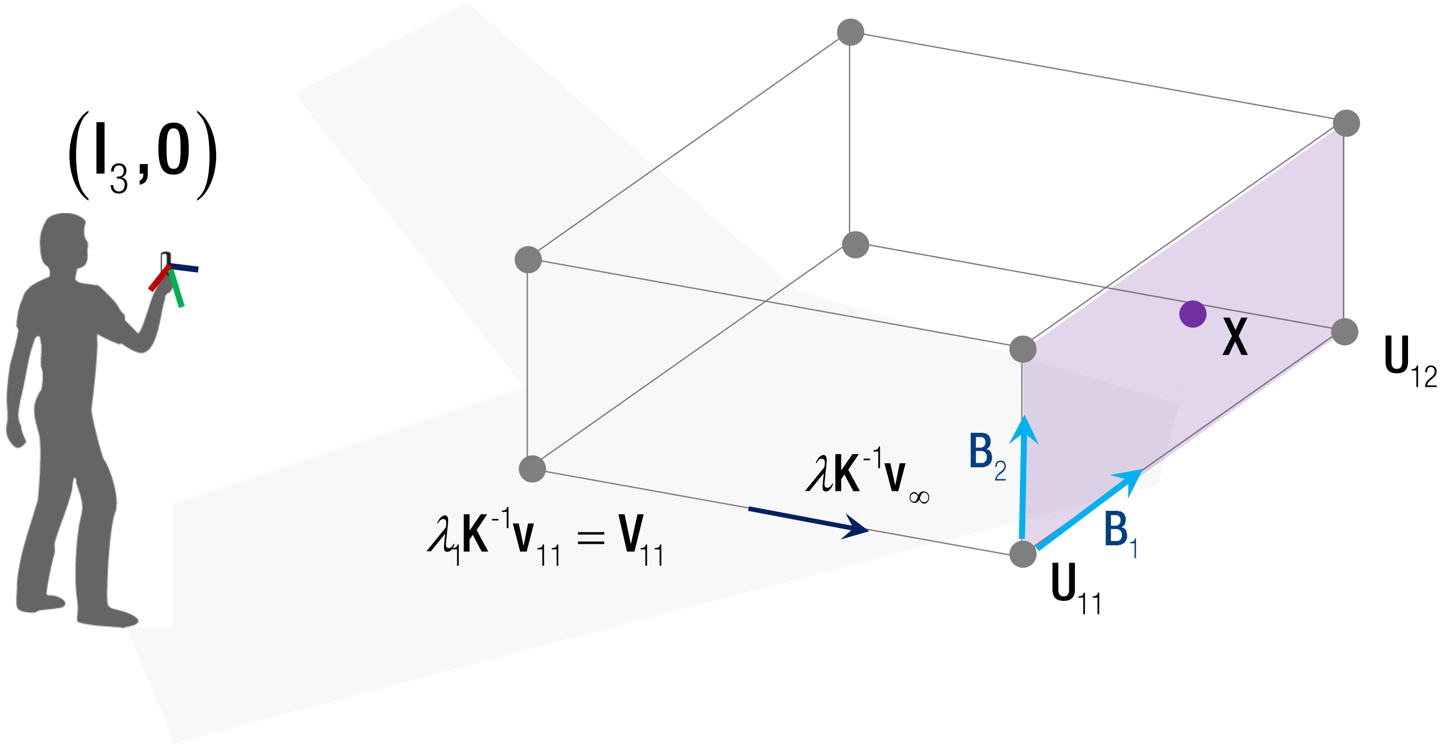
Spatial Rotation

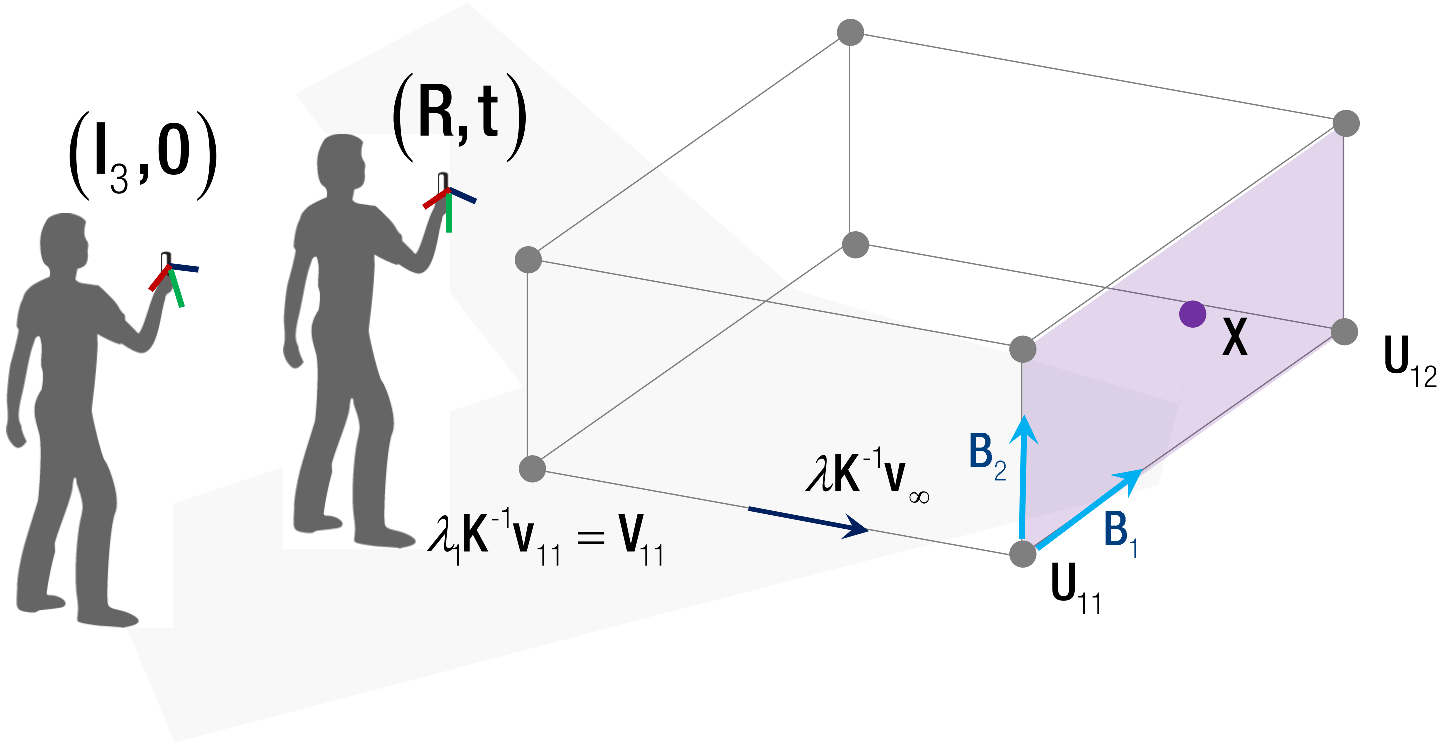
Announcement

- HW #3 deadline is extended (March 9)
- Paper assignment will be out today.

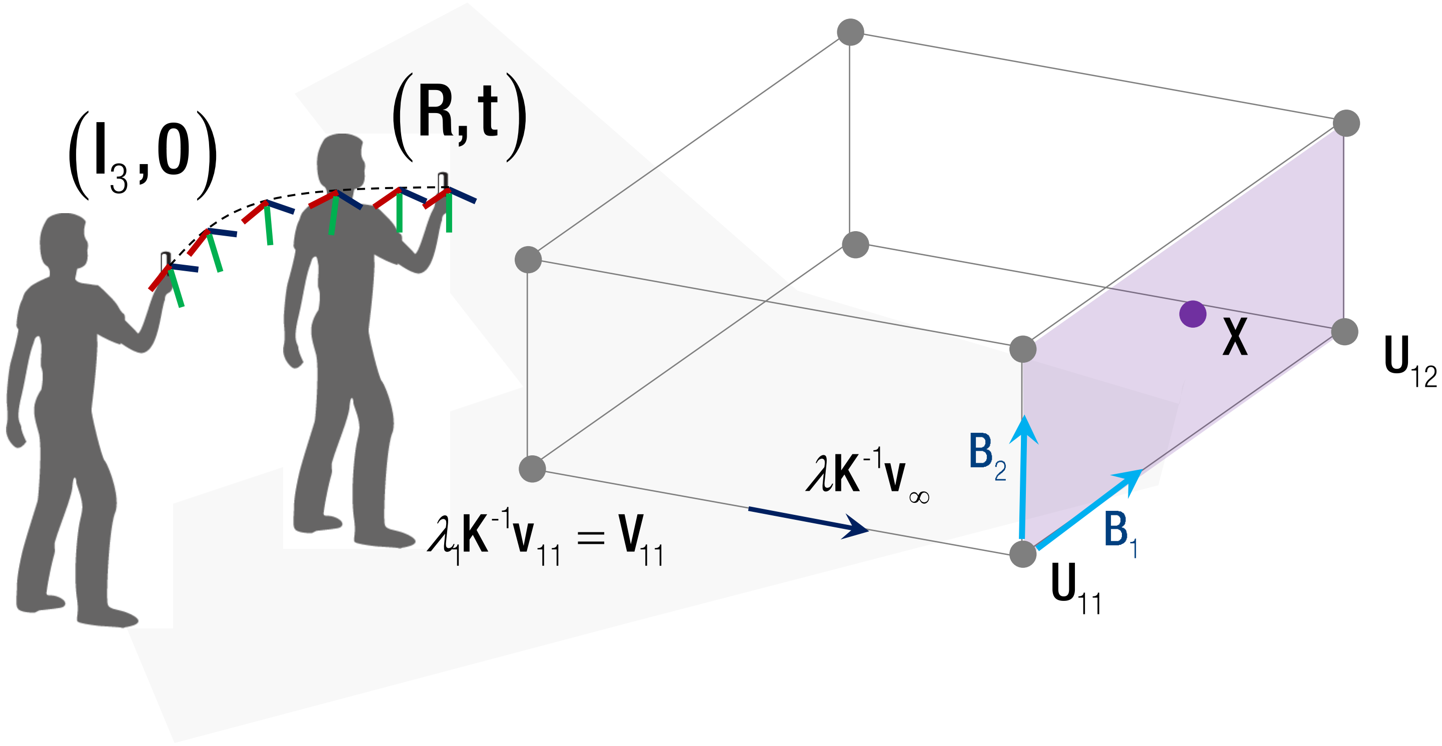
Interpolation of Transformation



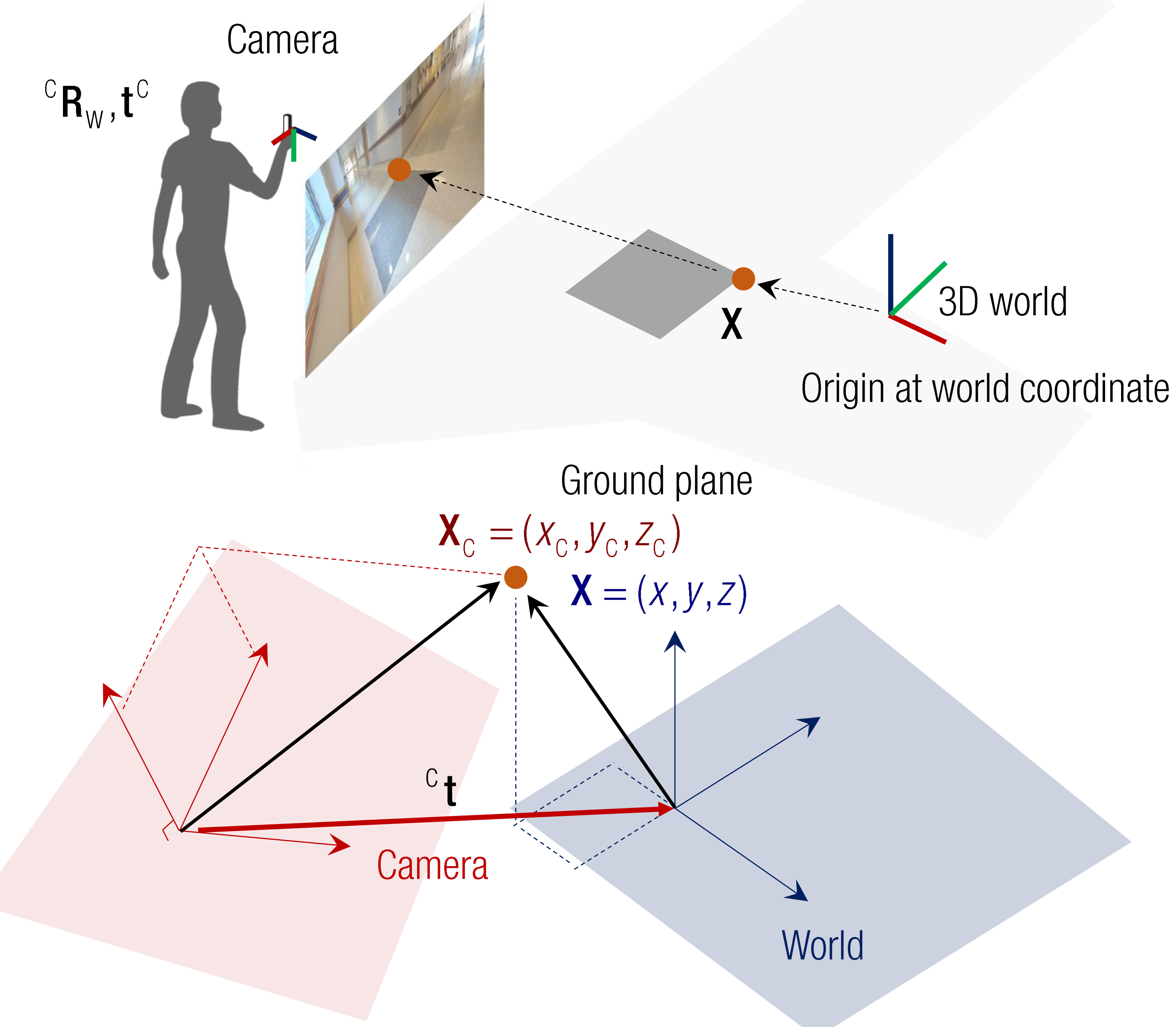
Interpolation of Transformation



Interpolation of Transformation



Recall: Rotate and then, Translate

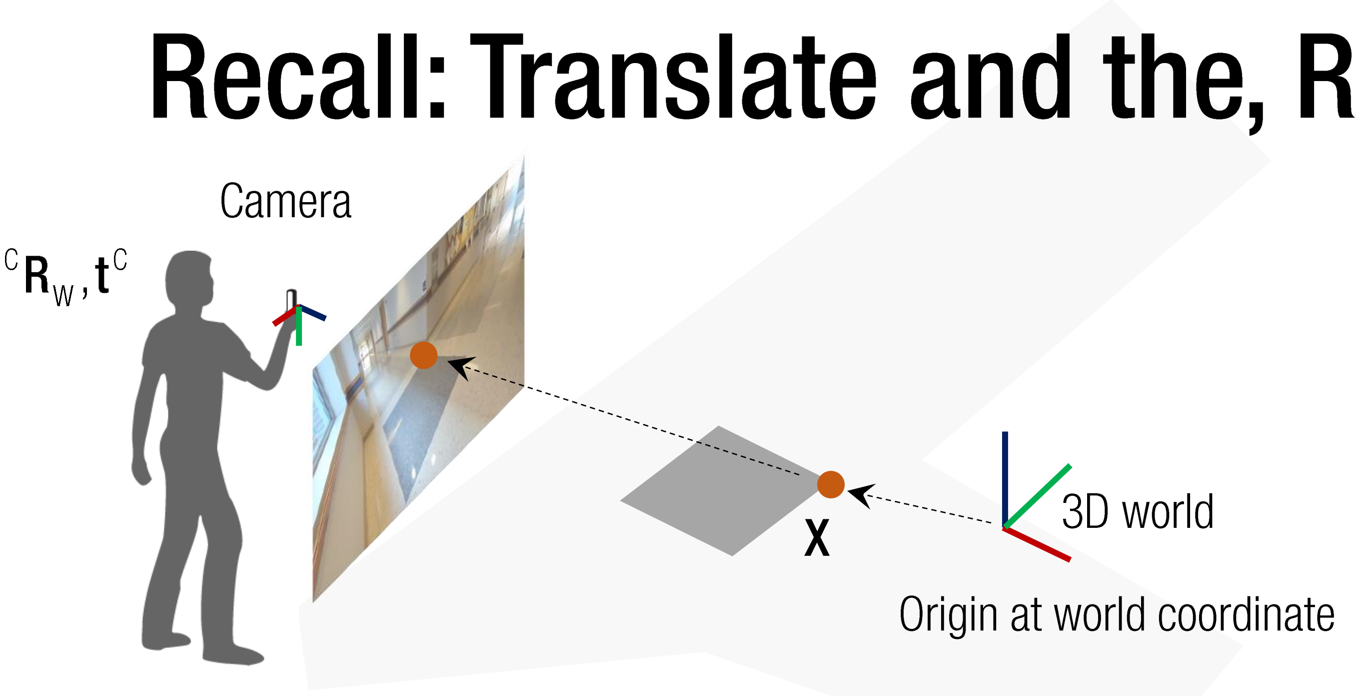


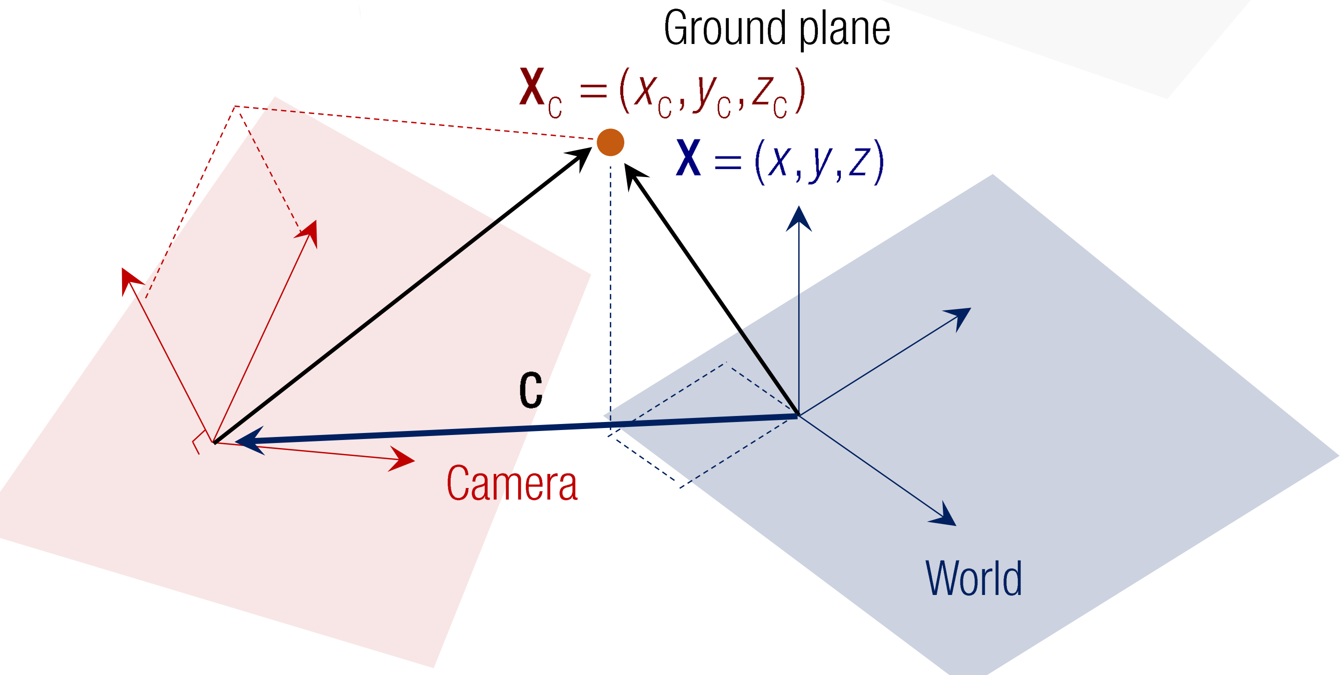
$$\mathbf{X}_C = {}^{C}\mathbf{R}_W \mathbf{X} + {}^{C}\mathbf{t} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where ${}^{C}\mathbf{t}$ is translation from world to camera seen from camera.

Rotate and then, translate.

Recall: Translate and the, Rotate





$$\mathbf{X}_C = {}^{C}\mathbf{R}_W \mathbf{X} + {}^{C}\mathbf{t} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where ${}^{C}\mathbf{t}$ is translation from world to camera seen from camera.

Rotate and then, translate.

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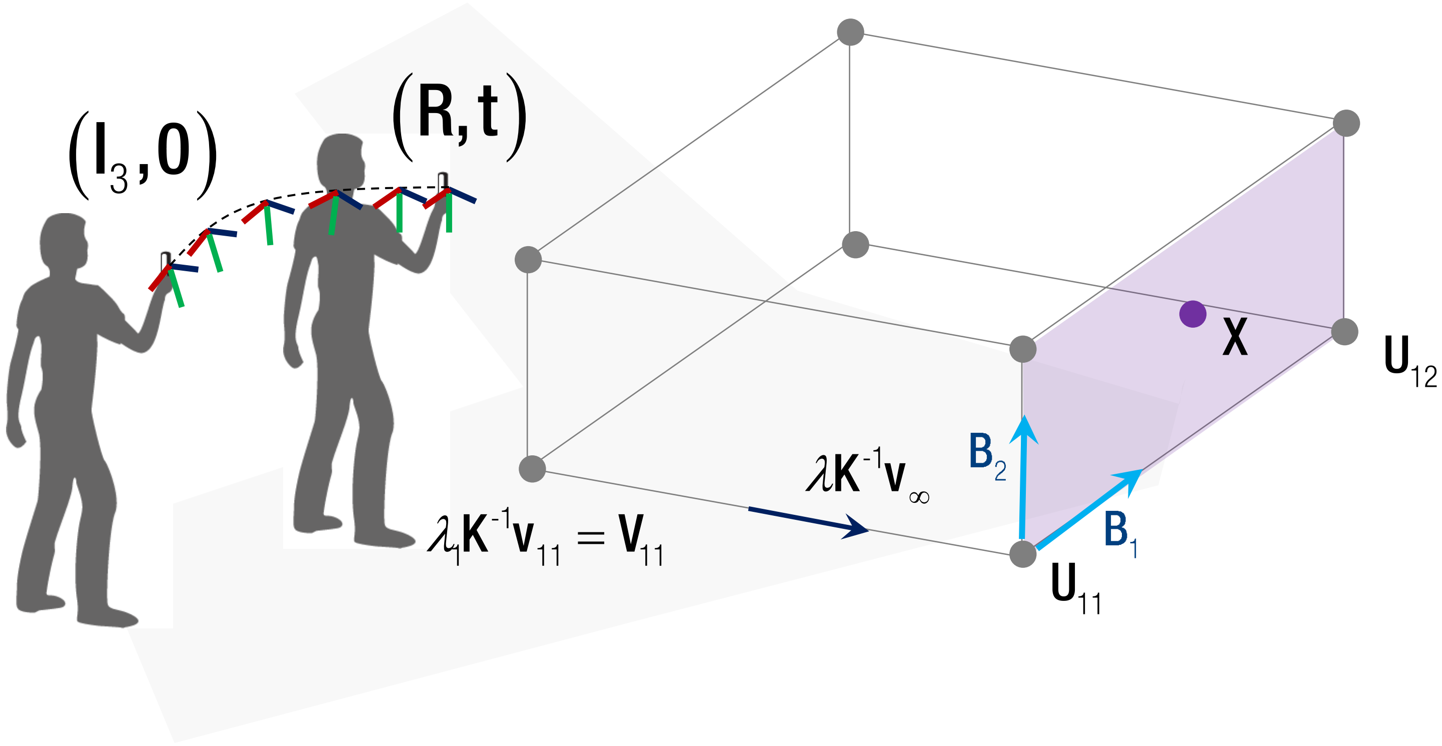
cf) **Translate and then, rotate.**

$$\mathbf{X}_C = {}^{C}\mathbf{R}_W (\mathbf{X} - \mathbf{C}) = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} 1 & & & -C_x \\ & 1 & & -C_y \\ & & 1 & -C_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

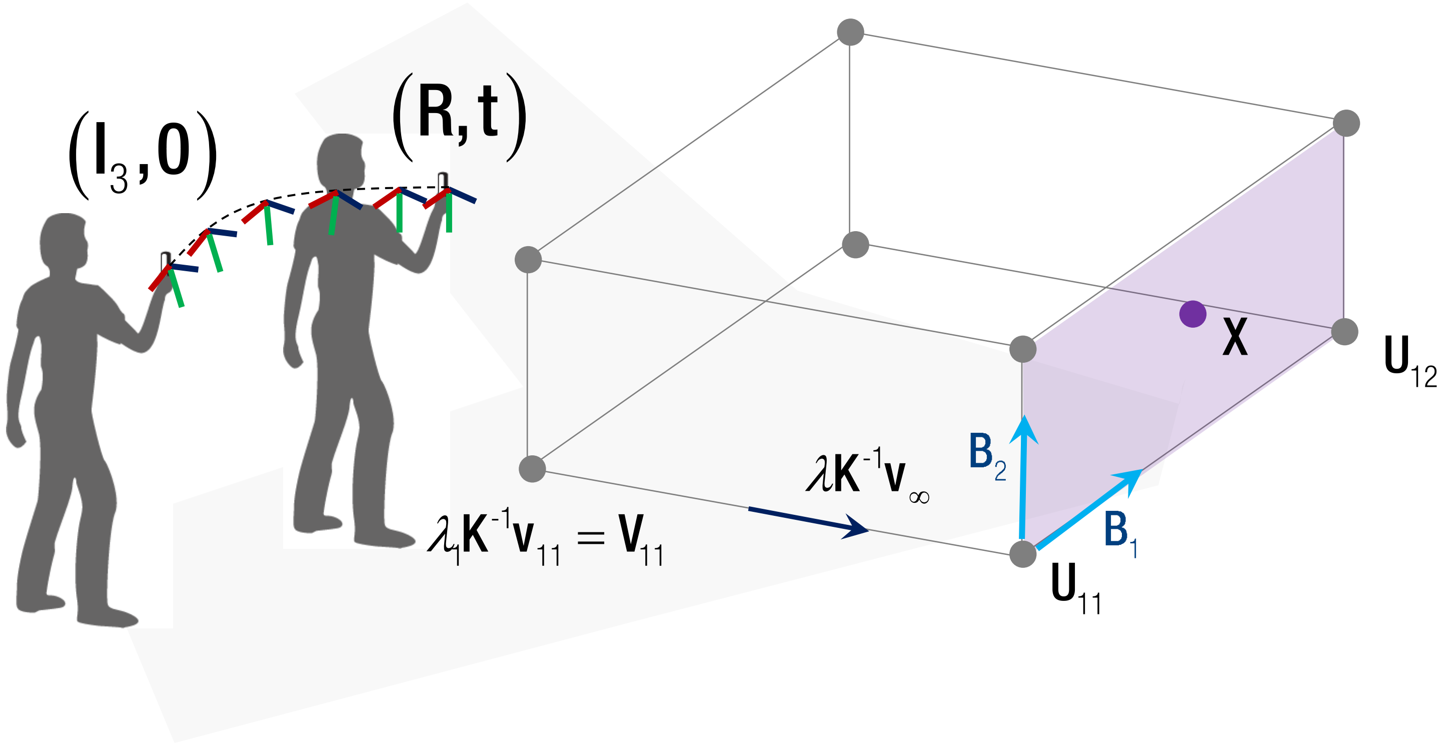
where $\mathbf{C}$ is translation from world to camera seen from world.

Interpolation of Translation



$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix} \mathbf{I}_3 & -\mathbf{C} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Interpolation of Translation



$$\lambda\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix} = \mathbf{KR}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{C}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

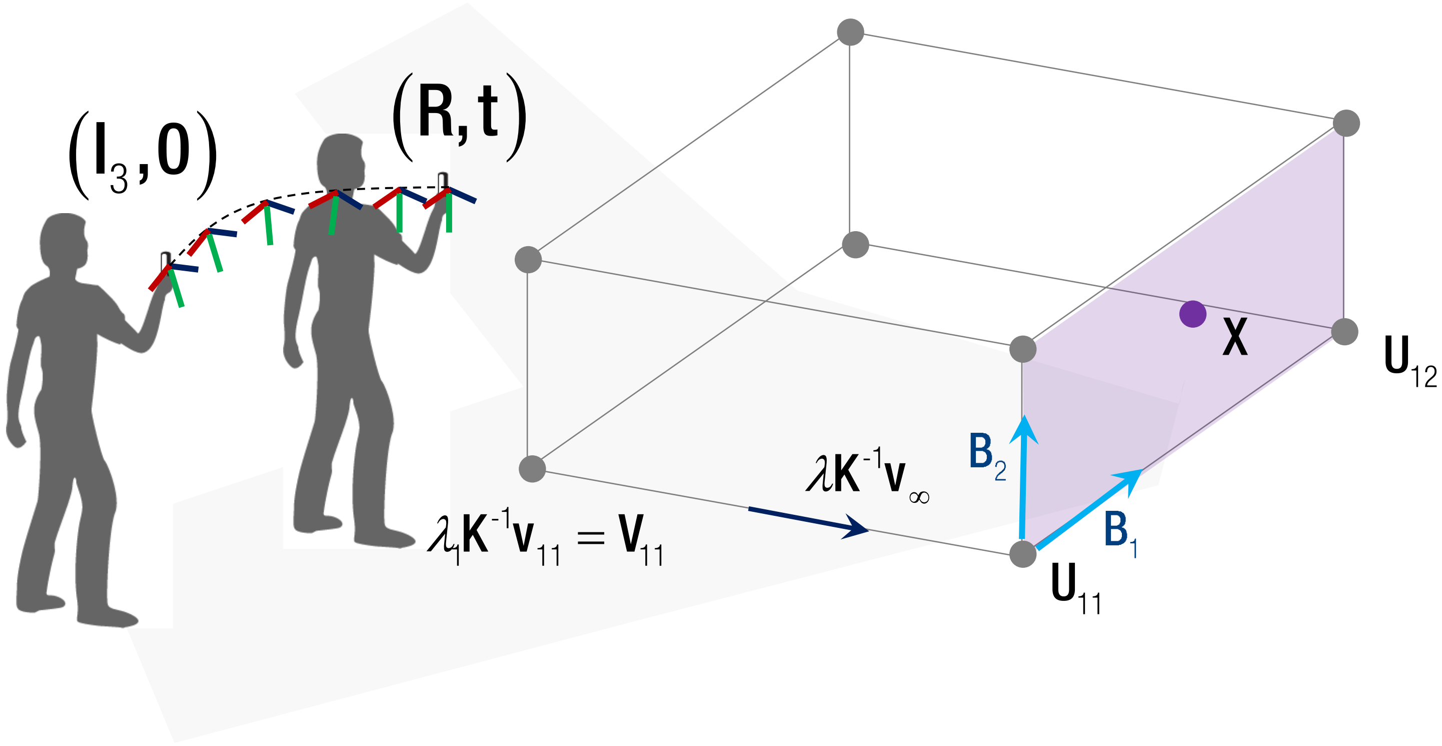
Rot. ➔ Trans. Trans. ➔ Rot.

Translation is independent on rotation.

How to interpolate translation?

$$\mathbf{C}_1 = \begin{bmatrix}C_1^x\\C_1^y\\C_1^z\end{bmatrix} \rightarrow \mathbf{C}_2 = \begin{bmatrix}C_2^x\\C_2^y\\C_2^z\end{bmatrix}$$

Interpolation of Translation



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Rot. ➔ Trans. Trans. ➔ Rot.

Translation is independent on rotation.

How to interpolate translation?

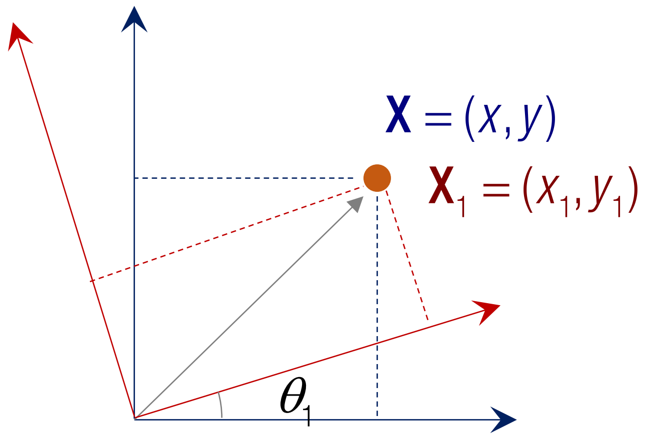
$$\mathbf{C}_1 = \begin{bmatrix} C_1^x \\ C_1^y \\ C_1^z \end{bmatrix} \rightarrow \mathbf{C}_2 = \begin{bmatrix} C_2^x \\ C_2^y \\ C_2^z \end{bmatrix}$$

Interpolated camera center:

$$\mathbf{C}_i = w\mathbf{C}_1 + (1-w)\mathbf{C}_2 \qquad w \in [0,1]$$

Interpolation of Rotation

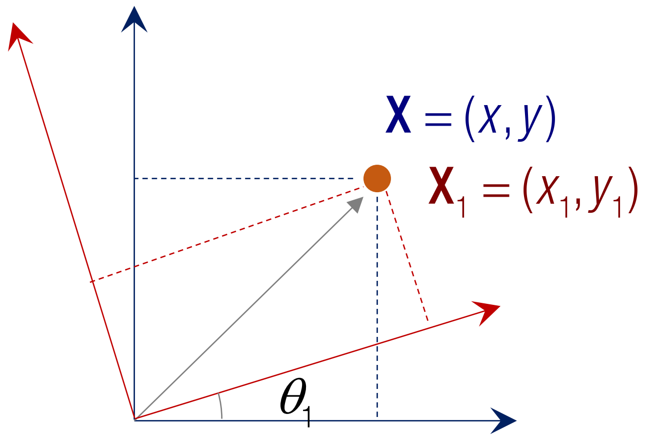
2D coordinate transform:



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \qquad\qquad \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation

2D coordinate transform:

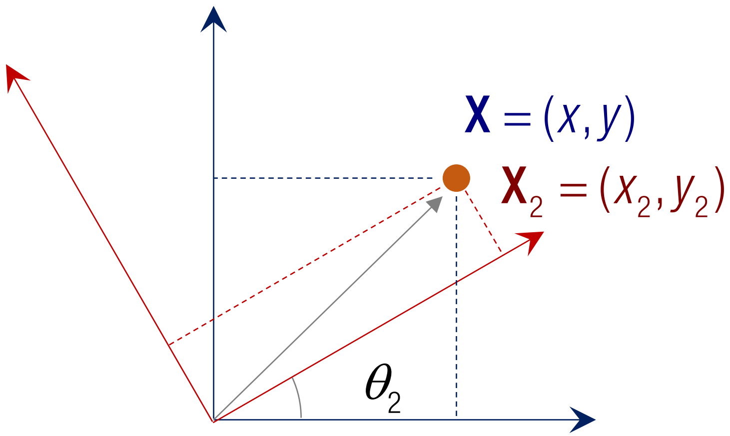


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \right) = \cos^2\theta_1 + \sin^2\theta_1 = 1$$

Interpolation of Rotation

2D coordinate transform:



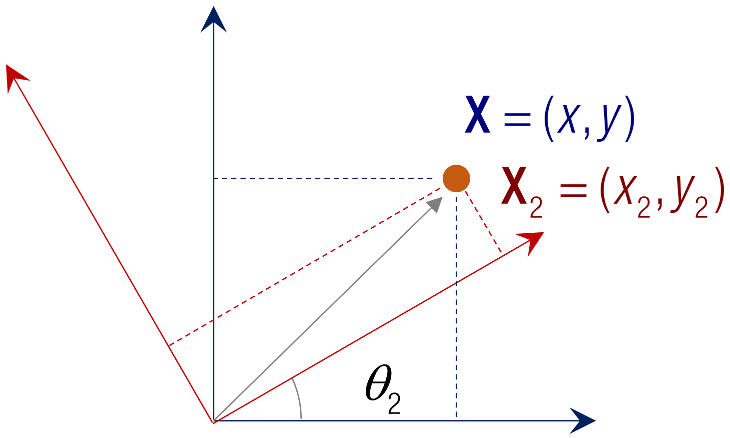
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation

2D coordinate transform:



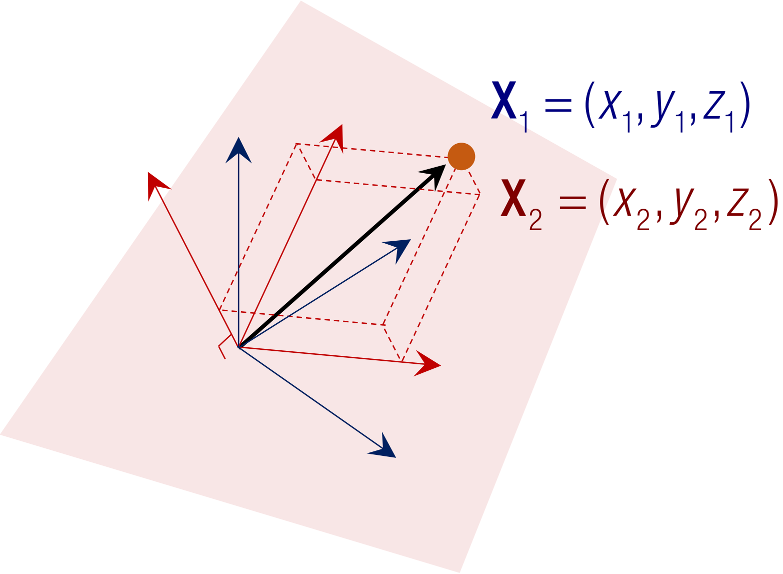
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\theta = w\theta_1 + (1-w)\theta_2$$

$$w \in [0,1]$$

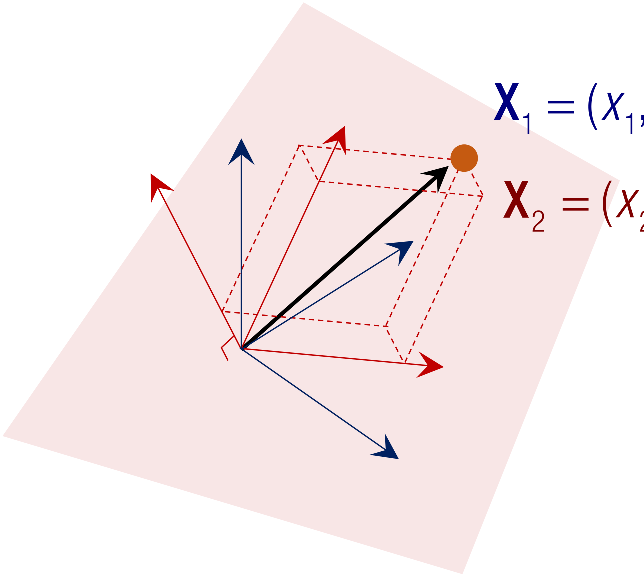
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation in 3D



$$\mathbf{X}_2 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathbf{X} = \mathbf{R}_1 \mathbf{X}_1$$

Interpolation of Rotation in 3D



$$\mathbf{X}_2 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathbf{X} = \mathbf{R}_1 \mathbf{X}_1$$

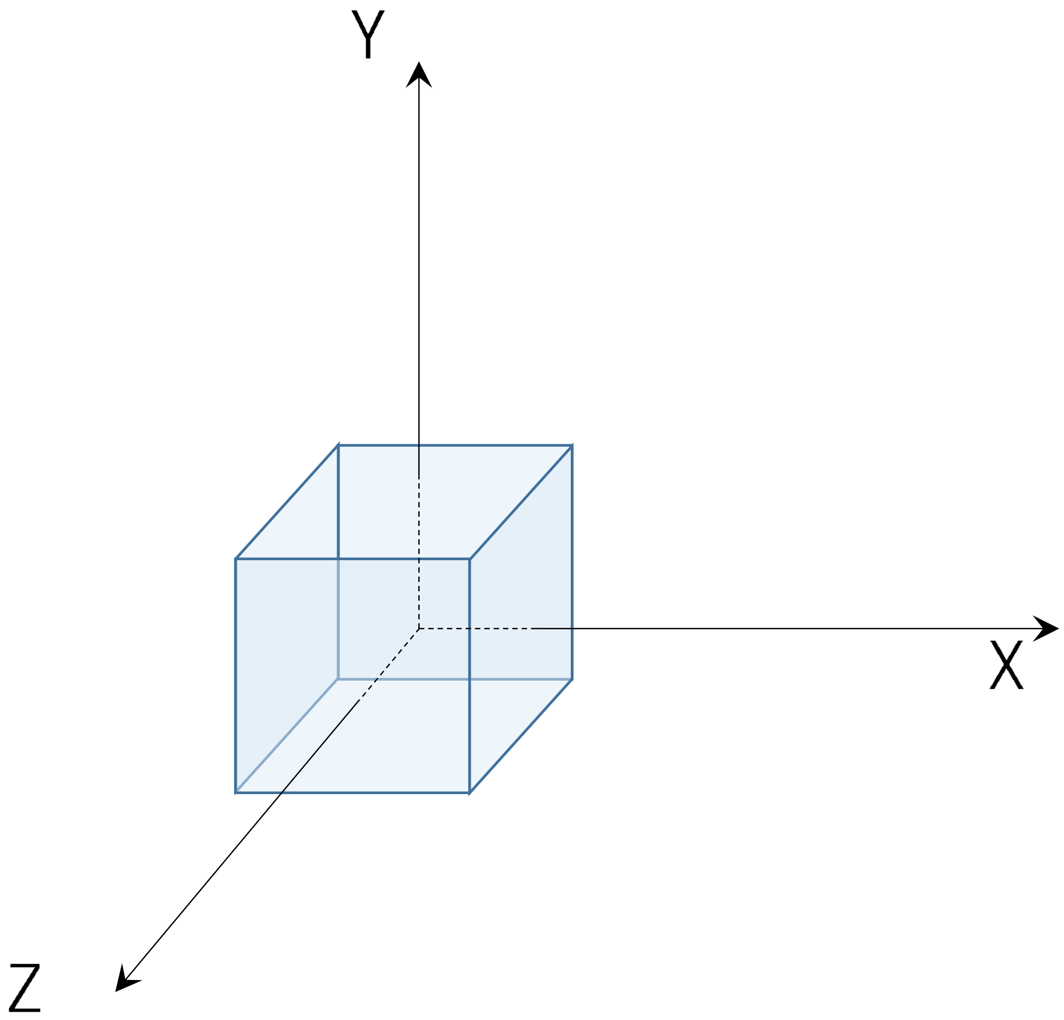
How to interpolate between two coordinates?

$$\mathbf{R}_1 \rightarrow \mathbf{R}_2$$

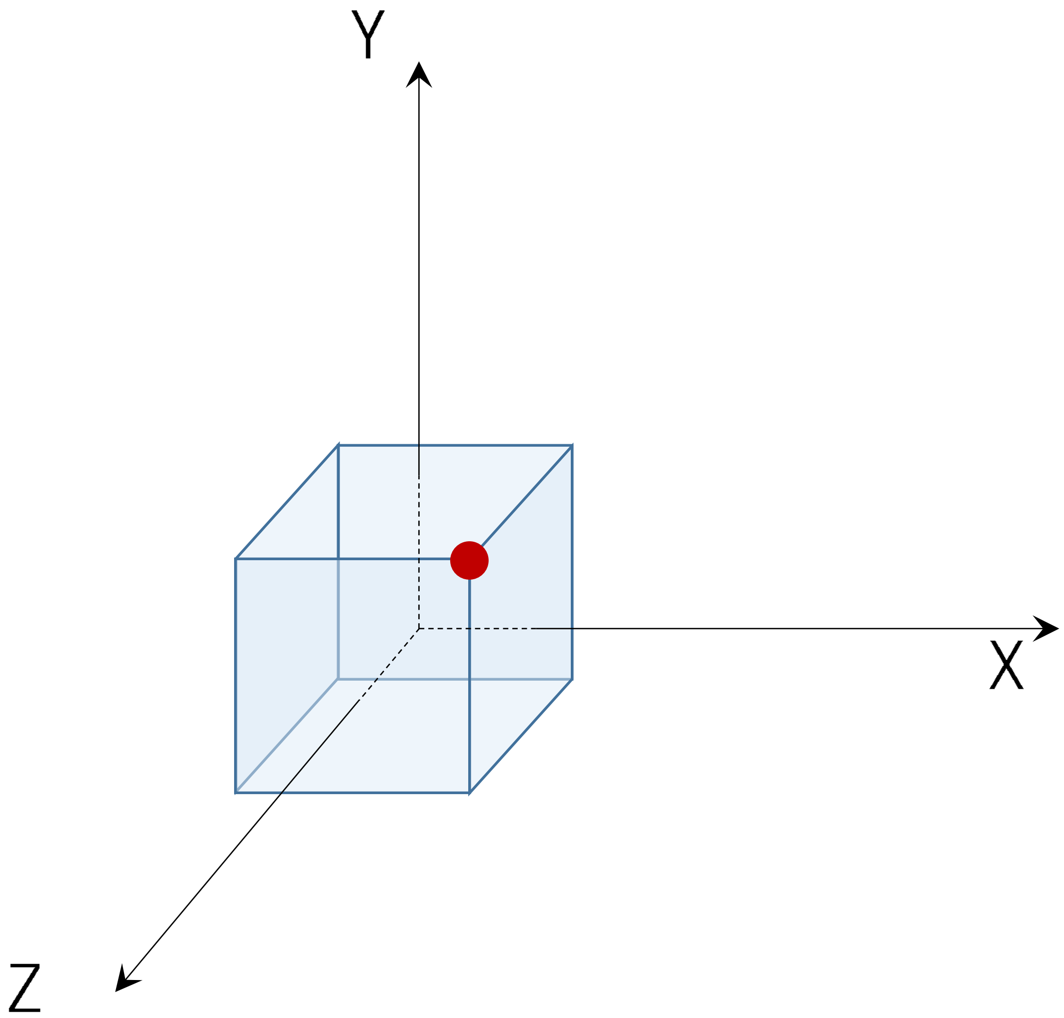
\# dof: 3

\# of parameters: 9

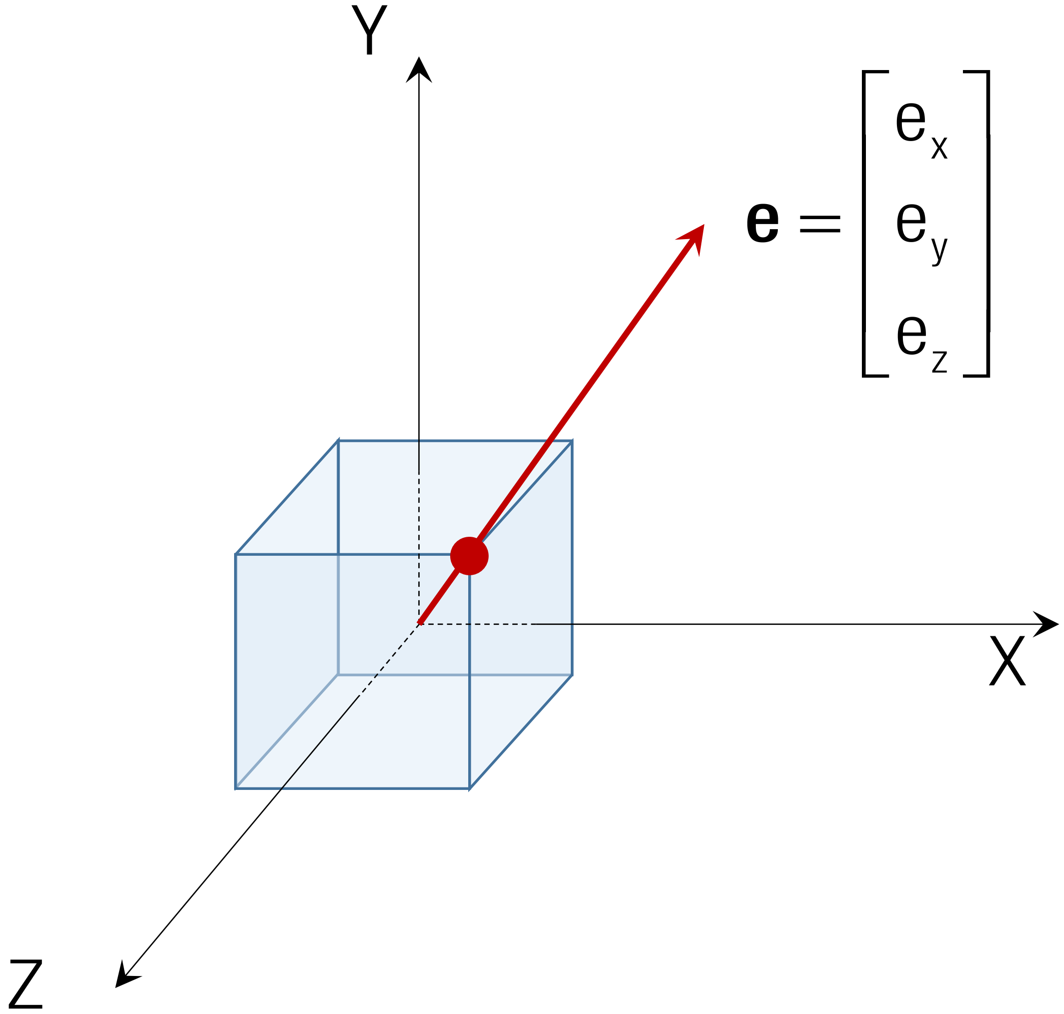
Axis Angle Representation



Axis Angle Representation

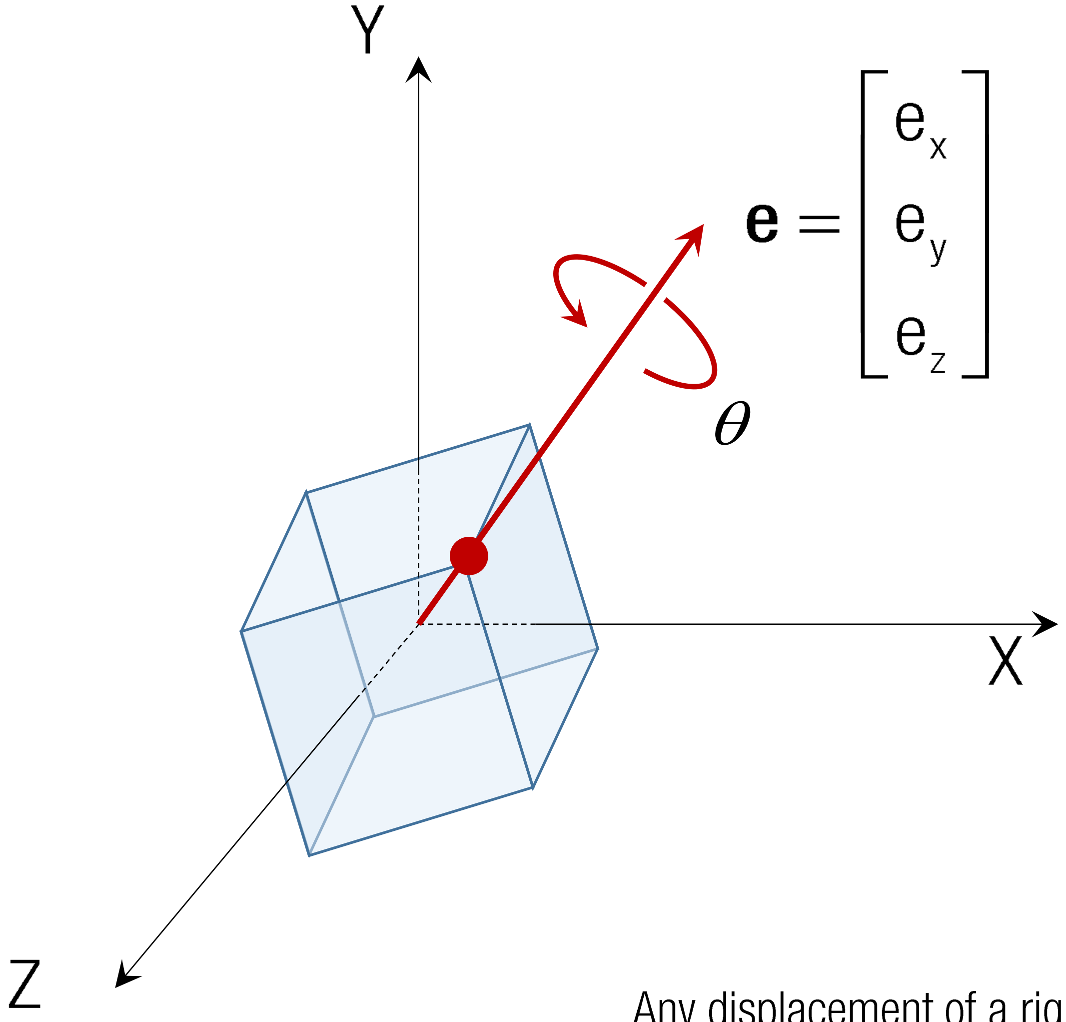


Axis Angle Representation



$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Axis Angle Representation

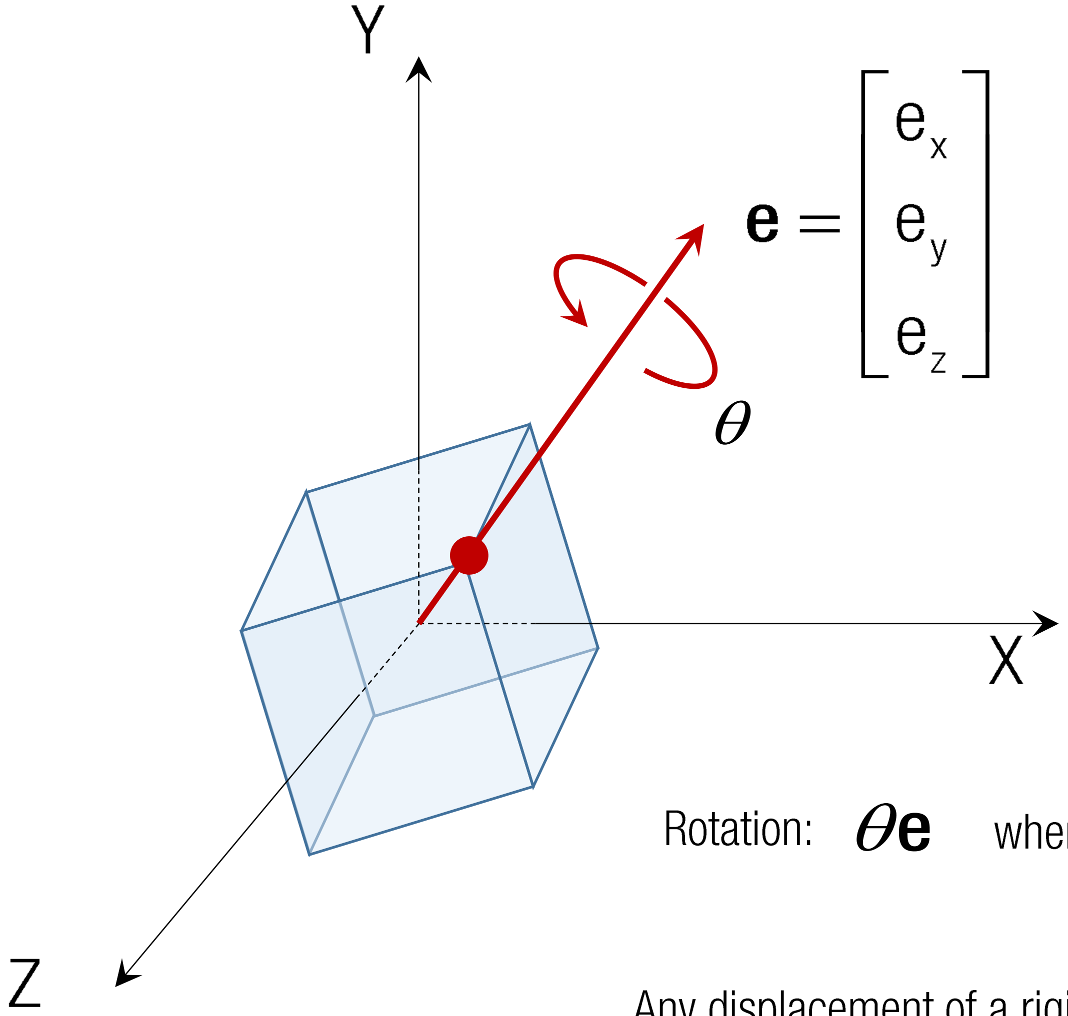


$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Euler's theorem

Any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that runs through the fixed point.

Axis Angle Representation



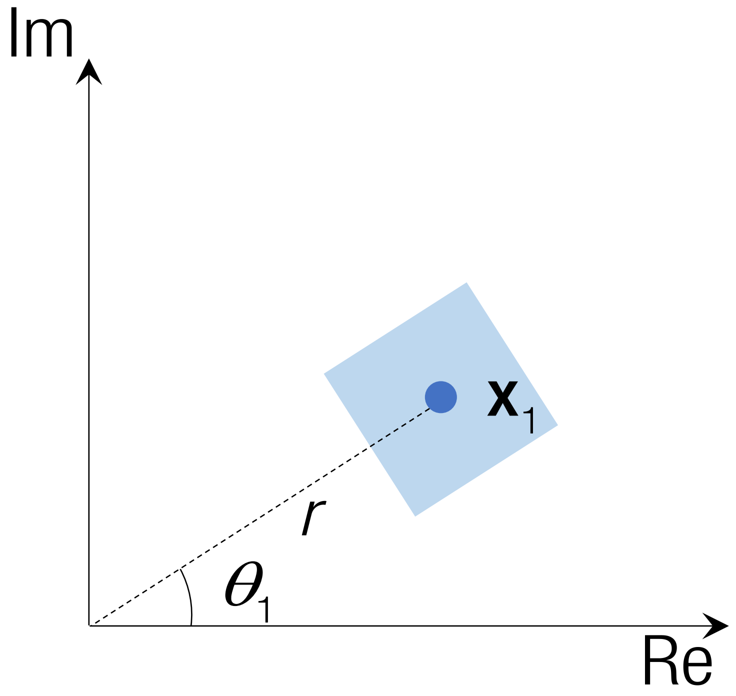
$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

Euler's theorem

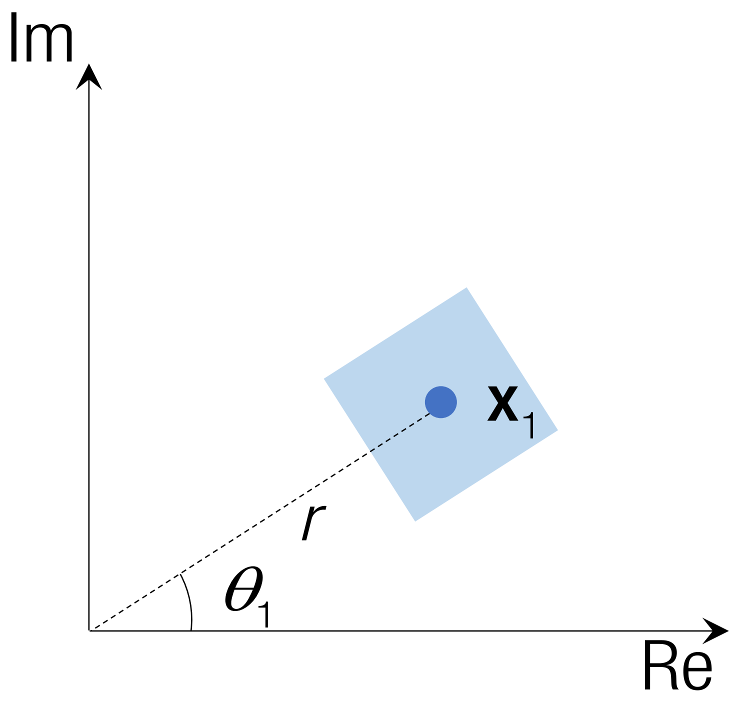
Any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that runs through the fixed point.

2D Exponential Map (Euler's Formula)



$\mathbf{x}_1 =$

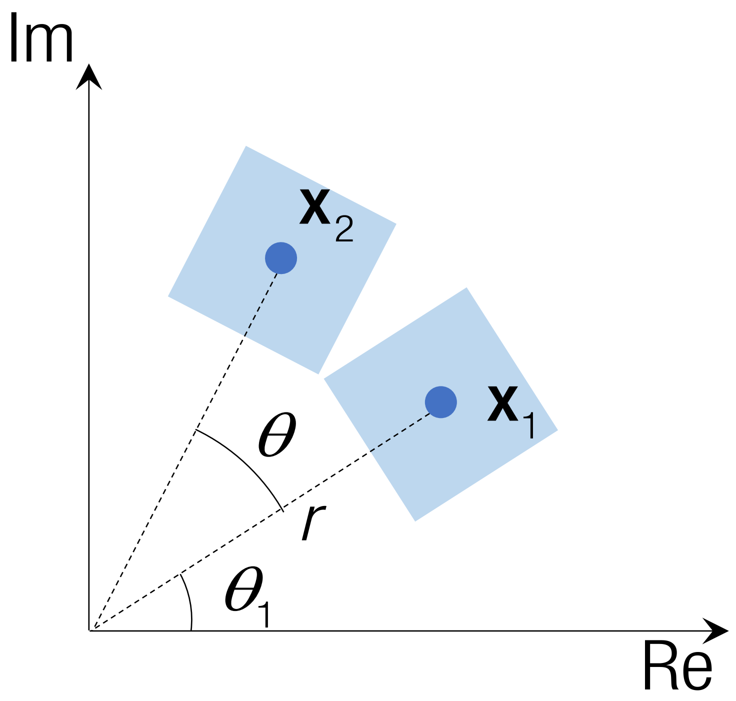
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

Ref) $$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

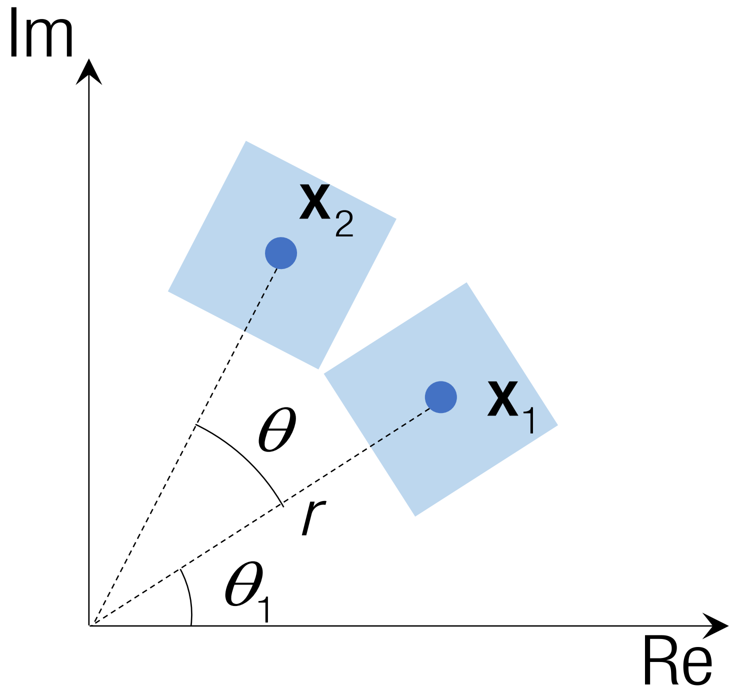
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$\mathbf{x}_2$

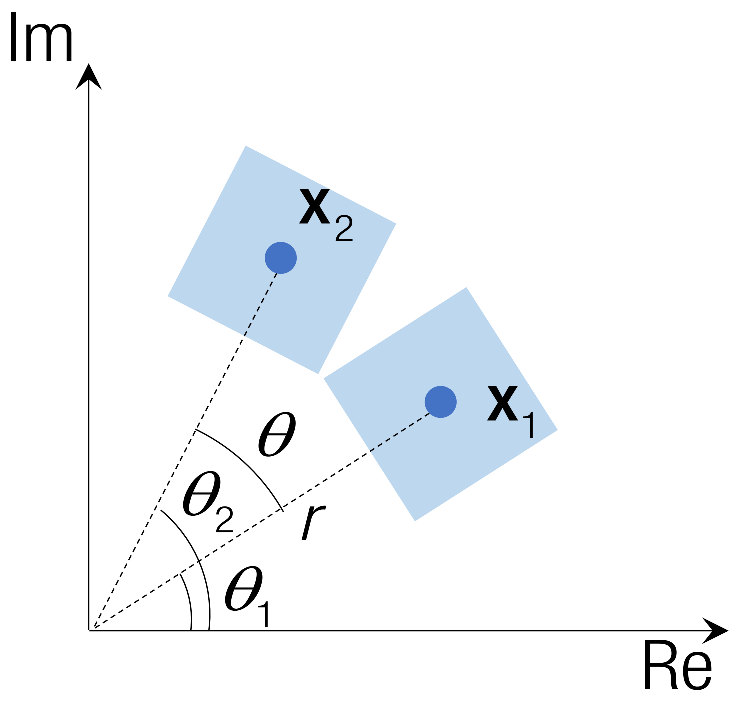
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$$\begin{aligned}\mathbf{x}_2 &= \exp(i\theta)\mathbf{x}_1 = r\left(\cos\theta + i\sin\theta\right)\left(\cos\theta_1 + i\sin\theta_1\right)\\ &= r\left(\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 + i\left(\cos\theta\sin\theta_1 + \sin\theta\cos\theta_1\right)\right)\\ &= r\left(\cos(\theta+\theta_1) + i\sin(\theta+\theta_1)\right)\end{aligned}$$

2D Exponential Map (Euler's Formula)

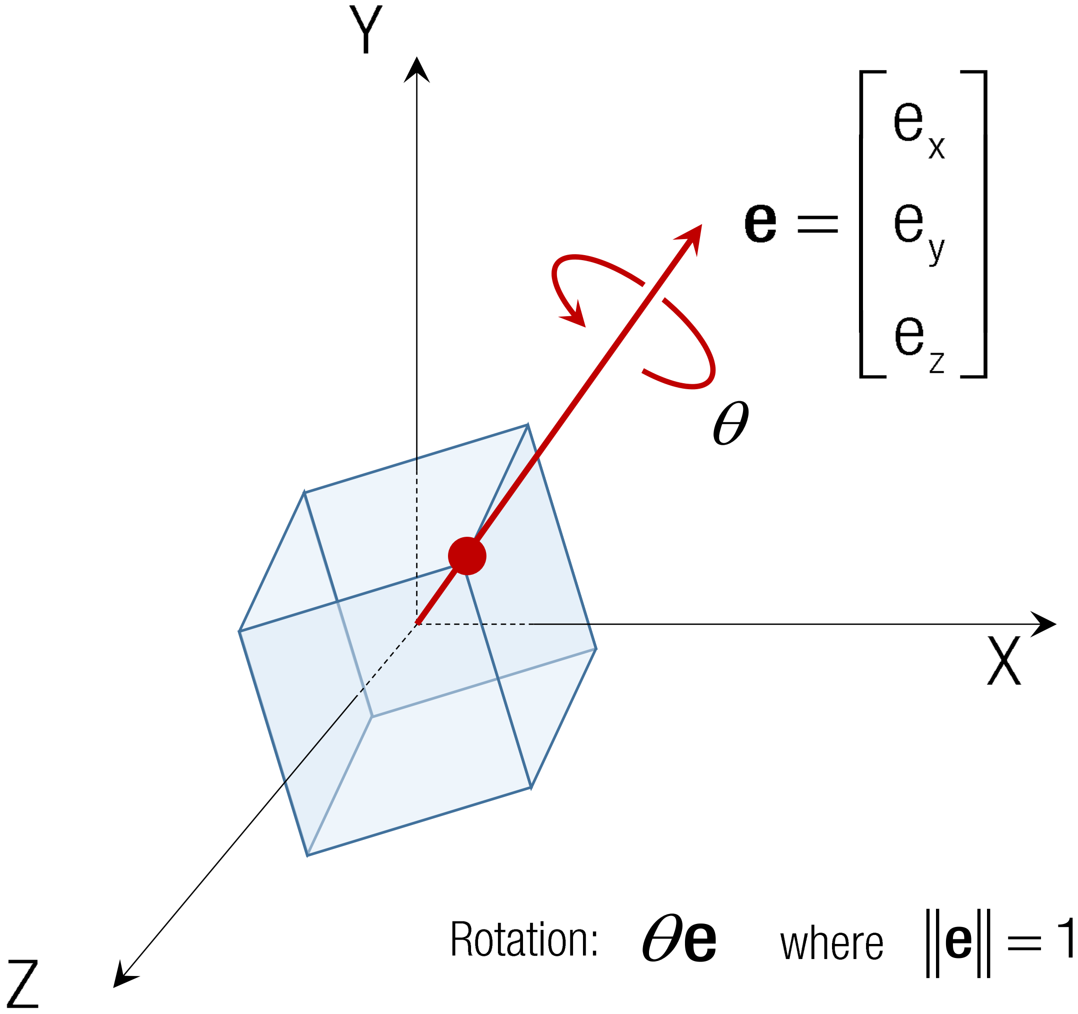


$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$$\begin{aligned}
\mathbf{x}_2 &= \exp(i\theta)\mathbf{x}_1 = r\left(\cos\theta + i\sin\theta\right)\left(\cos\theta_1 + i\sin\theta_1\right) \\
&= r\left(\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 + i\left(\cos\theta\sin\theta_1 + \sin\theta\cos\theta_1\right)\right) \\
&= r\left(\cos(\theta+\theta_1) + i\sin(\theta+\theta_1)\right) \\
&= r\left(\cos\theta_2 + i\sin\theta_2\right)
\end{aligned}$$

$$\theta_2 = \theta_1 + \theta$$

3D Exponential Map: Quaternion

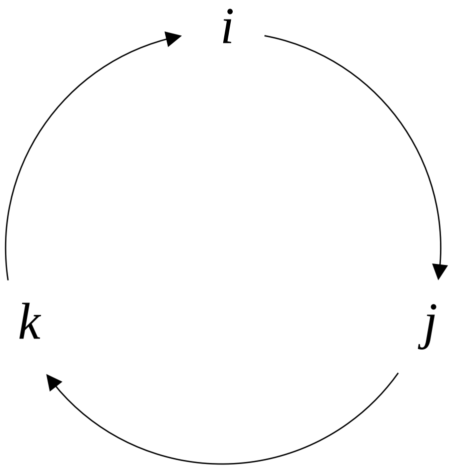


$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

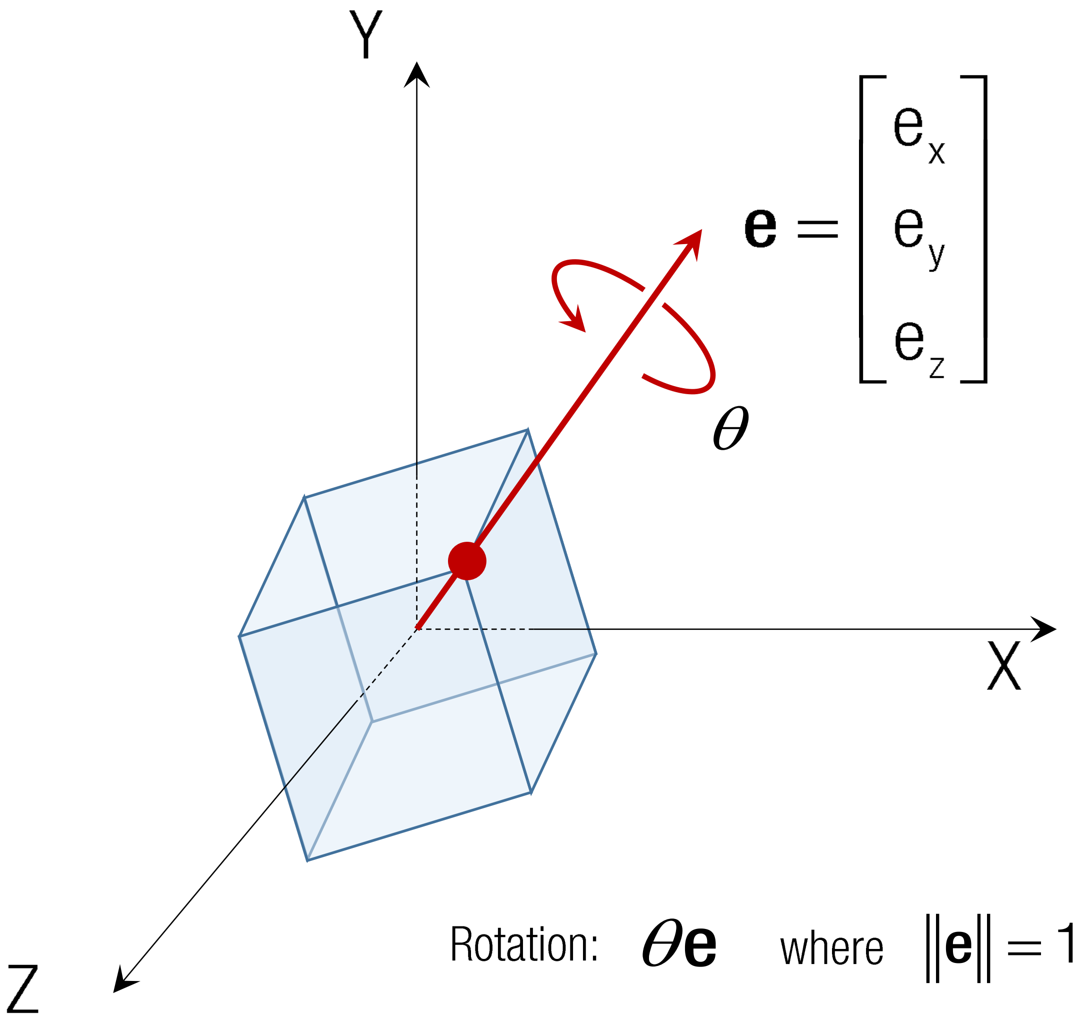
$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

$$i^2 = j^2 = k^2 = ijk = -1$$



$$ij = k$$
$$jk = i$$
$$ki = j$$

Exercise

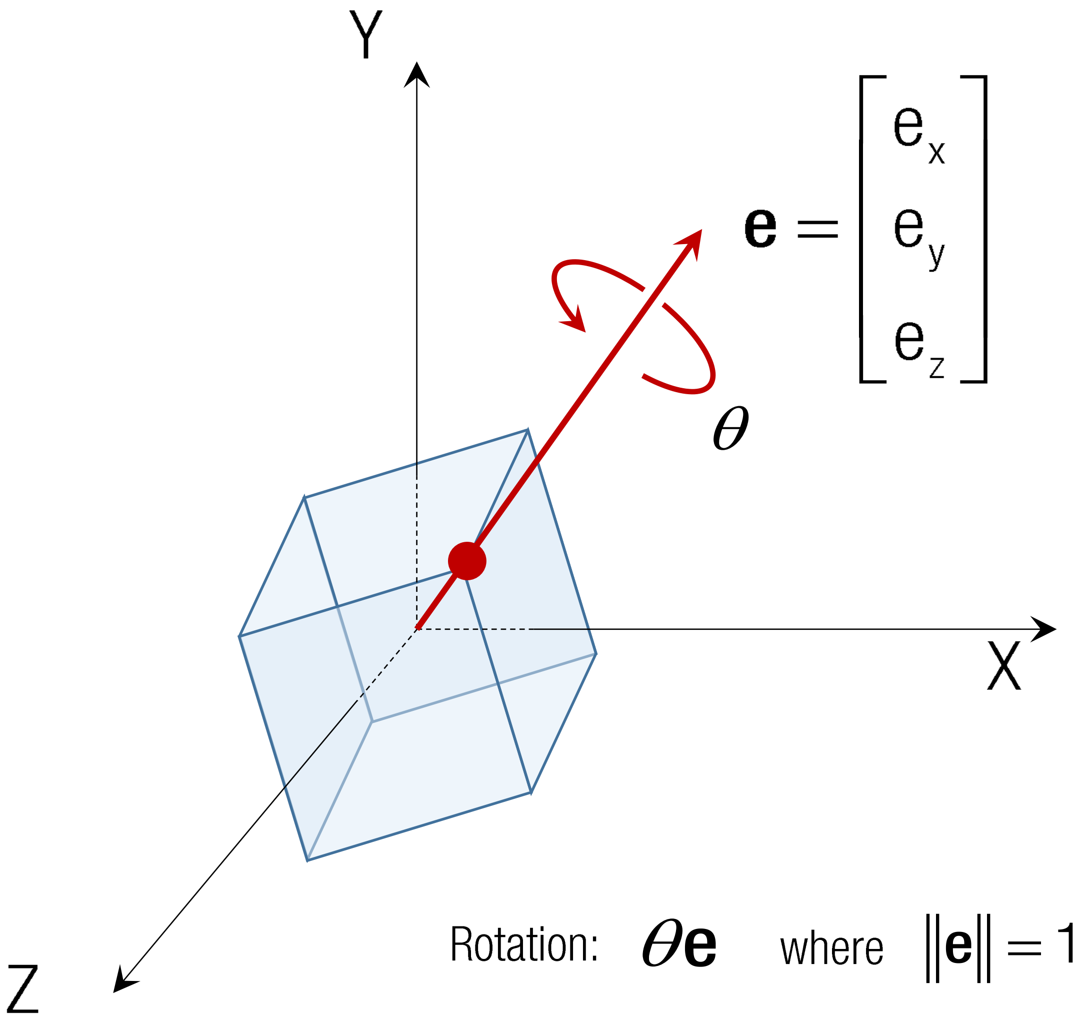


Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

Find a quaternion **q** such that it describes a rotation of 60 degrees about the axis **a**=[3, 4, 0].

Exercise



Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

Find a quaternion **q** such that it describes a rotation of 60 degrees about the axis **a**=[3, 4, 0].

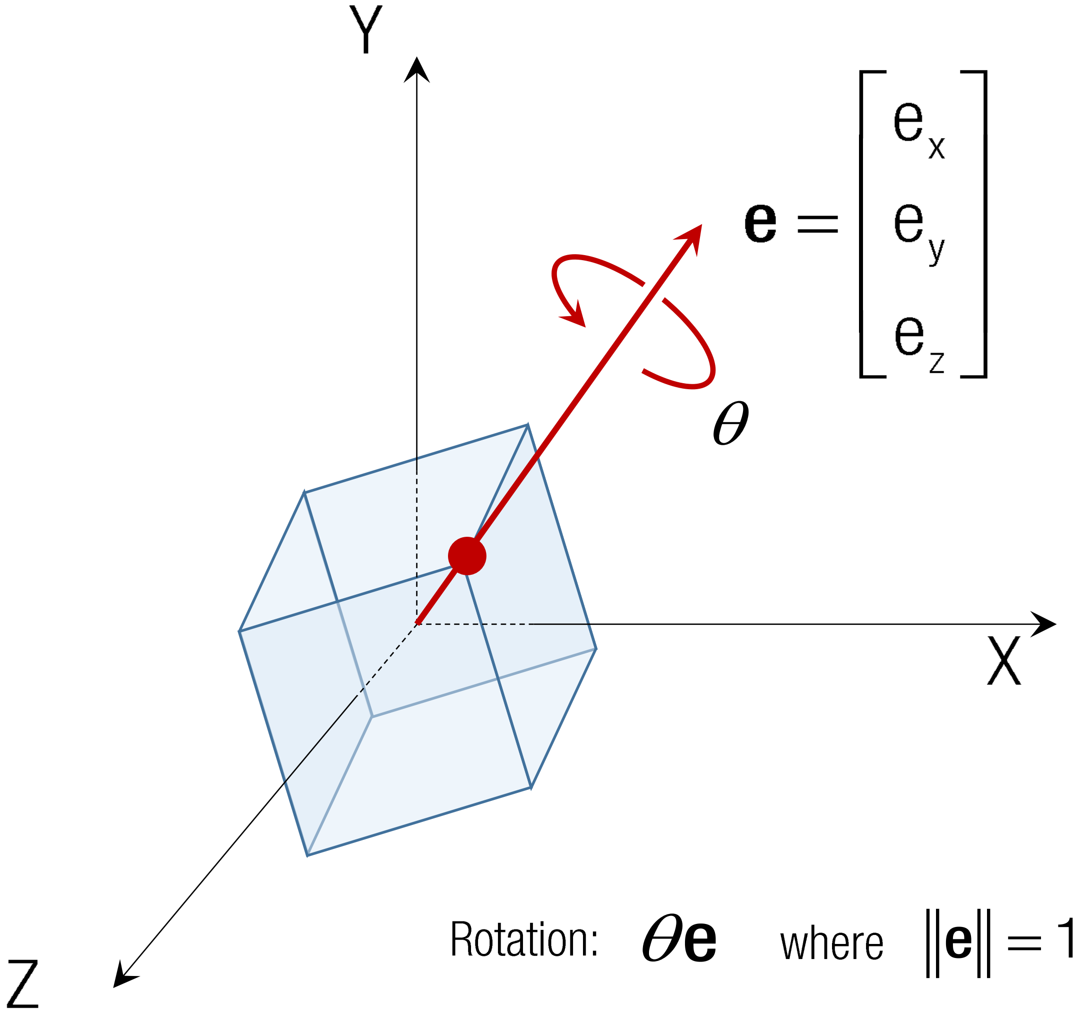
$$\mathbf{e} = \mathbf{a} / \|\mathbf{a}\| = \underbrace{i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}}_{\text{Unit vector}}$$

$$\mathbf{q} = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right) \qquad \theta = \frac{\pi}{3}$$

$$= \cos\frac{\pi}{3\cdot 2} + \sin\frac{\pi}{3\cdot 2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right)$$

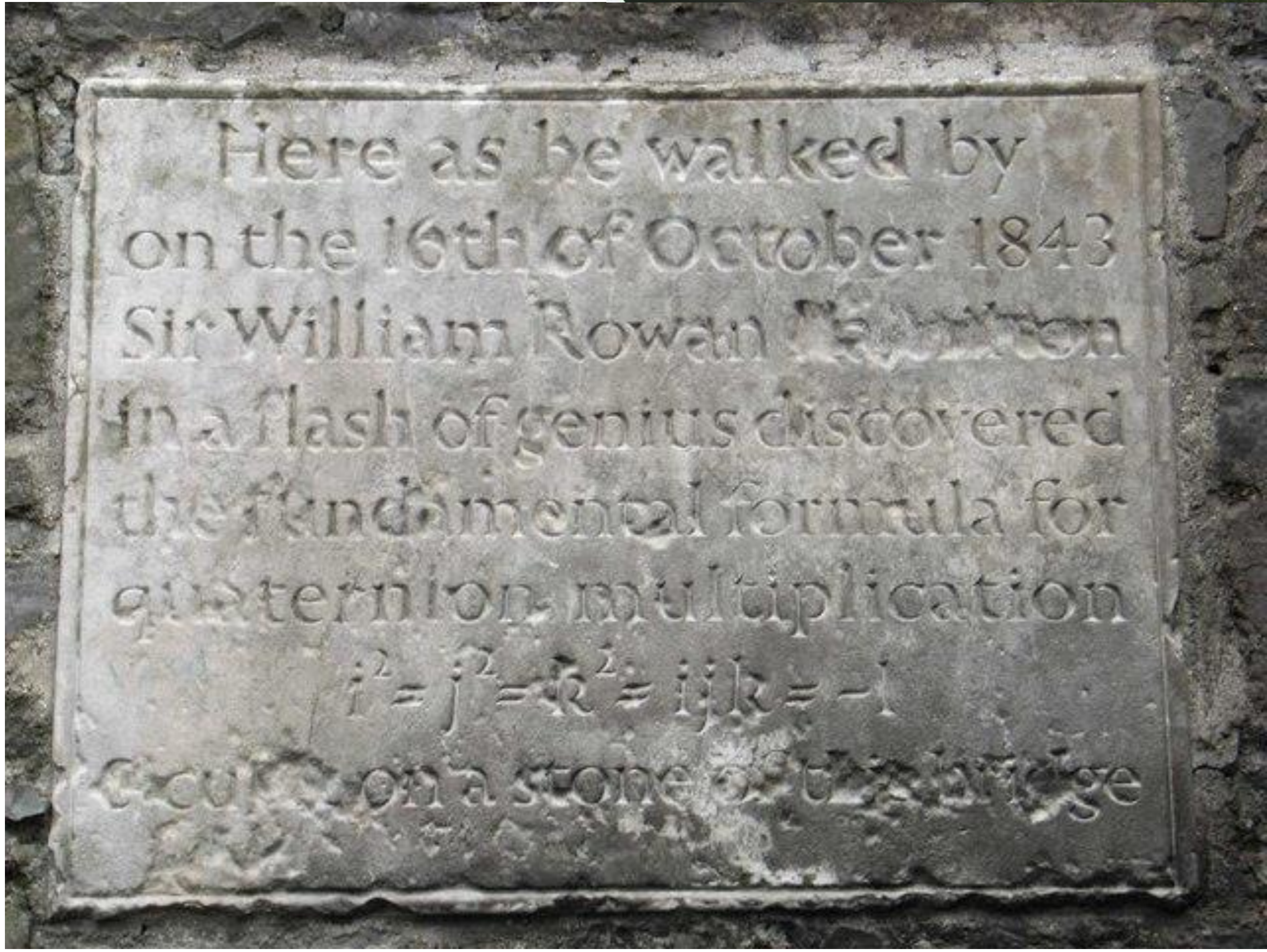
3D Exponential Map: Quaternion



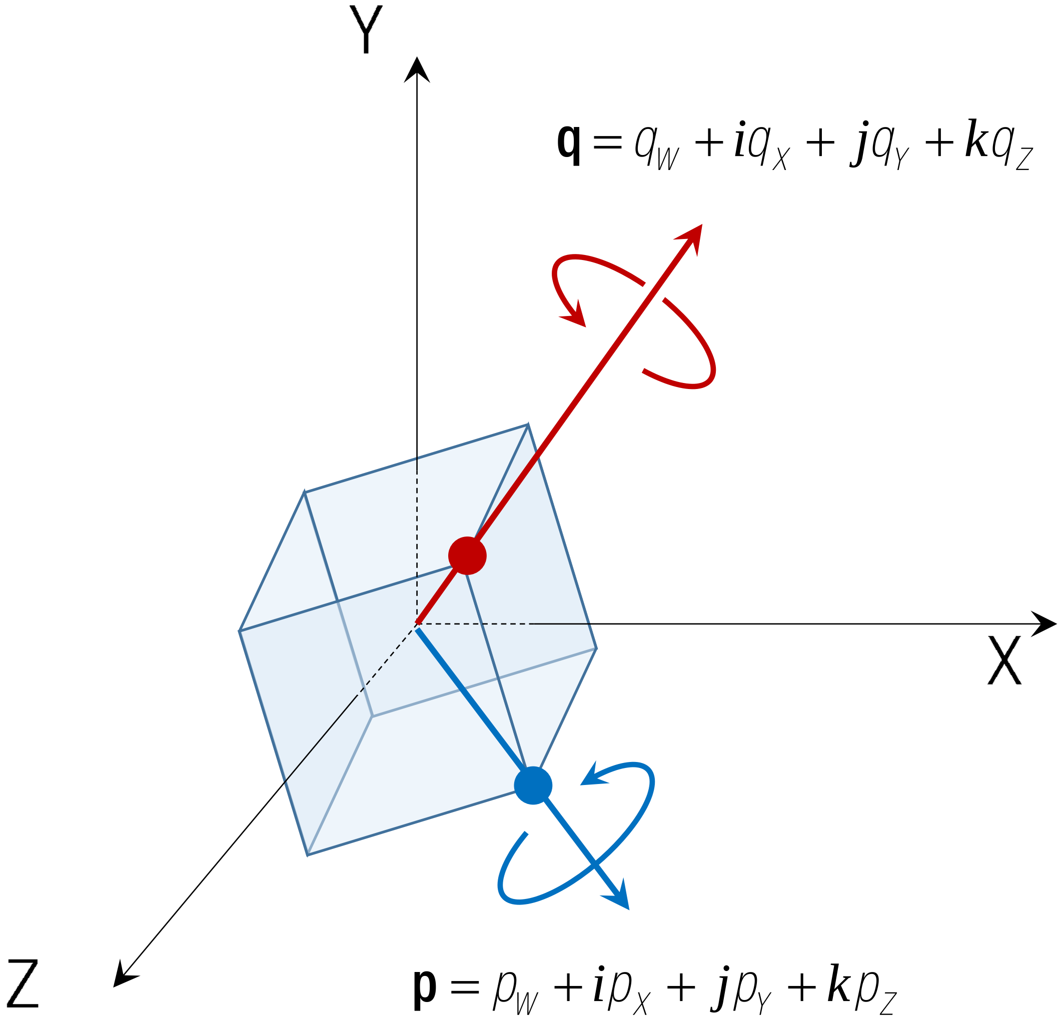
$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

Broom Bridge, Dublin, Ireland



Quaternion Product



$\mathbf{q} = q_W + \boldsymbol{i} q_X + \boldsymbol{j} q_Y + \boldsymbol{k} q_Z$

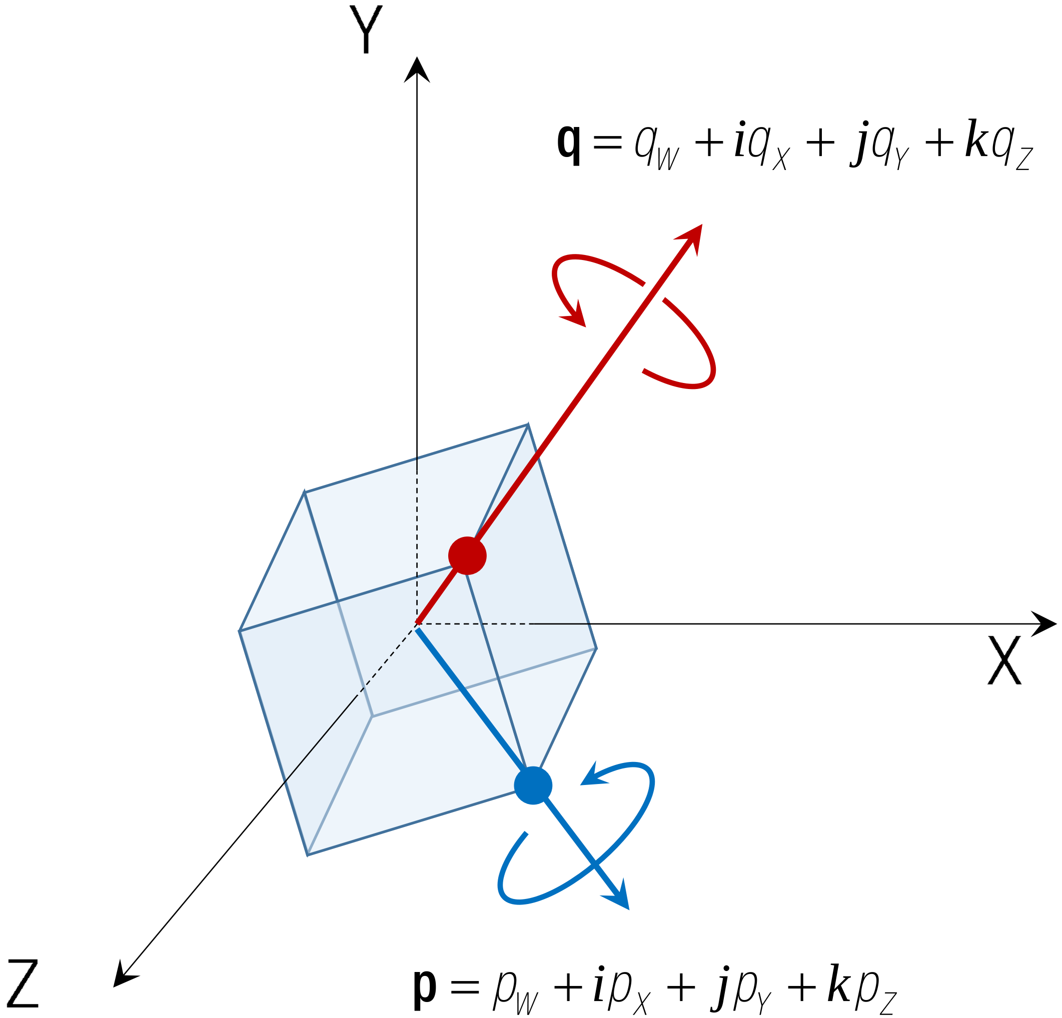
$\mathbf{p} = p_W + \boldsymbol{i} p_X + \boldsymbol{j} p_Y + \boldsymbol{k} p_Z$

Rotate **q** and then, **p**:

$$\mathbf{qp} = \left(q_W + \boldsymbol{i} q_X + \boldsymbol{j} q_Y + \boldsymbol{k} q_Z\right)\left(p_W + \boldsymbol{i} p_X + \boldsymbol{j} p_Y + \boldsymbol{k} p_Z\right)$$

where $\hat{\mathbf{q}} = \boldsymbol{i} q_X + \boldsymbol{j} q_Y + \boldsymbol{k} q_Z$

Quaternion Product



$$\mathbf{q} = q_W + \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z$$

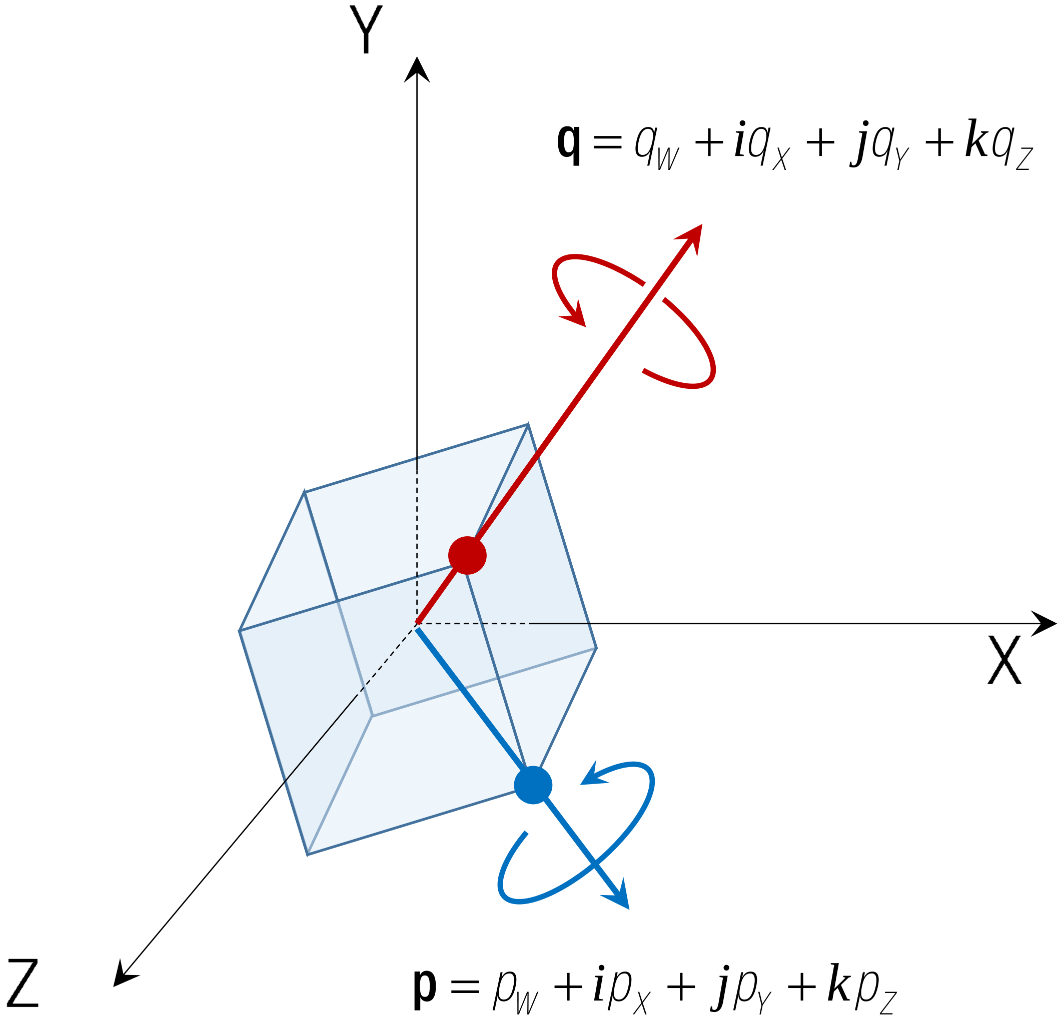
$$\mathbf{p} = p_W + \boldsymbol{i}p_X + \boldsymbol{j}p_Y + \boldsymbol{k}p_Z$$

Rotate **q** and then, **p**:

$$\begin{aligned}\mathbf{qp} &= \left(q_W + \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z\right)\left(p_W + \boldsymbol{i}p_X + \boldsymbol{j}p_Y + \boldsymbol{k}p_Z\right) \\ &= \left(q_W p_W - q_X p_X - q_Y p_Y - q_Z p_Z\right) + \boldsymbol{i}\left(q_W p_X + q_X p_W + q_Y p_Z - q_Z p_Y\right) \\ &\quad + \boldsymbol{j}\left(q_W p_Y - q_X p_Z + q_Y p_W + q_Z p_X\right) + \boldsymbol{k}\left(q_W p_Z + q_X p_Y - q_Y p_X + q_Z p_W\right)\end{aligned}$$

where $\hat{\mathbf{q}} = \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z$

Quaternion Product



$$\mathbf{q} = q_W + \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z$$

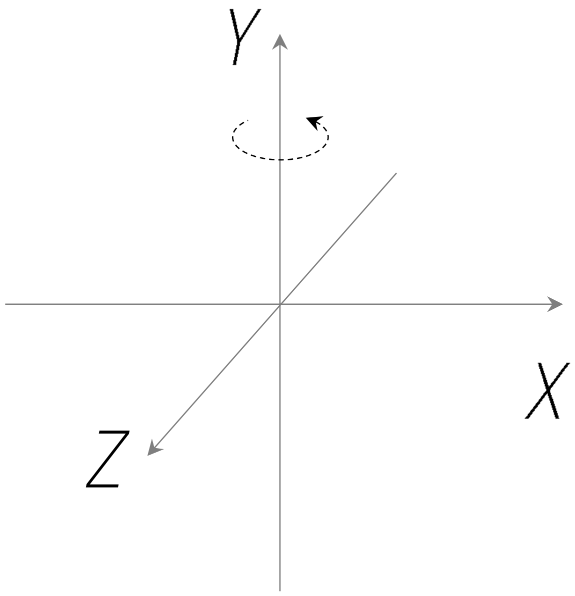
$$\mathbf{p} = p_W + \boldsymbol{i}p_X + \boldsymbol{j}p_Y + \boldsymbol{k}p_Z$$

Rotate **q** and then, **p**:

$$\begin{aligned}
\mathbf{qp} &= \left(q_W + \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z\right)\left(p_W + \boldsymbol{i}p_X + \boldsymbol{j}p_Y + \boldsymbol{k}p_Z\right) \\
&= \left(q_W p_W - q_X p_X - q_Y p_Y - q_Z p_Z\right) + \boldsymbol{i}\left(q_W p_X + q_X p_W + q_Y p_Z - q_Z p_Y\right) \\
&\quad + \boldsymbol{j}\left(q_W p_Y - q_X p_Z + q_Y p_W + q_Z p_X\right) + \boldsymbol{k}\left(q_W p_Z + q_X p_Y - q_Y p_X + q_Z p_W\right) \\
&= \left(q_W p_W - \hat{\mathbf{q}}\cdot\hat{\mathbf{p}}\right) + \left(q_W \hat{\mathbf{p}} + p_W \hat{\mathbf{q}} + \hat{\mathbf{q}}\times\hat{\mathbf{p}}\right)
\end{aligned}$$

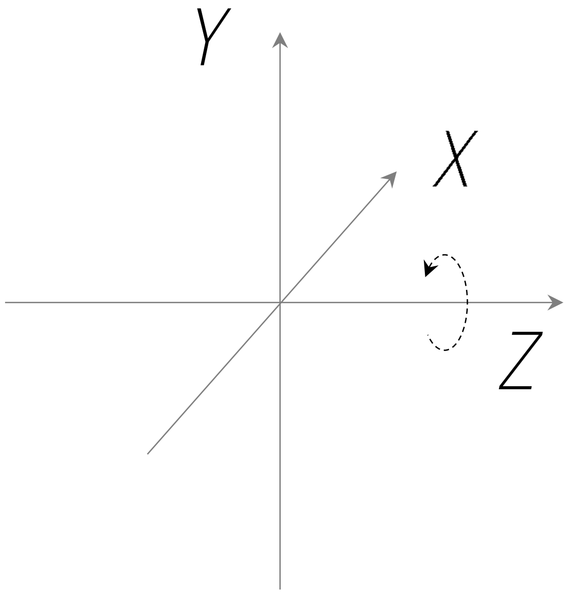
where $\hat{\mathbf{q}} = \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z$

Quaternion Product Example



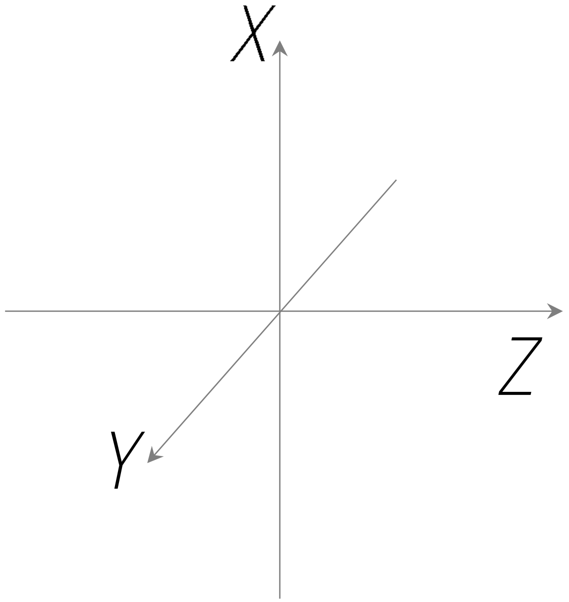
Rotating 90 degrees about *Y* axis.

$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2}$$



Rotating 90 degrees about *Z* axis.

$$\mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$



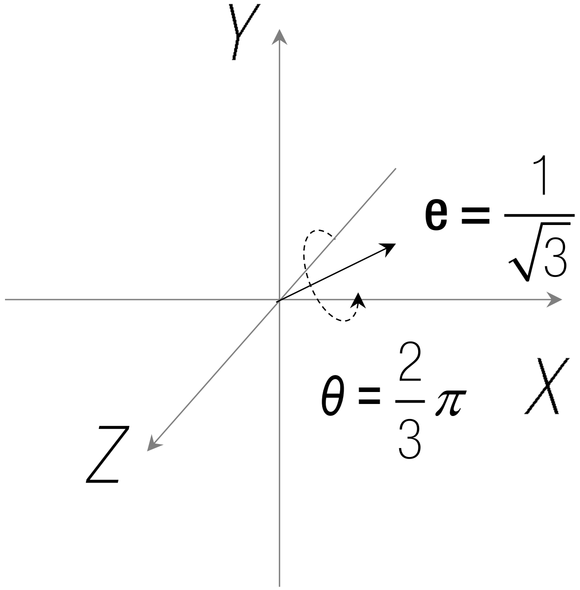
$$\mathbf{q}_{12} = \mathbf{q}_1\mathbf{q}_2 \qquad ?$$

Quaternion Product Example

$$\mathbf{qp} = \left(q_W p_W - \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}\right) + \left(q_W \hat{\mathbf{p}} + p_W \hat{\mathbf{q}} + \hat{\mathbf{q}} \times \hat{\mathbf{p}}\right)$$

$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2} \qquad \mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$

Quaternion Product Example



$\mathbf{e} = \frac{1}{\sqrt{3}}(i + j + k)$

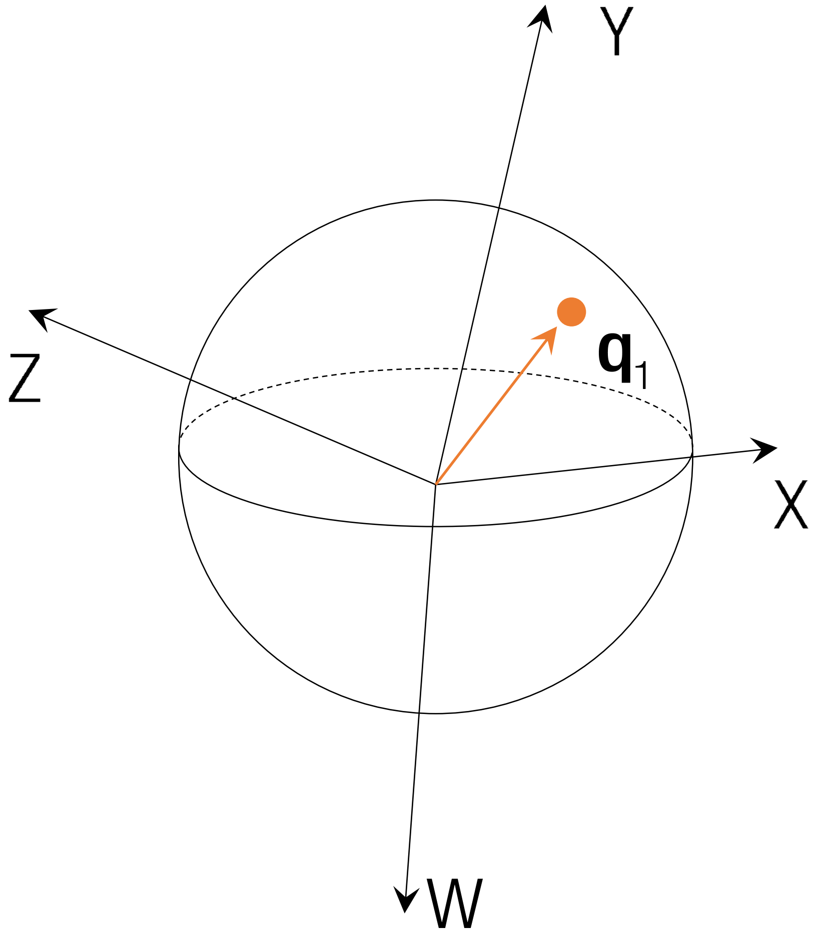
$\theta = \frac{2}{3}\pi$

$$\mathbf{qp} = \left(q_W p_W - \hat{\mathbf{q}}\cdot\hat{\mathbf{p}}\right) + \left(q_W\hat{\mathbf{p}} + p_W\hat{\mathbf{q}} + \hat{\mathbf{q}}\times\hat{\mathbf{p}}\right)$$

$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2} \qquad \mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$

$$\begin{aligned}
\mathbf{q}_{12} &= \mathbf{q}_1\mathbf{q}_2 \\
&= \left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2} + \boldsymbol{j}\frac{\sqrt{2}}{2}\right) \\
&= \frac{1}{2} + \frac{1}{2}\boldsymbol{i} + \frac{1}{2}\boldsymbol{j} + \frac{1}{2}\boldsymbol{k} \\
&= \frac{1}{2} + \frac{1}{2}\boldsymbol{i} + \frac{1}{2}\boldsymbol{j} + \frac{1}{2}\boldsymbol{k} \\
&= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}}\boldsymbol{i} + \frac{1}{\sqrt{3}}\boldsymbol{j} + \frac{1}{\sqrt{3}}\boldsymbol{k}\right)
\end{aligned}$$

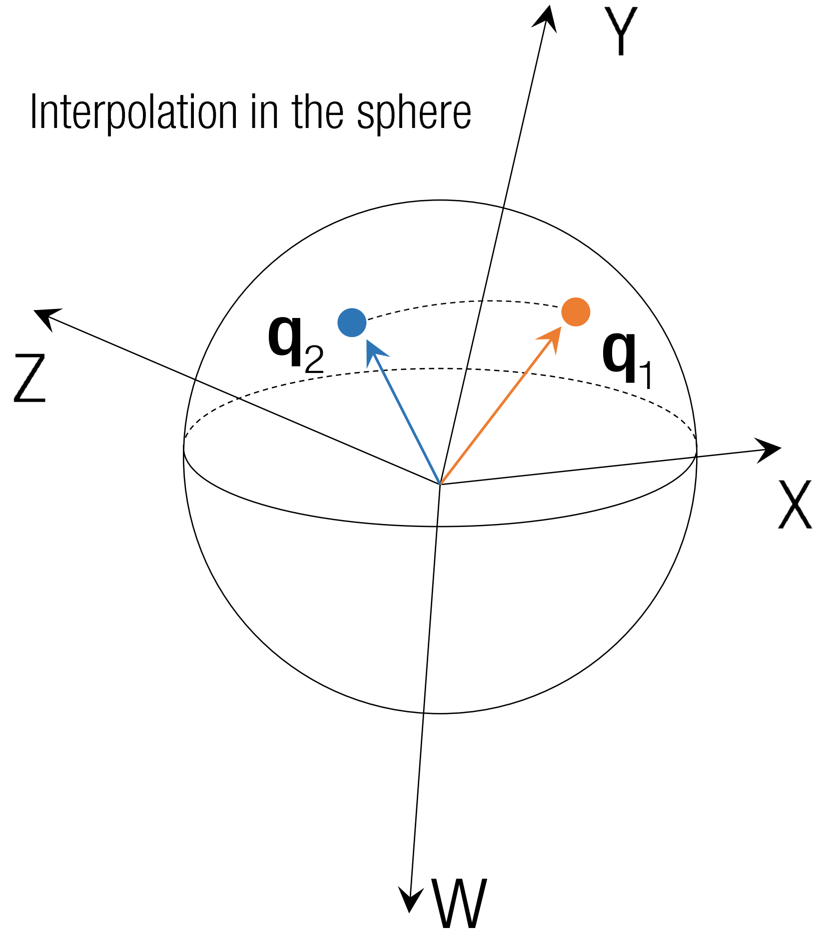
Quaternion in 4D Sphere



$$\mathbf{q} = q_W + \boldsymbol{i}q_X + \boldsymbol{j}q_Y + \boldsymbol{k}q_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix}$$

$$q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

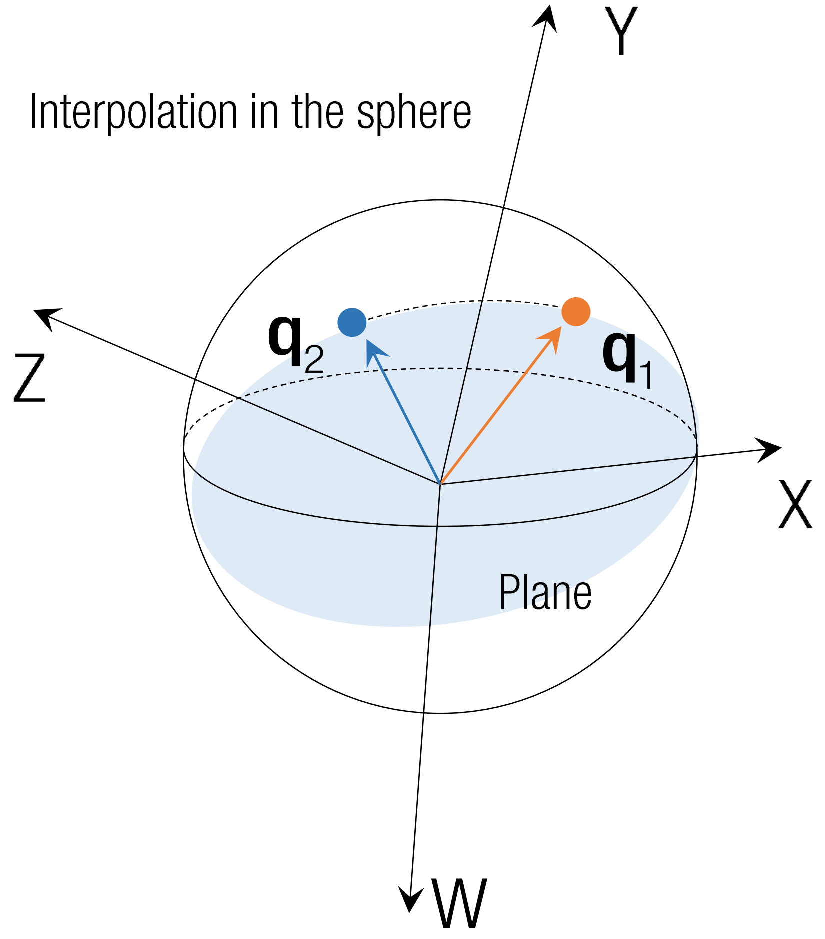
Quaternion in 4D Sphere



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix}$$

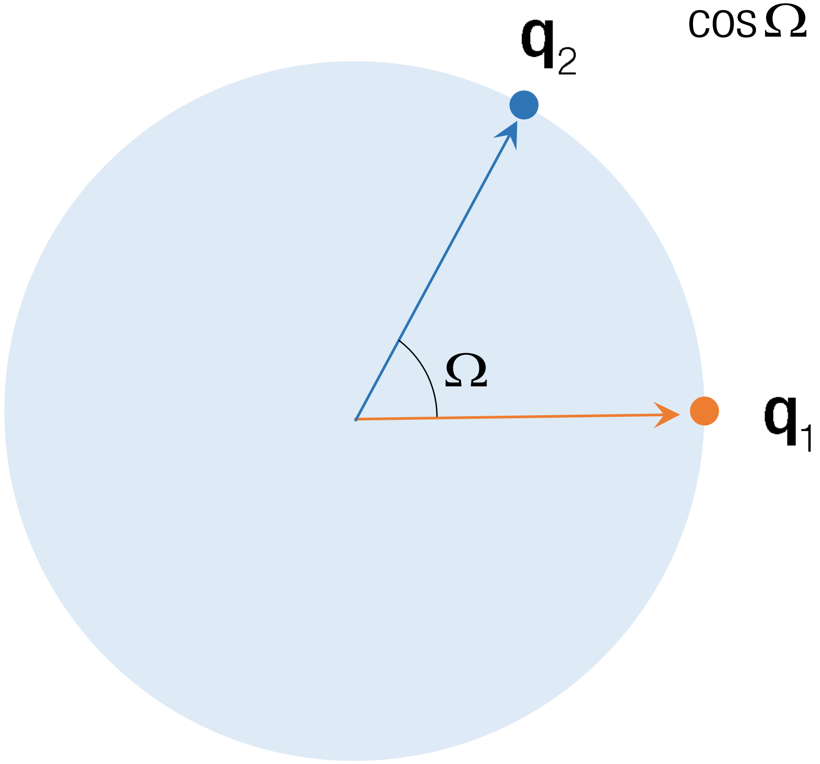
$$q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

Quaternion in 4D Sphere



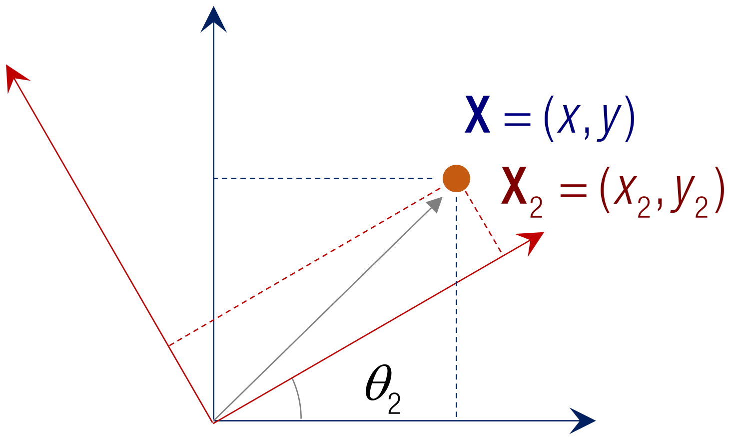
$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



Recall: Interpolation of Rotation

2D coordinate transform:



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

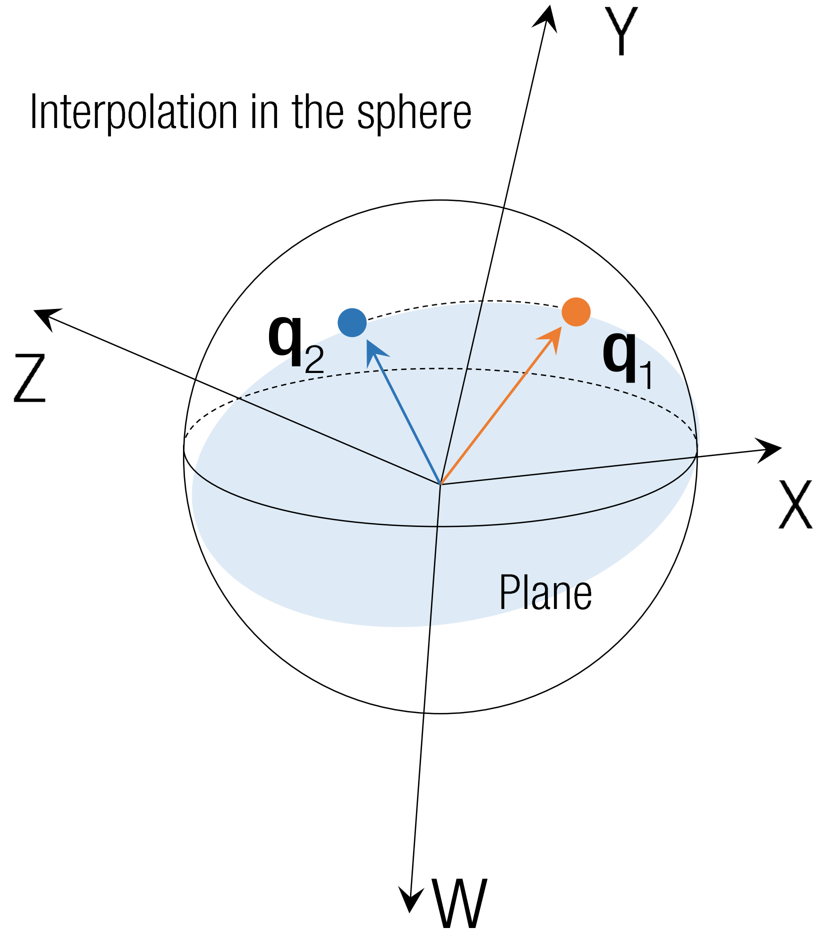
$$\theta = w\theta_1 + (1-w)\theta_2$$

$$w \in [0,1]$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

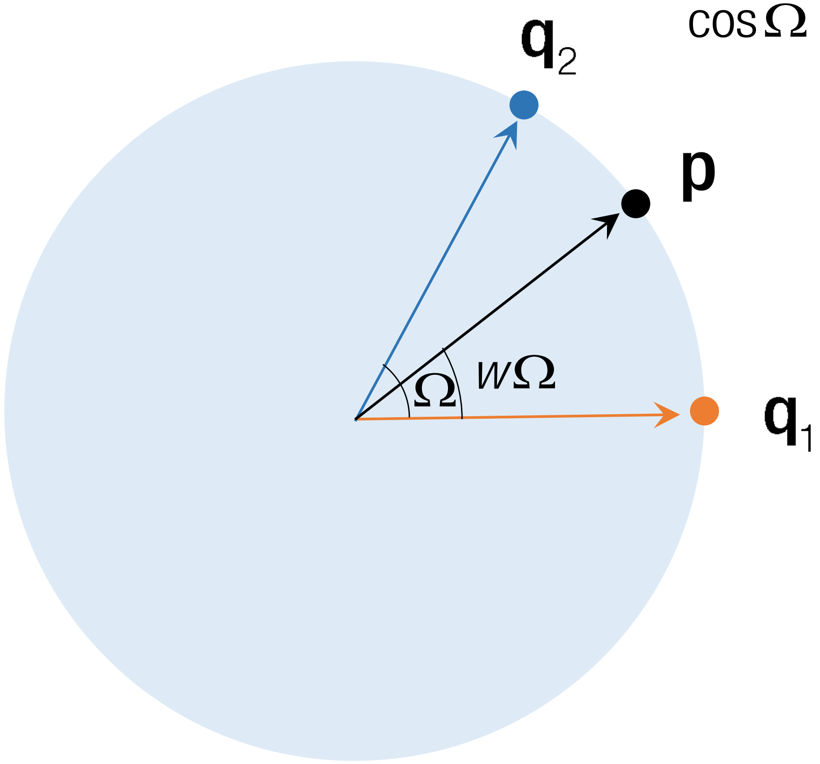
Quaternion Interpolation

Interpolation in the sphere



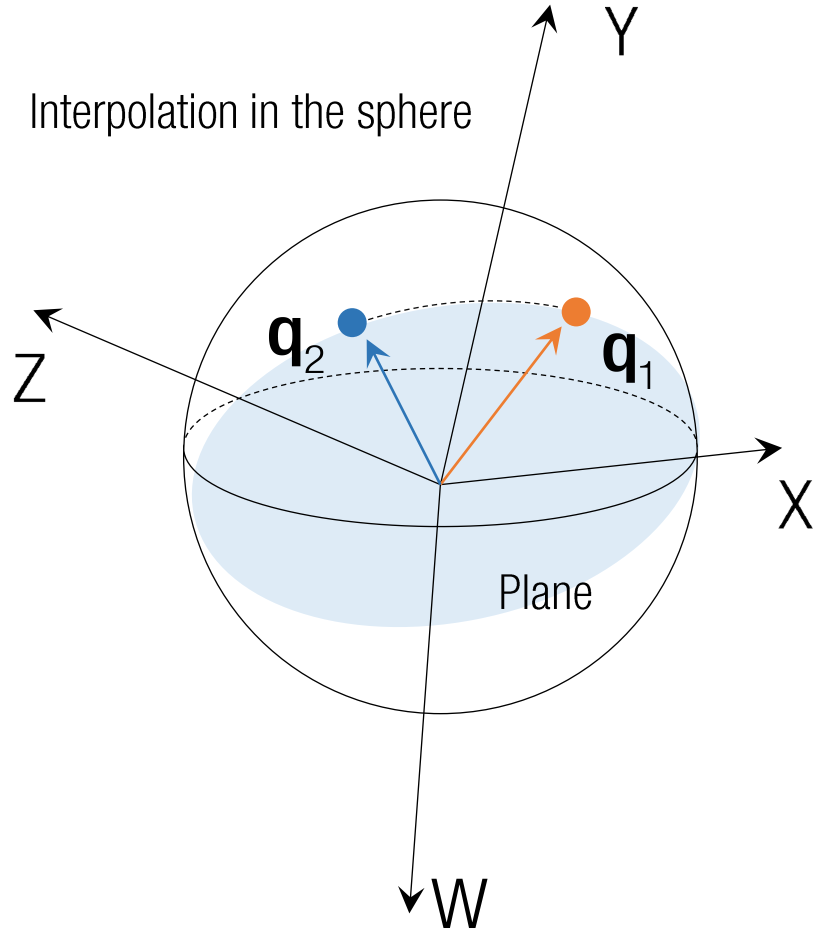
$$\mathbf{q} = q_W + i q_X + j q_Y + k q_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



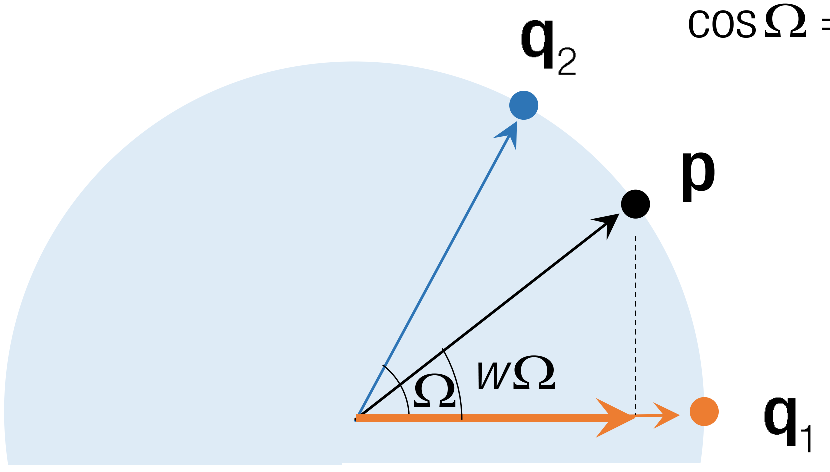
Quaternion Interpolation

Interpolation in the sphere



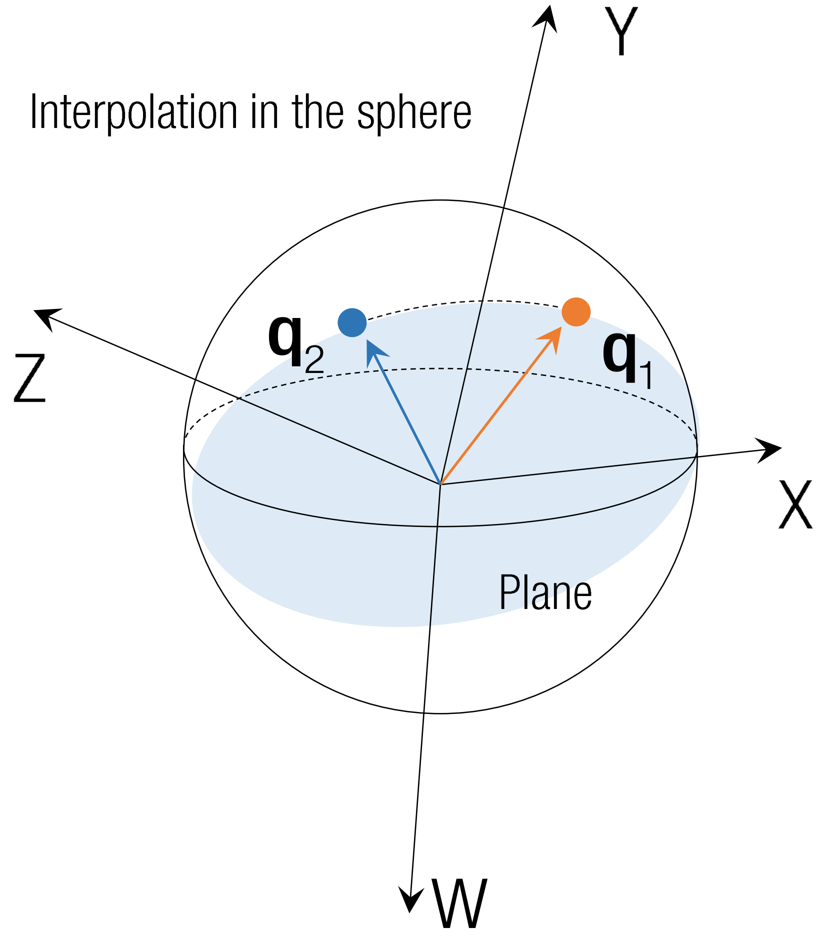
$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



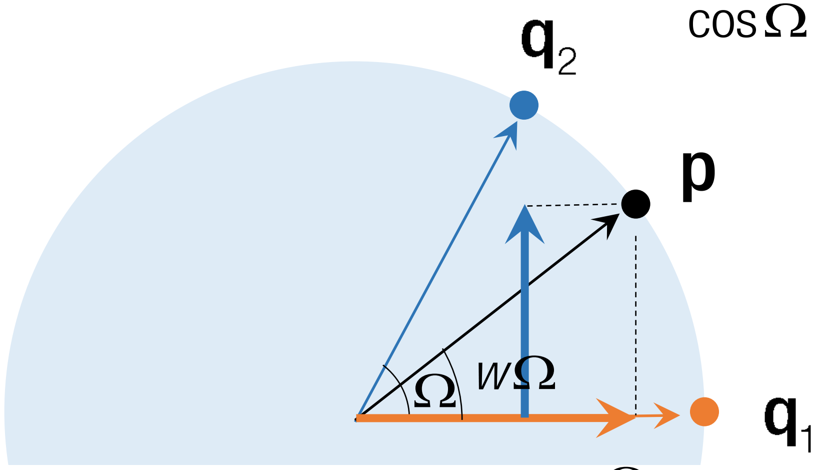
$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega$$

Quaternion Interpolation



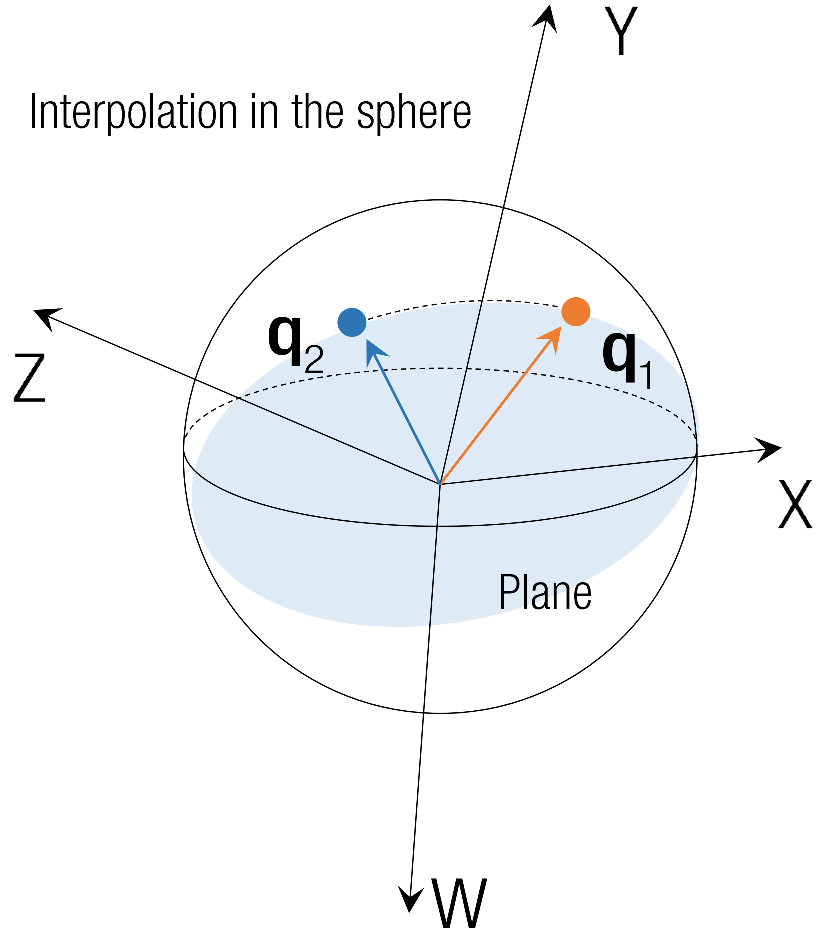
$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



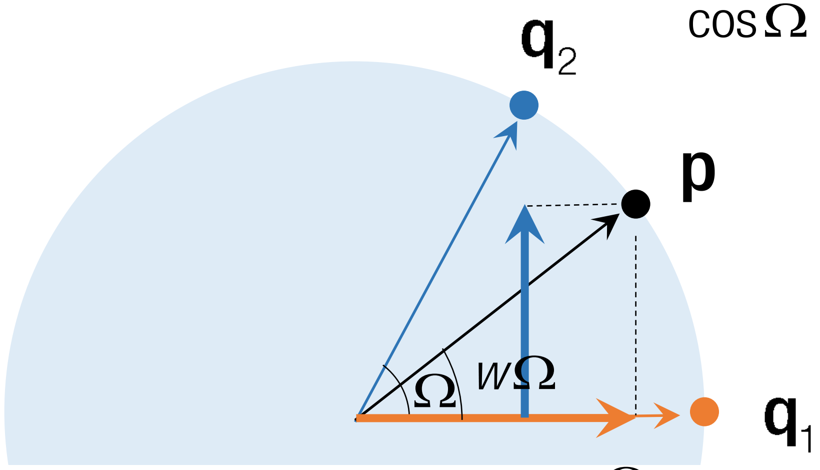
$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

Quaternion Interpolation



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$

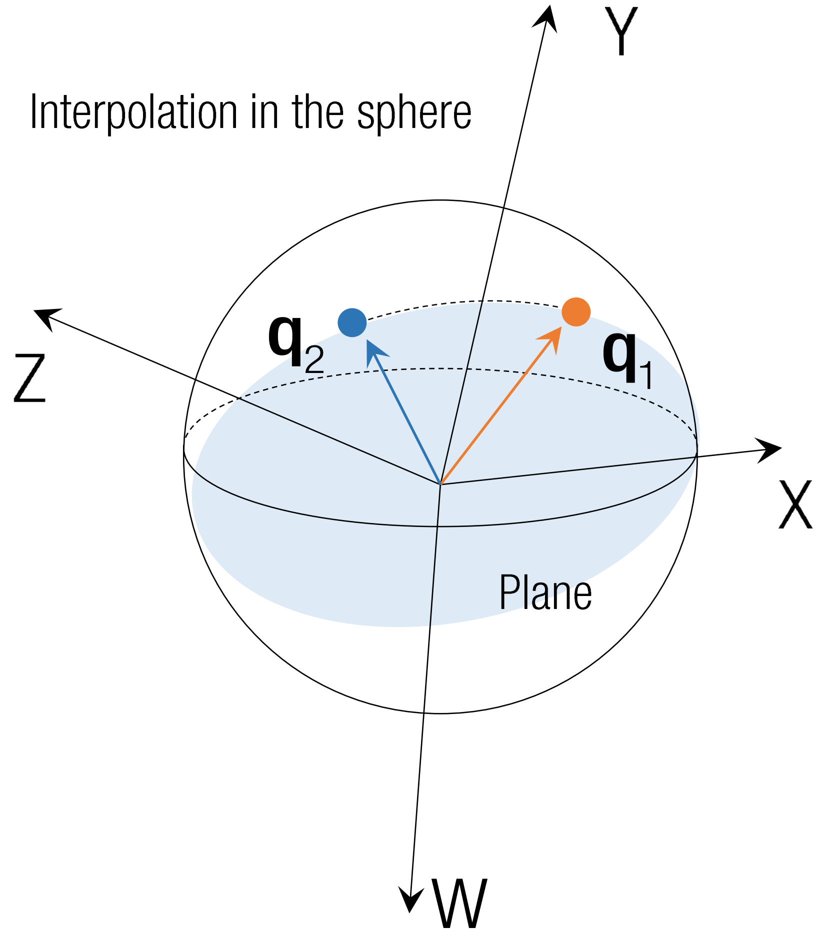


$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

$$= \frac{\mathbf{q}_1 \left(\sin\Omega \cos w\Omega - \cos\Omega \sin w\Omega\right) + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$

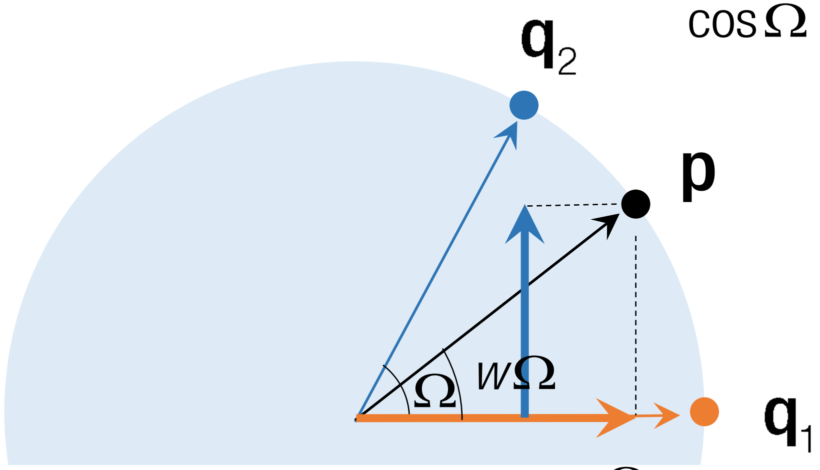
Quaternion Interpolation

Interpolation in the sphere



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

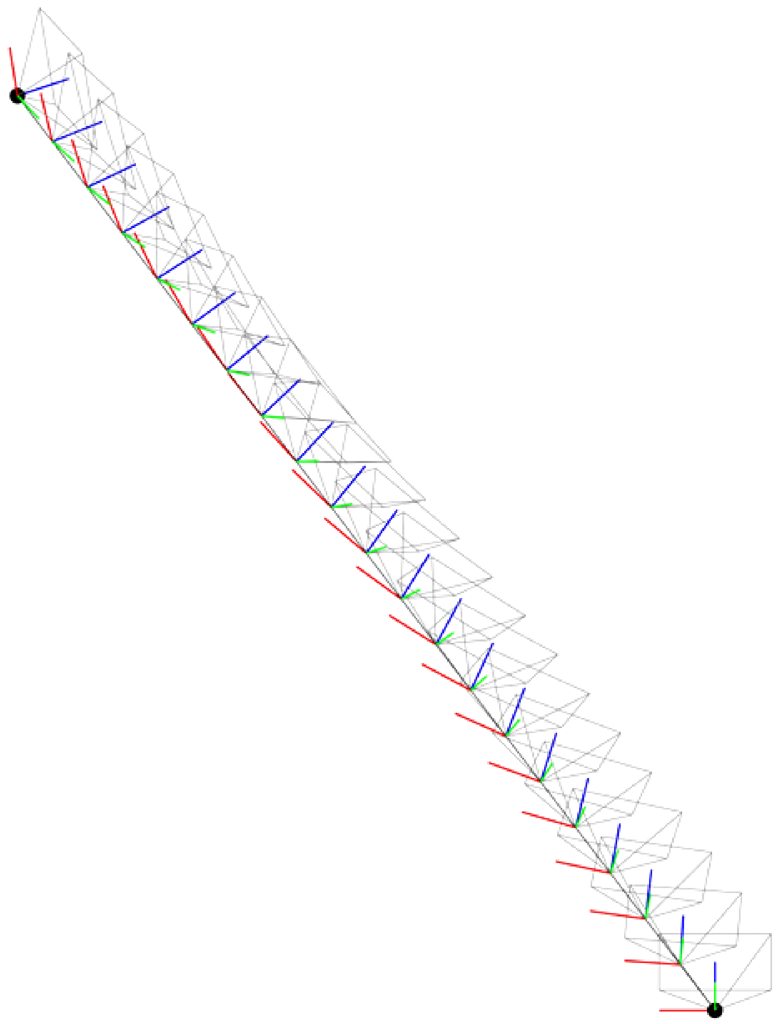
$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$

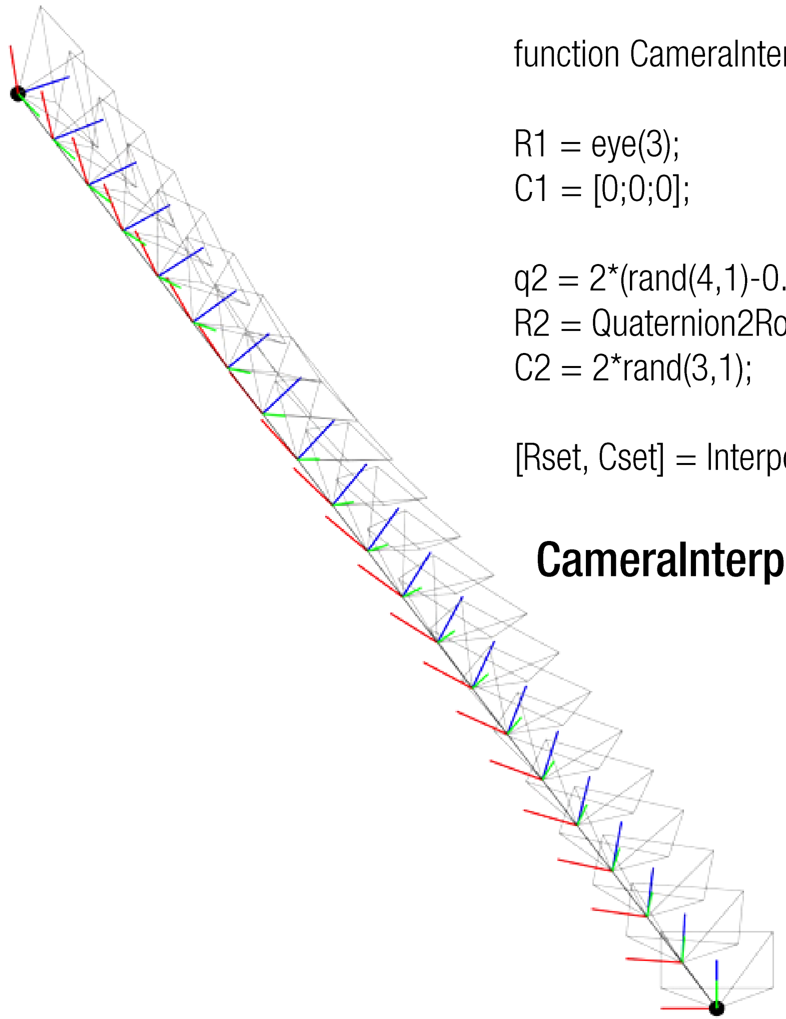


$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

$$= \frac{\mathbf{q}_1 \left(\sin\Omega \cos w\Omega - \cos\Omega \sin w\Omega\right) + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$

$$= \frac{\mathbf{q}_1 \sin(1-w)\Omega + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$





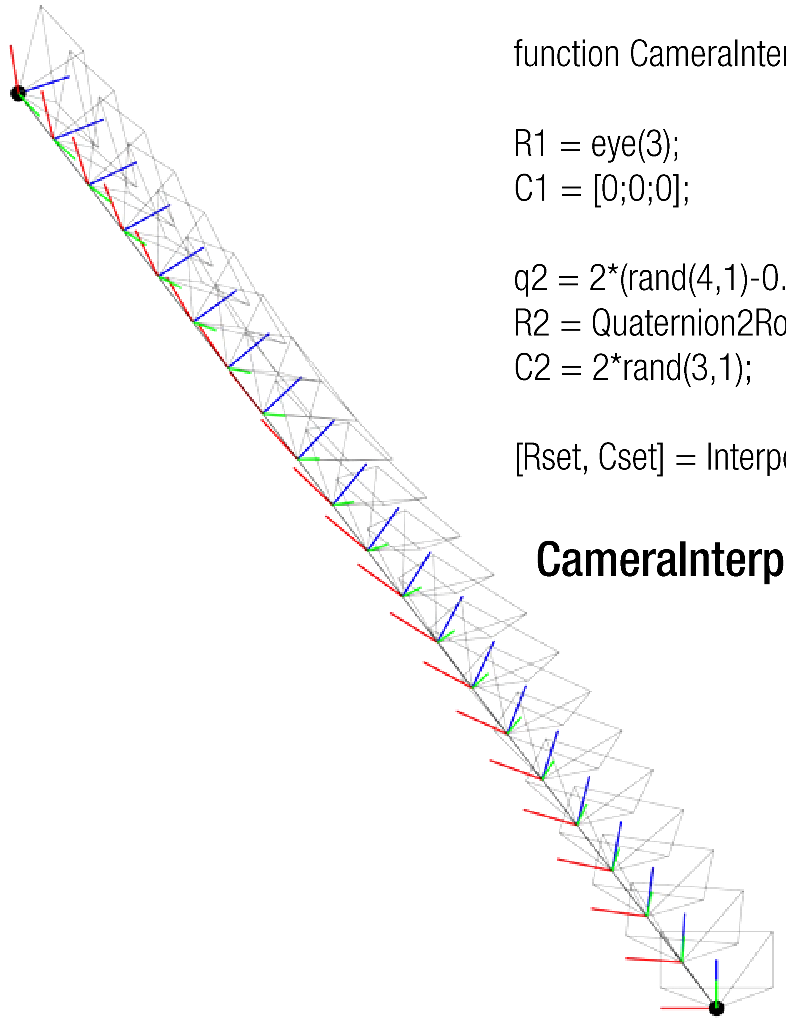
```
function CameraInterpolation

R1 = eye(3);
C1 = [0;0;0];

q2 = 2*(rand(4,1)-0.5);
R2 = Quaternion2Rotation(q2);
C2 = 2*rand(3,1);

[Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, 20);
```

CameraInterpolation.m



```
function CameraInterpolation

R1 = eye(3);
C1 = [0;0;0];

q2 = 2*(rand(4,1)-0.5);
R2 = Quaternion2Rotation(q2);
C2 = 2*rand(3,1);

[Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, 20);
```

CameraInterpolation.m

```
function [Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, n)

Cx = linspace(C1(1), C2(1), n+1);
Cy = linspace(C1(2), C2(2), n+1);
Cz = linspace(C1(3), C2(3), n+1);

Cset = [Cx; Cy; Cz];

w = 0 : 1/n : 1;
q1 = Rotation2Quaternion(R1);
q2 = Rotation2Quaternion(R2);

omega = acos(q1'*q2);
for i = 1 : length(w)
    q = sin(omega*(1-w(i)))/sin(omega) * q1 + sin(omega*w(i))/sin(omega) * q2;
    Rset{i} = Quaternion2Rotation(q);
end
```

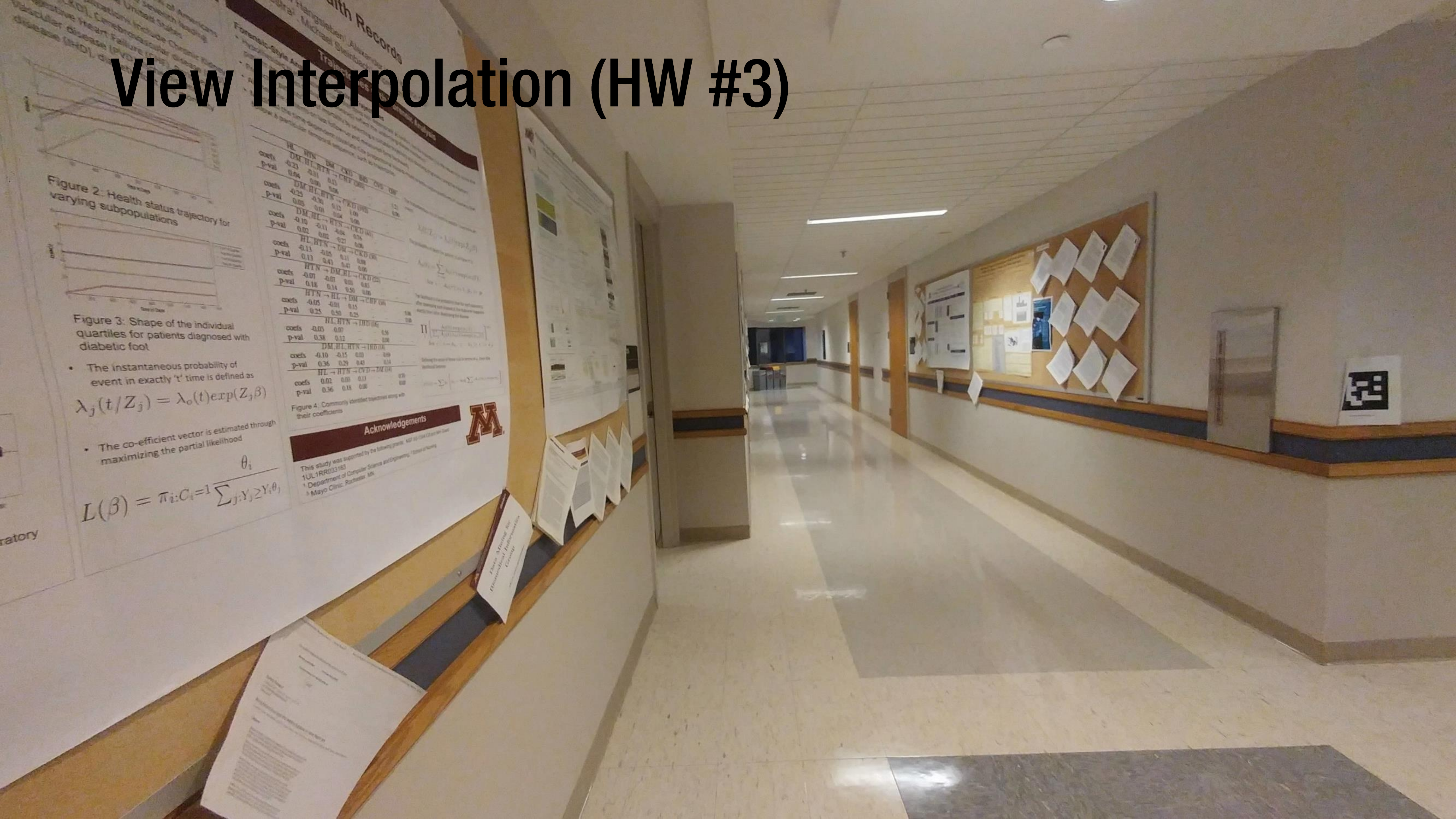
InterpolateCoordinate.m

View Interpolation

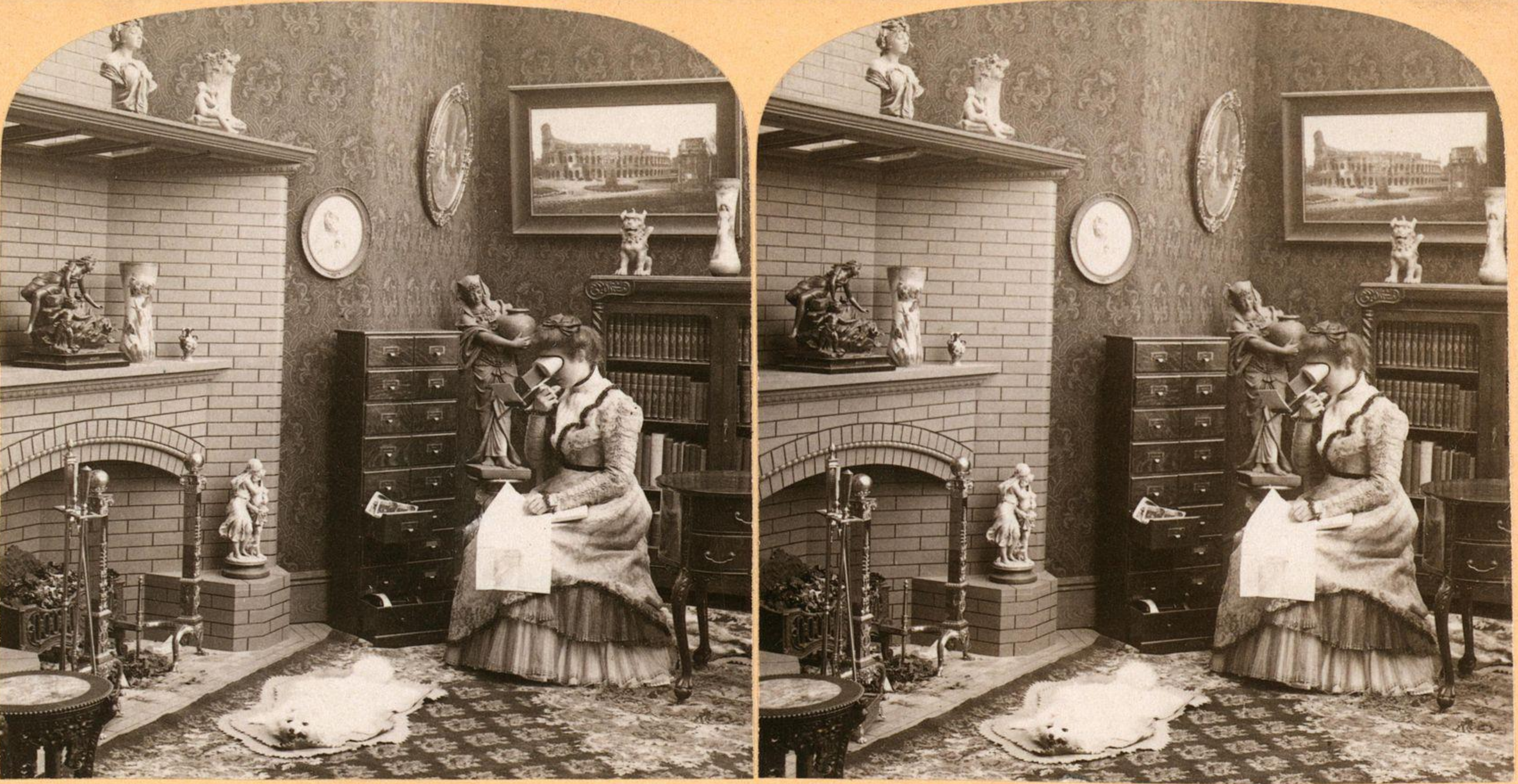
Looking left

Looking right

View Interpolation (HW #3)



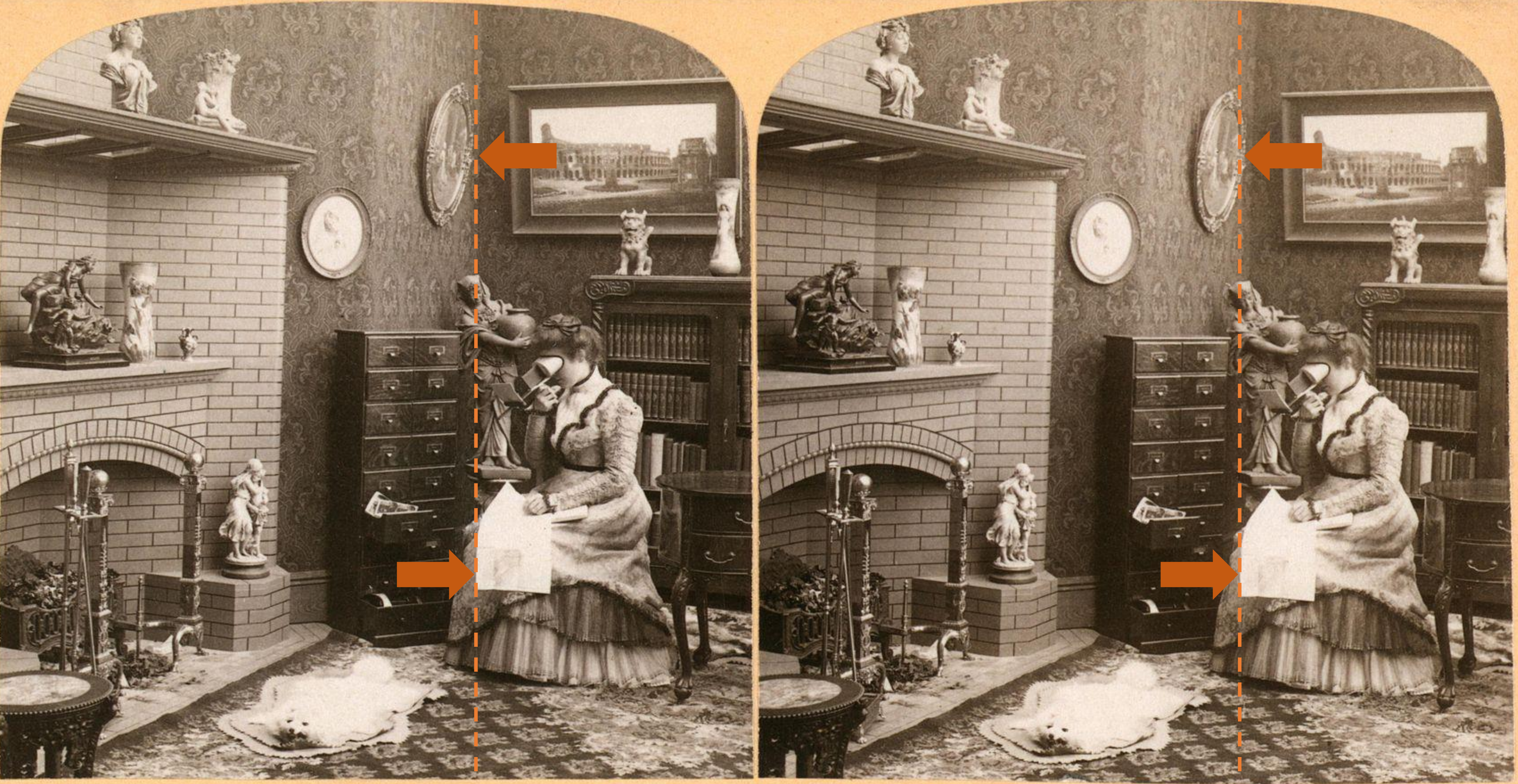
Two View (Epipolar) Geometry



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

(2

The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.



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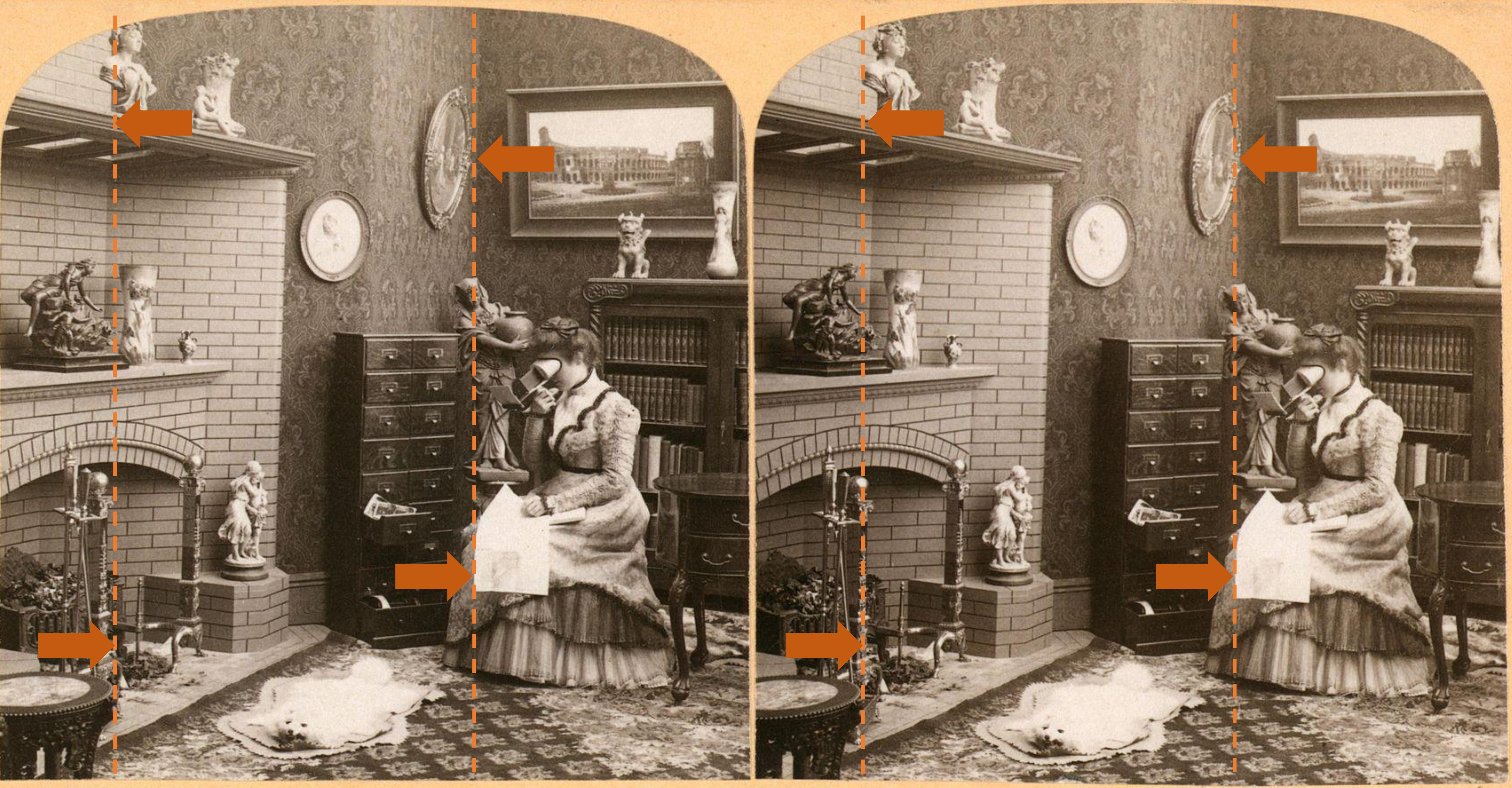

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Circa 1900

Stereo: Holmes Stereoscope


HERO3
HERO3


VITA COCO
Naked
3.99

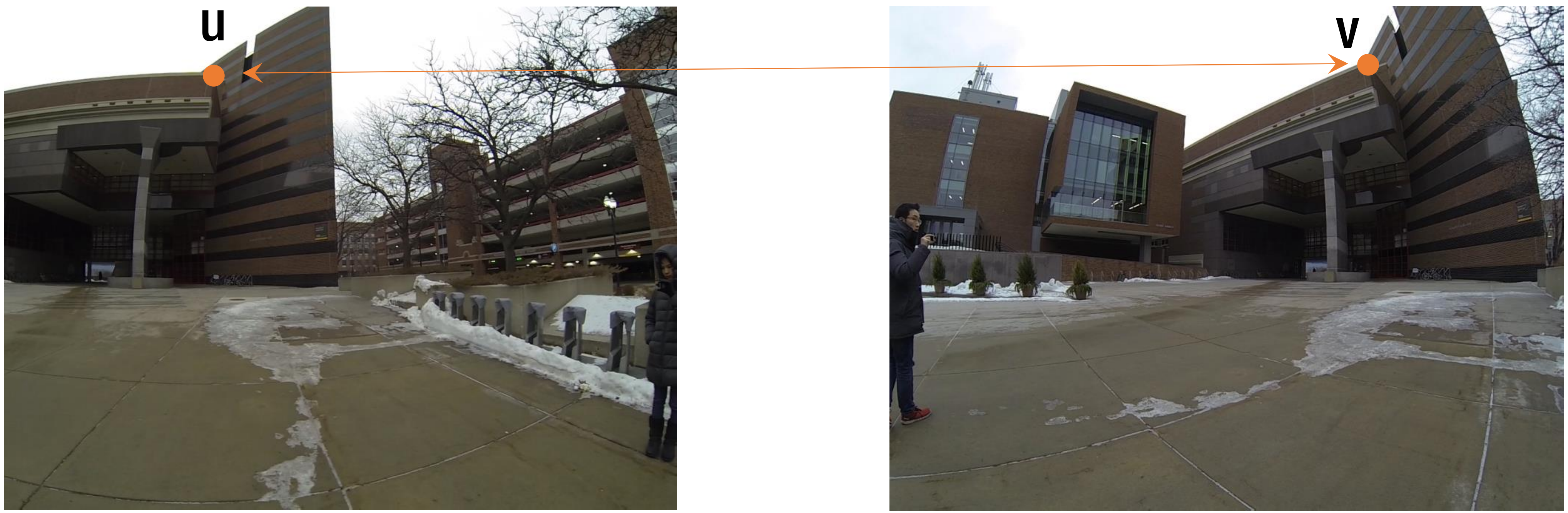
Naked
3.99


VITA
Naked

Left image (Bob)

Right image (Alice)

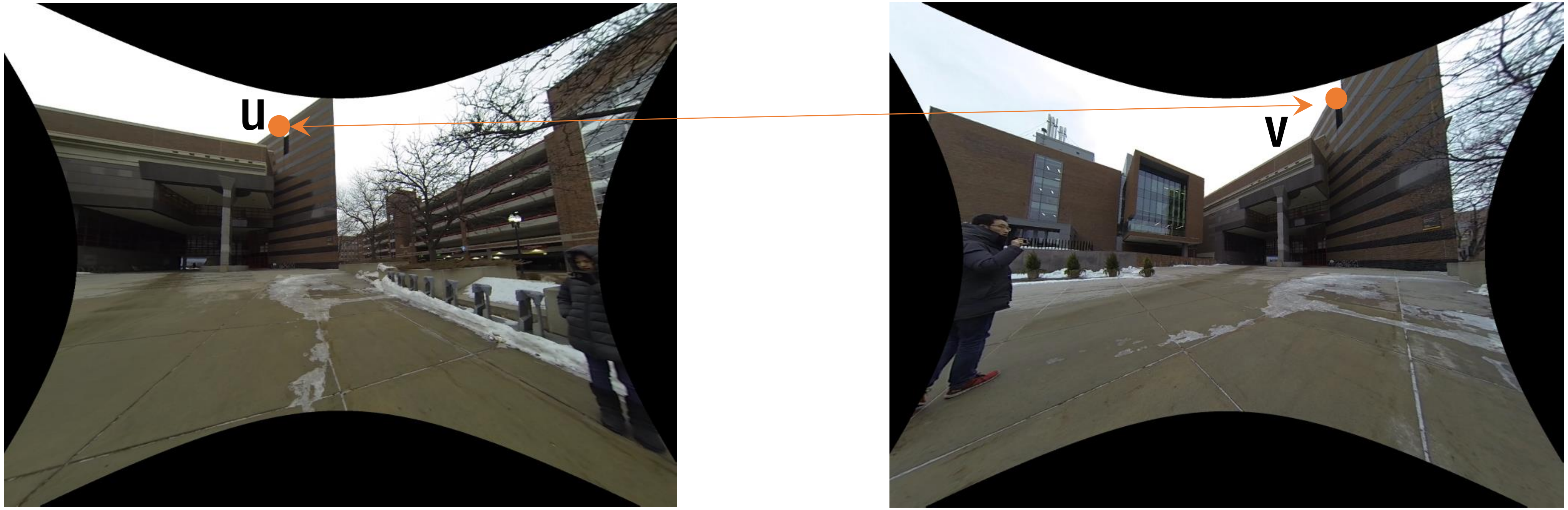
2D Correspondence



Left image (Bob)

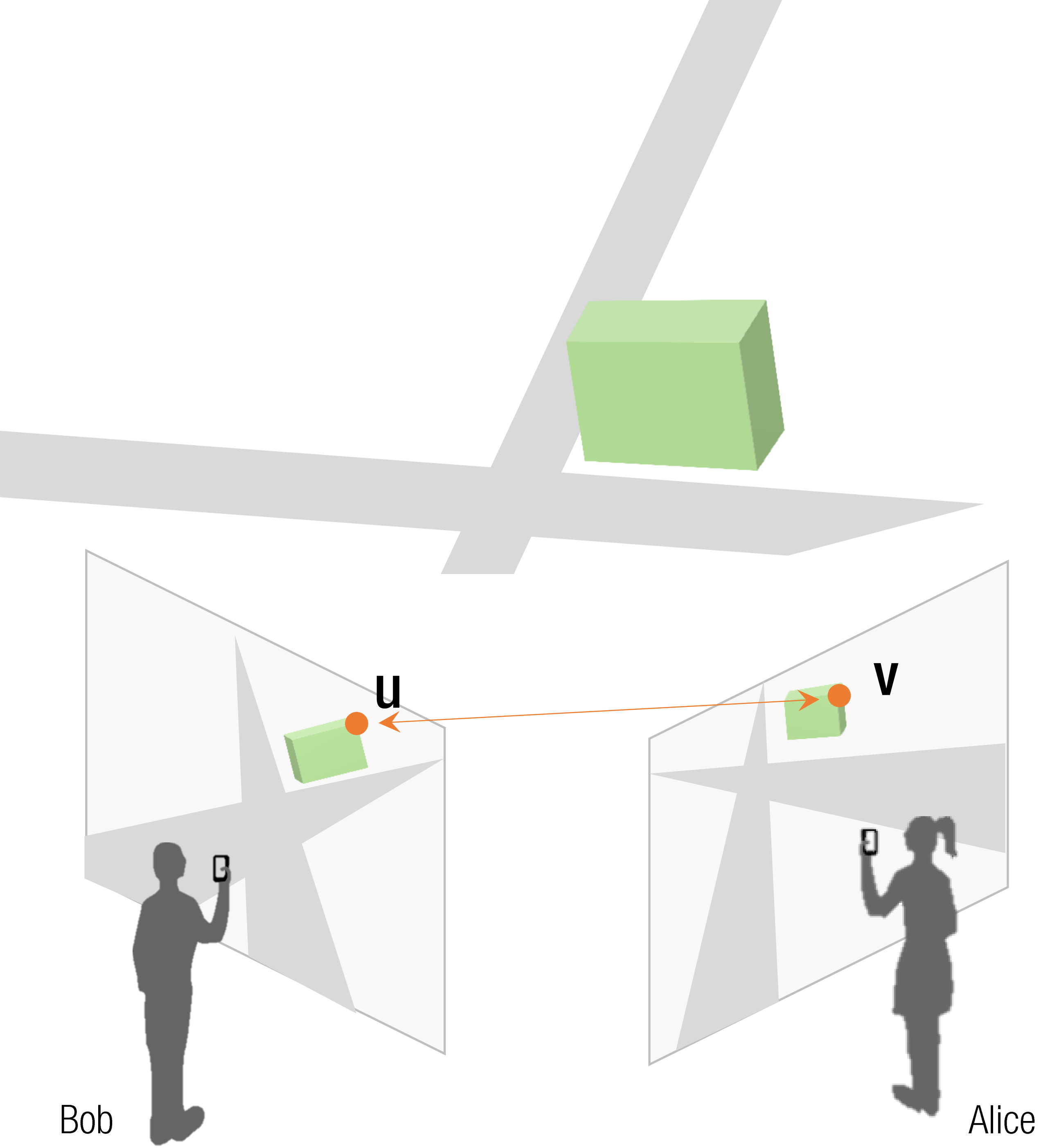
Right image (Alice)

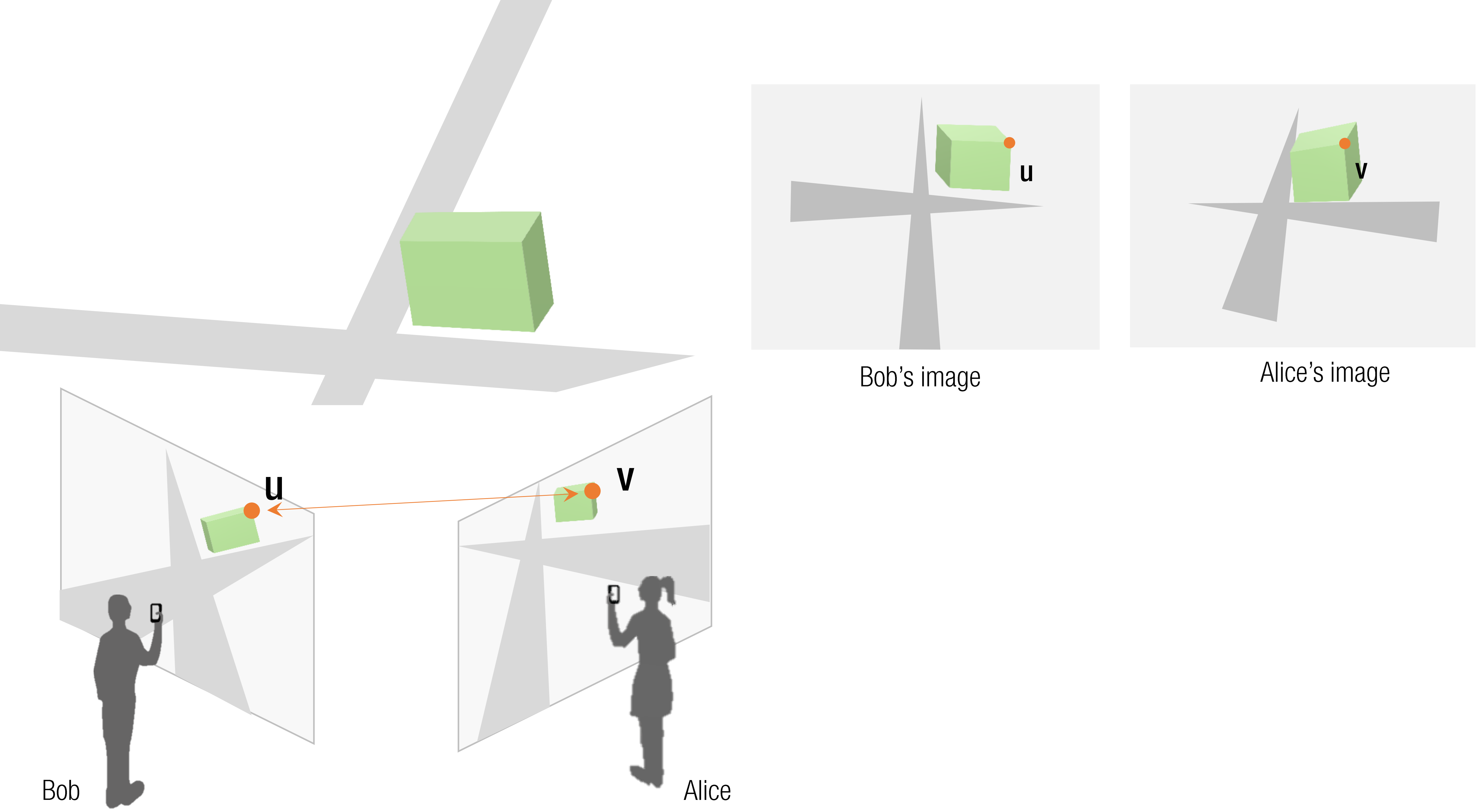
2D Correspondence



Left image (Bob)

Right image (Alice)


u
v
Bob
Alice



u
v
Bob
Alice
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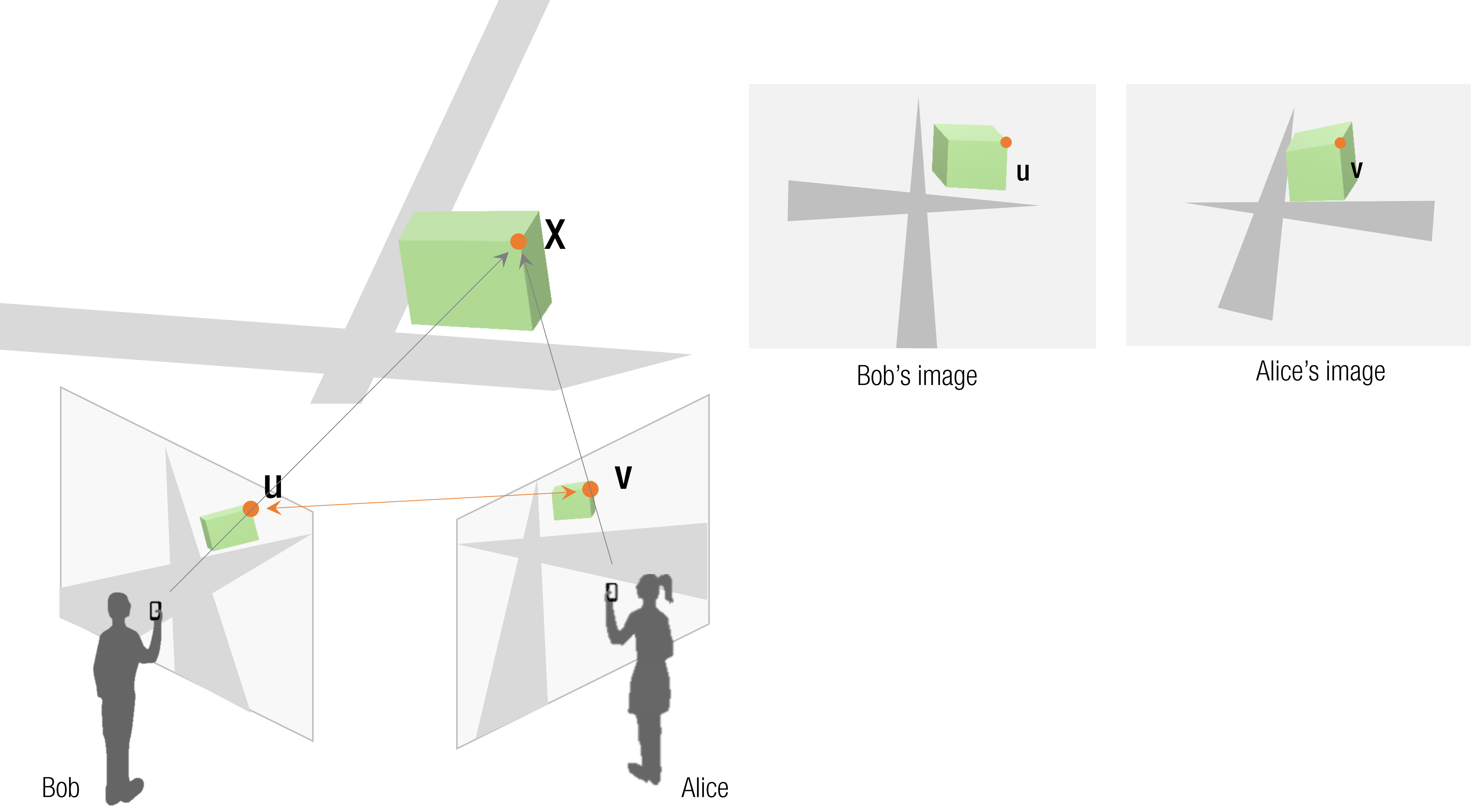
Bob's image
v
Alice's image


X
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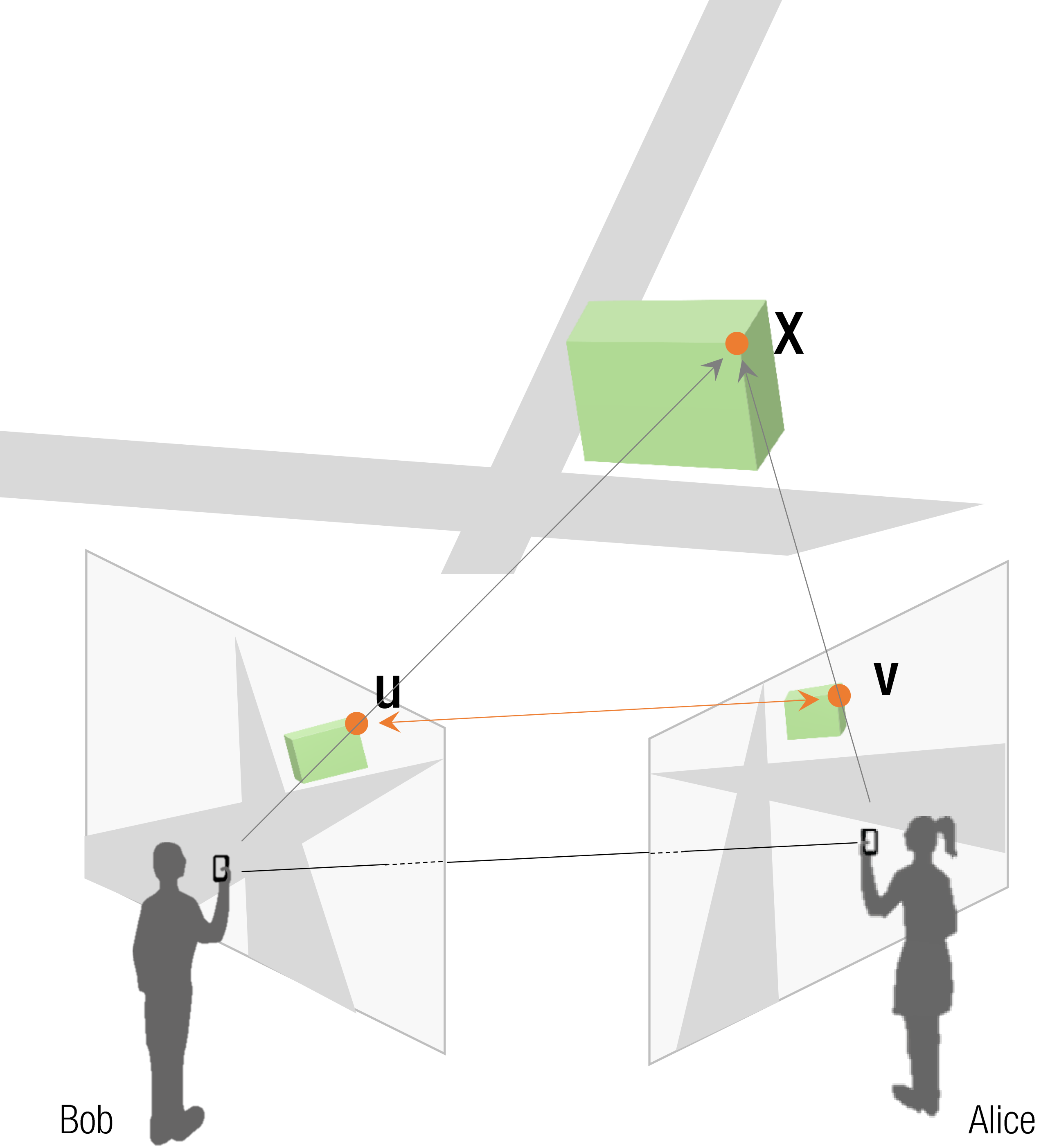
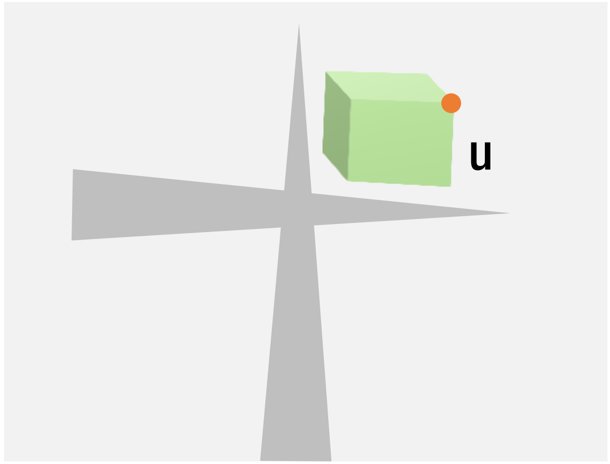
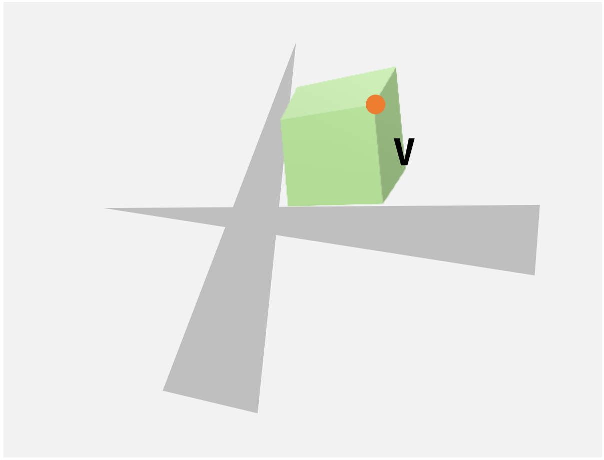
Bob
Alice
u
Bob's image
v
Alice's image

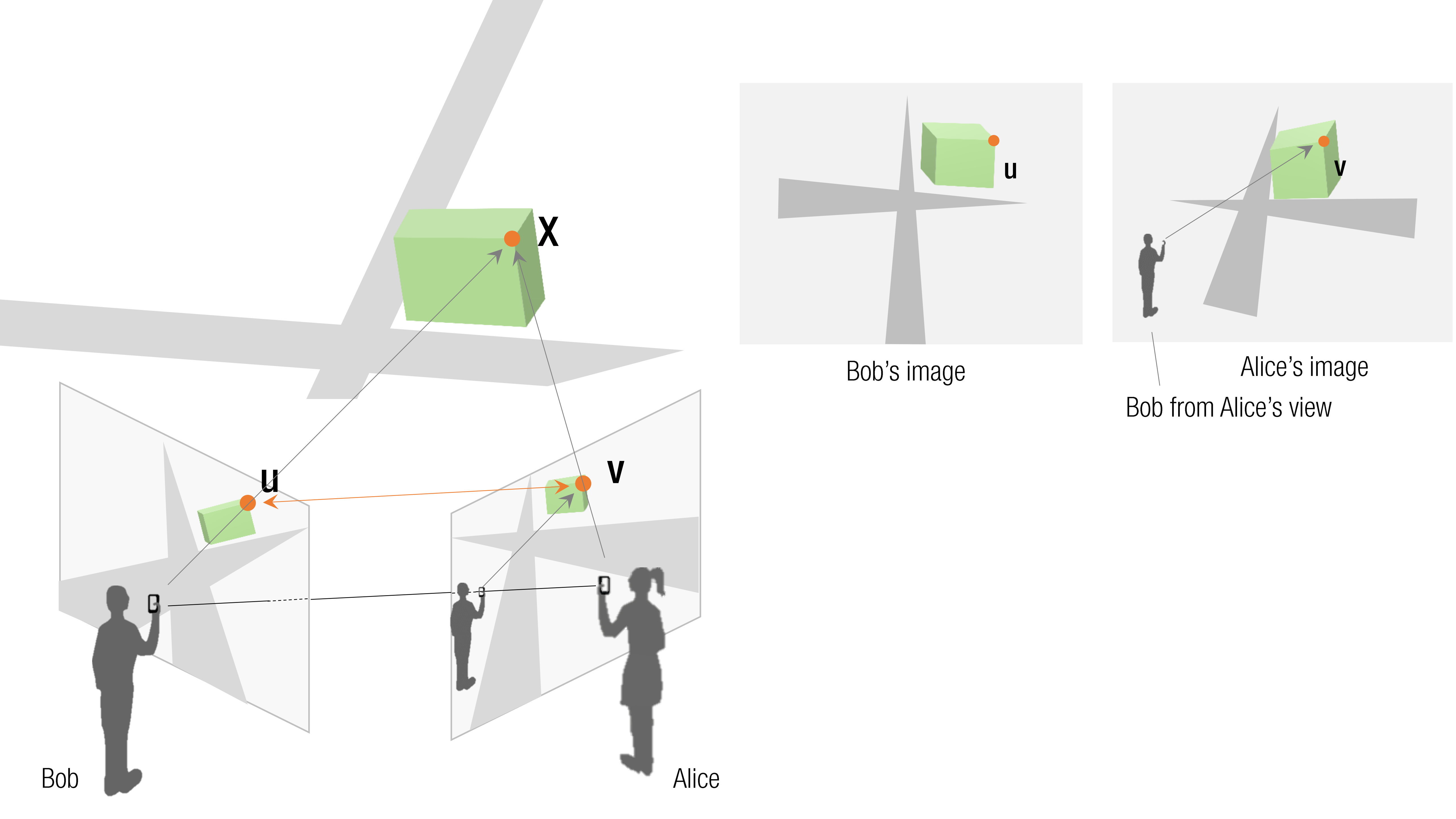




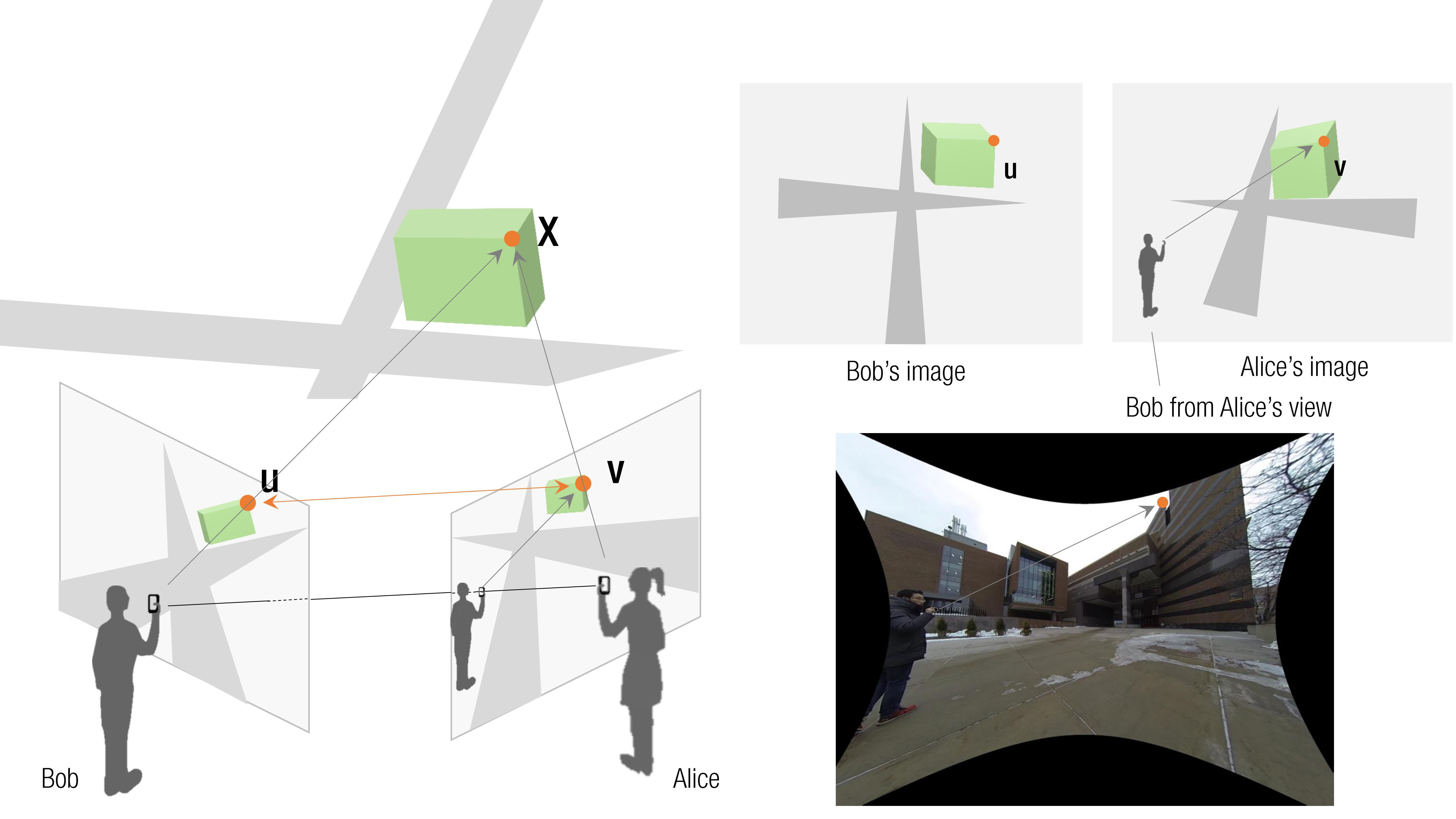
Bob's image



Alice's image



X
u
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view



X
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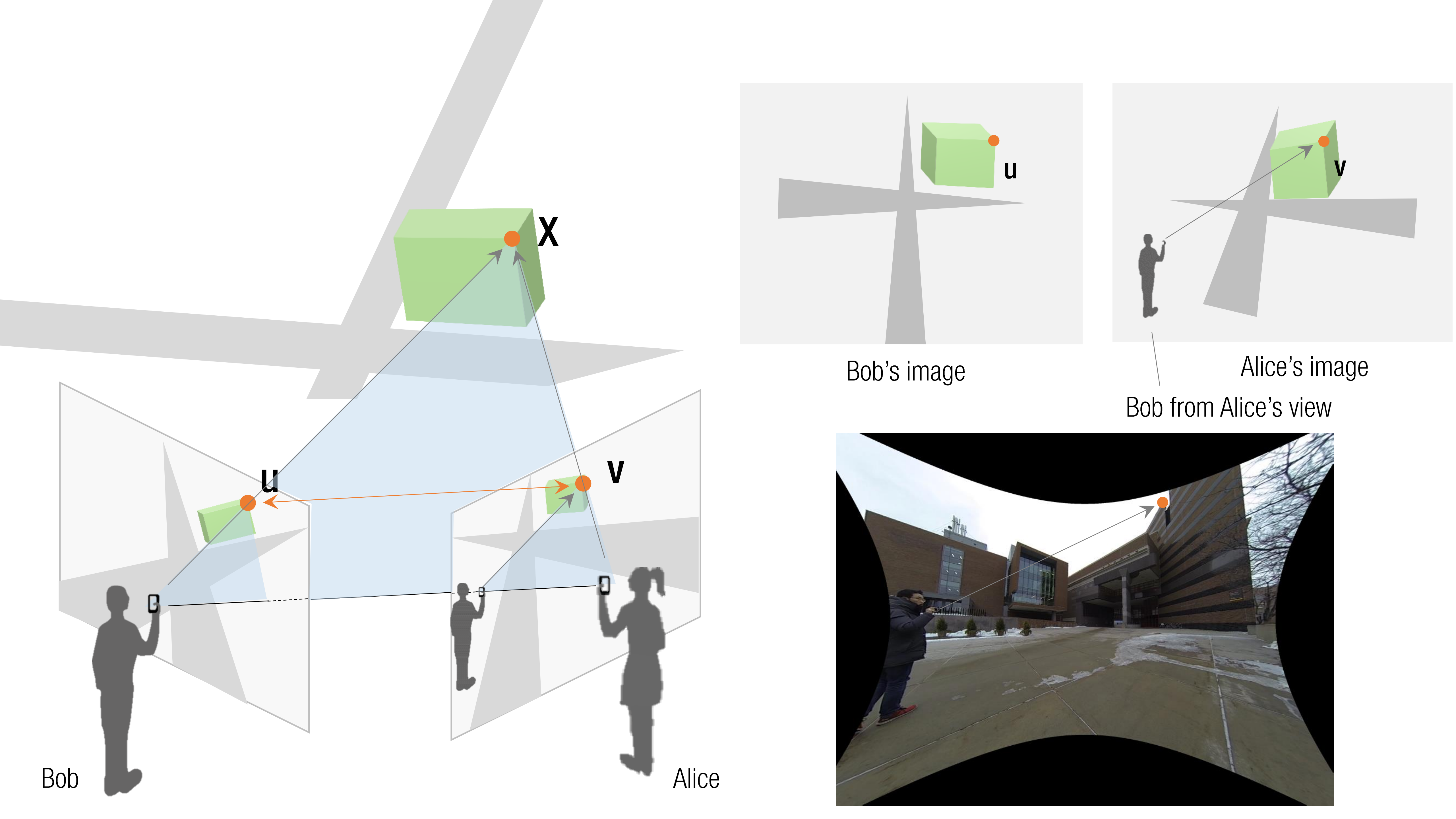
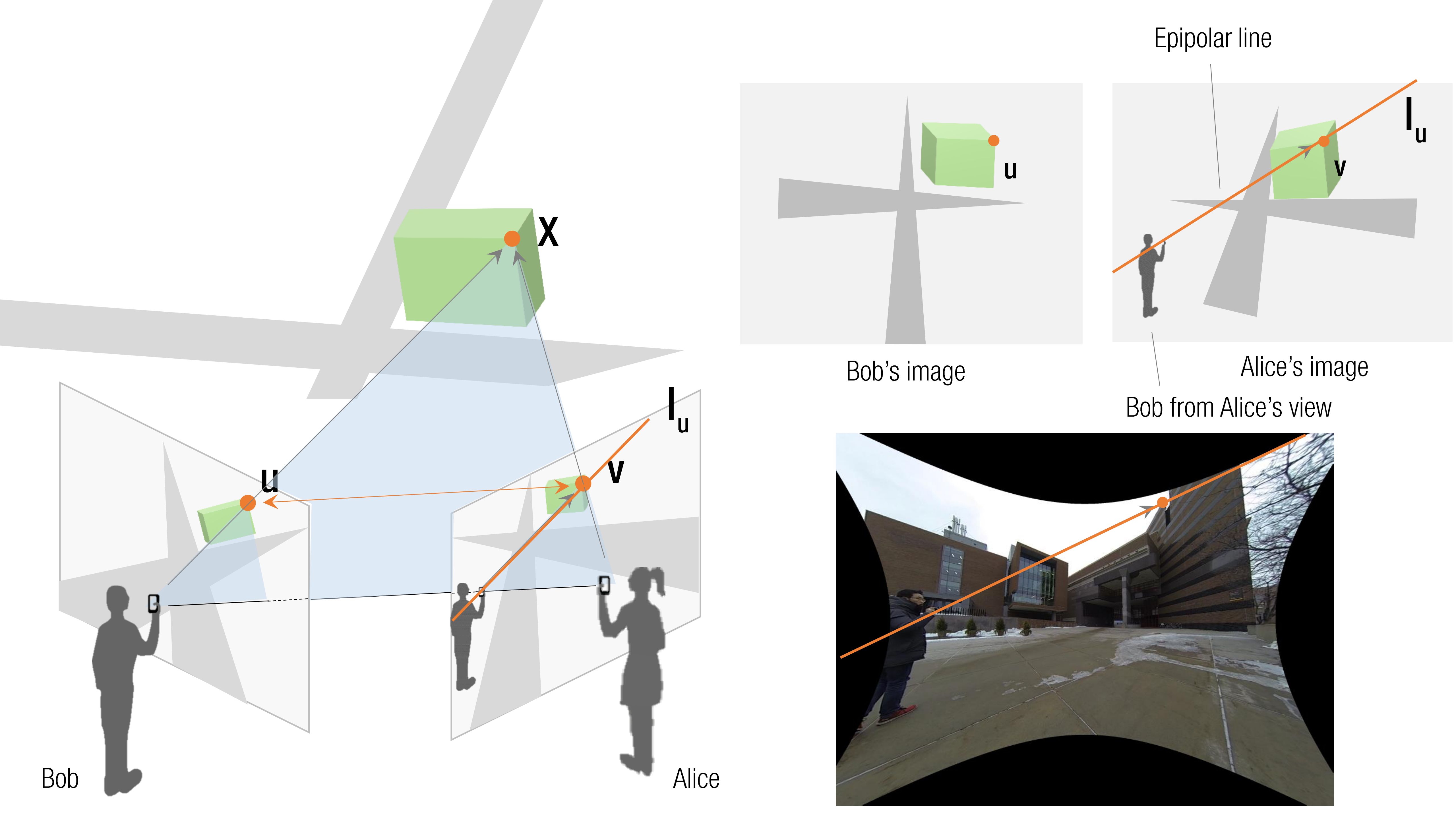
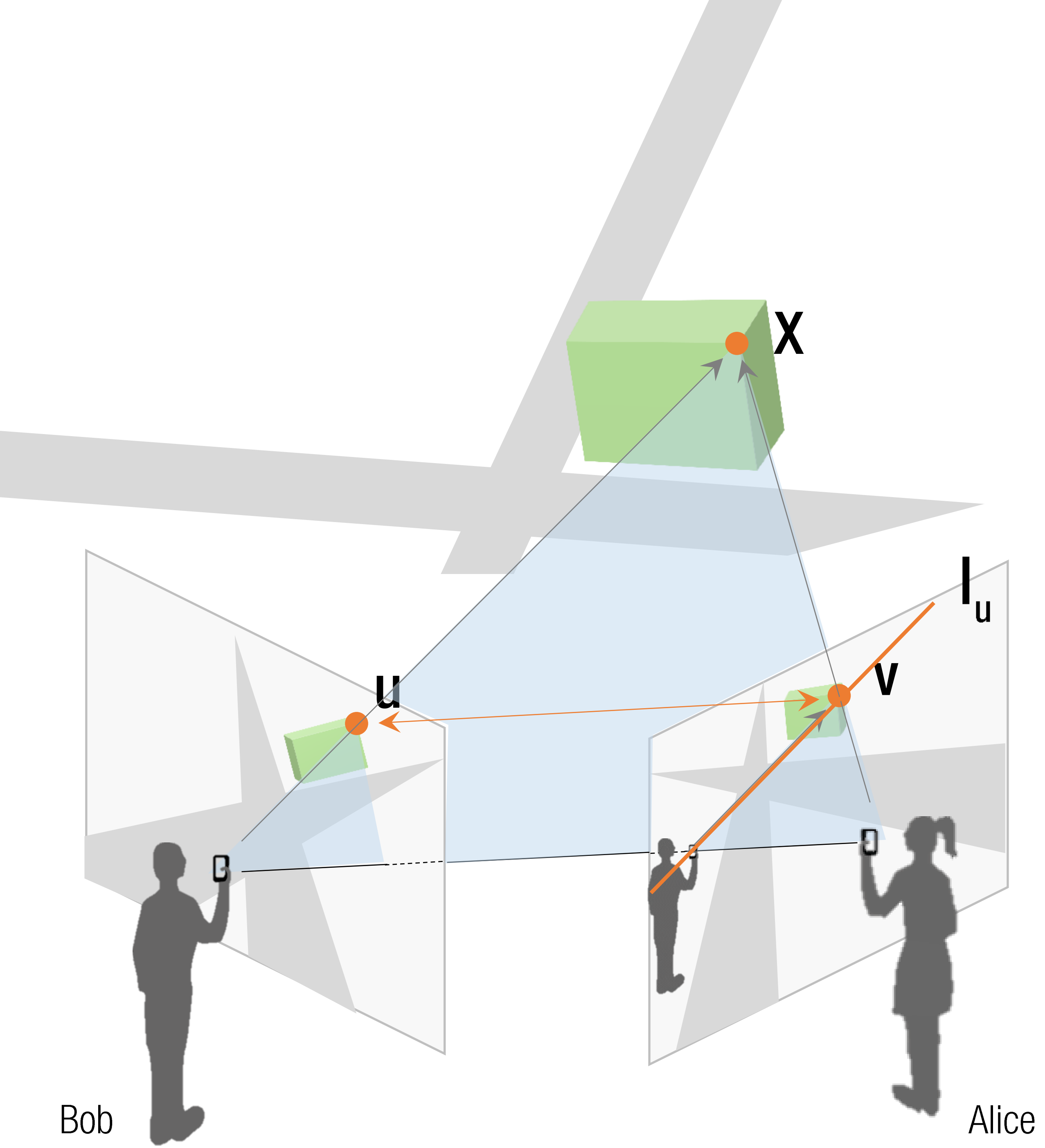
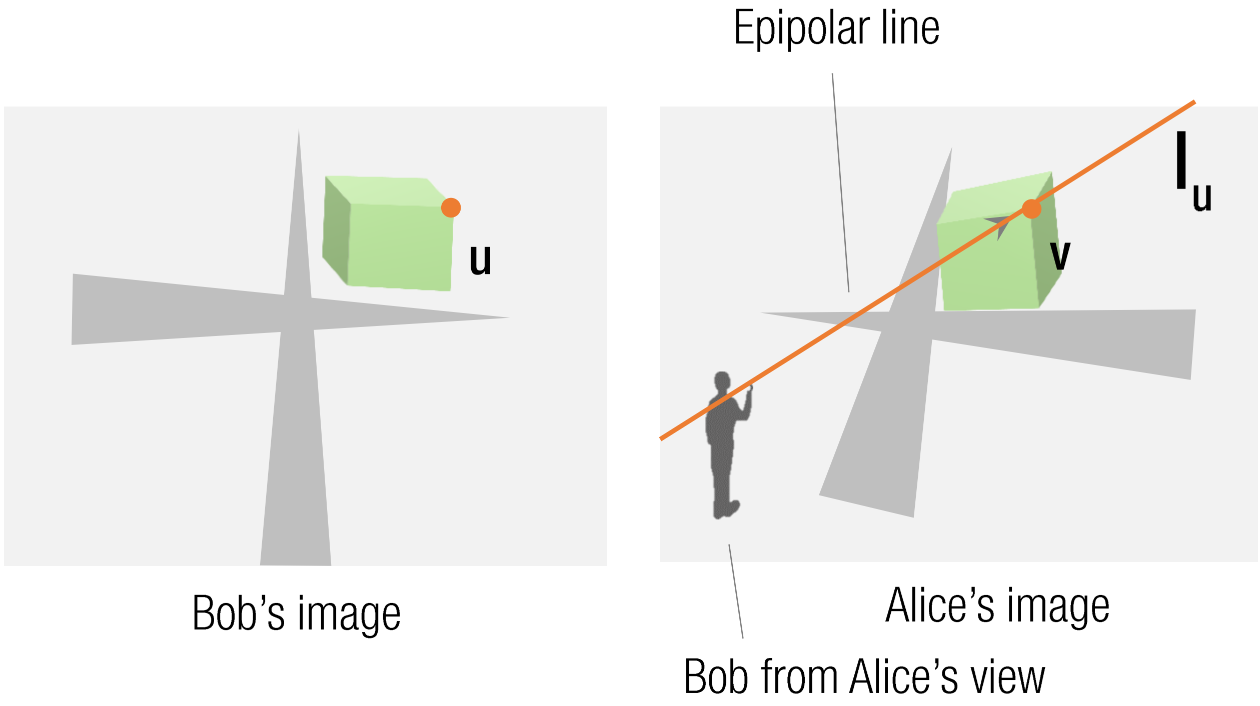
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view


X
u
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view

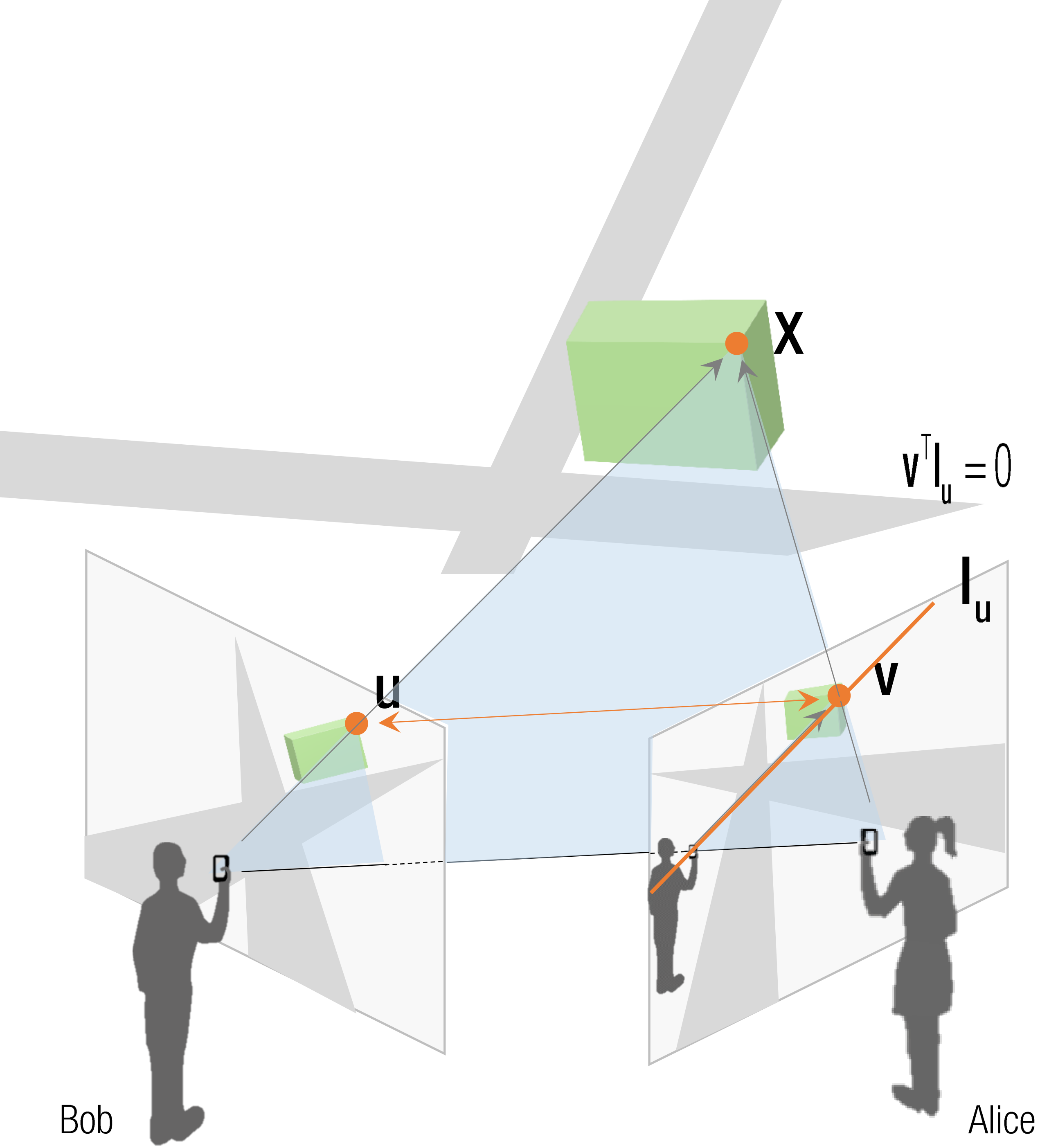

X
u
v
l_u
Bob
Alice
Epipolar line
u
v
l_u
Bob's image
Alice's image
Bob from Alice's view

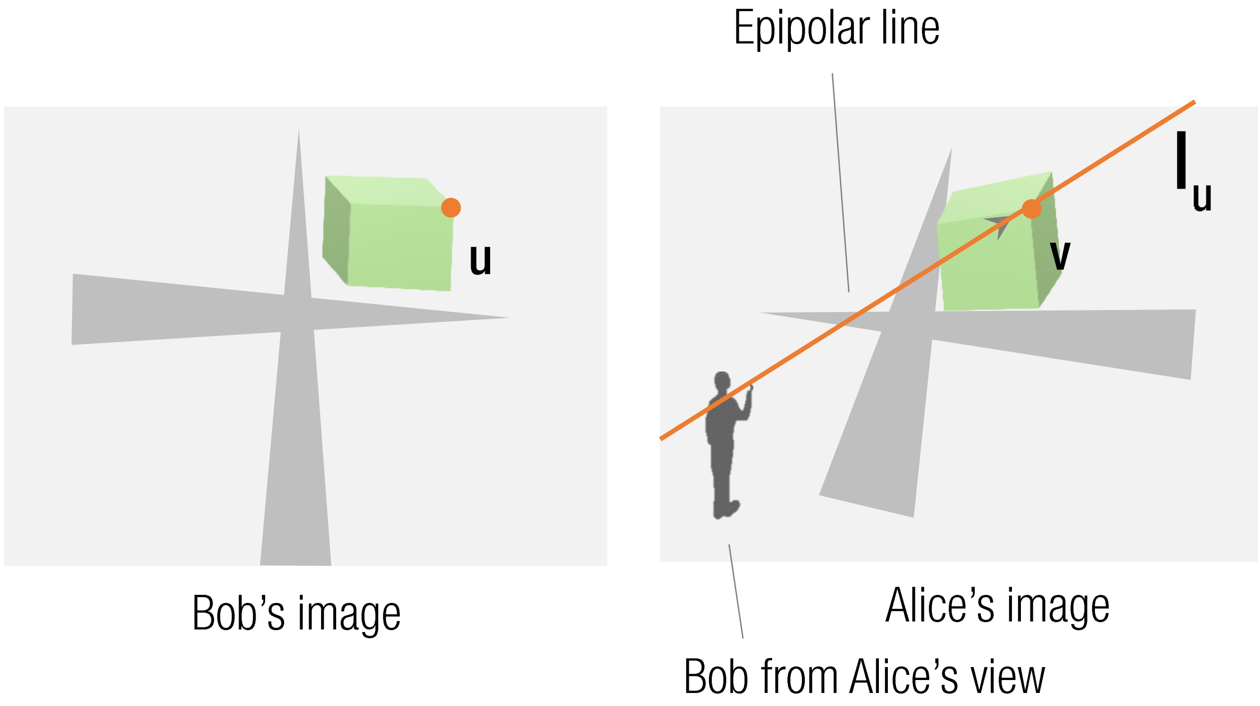




Epipolar constraint between two images:

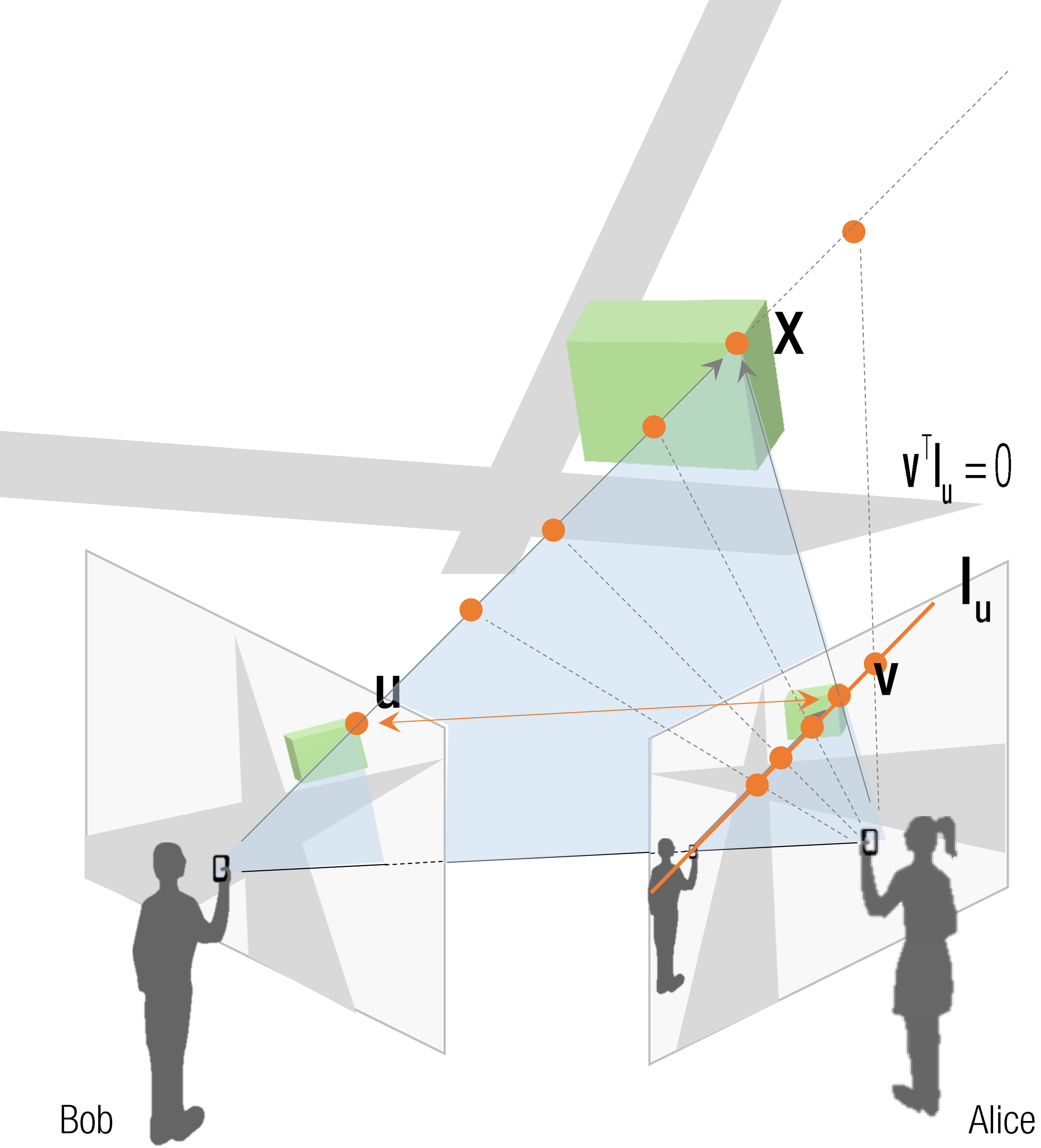
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

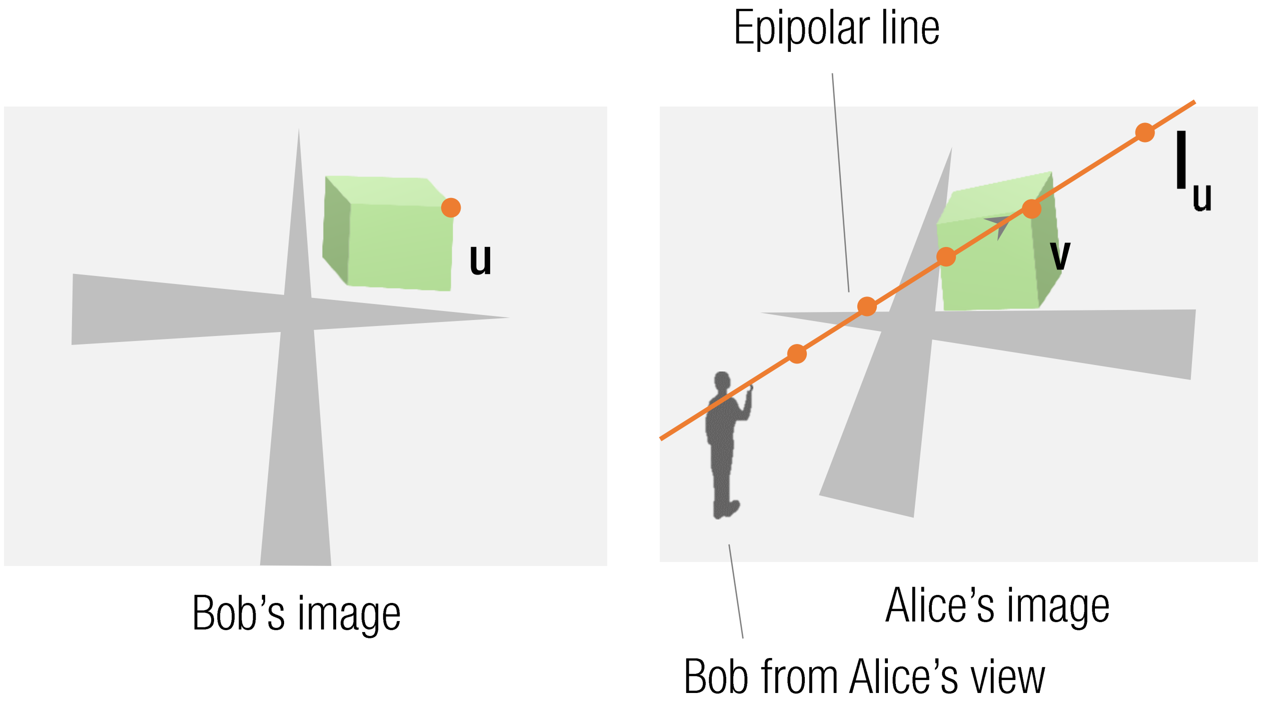




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$



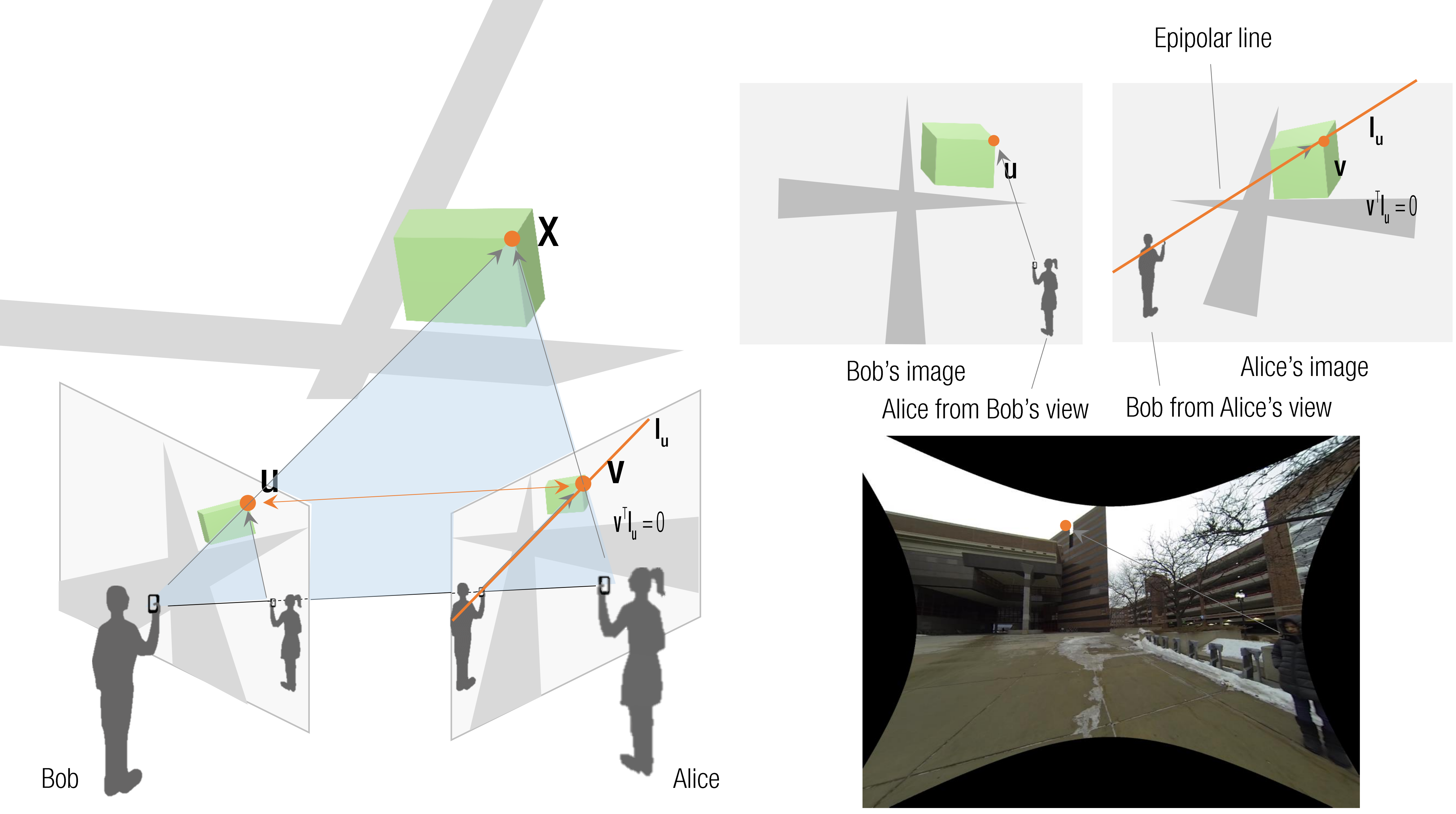


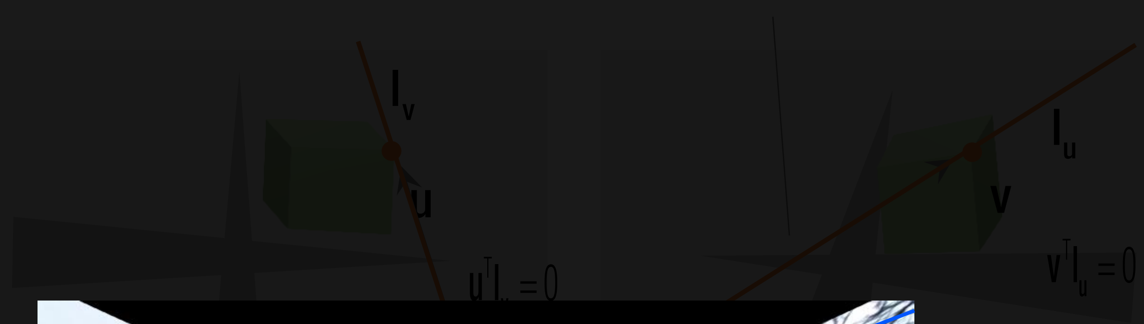
Epipolar constraint between two images:

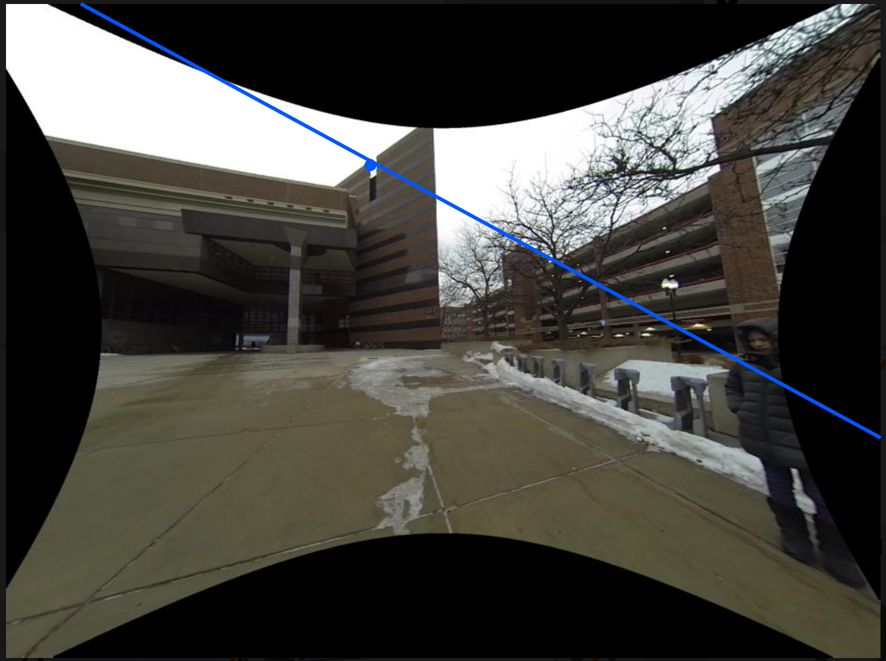
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

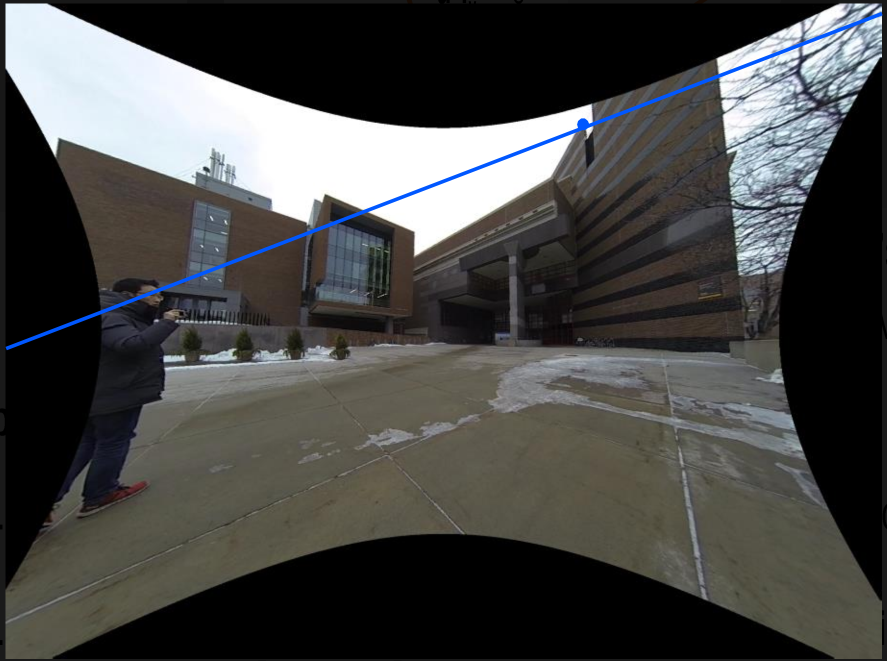
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$

3. Any point along the epipolar line can be a candidate of correspondences.


X
u
v
l_u
v^T l_u = 0
Bob
Alice
Epipolar line
Bob's image
Alice's image
Alice from Bob's view
Bob from Alice's view







s image

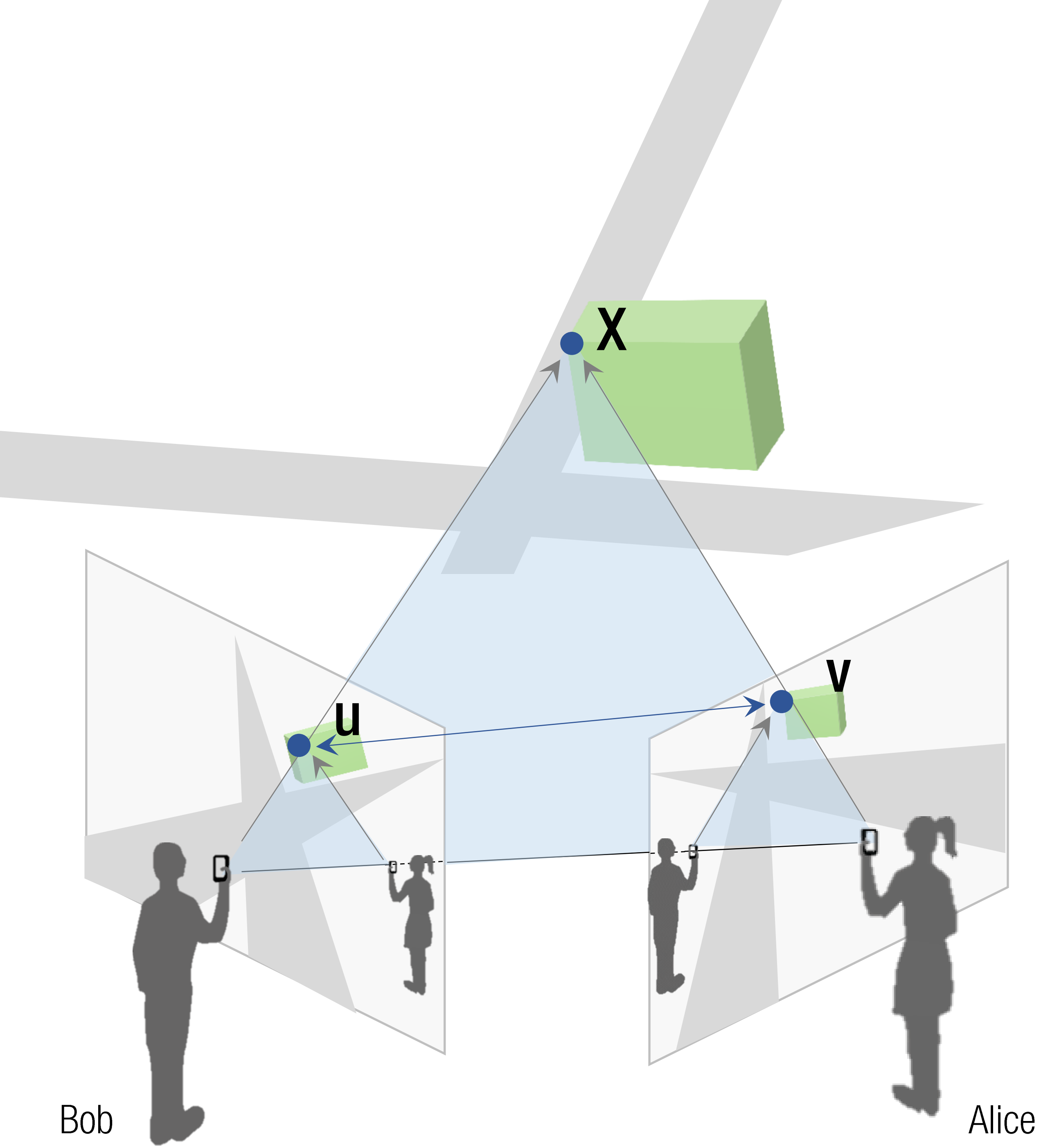
view

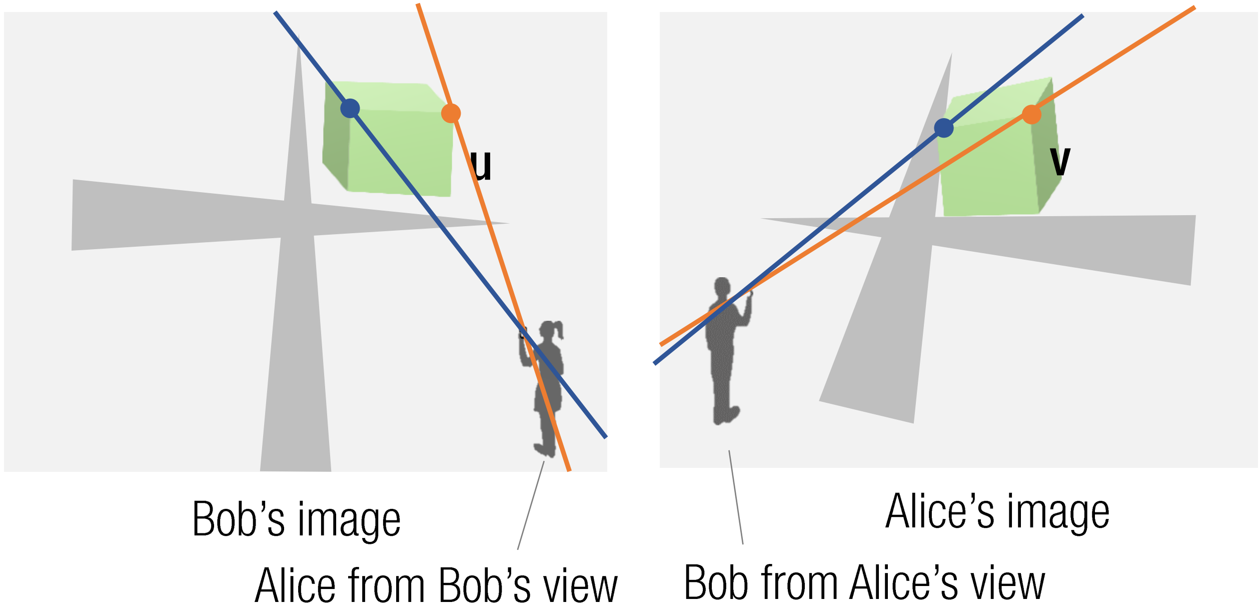
Ep

1. olar line in
2. in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$ $\mathbf{u}^T\mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

Bob

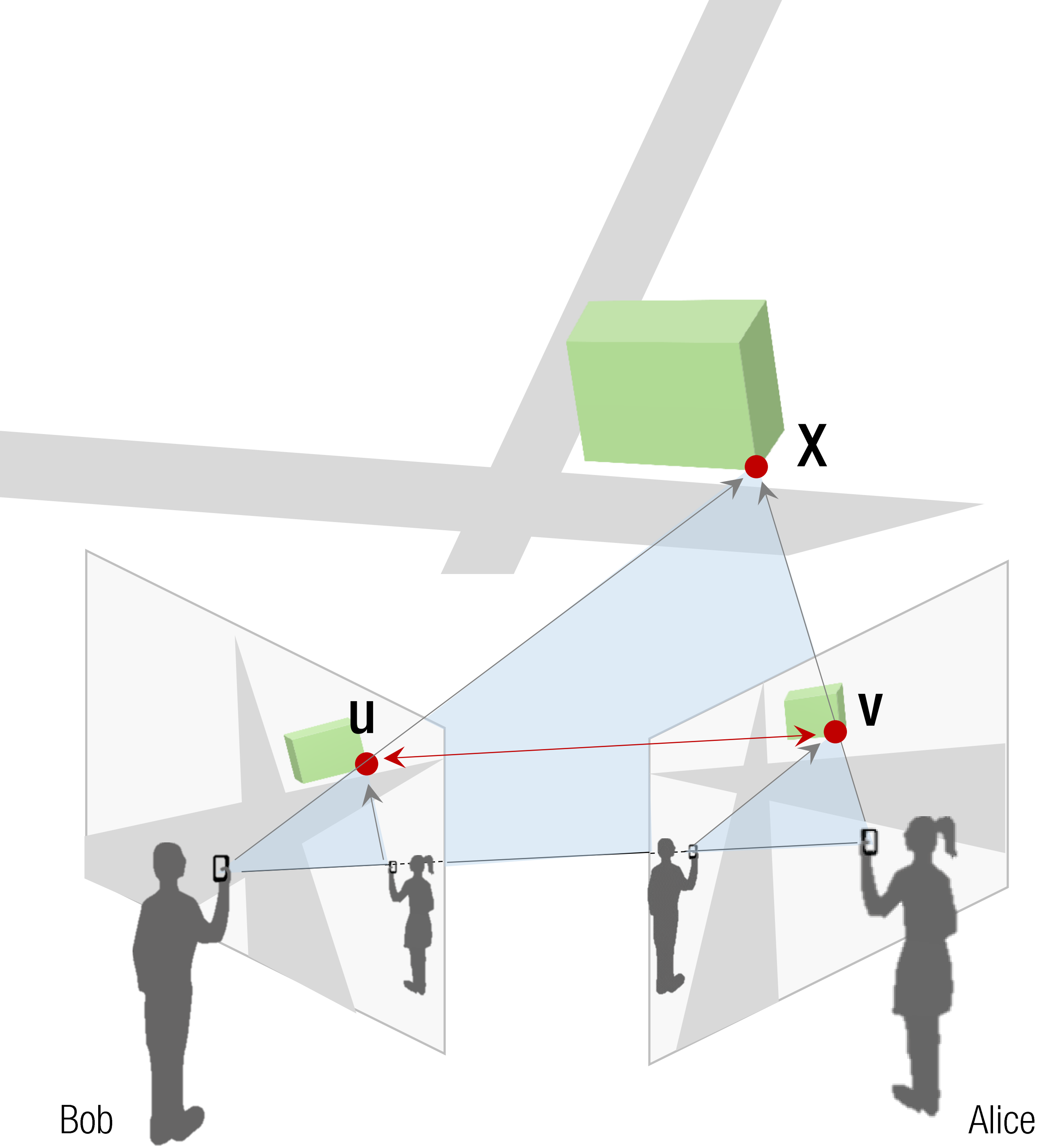
Alice

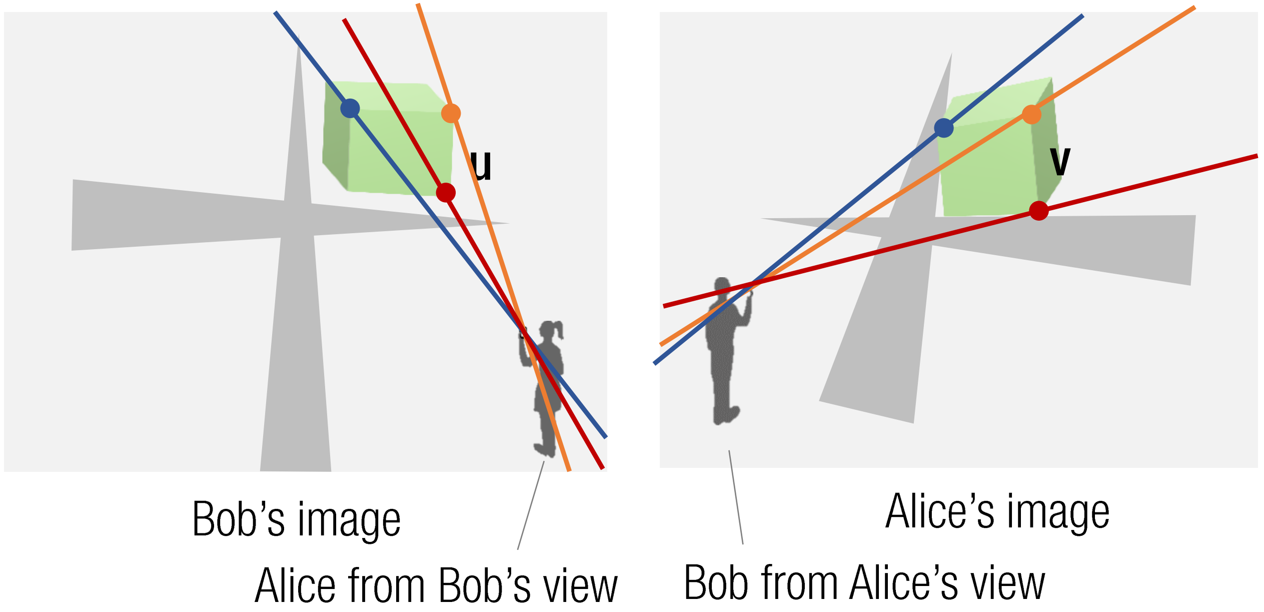




Epipolar constraint between two images:

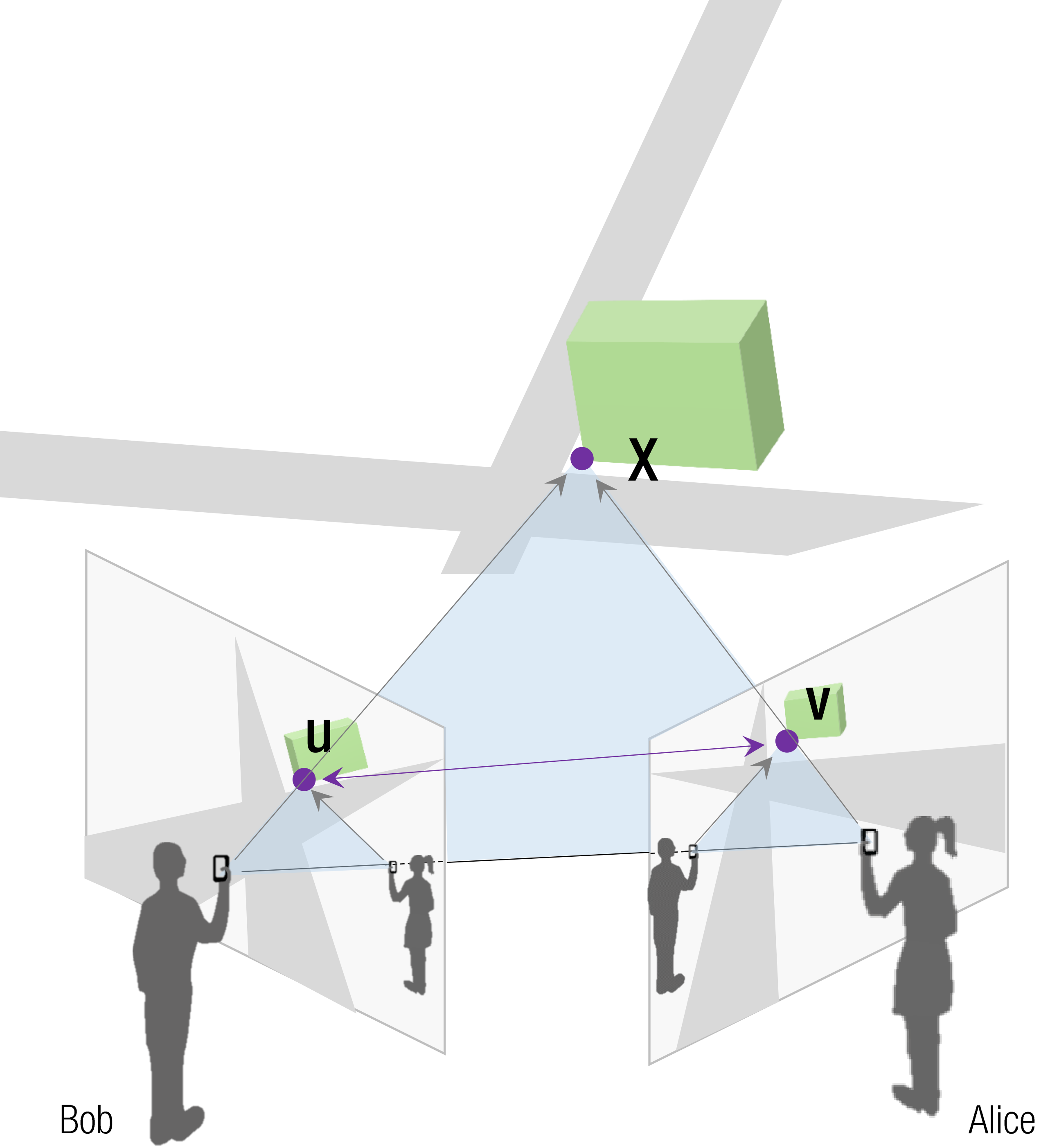
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T \mathbf{l}_u = 0 \quad \mathbf{u}^T \mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

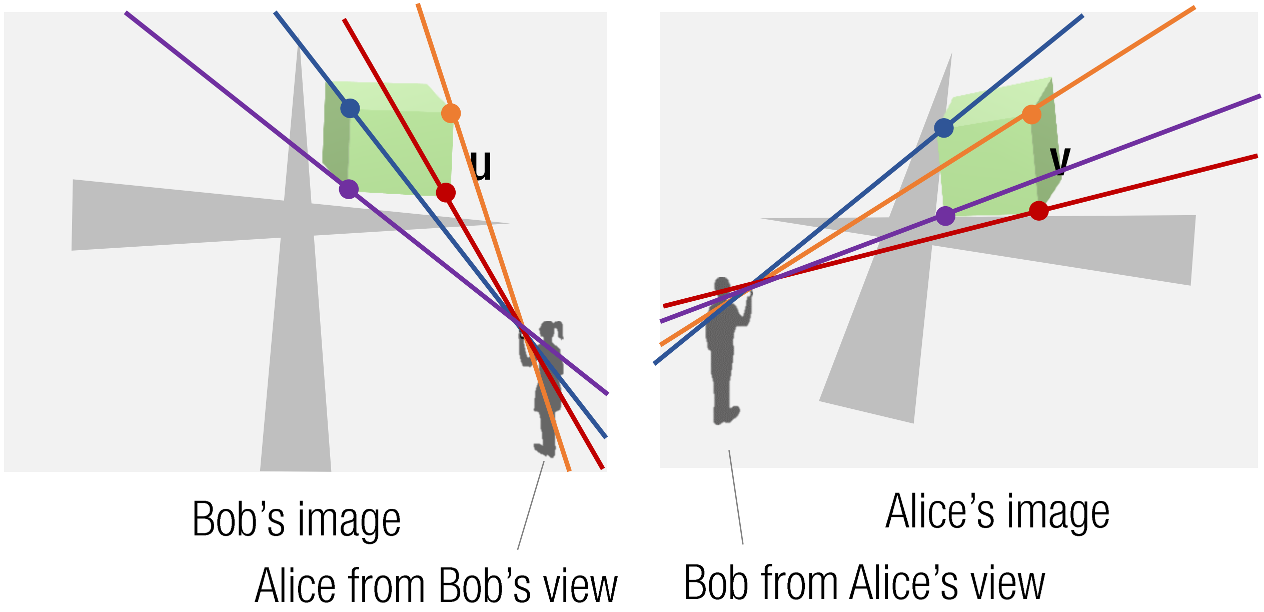




Epipolar constraint between two images:

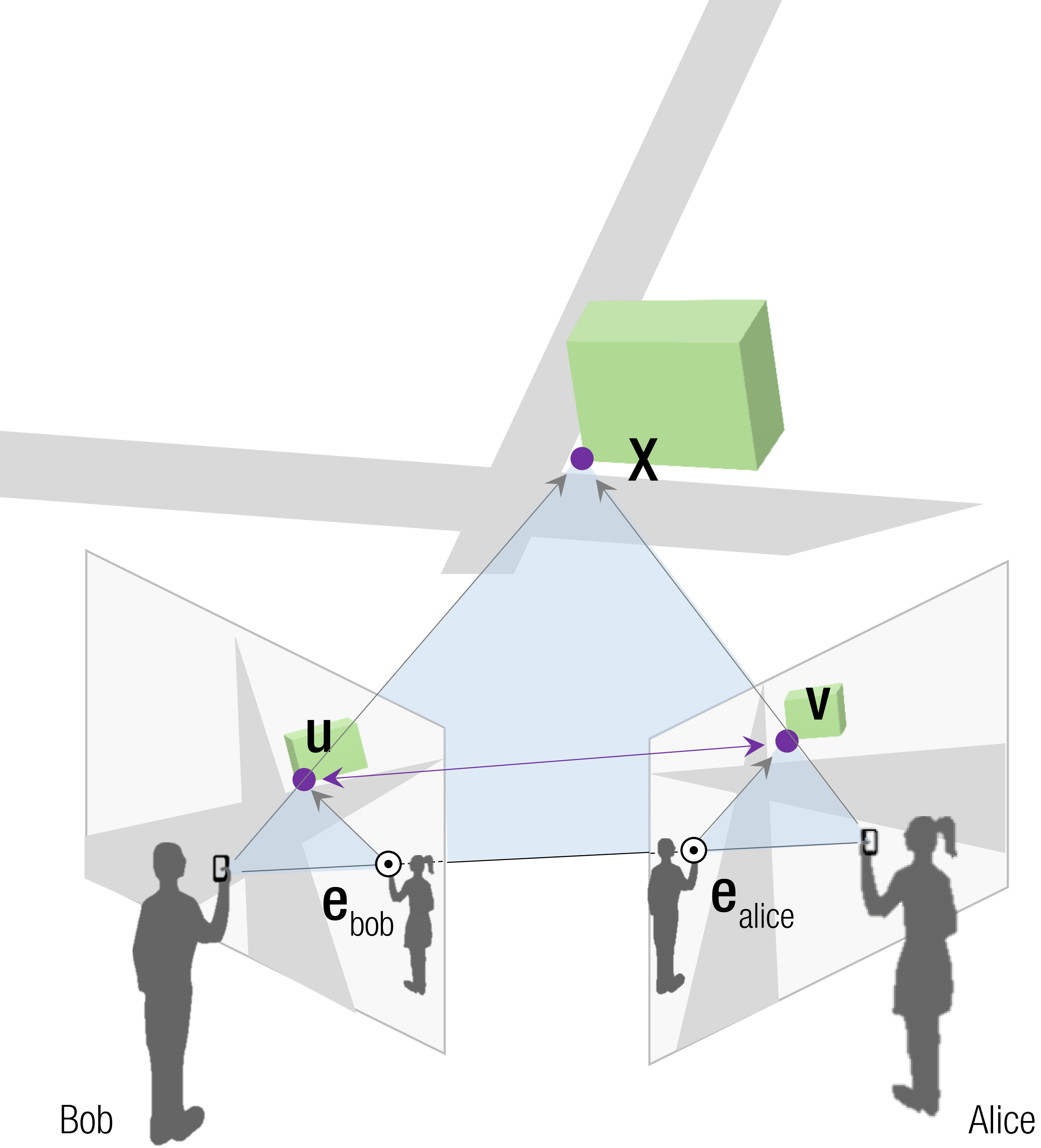
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

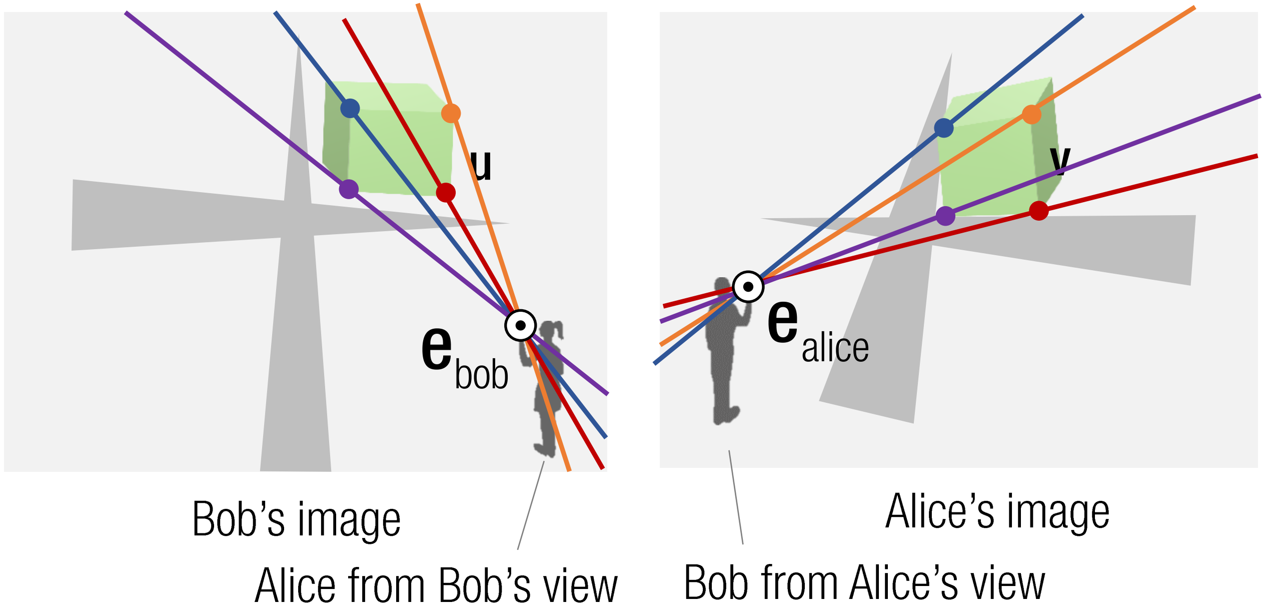




Epipolar constraint between two images:

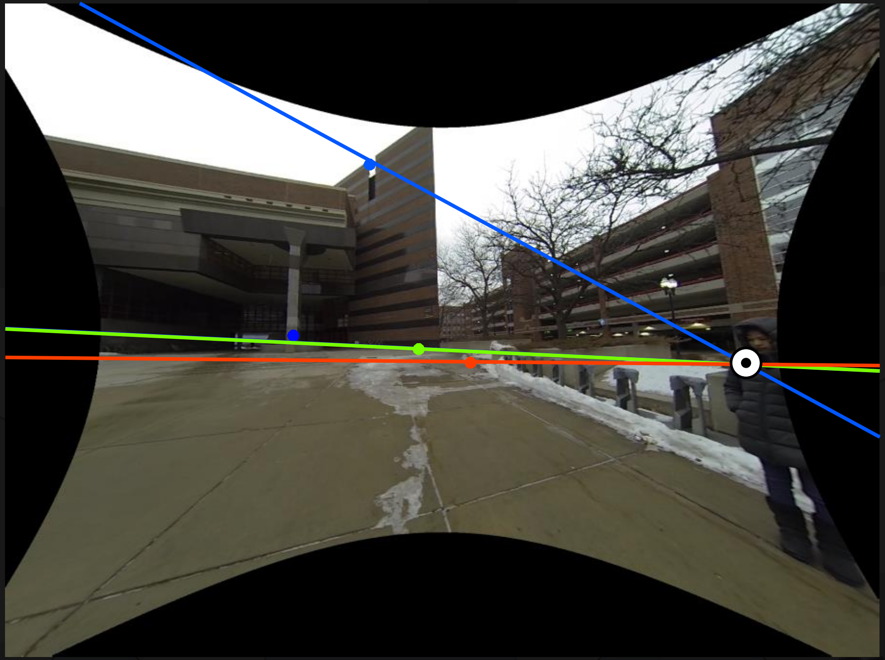
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^{\mathrm{T}}\mathbf{l}_{\mathbf{u}} = 0 \quad \mathbf{u}^{\mathrm{T}}\mathbf{l}_{\mathbf{v}} = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole.

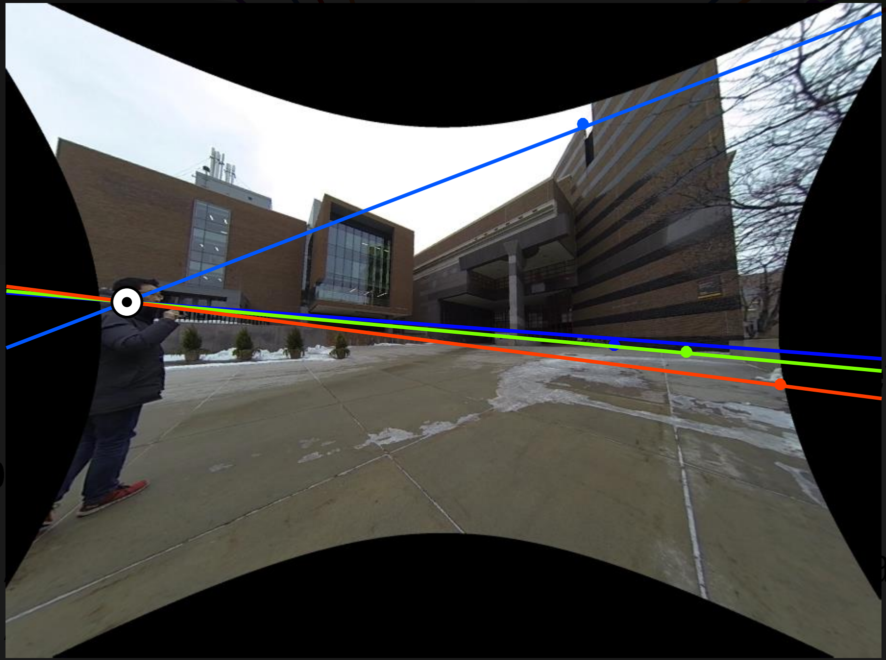




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $e_{bob}^T l_u = 0$ $e_{alice}^T l_v = 0$



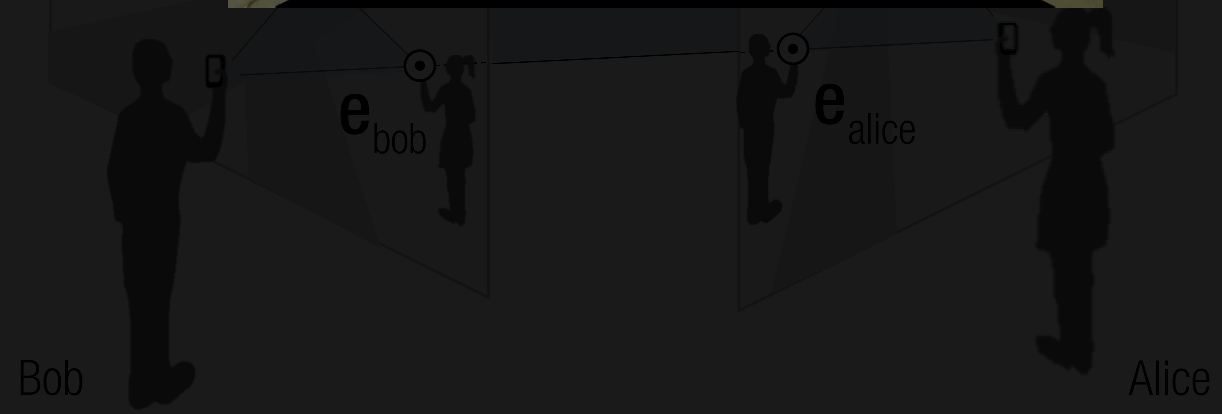


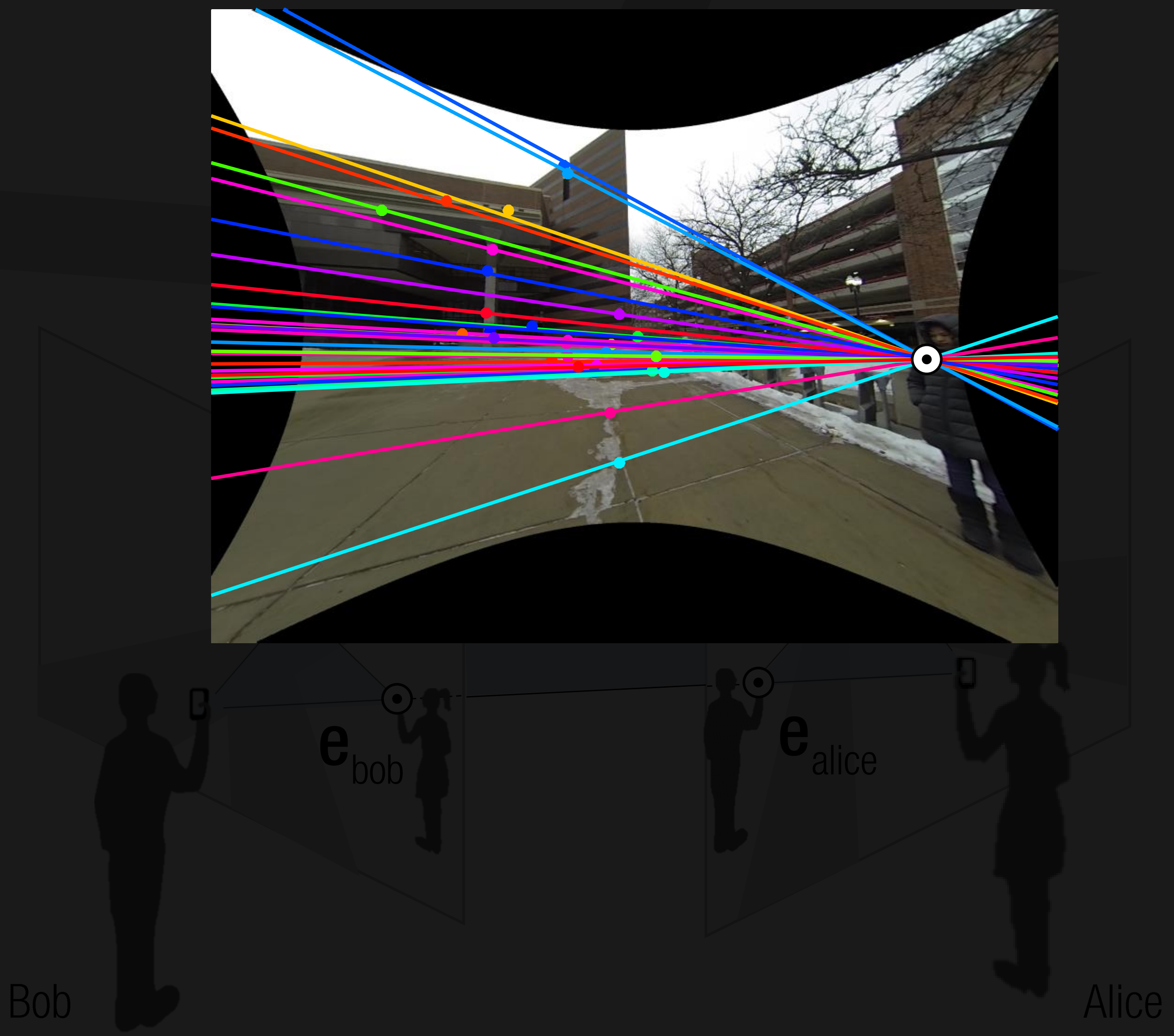
image

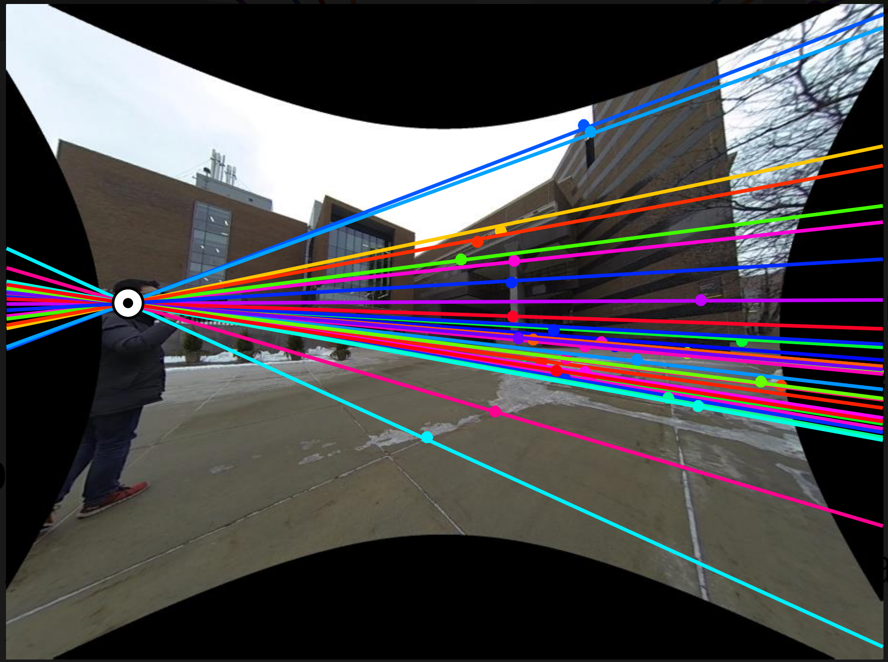
w

Epip

1. ar line in
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0 \quad \mathbf{u}^T\mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_u = 0 \quad \mathbf{e}_{alice}^T\mathbf{l}_v = 0$







mage

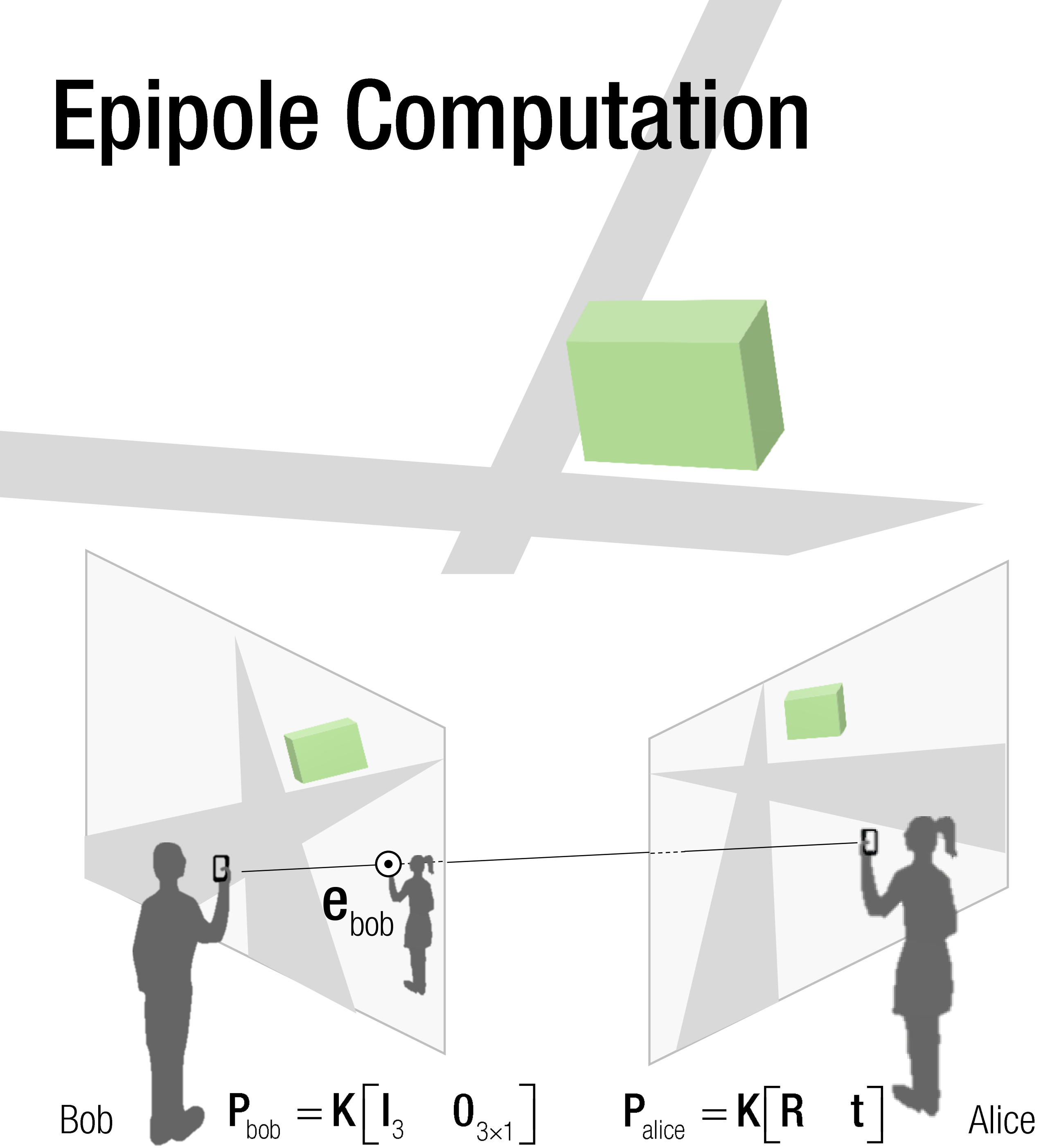
w

Epip

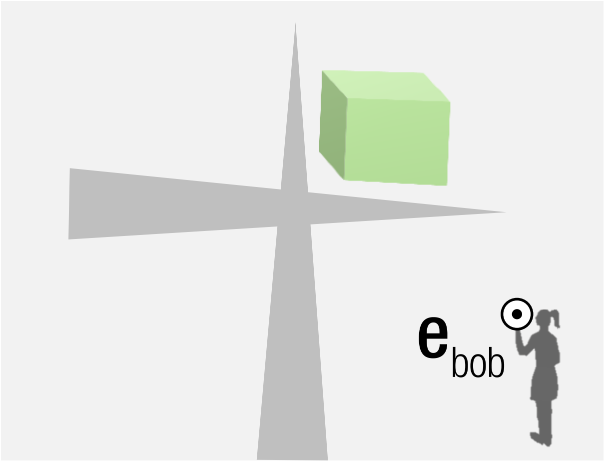
1. ar line in

2. The epipolar line passes the corresponding point in Alice's image, $\mathbf{v}$: $\mathbf{v}^T\mathbf{l}_u = 0 \quad \mathbf{u}^T\mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_u = 0 \quad \mathbf{e}_{alice}^T\mathbf{l}_v = 0$

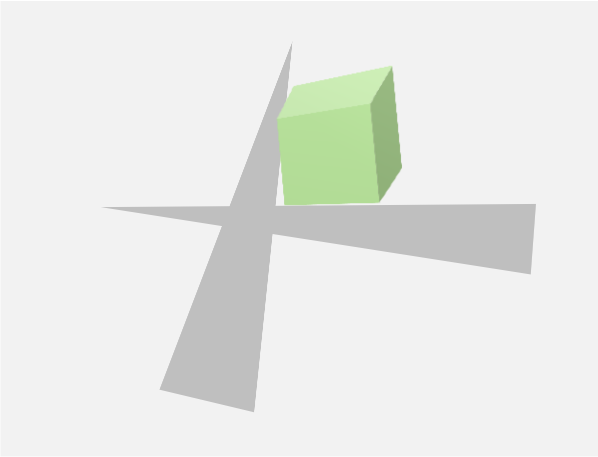
Epipole Computation



$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix} \qquad \mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$



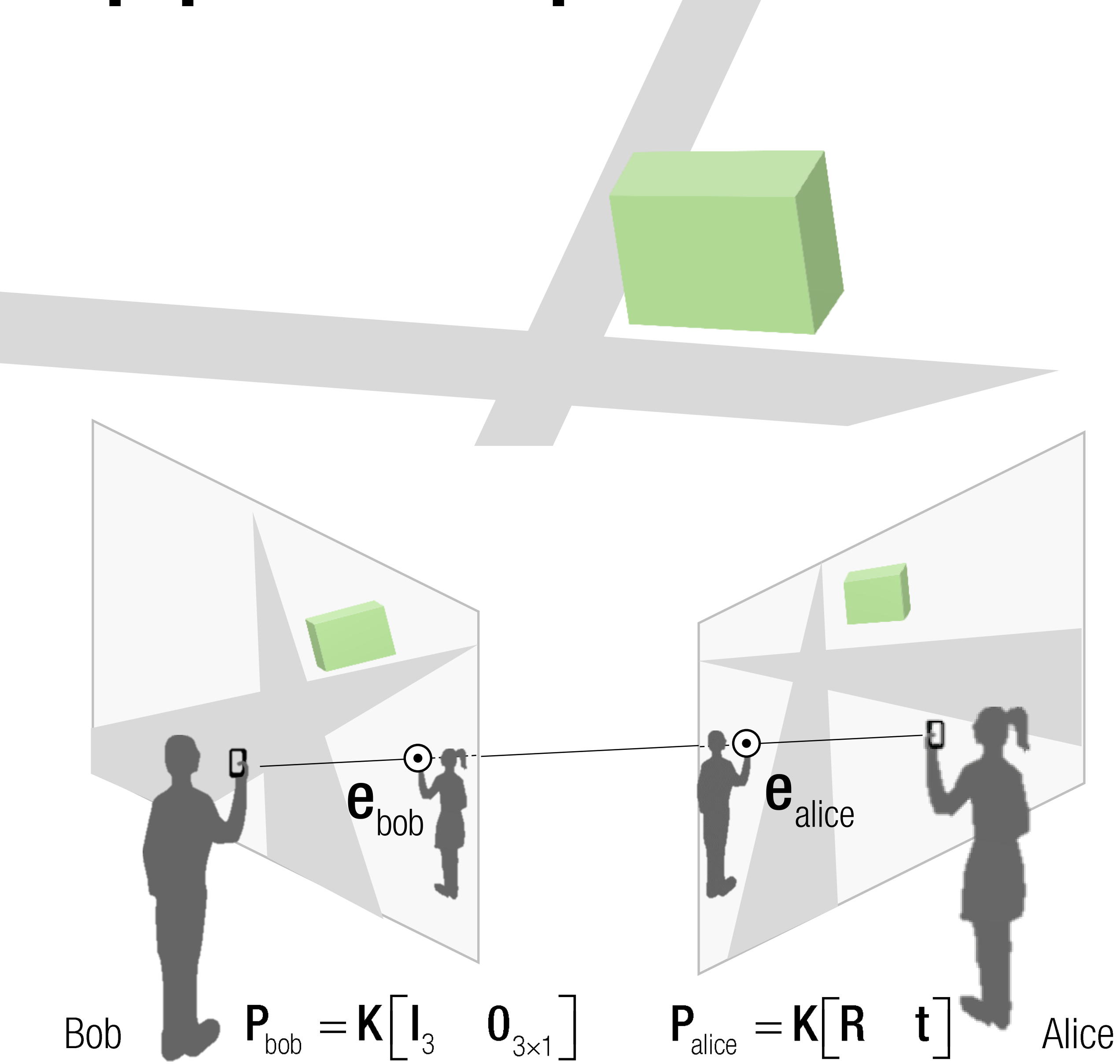
Bob's image

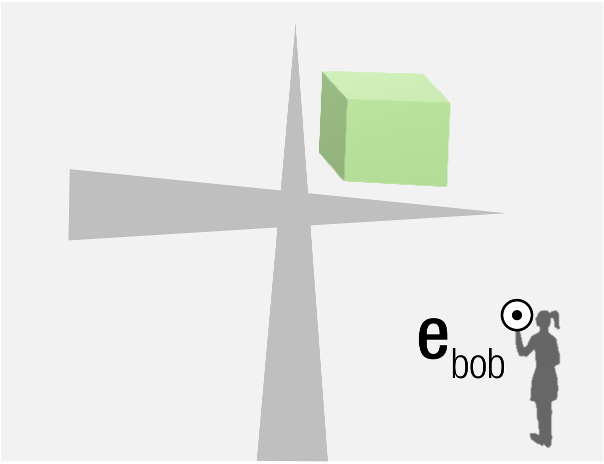


Alice's image

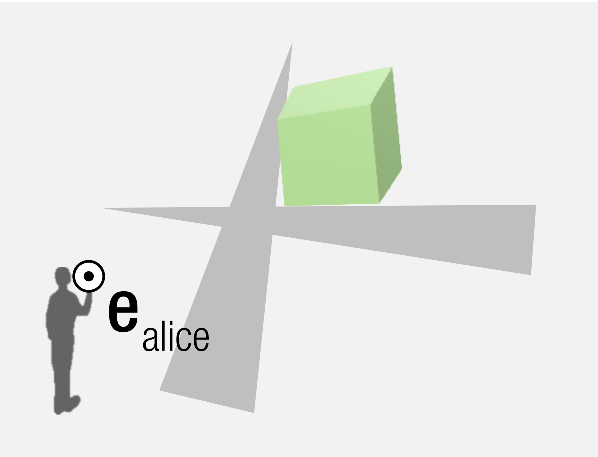
$$\lambda\mathbf{e}_{bob} = \underbrace{\mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}\end{bmatrix}}_{\text{Bob's camera projection matrix}}\begin{bmatrix}-\mathbf{R}^{\mathsf{T}}\mathbf{t}\\ 1\end{bmatrix} = -\mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

Epipole Computation





Bob's image



Alice's image

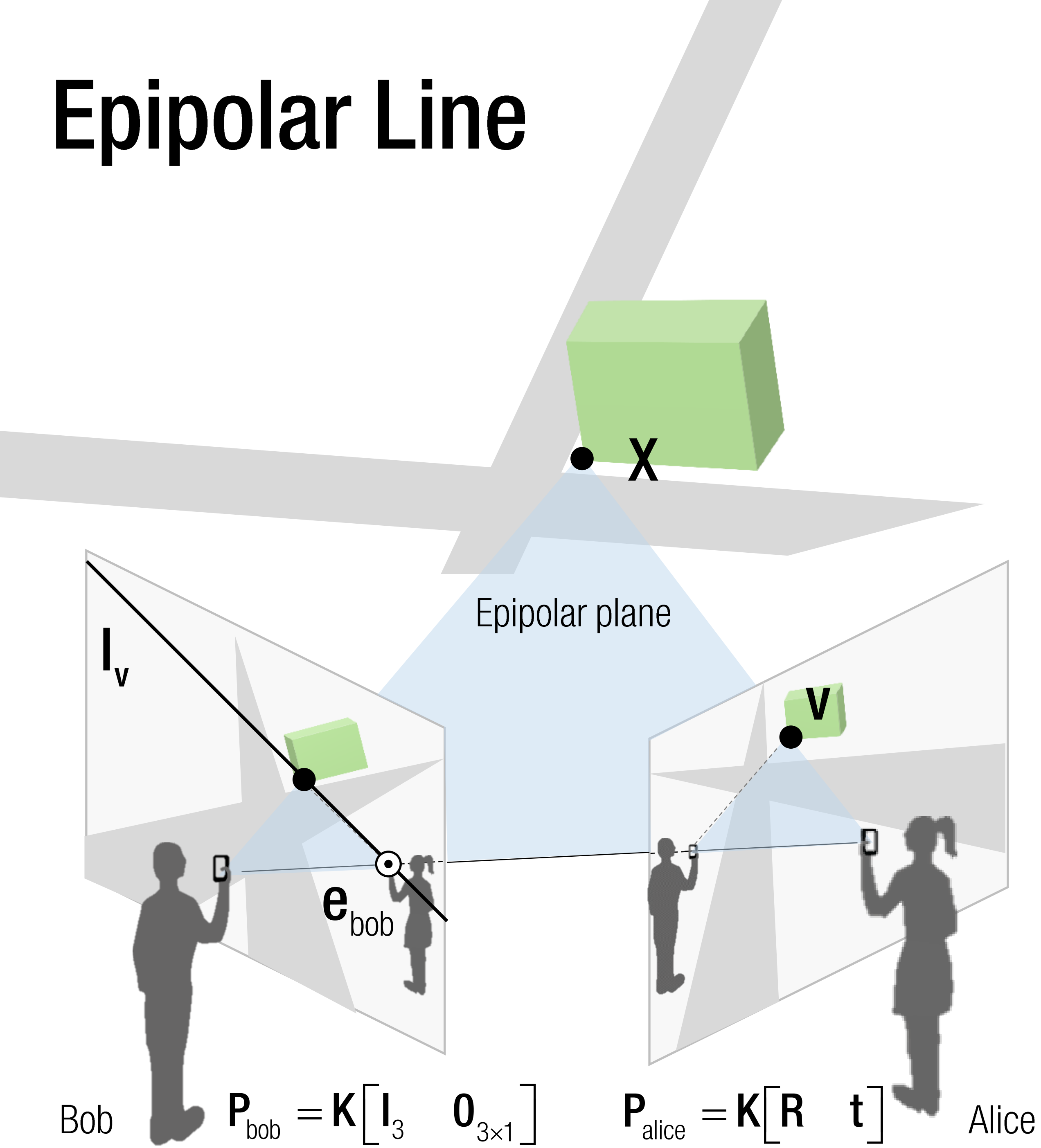
$$\lambda \mathbf{e}_{bob} = \underbrace{\mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}\end{bmatrix}}_{\text{Bob's camera projection matrix}} \begin{bmatrix} -\mathbf{R}^{\mathsf{T}}\mathbf{t} \\ 1 \end{bmatrix} = -\mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

$$\lambda \mathbf{e}_{alice} = \underbrace{\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}}_{\text{Alice's camera projection matrix}} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \mathbf{K}\mathbf{t}$$

Epipolar Line

$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\top}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} =$$



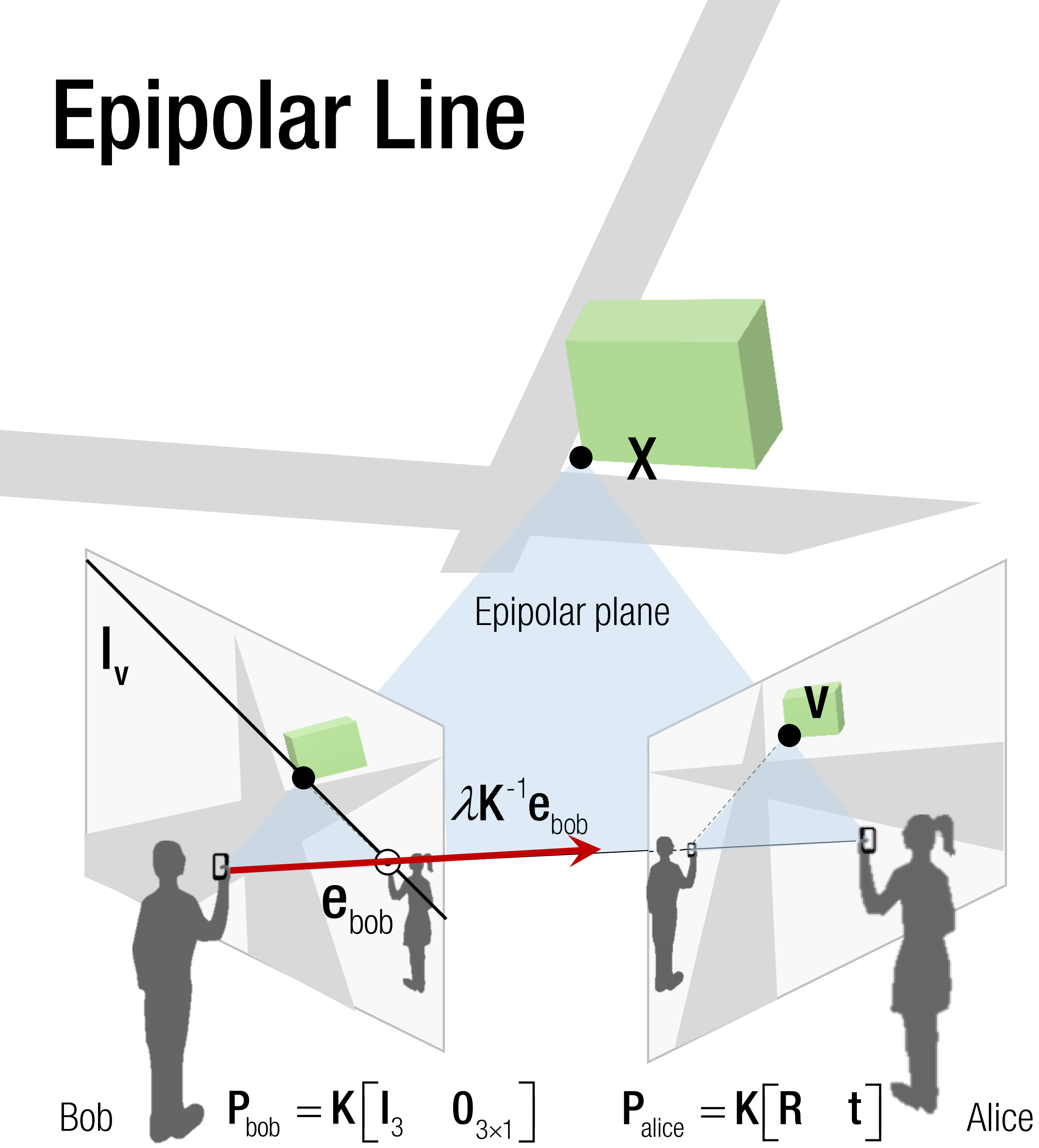
$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_{3\times 1} \end{bmatrix} \qquad \mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$

Epipolar Line

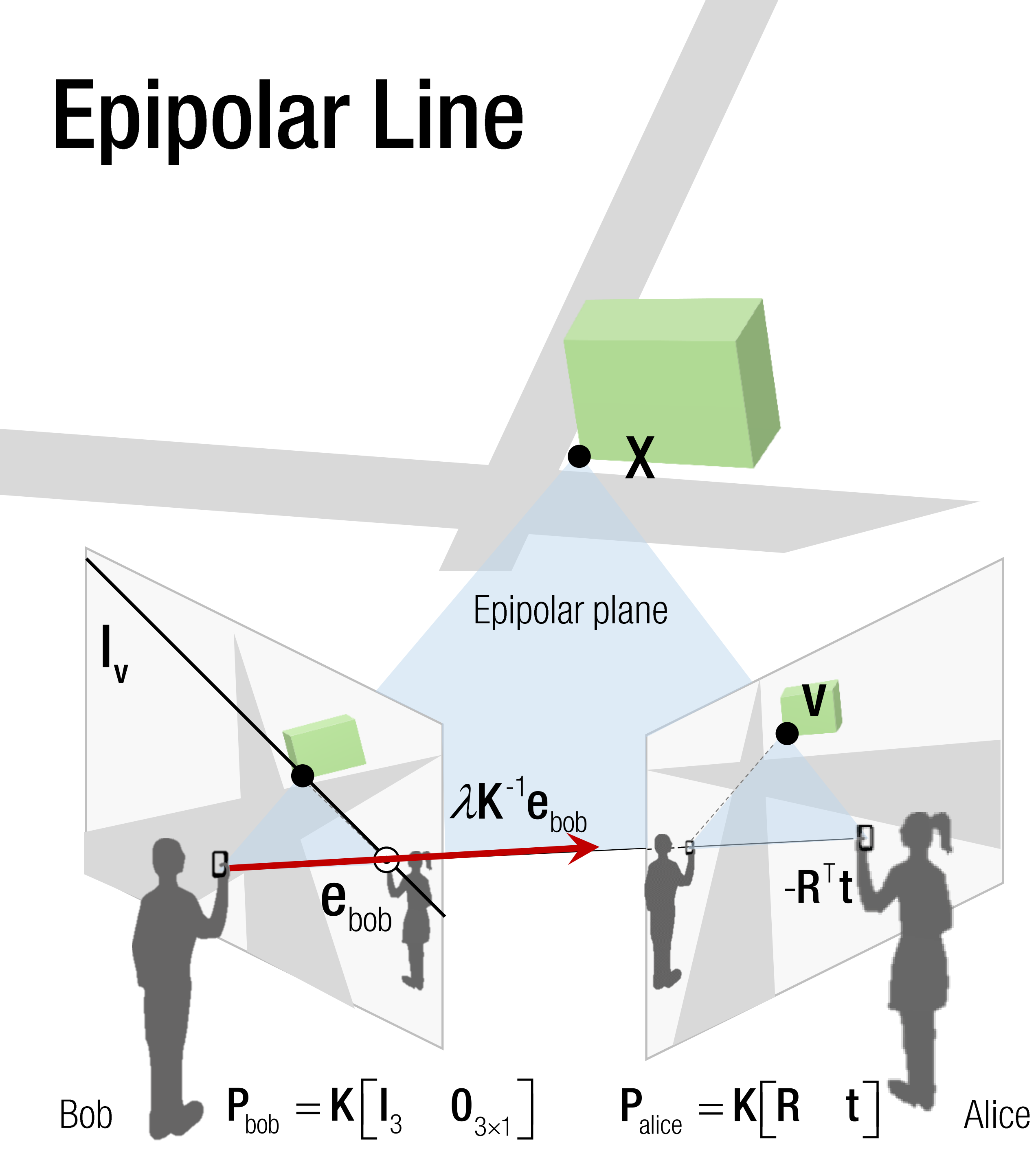
$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_{\mathbf{v}} =$$

$$\rightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^{\mathsf{T}}\mathbf{t}$$



Epipolar Line

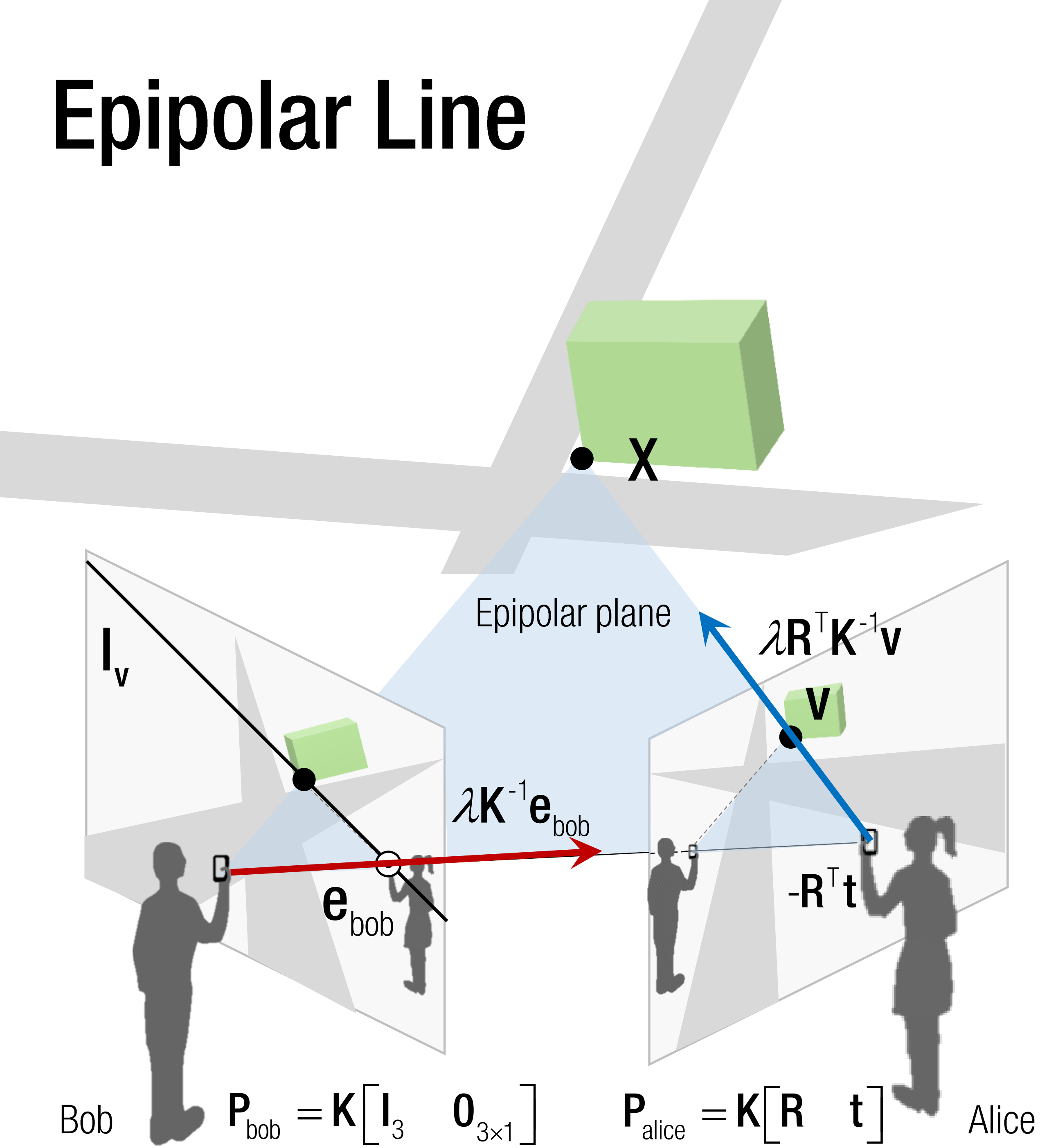


$$\lambda\mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^{\mathsf{T}}\mathbf{t} \qquad \lambda\mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_v =$$

$$\rightarrow \quad \lambda\mathbf{K}^{-1}\mathbf{e}_{bob} = \mathbf{R}^{\mathsf{T}}\mathbf{t}$$

Epipolar Line



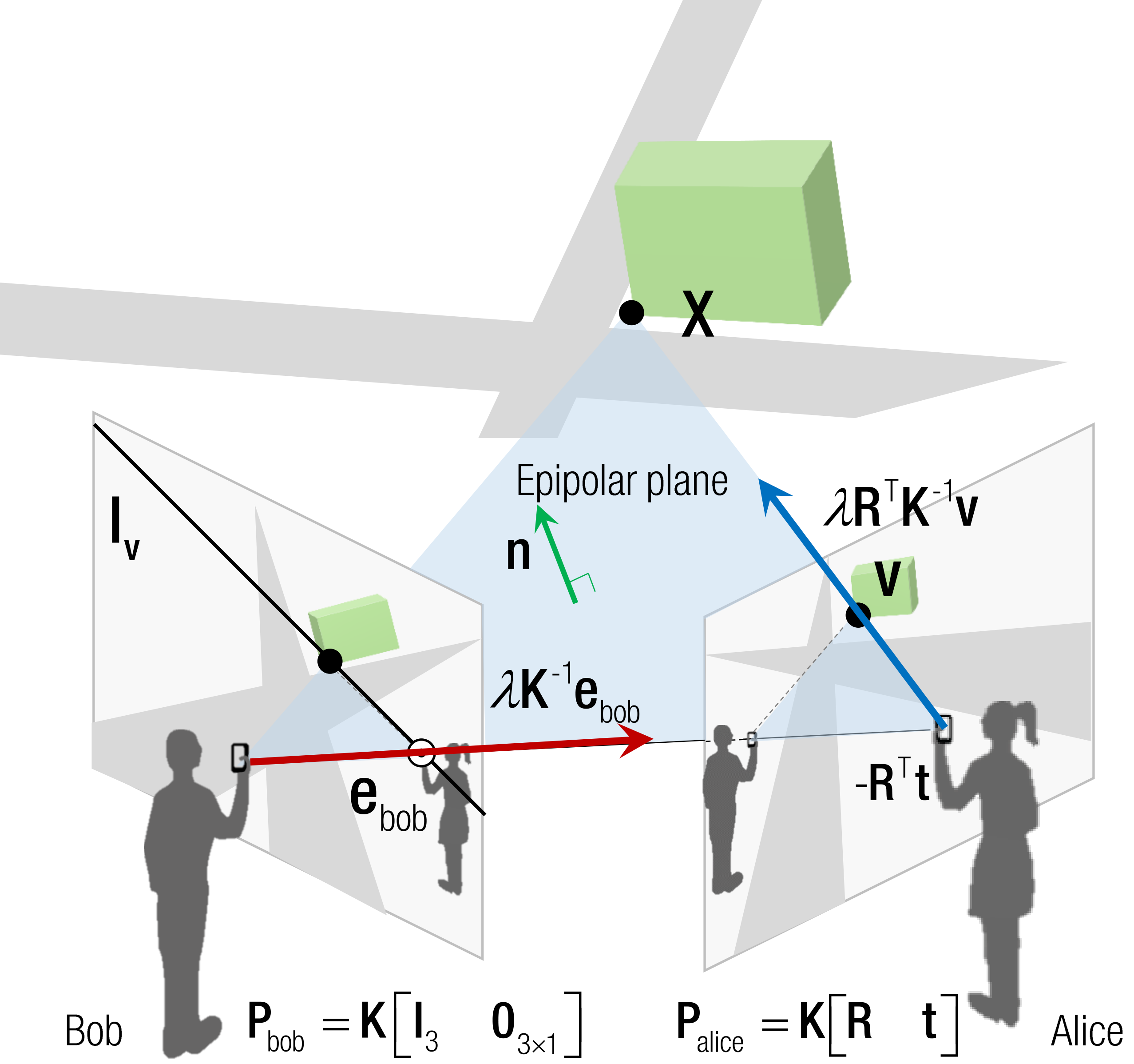
$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v =$$

$$\lambda K^{-1} e_{bob} = R^T t$$

$$\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$$

Epipolar Line



$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

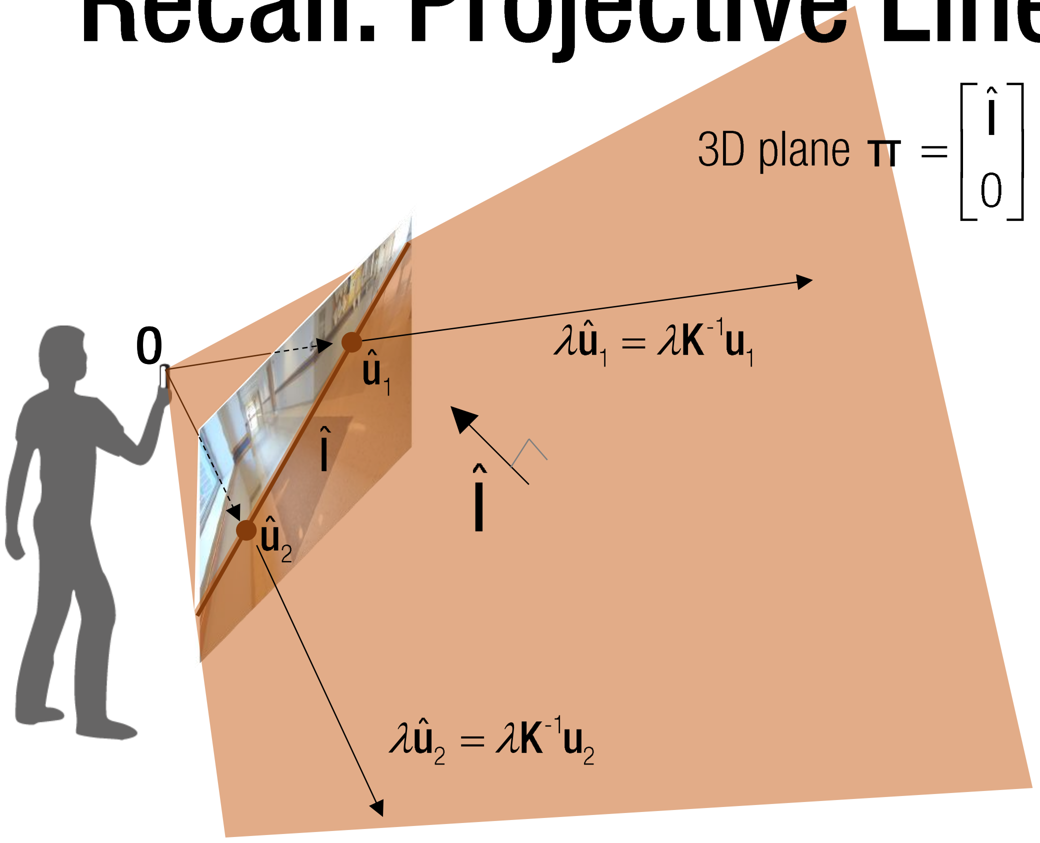
$$l_v =$$

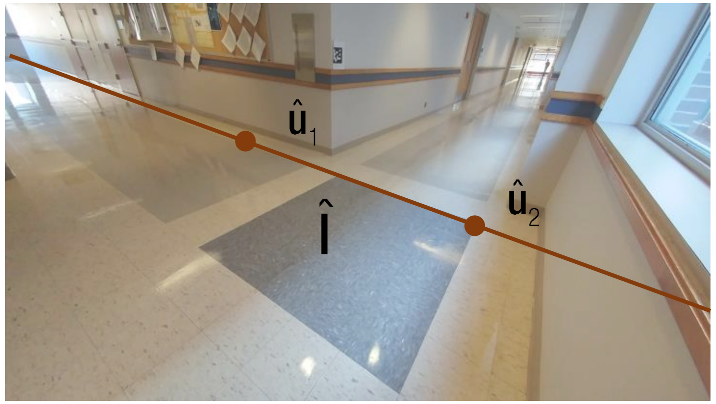
$$\lambda K^{-1} e_{bob} = R^T t$$

$$\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$$

$$\begin{aligned} n &= R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right) \\ &= R^T t \times R^T K^{-1} v \\ &= R^T \left(t \times K^{-1} v\right) \qquad \text{: only works for rotation} \\ &= R^T \left[t\right]_\times K^{-1} v \end{aligned}$$

Recall: Projective Line vs. Plane





Normalized coordinate:

$$\hat{\mathbf{u}}_1 = \mathbf{K}^{-1}\mathbf{u}_1 \qquad \hat{\mathbf{u}}_2 = \mathbf{K}^{-1}\mathbf{u}_2$$

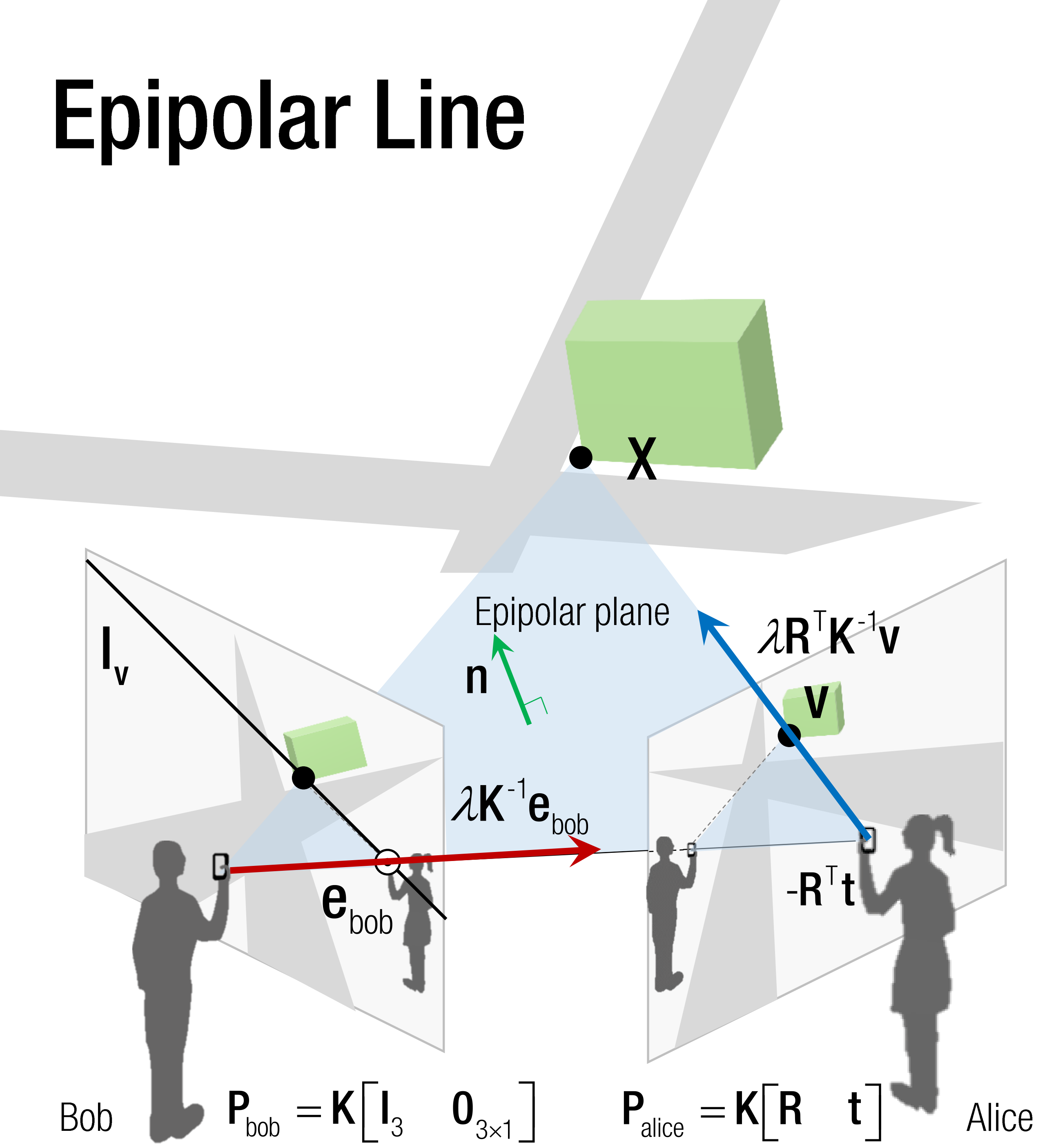
$$\longrightarrow \quad \hat{\mathbf{l}} = \hat{\mathbf{u}}_1 \times \hat{\mathbf{u}}_2$$

where $\hat{\mathbf{l}} = \left(\mathbf{K}^{-1}\right)^{-\mathsf{T}} \mathbf{l} = \mathbf{K}^{\mathsf{T}}\mathbf{l}$ due to duality

Plane normal: $\left(\lambda_1 \hat{\mathbf{u}}_1\right) \times \left(\lambda_2 \hat{\mathbf{u}}_2\right) = \lambda \hat{\mathbf{l}}$

$$\therefore \pi = \begin{bmatrix} \hat{\mathbf{l}} \\ 0 \end{bmatrix}$$

Epipolar Line



$\lambda e_{bob} = KR^T t$ $\quad\quad$ $\lambda e_{alice} = Kt$

$l_v =$

→ $\lambda K^{-1} e_{bob} = R^T t$

→ $\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$

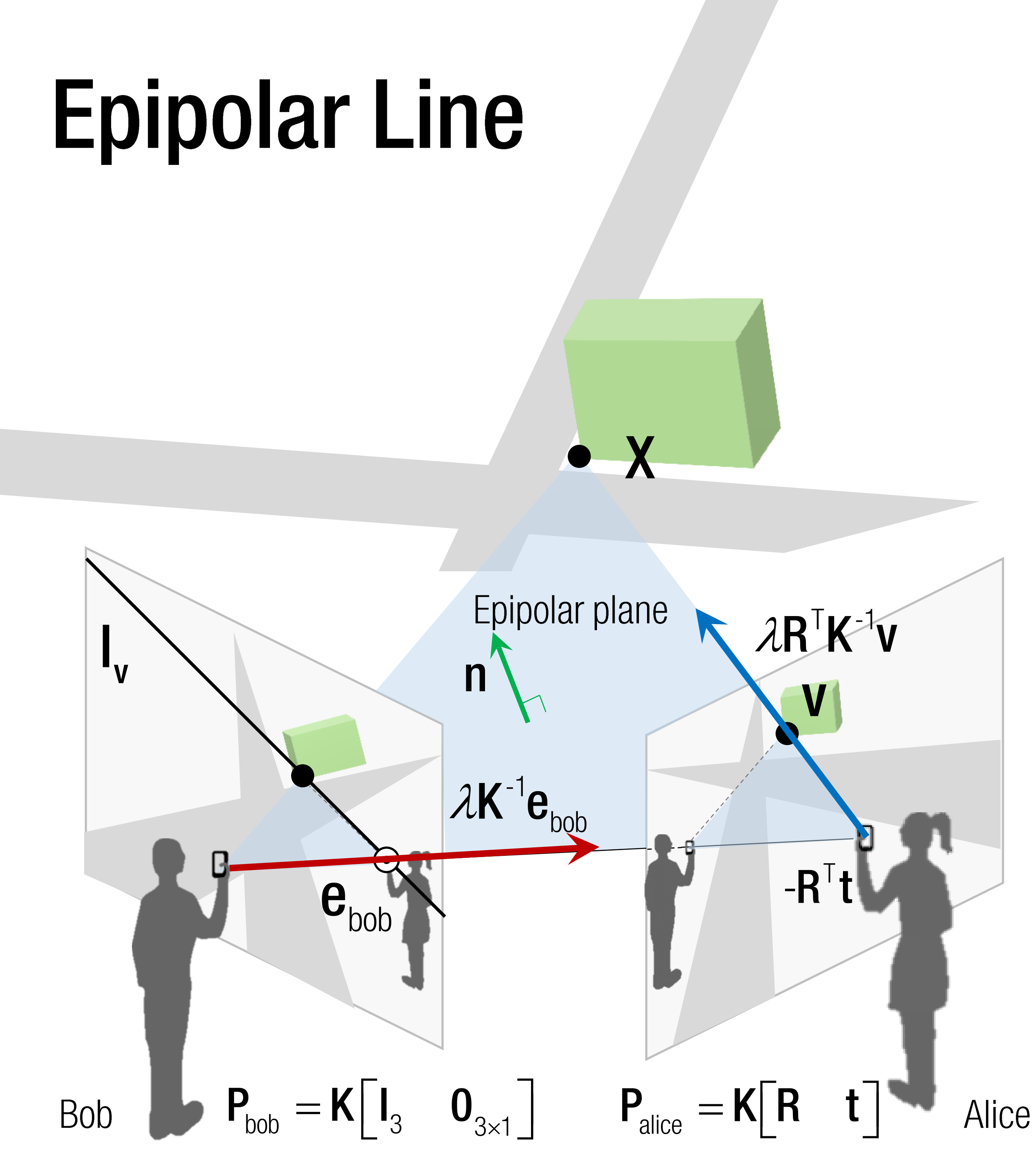
→ $n = R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right)$

$$= R^T t \times R^T K^{-1} v$$

$$= R^T \left(t \times K^{-1} v\right) \quad \text{: only works for rotation}$$

$$= R^T \left[t\right]_\times K^{-1} v$$

Epipolar Line



$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

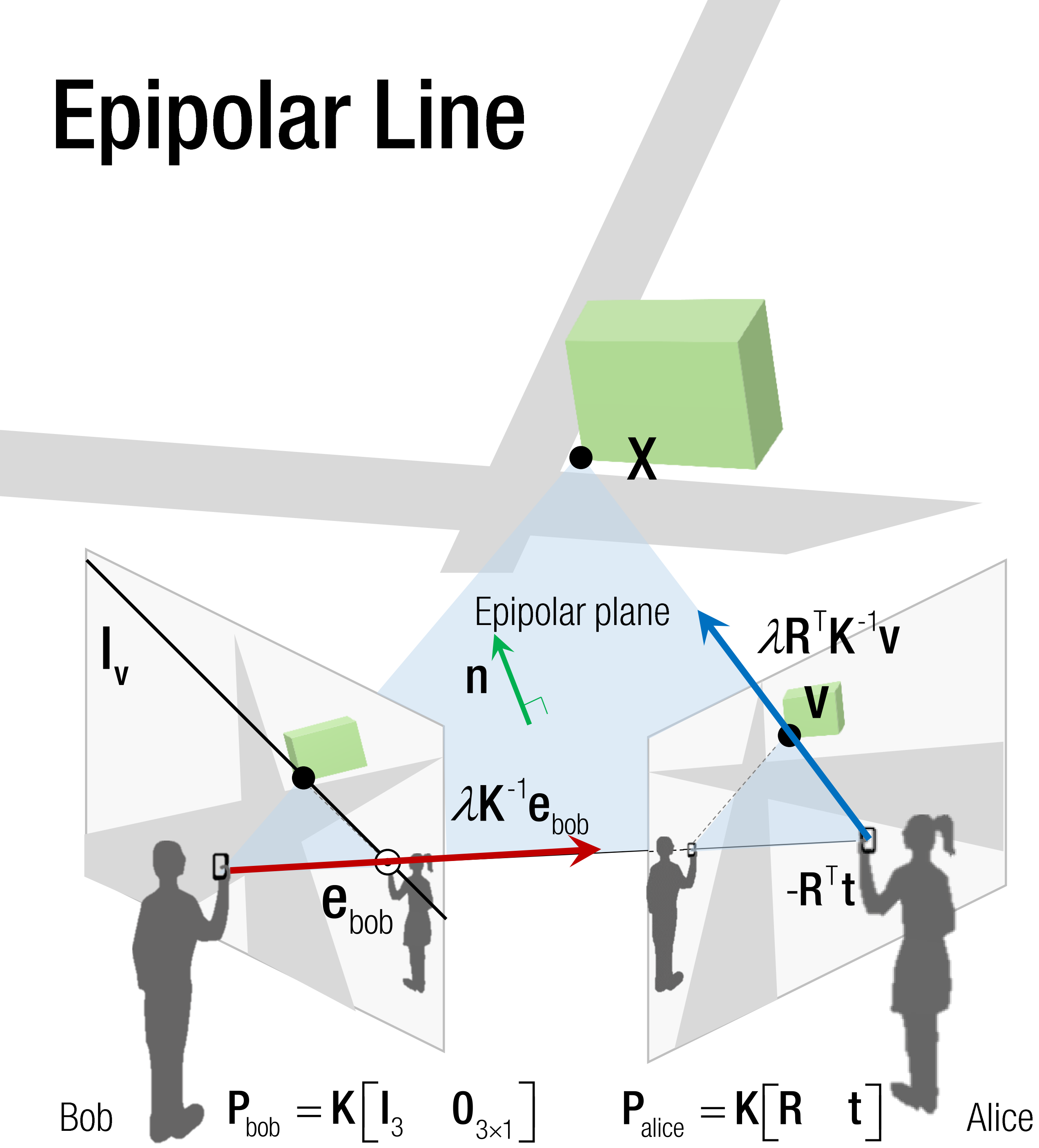
$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

$$\lambda K^{-1} e_{bob} = R^T t$$

$$\underbrace{\lambda_1 R^T K^{-1} v}_{\text{Direction}} - \underbrace{R^T t}_{\text{Alice's camer location}}$$

$$n = R^T t \times \left(\lambda_1 R^T K^{-1} v - R^T t\right)$$
$$= R^T t \times R^T K^{-1} v$$
$$= R^T \left(t \times K^{-1} v\right)$$: only works for rotation
$$= R^T [t]_\times K^{-1} v$$

Epipolar Line



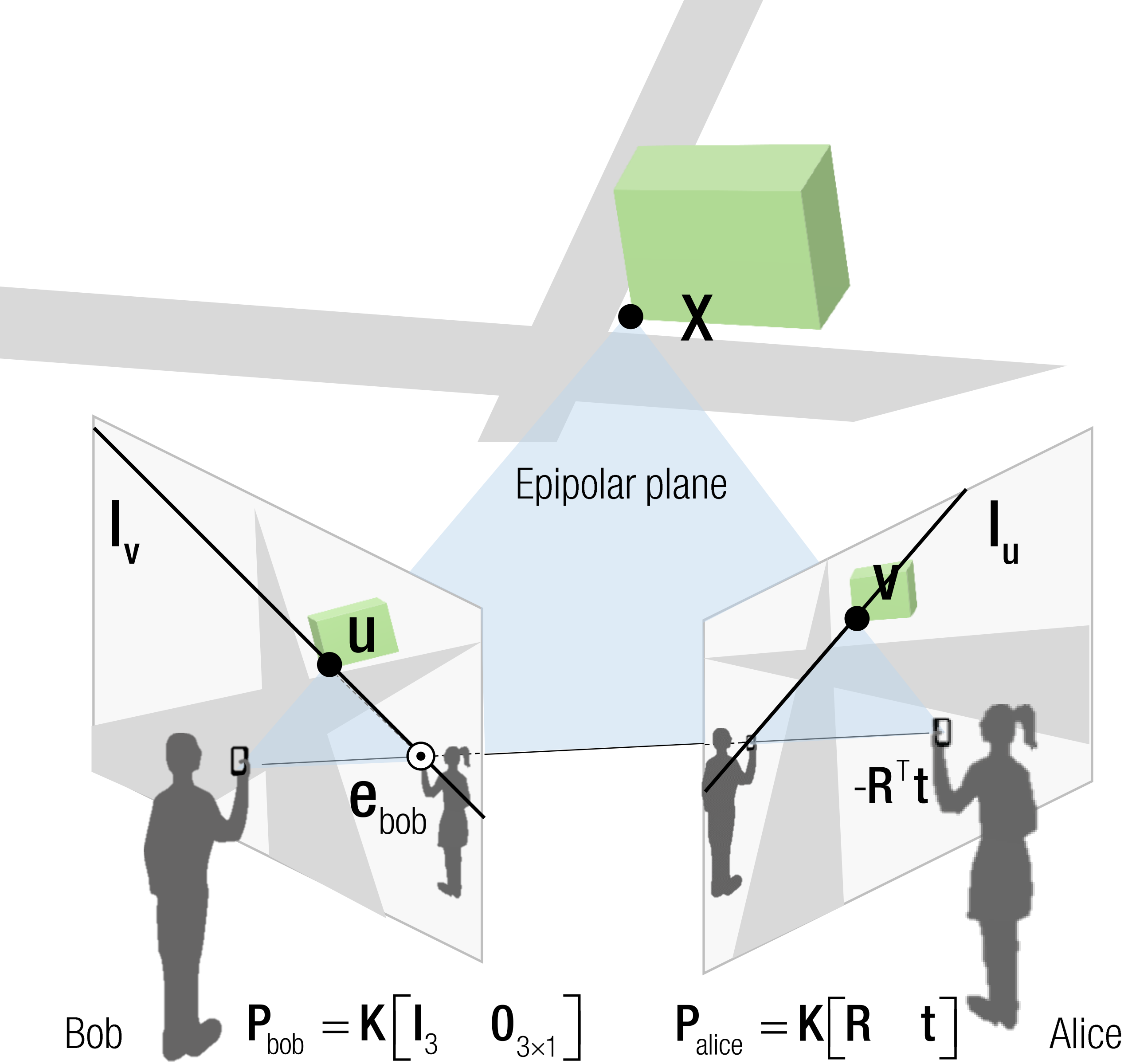
$$\lambda e_{bob} = KR^T t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T} n = K^{-T} R^T [t]_\times K^{-1} v$$: Epipolar line

Epipolar constraint:

$$u^T l_v = u^T K^{-T} R^T [t]_\times K^{-1} v = 0$$

Epipolar Line



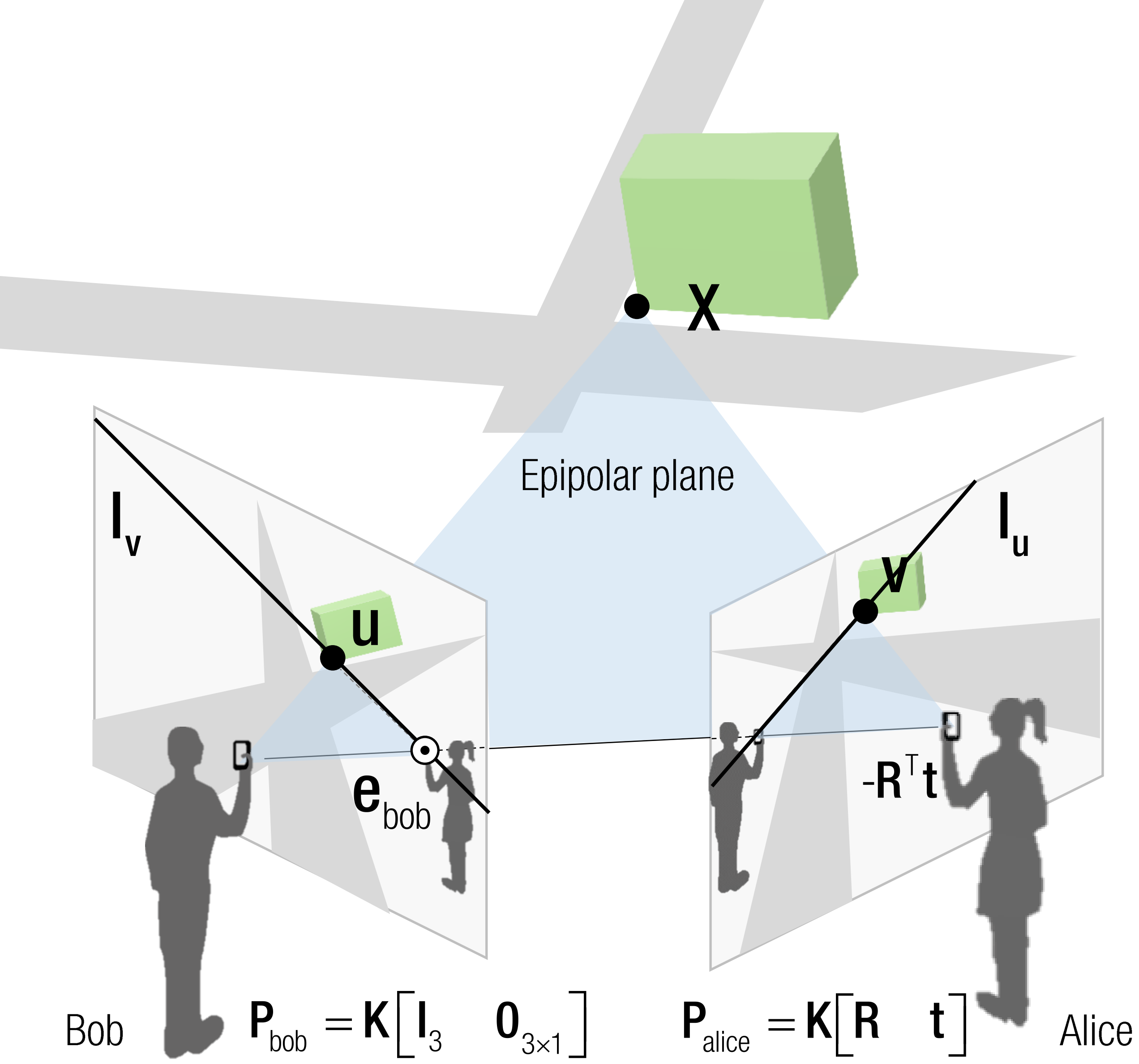
$$\lambda \mathbf{e}_{\text{bob}} = \mathbf{K}\mathbf{R}^\mathsf{T}\mathbf{t} \qquad \lambda \mathbf{e}_{\text{alice}} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_\mathbf{v} = \mathbf{K}^{-\mathsf{T}}\mathbf{n} = \mathbf{K}^{-\mathsf{T}}\mathbf{R}^\mathsf{T}\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v}$$: Epipolar line

Epipolar constraint:

$$\mathbf{u}^\mathsf{T}\mathbf{l}_\mathbf{v} = \mathbf{u}^\mathsf{T}\mathbf{K}^{-\mathsf{T}}\mathbf{R}^\mathsf{T}\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v} = 0$$

Epipolar Line



$$\lambda e_{bob} = KR^{T}t \qquad \lambda e_{alice} = Kt$$

$$l_v = K^{-T}n = K^{-T}R^{T}[t]_{\times}K^{-1}v$$: Epipolar line

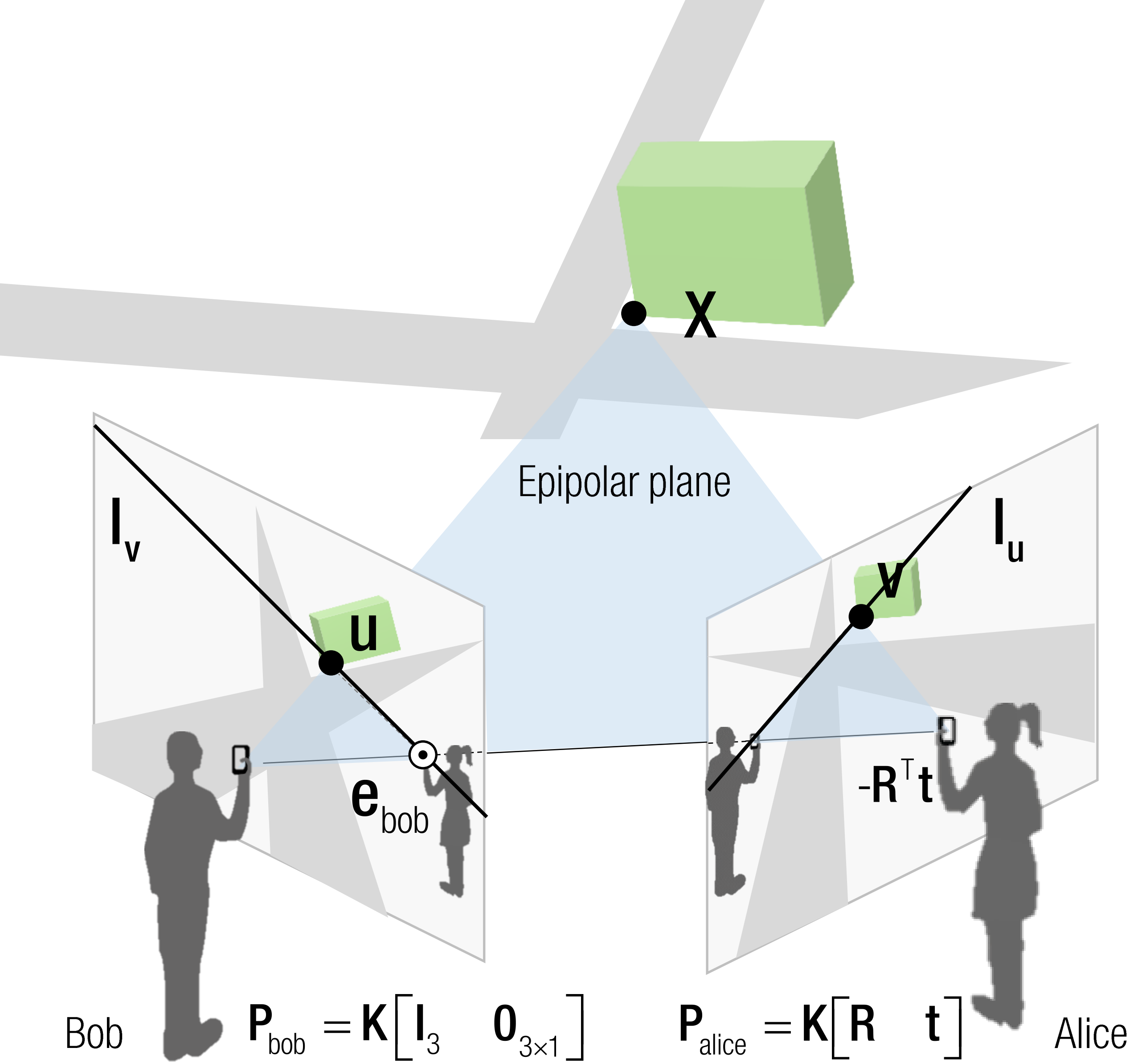
Epipolar constraint:

$$u^{T}l_v = \underbrace{u^{T}K^{-T}R^{T}[t]_{\times}K^{-1}}_{l_u^{T}}v = 0$$

$$l_u = -K^{-T}[t]_{\times}RK^{-1}u \qquad \because [t]_{\times}^{T} = -[t]_{\times}$$

Skew symetric matrix

Fundamental Matrix



$$\lambda \mathbf{e}_{bob} = \mathbf{K}\mathbf{R}^T\mathbf{t} \qquad \lambda \mathbf{e}_{alice} = \mathbf{K}\mathbf{t}$$

$$\mathbf{l}_v = \mathbf{K}^{-T}\mathbf{n} = \mathbf{K}^{-T}\mathbf{R}^T\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}\mathbf{v}$$: Epipolar line

Epipolar constraint:

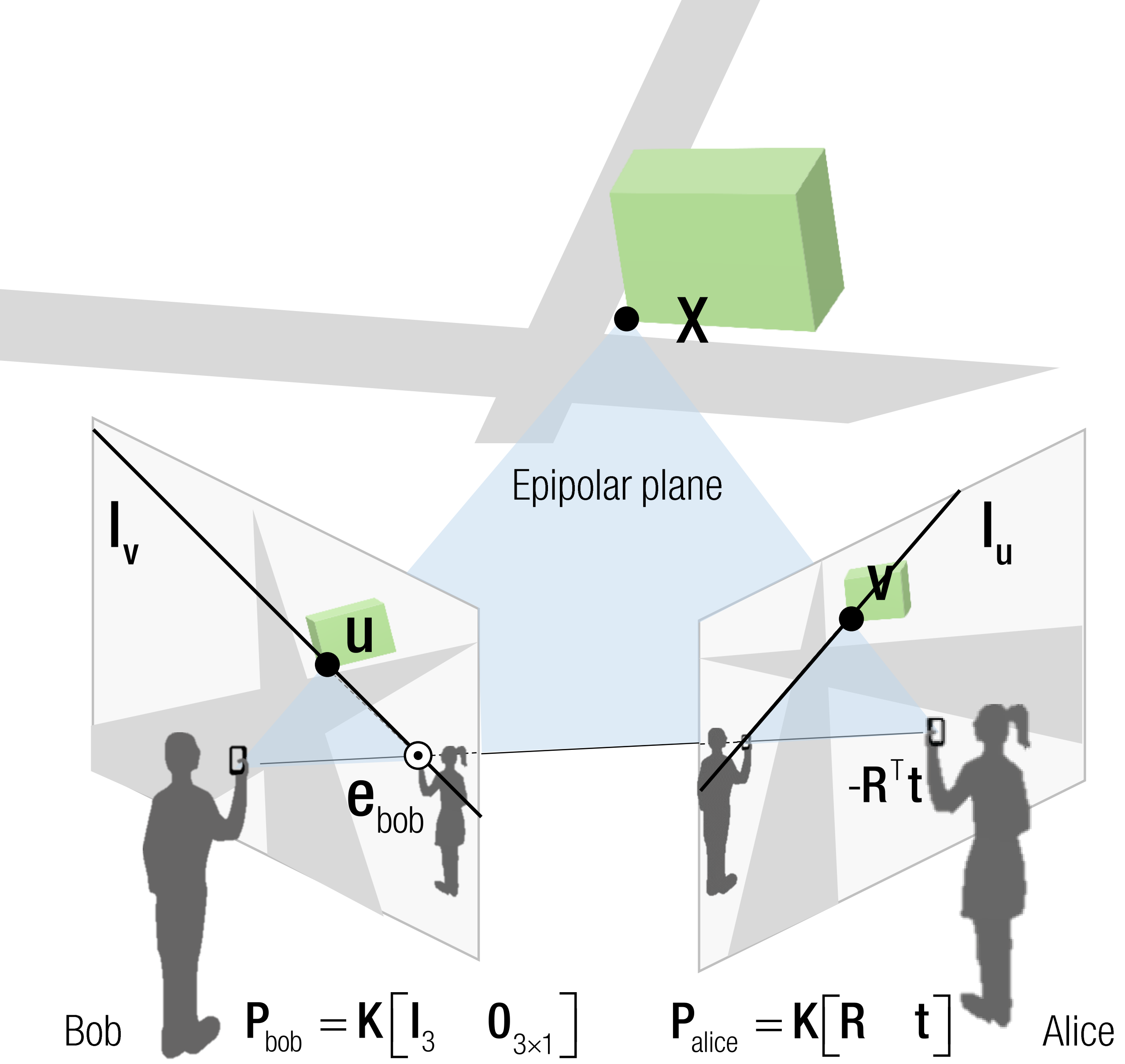
$$\mathbf{u}^T\mathbf{l}_v = \underbrace{\mathbf{u}^T\mathbf{K}^{-T}\mathbf{R}^T\left[\mathbf{t}\right]_\times \mathbf{K}^{-1}}_{\mathbf{l}_u^T}\mathbf{v} = 0$$

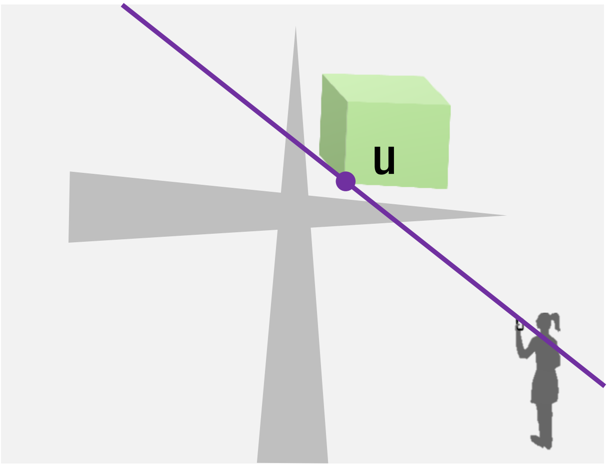
$$\mathbf{l}_u = -\underbrace{\mathbf{K}^{-T}\left[\mathbf{t}\right]_\times \mathbf{R}\mathbf{K}^{-1}}_{\text{Common for all points}}\mathbf{u}$$

$$\because \left[\mathbf{t}\right]_\times^T = -\left[\mathbf{t}\right]_\times$$

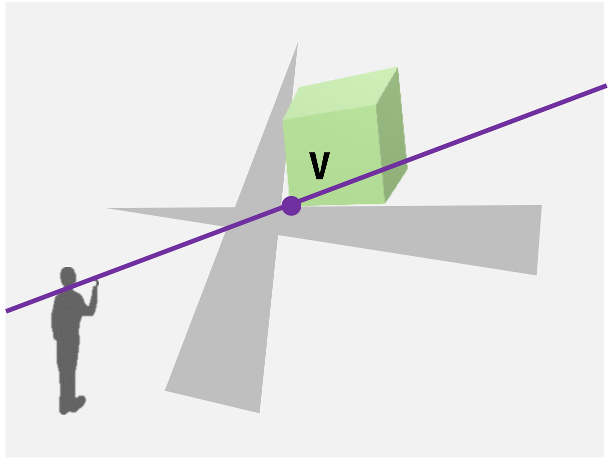
Skew symetric matrix

Fundamental Matrix





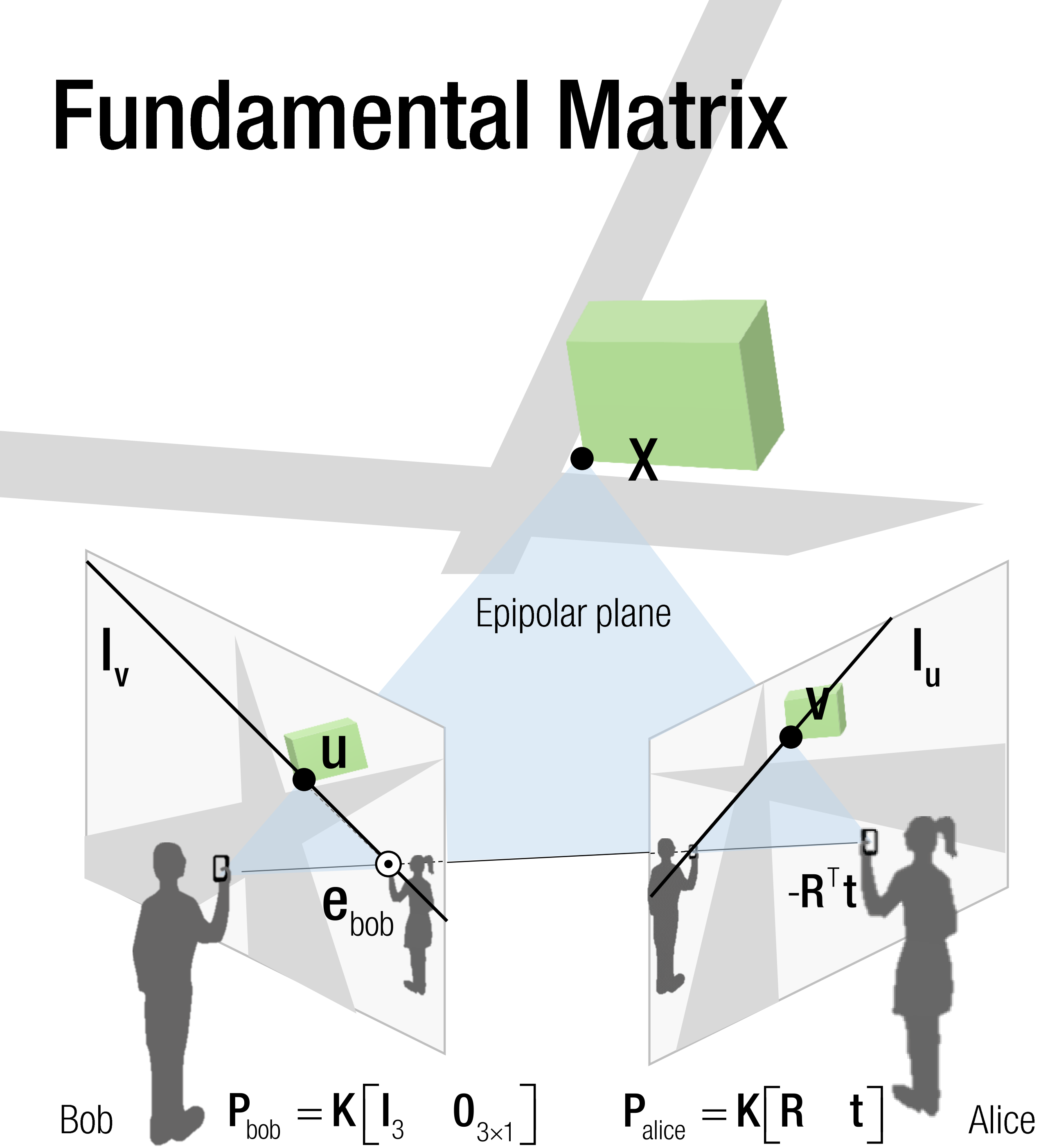
Bob's image

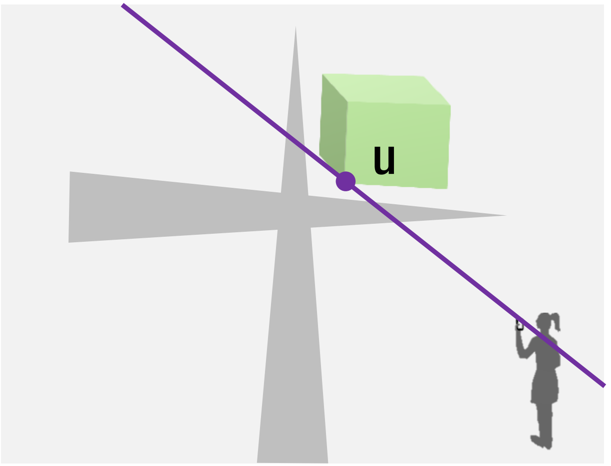


Alice's image

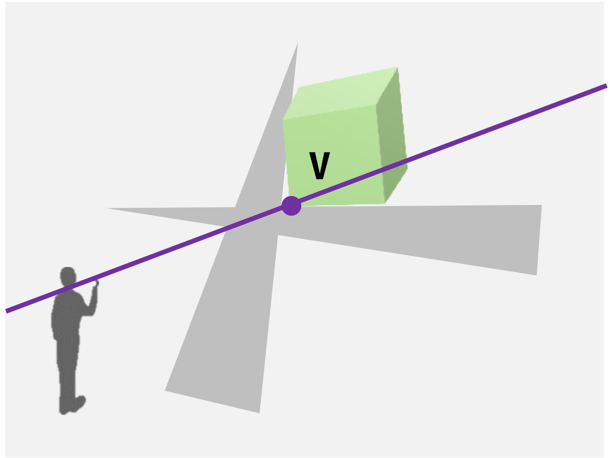
$$v^T l_u = v^T \underbrace{K^{-T}[t]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

Fundamental Matrix





Bob's image



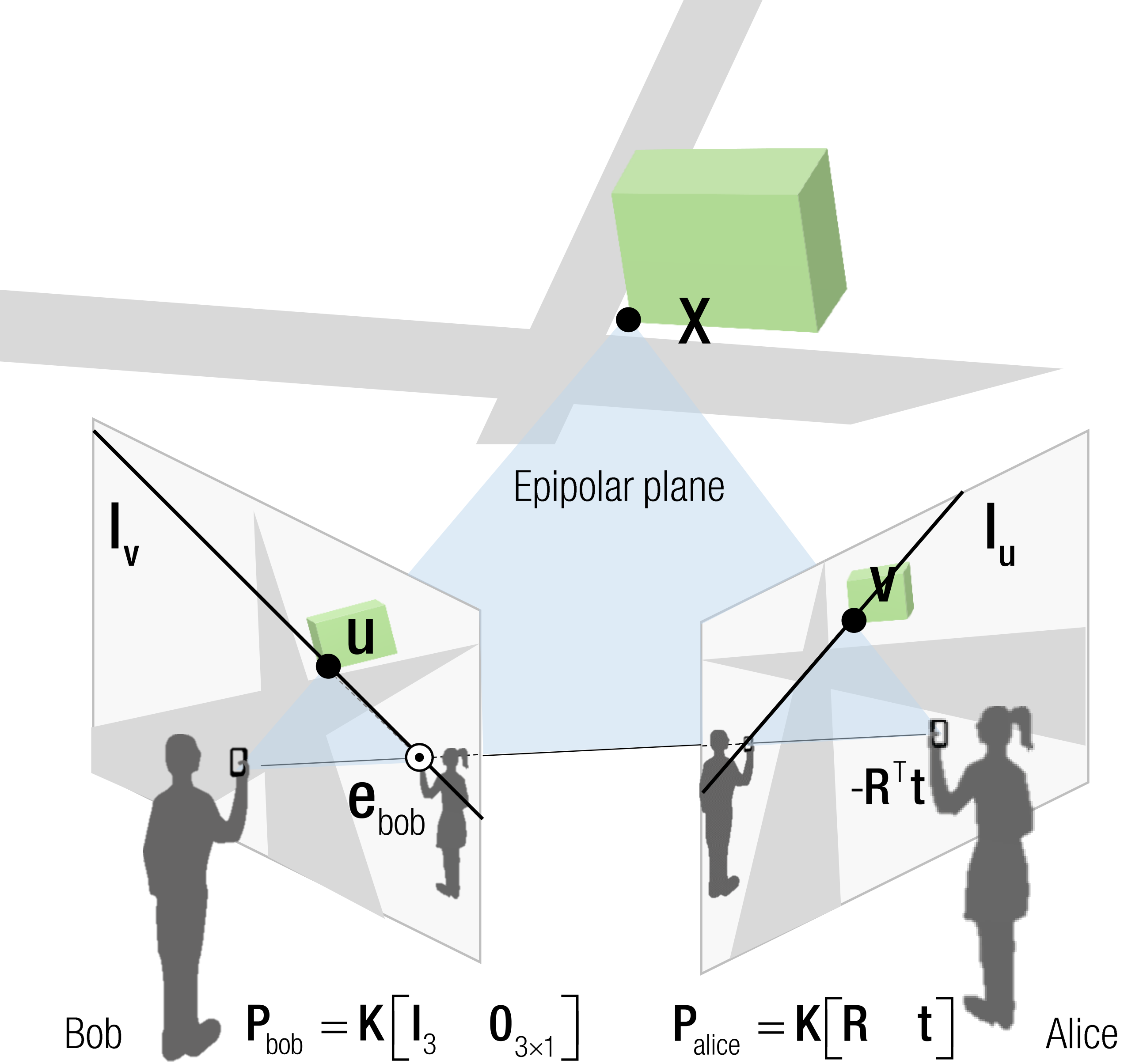
Alice's image

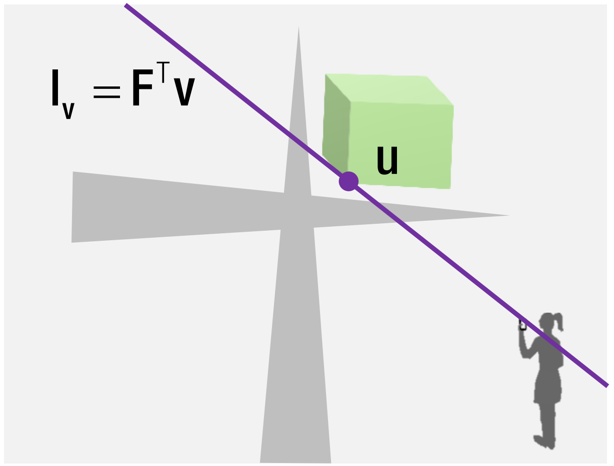
$$v^{T}l_u = \underbrace{v^{T}K^{-T}[t]_{\times}RK^{-1}}_{\text{Common for all points}}u = 0$$

$$= v^{T}Fu = 0$$

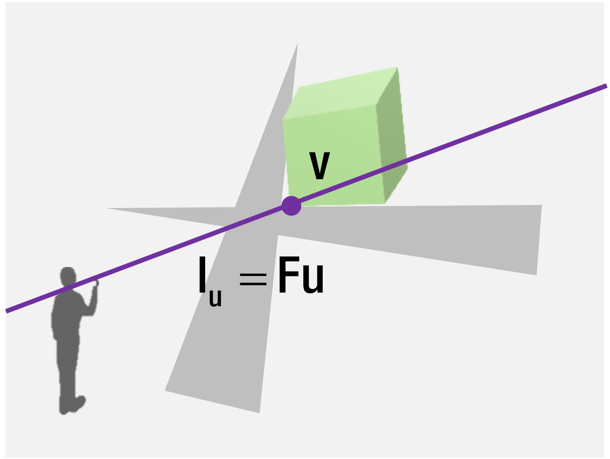
where $\underbrace{F = K^{-T}[t]_{\times}RK^{-1}}_{\text{Fundamental matrix}}$

Fundamental Matrix





Bob's image



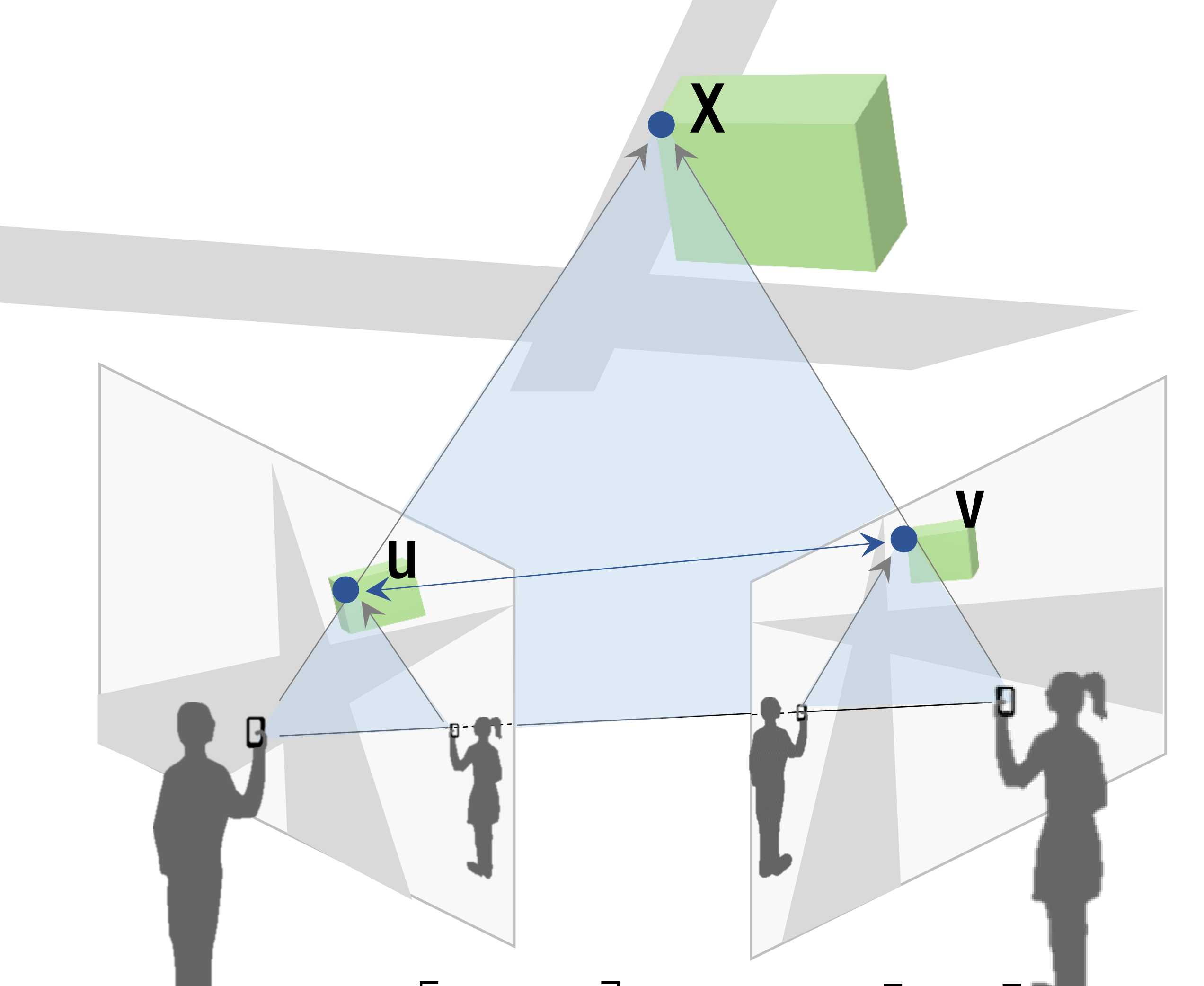
Alice's image

$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T}[\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = 0$$

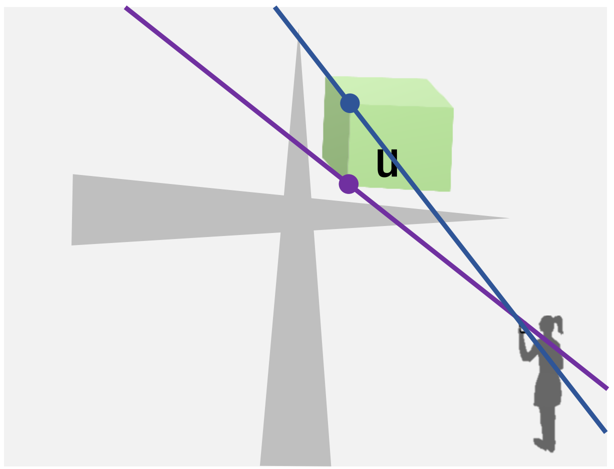
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

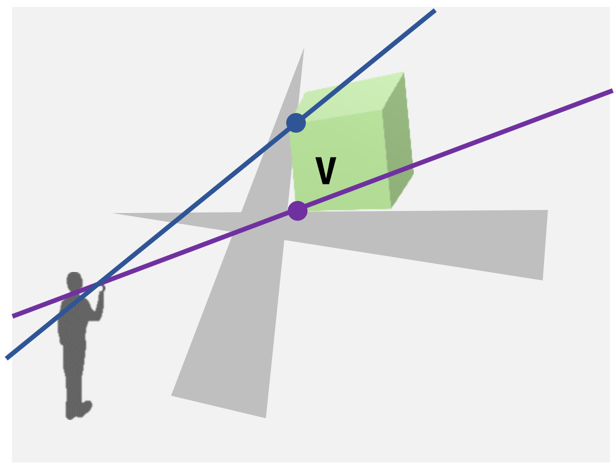
Fundamental Matrix



Bob $\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix}$ $\mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$ Alice



Bob's image



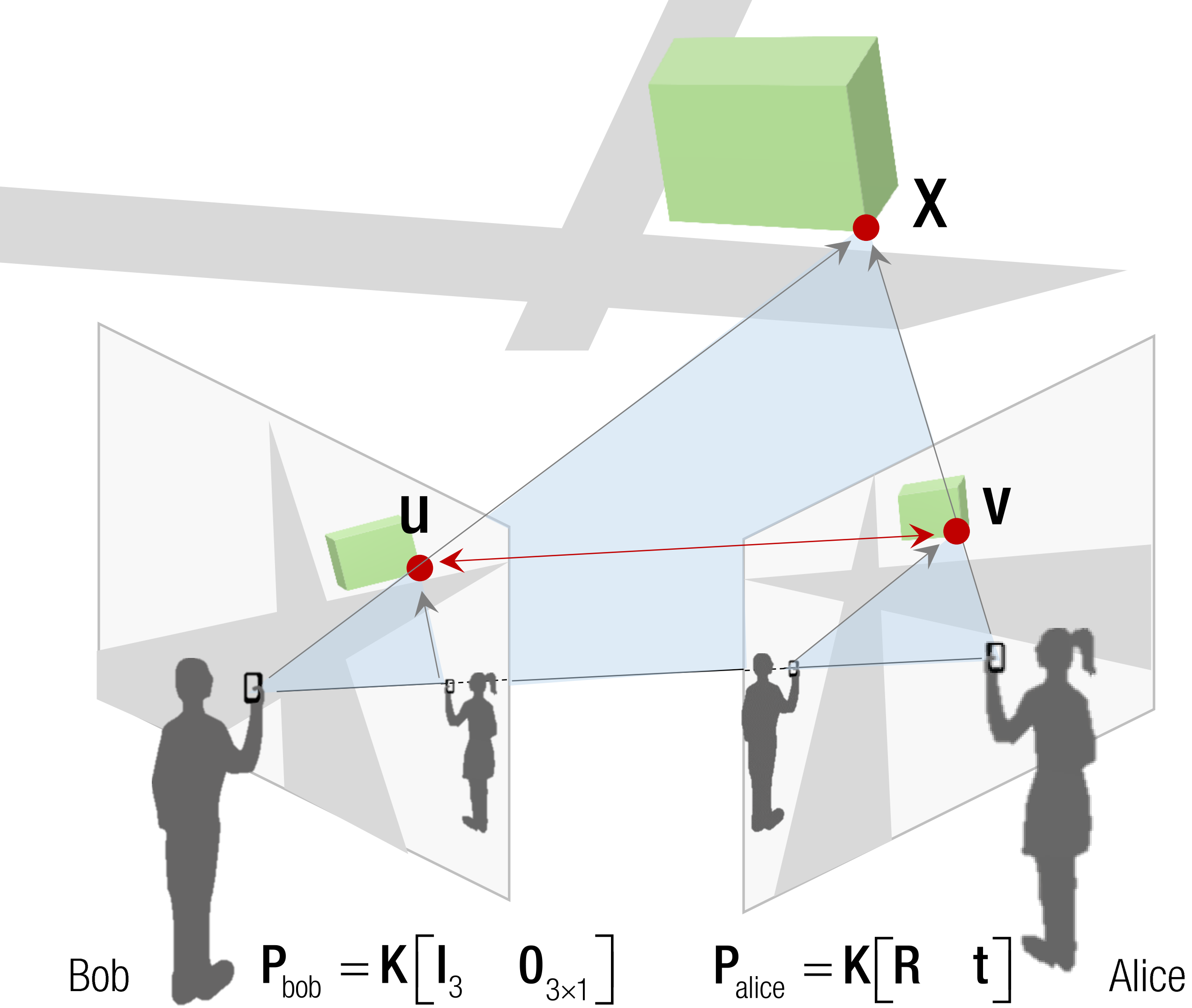
Alice's image

$$\mathbf{v}^{\mathsf{T}}\mathbf{l}_{\mathbf{u}} = \mathbf{v}^{\mathsf{T}}\underbrace{\mathbf{K}^{-\mathsf{T}}\left[\mathbf{t}\right]_{\times}\mathbf{R}\mathbf{K}^{-1}}_{\text{Common for all points}}\mathbf{u} = \mathbf{0}$$

$$= \mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

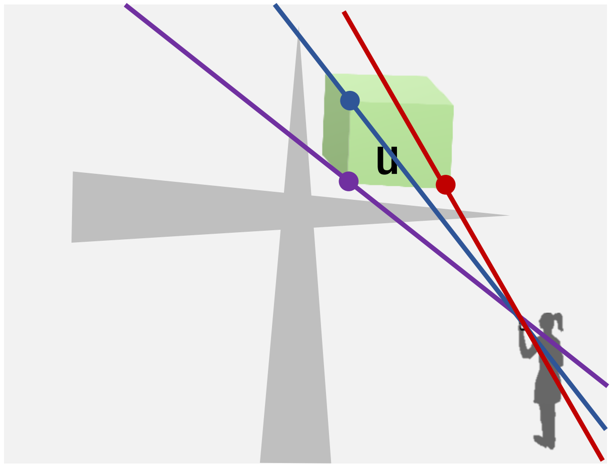
$$= \mathbf{v}^{\mathsf{T}}\left(\mathbf{F}\mathbf{u}\right) = \mathbf{u}^{\mathsf{T}}\left(\mathbf{F}^{\mathsf{T}}\mathbf{v}\right) = 0$$

Fundamental Matrix

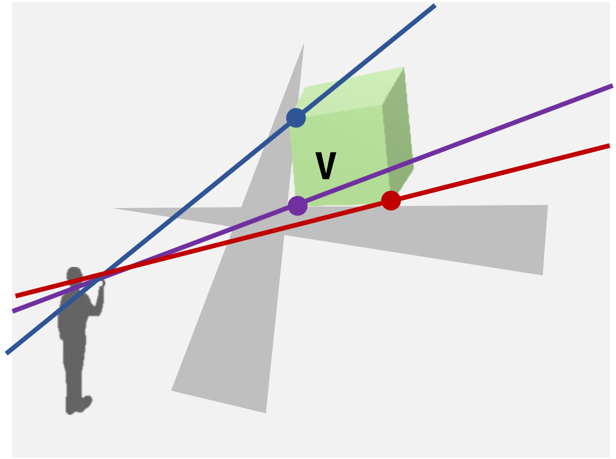


$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$$

$$P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



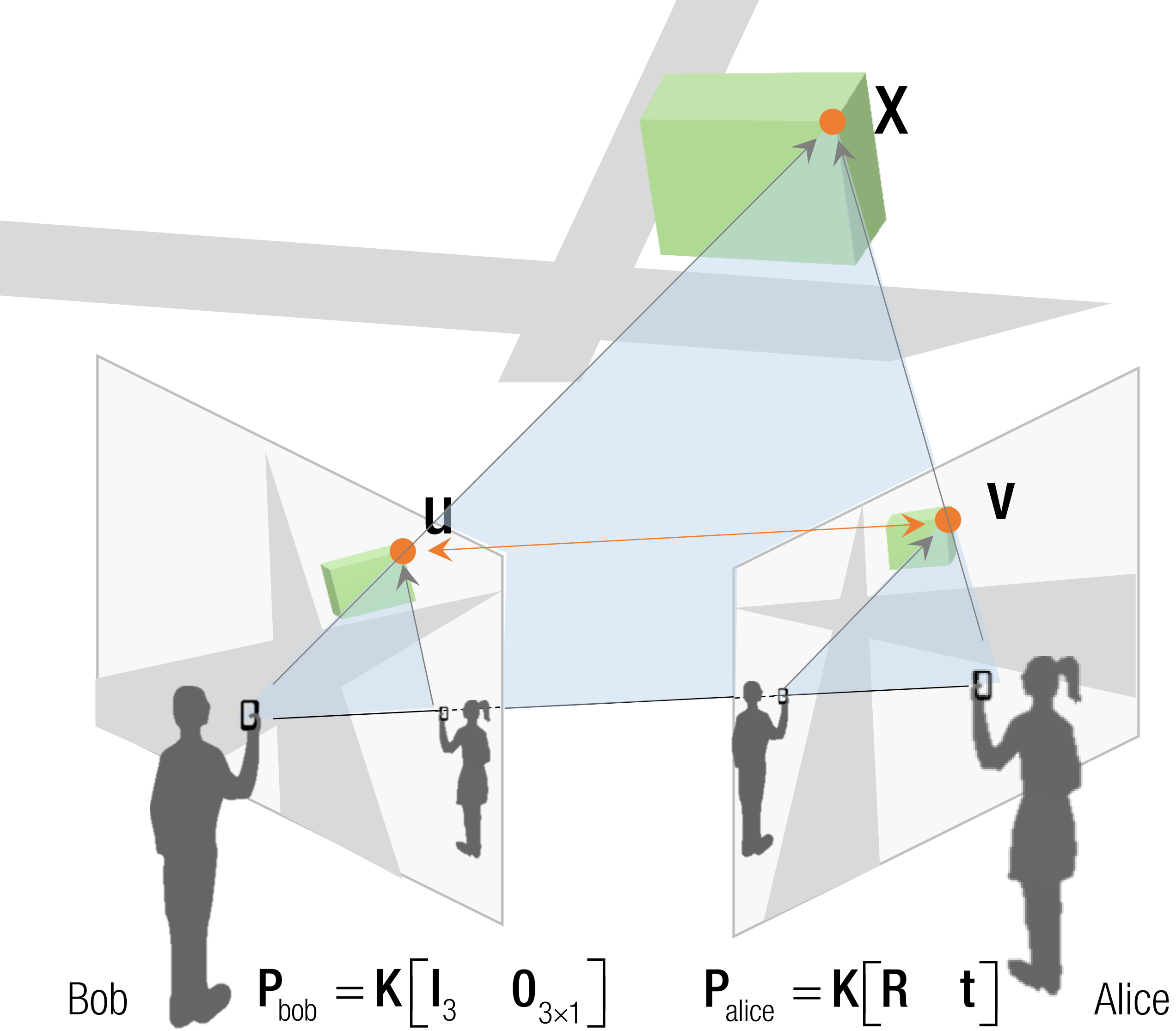
Alice's image

$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T}\left[\mathbf{t}\right]_\times \mathbf{R}\mathbf{K}^{-1}}_{\text{Common for all points}} \mathbf{u} = 0$$

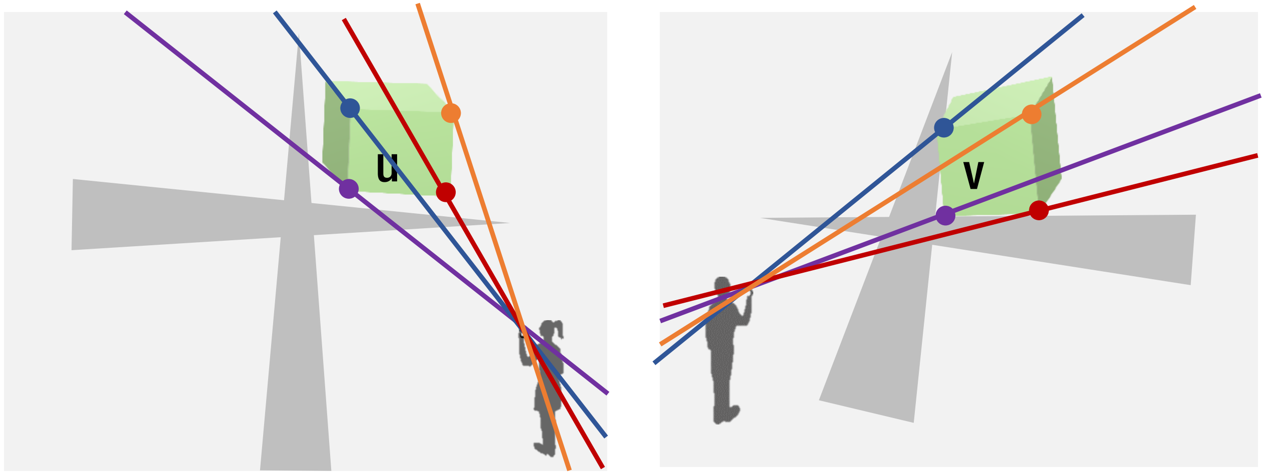
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T \left(\mathbf{F}\mathbf{u}\right) = \mathbf{u}^T \left(\mathbf{F}^T \mathbf{v}\right) = 0$$

Fundamental Matrix



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



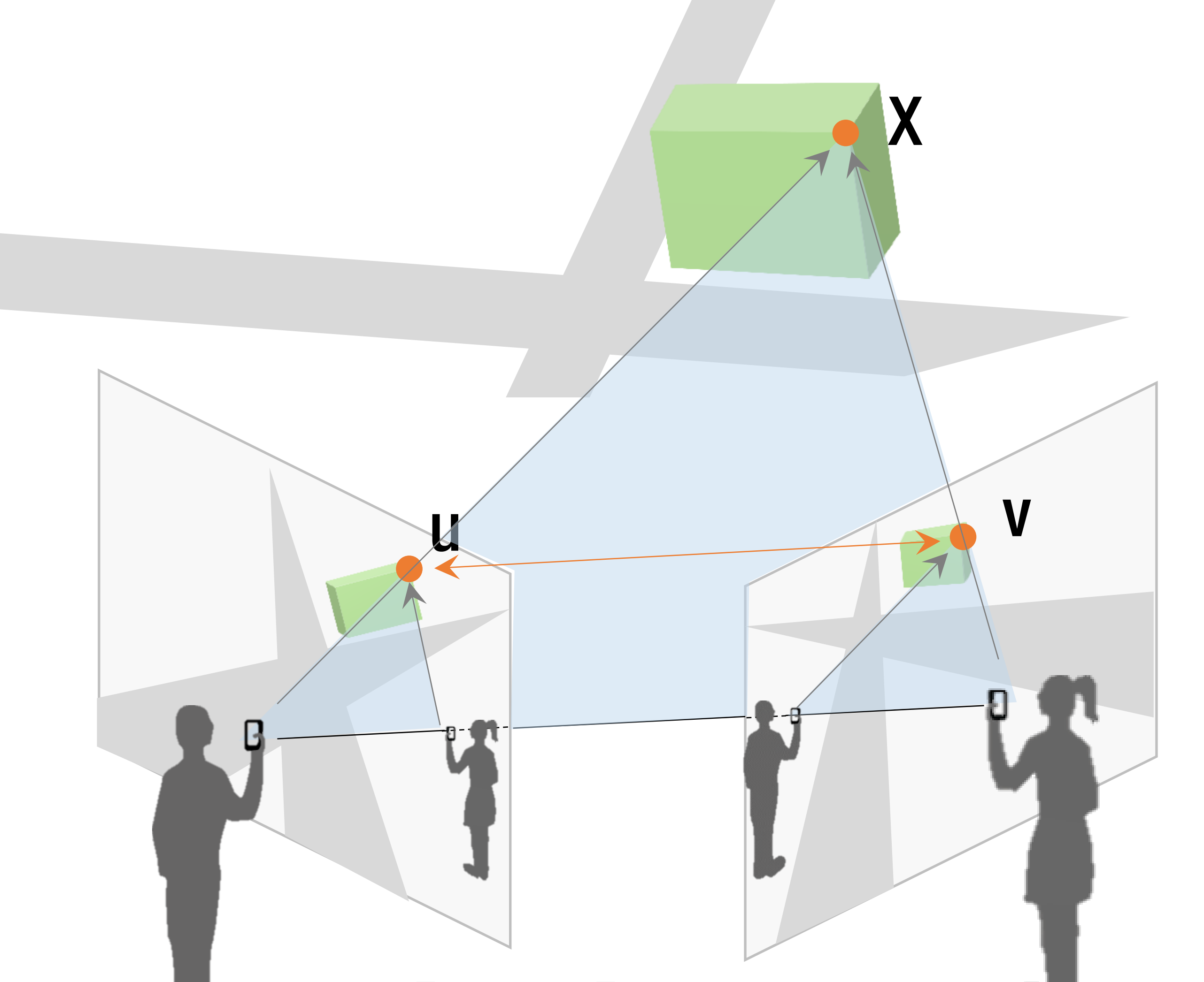
Bob's image   Alice's image

$$v^T l_u = \underbrace{v^T K^{-T}[t]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

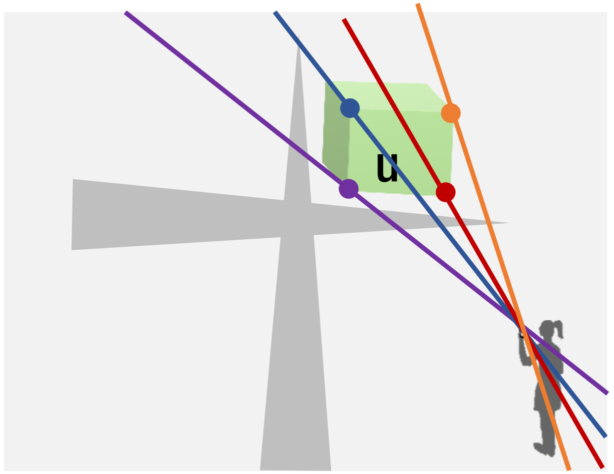
$$= v^T F u = 0$$

$$= v^T(Fu) = u^T(F^T v) = 0$$

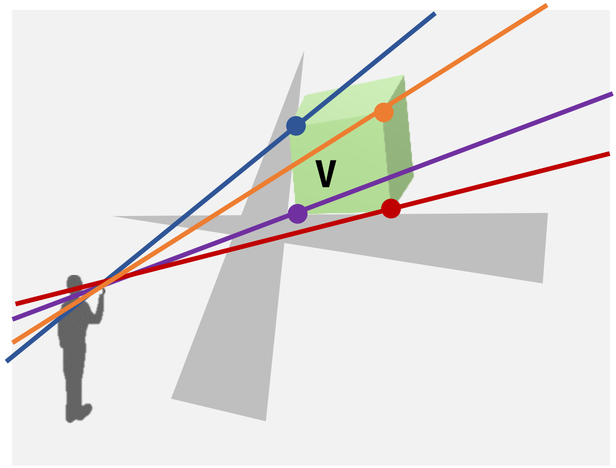
Fundamental Matrix



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



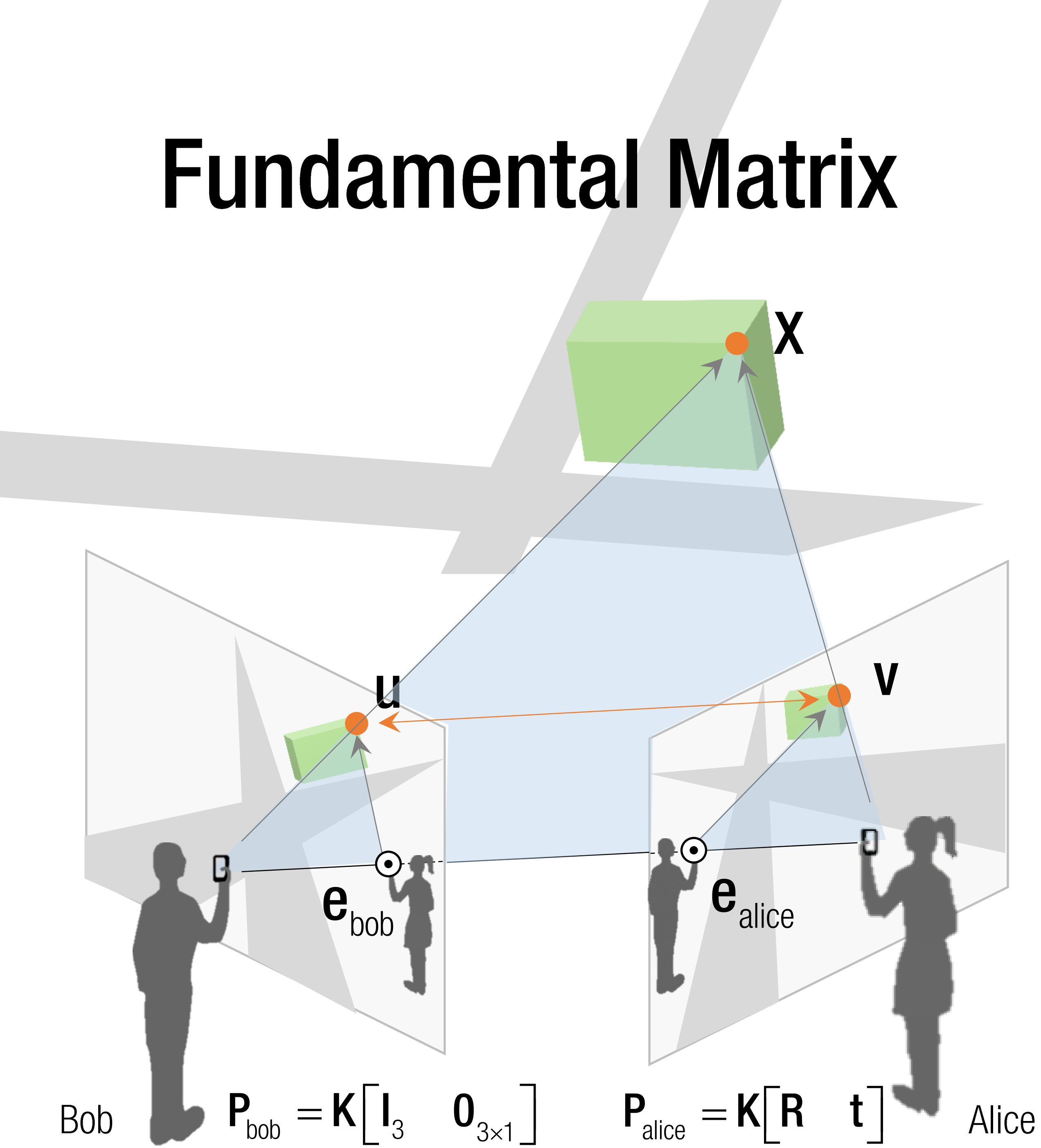
Alice's image

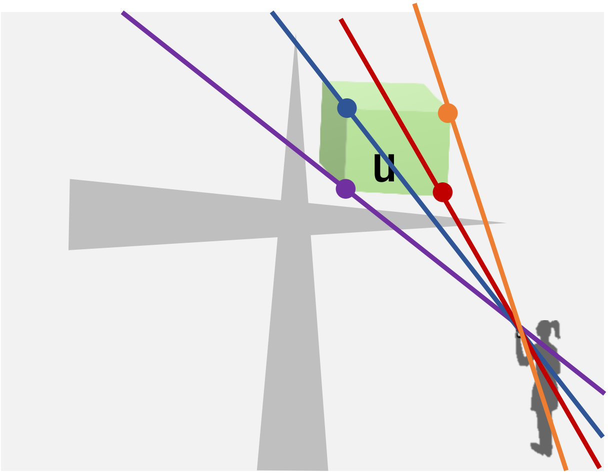
$$v^T l_u = v^T \underbrace{K^{-T}\left[t\right]_\times R K^{-1}}_{\text{Common for all points}} u = 0$$

$$= v^T F u = 0$$

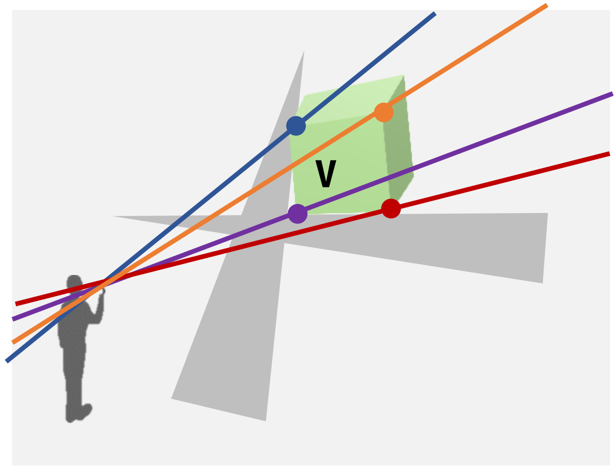
$$= v^T\left(Fu\right) = u^T\left(F^T v\right) = 0$$

Fundamental Matrix





Bob's image

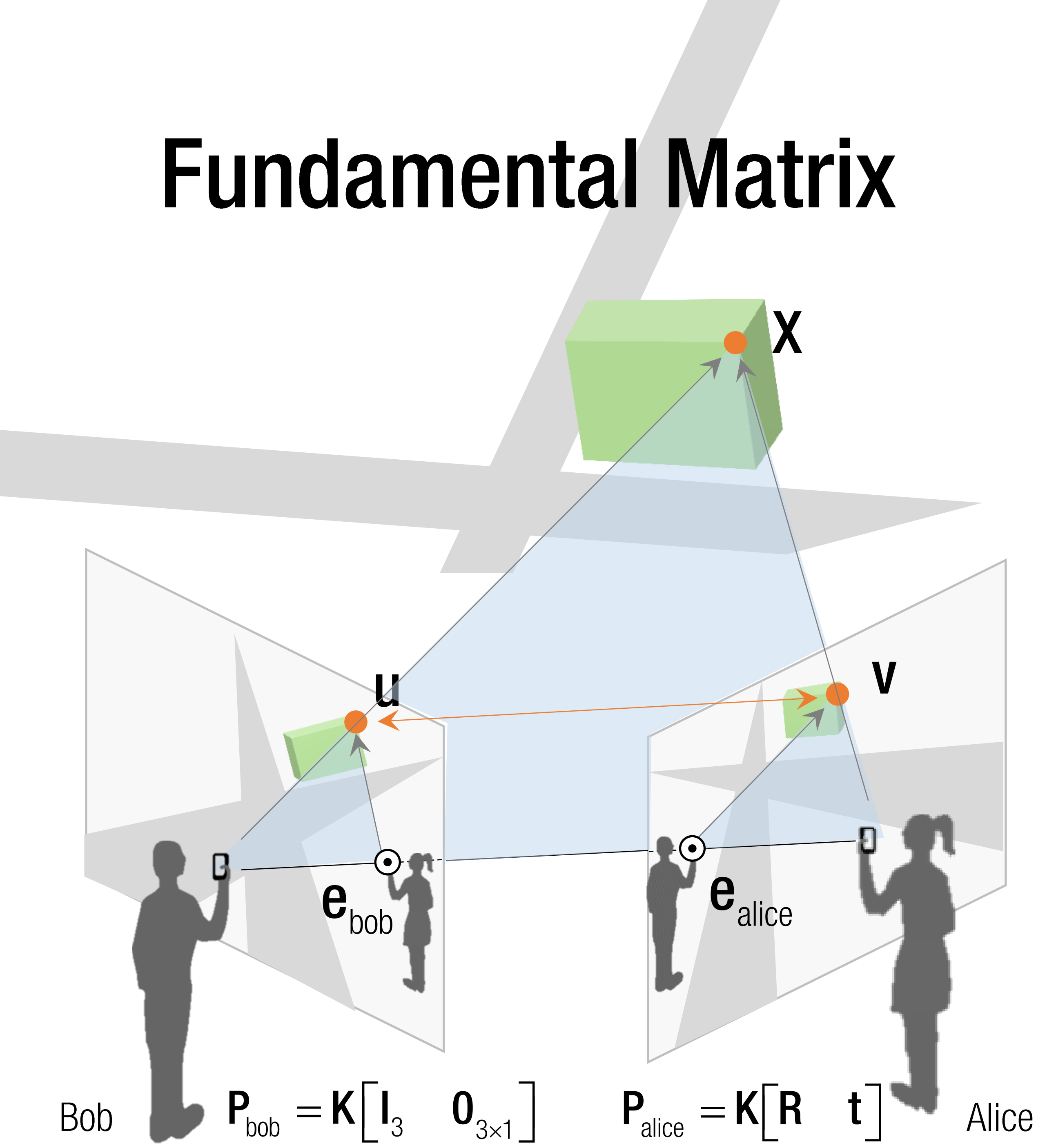


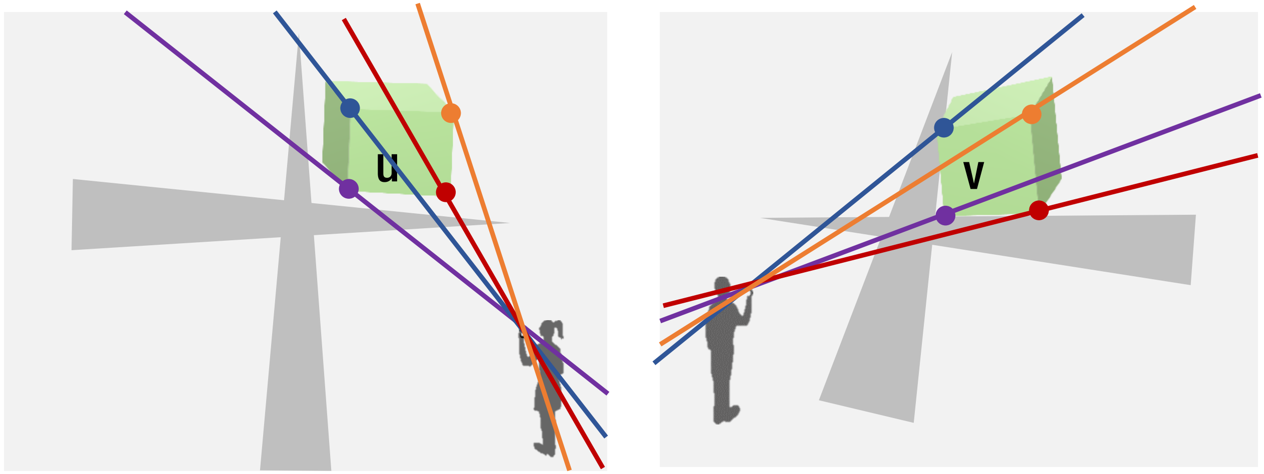
Alice's image

Properties of Fundamental Matrix

- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{\text{bob}}$, $\mathbf{P}_{\text{alice}}$, then $\mathbf{F}^{\mathrm{T}}$ is for $\mathbf{P}_{\text{alice}}$, $\mathbf{P}_{\text{bob}}$.

Fundamental Matrix





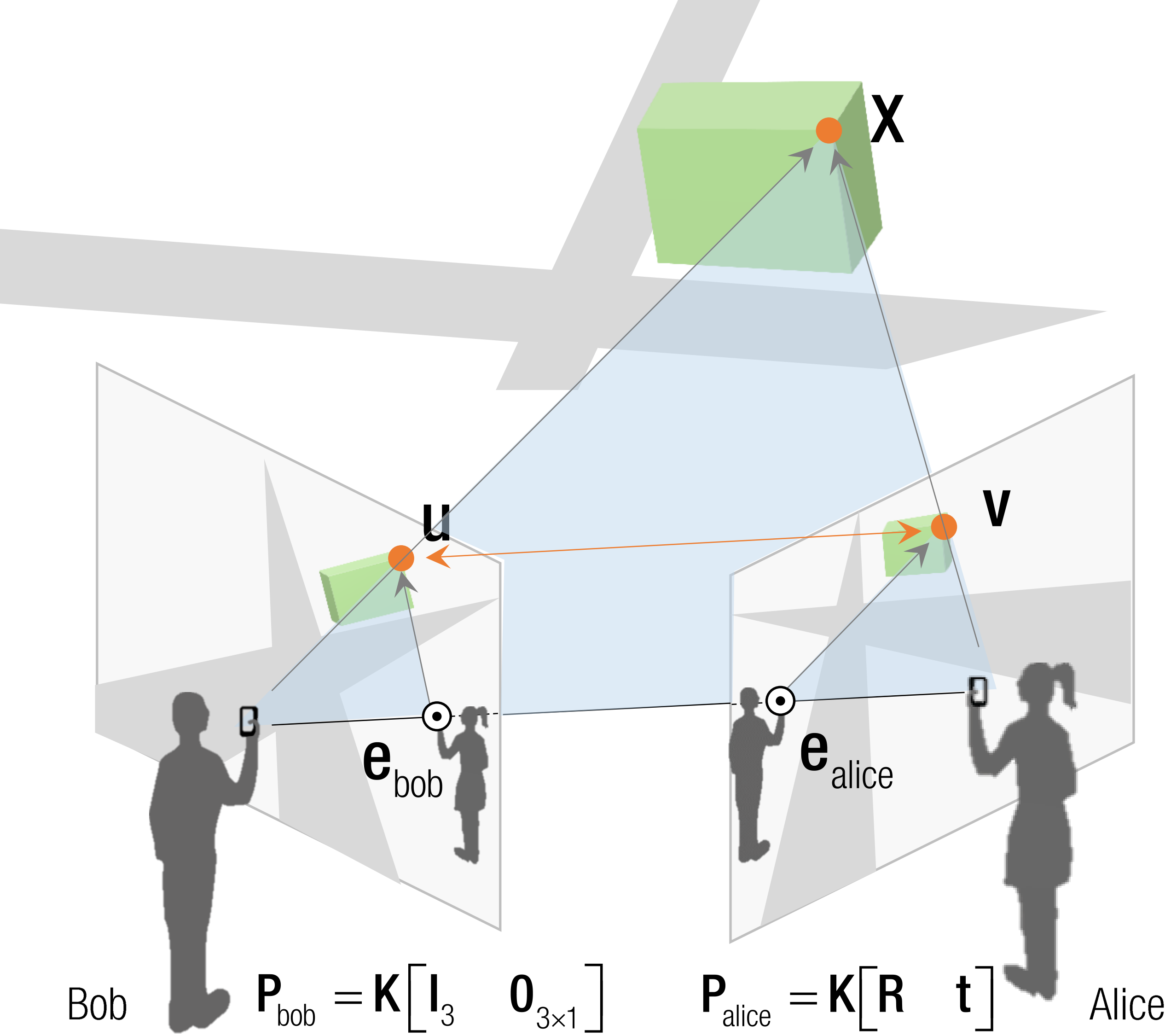
Bob's image

Alice's image

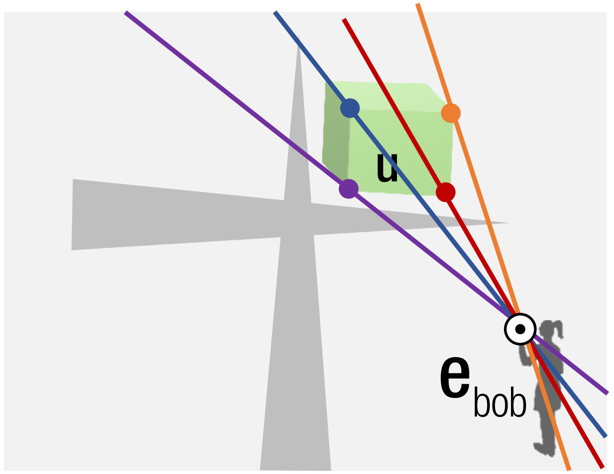
Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u}$ $\quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$

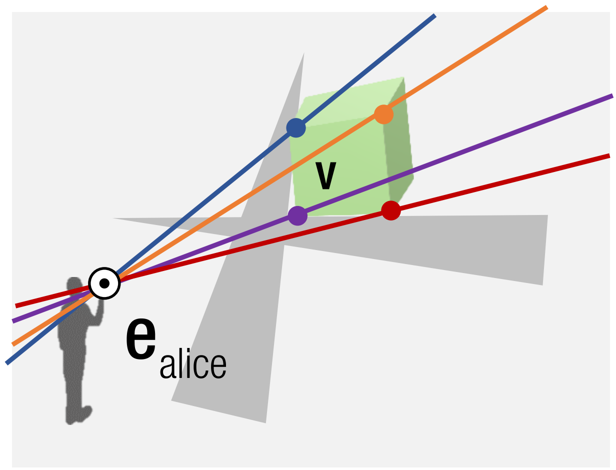
Fundamental Matrix



$\mathbf{P}_{\text{bob}} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix}$ $\mathbf{P}_{\text{alice}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$



Bob's image



Alice's image

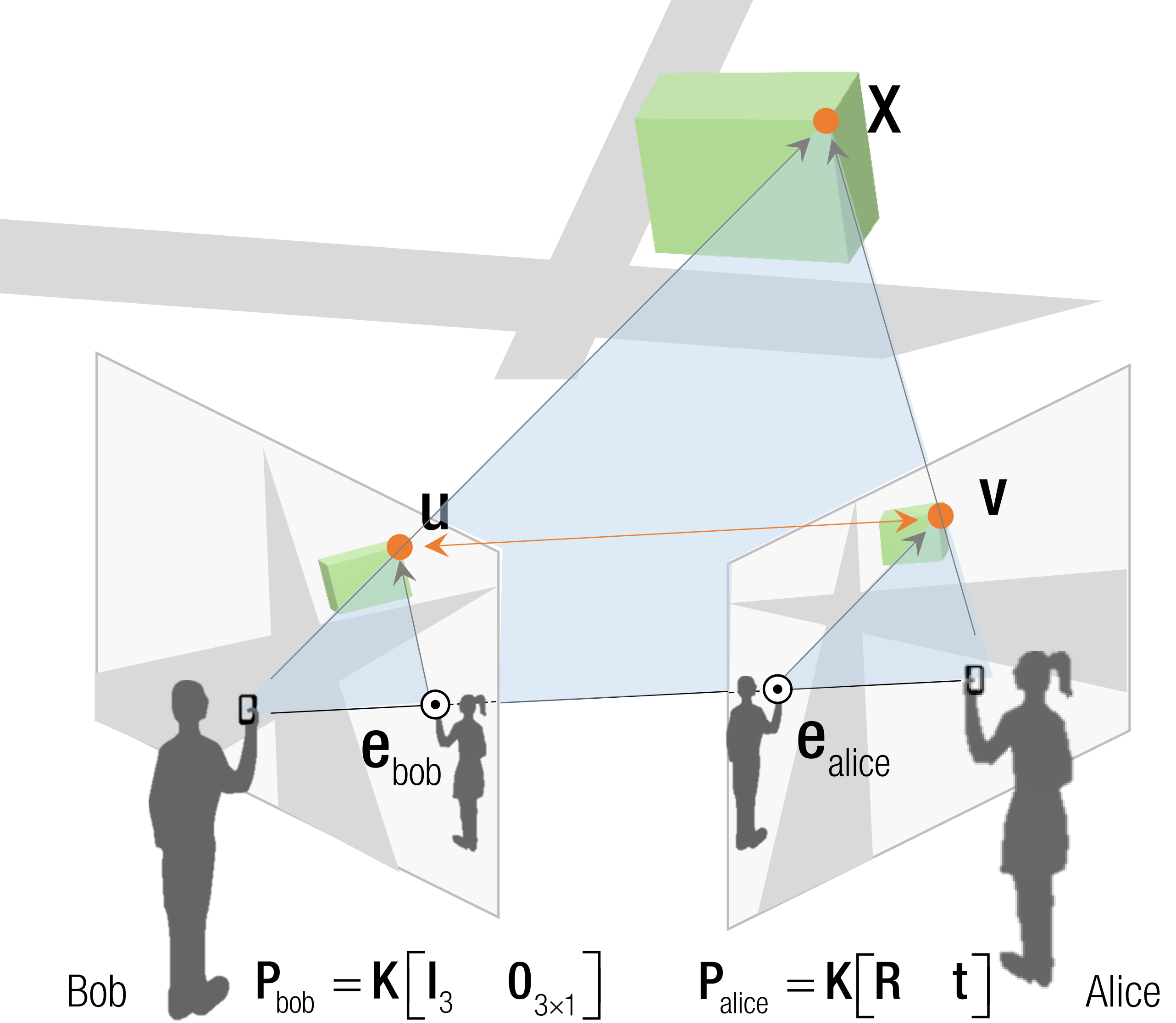
Properties of Fundamental Matrix

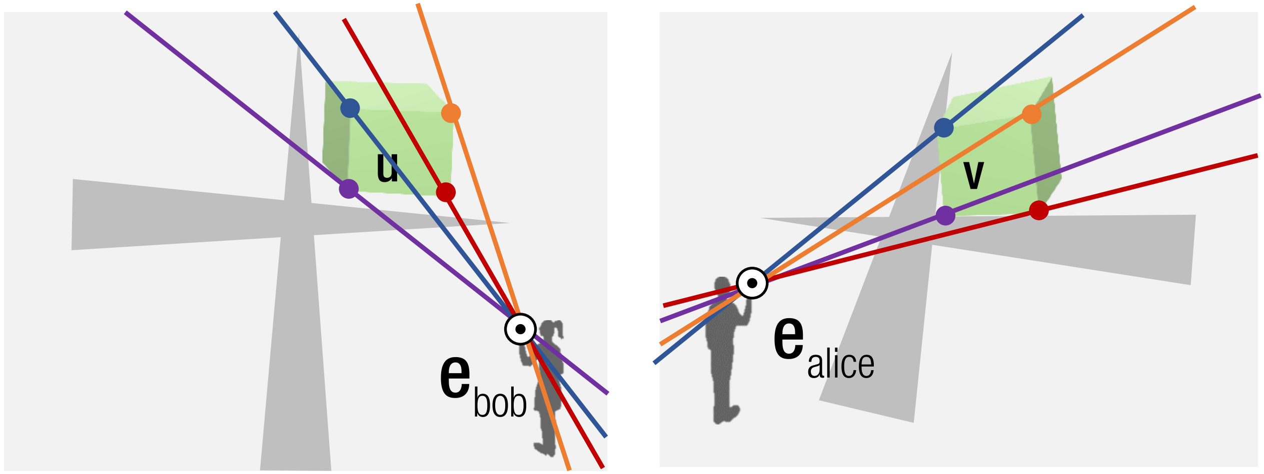
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{\text{bob}}, \mathbf{P}_{\text{alice}}$, then $\mathbf{F}^{\mathsf{T}}$ is for $\mathbf{P}_{\text{alice}}, \mathbf{P}_{\text{bob}}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^{\mathsf{T}}\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{\text{bob}} = \mathbf{0} \quad \mathbf{F}^{\mathsf{T}}\mathbf{e}_{\text{alice}} = \mathbf{0}$

$$\because \mathbf{v}_i^{\mathsf{T}}\mathbf{F}\mathbf{e}_{\text{bob}} = 0, \quad \mathbf{u}_i^{\mathsf{T}}\mathbf{F}^{\mathsf{T}}\mathbf{e}_{\text{alice}} = 0, \quad \forall i$$

$$\longrightarrow \mathbf{e}_{\text{bob}} = \text{null}(\mathbf{F}), \quad \mathbf{e}_{\text{alice}} = \text{null}(\mathbf{F}^{\mathsf{T}})$$

Fundamental Matrix





Bob's image

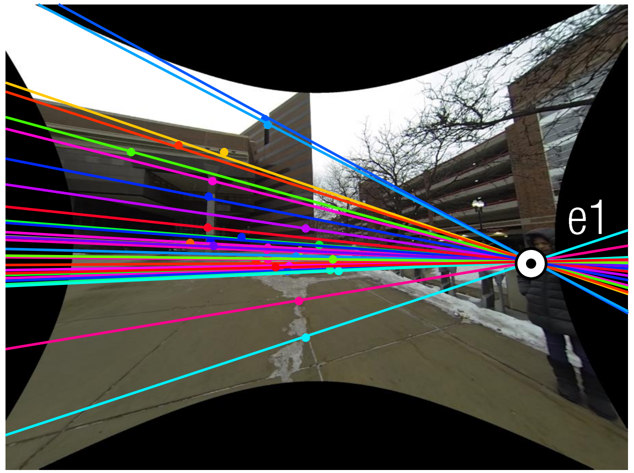
Alice's image

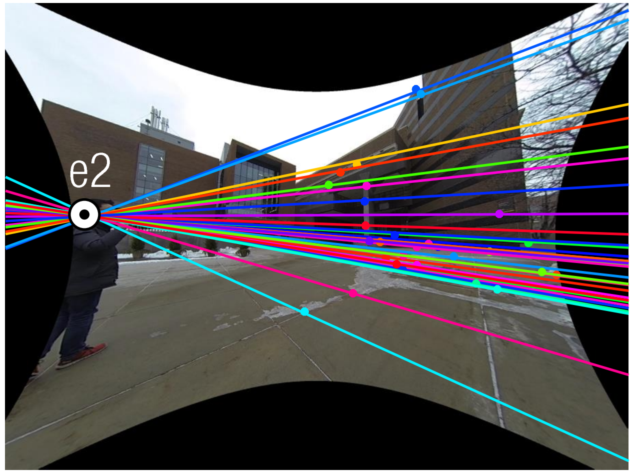
Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^{T}$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^{T}\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^{T}\mathbf{e}_{alice} = \mathbf{0}$
- rank(**F**)=2: degree of freedom 9 (3x3 matrix)-1 (scale)-1 (rank)=7

$$\mathbf{F} = \mathbf{K}_{alice}^{-T} \underbrace{\left[\mathbf{t}\right]_{\times}}_{\text{rank 2 matrix}} \mathbf{R}\mathbf{K}_{bob}^{-1}$$

Fundamental Matrix





K =

568.9961	0	643.2106
0	568.9884	477.9828
0	0	1.0000

R1 =

0.4344	0.0271	0.9003
-0.0139	0.9996	-0.0234
-0.9006	-0.0024	0.4346

t1 =

-1.8360
-0.1582
1.1219

```
F = inv(K)'*Vec2Skew(t1)*R1*inv(K);
```

F =

0.0000	-0.0000	0.0013
-0.0000	0.0000	0.0055
0.0016	-0.0010	-1.8055

Rank =
2

```
e1 = null(F);
e1 = e1(1:2)/e1(3)
```

e1 =
2167.6
1061.1

```
e2 = null(F');
e2 = e2(1:2)/e2(3);
```

e2 =
352.0323
877.7648

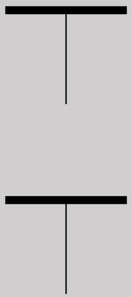
Camera Motion




Camera Motion

Image 2

Image 1



Forward motion

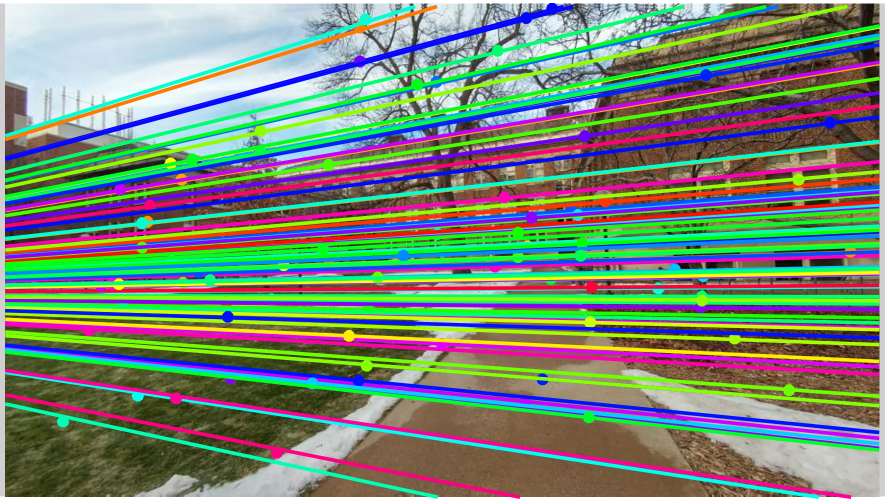


Image 2

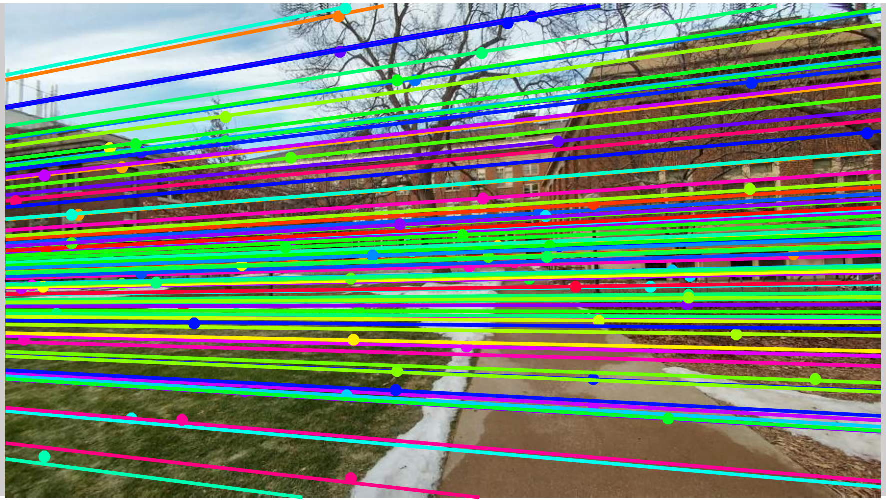


Image 1

Lateral motion