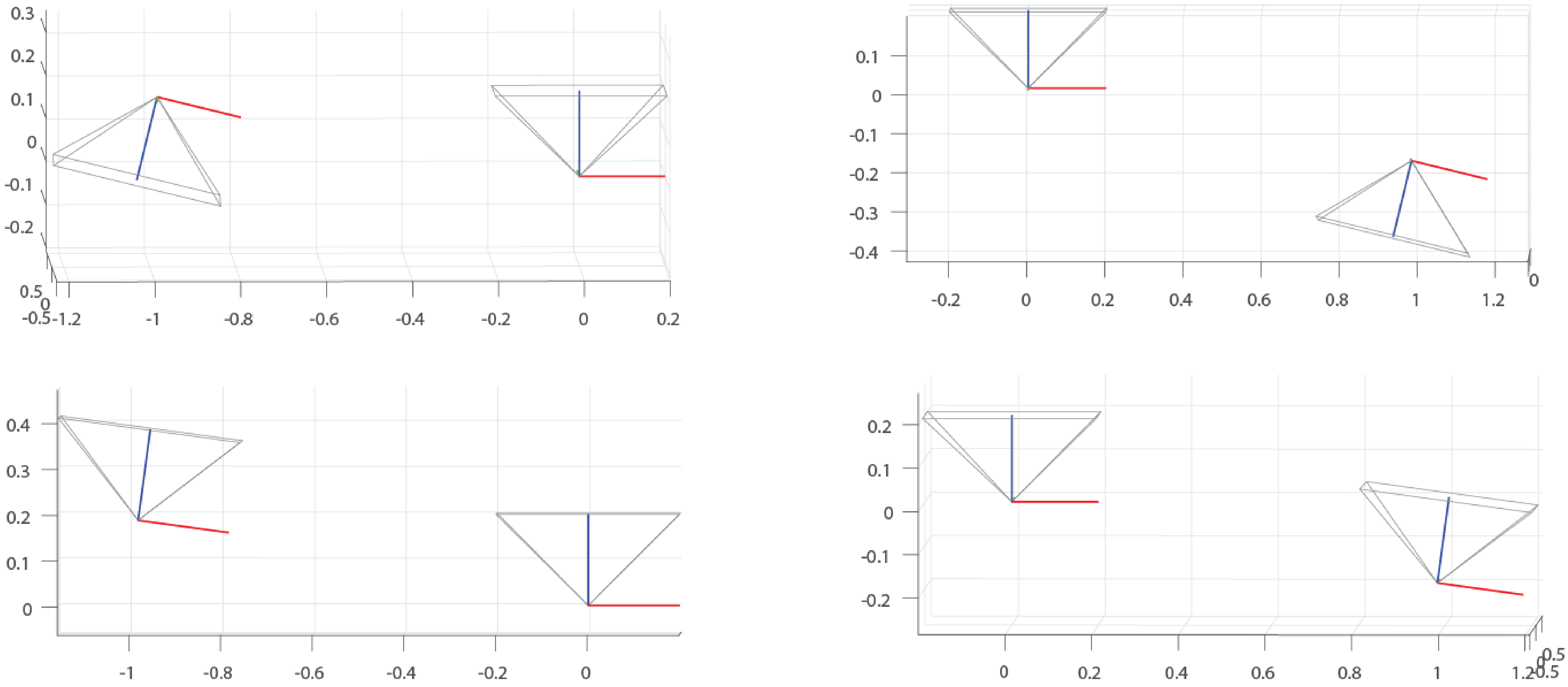
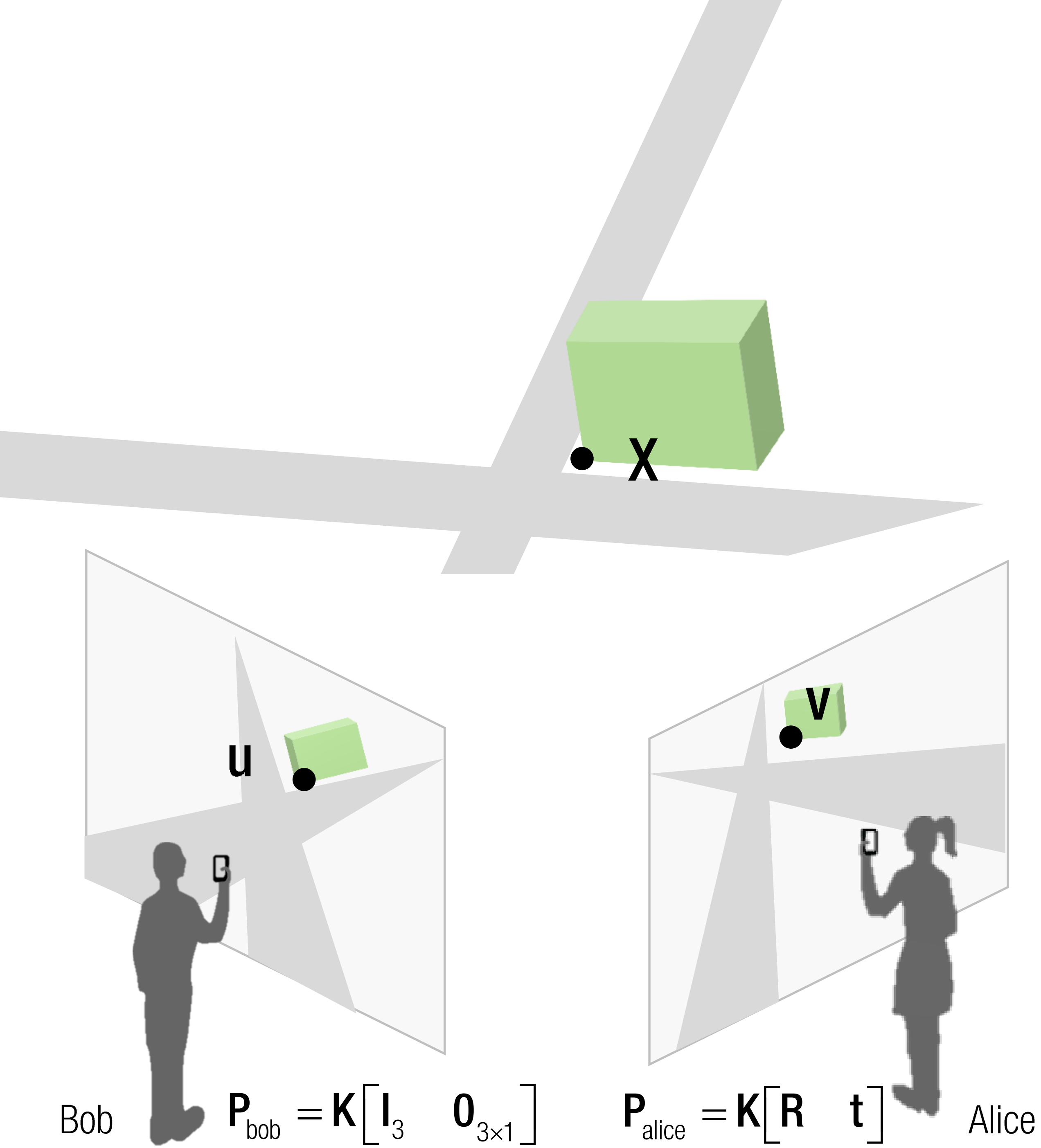
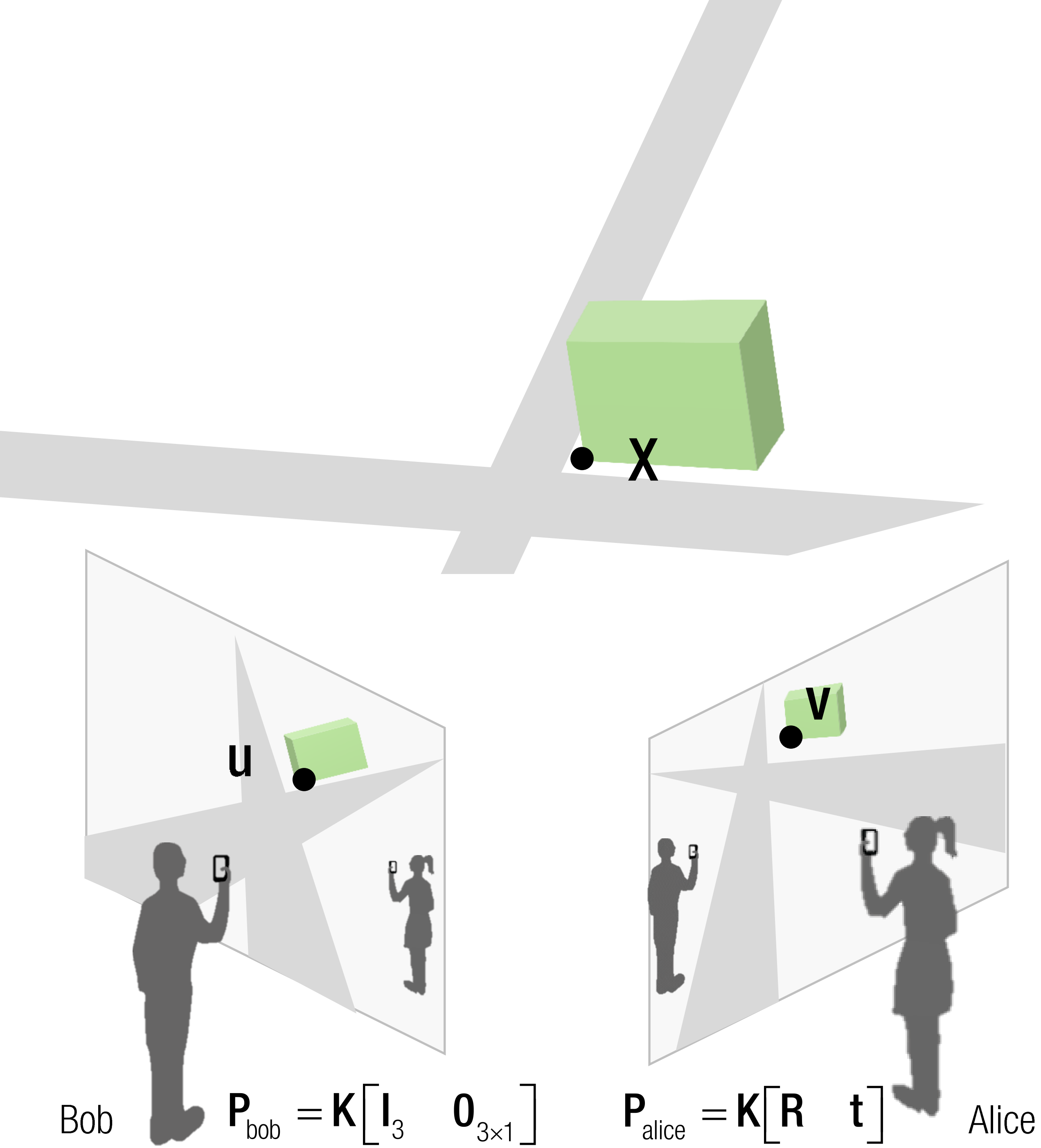
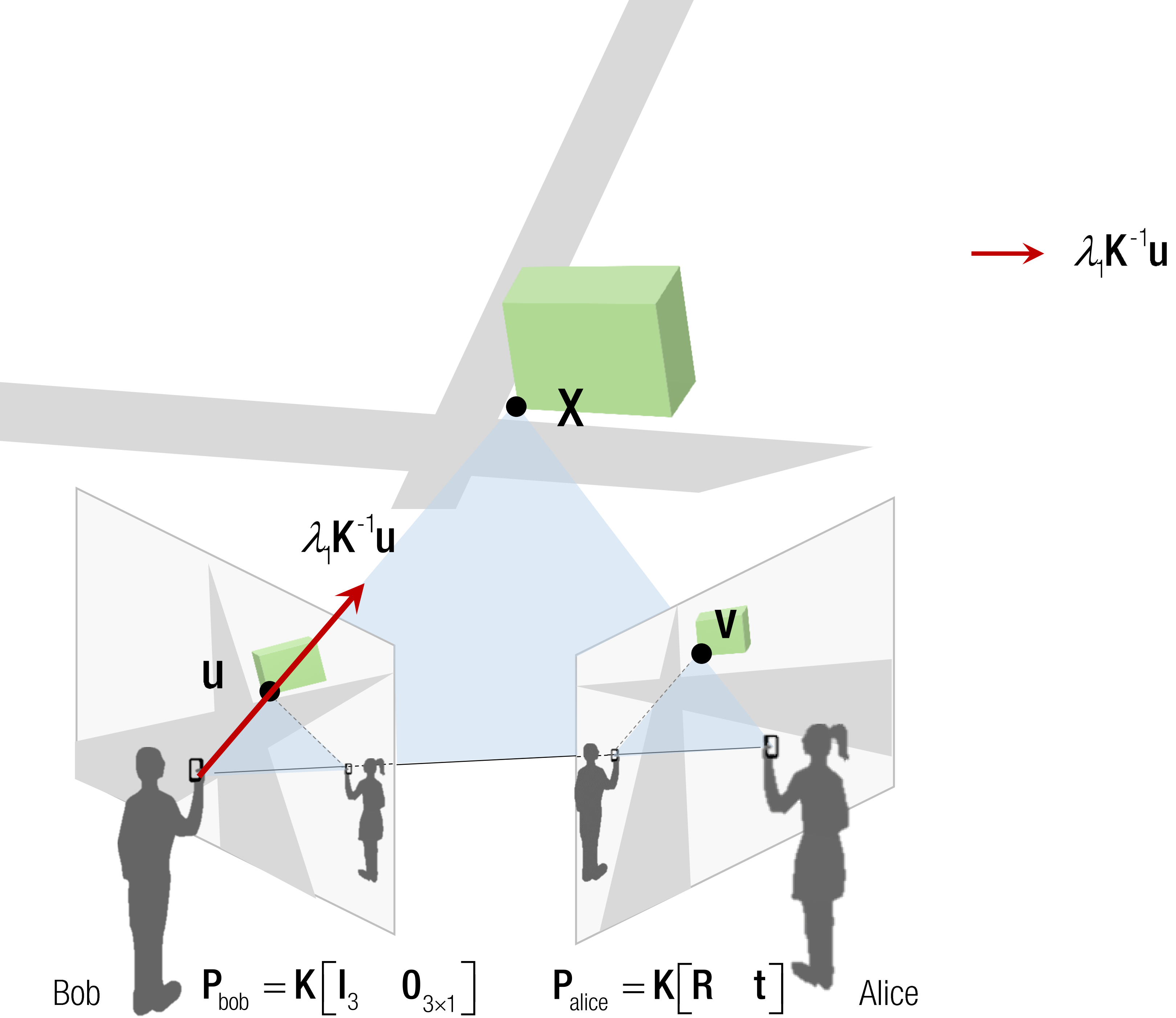
Triangulation

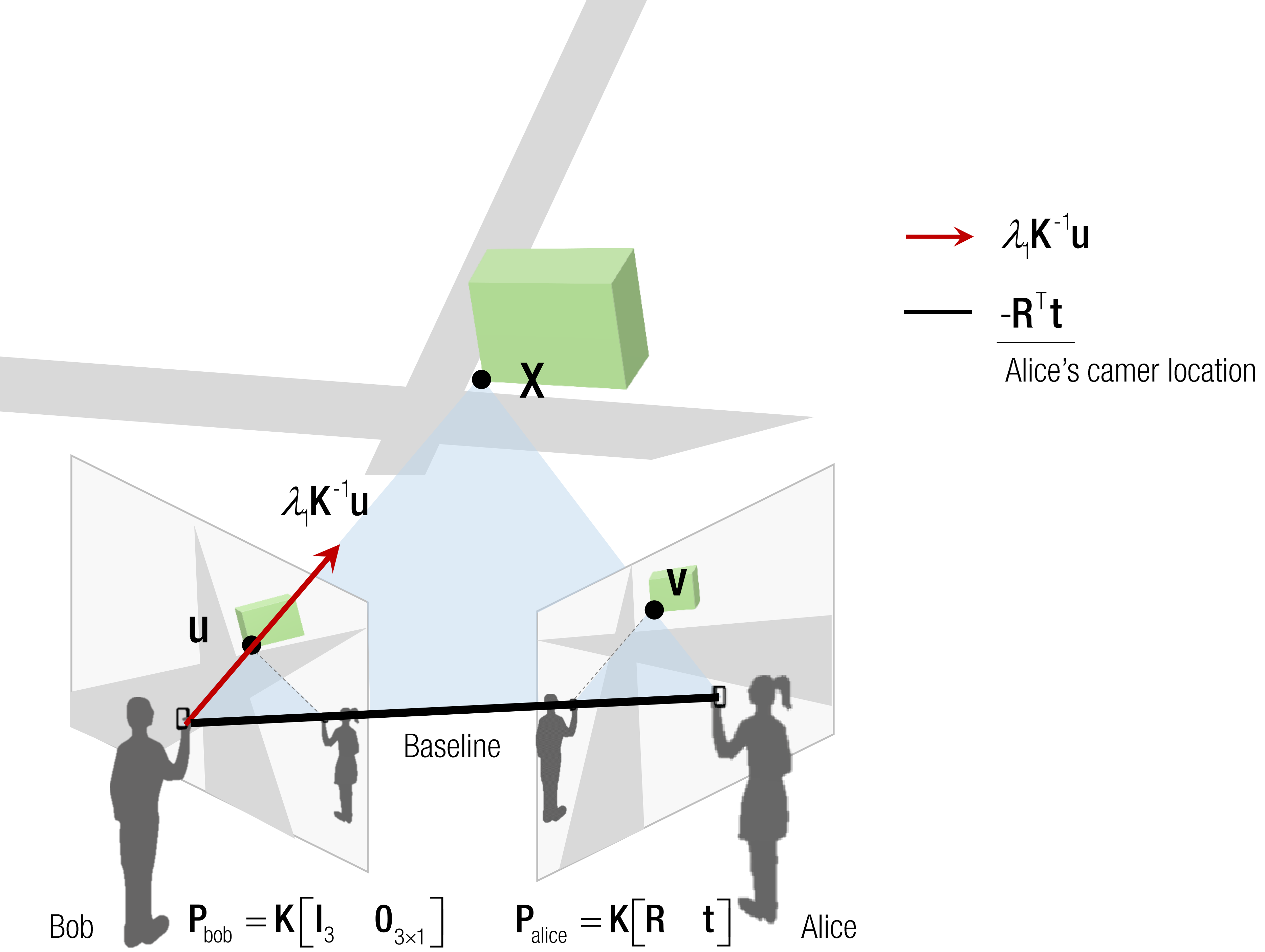
How to Disambiguate?

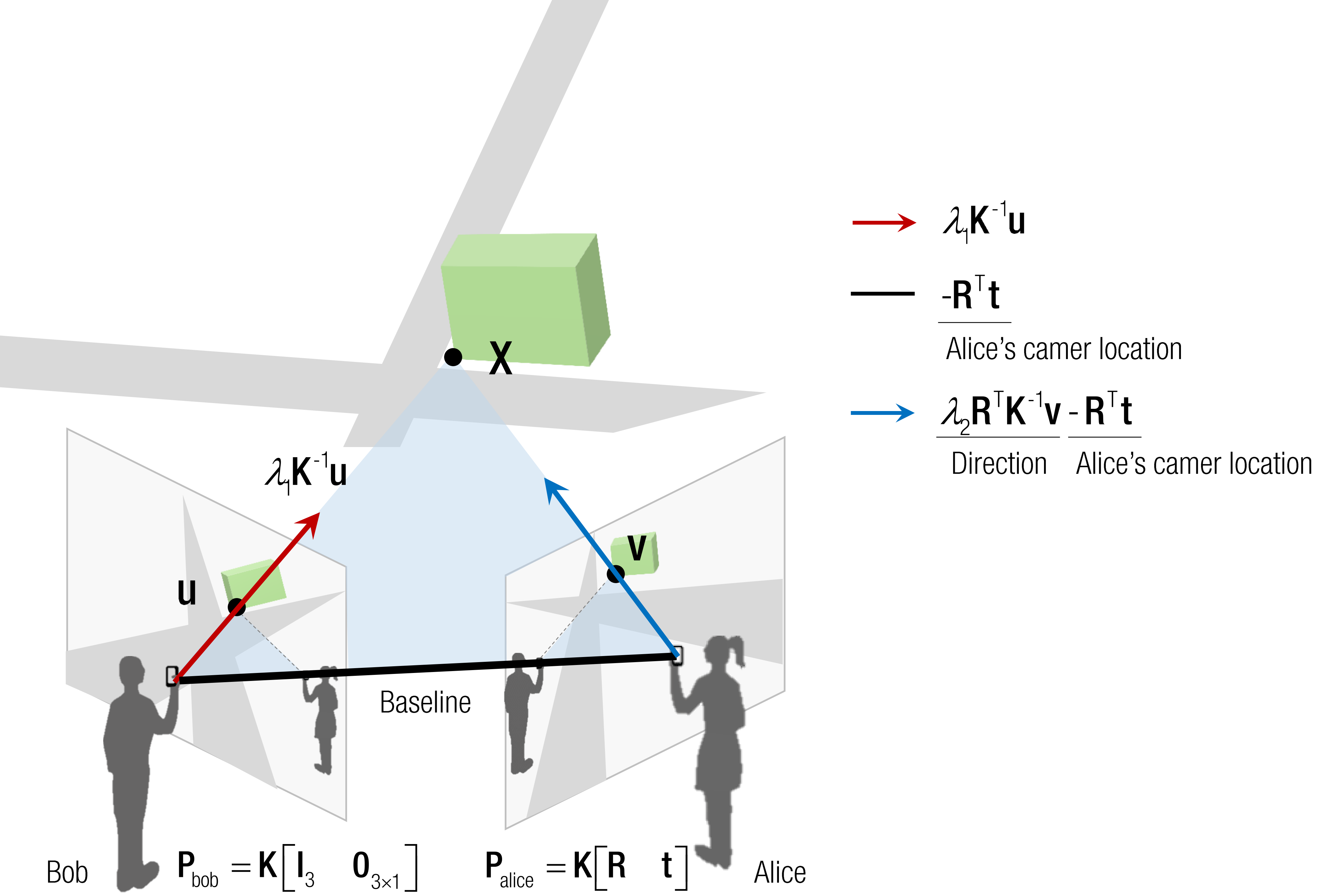



X
u
v
Bob
$\mathbf{P}_{\text{bob}} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix}$
$\mathbf{P}_{\text{alice}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Alice

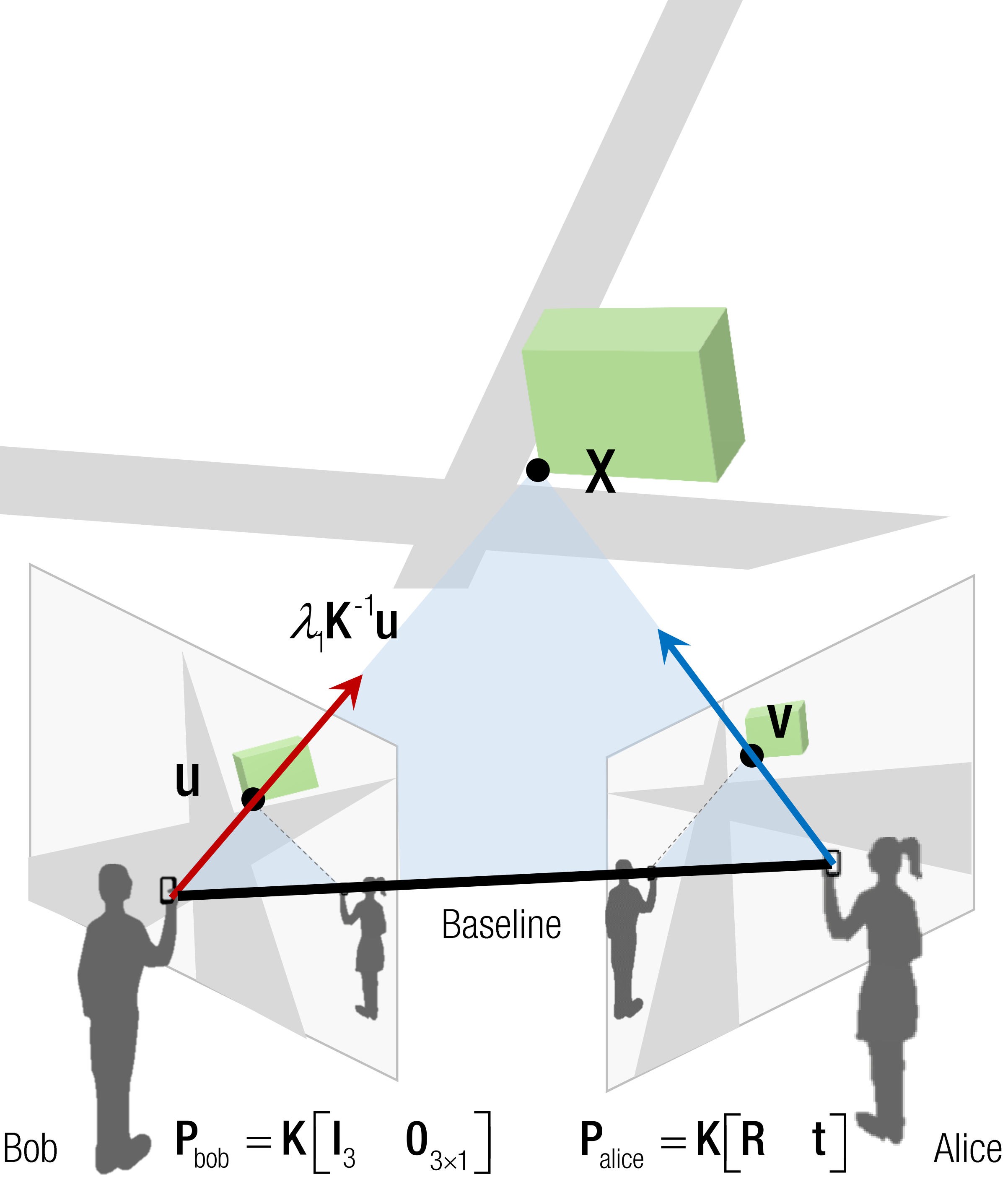

X
u
v
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice


$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$
X
$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$
u
v
Bob
$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_{3\times1}\end{bmatrix}$
$\mathbf{P}_{alice} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$
Alice


λ1K-1u
-RTt
Alice's camer location
X
λ1K-1u
u
v
Baseline
Bob
Pbob = K[I3 03×1]
Palice = K[R t]
Alice



$\lambda_1 K^{-1} u$
$-R^T t$
Alice's camer location
$\lambda_2 R^T K^{-1} v - R^T t$
Direction
Alice's camer location
X
$\lambda_1 K^{-1} u$
u
v
Baseline
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $\quad$ $P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$

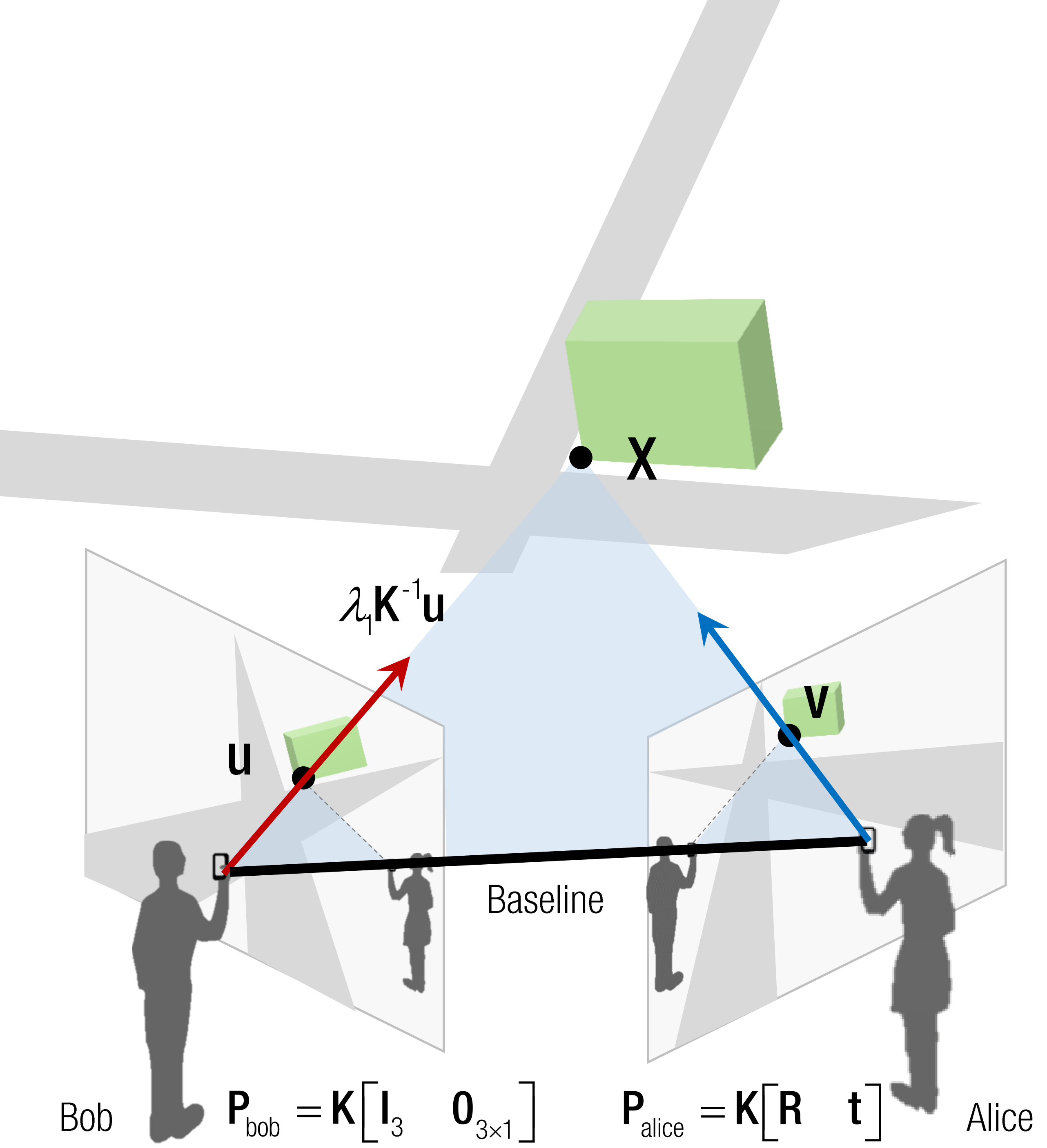
$\lambda_1 K^{-1} u$

$\underbrace{-R^T t}_{\text{Alice's camer location}}$

$\underbrace{\lambda_2 R^T K^{-1} v}_{\text{Direction}} \underbrace{- R^T t}_{\text{Alice's camer location}}$

$$\underbrace{X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v - R^T t}_{\text{3D point}}$$

# of unknowns: 2
# of equations: 3



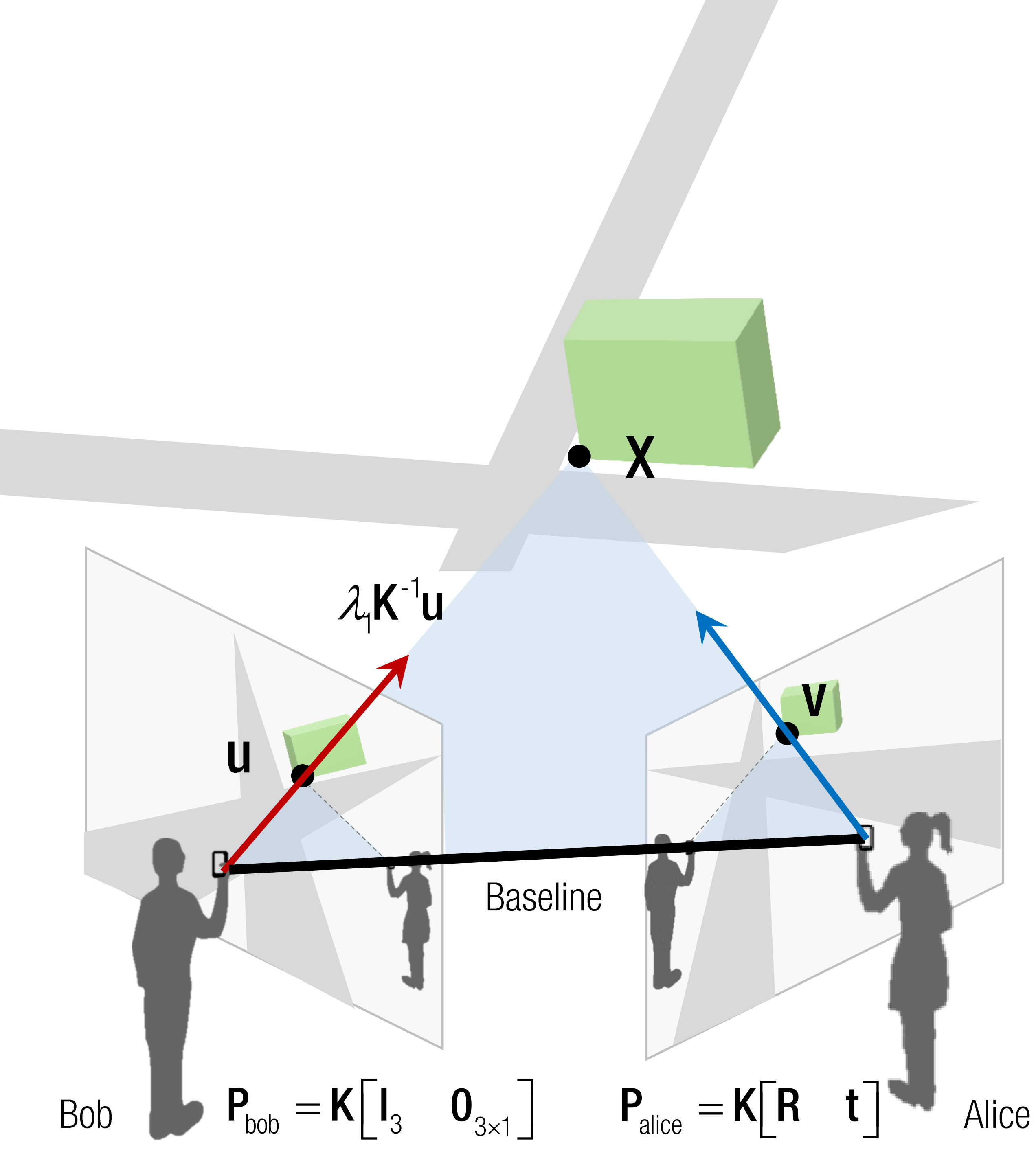
$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$

$\underbrace{-\mathbf{R}^{\mathrm{T}}\mathbf{t}}_{\text{Alice's camer location}}$

$\underbrace{\lambda_2 \mathbf{R}^{\mathrm{T}}\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}} \underbrace{-\mathbf{R}^{\mathrm{T}}\mathbf{t}}_{\text{Alice's camer location}}$

$$\underbrace{\mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathrm{T}}\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^{\mathrm{T}}\mathbf{t}}_{\text{3D point}}$$

$$\rightarrow \quad \lambda_1 \mathbf{K}^{-1}\mathbf{u} - \lambda_2 \mathbf{R}^{\mathrm{T}}\mathbf{K}^{-1}\mathbf{v} = -\mathbf{R}^{\mathrm{T}}\mathbf{t}$$



$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$

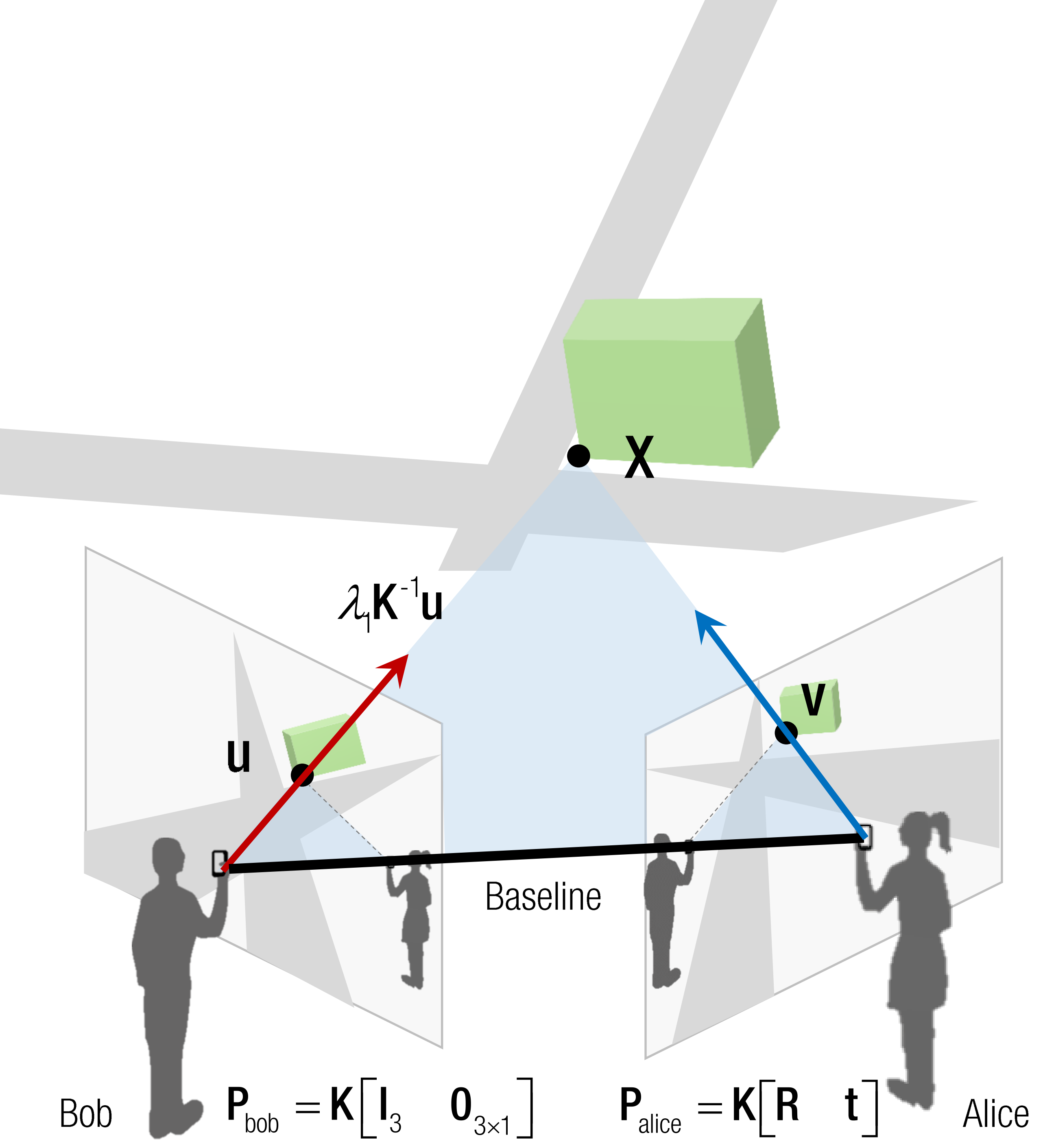
$\underbrace{-\mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{Alice's camer location}}$

$\underbrace{\lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}}\ \underbrace{-\mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{Alice's camer location}}$

$$\underbrace{\mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{3D point}}$$

$$\rightarrow \lambda_1 \mathbf{K}^{-1}\mathbf{u} - \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} = -\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

$$\rightarrow \begin{bmatrix} \mathbf{K}^{-1}\mathbf{u} & -\mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = -\mathbf{R}^{\mathsf{T}}\mathbf{t}$$



$\lambda_1 K^{-1} u$

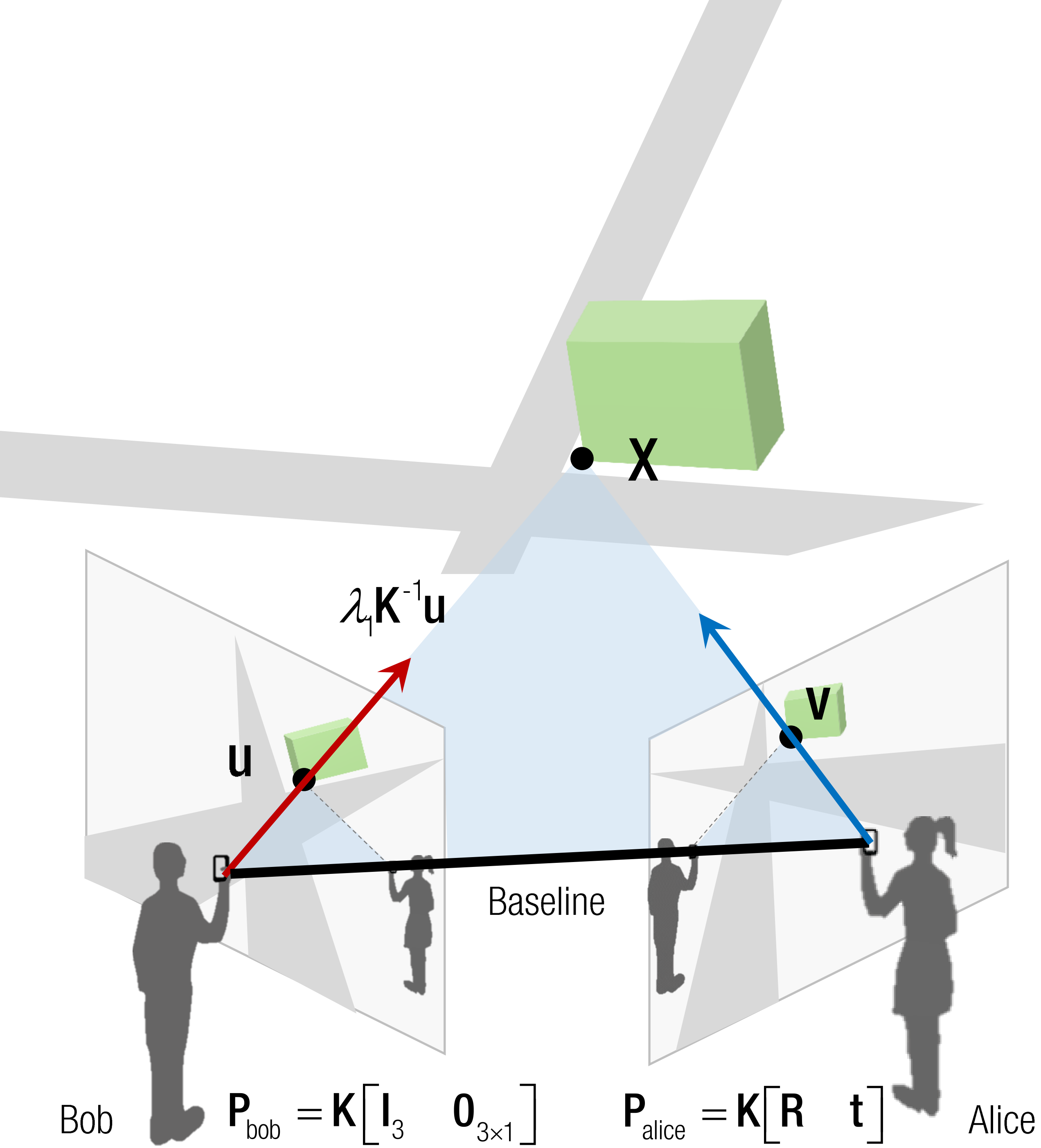
$\underbrace{-R^T t}_{\text{Alice's camer location}}$

$\underbrace{\lambda_2 R^T K^{-1} v}_{\text{Direction}} \underbrace{- R^T t}_{\text{Alice's camer location}}$

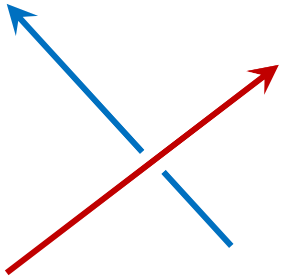
$$\underbrace{X = \lambda_1 K^{-1} u = \lambda_2 R^T K^{-1} v - R^T t}_{\text{3D point}}$$

$$\rightarrow \lambda_1 K^{-1} u - \lambda_2 R^T K^{-1} v = -R^T t$$

$$\rightarrow \underbrace{\begin{bmatrix} K^{-1}u & R^T K^{-1} v \end{bmatrix}}_{\text{3x2}}^{A} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}}_{x} = \underbrace{-R^T t}_{b}$$

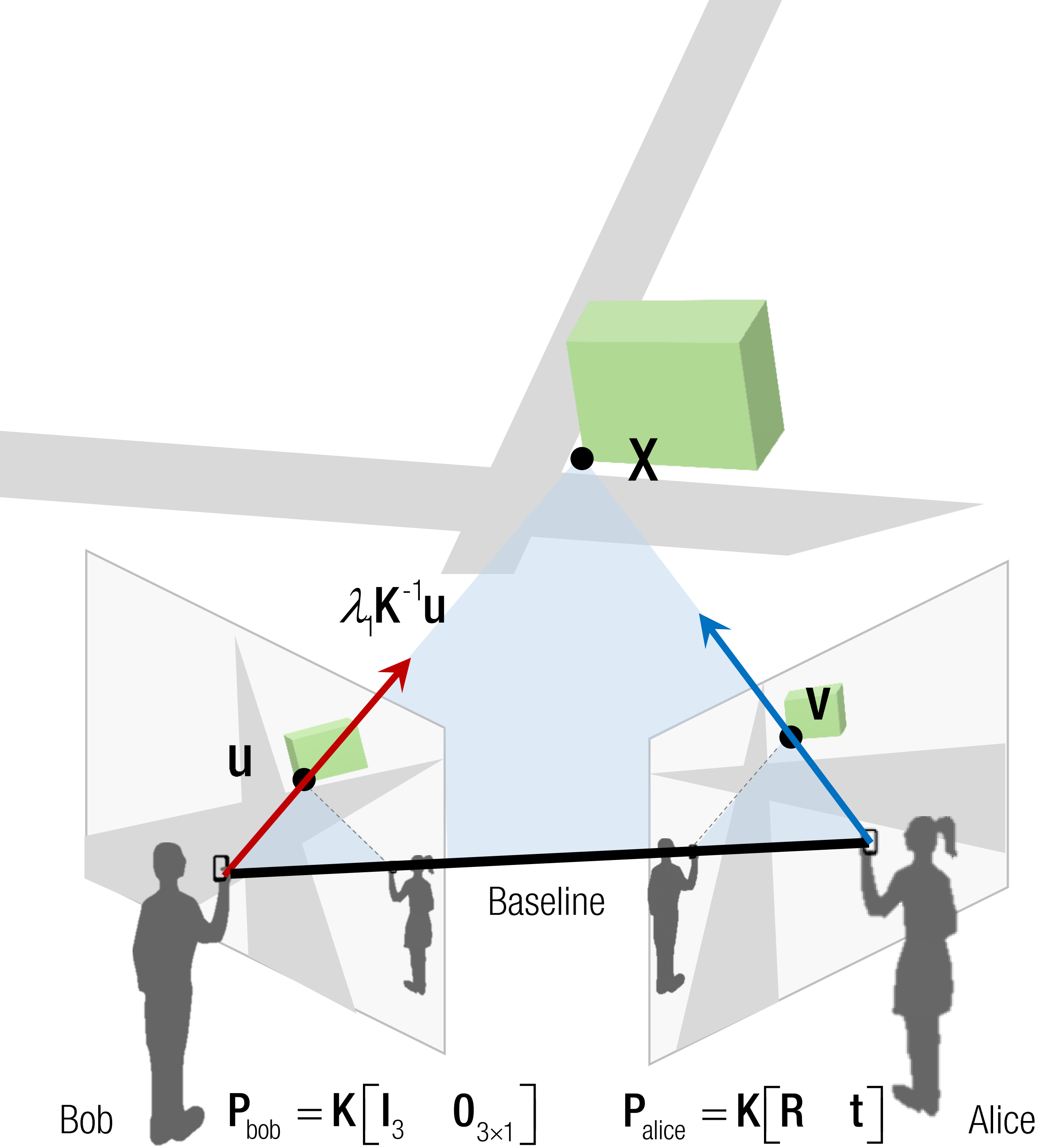


What if two does not meet at a point?



$$\to \underbrace{\begin{bmatrix} K^{-1}u & -R^{T}K^{-1}v \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{-R^{T}t}_{\mathbf{b}}$$

3x2



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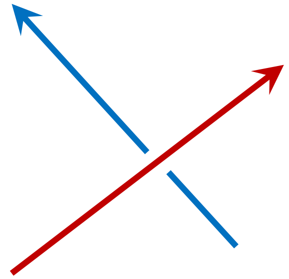
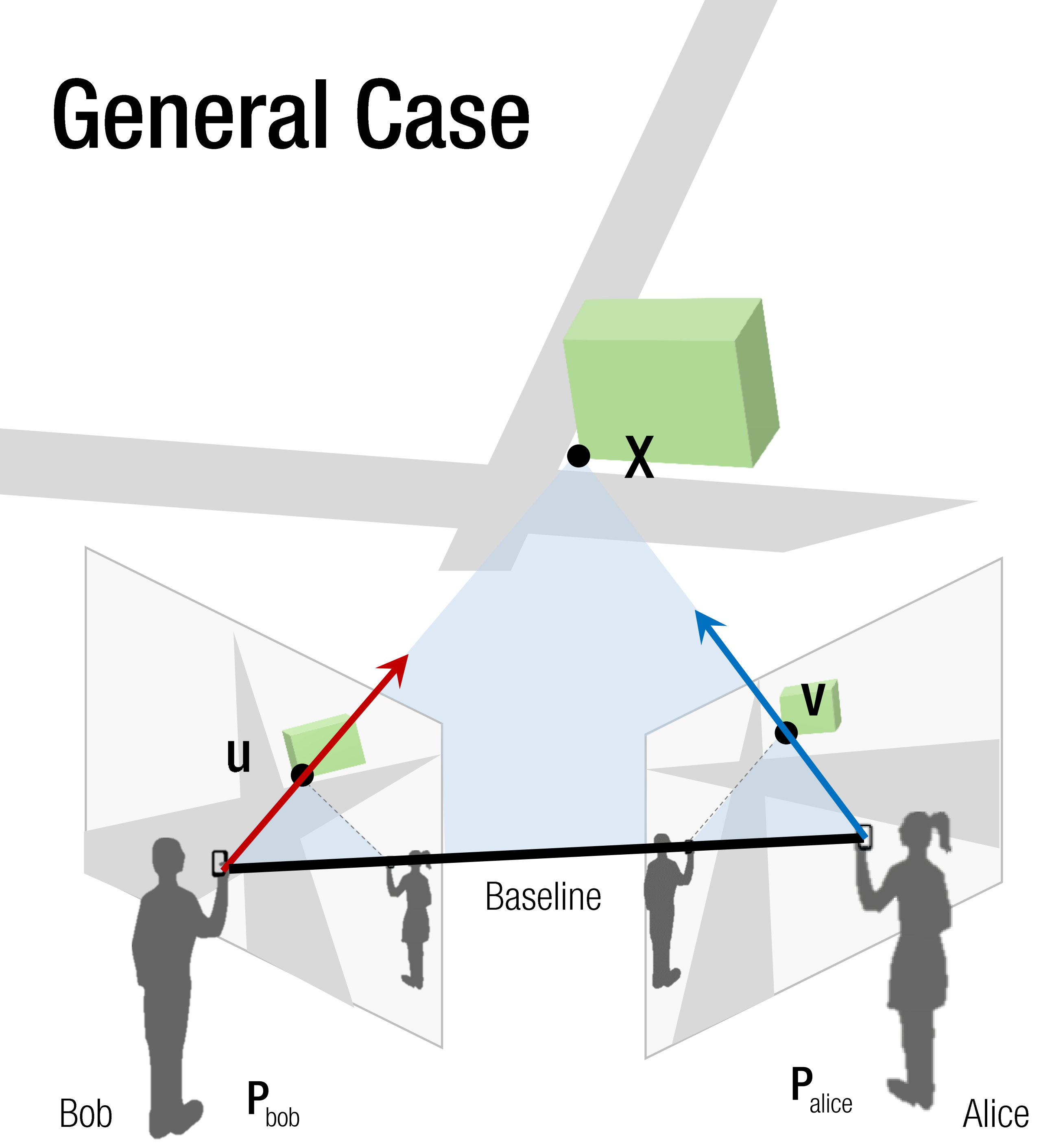


Least square solution finds *somewhere* in the middle.

$$\rightarrow \underbrace{\begin{bmatrix} K^{-1}u & R^{T}K^{-1}v \end{bmatrix}}_{\substack{\mathbf{A} \\ 3\text{x}2}} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{-R^{T}t}_{\mathbf{b}}$$

General Case



General camera pose

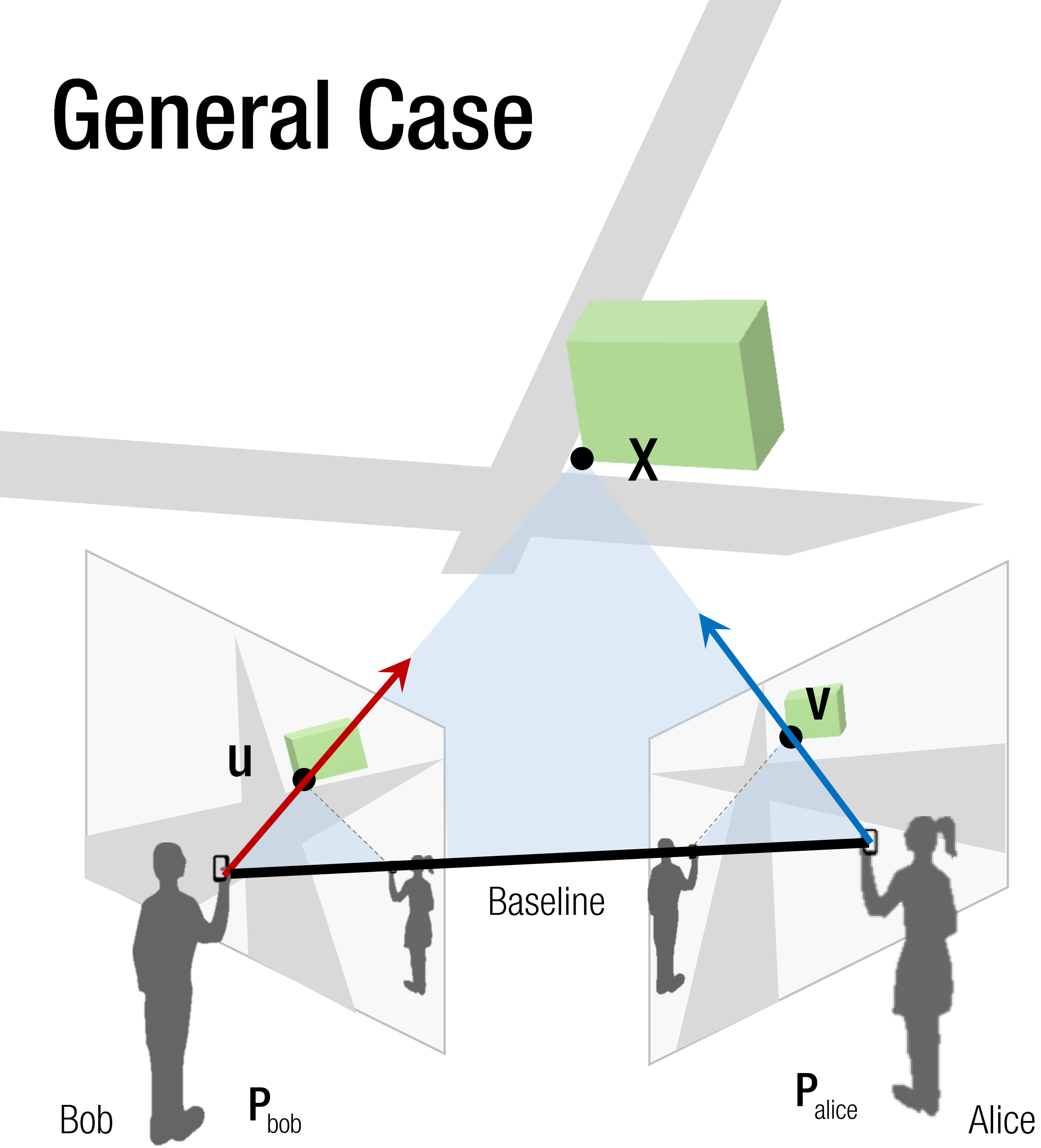
$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \underbrace{\begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times}}_{\text{Skew-symmetric matrix}} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

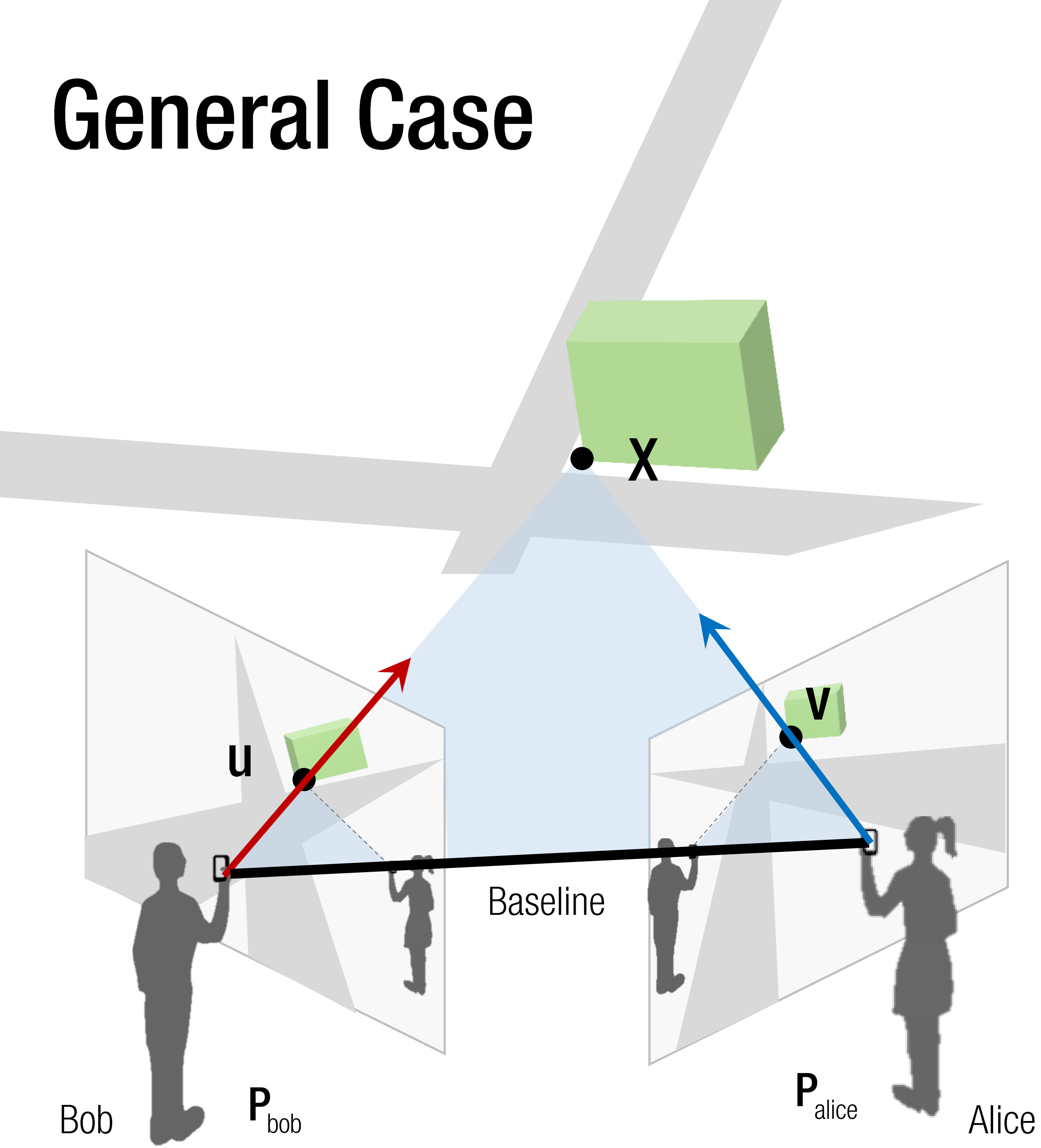
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Skew-symmetric matrix

: Knowns

: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

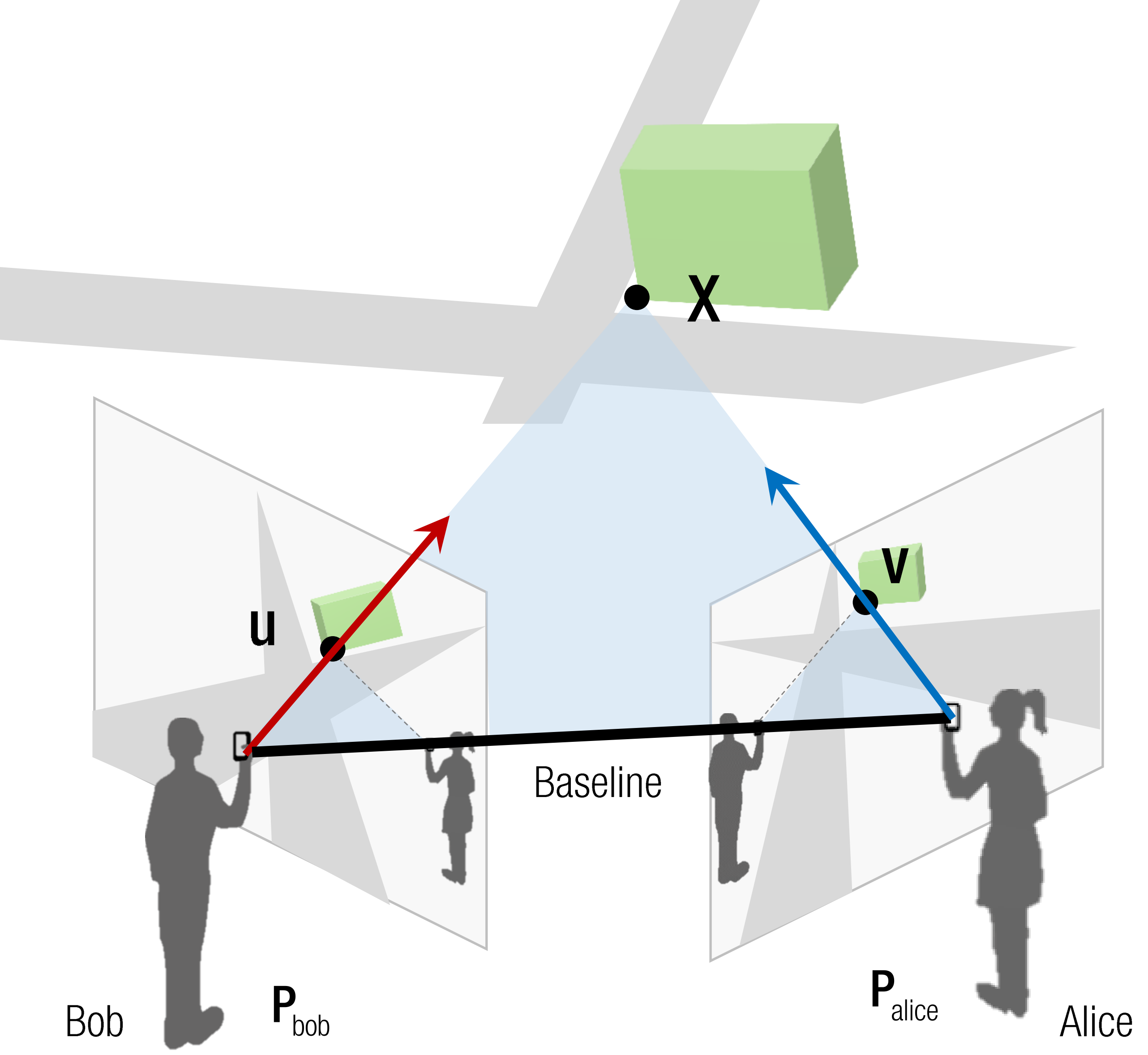
$$\rightarrow \underbrace{\begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob}}_{3\text{x}4} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

: Knowns

: Unknowns

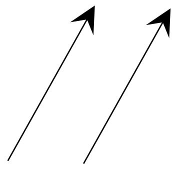
Can we solve for **X**? (single view reconstruction)
Why not?

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

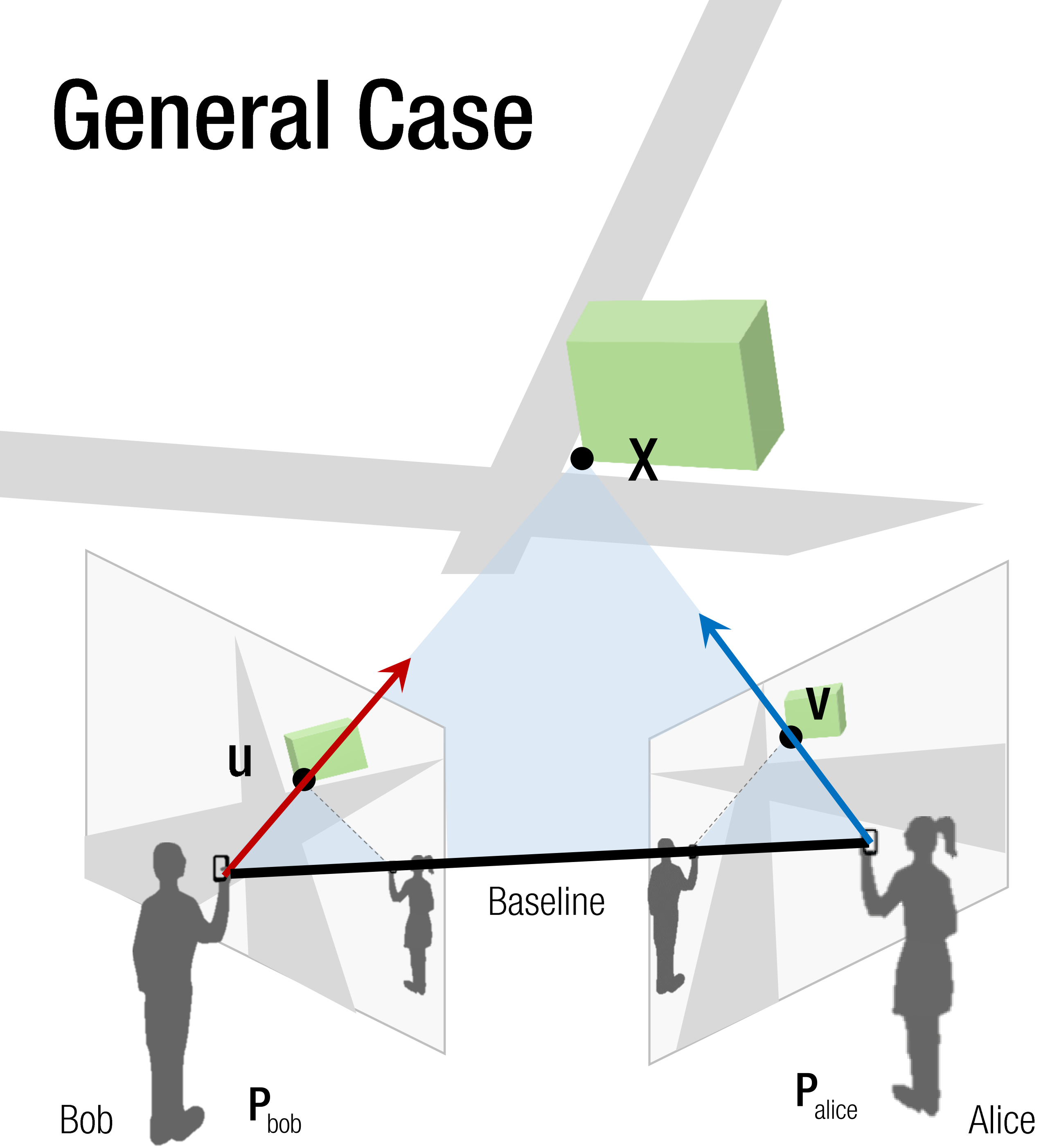
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \underbrace{\begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob}}_{2\text{x}4} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

: Knowns

: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

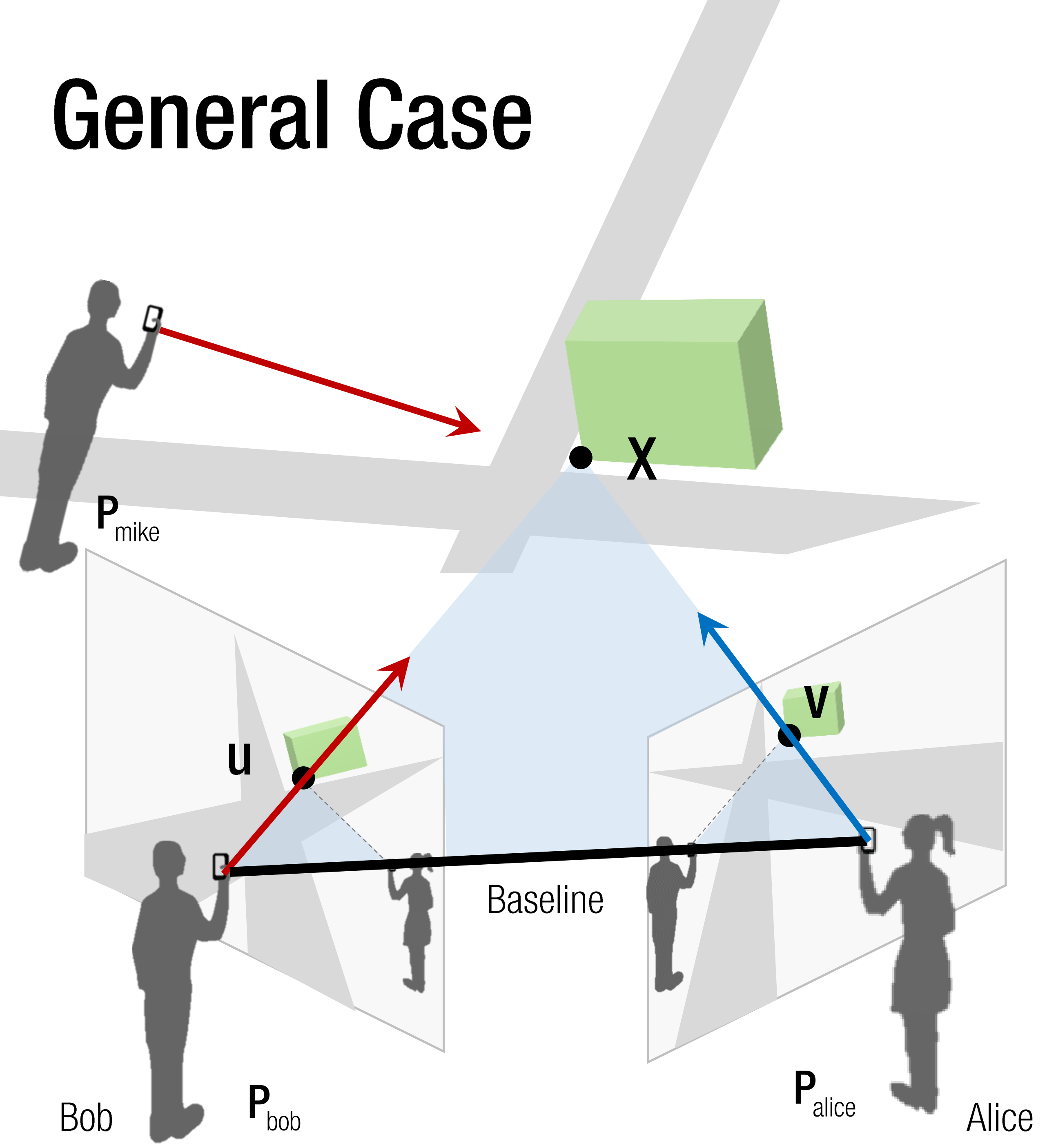
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \underbrace{\begin{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_\times \mathbf{P}_{\text{bob}} \\ \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_\times \mathbf{P}_{\text{alice}} \end{bmatrix}}_{4\text{x}4} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

: Knowns

: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\mathrm{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\mathrm{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

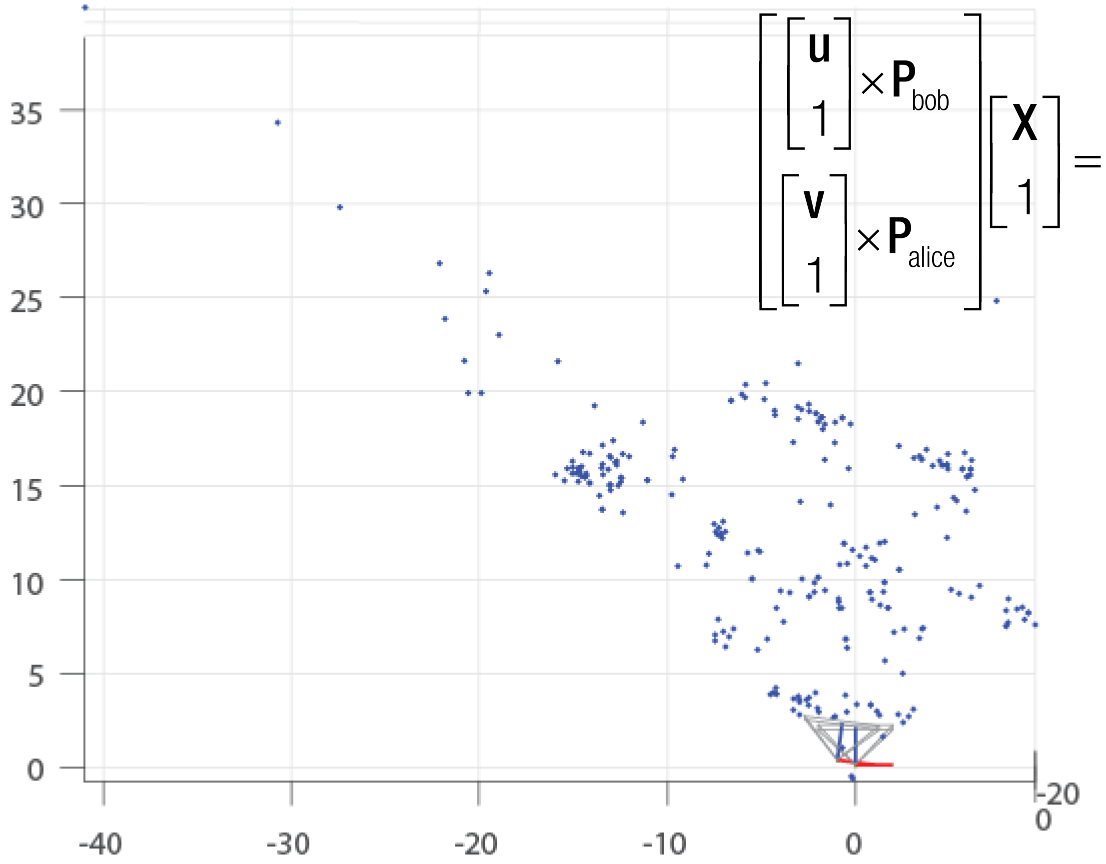
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\mathrm{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\mathrm{alice}}$$

$$\begin{bmatrix} \mathbf{w} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\mathrm{mike}}$$

: Knowns

: Unknowns



$$\begin{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\text{bob}} \\ \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\text{alice}} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Download **Triangulation.m** and **triangulation.mat** files

```
function Triangulation
%% Data loading
load('triangulation.mat');

% (C1, R1) and (C2, R2) are camera center and orientation of camera 1 and 2, respectively.
% u and v are Nx2 correspondences

%% Camera matrix build
K = [700/2 0 960/2;
     0 700/2 540/2;
     0 0 1];
% Build camera matrix 1 and 2
% P1
% P2

%% Triangulation
% Go to each correspondence and compute the 3D point X (3xN) matrix
for i = 1 : size(u,1)
    % Construct A matrix
    % Solve linear least squares to get 3D point
    % X(:,i) = point_3d;
end
```

Fill out

Fill out

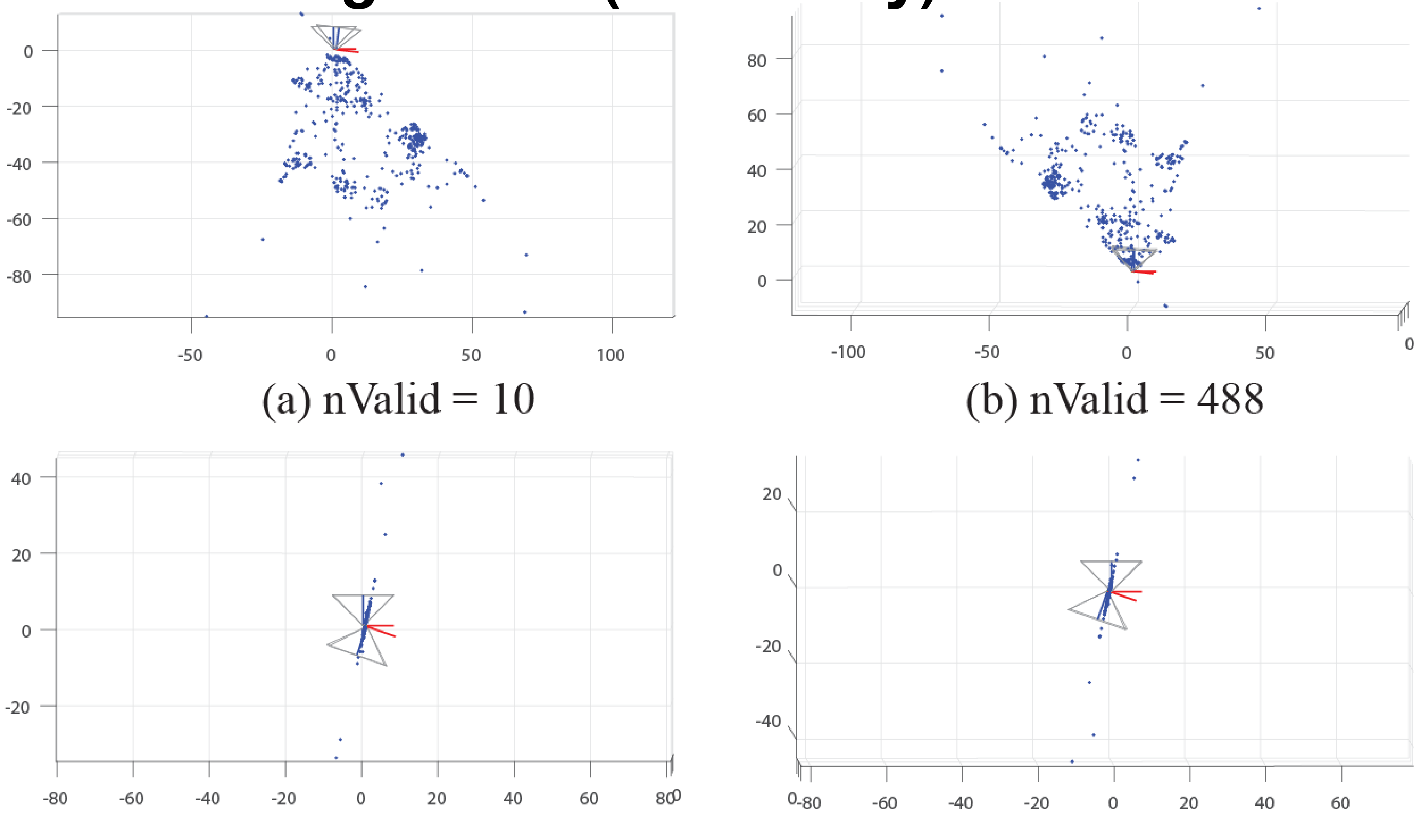
Camera Pose Disambiguation (Cheirality)

Cheirality condition:

$$\mathbf{r}_3^T (\mathbf{X} - \mathbf{C}) > 0$$

where

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}$$



(a) nValid = 10

(b) nValid = 488

(c) nValid = 0

(d) nValid = 0

Geometric Verification: Reprojection Error

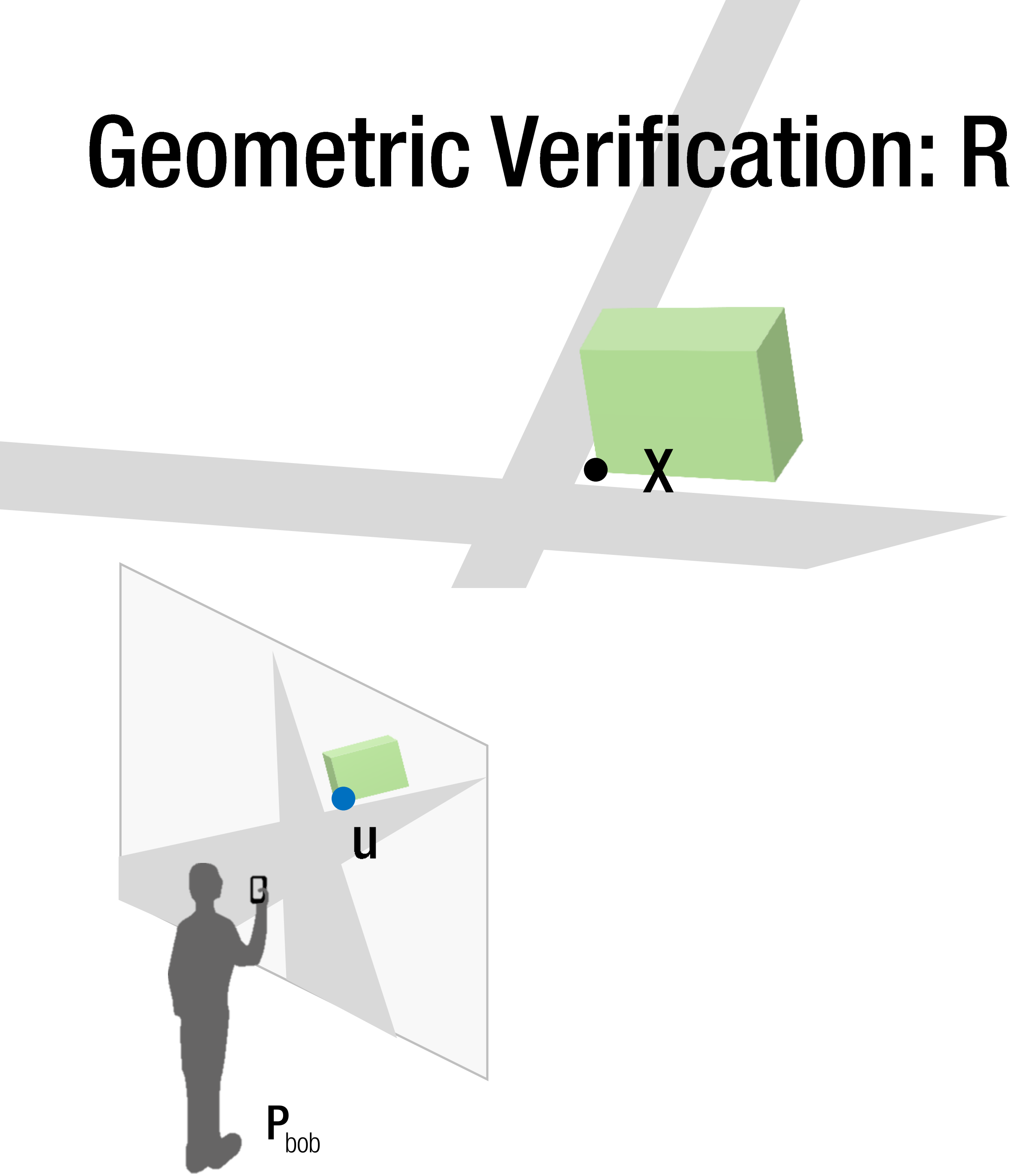


Image feature measurement, e.g. SIFT detection:

$\mathbf{u}$

Geometric Verification: Reprojection Error

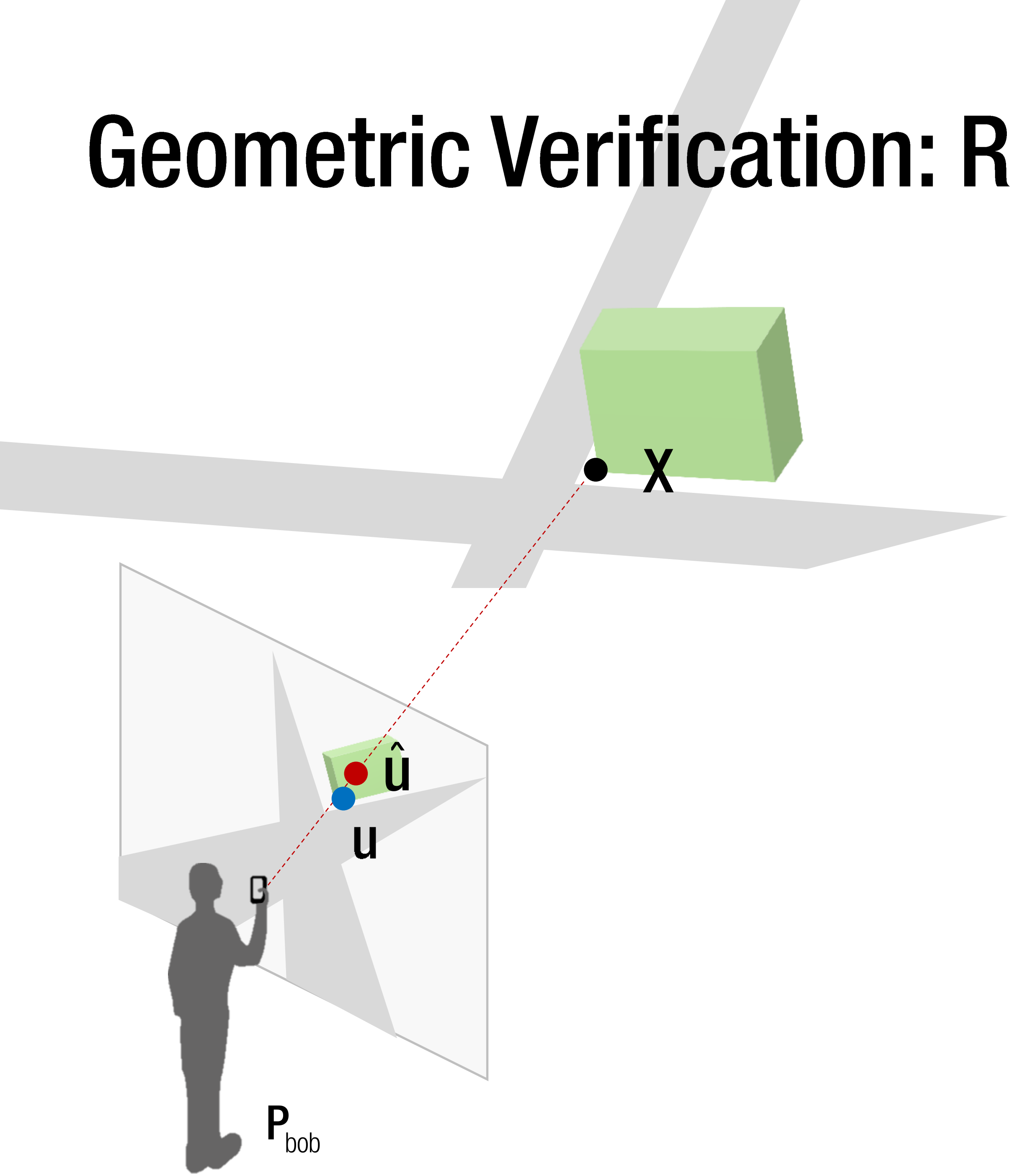


Image feature measurement, e.g. SIFT detection:

$\mathbf{u}$

3D point projection, or reprojection:

$$\lambda \hat{\mathbf{u}} = \mathbf{PX}$$

Geometric Verification: Reprojection Error

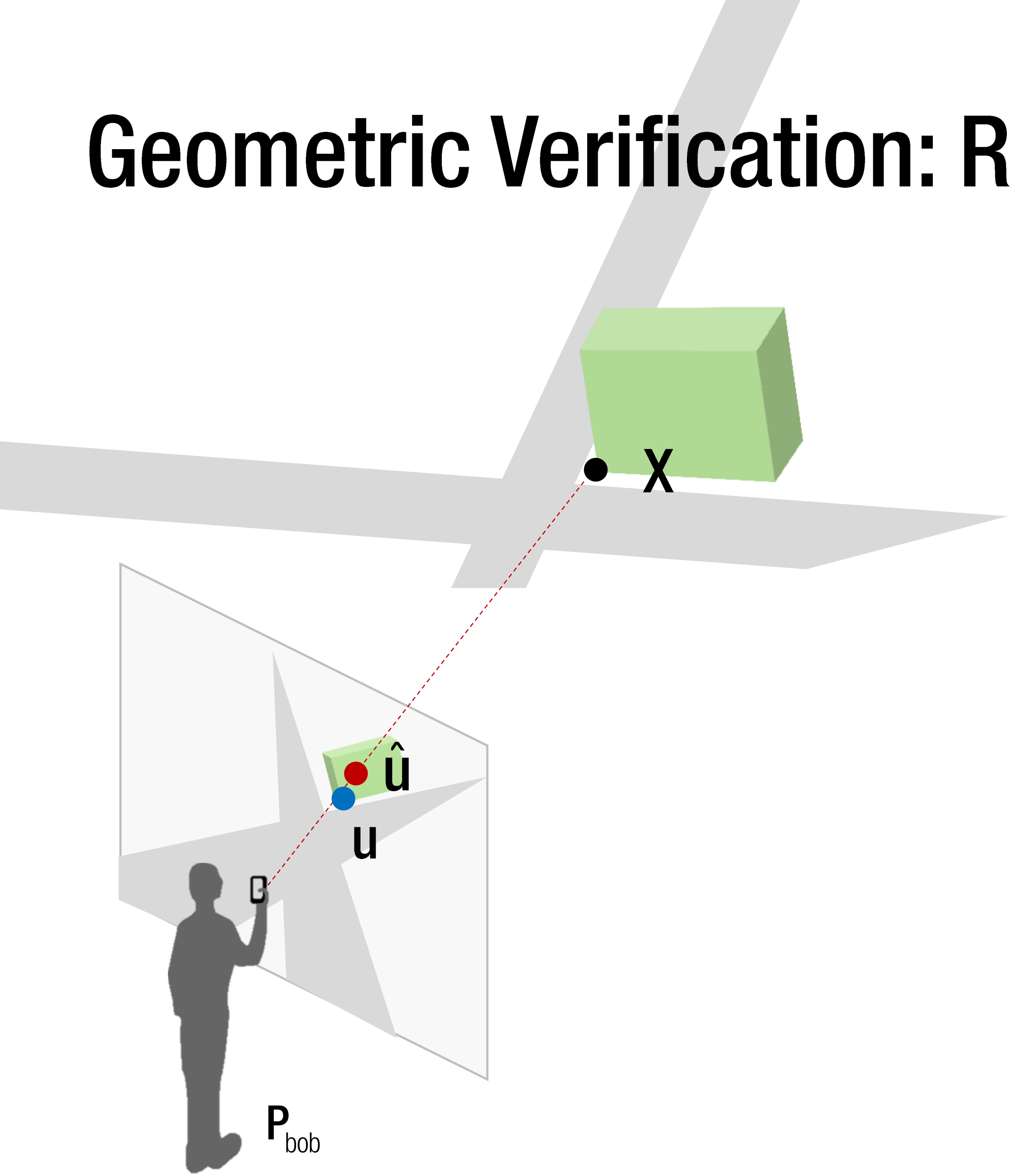


Image feature measurement, e.g. SIFT detection:

$$\mathbf{u}$$

3D point projection, or reprojection:

$$\lambda \hat{\mathbf{u}} = \mathbf{P}\mathbf{X}$$

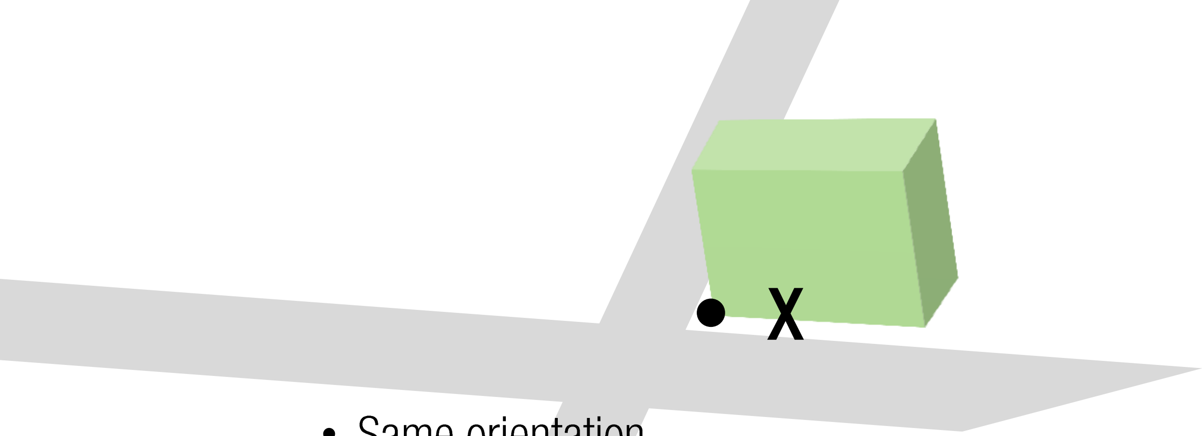
Reprojection error (geometric error):

$$e = \left\| \hat{\mathbf{u}} - \mathbf{u} \right\|$$

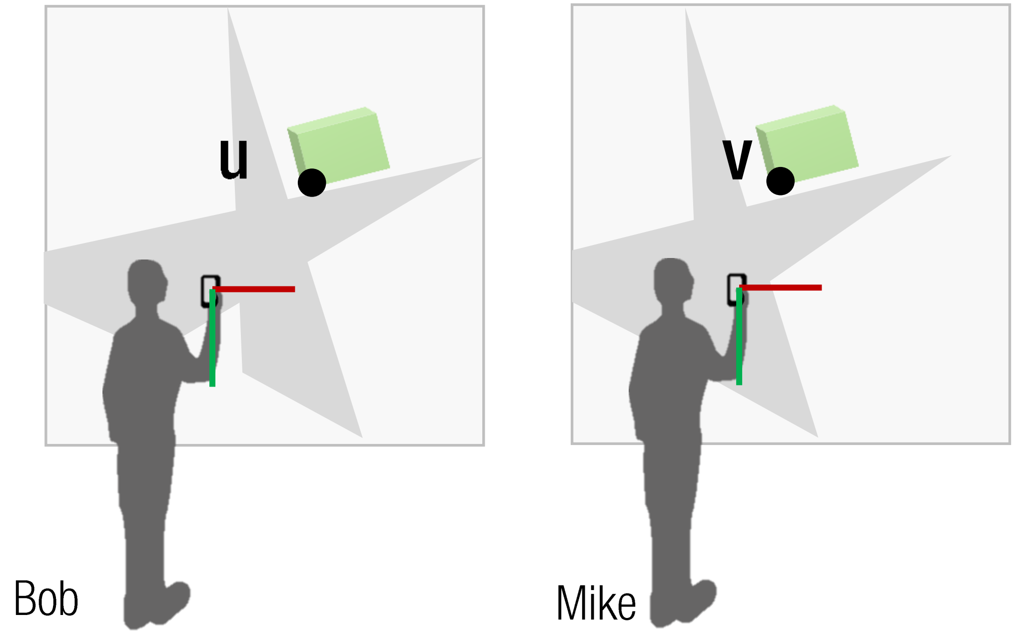

Reprojection
Measurement


Reprojection
Measurement

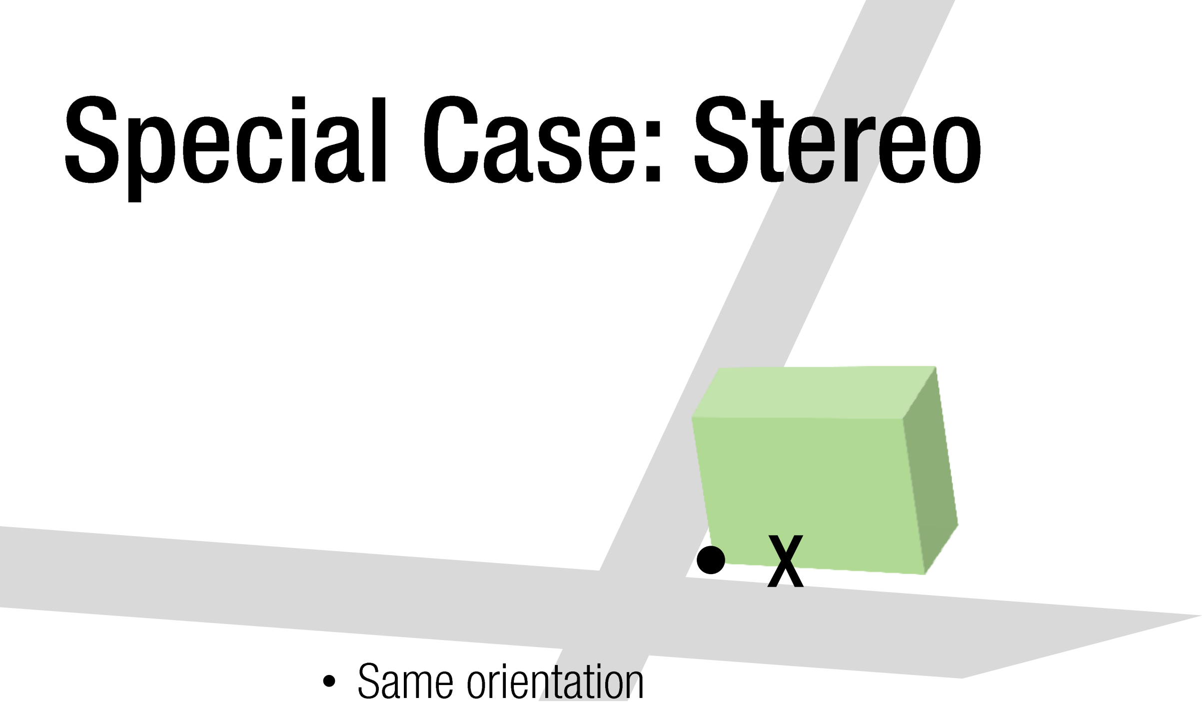
Special Case: Stereo



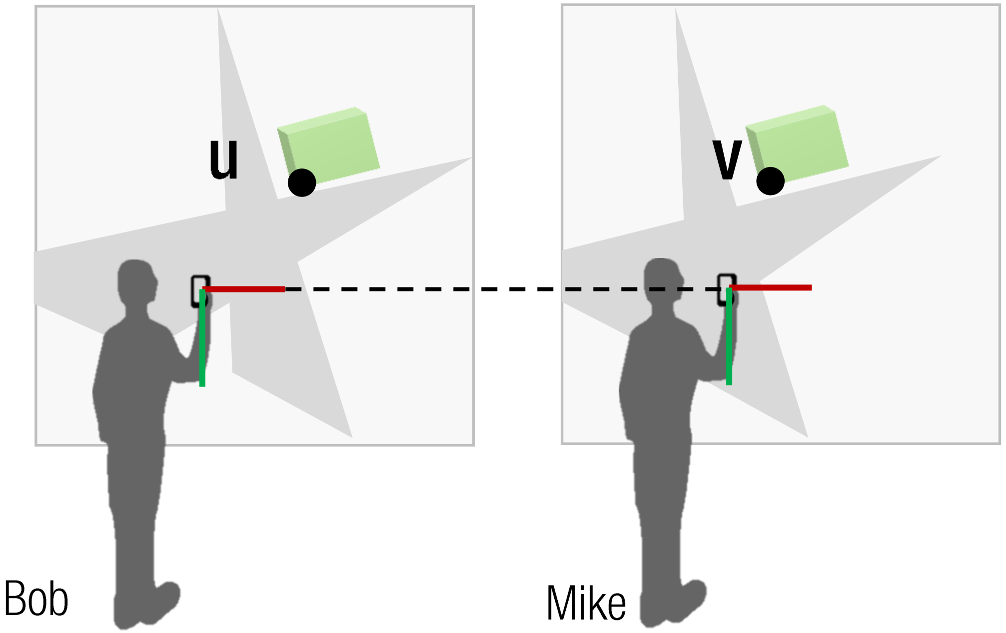
- Same orientation



Special Case: Stereo

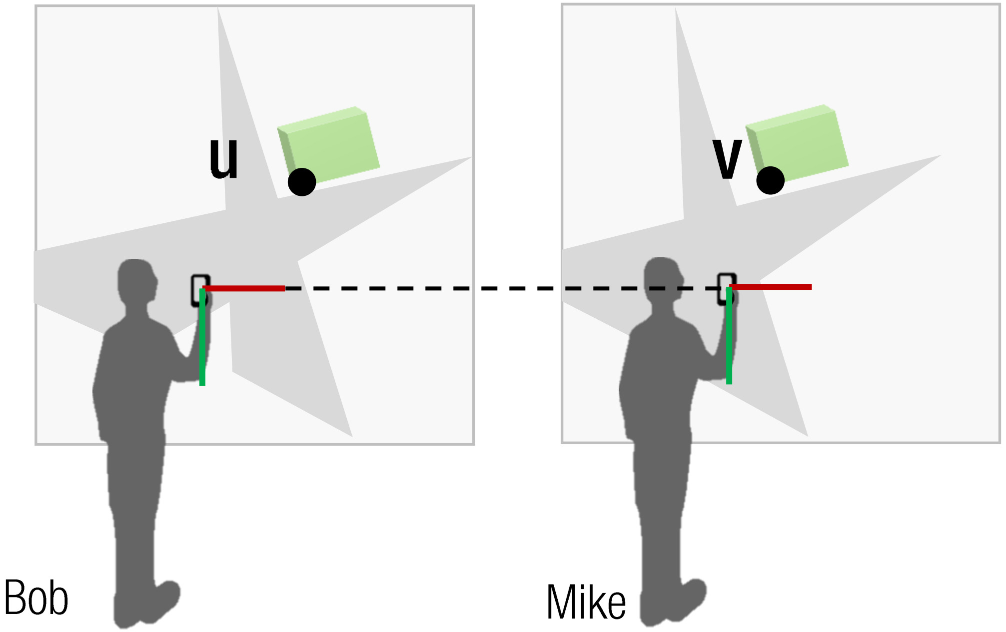


- Same orientation
- Alignment between X axis and baseline



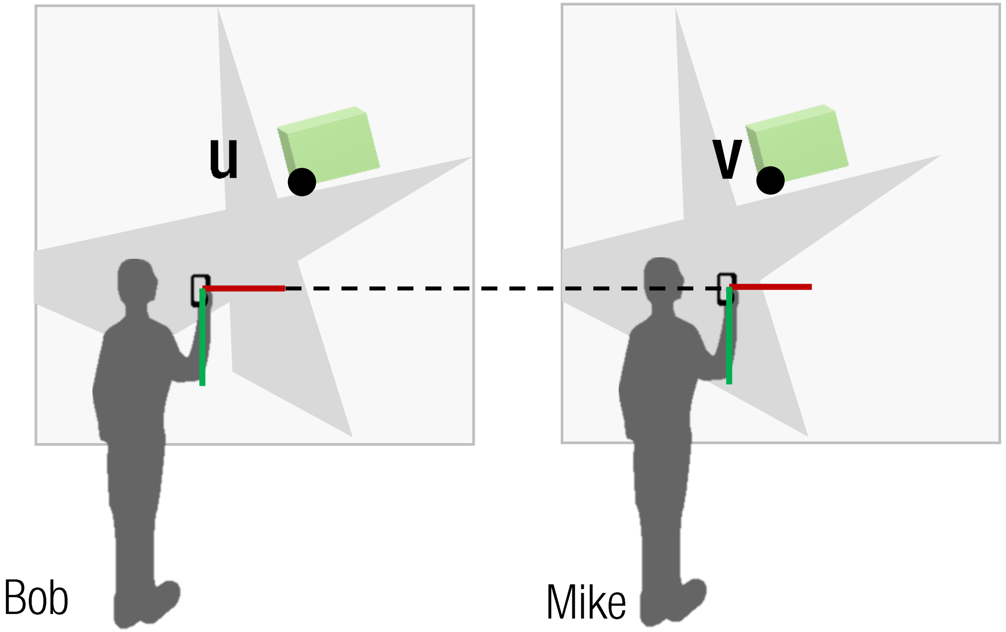
Special Case: Stereo

- Same orientation
- Alignment between X axis and baseline



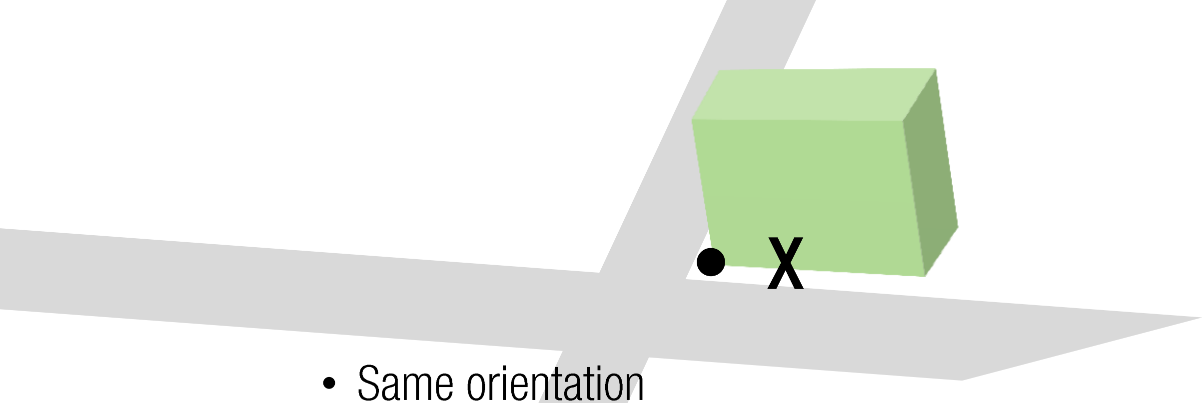
Special Case: Stereo

- Same orientation
- Alignment between X axis and baseline

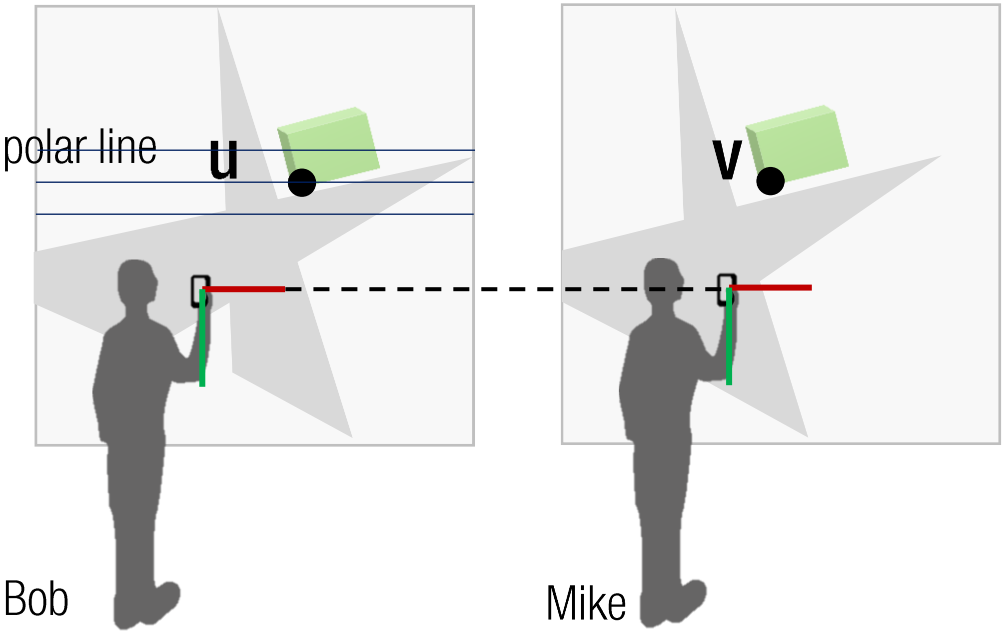


Top view

Special Case: Stereo



- Same orientation
- Alignment between X axis and baseline

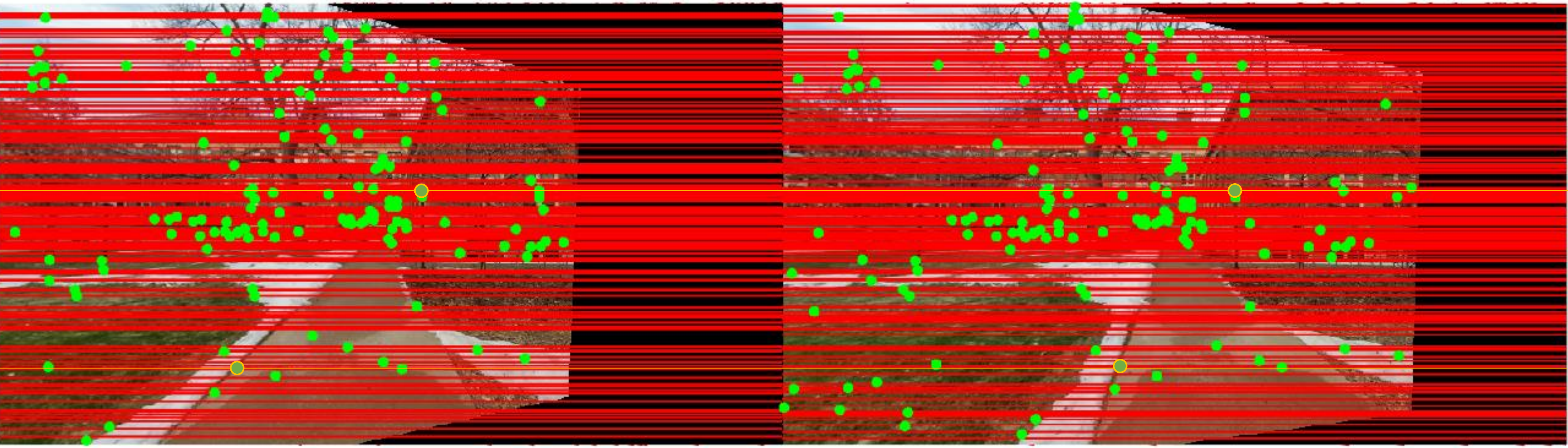


Epipole?
Point at infinity

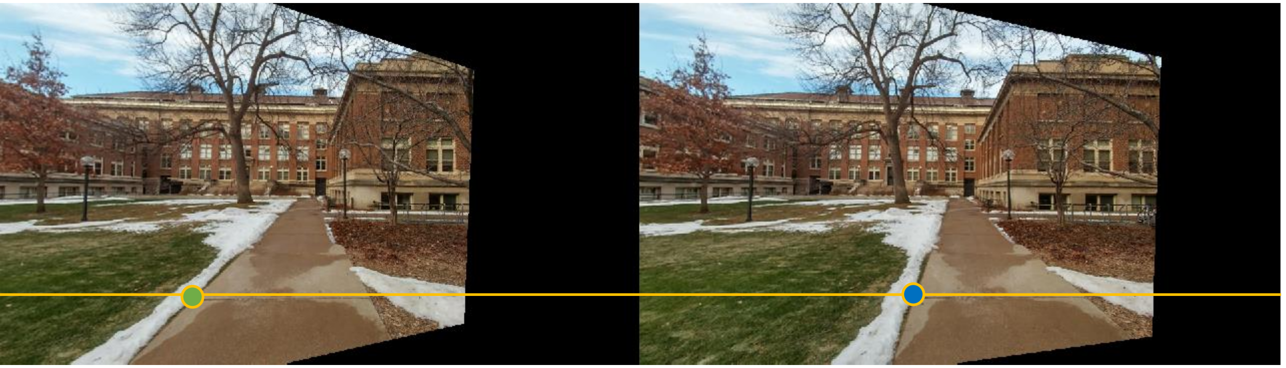
Special Case: Stereo

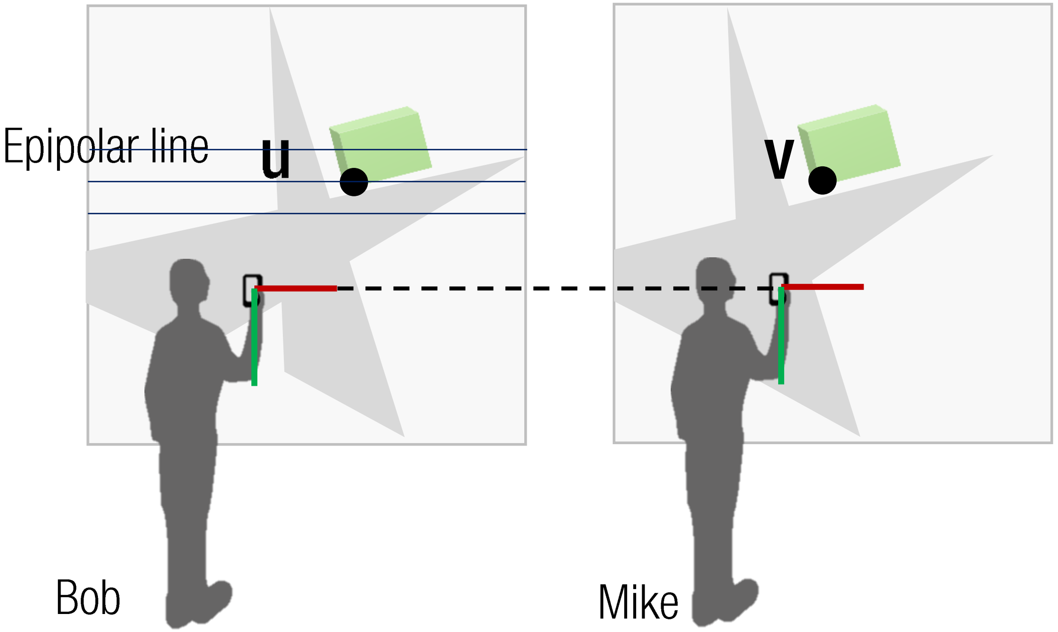
Special Case: Stereo

Special Case: Stereo

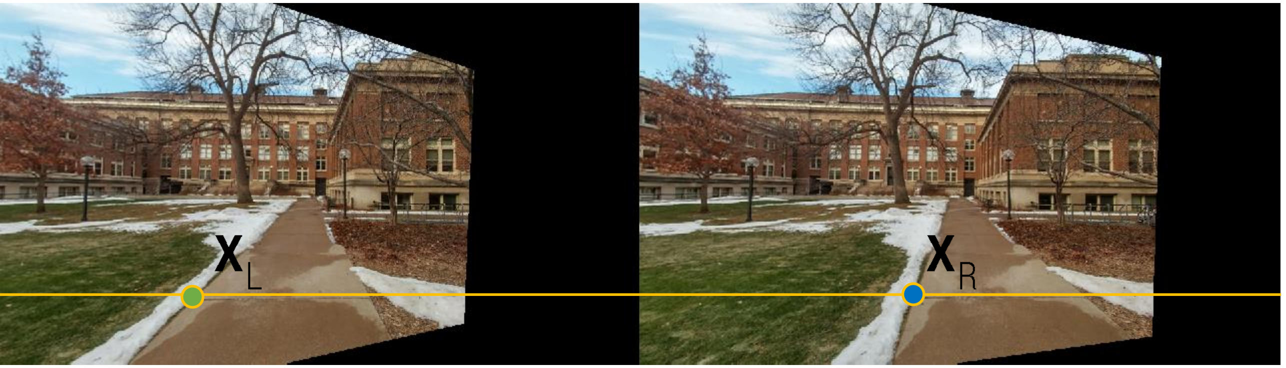


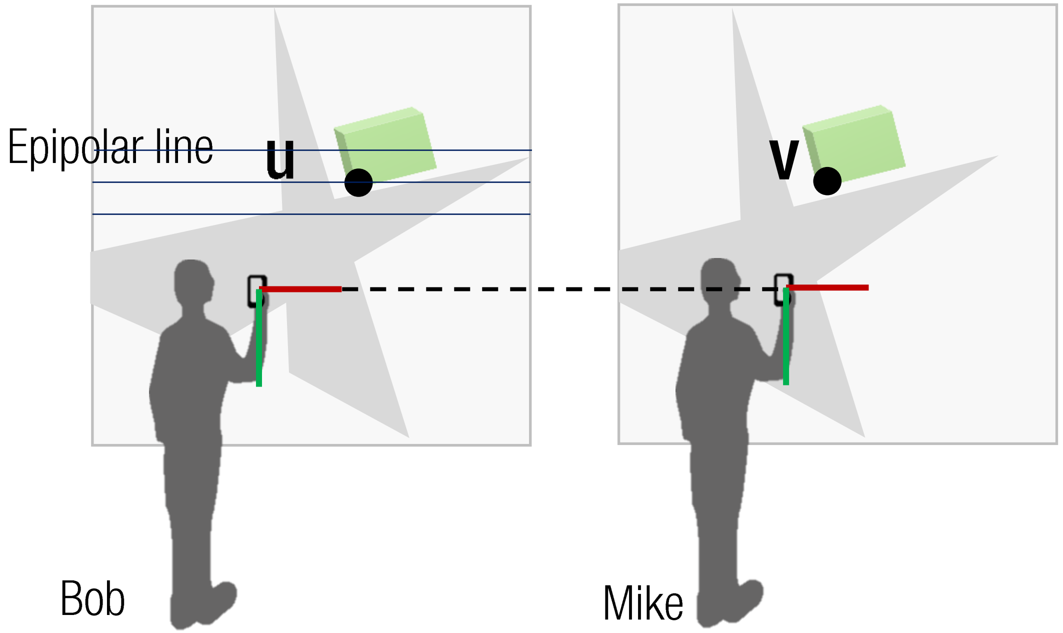
Special Case: Stereo

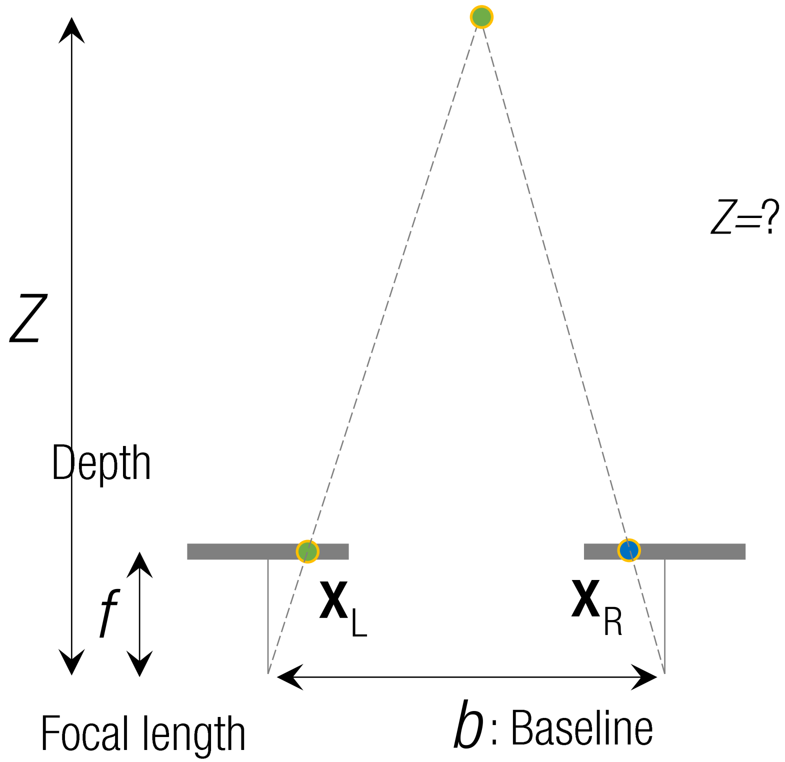




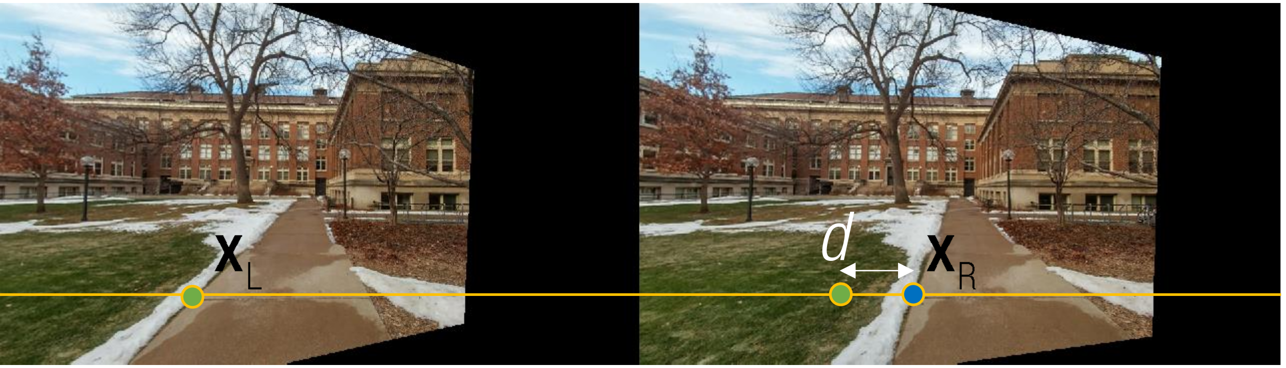
Special Case: Stereo

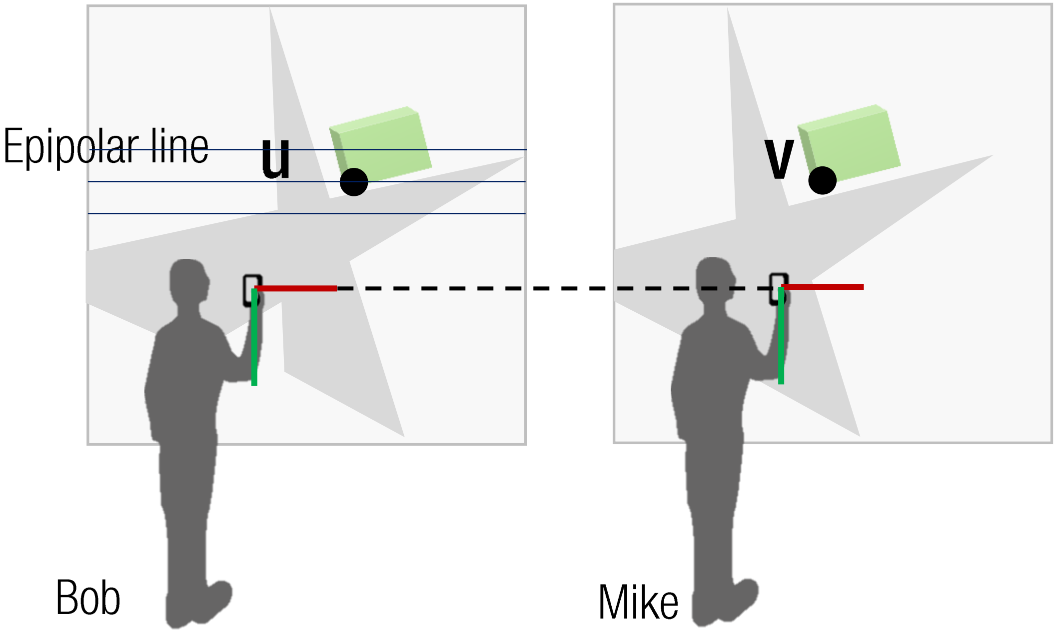


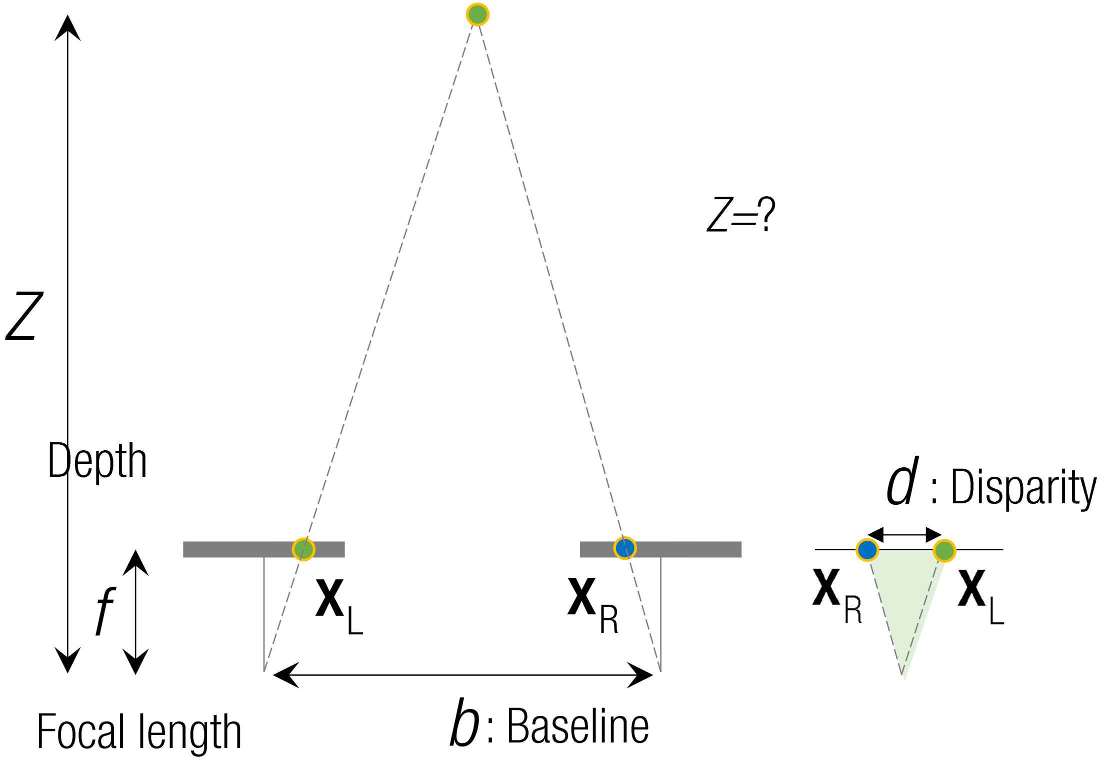




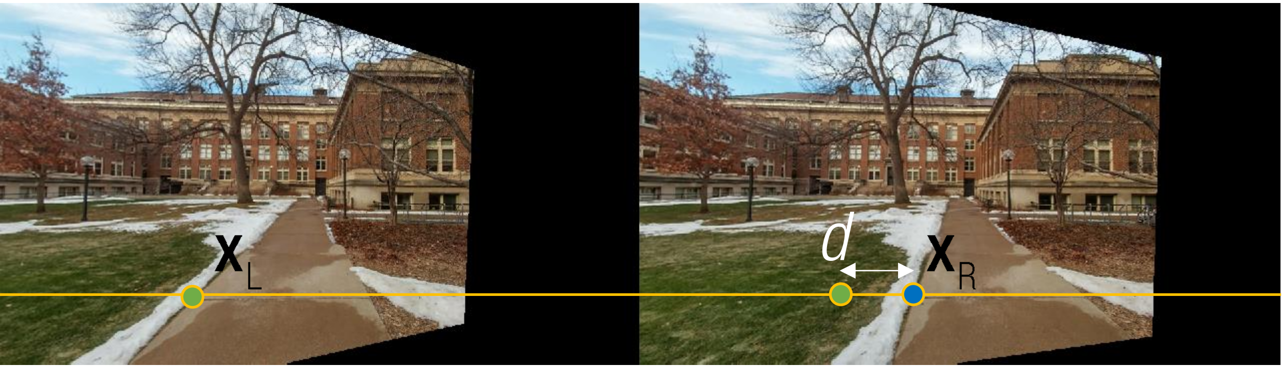
Special Case: Stereo

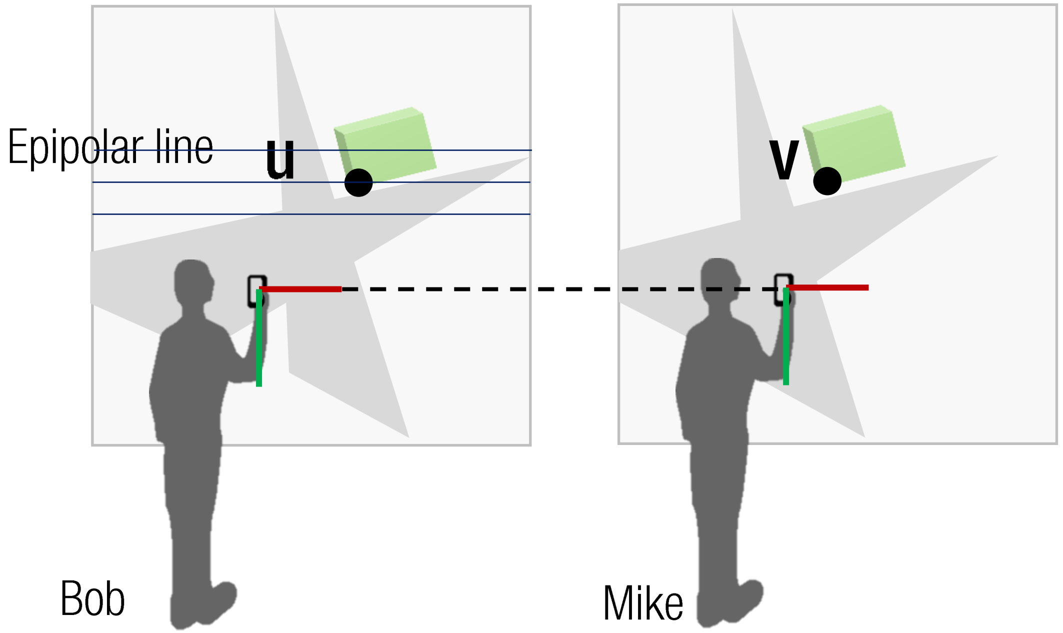


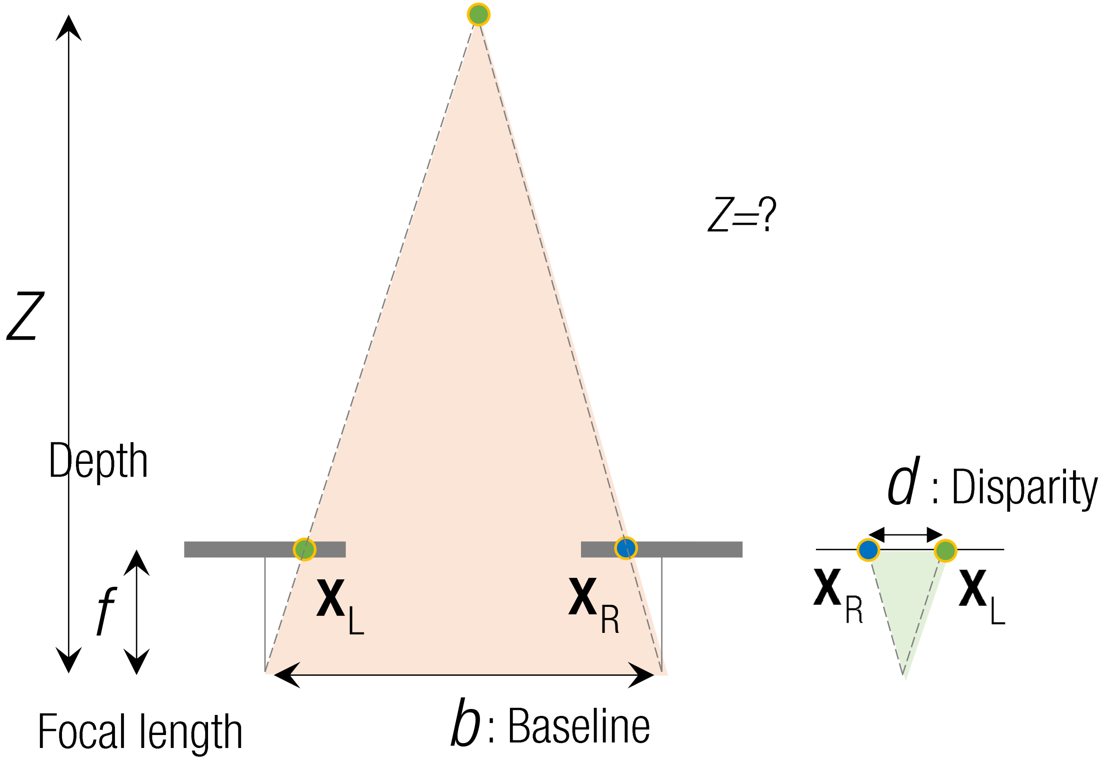




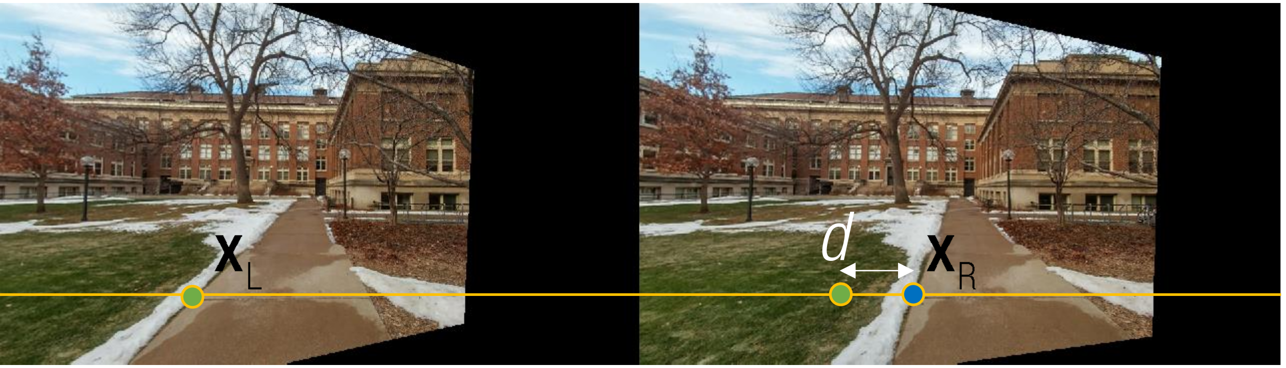
Special Case: Stereo

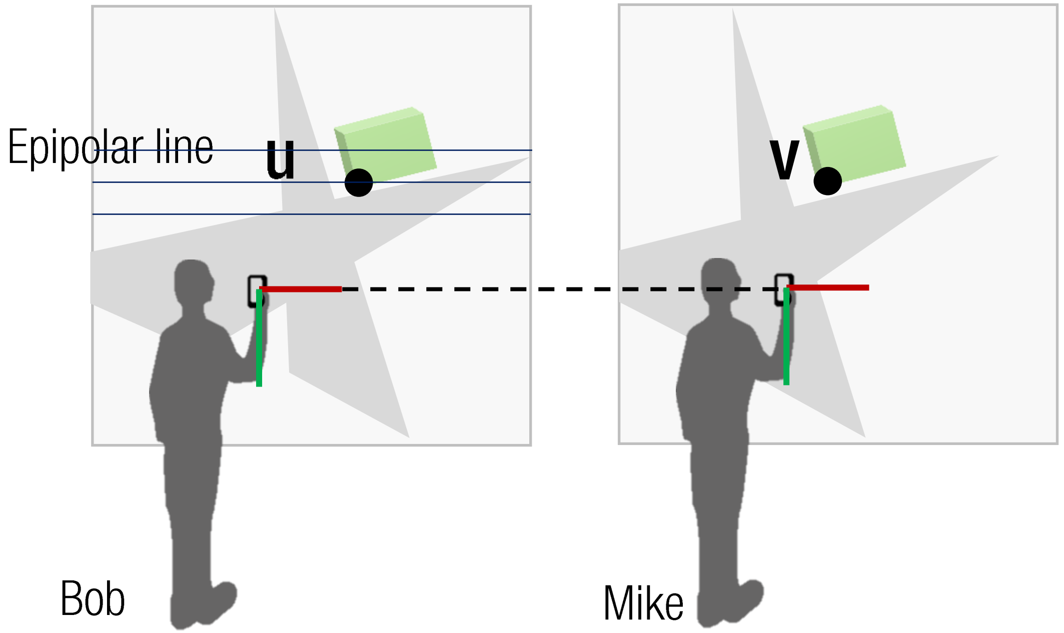


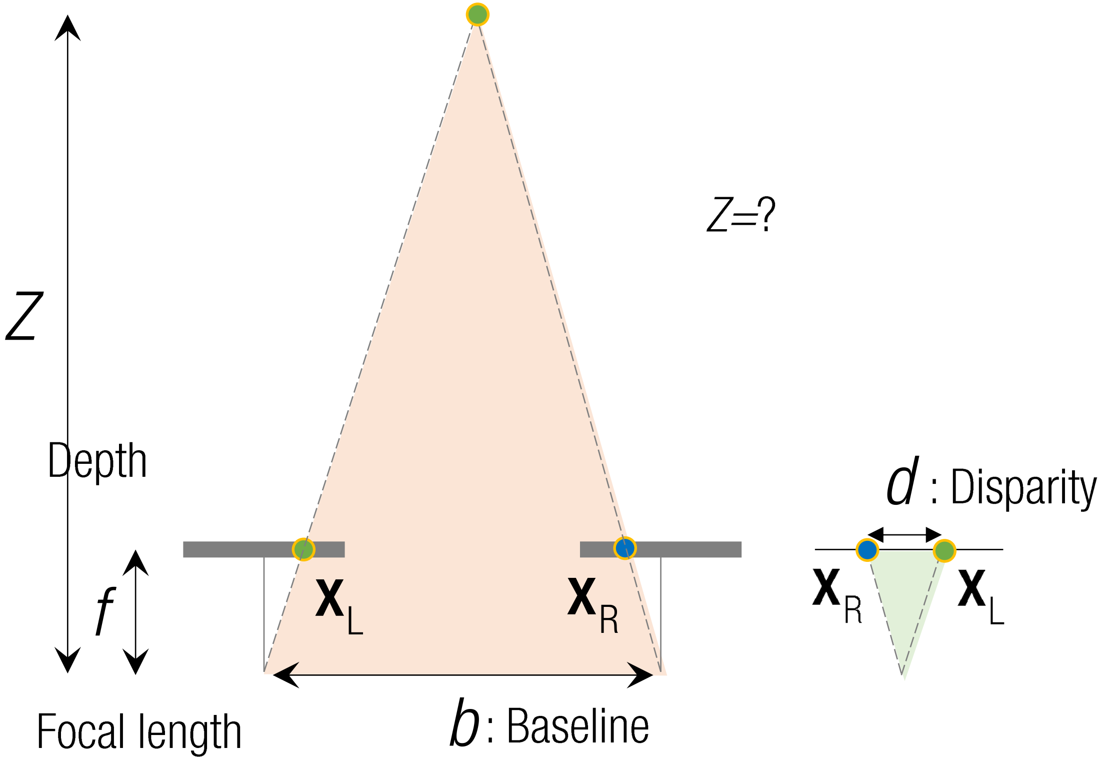


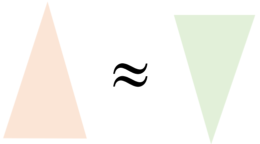


Special Case: Stereo







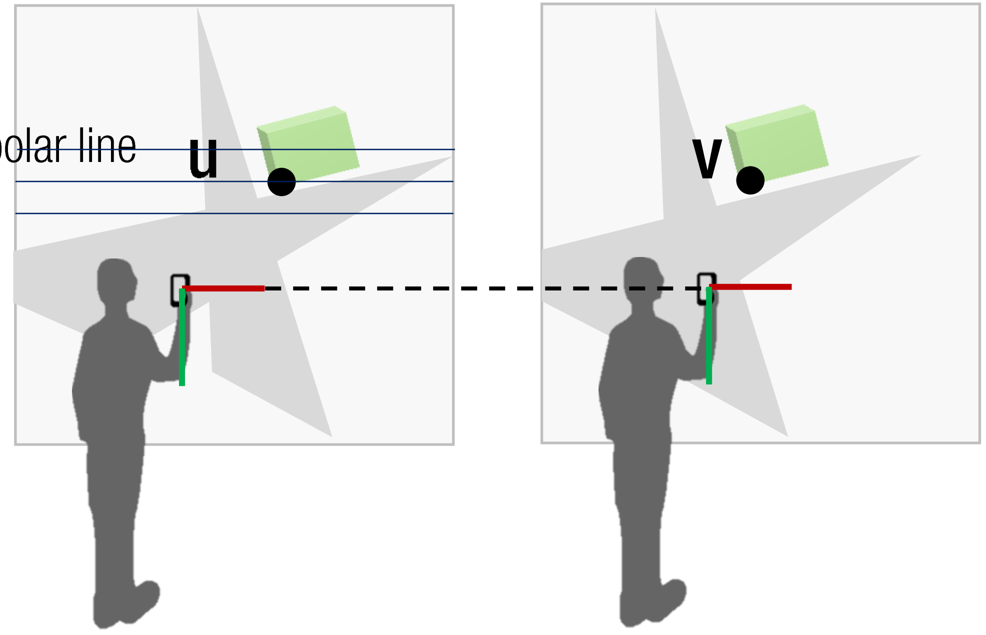


Two triangles are similar.

$$\frac{Z}{b} = \frac{f}{d} \rightarrow Z = b\frac{f}{d}$$

Stereo Rectification

Stereo Rectification



$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix} \qquad \mathbf{P}_{mike} = \mathbf{KR}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}$$

- Same orientation

$$\mathbf{R}_{rect} = \begin{bmatrix}\mathbf{r}_x^T \\ \mathbf{r}_y^T \\ \mathbf{r}_z^T\end{bmatrix}$$

- Alignment between X axis and baseline

$$\mathbf{r}_x = \frac{\mathbf{C}}{\|\mathbf{C}\|}$$

$$\mathbf{r}_z = \frac{\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x)\mathbf{r}_x}{\|\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x)\mathbf{r}_x\|}$$

: Orthogonal projection

$$\mathbf{r}_y = \mathbf{r}_z \times \mathbf{r}_x$$

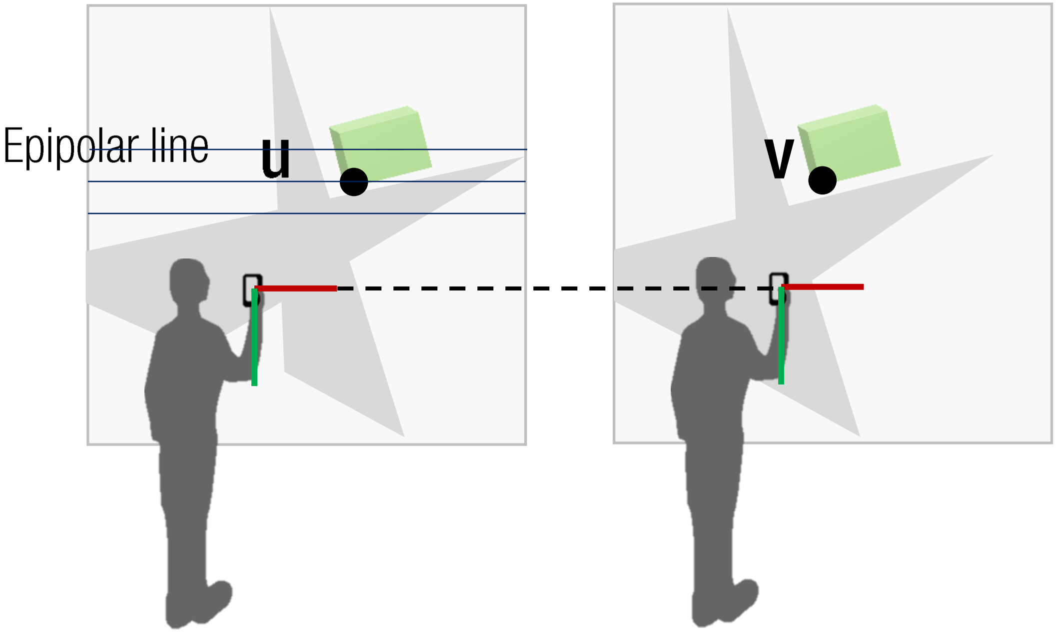
where $\tilde{\mathbf{r}}_z = \begin{bmatrix}0 \\ 0 \\ 1\end{bmatrix}$

Stereo Rectification

Homography by pure rotation: $\mathbf{R}_{rect}$

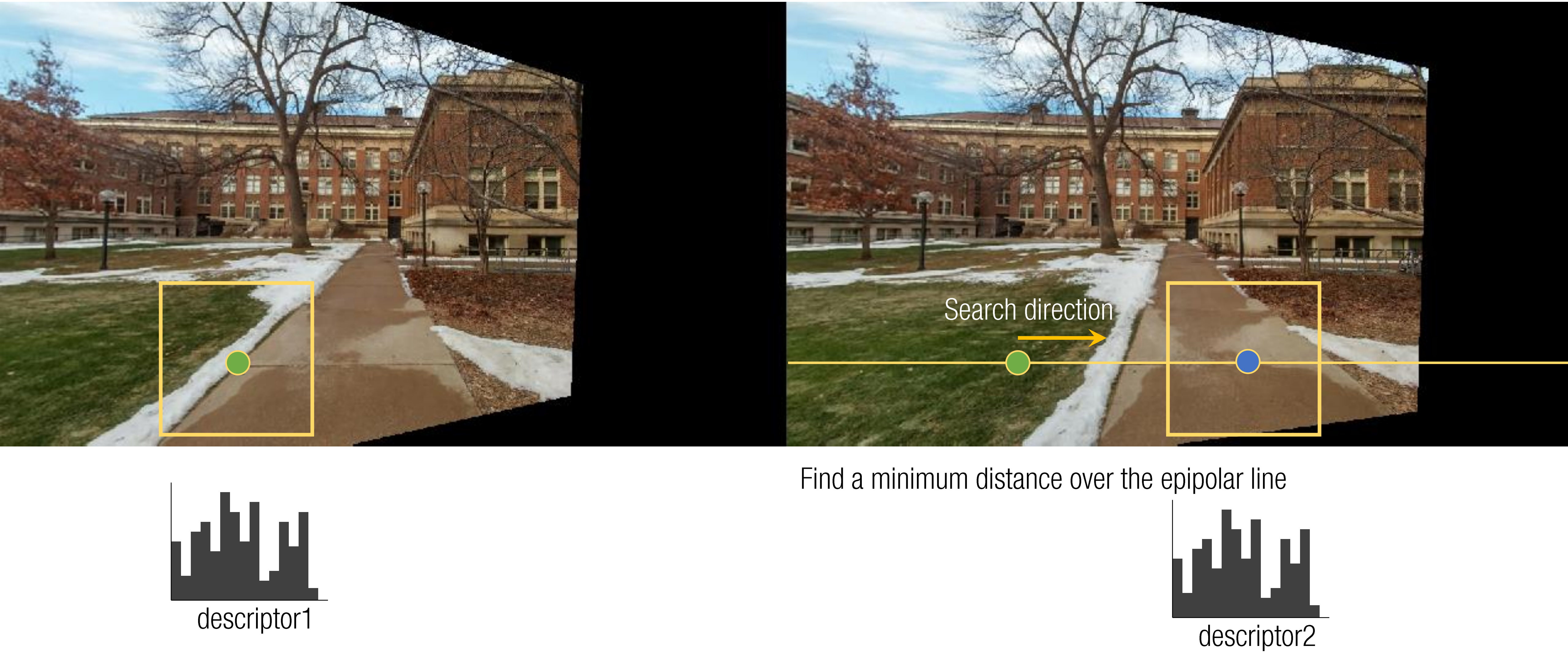
$$\mathbf{H}_{bob} = \mathbf{K}\mathbf{R}_{rect}\mathbf{K}^{-1}$$

$$\mathbf{H}_{mike} = \mathbf{K}\mathbf{R}_{rect}\mathbf{R}^{T}\mathbf{K}^{-1}$$



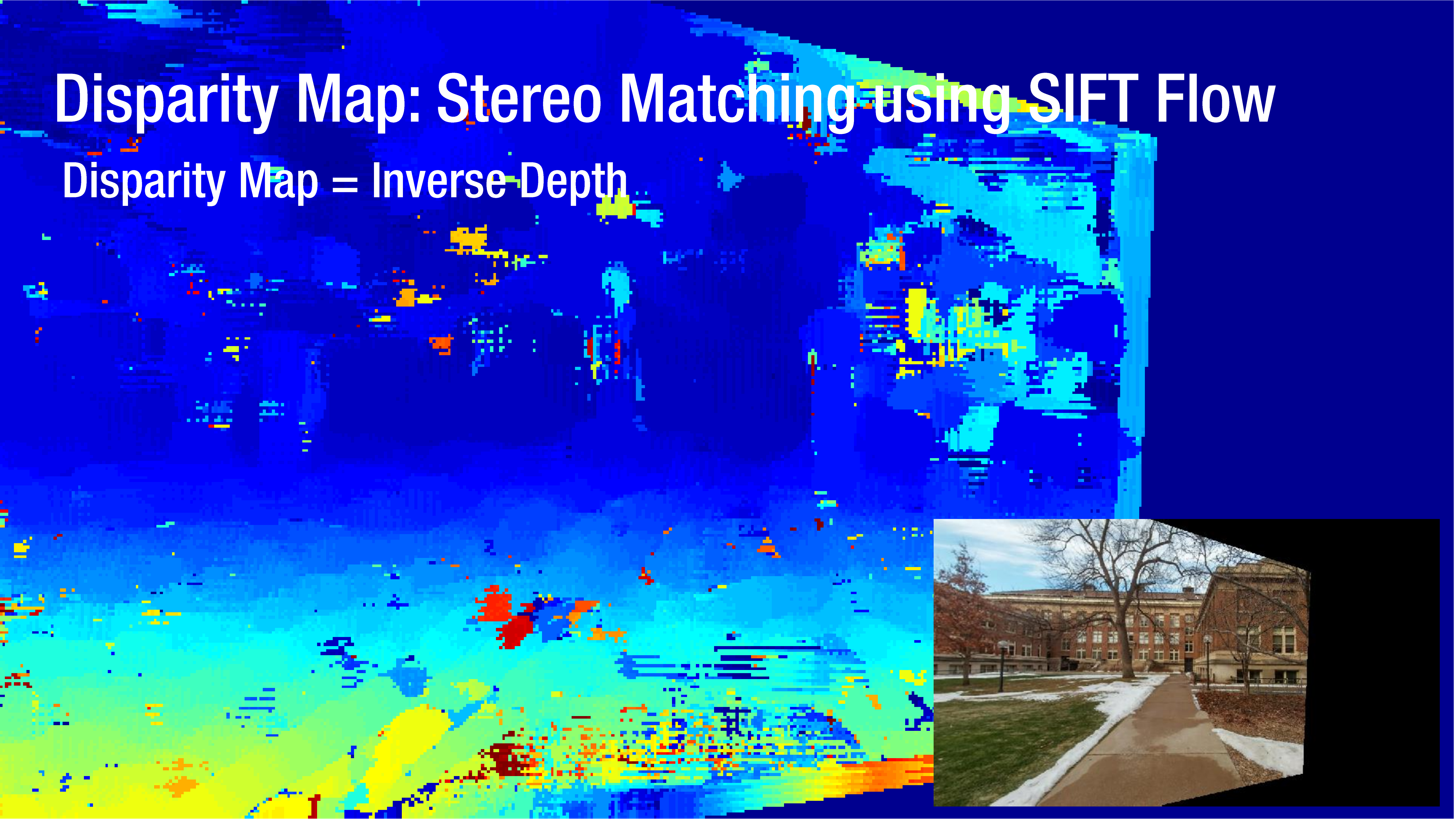
$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix} \qquad \mathbf{P}_{mike} = \mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}$$

Dense Feature Matching using SIFT Flow



Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth

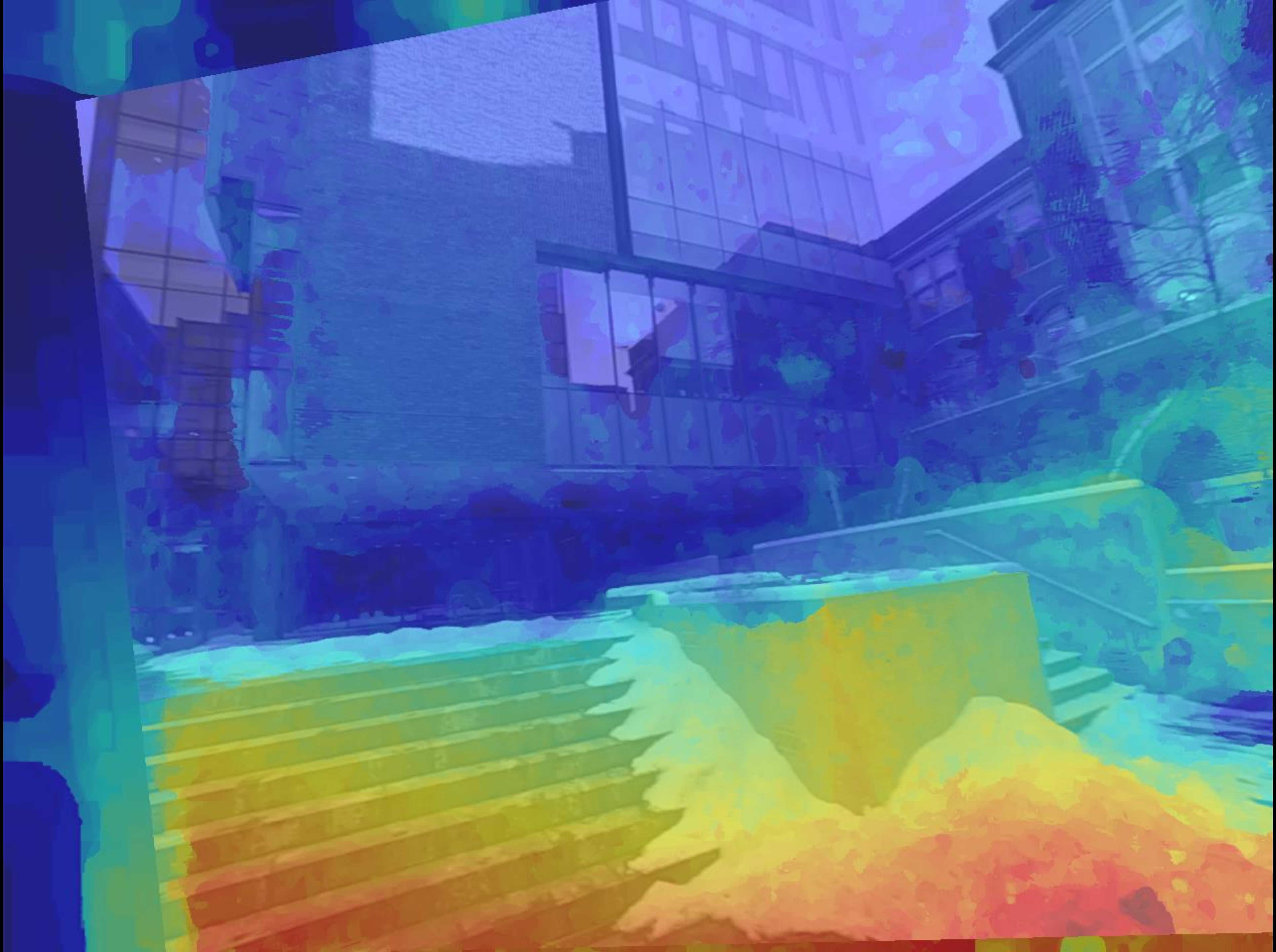


Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth

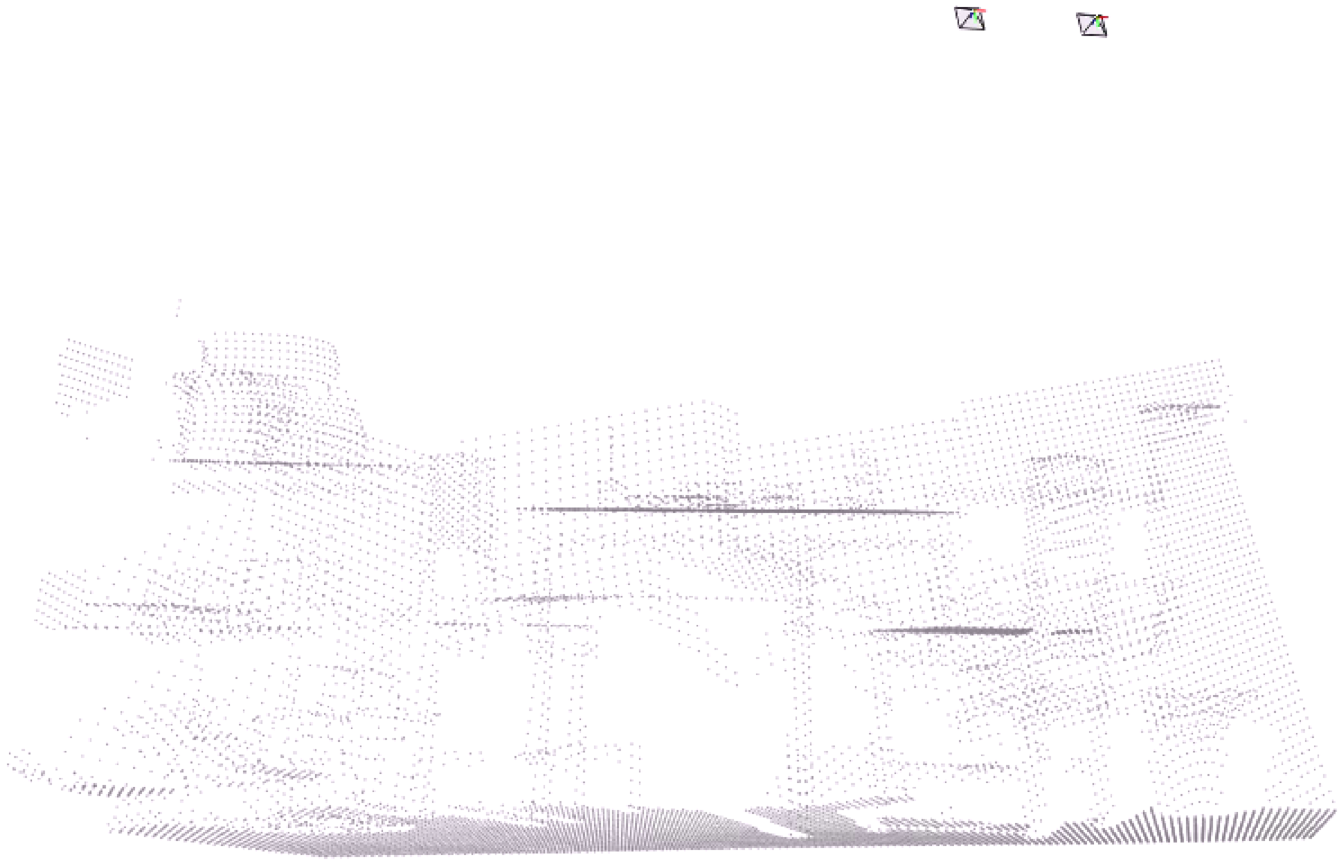
Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth


maxell
VIDEOCASSETTE

maxell
VIDEOCASSETTE

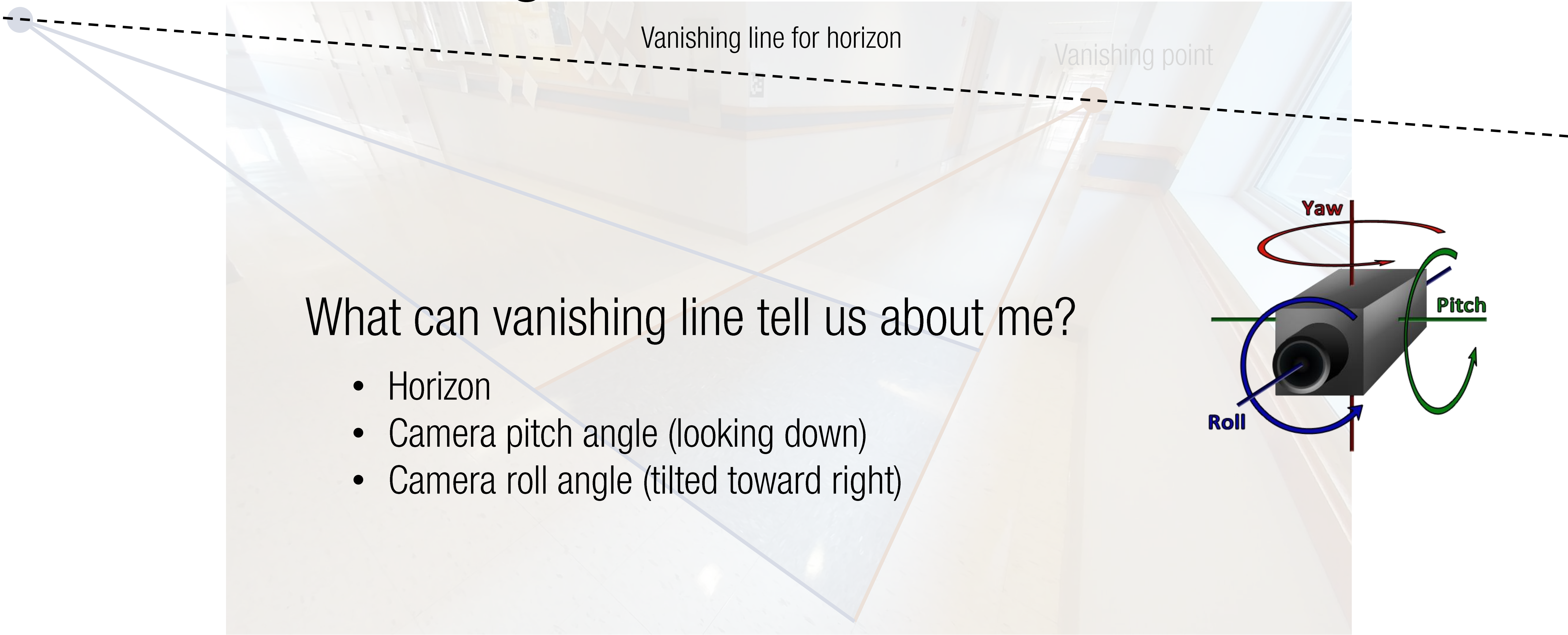



EgoMotion Dataset (outdoor)

Dense Reconstruction using a Monocular Camera

Where am I?
Perspective-n-Point

Recall: Vanishing Line

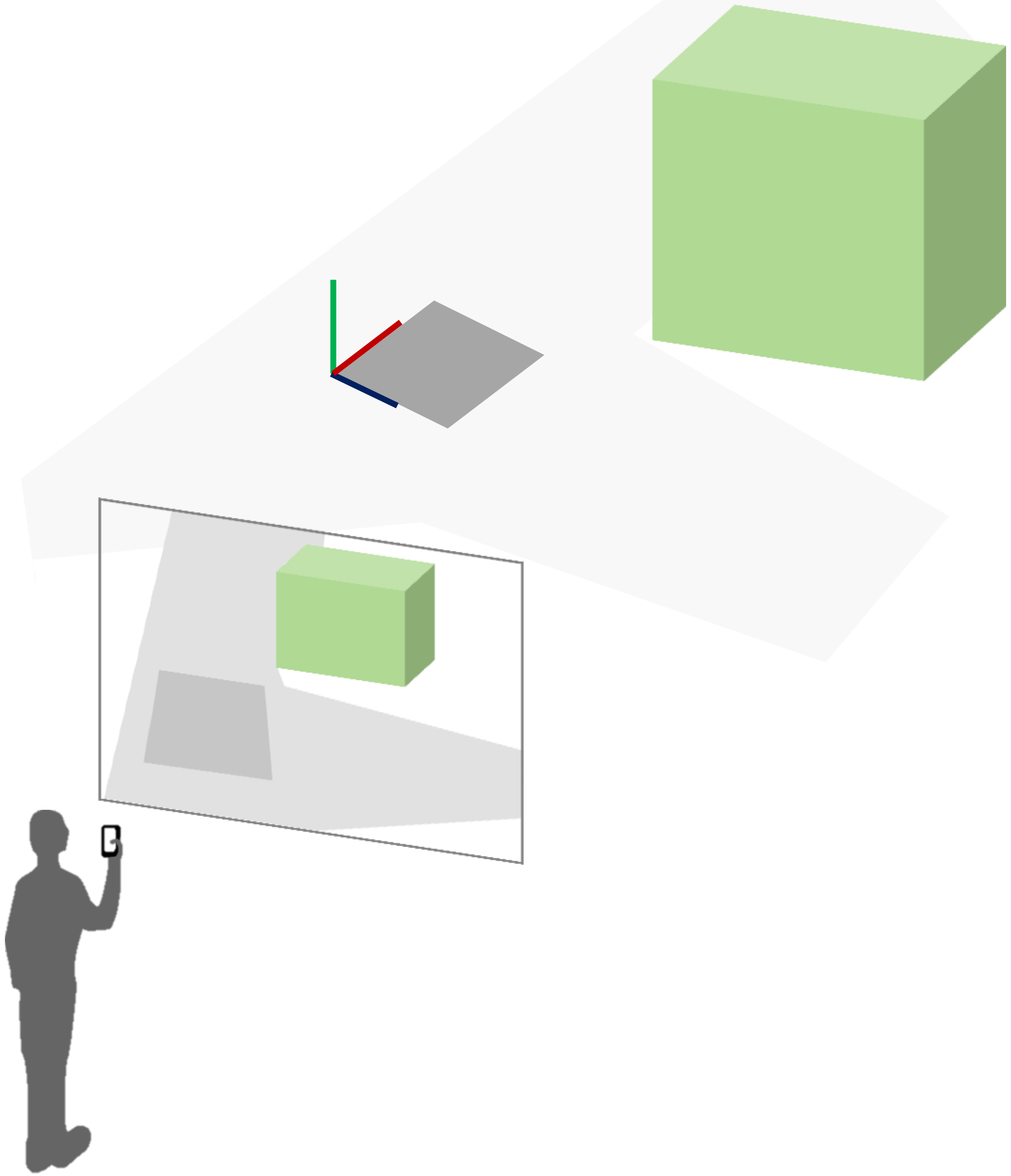


What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)

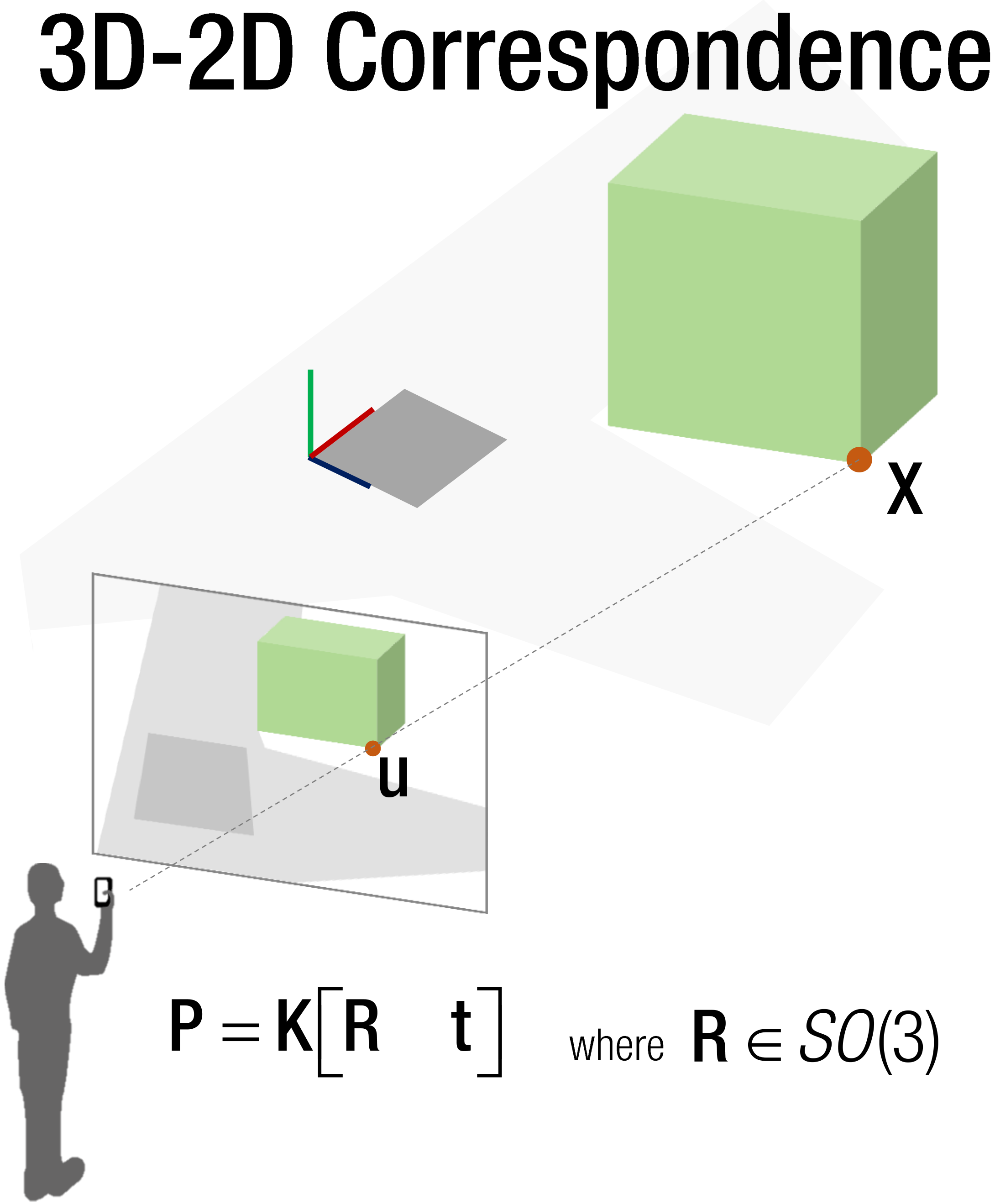
Keller Hall

What can 3D scene points tell us about?



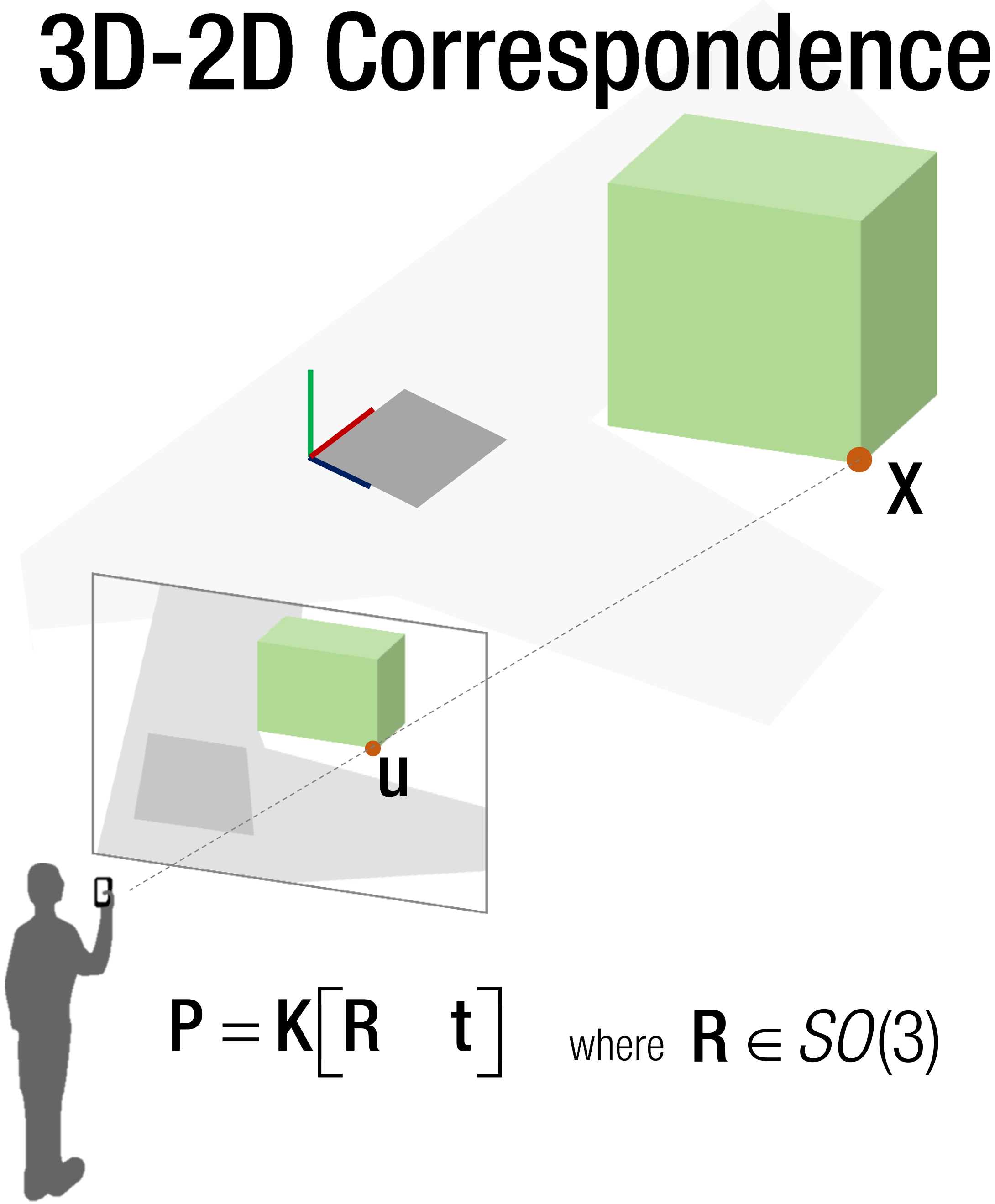
http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D Correspondence



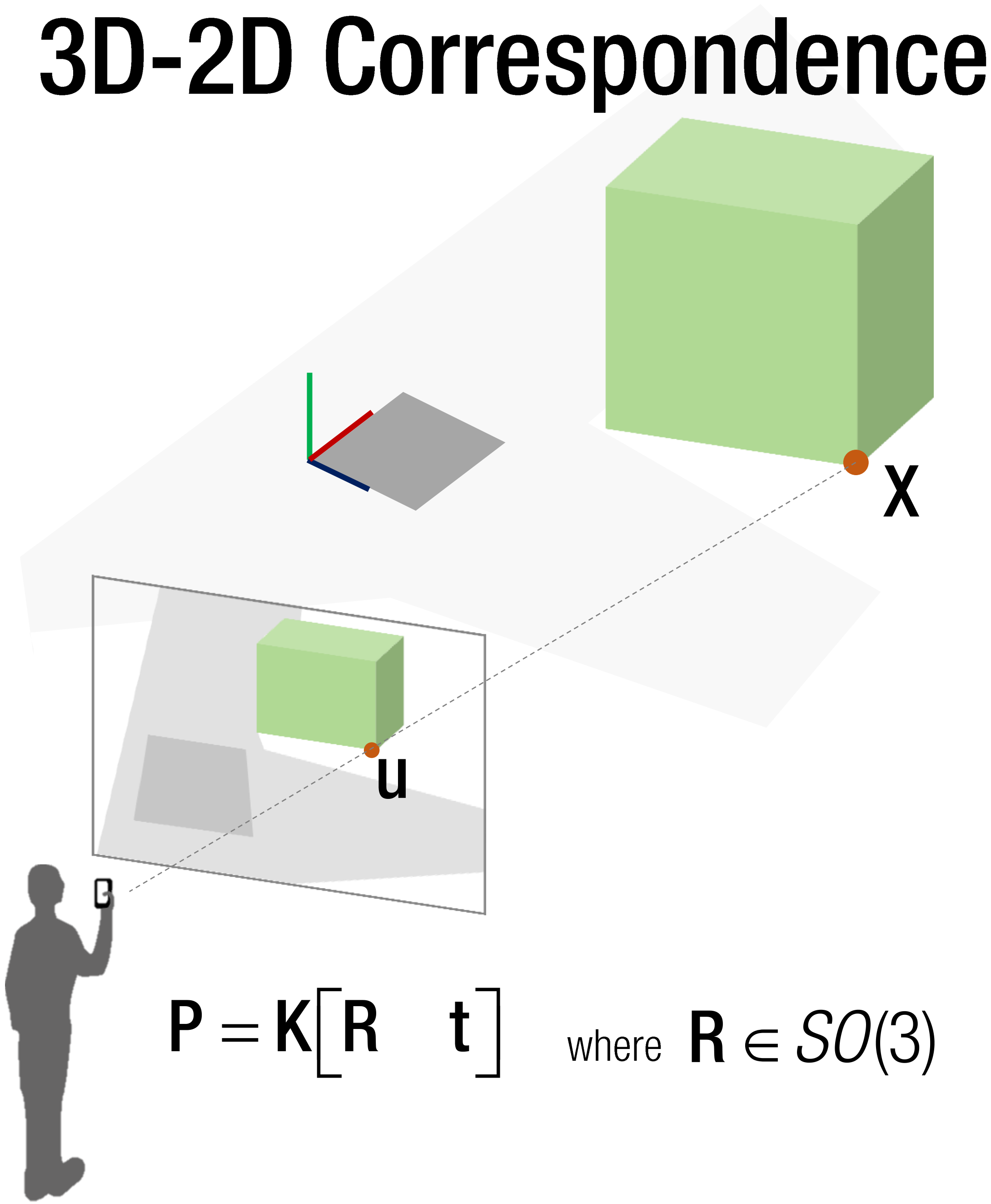
$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

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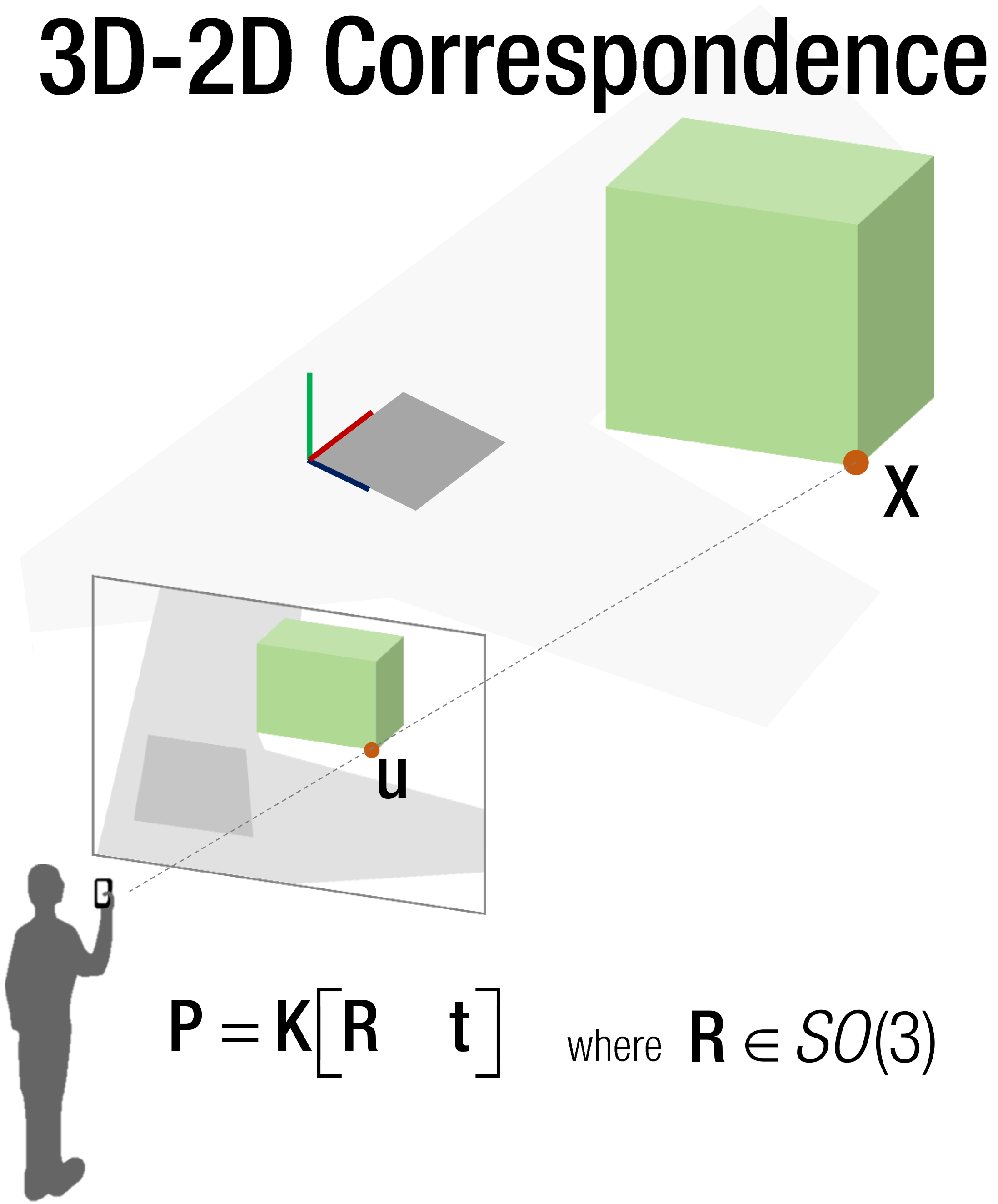
$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\text{Linear in camera matrix}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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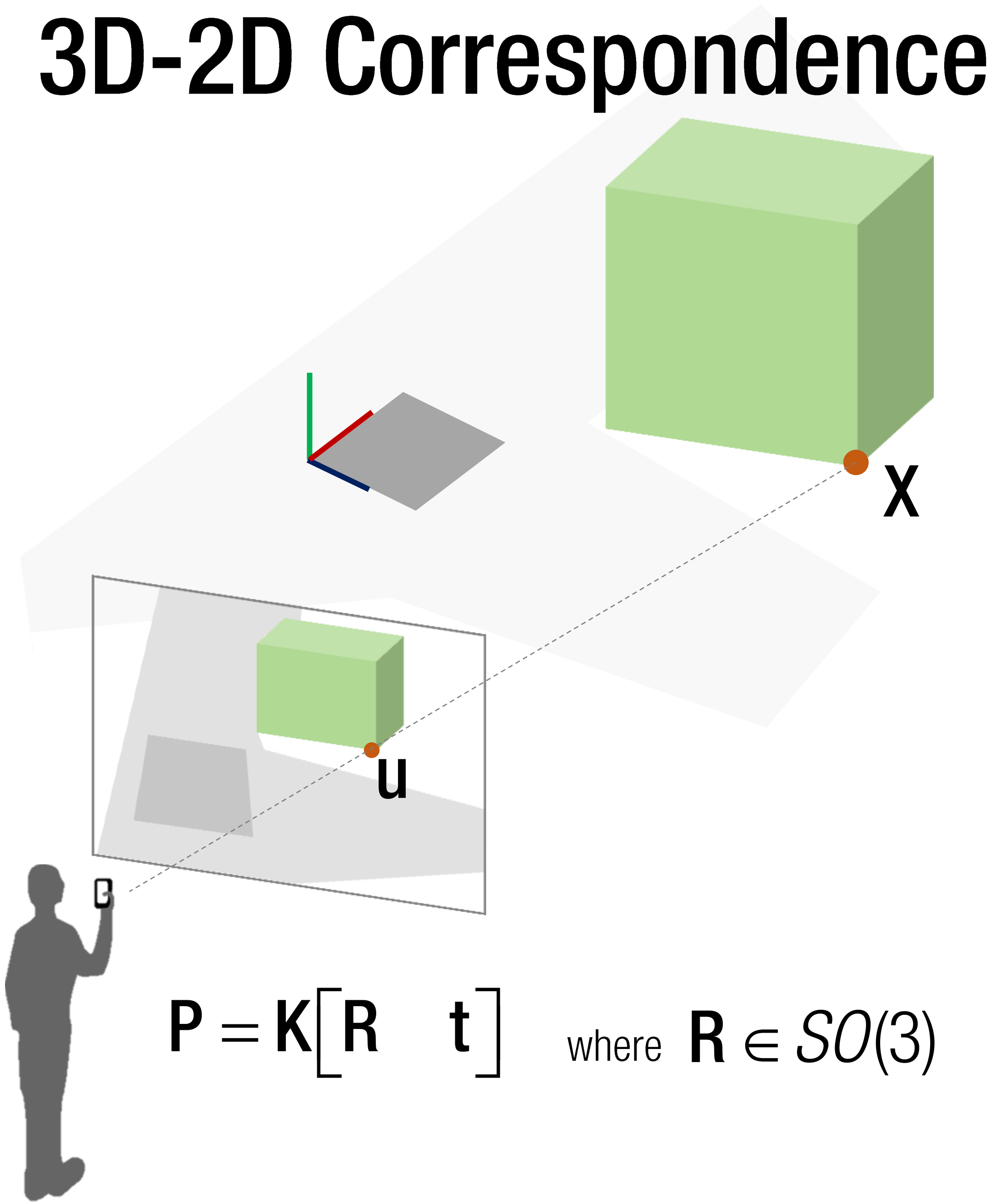
# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)
# of equations per correspondence: 2

3D-2D Correspondence

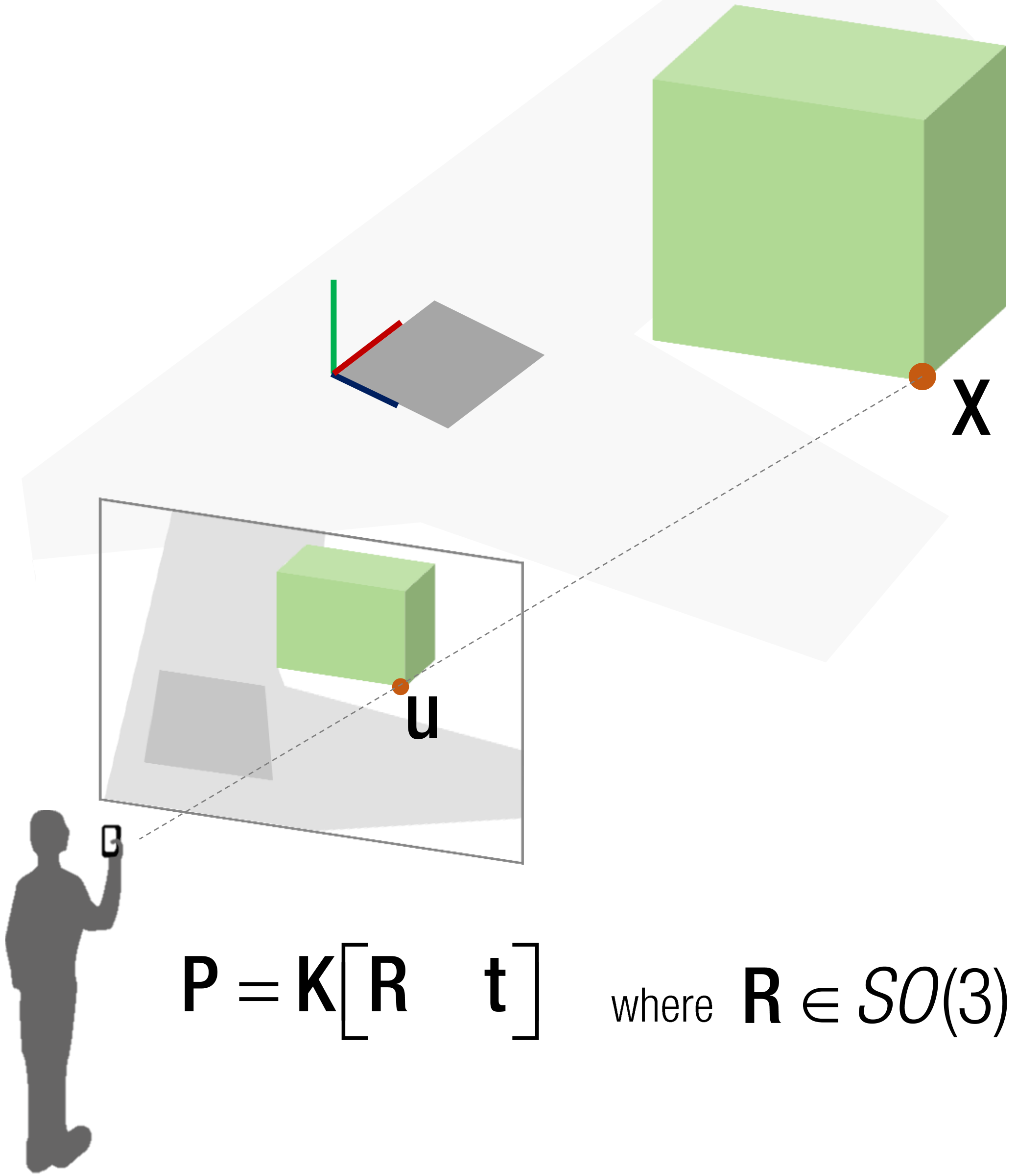


3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D Correspondence



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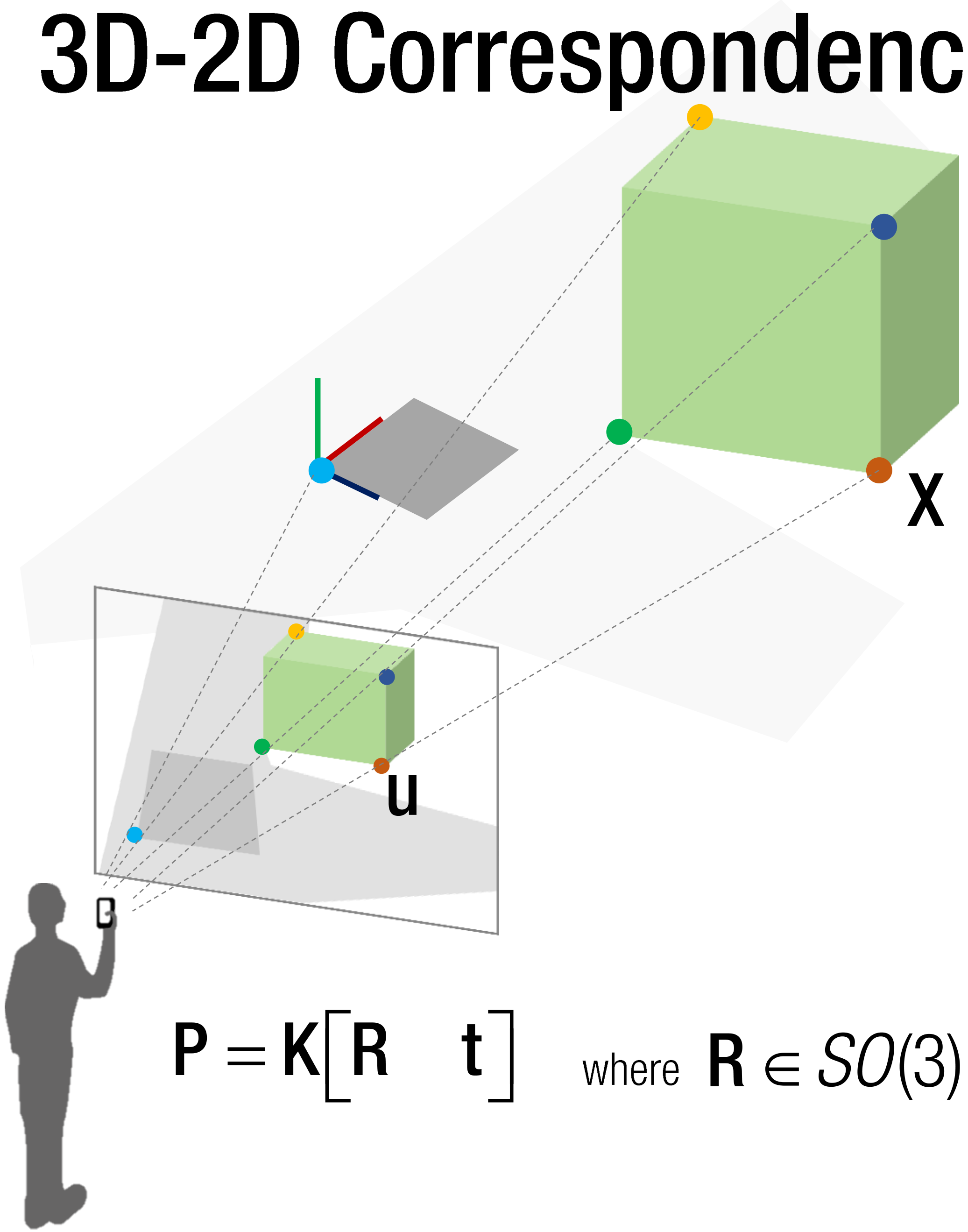
$$\underbrace{\begin{bmatrix} X & Y & Z & 1 & & & & & -u^xX & -u^xY & -u^xZ & -u^x \\ & & & & X & Y & Z & 1 & -u^yX & -u^yY & -u^yZ & -u^y \end{bmatrix}}_{2\text{x}12} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

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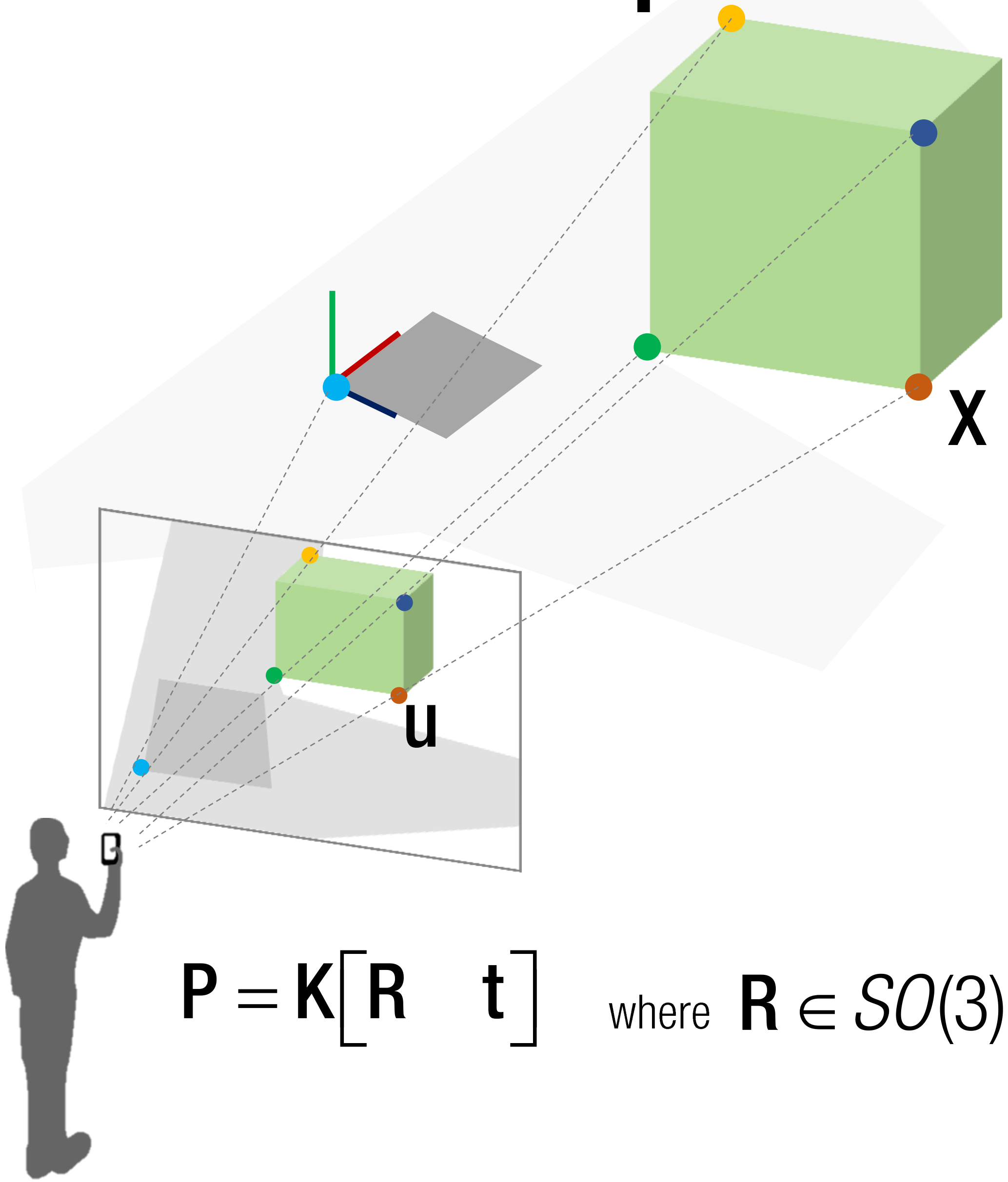
$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^xX & -u_1^xY & -u_1^xZ & -u_1^x \\ & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^yX & -u_1^yY & -u_1^yZ & -u_1^y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_m & Y_m & Z_m & 1 & & & & & -u_m^xX & -u_m^xY & -u_m^xZ & -u_m^x \\ & & & & X_m & Y_m & Z_m & 1 & -u_m^yX & -u_m^yY & -u_m^yZ & -u_m^y \end{bmatrix}}_{2m\text{x}12} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

3D-2D Correspondence



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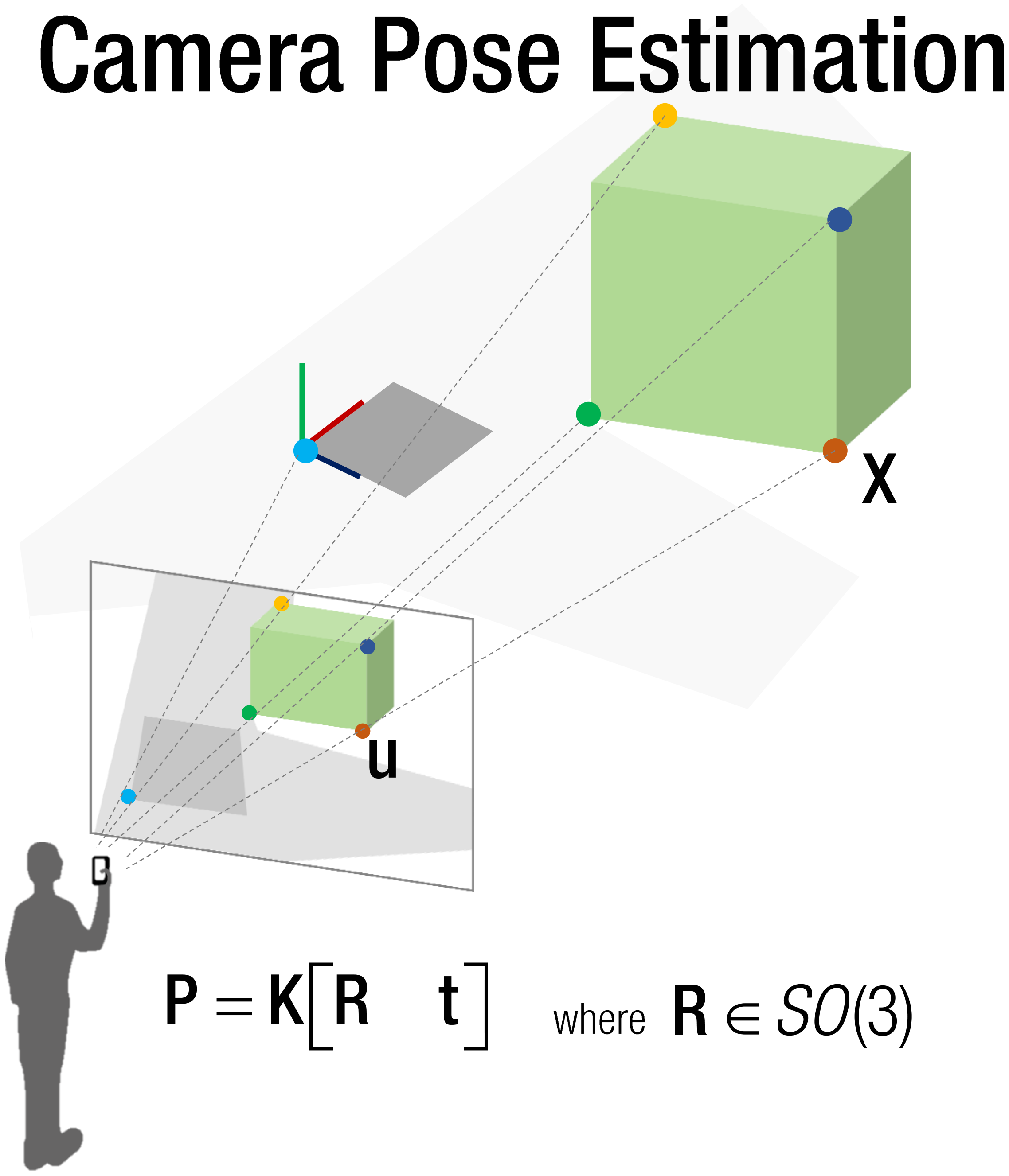
$$\underbrace{\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^x X & -u_1^x Y & -u_1^x Z & -u_1^x \\ & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^y X & -u_1^y Y & -u_1^y Z & -u_1^y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ X_m & Y_m & Z_m & 1 & & & & & -u_m^x X & -u_m^x Y & -u_m^x Z & -u_m^x \\ & & & & X_m & Y_m & Z_m & 1 & -u_m^y X & -u_m^y Y & -u_m^y Z & -u_m^y \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

$2m$x12

Camera Pose Estimation

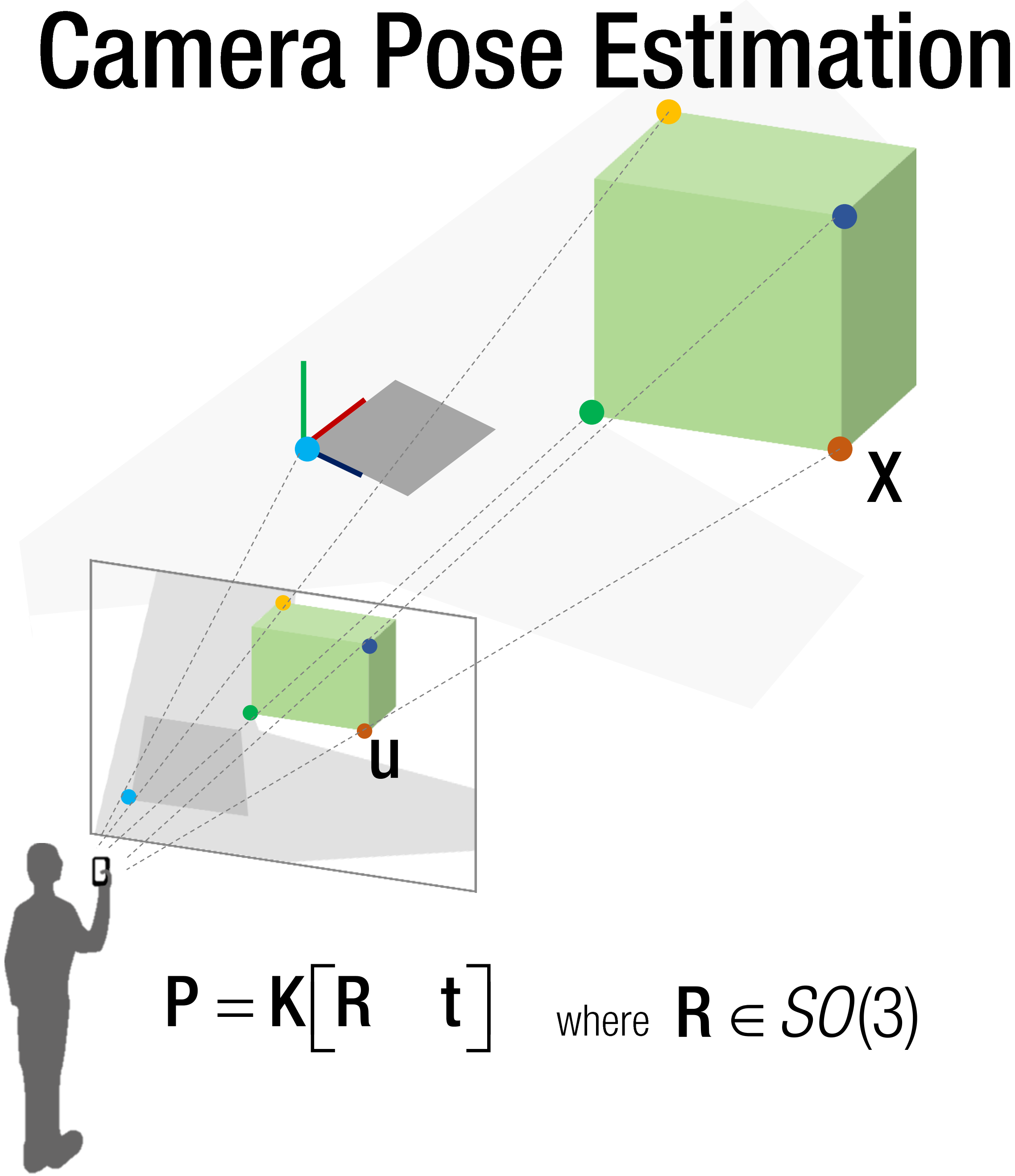


If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

Camera Pose Estimation



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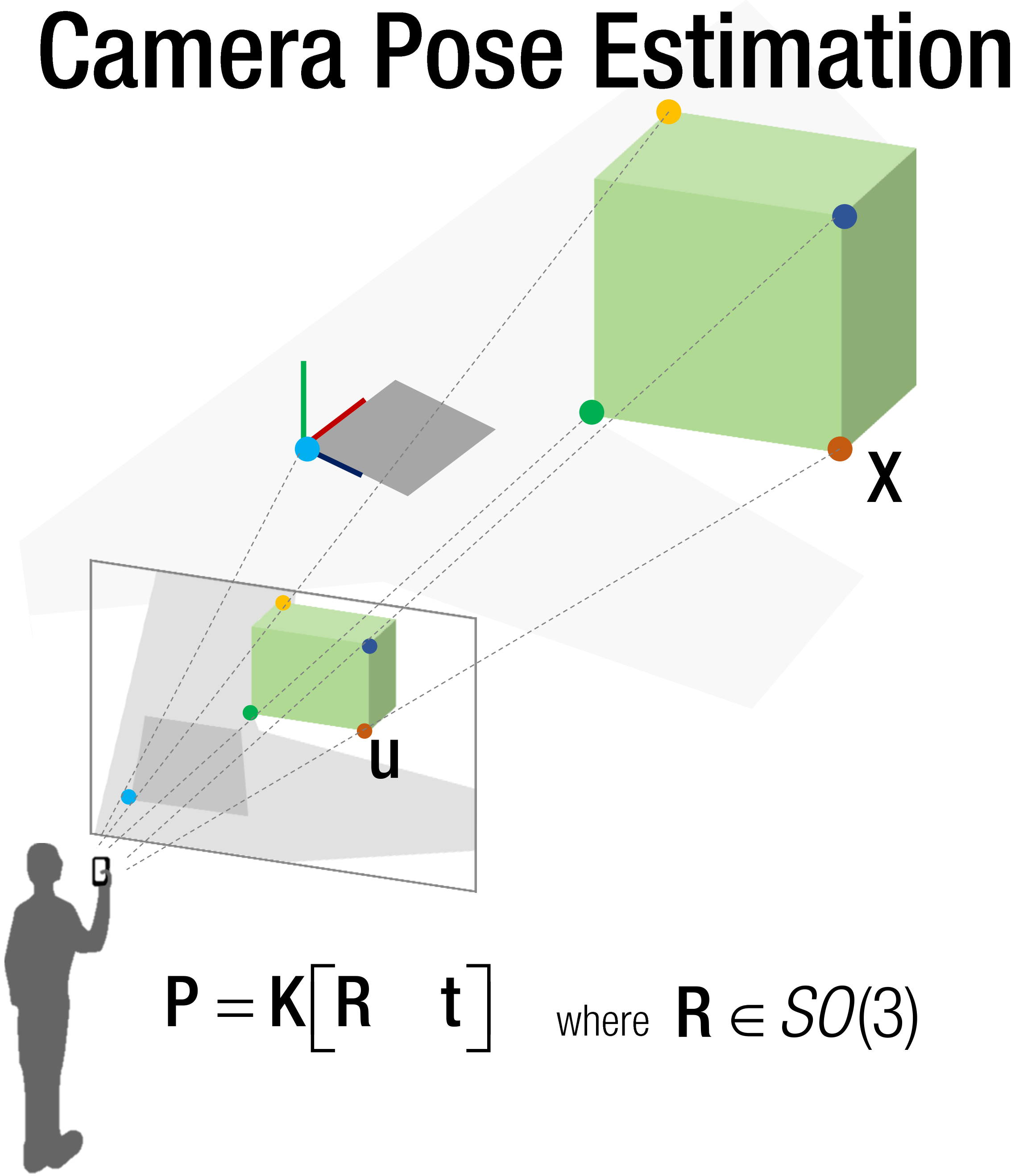
$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

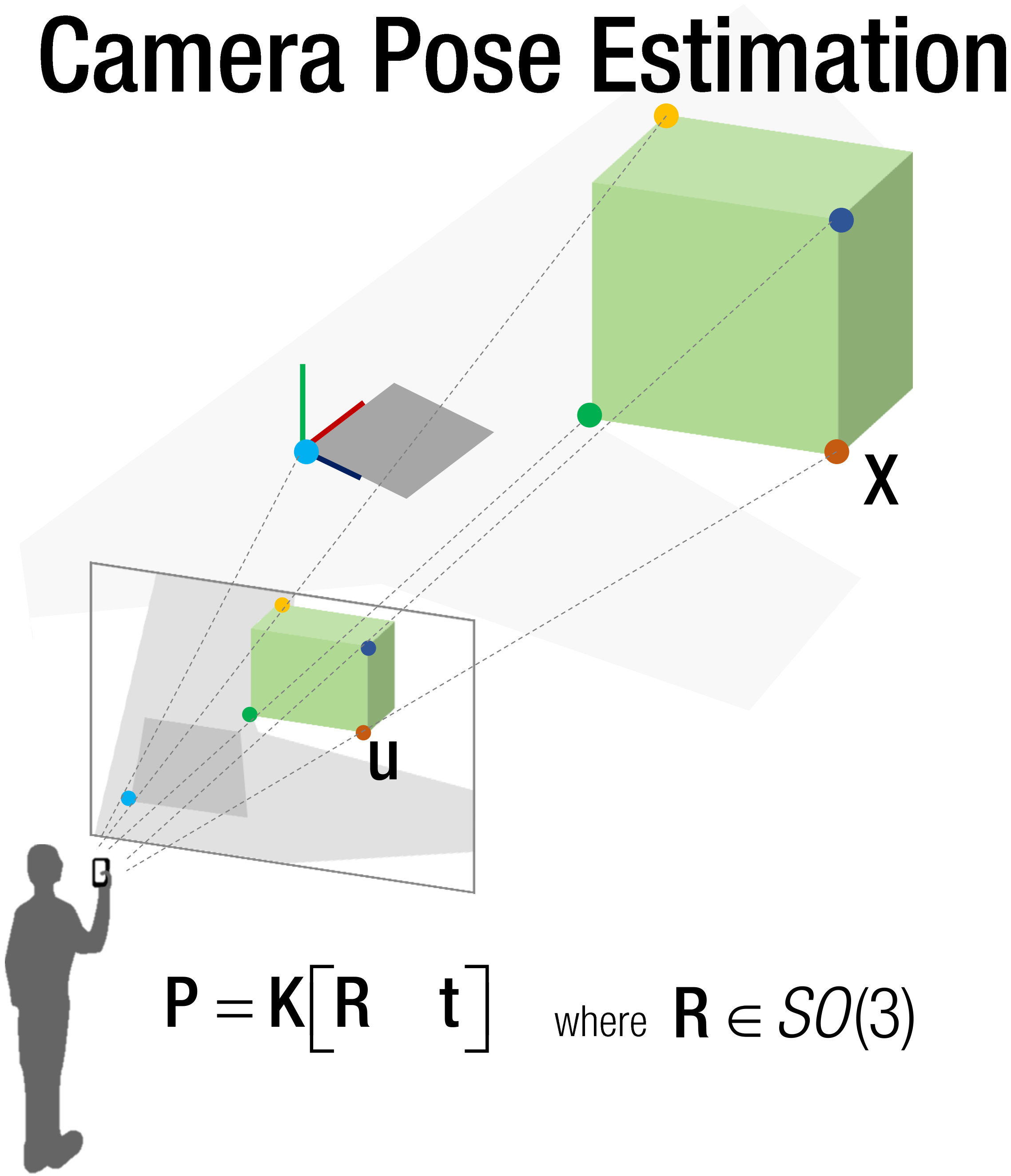
If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

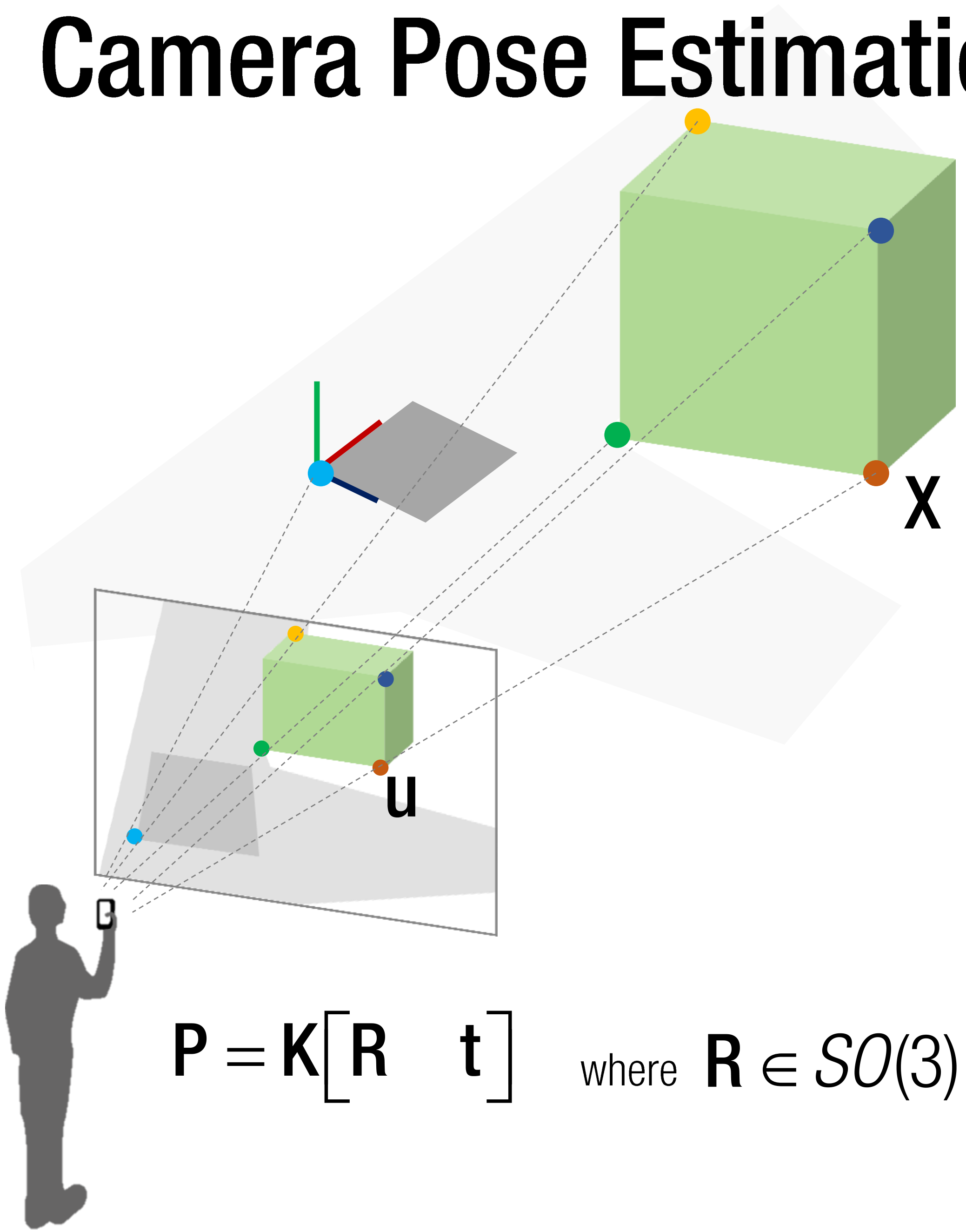
$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix}d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$$\longrightarrow \gamma \approx d_{11}$$

$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$: SVD cleanup

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

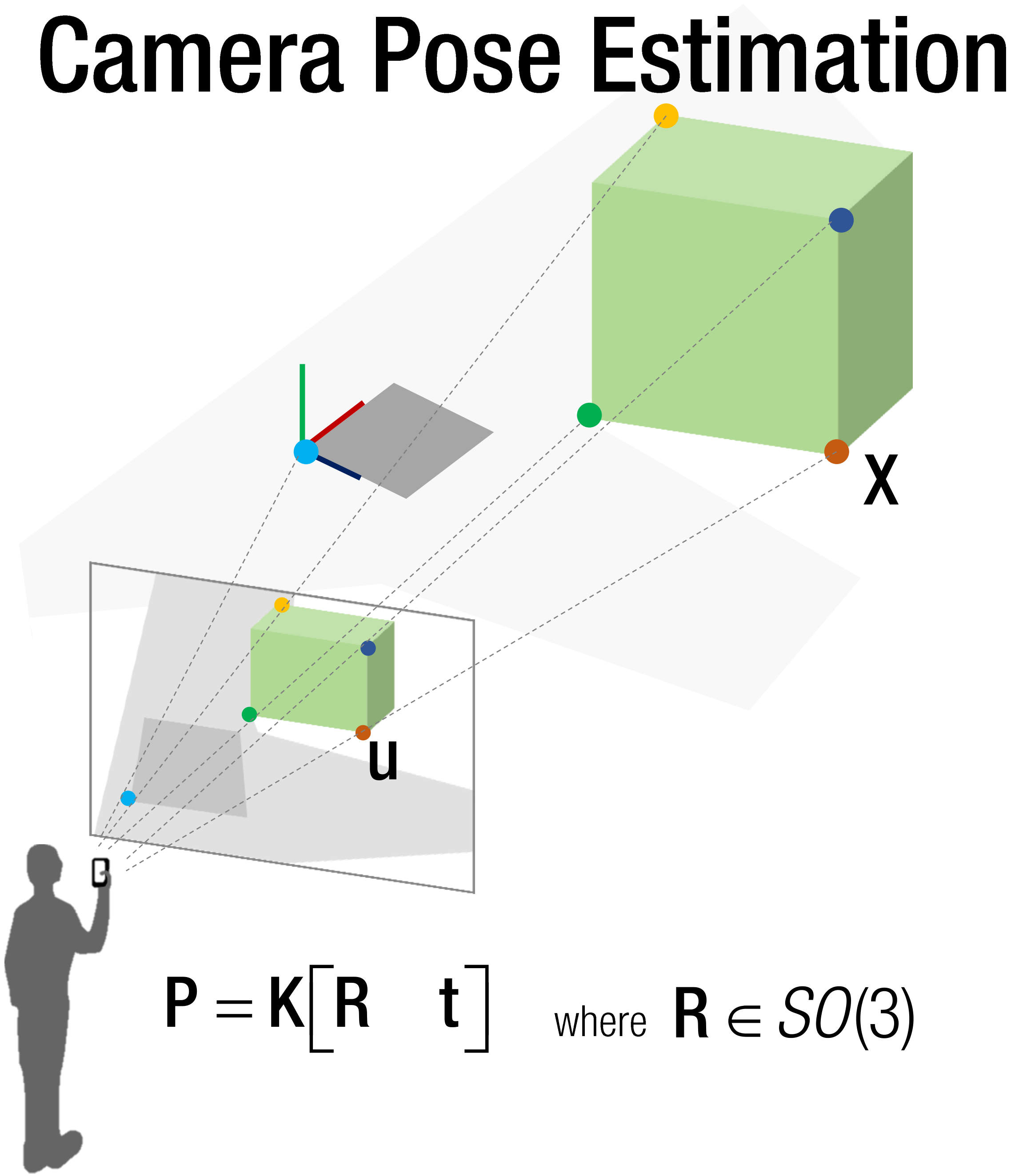
$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$$\longrightarrow \gamma \approx d_{11}$$

$$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$$: SVD cleanup

$$\longrightarrow \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{p}_4}{d_{11}}$$: Translation and scale recovery

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

```
function [R t] = LinearPnP(X, u, K)
A = [];
for i = 1 : size(X,1)
   %% Build A matrix here
End

[u d v] = svd(A);
P = v(:,end);
P = [P(1:4)'; P(5:8)'; P(9:12)'];
R = inv(K)*P(:,1:3);
t = inv(K)*P(:,4);

% SVD clean up
[u d v] = svd(R);
R = u * v';
t = t/d(1,1);

if det(R) < 0
   R = -R;
   t = -t;
end
```

$$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$$

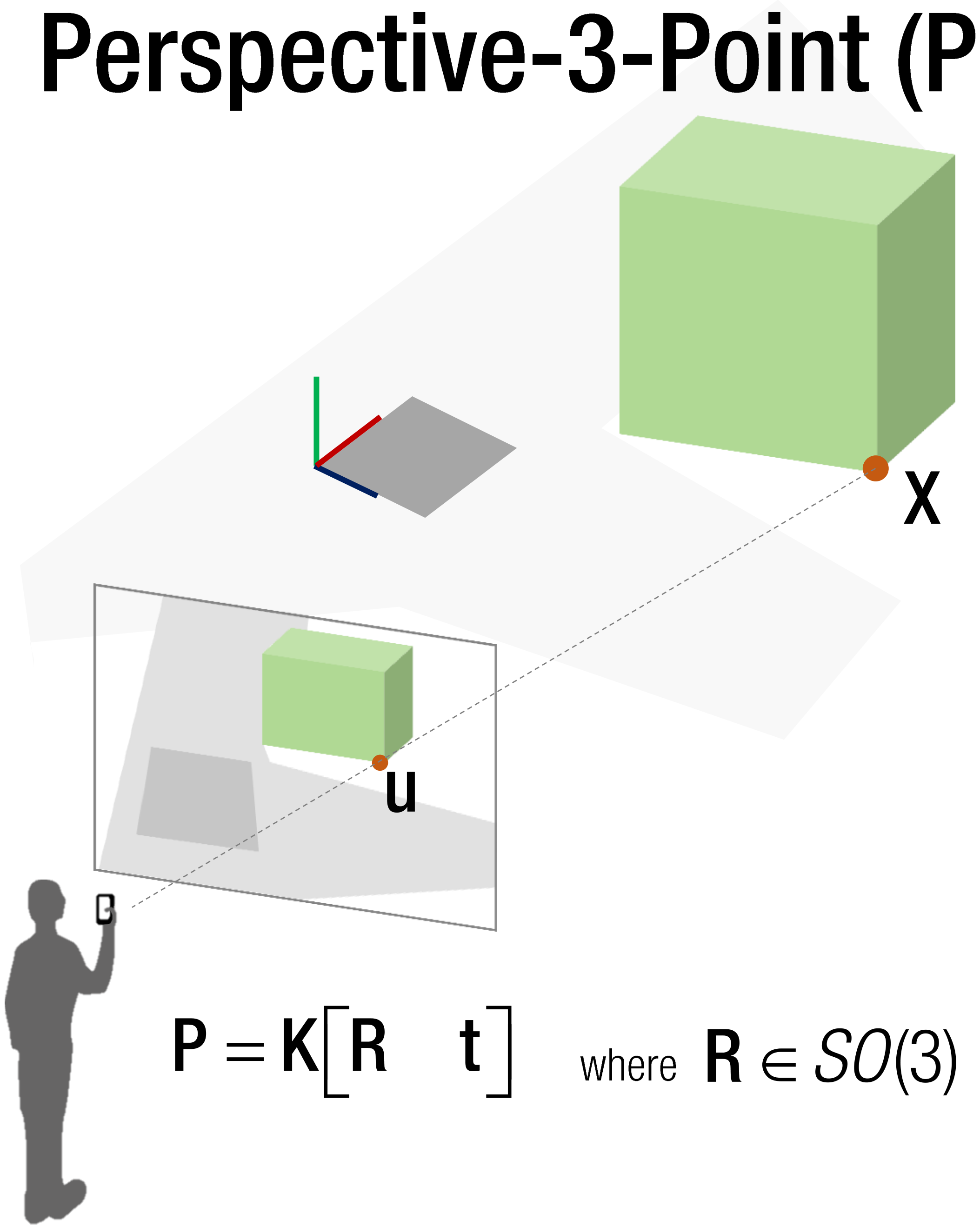
$$\mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{p}_4}{d_{11}}$$

RANSAC Camera Pose Estimation

Algorithm 1 PnP RANSAC

$nInliers \leftarrow 0$
for $i = 1 : M$ **do**
    Choose 6 correspondences, $\mathbf{X}_r$ and $\mathbf{w}_r$, randomly from $\mathbf{X}$ and $\mathbf{w}$.
    $[\mathbf{R}_r, \mathbf{t}_r]$ = `LinearPnP`$(\mathbf{X}_r, \mathbf{w}_r, \mathbf{K})$
    Compute the number of inliers, n_r, with respect to $\mathbf{R}_r$, $\mathbf{t}_r$.
    if $n_r > nInliers$ **then**
        $nInliers \leftarrow n_r$
        $\mathbf{R} = \mathbf{R}_r$ and $\mathbf{t} = \mathbf{t}_r$
    end if
end for

Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

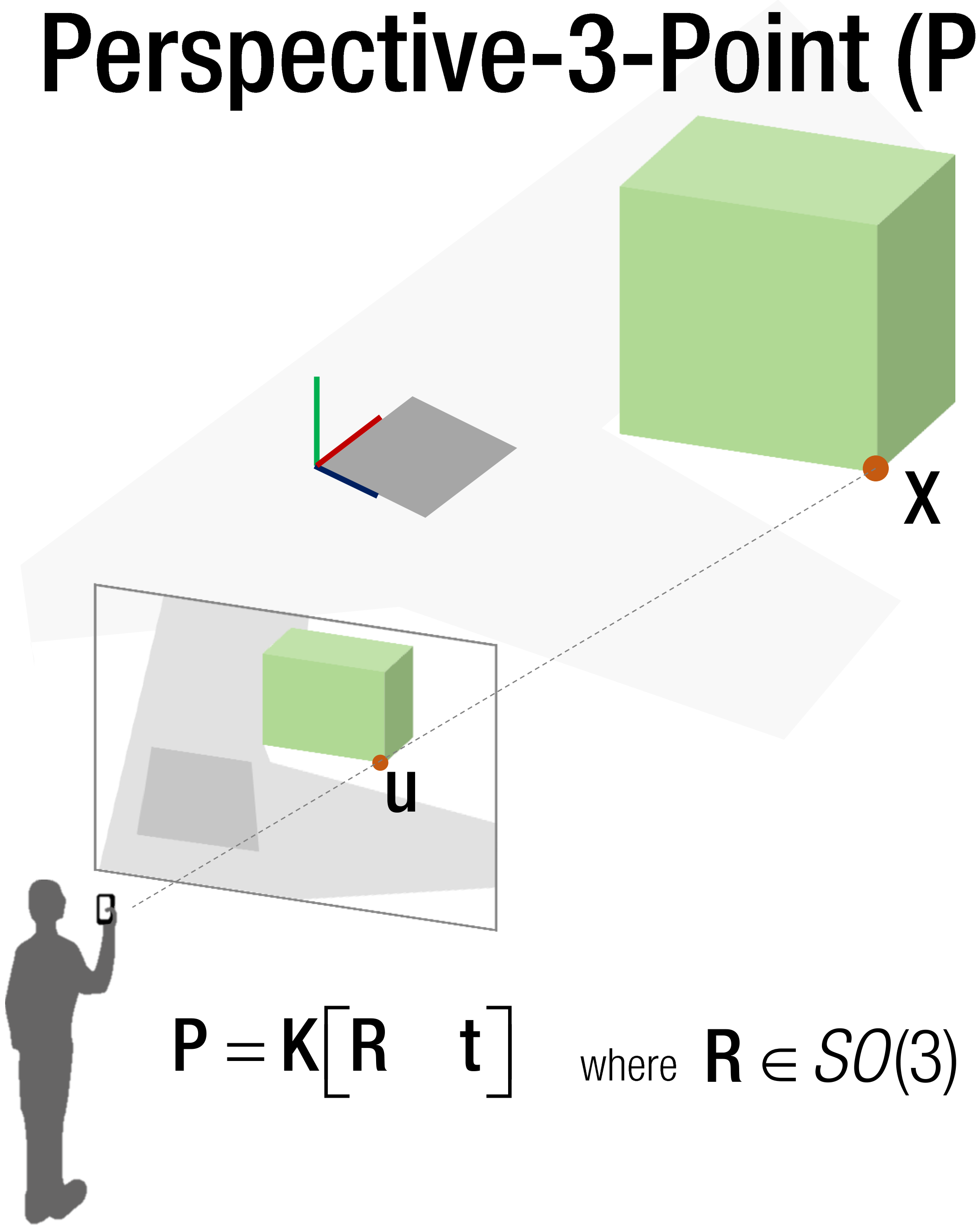
$$\lambda\mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

Perspective-3-Point (P3P)



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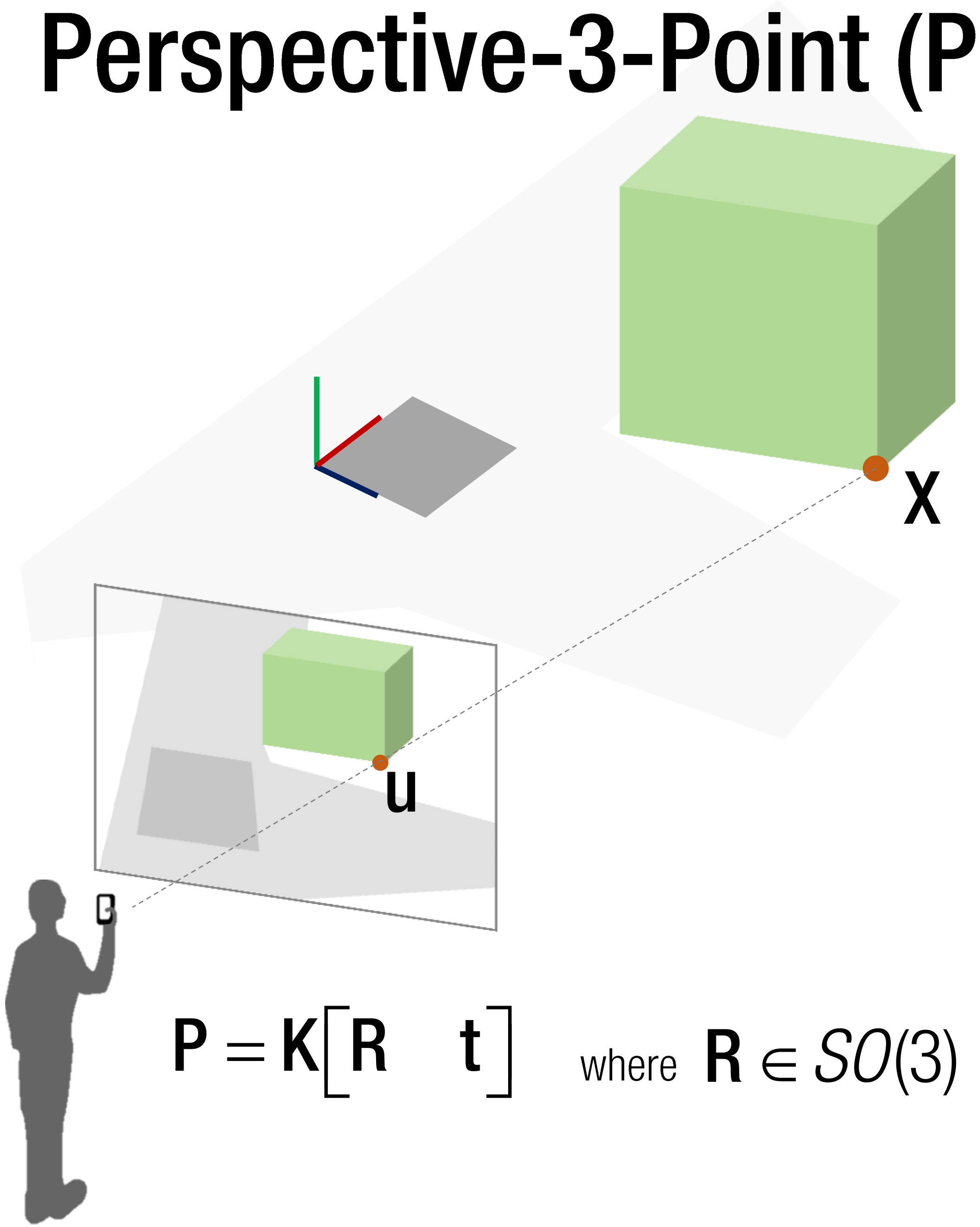
# of unknowns: $\underbrace{11 = 12 \text{ (3x4 matrix)} - 1 \text{ (scale)}}_{\text{6 dof when } \mathbf{K} \text{ is known.}}$

# of equations per correspondence: 2

Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$ where $\mathbf{R} \in SO(3)$

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$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\underline{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

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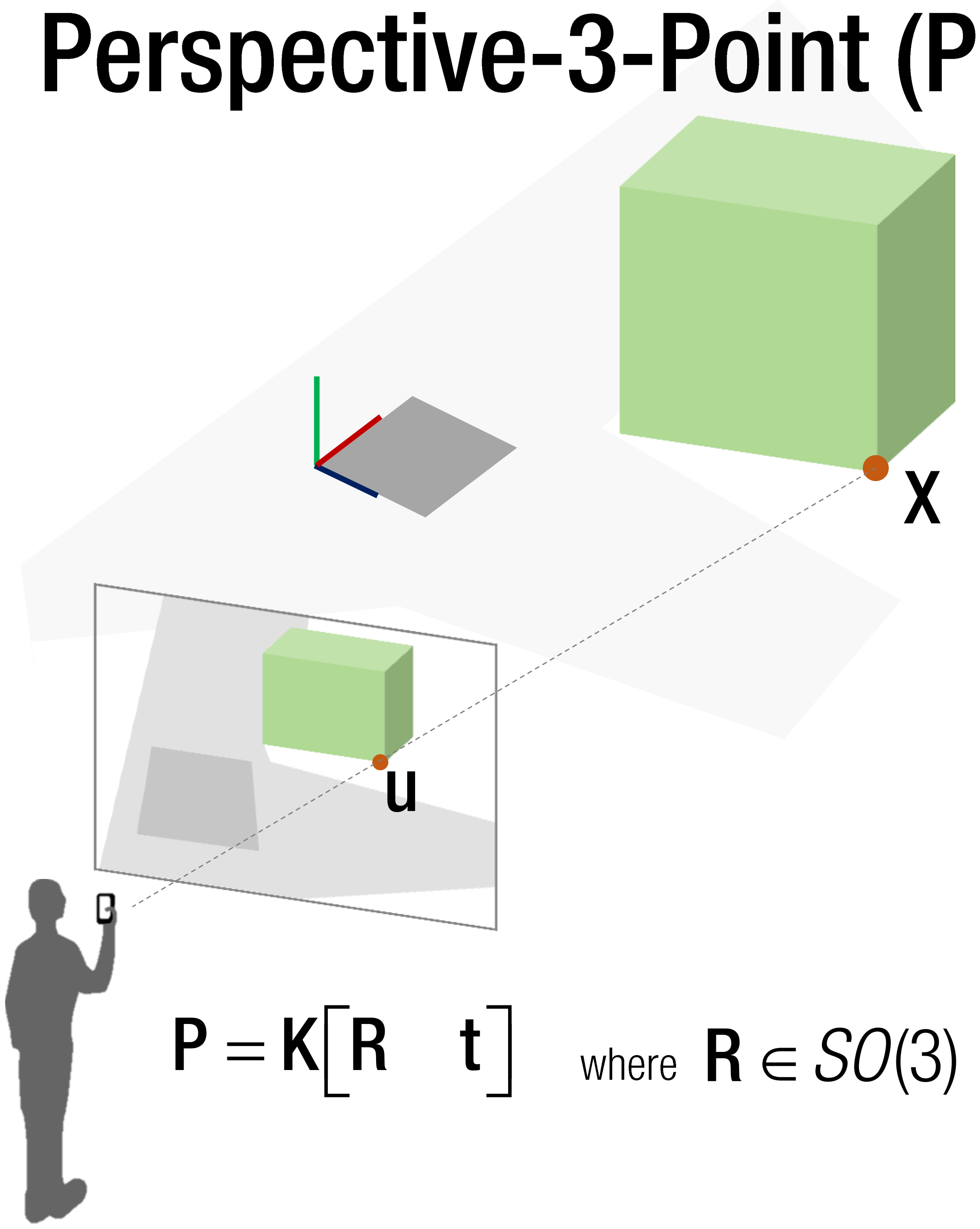
\# of unknowns: $\underline{11 = 12\ (3x4\ matrix) - 1\ (scale)}$

6 dof when **K** is known.

\# of equations per correspondence: 2

Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

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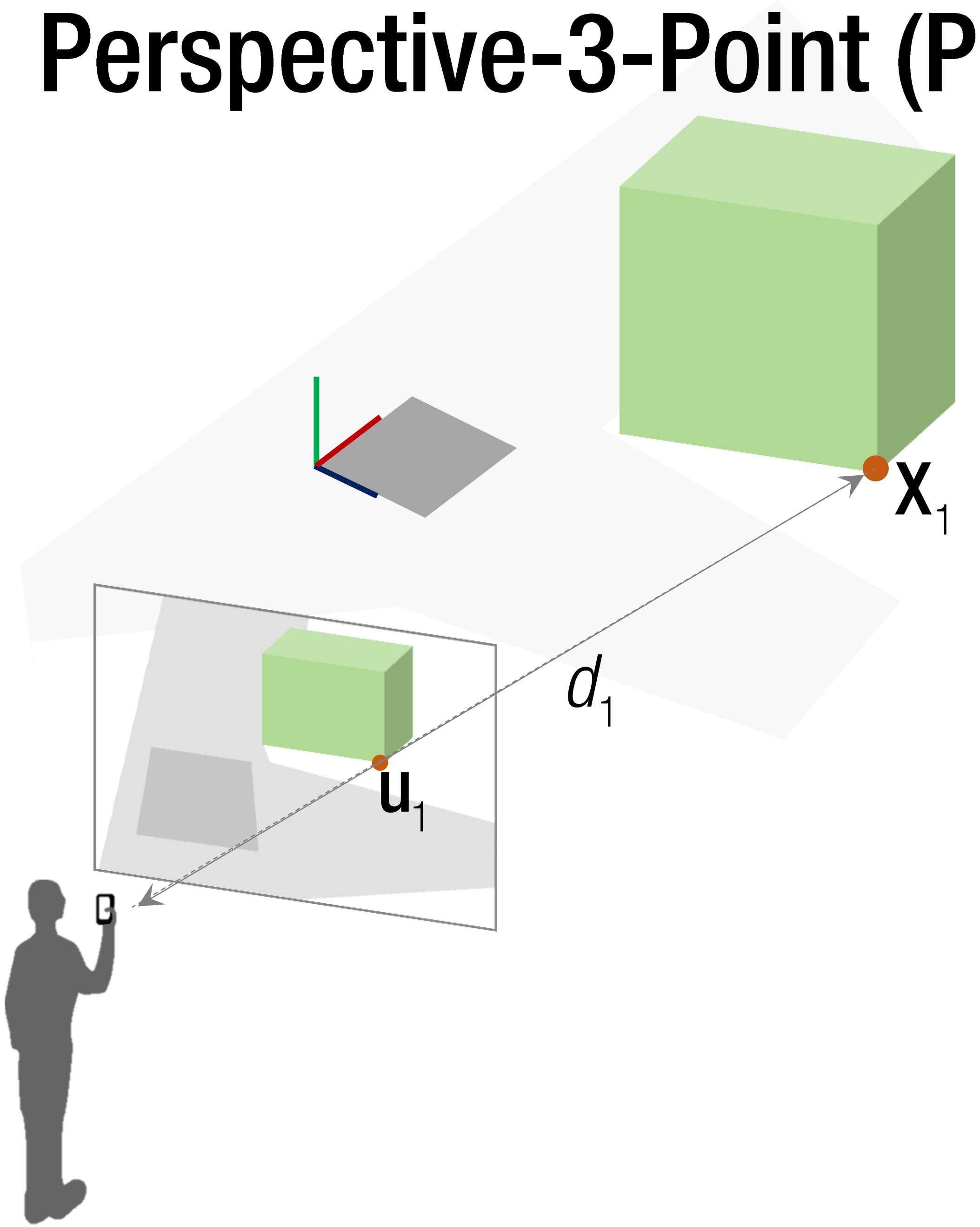
# of unknowns: $\underbrace{11 = 12 \text{ (3x4 matrix)} - 1 \text{ (scale)}}_{\text{6 dof when } \mathbf{K} \text{ is known.}}$

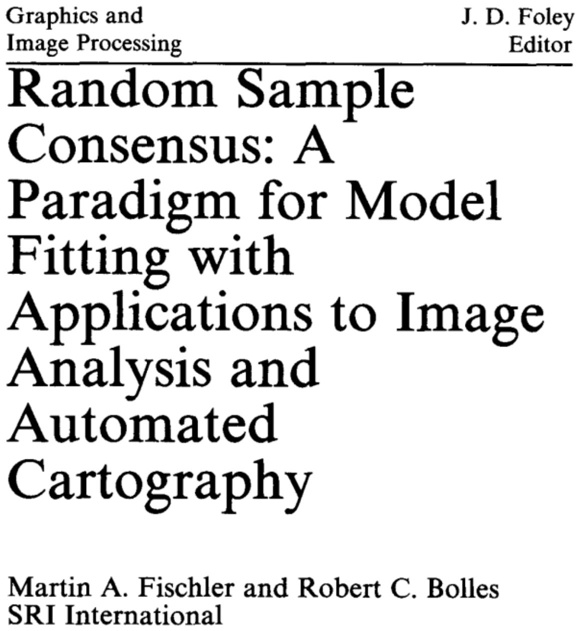
# of equations per correspondence: 2

3 correspondences should be enough.

Perspective-3-Point (P3P)

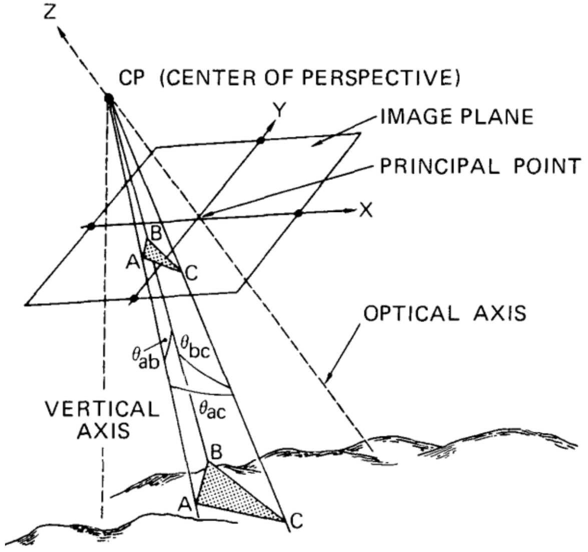


RANSAC with PnP

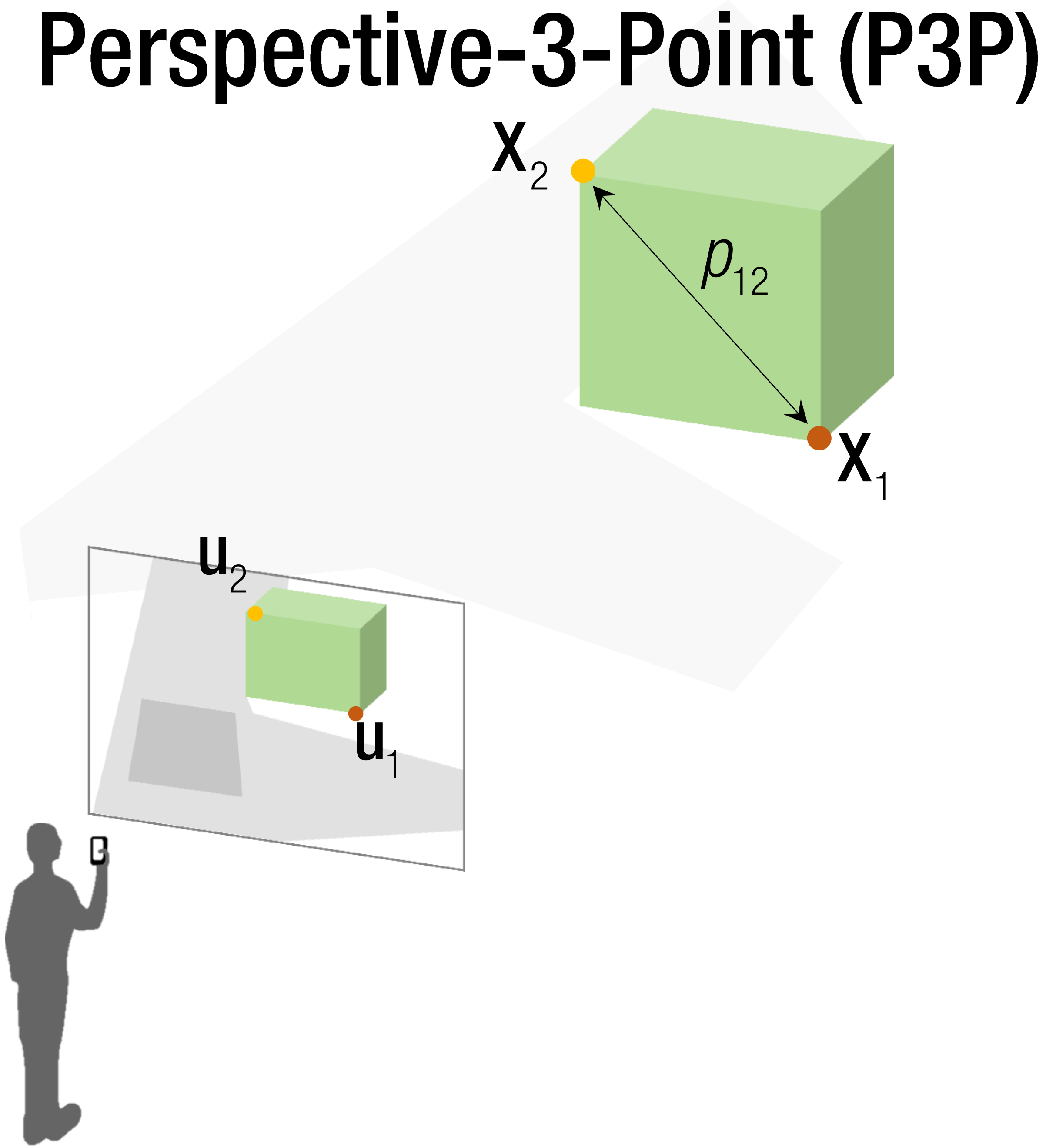

Graphics and Image Processing — J. D. Foley, Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

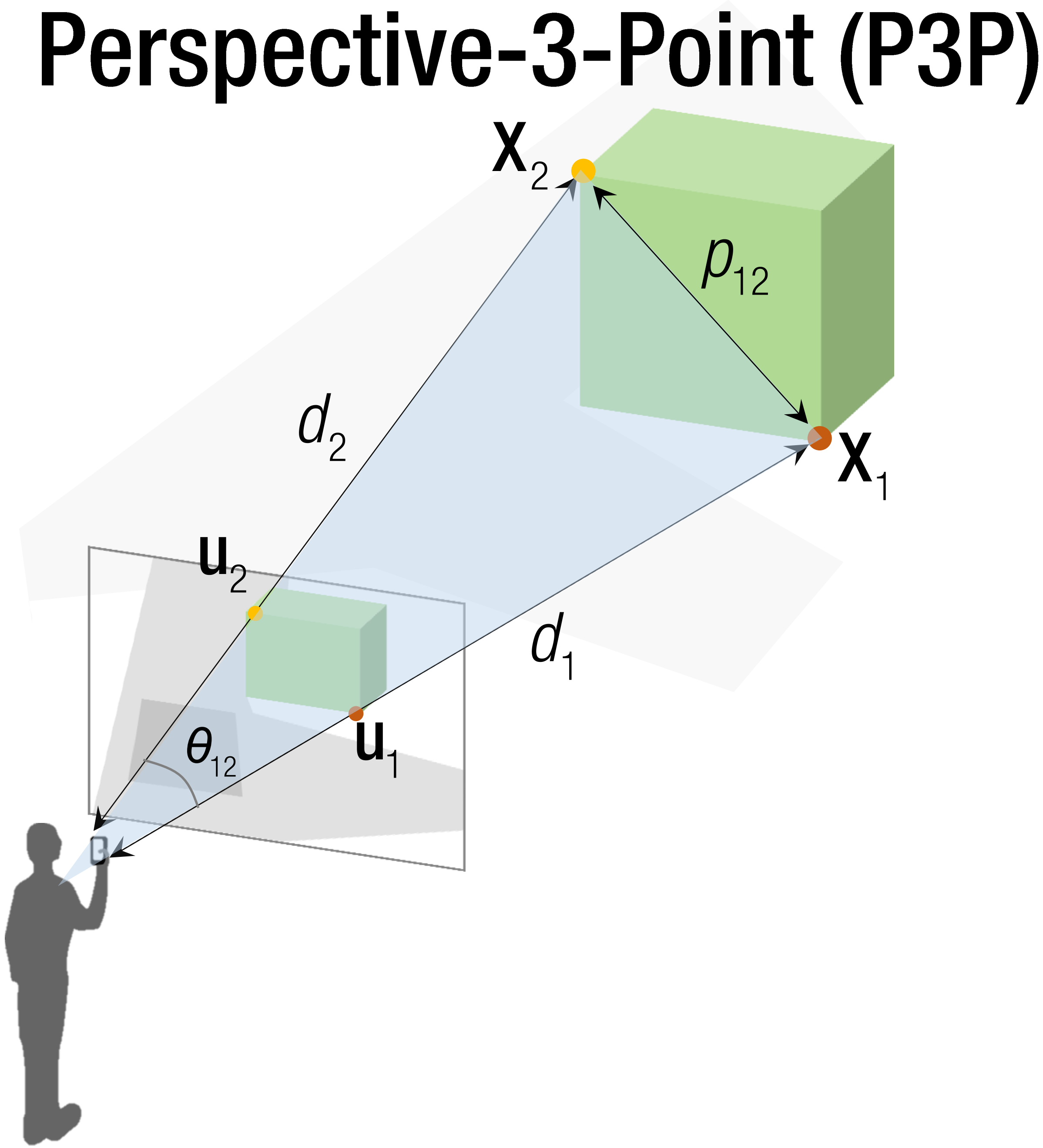
Martin A. Fischler and Robert C. Bolles
SRI International



Perspective-3-Point (P3P)



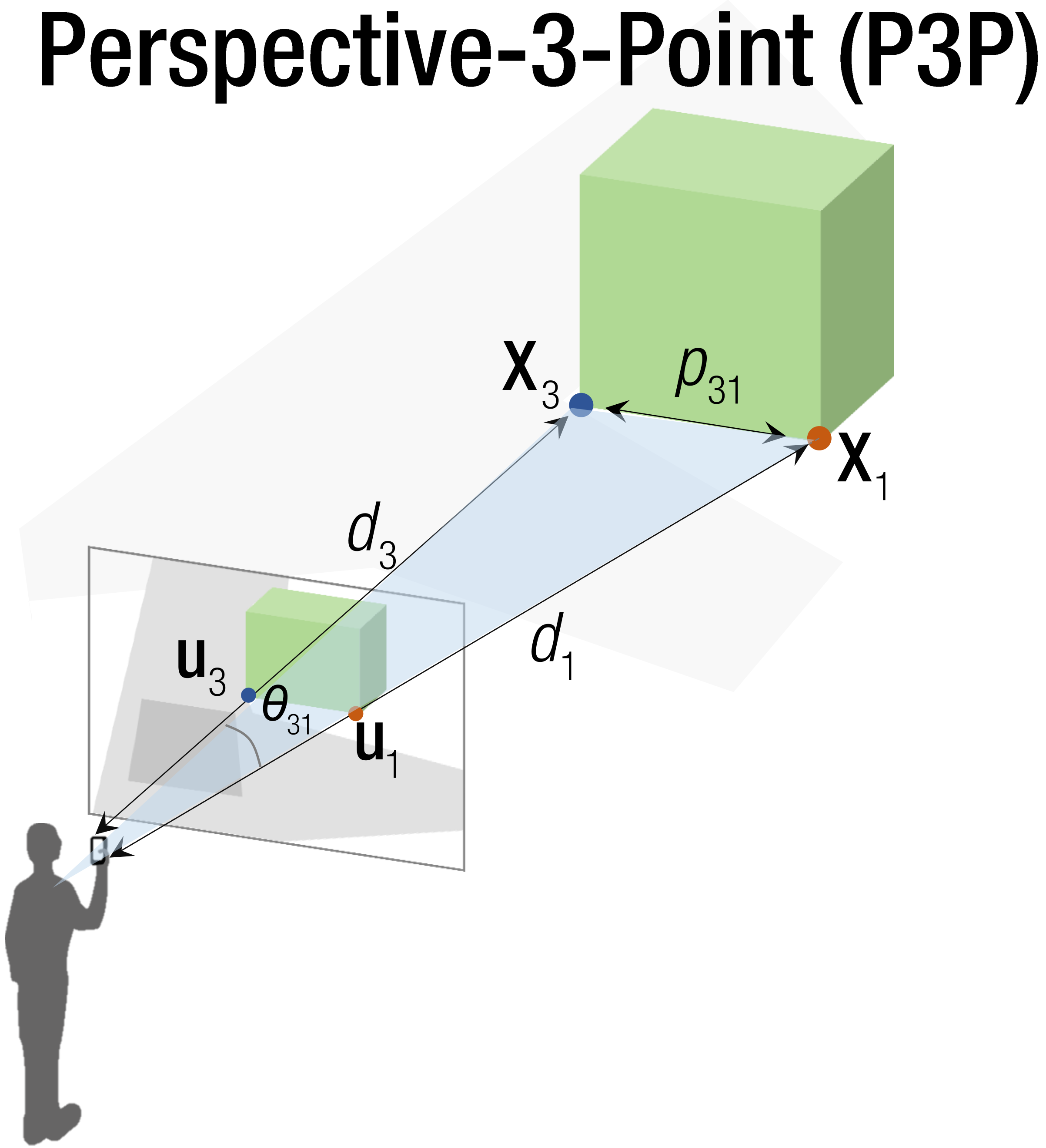
Perspective-3-Point (P3P)



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\boldsymbol{\theta}_{12} = p_{12}^2$$

Perspective-3-Point (P3P)

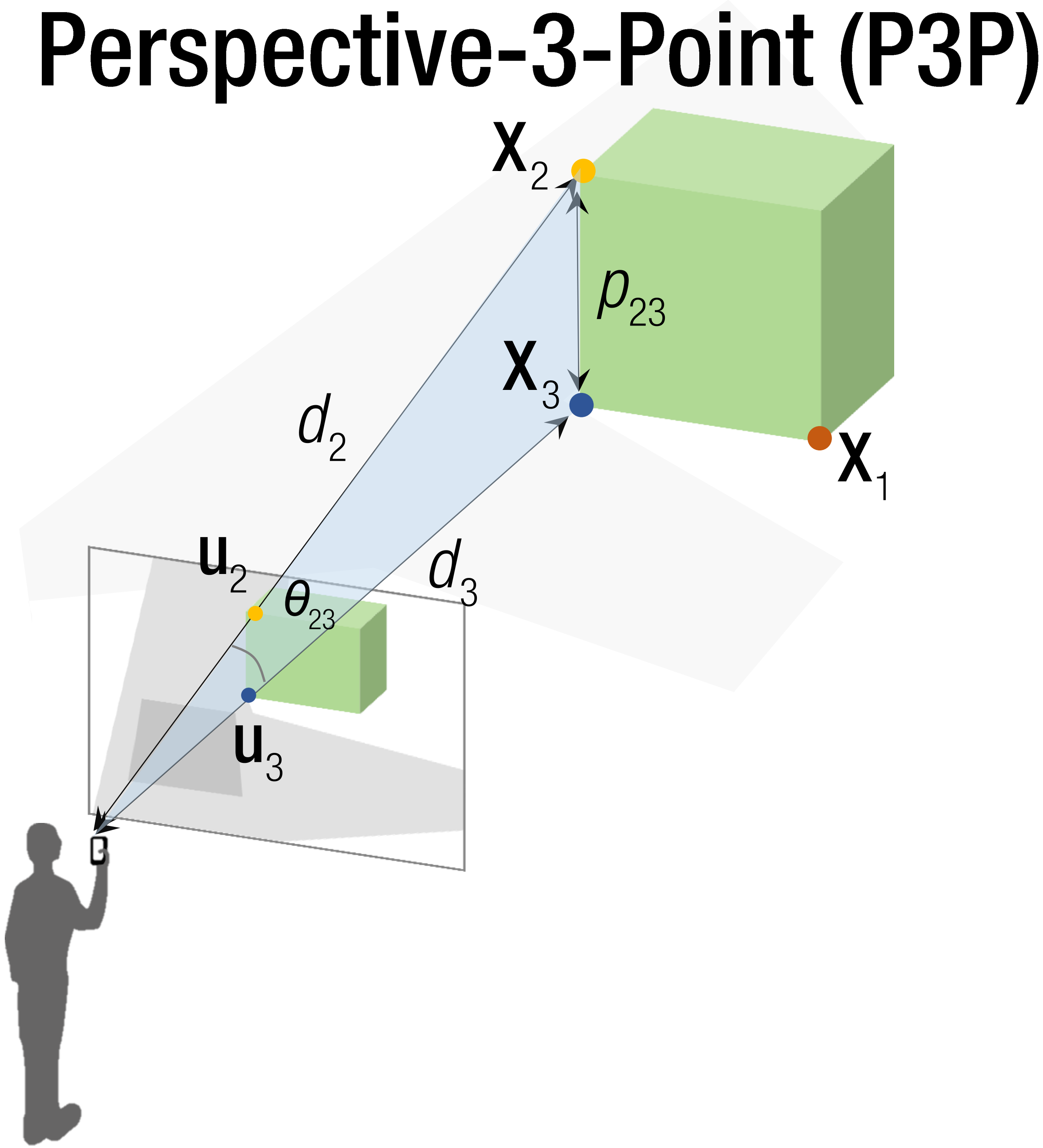


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\boldsymbol{\theta}_{12} = p_{12}^2$$

$$d_3^2 + d_1^2 - 2d_3 d_1 \cos\boldsymbol{\theta}_{31} = p_{31}^2$$

Perspective-3-Point (P3P)

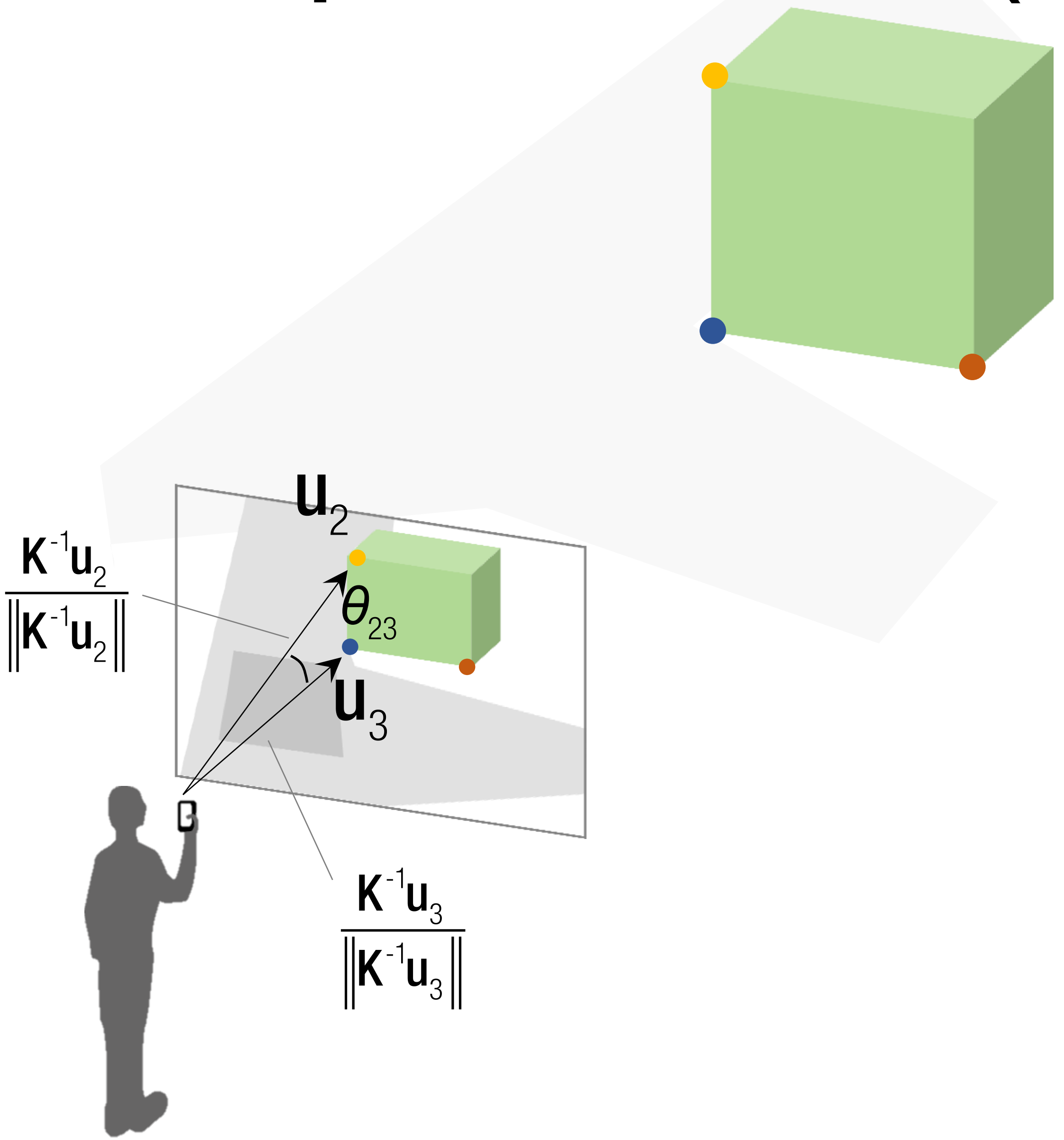


2nd Cosine law:

$$
\left.
\begin{aligned}
d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} &= p_{12}^2 \\
d_3^2 + d_1^2 - 2d_3 d_1 \cos\theta_{31} &= p_{31}^2 \\
d_2^2 + d_3^2 - 2d_2 d_3 \cos\theta_{23} &= p_{23}^2
\end{aligned}
\right| \quad \text{3 equations}
$$

Unknowns: d_1, d_2, d_3

Perspective-3-Point (P3P)



2nd Cosine law:

$$\left.\begin{aligned} d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} &= p_{12}^2 \\ d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} &= p_{31}^2 \\ d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} &= p_{23}^2 \end{aligned}\right| \text{ 3 equations}$$

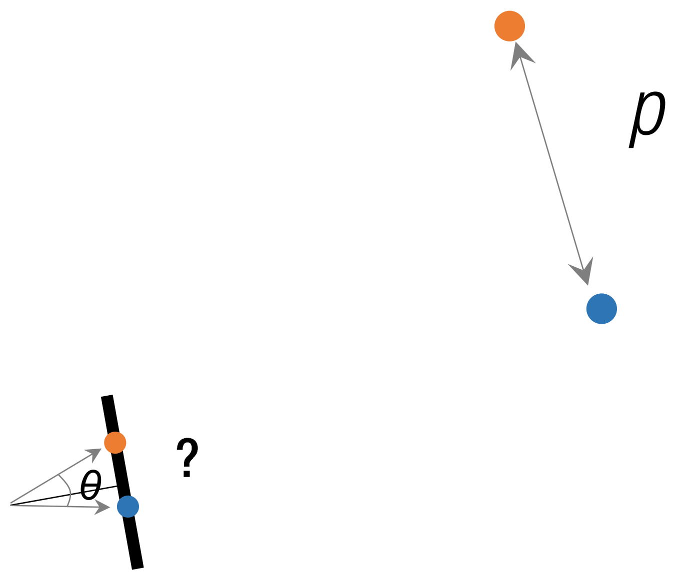
Unknowns: d_1, d_2, d_3

Note:

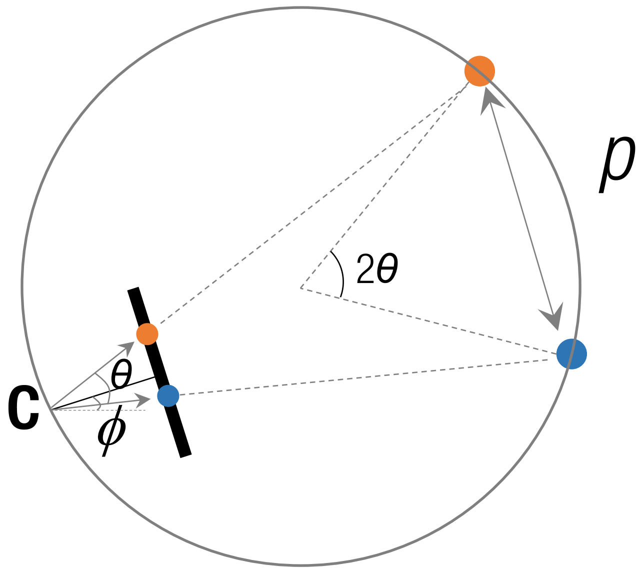
$$\cos\theta_{12} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_2\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|} \qquad \cos\theta_{23} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_2\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

$$\cos\theta_{31} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

Geometric Interpretation: 1D Camera

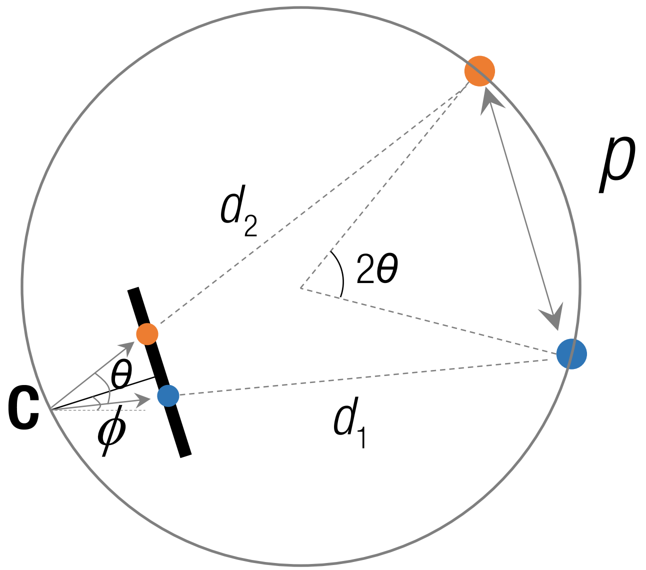


Geometric Interpretation: 1D Camera



Property of inscribed angle

Geometric Interpretation: 1D Camera

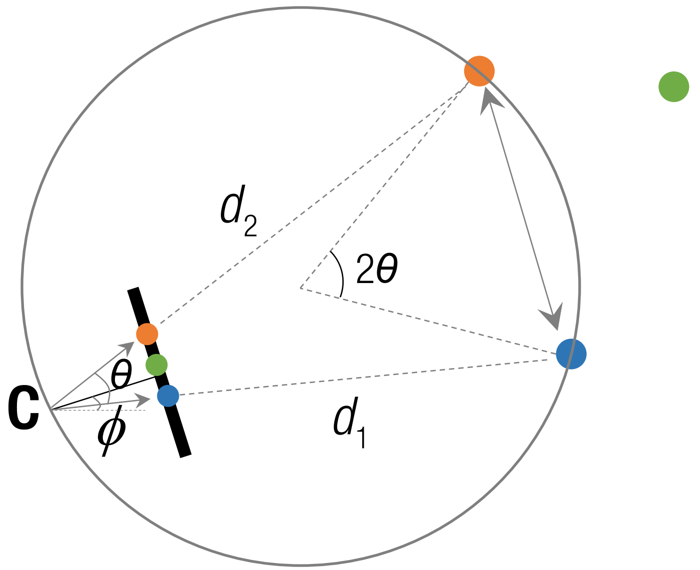


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\boldsymbol{\theta} = p^2$$

Infinite number of solutions

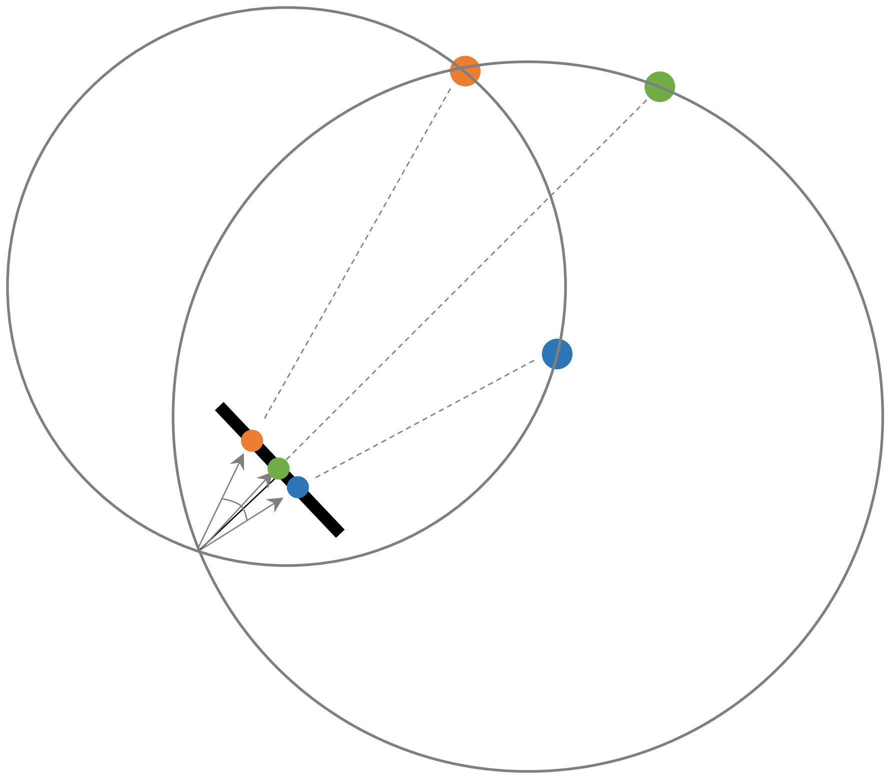
Geometric Interpretation: 1D Camera



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta = p^2$$

Geometric Interpretation: 1D Camera



Finite number of solutions

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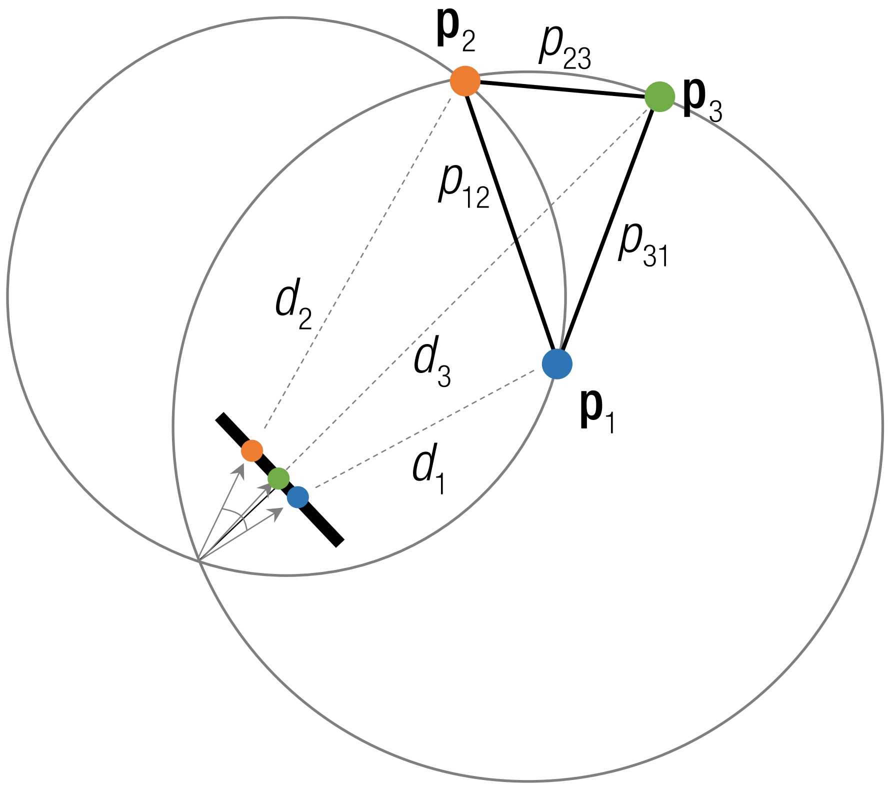
Geometric Interpretation: 1D Camera



2nd Cosine law:

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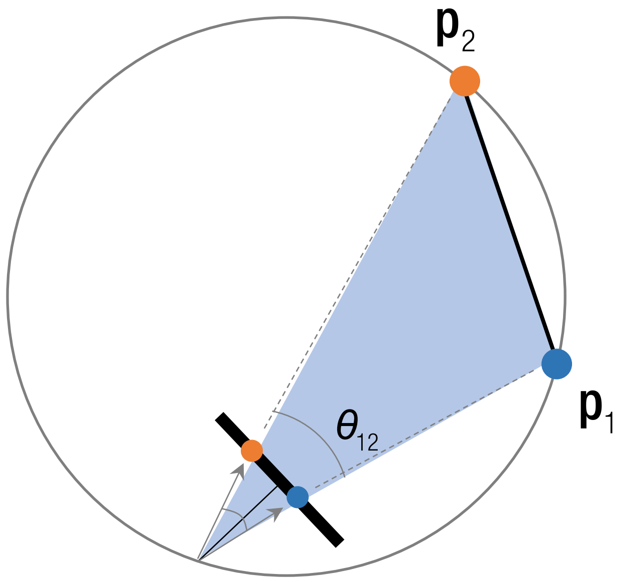
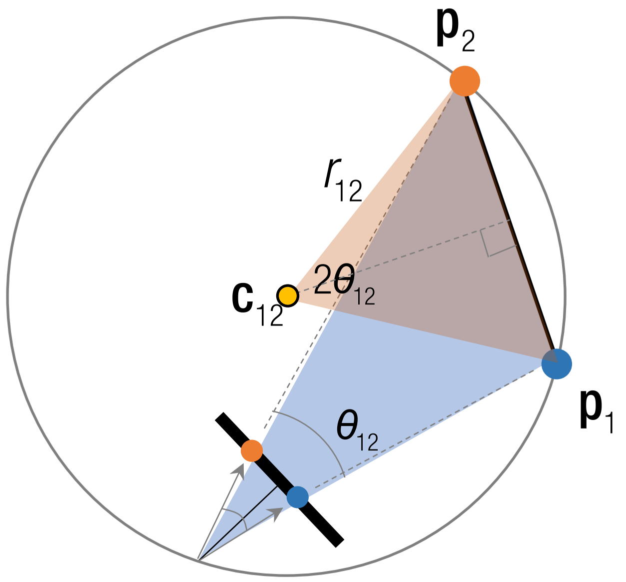
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} = p_{23}^2$$

Finite number of solutions

Geometric Interpretation: 1D Camera



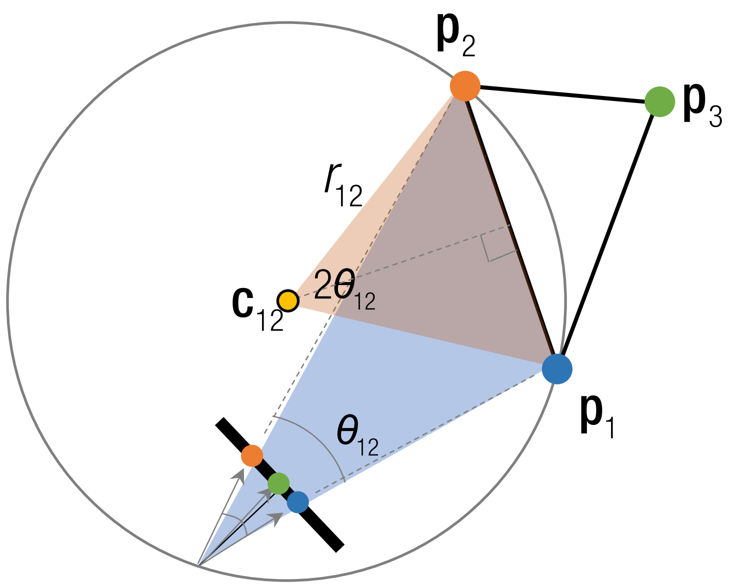
Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}$$

where $r_{12} = \dfrac{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}{2\sin\theta_{12}}$

Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}$$

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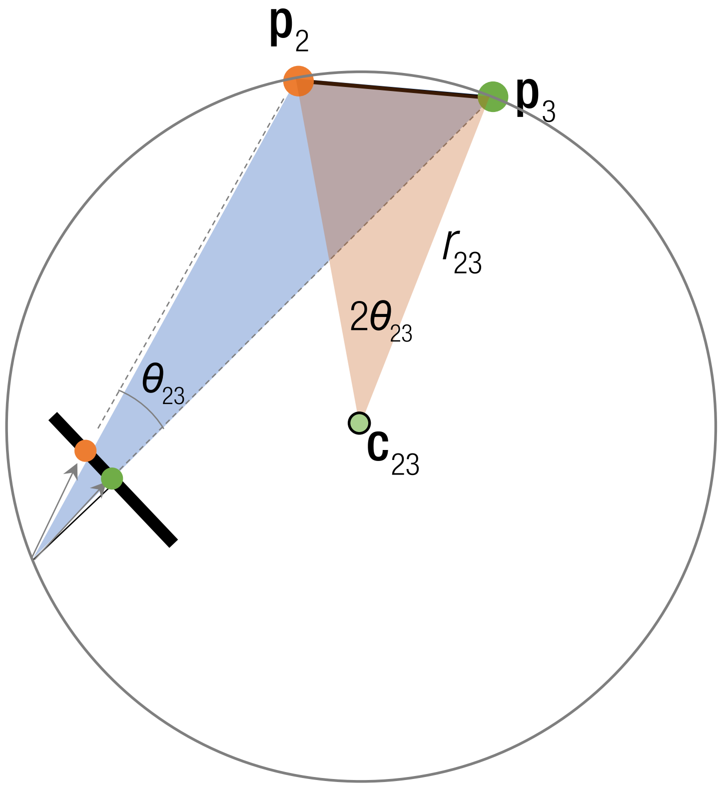
Geometric Interpretation: 1D Camera



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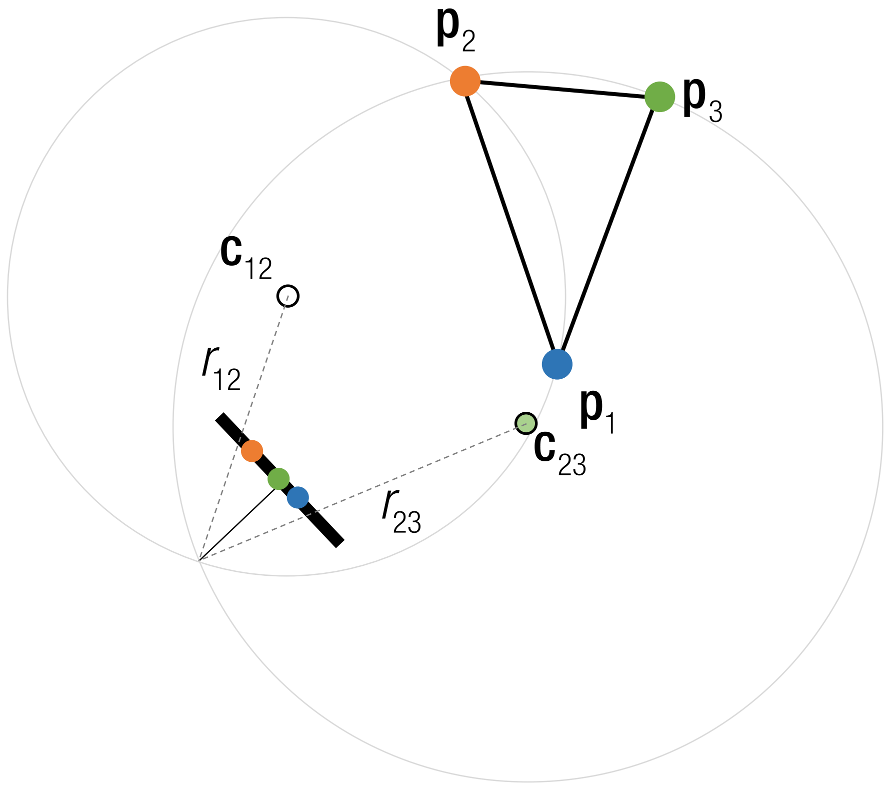
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\left\|\mathbf{p}_3 - \mathbf{p}_2\right\|}$$

where $r_{23} = \dfrac{\left\|\mathbf{p}_3 - \mathbf{p}_2\right\|}{2\sin\theta_{23}}$

Geometric Interpretation: 1D Camera



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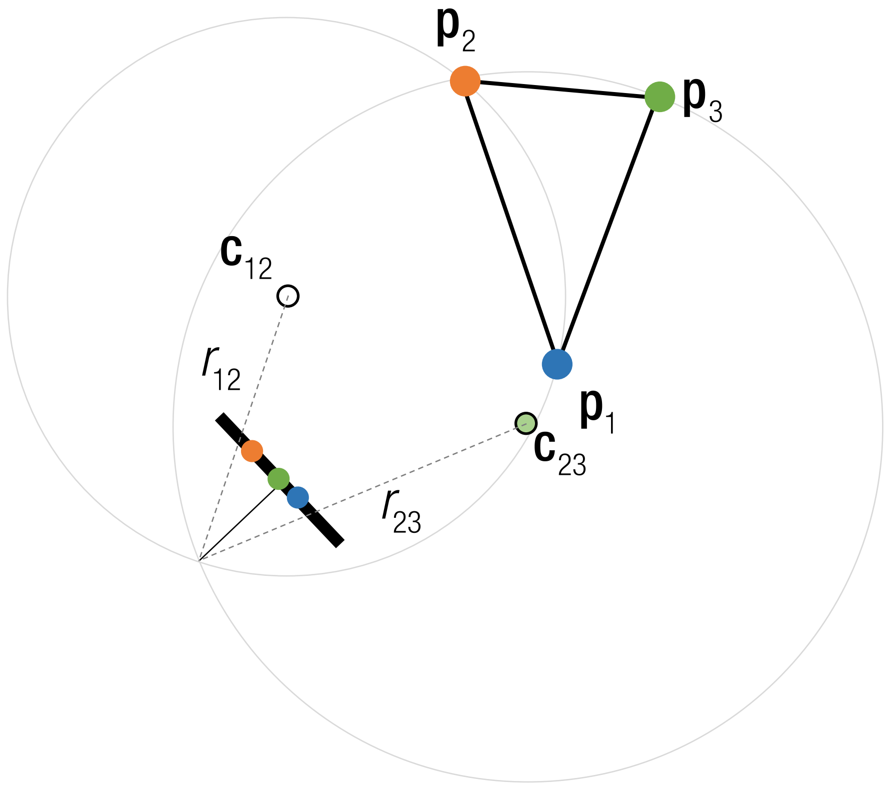
where $r_{23} = \dfrac{\left\|\mathbf{p}_3 - \mathbf{p}_2\right\|}{2\sin\theta_{23}}$

$$\left\|\mathbf{x} - \mathbf{c}_{12}\right\|^2 = r_{12}^2 \qquad \left\|\mathbf{x} - \mathbf{c}_{23}\right\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions



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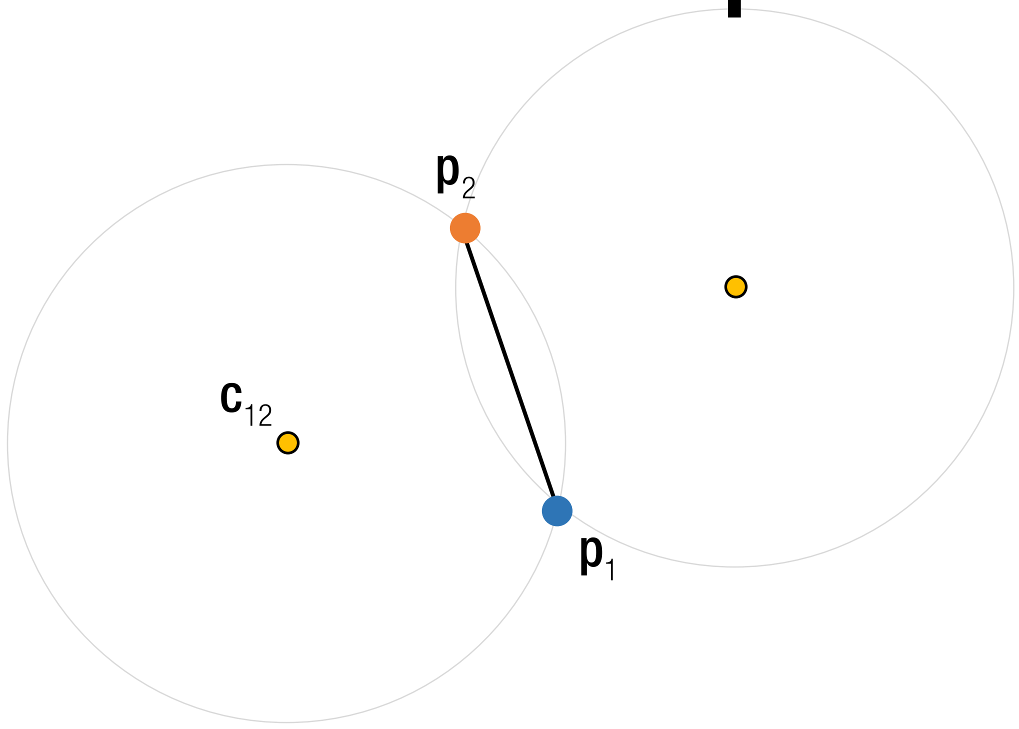
$$\left\|\mathbf{x} - \mathbf{c}_{12}\right\|^2 = r_{12}^2 \qquad \left\|\mathbf{x} - \mathbf{c}_{23}\right\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions



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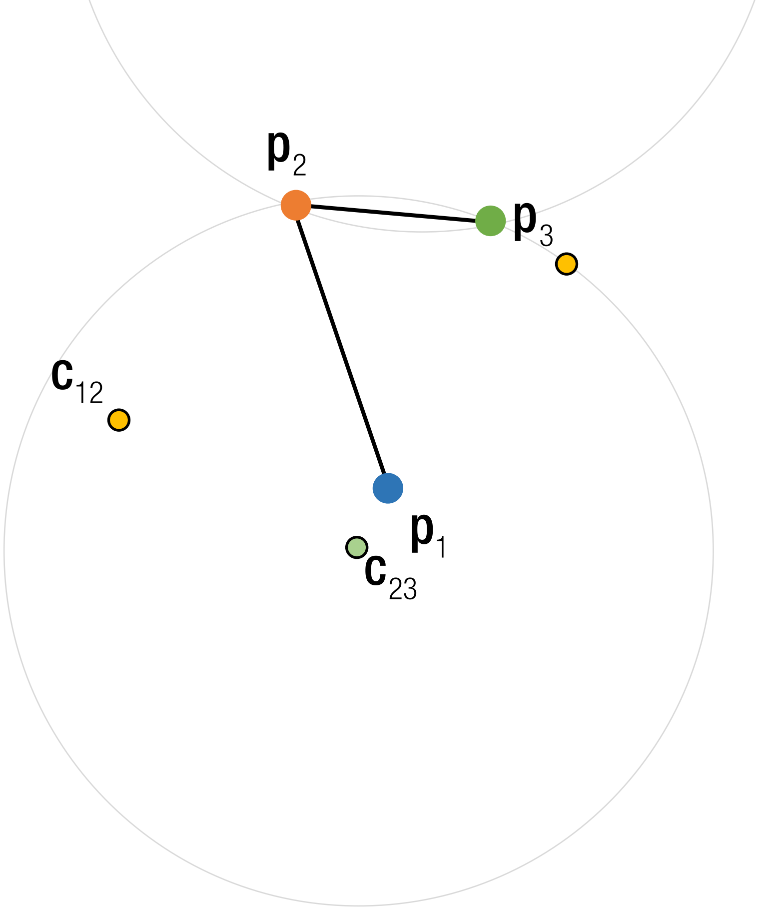
where $r_{12} = \dfrac{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}{2 \sin\theta_{12}}$

Geometric Interpretation: Family of Solutions



4 combinations

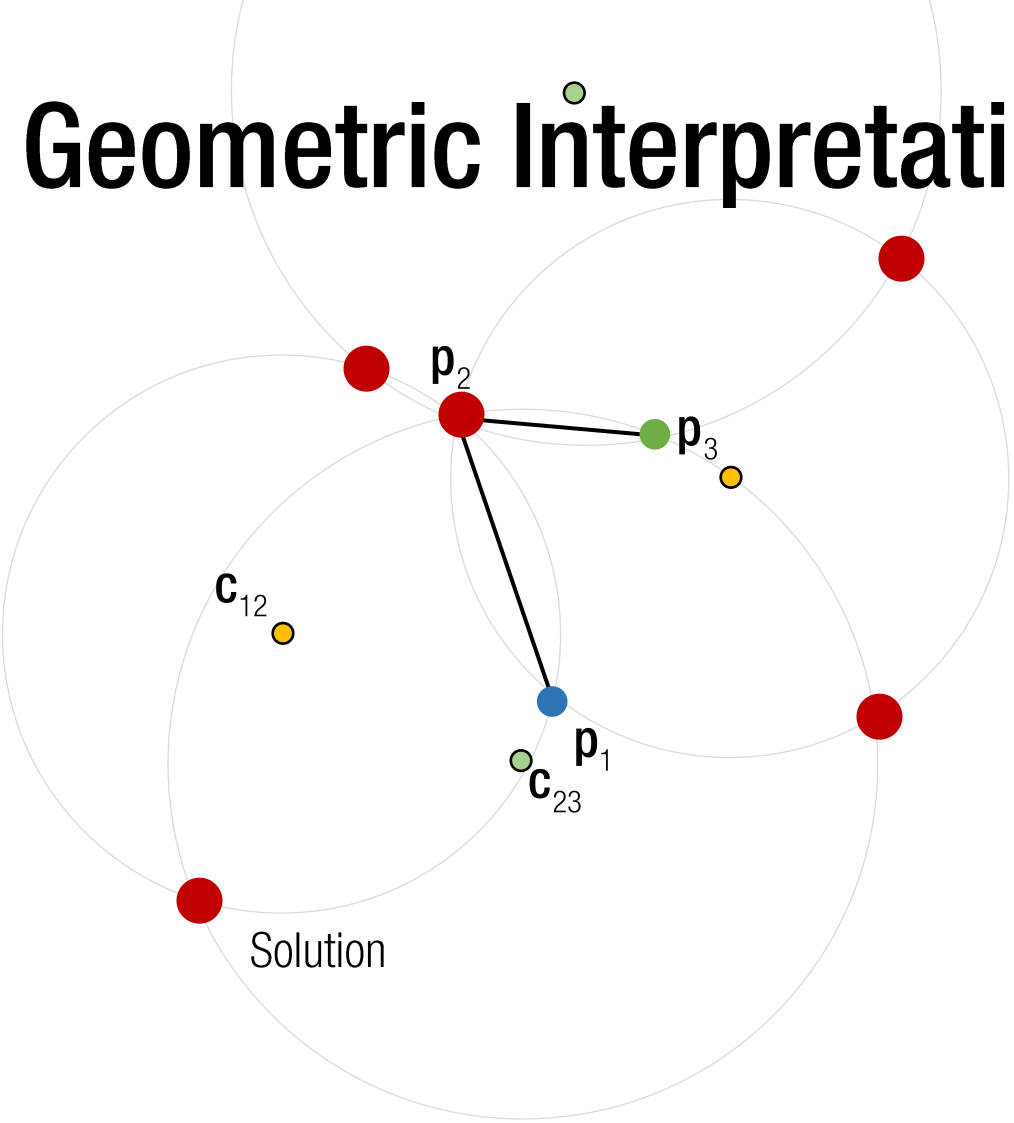
$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}$$

where $r_{12} = \dfrac{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}{2 \sin\theta_{12}}$

$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\left\| \mathbf{p}_3 - \mathbf{p}_2 \right\|}$$

where $r_{23} = \dfrac{\left\| \mathbf{p}_3 - \mathbf{p}_2 \right\|}{2 \sin\theta_{23}}$

Geometric Interpretation: Family of Solutions



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\|\mathbf{p}_2 - \mathbf{p}_1\|}$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

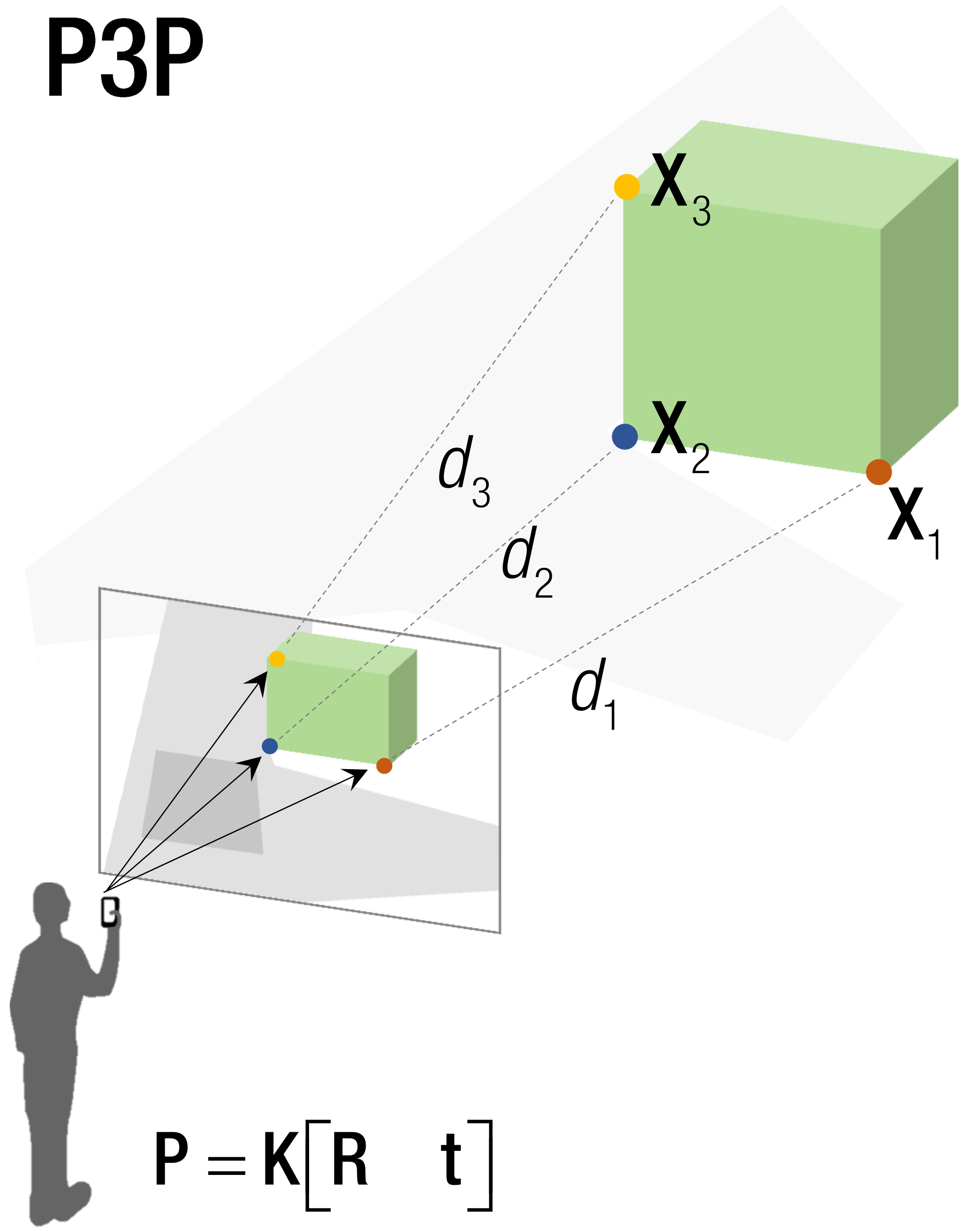
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\|\mathbf{p}_3 - \mathbf{p}_2\|}$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

4 combinations of circle centers

→ 4 solutions except for $\mathbf{p}_2$ ($\mathbf{p}_2$ is counted four times.).

P3P



2nd Cosine law:

$$\begin{aligned} d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} &= p_{12}^2 \\ d_3^2 + d_1^2 - 2d_3 d_1 \cos\theta_{31} &= p_{31}^2 \\ d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{23} &= p_{23}^2 \end{aligned}$$

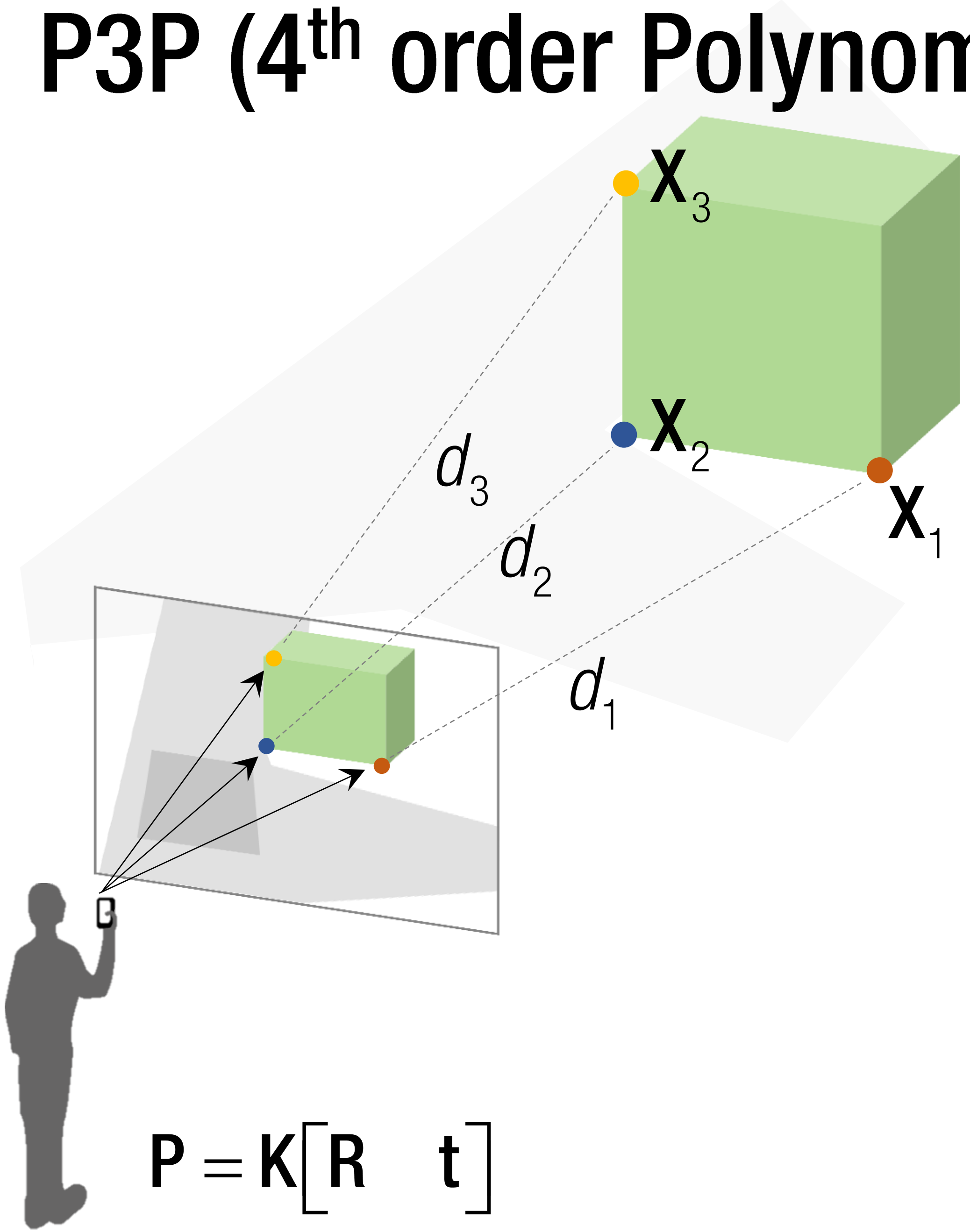
3 equations

The number of possible solutions: 8 = 2x2x2

$d_1 > 0 \quad d_2 > 0 \quad d_3 > 0 \quad : 4 = 2x2x2/2$

$\longrightarrow$ requires additional fourth point to verify the solution.

P3P (4th order Polynomial)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2nd Cosine law:

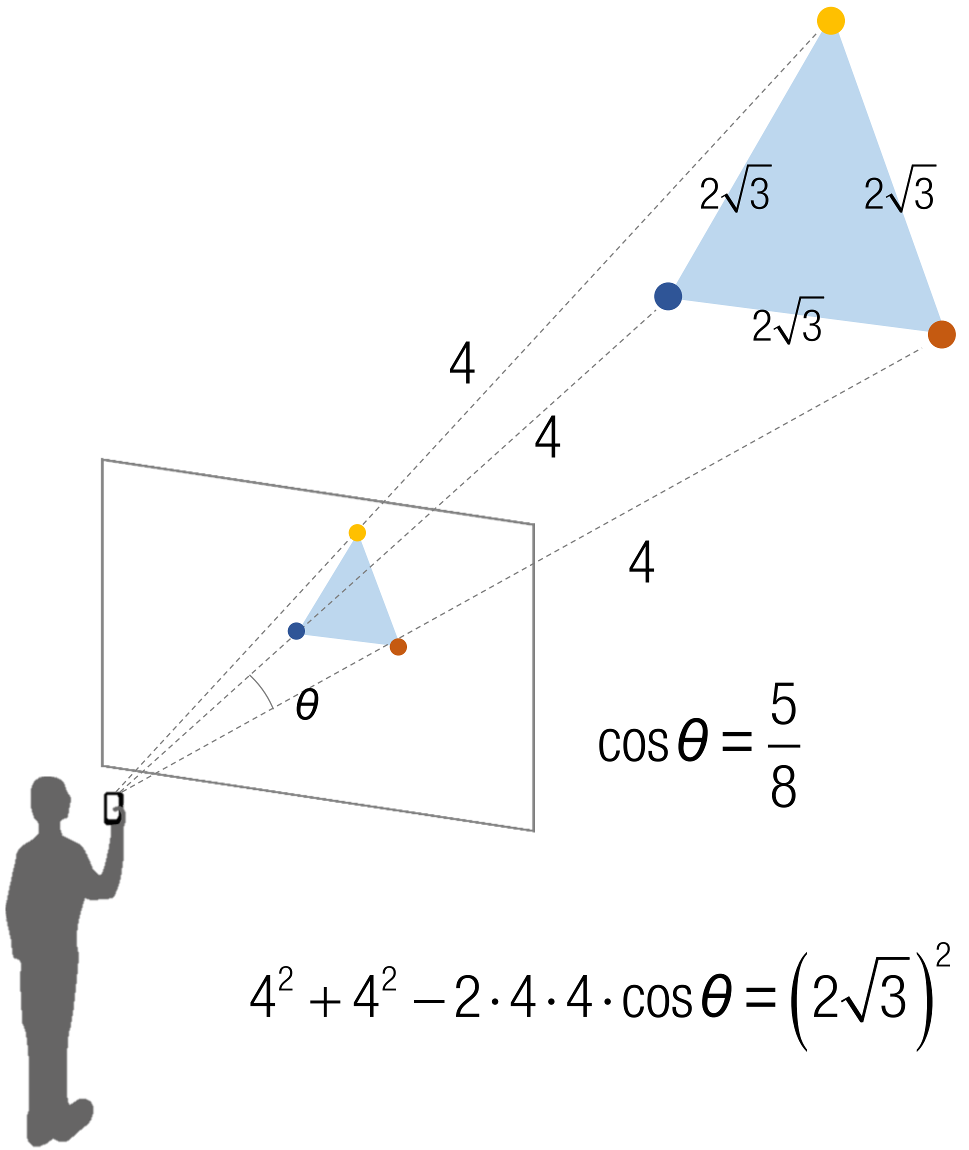
$$\left.\begin{aligned} d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} &= p_{12}^2 \\ d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} &= p_{31}^2 \\ d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} &= p_{23}^2 \end{aligned}\right| \text{ 3 equations}$$

4th order polynomial:

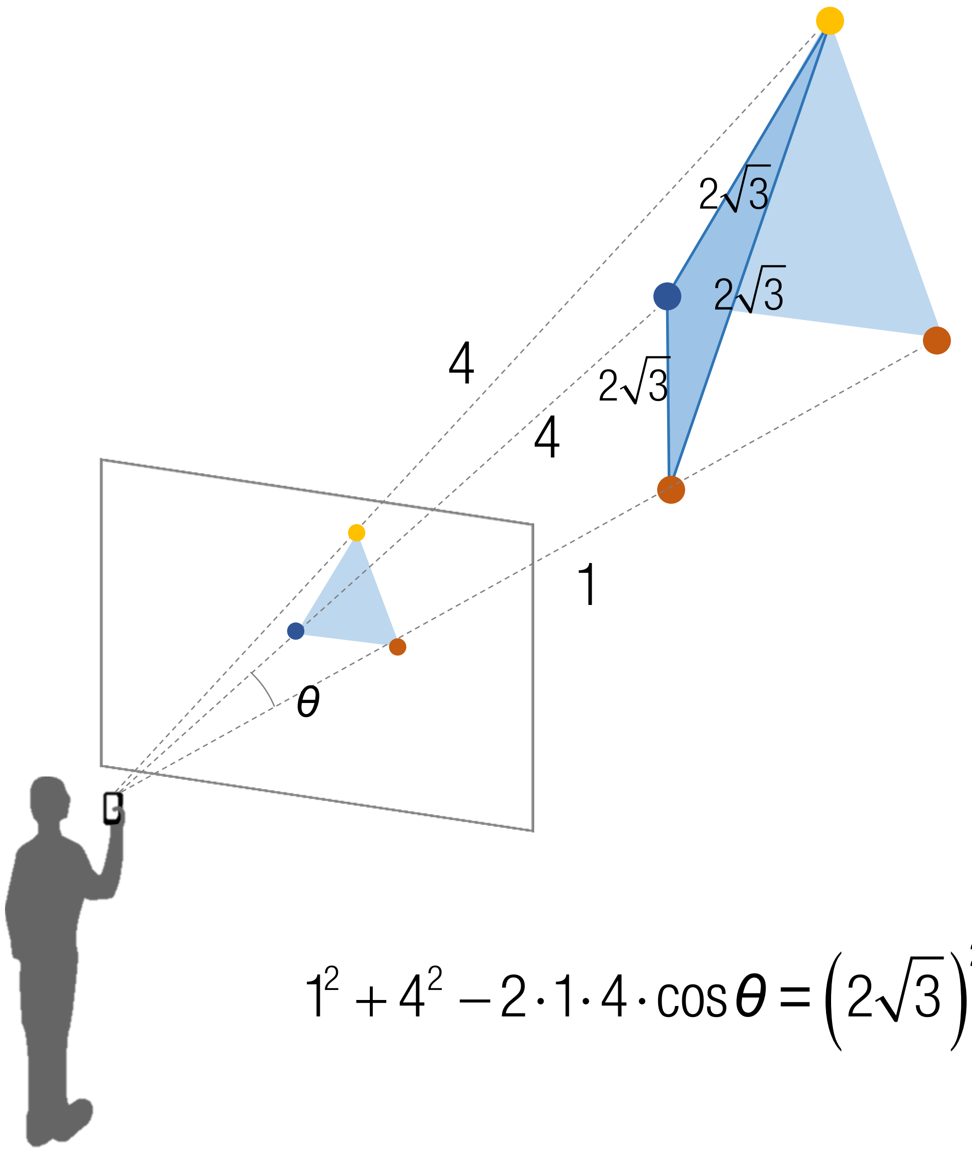
$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

Closed form solutions exist.

Four Solution Example

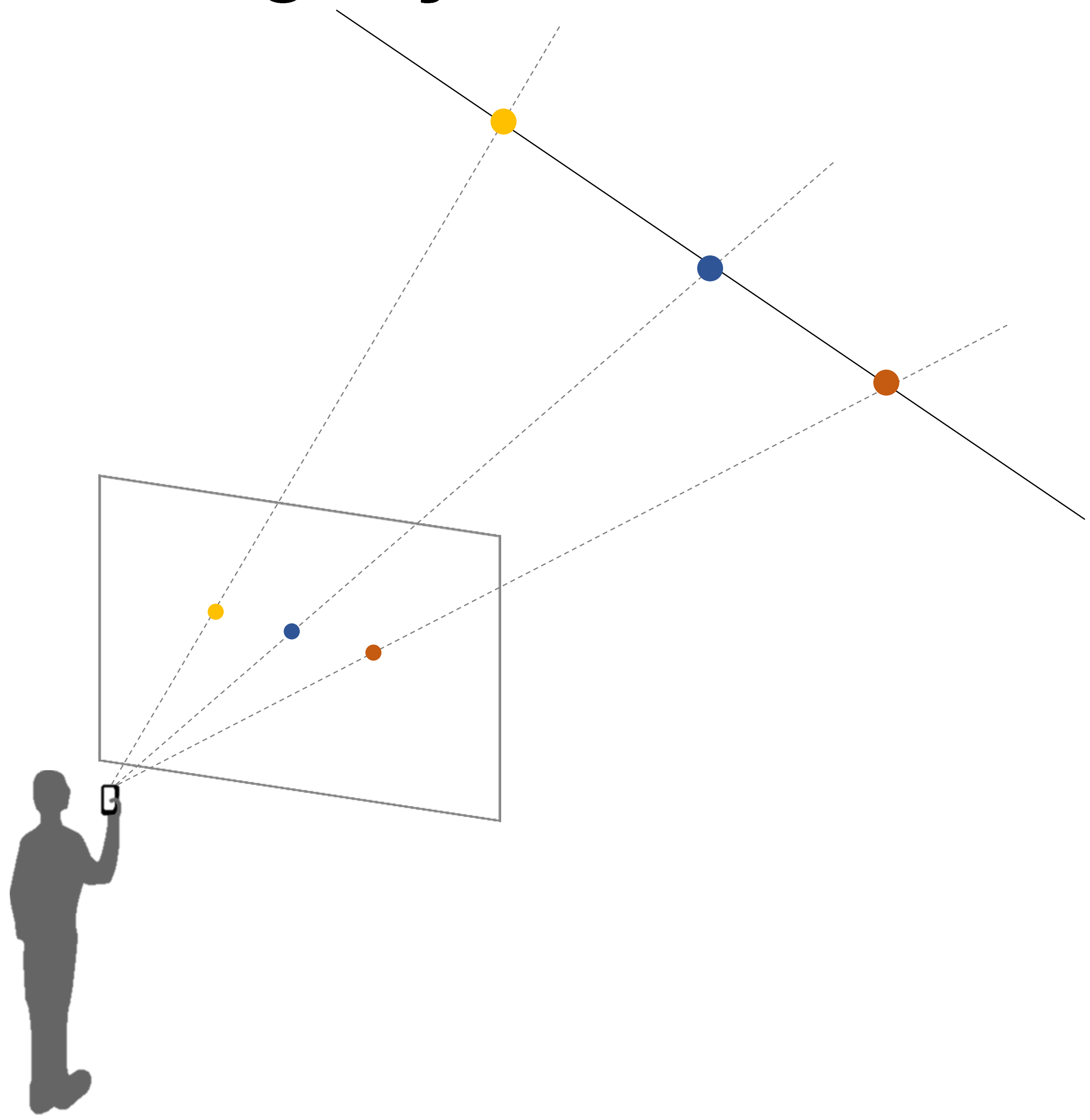


$$4^2 + 4^2 - 2 \cdot 4 \cdot 4 \cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

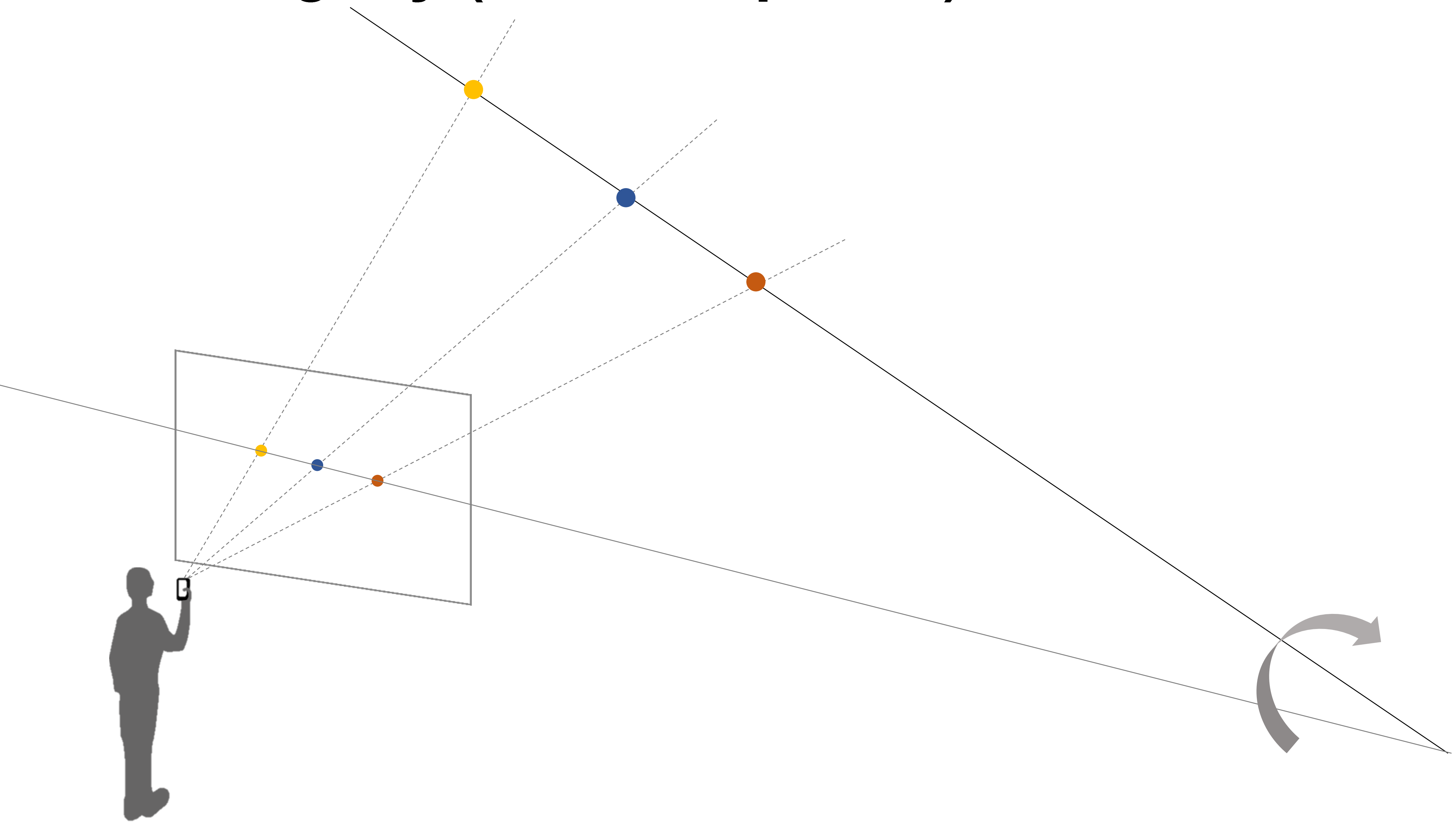


$$1^2 + 4^2 - 2 \cdot 1 \cdot 4 \cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

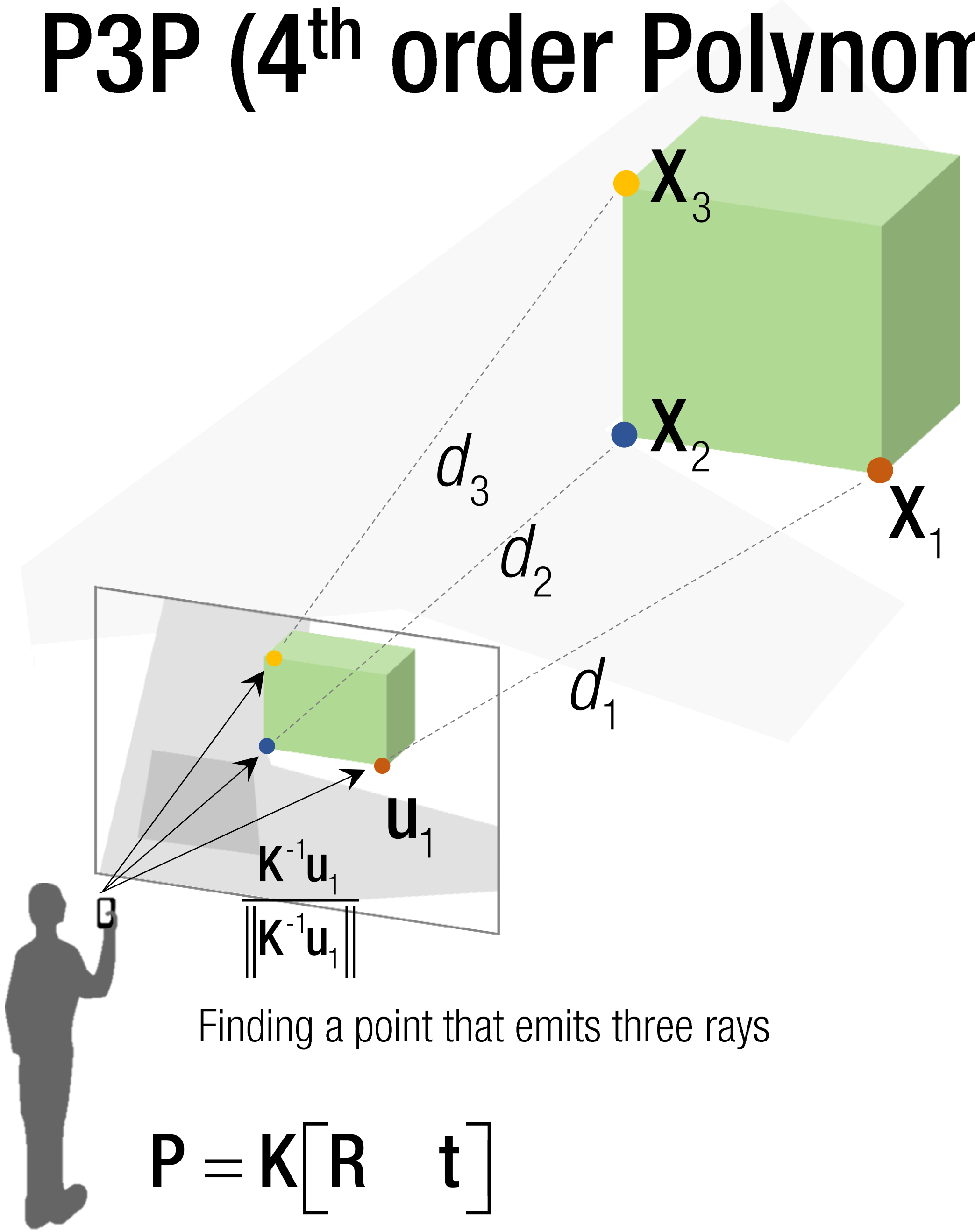
Ambiguity



Ambiguity (Colinear points)



P3P (4th order Polynomial)



Finding a point that emits three rays

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

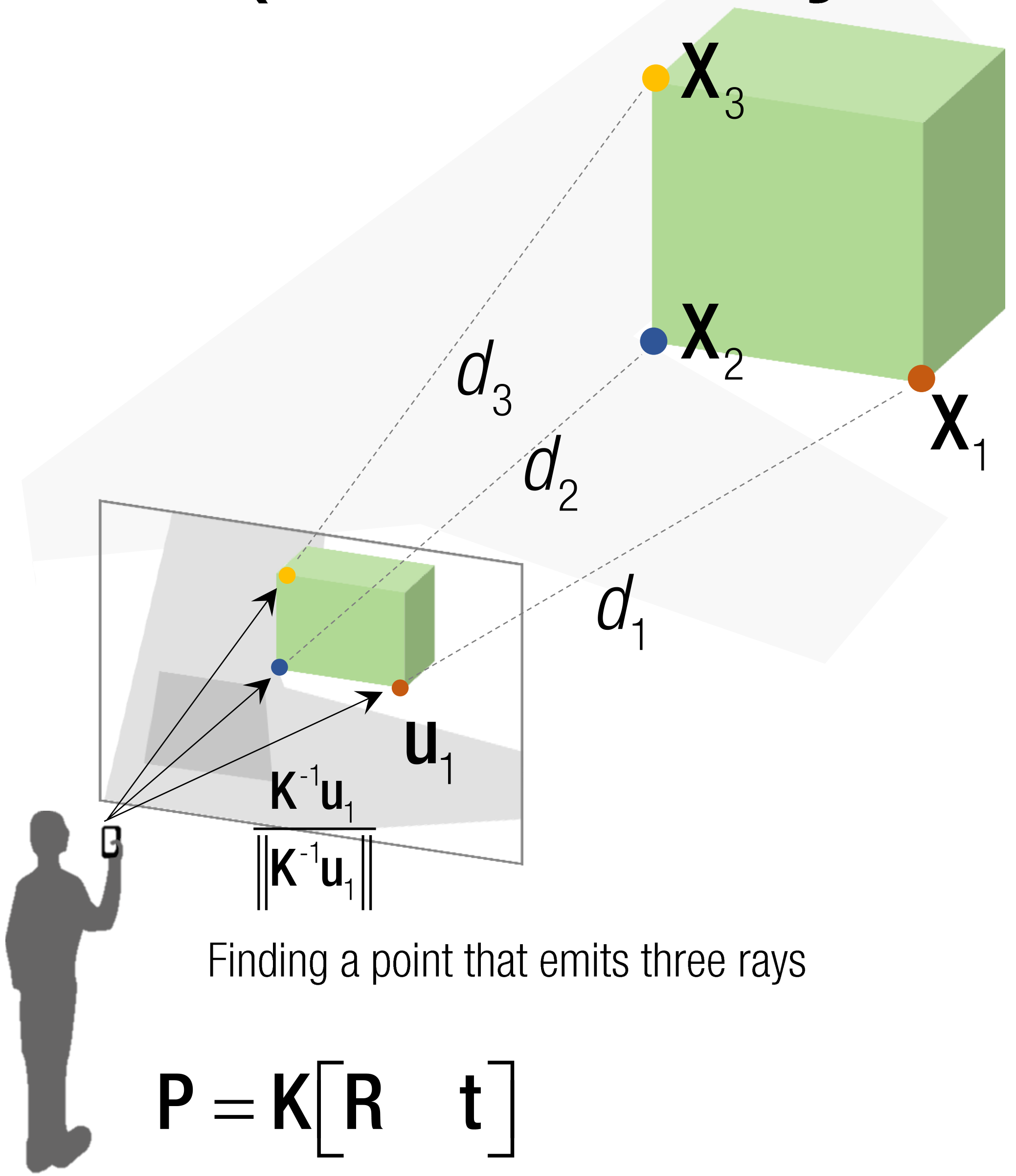
4th order polynomial:

$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

Closed form solutions exist.

→ Compute $\mathbf{t}$ using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

P3P (4th order Polynomial)



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3 equations

4th order polynomial:

$\rightarrow$ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

$\rightarrow$ Compute **t** using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

$\rightarrow$ $\begin{bmatrix} \tilde{\mathbf{X}}_1 & \tilde{\mathbf{X}}_2 & \tilde{\mathbf{X}}_3 \end{bmatrix} = \mathbf{R}\begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3 \end{bmatrix}$

Rotation matrix computation

where $\tilde{\mathbf{X}}_1 = d_1 \frac{\mathbf{K}^{-1}\mathbf{u}_1}{\|\mathbf{K}^{-1}\mathbf{u}_1\|}$ $\tilde{\mathbf{X}}_2 = d_2 \frac{\mathbf{K}^{-1}\mathbf{u}_2}{\|\mathbf{K}^{-1}\mathbf{u}_2\|}$ $\tilde{\mathbf{X}}_3 = d_3 \frac{\mathbf{K}^{-1}\mathbf{u}_3}{\|\mathbf{K}^{-1}\mathbf{u}_3\|}$