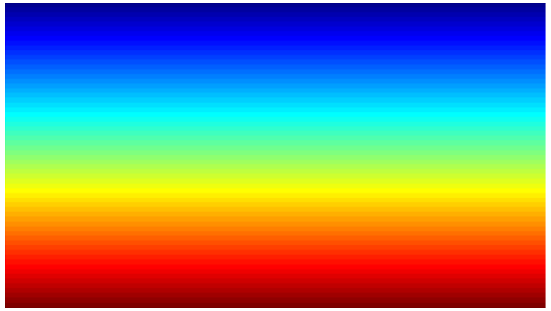
Assumption: Lens distortion is a function of distance from the principal point.

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

← XY coordinate



X



Y

$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\,\mathrm{pix}$$

$$p_{\mathrm{x}} = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920\,\mathrm{pix} \qquad p_{\mathrm{y}} = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080\,\mathrm{pix}$$

Radial Distortion Model (MATLAB Efficient)

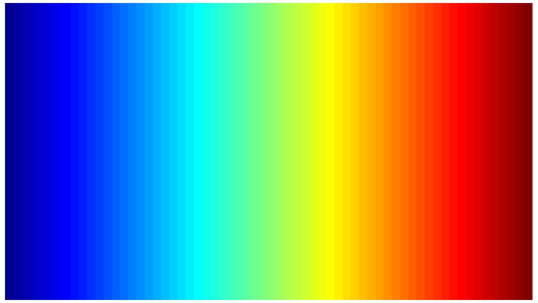
Assumption: Lens distortion is a function of distance from the principal point.

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

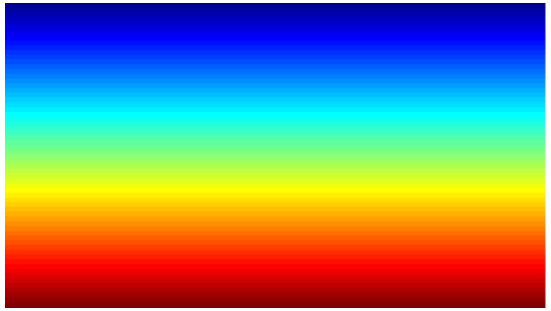
XY coordinate

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

$$\overline{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$



X



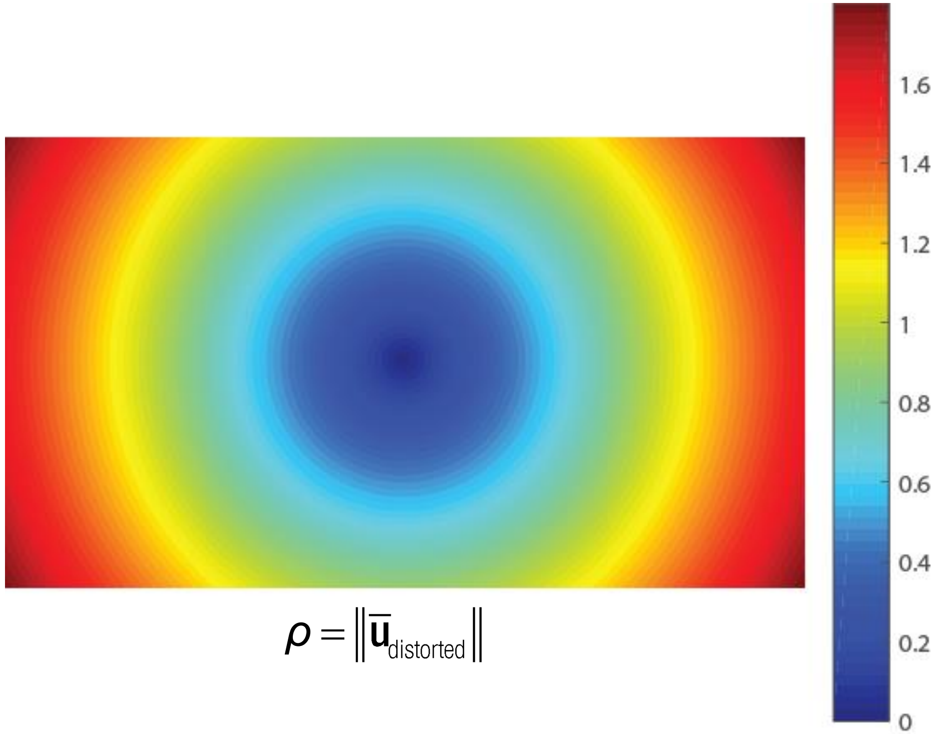
Y

$$f = f_{\text{m}}\frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153\frac{3840}{0.0048} = 1224\text{pix}$$

$$p_x = \frac{W_{\text{img}}}{2} = \frac{3840}{2} = 1920\text{pix} \qquad p_y = \frac{H_{\text{img}}}{2} = \frac{2160}{2} = 1080\text{pix}$$

Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



$\rho = \left\| \bar{\mathbf{u}}_{\text{distorted}} \right\|$

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);

X_n = (X-px)/f;
Y_n = (Y-py)/f;

r_u = sqrt(X_n.^2+Y_n.^2);
```

← XY coordinate

← $\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$

← $\rho = \left\| \bar{\mathbf{u}}_{\text{undistorted}} \right\|$

$$f = f_{\text{m}} \frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\,\text{pix}$$

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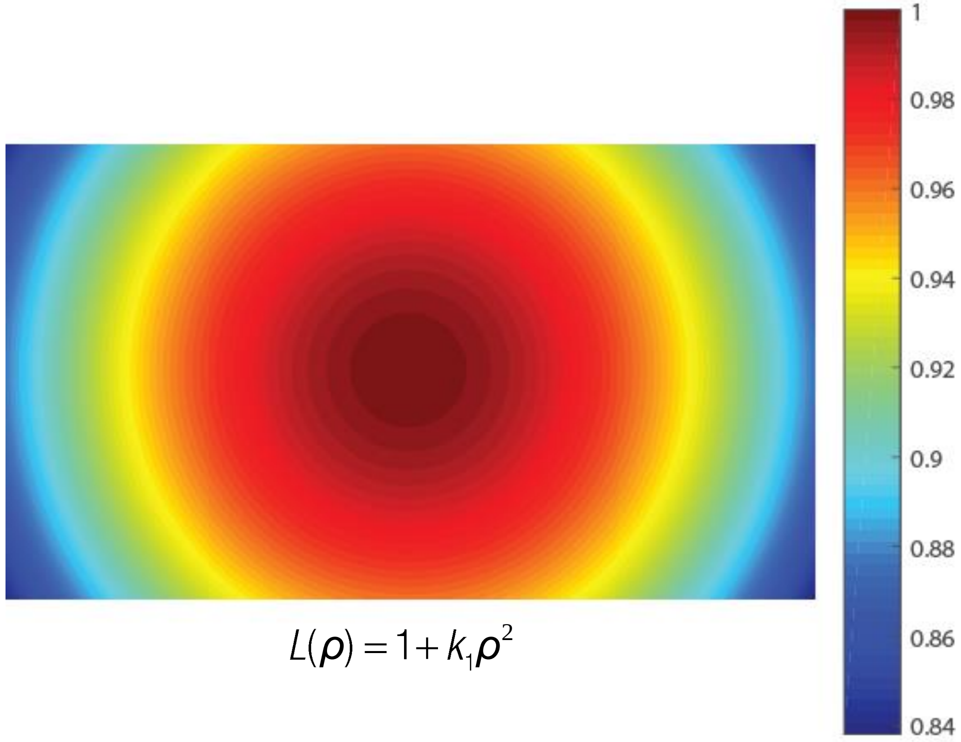
Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



$$L(\rho) = 1 + k_1\rho^2$$

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);

X_n = (X-px)/f;
Y_n = (Y-py)/f;

r_u = sqrt(X_n.^2+Y_n.^2);

L = 1 + k * r_u.^2;
```

XY coordinate

$$\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\rho = \left\|\bar{\mathbf{u}}_{\text{undistorted}}\right\|$$

$$L(\rho) = 1 + k_1\rho^2$$

$$f = f_{\text{m}}\frac{W_{\text{img}}}{W_{\text{ccd}}} = 0.00153\frac{3840}{0.0048} = 1224\,\text{pix}$$

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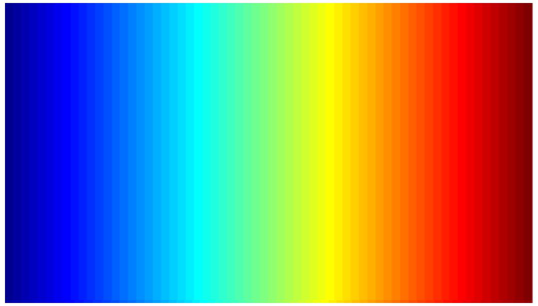
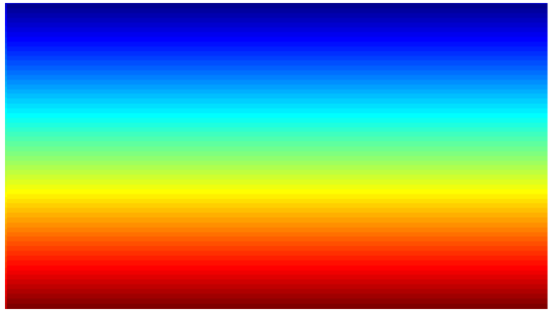
Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



X



Y

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

XY coordinate

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

$$\bar{\mathbf{u}}_{\mathrm{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\mathrm{undistorted}}$$

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

$$\rho = \left\| \bar{\mathbf{u}}_{\mathrm{undistorted}} \right\|$$

```
L = 1 + k * r_u.^2;
```

$$L(\rho) = 1 + k_1 \rho^2$$

```
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
```

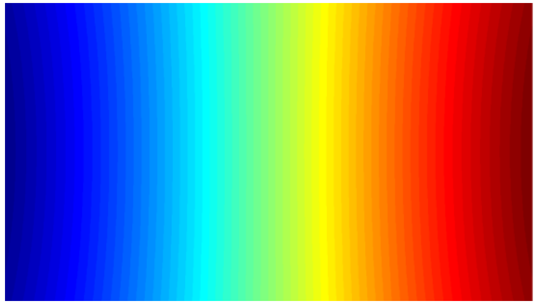
$$\mathbf{K}\bar{\mathbf{u}}_{\mathrm{distorted}} = \mathbf{u}_{\mathrm{distorted}}$$

$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\,\mathrm{pix}$$

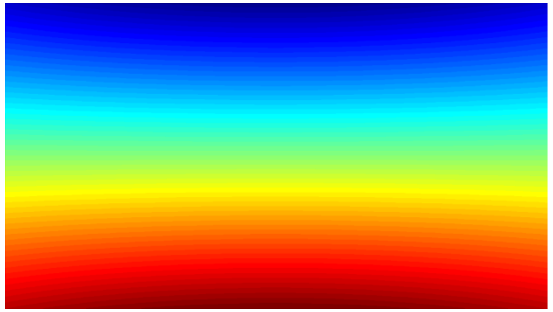
$$p_{\mathrm{x}} = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920\,\mathrm{pix} \qquad p_{\mathrm{y}} = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080\,\mathrm{pix}$$

Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



X_{dist}



Y_{dist}

```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

← XY coordinate

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

← $\bar{\mathbf{u}}_{\mathrm{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\mathrm{undistorted}}$

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

← $\rho = \left\| \bar{\mathbf{u}}_{\mathrm{undistorted}} \right\|$

```
L = 1 + k * r_u.^2;
```

← $L(\rho) = 1 + k_1\rho^2$

```
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
```

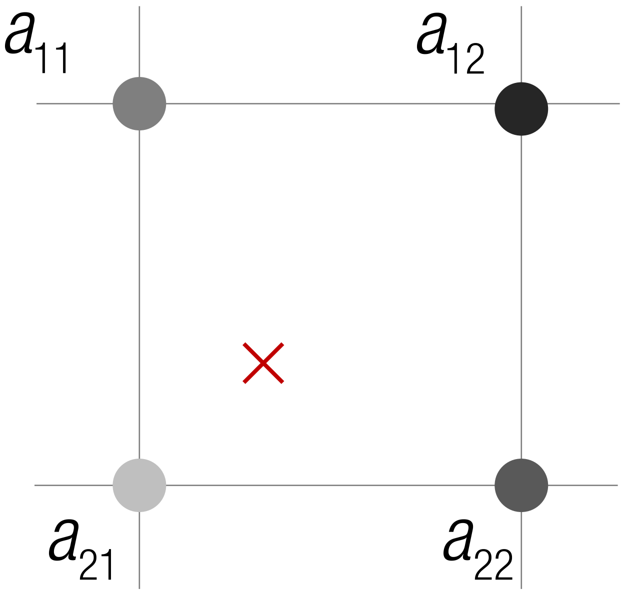
← $\mathbf{K}\bar{\mathbf{u}}_{\mathrm{distorted}} = \mathbf{u}_{\mathrm{distorted}}$

$$f = f_{\mathrm{m}} \frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153 \frac{3840}{0.0048} = 1224\mathrm{pix}$$

$$p_x = \frac{W_{\mathrm{img}}}{2} = \frac{3840}{2} = 1920\mathrm{pix} \qquad p_y = \frac{H_{\mathrm{img}}}{2} = \frac{2160}{2} = 1080\mathrm{pix}$$

Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



$$f = f_m \frac{W_{img}}{W_{ccd}} = 0.00153 \frac{3840}{0.0048} = 1224\text{pix}$$

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```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

← XY coordinate

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

← $\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

← $\rho = \left\|\bar{\mathbf{u}}_{\text{distorted}}\right\|$

```
L = 1 + k * r_u.^2;
```

← $L(\rho) = 1 + k_1\rho^2$

```
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
```

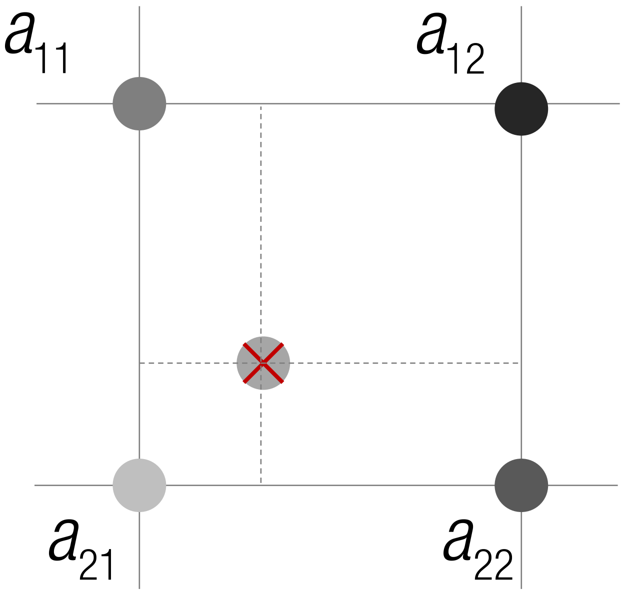
← $\mathbf{K}\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{u}_{\text{distorted}}$

```
imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), X_dist(:), Y_dist(:)), [h, w]);
```

$\underbrace{\texttt{interp2(im(:,:,3), X\_dist(:), Y\_dist(:))}}_{\text{Bilinear interpolation}}$

Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.



$$f = f_{\mathrm{m}}\frac{W_{\mathrm{img}}}{W_{\mathrm{ccd}}} = 0.00153\frac{3840}{0.0048} = 1224\,\mathrm{pix}$$

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```
[[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
```

← XY coordinate

```
X_n = (X-px)/f;
Y_n = (Y-py)/f;
```

← $\bar{\mathbf{u}}_{\mathrm{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\mathrm{undistorted}}$

```
r_u = sqrt(X_n.^2+Y_n.^2);
```

← $\rho = \left\|\bar{\mathbf{u}}_{\mathrm{distorted}}\right\|$

```
L = 1 + k * r_u.^2;
```

← $L(\rho) = 1 + k_1\rho^2$

```
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
```

← $\mathbf{K}\bar{\mathbf{u}}_{\mathrm{distorted}} = \mathbf{u}_{\mathrm{distorted}}$

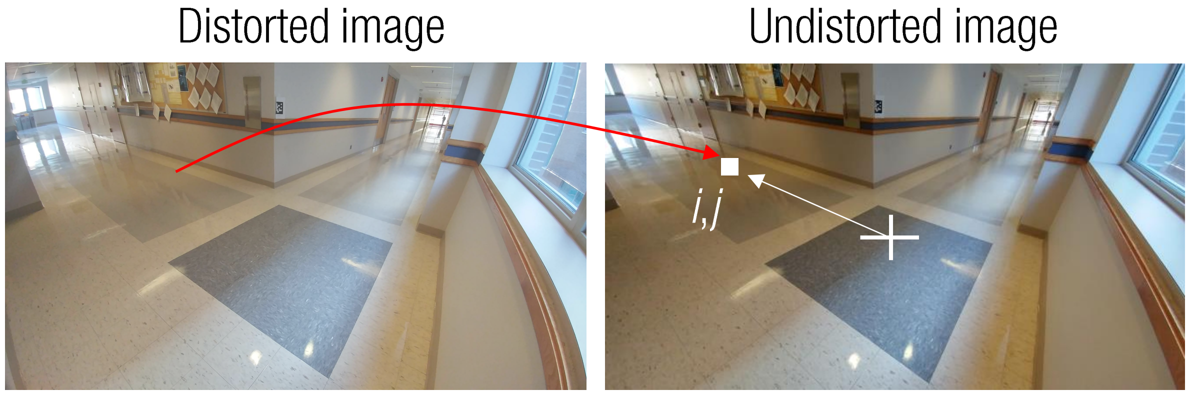
```
imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), X_dist(:), Y_dist(:)), [h, w]);
```

Bilinear interpolation

UndistortImageRadial.m

Radial Distortion Model (MATLAB Efficient)

Assumption: Lens distortion is a function of distance from the principal point.




```
[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);
X = X(:);
Y = Y(:);

pt = [X'; Y'];
pt = bsxfun(@minus, pt, [px;py]);
pt = bsxfun(@rdivide, pt, [f;f]);
r_u = sqrt(sum(pt.^2, 1));
pt = bsxfun(@times, pt, 1 + k * r_u.^2);
pt = bsxfun(@times, pt, [f;f]);
pt = bsxfun(@plus, pt, [px;py]);

imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), pt(1,:), pt(2,:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), pt(1,:), pt(2,:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), pt(1,:), pt(2,:)), [h, w]);
```

Bilinear interpolation

$$\bar{\mathbf{u}}_{\text{undistorted}} = \mathbf{K}^{-1}\mathbf{u}_{\text{undistorted}}$$

$$\rho = \|\bar{\mathbf{u}}_{\text{distorted}}\|$$

$$L(\rho) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots$$

$$\mathbf{K}\bar{\mathbf{u}}_{\text{distorted}} = \mathbf{u}_{\text{distorted}}$$

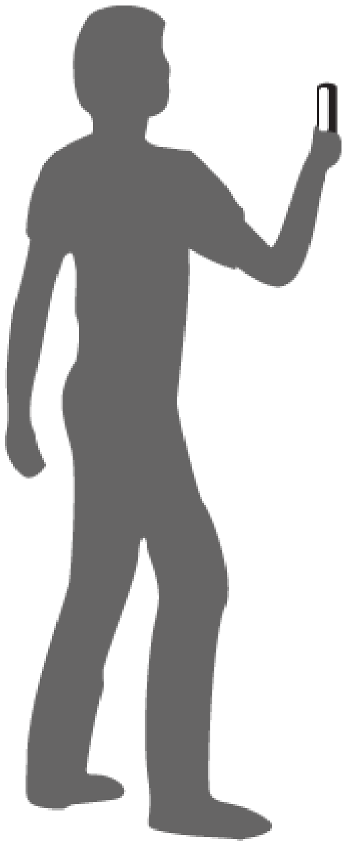
UndistortImageRadial.m

Lens Radial Distortion Correction

Practice with Your Cellphone Camera

Code download: http://www-users.cs.umn.edu/~hspark/CSci5980/code/

UndistortImageRadial.m

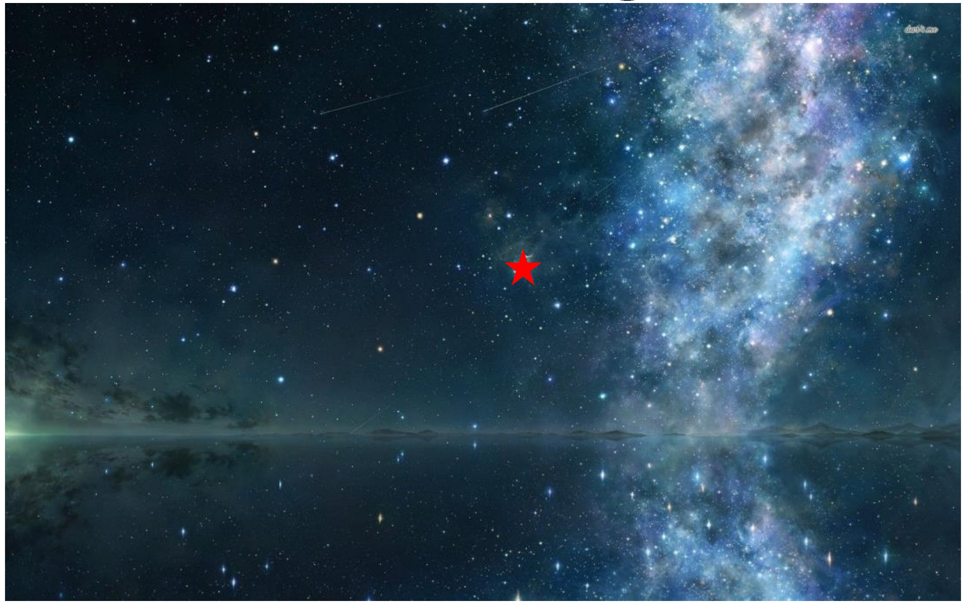


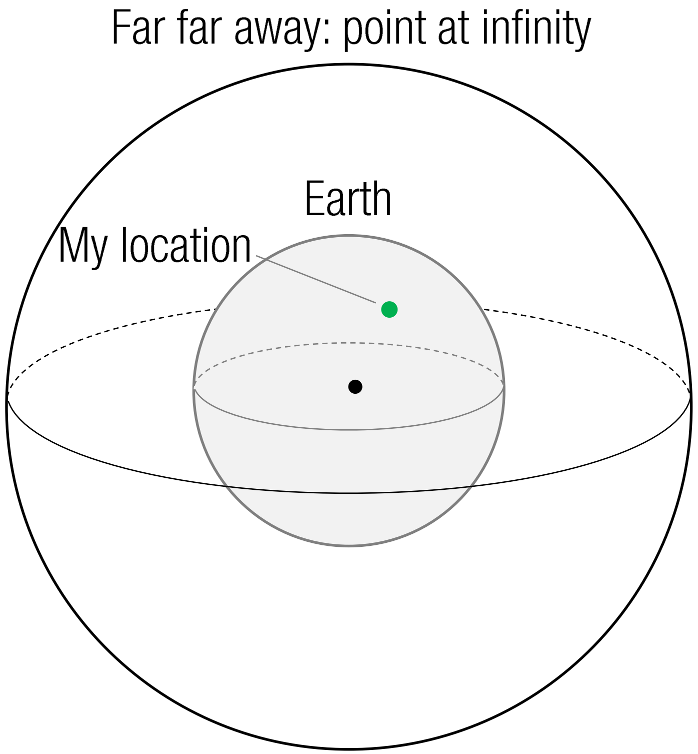
```
im = imread('1227161240_HDR.jpg');
im = double(im);
k = -0.05;
f = 1224;
px = size(im,2)/2;
py = size(im,1)/2;

[X, Y] = meshgrid(1:(size(im,2)), 1:(size(im,1)));
h = size(X, 1); w = size(X,2);

X_n = (X-px)/f;
Y_n = (Y-py)/f;
r_u = sqrt(X_n.^2+Y_n.^2);
L = 1 + k * r_u.^2;
X_dist_n = X_n.* L;
Y_dist_n = Y_n.* L;
X_dist = X_dist_n*f + px;
Y_dist = Y_dist_n*f + py;

imUndistortion(:,:,1) = reshape(interp2(im(:,:,1), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,2) = reshape(interp2(im(:,:,2), X_dist(:), Y_dist(:)), [h, w]);
imUndistortion(:,:,3) = reshape(interp2(im(:,:,3), X_dist(:), Y_dist(:)), [h, w]);
```

Projective Line

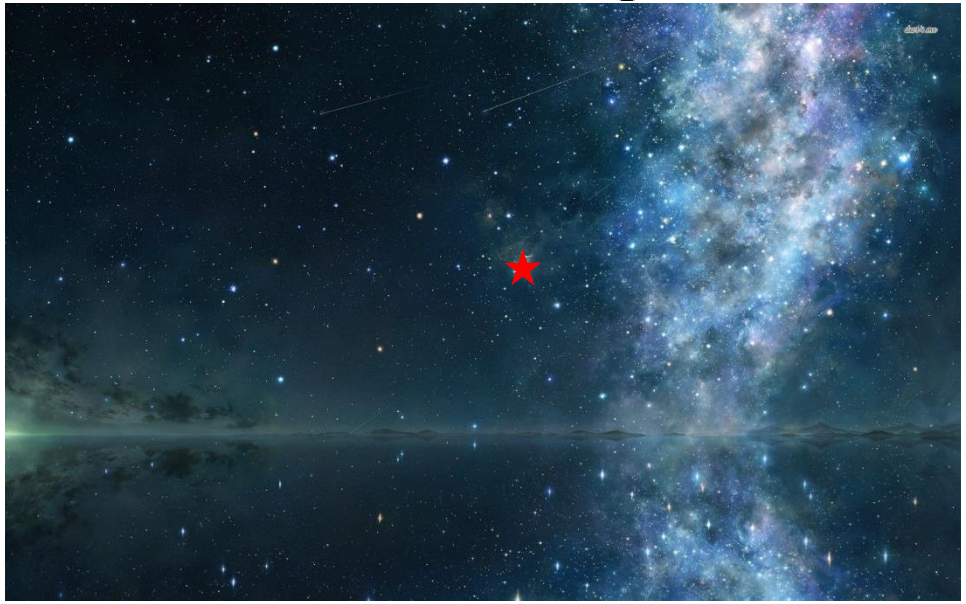

Where am I?

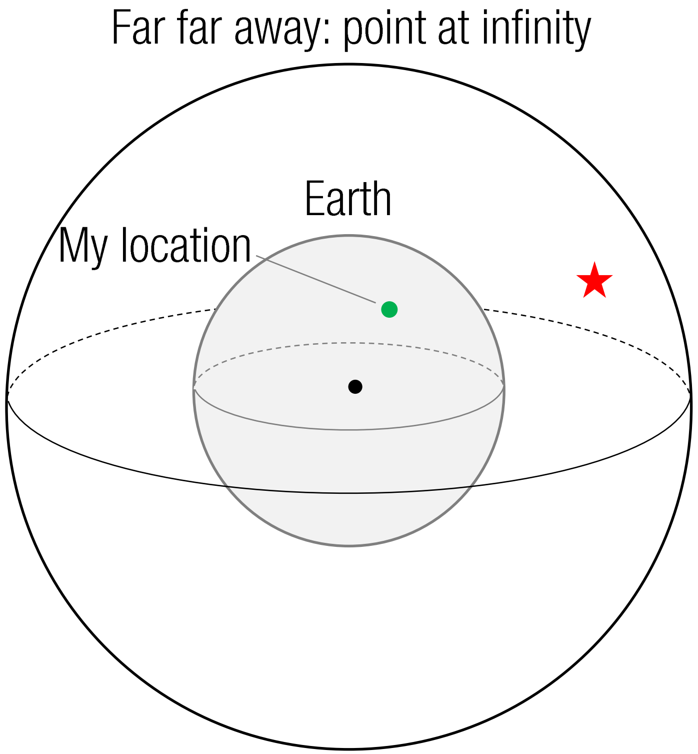
Celestial Navigation



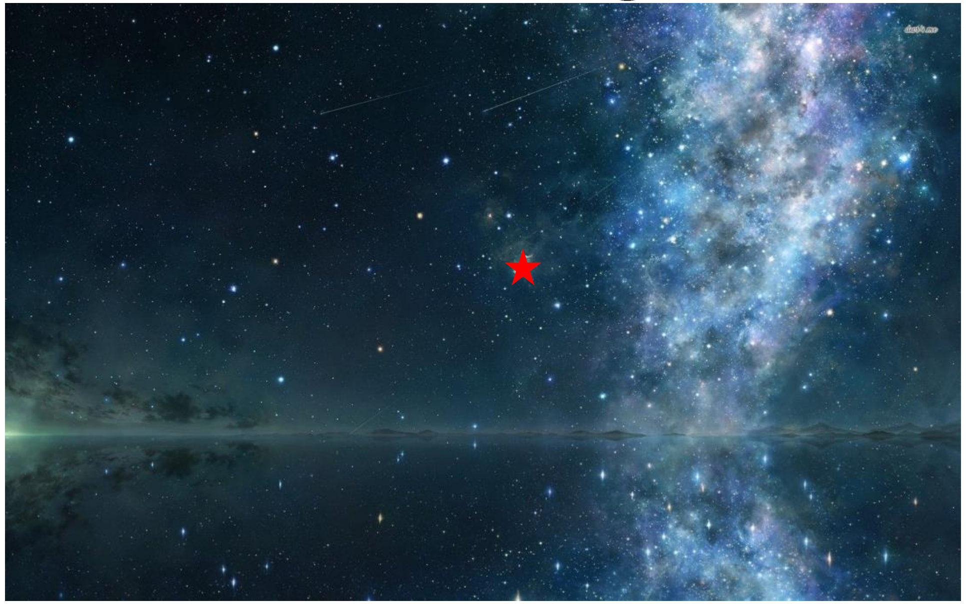


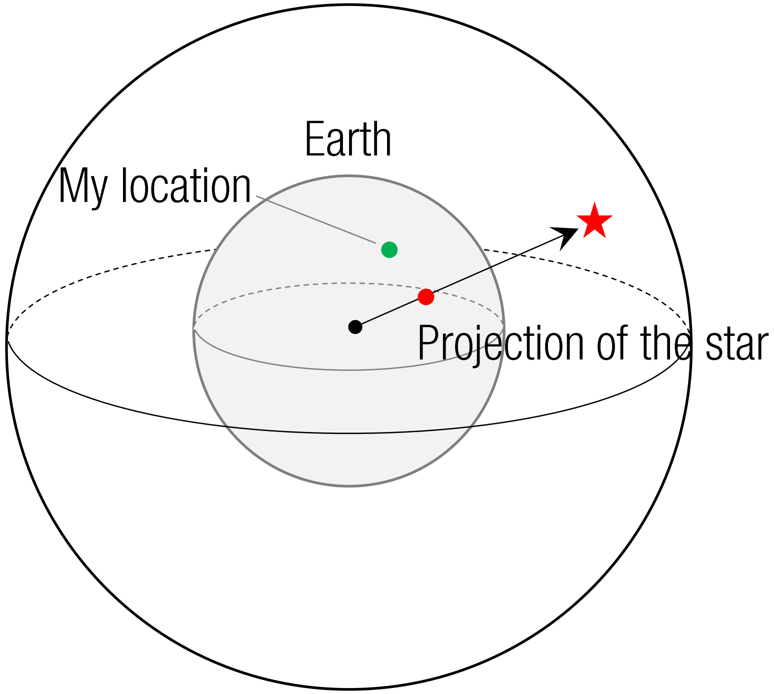
Celestial Navigation



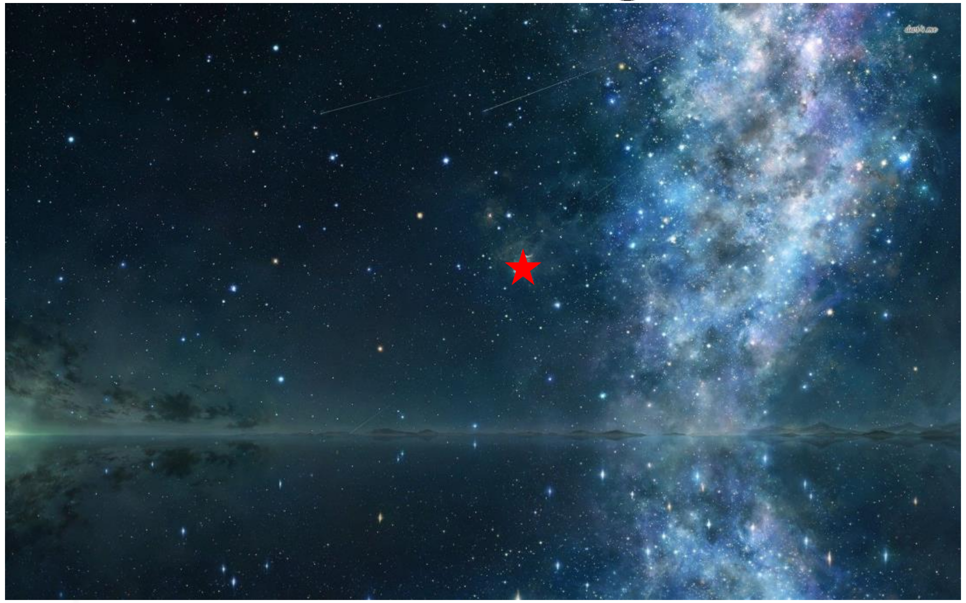


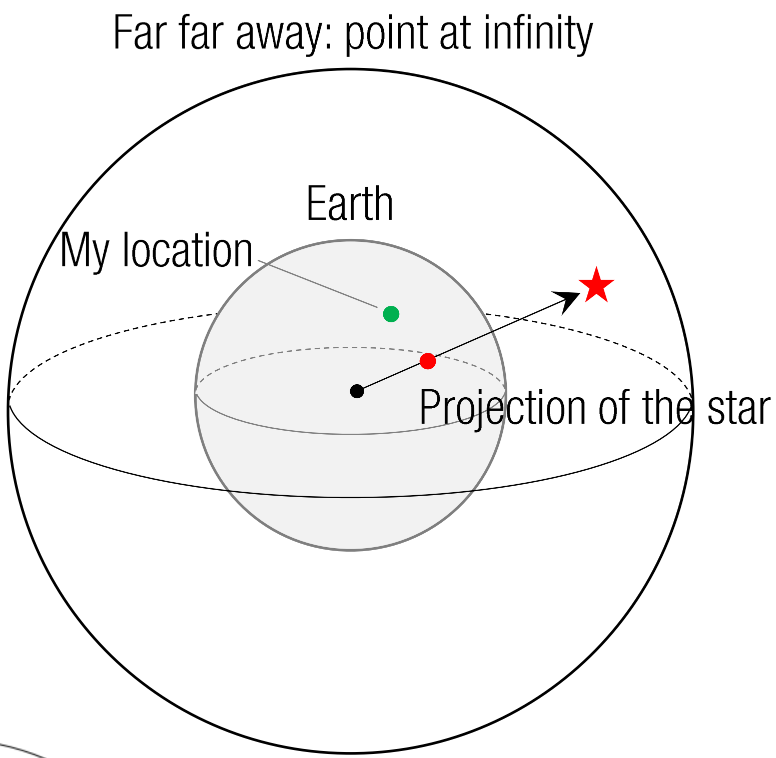
Celestial Navigation





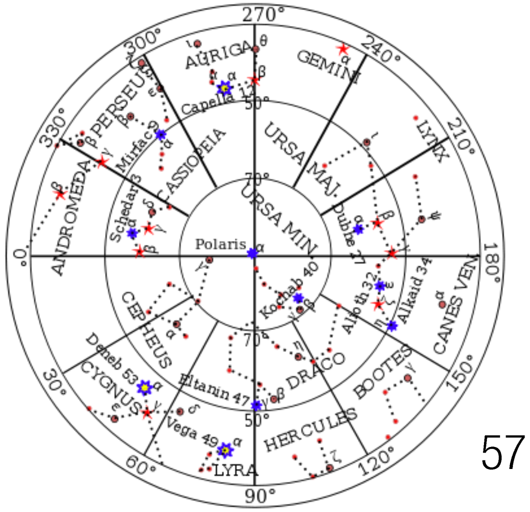
Celestial Navigation





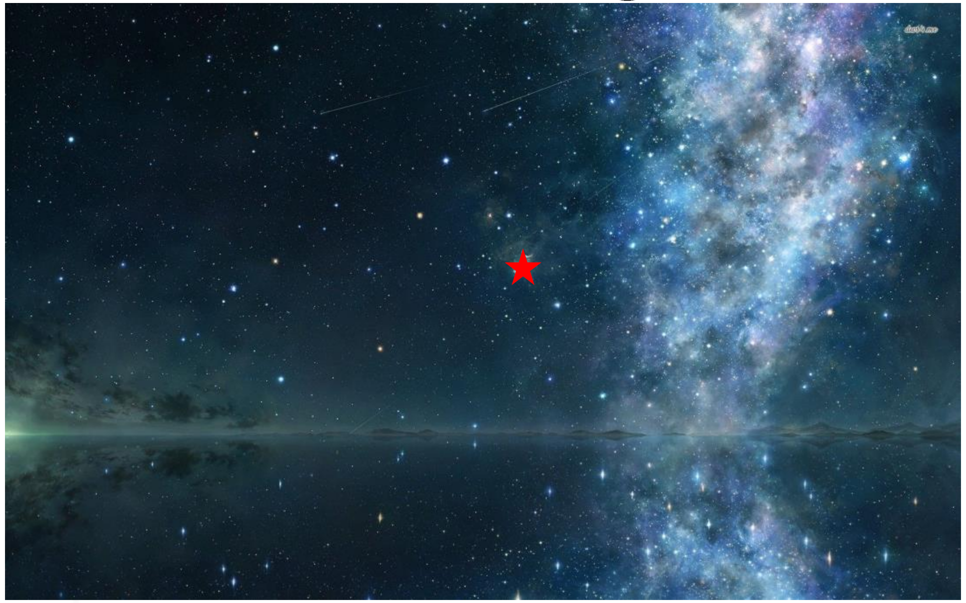
292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

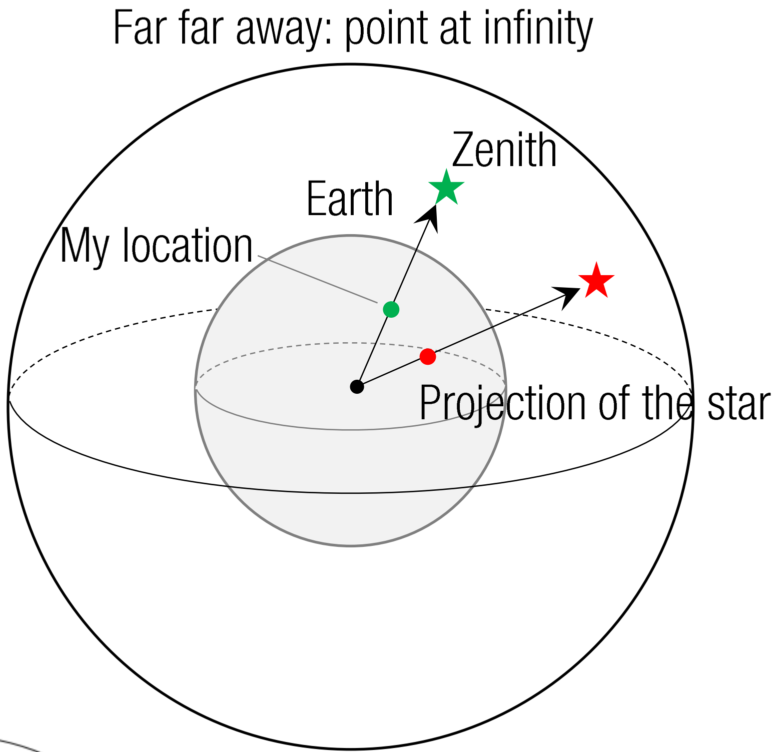
GCT	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS–3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER–1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twil.	Moon-rise	Diff.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12 00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
10	3 17		65 56	44 15		315 41		339 34		232 48	36					
20	5 47		68 26	46 45		318 11		342 04		235 13	38					
30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13 00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14 00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15 00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
10	48 17		111 03	89 15		0 43		24 40		276 15	09					
20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16 00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30

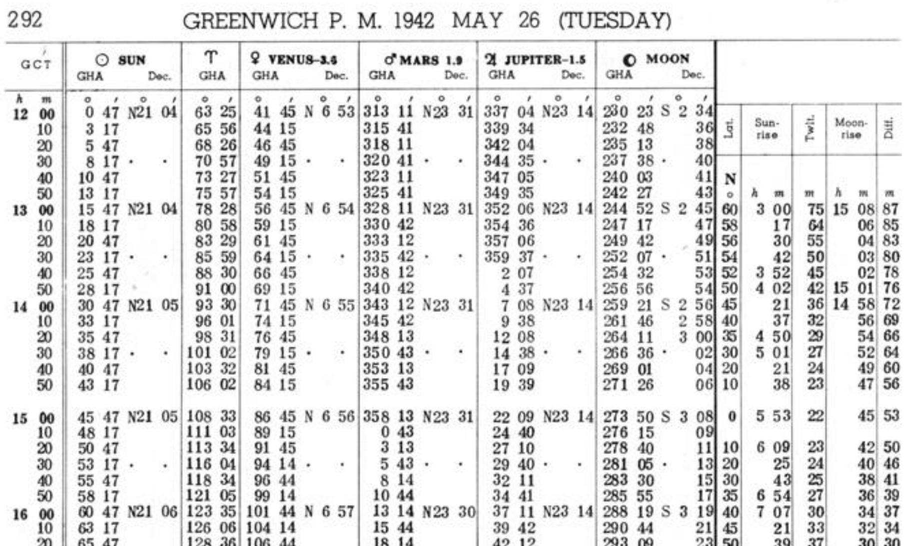


57 stars for navigation

Celestial Navigation

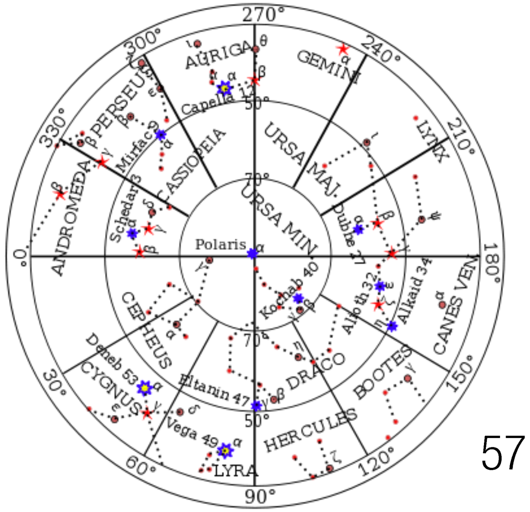






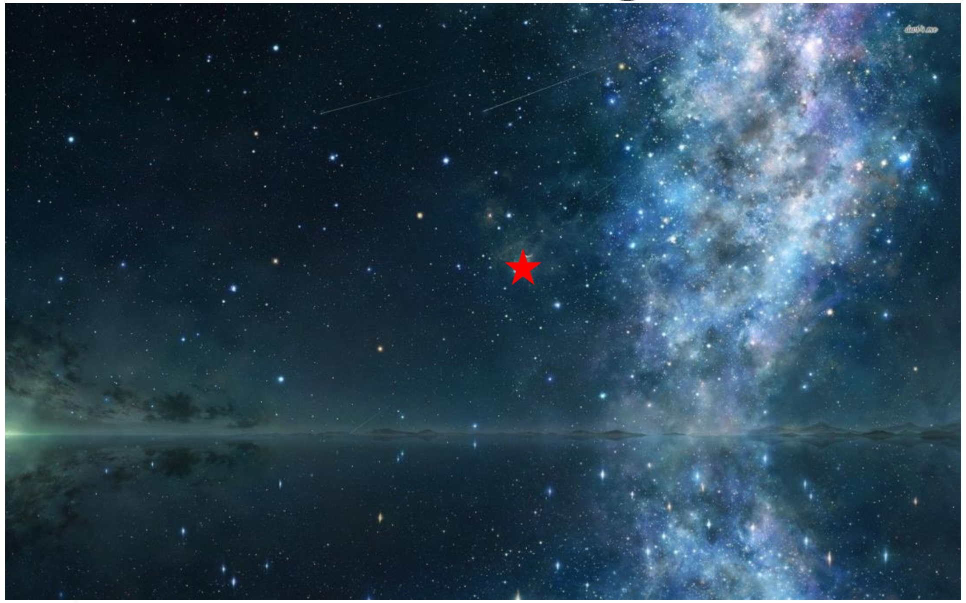
292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS–3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER–1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twilt.	Moon-rise	Diff.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12 00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
10	3 17		65 56	44 15		315 41		339 34		232 48	36					
20	5 47		68 26	46 45		318 11		342 04		235 13	38					
30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13 00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14 00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15 00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
10	48 17		111 03	89 15		0 43		24 40		276 15	09					
20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16 00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30



57 stars for navigation

Celestial Navigation



292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS-3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER-1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twilt.	Moon-rise	Diff.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12 00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
10	3 17		65 56	44 15		315 41		339 34		232 48	36					
20	5 47		68 26	46 45		318 11		342 04		235 13	38					
30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13 00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14 00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15 00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
10	48 17		111 03	89 15		0 43		24 40		276 15	09					
20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
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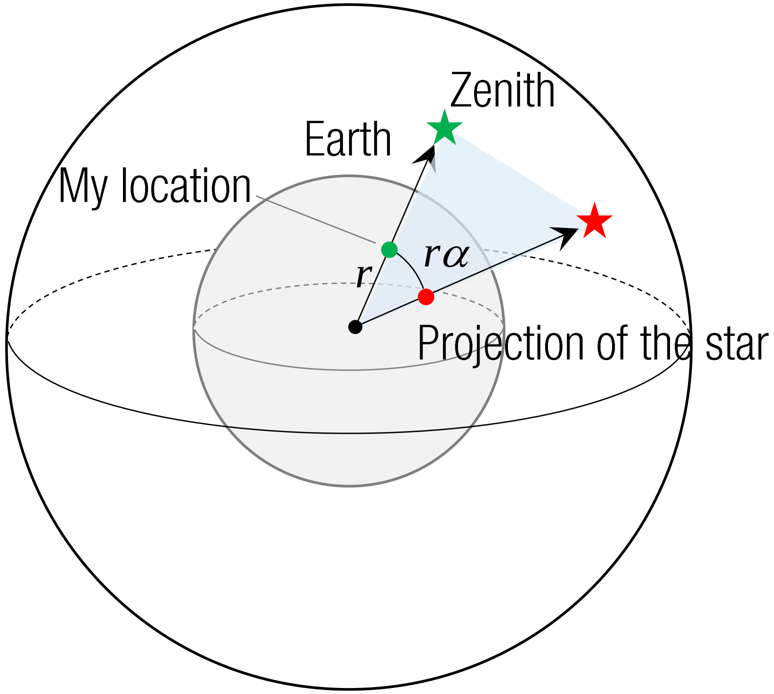
16 00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30

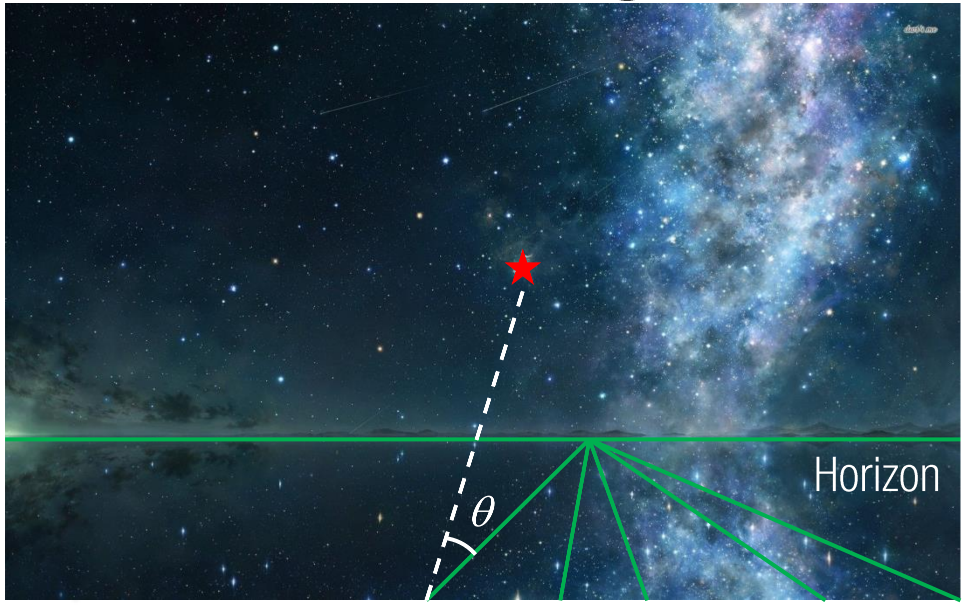
Far far away: point at infinity



r : radius of earth

α : angle between stars

Celestial Navigation



292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT h	m	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS–3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER–1.5 GHA	JUPITER Dec.	◐ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twil.	Moon-rise	Diff.
h	m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12	00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
	10	3 17		65 56	44 15		315 41		339 34		232 48	36					
	20	5 47		68 26	46 45		318 11		342 04		235 13	38					
	30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
	40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
	50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13	00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
	10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
	20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
	30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
	40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
	50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14	00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
	10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
	20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
	30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
	40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
	50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15	00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
	10	48 17		111 03	89 15		0 43		24 40		276 15	09					
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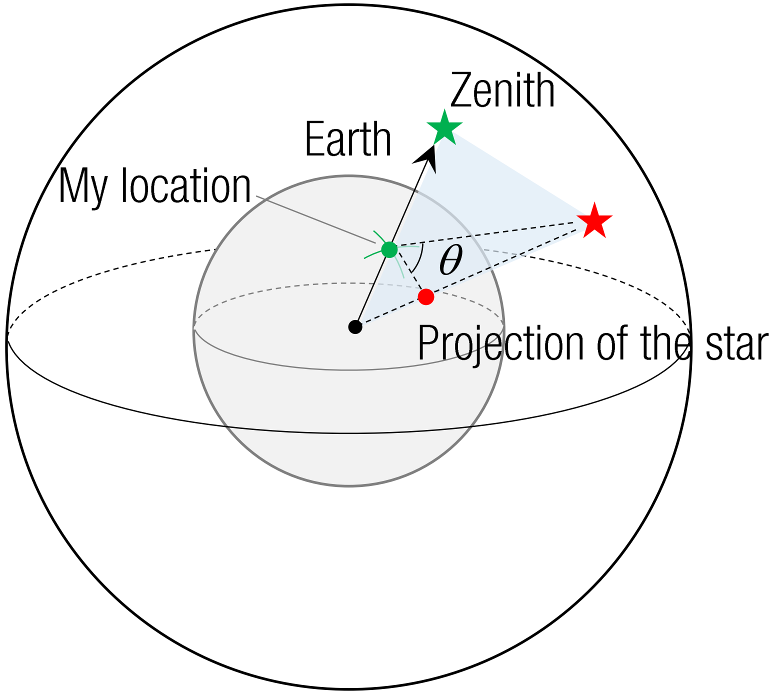
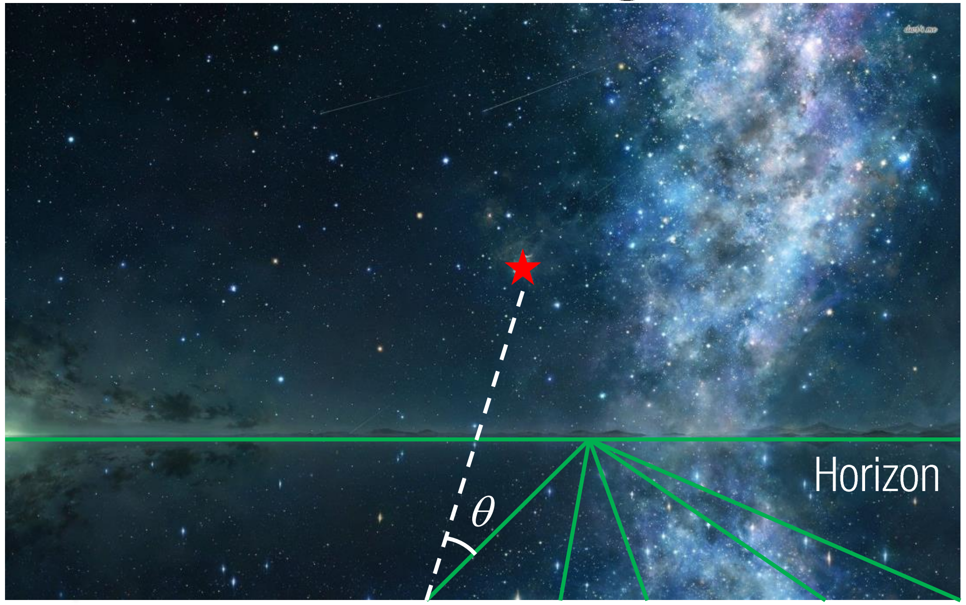
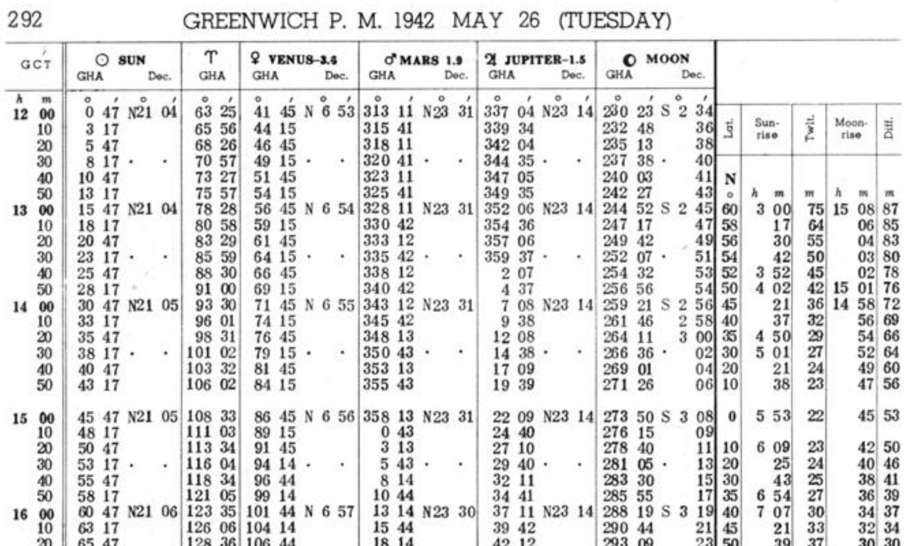
	30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
	40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
	50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16	00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
	10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
	20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30

Far far away: point at infinity



Celestial Navigation

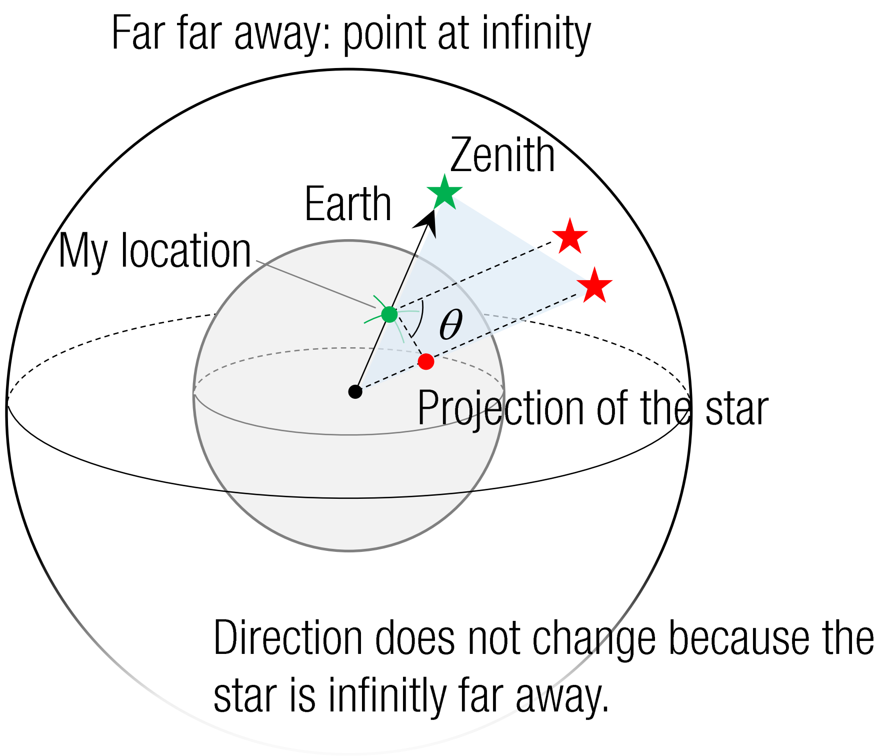




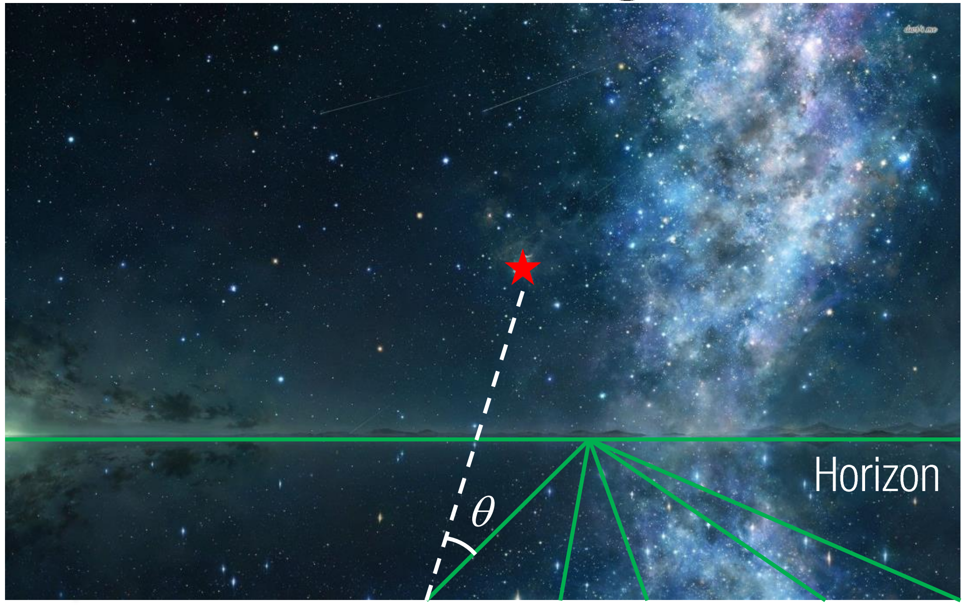
292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

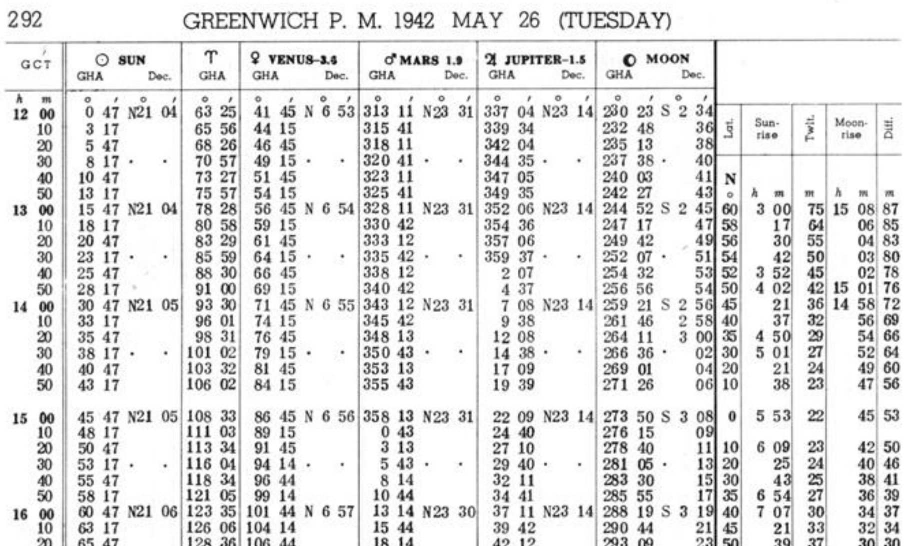
GCT h	m	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS–3.4 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER–1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.
h	m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
12	00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34
	10	3 17		65 56	44 15		315 41		339 34		232 48	36
	20	5 47		68 26	46 45		318 11		342 04		235 13	38
	30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40
	40	10 47		73 27	51 45		323 11		347 05		240 03	41
	50	13 17		75 57	54 15		325 41		349 35		242 27	43
13	00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45
	10	18 17		80 58	59 15		330 42		354 36		247 17	47
	20	20 47		83 29	61 45		333 12		357 06		249 42	49
	30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51
	40	25 47		88 30	66 45		338 12		2 07		254 32	53
	50	28 17		91 00	69 15		340 42		4 37		256 56	54
14	00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56
	10	33 17		96 01	74 15		345 42		9 38		261 46	2 58
	20	35 47		98 31	76 45		348 13		12 08		264 11	3 00
	30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02
	40	40 47		103 32	81 45		353 13		17 09		269 01	04
	50	43 17		106 02	84 15		355 43		19 39		271 26	06
15	00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08
	10	48 17		111 03	89 15		0 43		24 40		276 15	09
	20	50 47		113 34	91 45		3 13		27 10		278 40	11
	30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13
	40	55 47		118 34	96 44		8 14		32 11		283 30	15
	50	58 17		121 05	99 14		10 44		34 41		285 55	17
16	00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19
	10	63 17		126 06	104 14		15 44		39 42		290 44	21
	20	65 47		128 36	106 44		18 14		42 12		293 09	23

Lat.	Sun-rise	Twilt.	Moon-rise	Diff.
N °	h m	m	h m	m
60	3 00	75	15 08	87
58	17	64	06	85
56	30	55	04	83
54	42	50	03	80
52	3 52	45	02	78
50	4 02	42	15 01	76
45	21	36	14 58	72
40	37	32	56	69
35	4 50	29	54	66
30	5 01	27	52	64
20	21	24	49	60
10	38	23	47	56
0	5 53	22	45	53
10	6 09	23	42	50
20	25	24	40	46
30	43	25	38	41
35	6 54	27	36	39
40	7 07	30	34	37
45	21	33	32	34
50	39	37	30	30



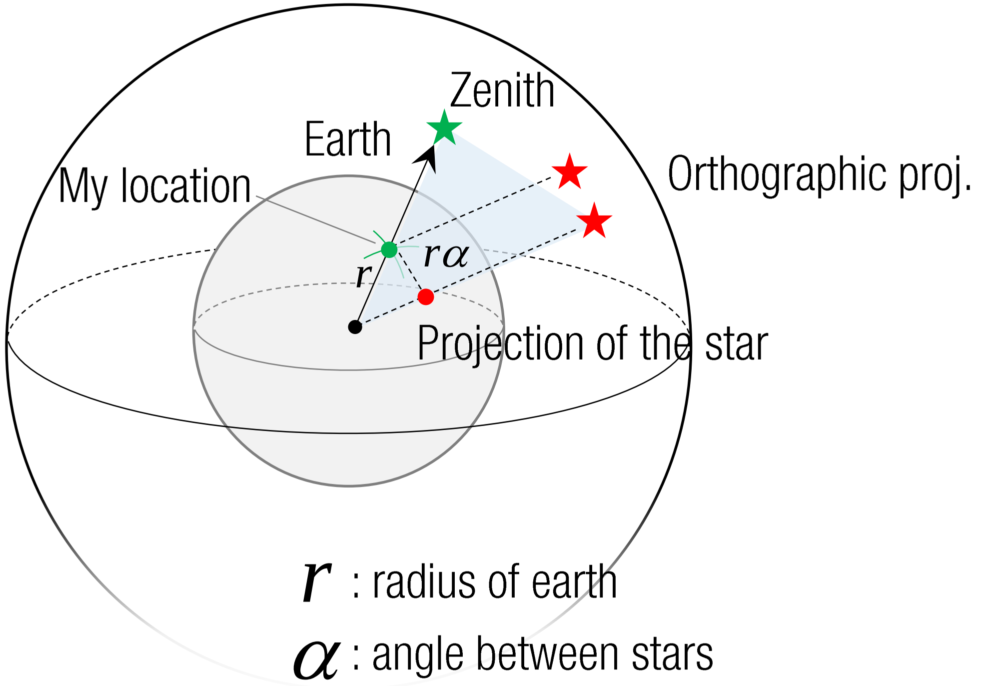
Celestial Navigation





292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

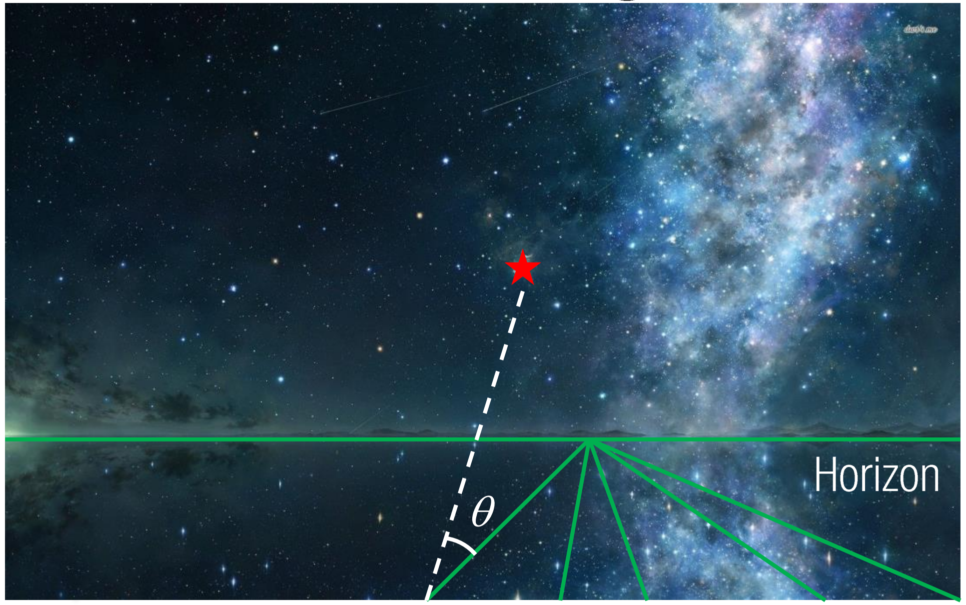
GCT	☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS-3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER-1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twil.	Moon-rise	Diff.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12 00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
10	3 17		65 56	44 15		315 41		339 34		232 48	36					
20	5 47		68 26	46 45		318 11		342 04		235 13	38					
30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13 00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14 00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15 00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
10	48 17		111 03	89 15		0 43		24 40		276 15	09					
20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16 00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30



r : radius of earth

α : angle between stars

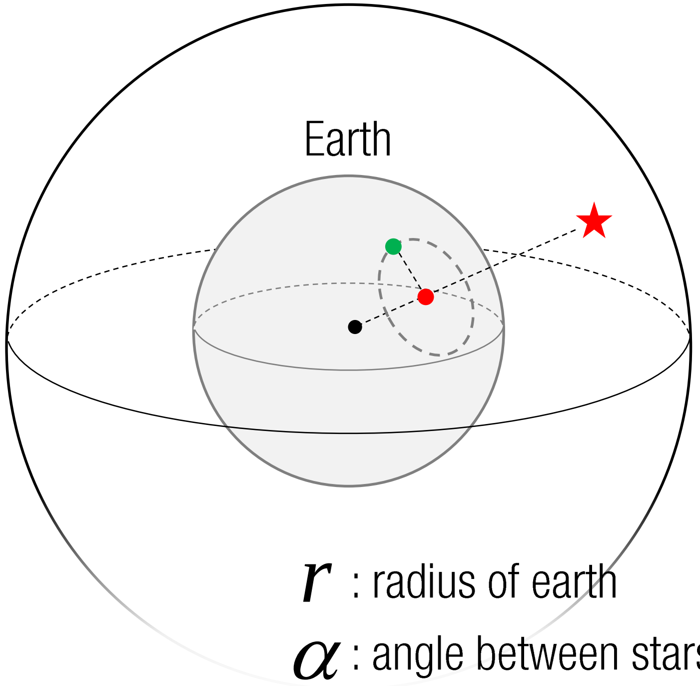
Celestial Navigation



292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

GCT h m	☉ SUN GHA ° ′	SUN Dec. ° ′	♈ GHA ° ′	♀ VENUS-3.6 GHA ° ′	VENUS Dec. ° ′	♂ MARS 1.9 GHA ° ′	MARS Dec. ° ′	♃ JUPITER-1.5 GHA ° ′	JUPITER Dec. ° ′	◐ MOON GHA ° ′	MOON Dec. ° ′	Lat. N °	Sun-rise h m	Twil. m	Moon-rise h m	Diff. m
12 00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
10	3 17		65 56	44 15		315 41		339 34		232 48	36					
20	5 47		68 26	46 45		318 11		342 04		235 13	38					
30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
40	10 47		73 27	51 45		323 11		347 05		240 03	41					
50	13 17		75 57	54 15		325 41		349 35		242 27	43					
13 00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14 00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15 00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
10	48 17		111 03	89 15		0 43		24 40		276 15	09					
20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16 00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30

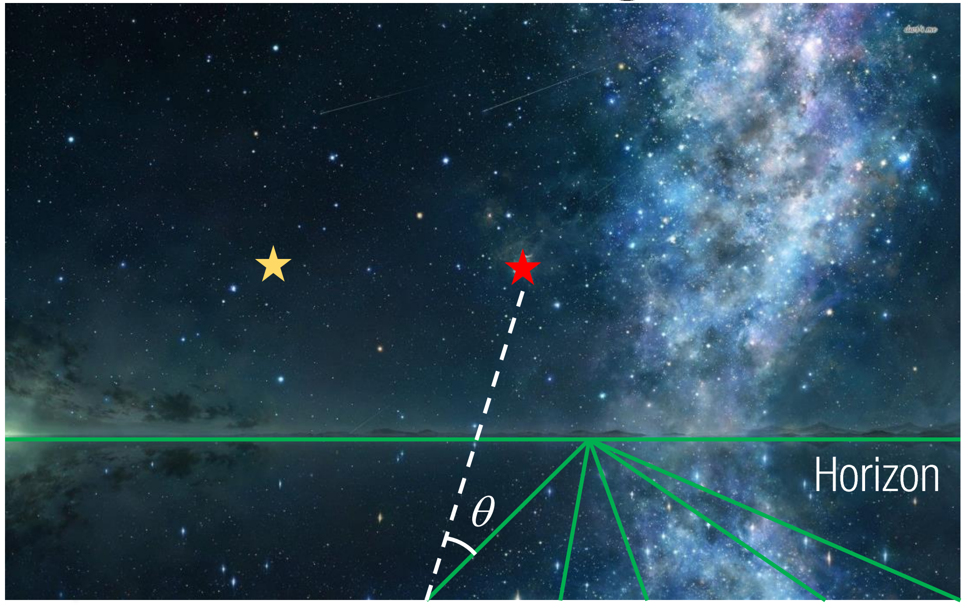
Far far away: point at infinity

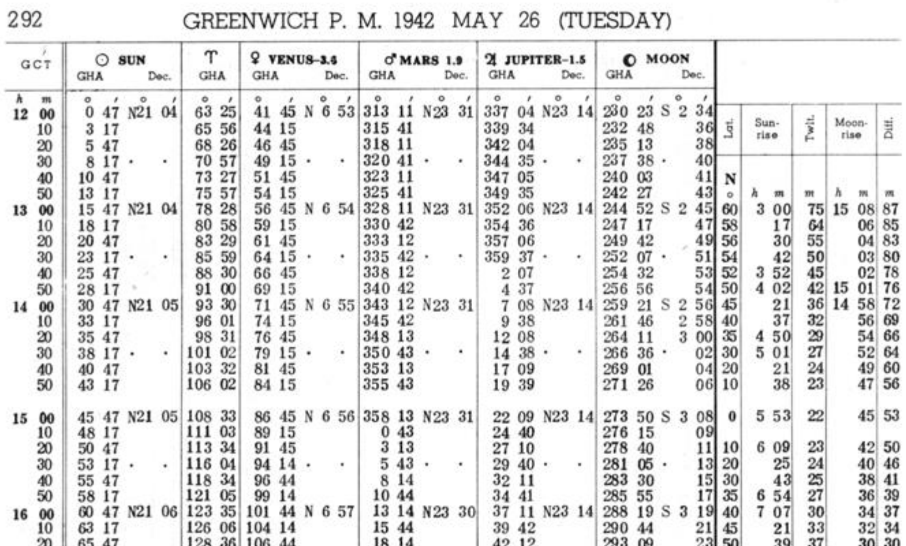


r : radius of earth

α : angle between stars

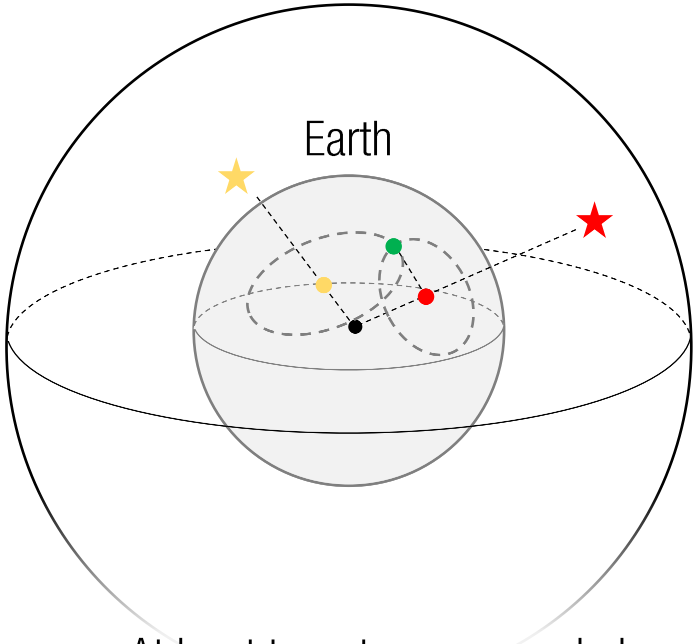
Celestial Navigation



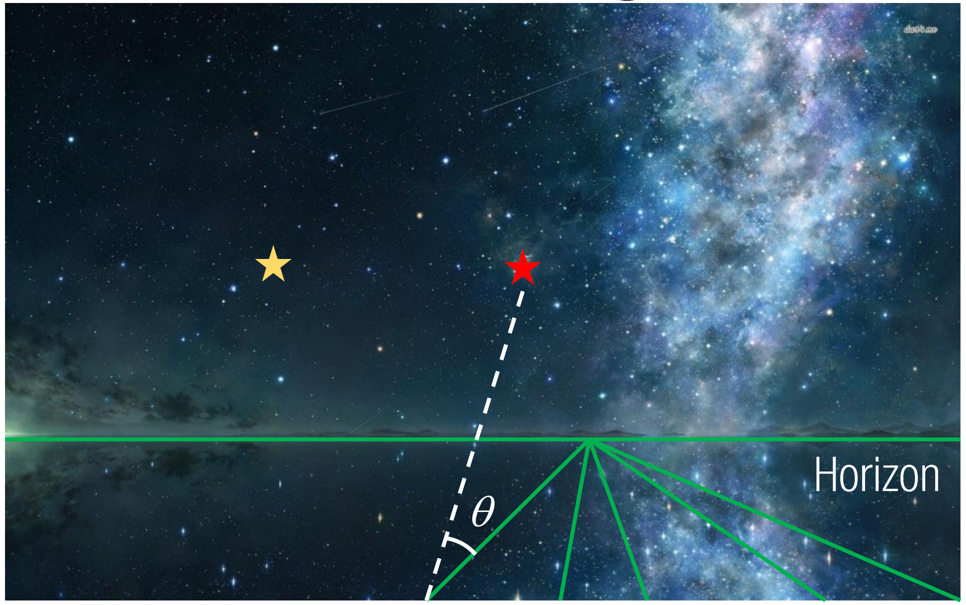


292 GREENWICH P. M. 1942 MAY 26 (TUESDAY)

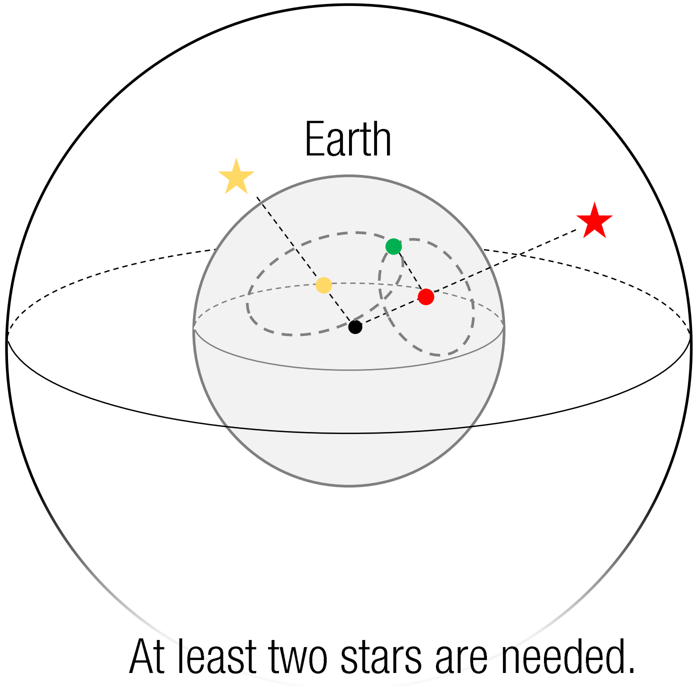
GCT		☉ SUN GHA	SUN Dec.	♈ GHA	♀ VENUS–3.6 GHA	VENUS Dec.	♂ MARS 1.9 GHA	MARS Dec.	♃ JUPITER–1.5 GHA	JUPITER Dec.	☾ MOON GHA	MOON Dec.	Lat.	Sun-rise	Twil.	Moon-rise	Diff.
h	m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′					
12	00	0 47	N21 04	63 25	41 45	N 6 53	313 11	N23 31	337 04	N23 14	230 23	S 2 34					
	10	3 17		65 56	44 15		315 41		339 34		232 48	36					
	20	5 47		68 26	46 45		318 11		342 04		235 13	38					
	30	8 17	· ·	70 57	49 15	· ·	320 41	· ·	344 35	· ·	237 38	· 40					
	40	10 47		73 27	51 45		323 11		347 05		240 03	41	N				
	50	13 17		75 57	54 15		325 41		349 35		242 27	43	°	h m	m	h m	m
13	00	15 47	N21 04	78 28	56 45	N 6 54	328 11	N23 31	352 06	N23 14	244 52	S 2 45	60	3 00	75	15 08	87
	10	18 17		80 58	59 15		330 42		354 36		247 17	47	58	17	64	06	85
	20	20 47		83 29	61 45		333 12		357 06		249 42	49	56	30	55	04	83
	30	23 17	· ·	85 59	64 15	· ·	335 42	· ·	359 37	· ·	252 07	· 51	54	42	50	03	80
	40	25 47		88 30	66 45		338 12		2 07		254 32	53	52	3 52	45	02	78
	50	28 17		91 00	69 15		340 42		4 37		256 56	54	50	4 02	42	15 01	76
14	00	30 47	N21 05	93 30	71 45	N 6 55	343 12	N23 31	7 08	N23 14	259 21	S 2 56	45	21	36	14 58	72
	10	33 17		96 01	74 15		345 42		9 38		261 46	2 58	40	37	32	56	69
	20	35 47		98 31	76 45		348 13		12 08		264 11	3 00	35	4 50	29	54	66
	30	38 17	· ·	101 02	79 15	· ·	350 43	· ·	14 38	· ·	266 36	· 02	30	5 01	27	52	64
	40	40 47		103 32	81 45		353 13		17 09		269 01	04	20	21	24	49	60
	50	43 17		106 02	84 15		355 43		19 39		271 26	06	10	38	23	47	56
15	00	45 47	N21 05	108 33	86 45	N 6 56	358 13	N23 31	22 09	N23 14	273 50	S 3 08	0	5 53	22	45	53
	10	48 17		111 03	89 15		0 43		24 40		276 15	09					
	20	50 47		113 34	91 45		3 13		27 10		278 40	11	10	6 09	23	42	50
	30	53 17	· ·	116 04	94 14	· ·	5 43	· ·	29 40	· ·	281 05	· 13	20	25	24	40	46
	40	55 47		118 34	96 44		8 14		32 11		283 30	15	30	43	25	38	41
	50	58 17		121 05	99 14		10 44		34 41		285 55	17	35	6 54	27	36	39
16	00	60 47	N21 06	123 35	101 44	N 6 57	13 14	N23 30	37 11	N23 14	288 19	S 3 19	40	7 07	30	34	37
	10	63 17		126 06	104 14		15 44		39 42		290 44	21	45	21	33	32	34
	20	65 47		128 36	106 44		18 14		42 12		293 09	23	50	39	37	30	30



Celestial Navigation

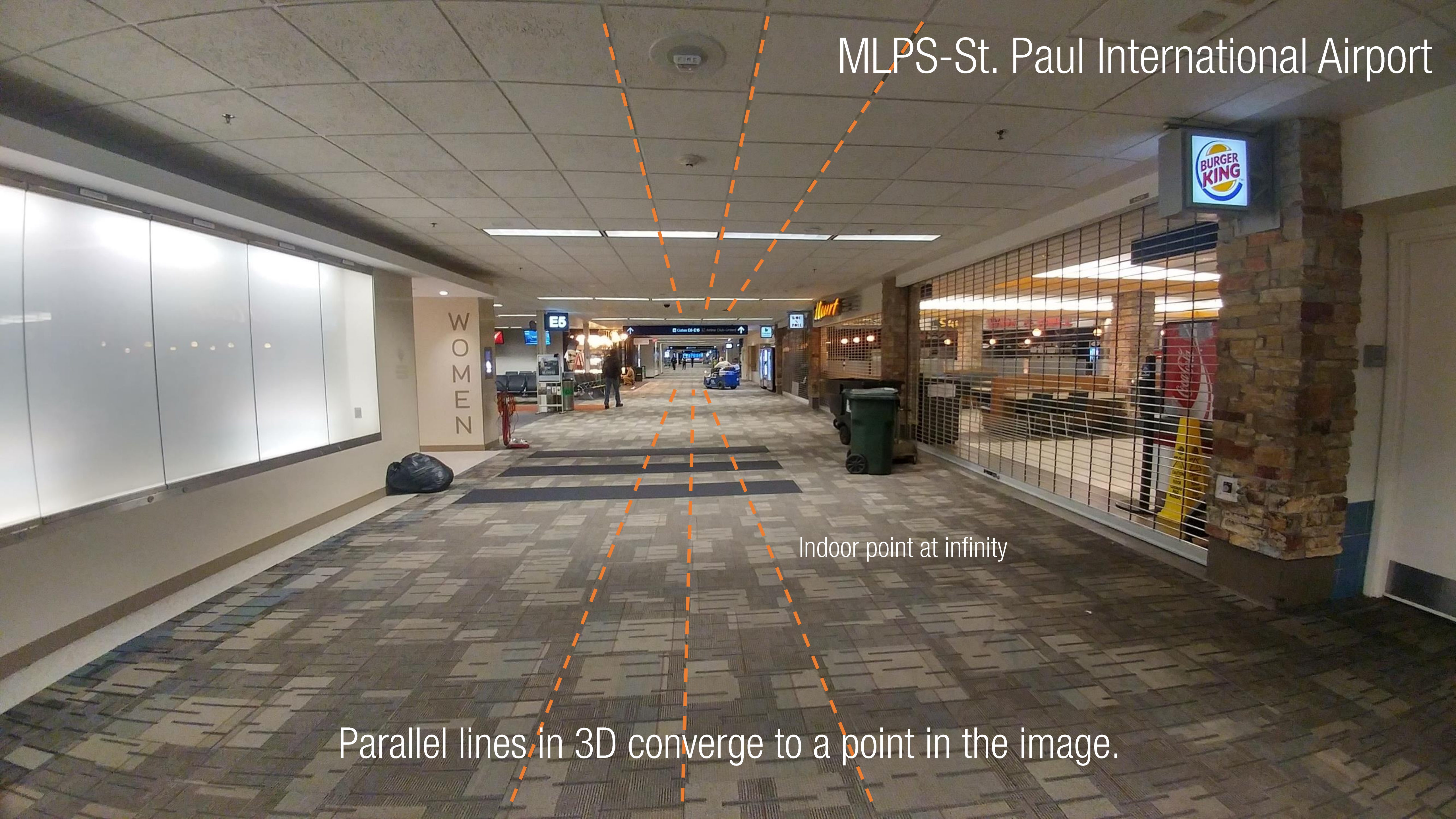


Far far away: point at infinity

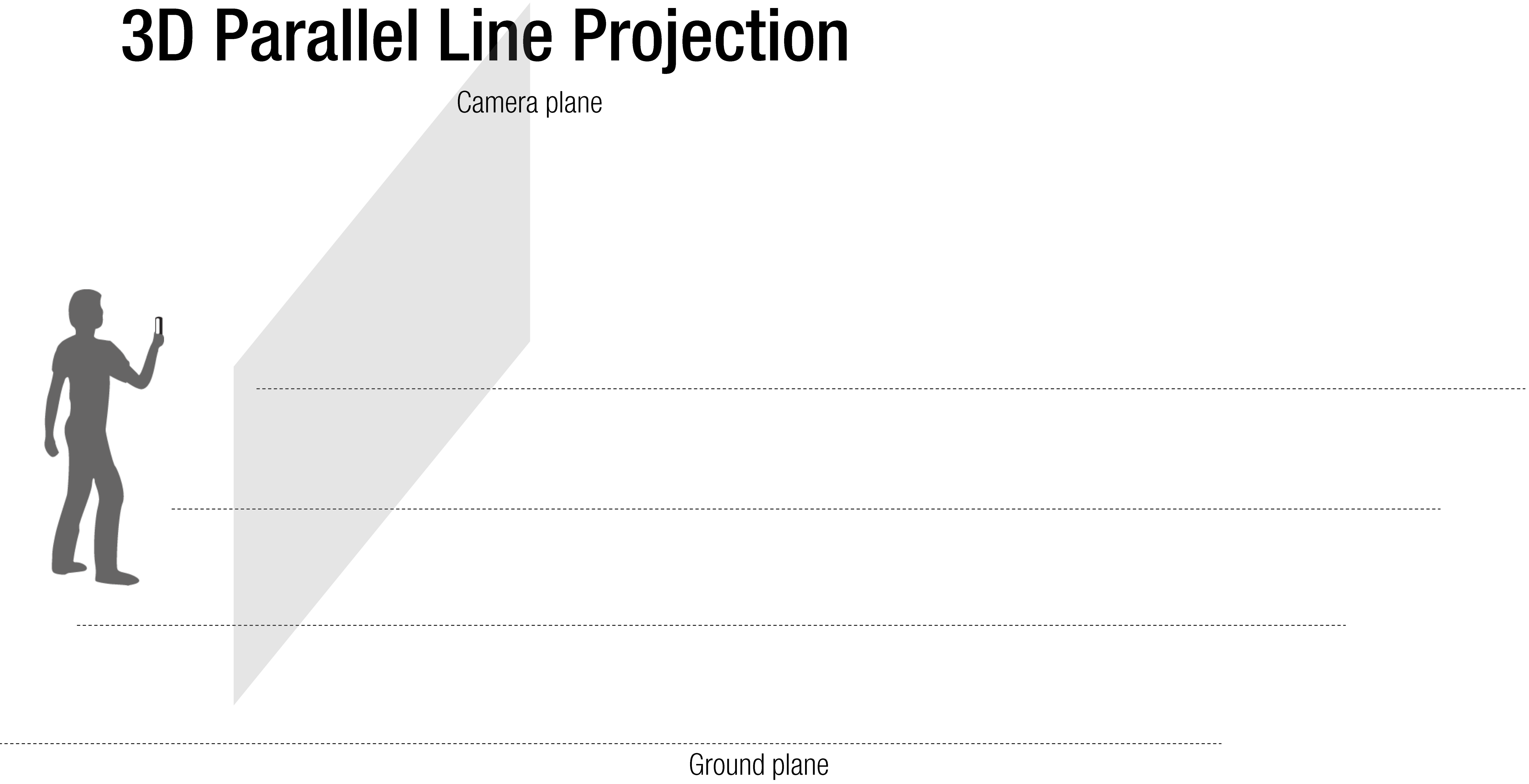


At least two stars are needed.

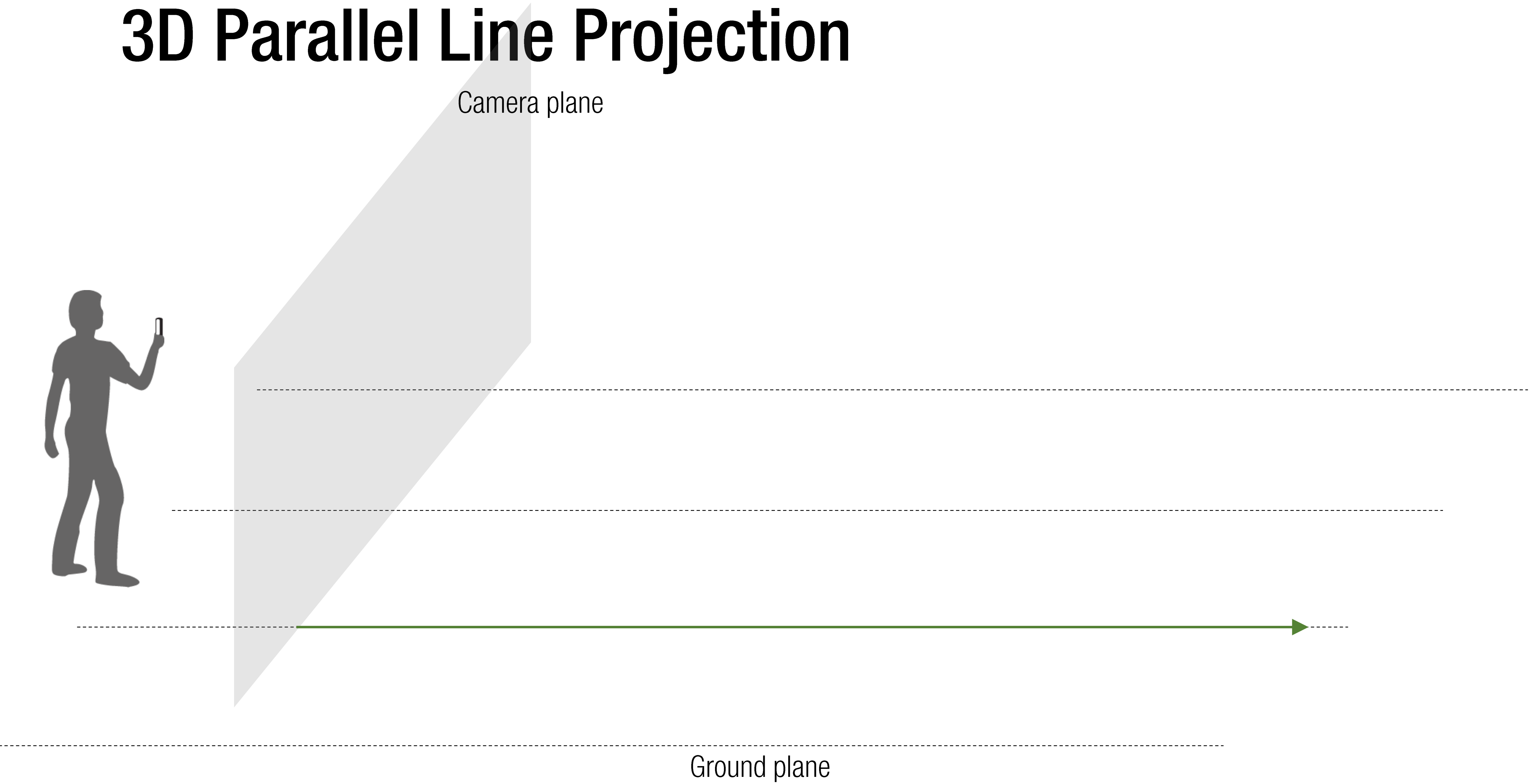
Two points at infinity (vanishing points) tells us about where I am.


MLPS-St. Paul International Airport
BURGER KING
WOMEN
E5
Indoor point at infinity
Parallel lines in 3D converge to a point in the image.

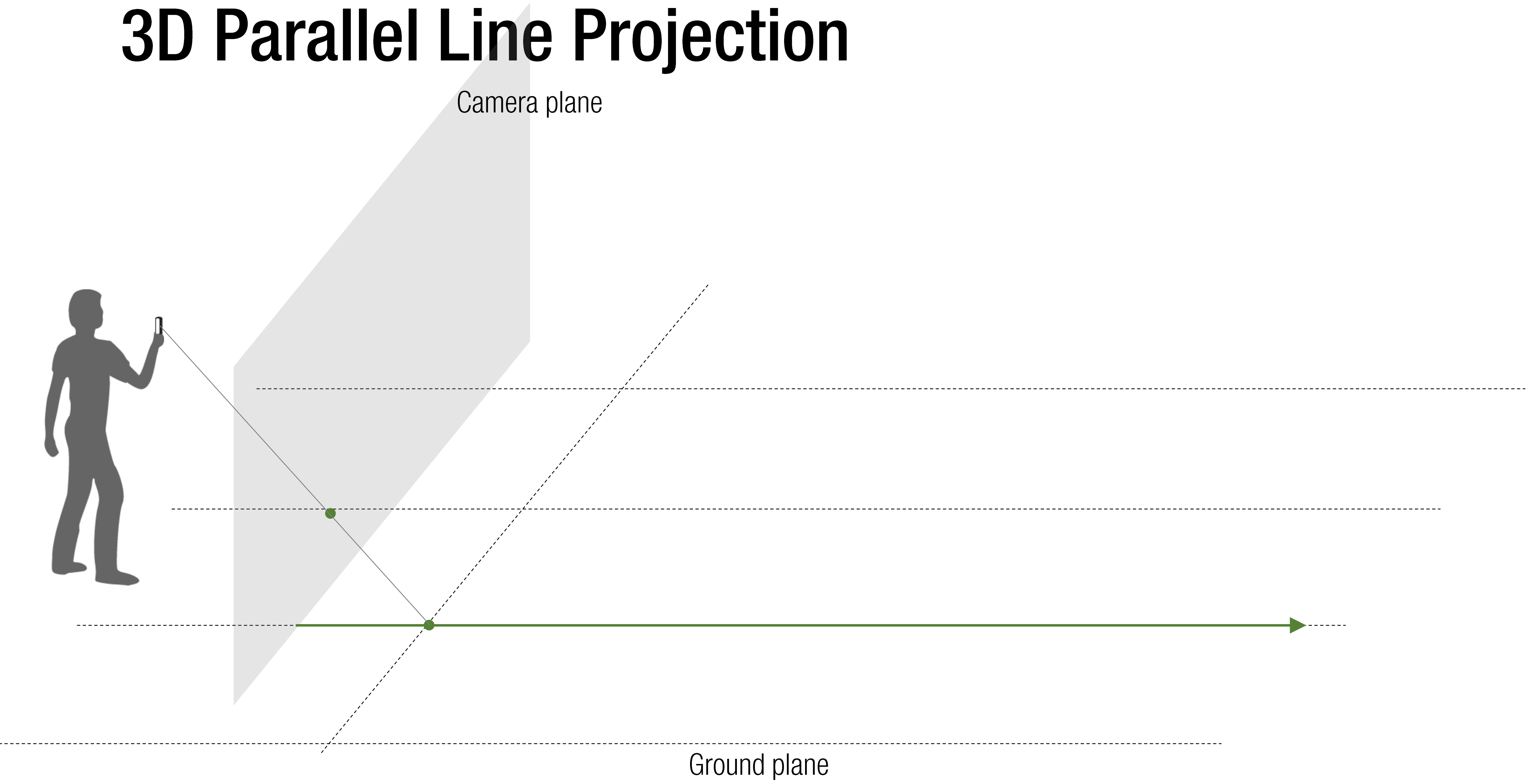
3D Parallel Line Projection



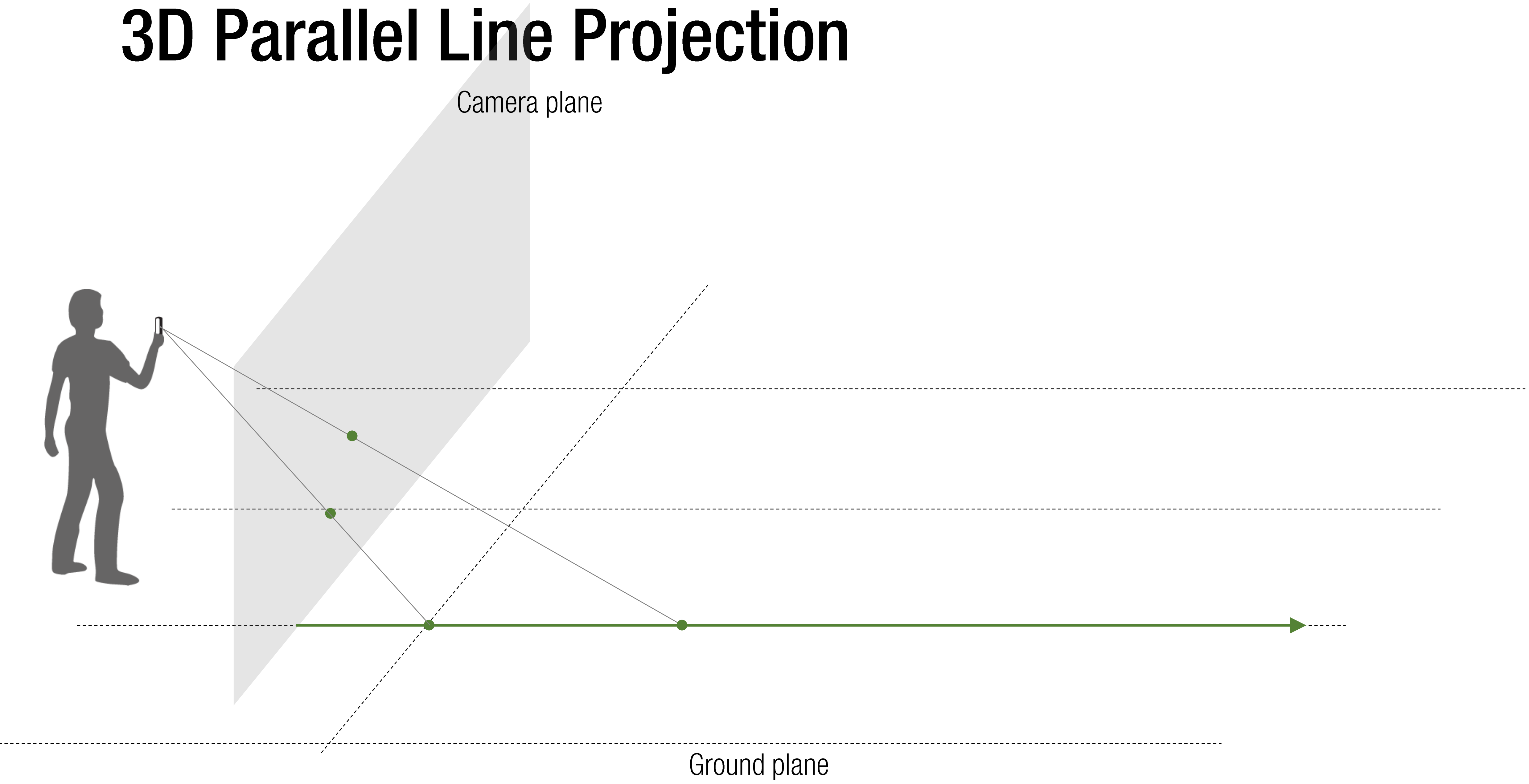
3D Parallel Line Projection



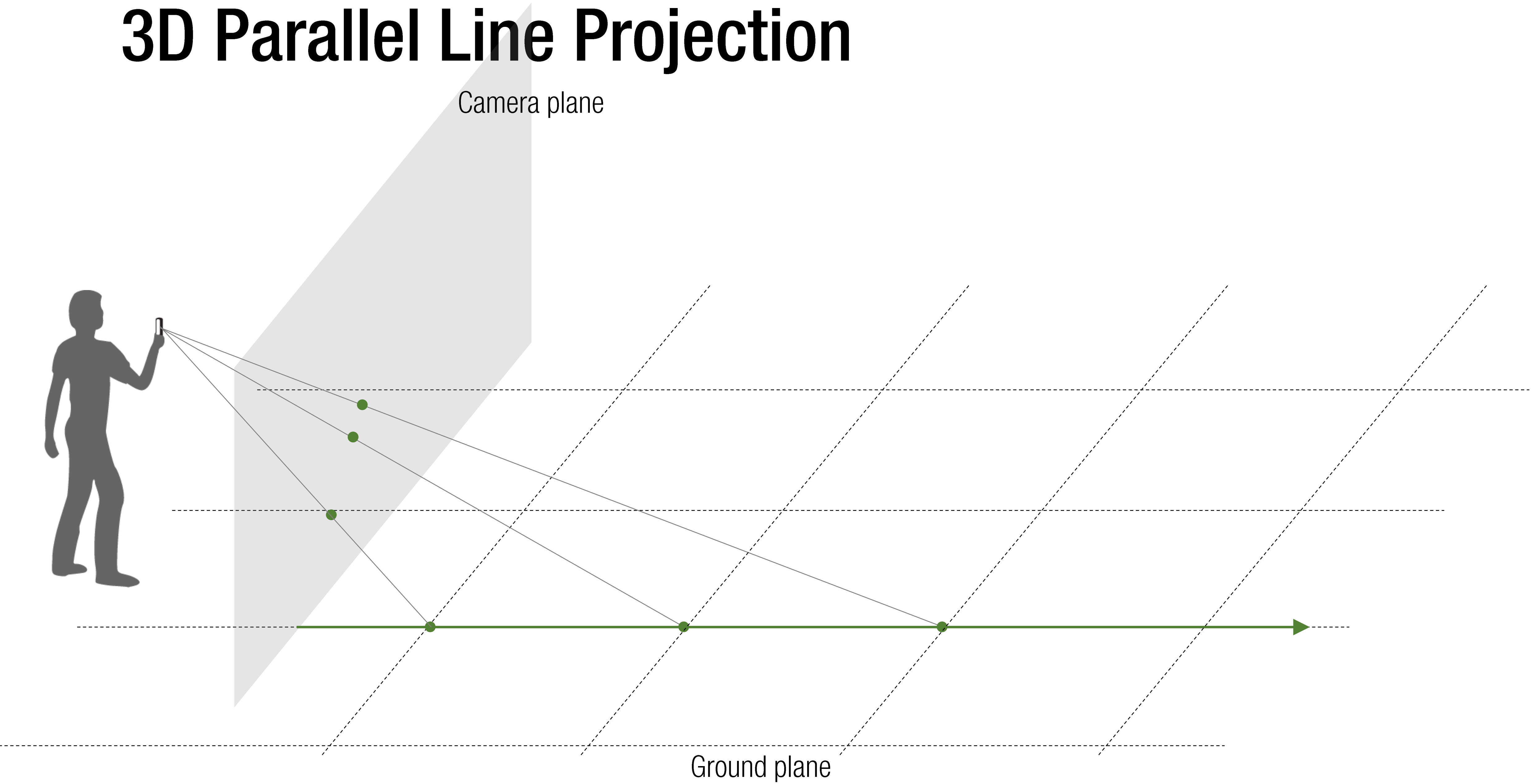
3D Parallel Line Projection



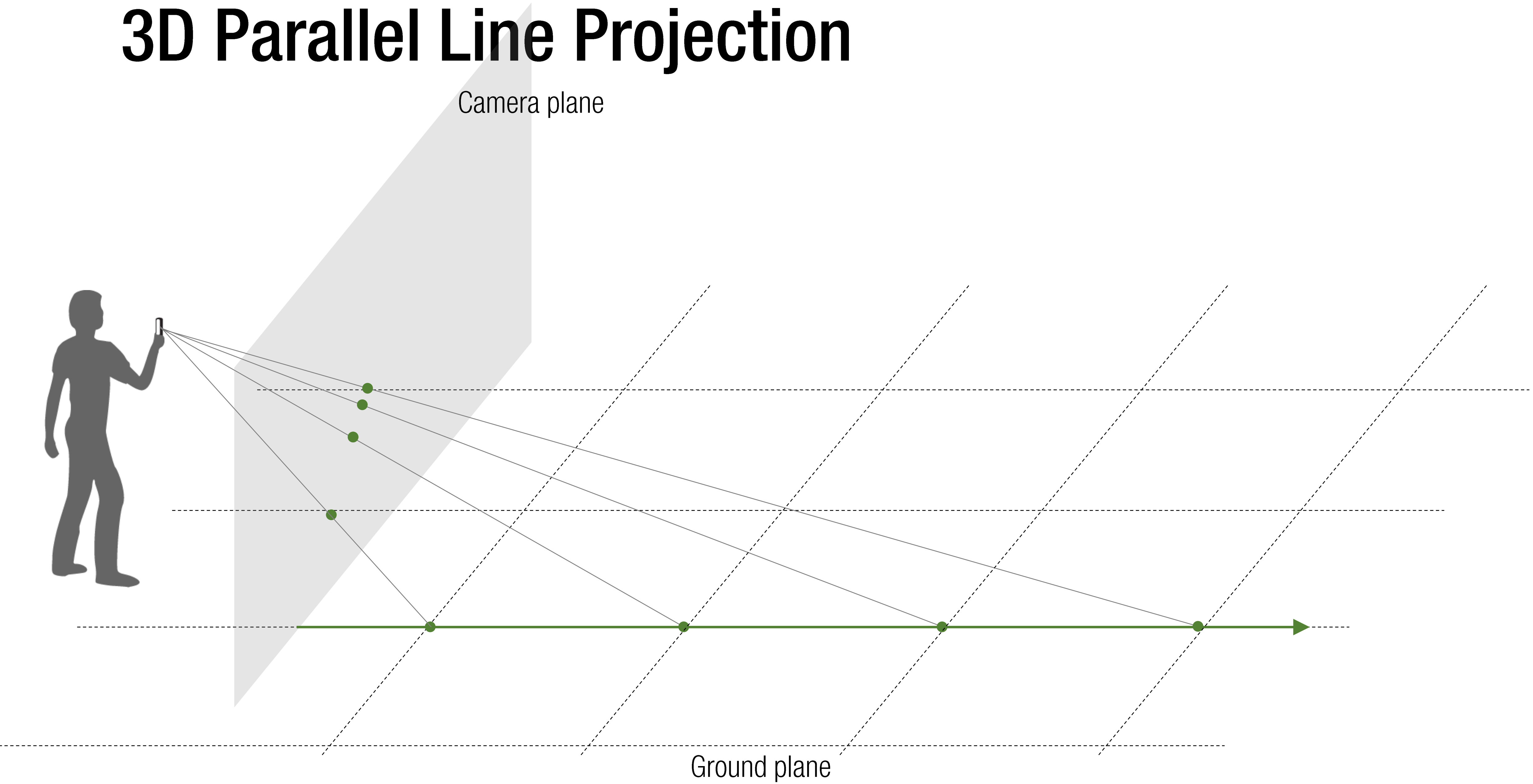
3D Parallel Line Projection



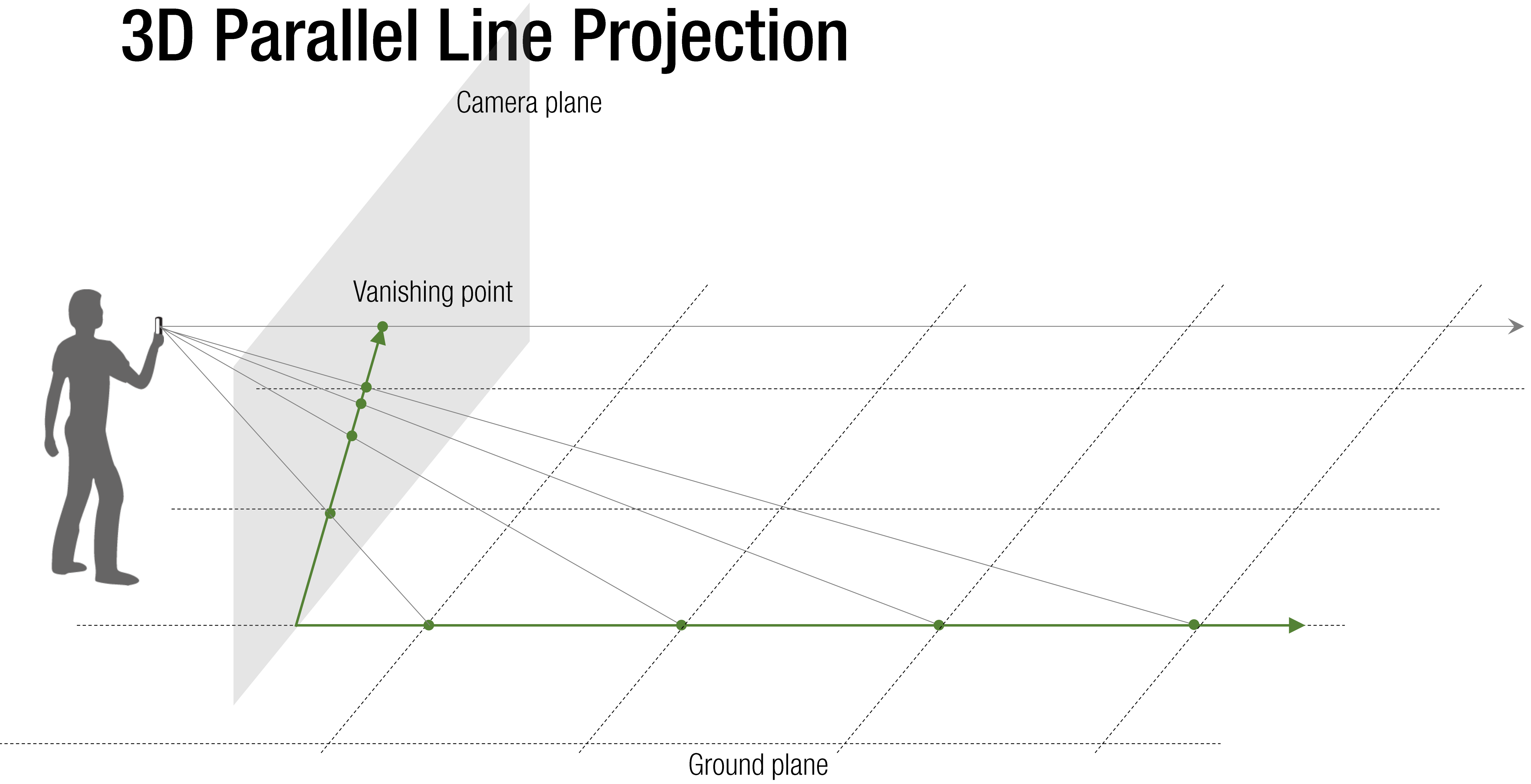
3D Parallel Line Projection



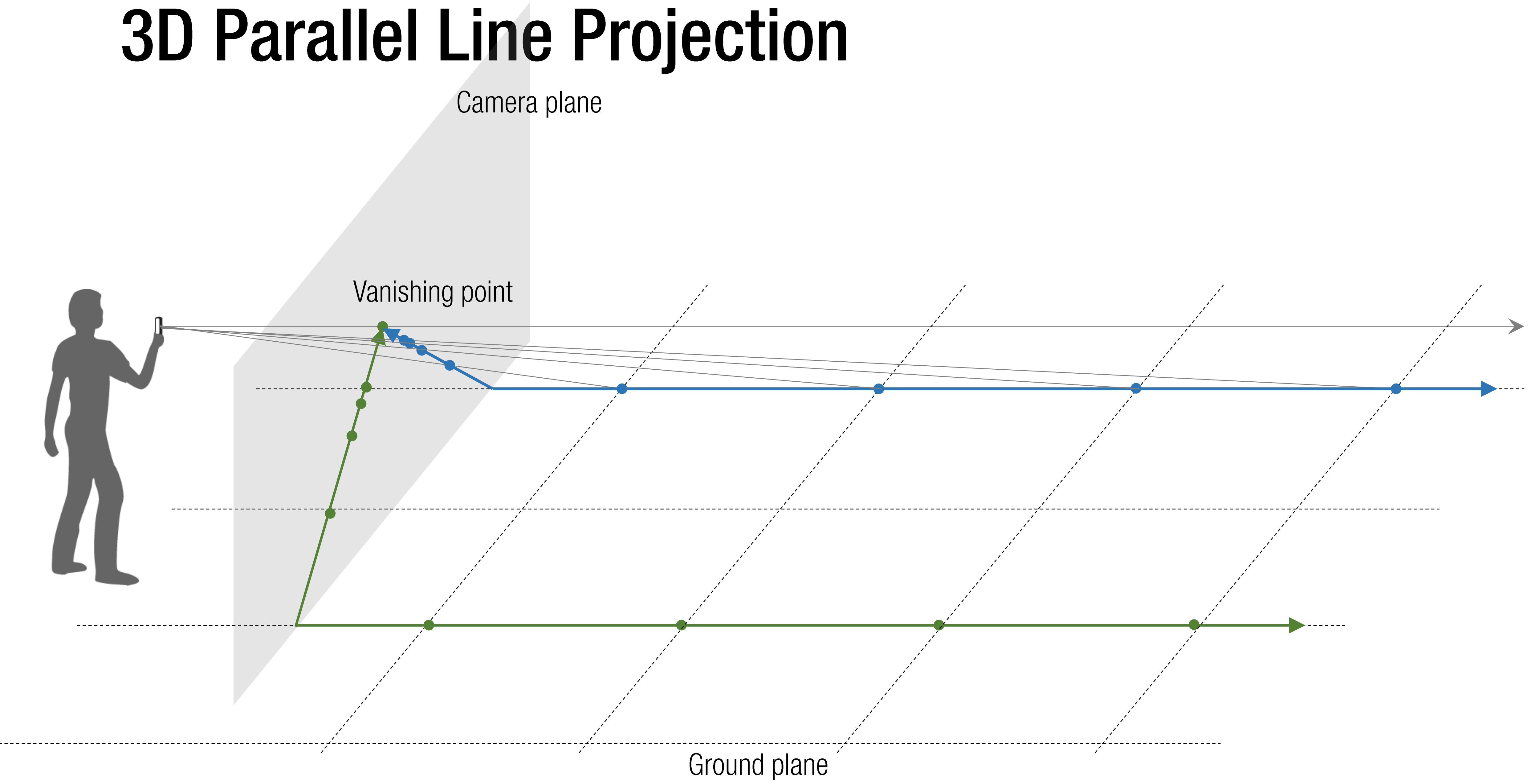
3D Parallel Line Projection



3D Parallel Line Projection

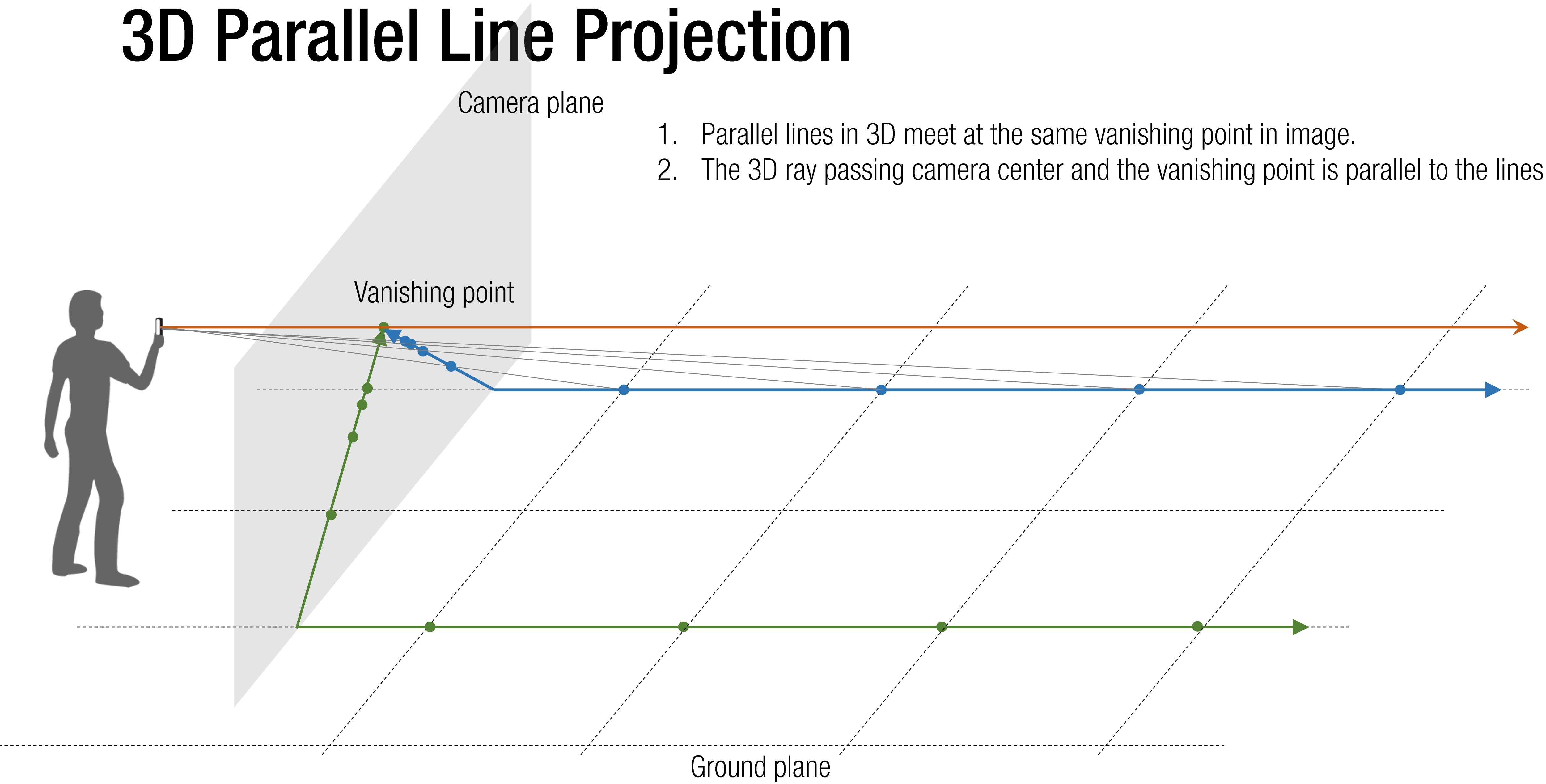


3D Parallel Line Projection



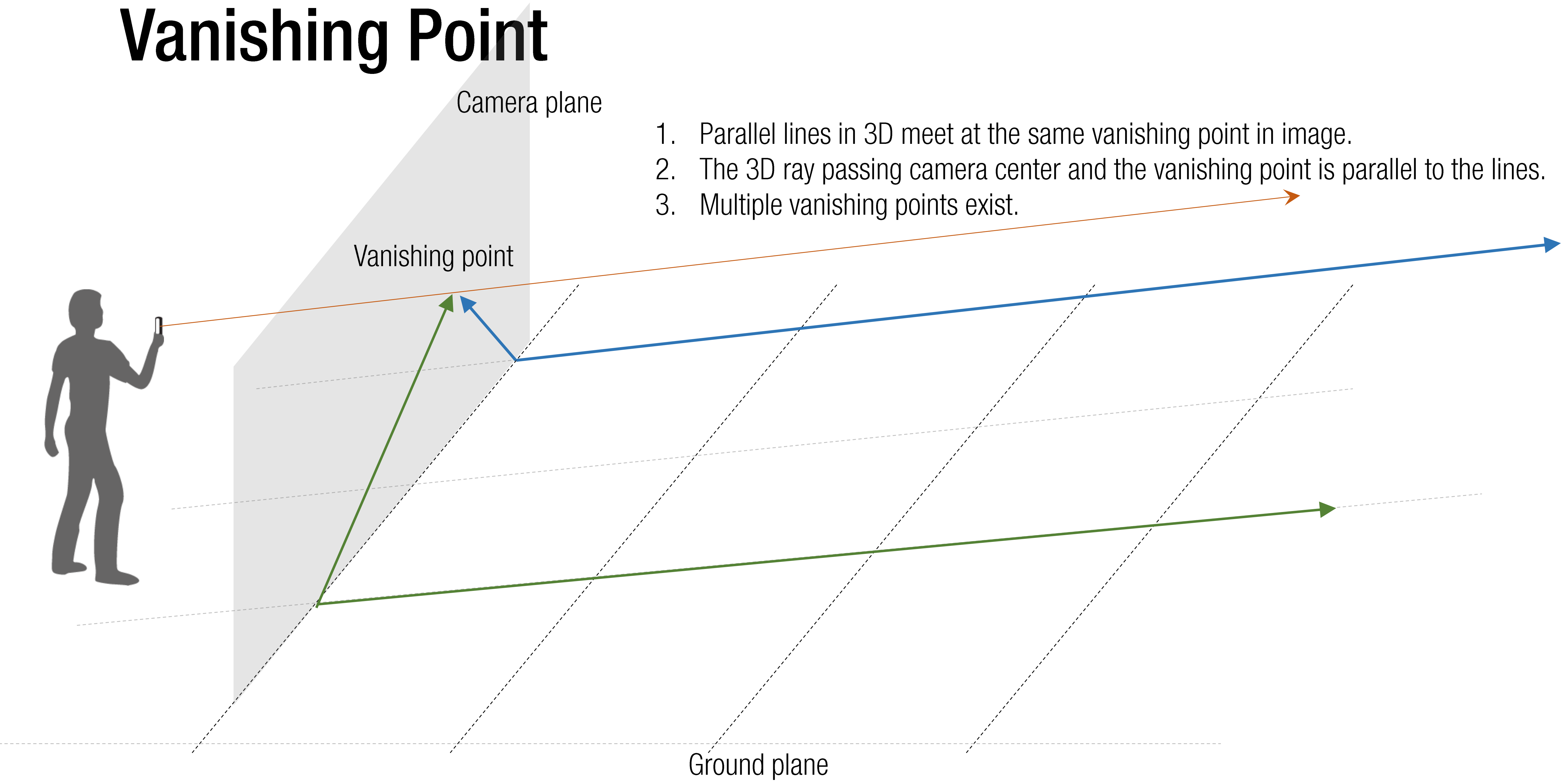
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.



Vanishing Point

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.



Keller Hall

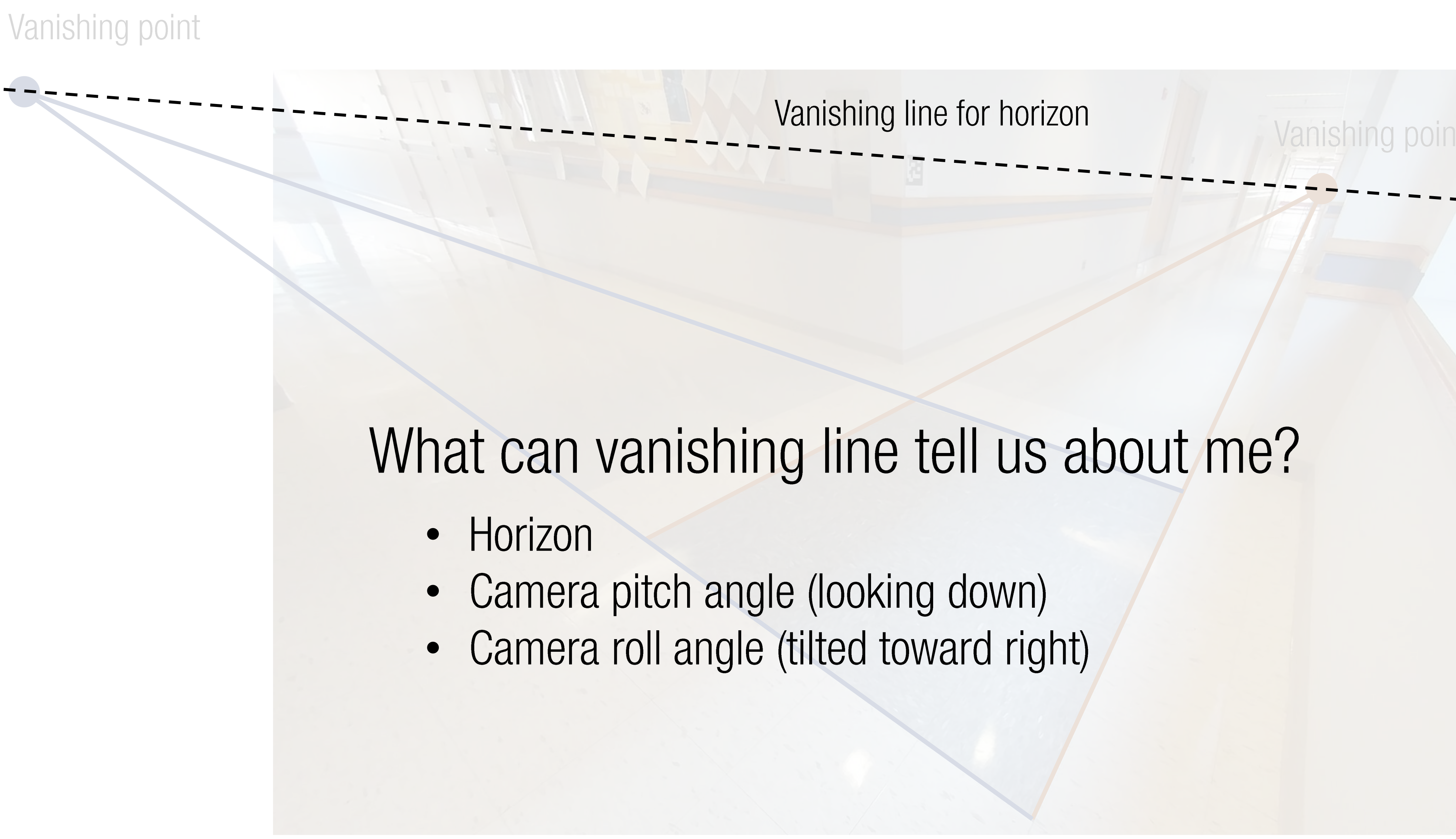
Keller Hall

Vanishing point

Keller Hall

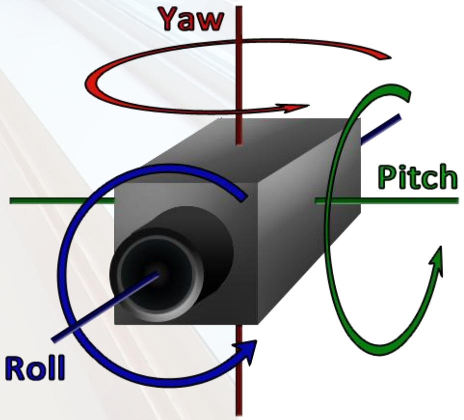
Vanishing line: Horizon

Keller Hall



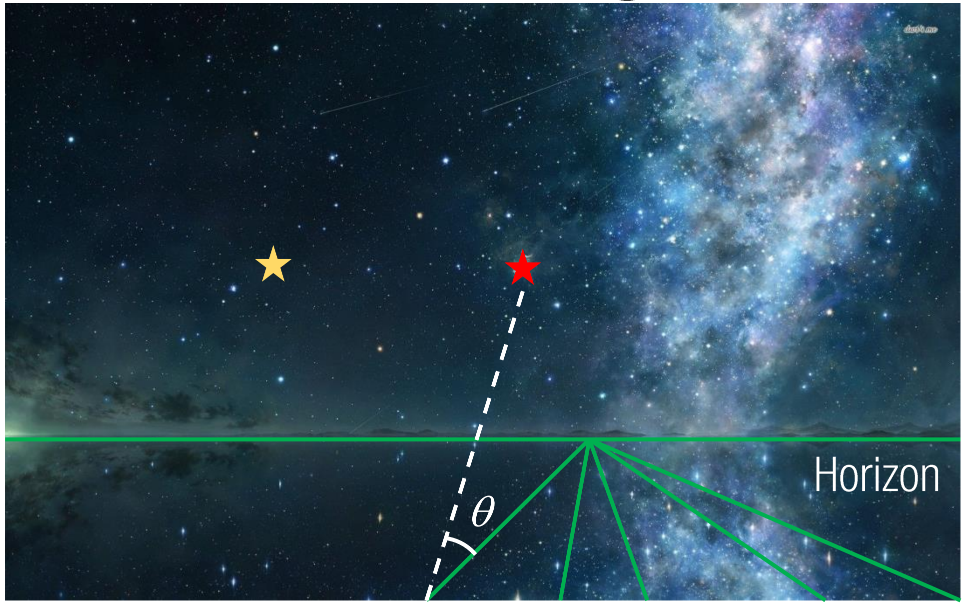
What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)

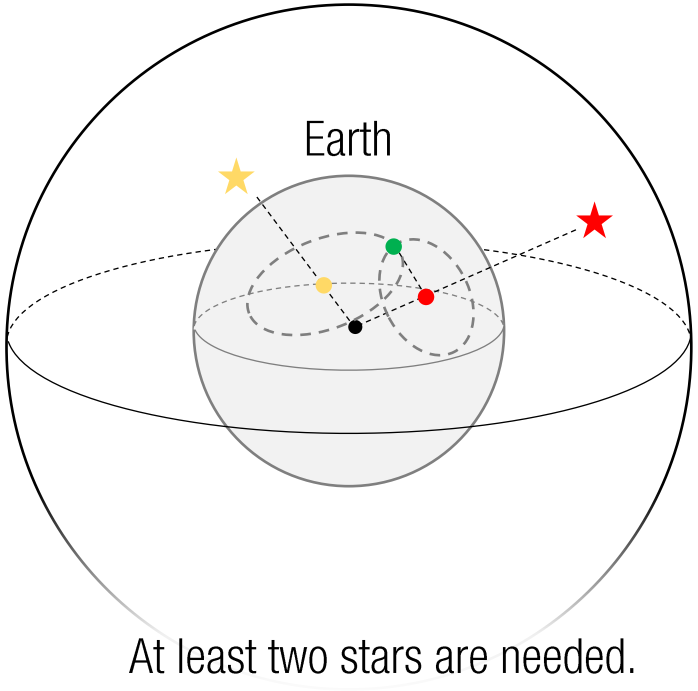


Keller Hall

Celestial Navigation



Far far away: point at infinity

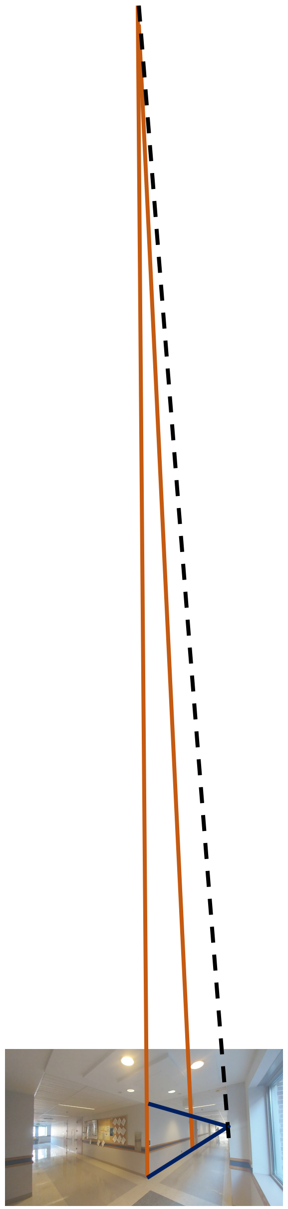


At least two stars are needed.

Two points at infinity (vanishing points) tells us about where I am.

Parallel 3D planes share the vanishing line.

Different plane produces different vanishing line.

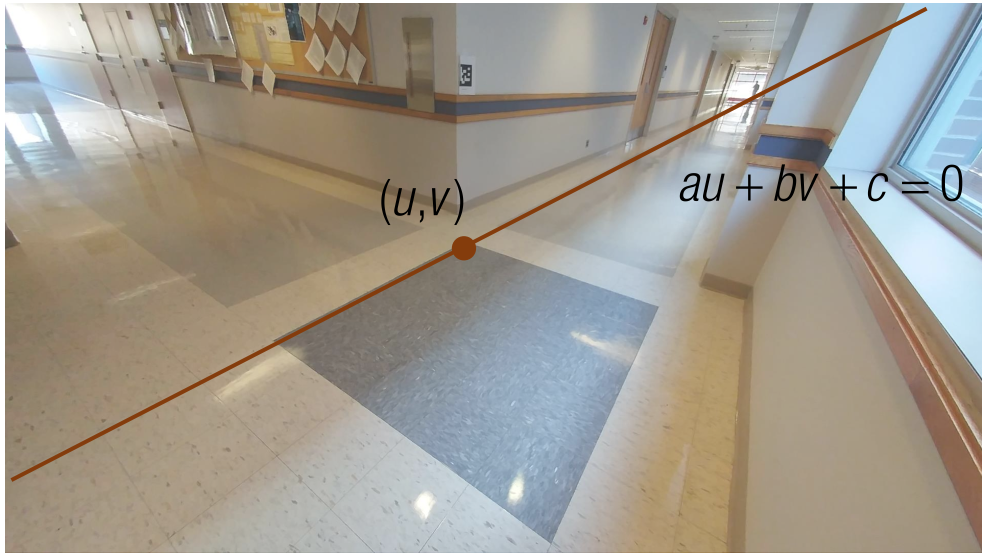


Different plane produces different vanishing line.

How to compute a vanishing point?

Different plane produces different vanishing line.

Point-Line in Image

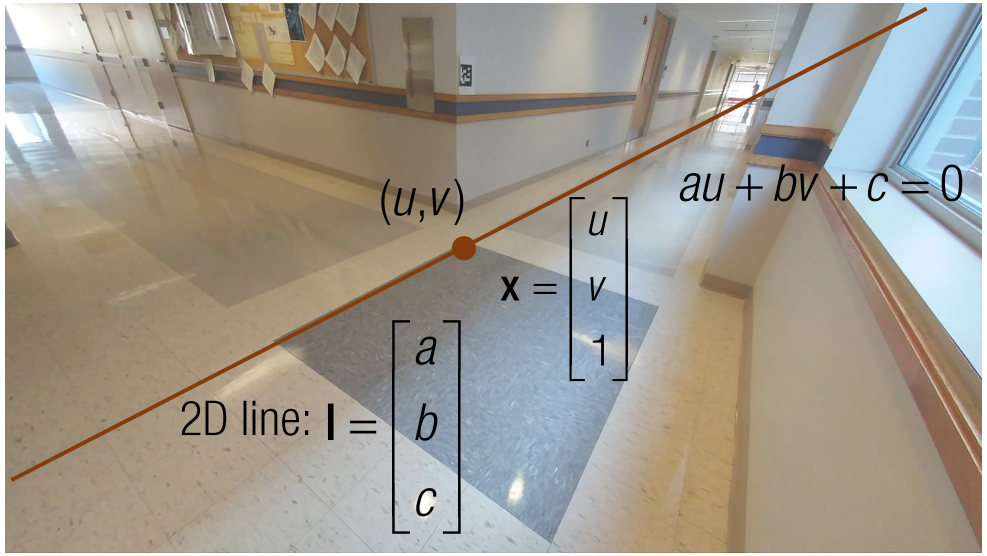


A 2D line passing through 2D point (u,v):

$$au + bv + c = 0$$

Line parameter: (a,b,c)

Point-Line in Image



A 2D line passing through 2D point (u,v):

$$au+bv+c=0$$

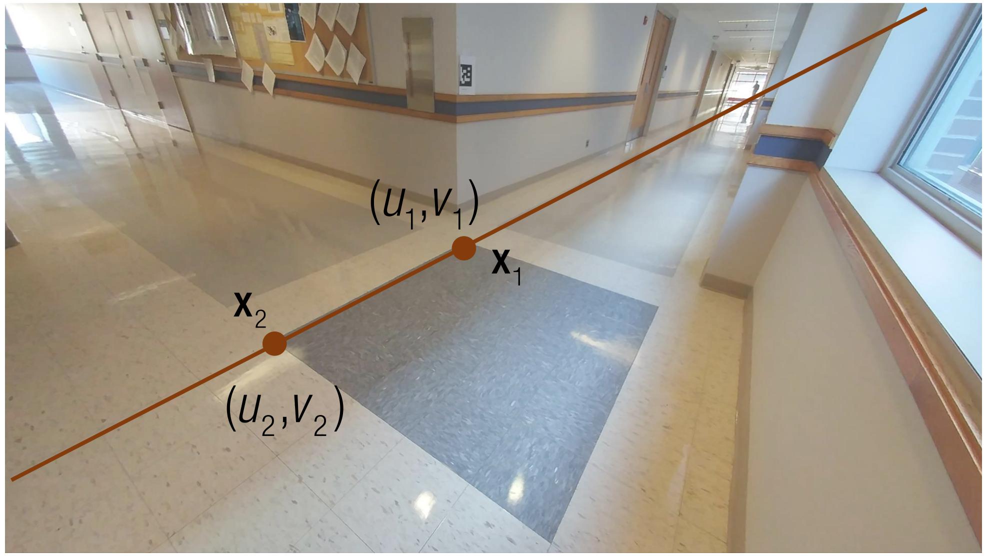
Line parameter: (a,b,c)

$$au+bv+c=0 \longrightarrow \begin{bmatrix}a & b & c\end{bmatrix}\begin{bmatrix}u\\v\\1\end{bmatrix}=\mathbf{l}^{\mathsf{T}}\mathbf{x}=0$$

where $\mathbf{x}=\begin{bmatrix}u\\v\\1\end{bmatrix}$ and $\mathbf{l}=\begin{bmatrix}a\\b\\c\end{bmatrix}$

2D point Line parameter

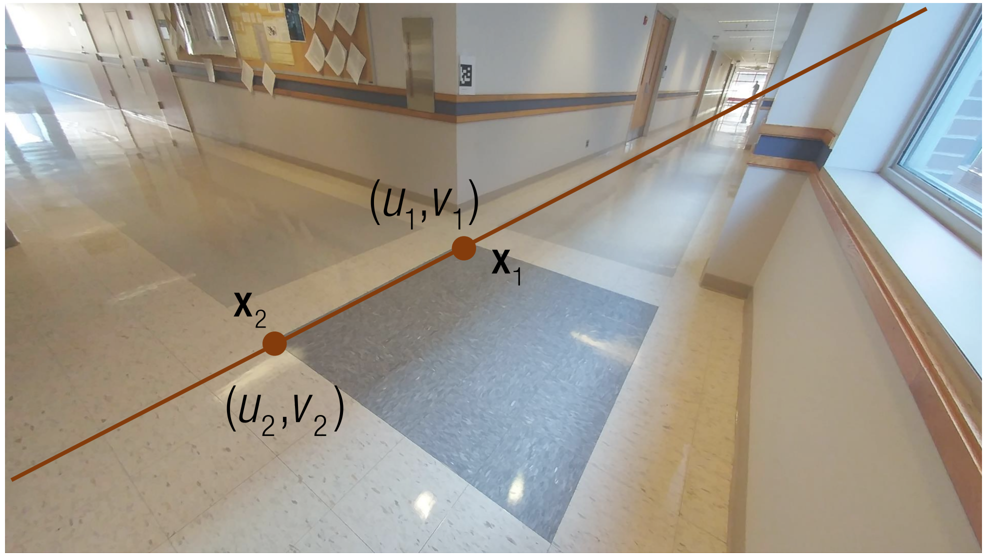
Point-Point in Image



A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

Point-Point in Image



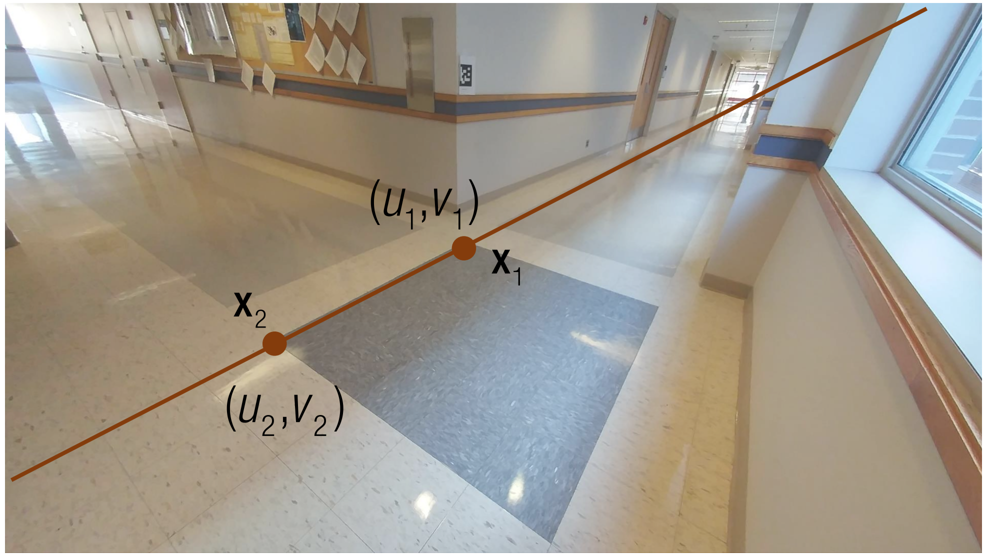
A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

$$\mathbf{x}_1^\top \mathbf{l} = 0 \qquad \mathbf{x}_2^\top \mathbf{l} = 0$$

where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Point-Point in Image



A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

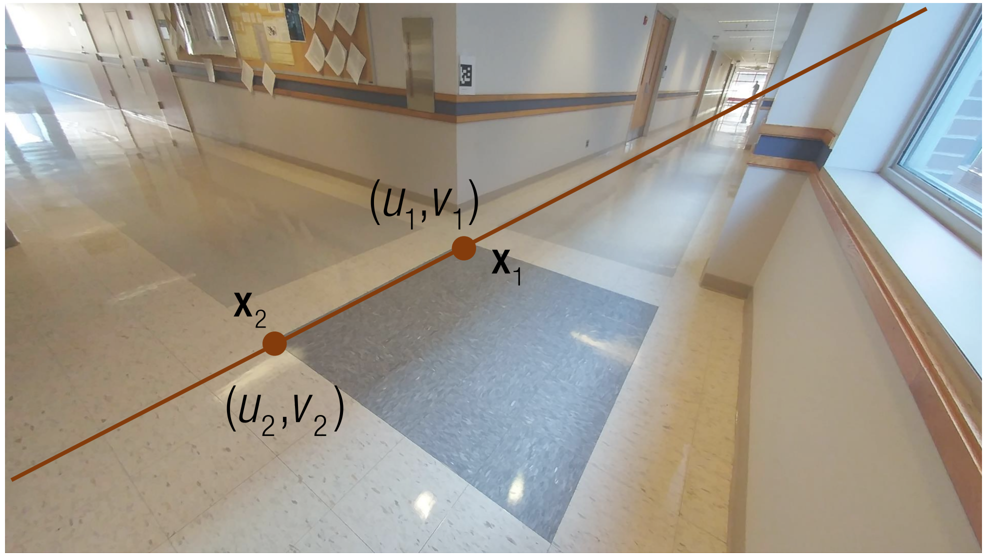
$$\mathbf{x}_1^{\mathrm{T}}\mathbf{l} = 0 \qquad \mathbf{x}_2^{\mathrm{T}}\mathbf{l} = 0$$

where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad \mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \mathbf{x}_1^{\mathrm{T}} \\ \mathbf{x}_2^{\mathrm{T}} \end{bmatrix} \mathbf{l} = \mathbf{0}$$

$$\underbrace{\mathbf{A}}_{3\mathrm{x}2} \; \blacksquare = \begin{matrix} 0 \\ 0 \end{matrix}$$

Point-Point in Image



A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

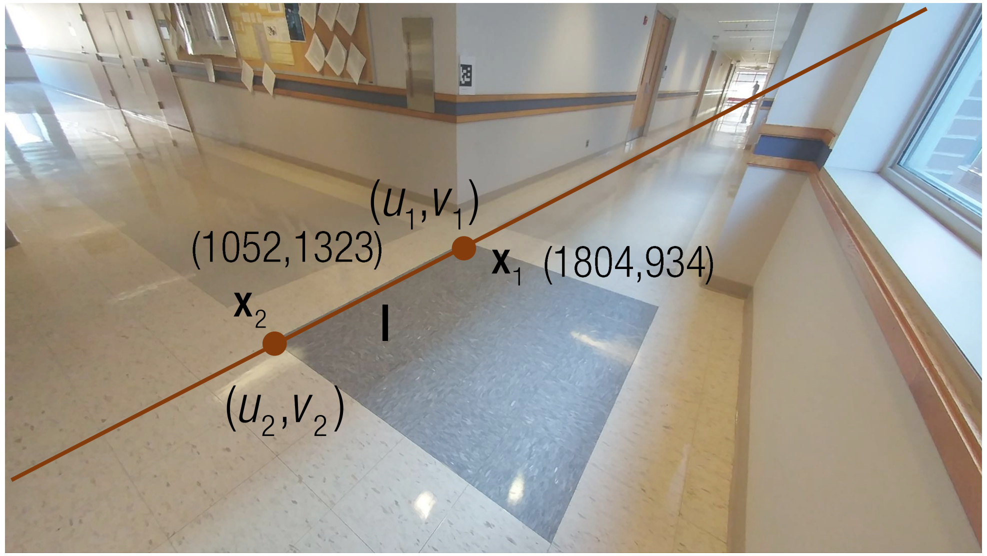
$$\mathbf{x}_1^\mathsf{T}\mathbf{l} = 0 \qquad \mathbf{x}_2^\mathsf{T}\mathbf{l} = 0$$

where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \mathbf{x}_1^\mathsf{T} \\ \mathbf{x}_2^\mathsf{T} \end{bmatrix} \mathbf{l} = \mathbf{0}$$

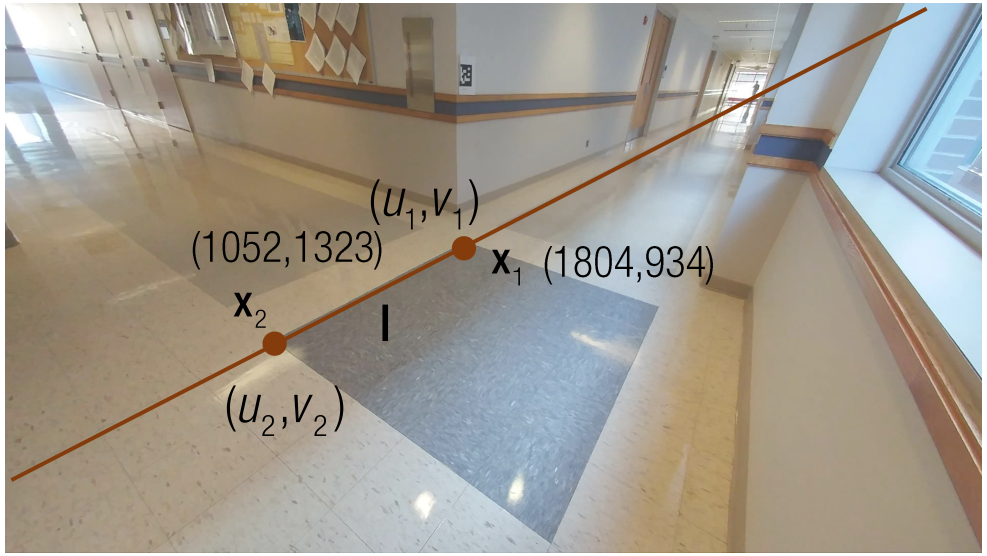
$$\underbrace{\mathbf{A}}_{3\text{x}2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \rightarrow \quad = \text{null}\left(\mathbf{A}\right) \qquad \text{or} \qquad \mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

Point-Point in Image




```
function GetLineFromTwoPoints

x1 = [1804;934;1];
x2 = [1052;1323;1];
```

Point-Point in Image




```
function GetLineFromTwoPoints

x1 = [1804;934;1];
x2 = [1052;1323;1];

l = Vec2Skew(x1)*x2;
```

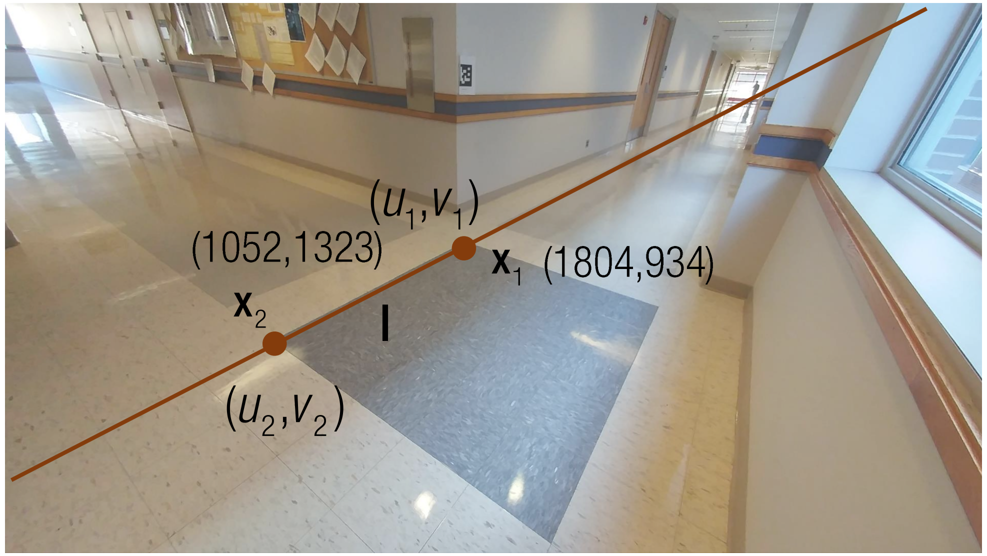
$\underbrace{\text{Vec2Skew(x1)*x2}}_{\text{Cross product}}$

Cross product with skew-symmetric matrix representation:

$$\mathbf{a}\times\mathbf{b} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [\mathbf{a}]_\times \mathbf{b}$$

```
function skew = Vec2Skew(v)
skew = [0 -v(3) v(2);
       v(3) 0 -v(1);
       -v(2) v(1) 0];
```

Point-Point in Image




```
function GetLineFromTwoPoints

x1 = [1804;934;1];
x2 = [1052;1323;1];

l = Vec2Skew(x1)*x2;
```

Cross product

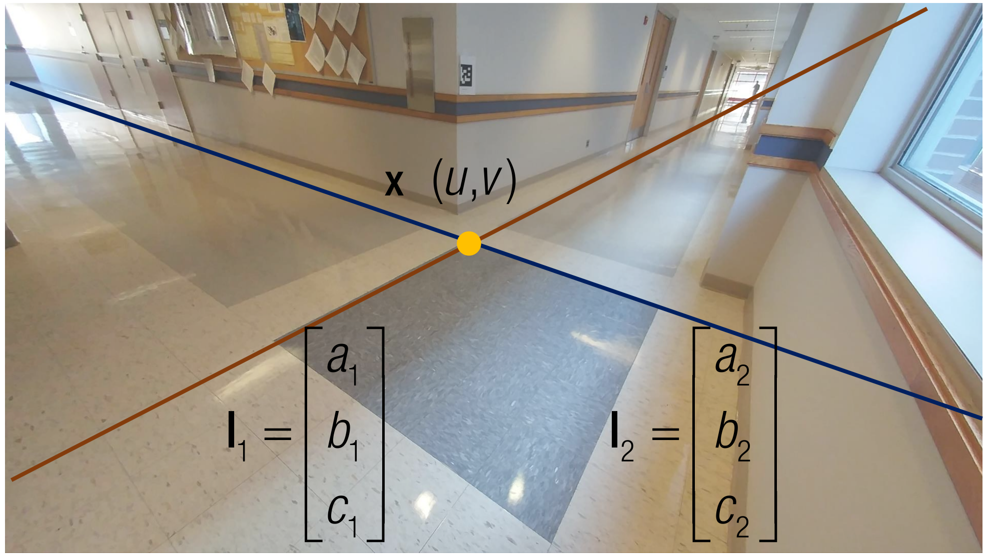
```
l =

     -389
     -752
  1404124
```

Cross product with skew-symmetric matrix representation:

$$\mathbf{a}\times\mathbf{b} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [\mathbf{a}]_\times \mathbf{b}$$

```
function skew = Vec2Skew(v)
skew = [0 -v(3) v(2);
     v(3) 0 -v(1);
     -v(2) v(1) 0];
```

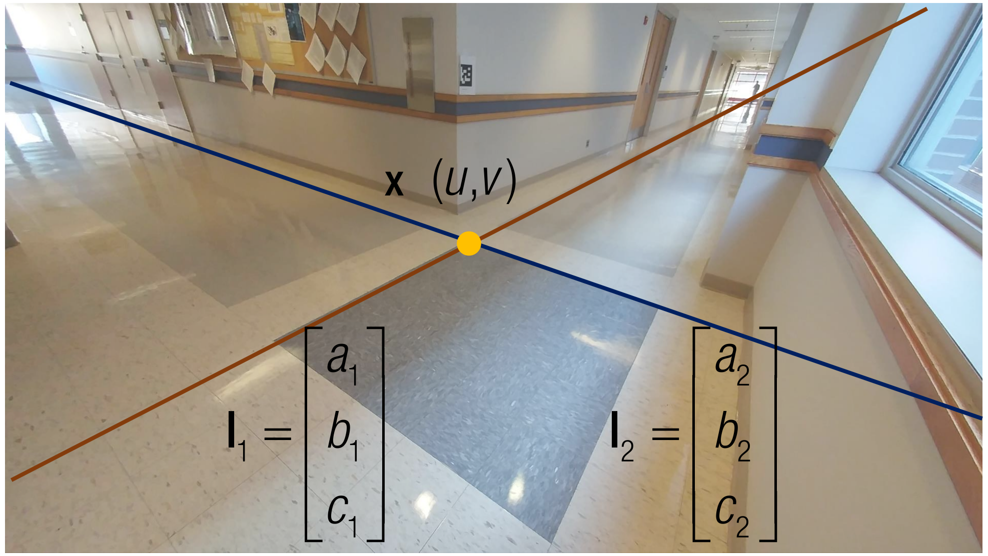
Line-Line in Image



Two 2D lines in an image intersect at a 2D point:

$$a_1 u + b_1 v + c_1 = 0 \qquad a_2 u + b_2 v + c_2 = 0$$

Line-Line in Image



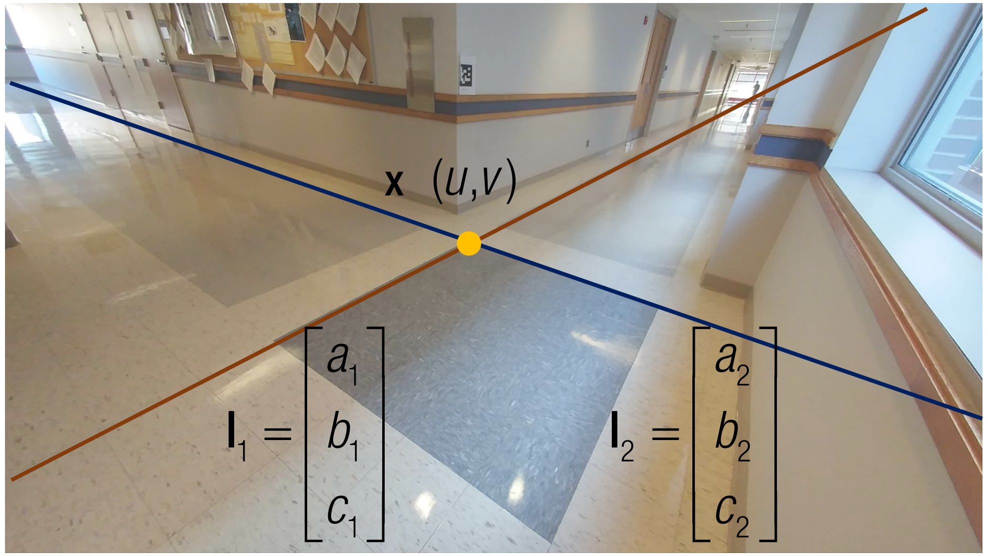
Two 2D lines in an image intersect at a 2D point:

$$a_1 u + b_1 v + c_1 = 0 \qquad a_2 u + b_2 v + c_2 = 0$$

$$\mathbf{l}_1^T \mathbf{x} = 0 \qquad \mathbf{l}_2^T \mathbf{x} = 0$$

where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ $\quad \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ $\quad \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

Line-Line in Image



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$$a_1 u + b_1 v + c_1 = 0 \qquad a_2 u + b_2 v + c_2 = 0$$

$$\mathbf{l}_1^{\mathsf{T}} \mathbf{x} = 0 \qquad \mathbf{l}_2^{\mathsf{T}} \mathbf{x} = 0$$

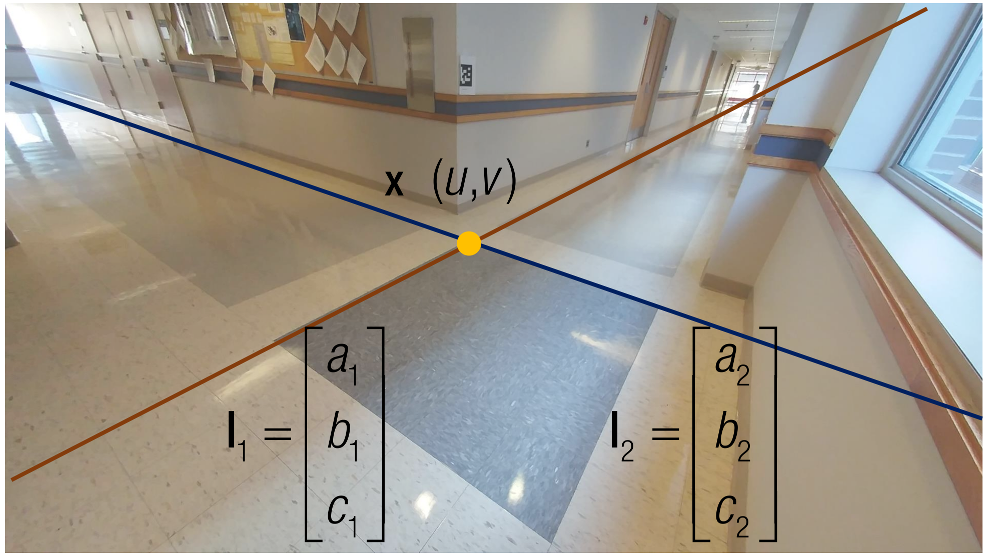
where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ $\quad \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ $\quad \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \mathbf{l}_1^{\mathsf{T}} \\ \mathbf{l}_2^{\mathsf{T}} \end{bmatrix} \mathbf{x} = \mathbf{0}$$

$$\underset{3\text{x}2}{\mathbf{A}} \, \blacksquare = \begin{matrix} 0 \\ 0 \end{matrix} \qquad \rightarrow \qquad \blacksquare = \text{null}\left(\mathbf{A} \right)$$

or $\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$

Line-Line in Image




```
function GetPointFromTwoLines

l1 = [-398;-752;1404124];
l2 = [310;-924;303790];
x = Vec2Skew(l1)*l2;
x = x/x(3)


x =

   1779.0
   925.6
   1
```

similar to (1804,934)

2D Point and Line Duality

The 2D line joining two points:

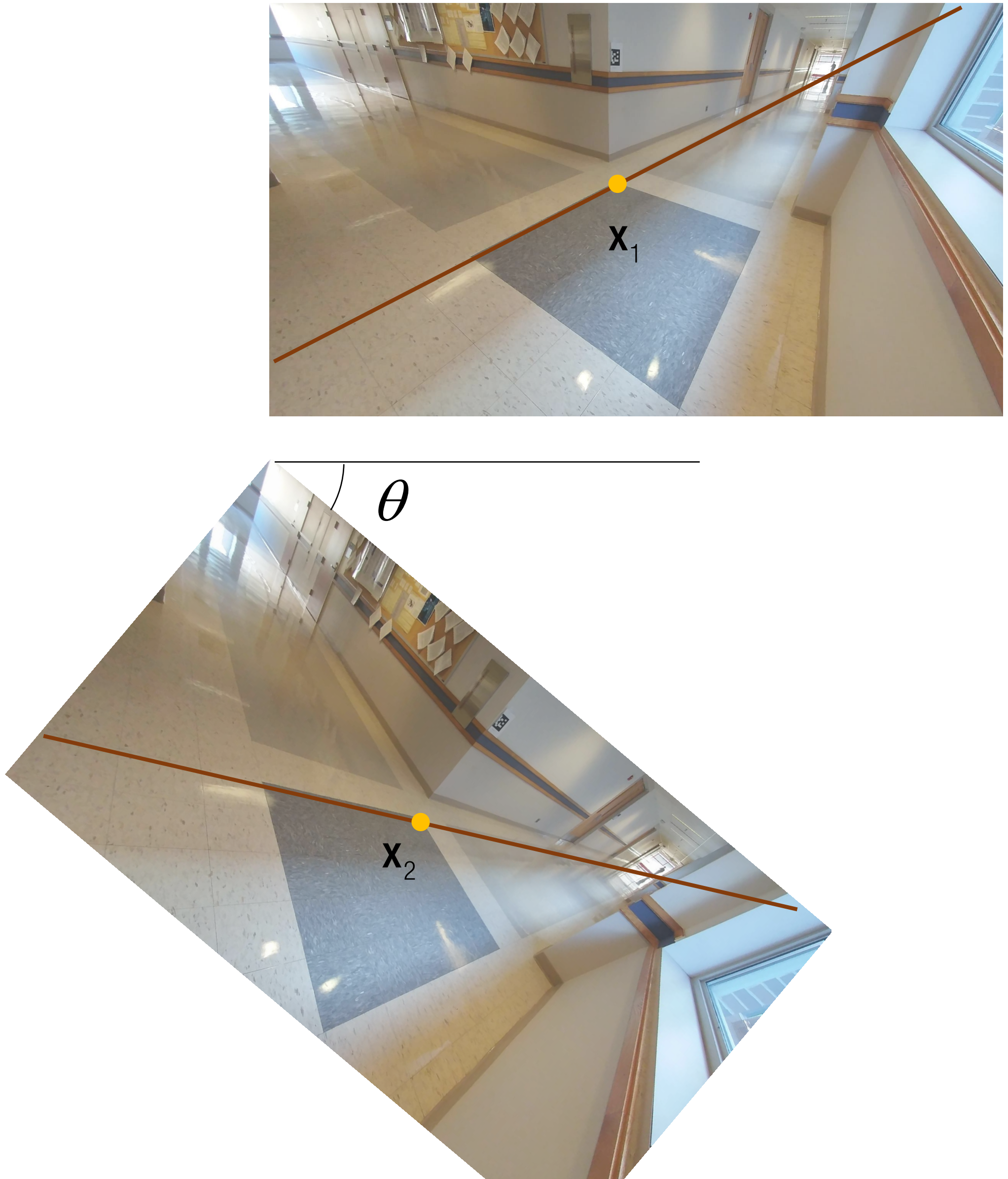
$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

The intersection between two lines:

$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$$

Given any formula, we can switch the meaning of point and line to get another formula.

2D Point and Line Duality



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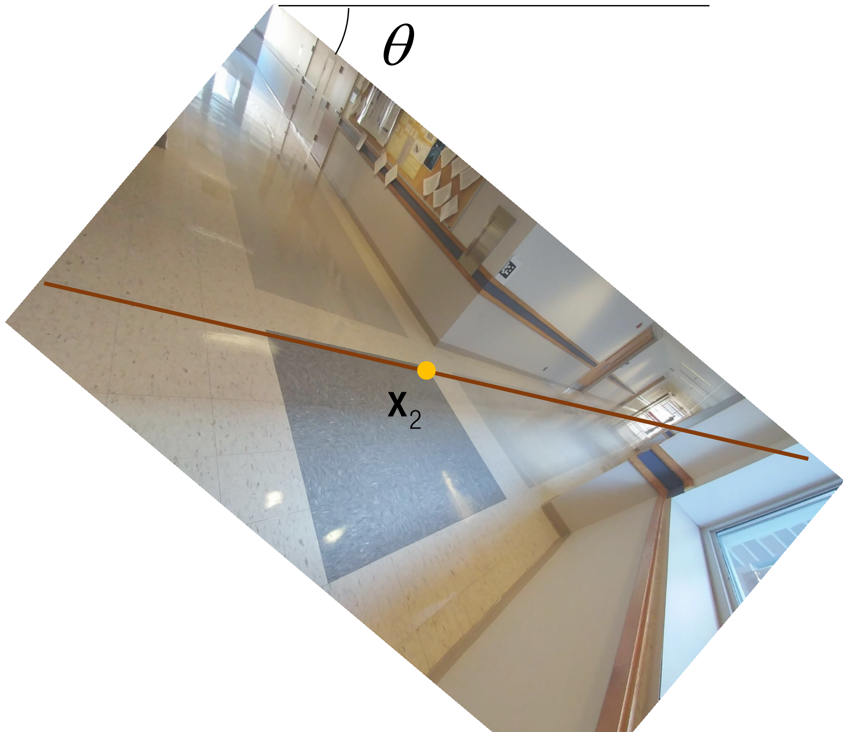
Given any formula, we can switch the meaning of point and line to get another formula.

$$\mathbf{x}_2 = \mathbf{T}\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^{-\mathrm{T}}\mathbf{l}_1$$

T: Transformation

2D Point and Line Duality



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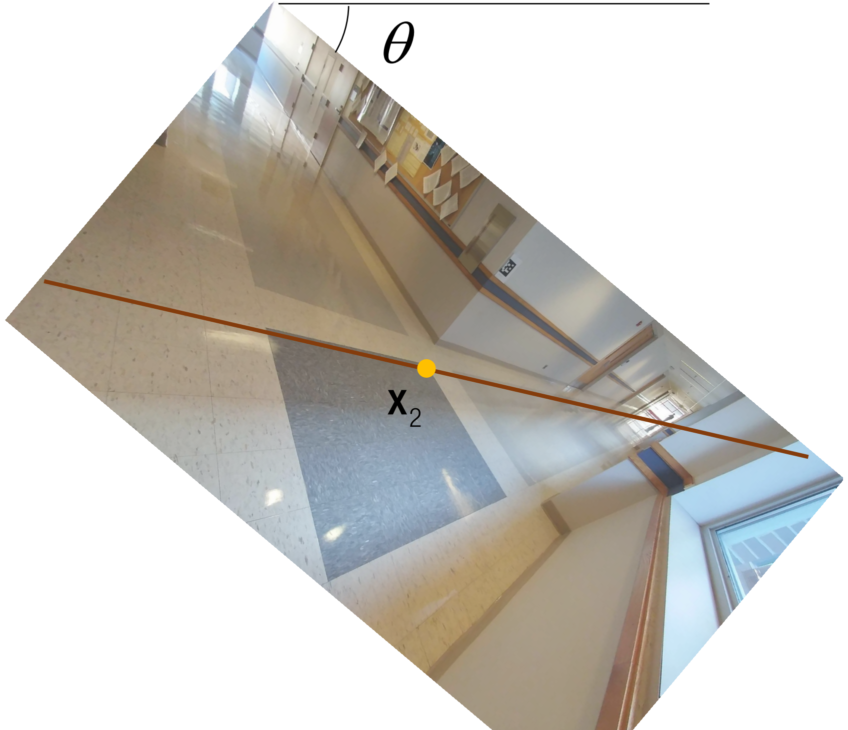
T: Transformation

$$\because \mathbf{l}_1^{\mathsf{T}}\mathbf{x}_1 = 0$$

2D Point and Line Duality



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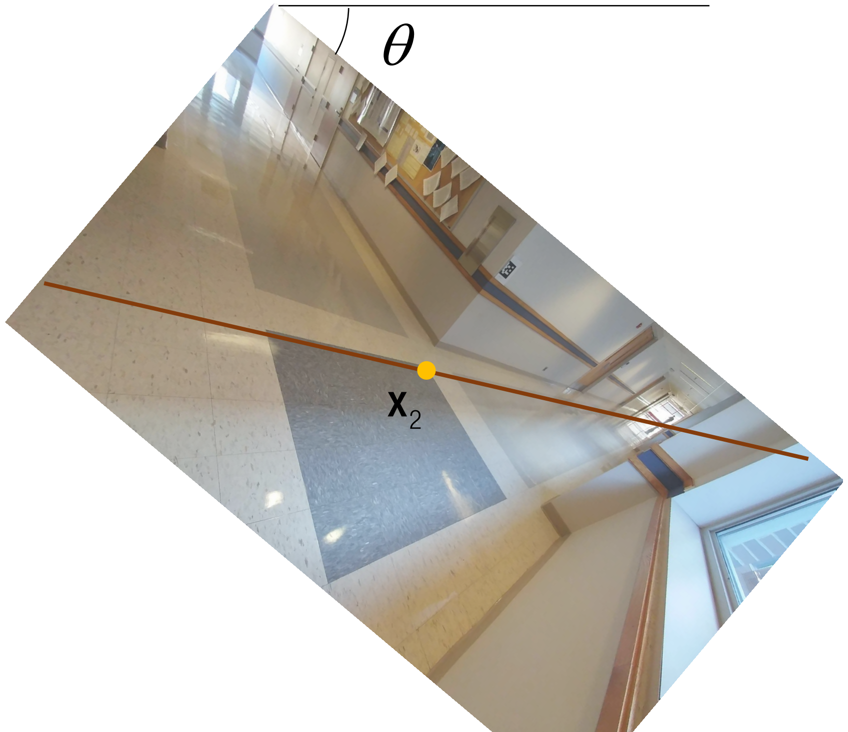
$$\mathbf{x}_2 = \mathbf{T}\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^{-T}\mathbf{l}_1 \qquad \mathbf{T}\text{: Transformation}$$

$$\because \mathbf{l}_1^T\mathbf{x}_1 = \left(\mathbf{l}_1^T\mathbf{T}^{-1}\right)\left(\mathbf{T}\mathbf{x}_1\right) = 0$$

2D Point and Line Duality



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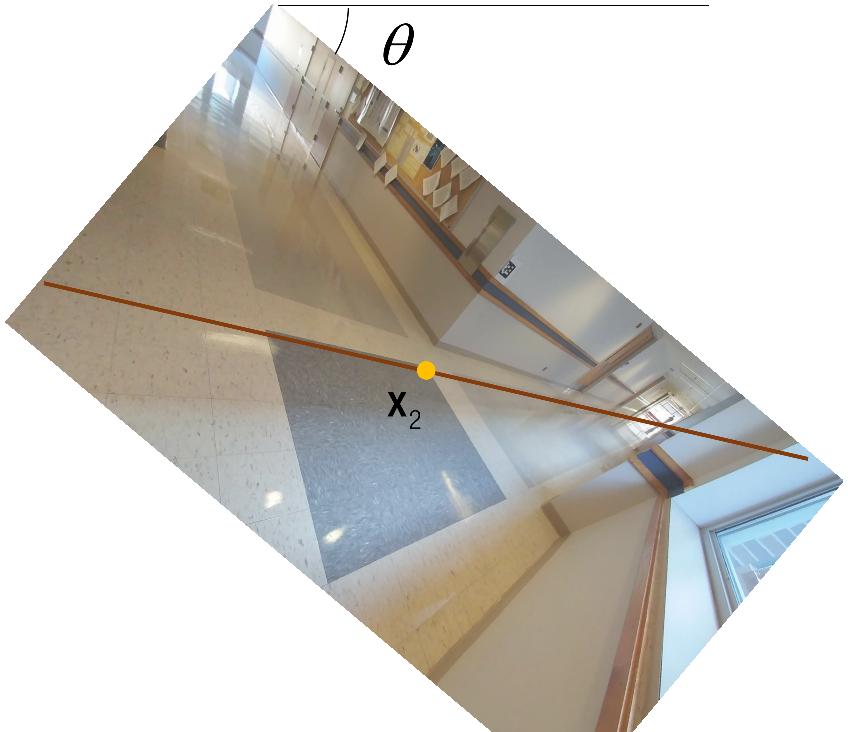
$$\because \mathbf{l}_1^{\mathrm{T}}\mathbf{x}_1 = \left(\mathbf{l}_1^{\mathrm{T}}\mathbf{T}^{-1}\right)\left(\mathbf{T}\mathbf{x}_1\right) = \left(\mathbf{T}^{-\mathrm{T}}\mathbf{l}_1\right)^{\mathrm{T}}\left(\mathbf{T}\mathbf{x}_1\right) = \mathbf{l}_2^{\mathrm{T}}\mathbf{x}_2$$

2D Point and Line Duality



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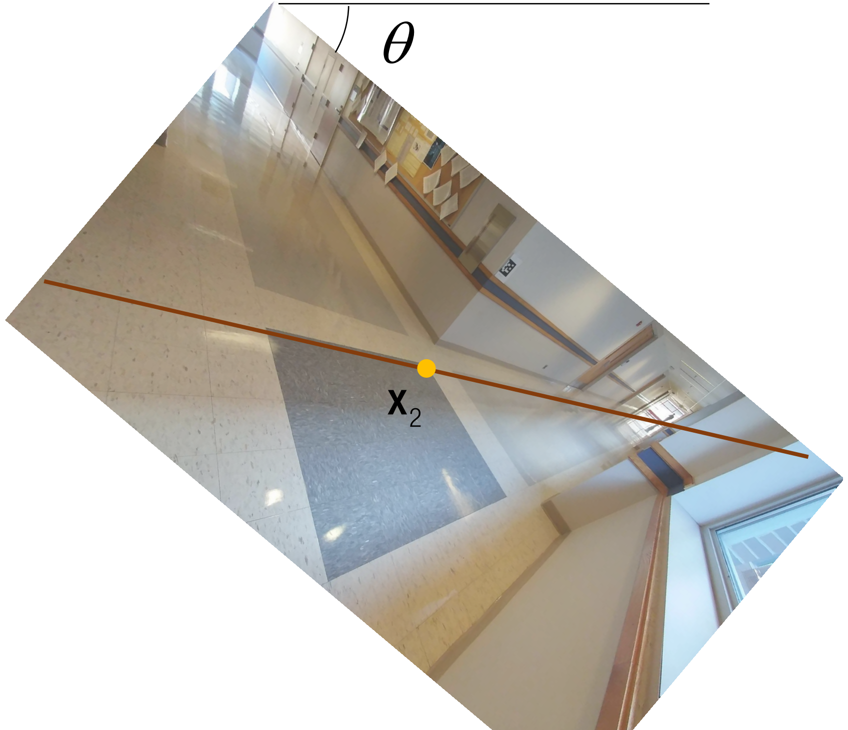
Given any formula, we can switch the meaning of point and line to get another formula.

$$\mathbf{x}_2 = \mathbf{T}\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^{-\mathsf{T}}\mathbf{l}_1 \qquad \mathbf{T}\text{: Transformation}$$

$$\because \mathbf{l}_1^{\mathsf{T}}\mathbf{x}_1 = \left(\mathbf{l}_1^{\mathsf{T}}\mathbf{T}^{-1}\right)\left(\mathbf{T}\mathbf{x}_1\right) = \left(\mathbf{T}^{-\mathsf{T}}\mathbf{l}_1\right)^{\mathsf{T}}\left(\mathbf{T}\mathbf{x}_1\right) = \mathbf{l}_2^{\mathsf{T}}\mathbf{x}_2$$

$$\mathbf{x}_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_1 \longleftrightarrow \qquad \textbf{?}$$

2D Point and Line Duality



The 2D line joining two points:

$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

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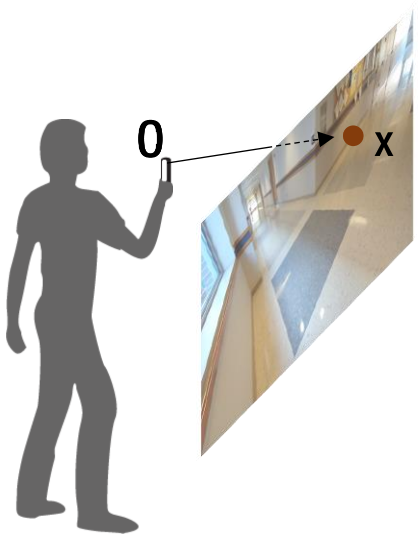
$$\mathbf{x}_2 = \mathbf{T}\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^{-T}\mathbf{l}_1 \qquad \mathbf{T}\text{: Transformation}$$

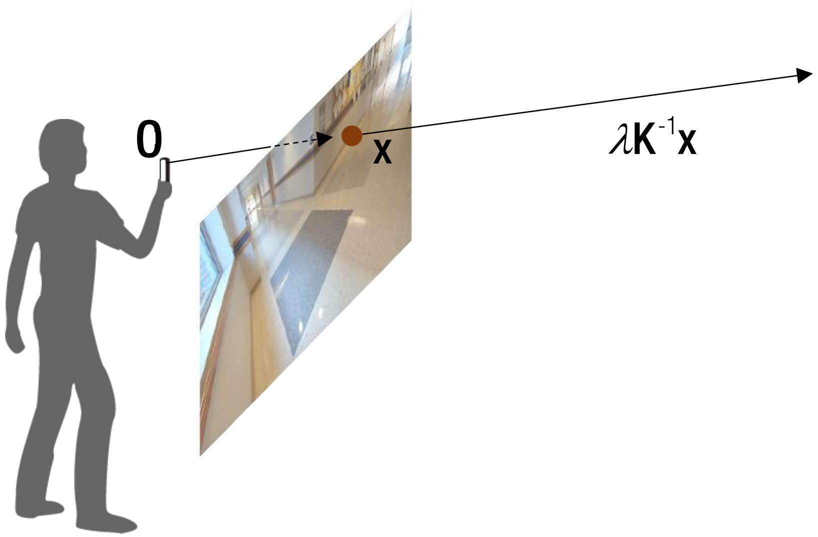
$$\because \mathbf{l}_1^T\mathbf{x}_1 = \left(\mathbf{l}_1^T\mathbf{T}^{-1}\right)\left(\mathbf{T}\mathbf{x}_1\right) = \left(\mathbf{T}^{-T}\mathbf{l}_1\right)^T\left(\mathbf{T}\mathbf{x}_1\right) = \mathbf{l}_2^T\mathbf{x}_2$$

$$\mathbf{x}_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}_1 \longleftrightarrow \mathbf{l}_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-T} \mathbf{l}_1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{l}_1$$

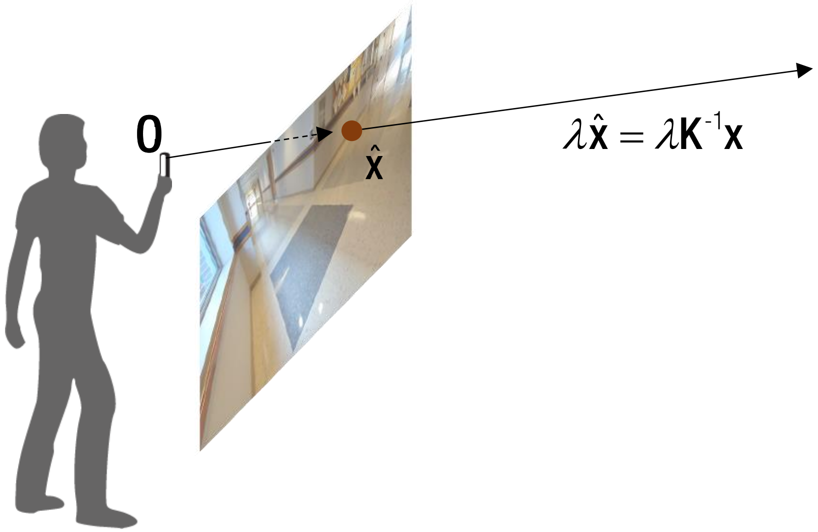
Geometric Interpretation (Point)



Geometric Interpretation (Point)



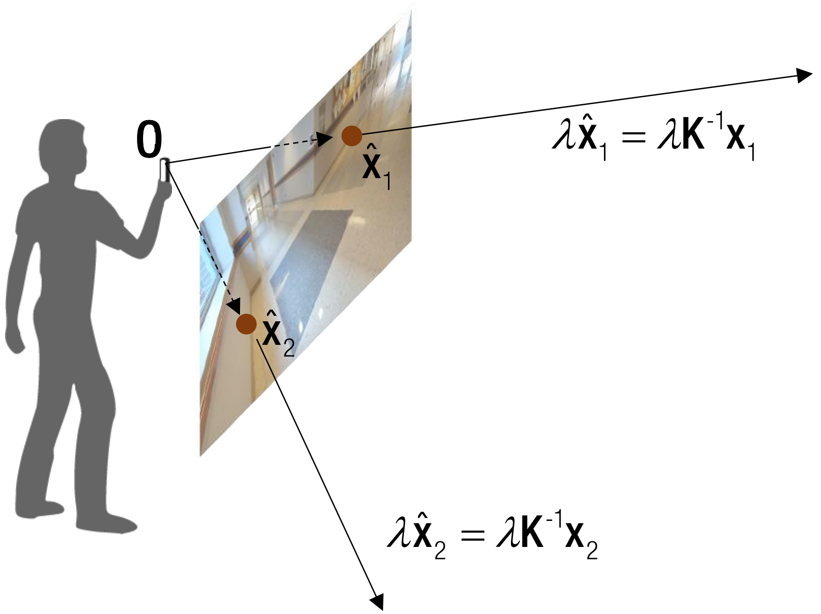
Geometric Interpretation (Point)

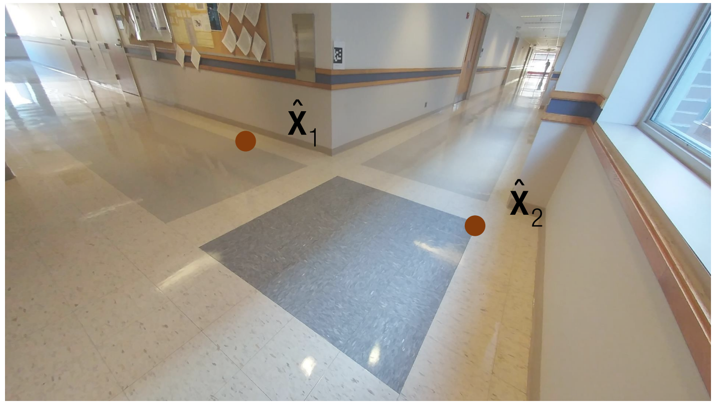


Normalized coordinate:

$$\hat{\mathbf{x}} = \mathbf{K}^{-1}\mathbf{x}$$

Geometric Interpretation (Point)

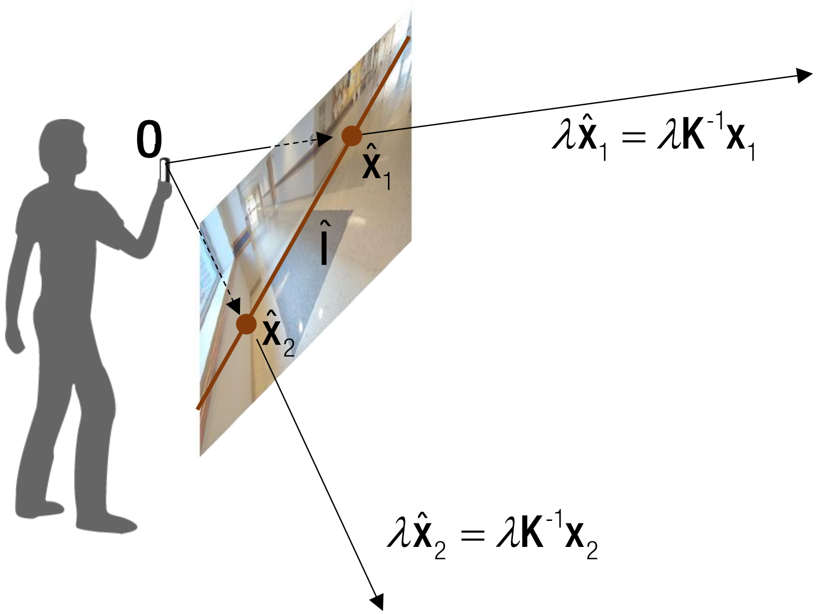


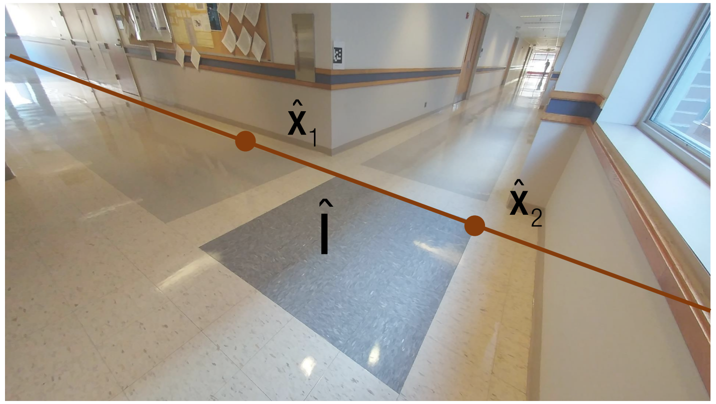


Normalized coordinate:

$$\hat{\mathbf{x}}_1 = \mathbf{K}^{-1}\mathbf{x}_1 \qquad \hat{\mathbf{x}}_2 = \mathbf{K}^{-1}\mathbf{x}_2$$

Geometric Interpretation (Line)





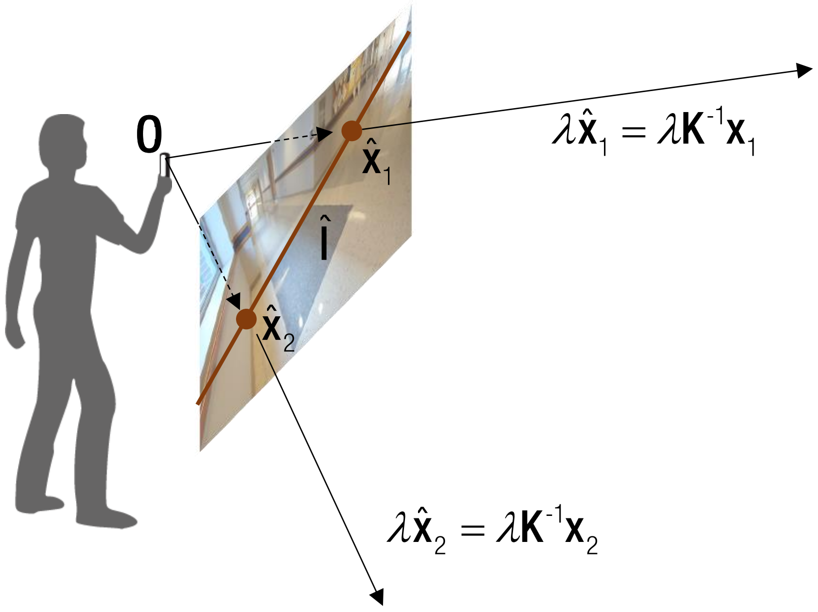
Normalized coordinate:

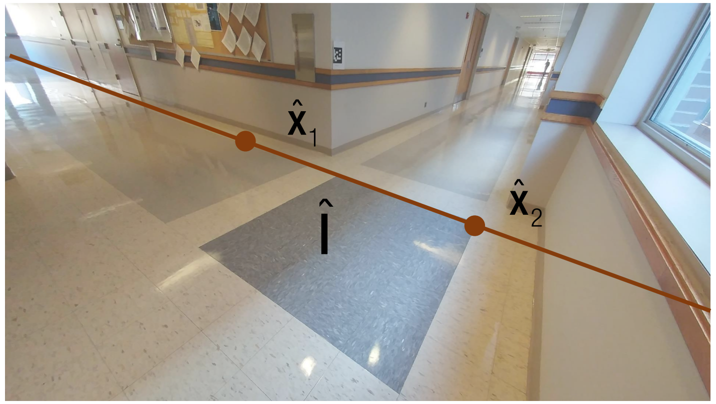
$$\hat{\mathbf{x}}_1 = \mathbf{K}^{-1}\mathbf{x}_1 \qquad \hat{\mathbf{x}}_2 = \mathbf{K}^{-1}\mathbf{x}_2$$

$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2$$

where $\hat{\mathbf{l}} = \ ?$

Geometric Interpretation (Line)





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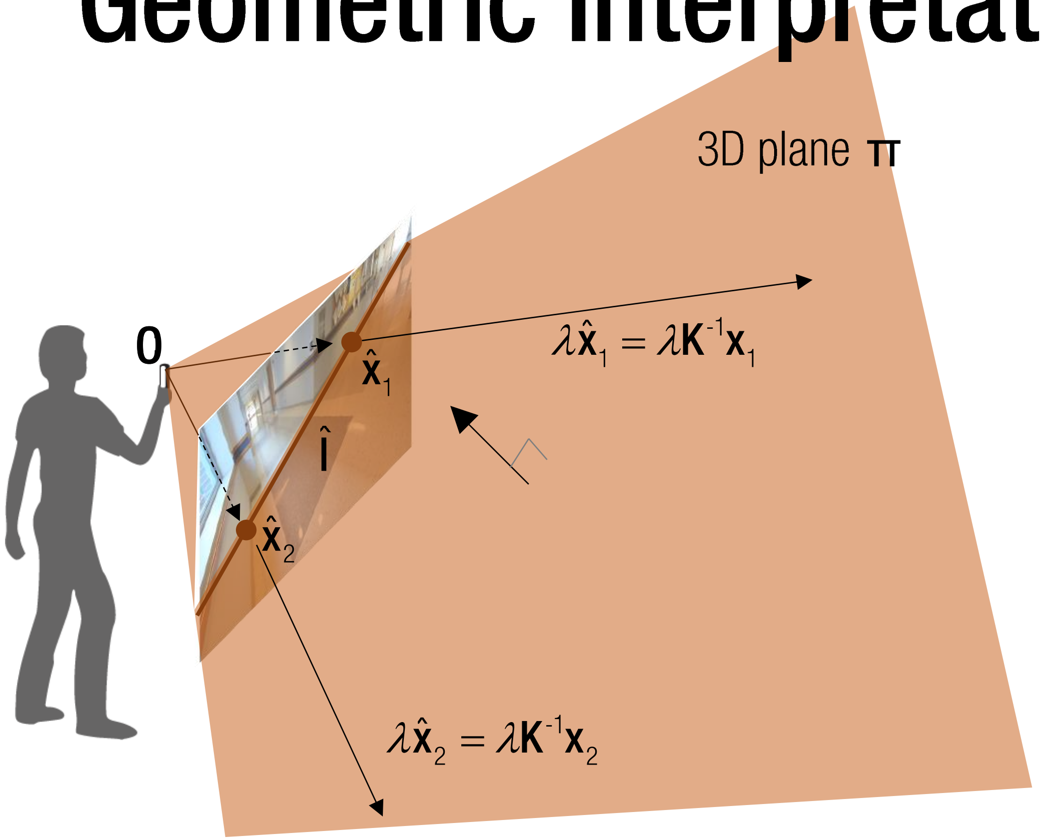
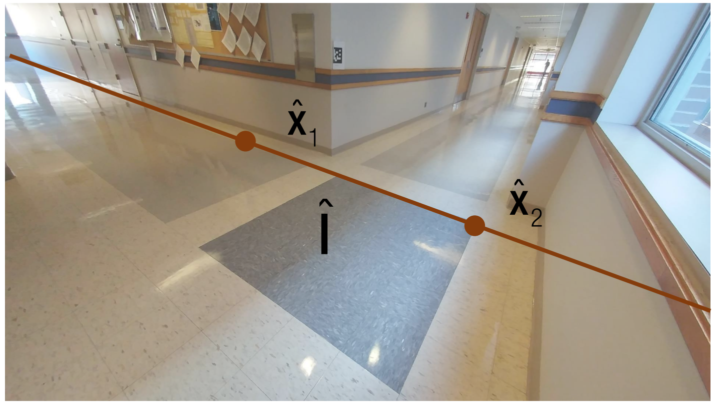
$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2$$

where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathsf{T}} \mathbf{l} = \mathbf{K}^{\mathsf{T}}\mathbf{l}$ due to duality

Geometric Interpretation (Line)





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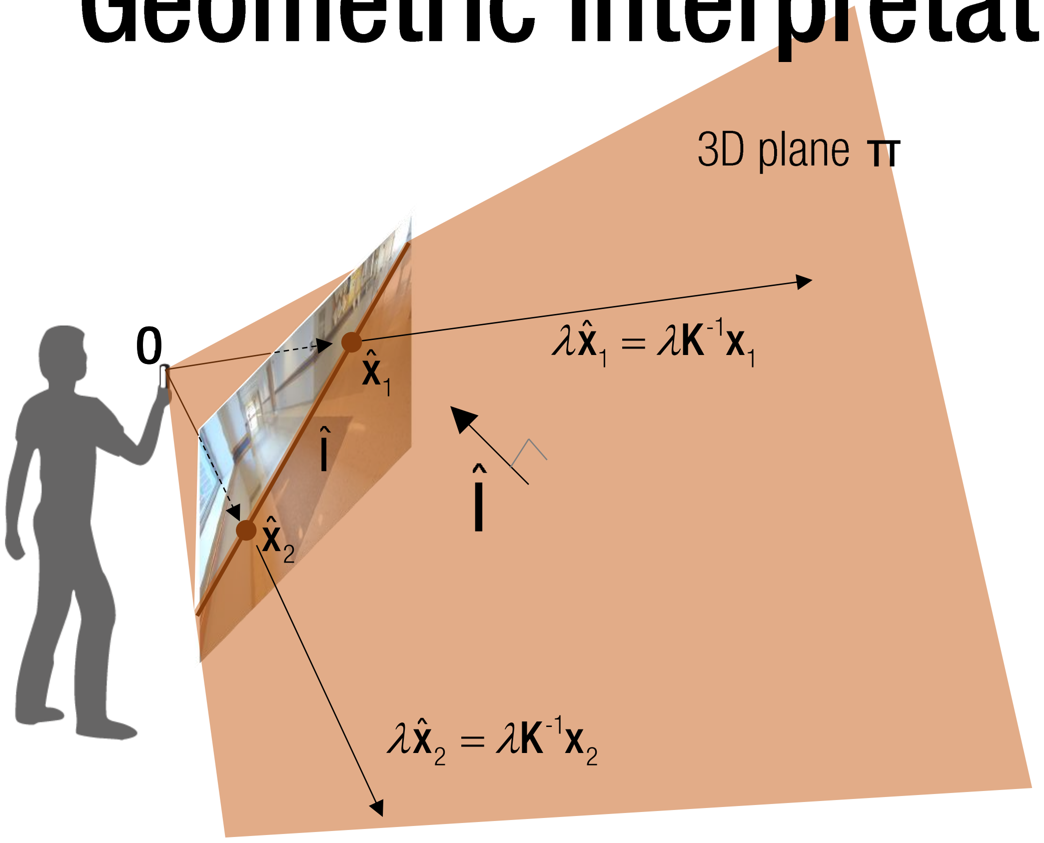
$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2$$

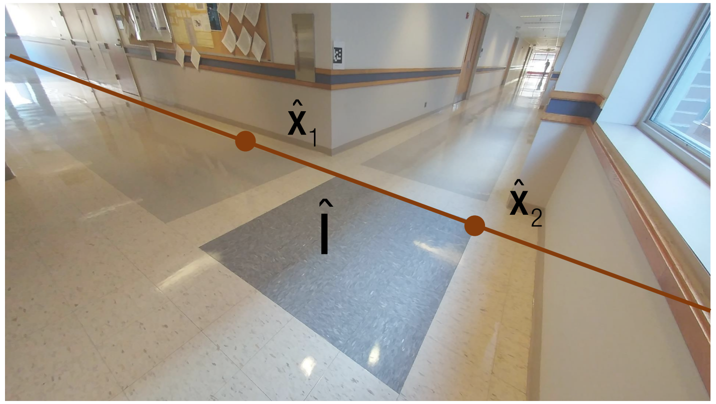
where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathrm{T}}\mathbf{l} = \mathbf{K}^{\mathrm{T}}\mathbf{l}$ due to duality

A 2D line in an image defines to a 3D plane passing the camera center:

$$\hat{\mathbf{l}} \rightarrow \pi$$

Geometric Interpretation (Line)





Normalized coordinate:

$$\hat{\mathbf{x}}_1 = \mathbf{K}^{-1}\mathbf{x}_1 \qquad \hat{\mathbf{x}}_2 = \mathbf{K}^{-1}\mathbf{x}_2$$

$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2$$

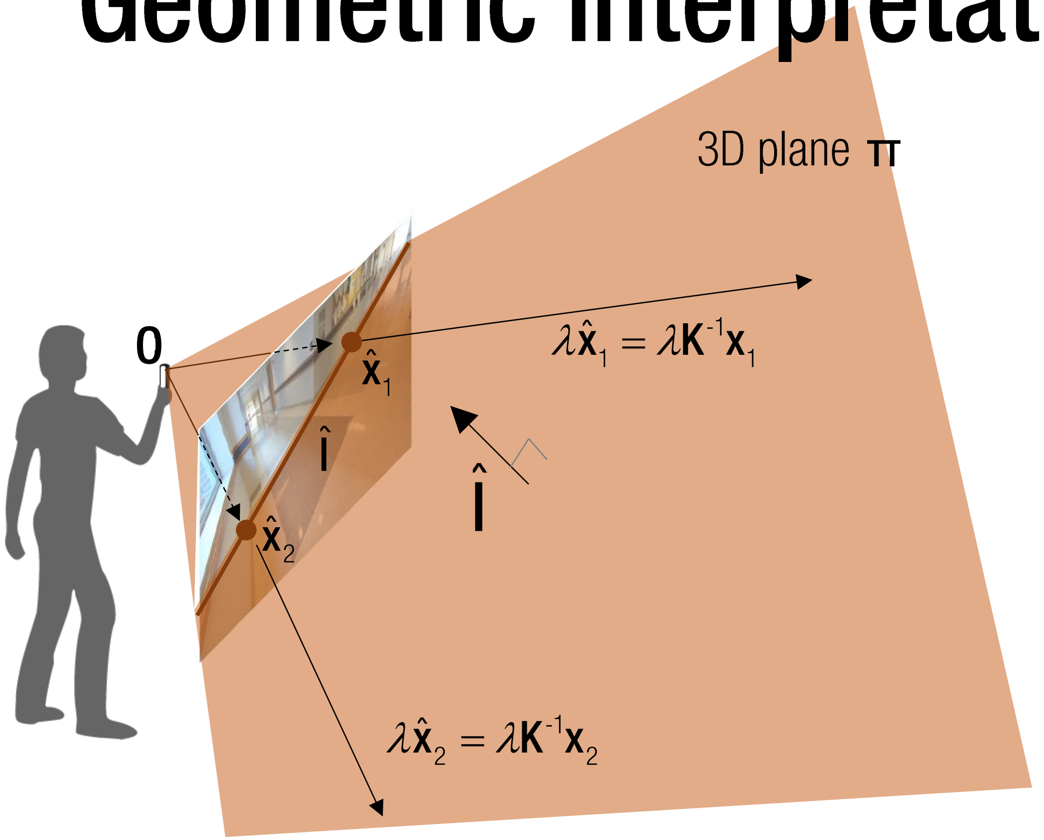
where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathrm{T}}\mathbf{l} = \mathbf{K}^{\mathrm{T}}\mathbf{l}$ due to duality

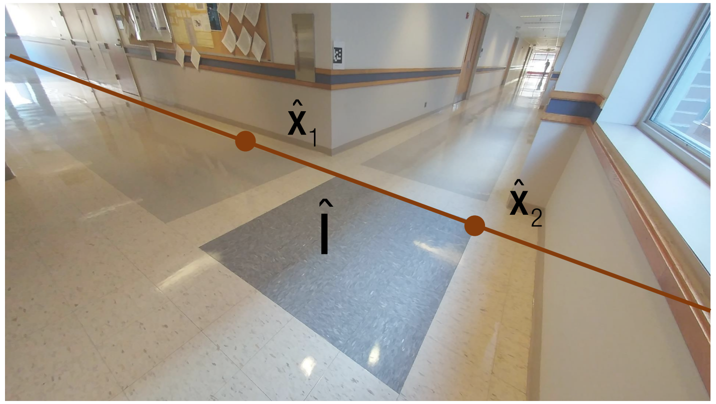
A 2D line in an image defines to a 3D plane passing the camera center:

$$\hat{\mathbf{l}} \rightarrow \pi$$

Plane normal: $\quad$ **?** $\quad = \lambda\hat{\mathbf{l}}$

Geometric Interpretation (Line)





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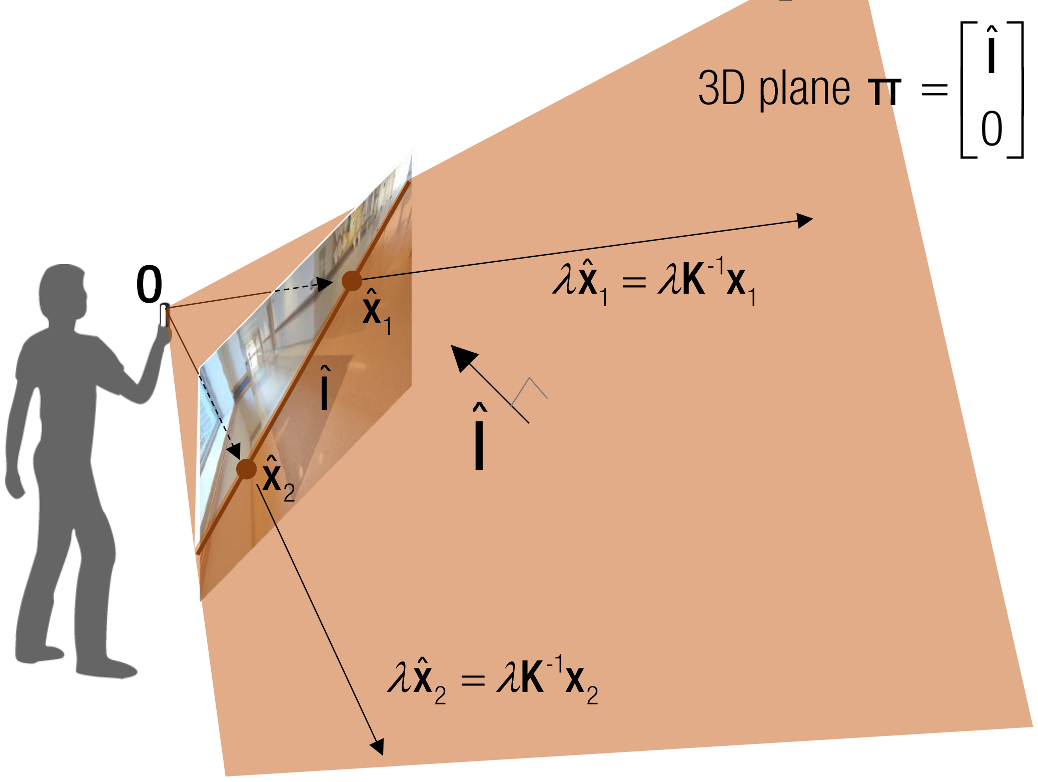

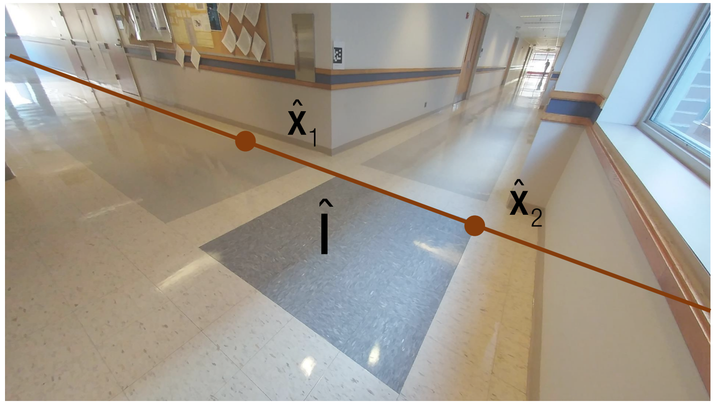

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Plane normal: $\left(\lambda_1 \hat{\mathbf{x}}_1\right) \times \left(\lambda_2 \hat{\mathbf{x}}_2\right) = \lambda \hat{\mathbf{l}}$

$$\therefore \pi = \begin{bmatrix} \hat{\mathbf{l}} \\ 0 \end{bmatrix}$$

Geometric Interpretation (Line-Line)

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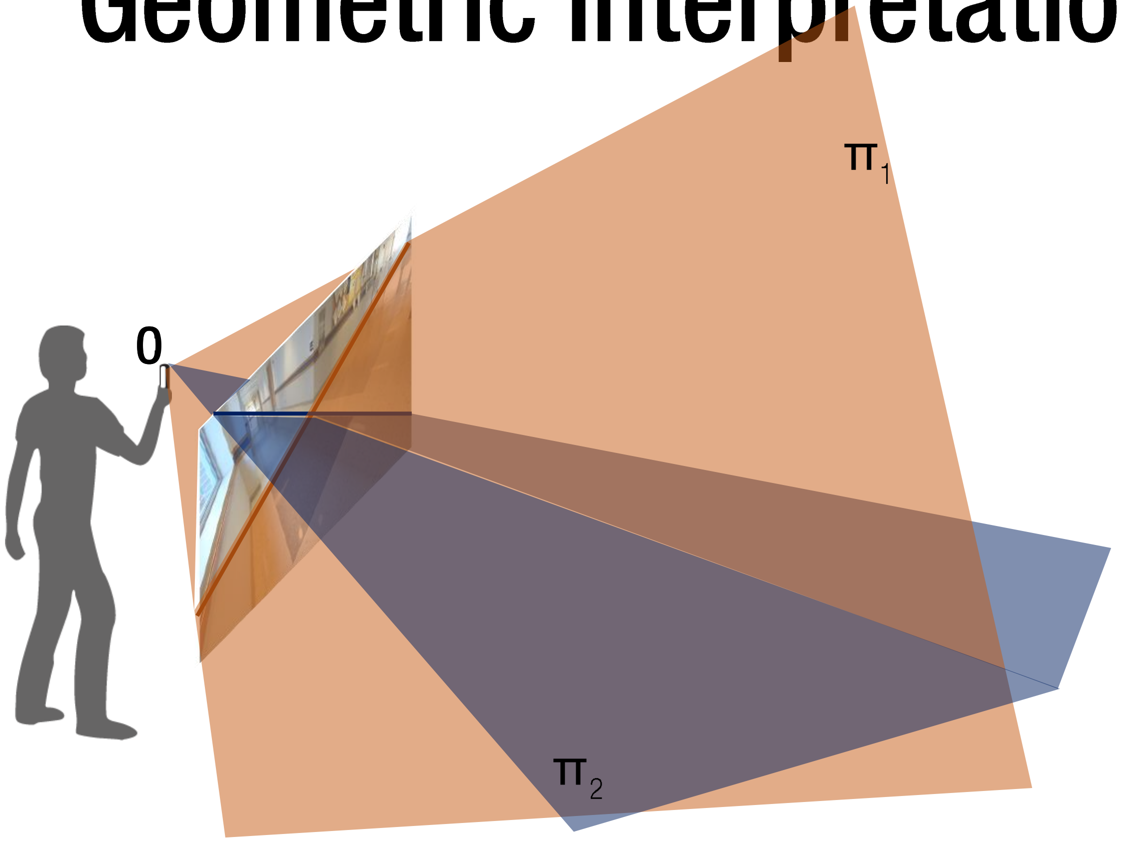
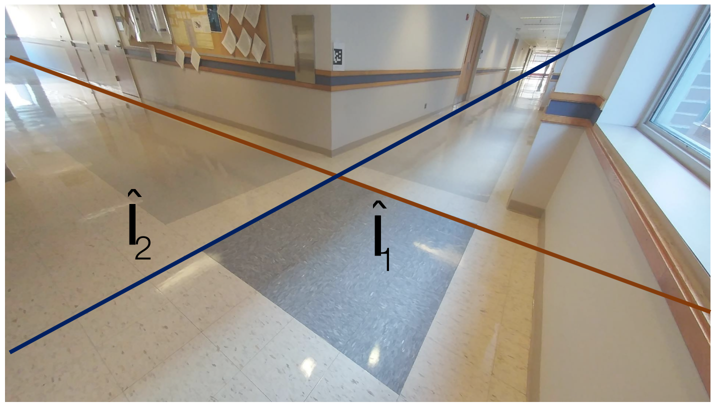
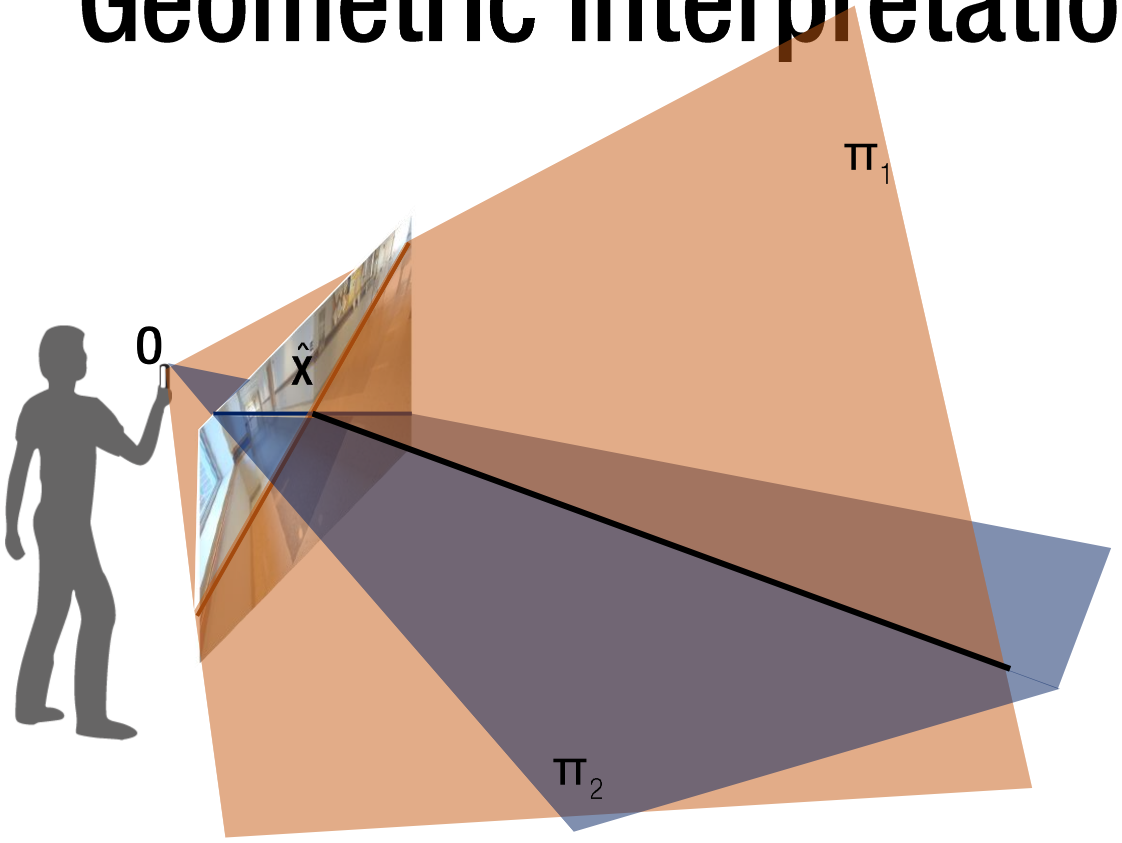
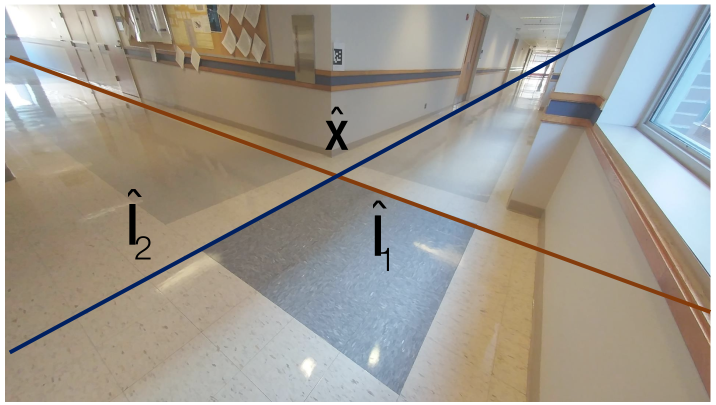




Geometric Interpretation (Line-Line)

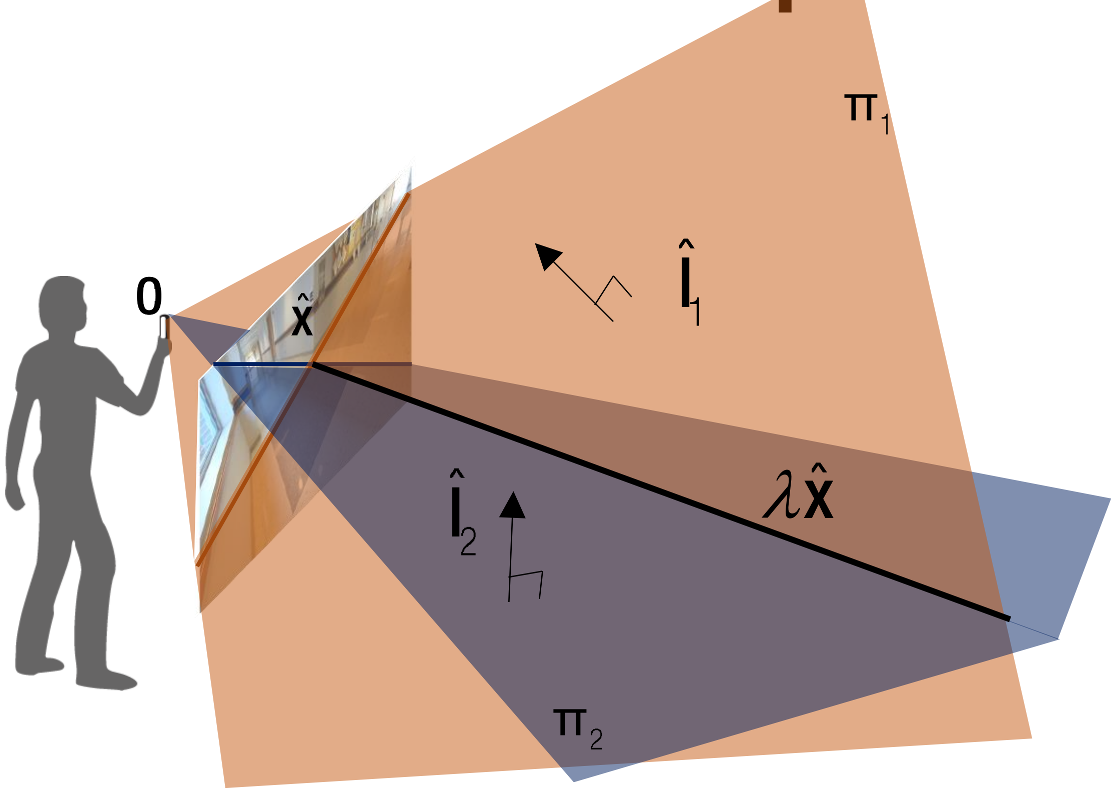


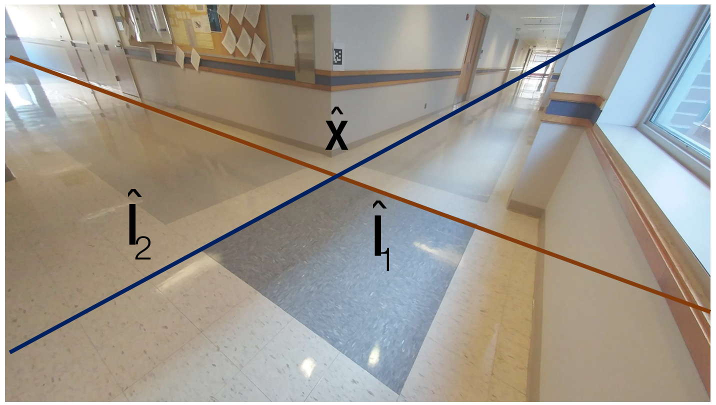


2D lines in an image intersect a 2D point corresponding to a 3D ray:

$$\hat{\mathbf{x}} = \hat{\mathbf{l}}_1 \times \hat{\mathbf{l}}_2$$

Geometric Interpretation (Line-Line)





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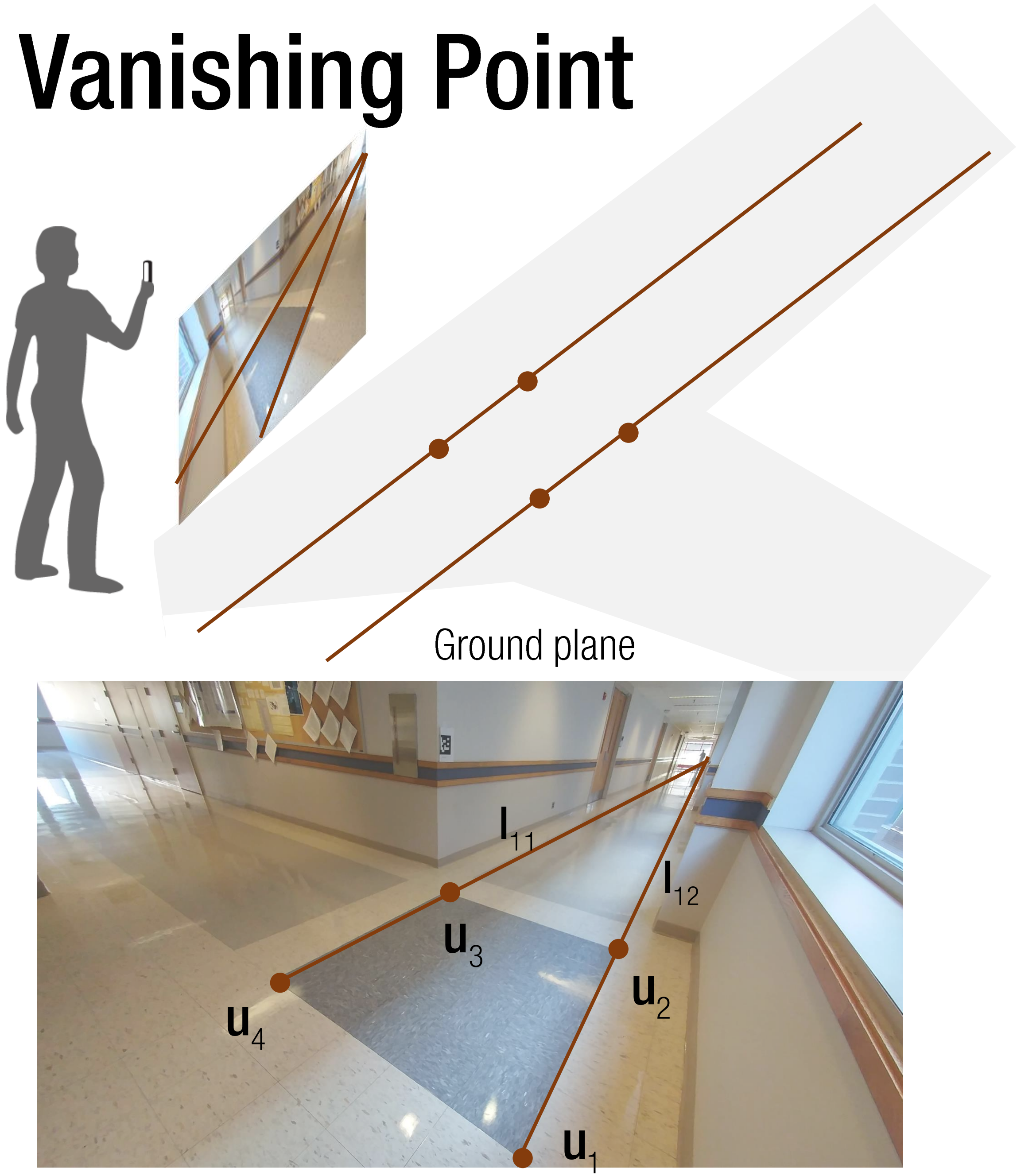
$$\hat{\mathbf{x}} = \hat{\mathbf{l}}_1 \times \hat{\mathbf{l}}_2$$

: the 3D ray is perpendicular to two plane normals.

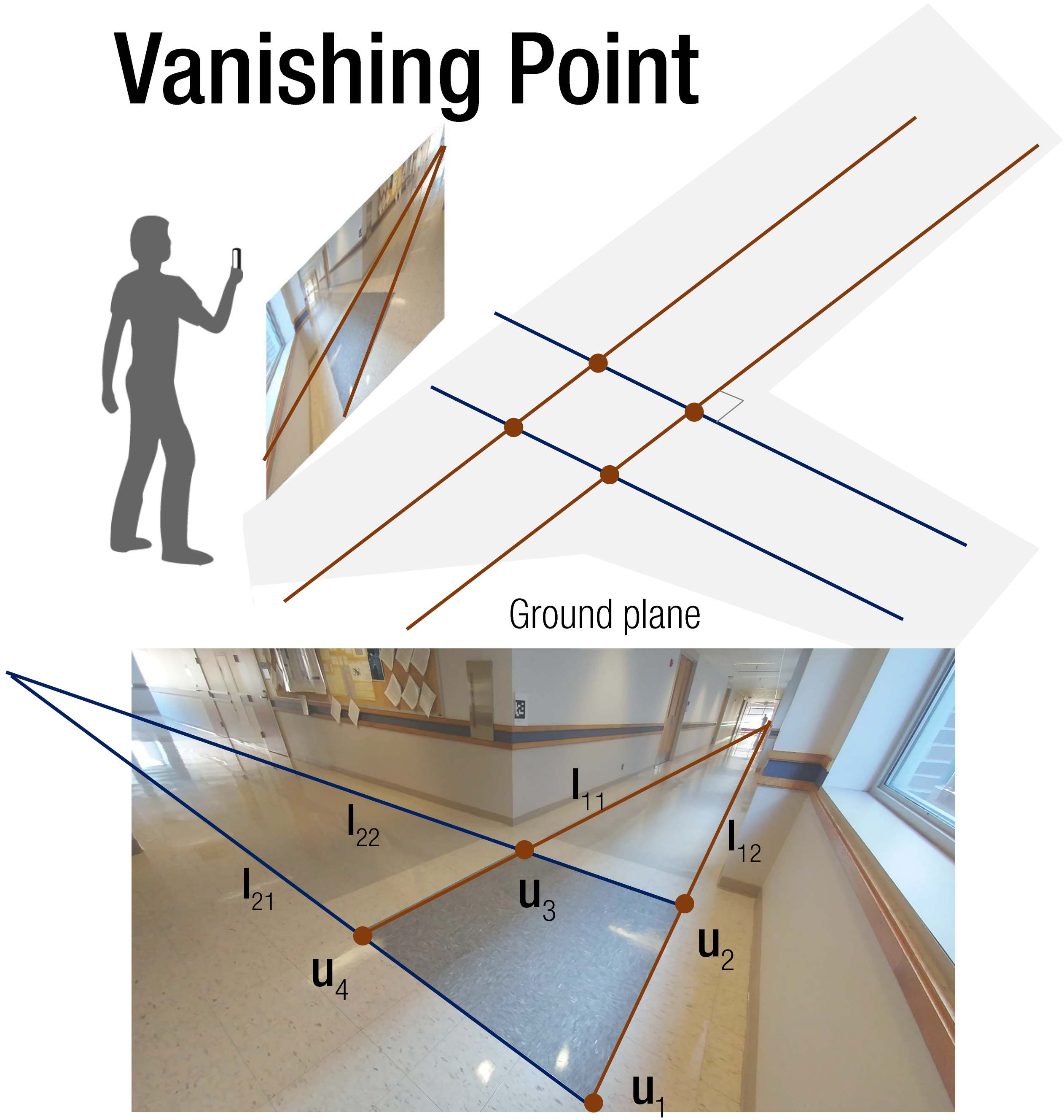
Vanishing Point



Parallel lines:

$$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \qquad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$$

Vanishing Point



Parallel lines:

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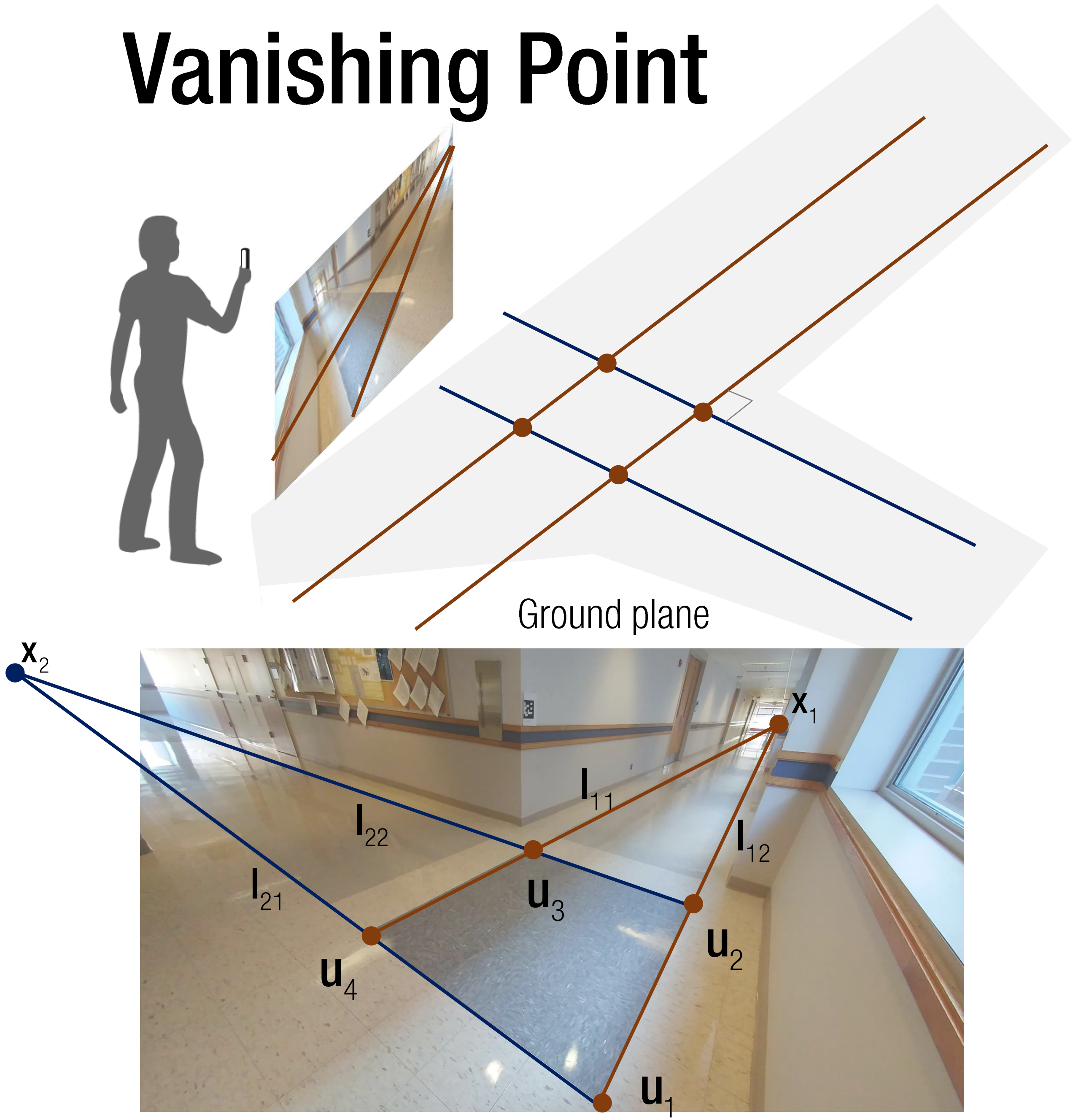
$$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \qquad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$$

Vanishing Point



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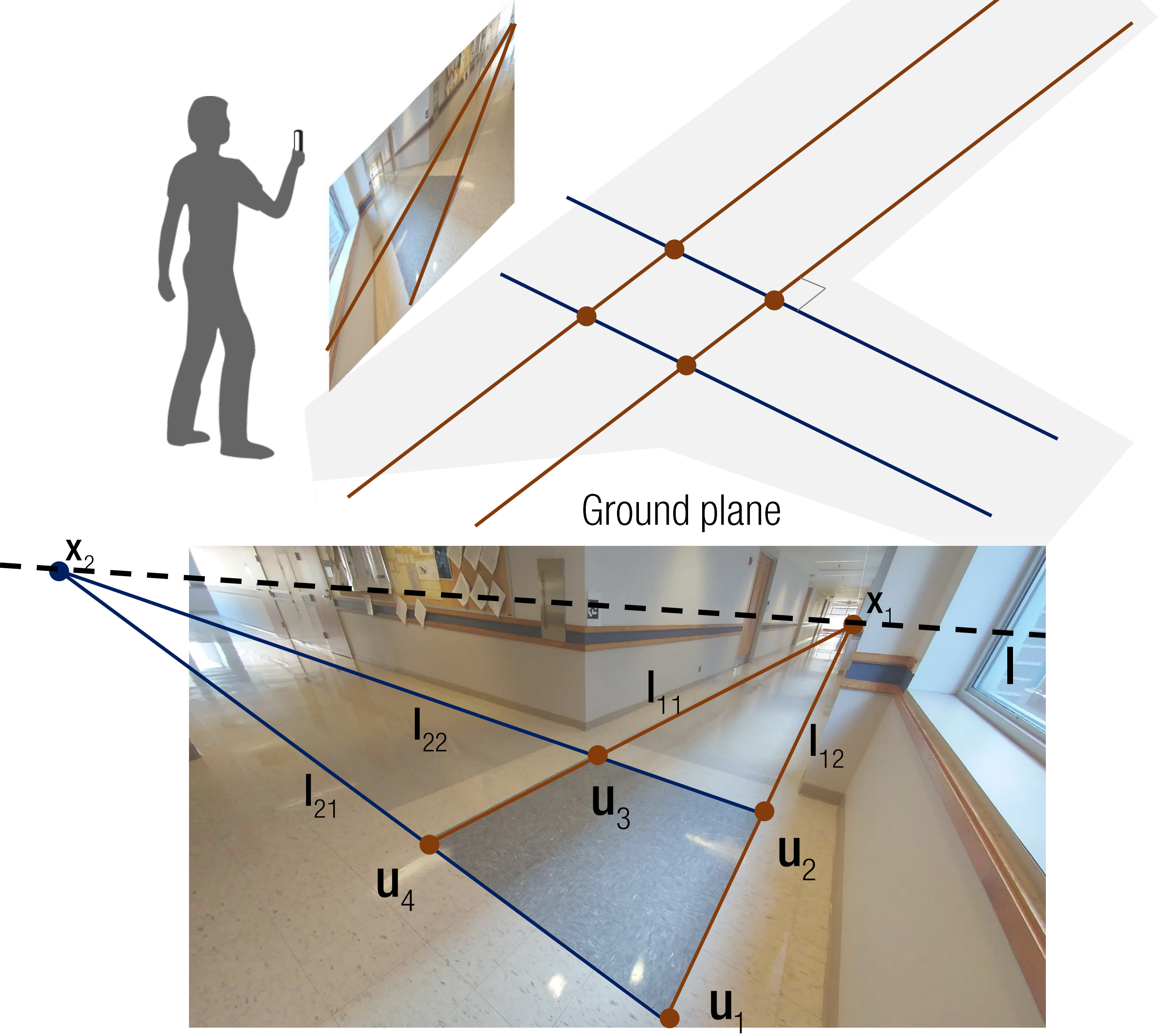
Vanishing points:

$$\mathbf{x}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \qquad \mathbf{x}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$$

Vanishing Point



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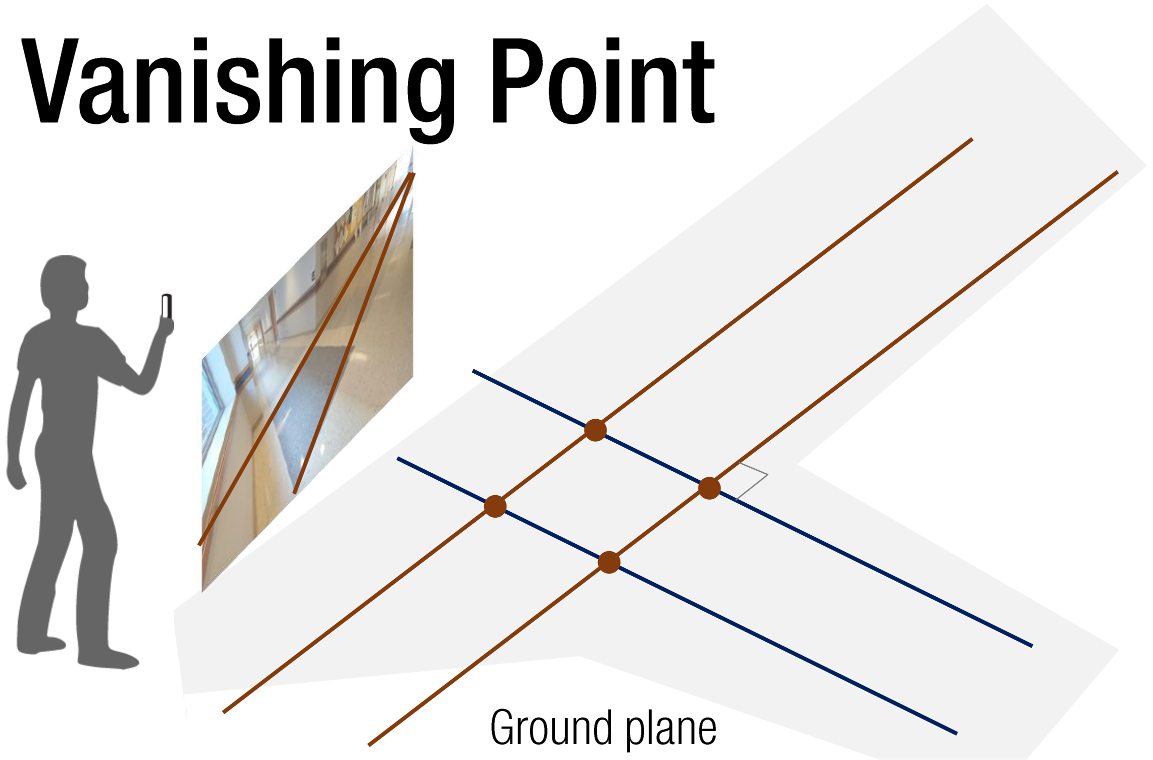
Vanishing points:

$$\mathbf{x}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \qquad \mathbf{x}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$$

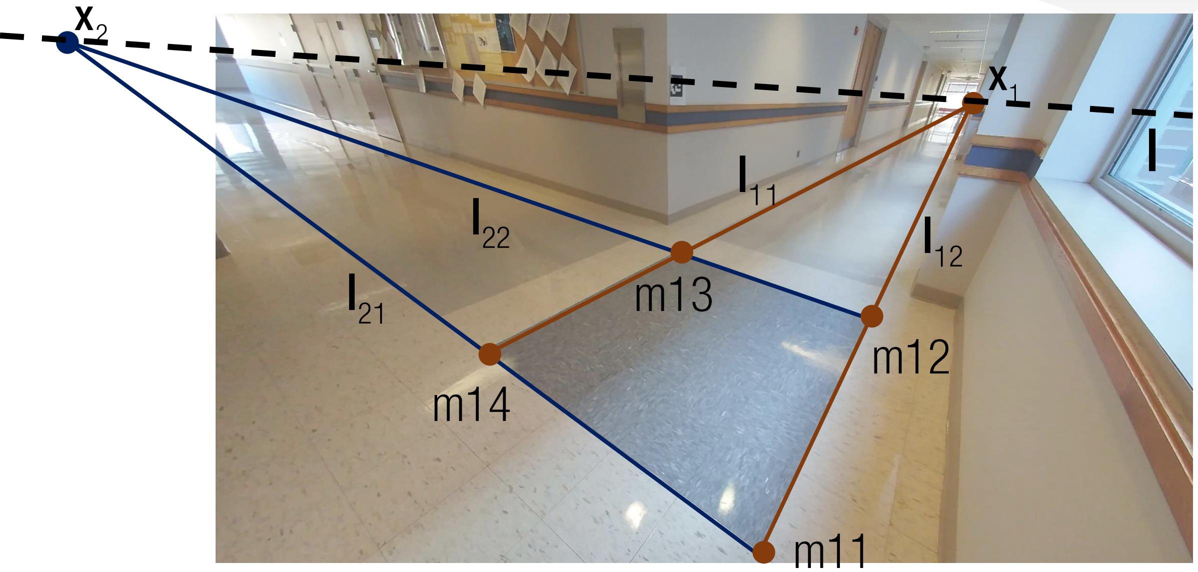
Vanishing line:

$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

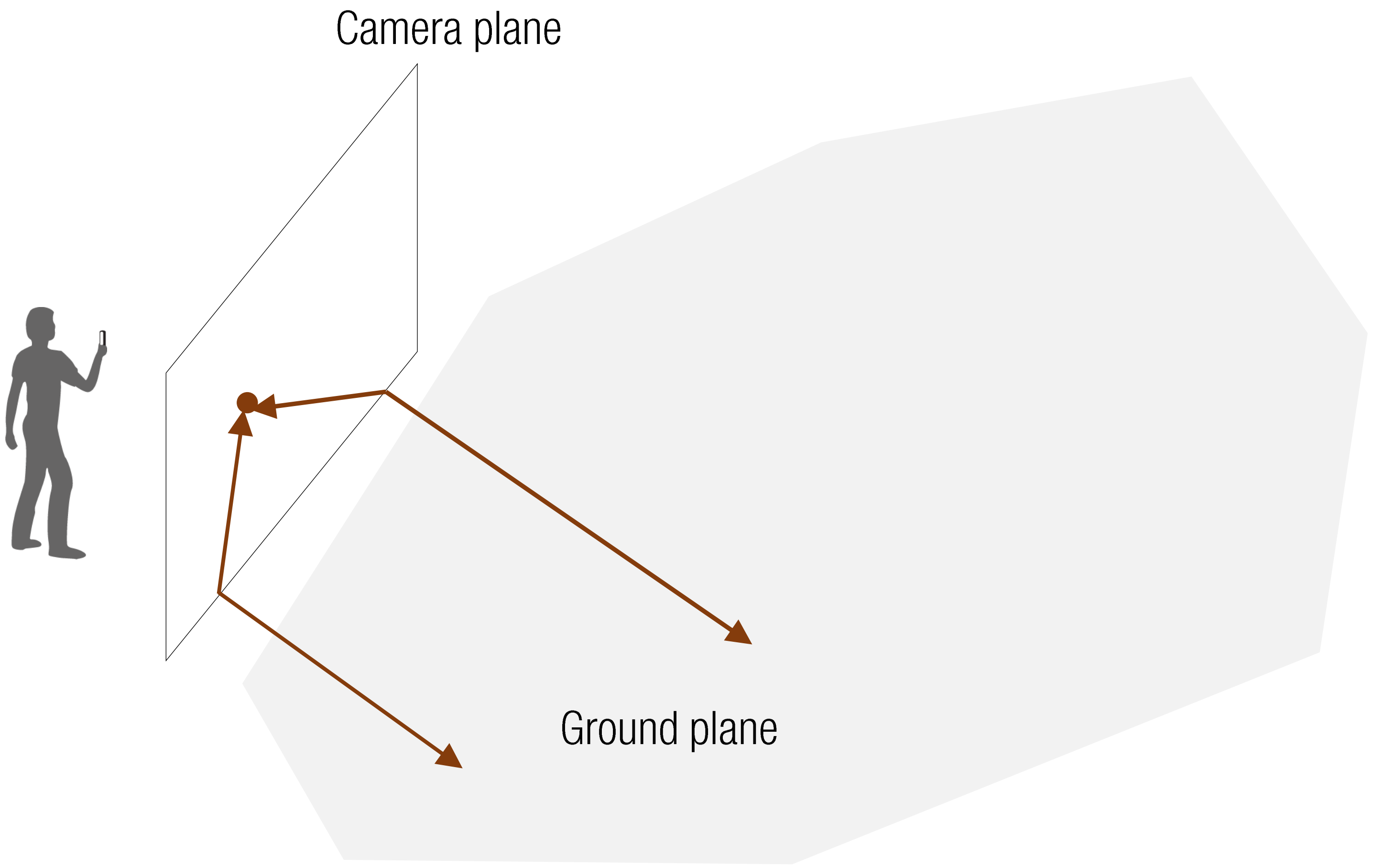
Vanishing Point




```
function ComputeVanishingLine

m11 = [2145;2120;1];m12 = [2566;1191;1];
m13 = [1804;935;1];m14 = [1050;1320;1];
m21 = m11;m22 = m14;m23 = m12;m24 = m13;

l11 = GetLineFromTwoPoints(m11,m12);
l12 = GetLineFromTwoPoints(m13,m14);

l21 = GetLineFromTwoPoints(m21,m22);
l22 = GetLineFromTwoPoints(m23,m24);

x1 = GetPointFromTwoLines(l11,l12);
x2 = GetPointFromTwoLines(l21,l22);

vanishing_line = GetLineFromTwoPoints(x1, x2);
```



Geometric Interpretation of Vanishing Line



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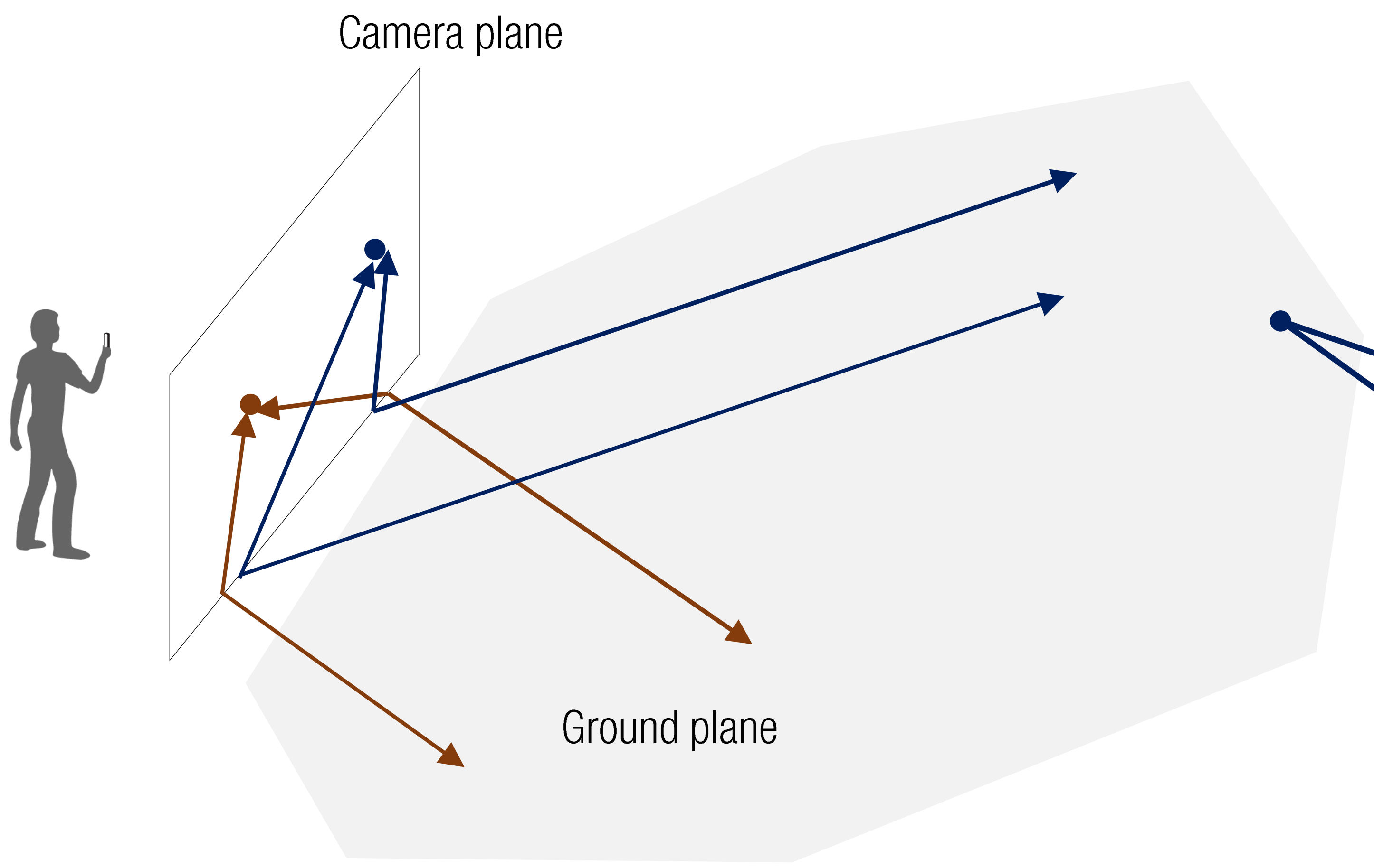

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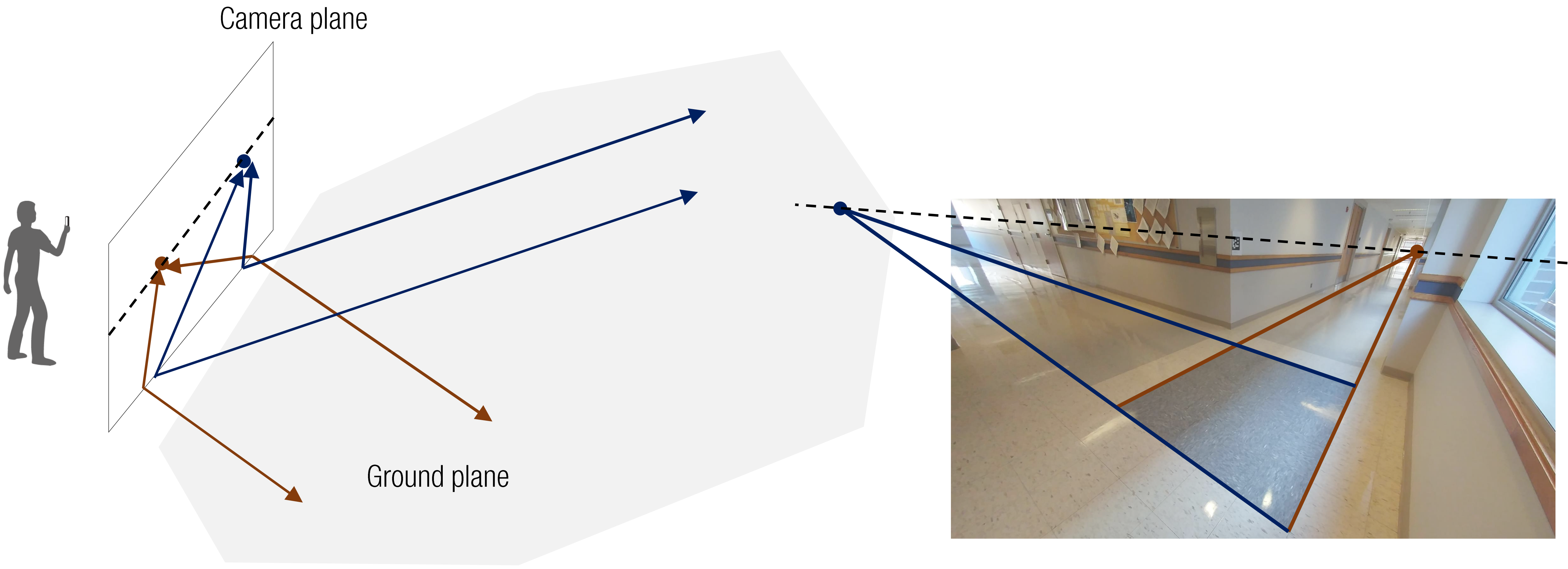

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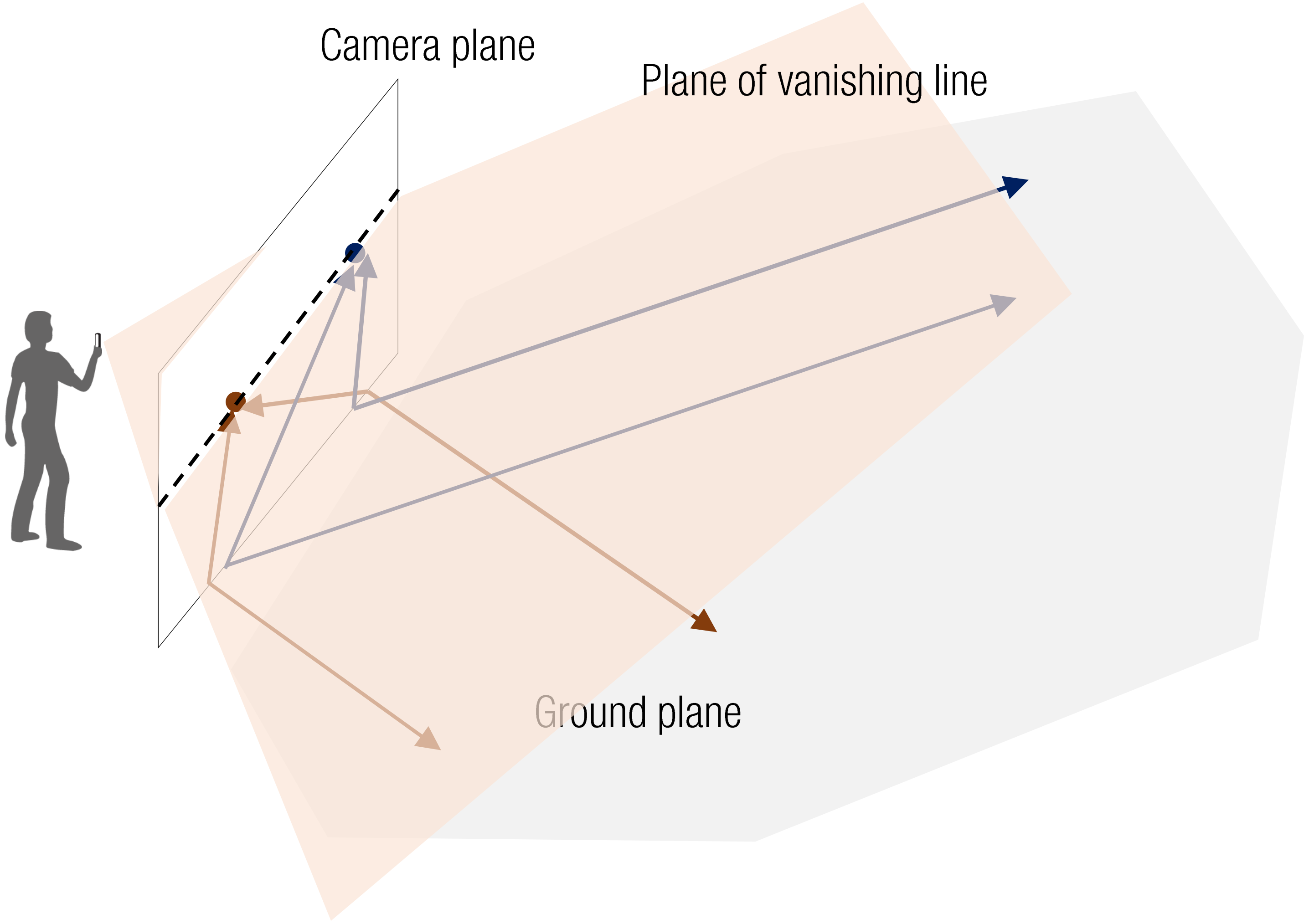
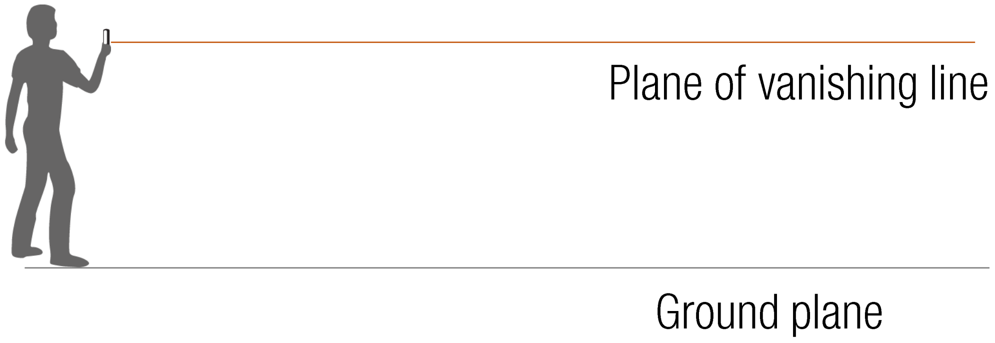
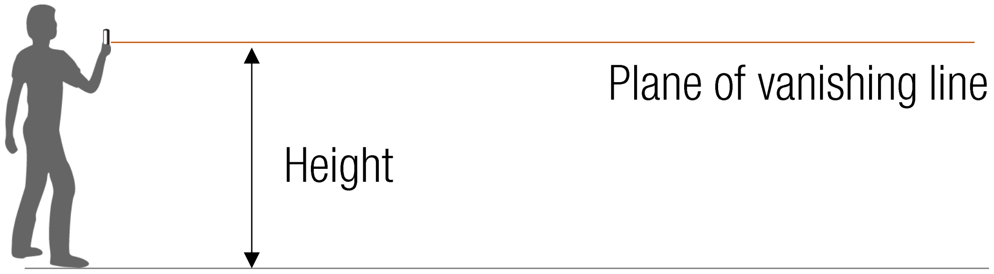




Side view

Geometric Interpretation of Vanishing Line



Side view

Where was I (how high)?

Where was I (how high)?

Taken from my hotel room (6^{th} floor)

Taken from beach

Where was I (how high)?

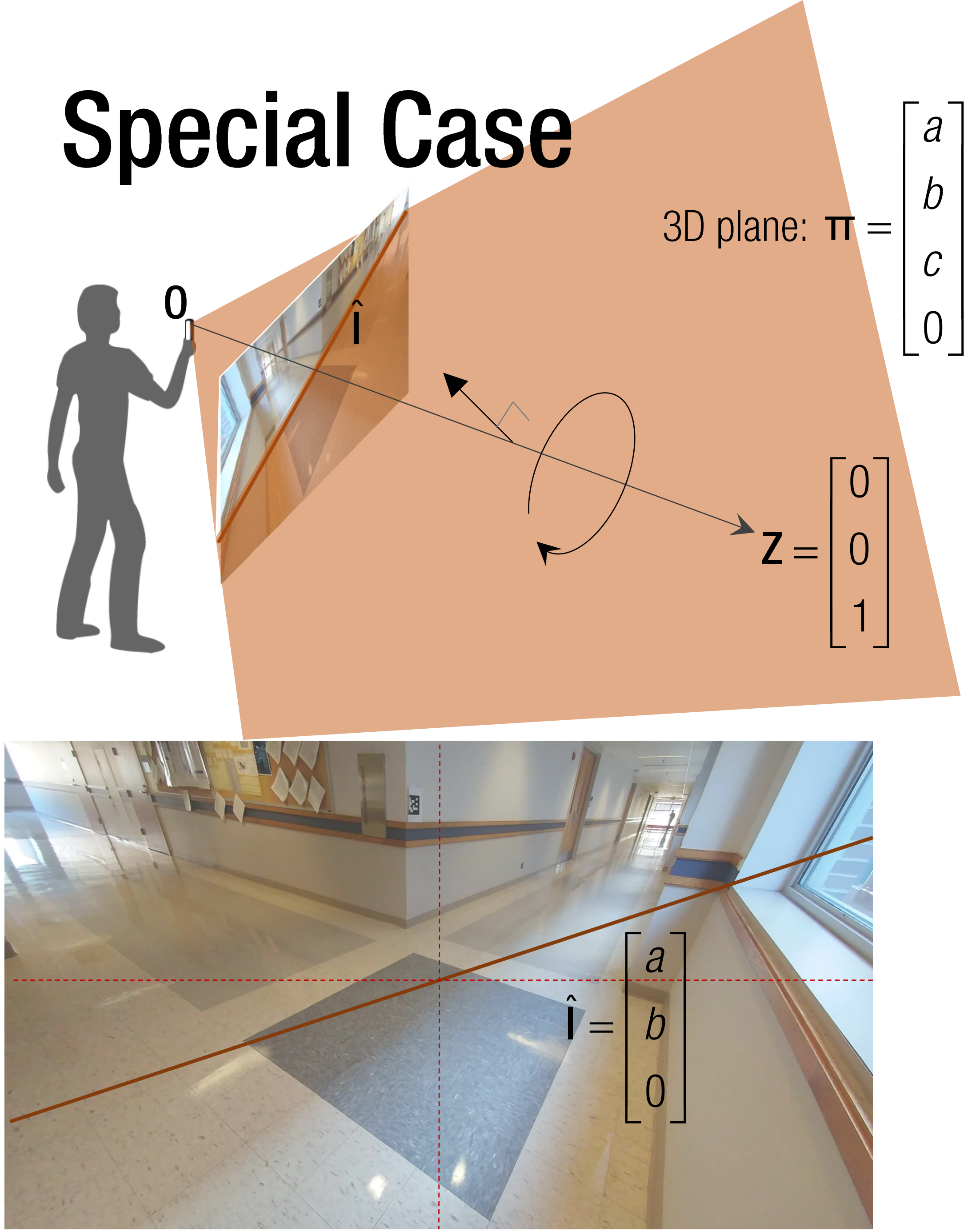
Taken from my hotel room (6th floor)

Taken from beach

First person video

Cylindrical projection

Special Case

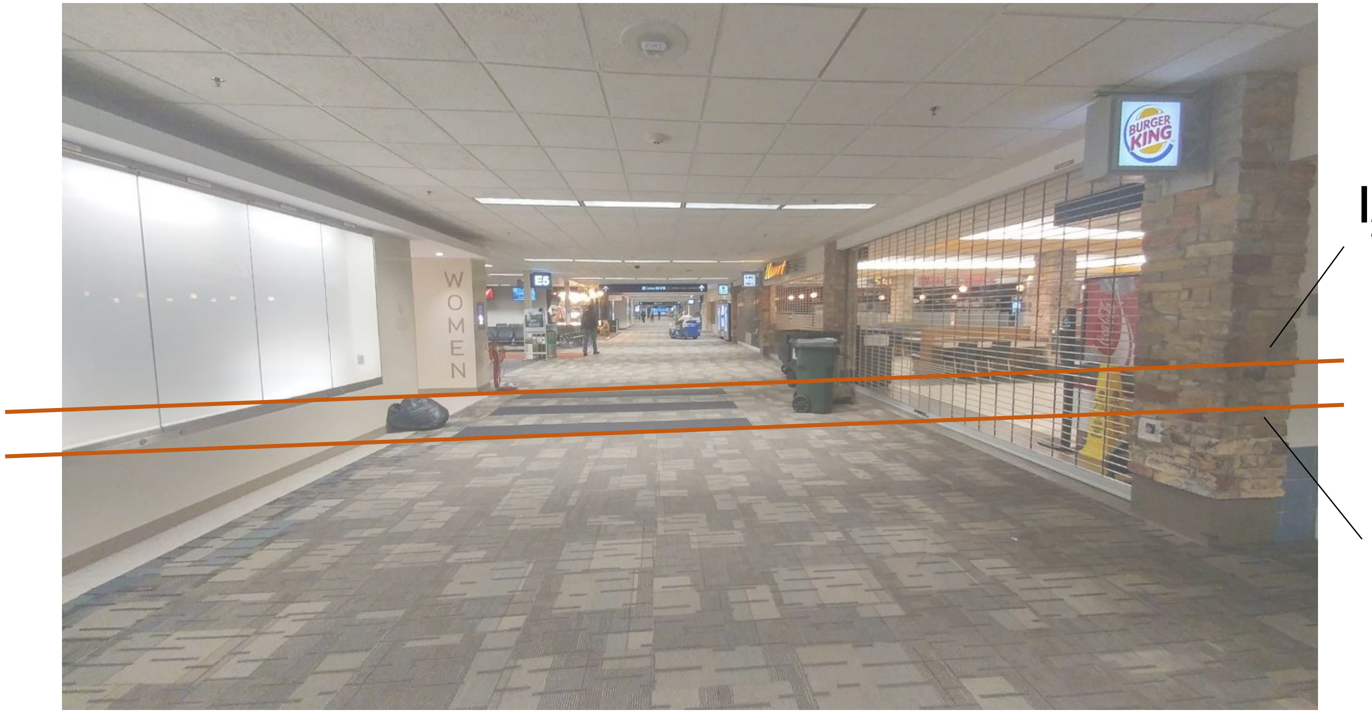


When the last element of line, **l** is zero:

- the line passes through the center of image.
- the surface normal of the 3D plane is perpendicular to Z axis
- the 3D plane is a plane rotating around Z axis

$$\begin{bmatrix} a & b & 0 \end{bmatrix} \mathbf{Z} = 0$$

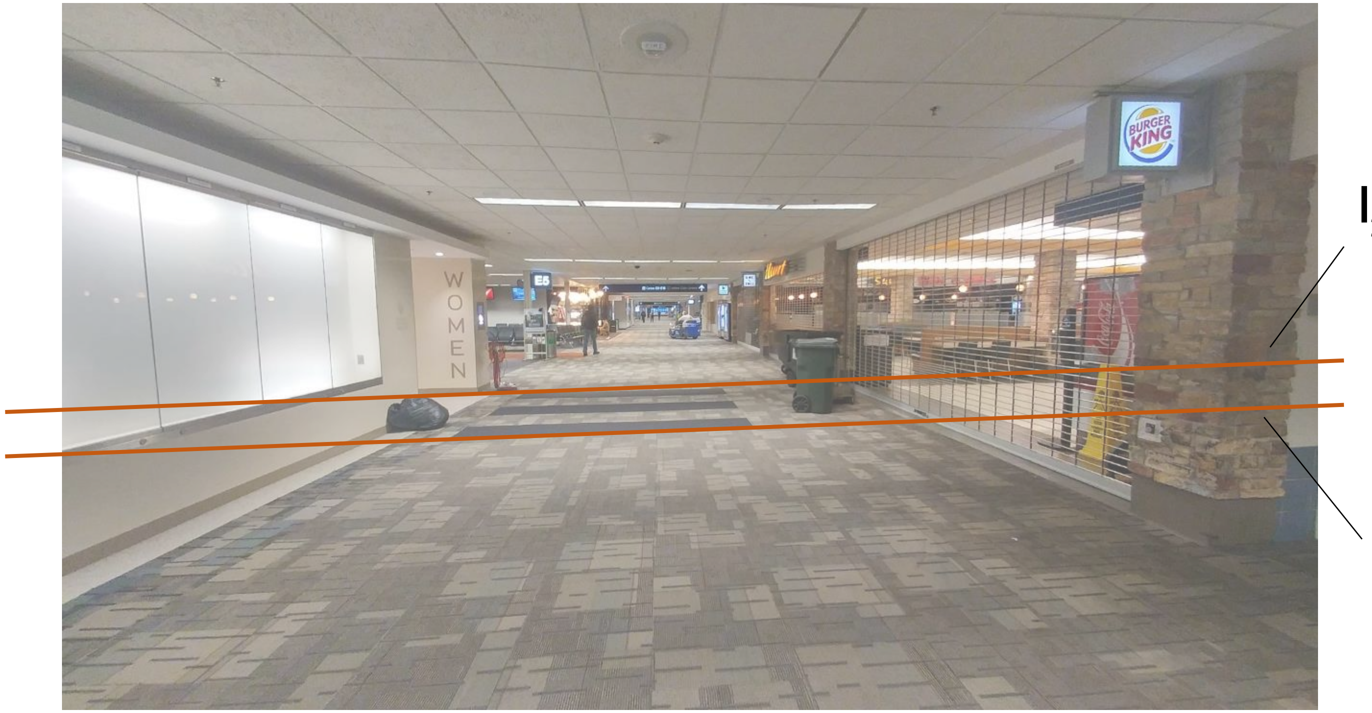
Special Case: 2D Point at Infinity



$$\mathbf{l}_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix}$$

$$\mathbf{l}_1 = \begin{bmatrix} a \\ b \\ c_1 \end{bmatrix}$$

Special Case: 2D Point at Infinity



$$\mathbf{l}_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix}$$

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The intersection of two parallel lines in an image:

$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 = (c_2 - c_1) \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$$

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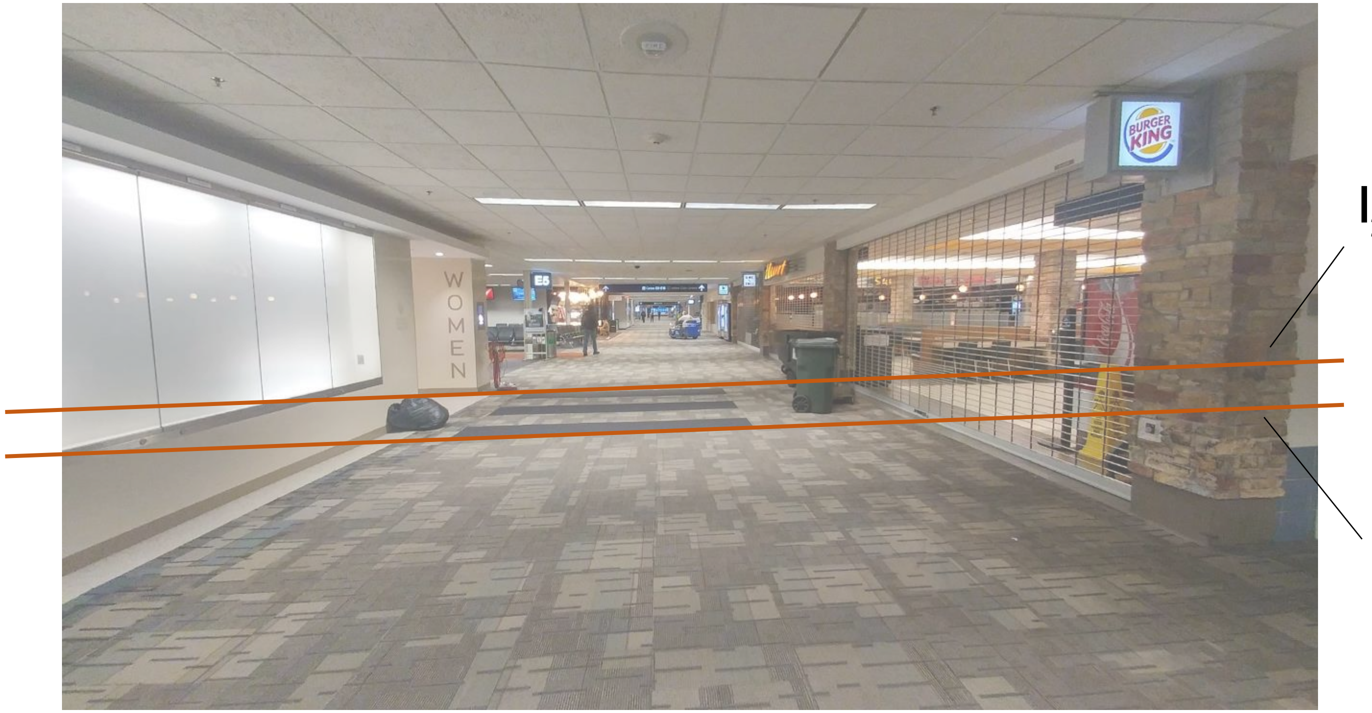

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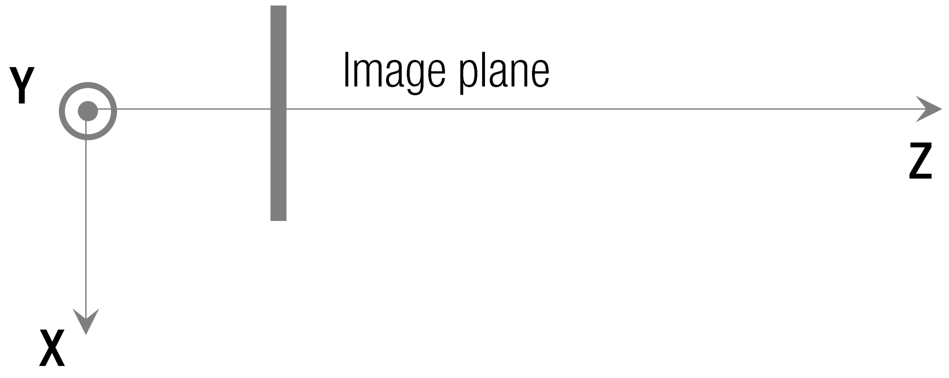
$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 = \left(c_2 - c_1\right) \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$$

This point does not correspond to a finite point because:

$$\mathbf{x} = \begin{bmatrix} -b/0 \\ a/0 \\ 0/0 \end{bmatrix}$$

Special Case: 2D Point at Infinity

Top view



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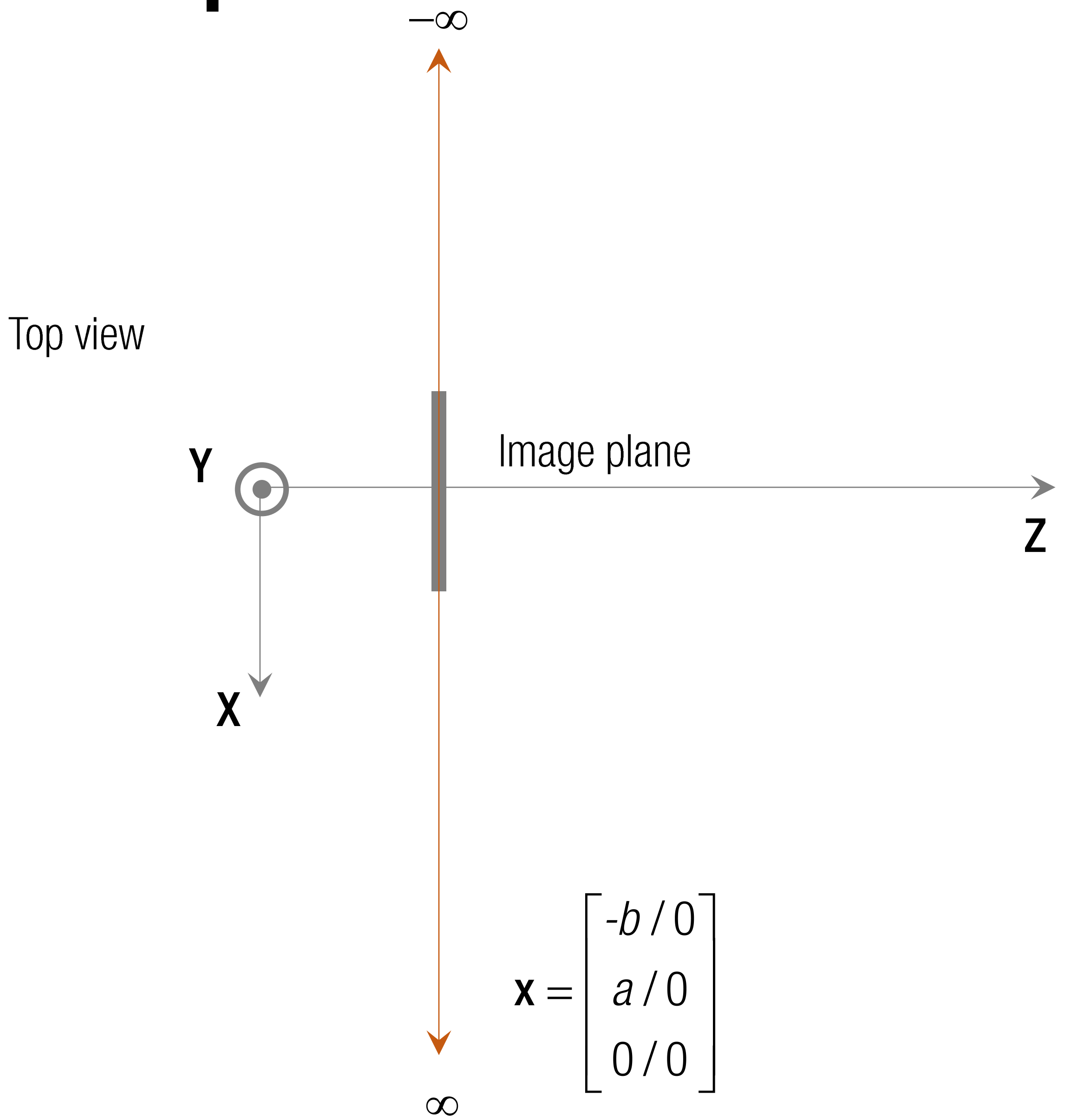

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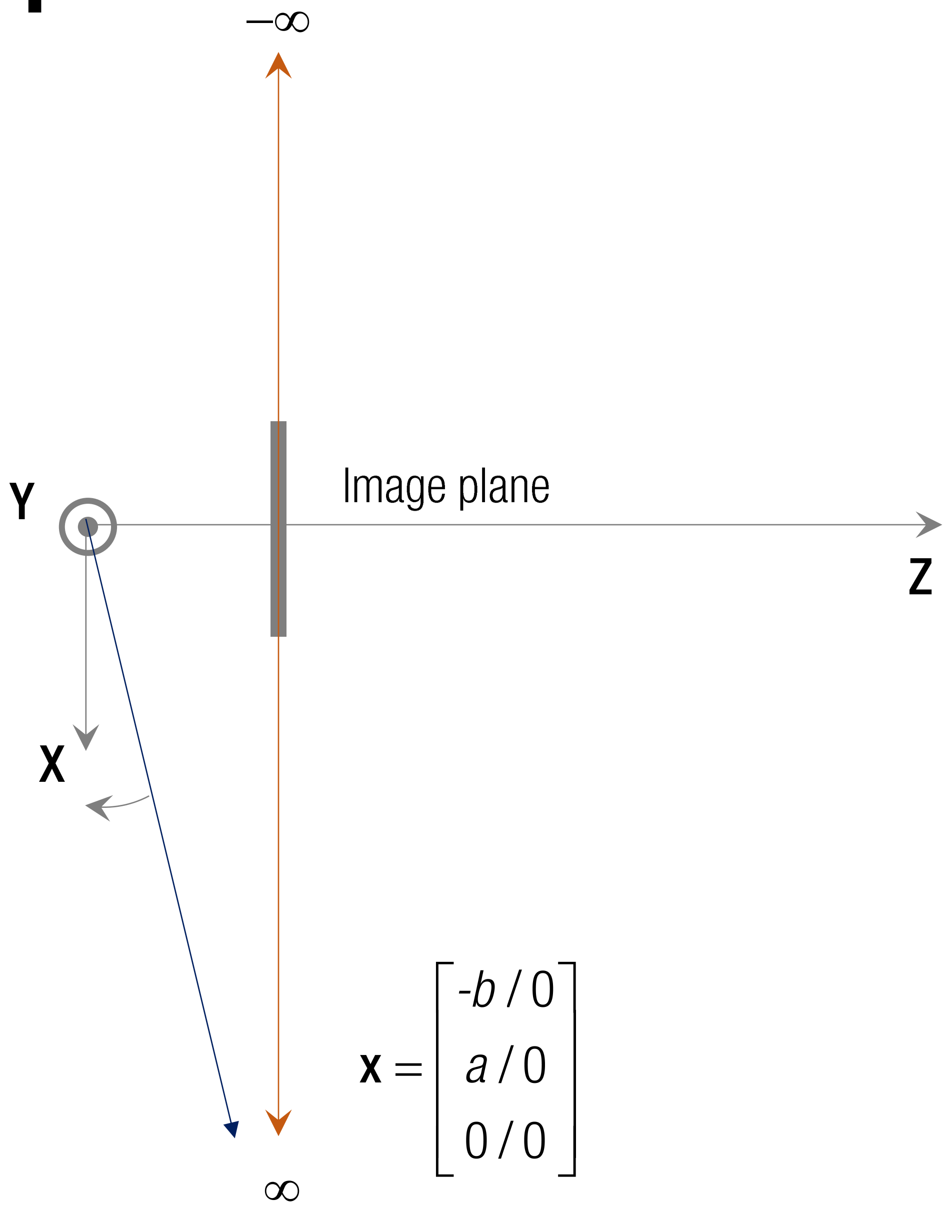

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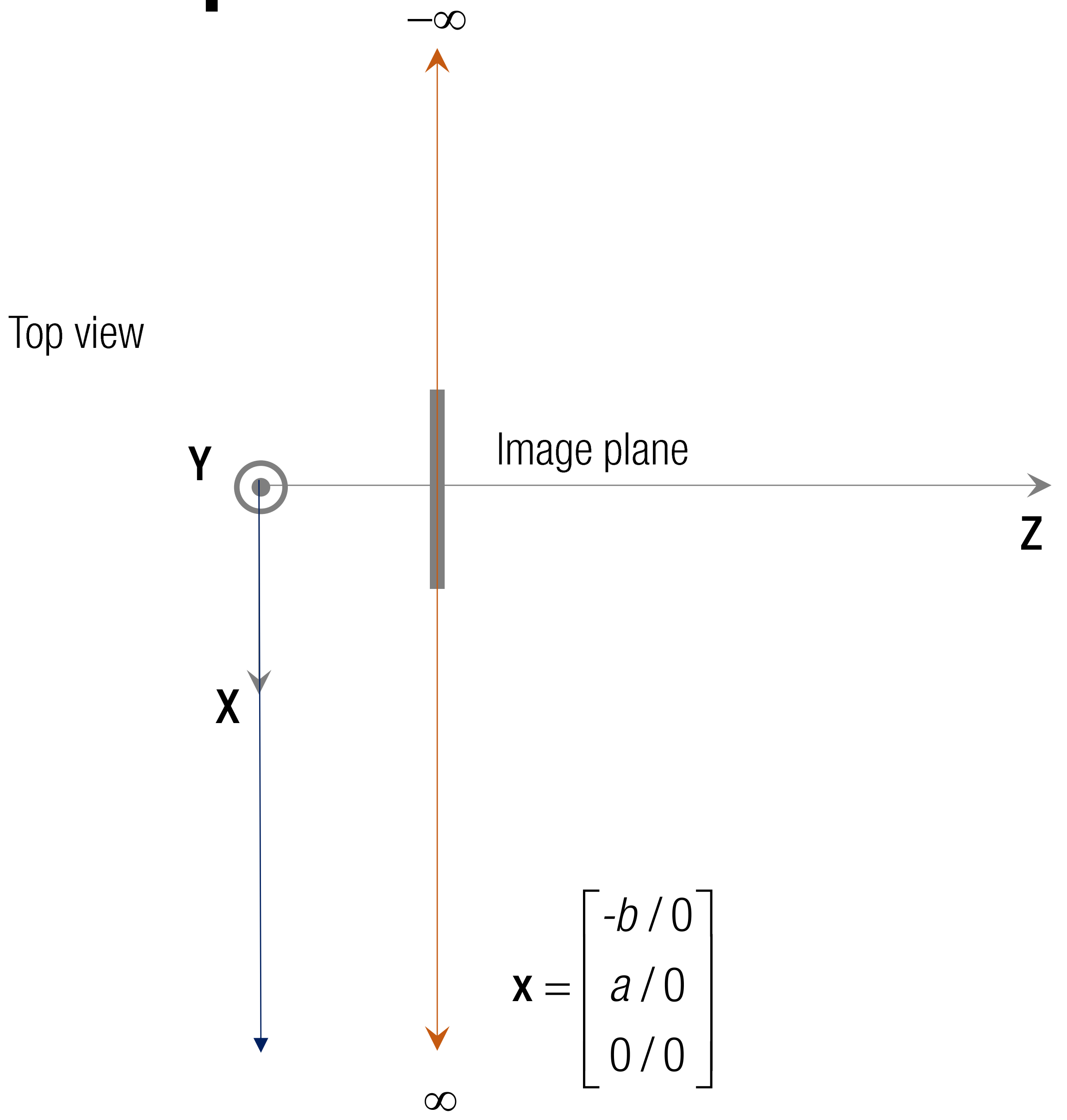

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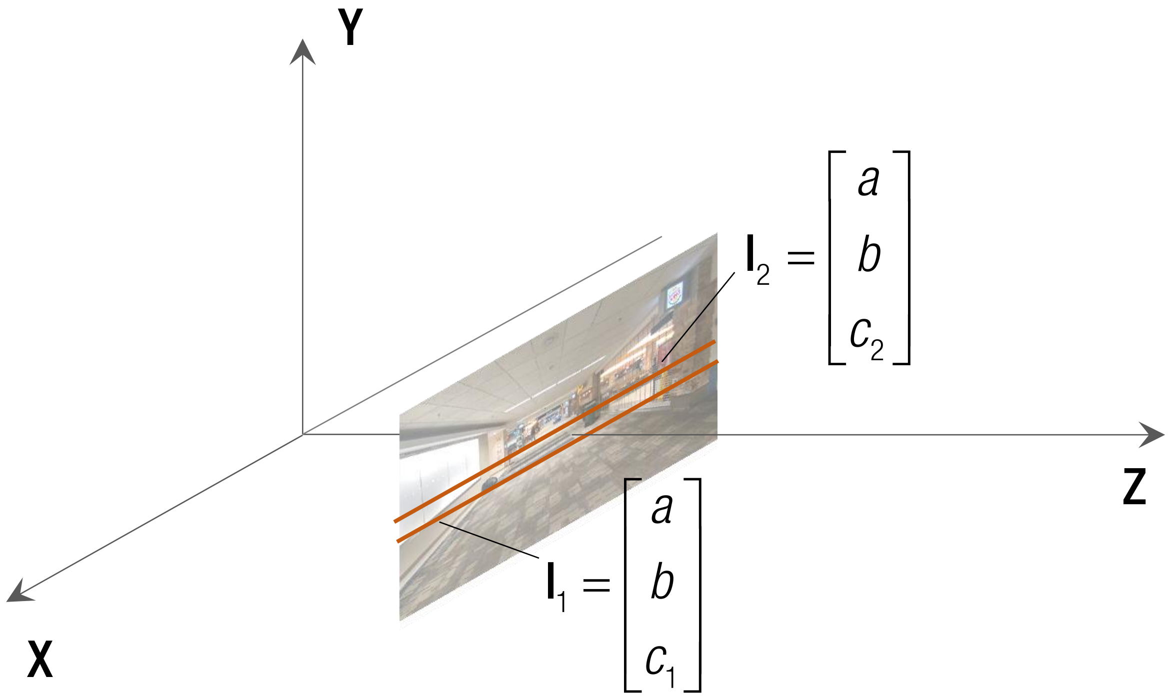
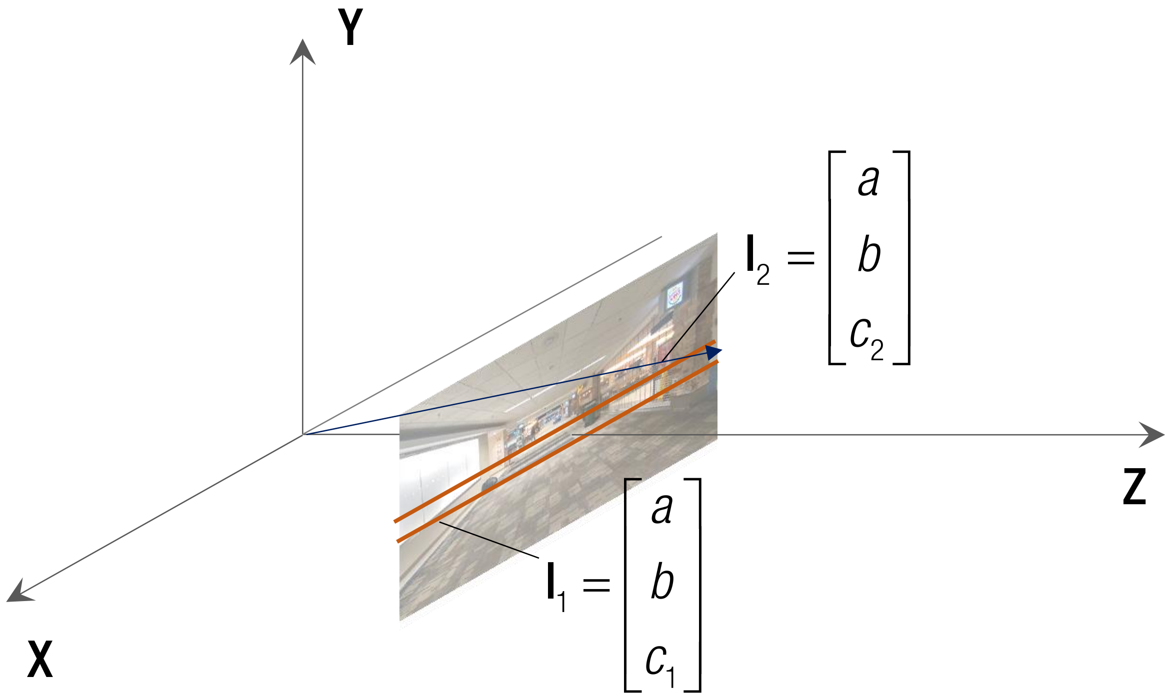
Point and Line at Infinity



Parallel lines intersect at the point at infinity:

Point at infinity: $\mathbf{x}_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

Point and Line at Infinity



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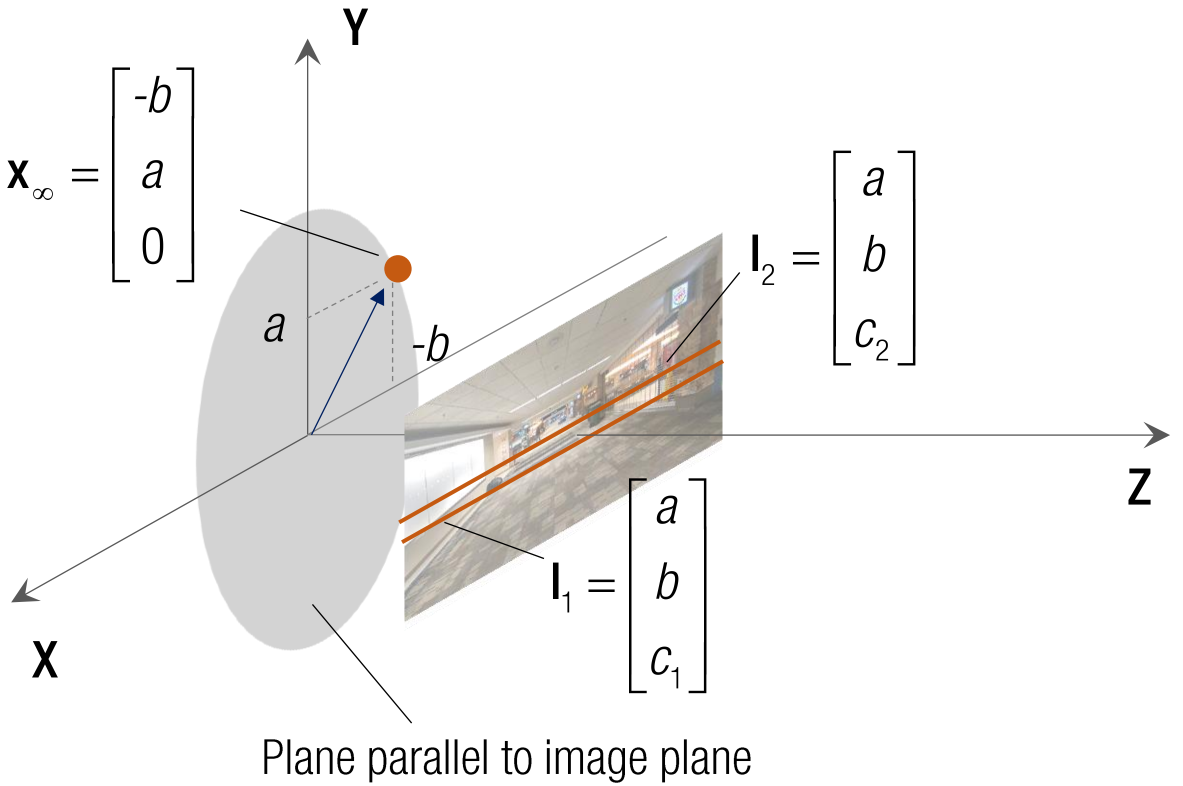
Point and Line at Infinity

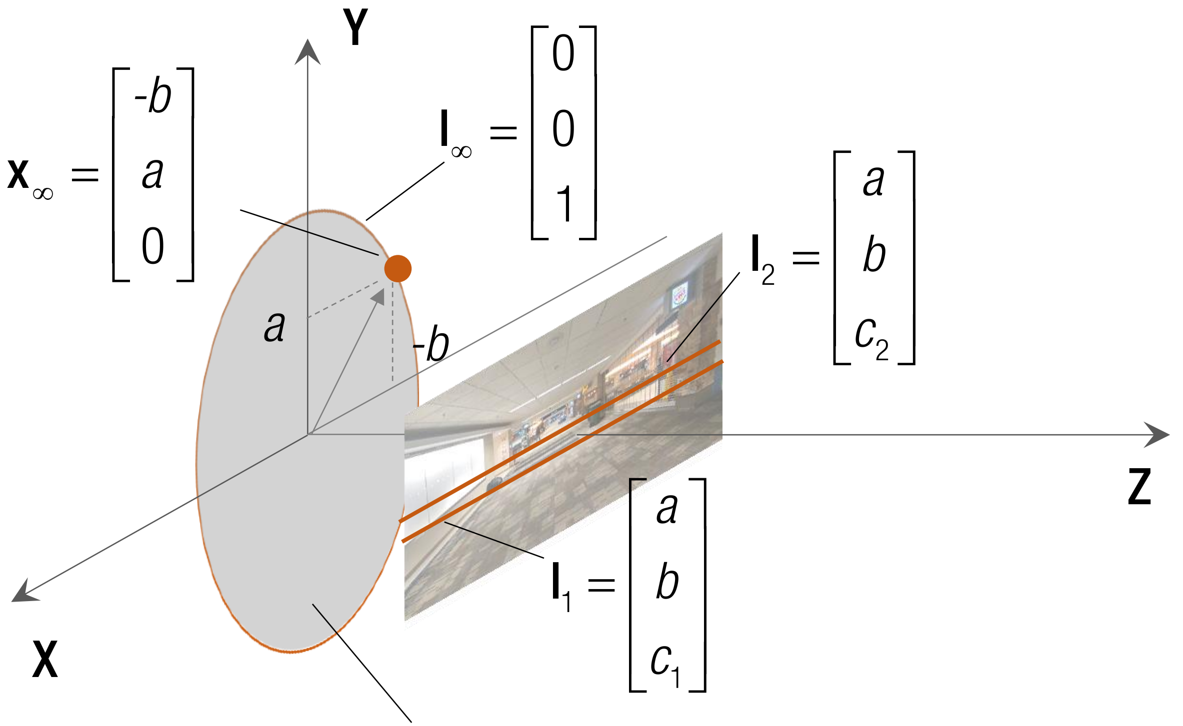


Parallel lines intersect at the point at infinity:

Point at infinity: $\mathbf{x}_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

A point at infinity corresponds to a direction.

Point and Line at Infinity



Parallel lines intersect at the point at infinity:

Point at infinity: $\mathbf{x}_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}$

A point at infinity corresponds to a direction.

All points at infinity lie in the line at infinity:

$$\mathbf{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{Z} \quad \because \mathbf{x}_{\text{ideal}}^{\mathsf{T}} \mathbf{l}_\infty = 0$$

where the surface normal is parallel to the Z axis.