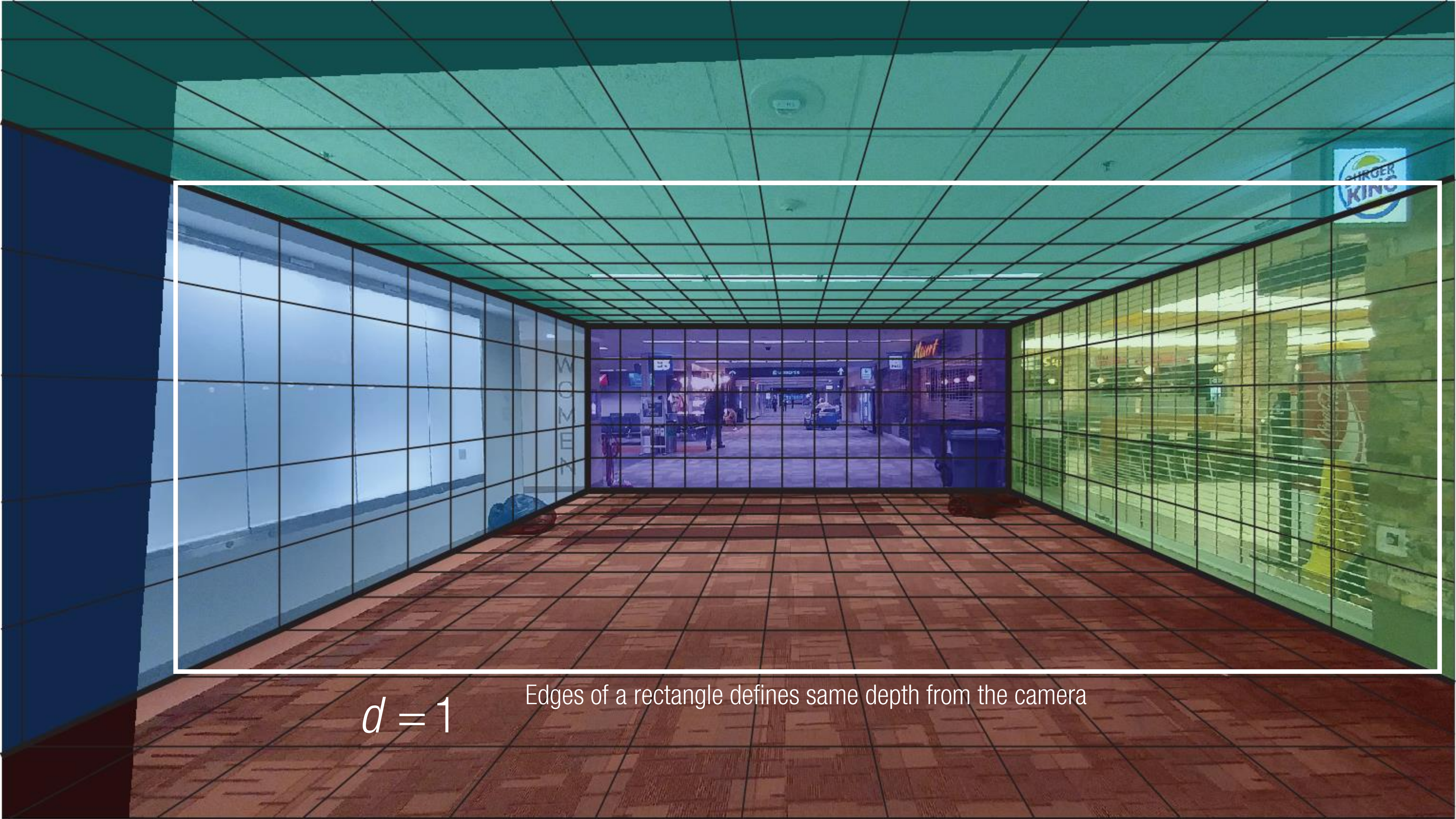
Where am I via Homography?

Announcement

- HW #3 deadline is extended (March 9)

- Paper selection by today (Send me at least two papers)


BURGER KING
WOMEN


Edges of a rectangle defines same depth from the camera
$d = 1$

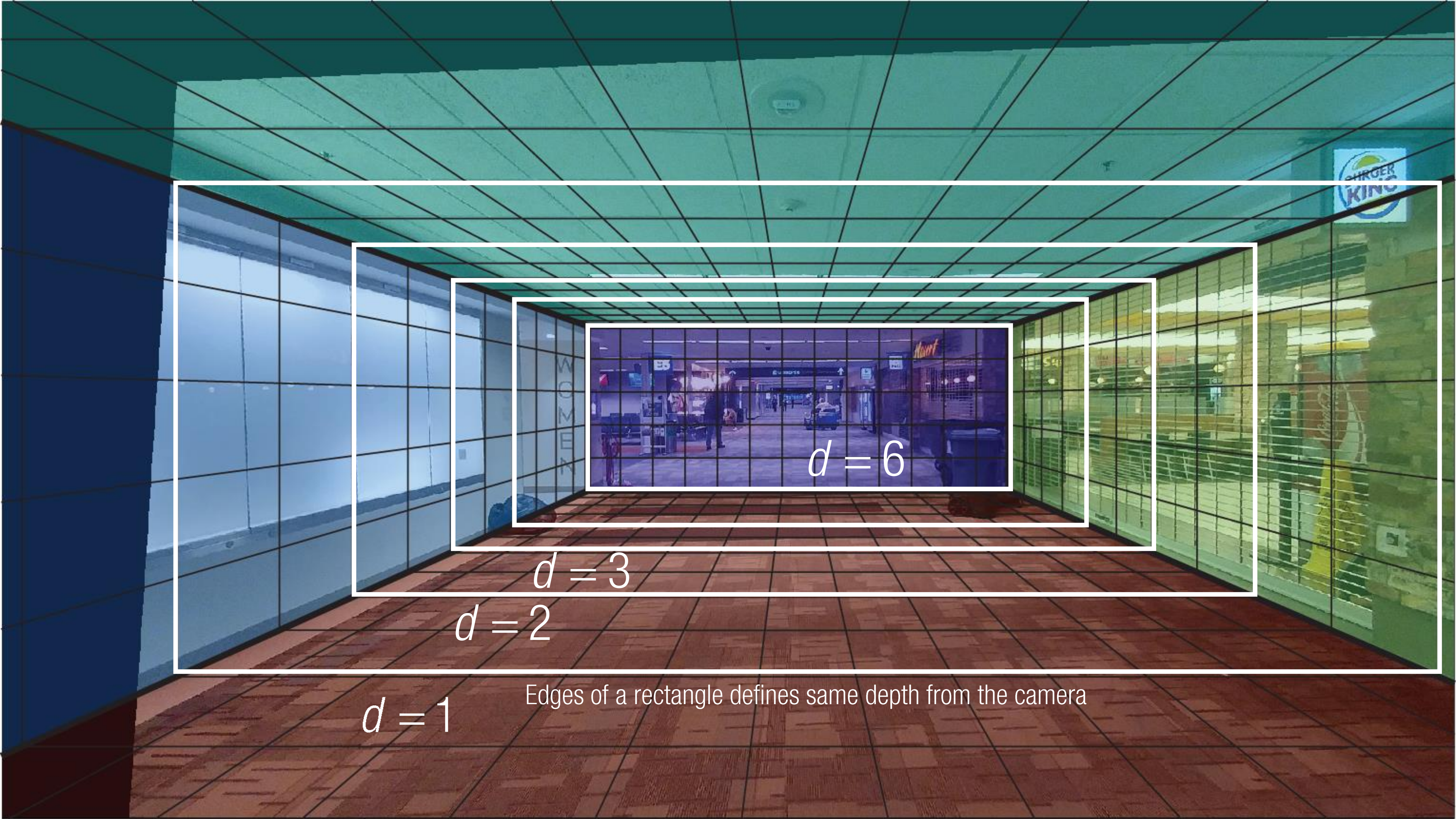
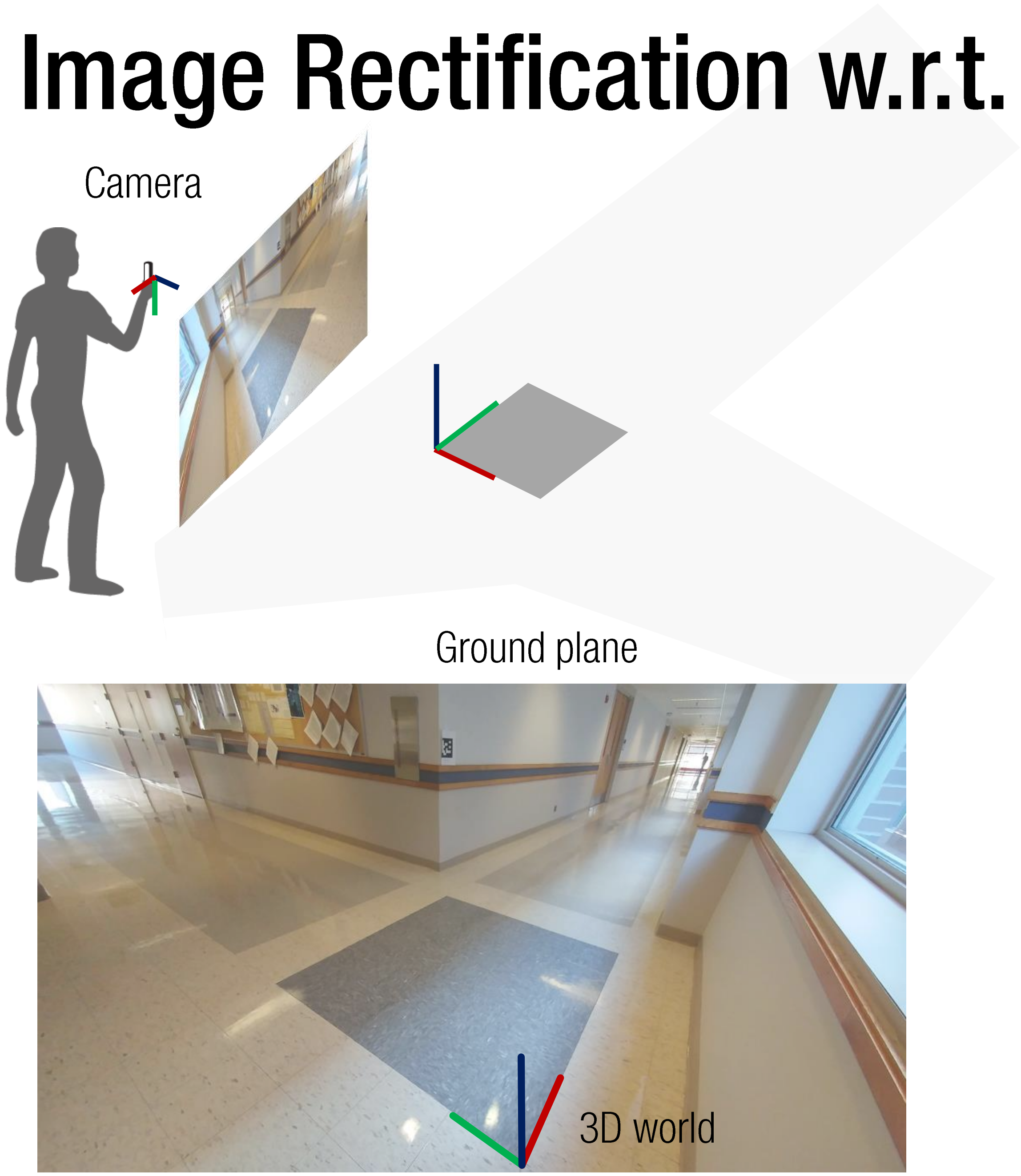

$d = 6$
$d = 3$
$d = 2$
$d = 1$
Edges of a rectangle defines same depth from the camera

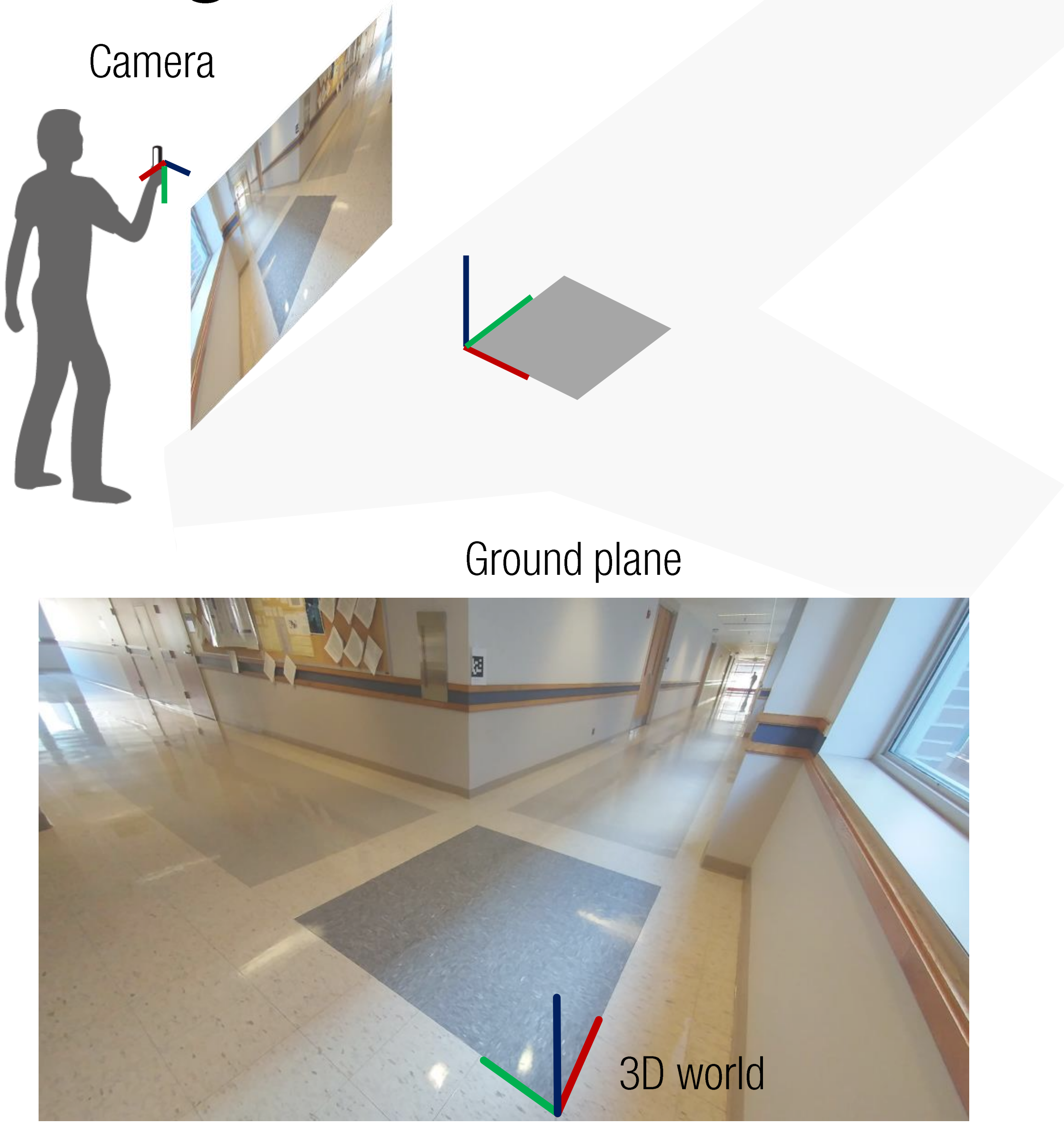
Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

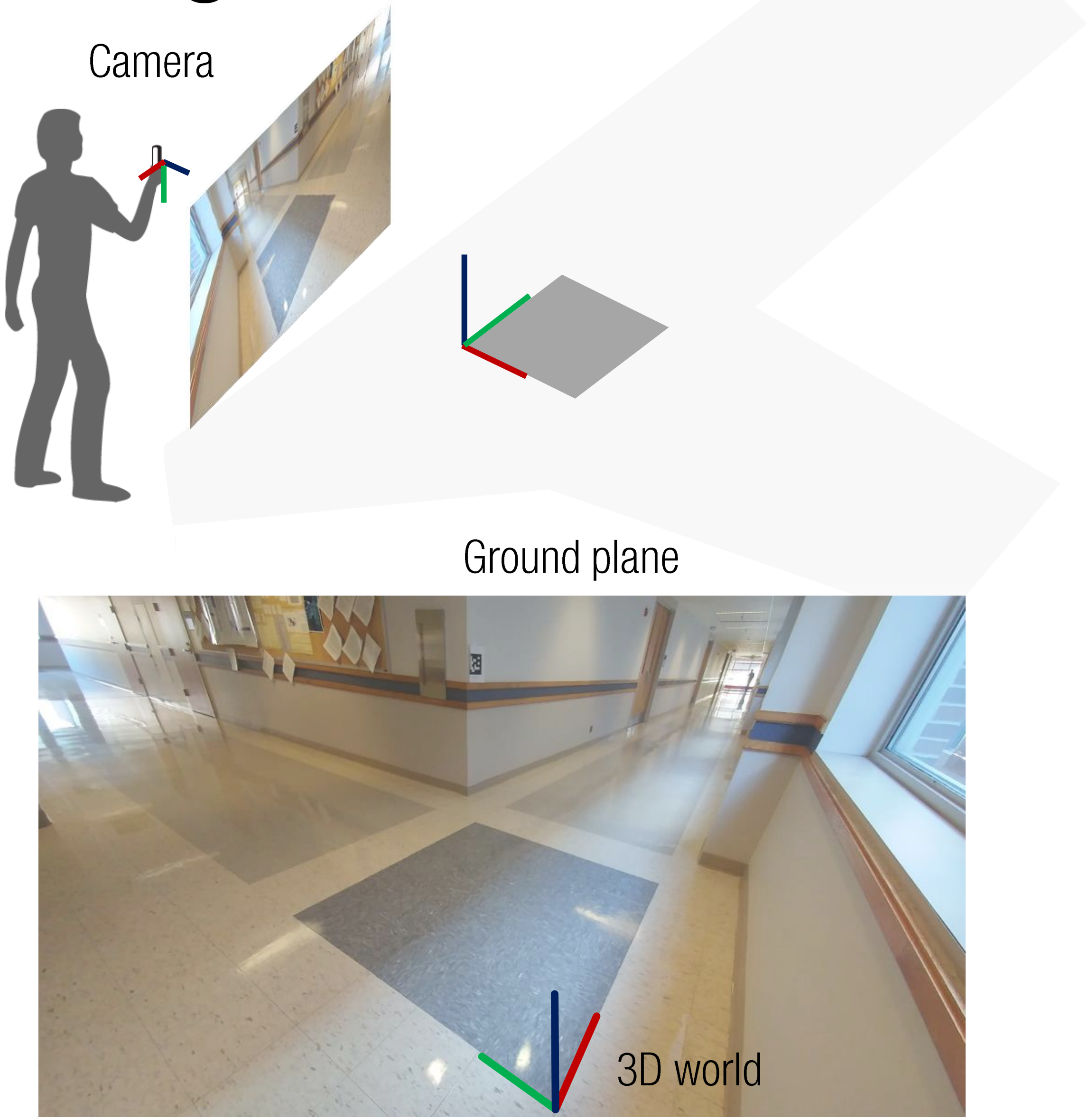
→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Camera pose from homography

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

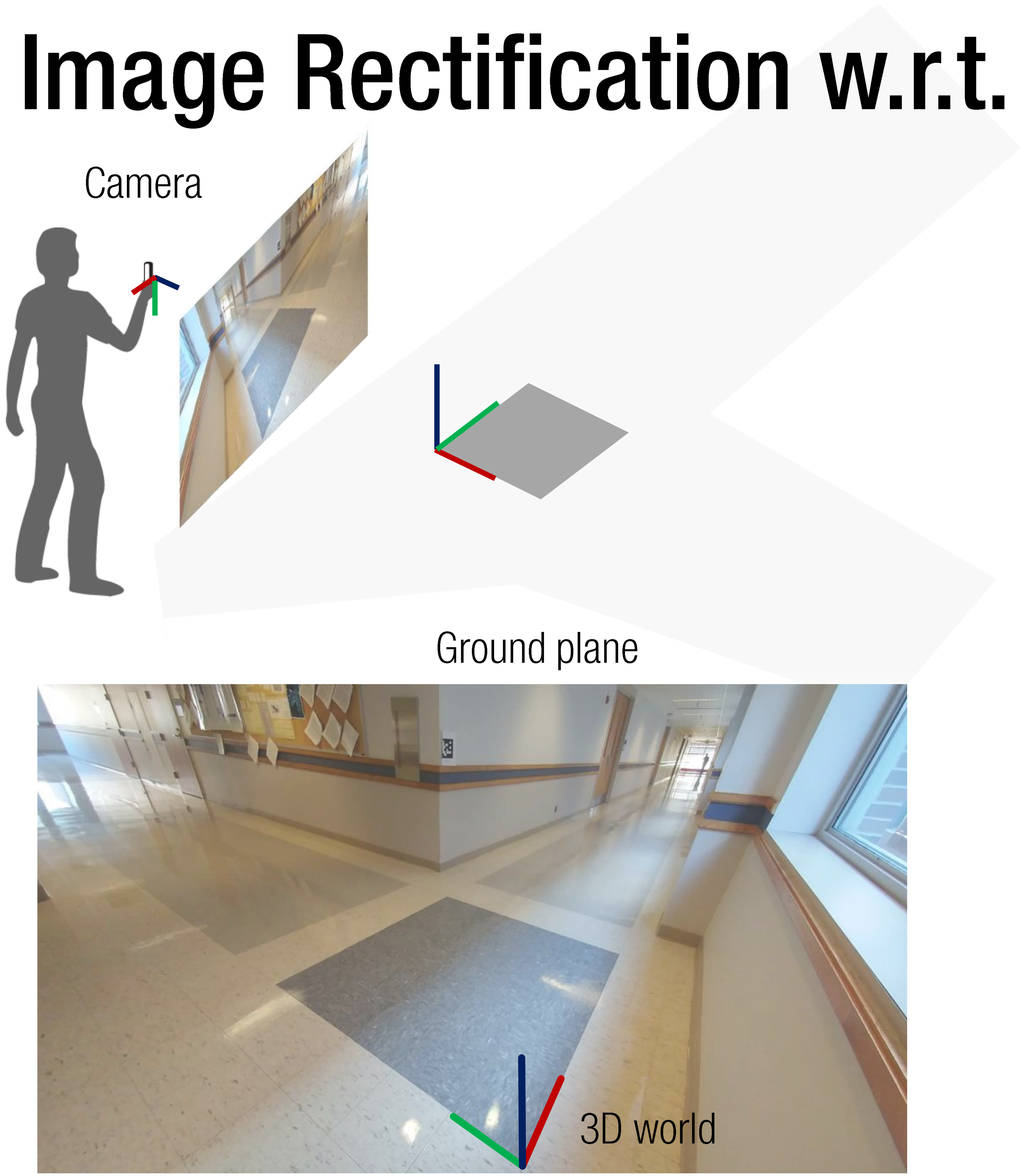
$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Camera pose from homography

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Image rotation

Image Rectification w.r.t. Ground Plane



How can I make my image upright?

→ Y axis of camera // Surface normal of ground plane

$$\lambda\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \mathbf{H}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

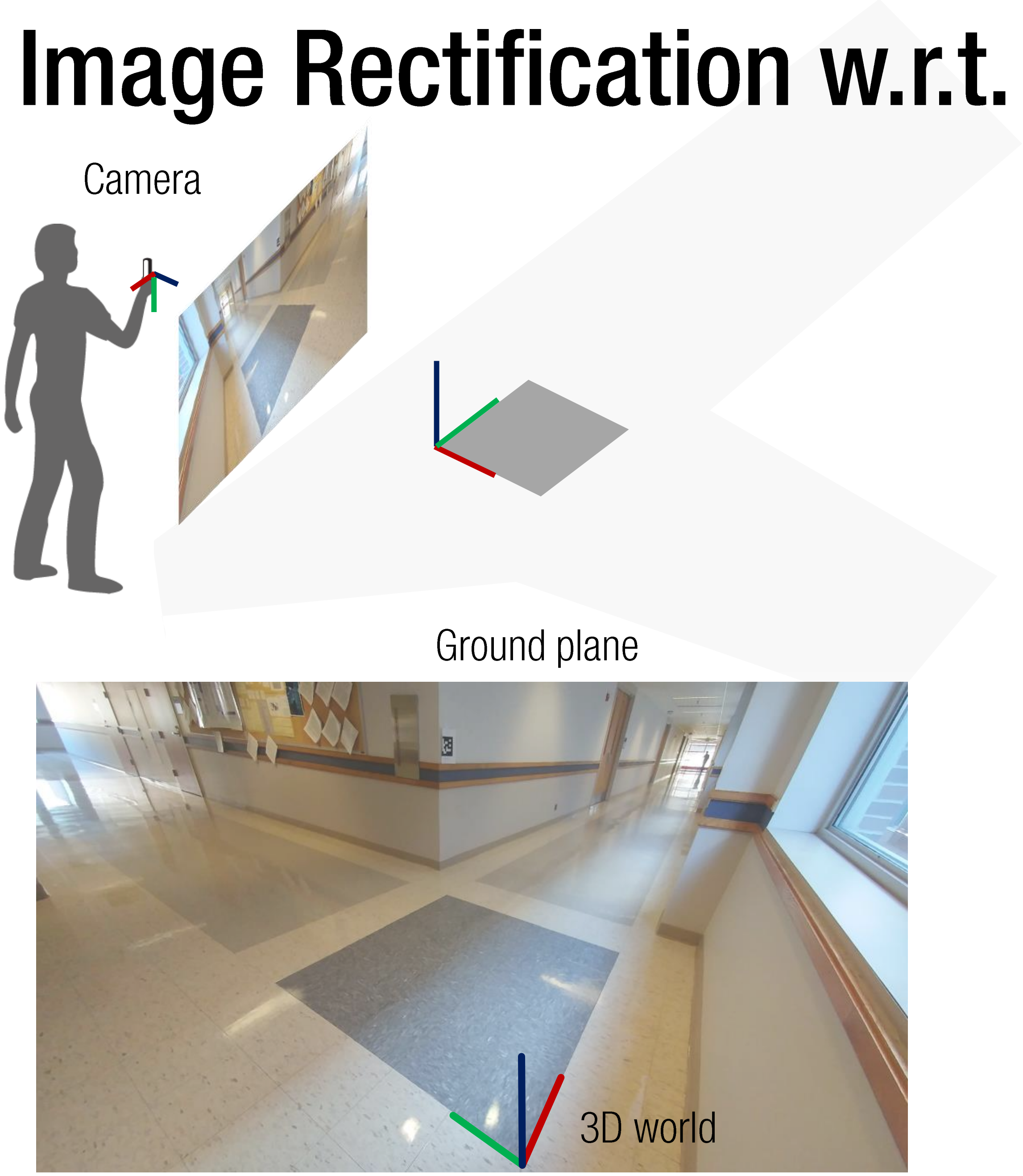
$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Camera pose from homography

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation   Rectified rotation

Image Rectification w.r.t. Ground Plane



$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

Rectified rotation

$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y \right\|}$$

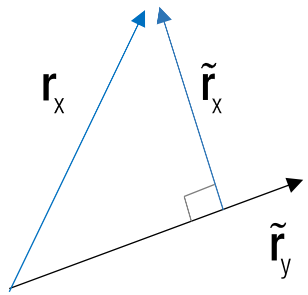
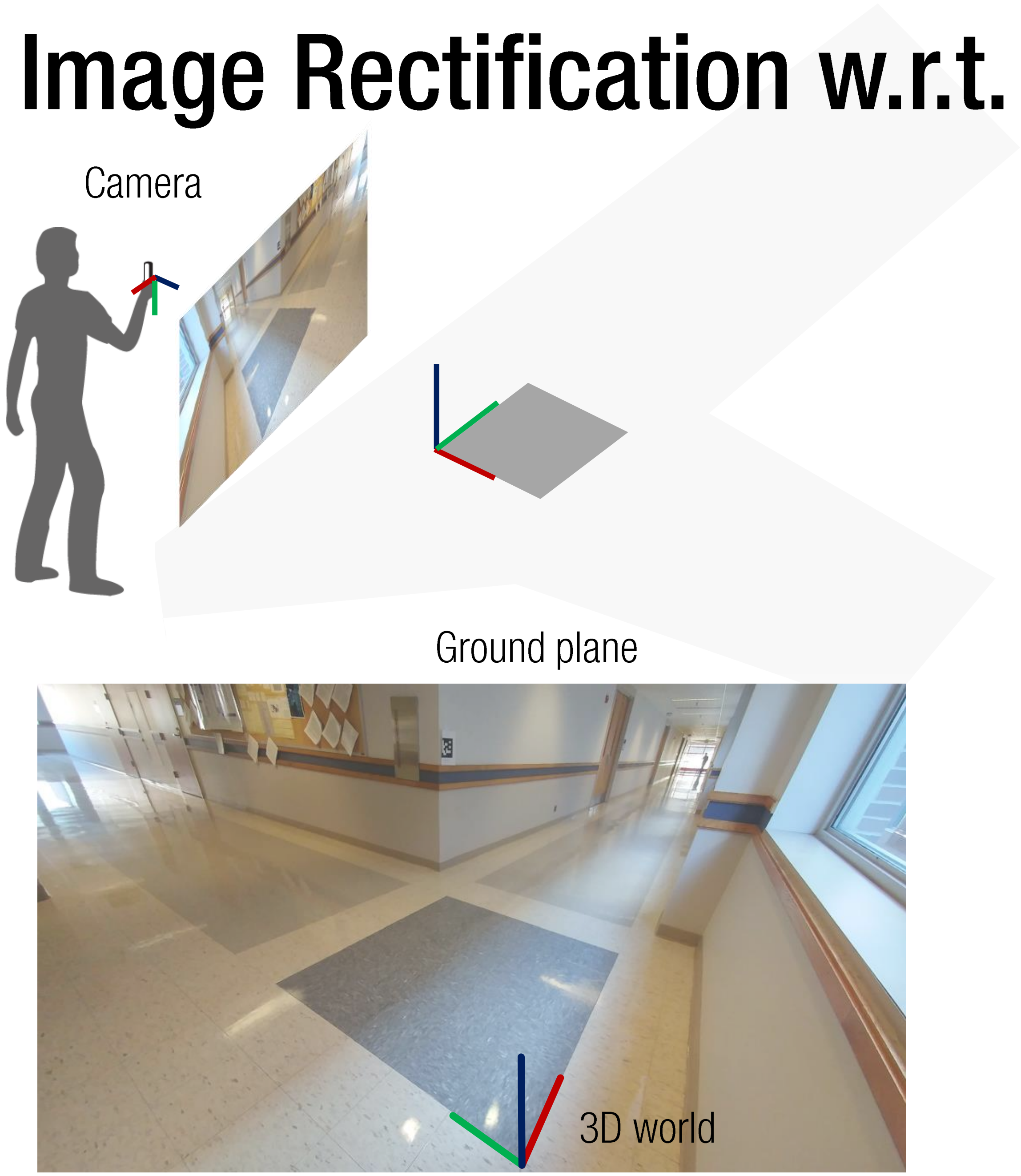


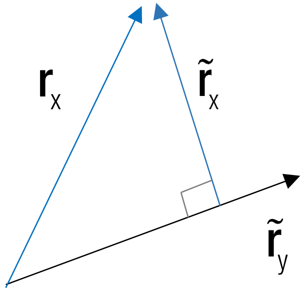
Image Rectification w.r.t. Ground Plane



$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

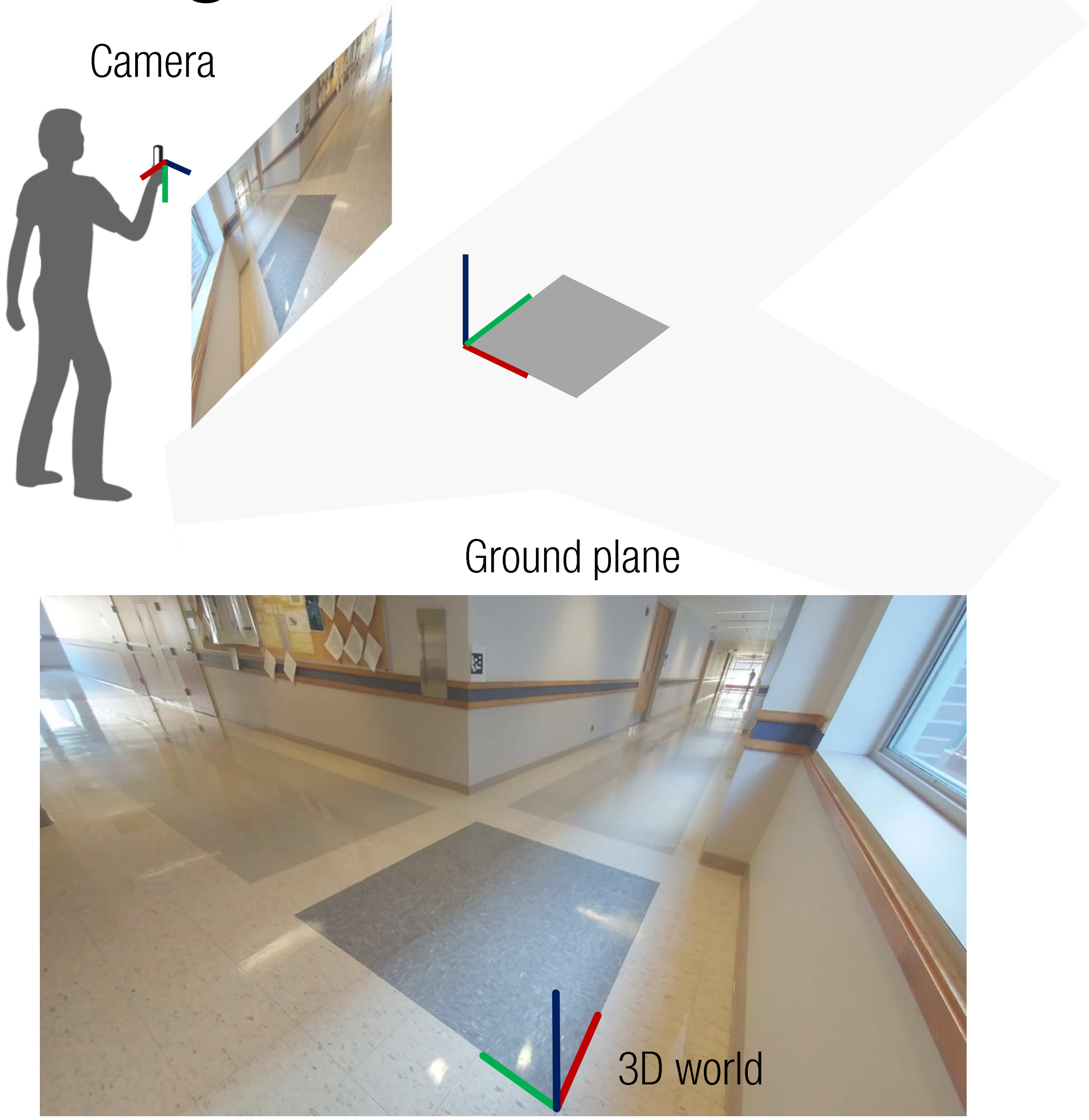
Rectified rotation



$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y \right\|}$$

$$\tilde{\mathbf{r}}_z = \tilde{\mathbf{r}}_x \times \tilde{\mathbf{r}}_y$$

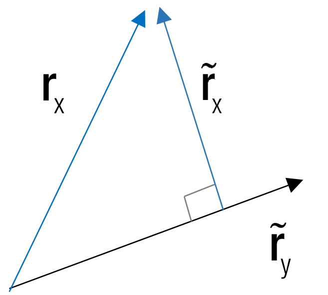
Image Rectification w.r.t. Ground Plane



$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow \tilde{\mathbf{R}} = \begin{bmatrix} & \tilde{\mathbf{r}}_x & \\ 0 & 0 & -1 \\ & \tilde{\mathbf{r}}_z & \end{bmatrix}$$

Image rotation

Rectified rotation



$$\tilde{\mathbf{r}}_x = \frac{\mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y}{\left\| \mathbf{r}_x - (\mathbf{r}_x \cdot \tilde{\mathbf{r}}_y)\tilde{\mathbf{r}}_y \right\|}$$

$$\tilde{\mathbf{r}}_z = \tilde{\mathbf{r}}_x \times \tilde{\mathbf{r}}_y$$

$$\lambda \tilde{\mathbf{R}}^T \mathbf{K}^{-1} \tilde{\mathbf{u}} = \mathbf{R}^T \mathbf{K}^{-1} \mathbf{u} \longrightarrow \lambda \tilde{\mathbf{u}} = \mathbf{K} \tilde{\mathbf{R}} \mathbf{R}^T \mathbf{K}^{-1} \mathbf{u}$$

Image Rectification w.r.t. Ground Plane

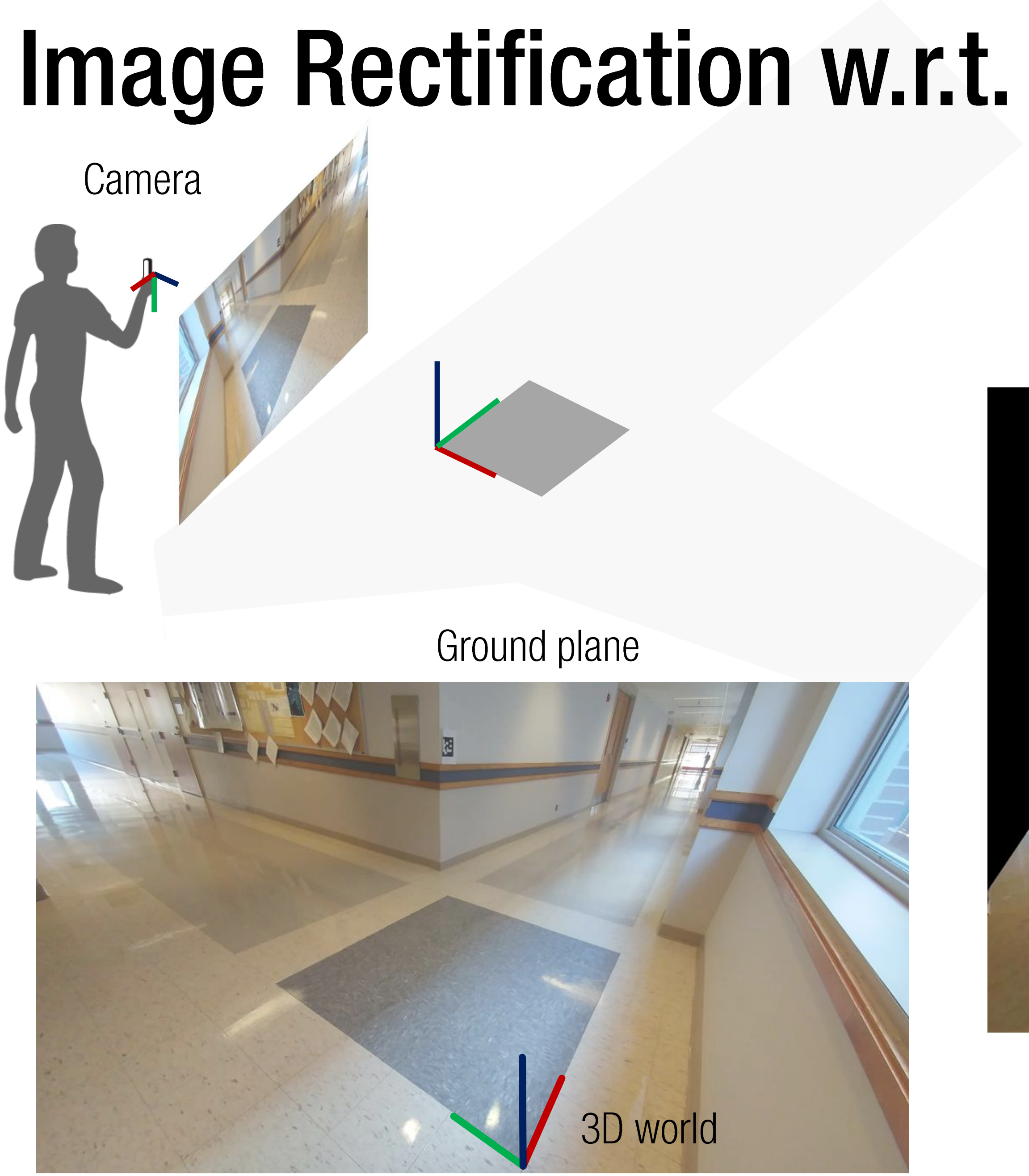
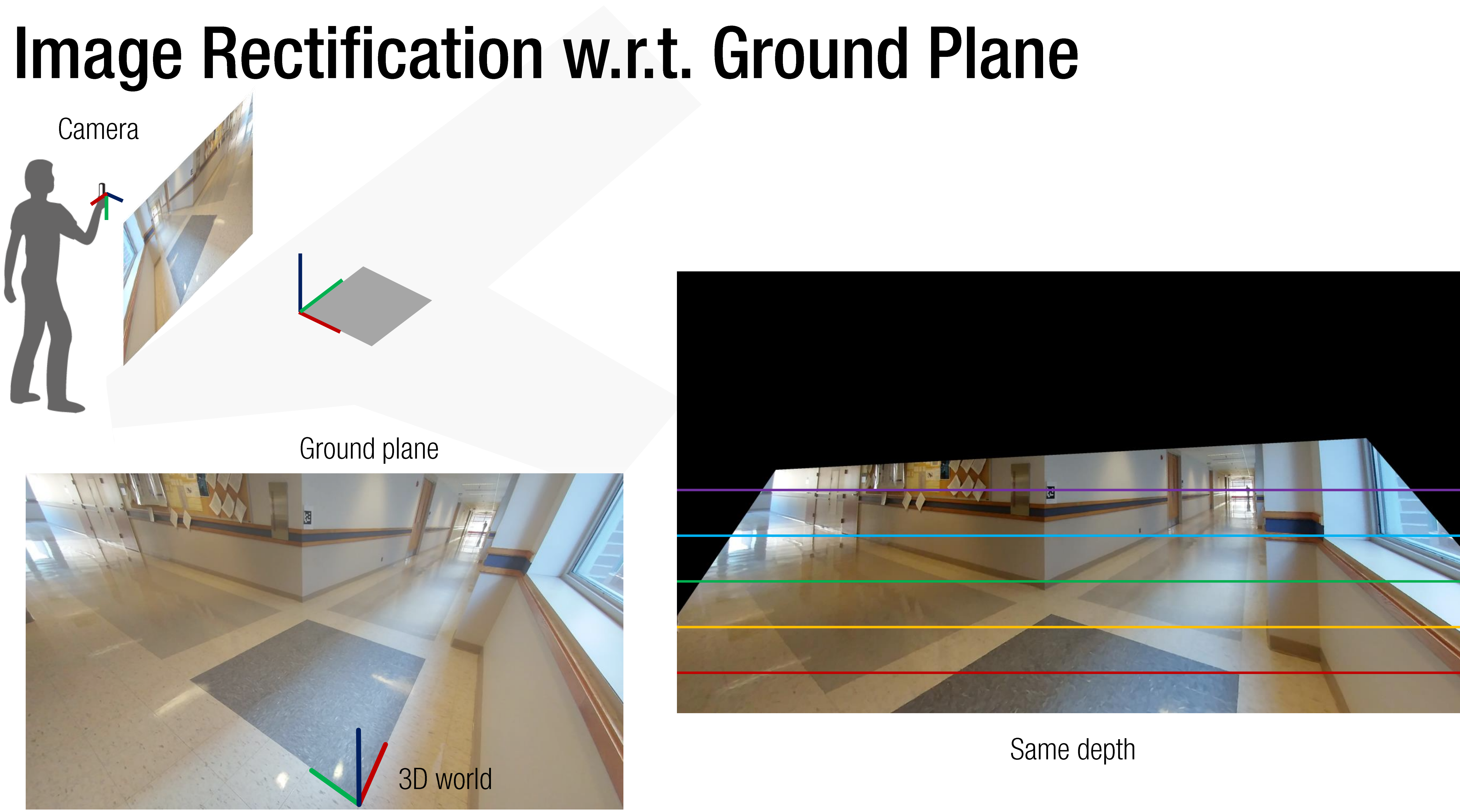


Image Rectification w.r.t. Ground Plane

```
im = imread('undistorted.png');
f = 1300;
K = [f 0 size(im,2)/2;
     0 f size(im,1)/2;
     0 0 1];
m11 = [2145;2120;1];m12 = [2566;1191;1];m13 = [1804;935;1];m14 = [1050;1320;1];

u = [m11(1:2)';m12(1:2)';m13(1:2)'; m14(1:2)'];
X = [0 0;1 0;1 1;0 1];
X = [X ones(4,1)]; % homogeneous coordinate

H = ComputeHomography(u, X);

denom = norm(inv(K)*H(:,1));
r1 = inv(K)*H(:,1)/denom;  r2 = inv(K)*H(:,2)/denom;  t = inv(K)*H(:,3)/denom;
r3 = Vec2Skew(r1)*r2;
R = [r1 r2 r3];

r2_n = [0 0 -1];
r1_n = (R(1,:) - (R(1,:)*r2_n')*r2_n);
r3_n = Vec2Skew(r1_n)*r2_n';

R_n = [r1_n; r2_n; r3_n'];
H_new = K * R_n * inv(R) * inv(K);

im_warped = ImageWarping(im, H_new);
```

RectificationFromHomography.m

Image Rectification w.r.t. Ground Plane



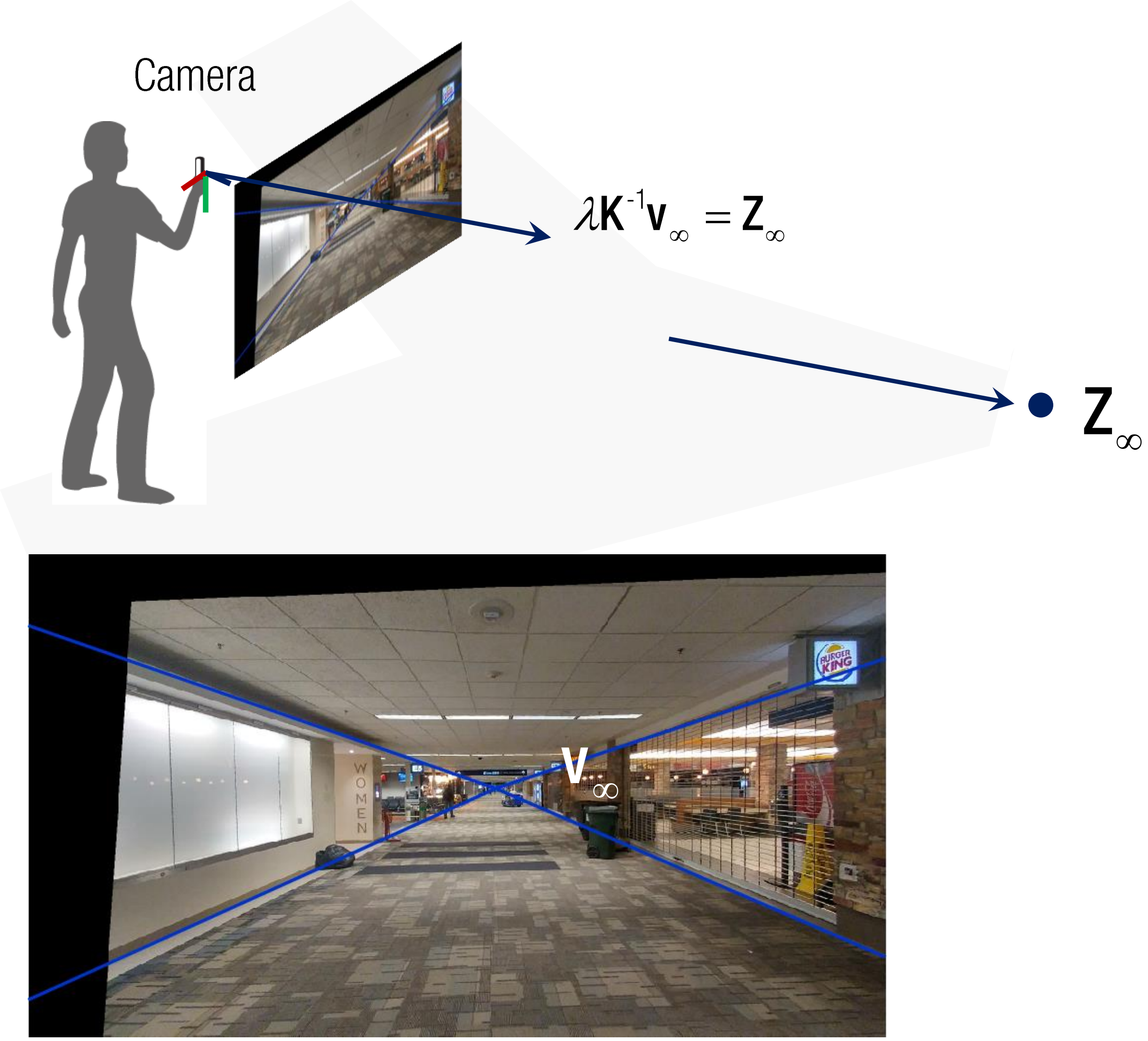

WOMEN
E5
BURGER KING


BURGER KING
WOMEN
E5


BURGER KING
WOMEN
E5


Vanishing point
WOMEN
BURGER KING
E5

Vanishing Point

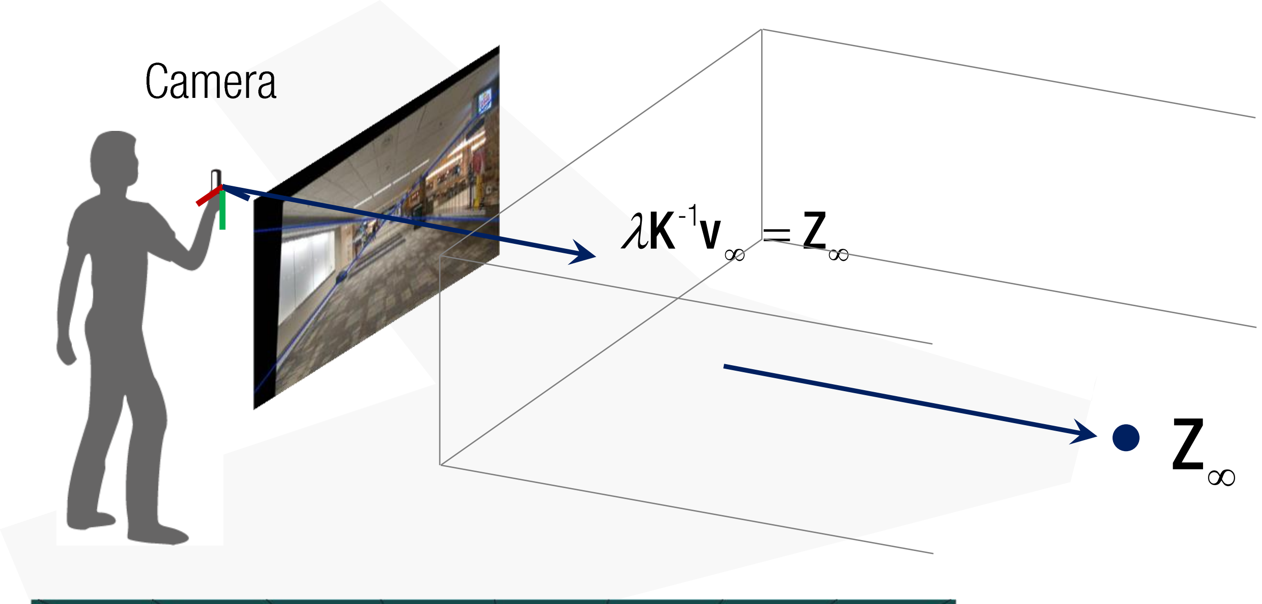


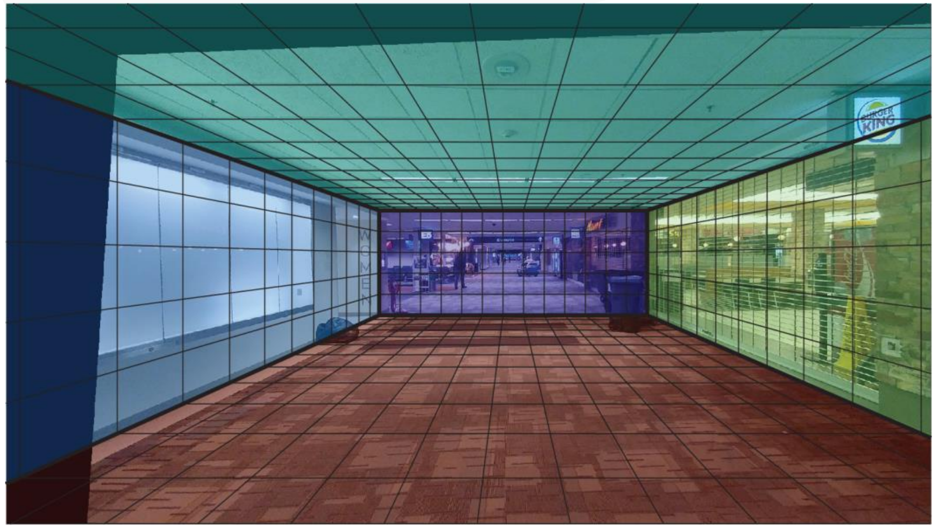
Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Box Representation



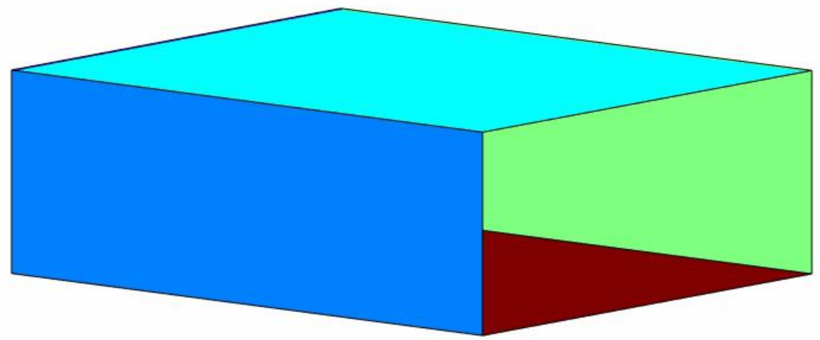


Vanishing point projection:

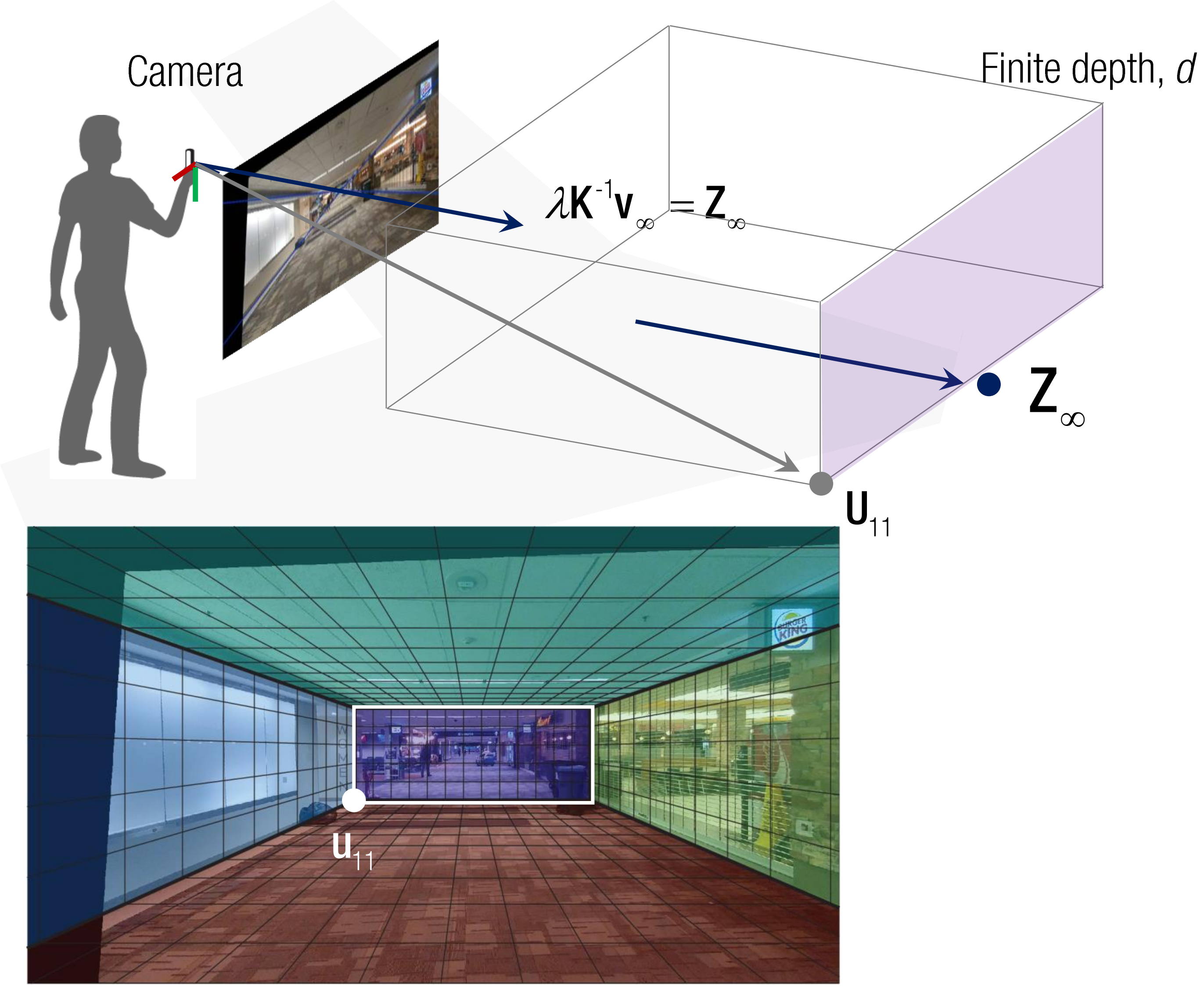
$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Define the direction of the box



Box Representation



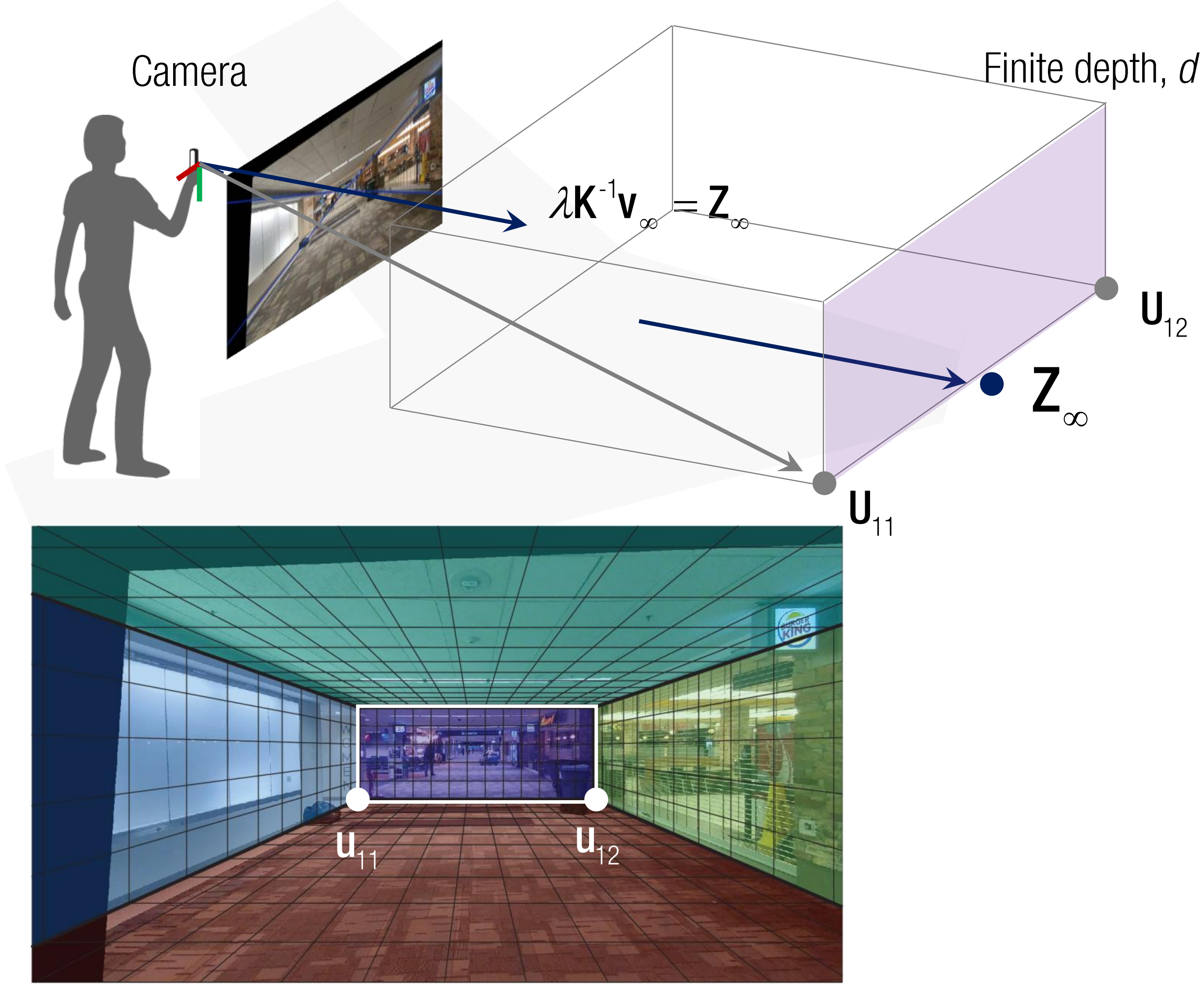
Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

$$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

Box Representation



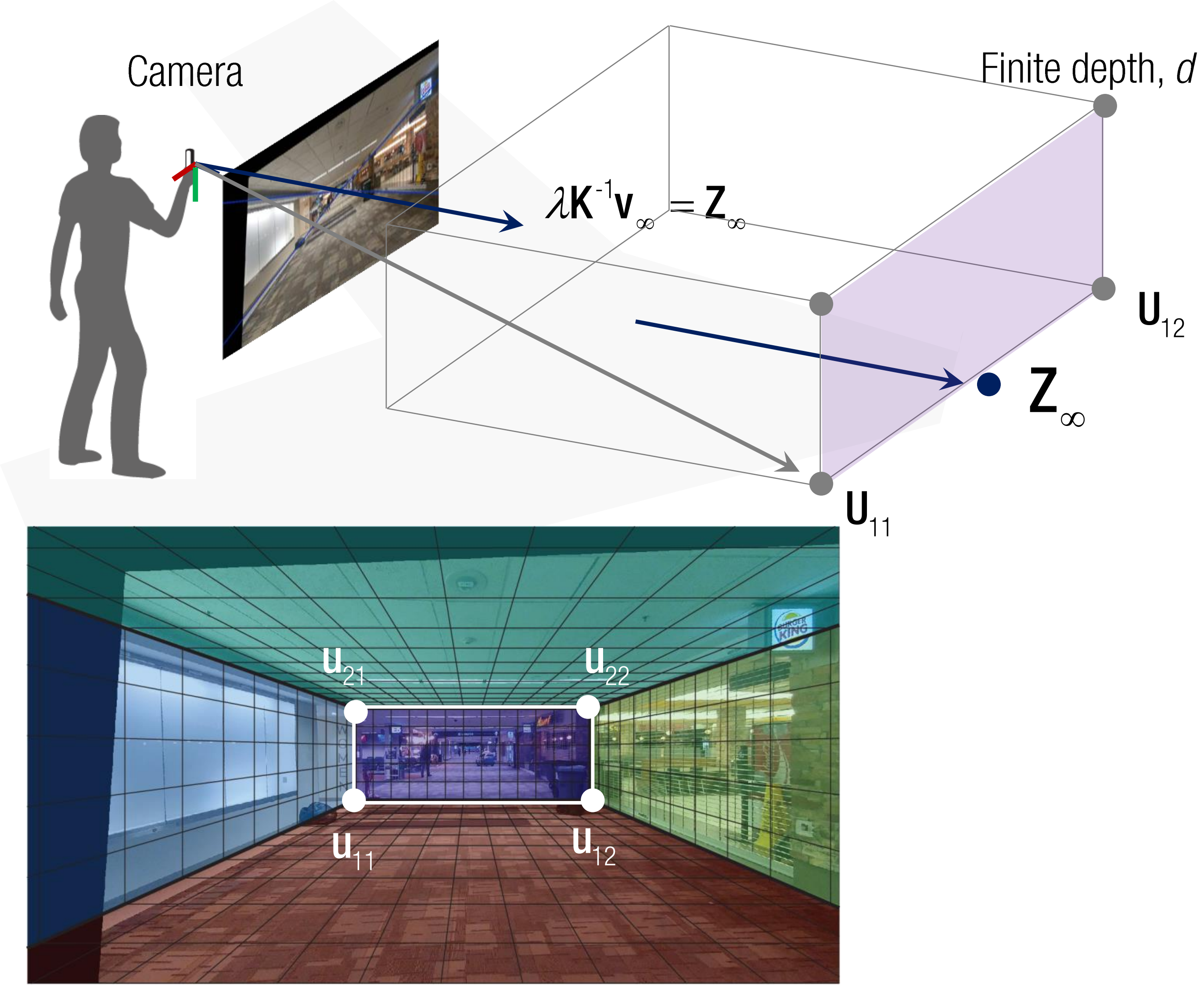
Vanishing point projection:

$$\lambda \mathbf{v}_{\infty} = \mathbf{K}\mathbf{Z}_{\infty}$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_{\infty} = \mathbf{Z}_{\infty}$$

$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$ $\quad$ $\mathbf{U}_{12} = d\mathbf{K}^{-1}\mathbf{u}_{12}$: Same x coord.

Box Representation



Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

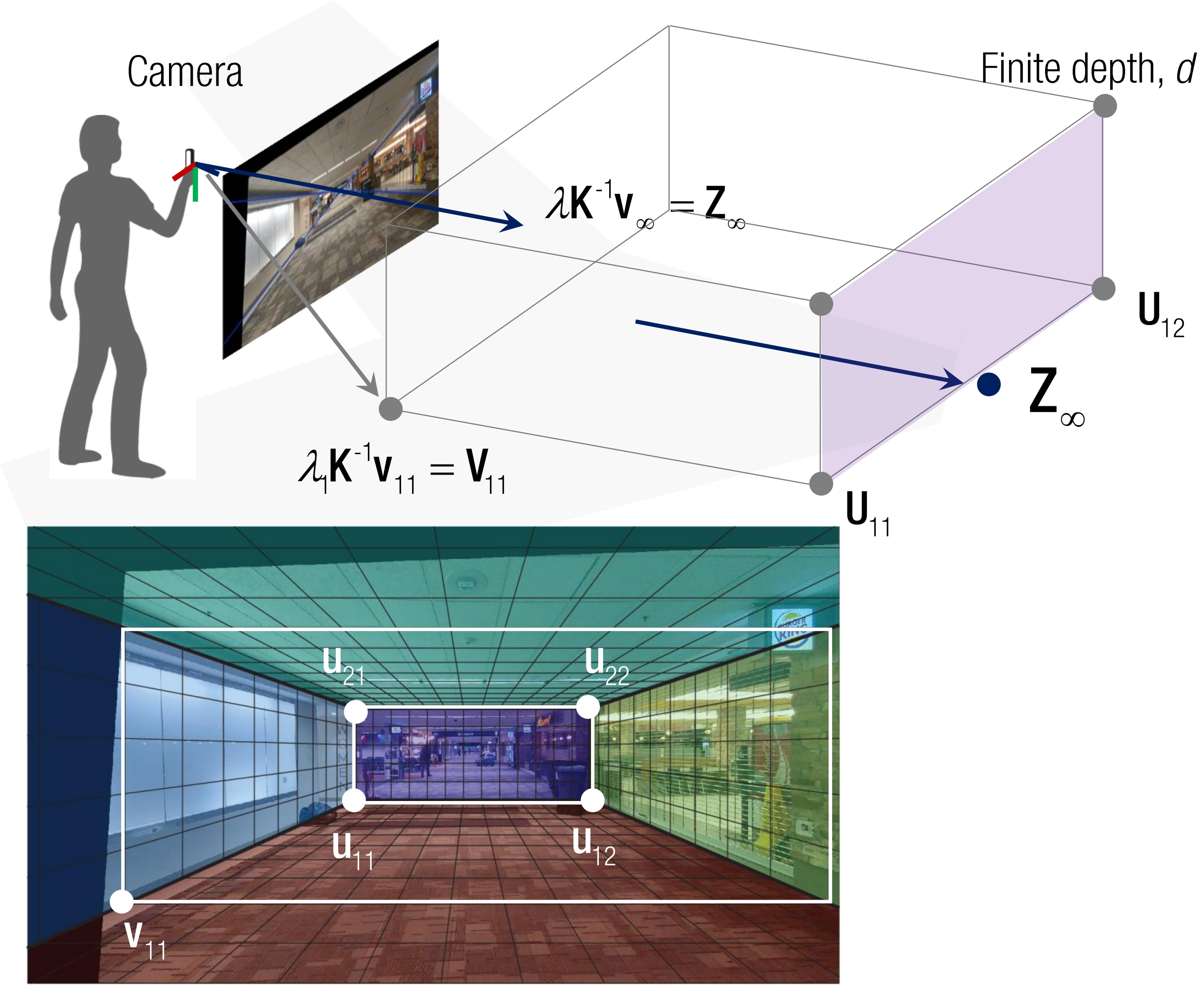
$$\mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11} \qquad \mathbf{U}_{12} = d\mathbf{K}^{-1}\mathbf{u}_{12}$$

: Same x coord.

$$\mathbf{U}_{21} = d\mathbf{K}^{-1}\mathbf{u}_{21} \qquad \mathbf{U}_{22} = d\mathbf{K}^{-1}\mathbf{u}_{22}$$

Same y coord.

Box Representation



Vanishing point projection:

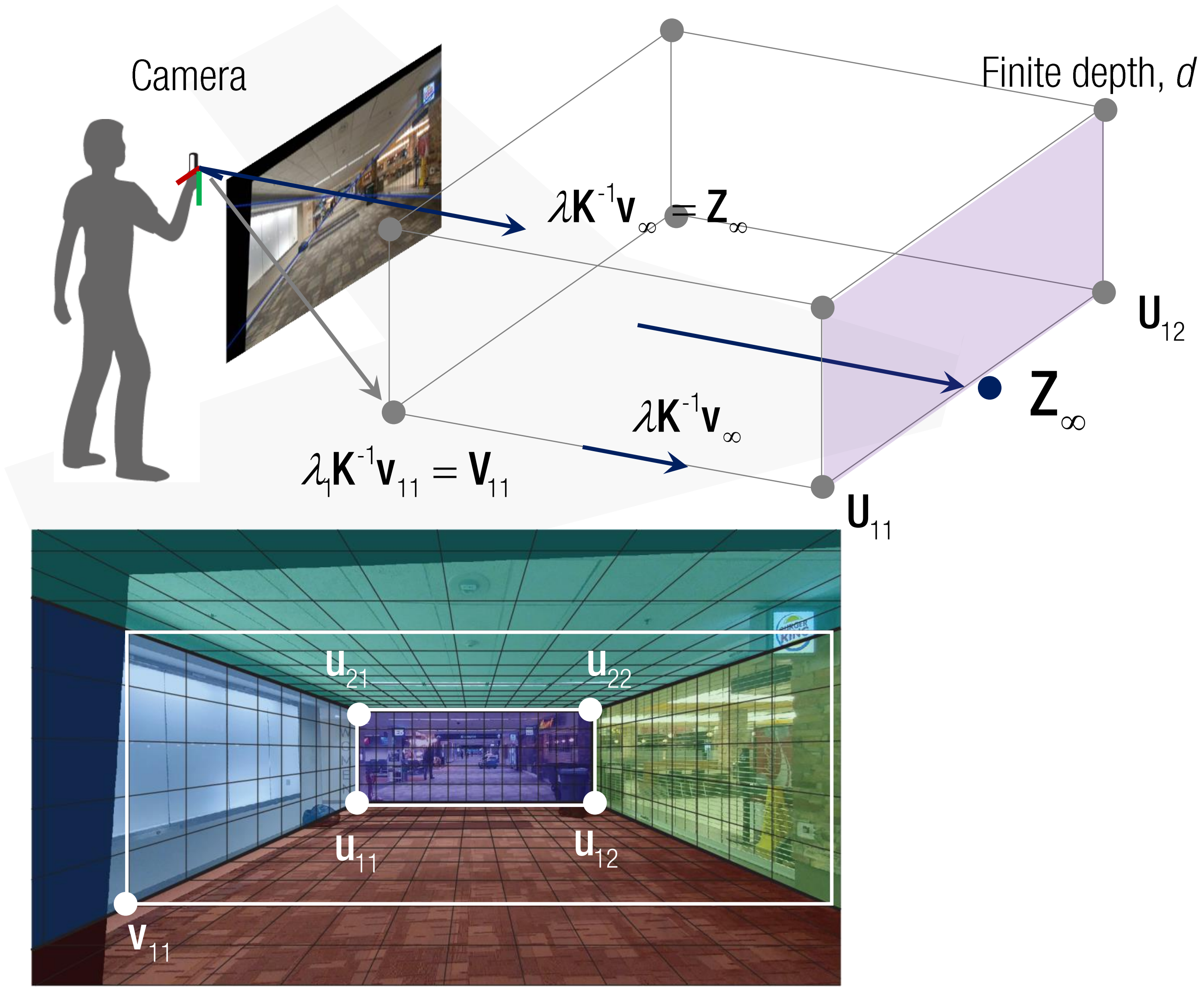
$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Depth of frontal surface?

$$\underline{\lambda_1} \mathbf{K}^{-1}\mathbf{v}_{11} = \mathbf{V}_{11}$$

Box Representation



Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Depth of frontal surface?

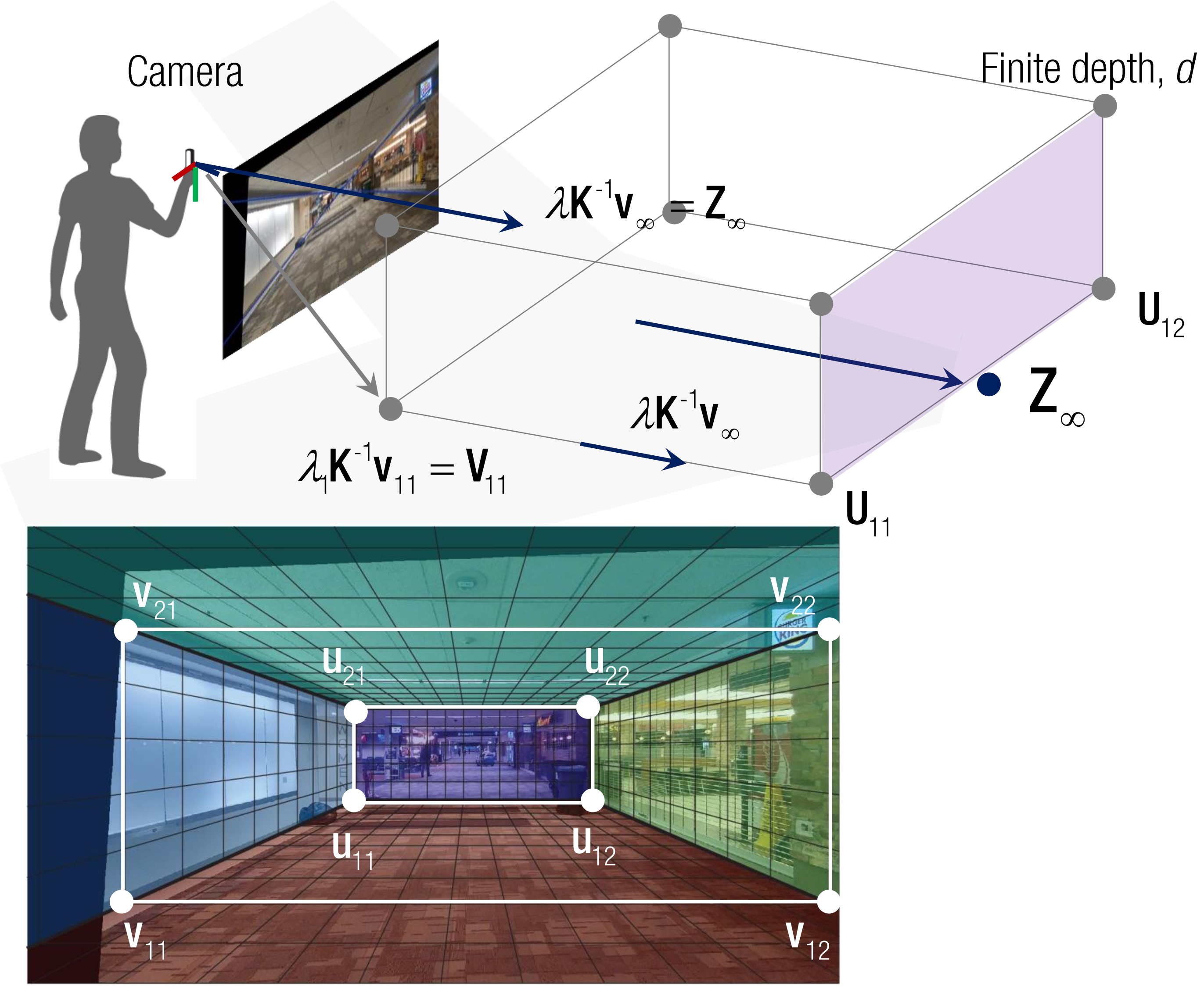
$$\underline{\lambda_1} \mathbf{K}^{-1}\mathbf{v}_{11} = \mathbf{V}_{11}$$

Line between $\mathbf{U}_{11}$ and $\mathbf{V}_{11}$ is parallel to the vanishing point direction.

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_{11} + \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

HW: express λ_1 using d.

Box Representation



Vanishing point projection:

$$\lambda \mathbf{v}_\infty = \mathbf{K}\mathbf{Z}_\infty$$

$$\longrightarrow \quad \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{Z}_\infty$$

Depth of frontal surface?

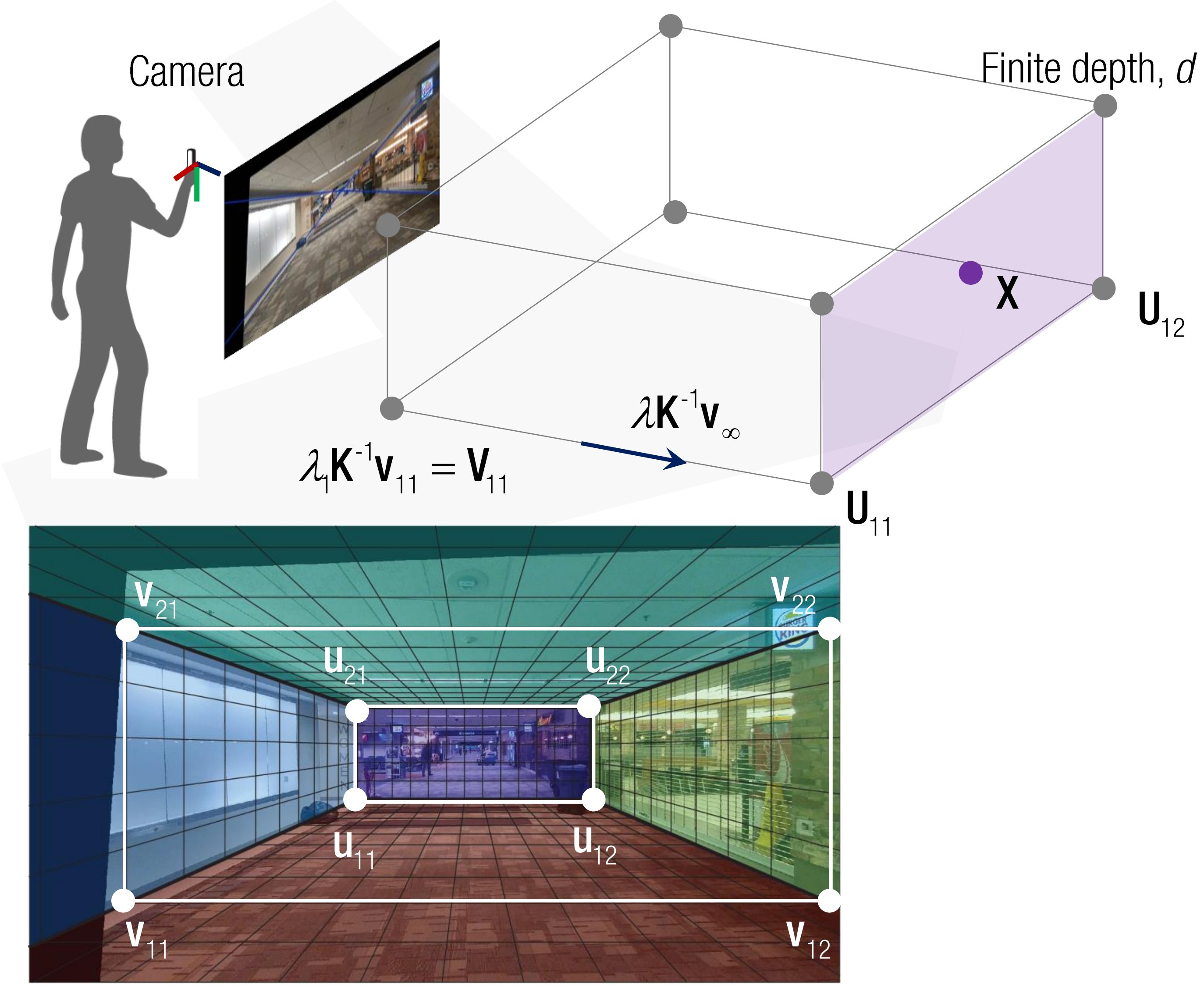
$$\underline{\lambda_1} \mathbf{K}^{-1}\mathbf{v}_{11} = \mathbf{V}_{11}$$

Line between $\mathbf{U}_{11}$ and $\mathbf{V}_{11}$ is parallel to the vanishing point direction.

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_{11} + \lambda \mathbf{K}^{-1}\mathbf{v}_\infty = \mathbf{U}_{11} = d\mathbf{K}^{-1}\mathbf{u}_{11}$$

HW: express λ_1 using d.

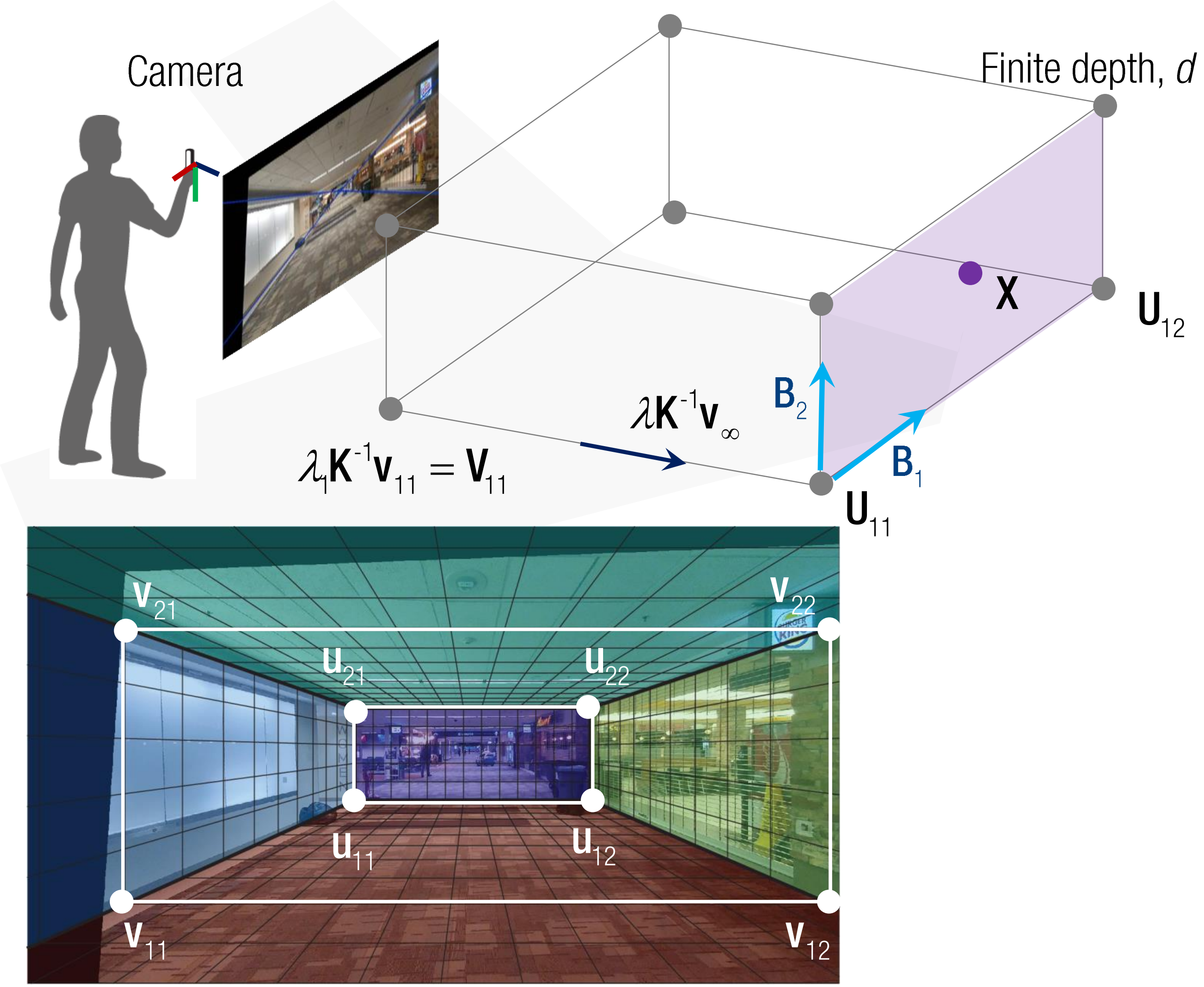
Box Representation



Point in a plane:

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Box Representation



Point in a plane:

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{U}_{11} + \mu_1 \mathbf{B}_1 + \mu_2 \mathbf{B}_2$$

$$= \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

2DOF

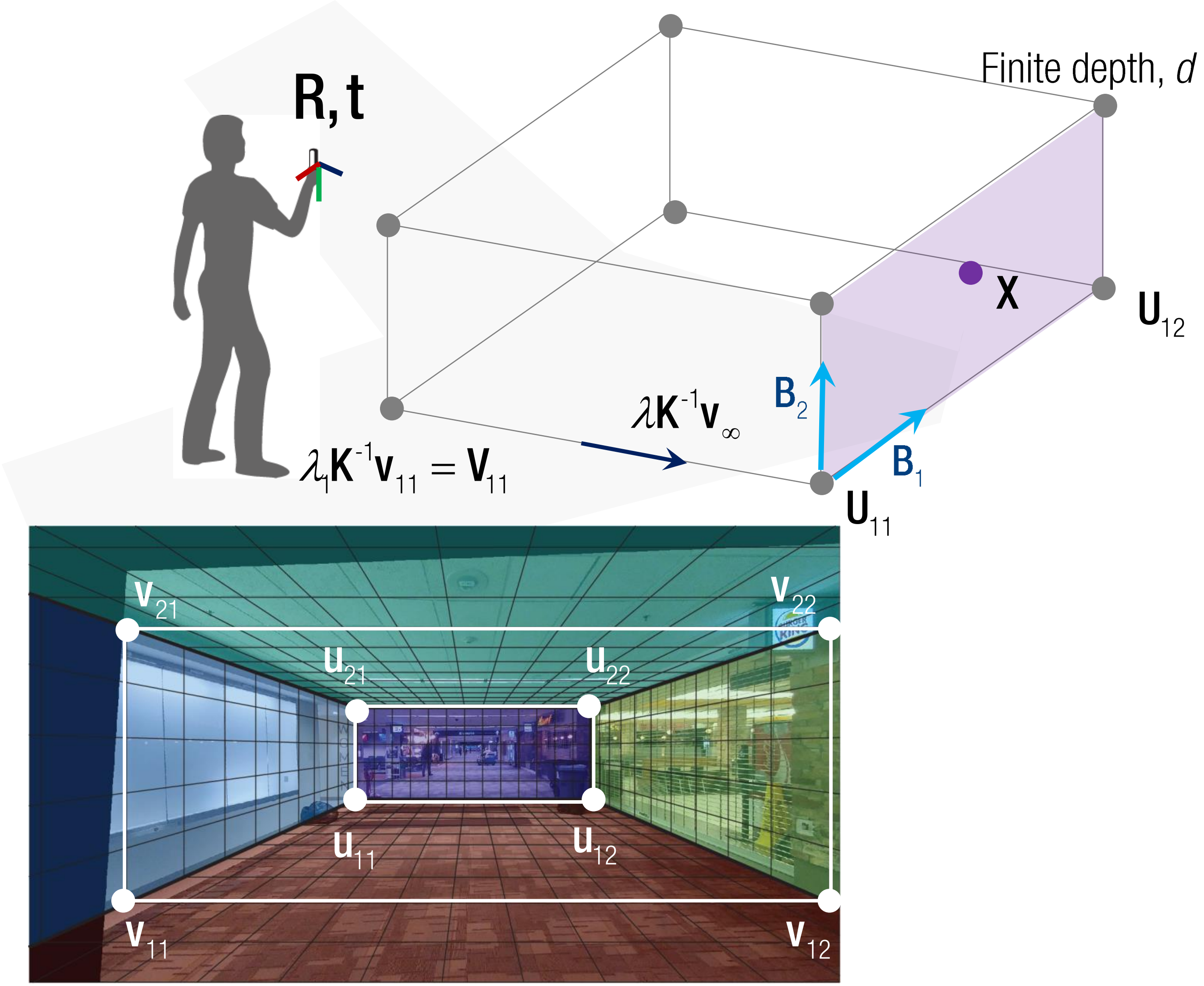
Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K} \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Box Representation

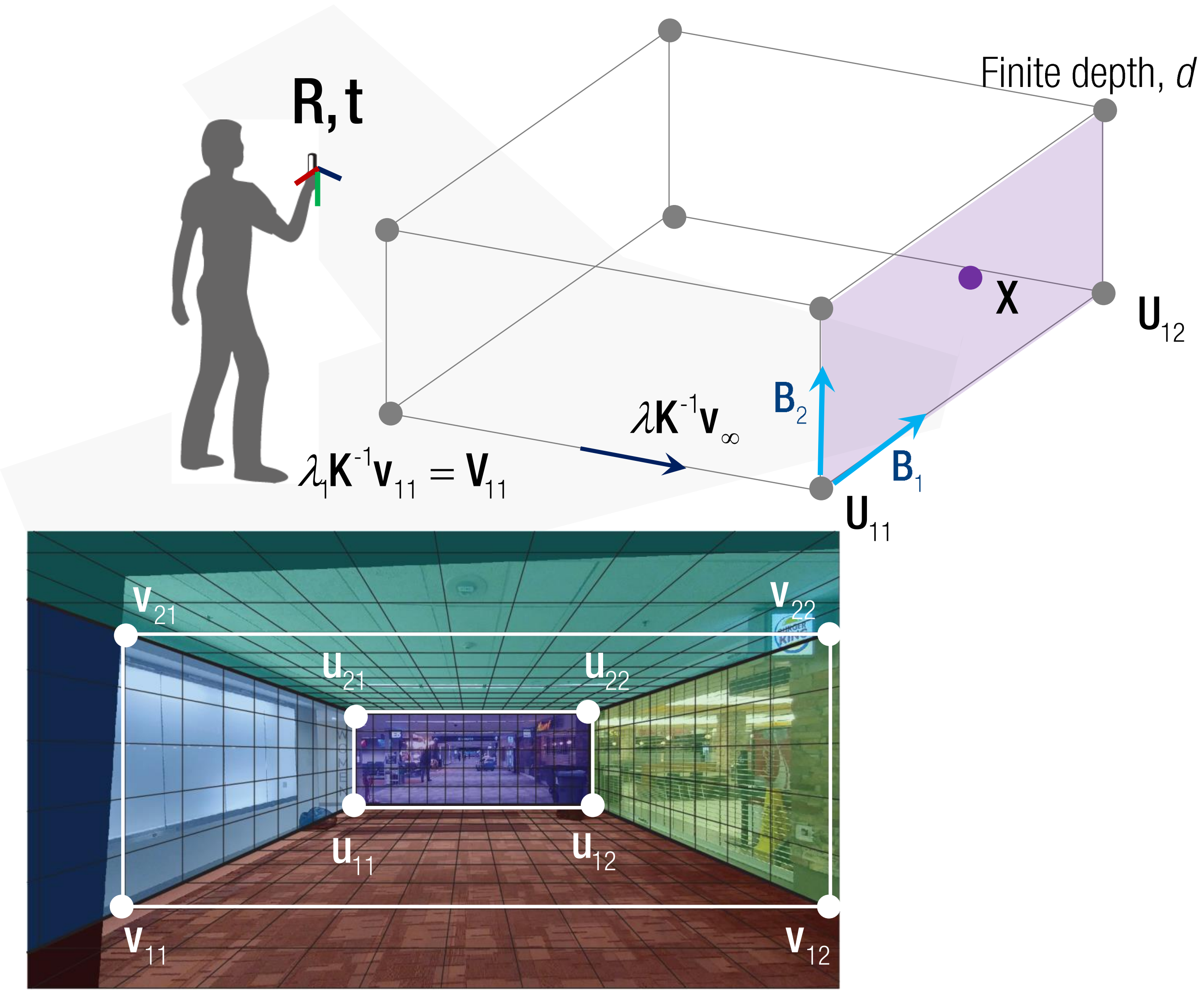
Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1 \\ \mu_2 \\ 1\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mu_1 \\ \mu_2 \\ 1\end{bmatrix}$$



Texture Mapping

Homography



Homography mapping from 3D plane to image:

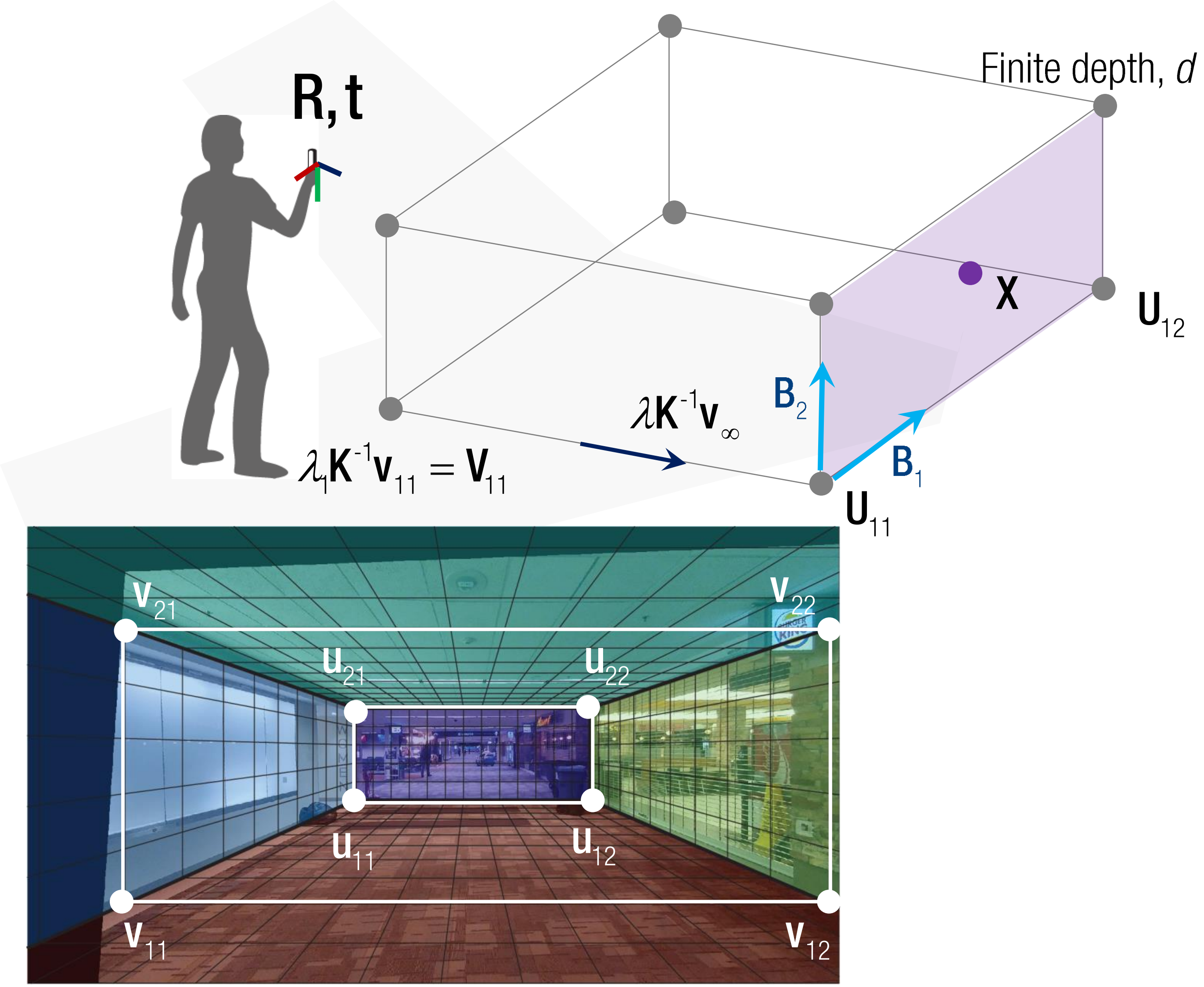
$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \mathbf{H}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}\mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}\\ 1\end{bmatrix}$$

$$= \mathbf{K}\begin{bmatrix}\mathbf{R}\mathbf{B}_1 & \mathbf{R}\mathbf{B}_2 & \mathbf{R}\mathbf{c}+\mathbf{t}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix} = \tilde{\mathbf{H}}\begin{bmatrix}\mu_1\\ \mu_2\\ 1\end{bmatrix}$$

Homography



Homography mapping from 3D plane to image:

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \mathbf{H}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

Homography mapping from 3D plane to target image:

$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{K}\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}\begin{bmatrix} \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{c} \end{bmatrix} \\ 1 \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

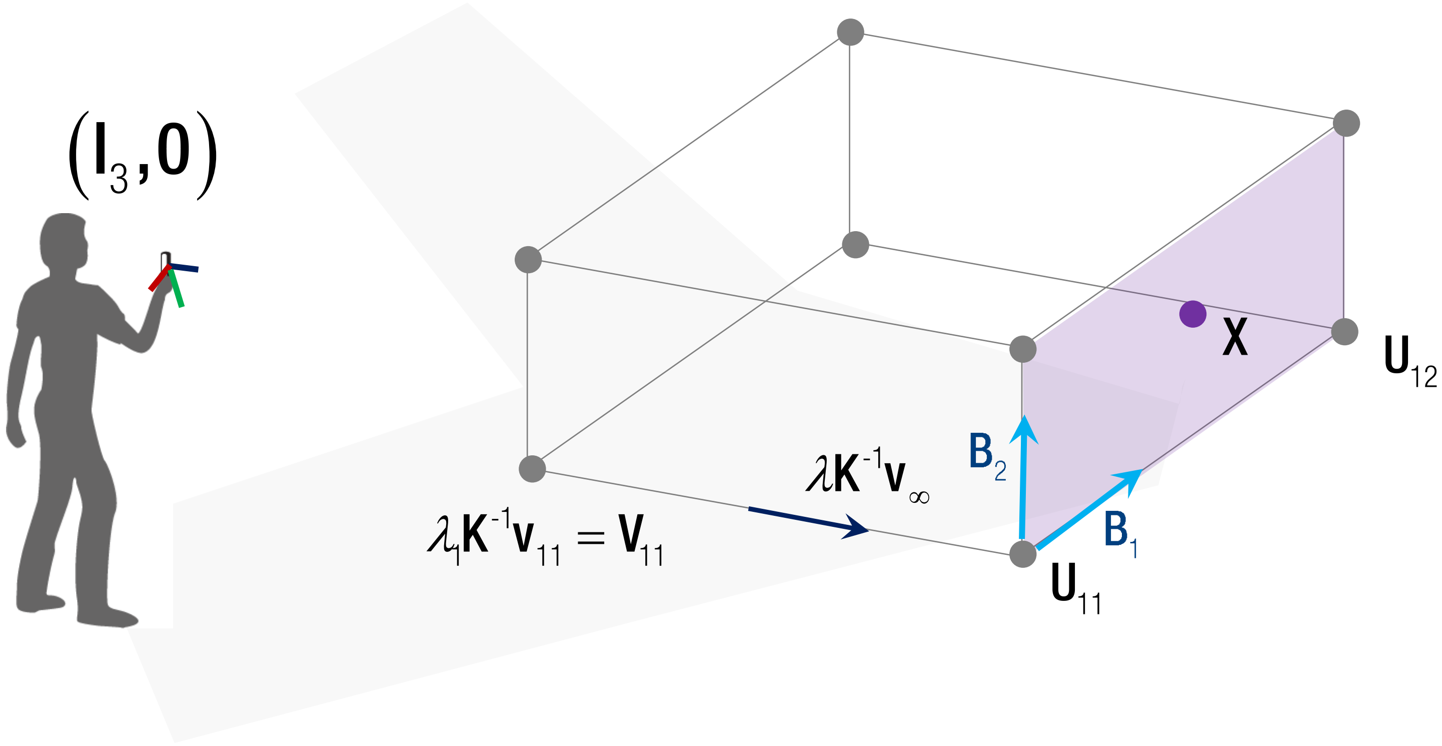
$$= \mathbf{K}\begin{bmatrix} \mathbf{R}\mathbf{B}_1 & \mathbf{R}\mathbf{B}_2 & \mathbf{R}\mathbf{c} + \mathbf{t} \end{bmatrix}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \tilde{\mathbf{H}}\begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix}$$

$$\longrightarrow \quad \lambda \tilde{\mathbf{H}}^{-1}\tilde{\mathbf{u}} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ 1 \end{bmatrix} = \lambda_1 \mathbf{H}^{-1}\mathbf{u} \quad \longrightarrow \quad \lambda \tilde{\mathbf{u}} = \tilde{\mathbf{H}}\mathbf{H}^{-1}\mathbf{u}$$

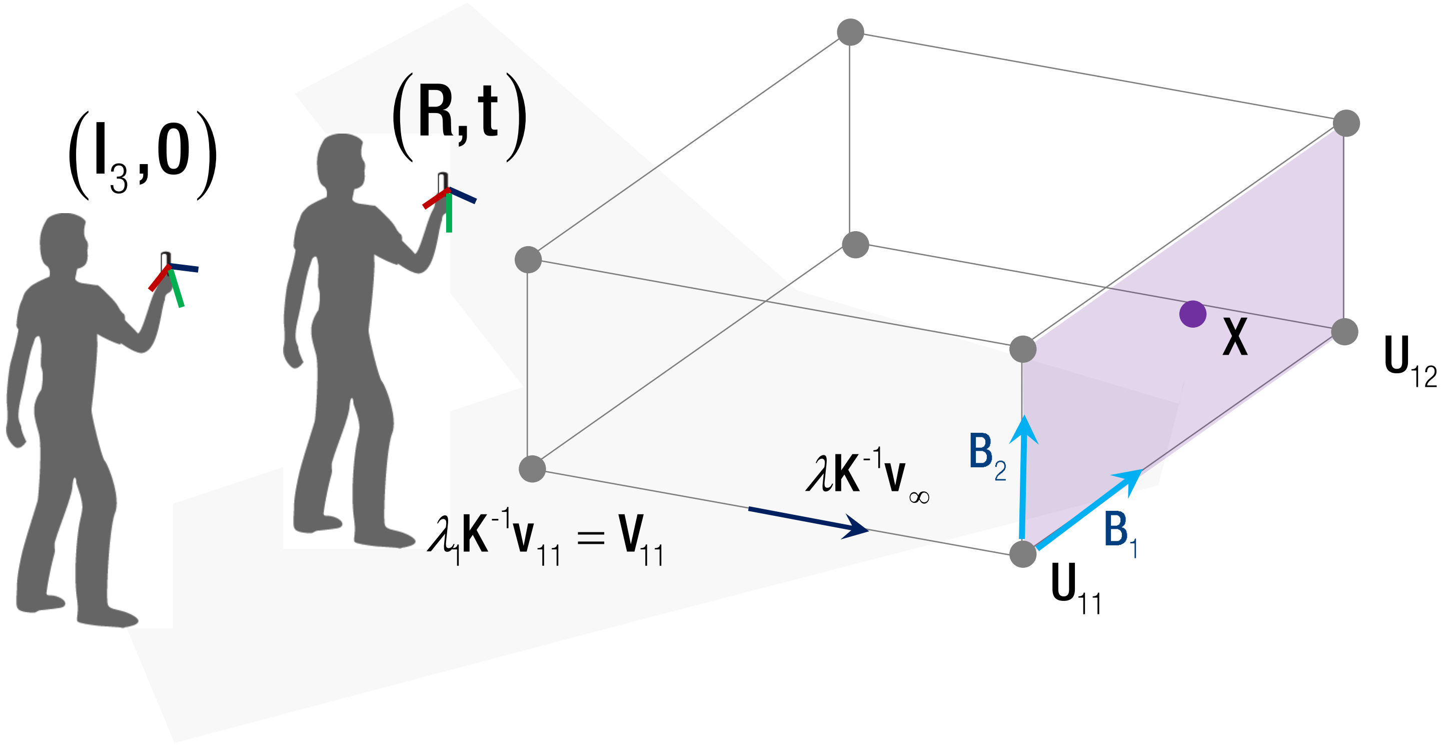
HW #3 Tour into your photo

Spatial Rotation

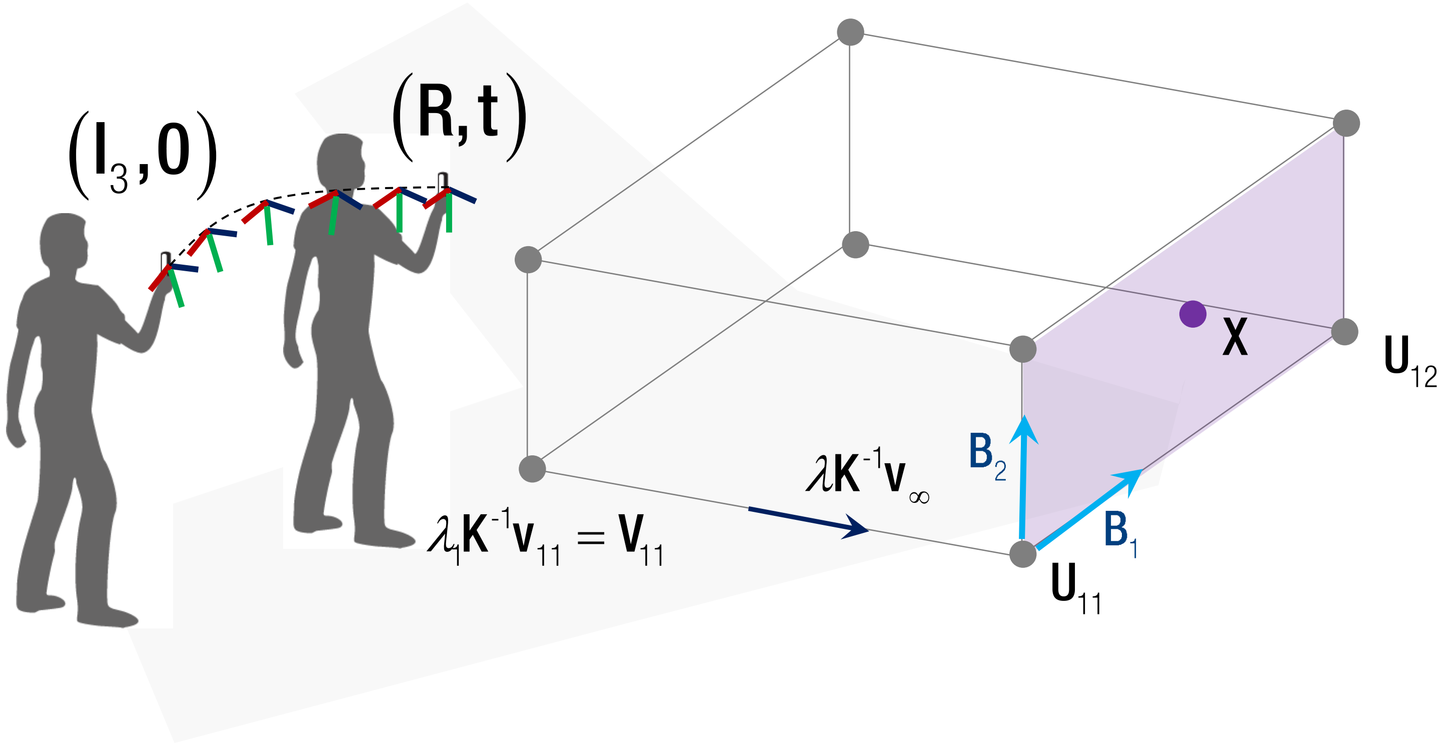
Interpolation of Transformation



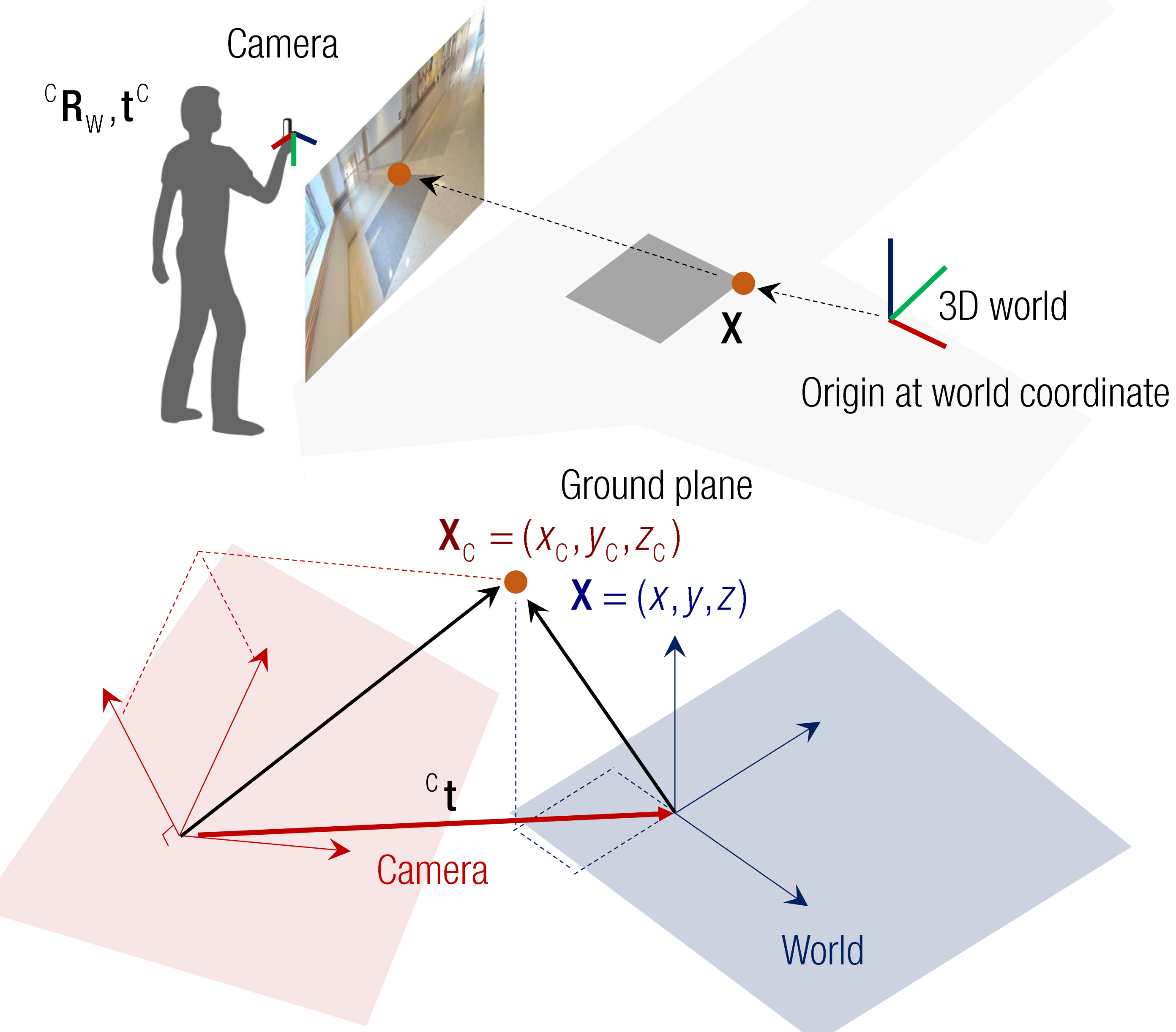
Interpolation of Transformation



Interpolation of Transformation



Recall: Rotate and then, Translate

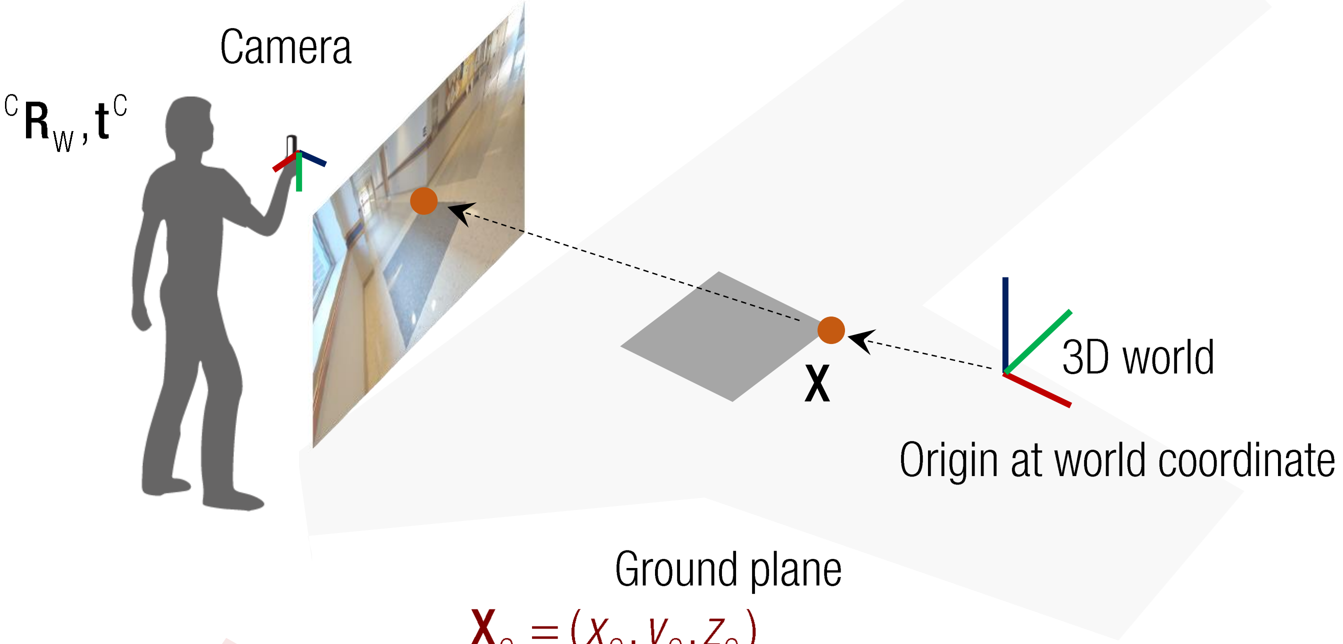


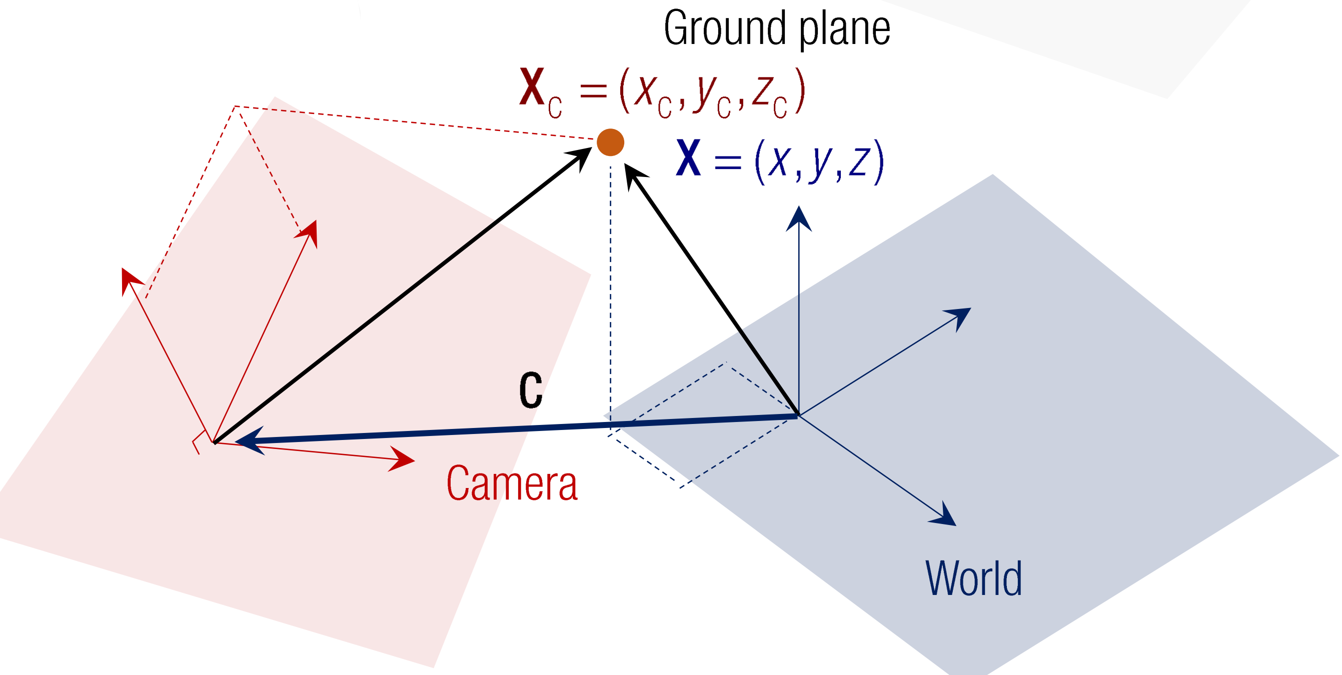
$$\mathbf{X}_{C} = {}^{C}\mathbf{R}_{W}\mathbf{X} + {}^{C}\mathbf{t} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_{x} \\ r_{y1} & r_{y2} & r_{y3} & t_{y} \\ r_{z1} & r_{z2} & r_{z3} & t_{z} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where $^{C}\mathbf{t}$ is translation from world to camera seen from camera.

Rotate and then, translate.

Recall: Translate and the, Rotate





$$\mathbf{X}_C = {}^{C}\mathbf{R}_W \mathbf{X} + {}^{C}\mathbf{t} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \\ r_{z1} & r_{z2} & r_{z3} & t_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

where ${}^{C}\mathbf{t}$ is translation from world to camera seen from camera.

Rotate and then, translate.

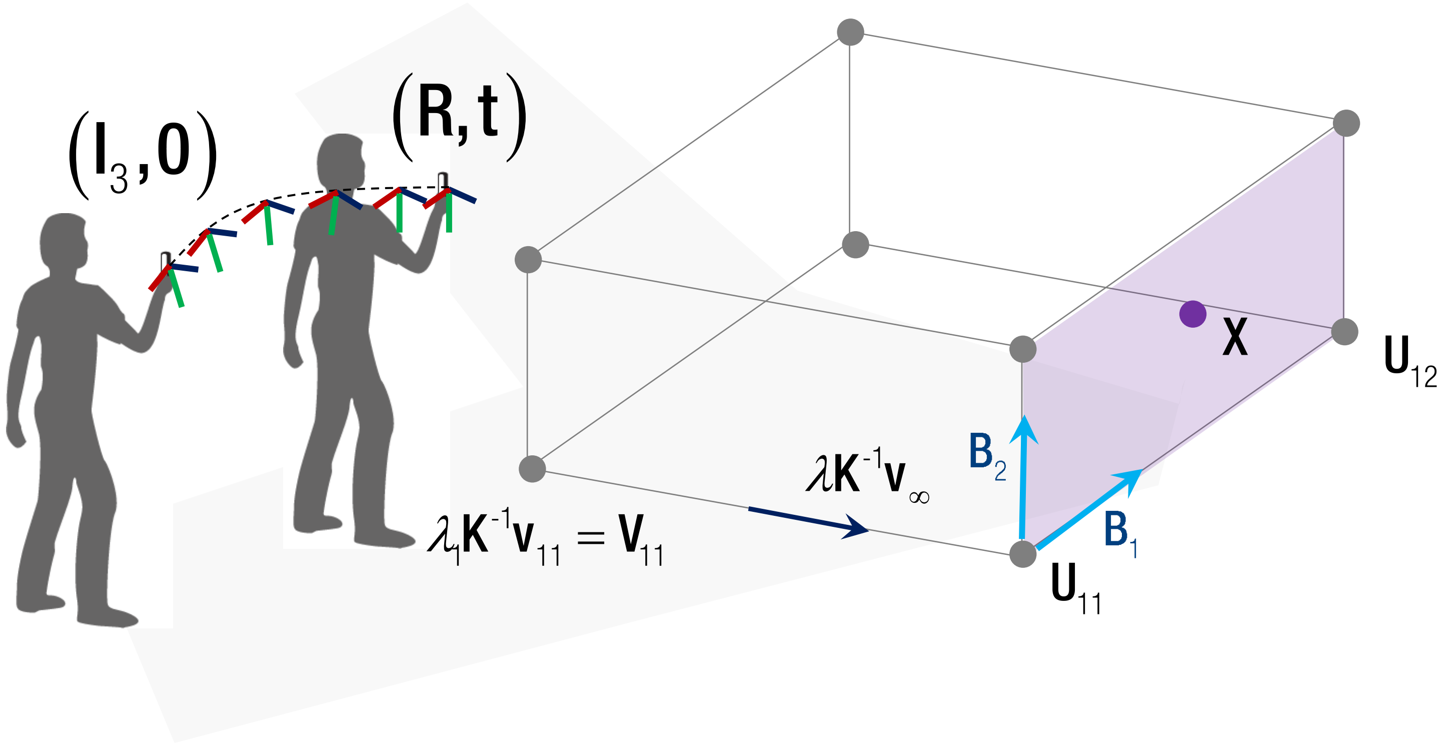
---

cf) **Translate and then, rotate.**

$$\mathbf{X}_C = {}^{C}\mathbf{R}_W (\mathbf{X} - \mathbf{C}) = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \begin{bmatrix} 1 & & & -C_x \\ & 1 & & -C_y \\ & & 1 & -C_z \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

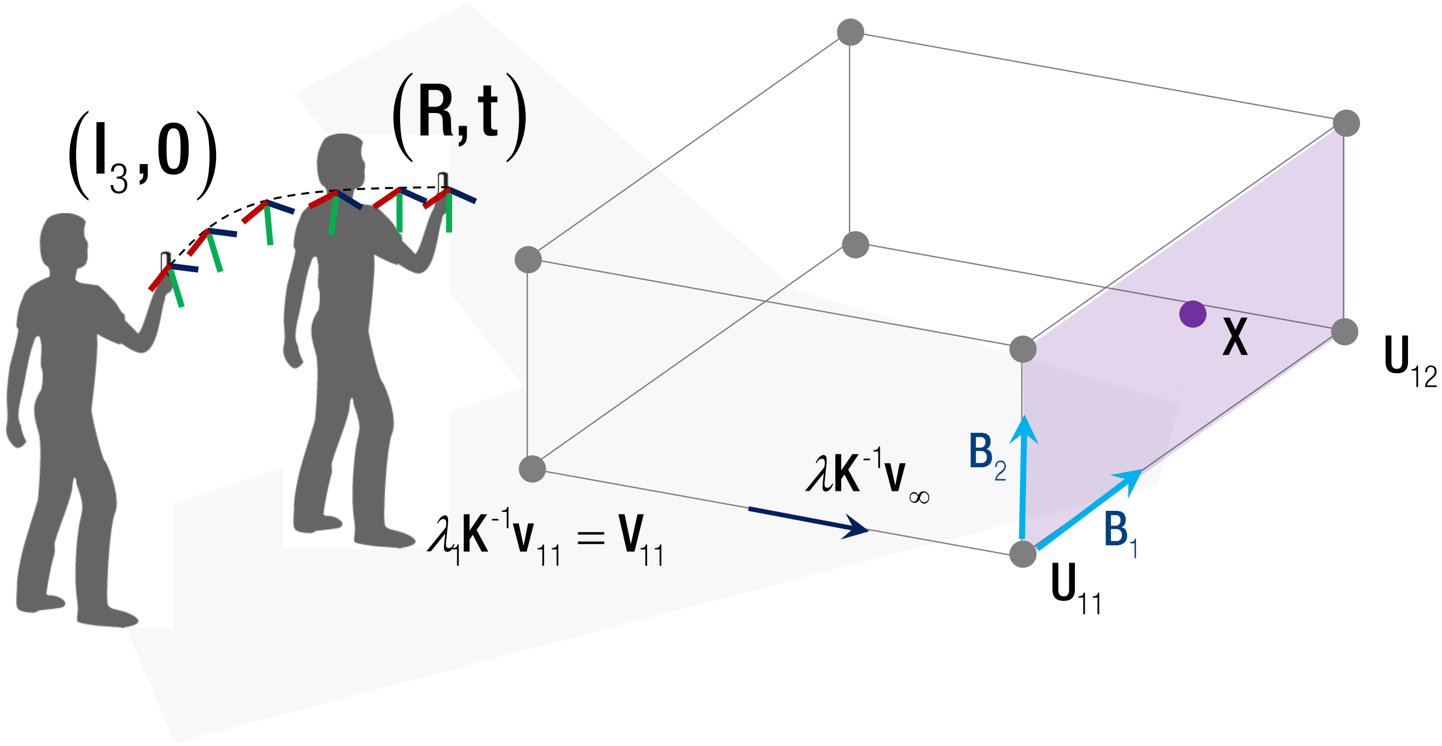
where $\mathbf{C}$ is translation from world to camera seen from world.

Interpolation of Translation



$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X \\ Y \\ Z \\ 1\end{bmatrix} = \mathbf{KR}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{C}\end{bmatrix}\begin{bmatrix}X \\ Y \\ Z \\ 1\end{bmatrix}$$

Interpolation of Translation



$$\lambda \tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X \\ Y \\ Z \\ 1\end{bmatrix} = \mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{C}\end{bmatrix}\begin{bmatrix}X \\ Y \\ Z \\ 1\end{bmatrix}$$

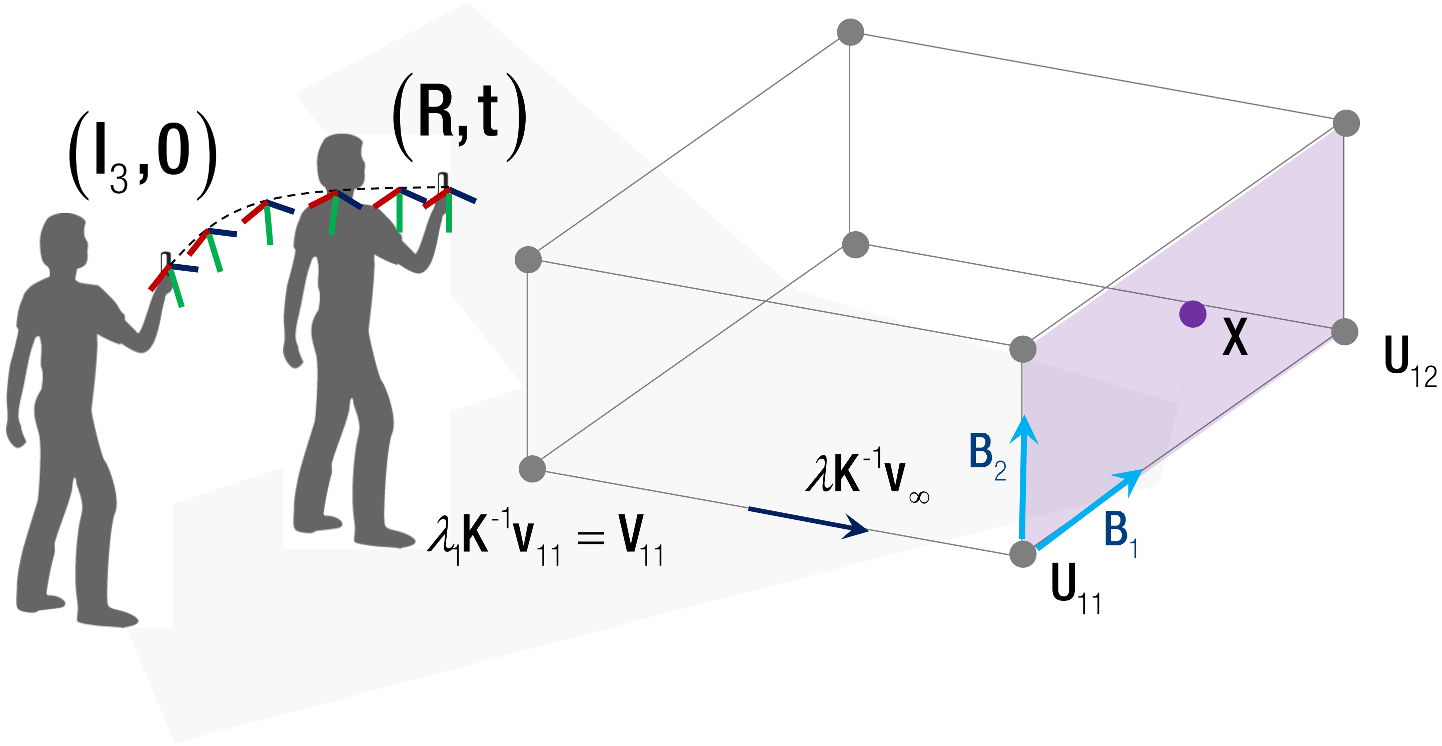
Rot. ➔ Trans. Trans. ➔ Rot.

Translation is independent on rotation.

How to interpolate translation?

$$\mathbf{C}_1 = \begin{bmatrix} C_1^x \\ C_1^y \\ C_1^z \end{bmatrix} \rightarrow \mathbf{C}_2 = \begin{bmatrix} C_2^x \\ C_2^y \\ C_2^z \end{bmatrix}$$

Interpolation of Translation



$$\lambda\tilde{\mathbf{u}} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix} = \mathbf{KR}\begin{bmatrix}\mathbf{I}_3 & -\mathbf{C}\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

Rot. ➔ Trans. Trans. ➔ Rot.

Translation is independent on rotation.

How to interpolate translation?

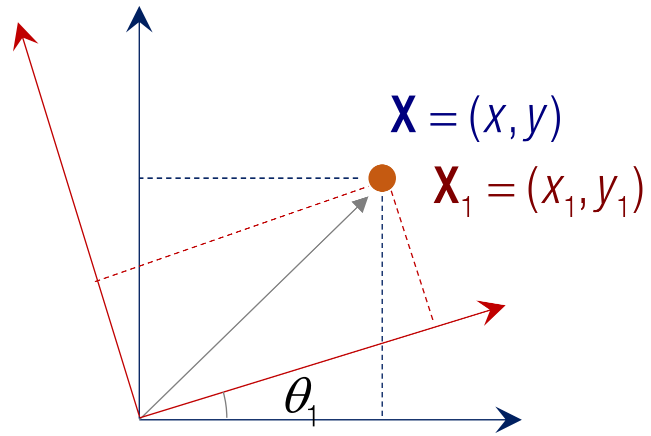
$$\mathbf{C}_1 = \begin{bmatrix}C_1^x\\C_1^y\\C_1^z\end{bmatrix} \rightarrow \mathbf{C}_2 = \begin{bmatrix}C_2^x\\C_2^y\\C_2^z\end{bmatrix}$$

Interpolated camera center:

$$\mathbf{C}_i = w\mathbf{C}_1 + (1-w)\mathbf{C}_2 \qquad w \in [0,1]$$

Interpolation of Rotation

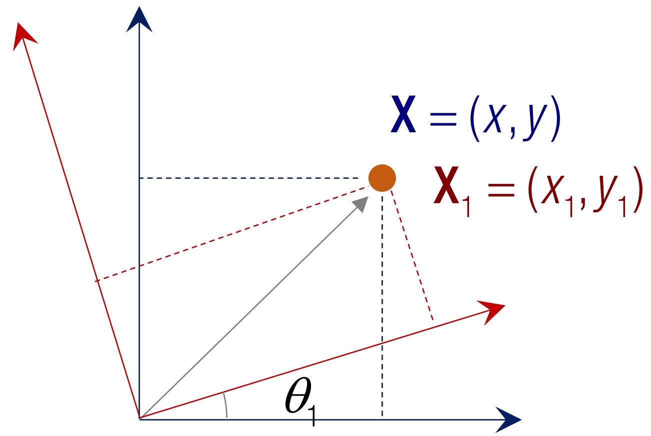
2D coordinate transform:



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \qquad \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation

2D coordinate transform:

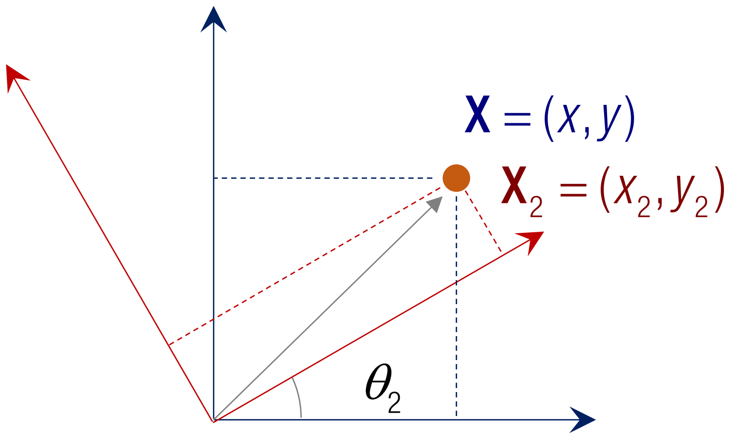


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \right) = \cos^2\theta_1 + \sin^2\theta_1 = 1$$

Interpolation of Rotation

2D coordinate transform:



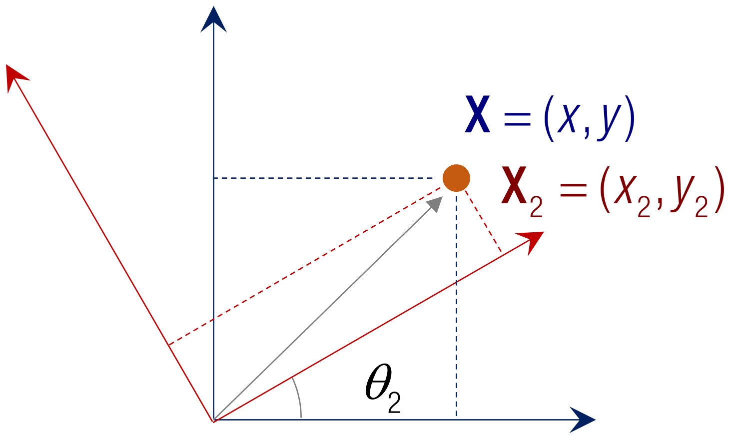
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation

2D coordinate transform:



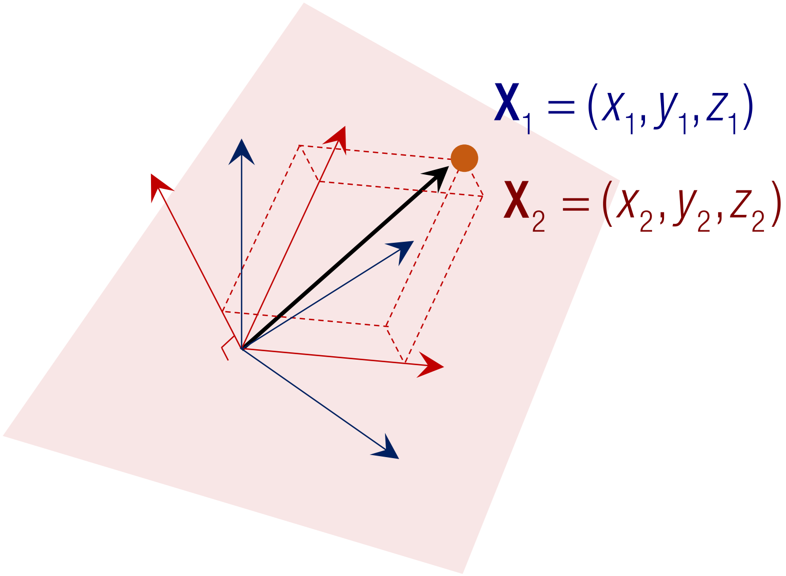
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\theta = w\theta_1 + (1-w)\theta_2$$

$$w \in [0,1]$$

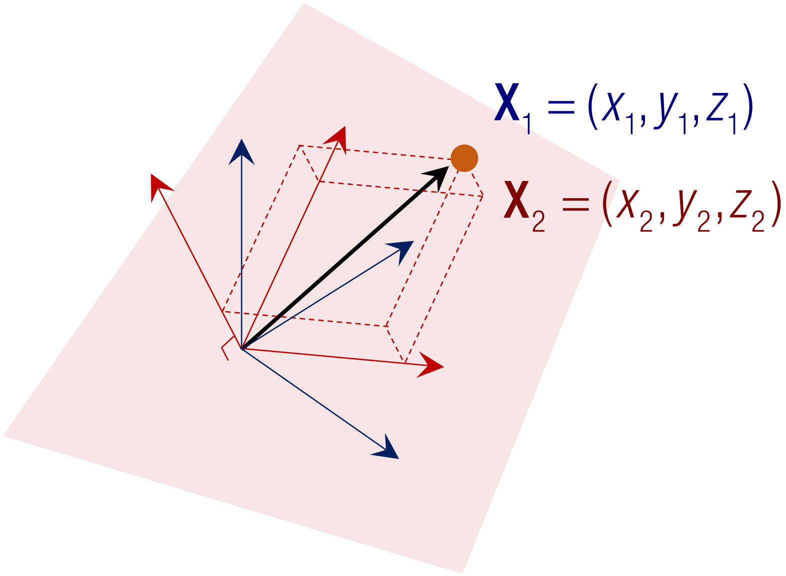
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Interpolation of Rotation in 3D



$$\mathbf{X}_2 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathbf{X} = \mathbf{R}_1 \mathbf{X}_1$$

Interpolation of Rotation in 3D



$$\mathbf{X}_2 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \\ r_{z1} & r_{z2} & r_{z3} \end{bmatrix} \mathbf{X} = \mathbf{R}_1 \mathbf{X}_1$$

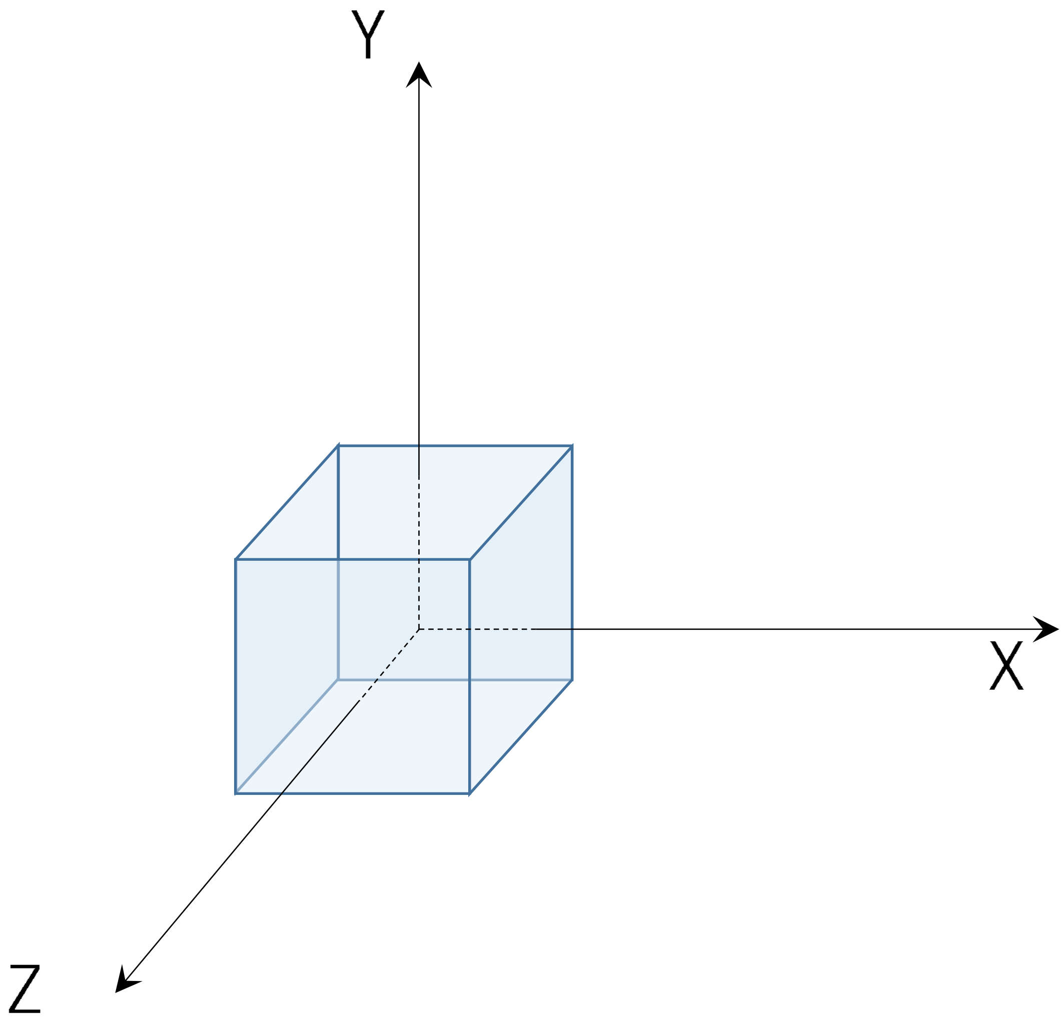
How to interpolate between two coordinates?

$$\mathbf{R}_1 \rightarrow \mathbf{R}_2$$

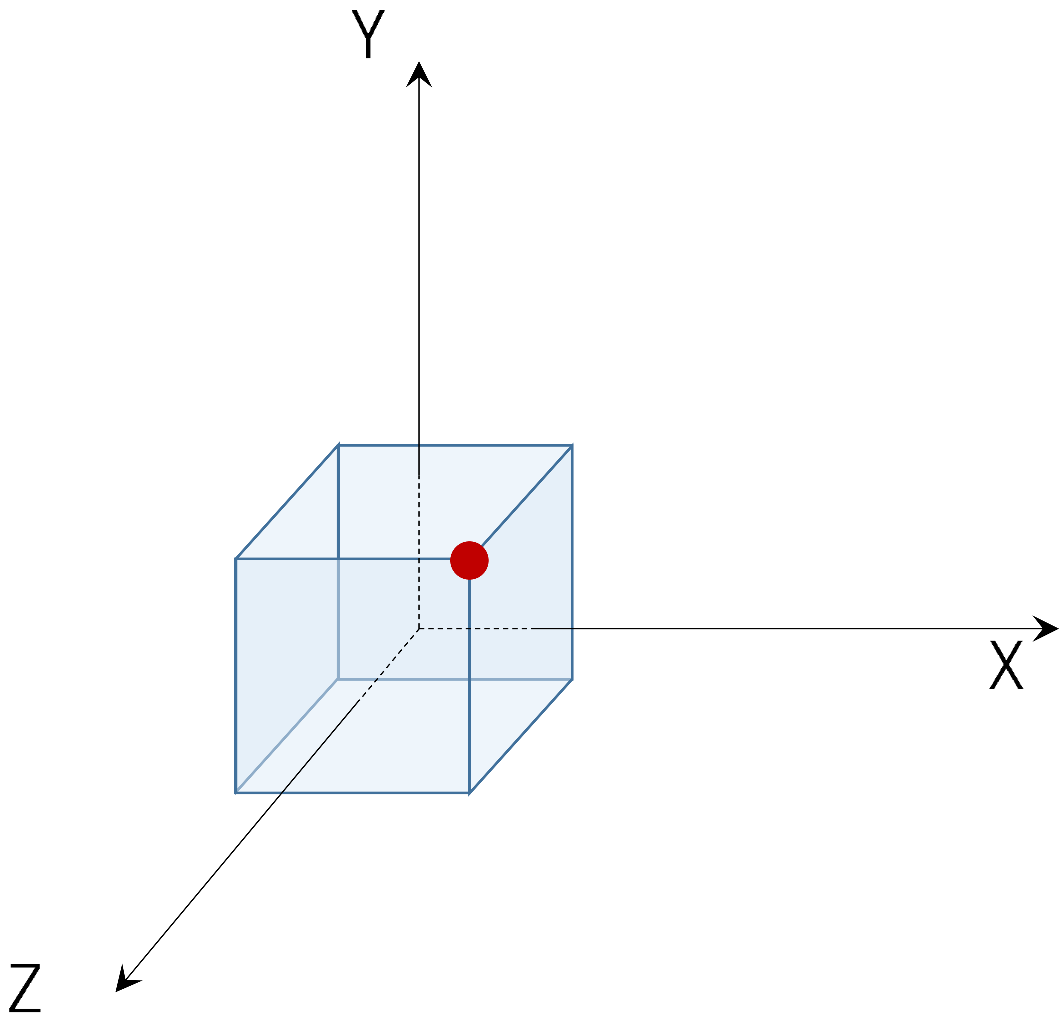
dof: 3

of parameters: 9

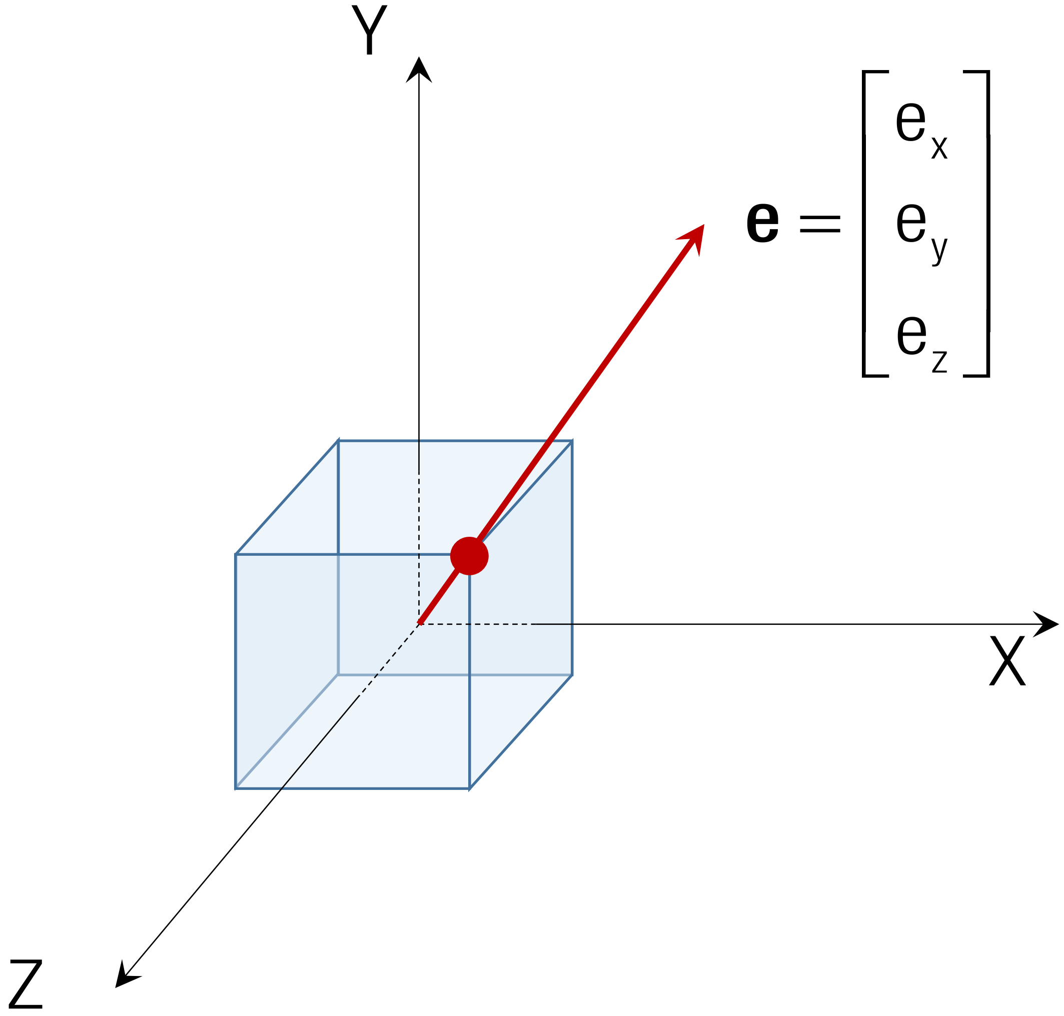
Axis Angle Representation



Axis Angle Representation

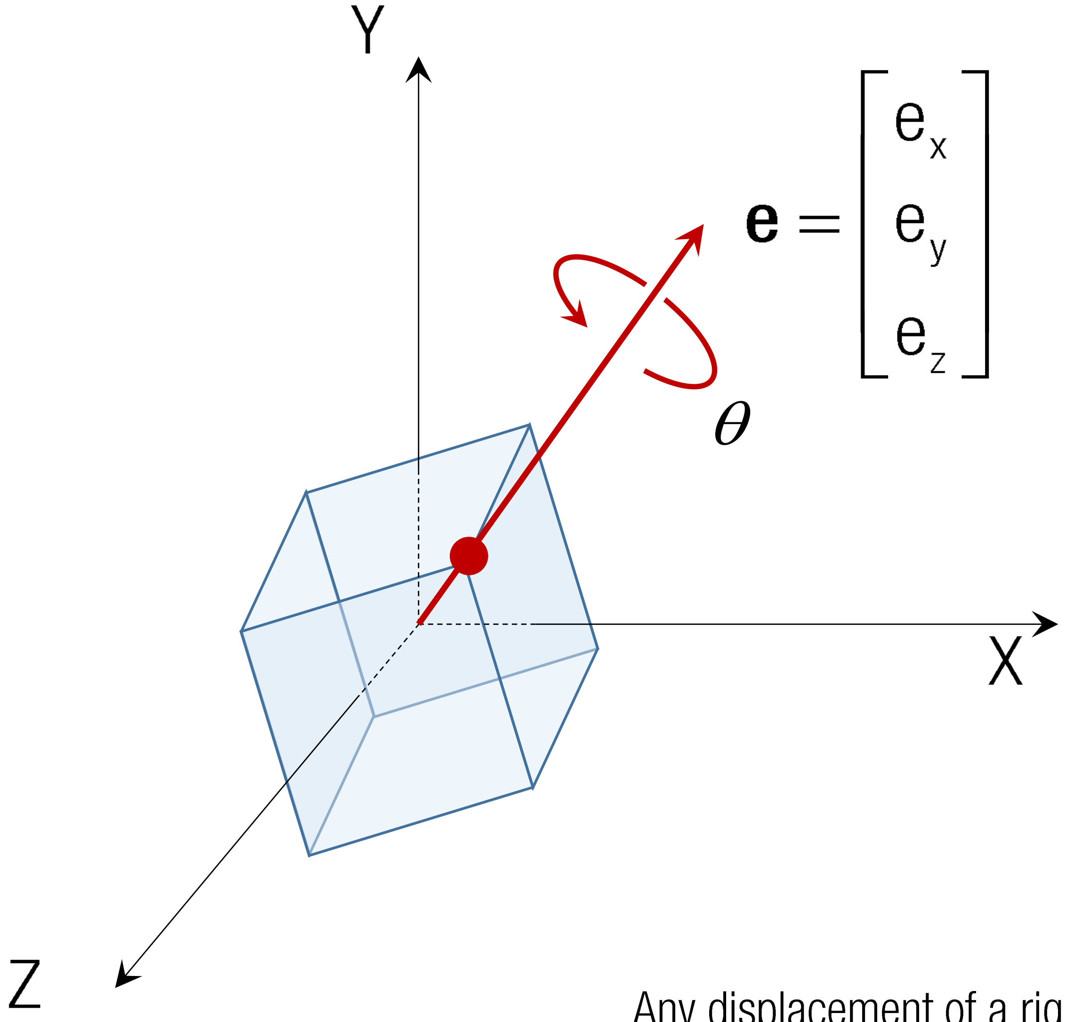


Axis Angle Representation



$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

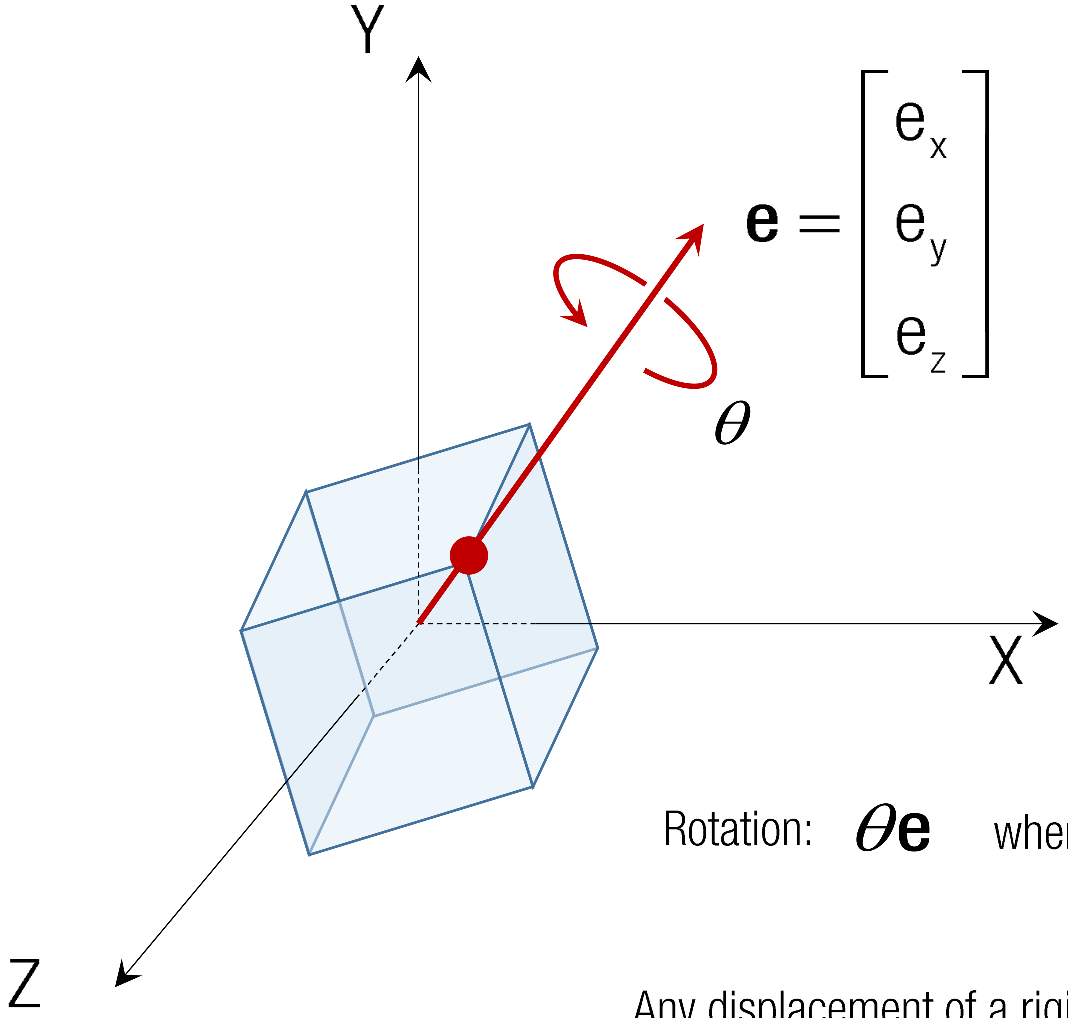
Axis Angle Representation



Euler's theorem

Any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that runs through the fixed point.

Axis Angle Representation



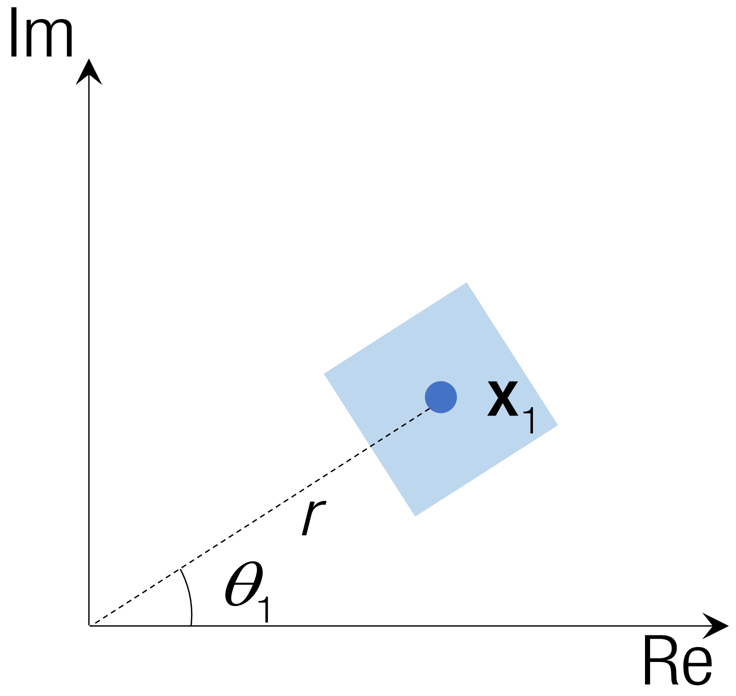
$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Rotation: $\theta \mathbf{e}$ where $\|\mathbf{e}\| = 1$

Euler's theorem

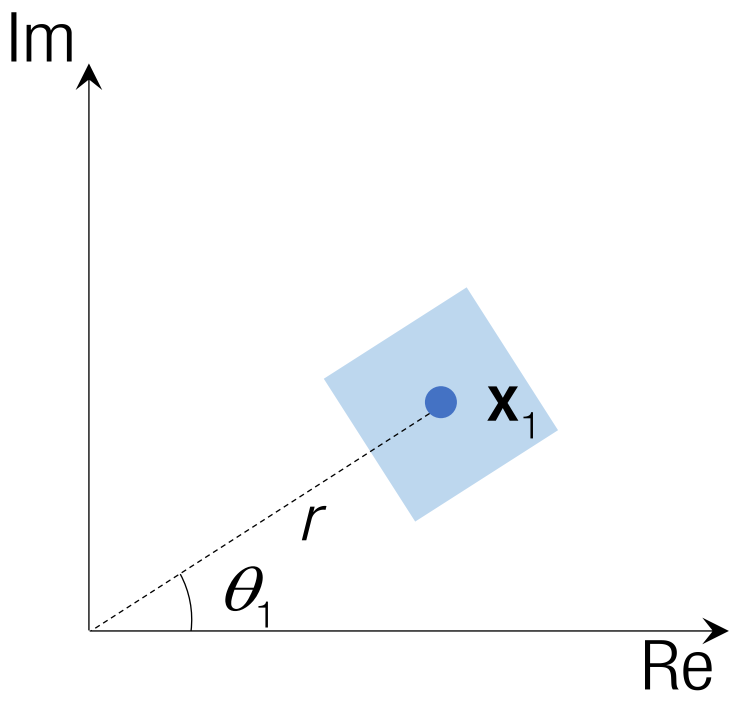
Any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that runs through the fixed point.

2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 =$$

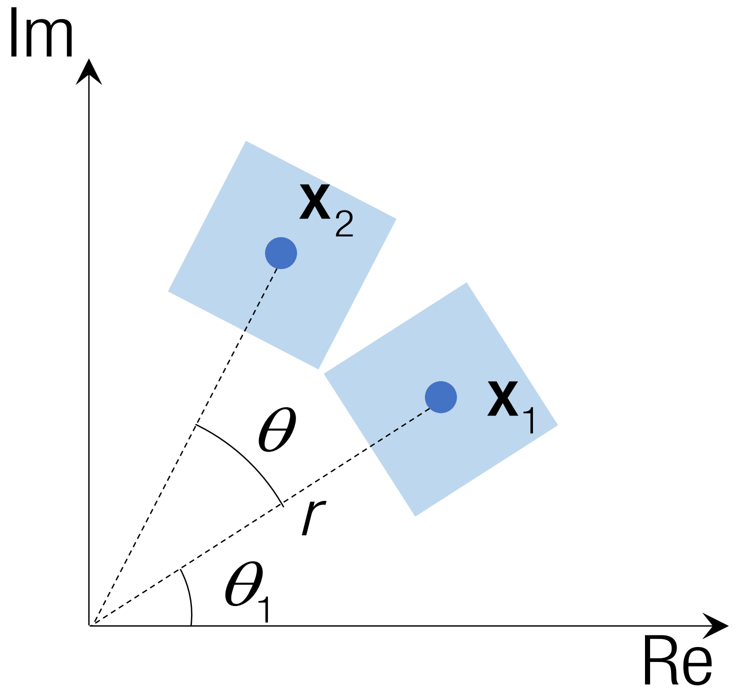
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

Ref) $$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

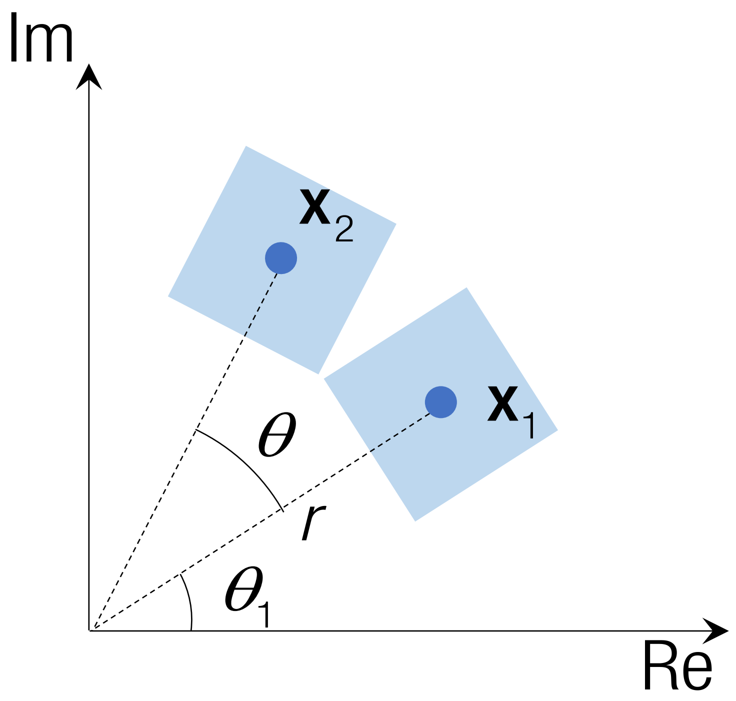
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$\mathbf{x}_2$

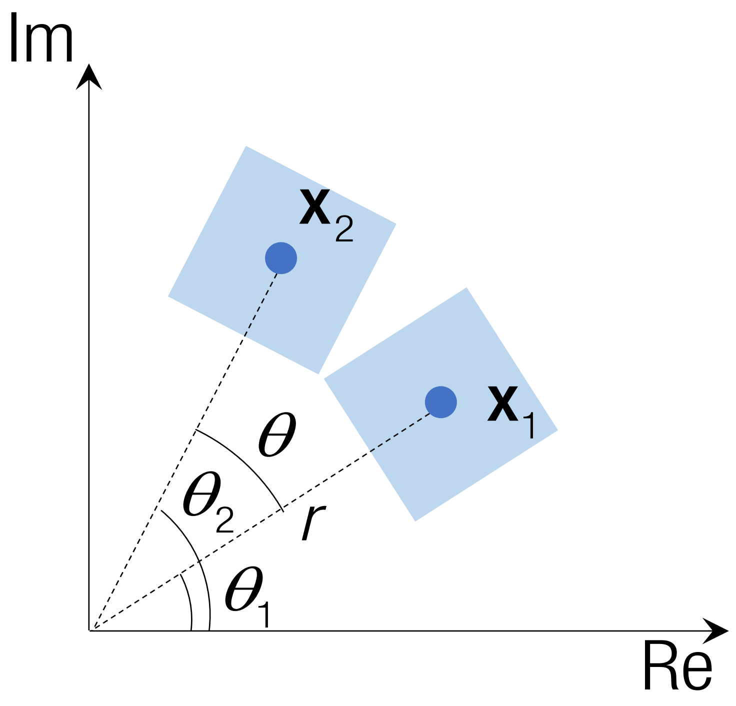
2D Exponential Map (Euler's Formula)



$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$$\begin{aligned}\mathbf{x}_2 &= \exp(i\theta)\mathbf{x}_1 = r\left(\cos\theta + i\sin\theta\right)\left(\cos\theta_1 + i\sin\theta_1\right)\\ &= r\left(\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 + i\left(\cos\theta\sin\theta_1 + \sin\theta\cos\theta_1\right)\right)\\ &= r\left(\cos(\theta+\theta_1) + i\sin(\theta+\theta_1)\right)\end{aligned}$$

2D Exponential Map (Euler's Formula)

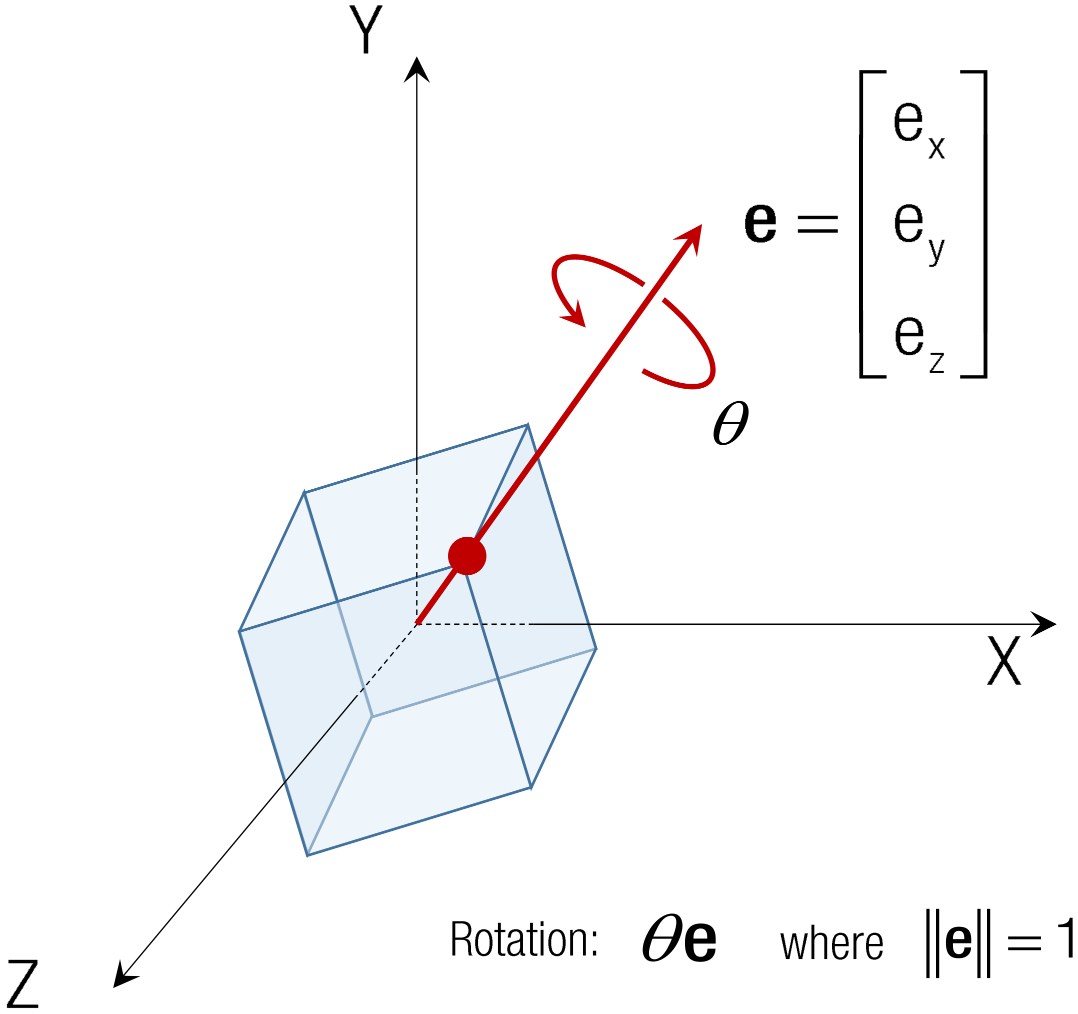


$$\mathbf{x}_1 = r\exp(i\theta_1) = r\left(\cos\theta_1 + i\sin\theta_1\right)$$

$$\begin{aligned}\mathbf{x}_2 &= \exp(i\theta)\mathbf{x}_1 = r\left(\cos\theta + i\sin\theta\right)\left(\cos\theta_1 + i\sin\theta_1\right)\\ &= r\left(\cos\theta\cos\theta_1 - \sin\theta\sin\theta_1 + i\left(\cos\theta\sin\theta_1 + \sin\theta\cos\theta_1\right)\right)\\ &= r\left(\cos(\theta+\theta_1) + i\sin(\theta+\theta_1)\right)\\ &= r\left(\cos\theta_2 + i\sin\theta_2\right)\end{aligned}$$

$$\theta_2 = \theta_1 + \theta$$

3D Exponential Map: Quaternion

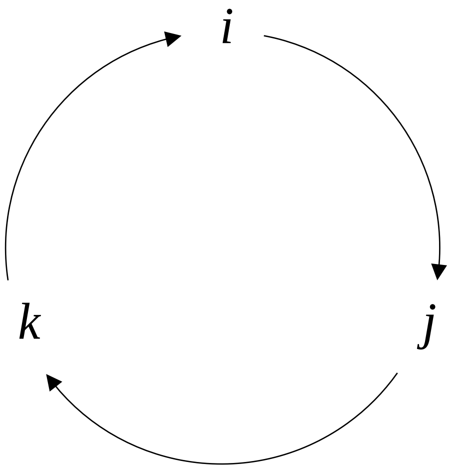


$$\mathbf{e} = \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix}$$

Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

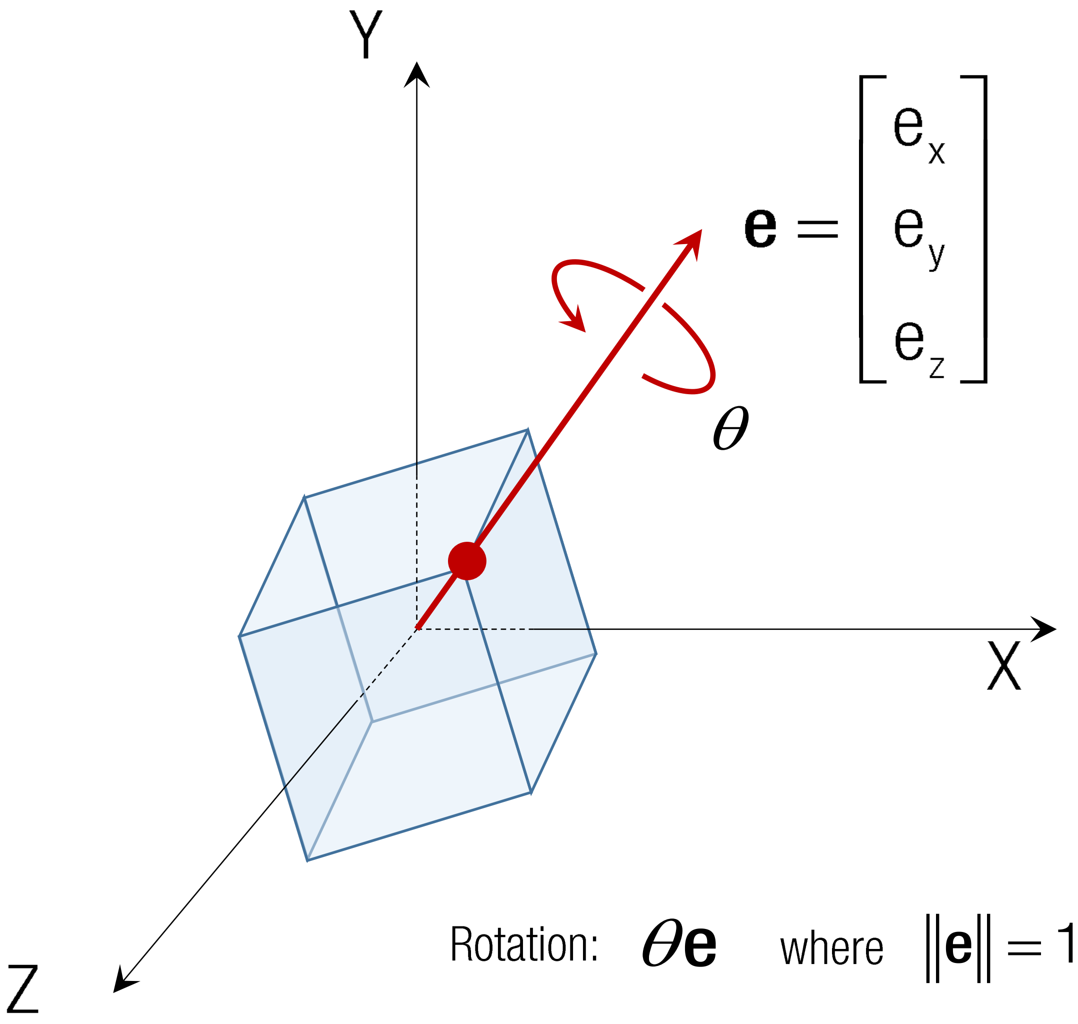
$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

$$i^2 = j^2 = k^2 = ijk = -1$$



$$ij = k$$
$$jk = i$$
$$ki = j$$

Exercise

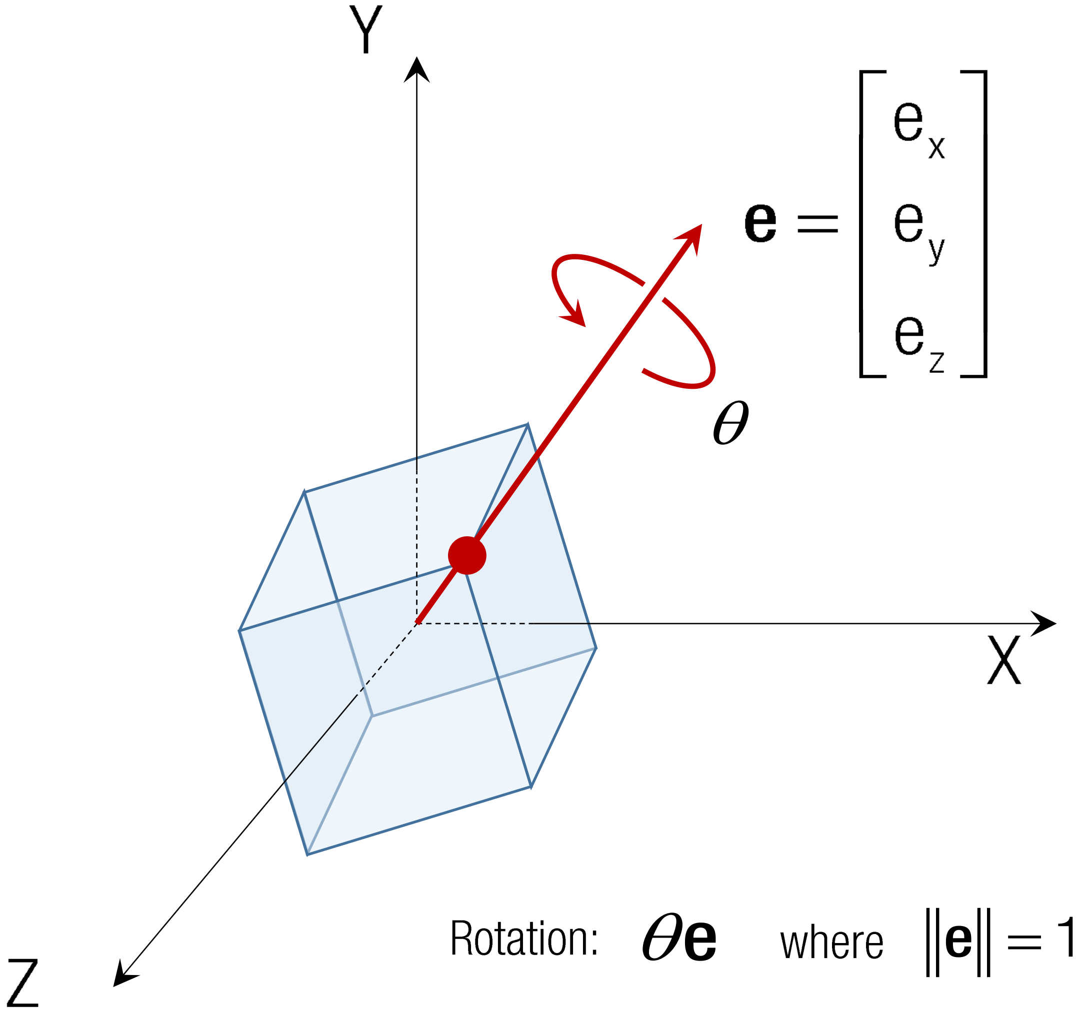


Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

Find a quaternion **q** such that it describes a rotation of 60 degrees about the axis **a**=[3, 4, 0].

Exercise



Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$

$$\exp\left(\frac{\theta}{2}\mathbf{e}\right) = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\,e_x + j\,e_y + k\,e_z\right)$$

Find a quaternion **q** such that it describes a rotation of 60 degrees about the axis **a**=[3, 4, 0].

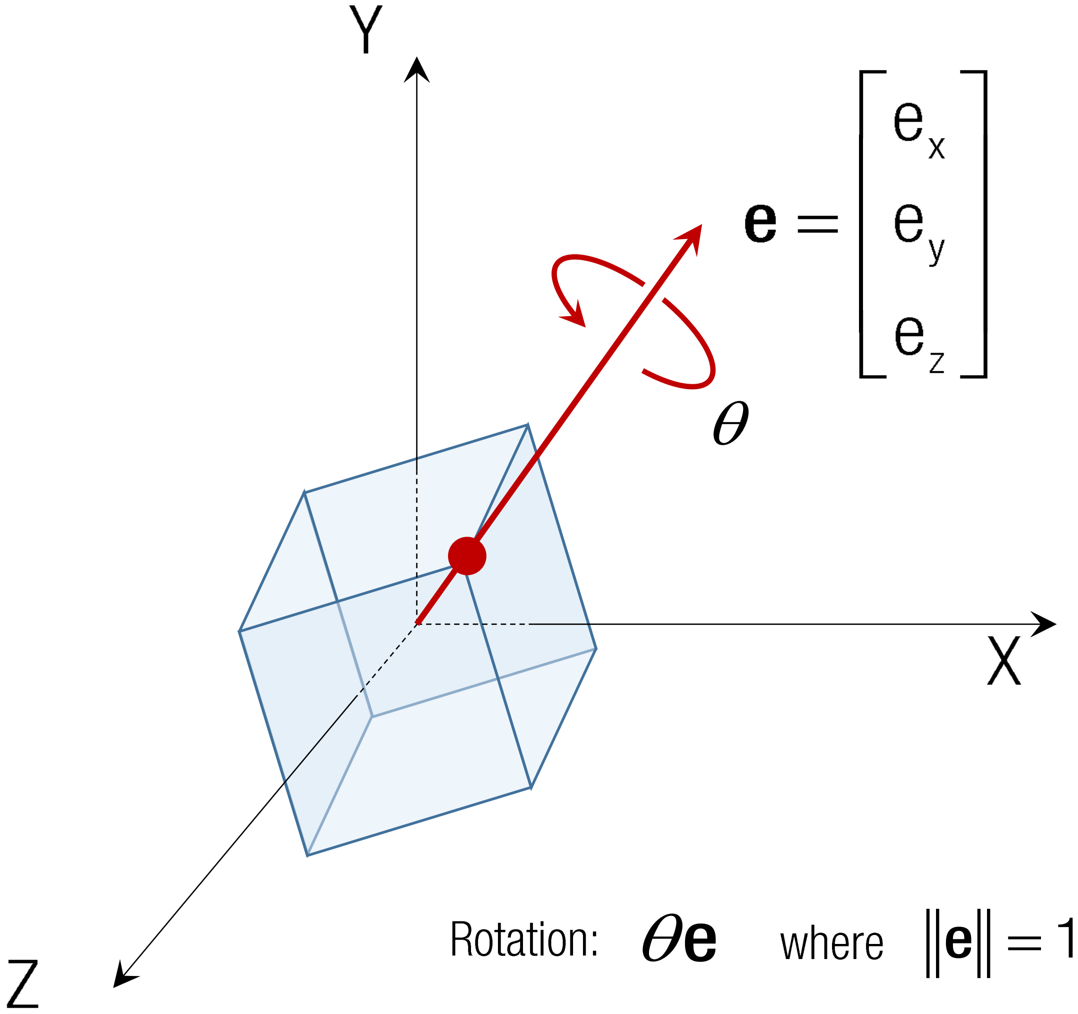
$$\mathbf{e} = \mathbf{a} / \|\mathbf{a}\| = \underbrace{i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}}_{\text{Unit vector}}$$

$$\mathbf{q} = \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right) \qquad \theta = \frac{\pi}{3}$$

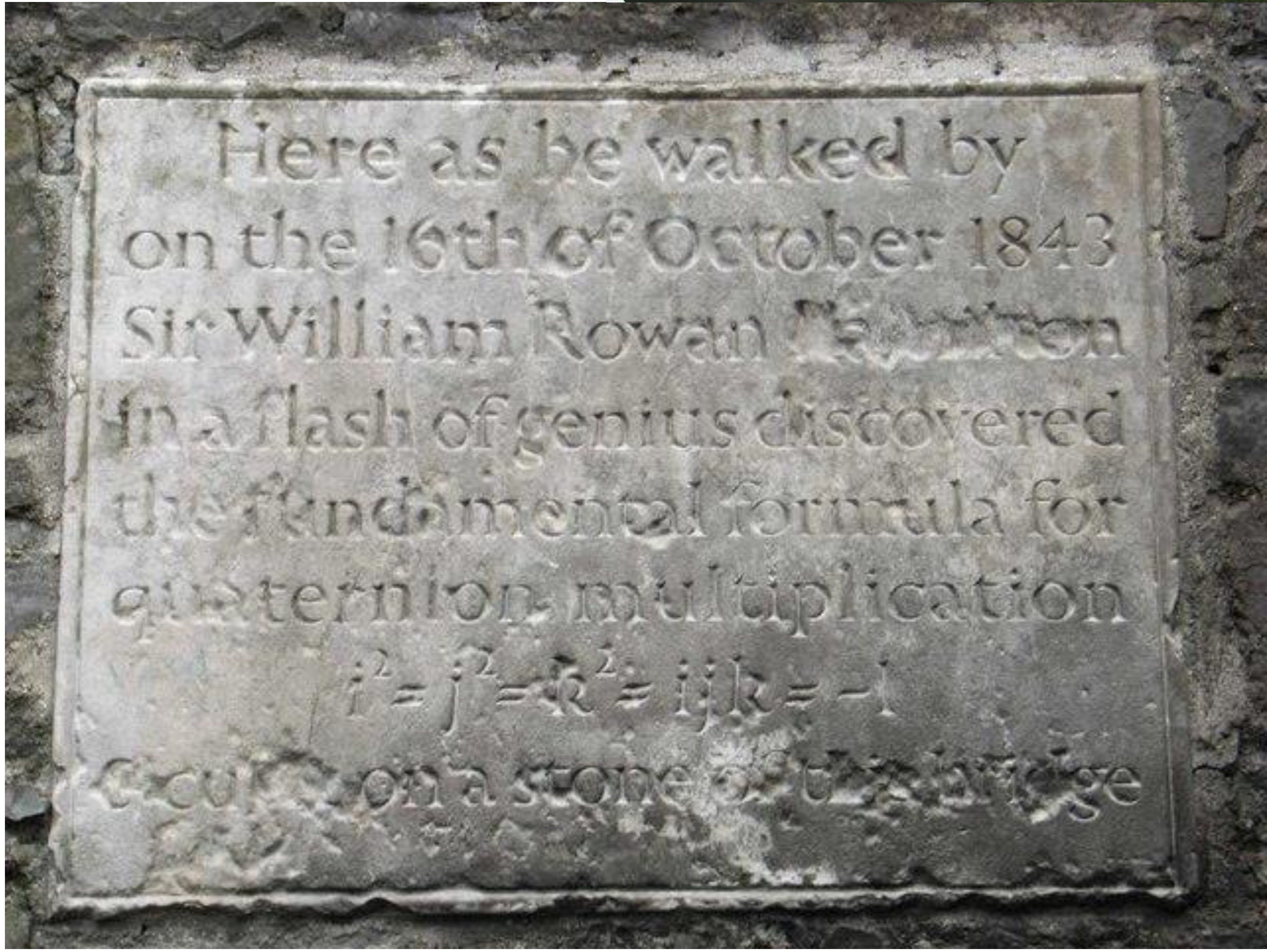
$$= \cos\frac{\pi}{3\cdot 2} + \sin\frac{\pi}{3\cdot 2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(i\frac{3}{5} + j\frac{4}{5} + k\frac{0}{5}\right)$$

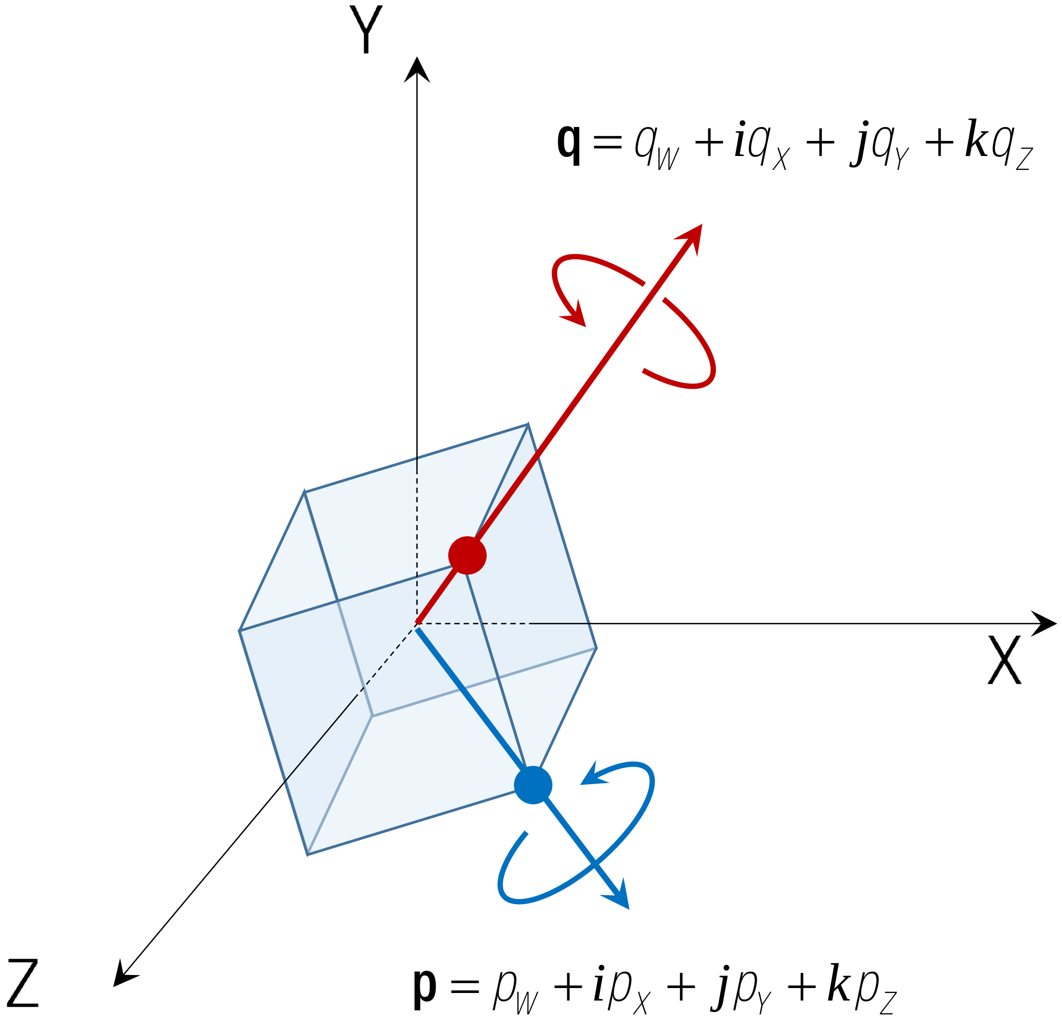
3D Exponential Map: Quaternion



Rotation: $\theta\mathbf{e}$ where $\|\mathbf{e}\| = 1$



Quaternion Product

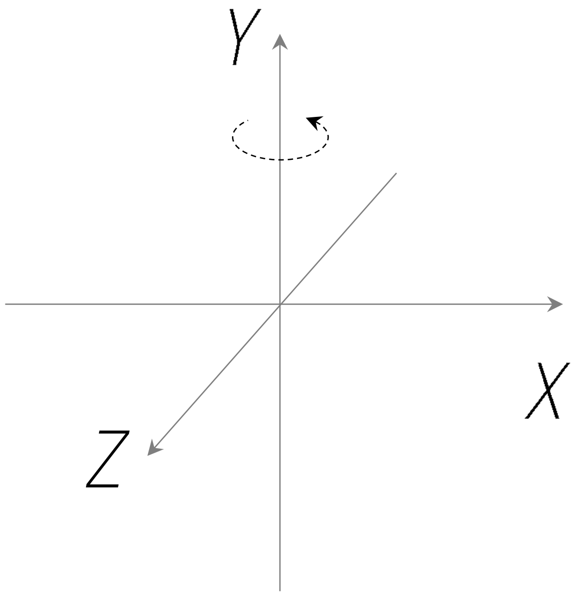


Rotate **q** and then, **p**:

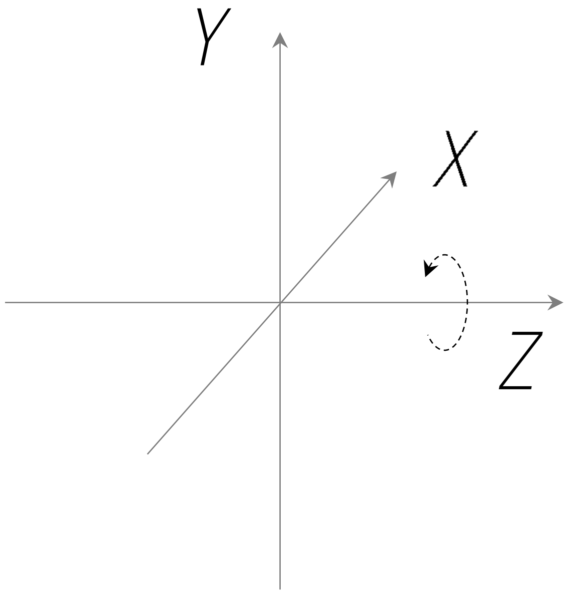
$$
\begin{aligned}
\mathbf{qp} &= \left(q_W + i q_X + j q_Y + k q_Z\right)\left(p_W + i p_X + j p_Y + k p_Z\right) \\
&= \left(q_W p_W - q_X p_X - q_Y p_Y - q_Z p_Z\right) + i\left(q_W p_X + q_X p_W + q_Y p_Z - q_Z p_Y\right) \\
&\quad + j\left(q_W p_Y - q_X p_Z + q_Y p_W + q_Z p_X\right) + k\left(q_W p_Z + q_X p_Y - q_Y p_X + q_Z p_W\right) \\
&= \left(q_W p_W - \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}\right) + \left(q_W \hat{\mathbf{p}} + p_W \hat{\mathbf{q}} + \hat{\mathbf{q}} \times \hat{\mathbf{p}}\right)
\end{aligned}
$$

where $\hat{\mathbf{q}} = i q_X + j q_Y + k q_Z$

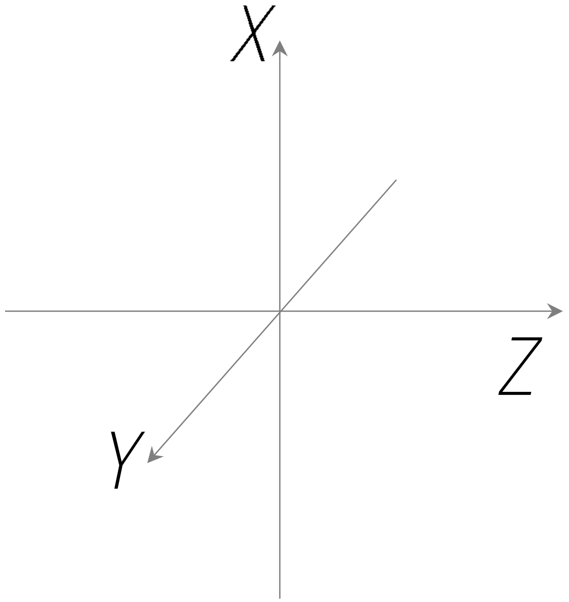
Quaternion Product Example



Rotating 90 degrees about *Y* axis.



Rotating 90 degrees about *Z* axis.



$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2}$$

$$\mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$

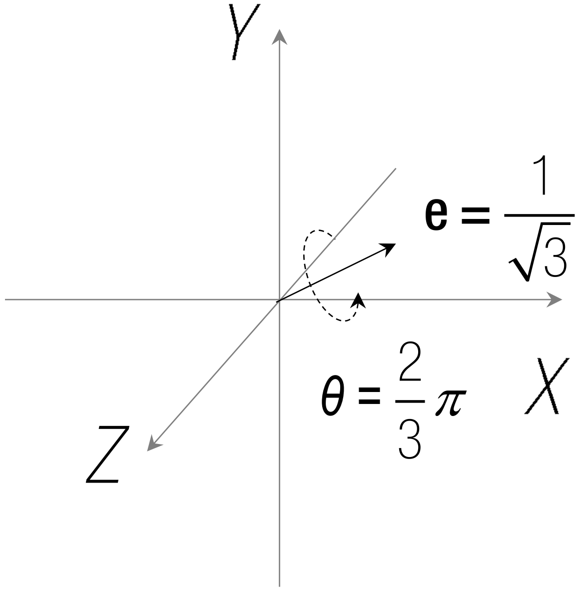
$$\mathbf{q}_{12} = \mathbf{q}_1\mathbf{q}_2 \qquad ?$$

Quaternion Product Example

$$\mathbf{qp} = \left(q_W p_W - \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}\right) + \left(q_W \hat{\mathbf{p}} + p_W \hat{\mathbf{q}} + \hat{\mathbf{q}} \times \hat{\mathbf{p}}\right)$$

$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2} \qquad \mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$

Quaternion Product Example



$\mathbf{e} = \frac{1}{\sqrt{3}}(i + j + k)$

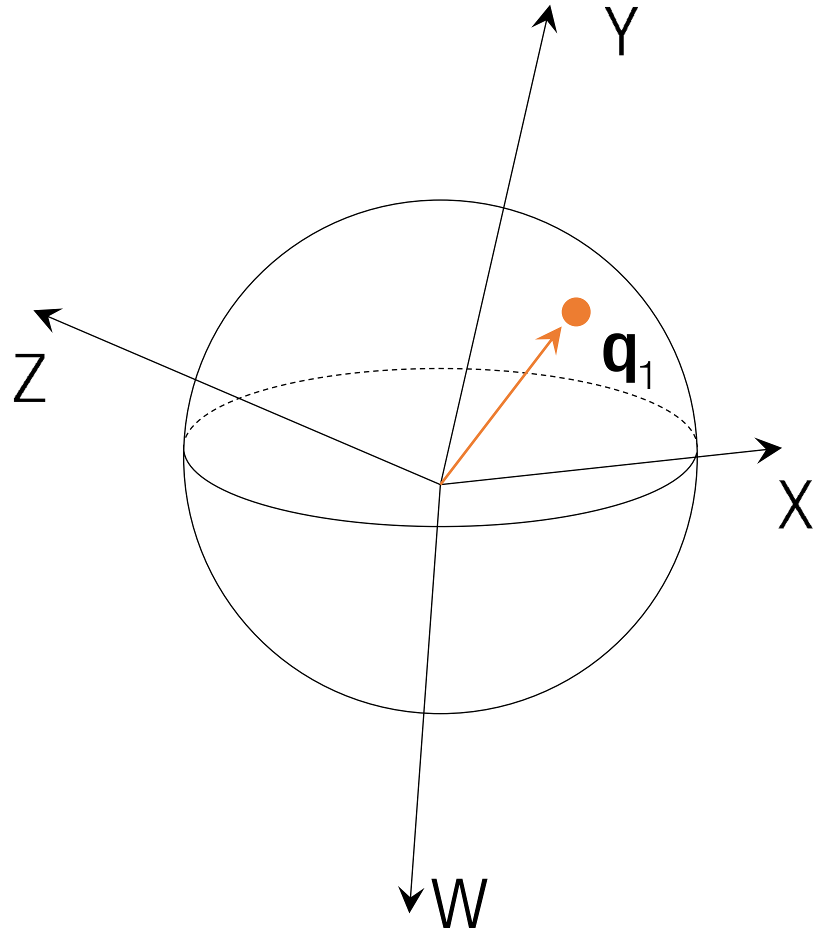
$\theta = \frac{2}{3}\pi$

$$\mathbf{qp} = \left(q_W p_W - \hat{\mathbf{q}} \cdot \hat{\mathbf{p}}\right) + \left(q_W \hat{\mathbf{p}} + p_W \hat{\mathbf{q}} + \hat{\mathbf{q}} \times \hat{\mathbf{p}}\right)$$

$$\mathbf{q}_1 = \cos\frac{\pi/2}{2} + j\sin\frac{\pi/2}{2} \qquad \mathbf{q}_2 = \cos\frac{\pi/2}{2} + k\sin\frac{\pi/2}{2}$$

$$\begin{aligned}
\mathbf{q}_{12} &= \mathbf{q}_1\mathbf{q}_2 \\
&= \left(\frac{\sqrt{2}}{2} + \boldsymbol{i}\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2} + \boldsymbol{j}\frac{\sqrt{2}}{2}\right) \\
&= \frac{1}{2} + \frac{1}{2}\boldsymbol{i} + \frac{1}{2}\boldsymbol{j} + \frac{1}{2}\boldsymbol{k} \\
&= \frac{1}{2} + \frac{1}{2}\boldsymbol{i} + \frac{1}{2}\boldsymbol{j} + \frac{1}{2}\boldsymbol{k} \\
&= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}}\boldsymbol{i} + \frac{1}{\sqrt{3}}\boldsymbol{j} + \frac{1}{\sqrt{3}}\boldsymbol{k}\right)
\end{aligned}$$

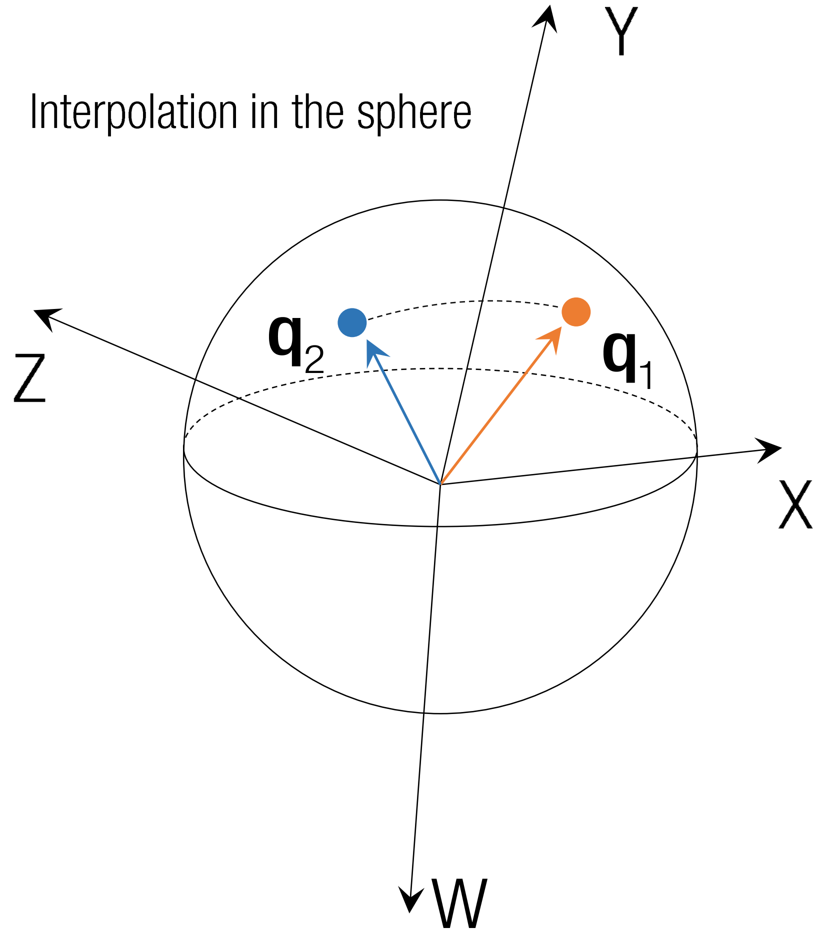
Quaternion in 4D Sphere



$$\mathbf{q} = q_W + \boldsymbol{i} q_X + \boldsymbol{j} q_Y + \boldsymbol{k} q_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

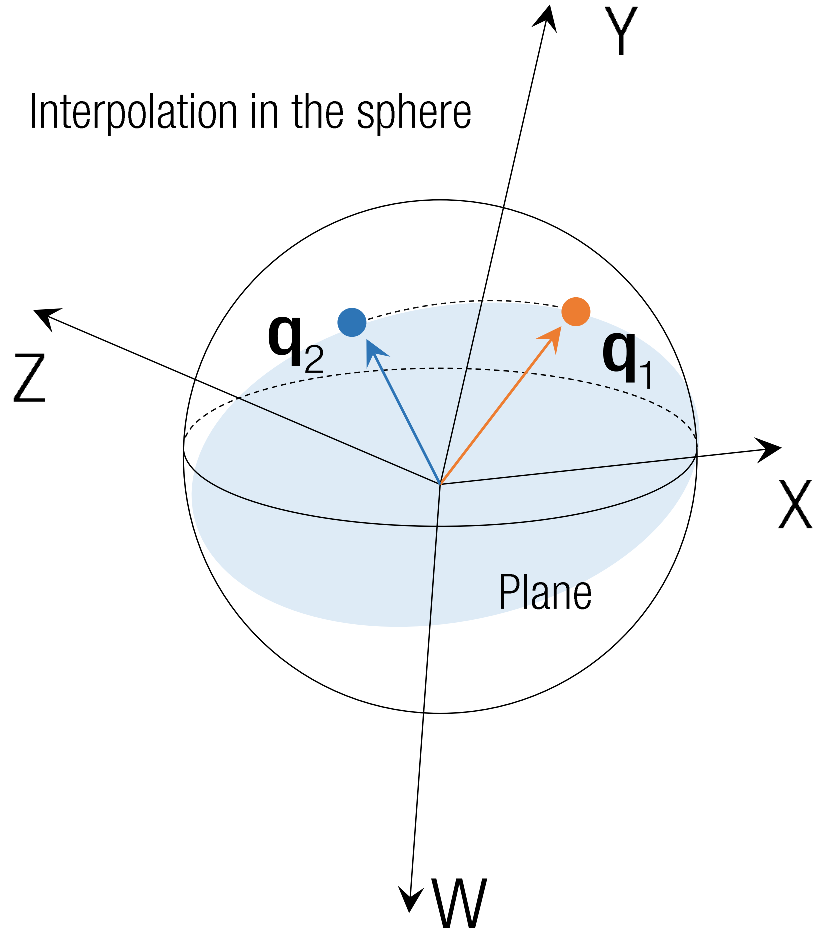
Quaternion in 4D Sphere

Interpolation in the sphere



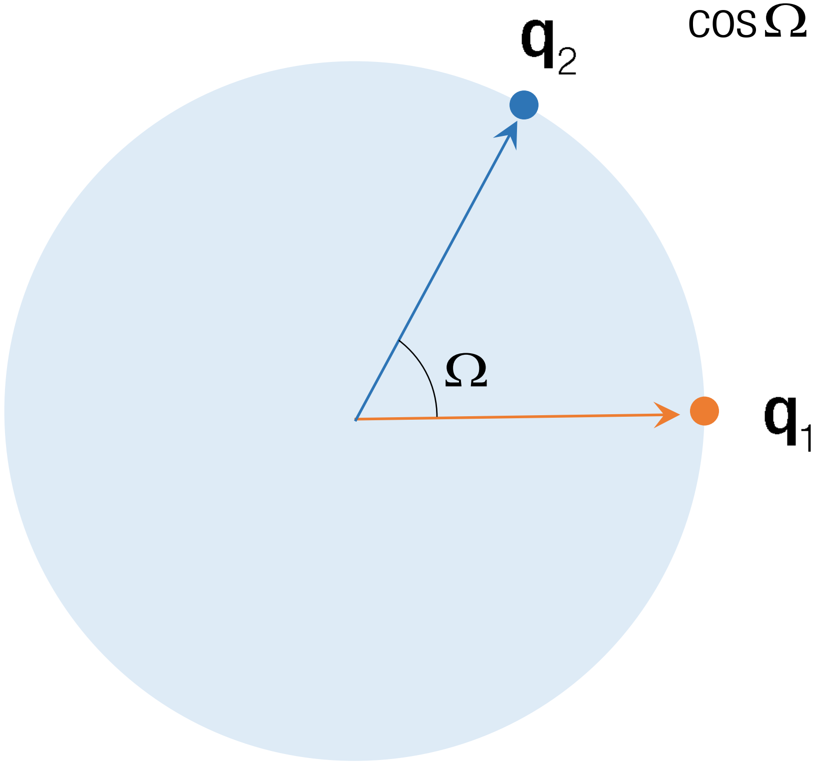
$$\mathbf{q} = q_W + \mathbf{i}q_X + \mathbf{j}q_Y + \mathbf{k}q_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

Quaternion in 4D Sphere



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

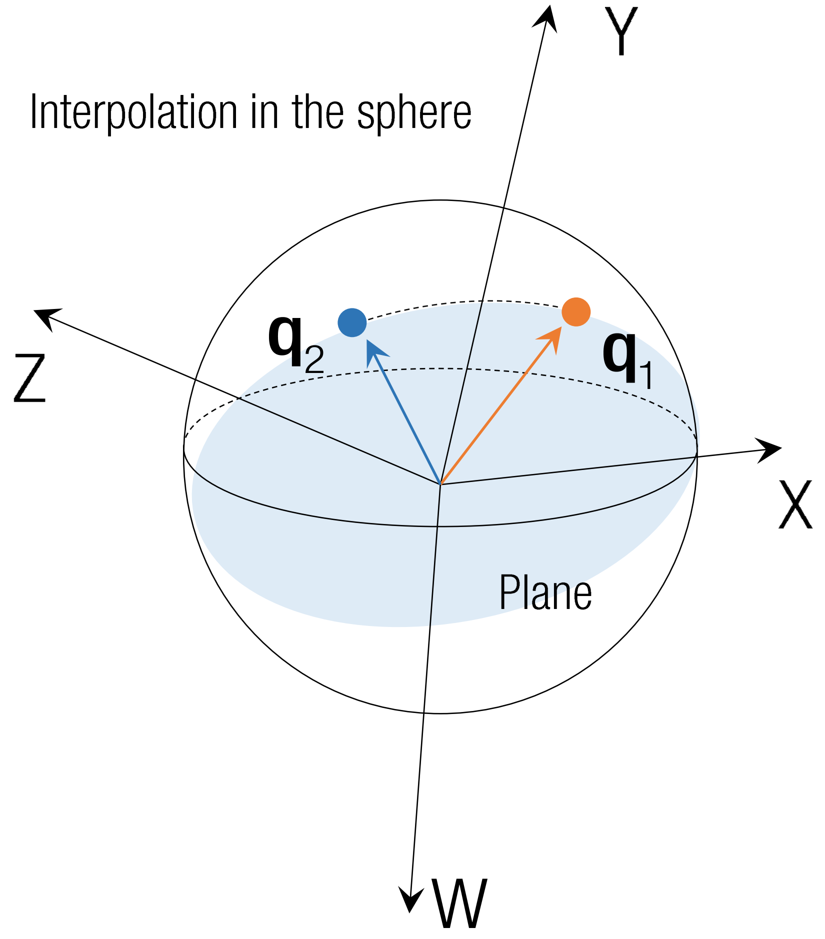
$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



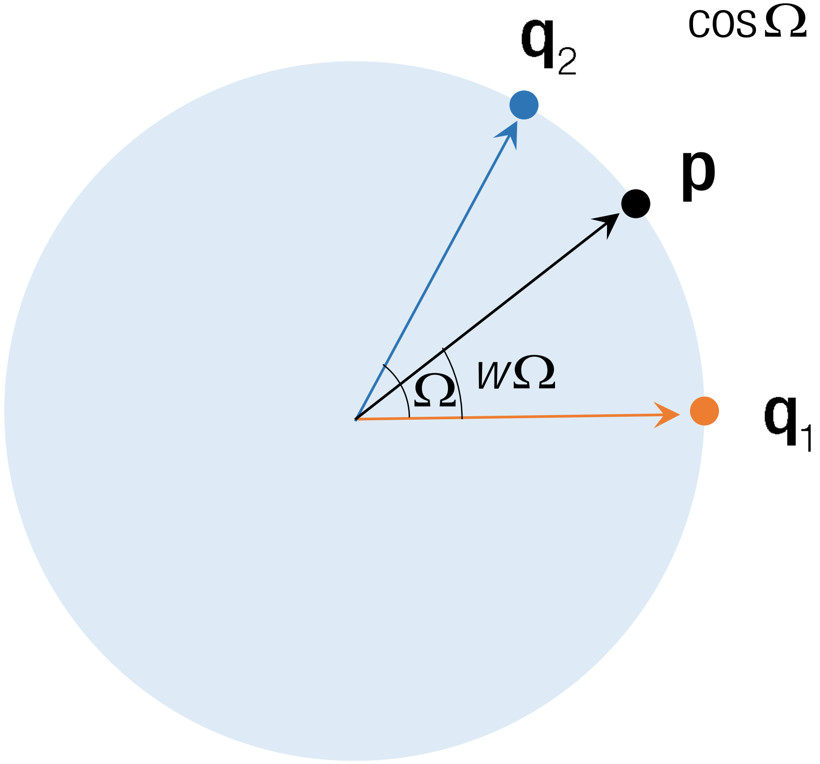
Quaternion Interpolation

$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

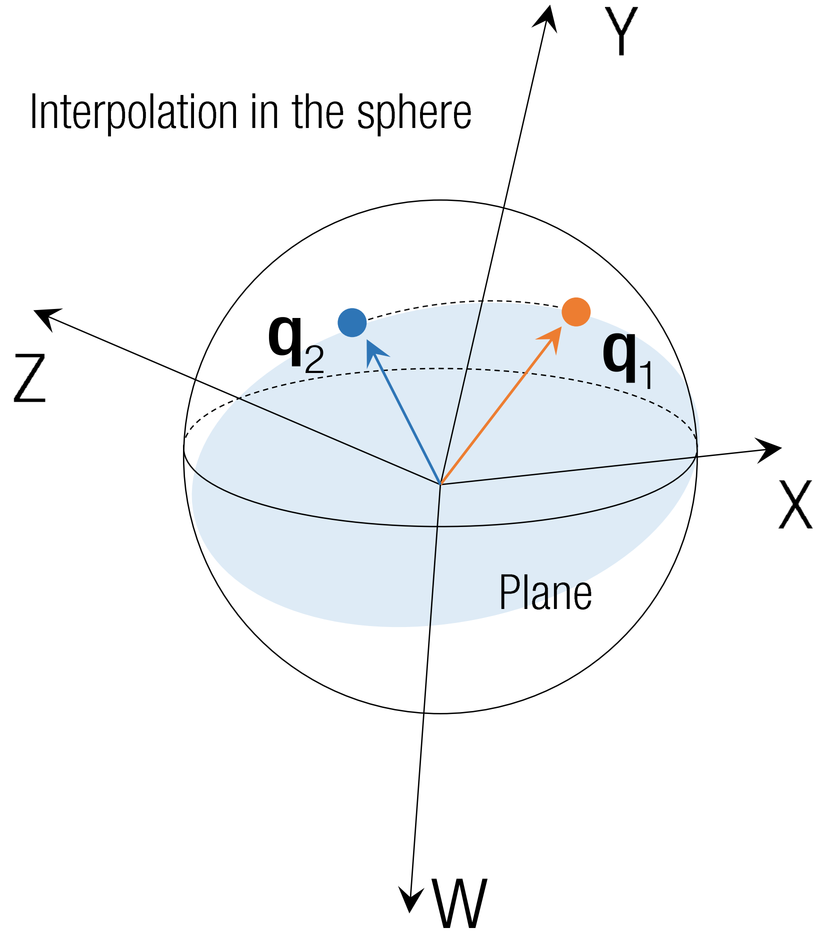
Interpolation in the sphere



$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$

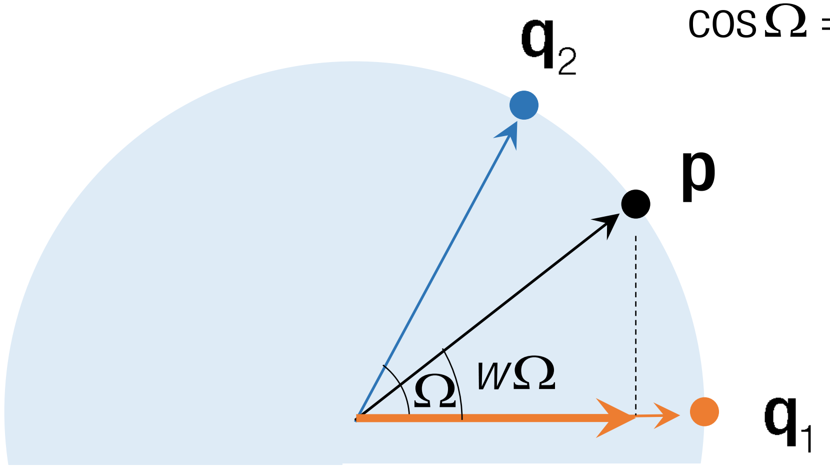


Quaternion Interpolation



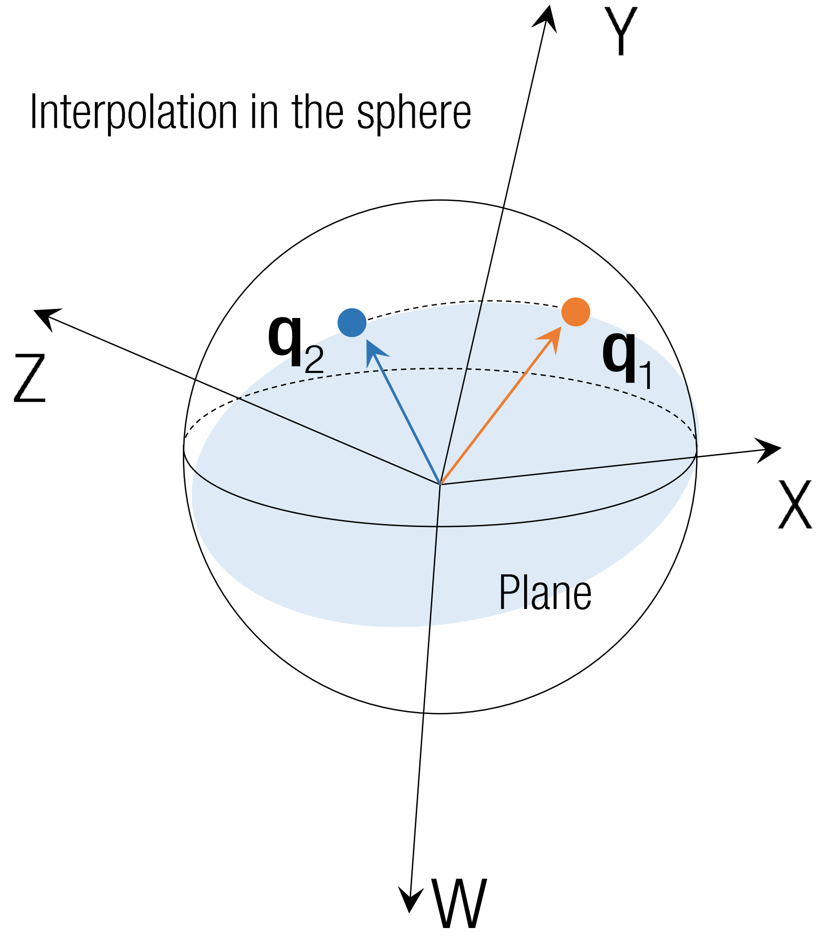
$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



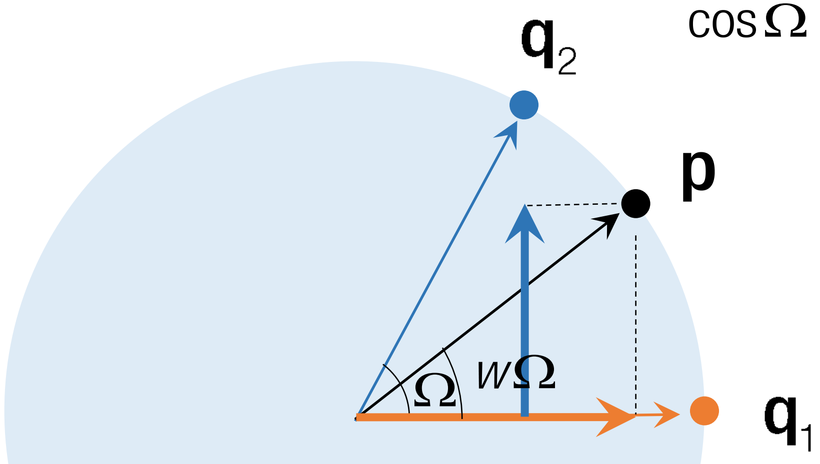
$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega$$

Quaternion Interpolation



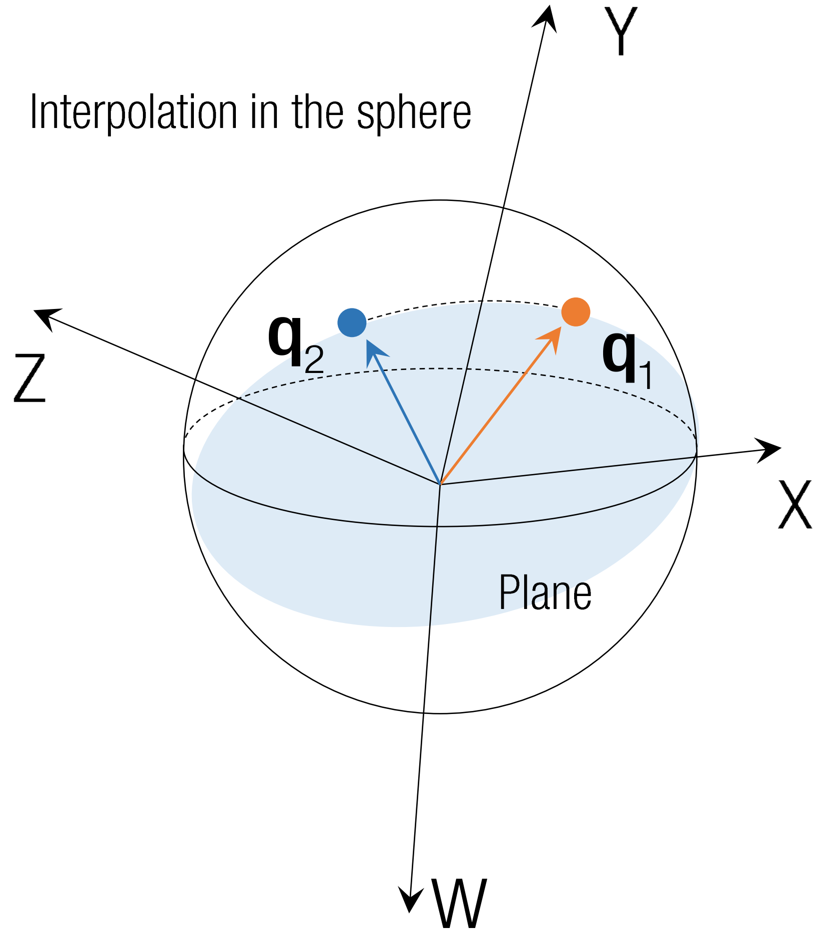
$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



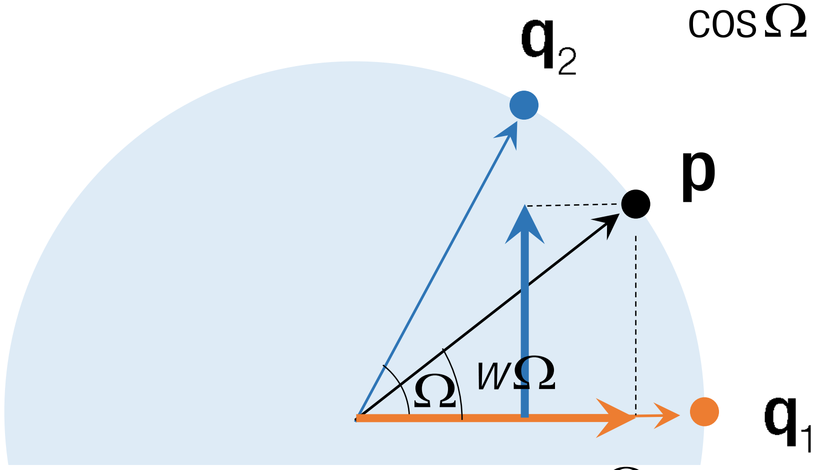
$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

Quaternion Interpolation



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

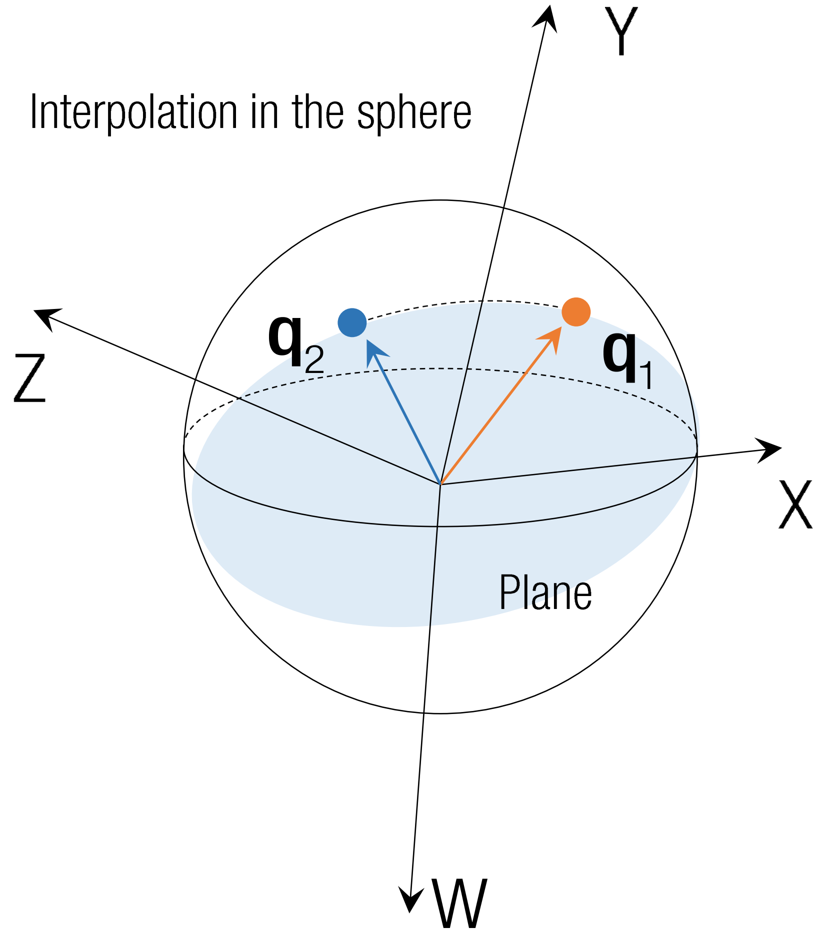
$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$



$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

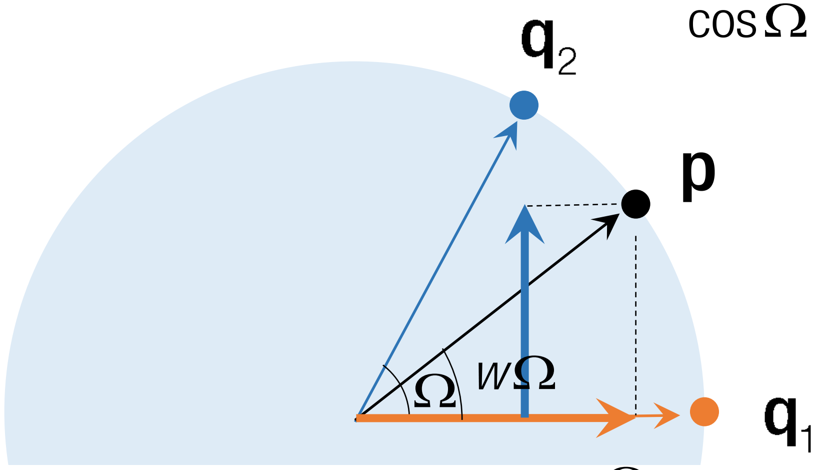
$$= \frac{\mathbf{q}_1 \left(\sin\Omega \cos w\Omega - \cos\Omega \sin w\Omega\right) + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$

Quaternion Interpolation



$$\mathbf{q} = q_W + iq_X + jq_Y + kq_Z = \begin{bmatrix} q_W \\ q_X \\ q_Y \\ q_Z \end{bmatrix} \qquad q_W^2 + q_X^2 + q_Y^2 + q_Z^2 = 1$$

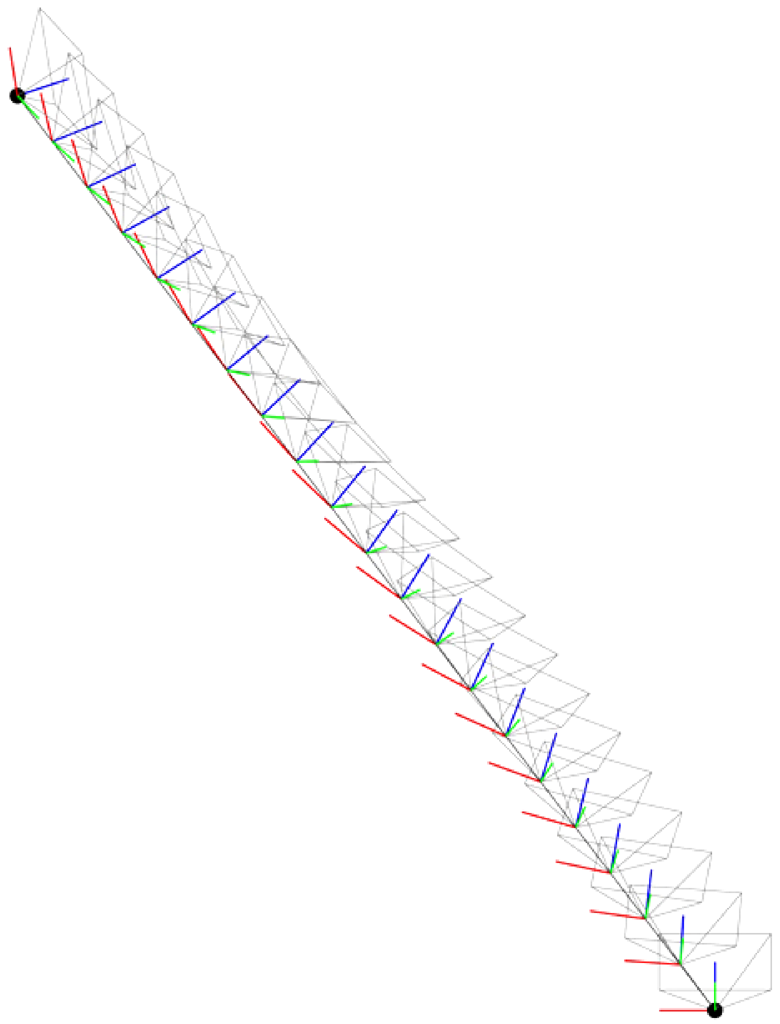
$$\cos\Omega = \mathbf{q}_1 \cdot \mathbf{q}_2$$

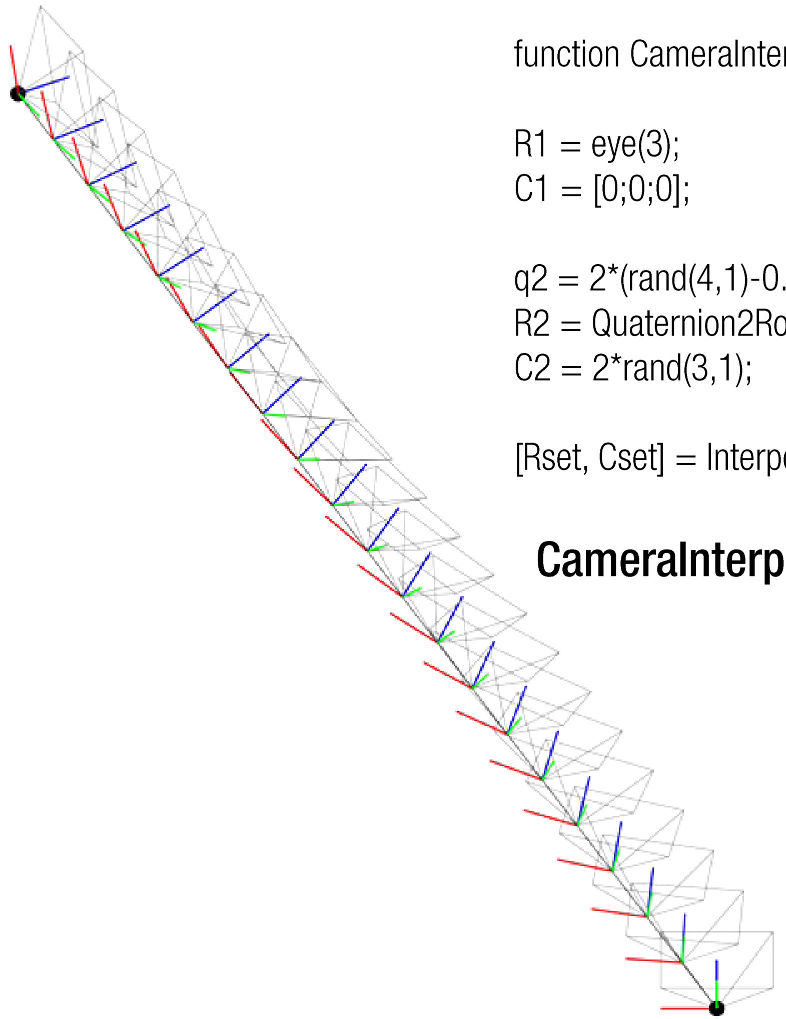


$$\mathbf{p} = \mathbf{q}_1 \cos w\Omega + \frac{\mathbf{q}_2 - \mathbf{q}_1 \cos\Omega}{\sin\Omega} \sin w\Omega$$

$$= \frac{\mathbf{q}_1 \left(\sin\Omega \cos w\Omega - \cos\Omega \sin w\Omega\right) + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$

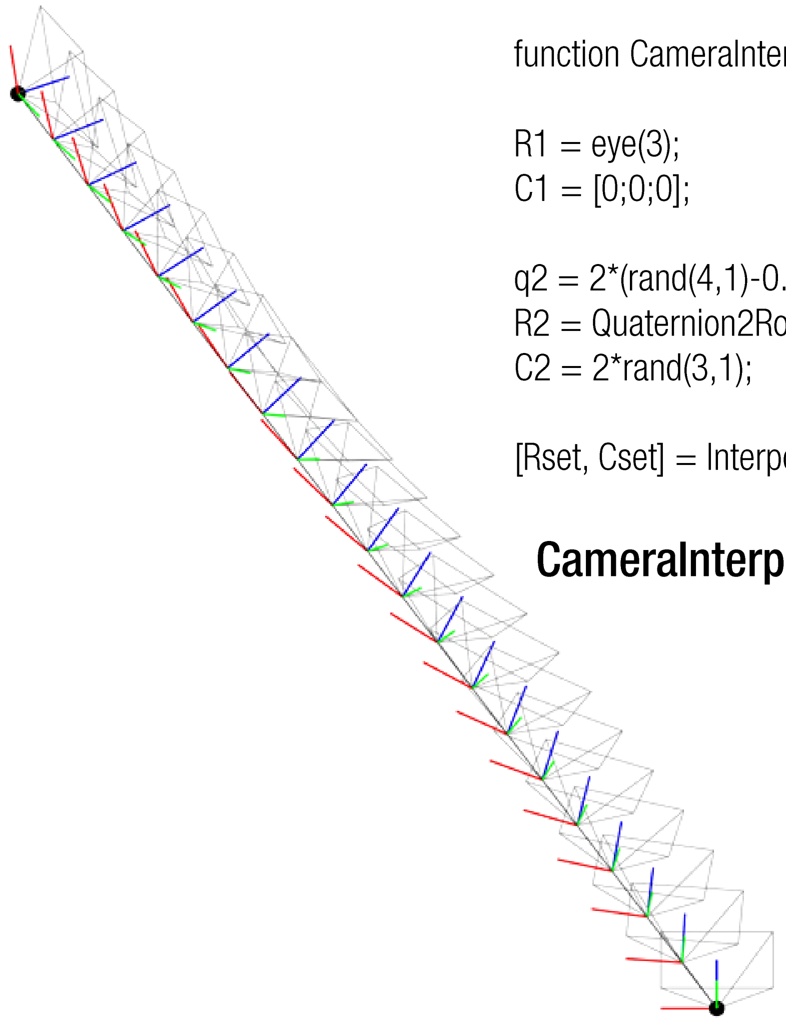
$$= \frac{\mathbf{q}_1 \sin(1-w)\Omega + \mathbf{q}_2 \sin w\Omega}{\sin\Omega}$$





```
function CameraInterpolation

R1 = eye(3);
C1 = [0;0;0];

q2 = 2*(rand(4,1)-0.5);
R2 = Quaternion2Rotation(q2);
C2 = 2*rand(3,1);

[Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, 20);
```

CameraInterpolation.m



```
function CameraInterpolation

R1 = eye(3);
C1 = [0;0;0];

q2 = 2*(rand(4,1)-0.5);
R2 = Quaternion2Rotation(q2);
C2 = 2*rand(3,1);

[Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, 20);
```

CameraInterpolation.m

```
function [Rset, Cset] = InterpolateCoordinate(R1, C1, R2, C2, n)

Cx = linspace(C1(1), C2(1), n+1);
Cy = linspace(C1(2), C2(2), n+1);
Cz = linspace(C1(3), C2(3), n+1);

Cset = [Cx; Cy; Cz];

w = 0 : 1/n : 1;
q1 = Rotation2Quaternion(R1);
q2 = Rotation2Quaternion(R2);

omega = acos(q1'*q2);
for i = 1 : length(w)
    q = sin(omega*(1-w(i)))/sin(omega) * q1 + sin(omega*w(i))/sin(omega) * q2;
    Rset{i} = Quaternion2Rotation(q);
end
```

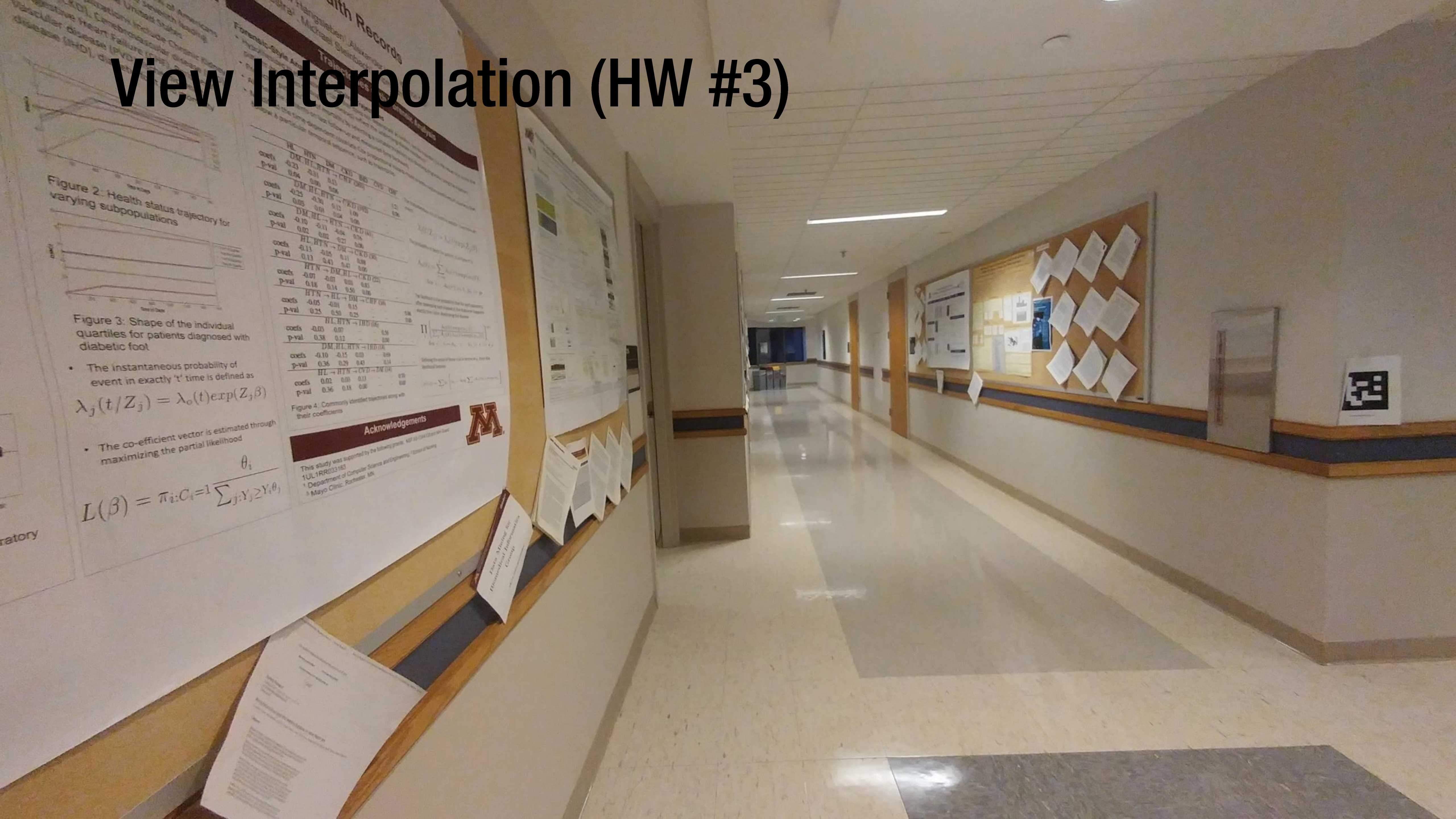
InterpolateCoordinate.m

View Interpolation

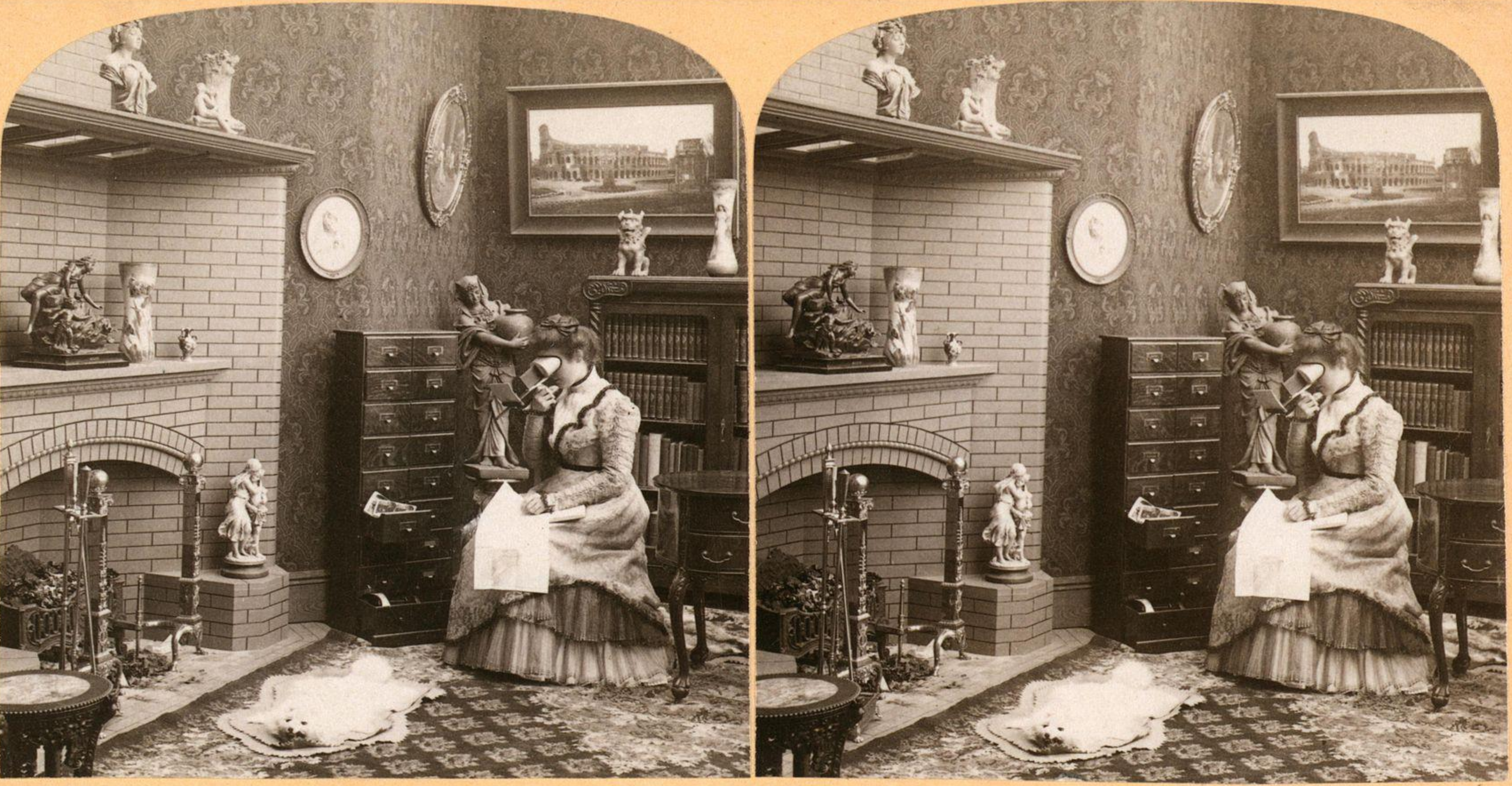
Looking left

Looking right

View Interpolation (HW #3)

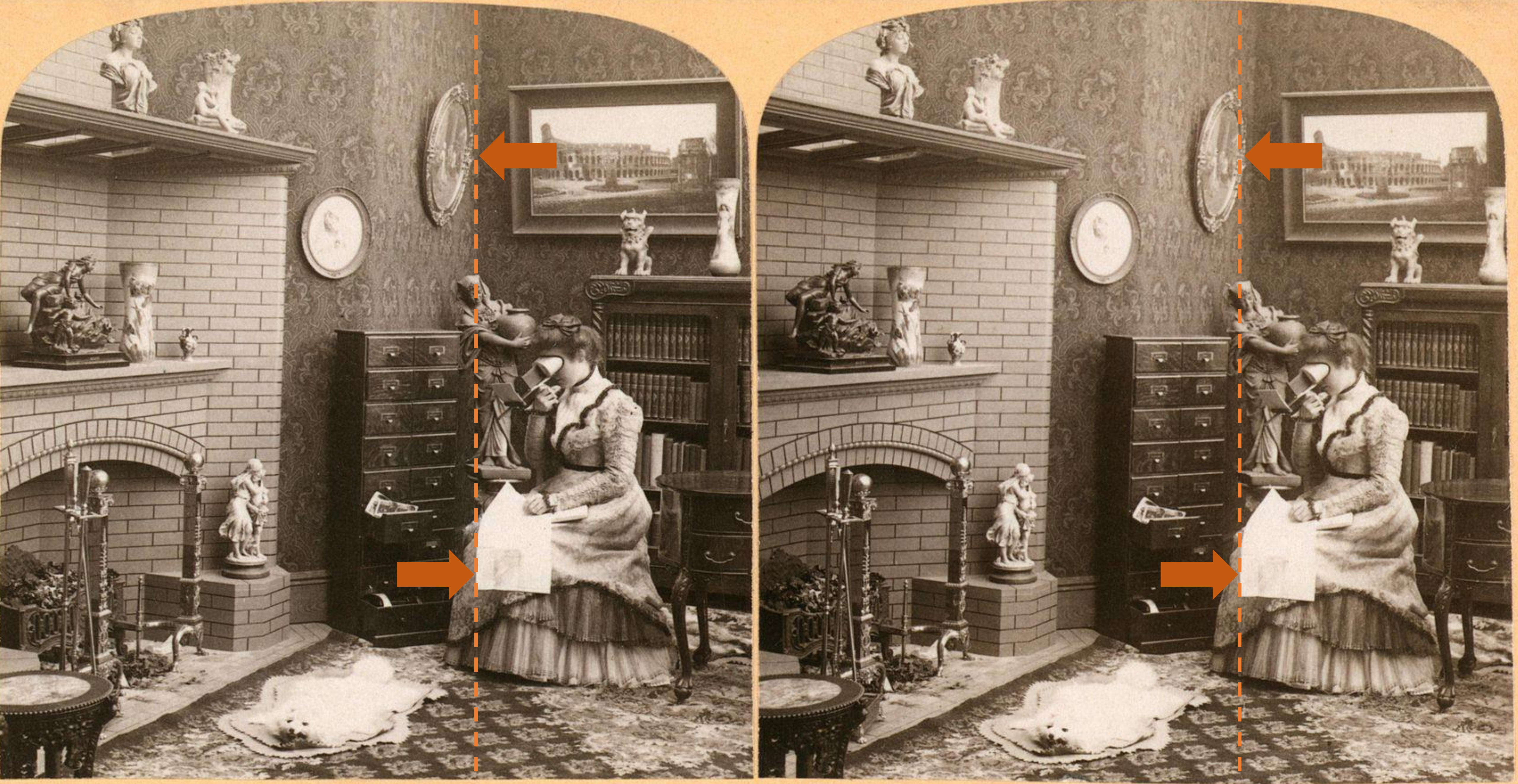


Two View (Epipolar) Geometry

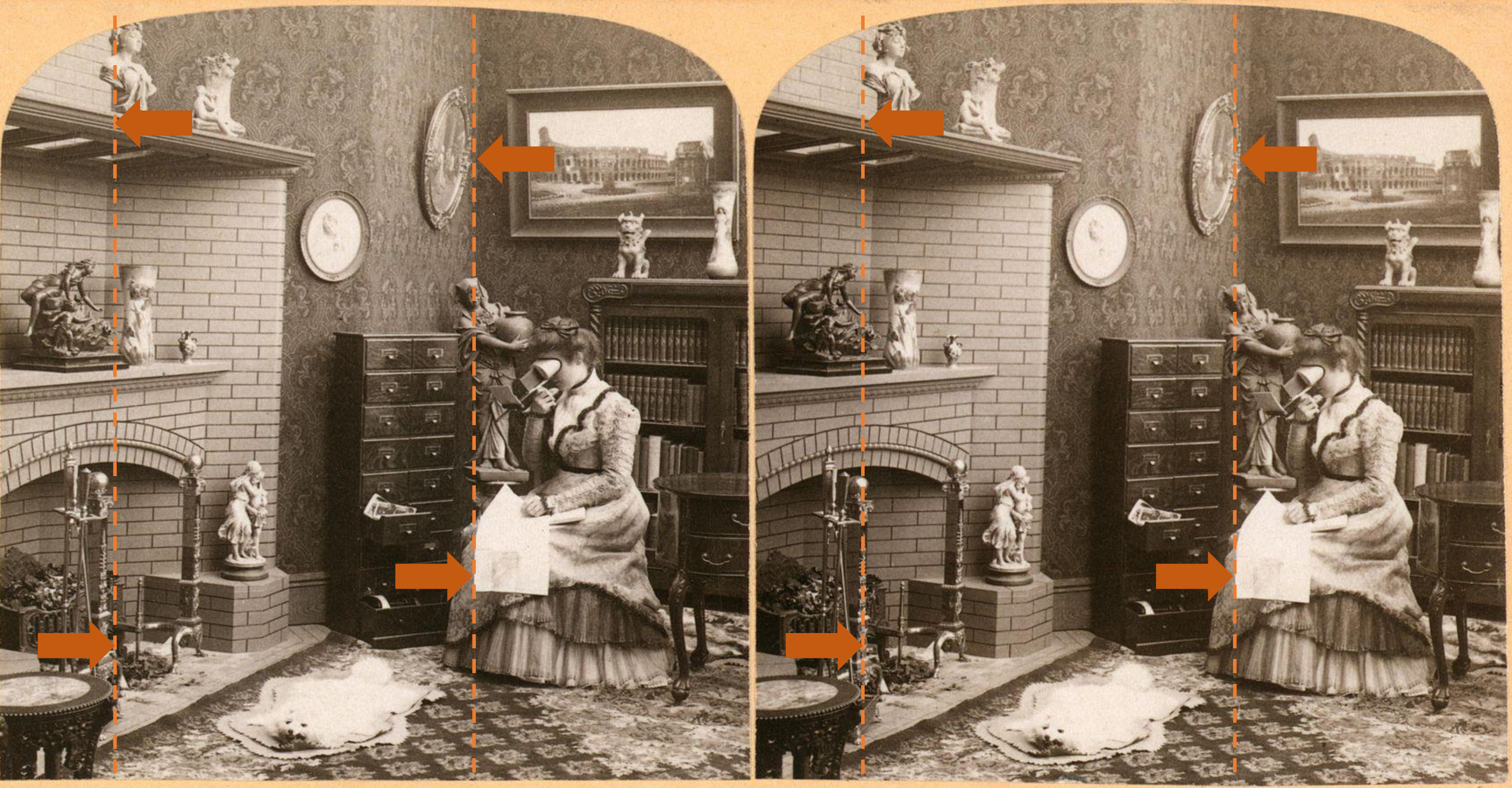


The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.
Copyright 1901 by Underwood & Underwood.



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library.

Circa 1900

Stereo: Holmes Stereoscope


HERO3
mode
HERO3


VITA COCO
Naked
3.99

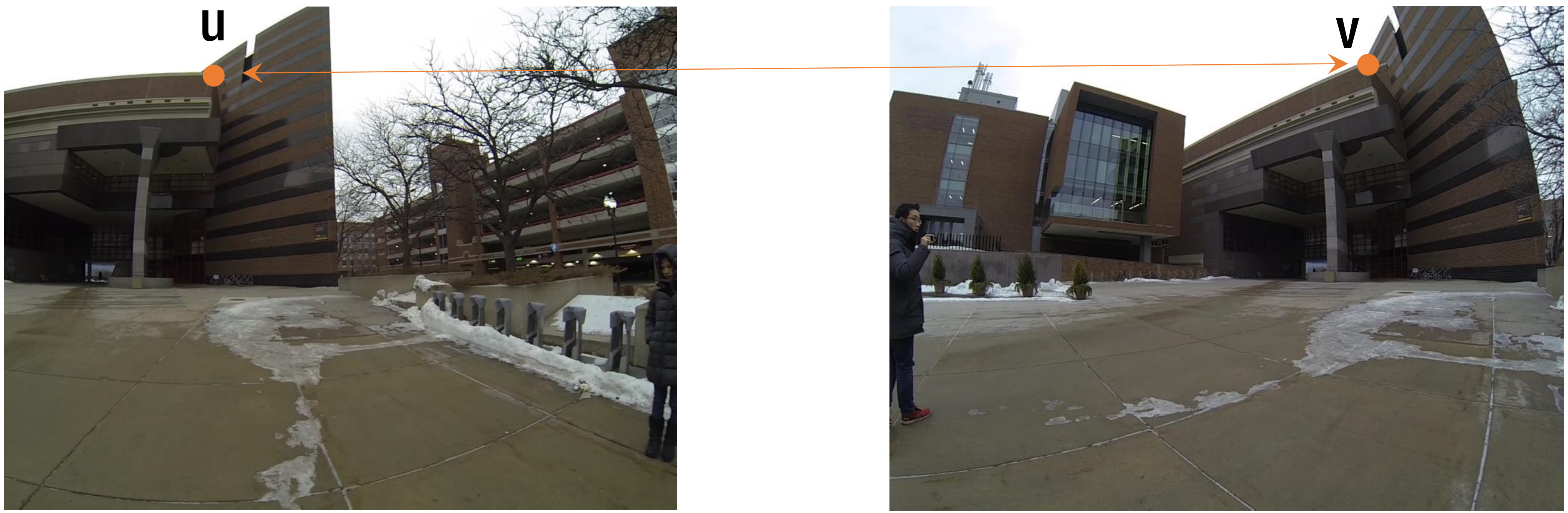
Naked
3.99


VITA
Naked
3.99

Left image (Bob)

Right image (Alice)

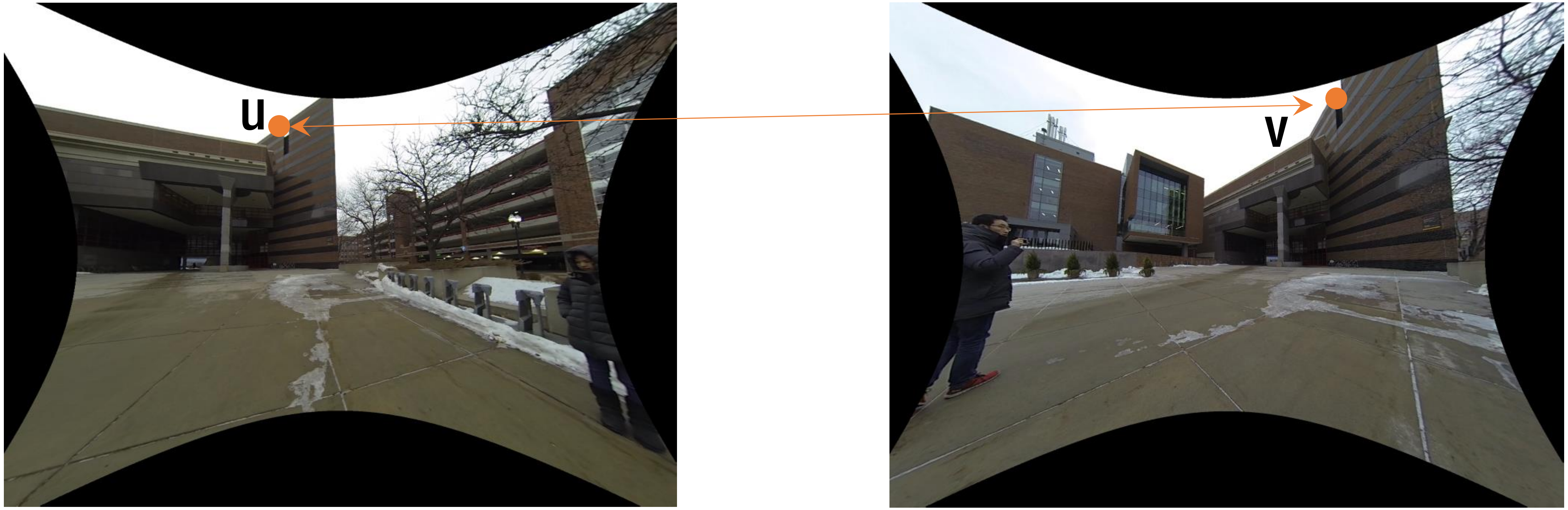
2D Correspondence



Left image (Bob)

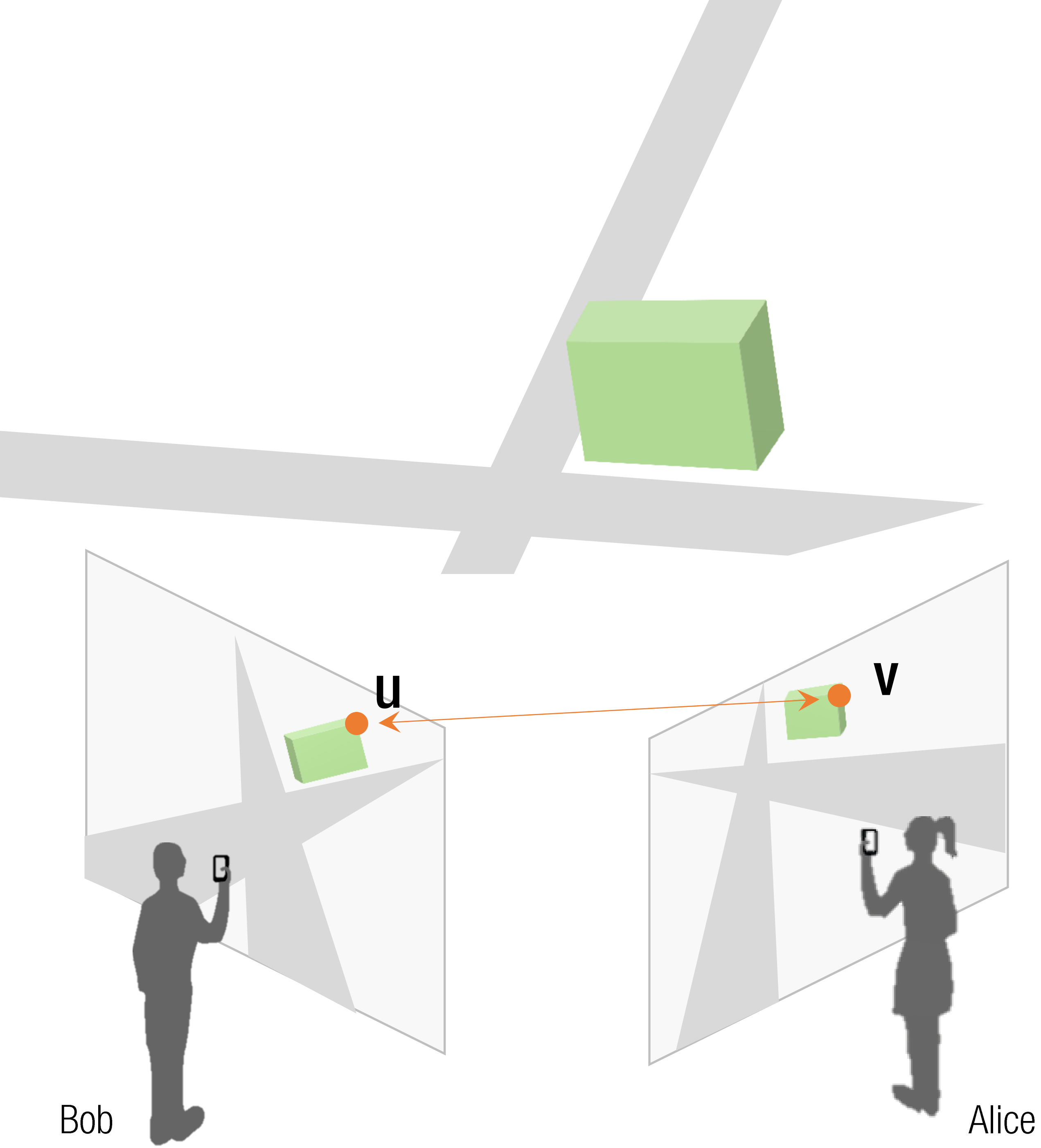
Right image (Alice)

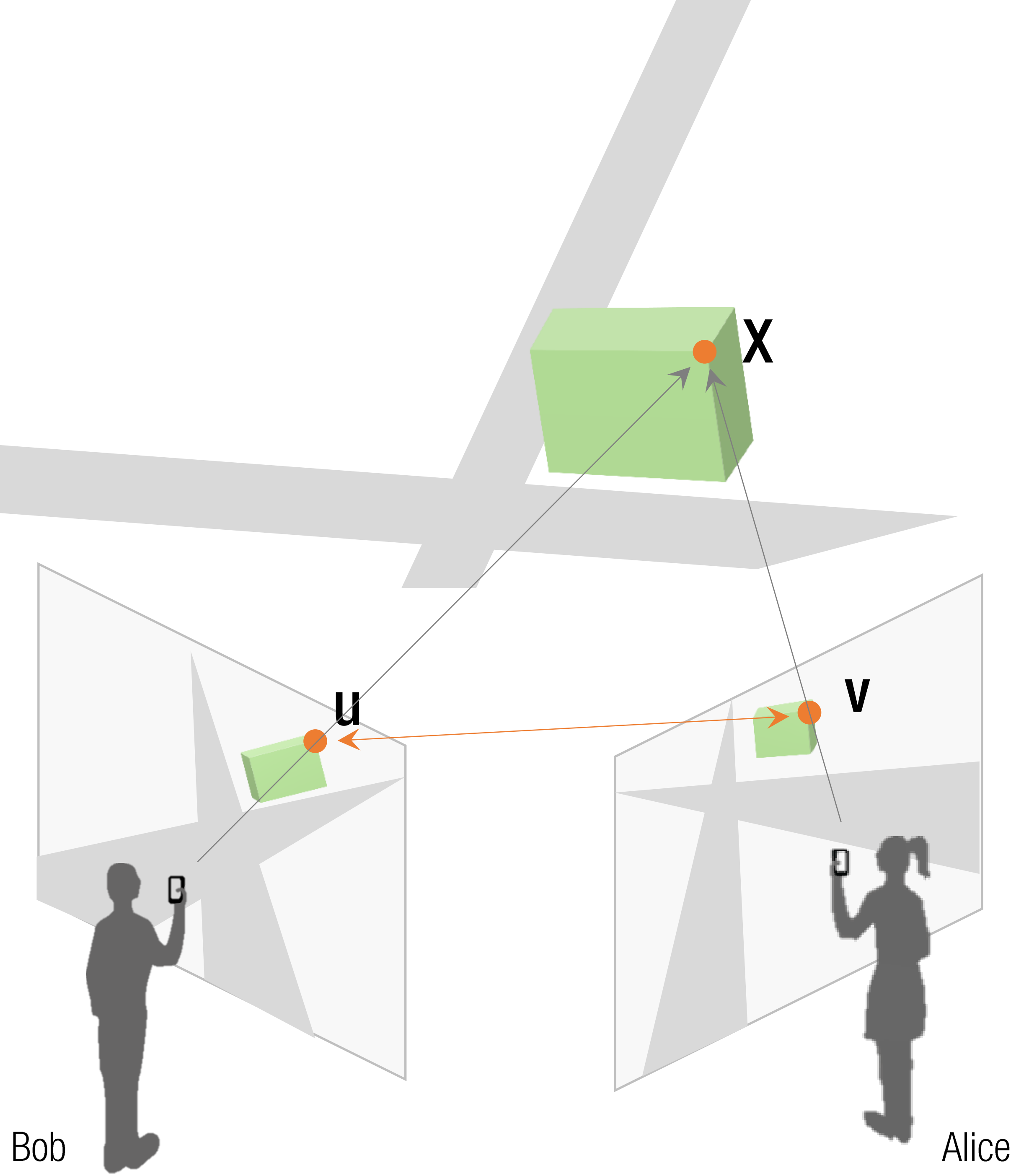
2D Correspondence

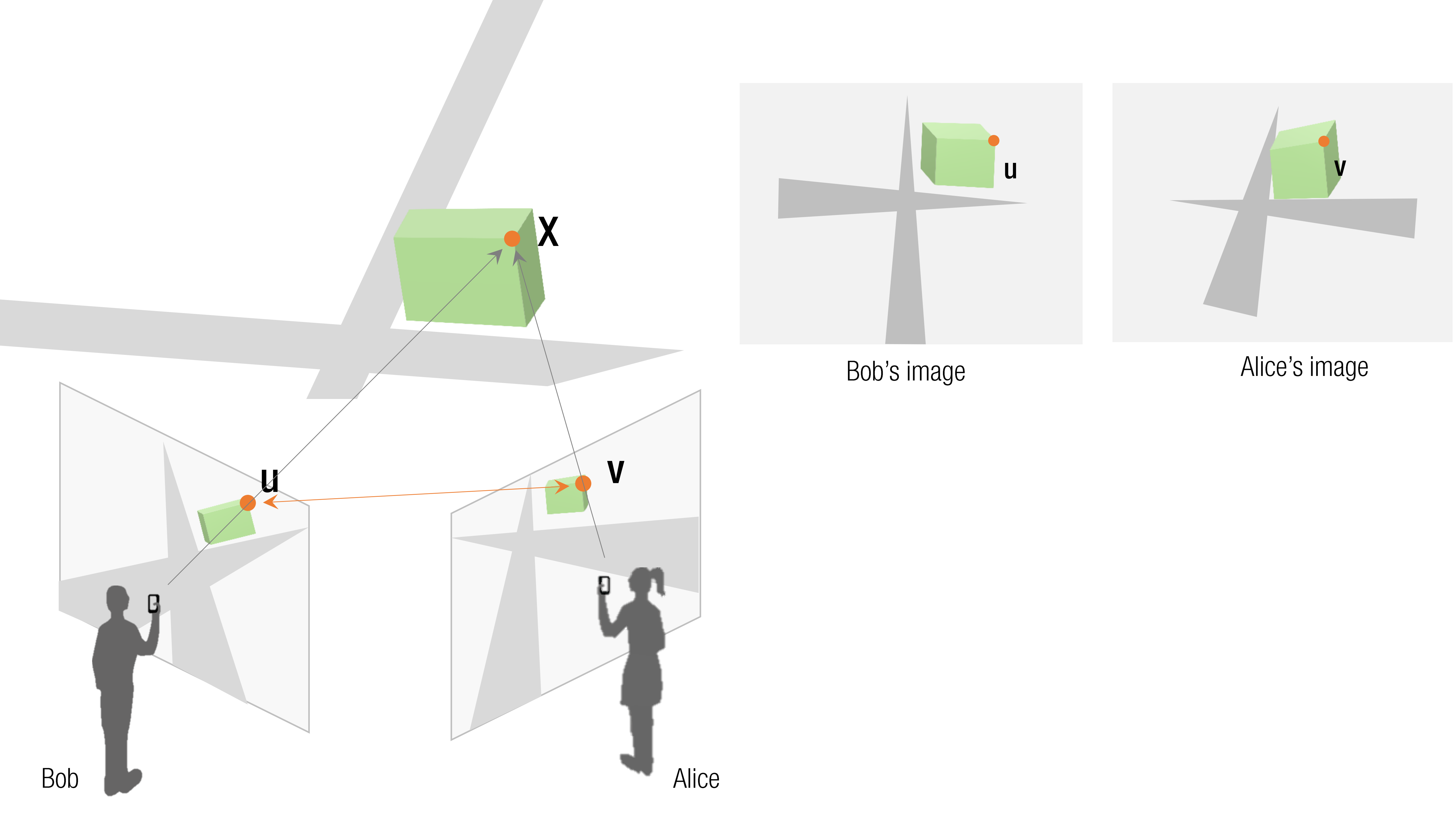


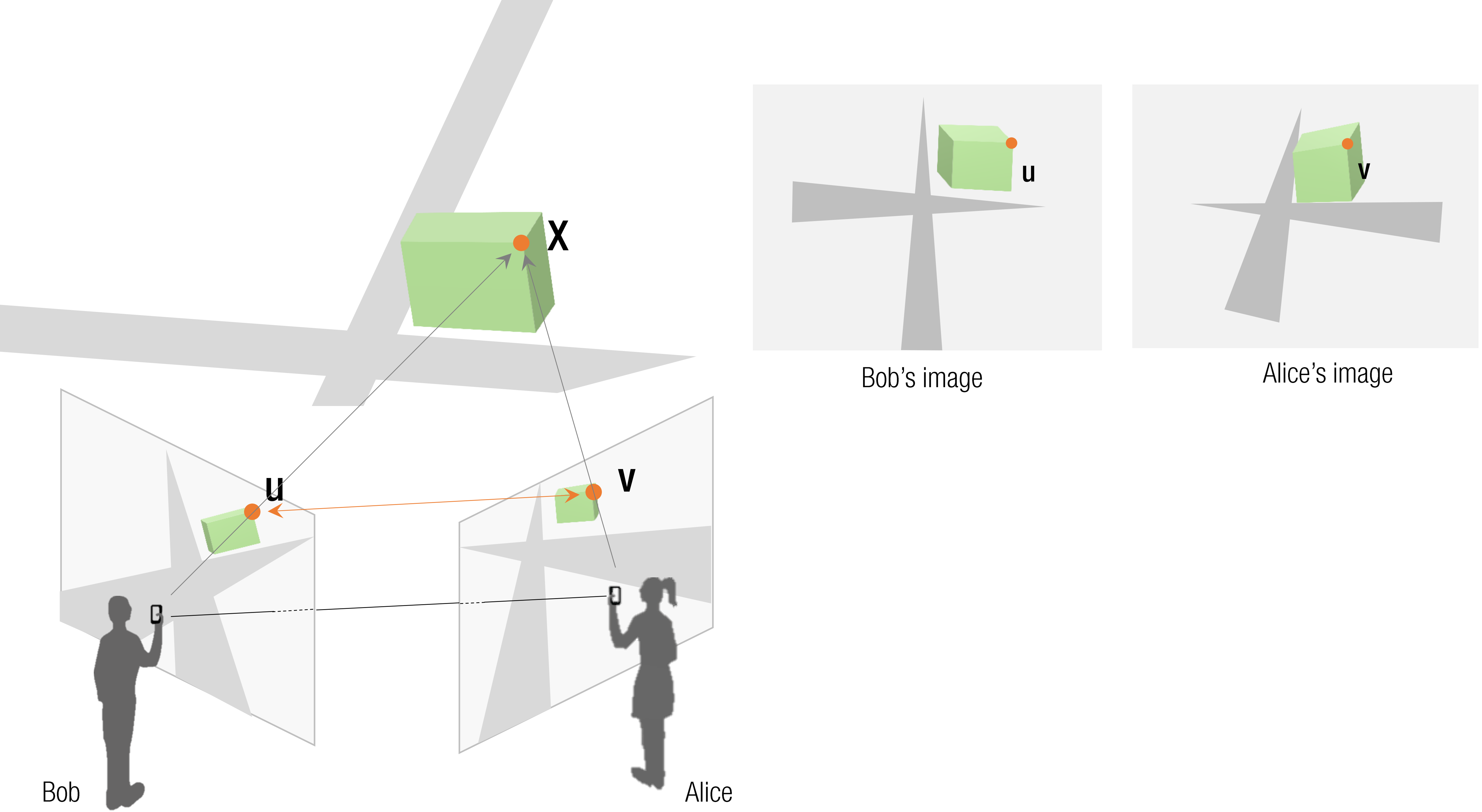
Left image (Bob)

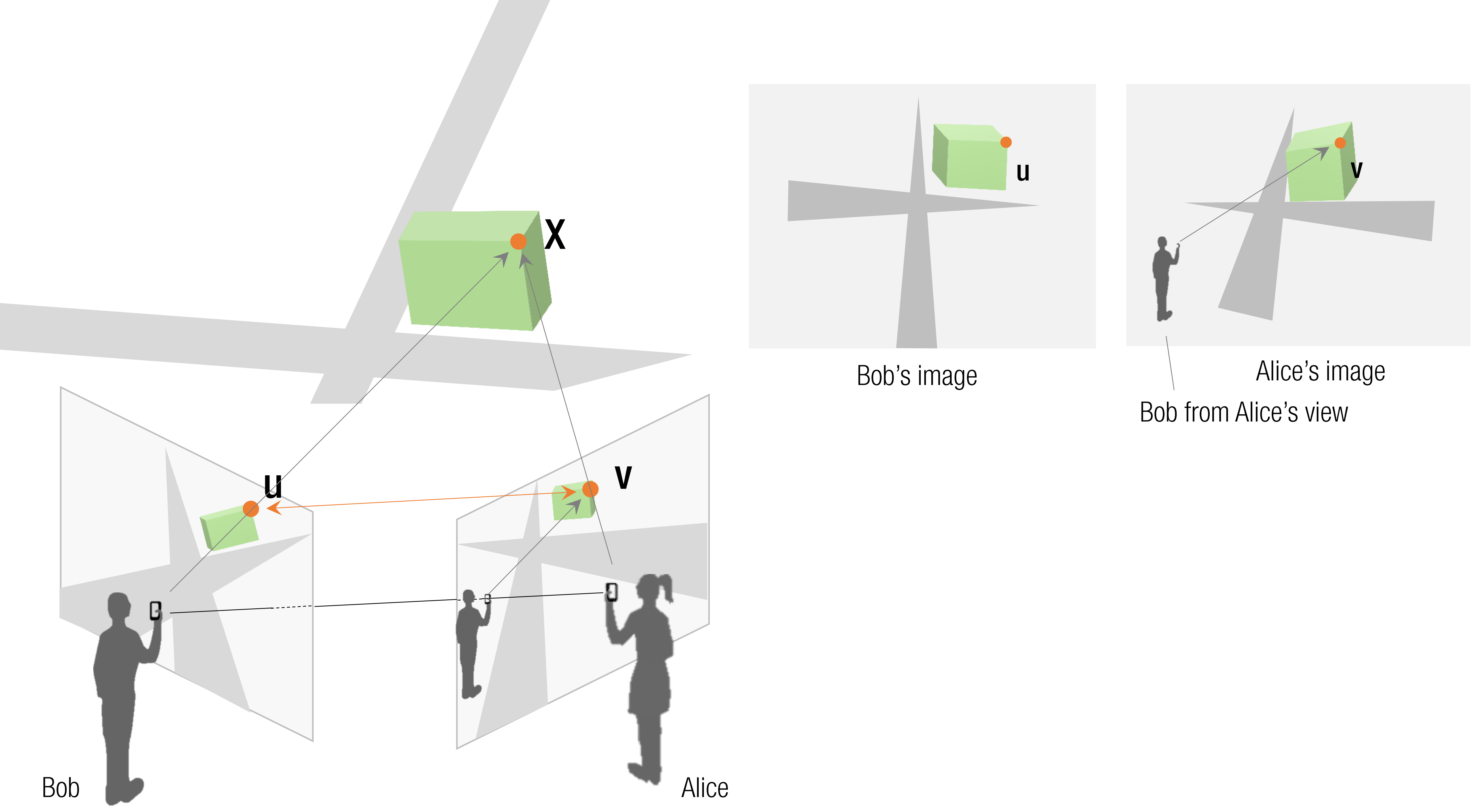
Right image (Alice)

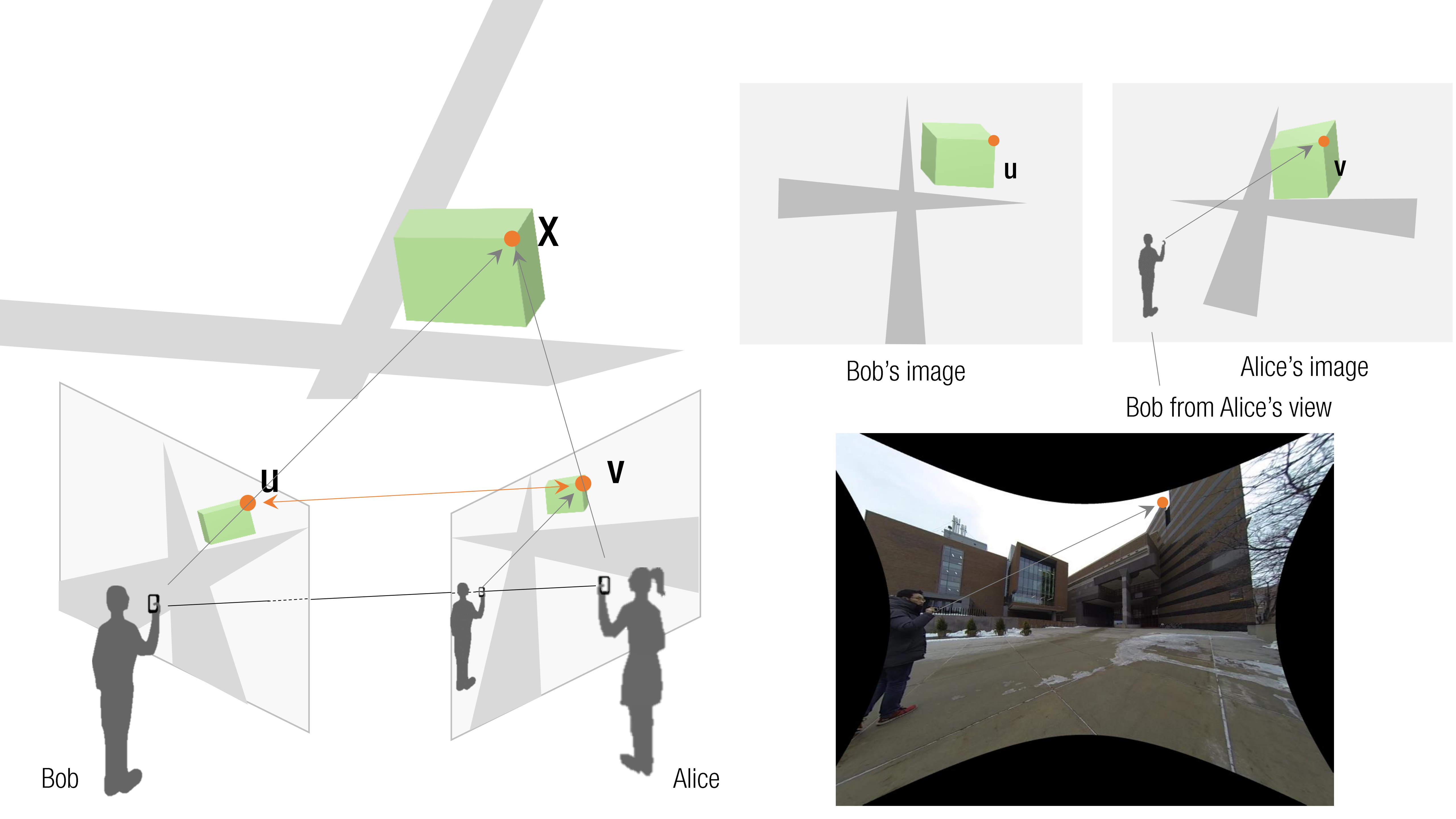

u
v
Bob
Alice


X
u
v
Bob
Alice

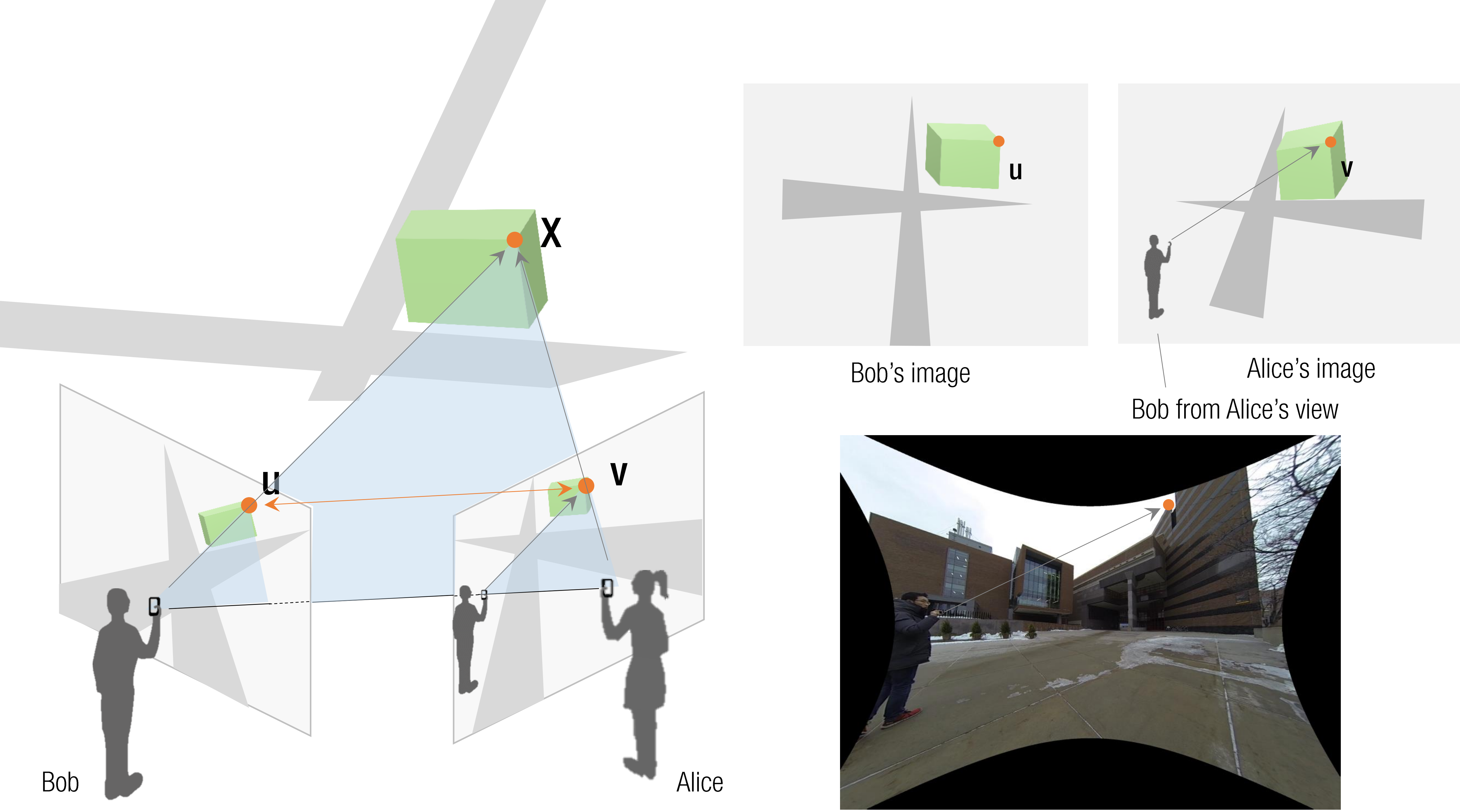

X
u
v
Bob
Alice
u
v
Bob's image
Alice's image

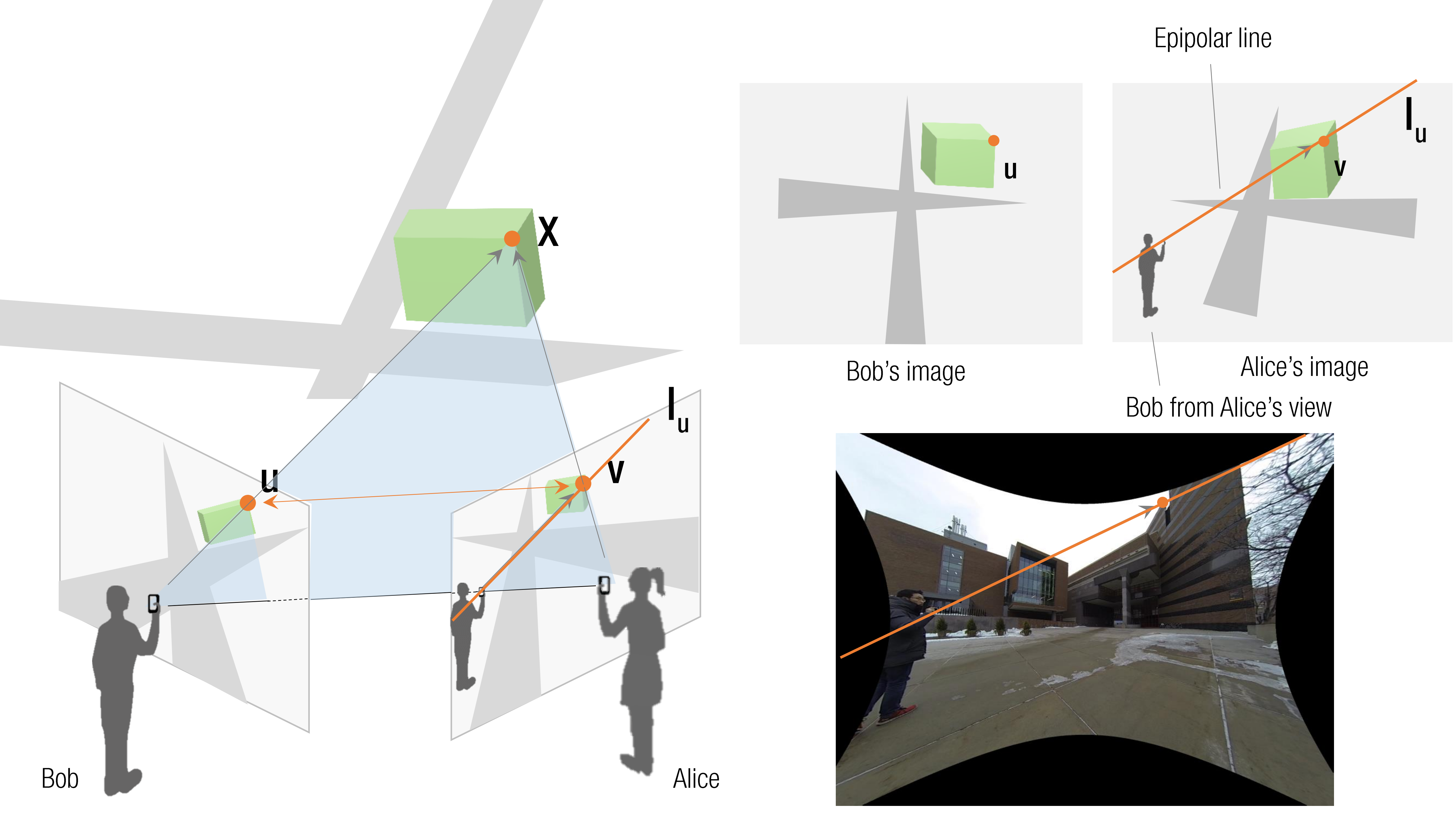

X
u
v
Bob
Alice
u
v
Bob's image
Alice's image


X
u
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view

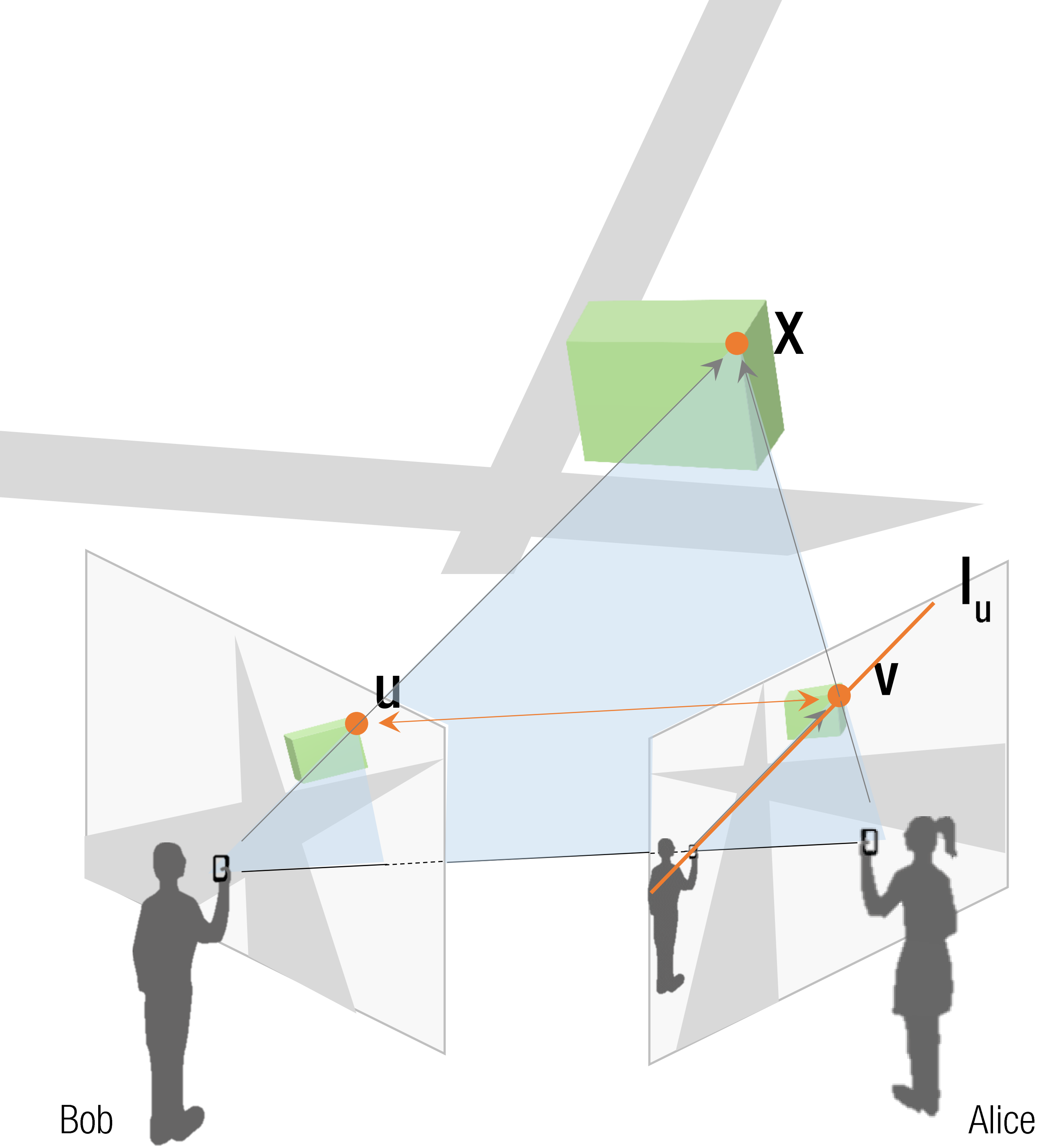


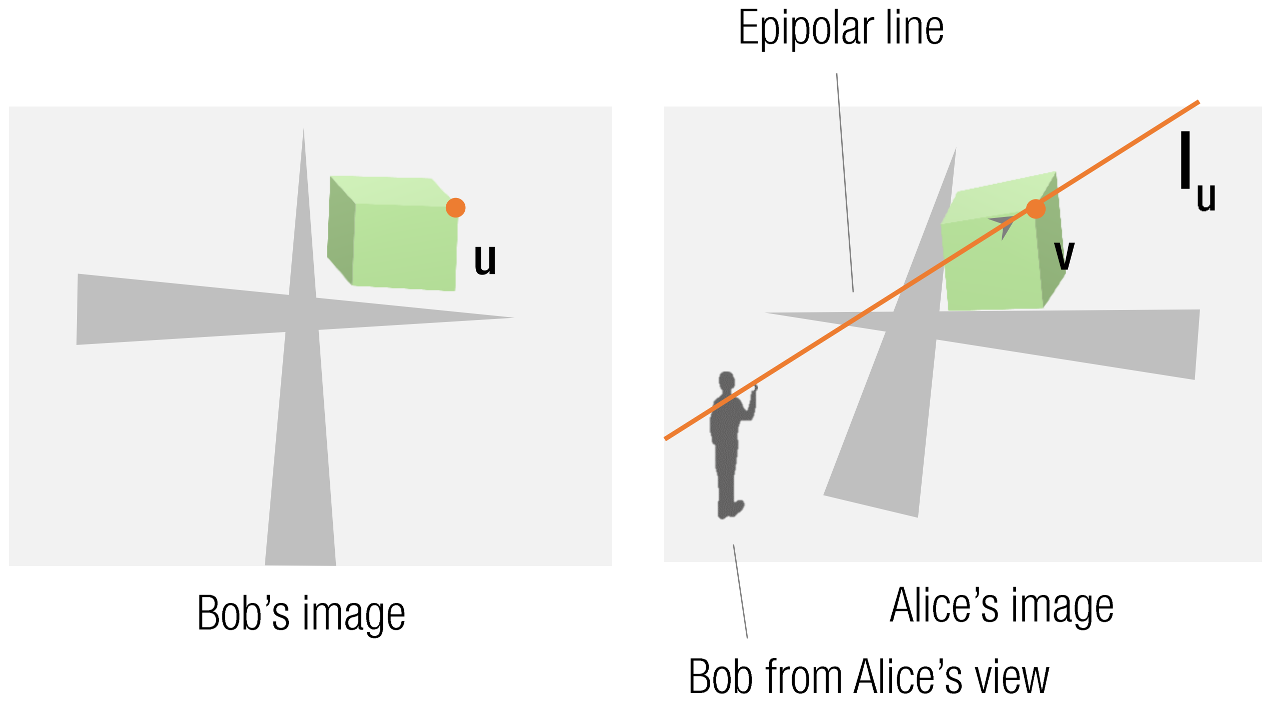
X
u
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view


X
u
v
Bob
Alice
u
Bob's image
v
Alice's image
Bob from Alice's view



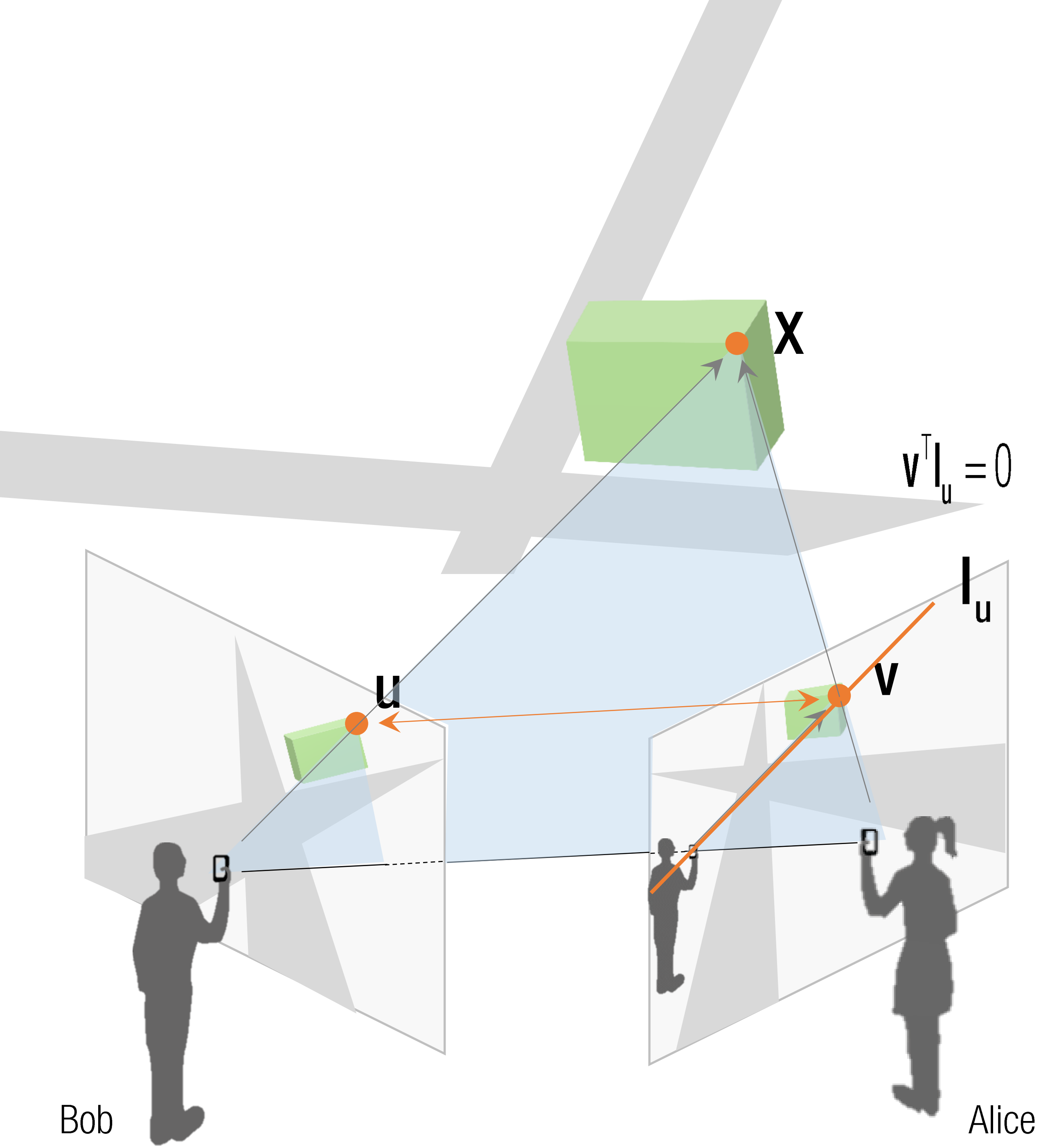
X
u
v
lu
Bob
Alice
Epipolar line
u
v
lu
Bob’s image
Alice’s image
Bob from Alice’s view

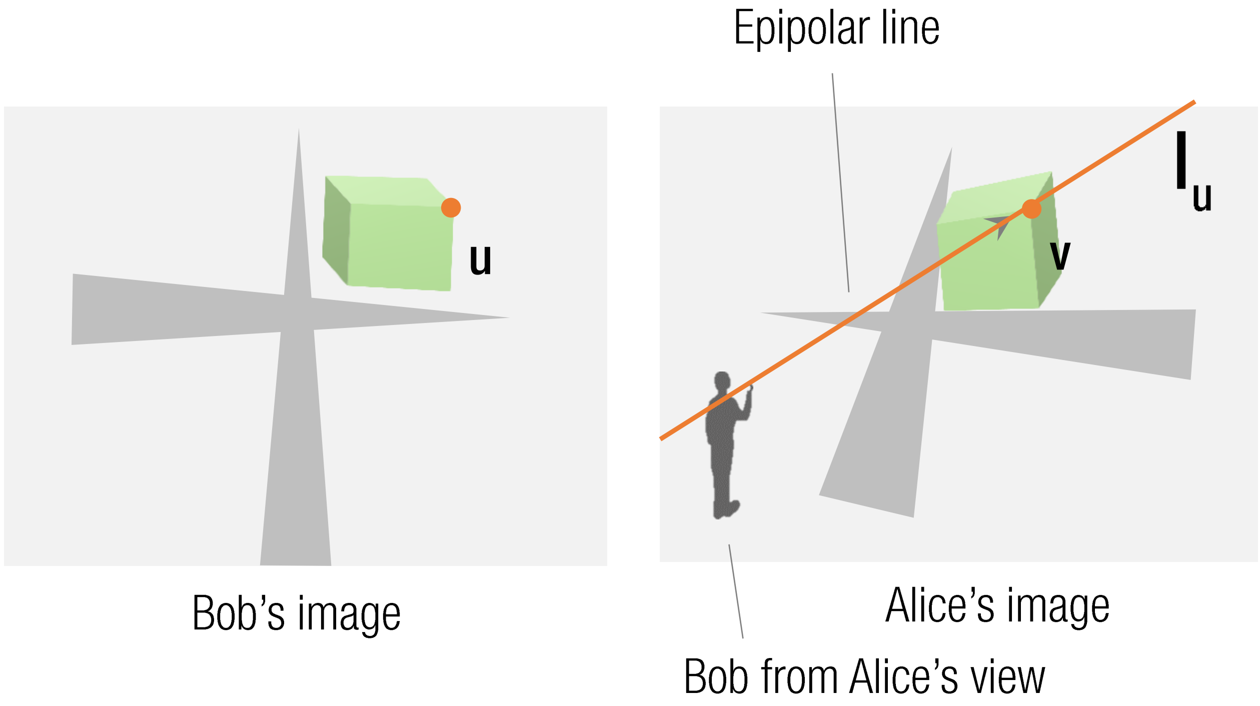




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

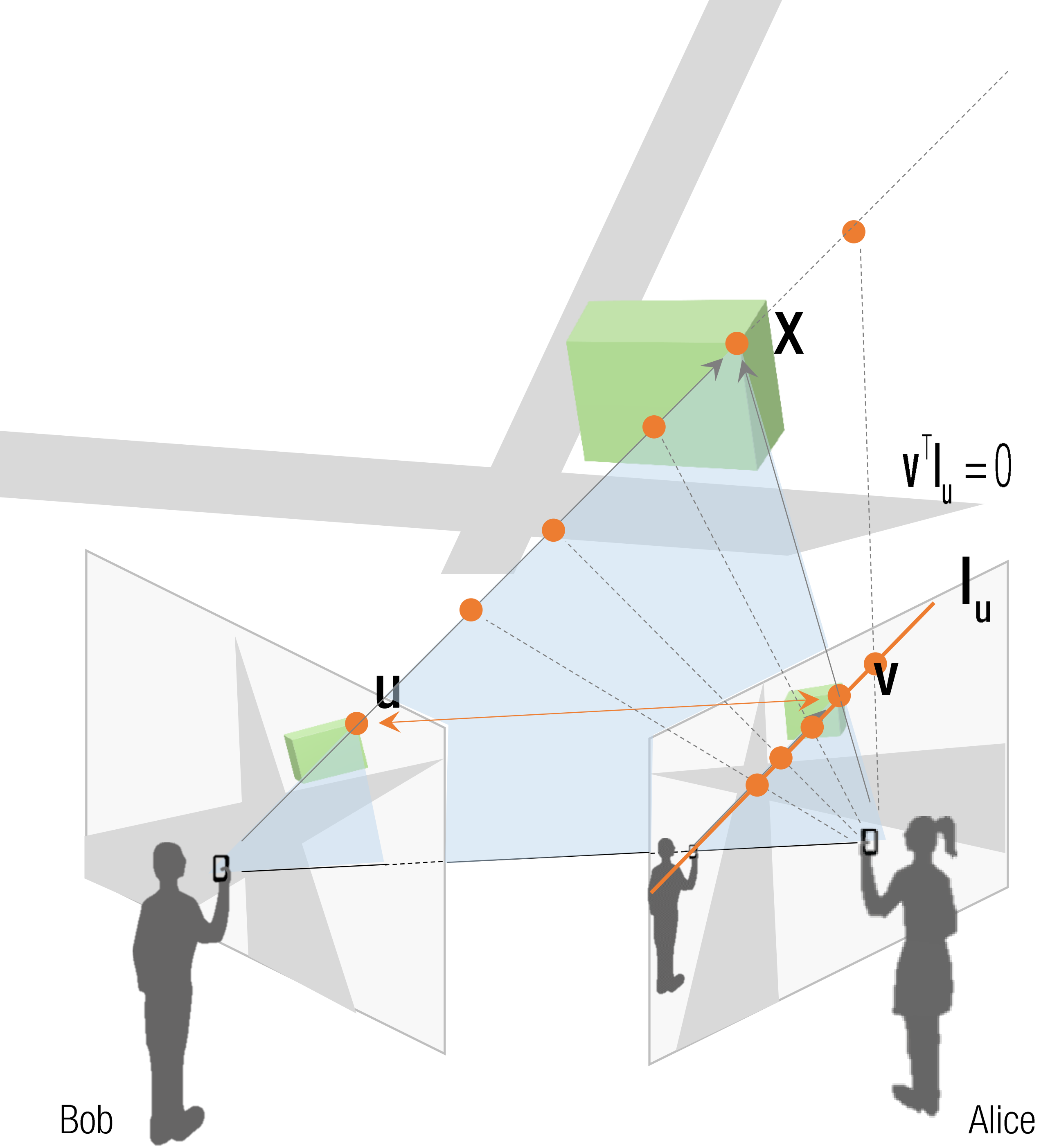


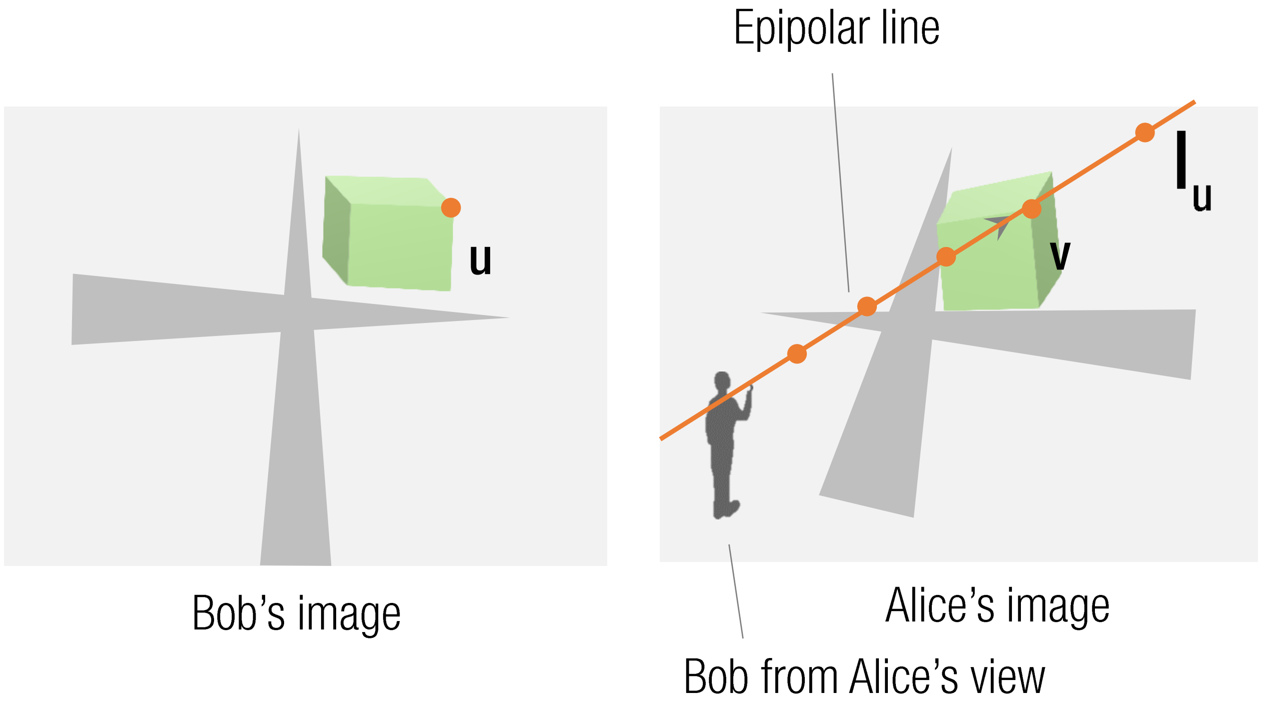


Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$



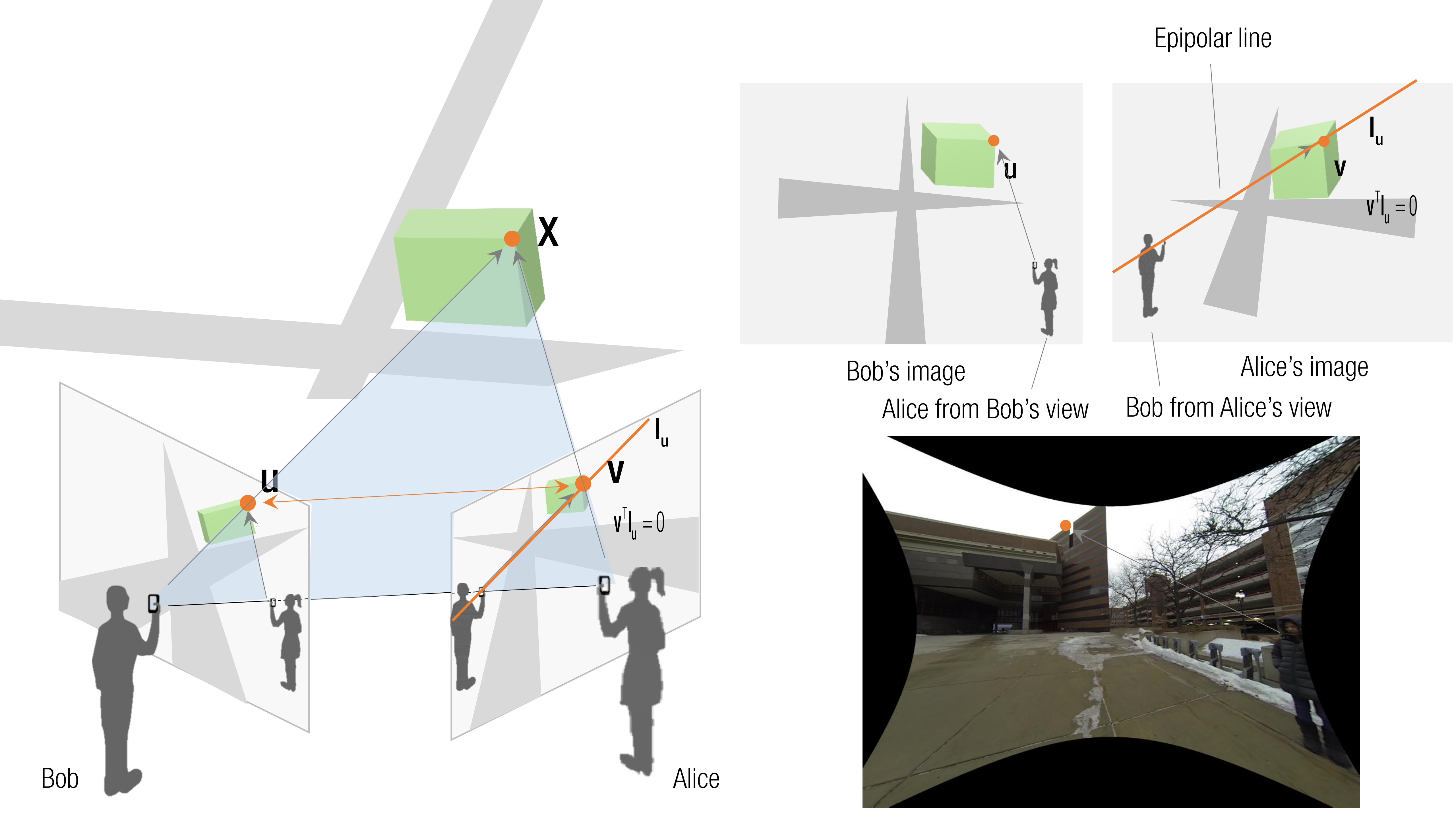


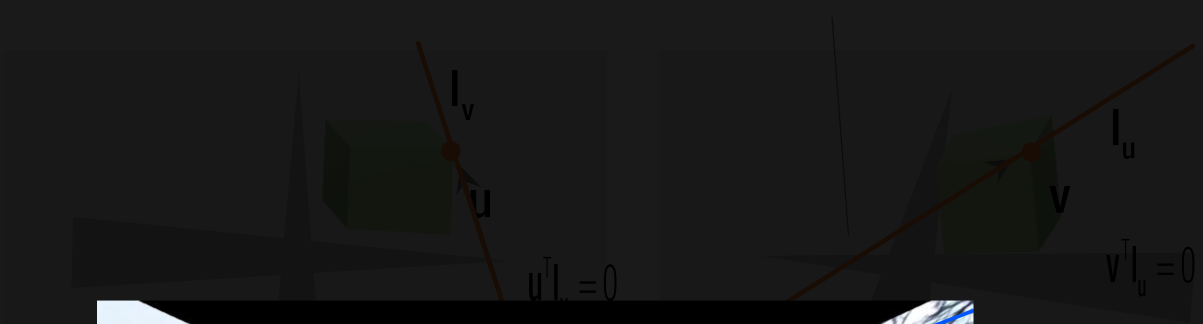
Epipolar constraint between two images:

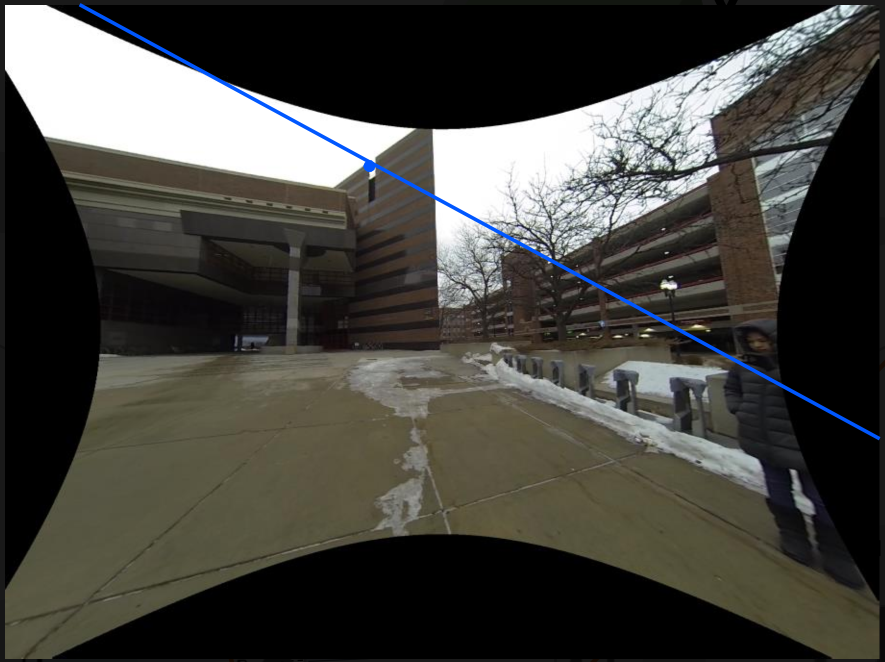
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.

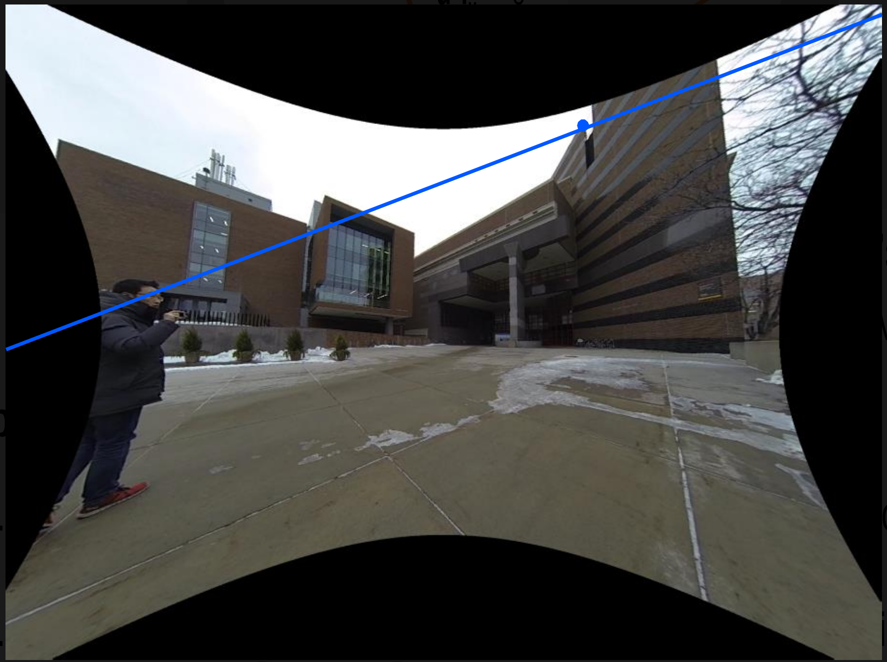
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$

3. Any point along the epipolar line can be a candidate of correspondences.


Epipolar line
X
u
v
l_u
$v^T l_u = 0$
Bob's image
Alice from Bob's view
Alice's image
Bob from Alice's view
Bob
Alice



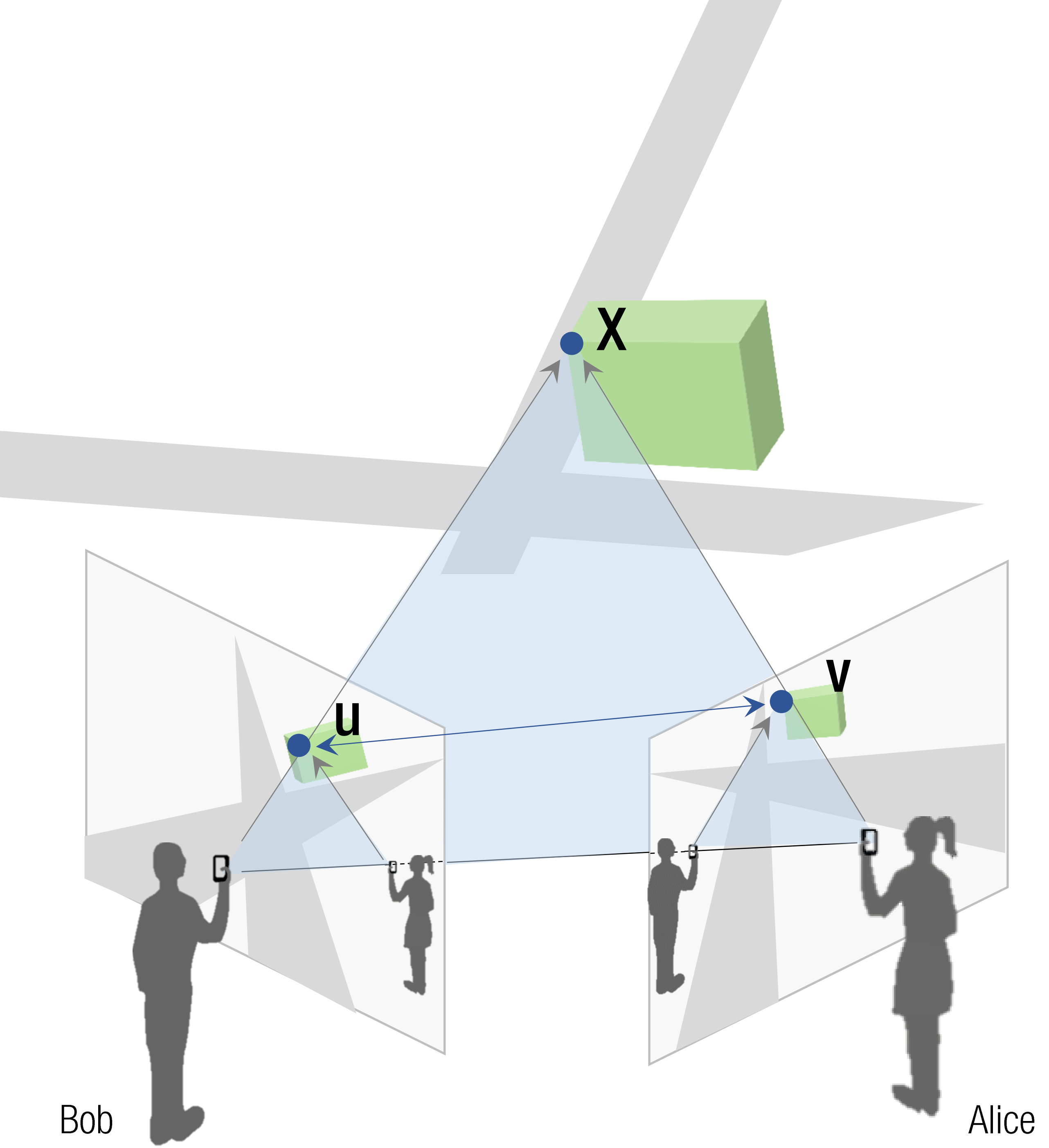


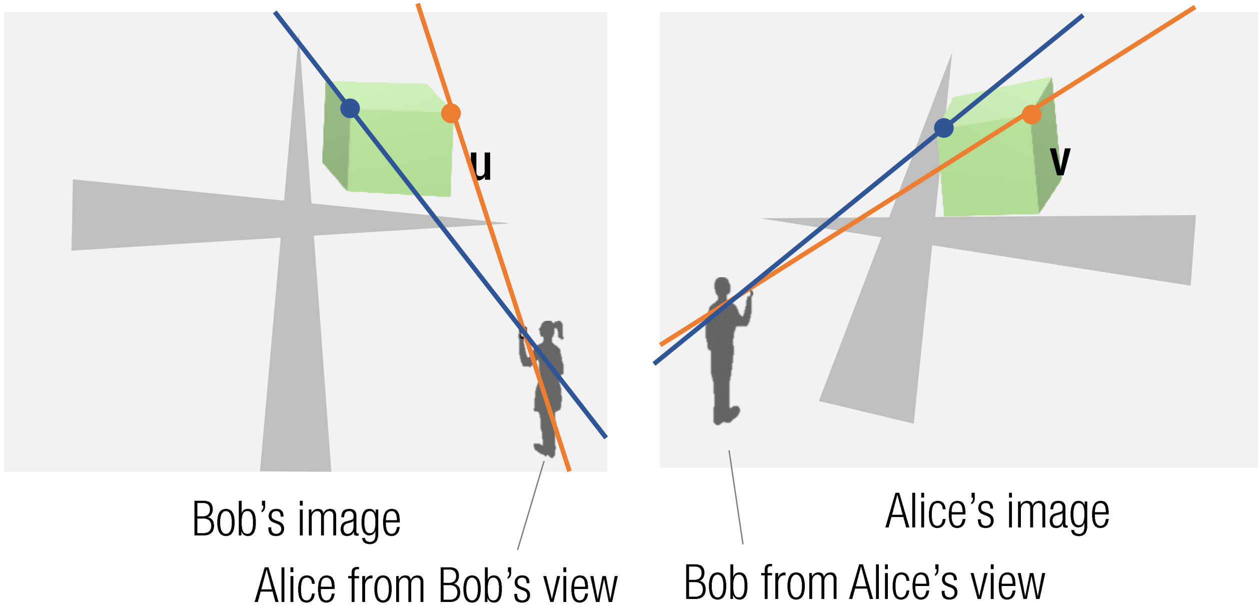


Ep…

…s image
…view

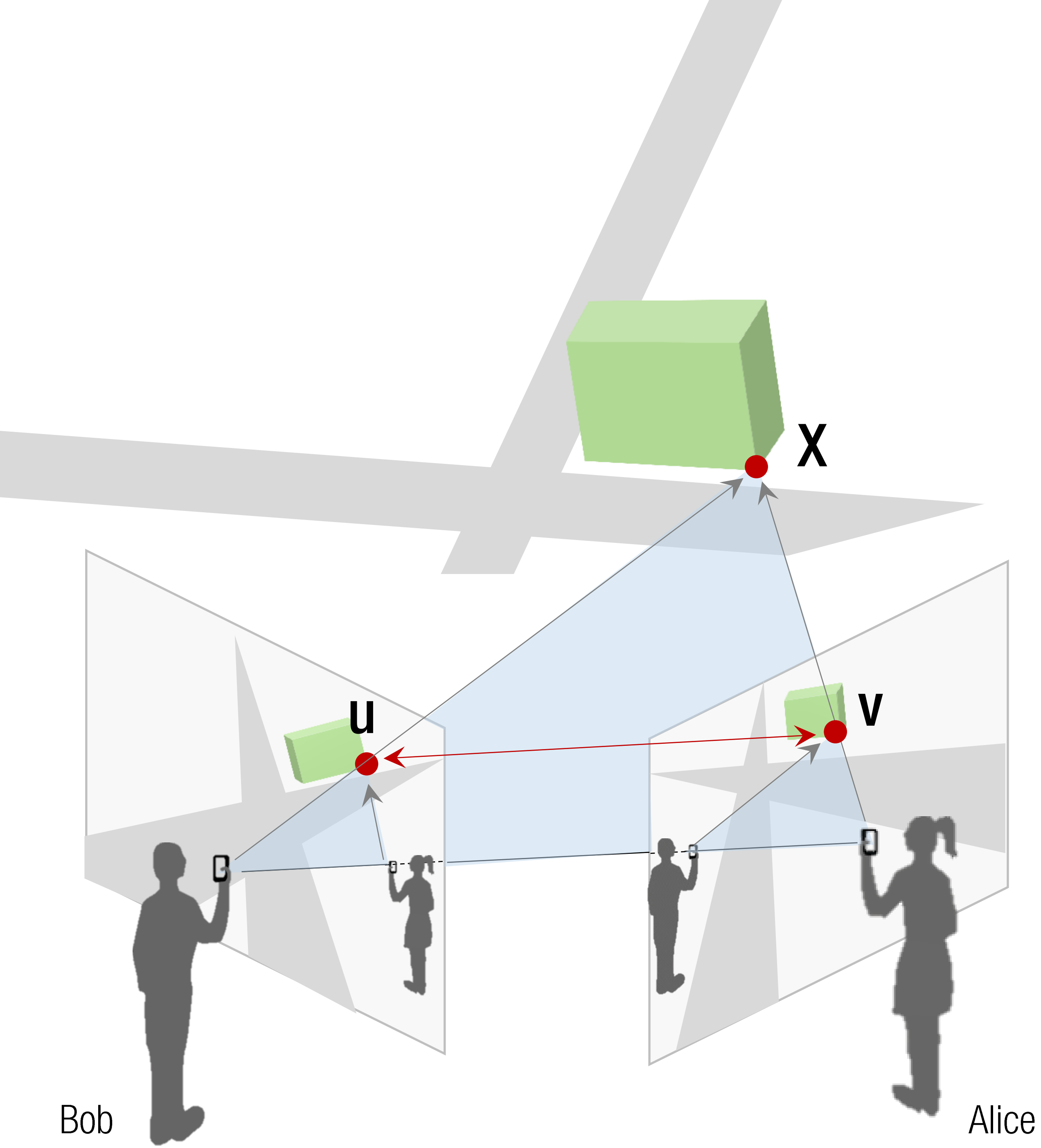
1. … …olar line in
2. … in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$ $\mathbf{u}^T\mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

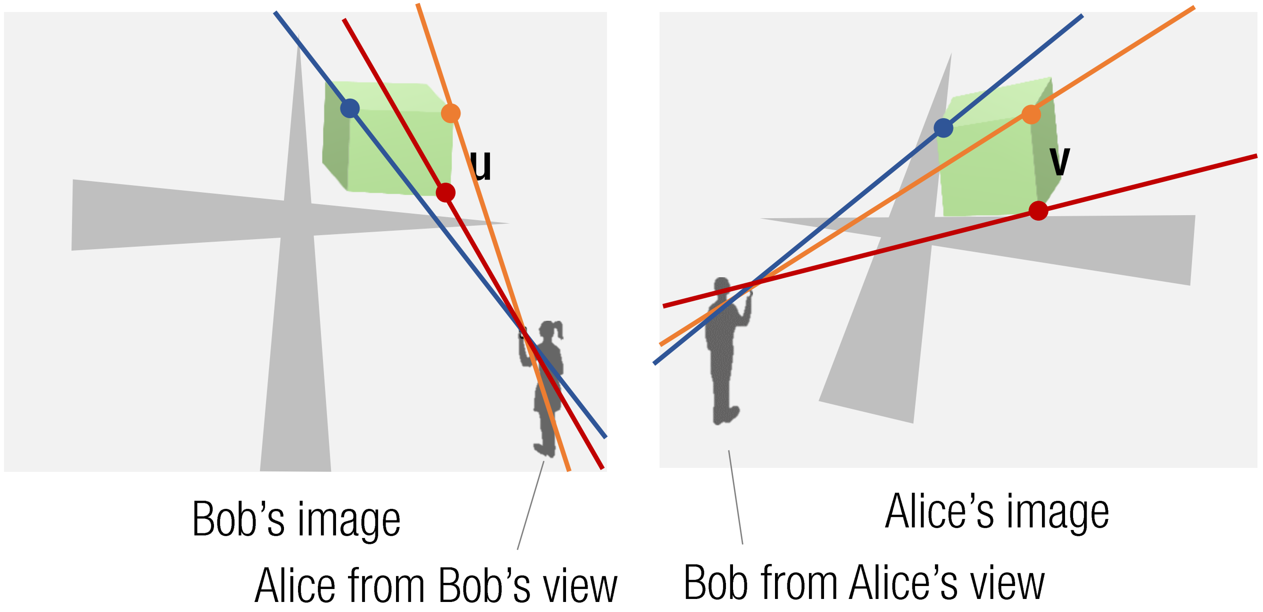




Epipolar constraint between two images:

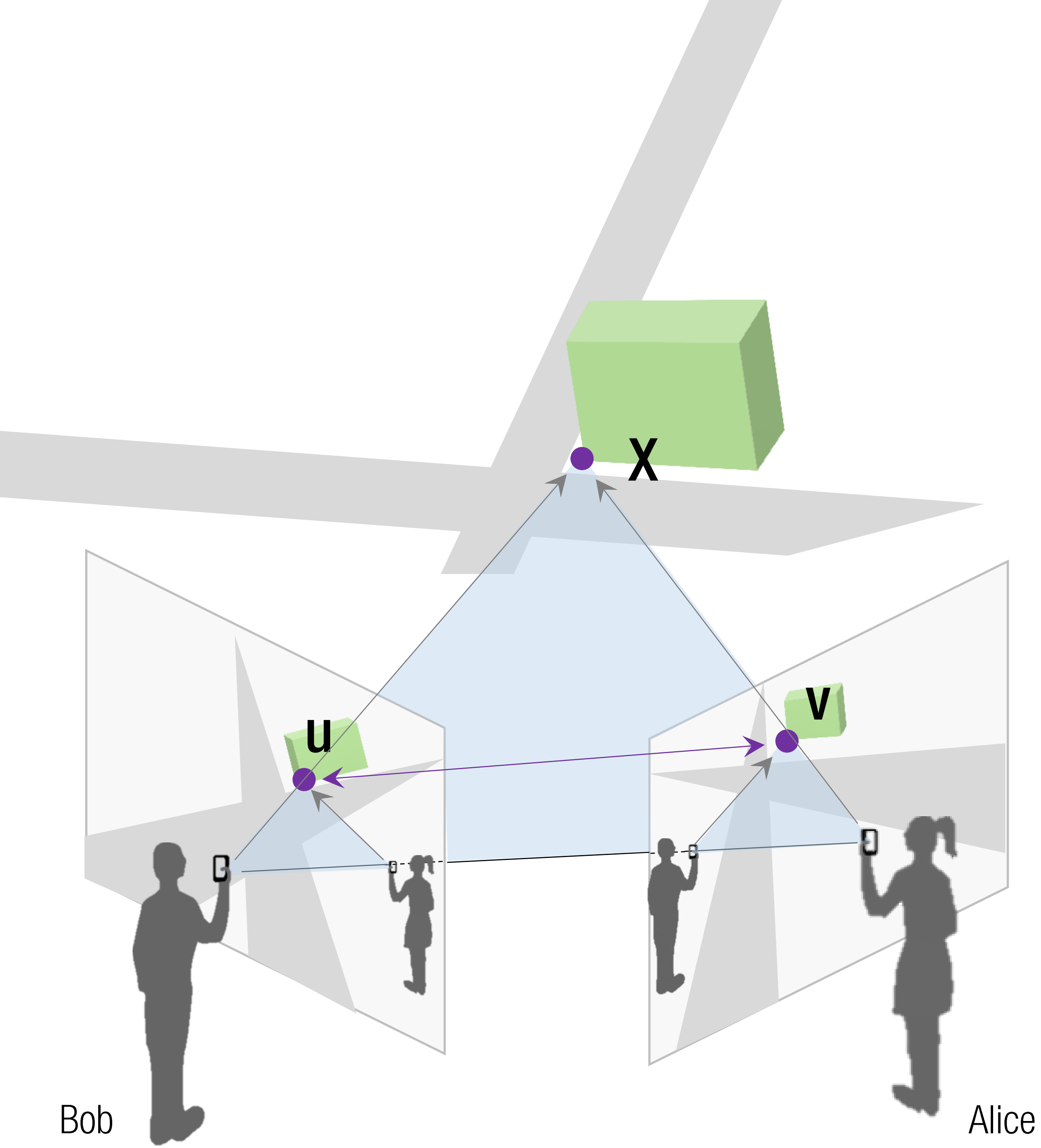
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

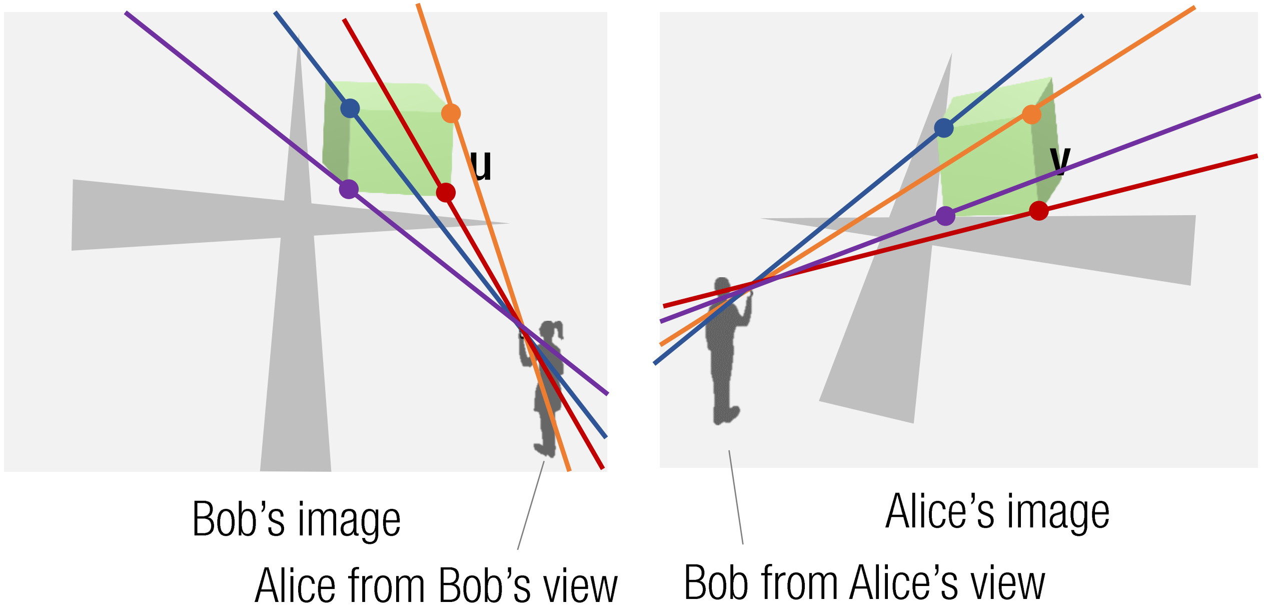




Epipolar constraint between two images:

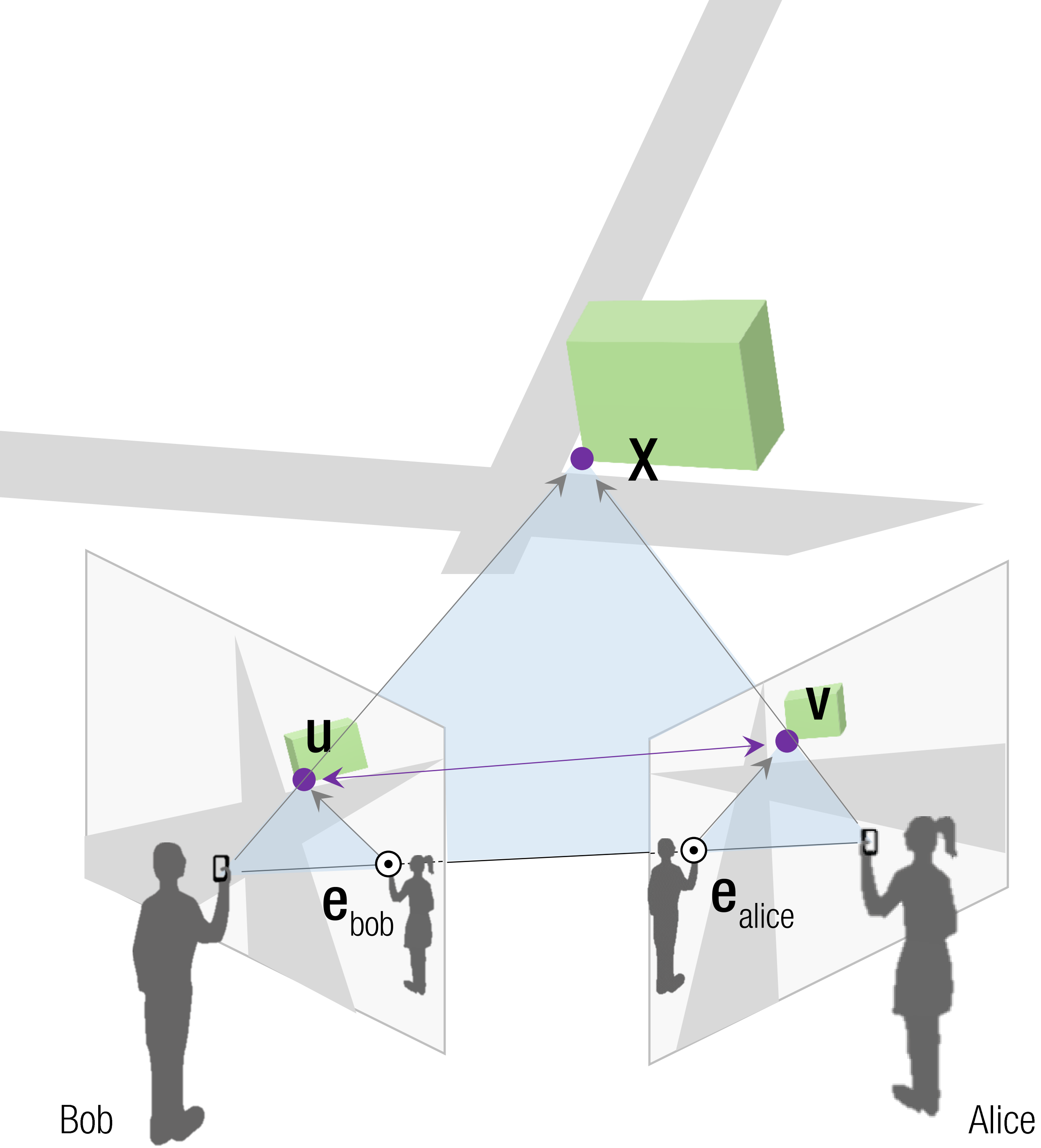
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

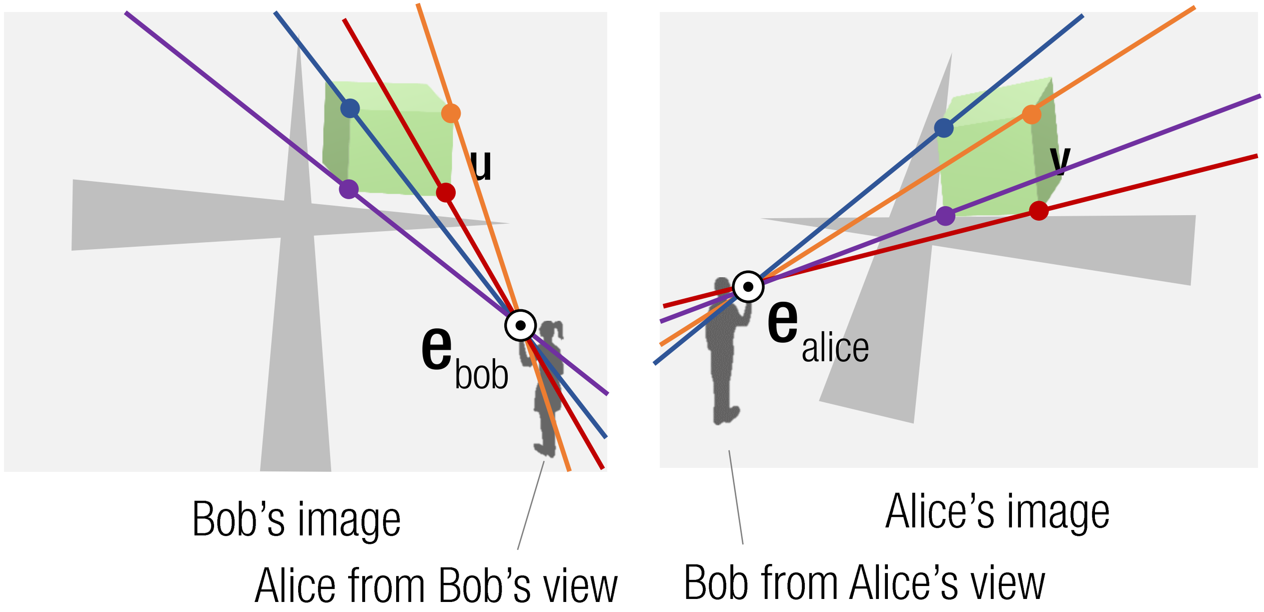




Epipolar constraint between two images:

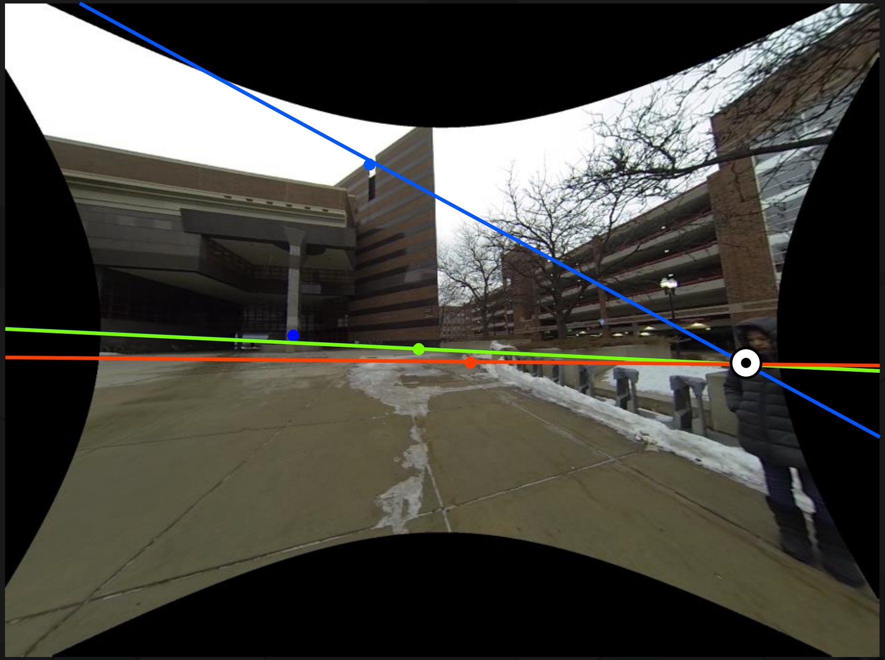
1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0 \quad u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole.

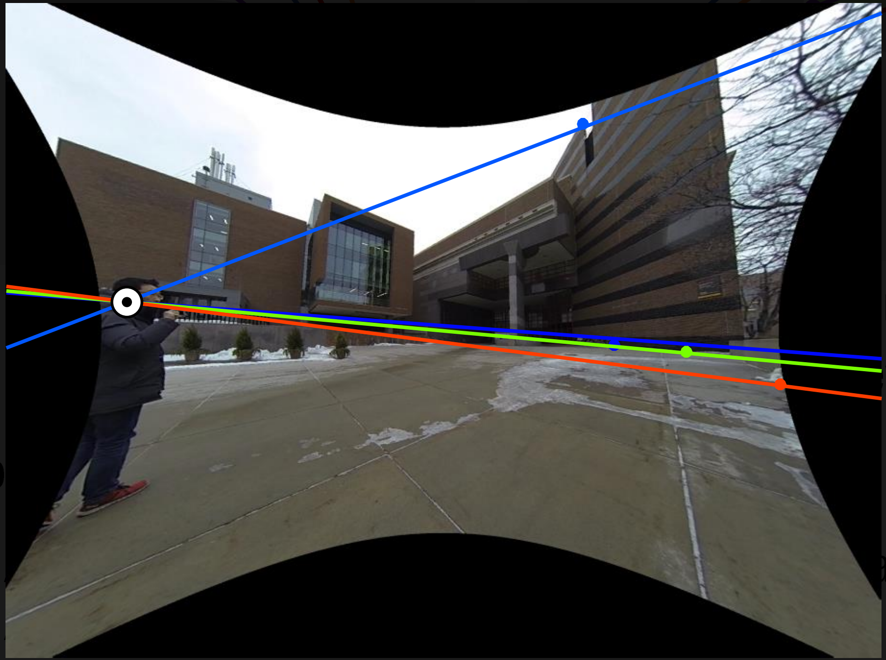




Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T \mathbf{l}_u = 0 \quad \mathbf{u}^T \mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T \mathbf{l}_u = 0 \quad \mathbf{e}_{alice}^T \mathbf{l}_v = 0$





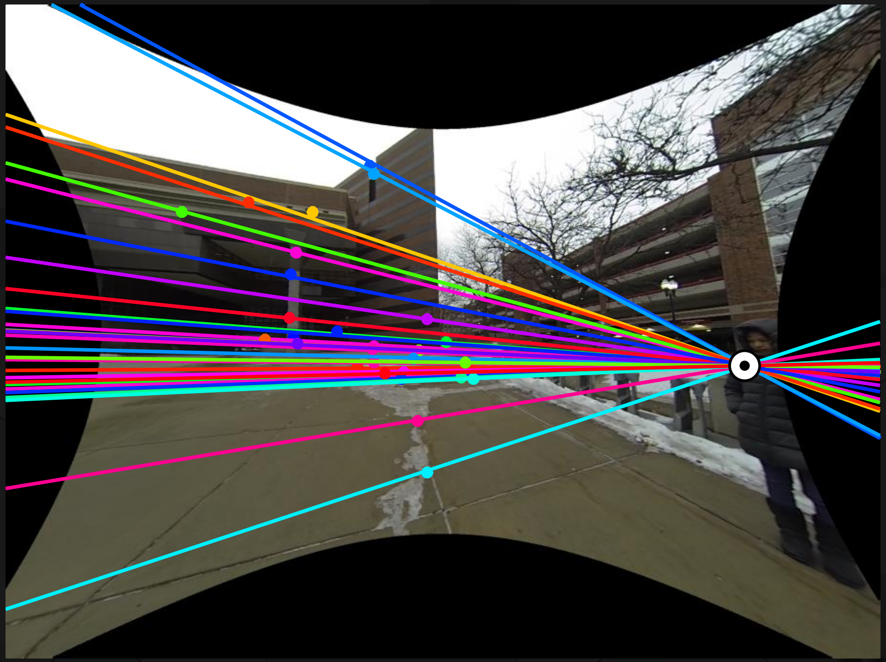
...mage

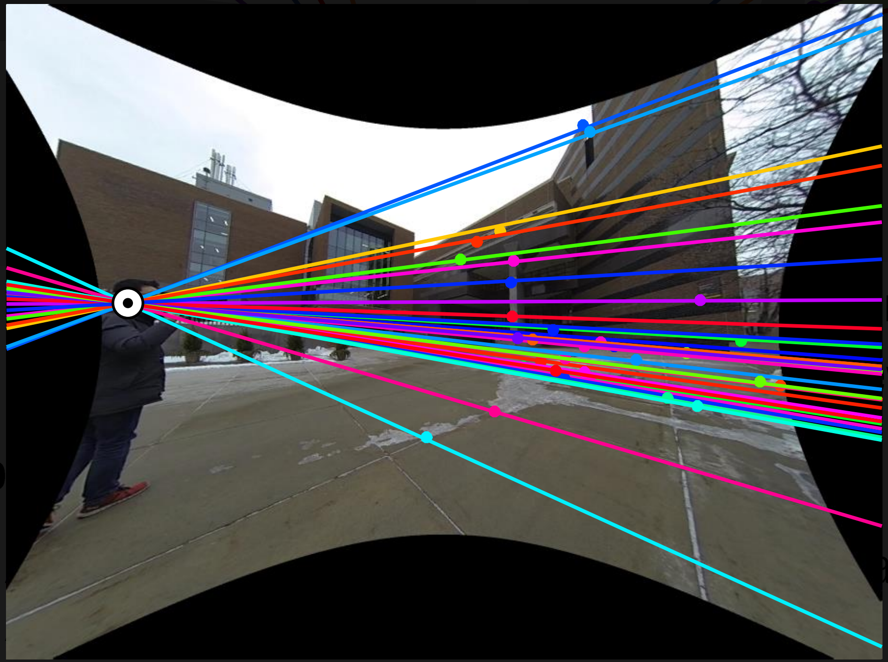
...w

Epip...

1. ... ...ar line in

2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0 \quad \mathbf{u}^T\mathbf{l}_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_u = 0 \quad \mathbf{e}_{alice}^T\mathbf{l}_v = 0$





mage

w

Epip

1. ar line in

2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $e_{bob}^T l_u = 0$ $e_{alice}^T l_v = 0$