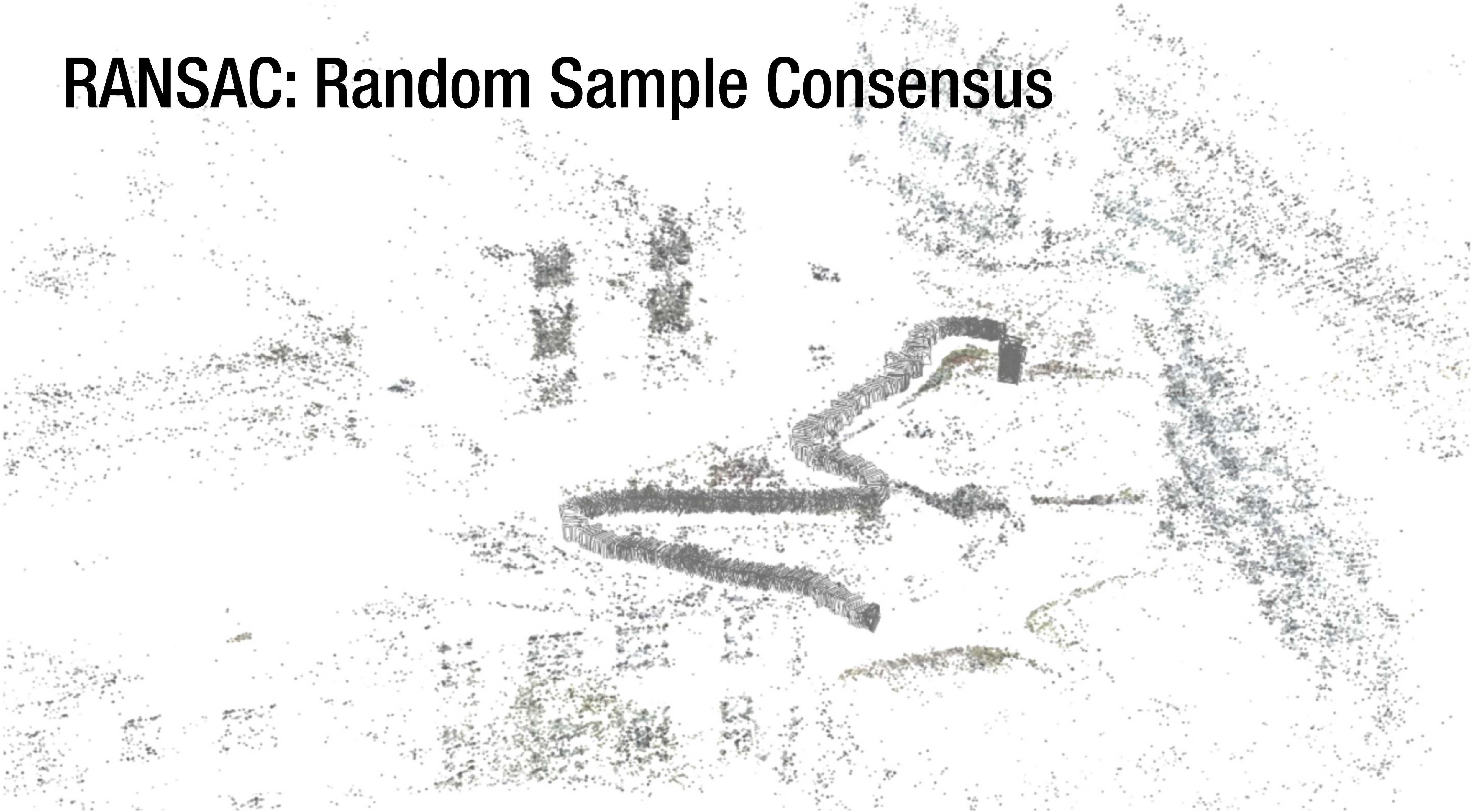
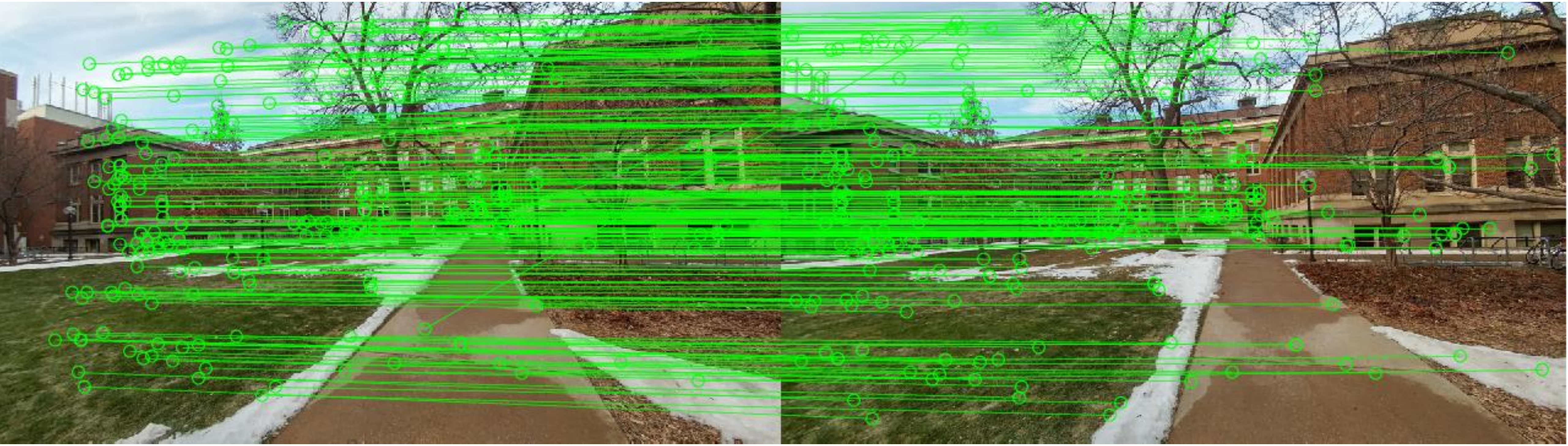
RANSAC: Random Sample Consensus



Fundamental Matrix Computation: Linear Least Squares

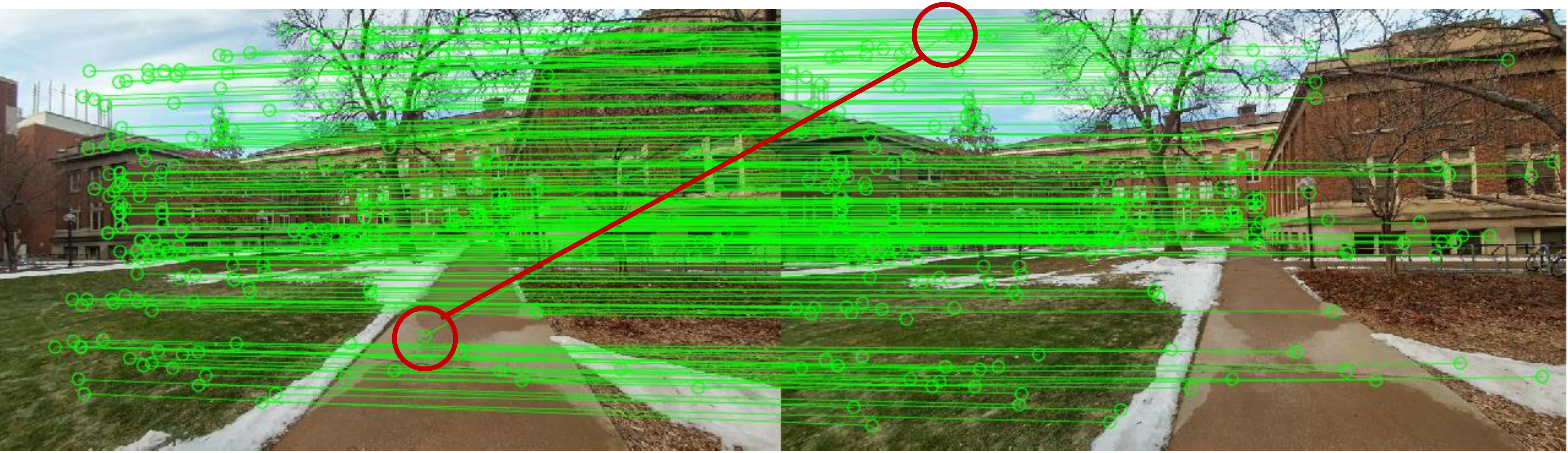
$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

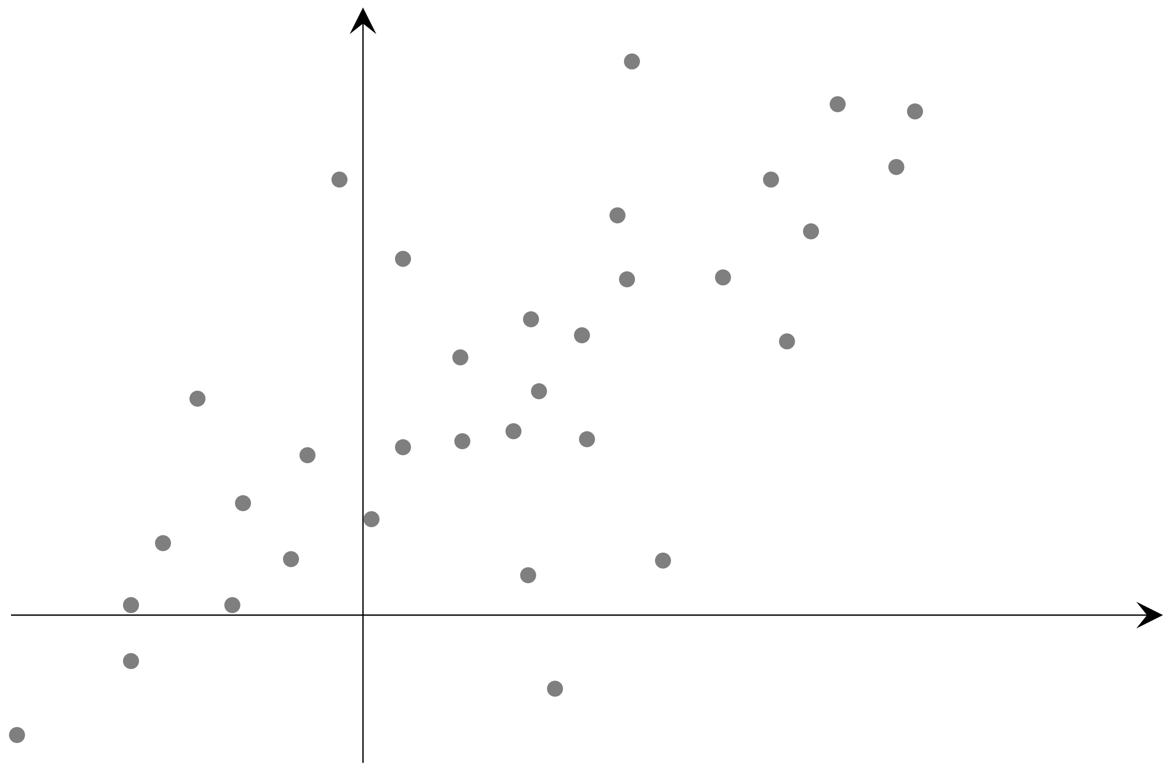


Fundamental Matrix Computation: Linear Least Squares

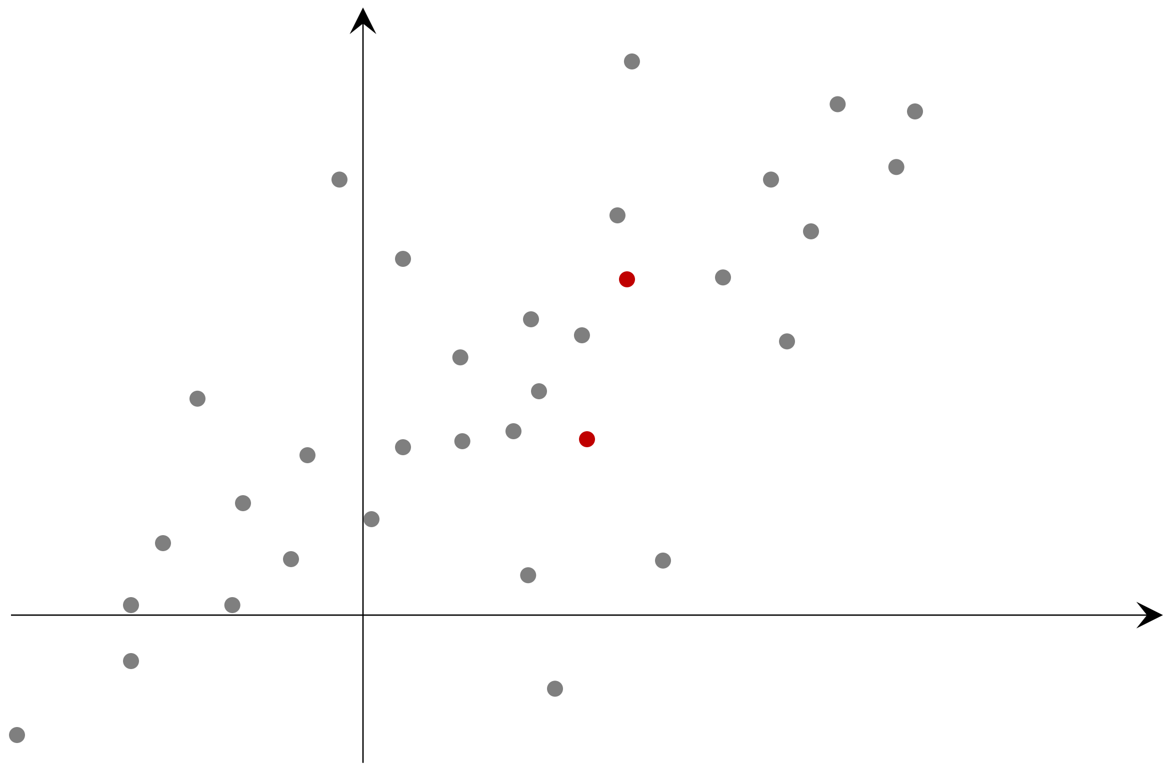
$$\underbrace{\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{x}} = \mathbf{0}$$

Outlier?



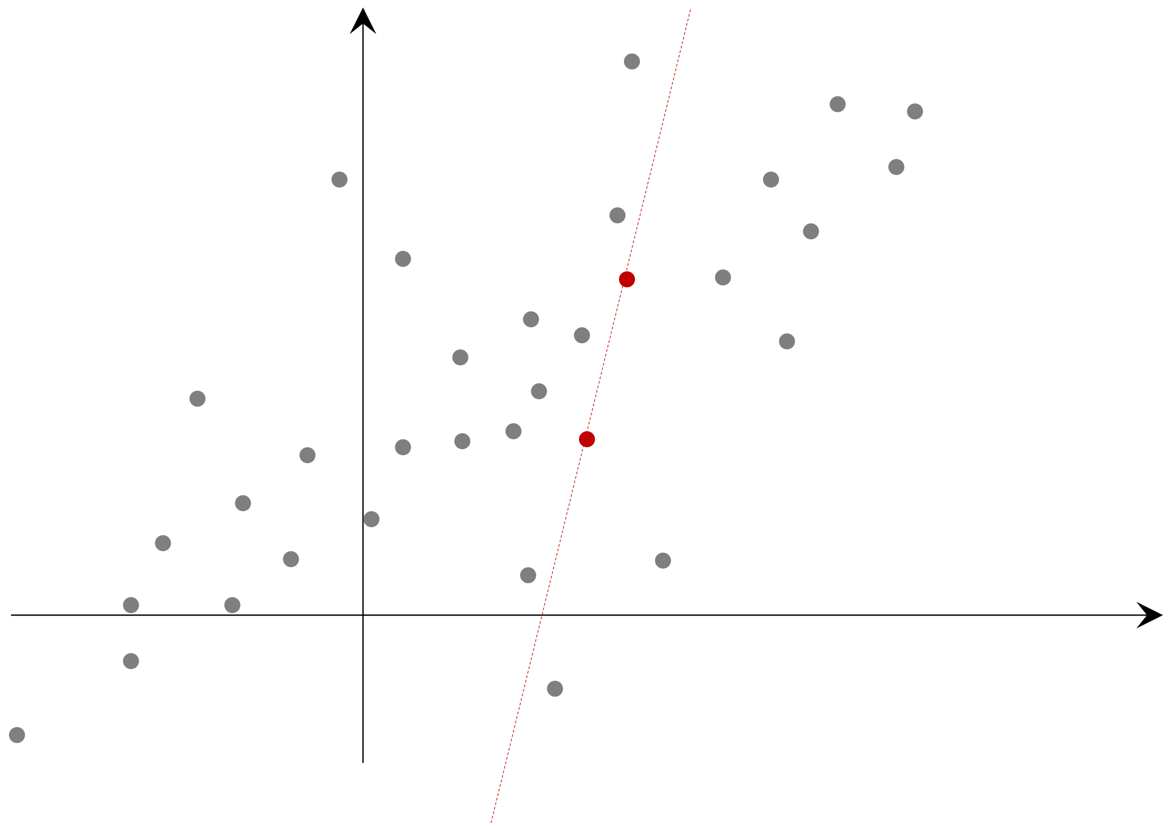


RANSAC: Random Sample Consensus



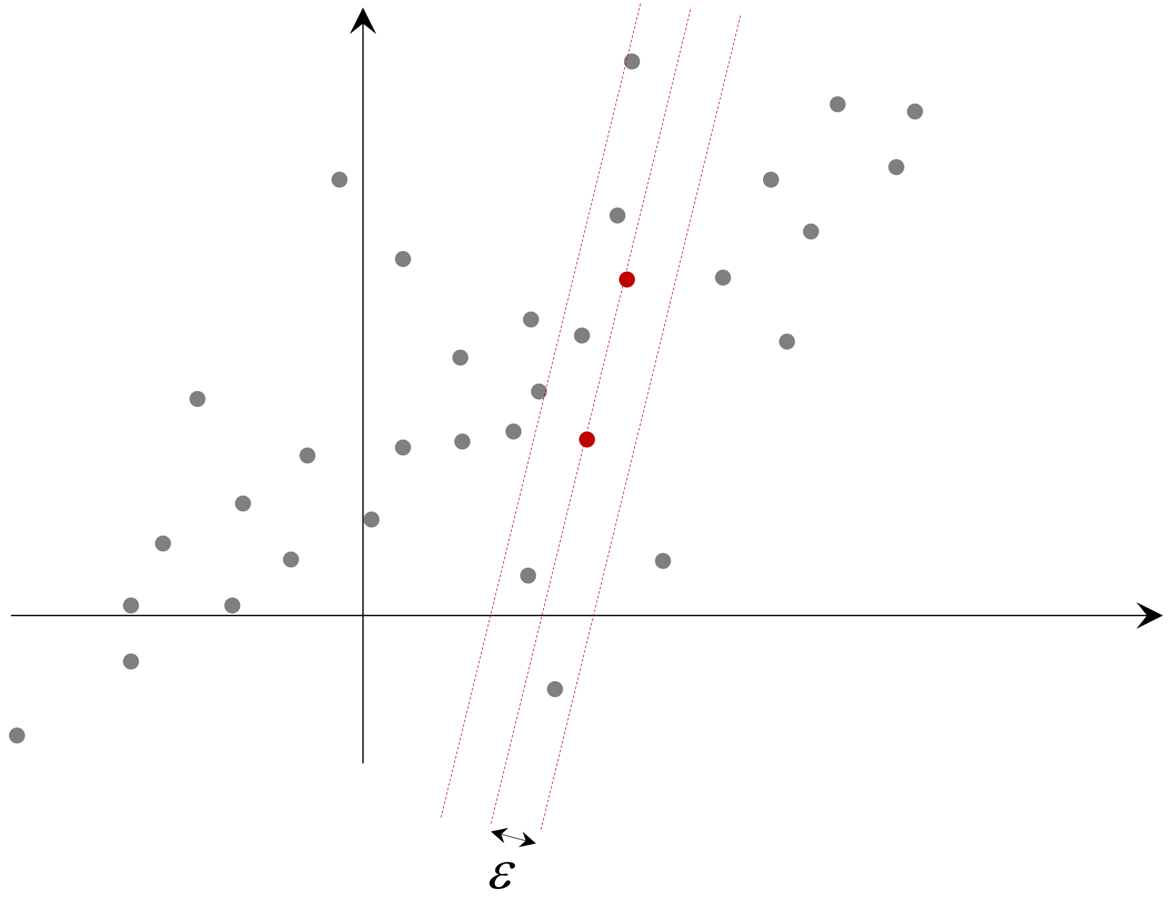
1. Random sampling

RANSAC: Random Sample Consensus



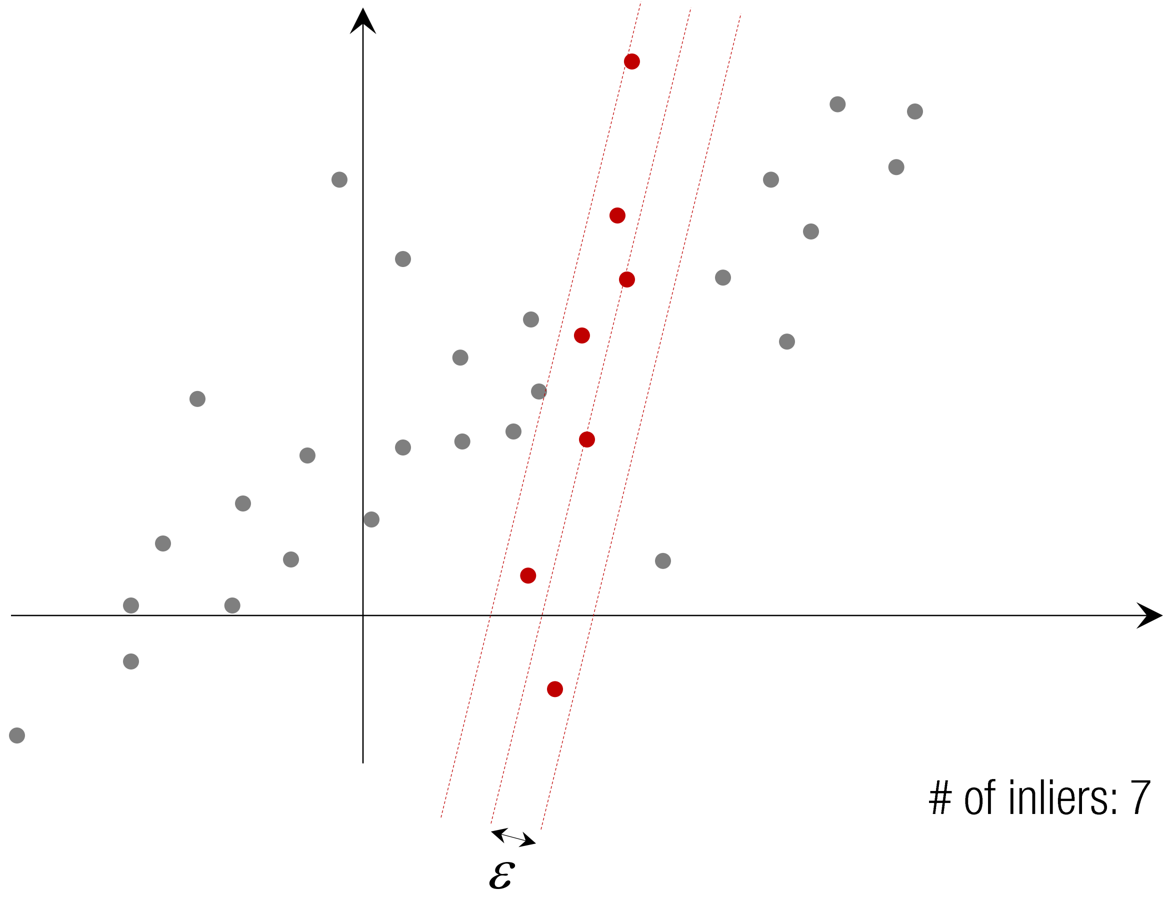
1. Random sampling
2. Model building

RANSAC: Random Sample Consensus



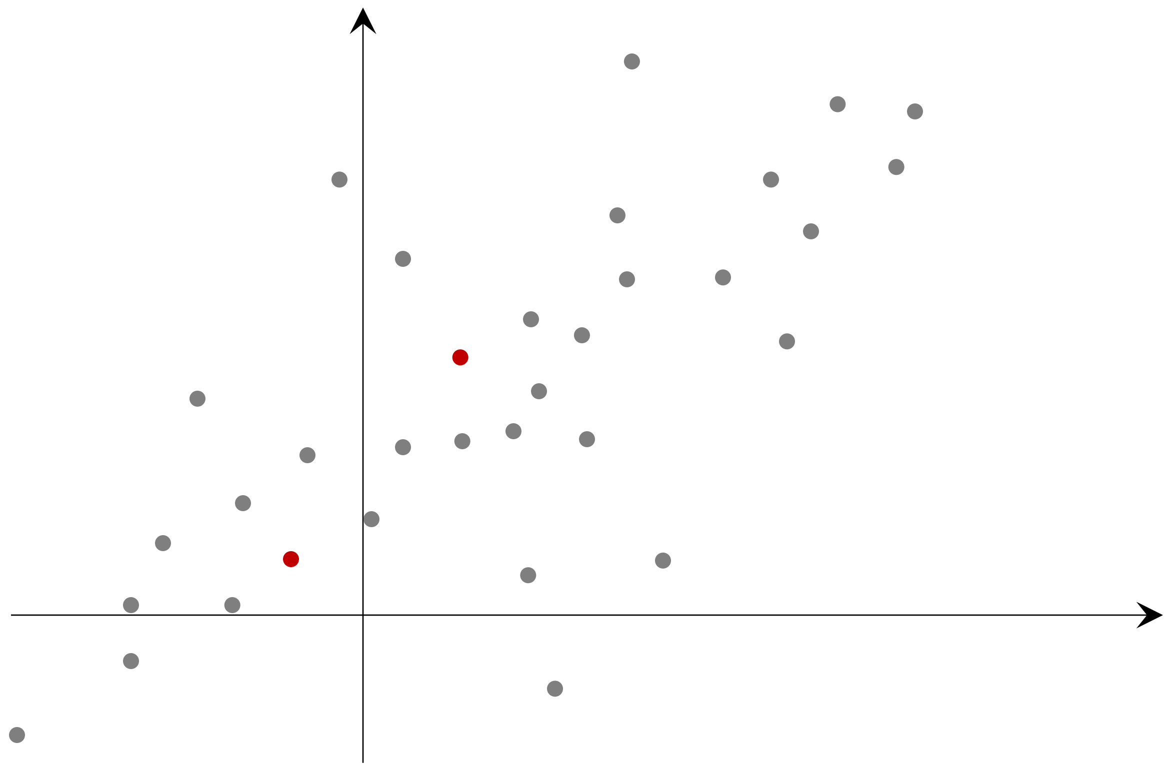
1. Random sampling
2. Model building
3. Thresholding

RANSAC: Random Sample Consensus



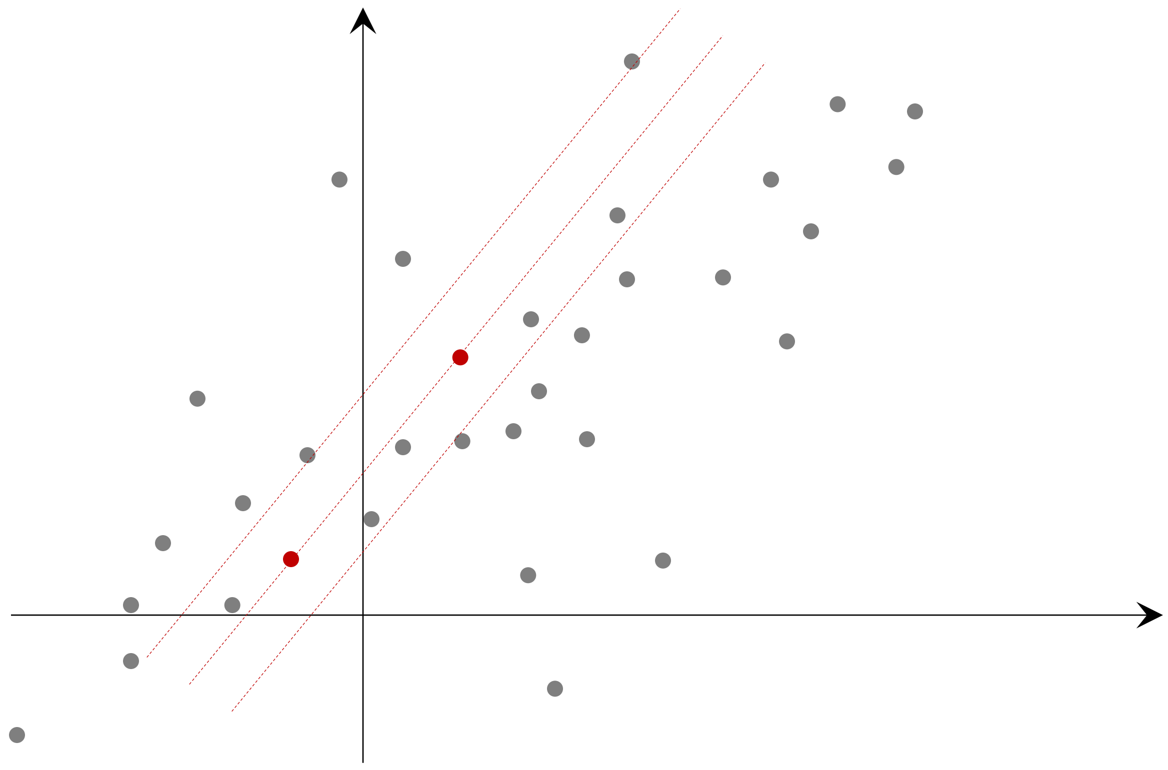
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



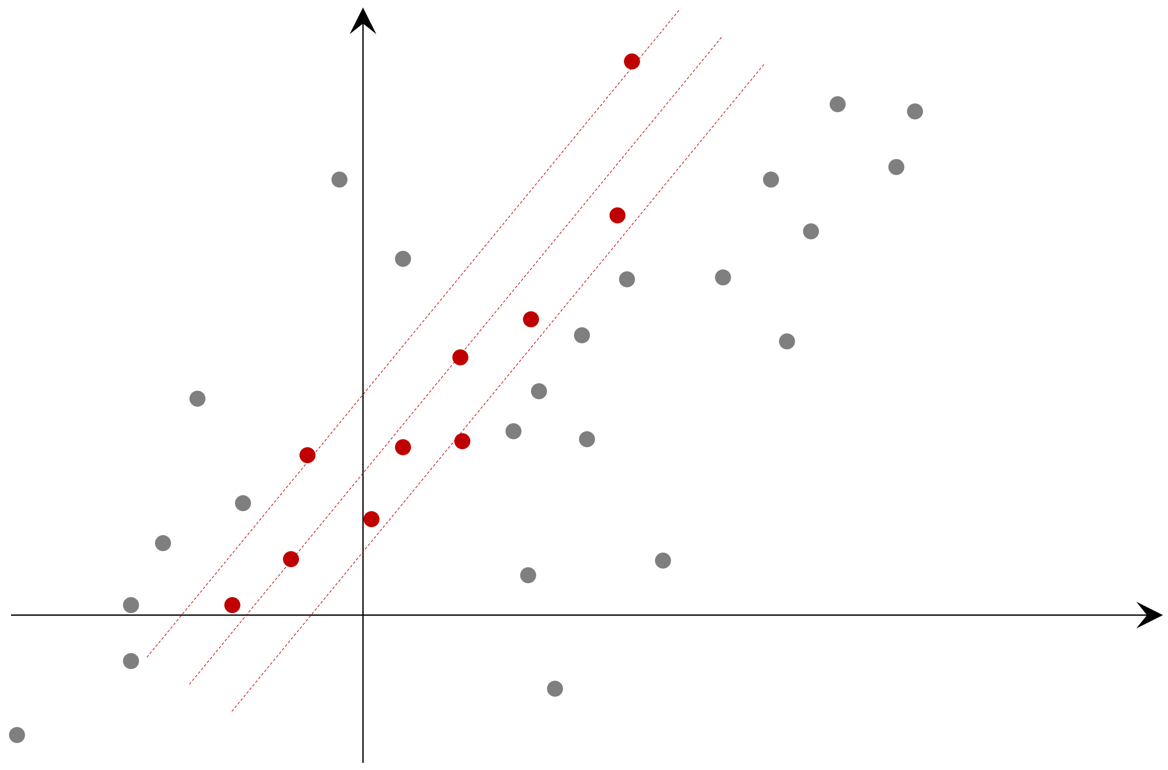
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
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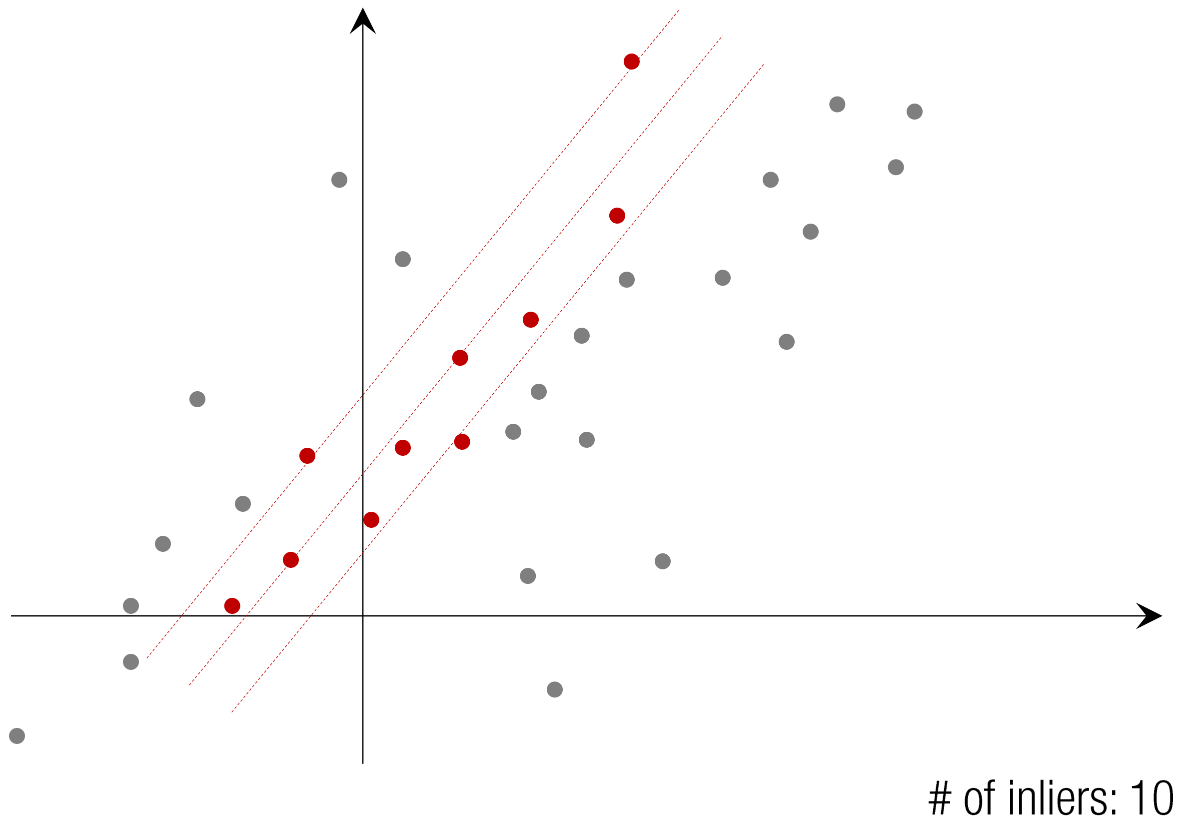
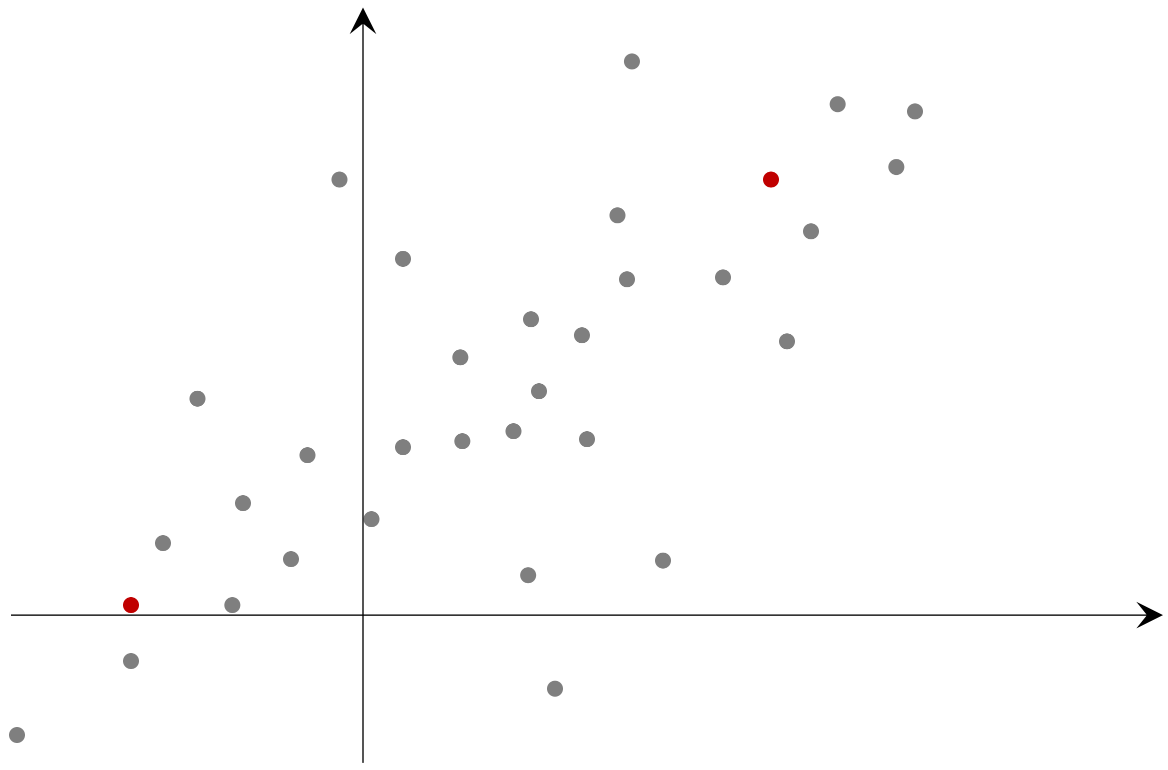
4. Inlier counting

RANSAC: Random Sample Consensus



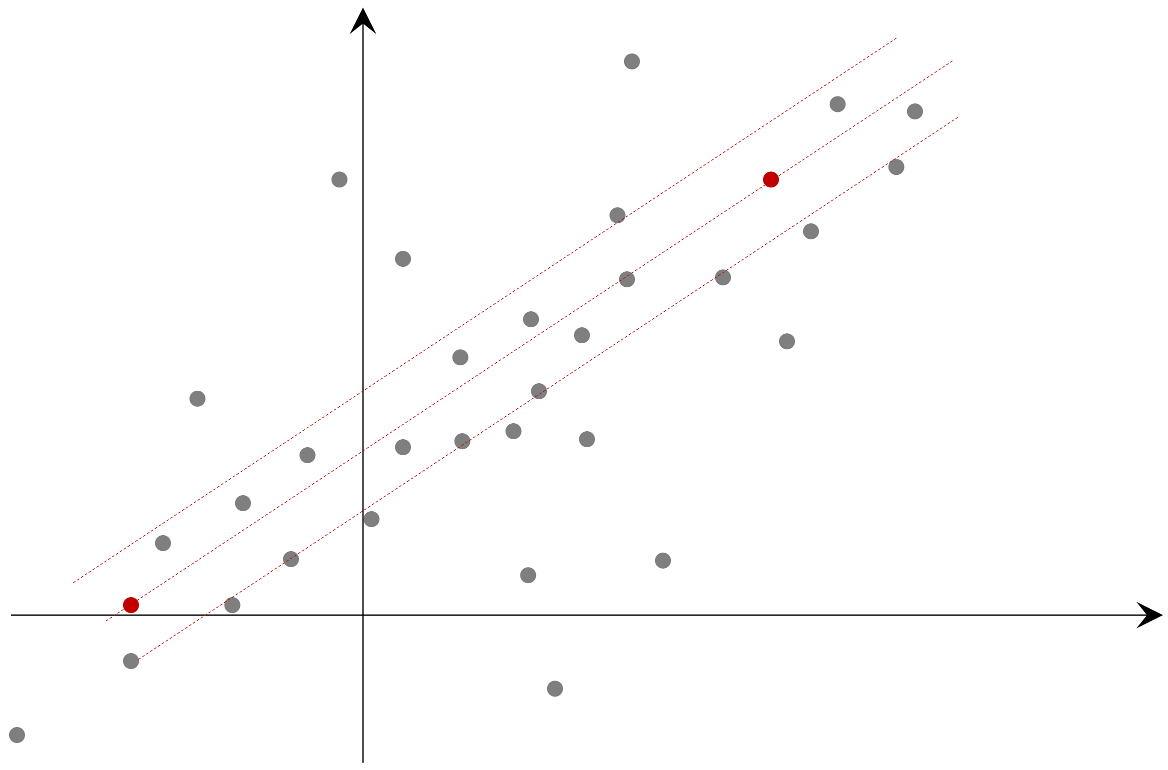
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



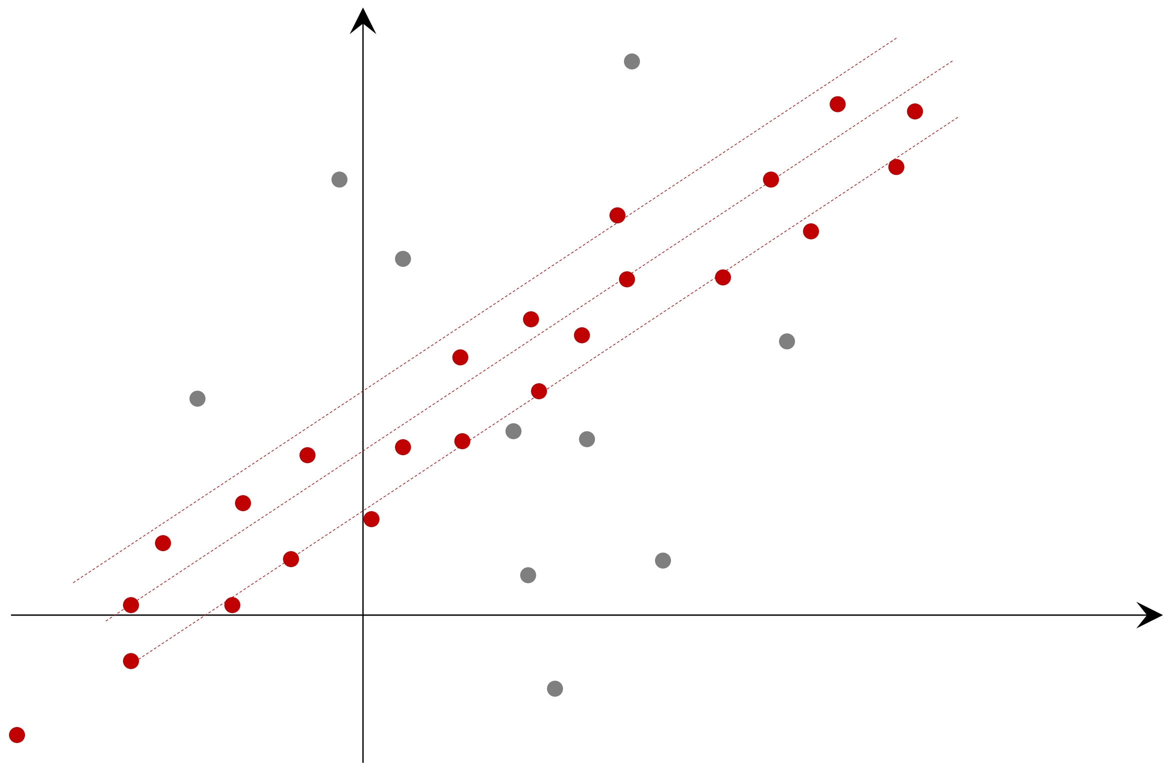
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



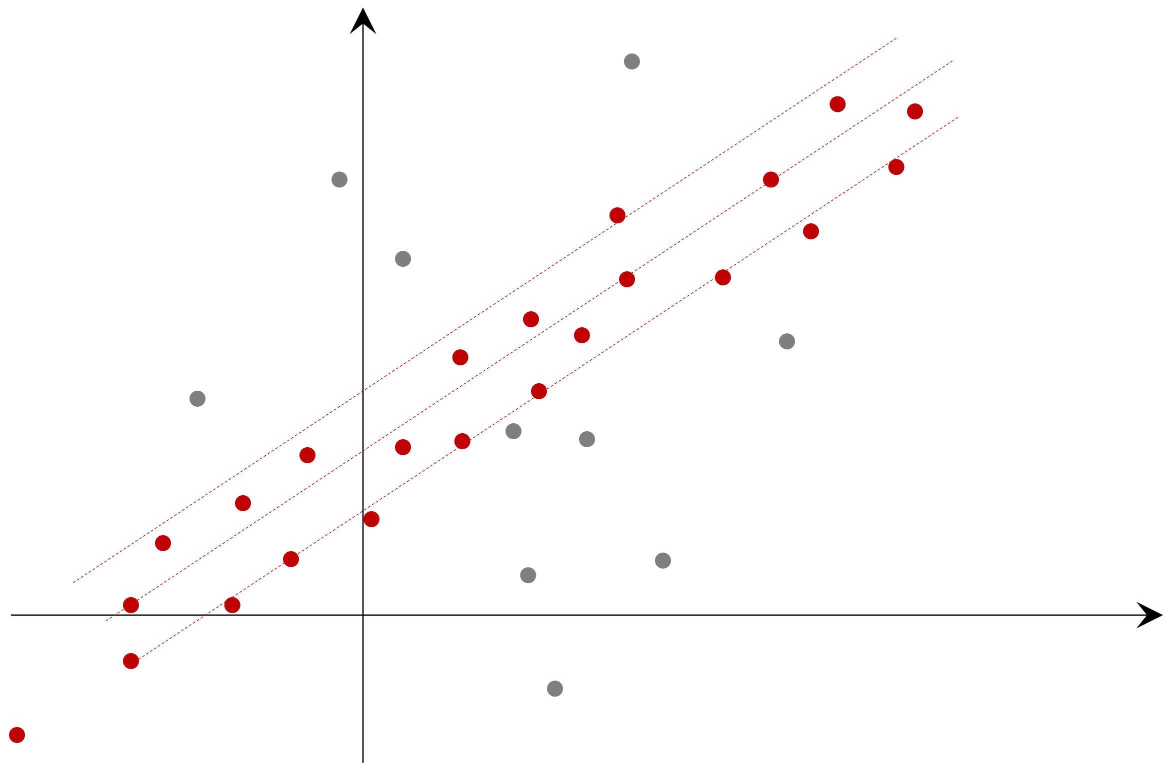
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus



1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: Random Sample Consensus

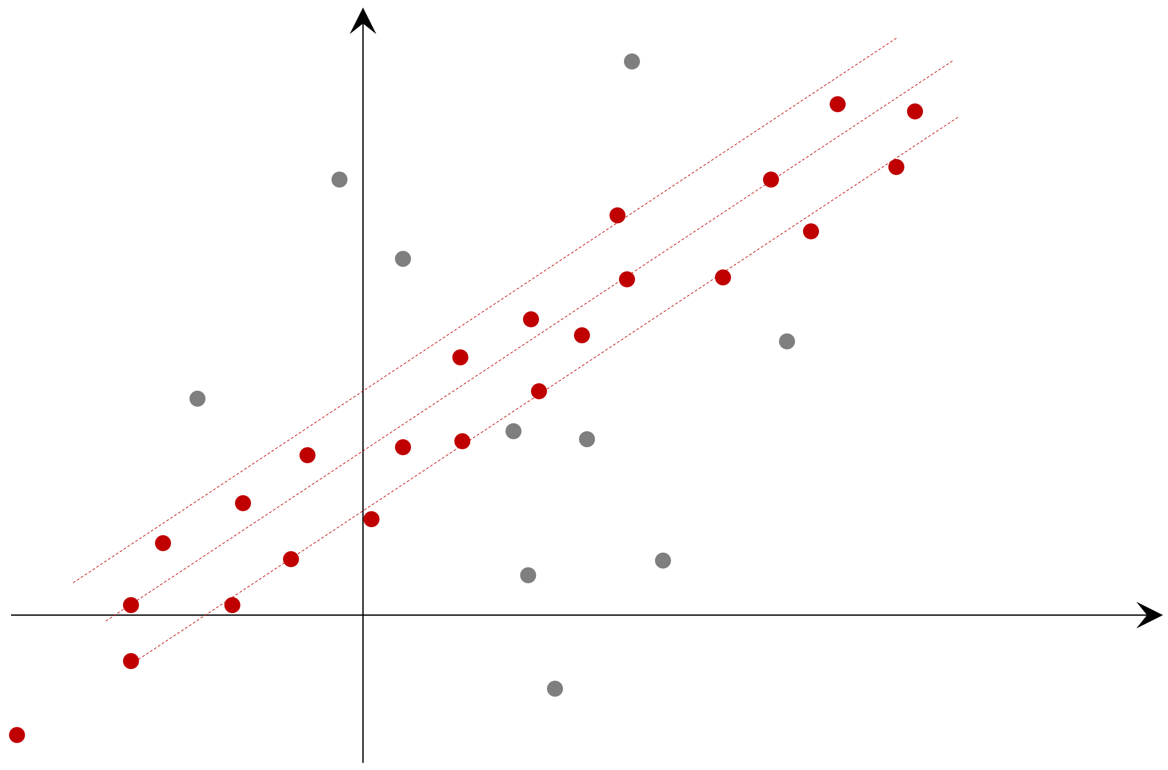


1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

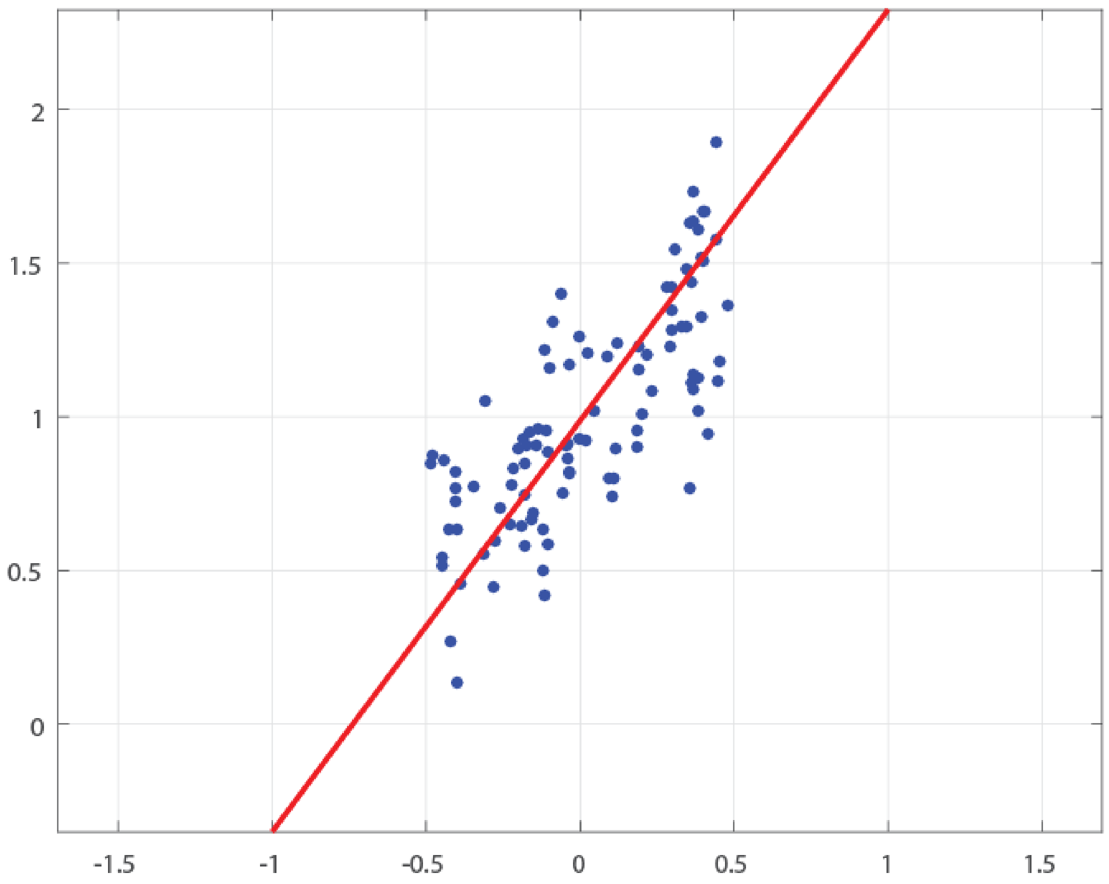
# of inliers: 23

Maximum number of inliers

RANSAC: Random Sample Consensus

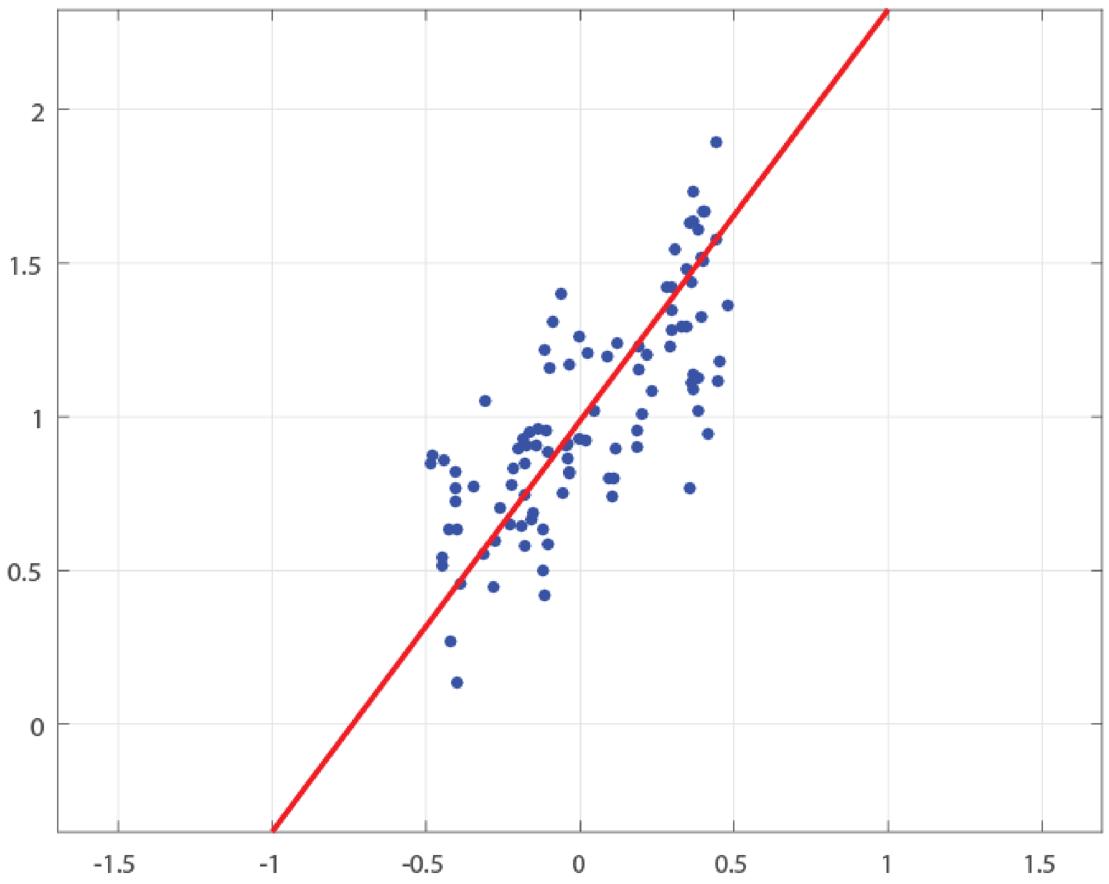


Required number of iterations with *p* success rate:




```
function RANSAC_line

a = 1;
b = 1;

nPoints = 100;
x = rand(nPoints,1)-0.5;
y = a*x + b + 0.2*randn(nPoints,1);
```

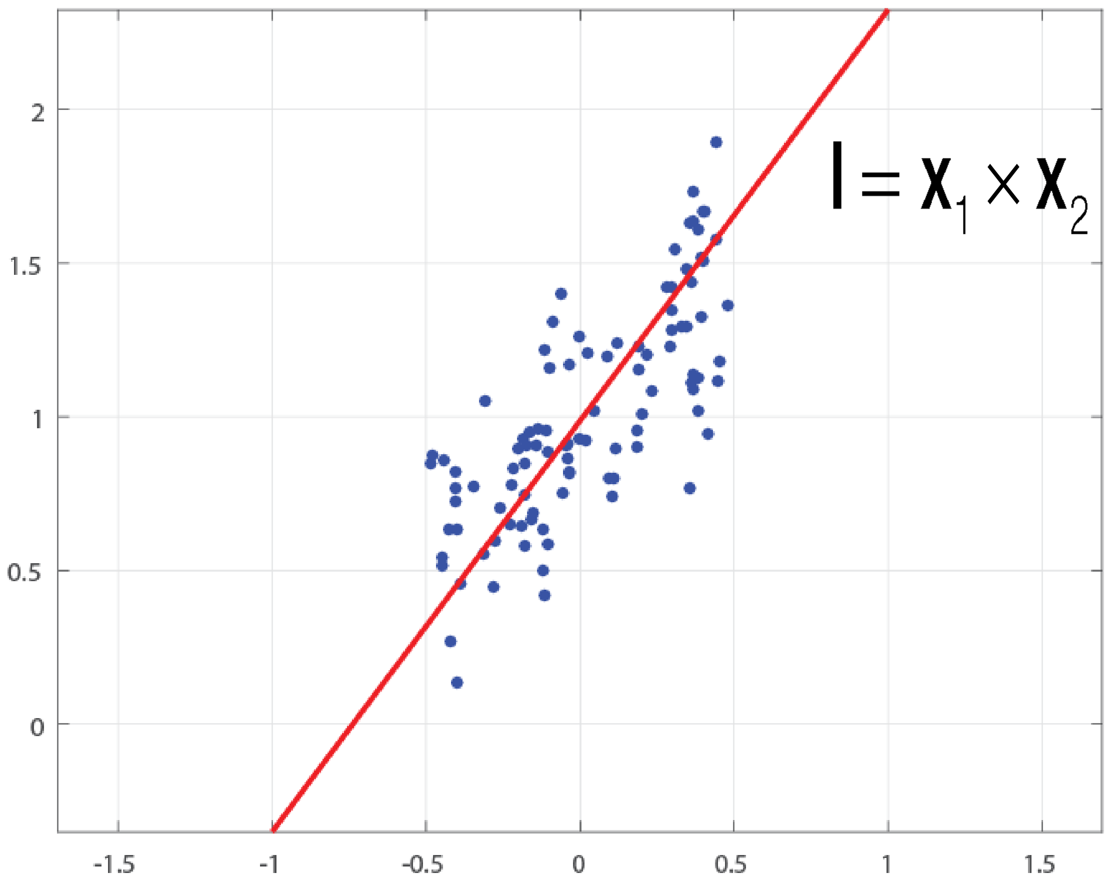
Generating samples




```
function RANSAC_line

a = 1;
b = 1;

nPoints = 100;
x = rand(nPoints,1)-0.5;
y = a*x + b + 0.2*randn(nPoints,1);

nRansacIter = 500;
threshold = 0.1;
max_nInliers = 0;
for i = 1 : nRansacIter
   r = randperm(nPoints);
   s1 = [x(r(1)); y(r(1)); 1];
   s2 = [x(r(2)); y(r(2)); 1];
```

Generating samples

Random sampling

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y = a*x + b + 0.2*randn(nPoints,1);

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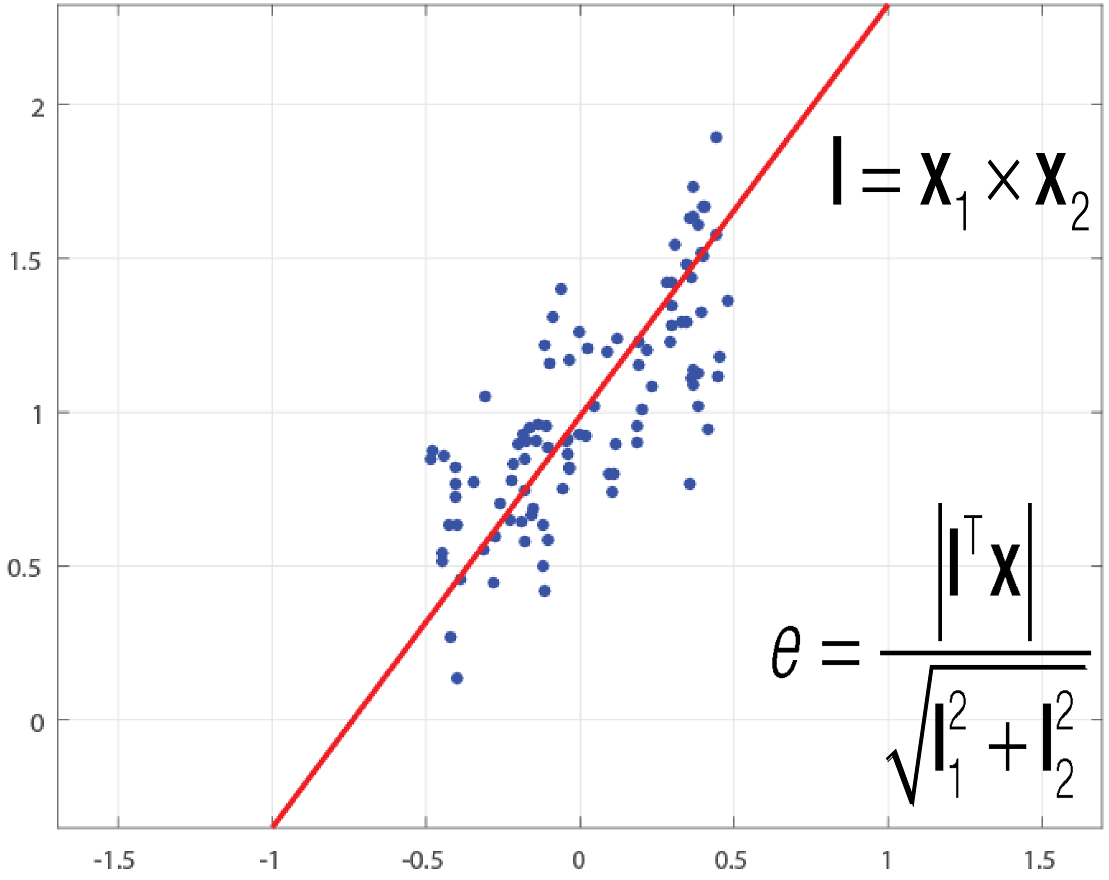
for i = 1 : nRansacIter
    r = randperm(nPoints);
    s1 = [x(r(1)); y(r(1)); 1];
    s2 = [x(r(2)); y(r(2)); 1];

    l = GetLineFromTwoPoints(s1, s2);
```

Generating samples

Random sampling

Model building

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x = rand(nPoints,1)-0.5;
y = a*x + b + 0.2*randn(nPoints,1);

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    r = randperm(nPoints);
    s1 = [x(r(1)); y(r(1)); 1];
    s2 = [x(r(2)); y(r(2)); 1];

    l = GetLineFromTwoPoints(s1, s2);

    nInliers = 0;
    for j = 1 : nPoints
        e = abs(l' * [x(j); y(j); 1])/norm(l(1:2));
        if (e < threshold)
            nInliers = nInliers + 1;
        end
    end
```

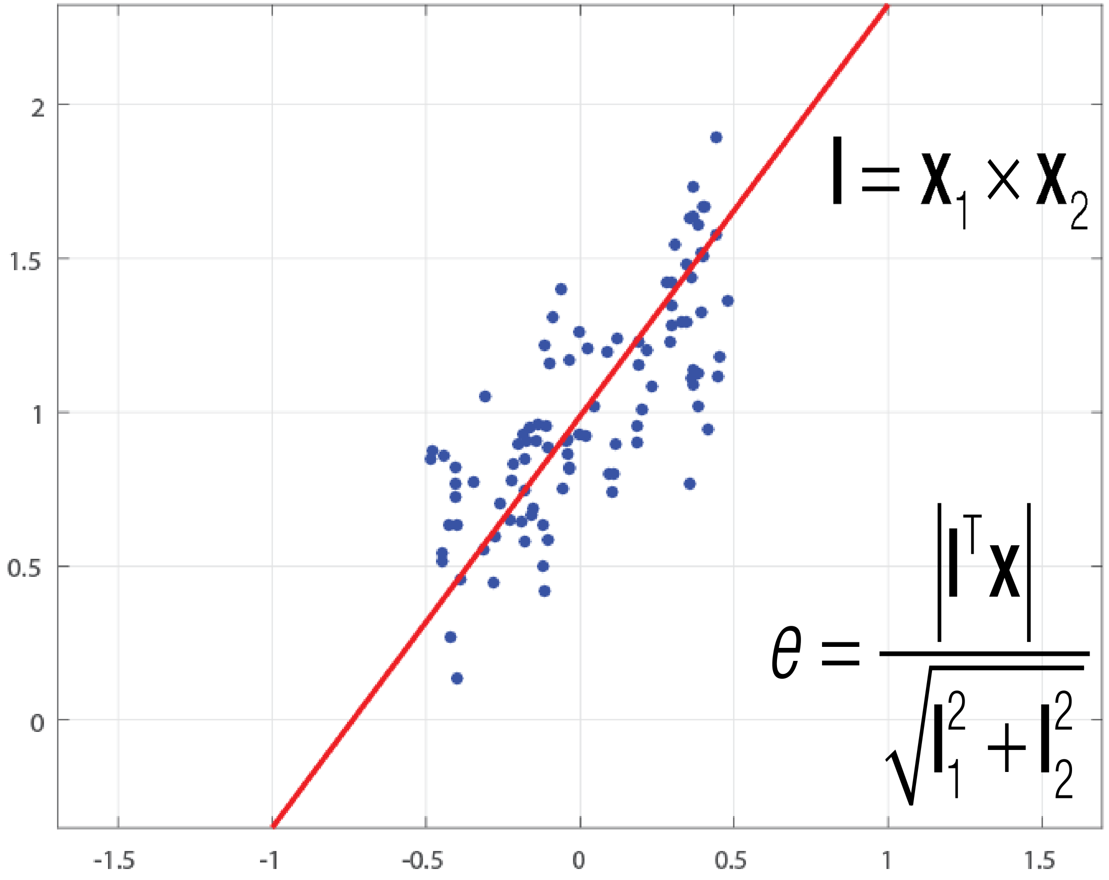
Generating samples

Random sampling

Model building

Thresholding

Inlier counting



$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

$$e = \frac{\left|\mathbf{l}^T \mathbf{x}\right|}{\sqrt{l_1^2 + l_2^2}}$$

```
function RANSAC_line

a = 1;
b = 1;

nPoints = 100;
x = rand(nPoints,1)-0.5;
y = a*x + b + 0.2*randn(nPoints,1);

nRansacIter = 500;
threshold = 0.1;
max_nInliers = 0;
for i = 1 : nRansacIter
    r = randperm(nPoints);
    s1 = [x(r(1)); y(r(1)); 1];
    s2 = [x(r(2)); y(r(2)); 1];

    l = GetLineFromTwoPoints(s1, s2);

    nInliers = 0;
    for j = 1 : nPoints
        e = abs(l' * [x(j); y(j); 1])/norm(l(1:2));
        if (e < threshold)
            nInliers = nInliers + 1;
        end
    end
     if (nInliers > max_nInliers)
        max_nInliers = nInliers;
        L = l;
    end
end
```

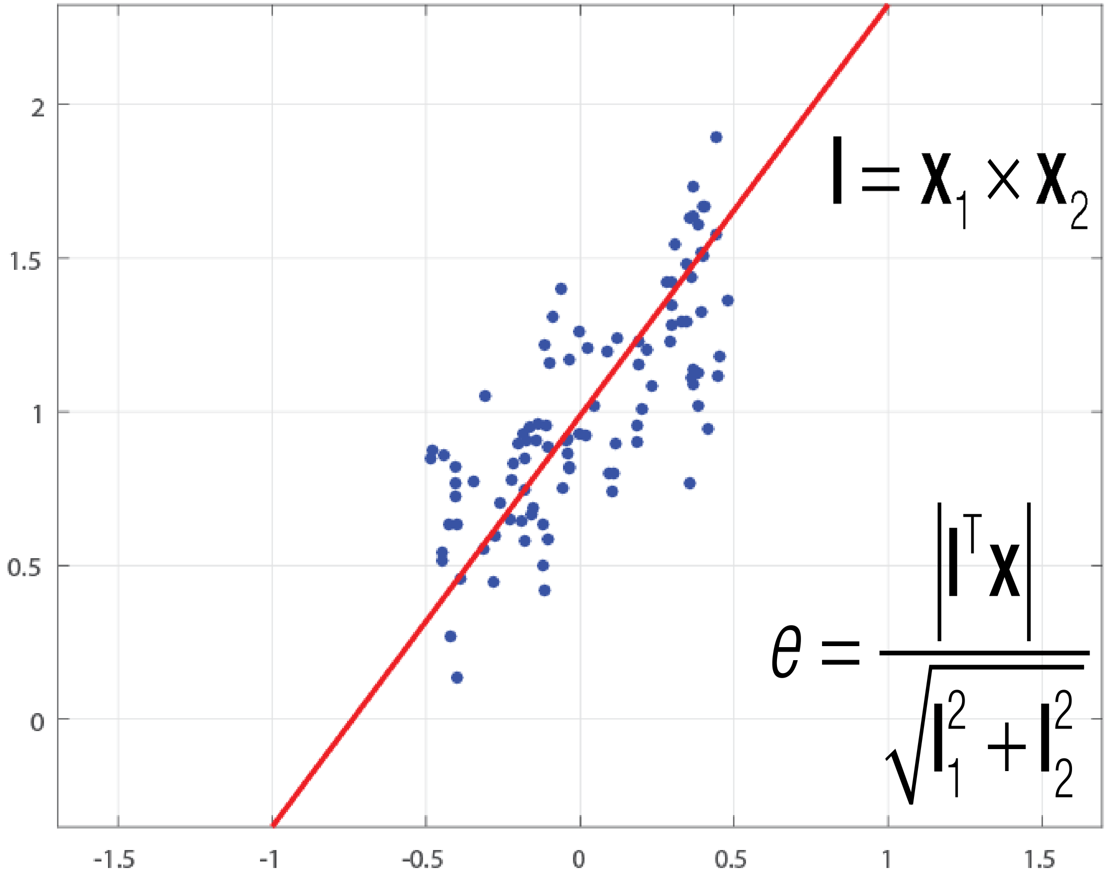
Generating samples

Random sampling

Model building

Thresholding
Inlier counting

Model updating



$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

$$e = \frac{\left|\mathbf{l}^T \mathbf{x}\right|}{\sqrt{\mathbf{l}_1^2 + \mathbf{l}_2^2}}$$

```
function RANSAC_line

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b = 1;

nPoints = 100;
x = rand(nPoints,1)-0.5;
y = a*x + b + 0.2*randn(nPoints,1);

nRansacIter = 500;
threshold = 0.1;
max_nInliers = 0;
for i = 1 : nRansacIter
    r = randperm(nPoints);
    s1 = [x(r(1)); y(r(1)); 1];
    s2 = [x(r(2)); y(r(2)); 1];

    l = GetLineFromTwoPoints(s1, s2);

    nInliers = 0;
    for j = 1 : nPoints
        e = abs(l' * [x(j); y(j); 1])/norm(l(1:2));
        if (e < threshold)
            nInliers = nInliers + 1;
        end
    end
     if (nInliers > max_nInliers)
        max_nInliers = nInliers;
        L = l;
    end
end
```

Generating samples

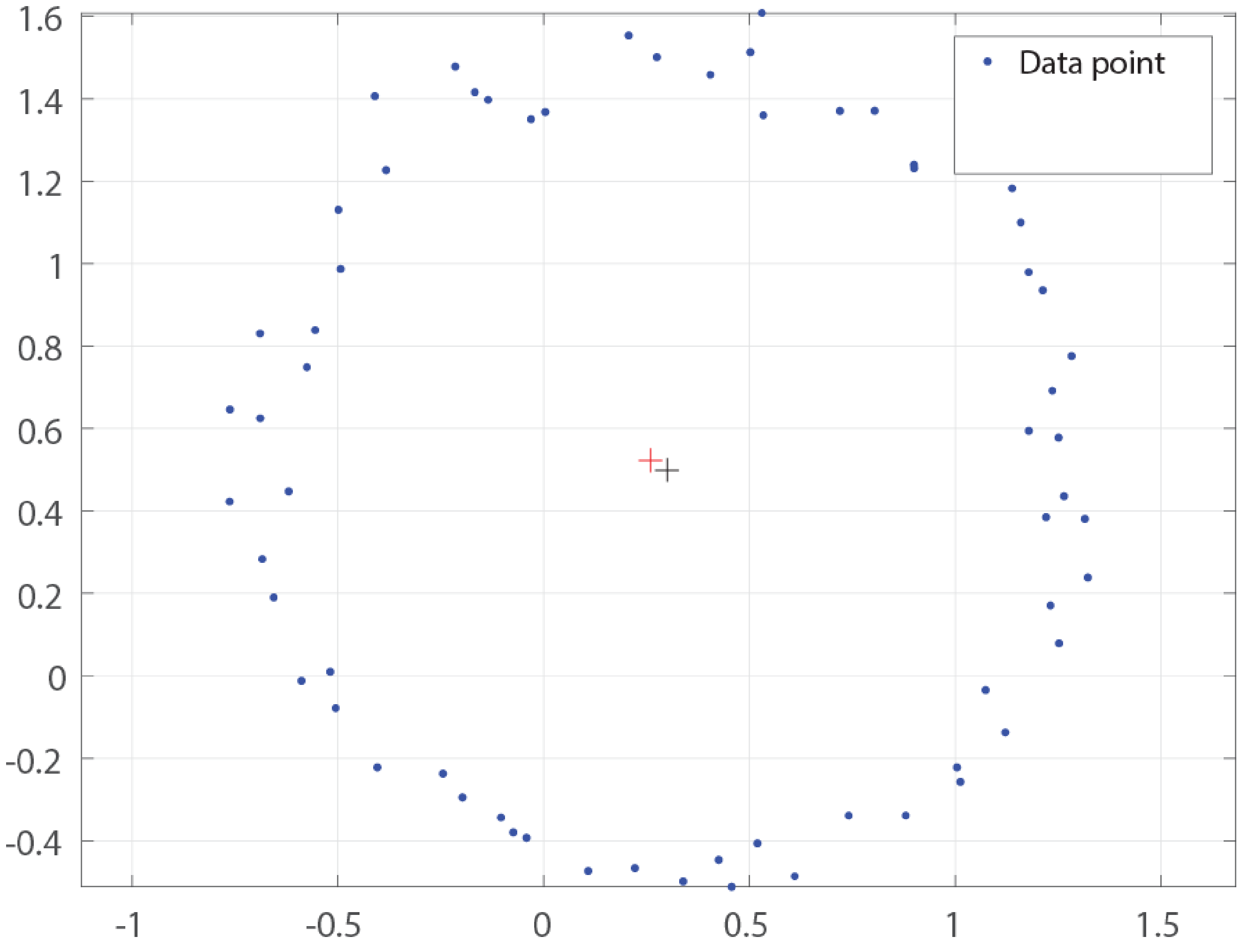
Random sampling

Model building

Thresholding
Inlier counting

Model updating

Recall: Circle Fitting (Ax=b)



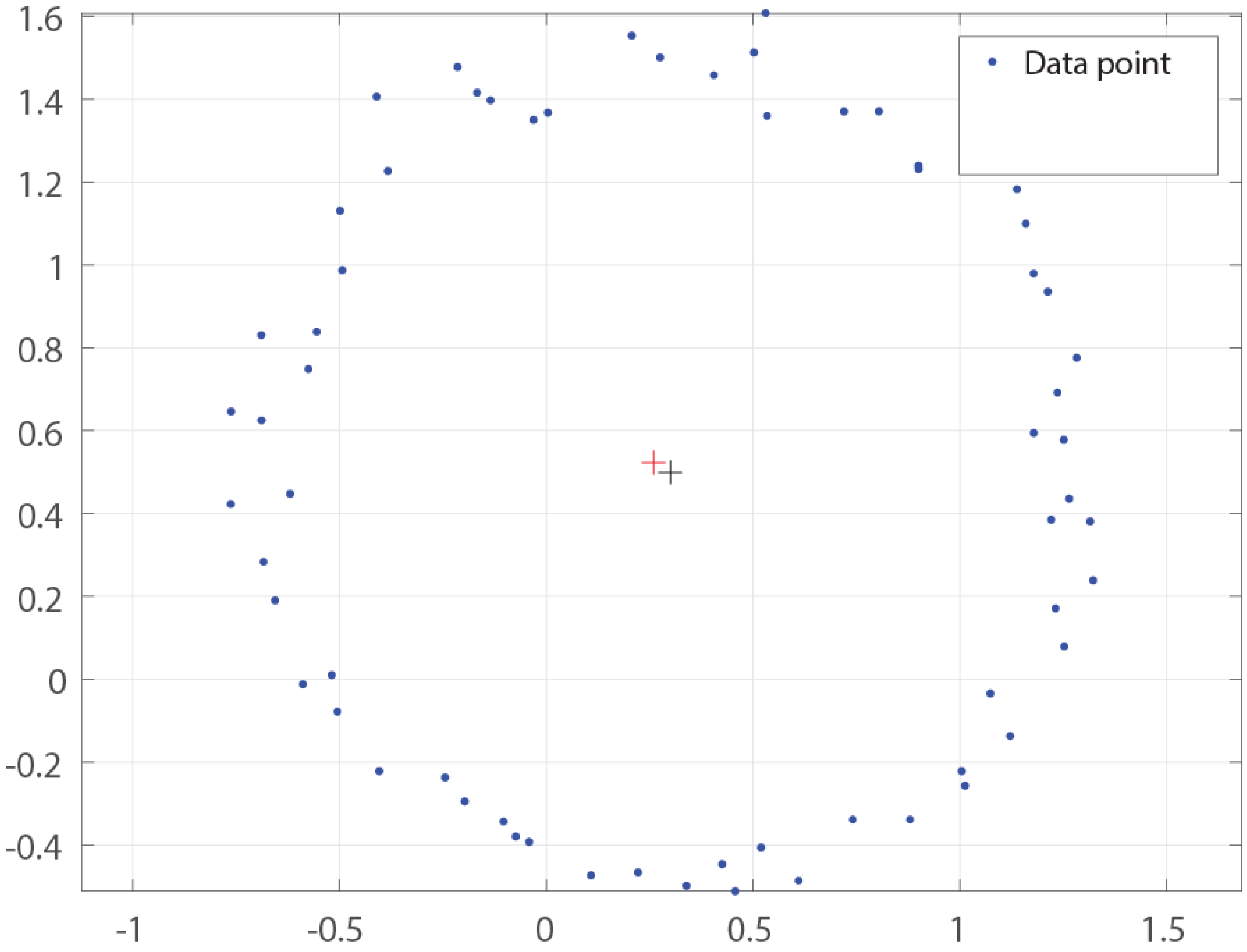
$$(x_1 - c_x)^2 + (y_1 - c_y)^2 = r^2$$

$$\vdots$$

$$(x_n - c_x)^2 + (y_n - c_y)^2 = r^2$$

Unkowns: c_x, c_y, r

Recall: Circle Fitting (Ax=b)

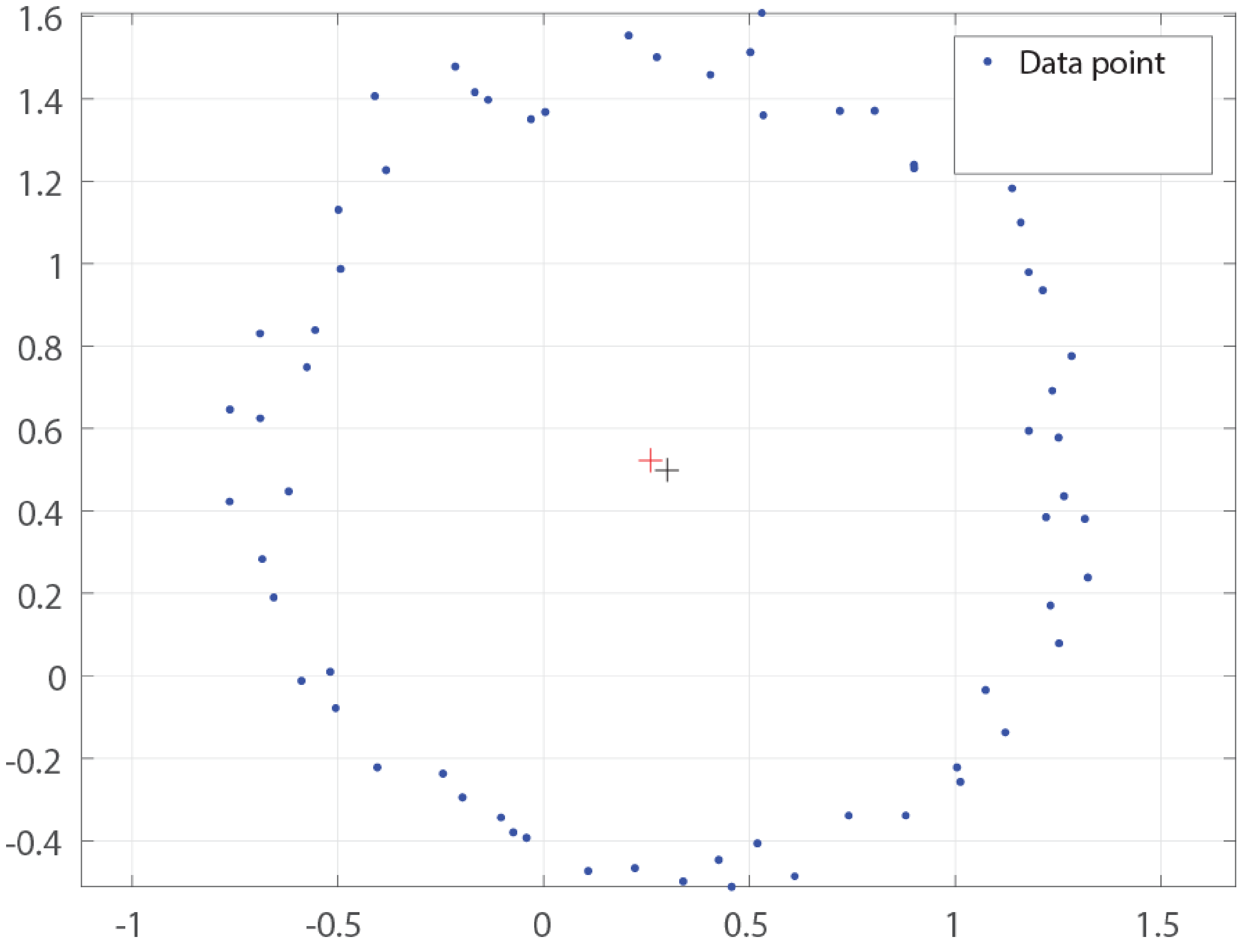


$$x_1^2 - 2x_1 c_x + c_x^2 + y_1^2 - 2y_1 c_y + c_y^2 = r^2$$

$$\vdots$$

$$x_n^2 - 2x_n c_x + c_x^2 + y_n^2 - 2y_n c_y + c_y^2 = r^2$$

Recall: Circle Fitting (Ax=b)



$$x_1^2 - 2x_1 c_x + c_x^2 + y_1^2 - 2y_1 c_y + c_y^2 = r^2$$

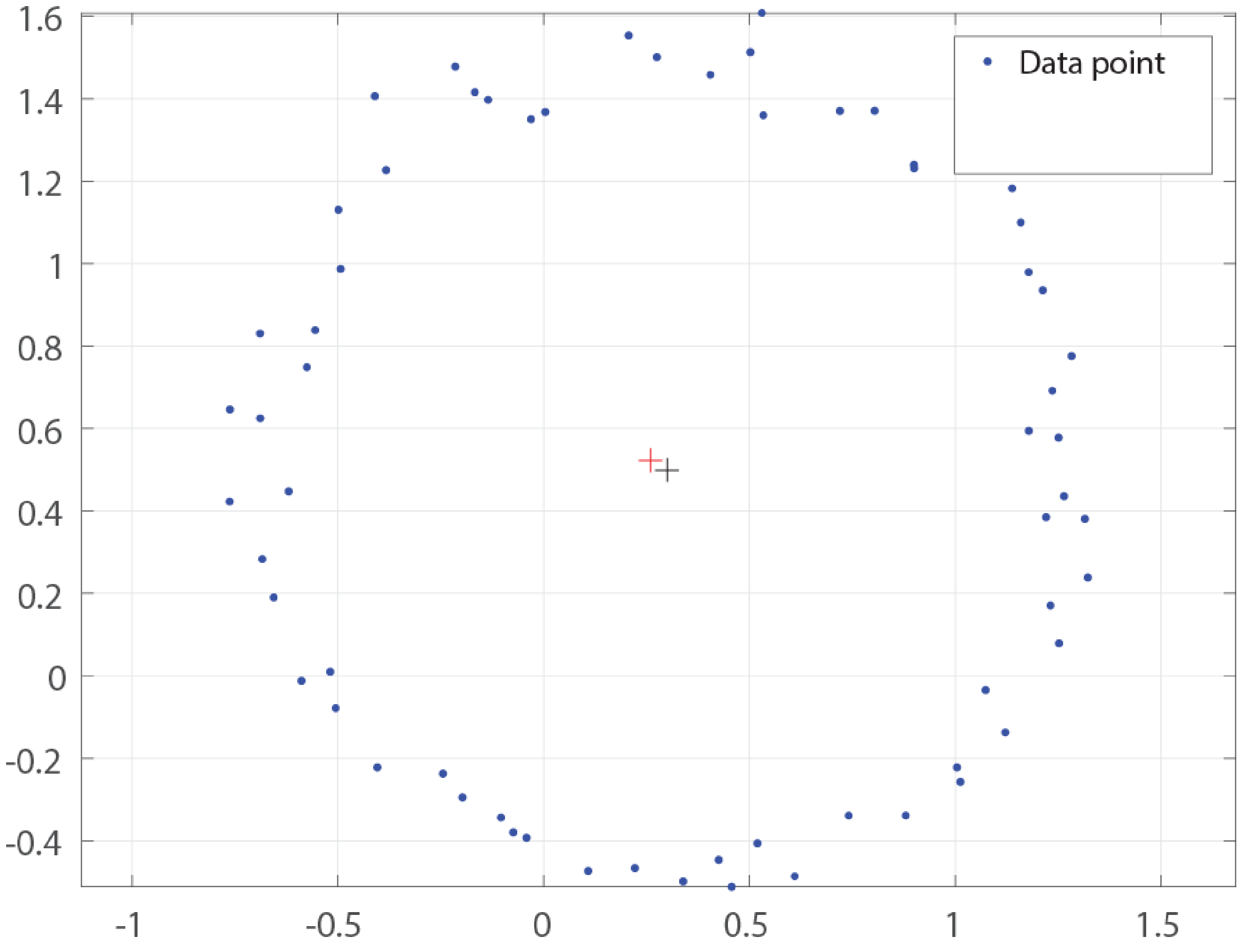
$$\vdots$$

$$x_n^2 - 2x_n c_x + c_x^2 + y_n^2 - 2y_n c_y + c_y^2 = r^2$$

$$\downarrow$$

$$x_i^2 - x_1^2 - 2c_x\left(x_i - x_1\right) + y_i^2 - y_1^2 - 2\left(y_i - y_1\right)c_y = 0$$

Recall: Circle Fitting (Ax=b)



$$x_1^2 - 2x_1 c_x + c_x^2 + y_1^2 - 2y_1 c_y + c_y^2 = r^2$$

$$\vdots$$

$$x_n^2 - 2x_n c_x + c_x^2 + y_n^2 - 2y_n c_y + c_y^2 = r^2$$

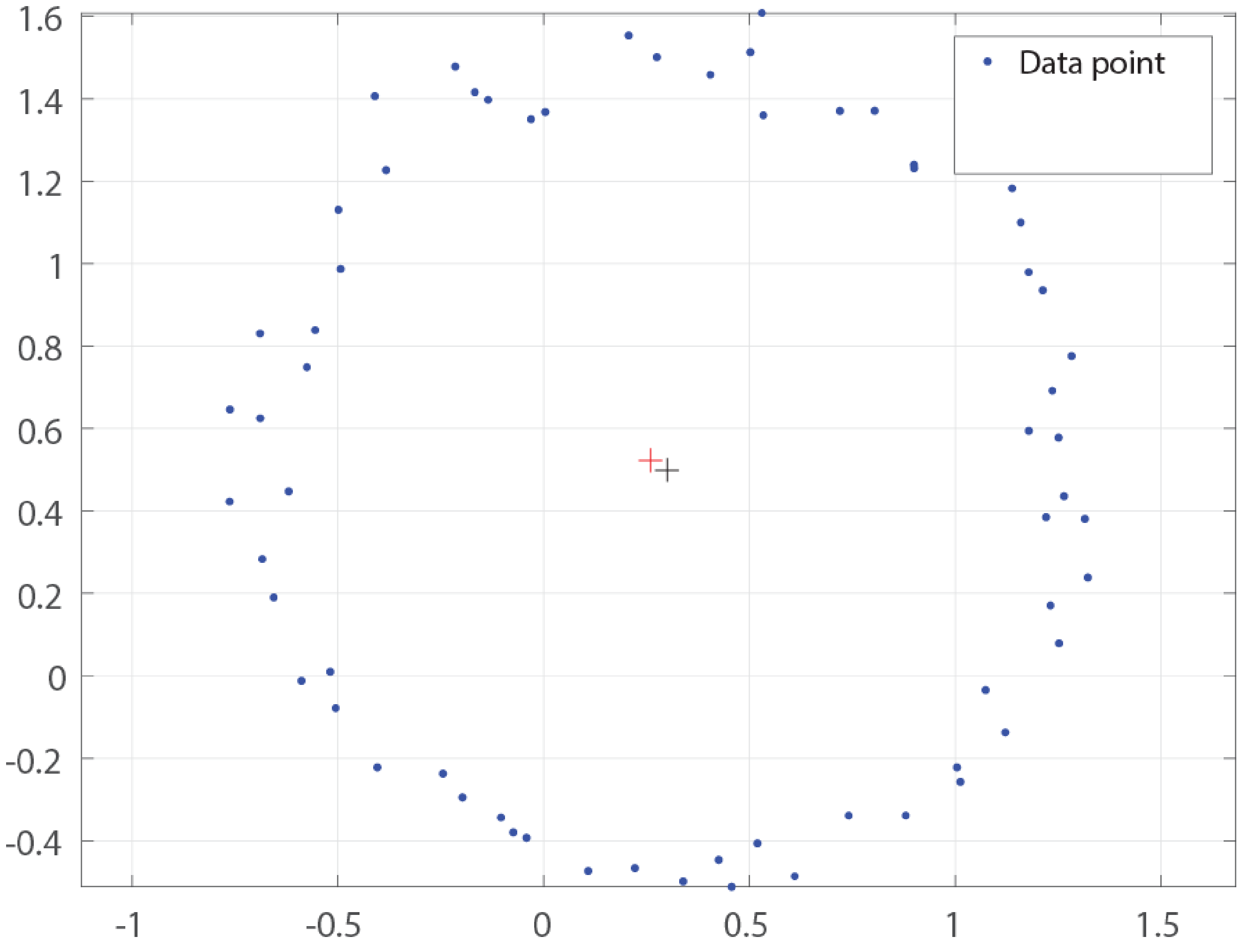
$$\downarrow$$

$$x_i^2 - x_1^2 - 2c_x\left(x_i - x_1\right) + y_i^2 - y_1^2 - 2\left(y_i - y_1\right)c_y = 0$$

$$\downarrow$$

$$\begin{bmatrix} 2\left(x_2 - x_1\right) & 2\left(y_2 - y_1\right) \\ \vdots & \vdots \\ 2\left(x_n - x_1\right) & 2\left(y_n - y_1\right) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \\ x_n^2 - x_1^2 + y_n^2 - y_1^2 \end{bmatrix}$$

Circle Fitting (Ax=b)



$$x_1^2 - 2x_1 c_x + c_x^2 + y_1^2 - 2y_1 c_y + c_y^2 = r^2$$

$$\vdots$$

$$x_n^2 - 2x_n c_x + c_x^2 + y_n^2 - 2y_n c_y + c_y^2 = r^2$$

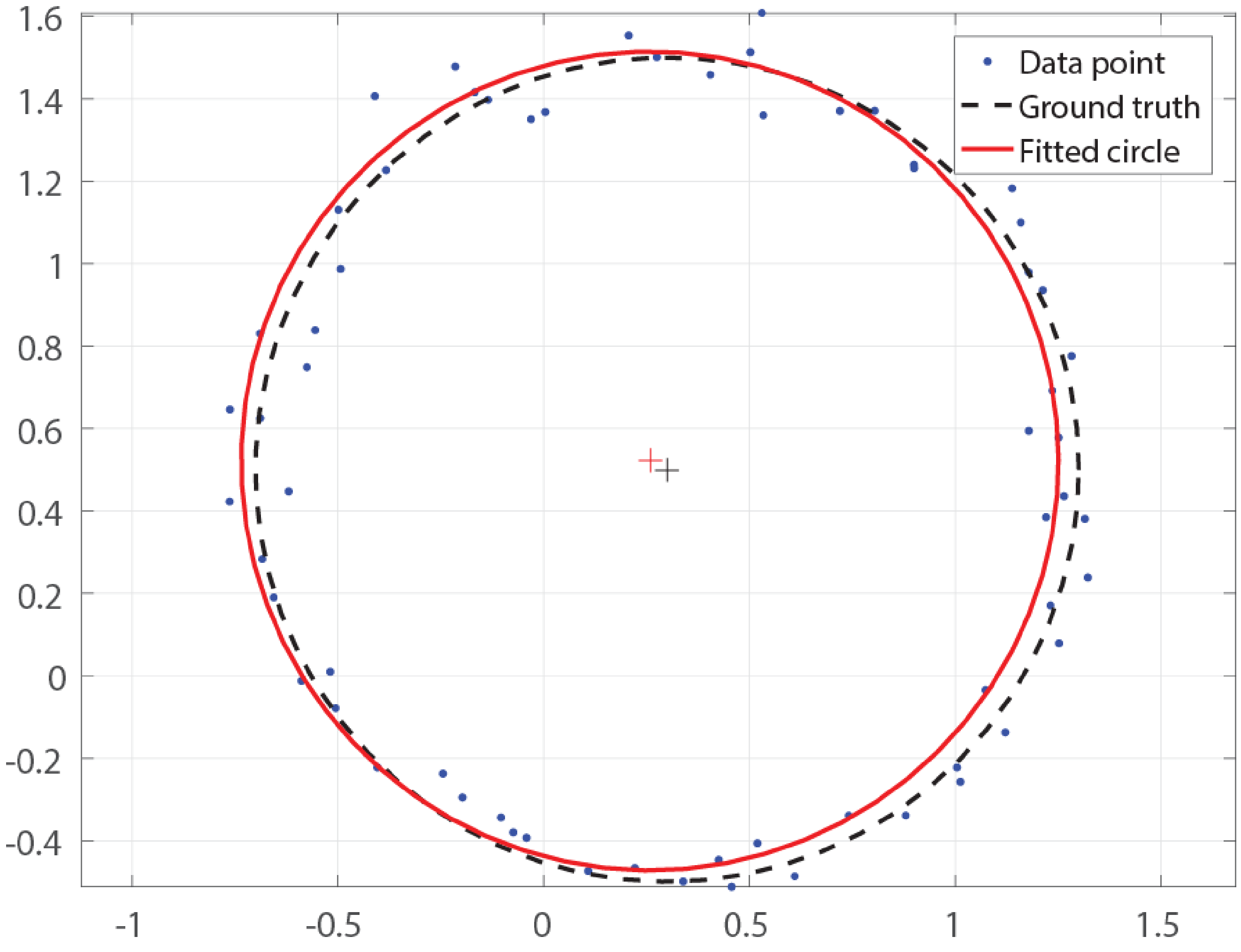
$$\downarrow$$

$$x_i^2 - x_1^2 - 2c_x\left(x_i - x_1\right) + y_i^2 - y_1^2 - 2\left(y_i - y_1\right)c_y = 0$$

$$\downarrow$$

$$\underbrace{\begin{bmatrix} 2\left(x_2 - x_1\right) & 2\left(y_2 - y_1\right) \\ \vdots & \vdots \\ 2\left(x_n - x_1\right) & 2\left(y_n - y_1\right) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} c_x \\ c_y \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \vdots \\ x_n^2 - x_1^2 + y_n^2 - y_1^2 \end{bmatrix}}_{\mathbf{b}}$$

Recall: Circle Fitting (Ax=b)



$$x_1^2 - 2x_1 c_x + c_x^2 + y_1^2 - 2y_1 c_y + c_y^2 = r^2$$

$$\vdots$$

$$x_n^2 - 2x_n c_x + c_x^2 + y_n^2 - 2y_n c_y + c_y^2 = r^2$$

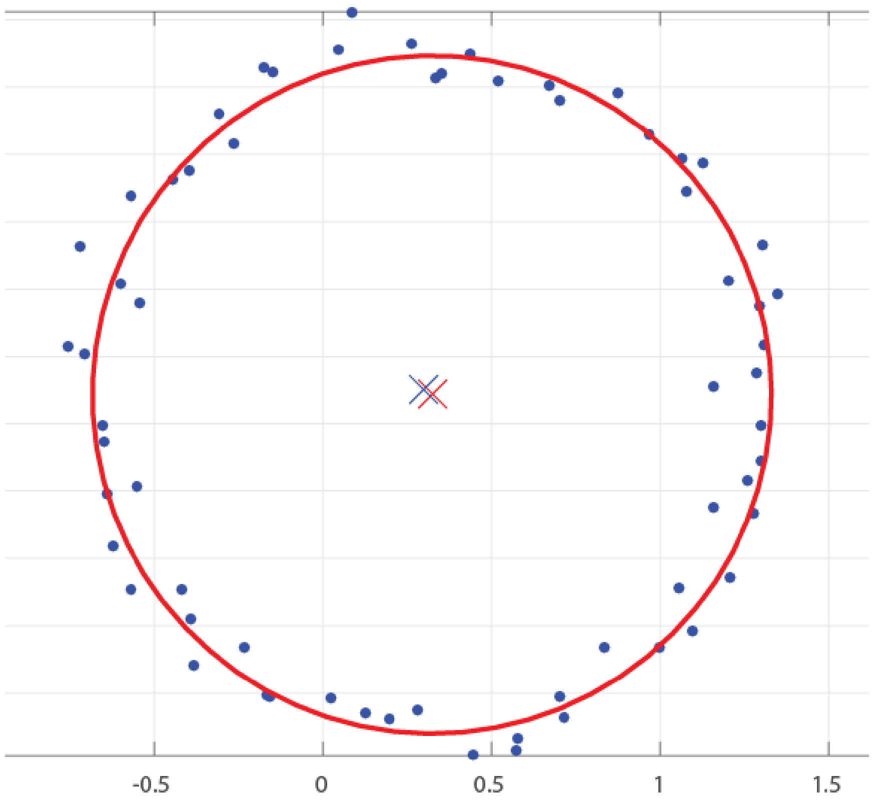
$$\downarrow$$

$$x_i^2 - x_1^2 - 2c_x\left(x_i - x_1\right) + y_i^2 - y_1^2 - 2\left(y_i - y_1\right)c_y = 0$$

$$\downarrow$$

$$\underbrace{\begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ \vdots & \vdots \\ 2(x_n - x_1) & 2(y_n - y_1) \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} c_x \\ c_y \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \vdots \\ x_n^2 - x_1^2 + y_n^2 - y_1^2 \end{bmatrix}}_{\mathbf{b}}$$

Circle Fitting RANSAC



$$\begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ \vdots & \vdots \\ 2(x_n - x_1) & 2(y_n - y_1) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 \\ \\ x_n^2 - x_1^2 + y_n^2 - y_1^2 \end{bmatrix}$$

Download **RANSAC_circle.m**

```
function RANSAC_circle

cx = 0.3;
cy = 0.5;
r = 1;

theta = 0:0.1:2*pi+0.1;
theta = theta';
x = cos(theta) + cx + 0.05*randn(size(theta));
y = sin(theta) + cy + 0.05*randn(size(theta));

figure(1)
clf;
plot(x,y, 'b.');
axis equal
grid on

nRansacIter = 500;
threshold = 0.1;
max_nInliers = 0;
%% RANSAC circle
% U: Estimated center of circle
% R: Radius

%%
hold on
plot(R*cos(theta)+U(1), R*sin(theta)+U(2), 'r-');
hold on
plot(U(1), U(2), 'rx');
hold on
plot(cx, cy, 'bx');
```

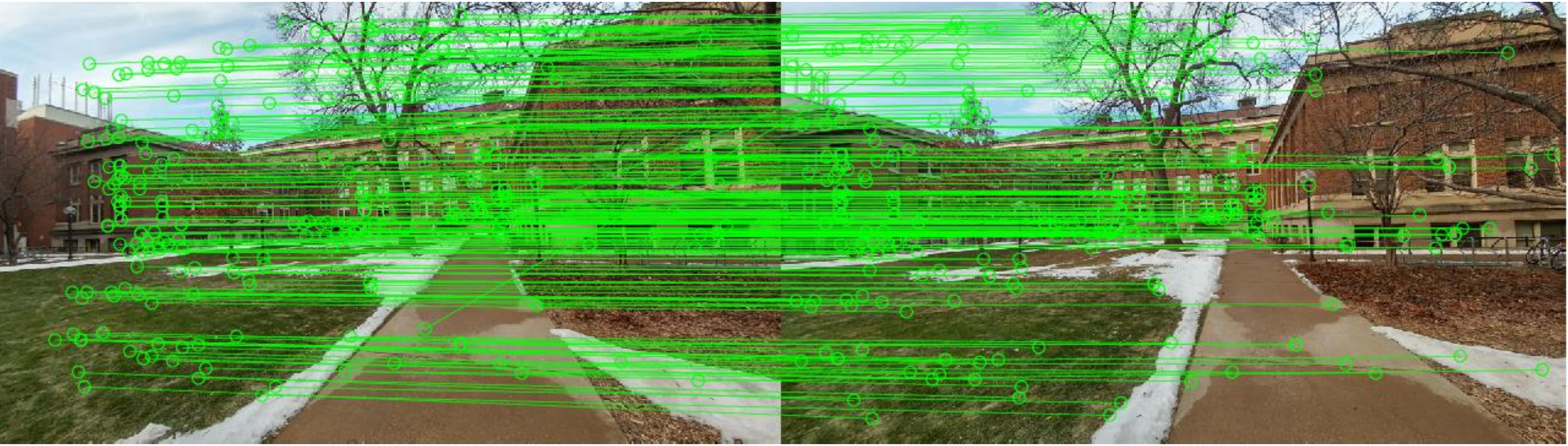
Fill out this part

HW #4 RANSAC Fundamental matrix

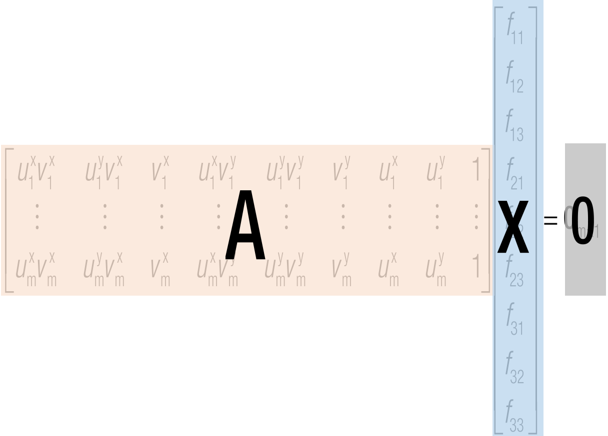
Algorithm 1 GetInliersRANSAC

$n \leftarrow 0$
for $i = 1 : M$ **do**
  Choose 8 correspondences, $\mathbf{u}_r$ and $\mathbf{v}_r$, randomly from $\mathbf{u}$ and $\mathbf{v}$.
  $\mathbf{F}_r$ = `ComputeFundamentalMatrix(`$\mathbf{u}_r$`, `$\mathbf{v}_r$`)`
  Compute the number of inliers, n_r, with respect to `F`.
  if $n_r > n$ **then**
    $n \leftarrow n_r$
    $\mathbf{F} = \mathbf{F}_r$
  end if
end for

Fundamental Matrix Computation via RANSAC



Fundamental Matrix Computation via RANSAC



Fundamental Matrix Computation via RANSAC

Epipolar line: $\mathbf{l}_{\mathbf{u}} = \mathbf{F}\mathbf{u}$

Distance: $d = \frac{|au_x + bu_y + c|}{\sqrt{a^2 + b^2}} = \frac{|\mathbf{F}\mathbf{u}|}{\sqrt{(\mathbf{F}_{1,:}\mathbf{u})^2 + (\mathbf{F}_{2,:}\mathbf{u})^2}}$

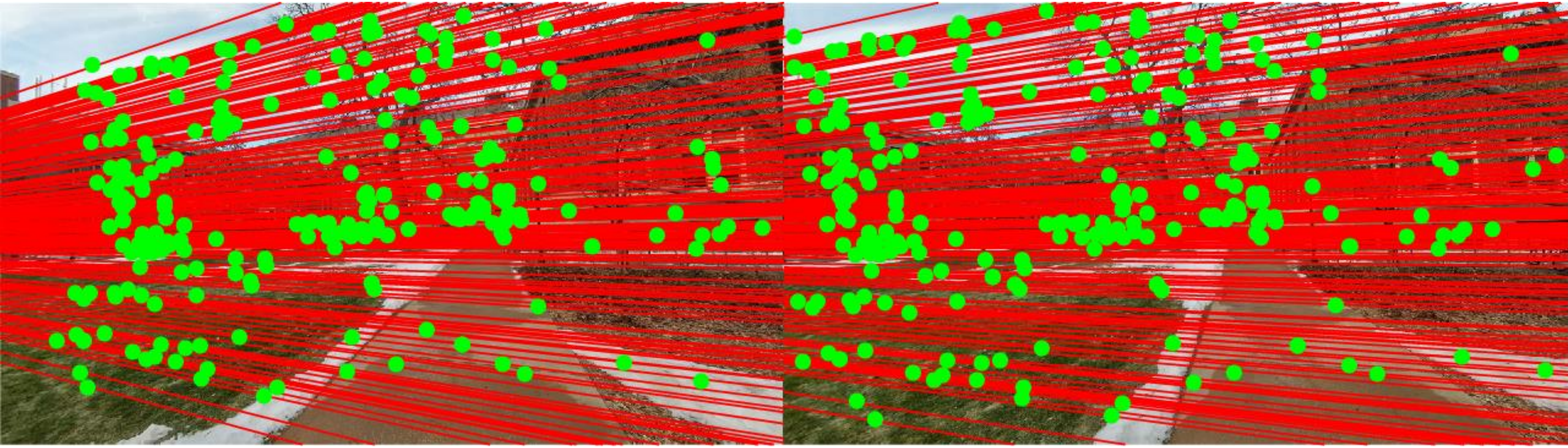
Fundamental Matrix Computation via RANSAC

# of inliers: 65 out of 260

Fundamental Matrix Computation via RANSAC

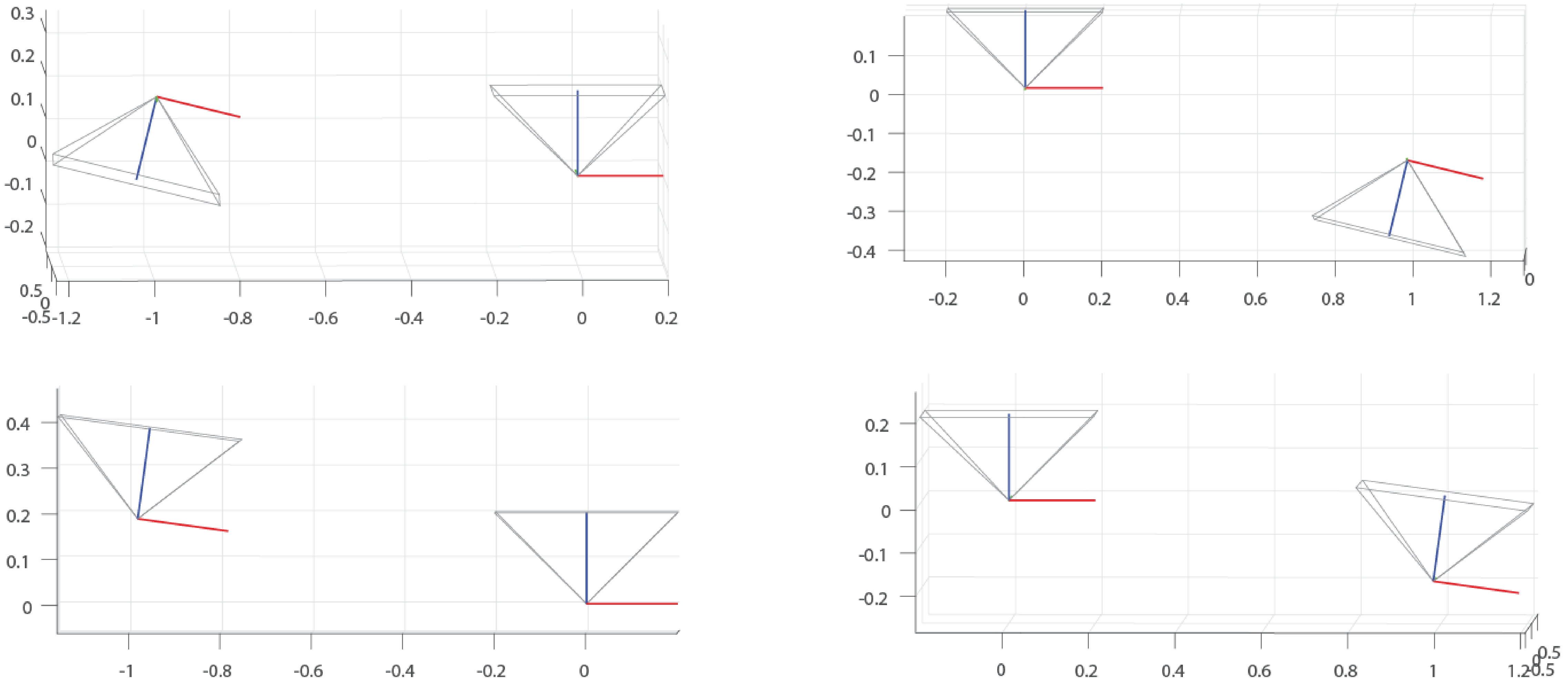
# of inliers: 65 out of 260

Fundamental Matrix Computation via RANSAC



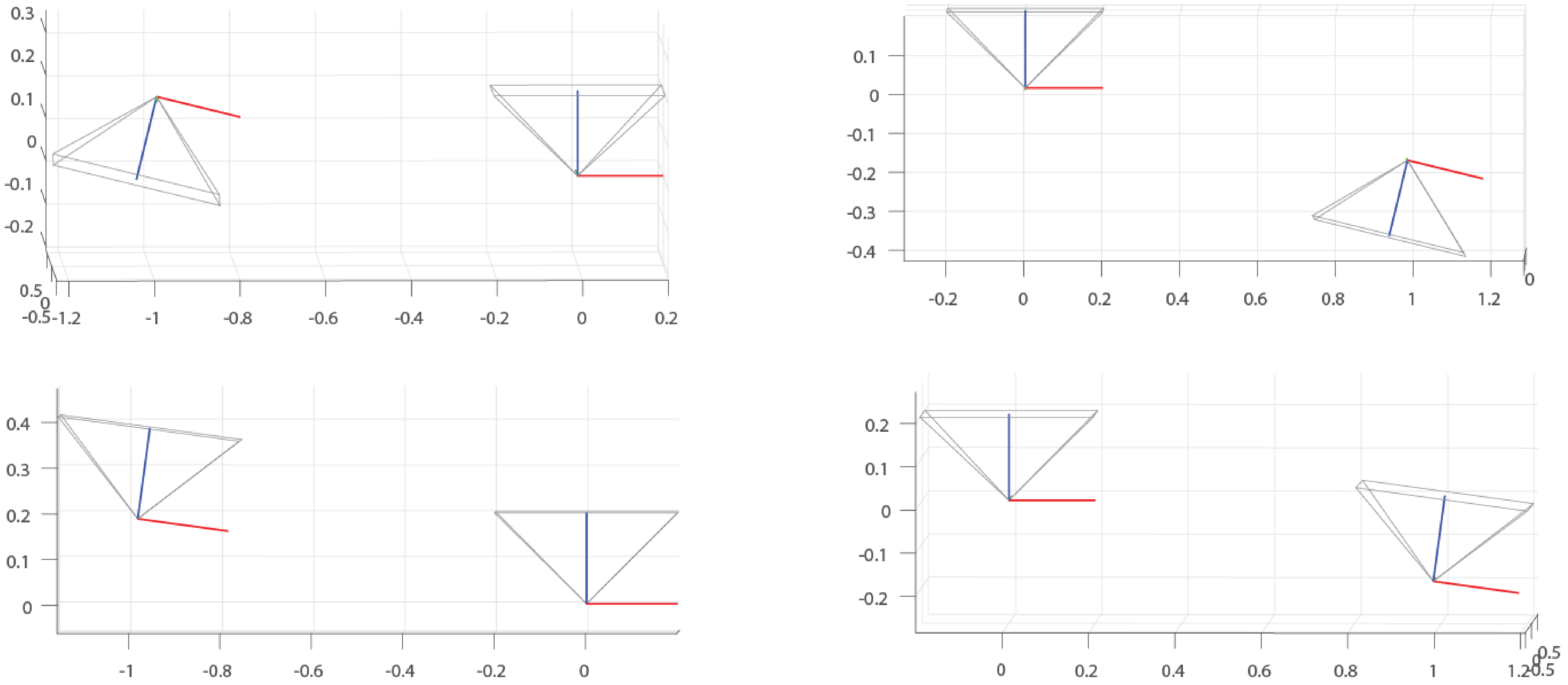
# of inliers: 186 out of 260

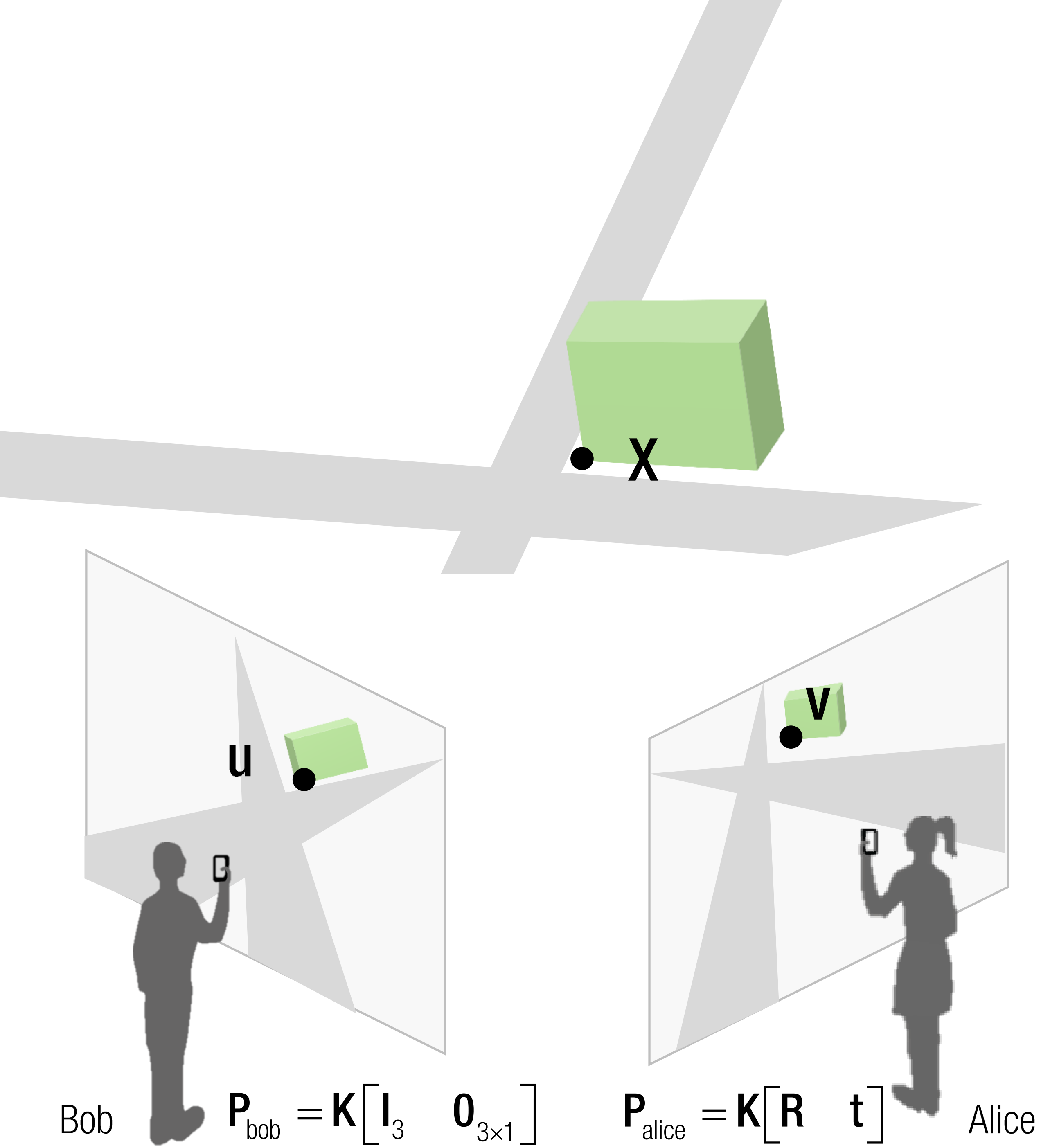
Four Camera Pose Config. From Essential Matrix



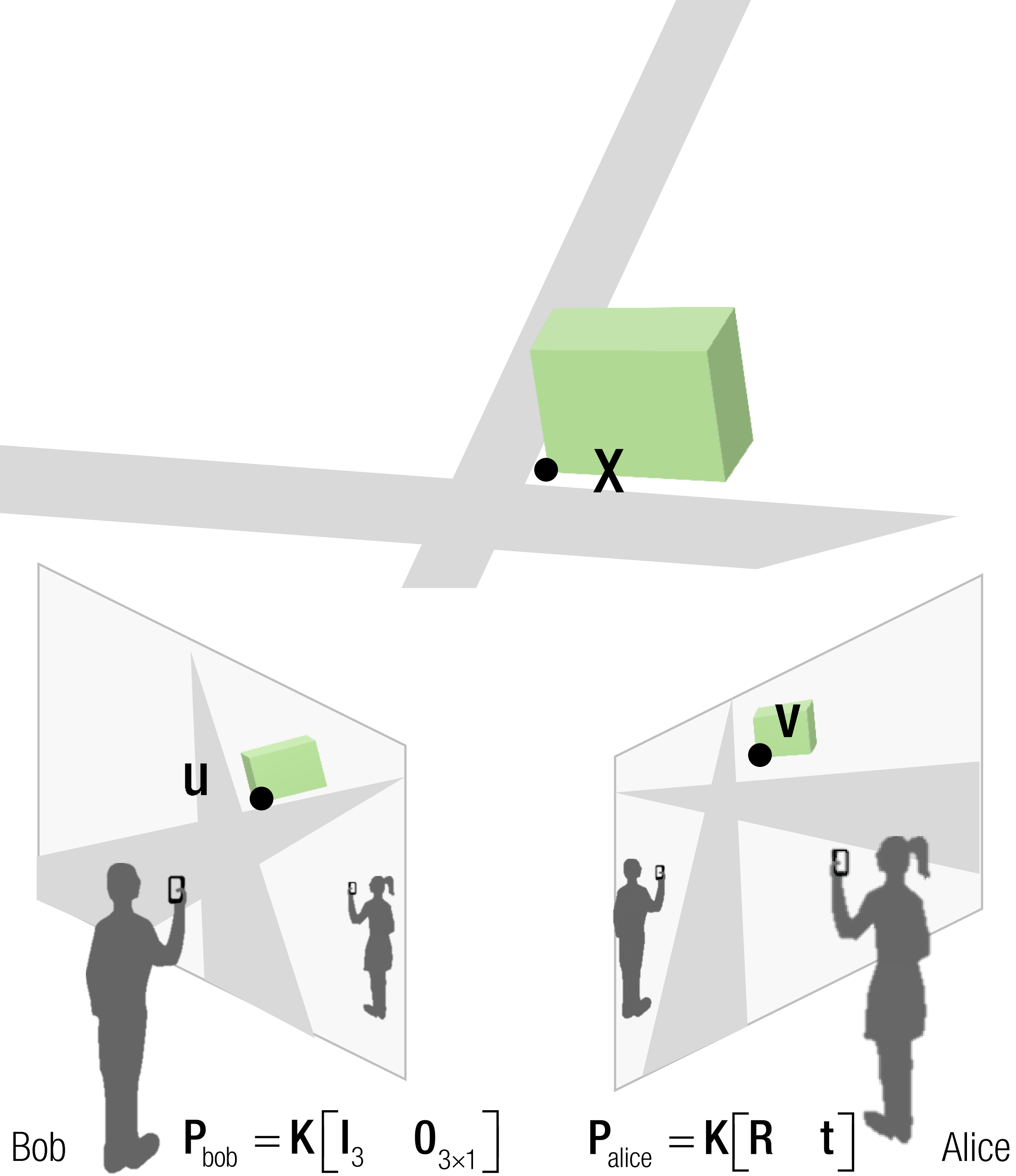
Triangulation

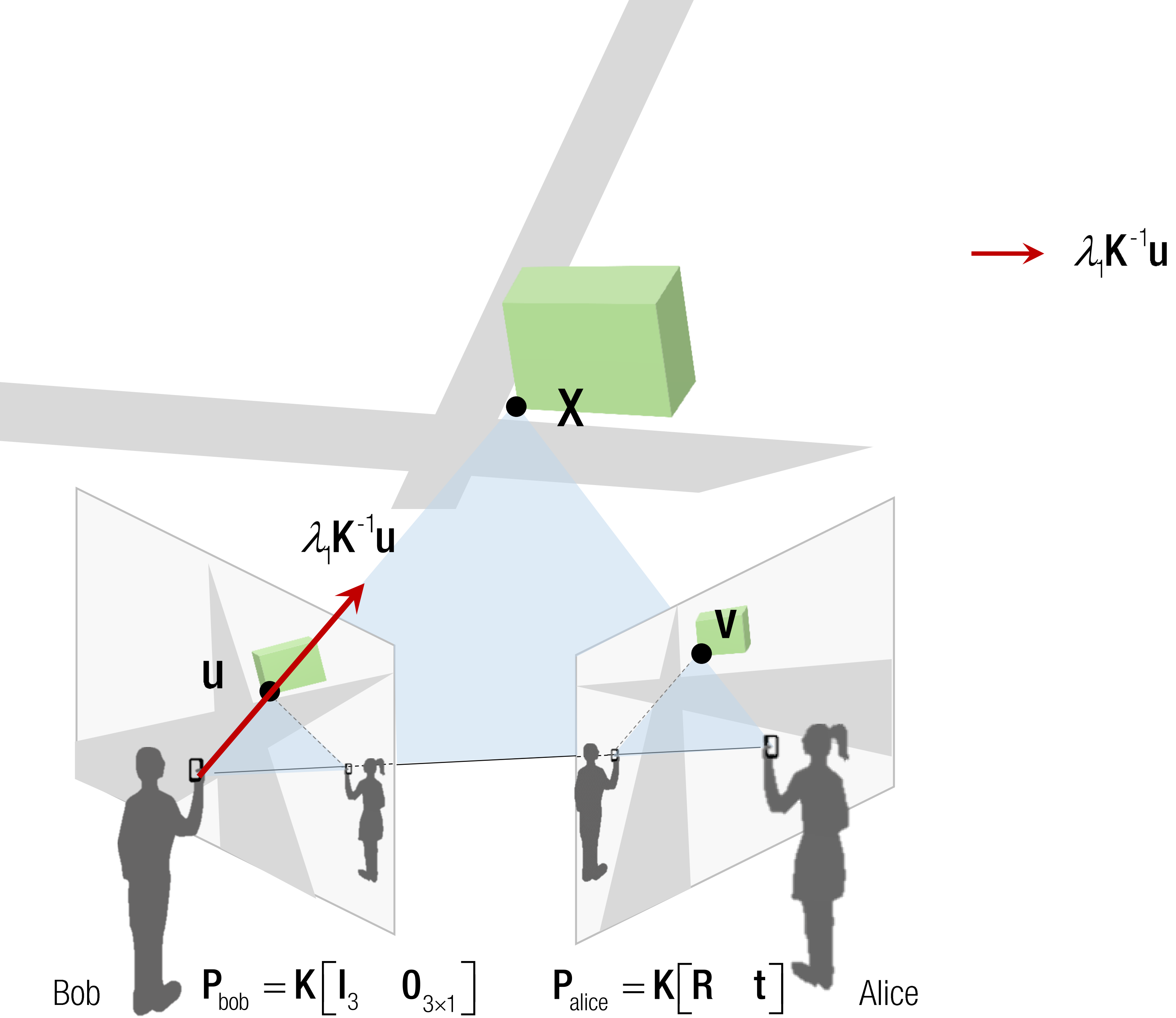
How to Disambiguate?

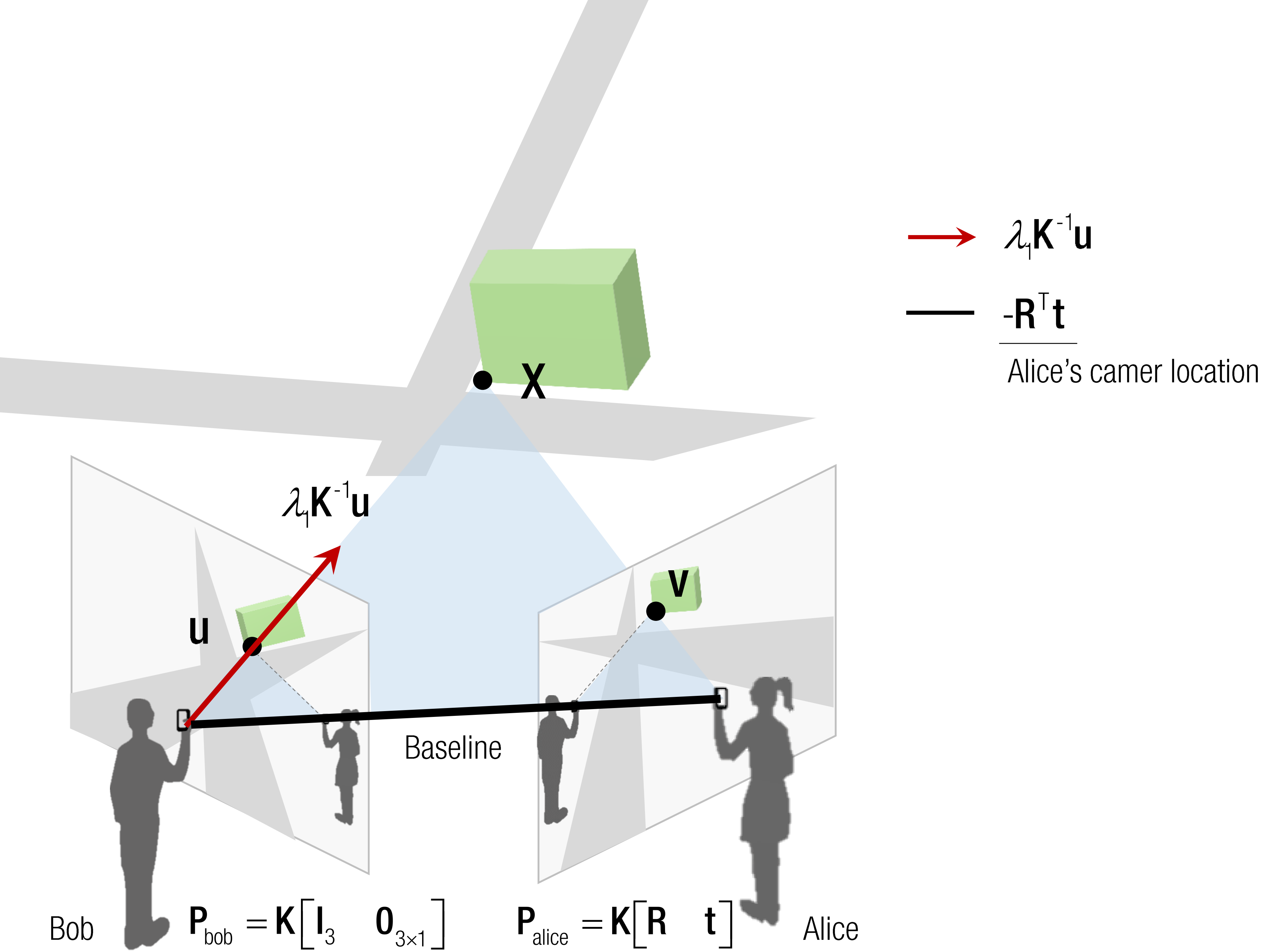




X
u
v
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice

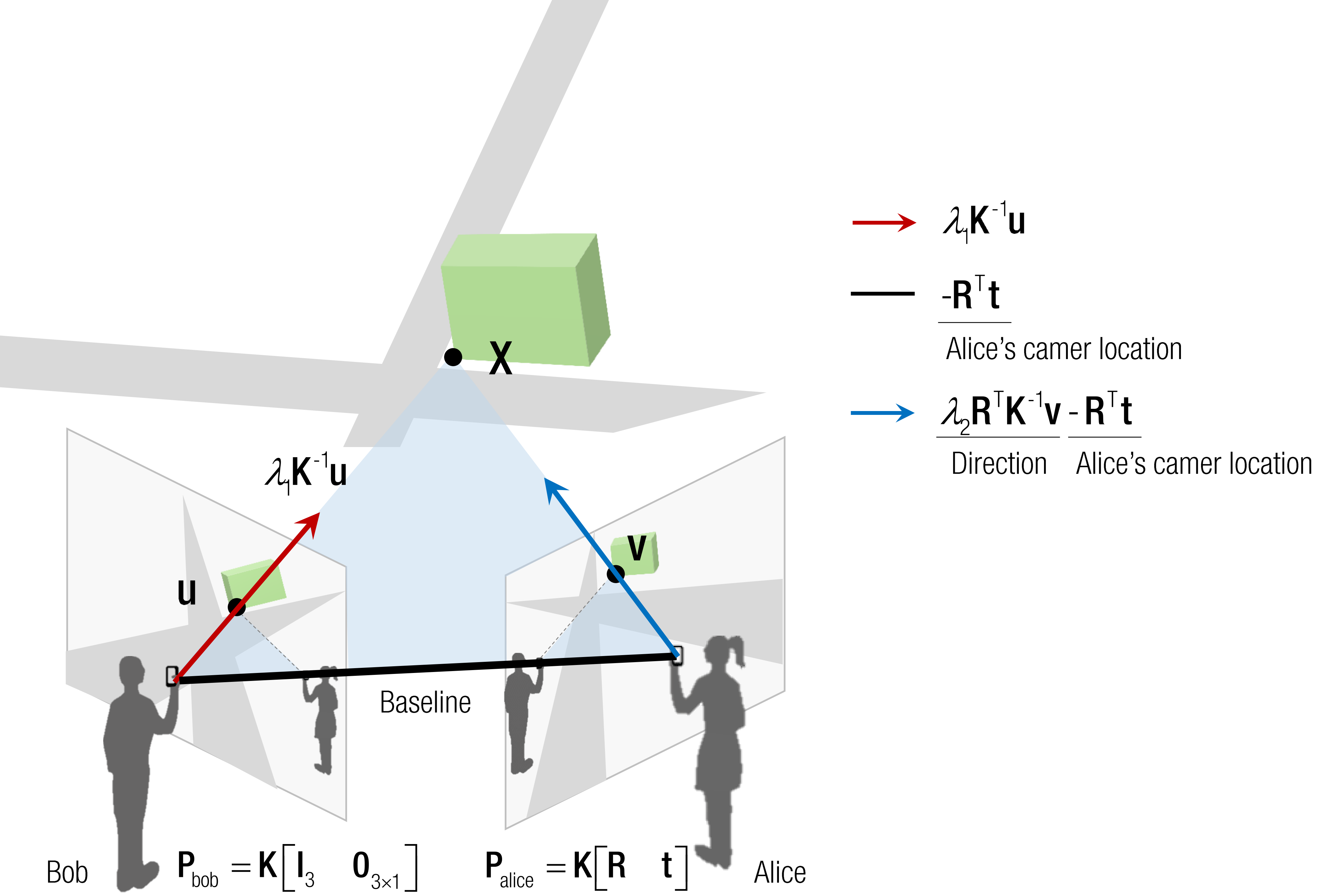

X
u
v
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice


X
$\lambda_1 K^{-1} u$
u
v
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice
$\lambda_1 K^{-1} u$

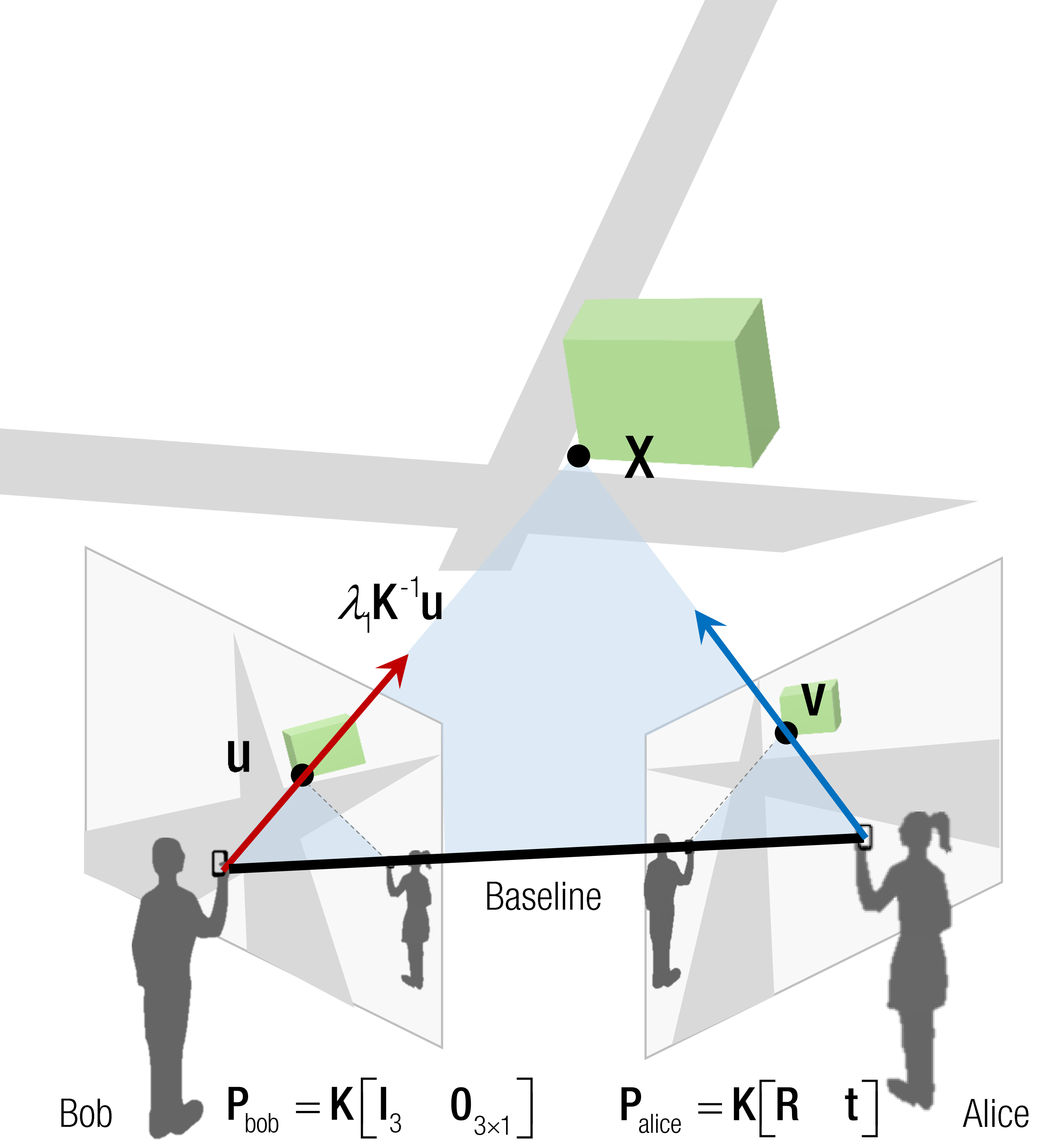


X
$\lambda_1 K^{-1} u$
u
v
Baseline
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice

$\lambda_1 K^{-1} u$
$-R^T t$
Alice's camer location



X
$\lambda_1 K^{-1} u$
u
v
Baseline
Bob
$P_{bob} = K[I_3 \quad 0_{3\times1}]$
$P_{alice} = K[R \quad t]$
Alice
$\lambda_1 K^{-1} u$
$-R^T t$
Alice's camer location
$\lambda_2 R^T K^{-1} v - R^T t$
Direction
Alice's camer location



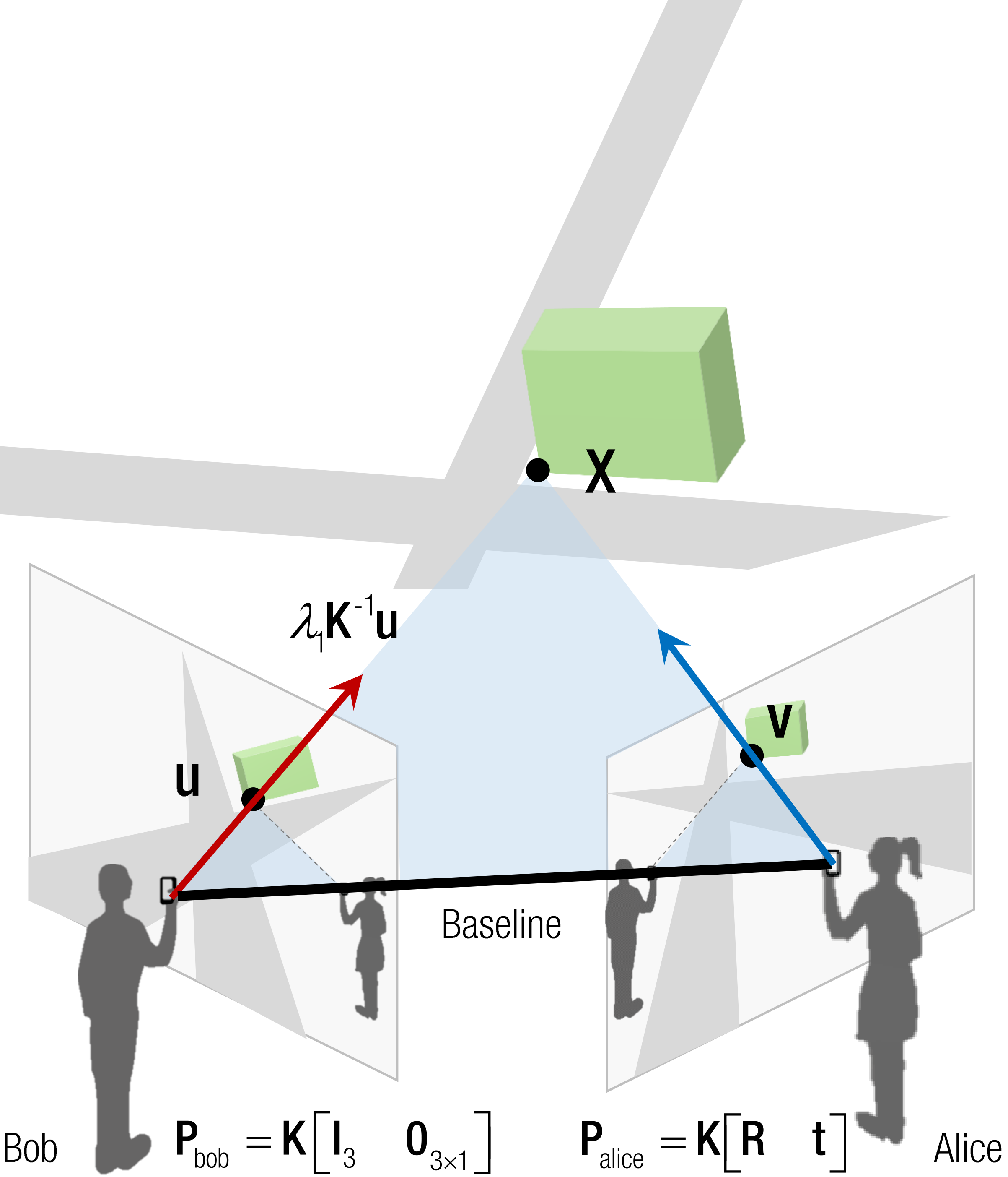
$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$

$\underbrace{-\mathbf{R}^{T}\mathbf{t}}_{\text{Alice's camer location}}$

$\underbrace{\lambda_2 \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}} \underbrace{-\mathbf{R}^{T}\mathbf{t}}_{\text{Alice's camer location}}$

$$\underbrace{\mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^{T}\mathbf{t}}_{\text{3D point}}$$

# of unknowns: 2
# of equations: 3



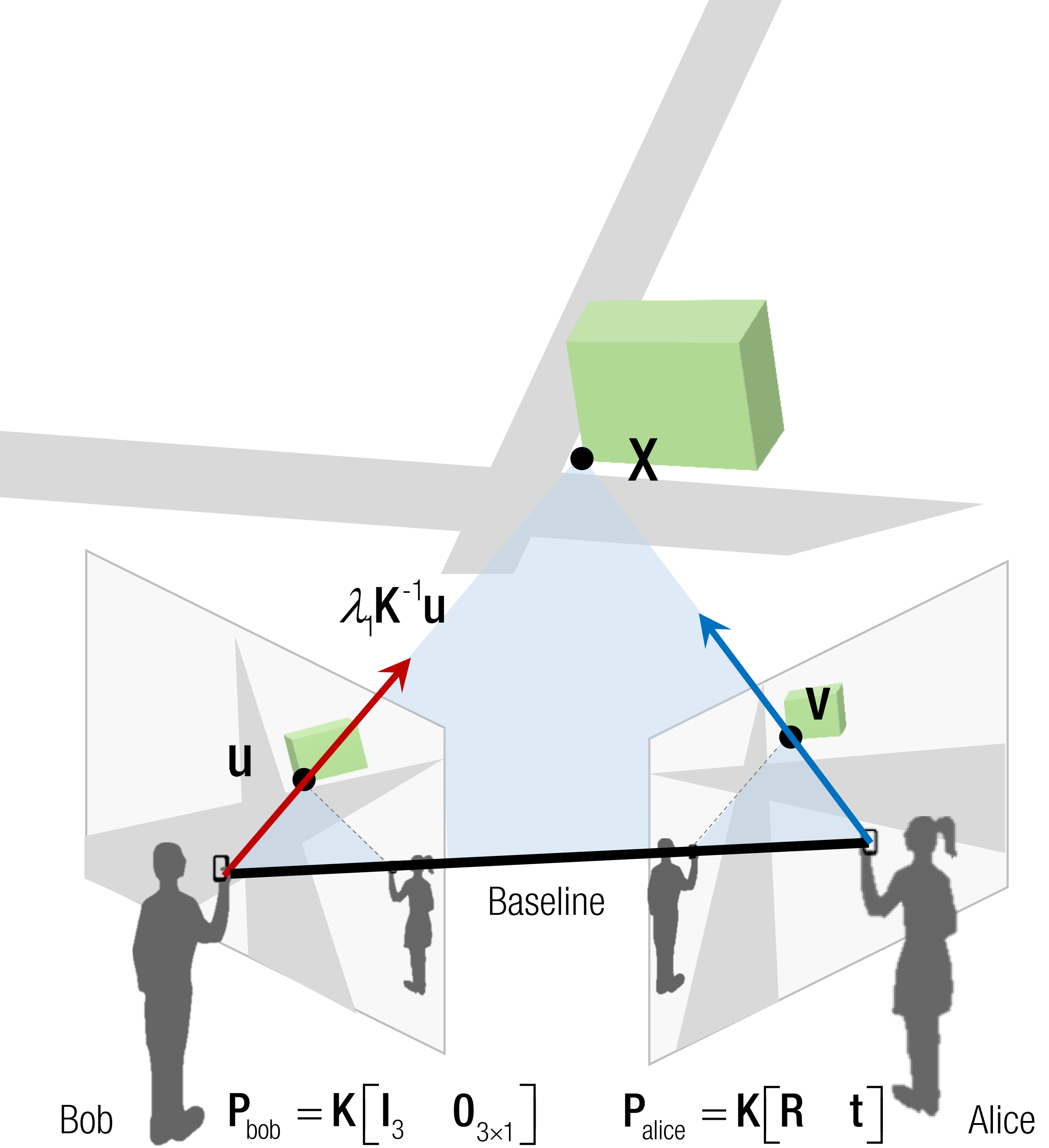
$\lambda_1 \mathbf{K}^{-1}\mathbf{u}$ (red arrow)

$\underbrace{-\mathbf{R}^{T}\mathbf{t}}_{\text{Alice's camer location}}$ (black line)

$\underbrace{\lambda_2 \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}} \underbrace{-\mathbf{R}^{T}\mathbf{t}}_{\text{Alice's camer location}}$ (blue arrow)

$$\underbrace{\mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^{T}\mathbf{t}}_{\text{3D point}}$$

$$\rightarrow \lambda_1 \mathbf{K}^{-1}\mathbf{u} - \lambda_2 \mathbf{R}^{T}\mathbf{K}^{-1}\mathbf{v} = -\mathbf{R}^{T}\mathbf{t}$$



→ $\lambda_1 \mathbf{K}^{-1}\mathbf{u}$

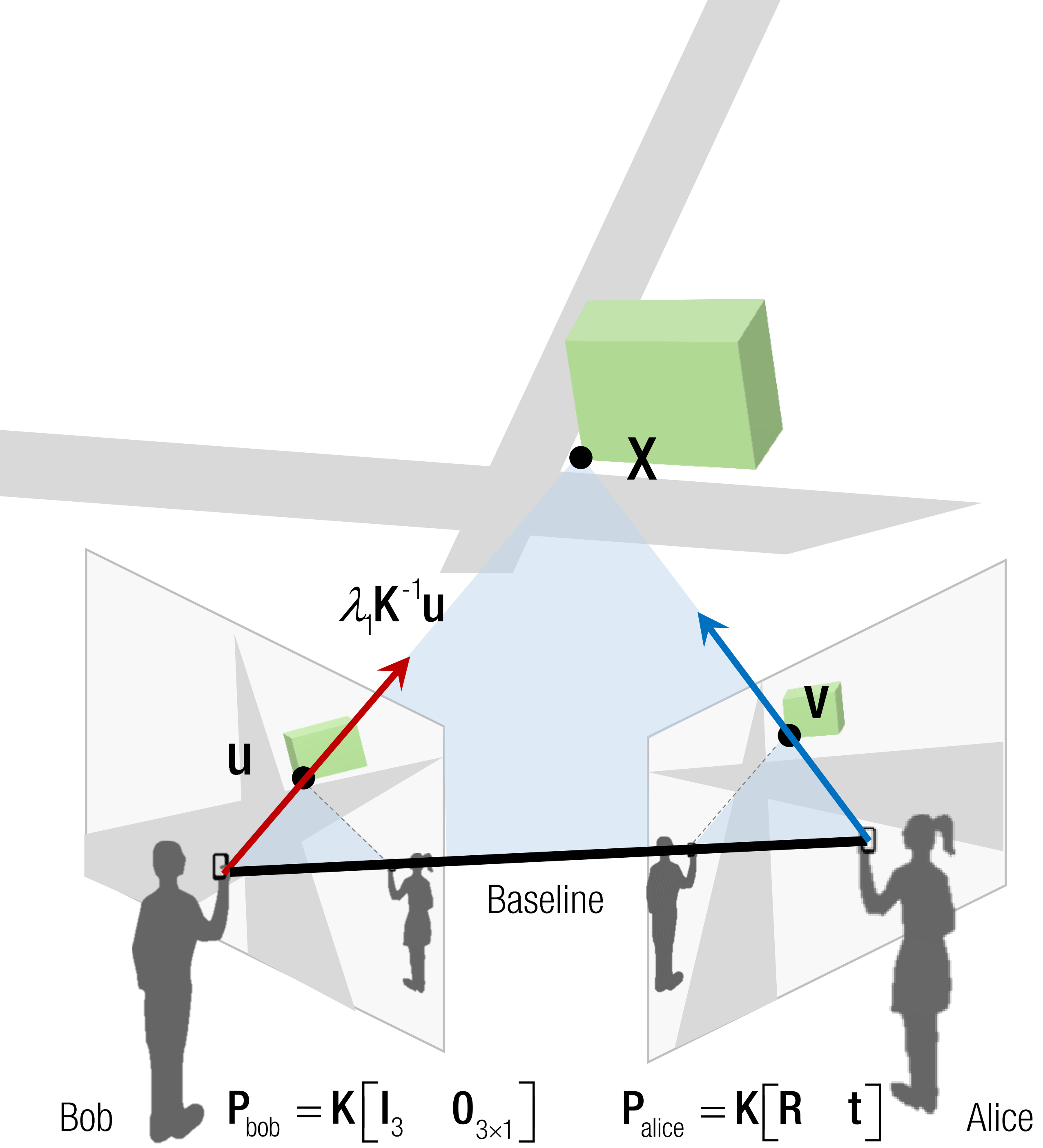
— $\underbrace{-\mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{Alice's camer location}}$

→ $\underbrace{\lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v}}_{\text{Direction}} \underbrace{-\mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{Alice's camer location}}$

$$\underbrace{\mathbf{X} = \lambda_1 \mathbf{K}^{-1}\mathbf{u} = \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} - \mathbf{R}^{\mathsf{T}}\mathbf{t}}_{\text{3D point}}$$

$$\rightarrow \lambda_1 \mathbf{K}^{-1}\mathbf{u} - \lambda_2 \mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} = -\mathbf{R}^{\mathsf{T}}\mathbf{t}$$

$$\rightarrow \begin{bmatrix} \mathbf{K}^{-1}\mathbf{u} & -\mathbf{R}^{\mathsf{T}}\mathbf{K}^{-1}\mathbf{v} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = -\mathbf{R}^{\mathsf{T}}\mathbf{t}$$



→ $\lambda_1 K^{-1}u$ (red arrow)

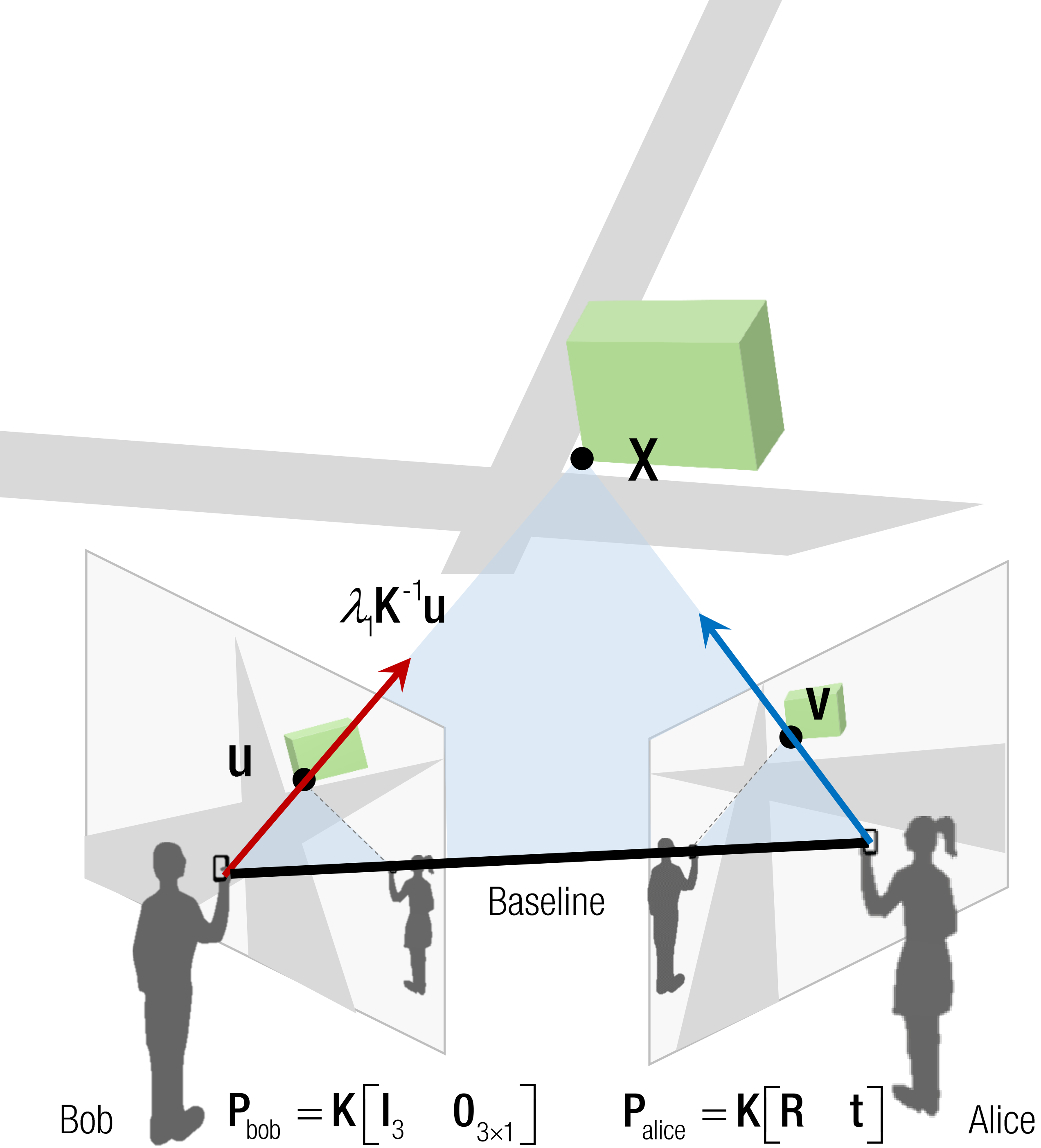
— $\underbrace{-R^Tt}_{\text{Alice's camer location}}$ (black line)

→ $\underbrace{\lambda_2 R^T K^{-1} v}_{\text{Direction}} \underbrace{- R^T t}_{\text{Alice's camer location}}$ (blue arrow)

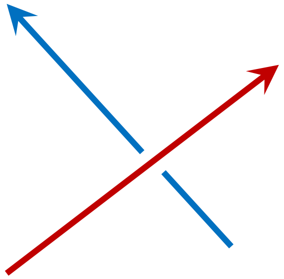
$$\underbrace{X = \lambda_1 K^{-1}u = \lambda_2 R^T K^{-1} v - R^T t}_{\text{3D point}}$$

$$\rightarrow \lambda_1 K^{-1}u - \lambda_2 R^T K^{-1} v = -R^T t$$

$$\rightarrow \underbrace{\begin{bmatrix} K^{-1}u & R^T K^{-1} v \end{bmatrix}}_{A \;(3\text{x}2)} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}}_{x} = \underbrace{-R^T t}_{b}$$



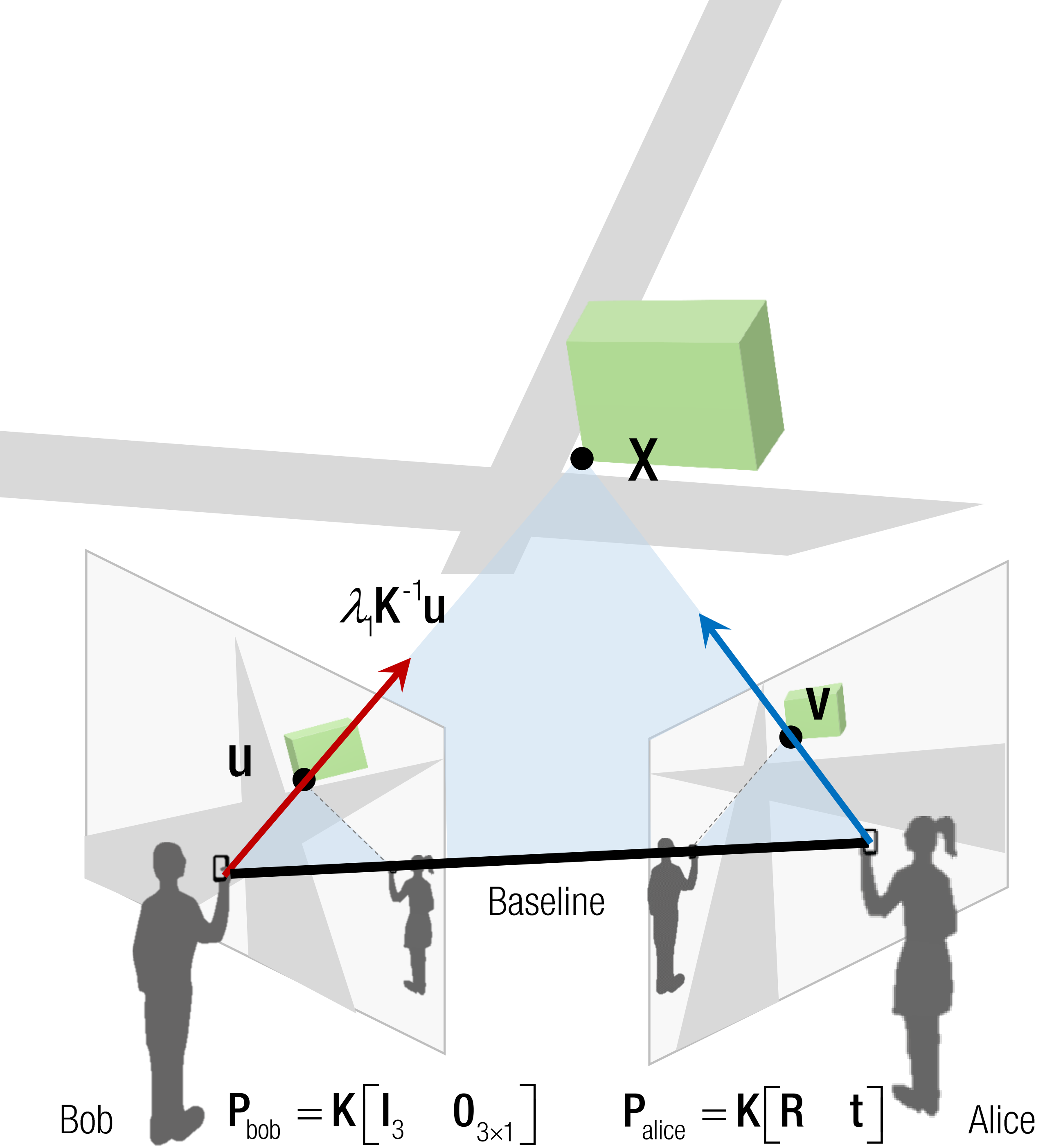
What if two does not meet at a point?



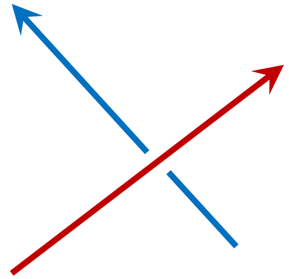
$$\rightarrow \begin{bmatrix} K^{-1}u & R^{T}K^{-1}v \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = -R^{T}t$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

3x2



What if two does not meet at a point?



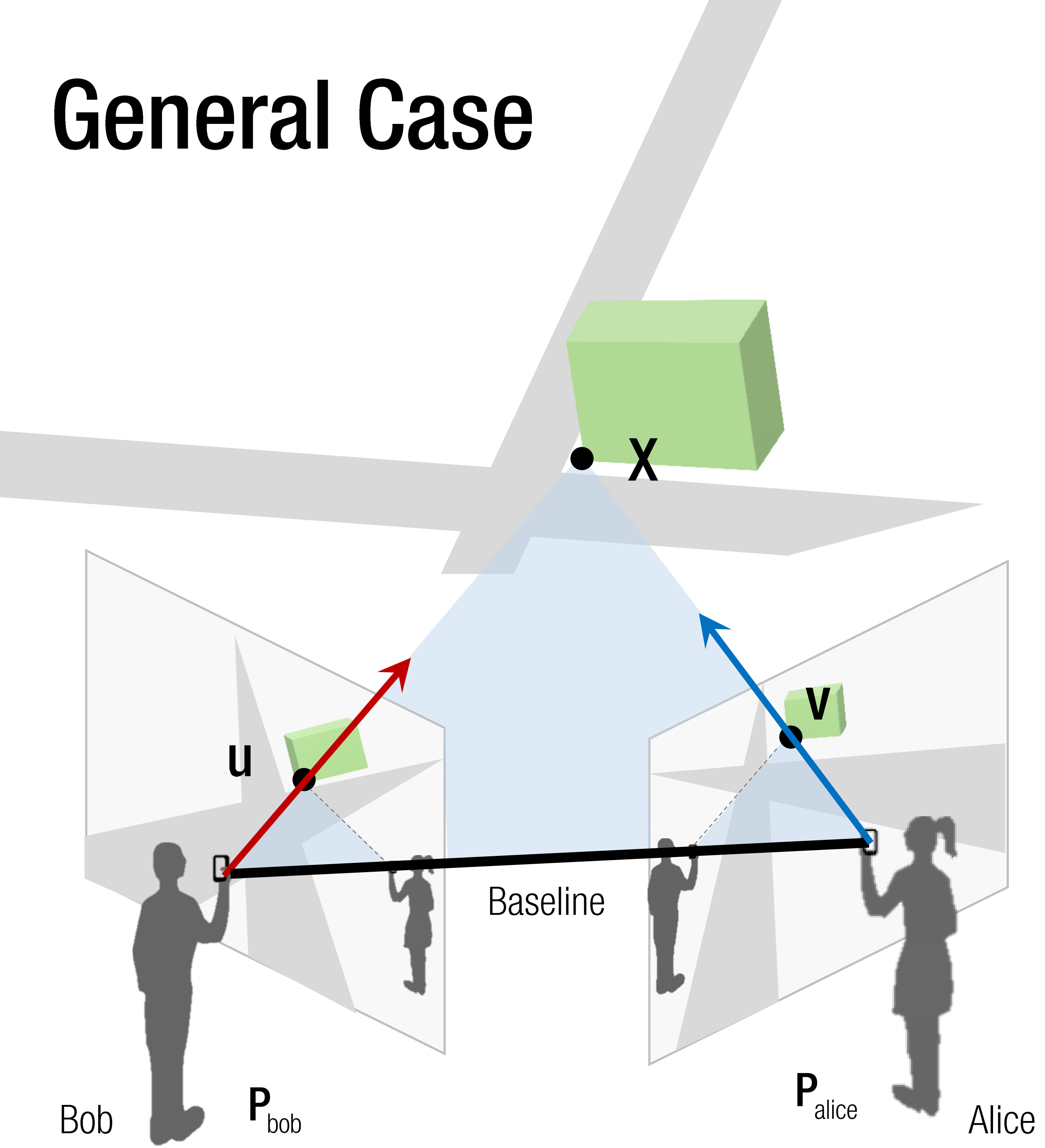
Least square solution finds *somewhere* in the middle.

$$\rightarrow \quad \begin{bmatrix} K^{-1}u & R^{T}K^{-1}v \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = -R^{T}t$$

$$A \, x = b$$

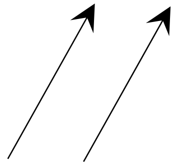
3x2

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



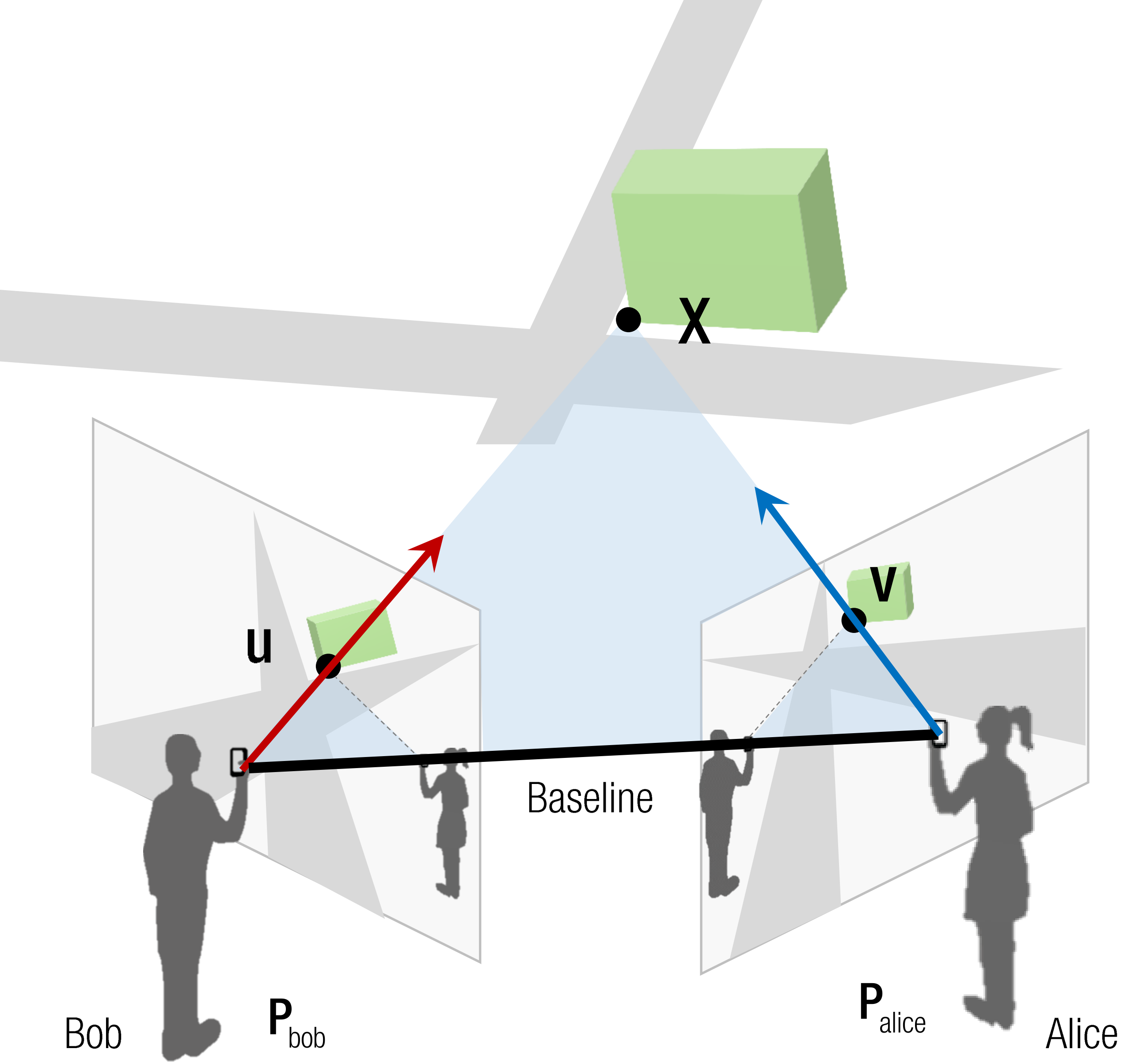
Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

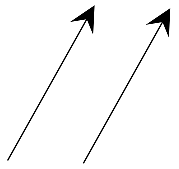
Skew-symmetric matrix

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

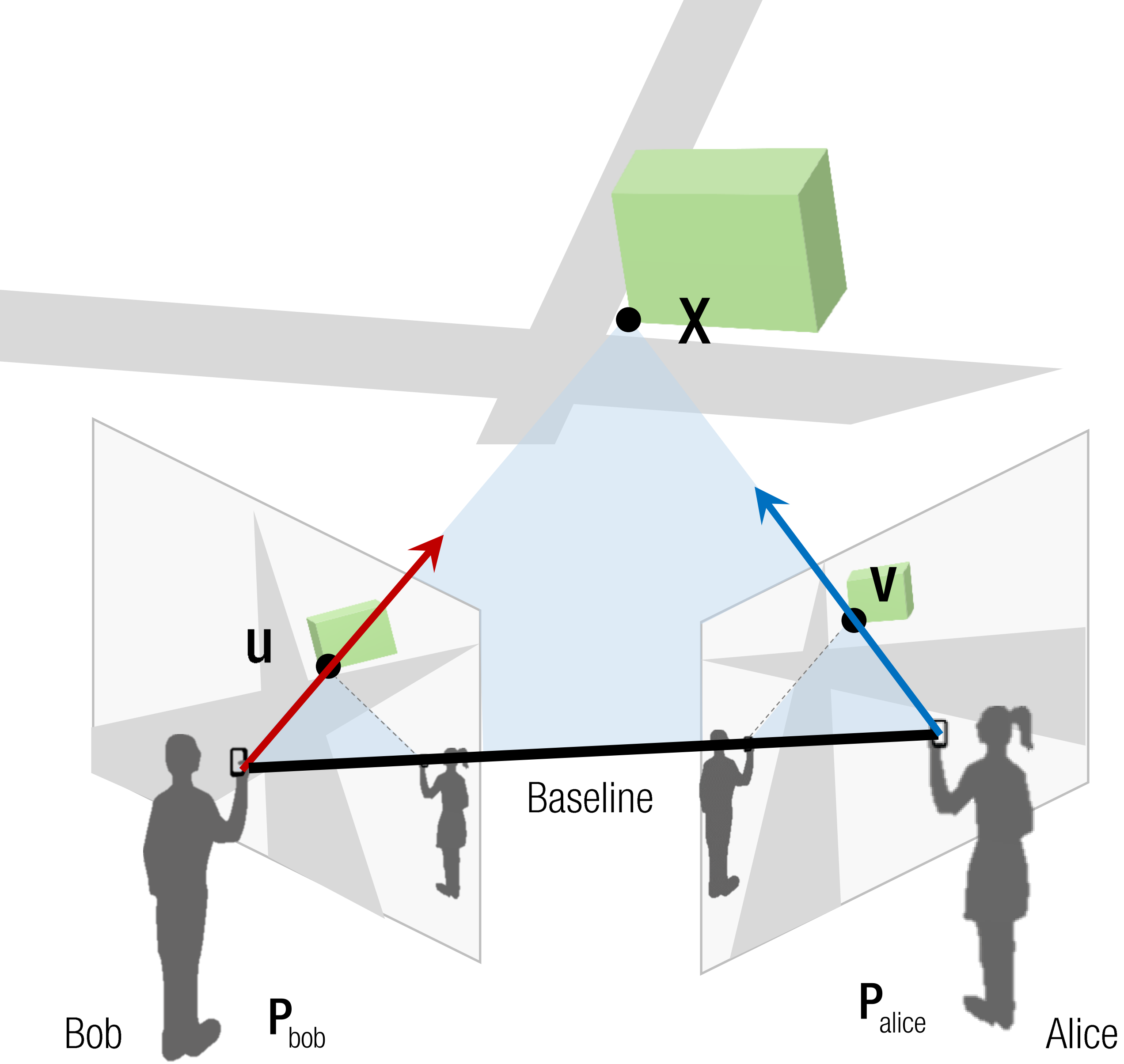
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

Skew-symmetric matrix

: Knowns

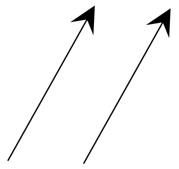
: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

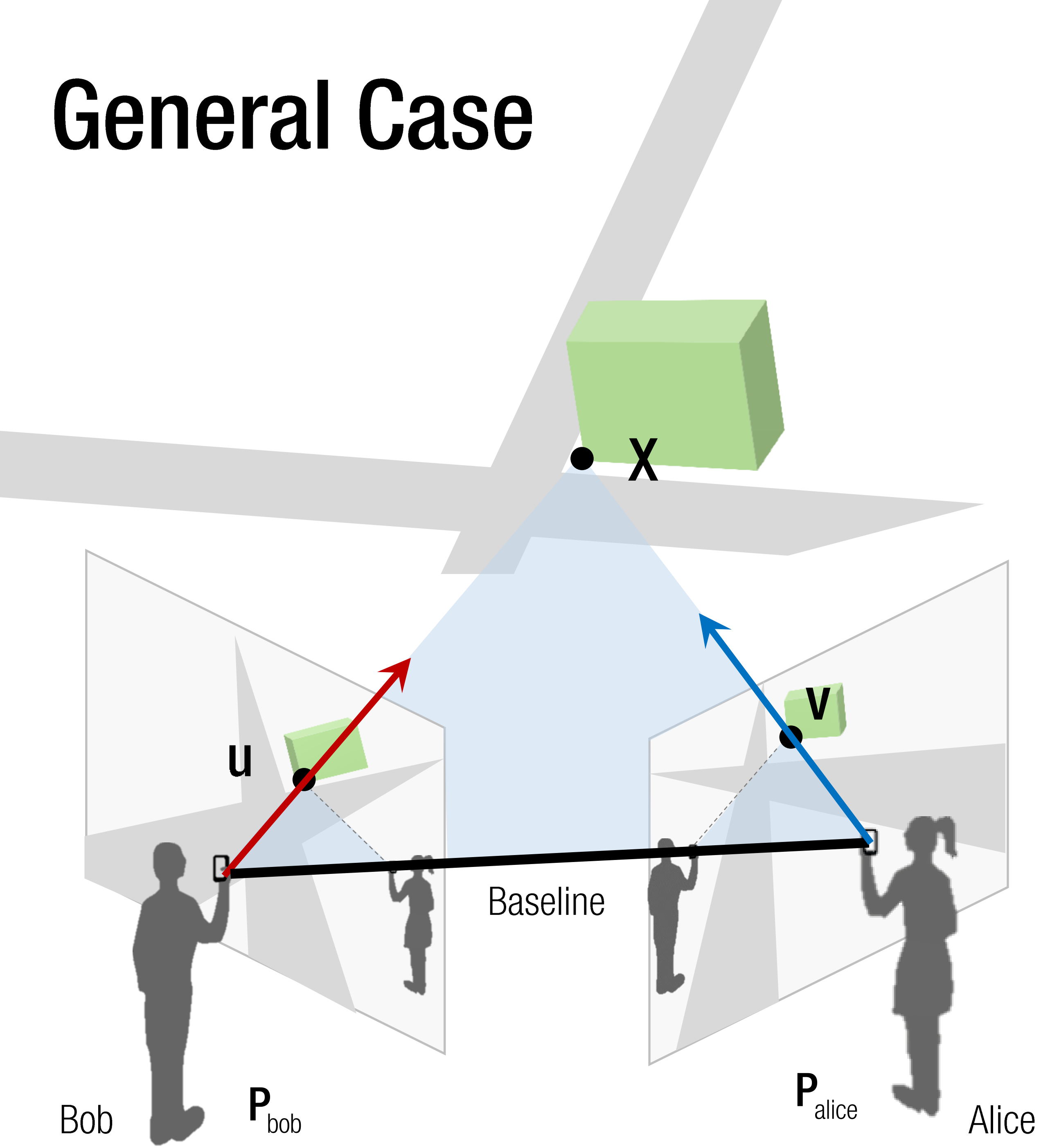
3x4

: Knowns

: Unknowns

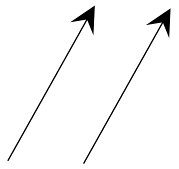
Can we solve for **X**? (single view reconstruction)
Why not?

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

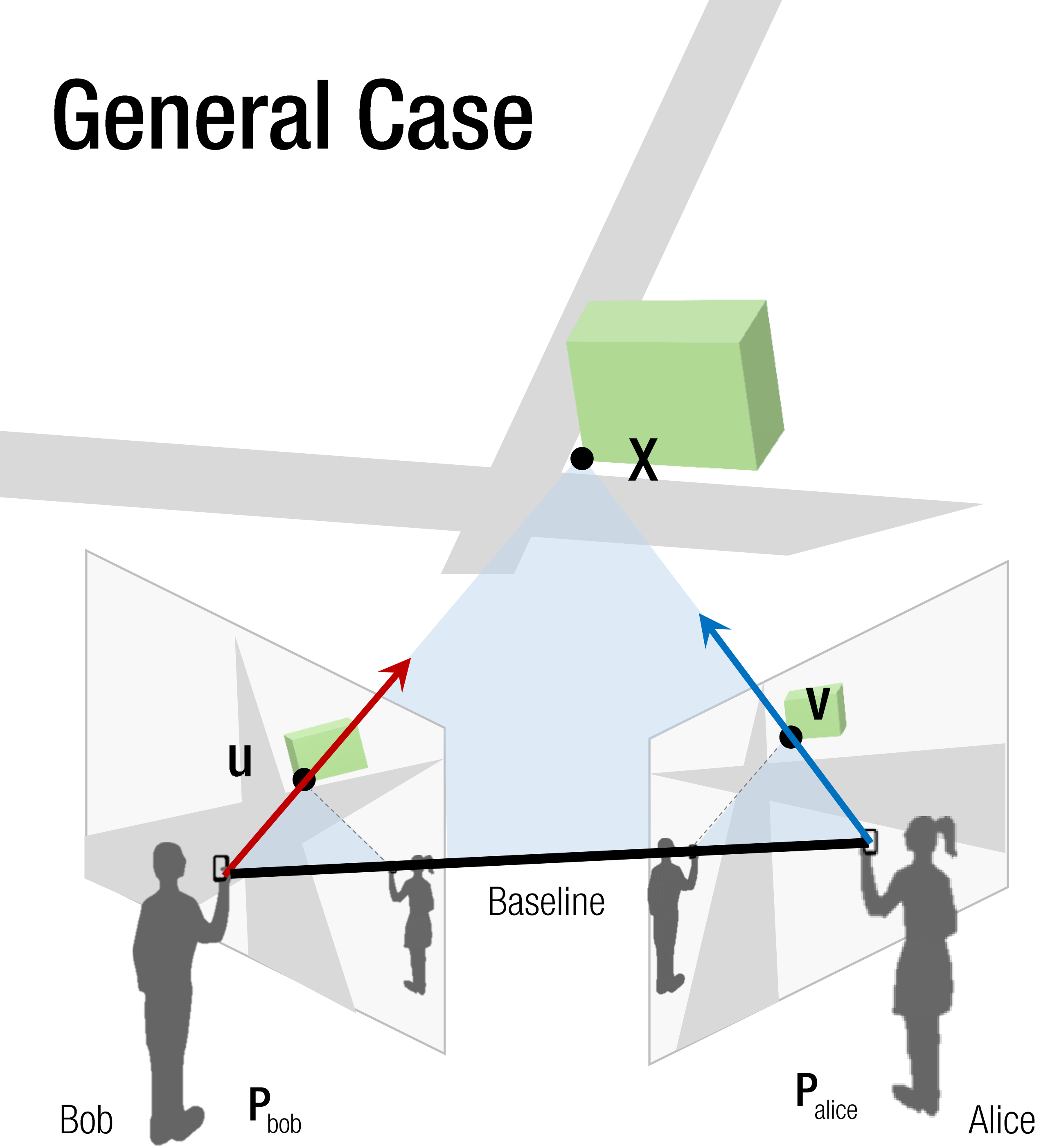
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

2x4

: Knowns

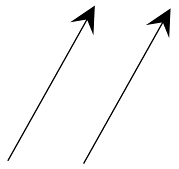
: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} u \\ 1 \end{bmatrix} = P_{bob} \begin{bmatrix} X \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} u \\ 1 \end{bmatrix} \times P_{bob} \begin{bmatrix} X \\ 1 \end{bmatrix} = 0$$

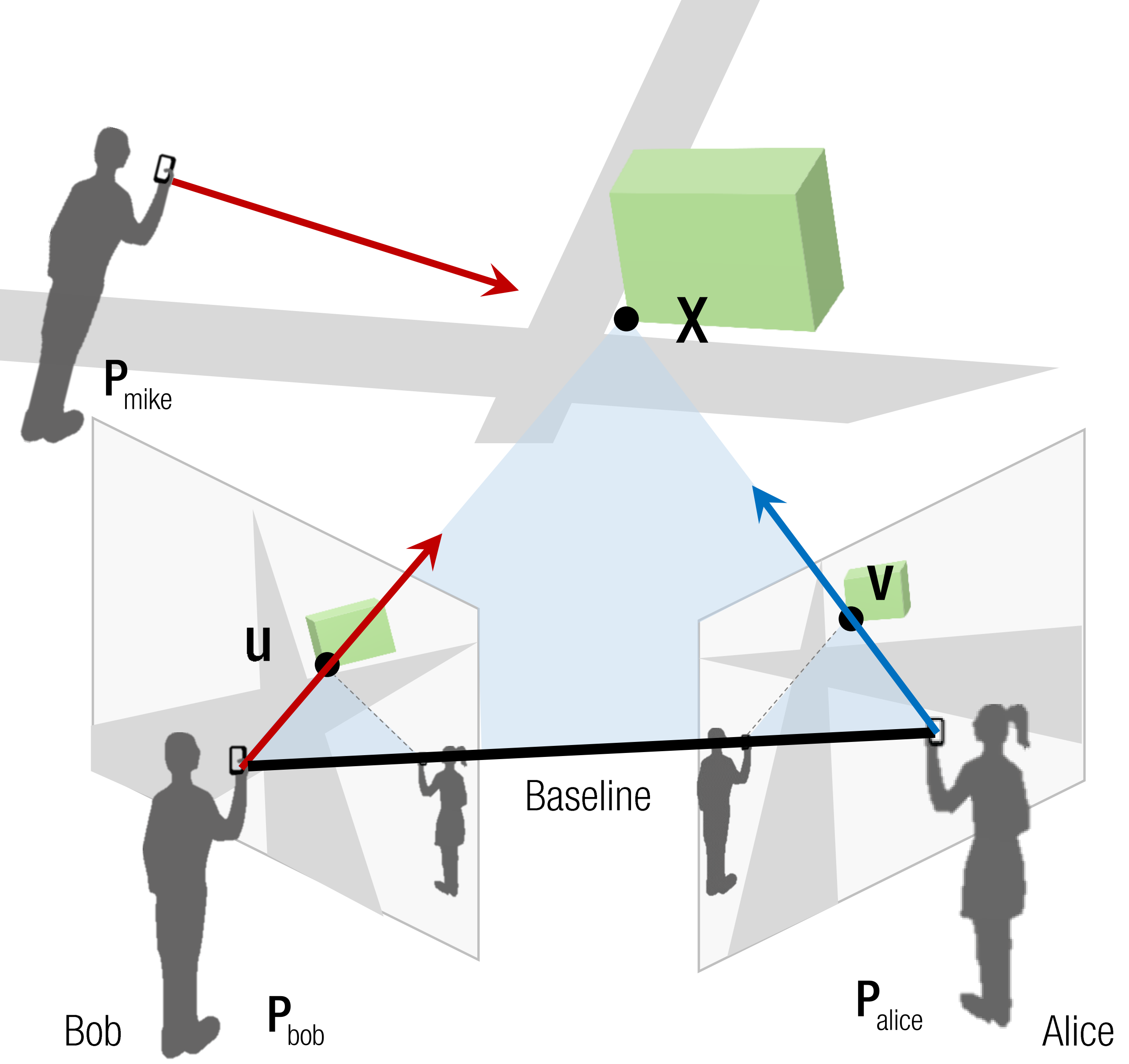
$$\rightarrow \begin{bmatrix} \begin{bmatrix} u \\ 1 \end{bmatrix}_{\times} P_{bob} \\ \begin{bmatrix} v \\ 1 \end{bmatrix}_{\times} P_{alice} \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix} = 0$$

4x4

: Knowns

: Unknowns

General Case



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

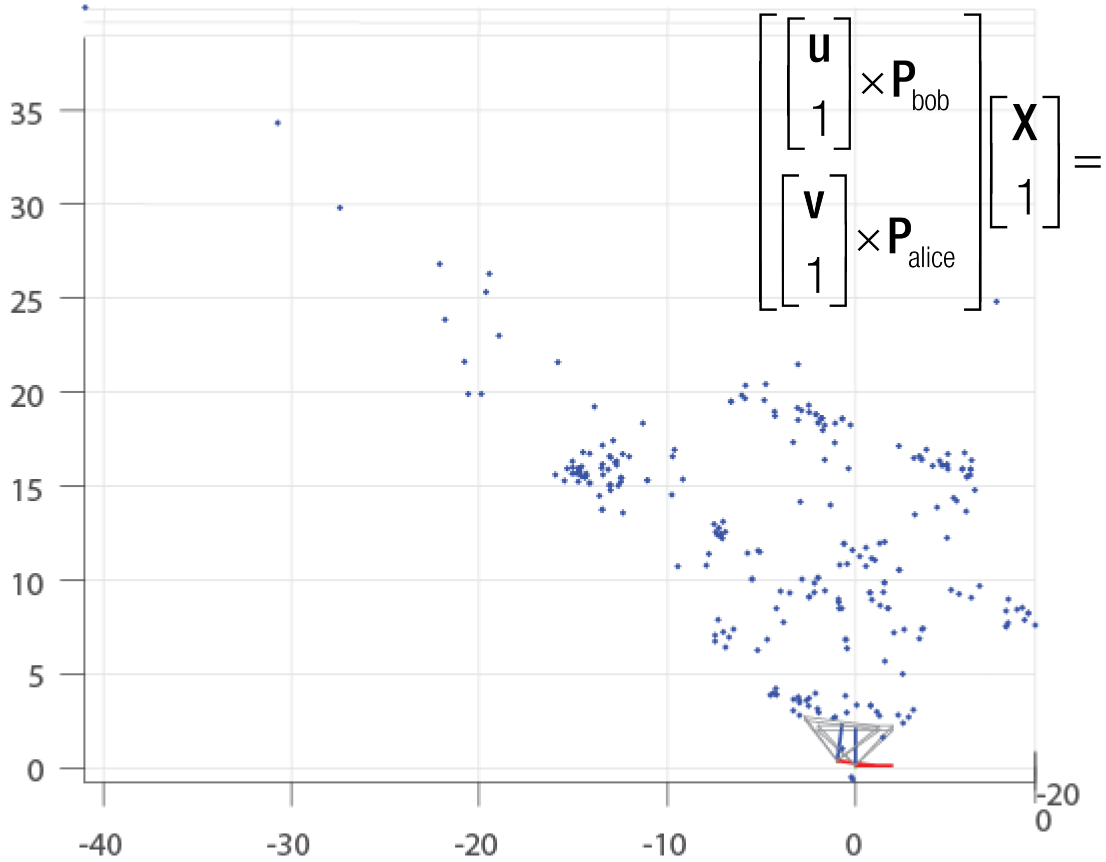
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_\times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_\times \mathbf{P}_{alice}$$

$$\begin{bmatrix} \mathbf{w} \\ 1 \end{bmatrix}_\times \mathbf{P}_{mike}$$

: Knowns

: Unknowns



$$\begin{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_\times \mathbf{P}_{bob} \\ \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_\times \mathbf{P}_{alice} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

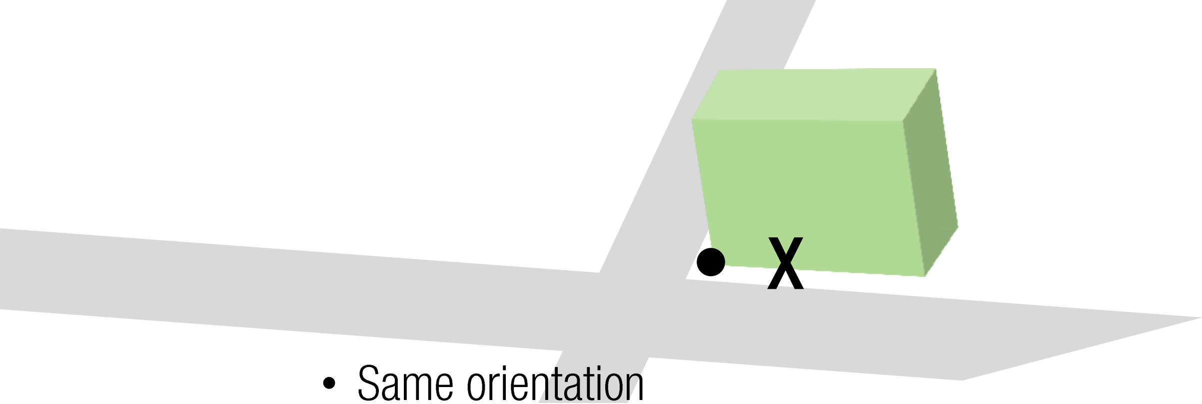
Download **Triangulation.m** and **triangulation.mat** files

```
function Triangulation
%% Data loading
load('triangulation.mat');

% (C1, R1) and (C2, R2) are camera center and orientation of camera 1 and 2, respectively.
% u and v are Nx2 correspondences

%% Camera matrix build
K = [700/2 0 960/2;
     0 700/2 540/2;
     0 0 1];
% Build camera matrix 1 and 2
% P1
% P2

%% Triangulation
% Go to each correspondence and compute the 3D point X (3xN) matrix
for i = 1 : size(u,1)
    % Construct A matrix
    % Solve linear least squares to get 3D point
    % X(:,i) = point_3d;
end
```

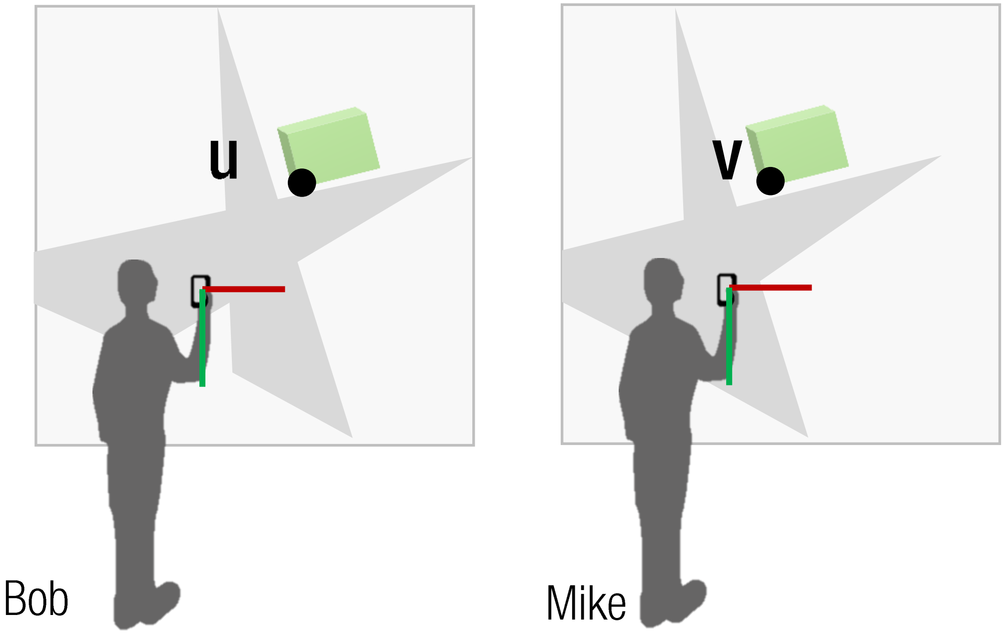
Fill out (for P1, P2)

Fill out (for the loop body)

Special Case: Stereo

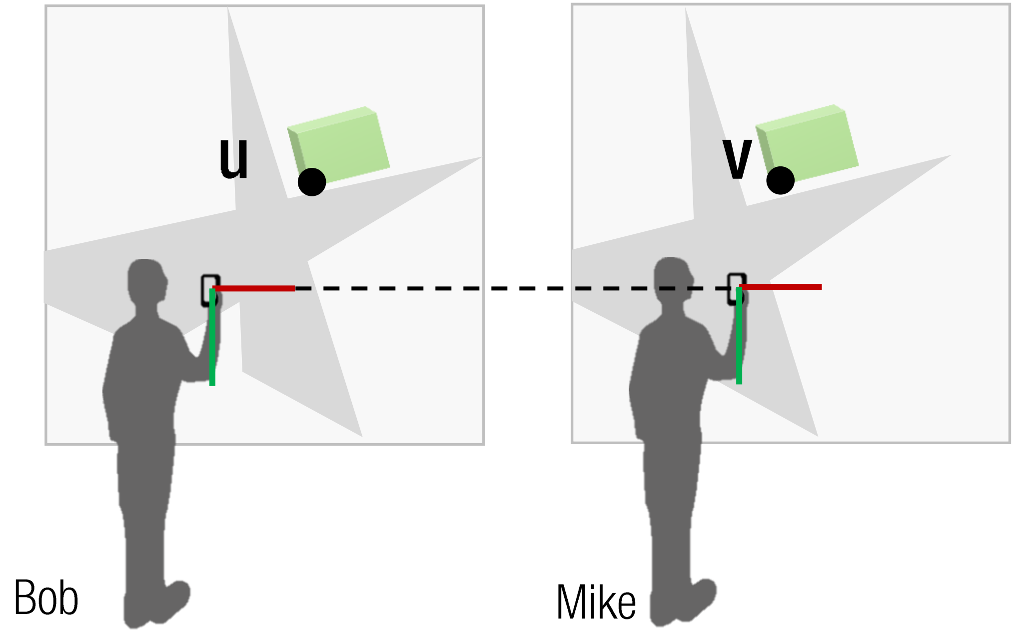


- Same orientation



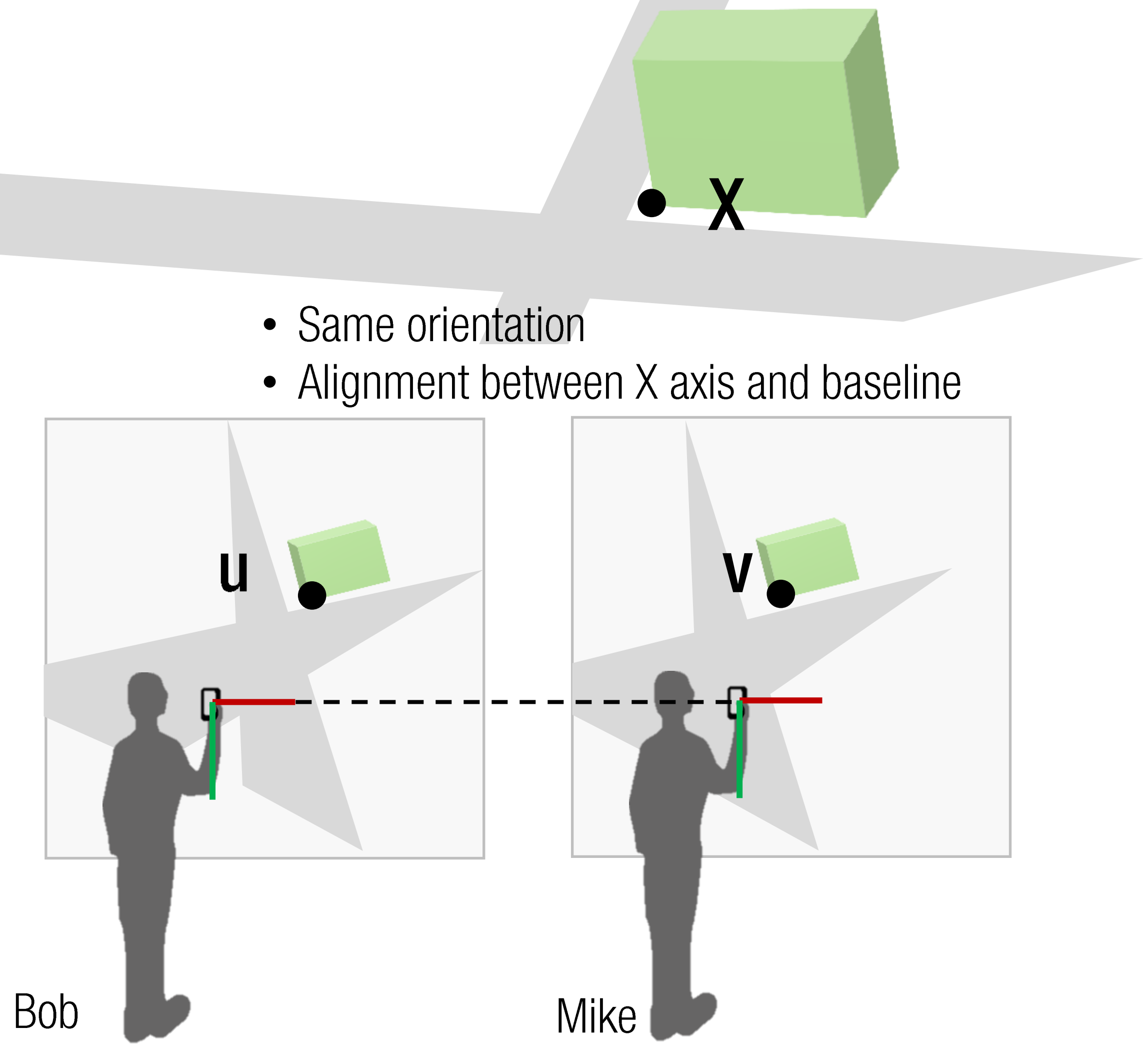
Special Case: Stereo

- Same orientation
- Alignment between X axis and baseline



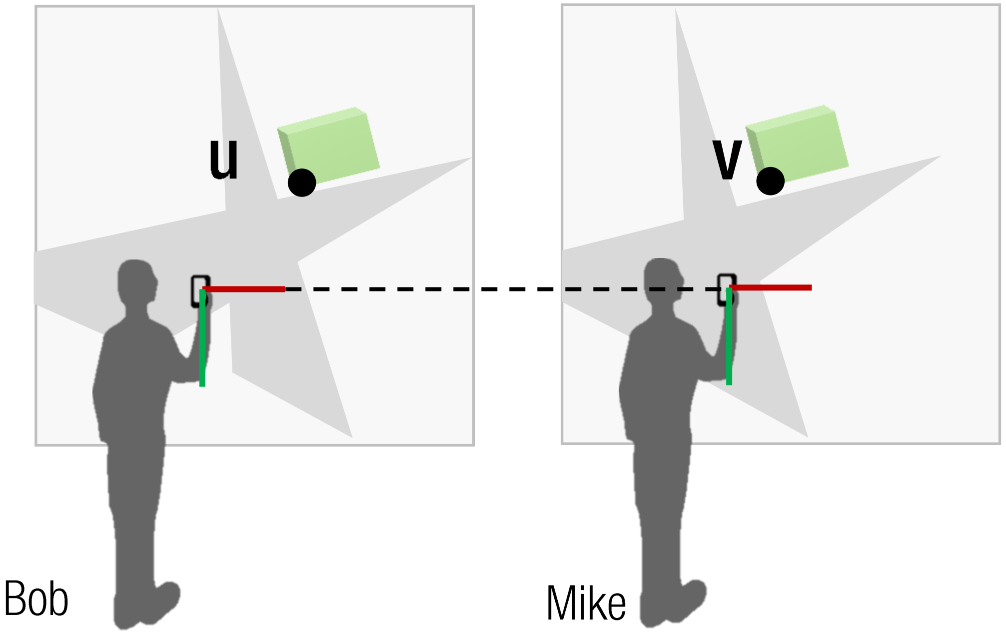
Special Case: Stereo

- Same orientation
- Alignment between X axis and baseline



Special Case: Stereo

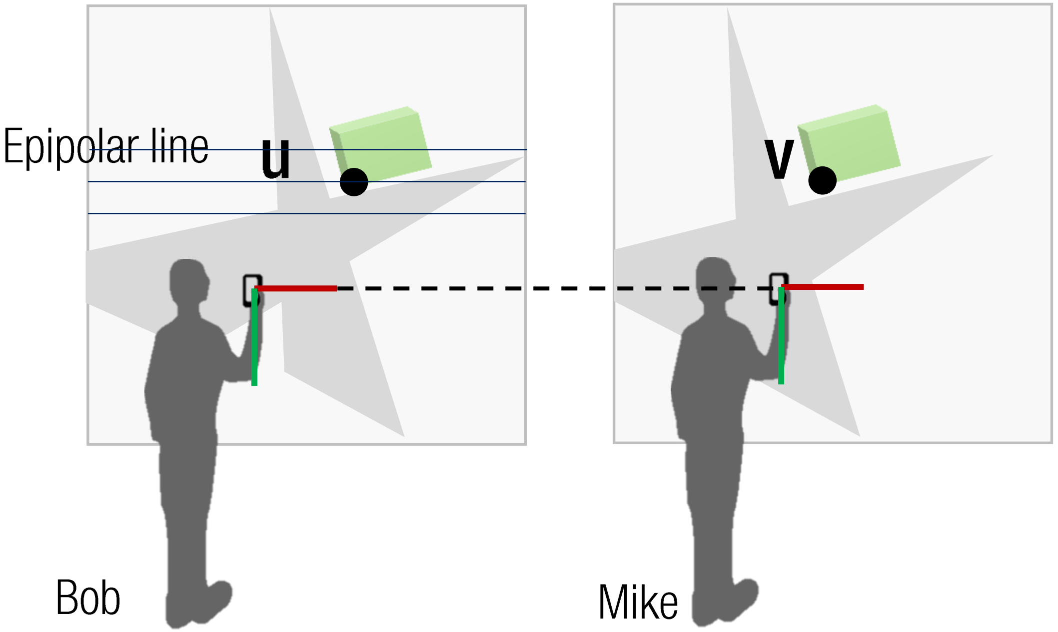
- Same orientation
- Alignment between X axis and baseline



Top view

Special Case: Stereo

- Same orientation
- Alignment between X axis and baseline

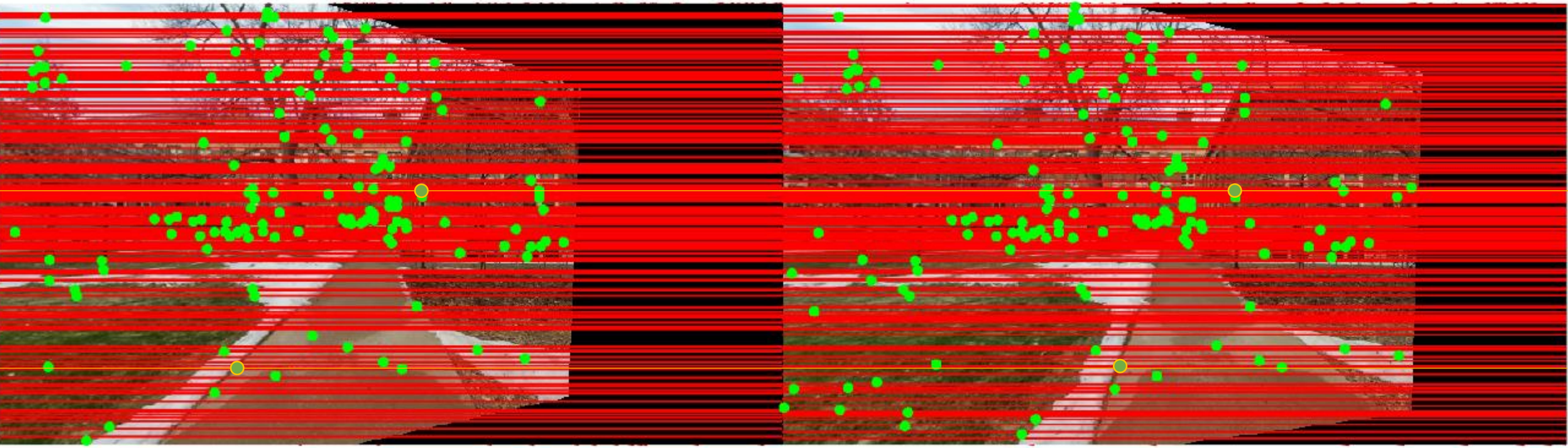


Epipole?
Point at infinity

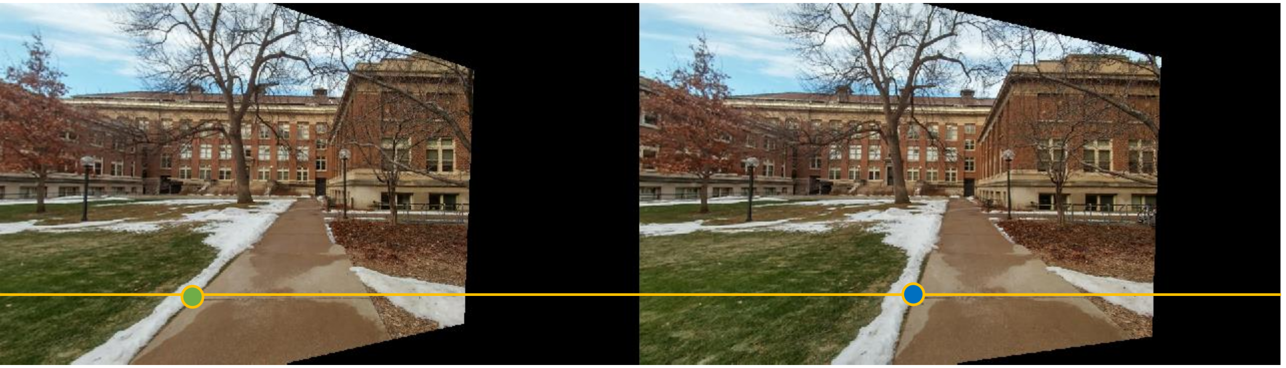
Special Case: Stereo

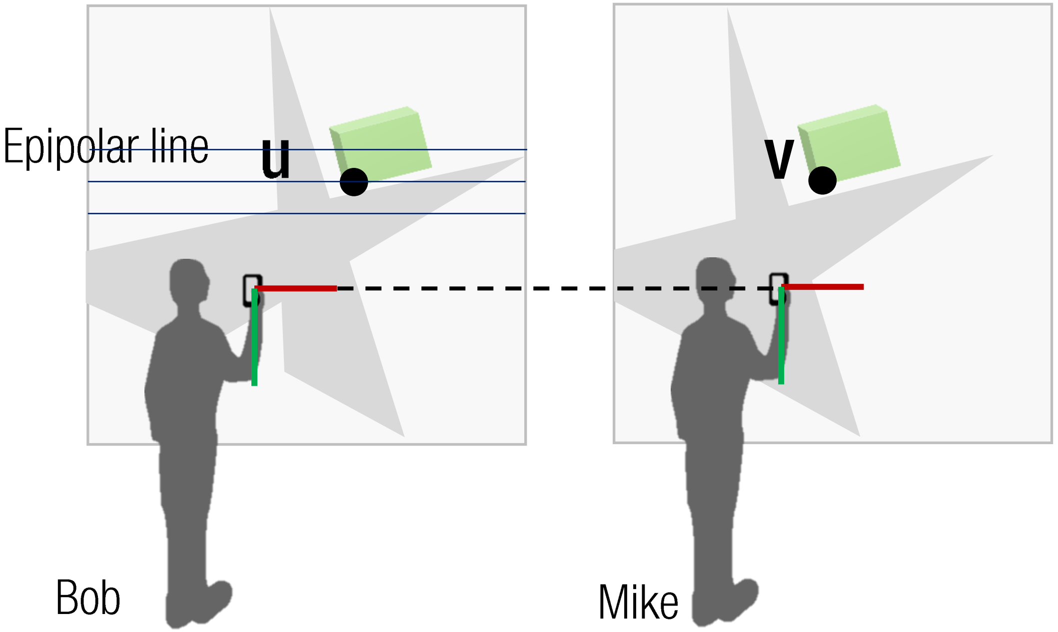
Special Case: Stereo

Special Case: Stereo

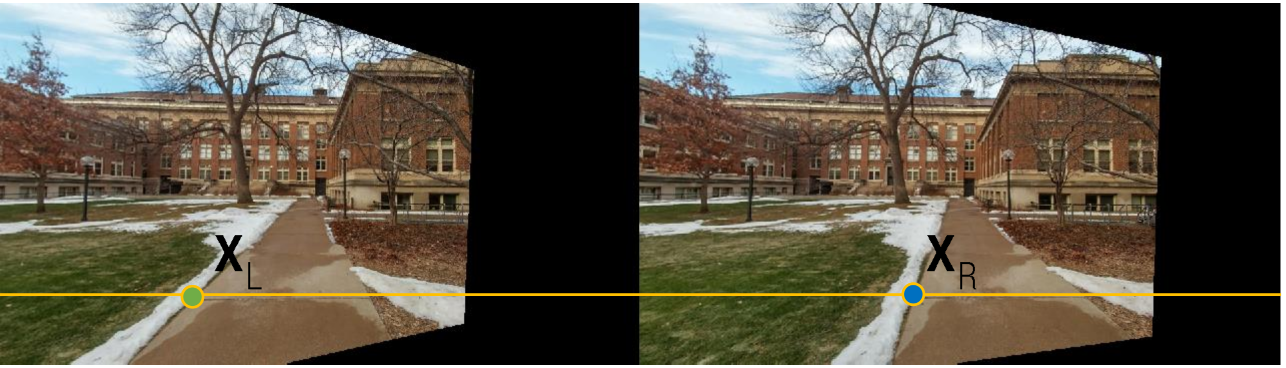


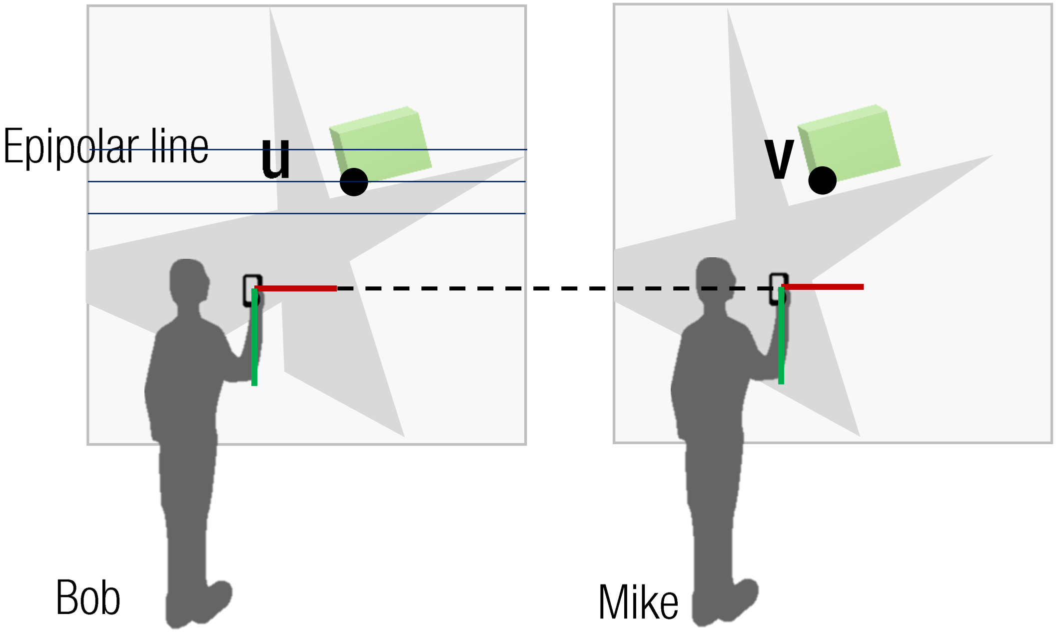
Special Case: Stereo

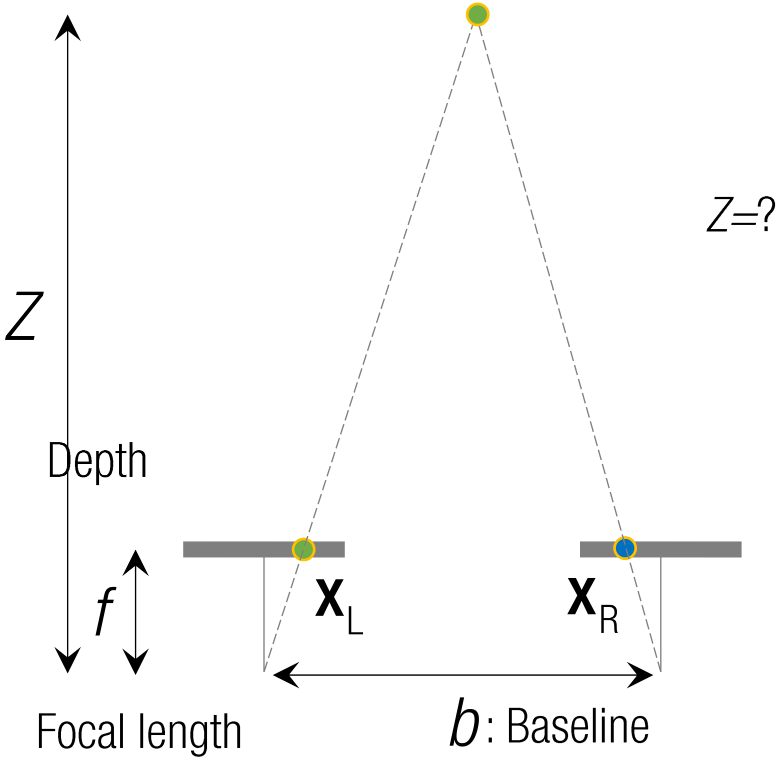




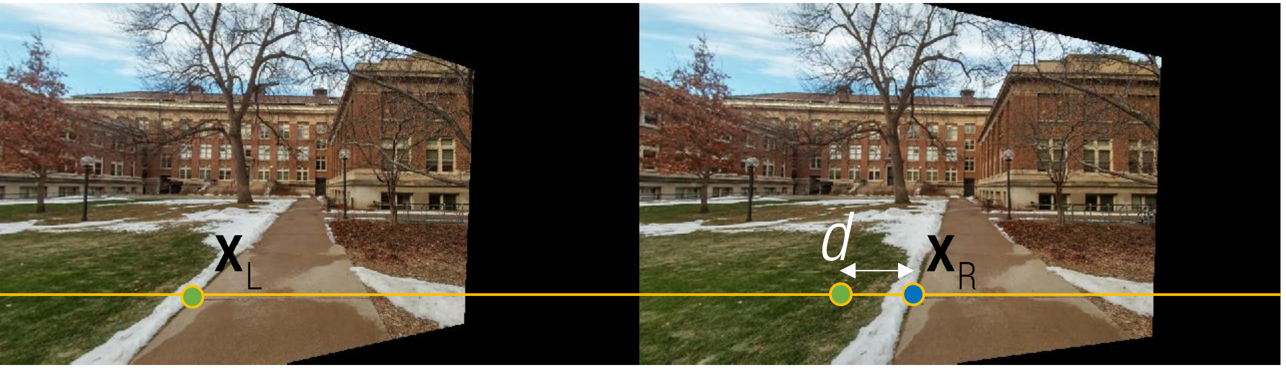
Special Case: Stereo

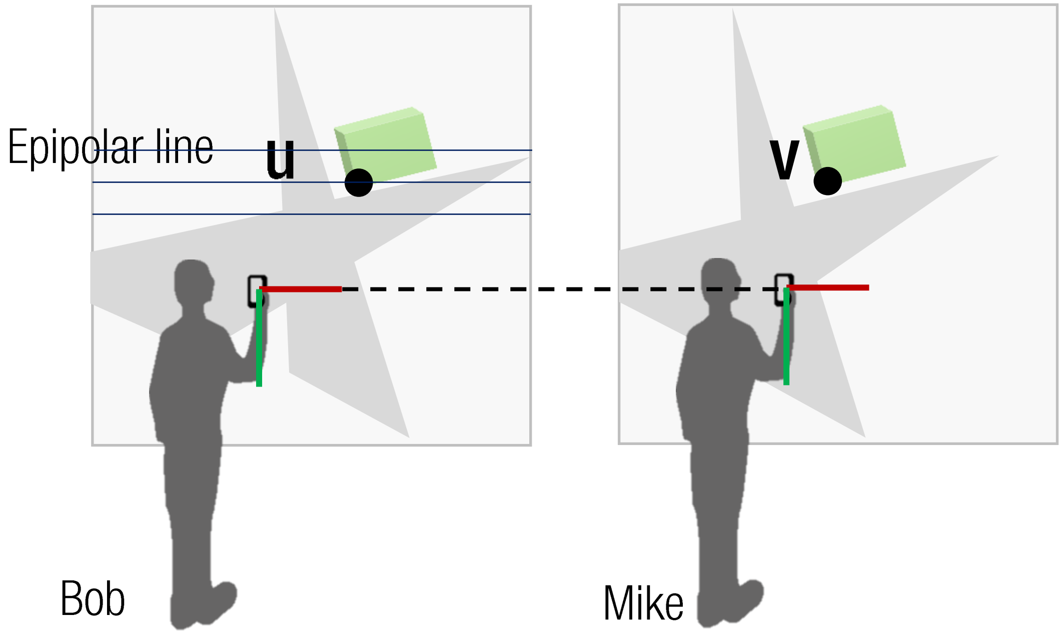


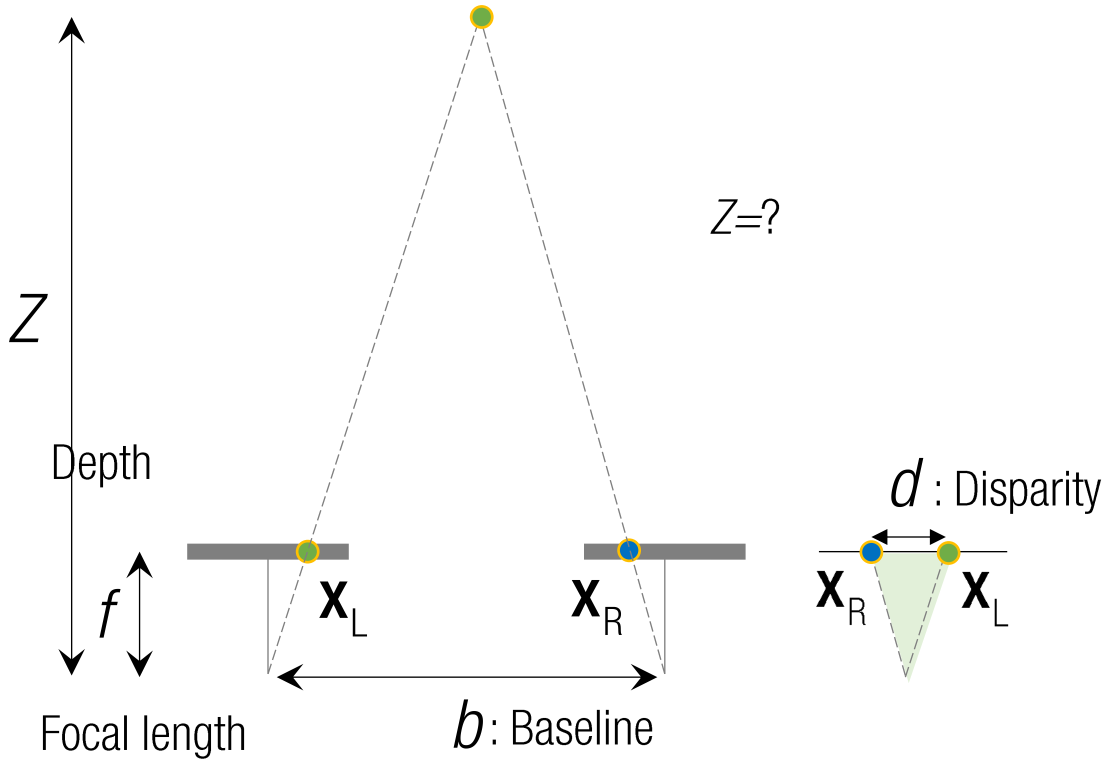




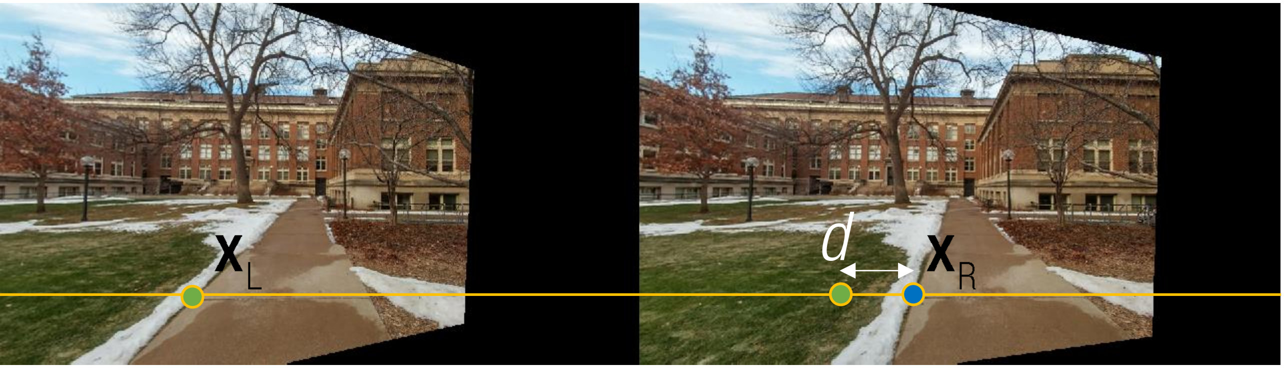
Special Case: Stereo

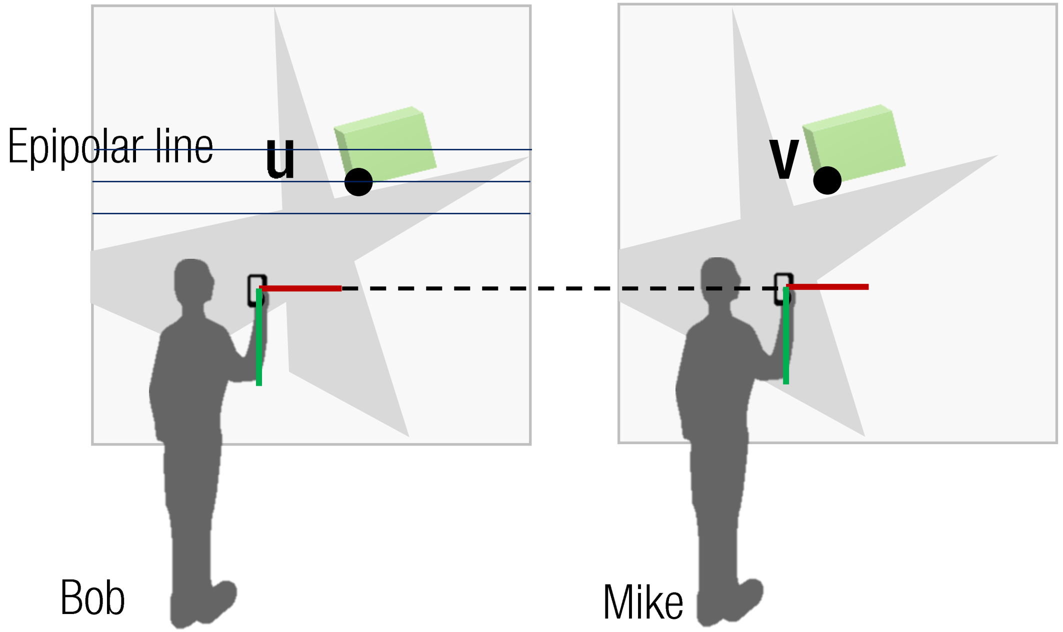


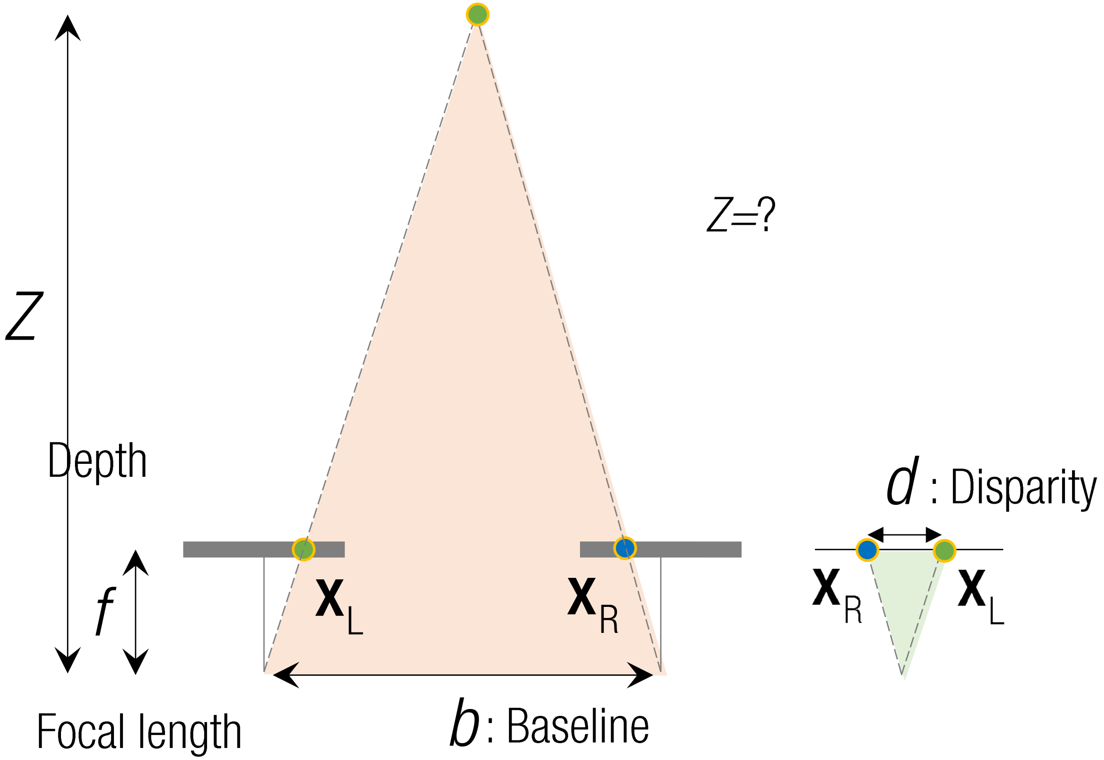




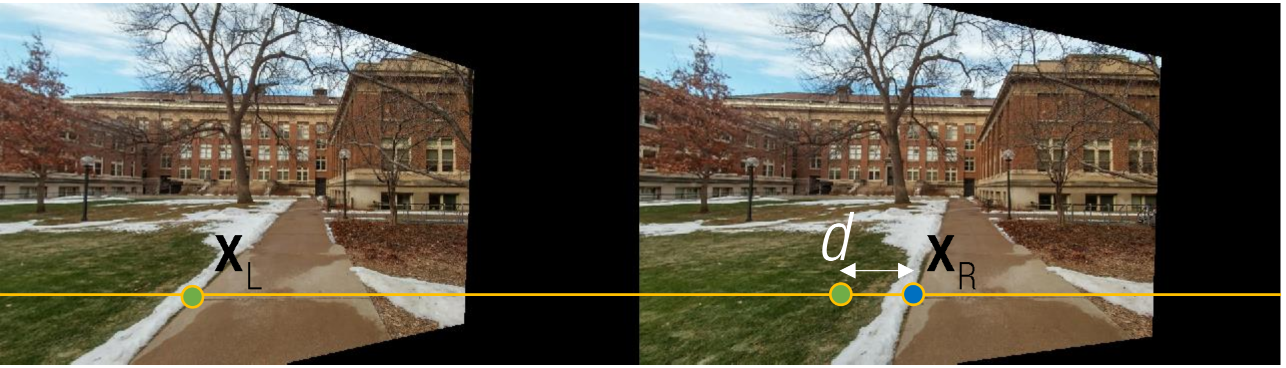
Special Case: Stereo

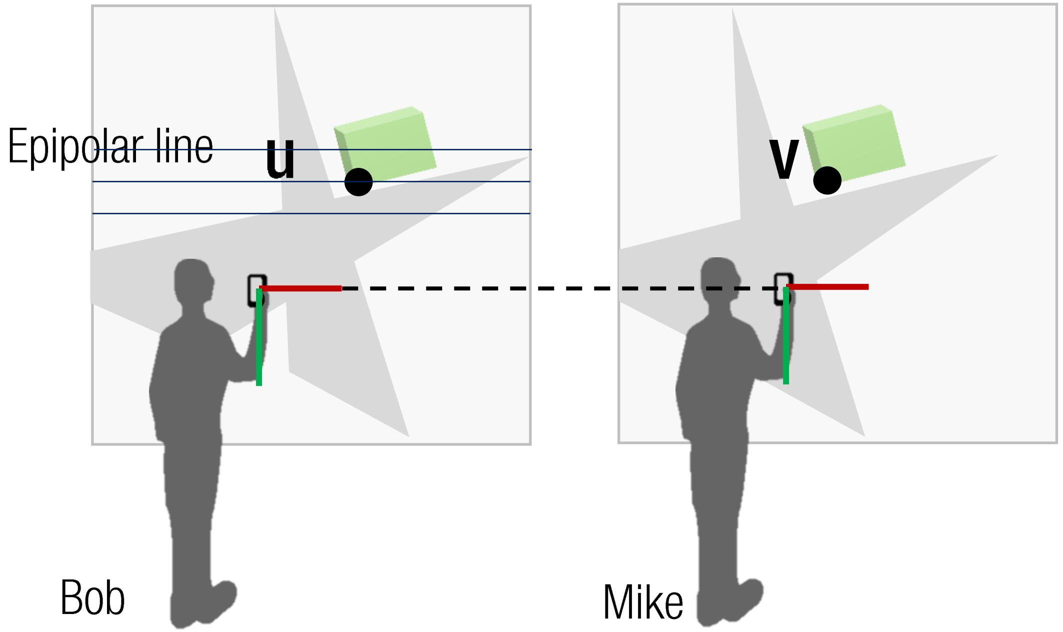


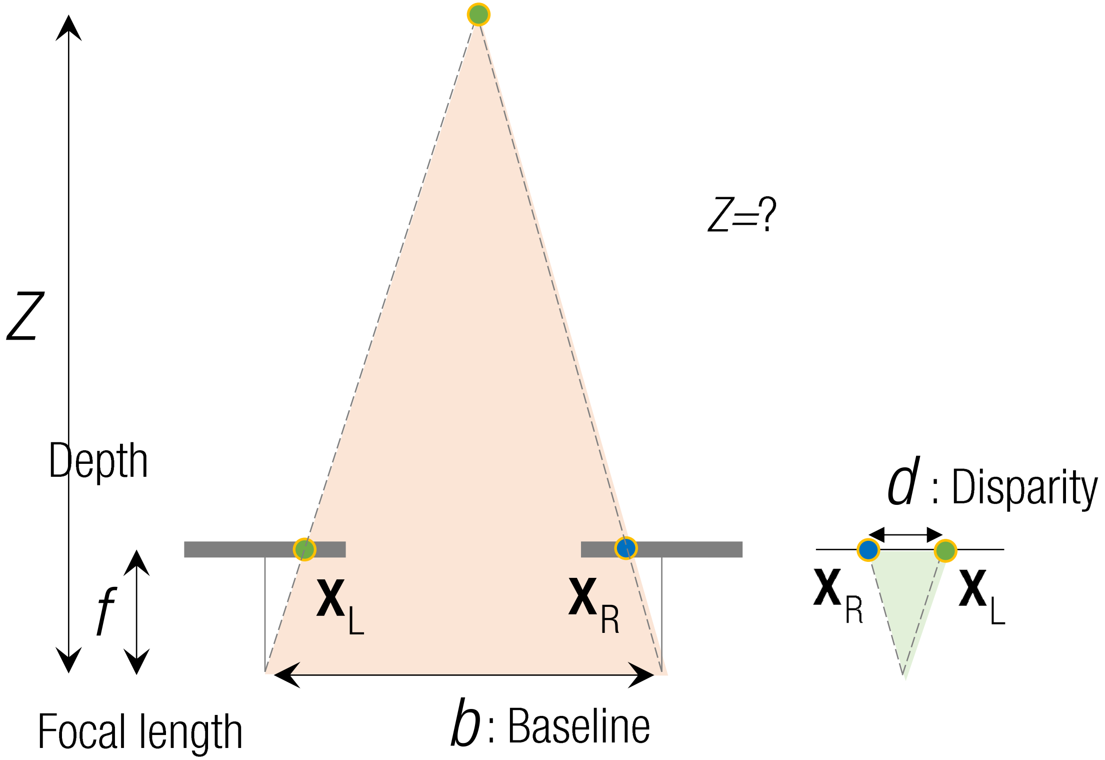


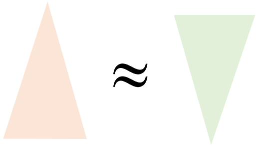


Special Case: Stereo







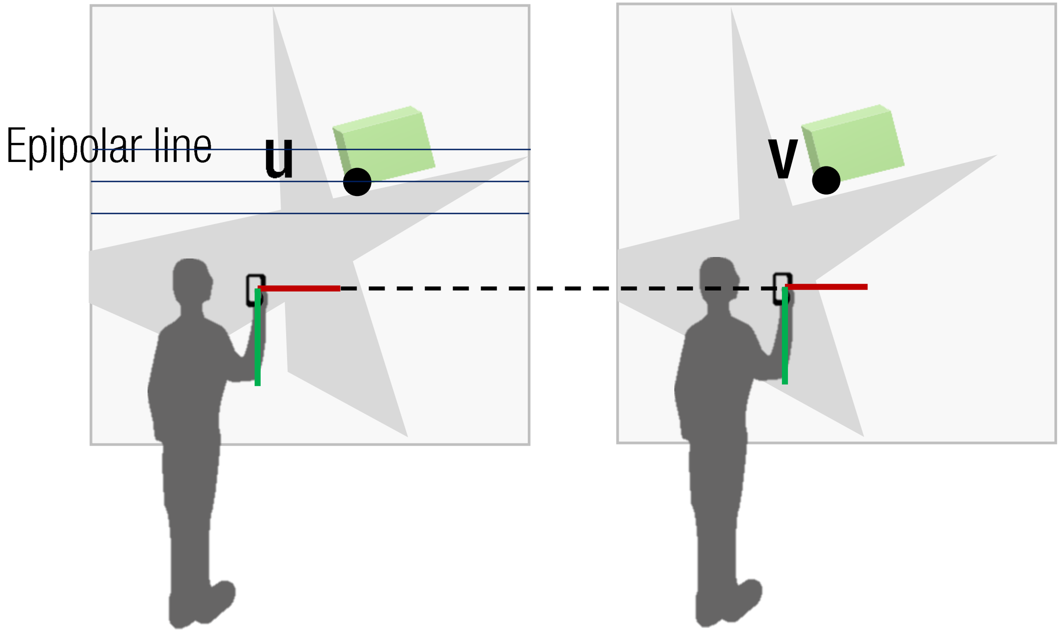


Two triangles are similar.

$$\frac{Z}{b} = \frac{f}{d} \rightarrow Z = b\frac{f}{d}$$

Stereo Rectification

Stereo Rectification



$$P_{bob} = K\begin{bmatrix} I & 0 \end{bmatrix} \qquad P_{mike} = KR\begin{bmatrix} I & -C \end{bmatrix}$$

- Same orientation

$$R_{rect} = \begin{bmatrix} \mathbf{r}_x^T \\ \mathbf{r}_y^T \\ \mathbf{r}_z^T \end{bmatrix}$$

- Alignment between X axis and baseline

$$\mathbf{r}_x = \frac{\mathbf{C}}{\|\mathbf{C}\|}$$

$$\mathbf{r}_z = \frac{\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x)\mathbf{r}_x}{\left\|\tilde{\mathbf{r}}_z - (\tilde{\mathbf{r}}_z \cdot \mathbf{r}_x)\mathbf{r}_x\right\|}$$

: Orthogonal projection

where $\tilde{\mathbf{r}}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

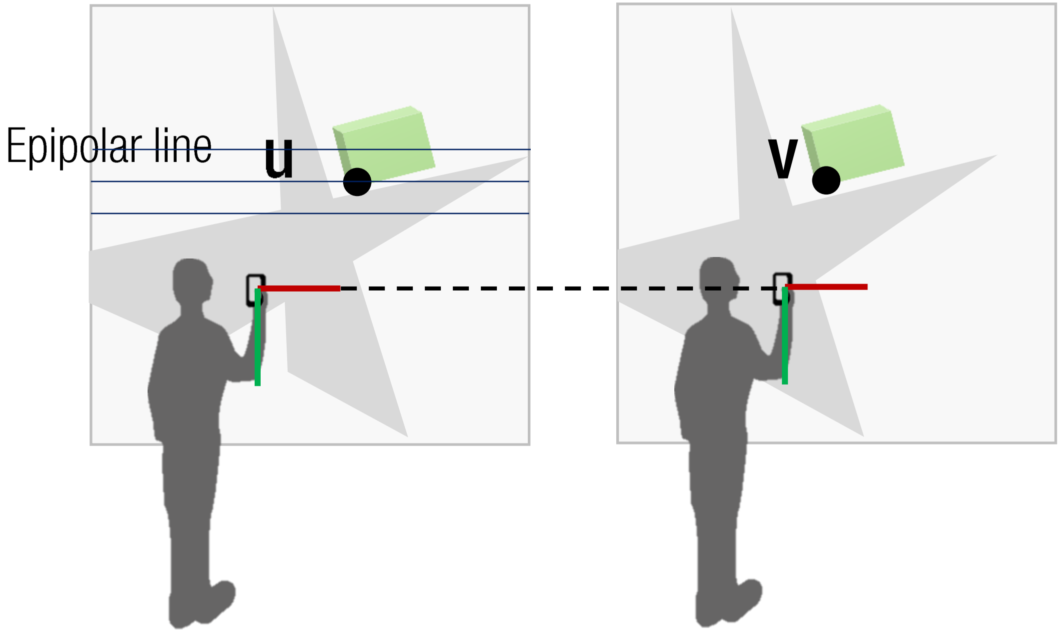
$$\mathbf{r}_y = \mathbf{r}_z \times \mathbf{r}_x$$

Stereo Rectification

Homography by pure rotation: $\mathbf{R}_{rect}$

$$\mathbf{H}_{bob} = \mathbf{K}\mathbf{R}_{rect}\mathbf{K}^{-1}$$

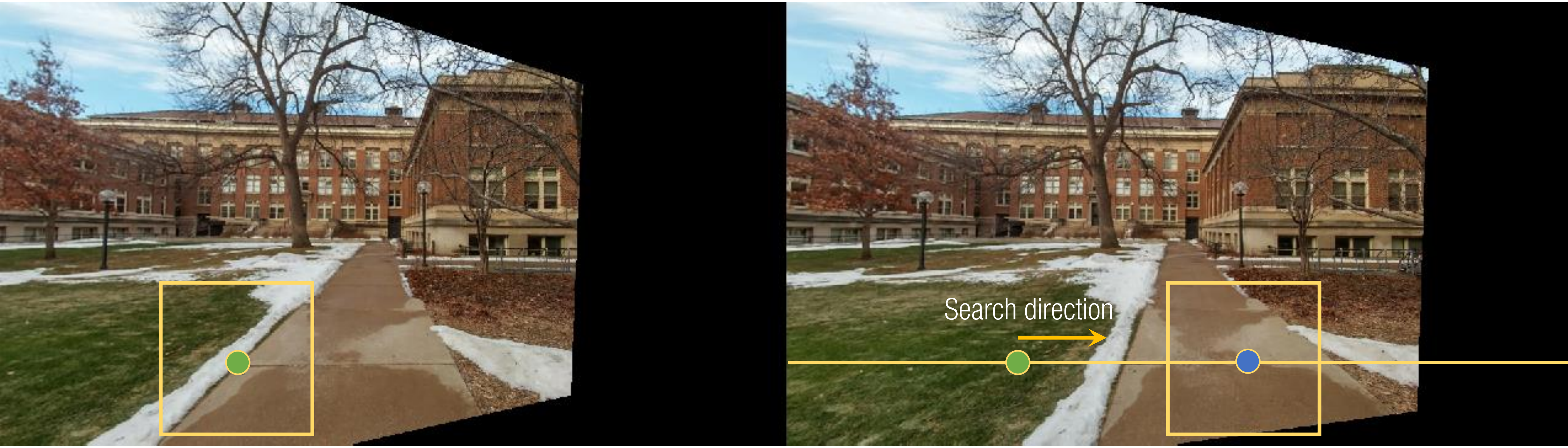
$$\mathbf{H}_{mike} = \mathbf{K}\mathbf{R}_{rect}\mathbf{R}^{T}\mathbf{K}^{-1}$$



$$\mathbf{P}_{bob} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$$

$$\mathbf{P}_{mike} = \mathbf{K}\mathbf{R}\begin{bmatrix}\mathbf{I} & -\mathbf{C}\end{bmatrix}$$

Dense Feature Matching using SIFT Flow



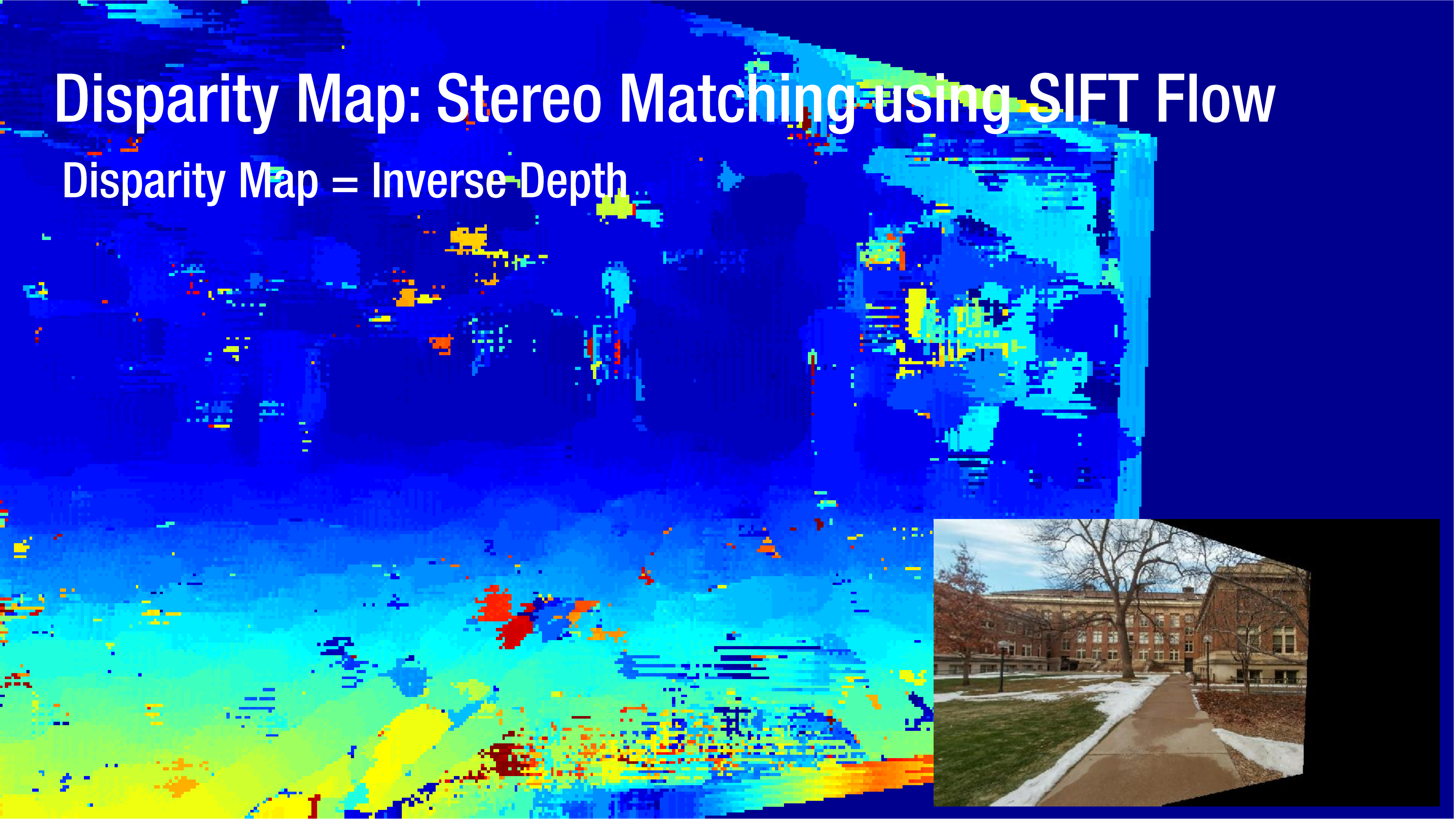
Find a minimum distance over the epipolar line

descriptor1

descriptor2

Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth

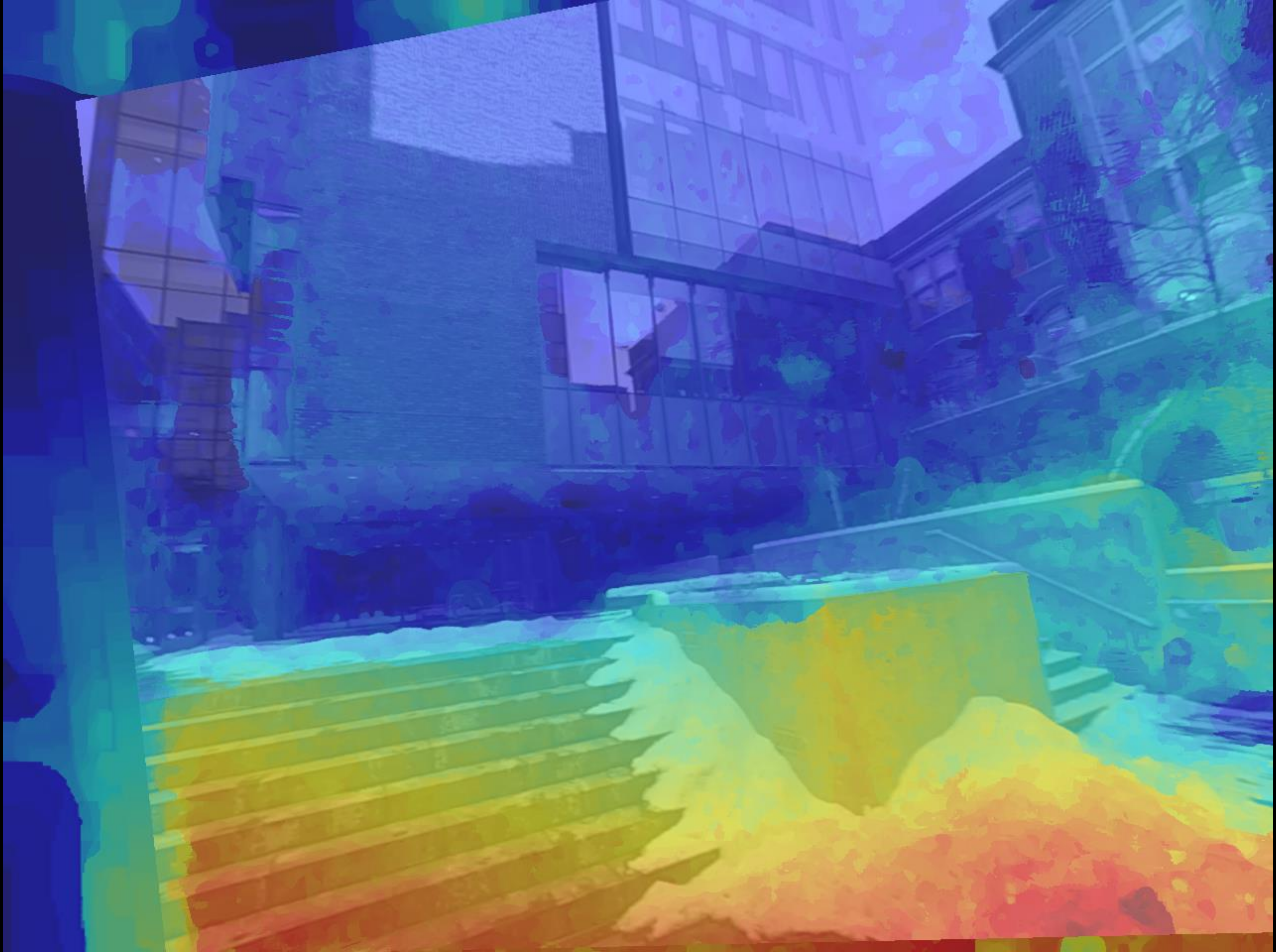


Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth

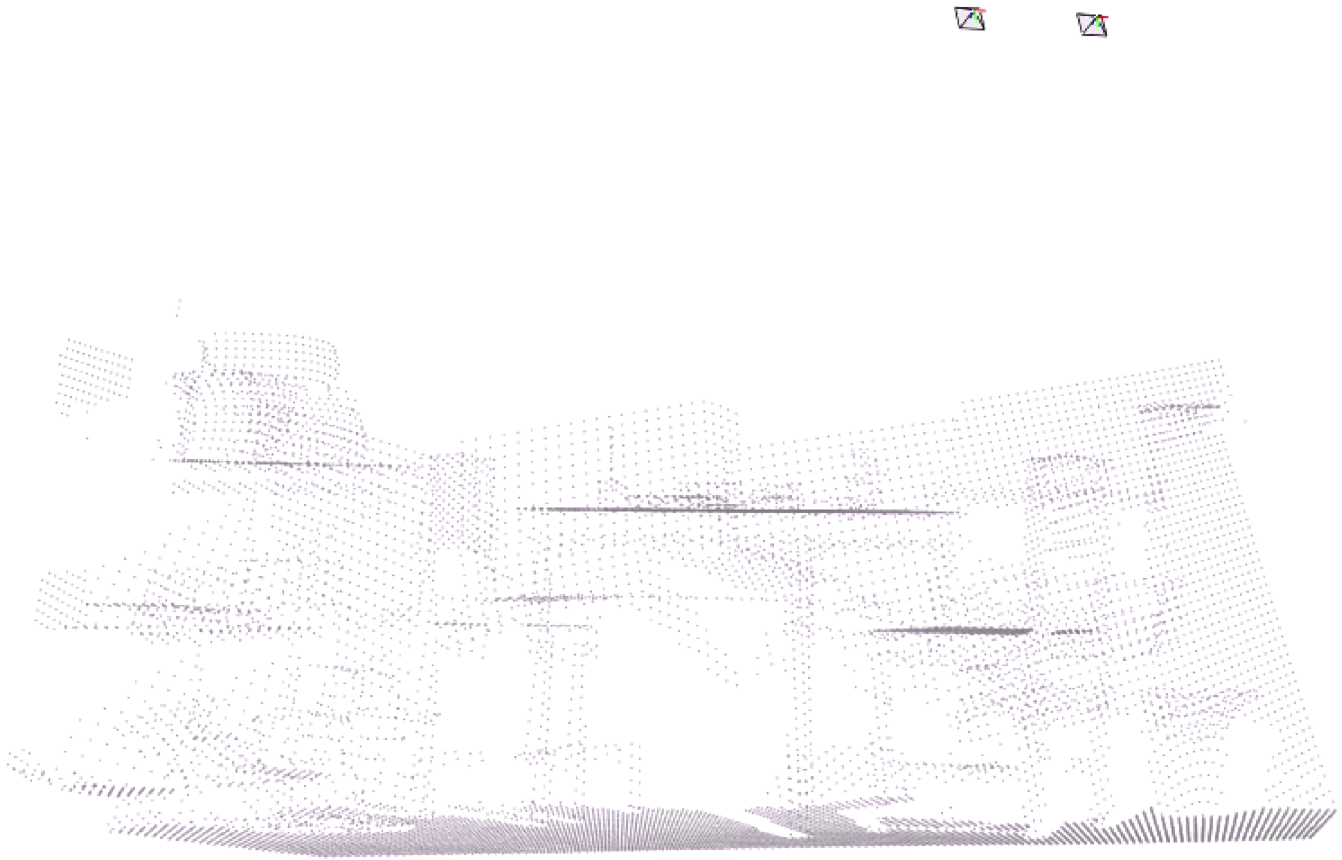
Disparity Map: Stereo Matching using SIFT Flow

Disparity Map = Inverse Depth


maxell
VIDEOCASSETTE

maxell
VIDEOCASSETTE

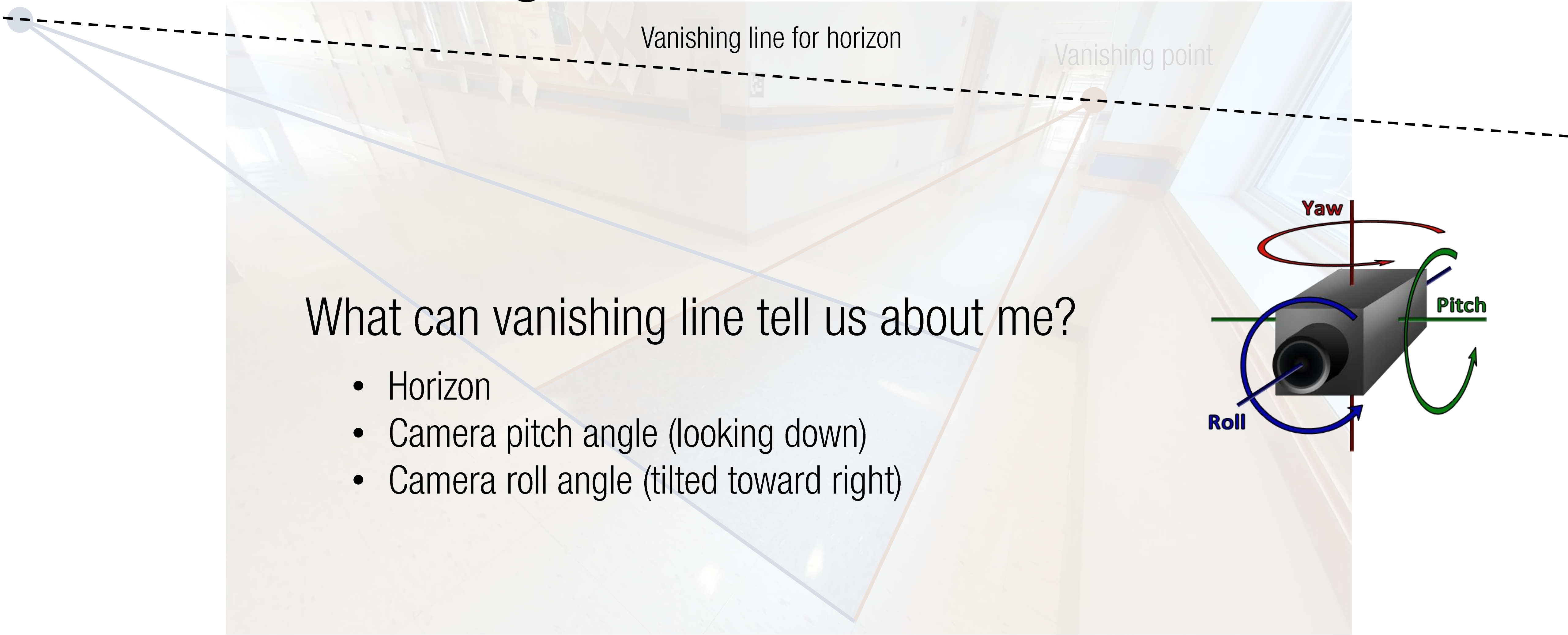



EgoMotion Dataset (out

Dense Reconstruction using a Monocular Camera

Where am I?
Perspective-n-Point

Recall: Vanishing Line

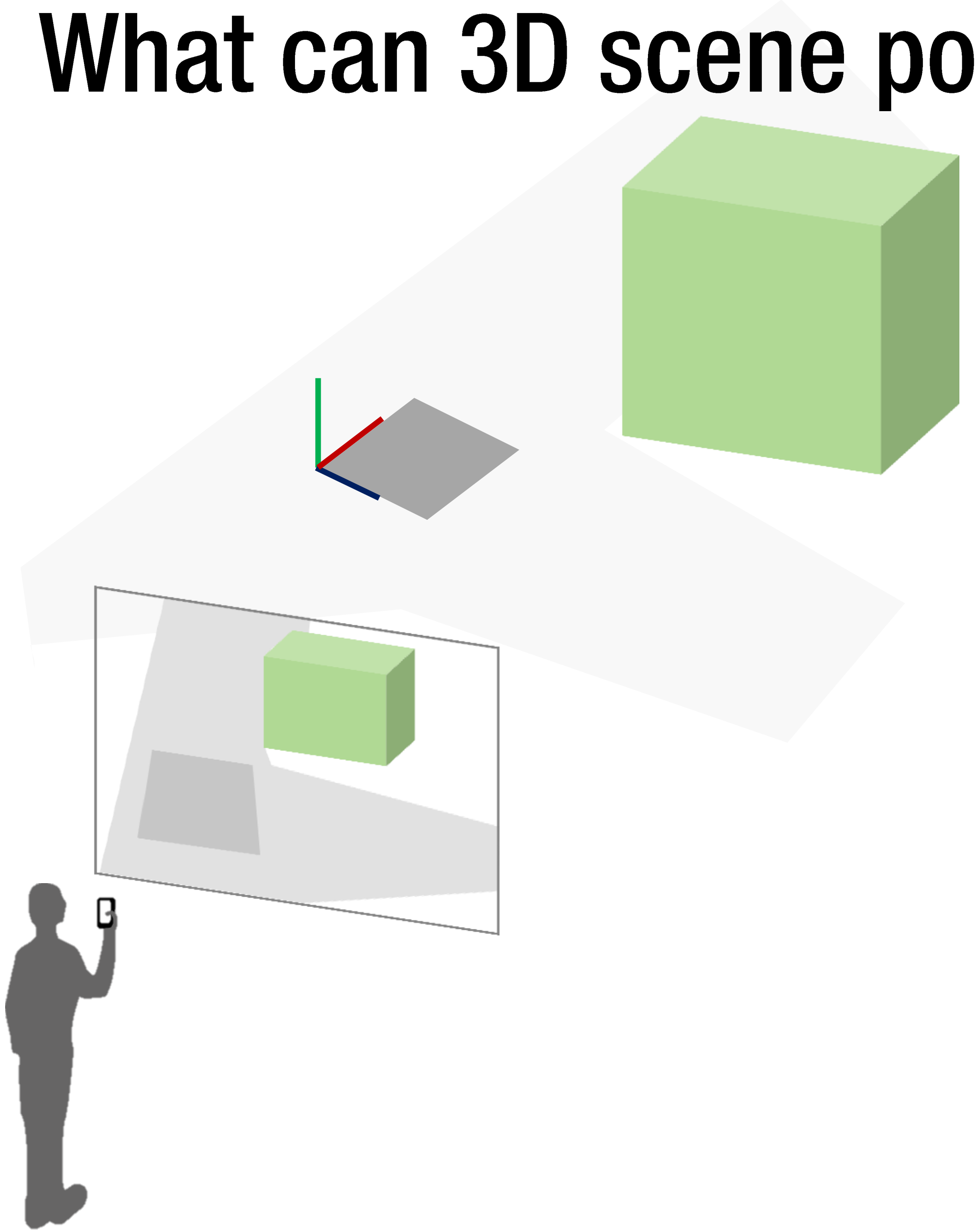


What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)

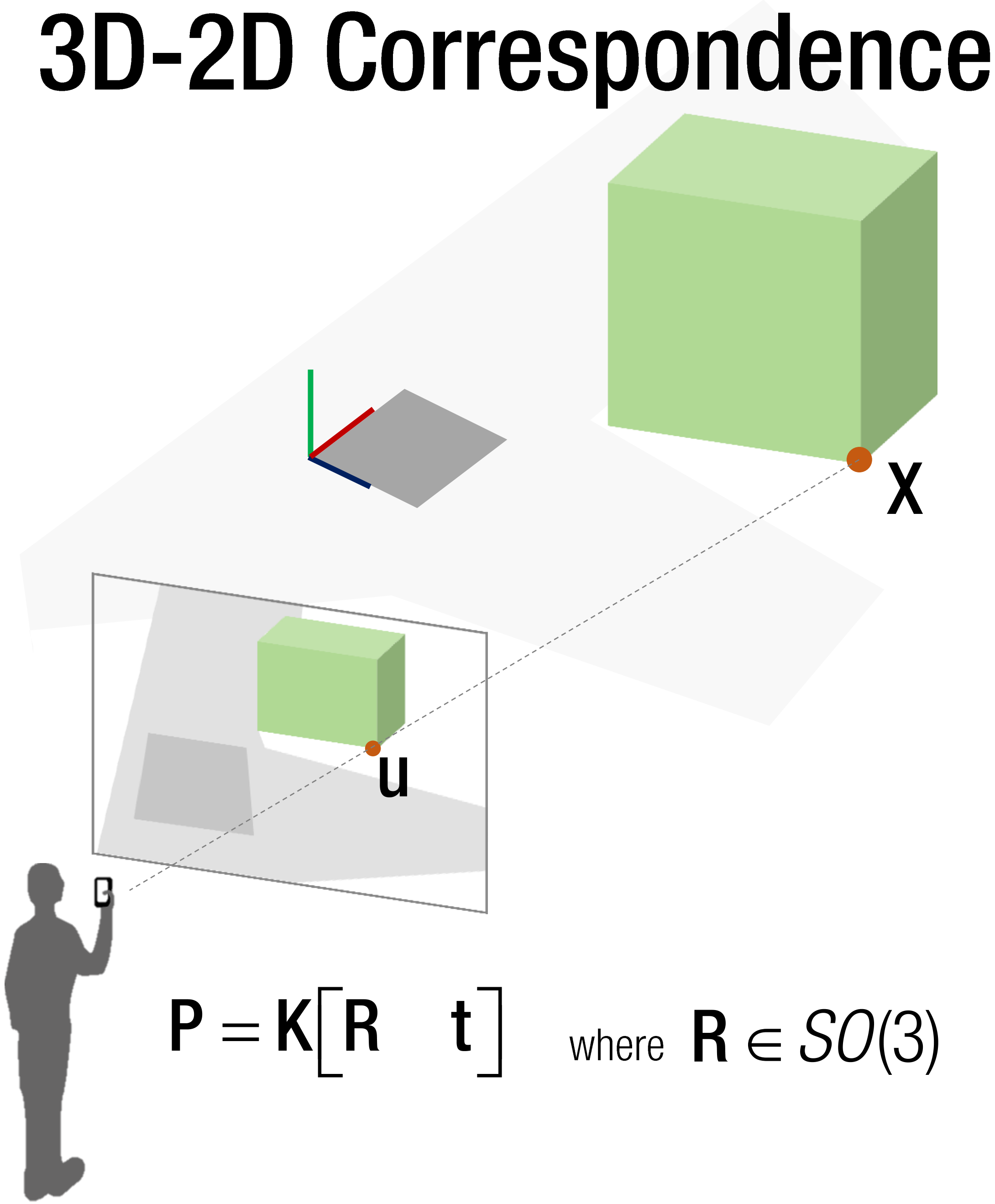
Keller Hall

What can 3D scene points tell us about?



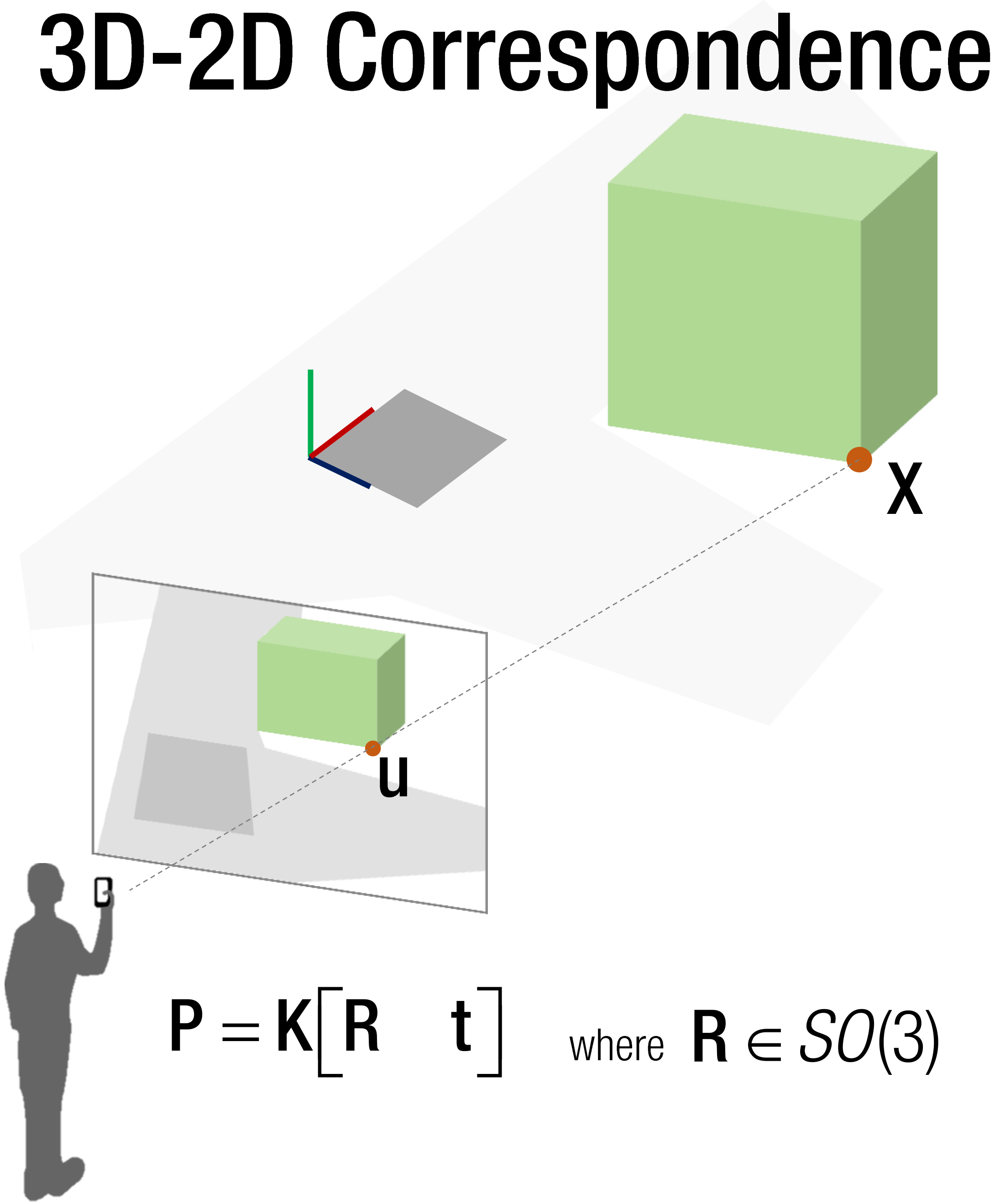
http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

3D-2D Correspondence



$$P = K\begin{bmatrix} R & t \end{bmatrix} \quad \text{where } R \in SO(3)$$

3D-2D Correspondence



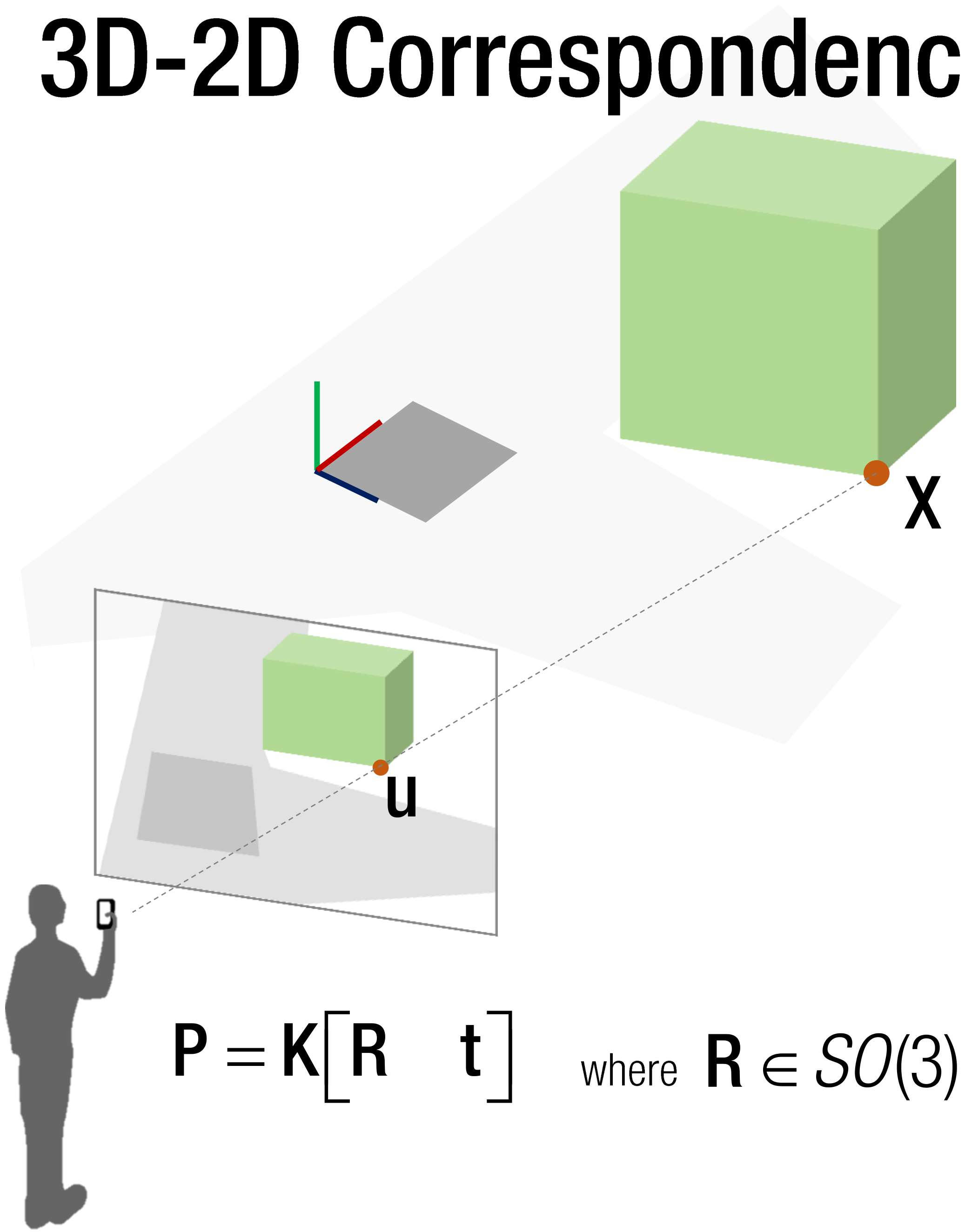
$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3D-2D Correspondence



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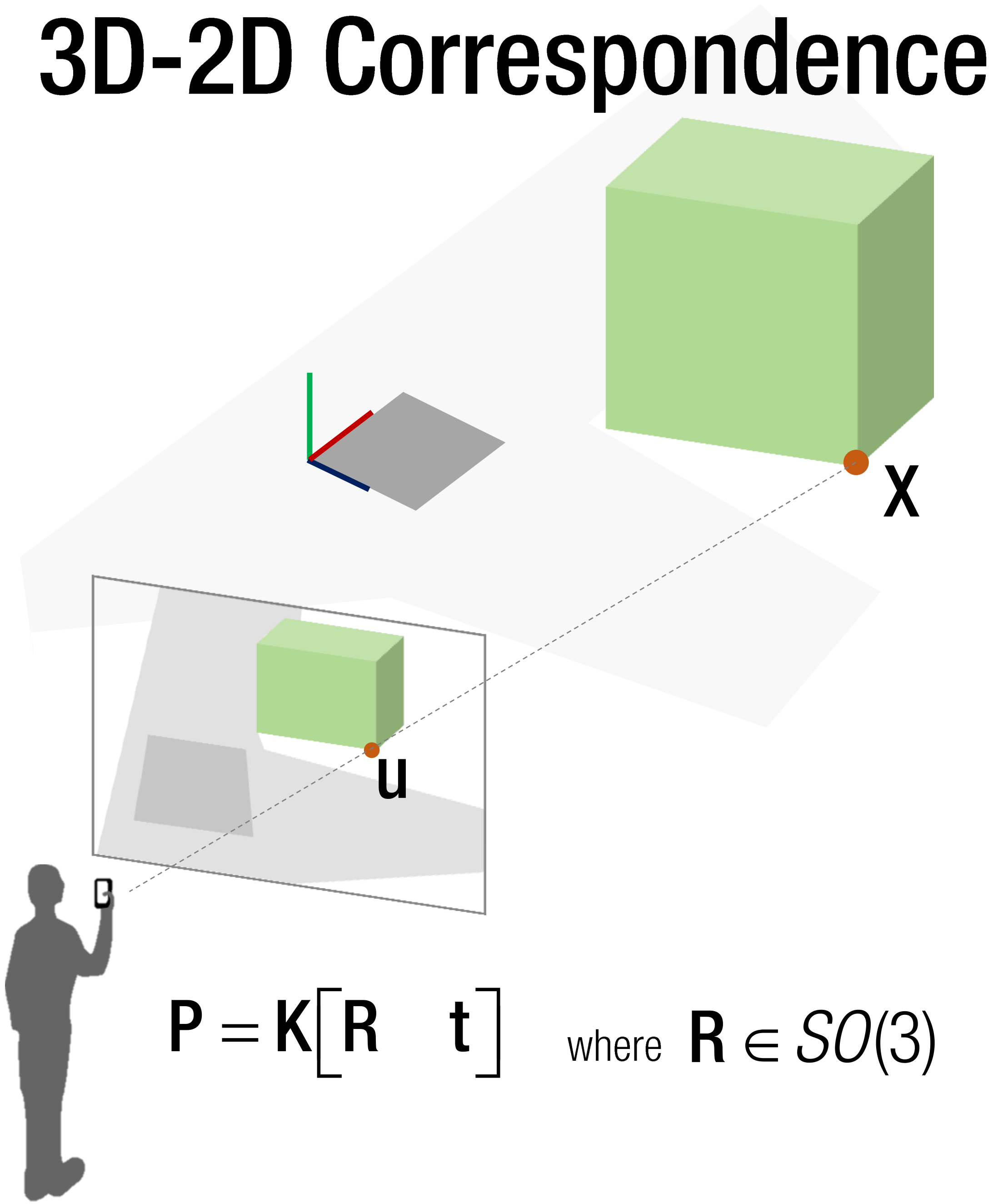
Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$ where $\mathbf{R} \in SO(3)$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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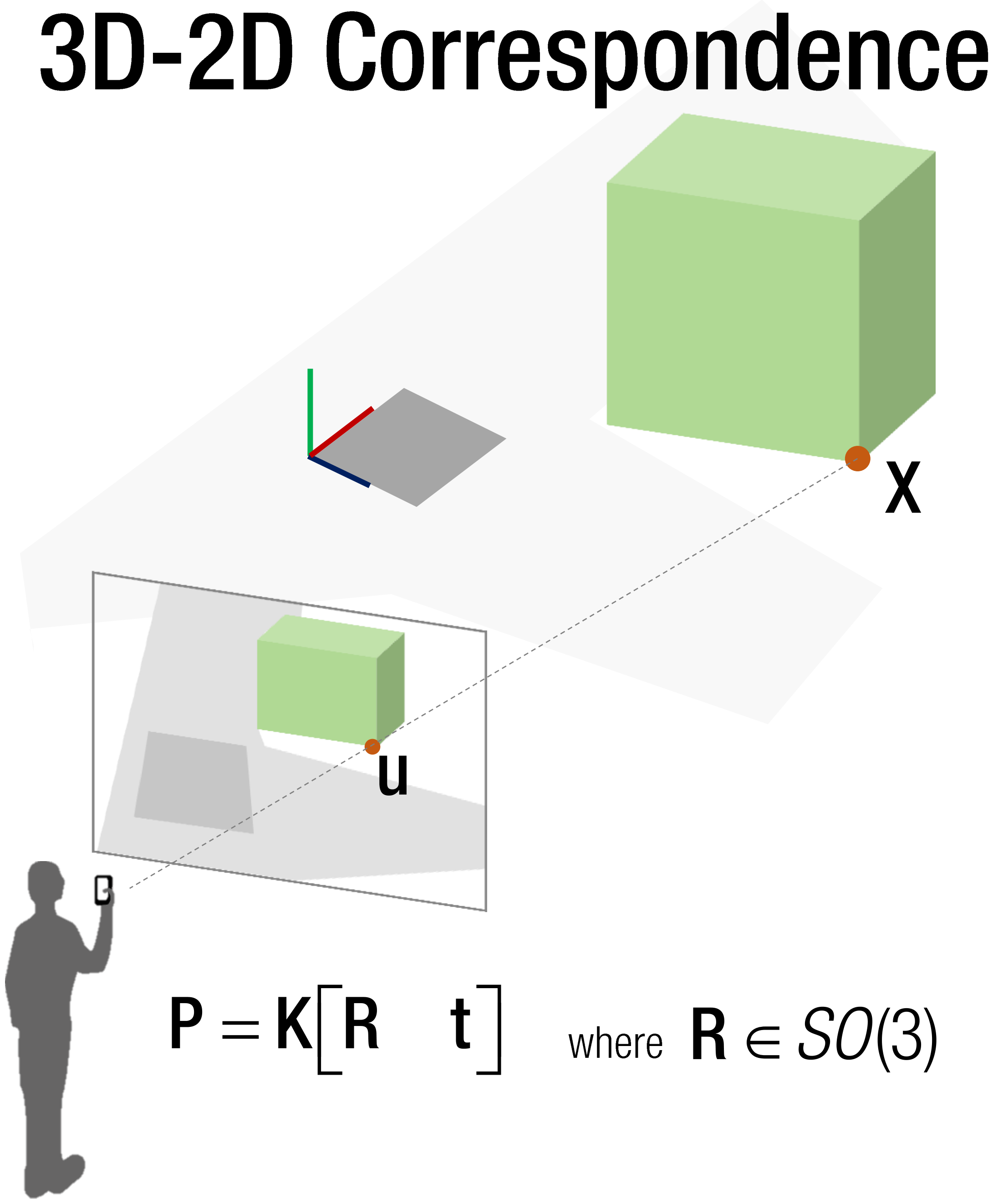
# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)
# of equations per correspondence: 2

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

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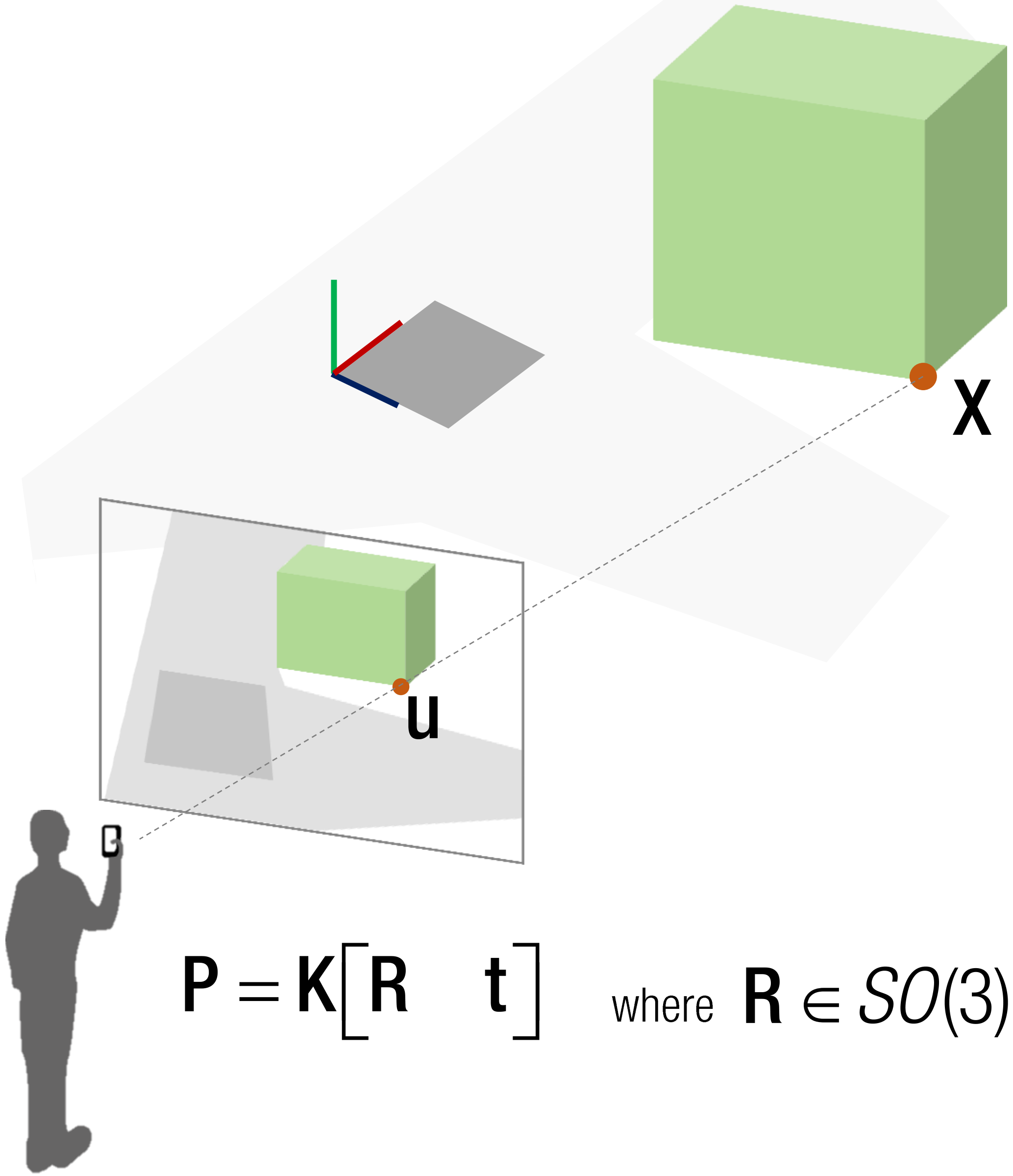
3D-2D Correspondence



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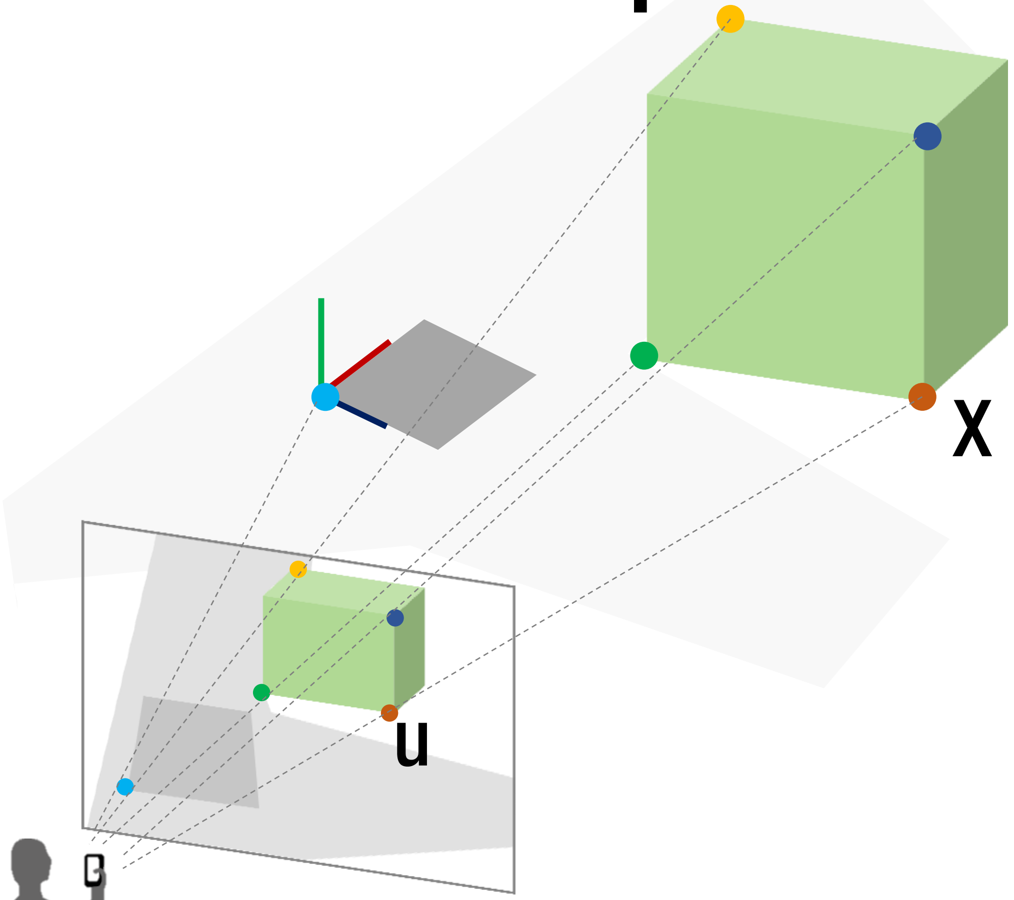
$$\begin{bmatrix} X & Y & Z & 1 & & & & & -u^x X & -u^x Y & -u^x Z & -u^x \\ & & & & X & Y & Z & 1 & -u^y X & -u^y Y & -u^y Z & -u^y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2x12

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

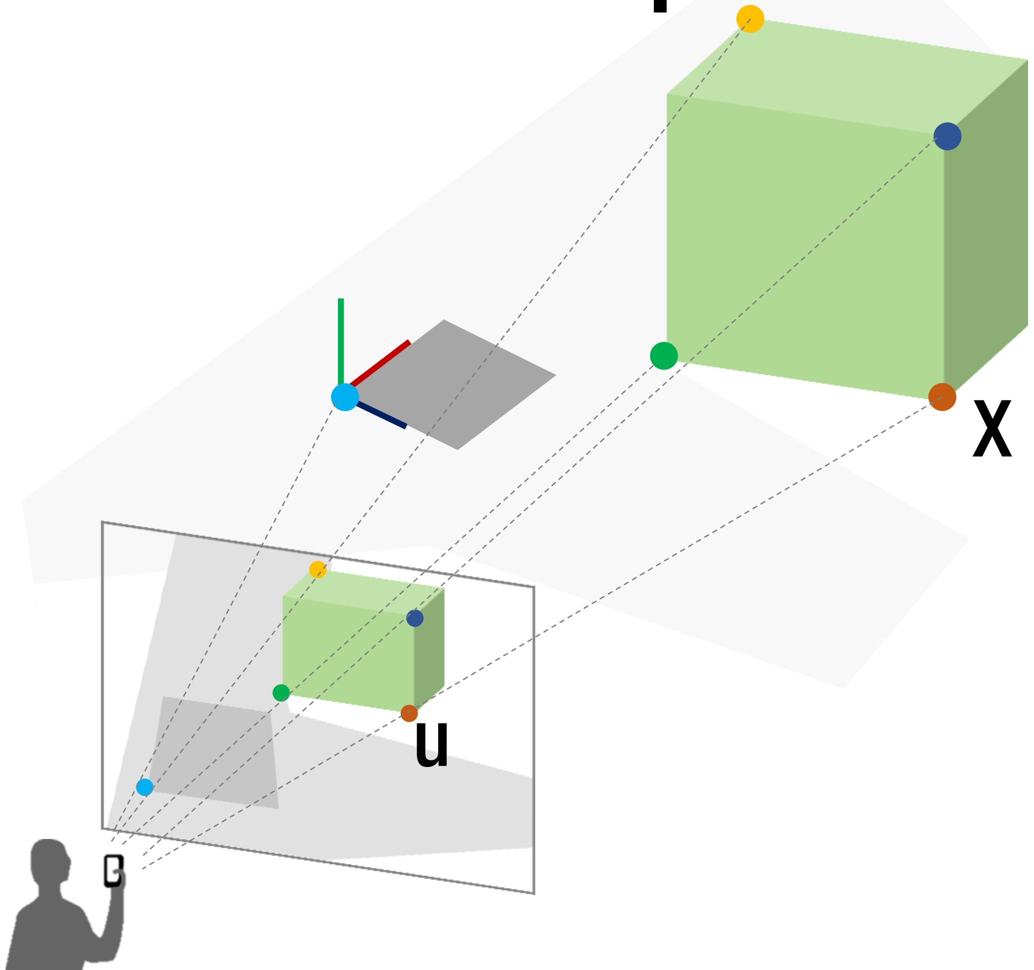
3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^x X & -u_1^x Y & -u_1^x Z & -u_1^x \\ & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^y X & -u_1^y Y & -u_1^y Z & -u_1^y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_m & Y_m & Z_m & 1 & & & & & -u_m^x X & -u_m^x Y & -u_m^x Z & -u_m^x \\ & & & & X_m & Y_m & Z_m & 1 & -u_m^y X & -u_m^y Y & -u_m^y Z & -u_m^y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$2m$x12

3D-2D Correspondence



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

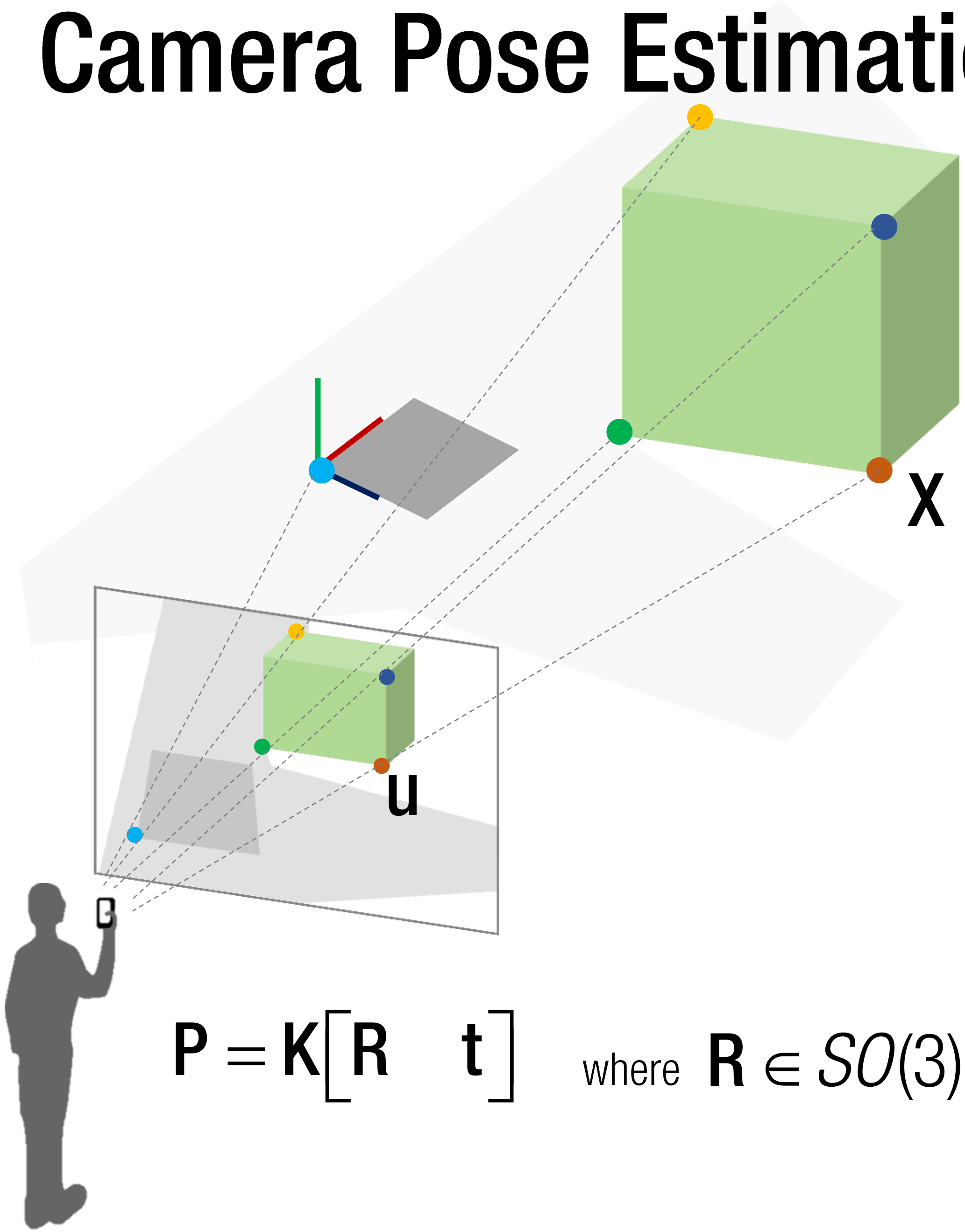
$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & & & & & -u_1^x X & -u_1^x Y & -u_1^x Z & -u_1^x \\ & & & & X_1 & Y_1 & Z_1 & 1 & -u_1^y X & -u_1^y Y & -u_1^y Z & -u_1^y \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ X_m & Y_m & Z_m & 1 & & & & & -u_m^x X & -u_m^x Y & -u_m^x Z & -u_m^x \\ & & & & X_m & Y_m & Z_m & 1 & -u_m^y X & -u_m^y Y & -u_m^y Z & -u_m^y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{0}$$

$2m$x12

Camera Pose Estimation

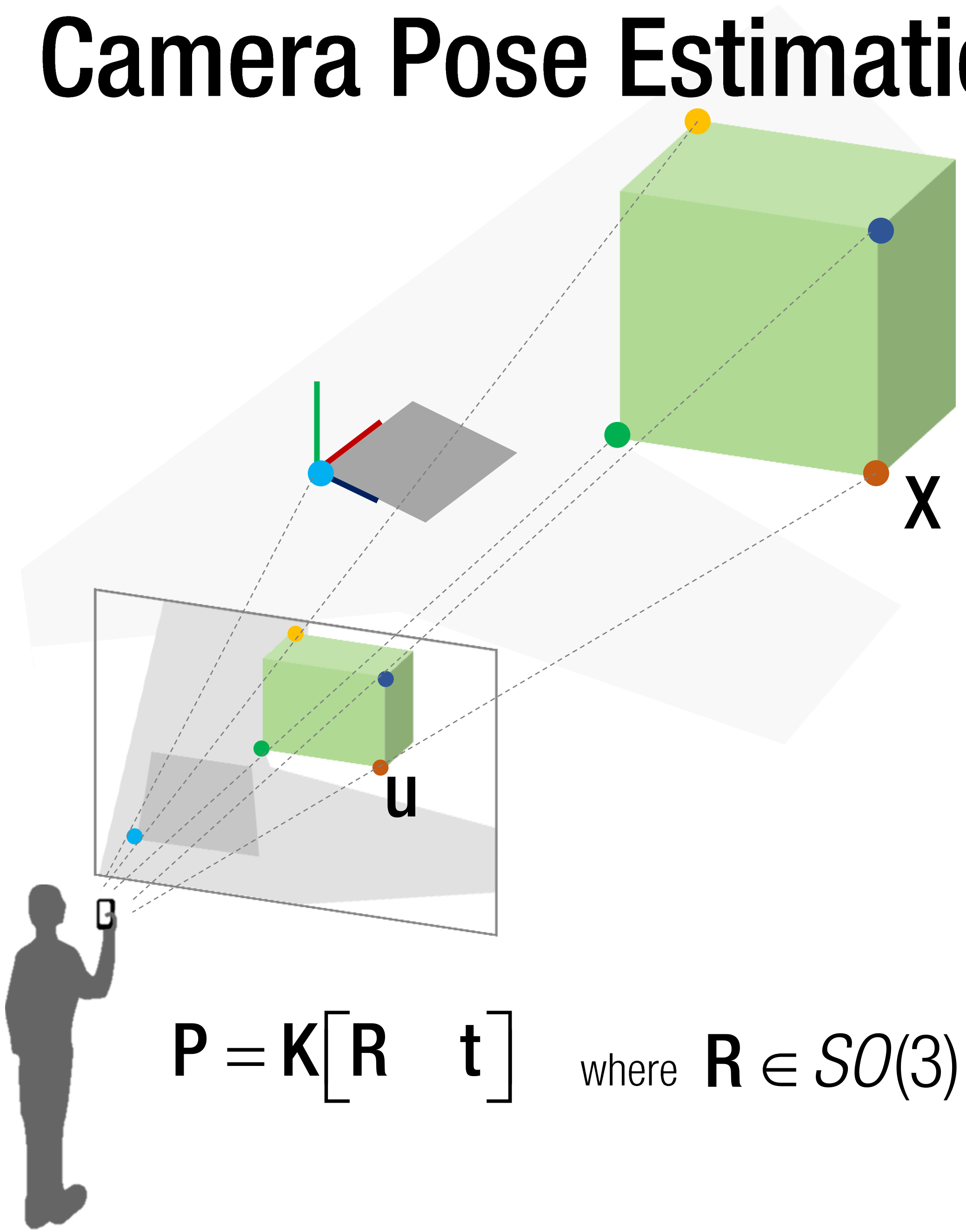


If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

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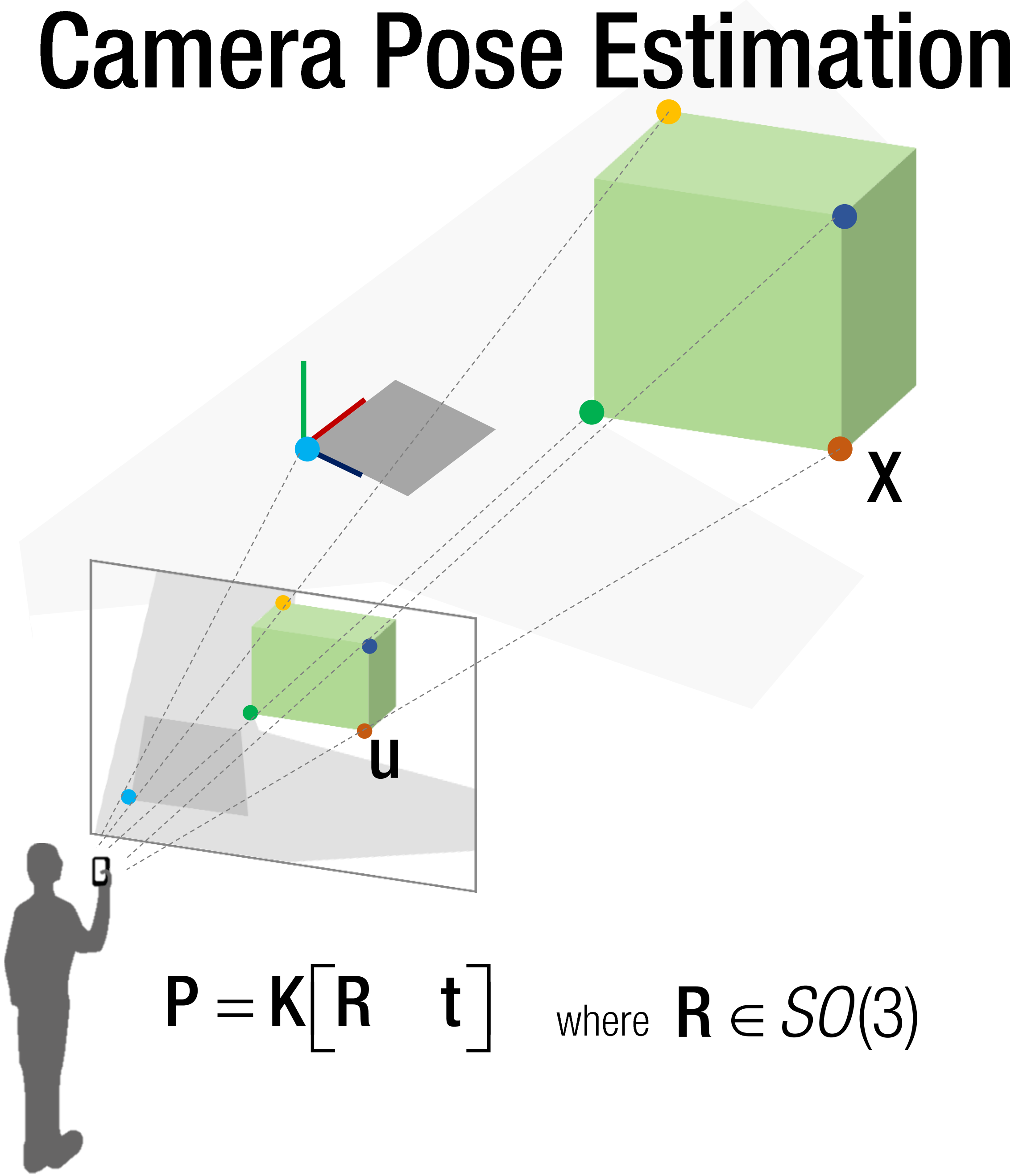
$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

If **K** is given,

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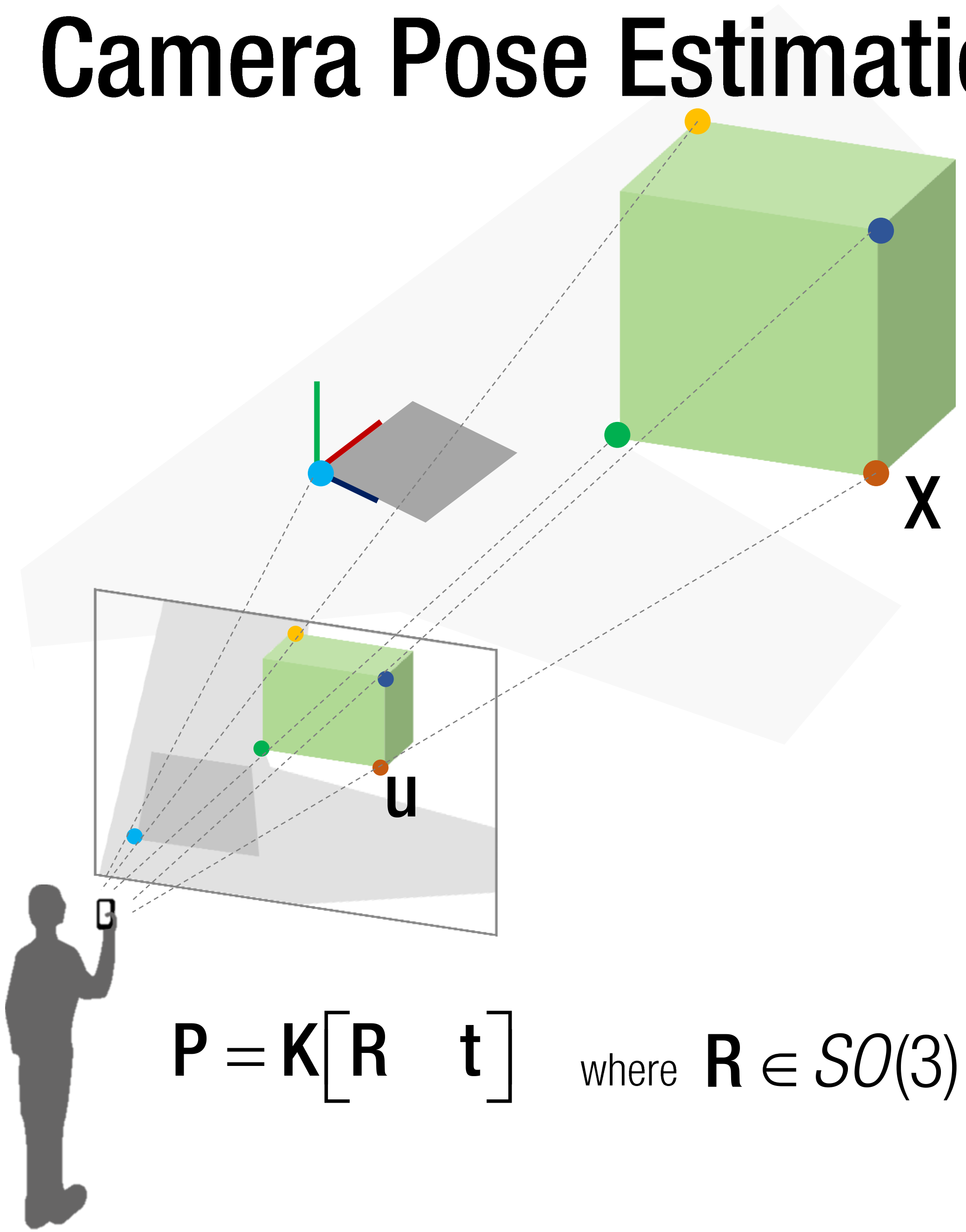
$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix}d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

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$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

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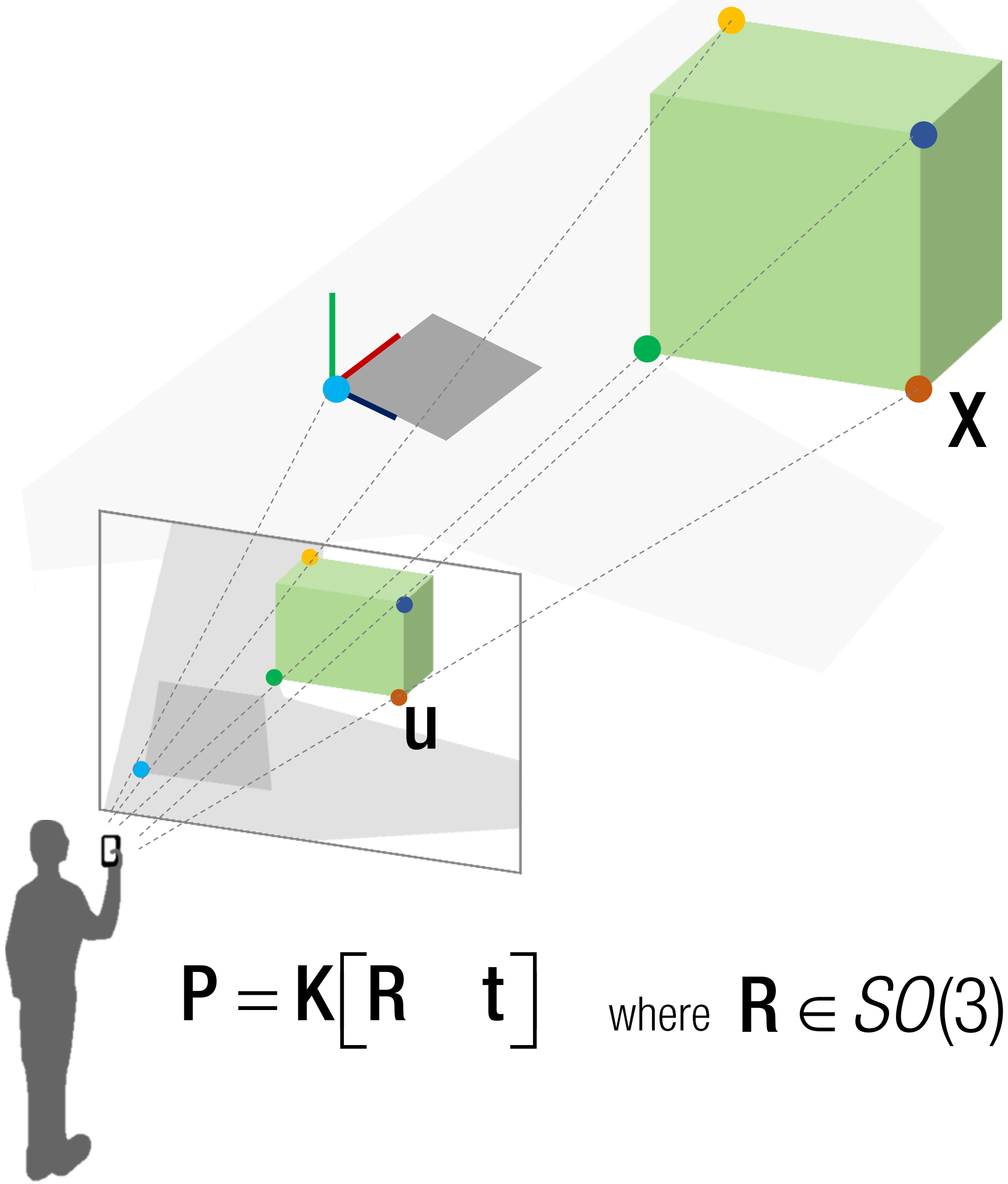
$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix}d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$$\longrightarrow \gamma \approx d_{11}$$

$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$: SVD cleanup

Camera Pose Estimation



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

If **K** is given,

$$\mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} = \gamma\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}$$

$$\longrightarrow \gamma\mathbf{R} = \mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix}$$

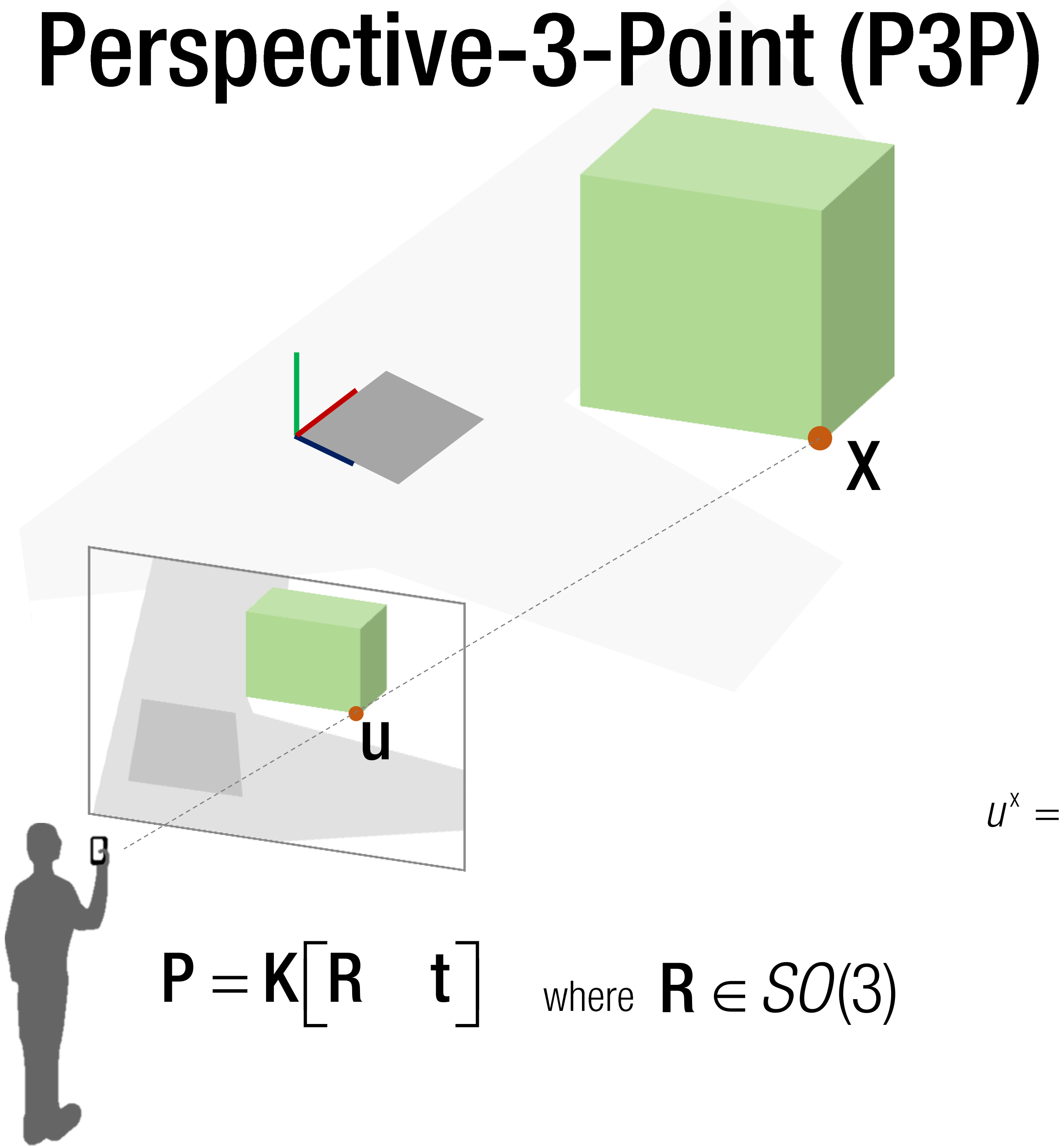
$$\mathbf{K}^{-1}\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3\end{bmatrix} = \mathbf{U}\begin{bmatrix}d_{11} & & \\ & d_{22} & \\ & & d_{33}\end{bmatrix}\mathbf{V}^{\mathsf{T}}$$

$$\longrightarrow \gamma \approx d_{11}$$

$$\mathbf{R} = \mathbf{U}\mathbf{V}^{\mathsf{T}}$$: SVD cleanup

$$\longrightarrow \mathbf{t} = \frac{\mathbf{K}^{-1}\mathbf{p}_4}{d_{11}}$$: Translation and scale recovery

Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

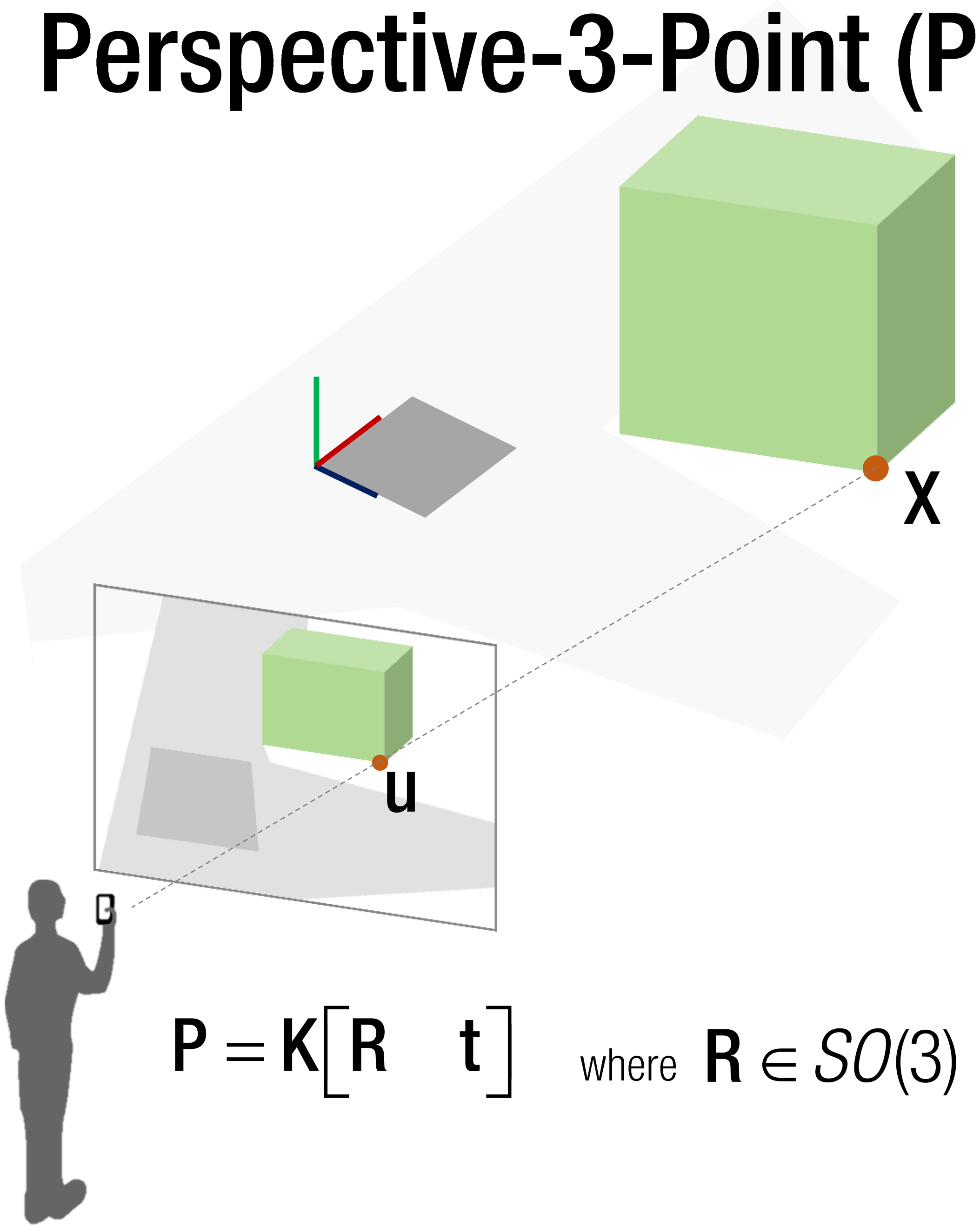
$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

\# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)

6 dof when **K** is known.

\# of equations per correspondence: 2

Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda\mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\text{Linear in camera matrix}}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

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6 dof when **K** is known.

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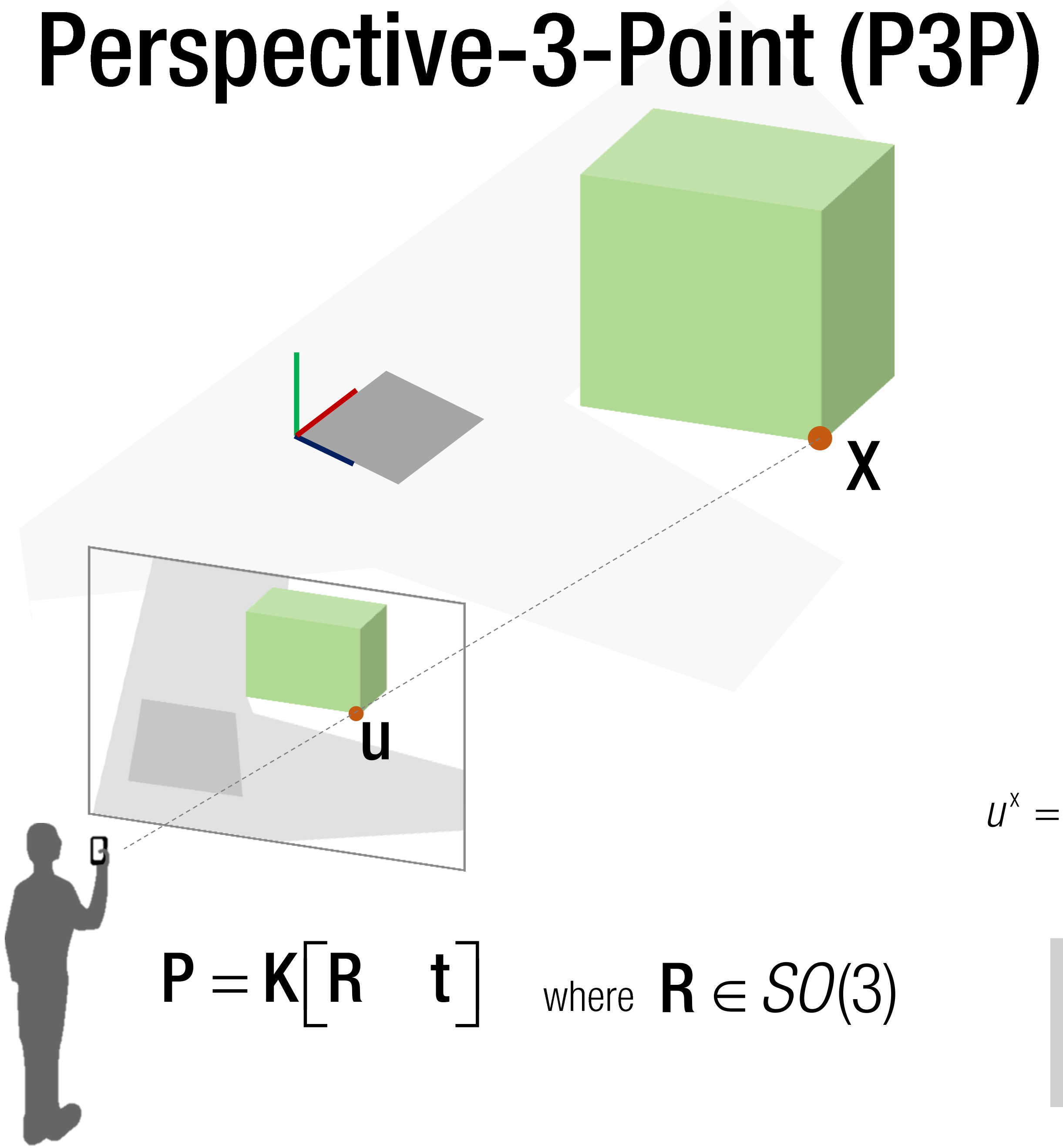
Perspective-3-Point (P3P)



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \quad \text{where } \mathbf{R} \in SO(3)$$

3D-2D correspondence: $\mathbf{u} \leftrightarrow \mathbf{X}$

$$\lambda \mathbf{u} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Linear in camera matrix

$$u^x = \frac{p_{11}X + p_{12}Y + p_{13}Z + p_{14}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}} \qquad u^y = \frac{p_{21}X + p_{22}Y + p_{23}Z + p_{24}}{p_{31}X + p_{32}Y + p_{33}Z + p_{34}}$$

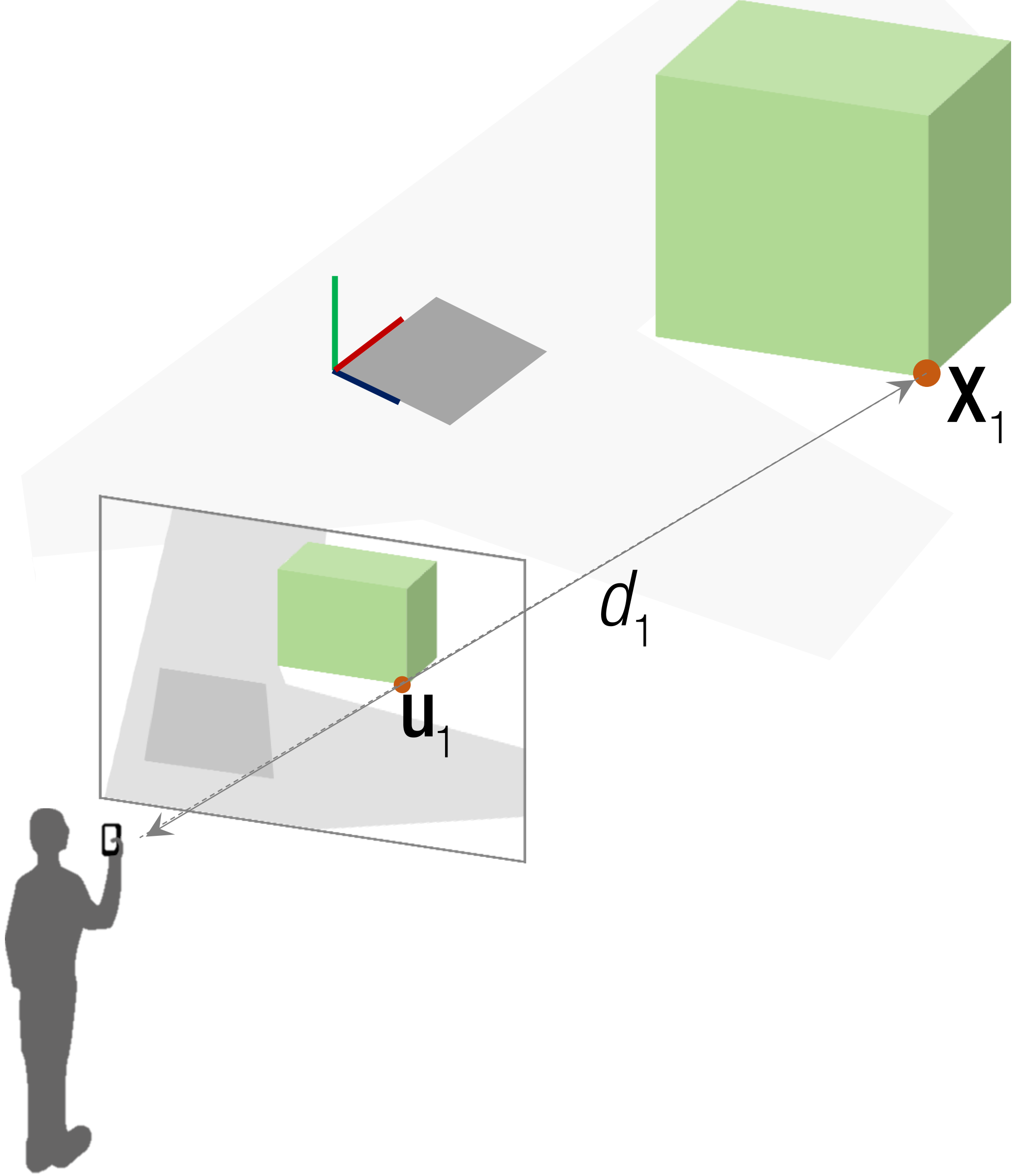
# of unknowns: 11 = 12 (3x4 matrix) – 1 (scale)

6 dof when **K** is known.

# of equations per correspondence: 2

3 correspondences should be enough.

Perspective-3-Point (P3P)

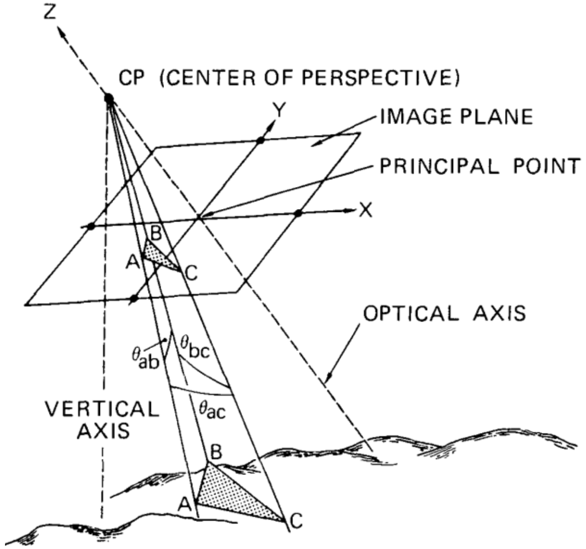


RANSAC with PnP

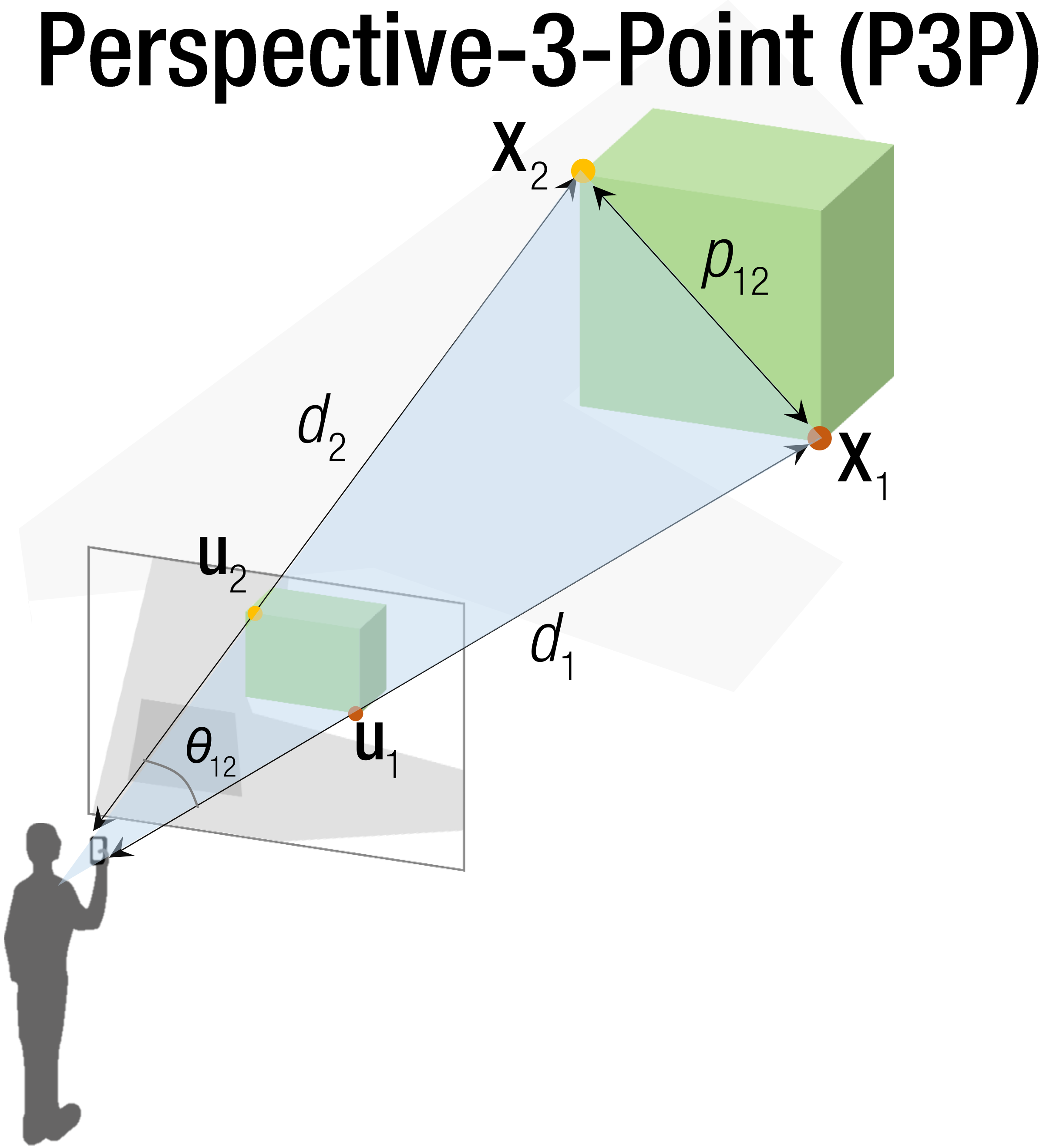
Graphics and Image Processing — J. D. Foley, Editor

Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography

Martin A. Fischler and Robert C. Bolles
SRI International



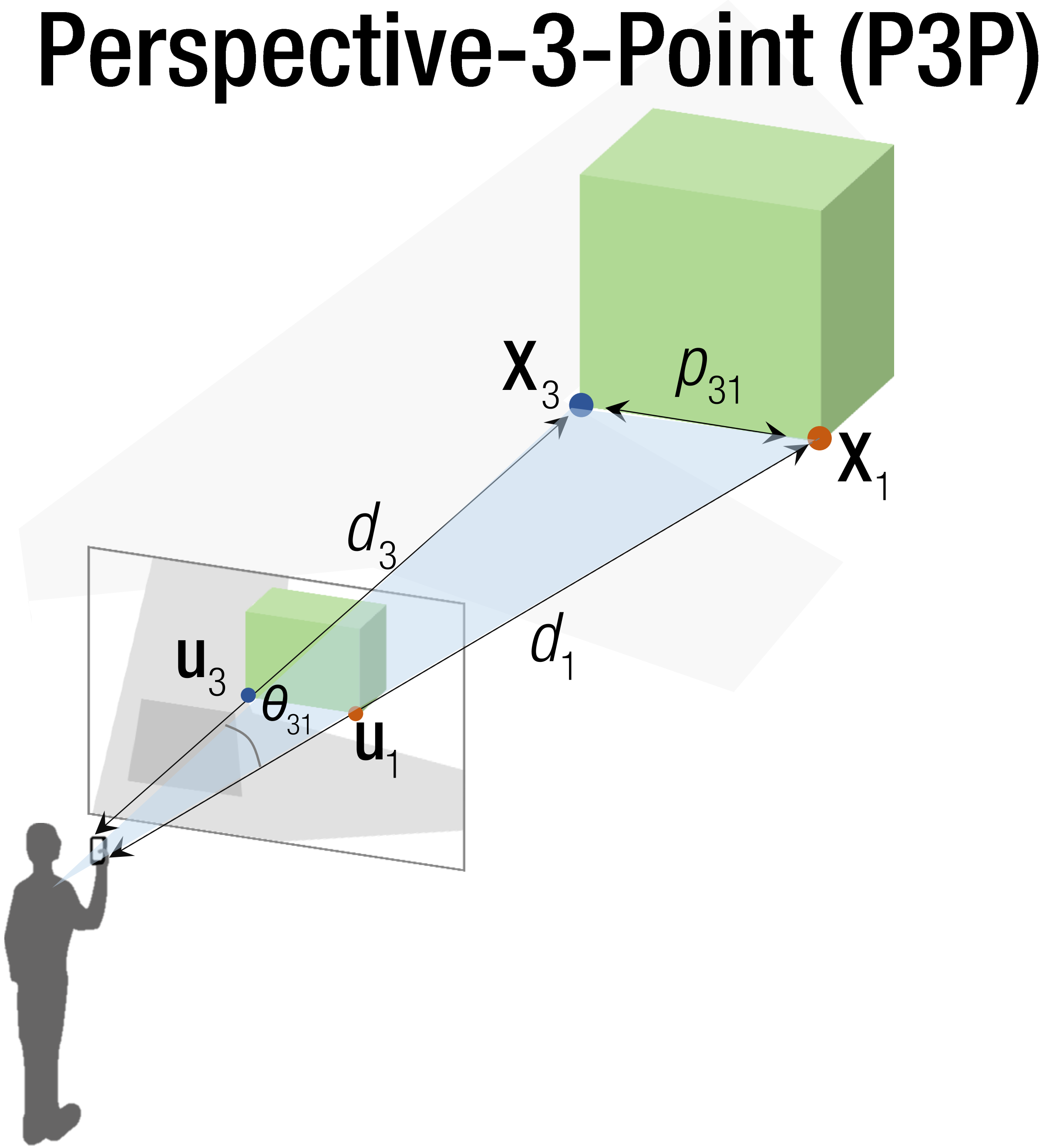
Perspective-3-Point (P3P)



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta_{12} = p_{12}^2$$

Perspective-3-Point (P3P)

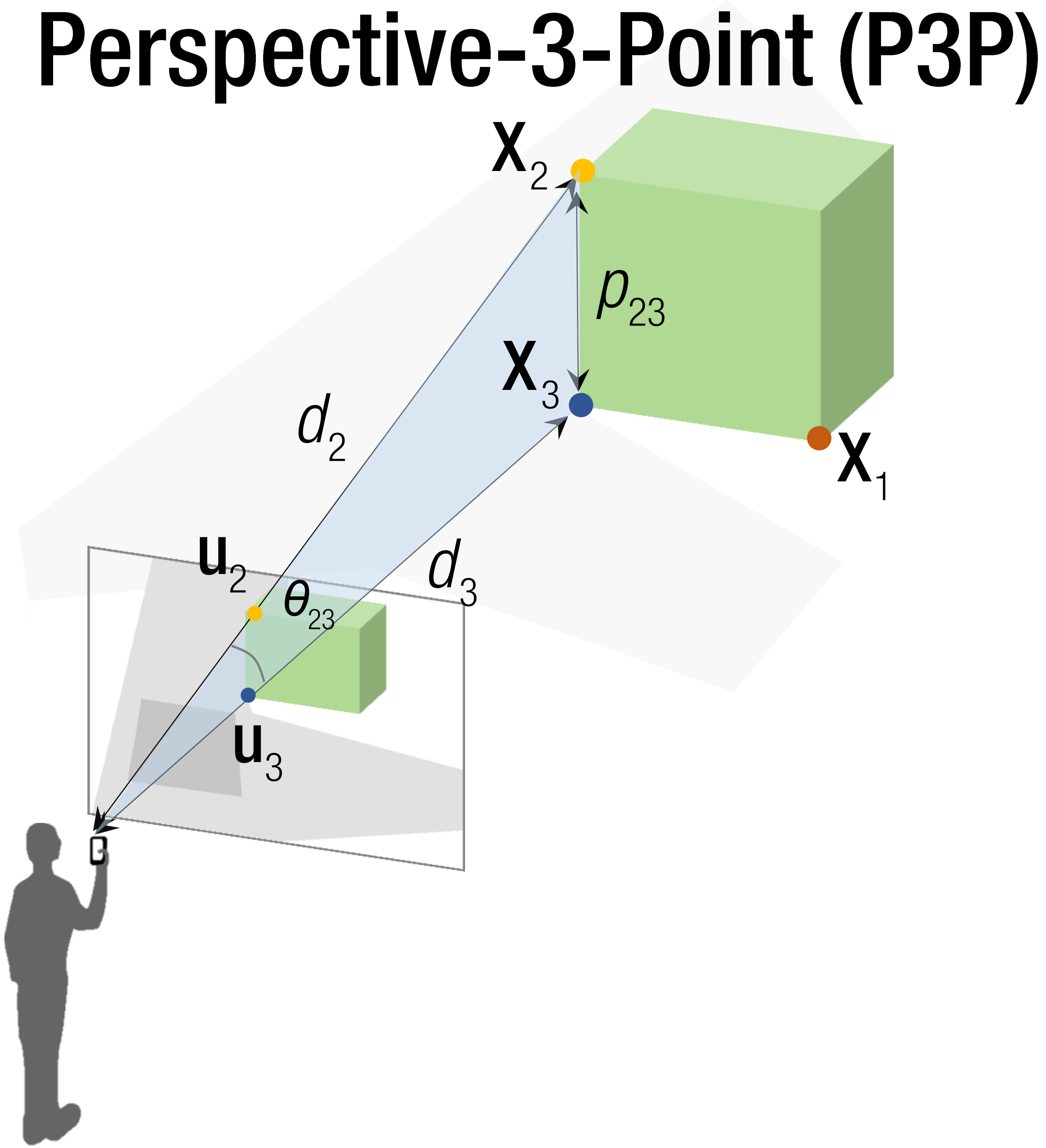


2[nd] Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$

$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$

Perspective-3-Point (P3P)



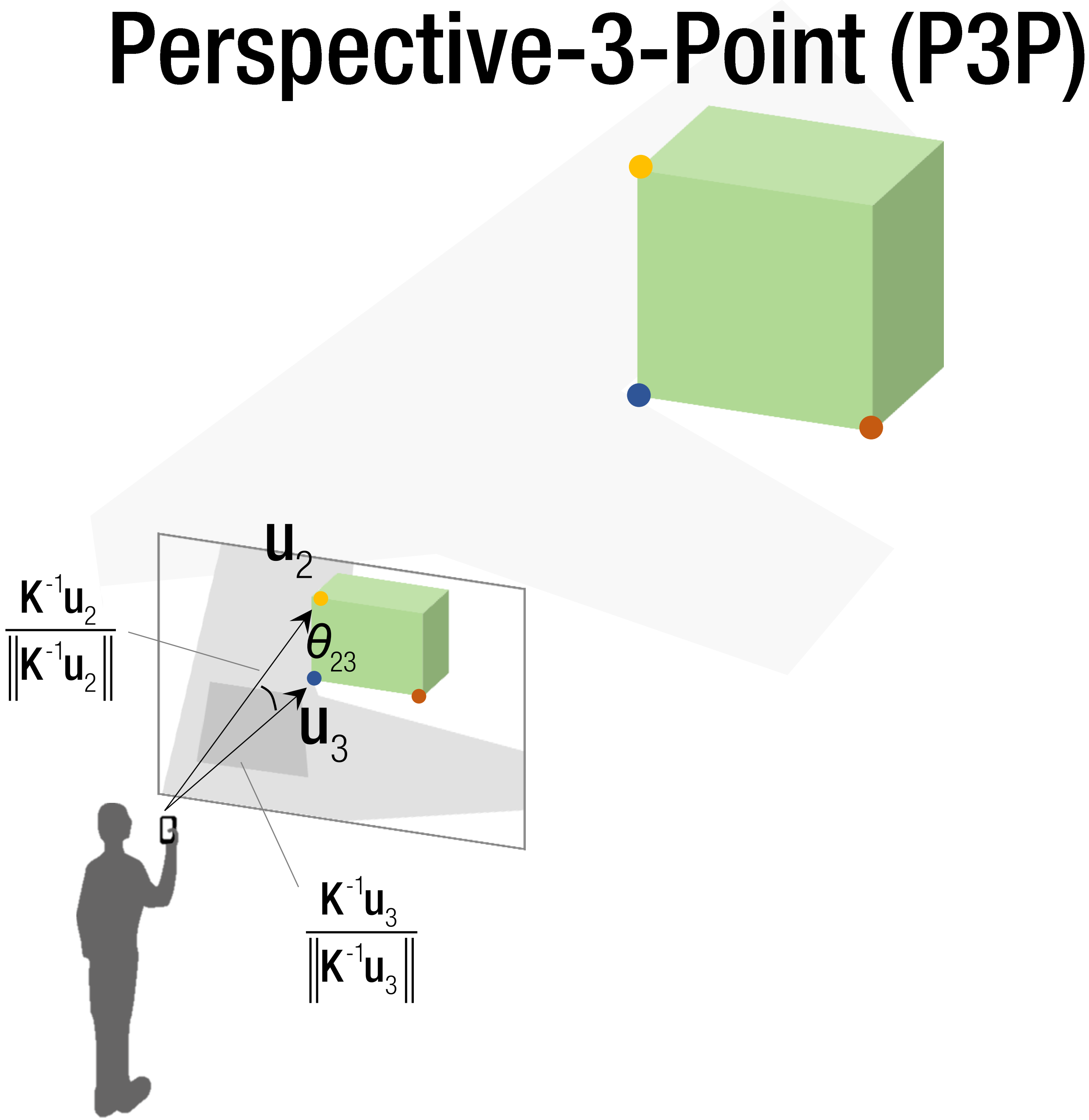
2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} = p_{23}^2$$

3 equations

Unknowns: d_1, d_2, d_3

Perspective-3-Point (P3P)



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_2^2 + d_3^2 - 2d_2d_3\cos\theta_{23} = p_{23}^2$$

3 equations

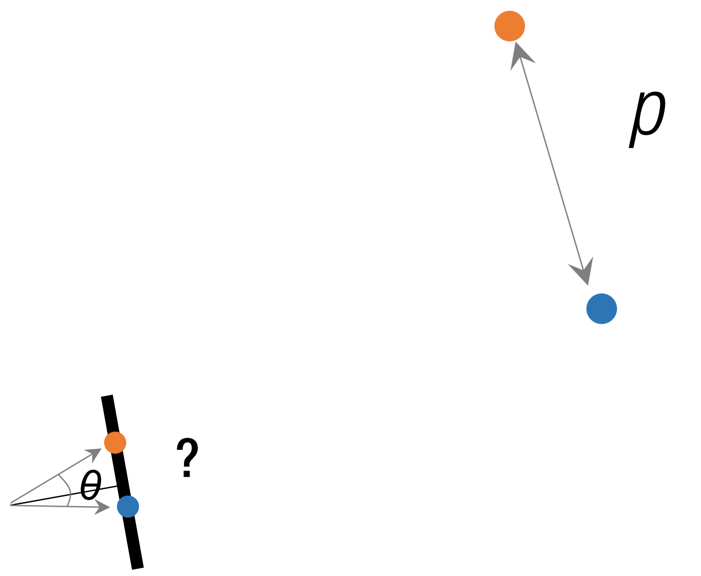
Unknowns: d_1, d_2, d_3

Note:

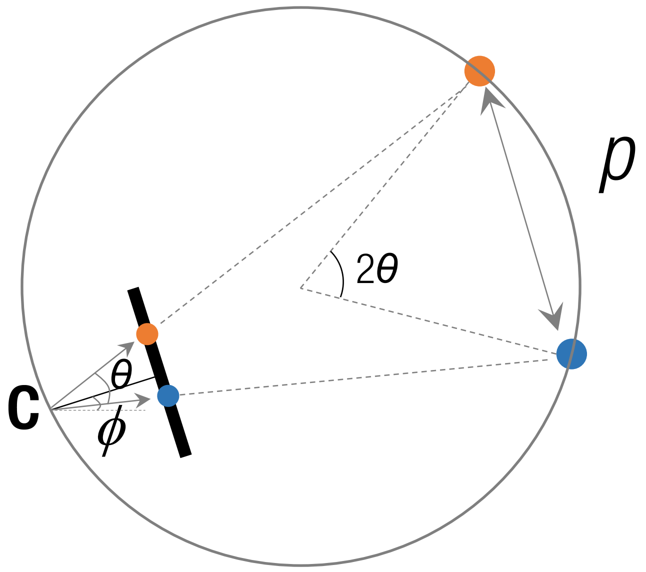
$$\cos\theta_{12} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_2\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|} \qquad \cos\theta_{23} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_2\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_2\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

$$\cos\theta_{31} = \frac{\left(\mathbf{K}^{-1}\mathbf{u}_1\right)^{\mathsf{T}}\left(\mathbf{K}^{-1}\mathbf{u}_3\right)}{\left\|\mathbf{K}^{-1}\mathbf{u}_1\right\|\left\|\mathbf{K}^{-1}\mathbf{u}_3\right\|}$$

Geometric Interpretation: 1D Camera

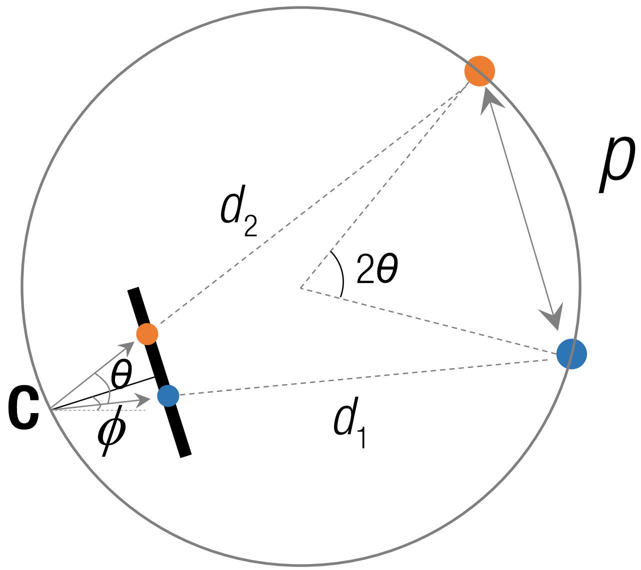


Geometric Interpretation: 1D Camera



Property of inscribed angle

Geometric Interpretation: 1D Camera

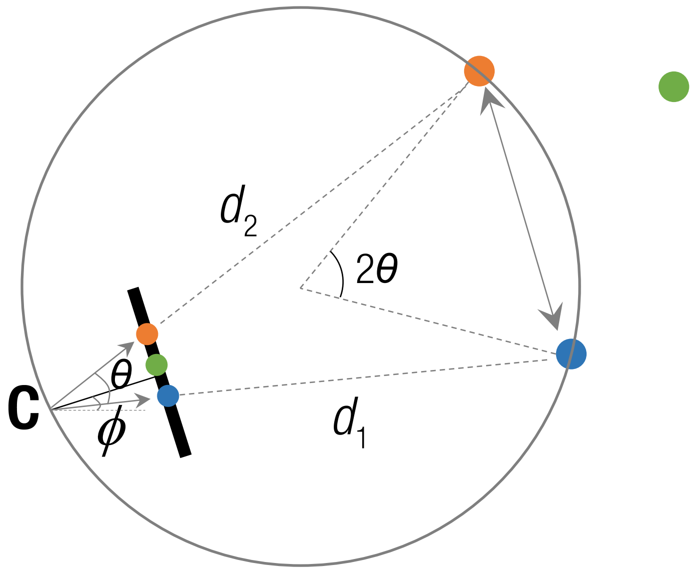


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta = p^2$$

Infinite number of solutions

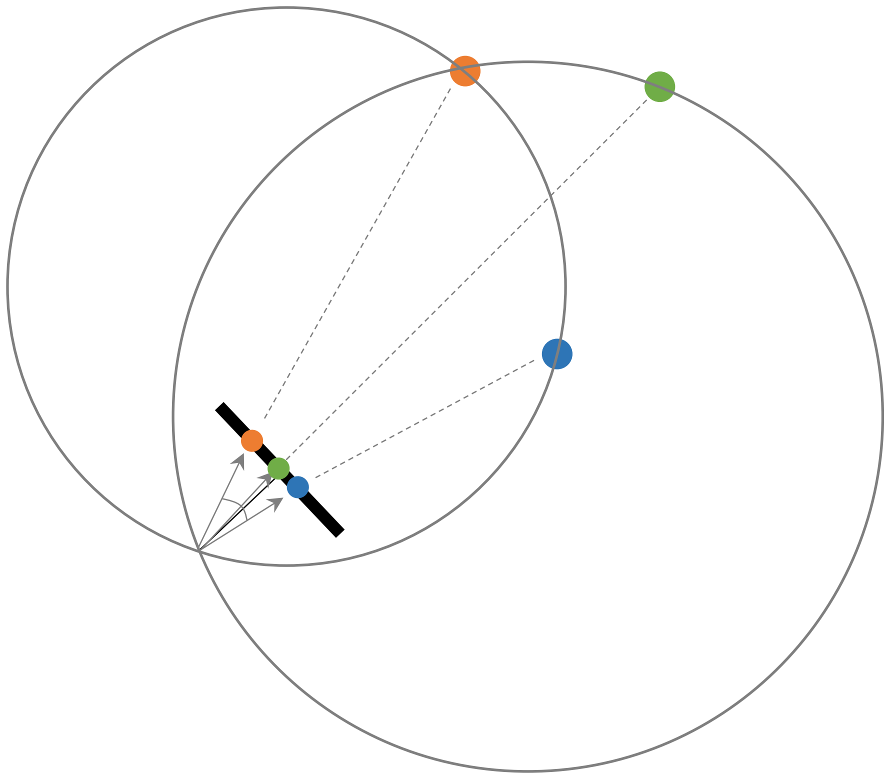
Geometric Interpretation: 1D Camera



2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\boldsymbol{\theta} = p^2$$

Geometric Interpretation: 1D Camera

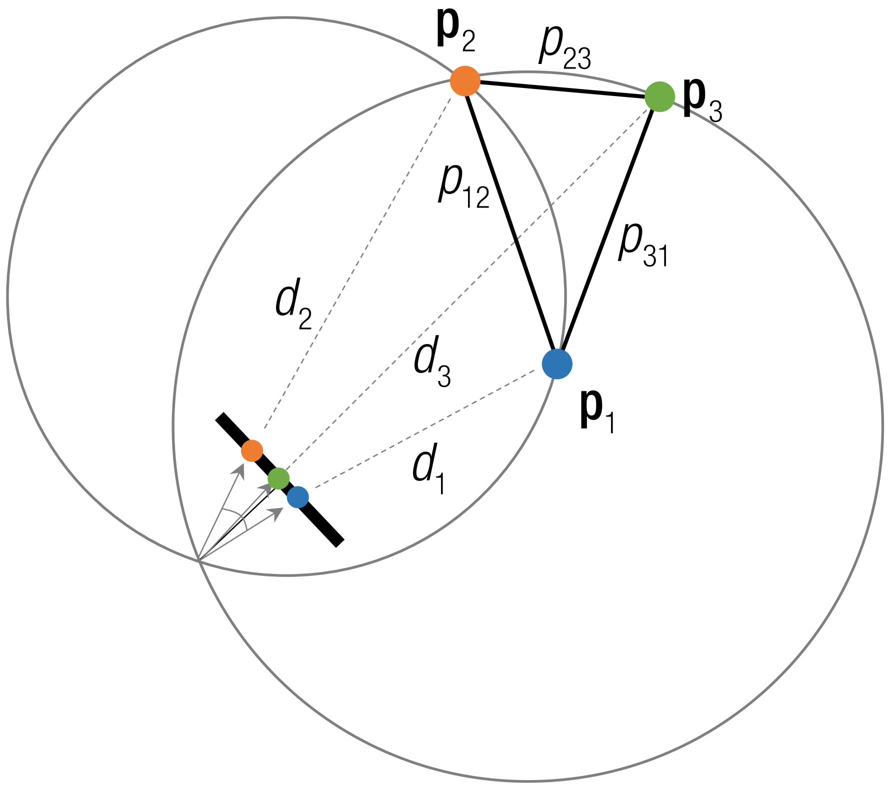


2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta = p^2$$

Finite number of solutions

Geometric Interpretation: 1D Camera



2nd Cosine law:

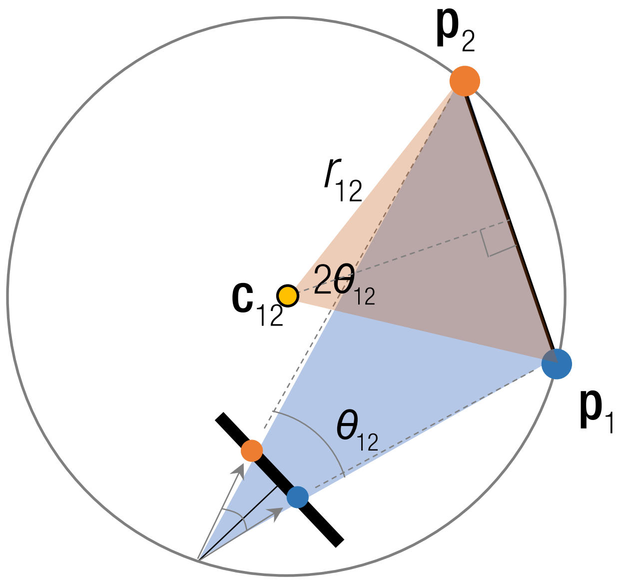
$$d_1^2 + d_2^2 - 2d_1 d_2 \cos\boldsymbol{\theta}_{12} = p_{12}^2$$

$$d_3^2 + d_1^2 - 2d_3 d_1 \cos\boldsymbol{\theta}_{31} = p_{31}^2$$

$$d_2^2 + d_3^2 - 2d_2 d_3 \cos\boldsymbol{\theta}_{23} = p_{23}^2$$

Finite number of solutions

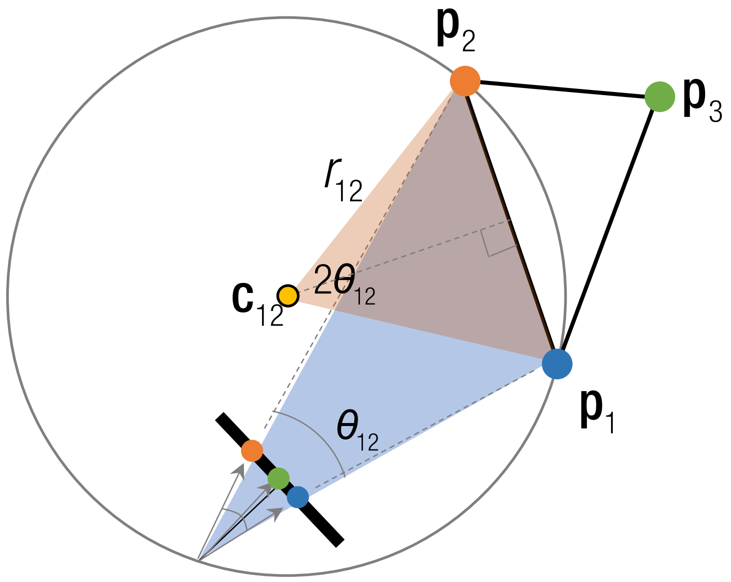
Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}$$

where $r_{12} = \dfrac{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}{2\sin\theta_{12}}$

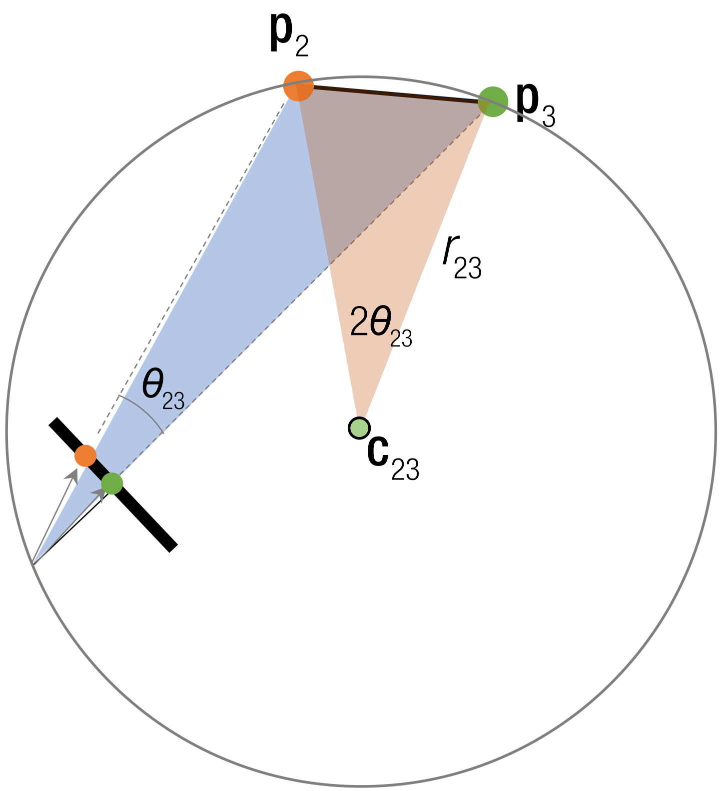
Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}$$

$$\text{where} \quad r_{12} = \frac{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}{2\sin\theta_{12}}$$

Geometric Interpretation: 1D Camera



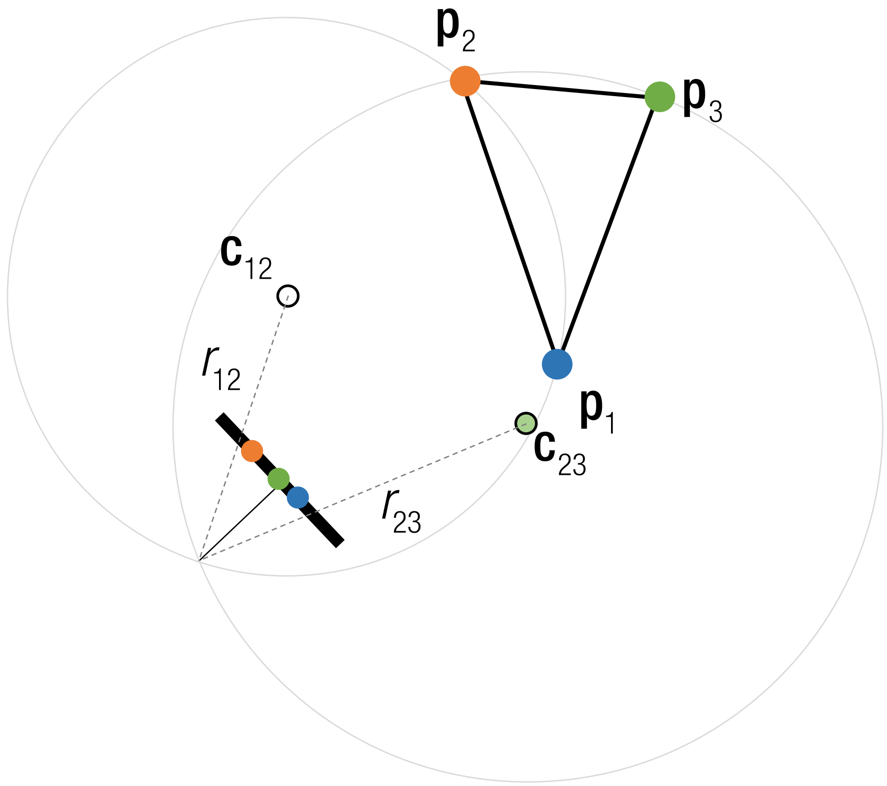
$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}$$

$$\text{where} \quad r_{12} = \frac{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}{2\sin\theta_{12}}$$

$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\left\|\mathbf{p}_3 - \mathbf{p}_2\right\|}$$

$$\text{where} \quad r_{23} = \frac{\left\|\mathbf{p}_3 - \mathbf{p}_2\right\|}{2\sin\theta_{23}}$$

Geometric Interpretation: 1D Camera



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\|\mathbf{p}_2 - \mathbf{p}_1\|}$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

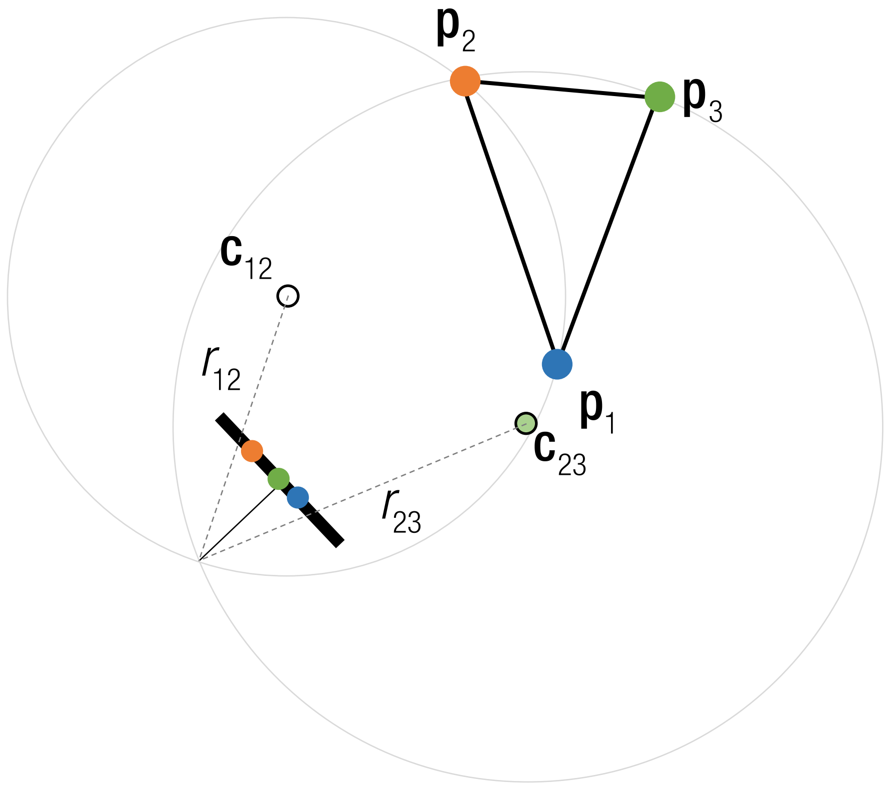
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\|\mathbf{p}_3 - \mathbf{p}_2\|}$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

$$\|\mathbf{x} - \mathbf{c}_{12}\|^2 = r_{12}^2 \qquad \|\mathbf{x} - \mathbf{c}_{23}\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\|\mathbf{p}_2 - \mathbf{p}_1\|}$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

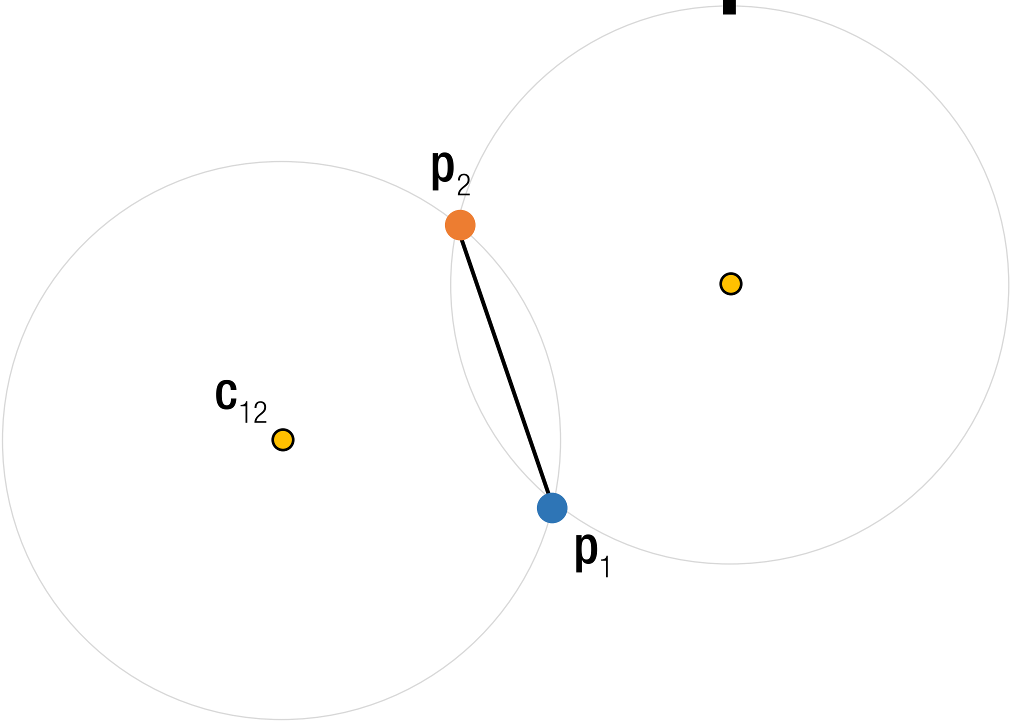
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\|\mathbf{p}_3 - \mathbf{p}_2\|}$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

$$\|\mathbf{x} - \mathbf{c}_{12}\|^2 = r_{12}^2 \qquad \|\mathbf{x} - \mathbf{c}_{23}\|^2 = r_{23}^2$$

HW: Drive **x** and orientation.

Geometric Interpretation: Family of Solutions



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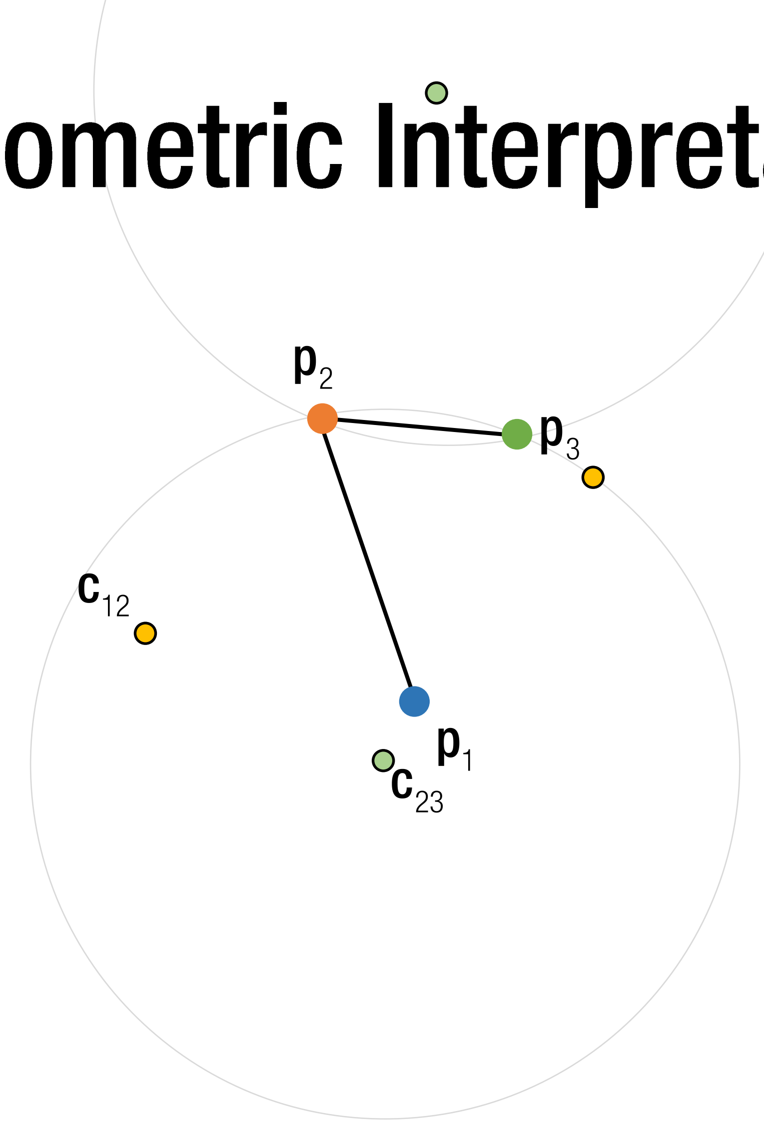
$$\text{where} \quad r_{12} = \frac{\left\|\mathbf{p}_2 - \mathbf{p}_1\right\|}{2\sin\theta_{12}}$$

Geometric Interpretation: Family of Solutions



4 combinations

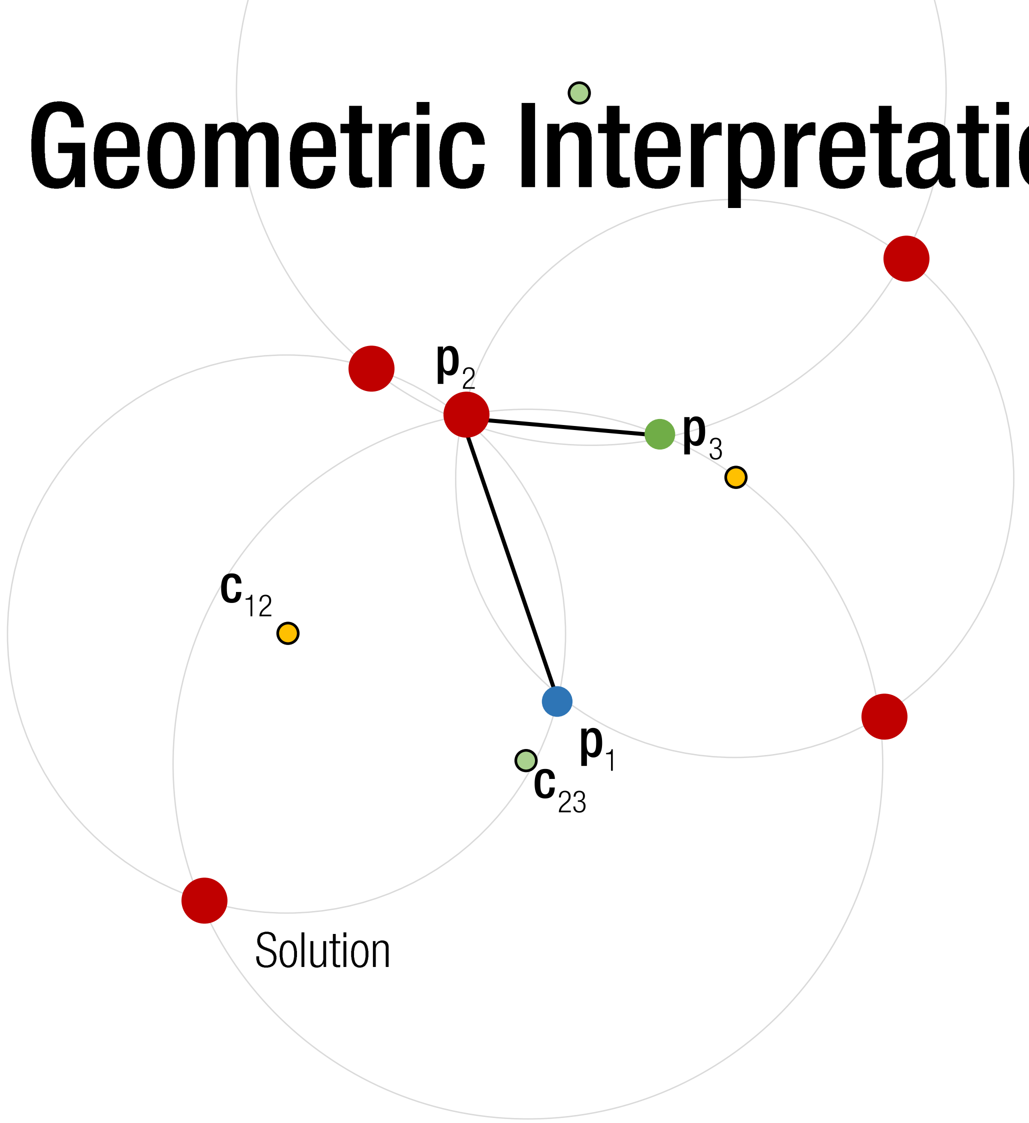
$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}$$

where $r_{12} = \frac{\left\| \mathbf{p}_2 - \mathbf{p}_1 \right\|}{2\sin\theta_{12}}$

$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\left\| \mathbf{p}_3 - \mathbf{p}_2 \right\|}$$

where $r_{23} = \frac{\left\| \mathbf{p}_3 - \mathbf{p}_2 \right\|}{2\sin\theta_{23}}$

Geometric Interpretation: Family of Solutions



$$\mathbf{c}_{12} = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2} \pm r_{12} \cos\theta_{12} \frac{\mathbf{p}_2 - \mathbf{p}_1}{\|\mathbf{p}_2 - \mathbf{p}_1\|}$$

where $r_{12} = \dfrac{\|\mathbf{p}_2 - \mathbf{p}_1\|}{2\sin\theta_{12}}$

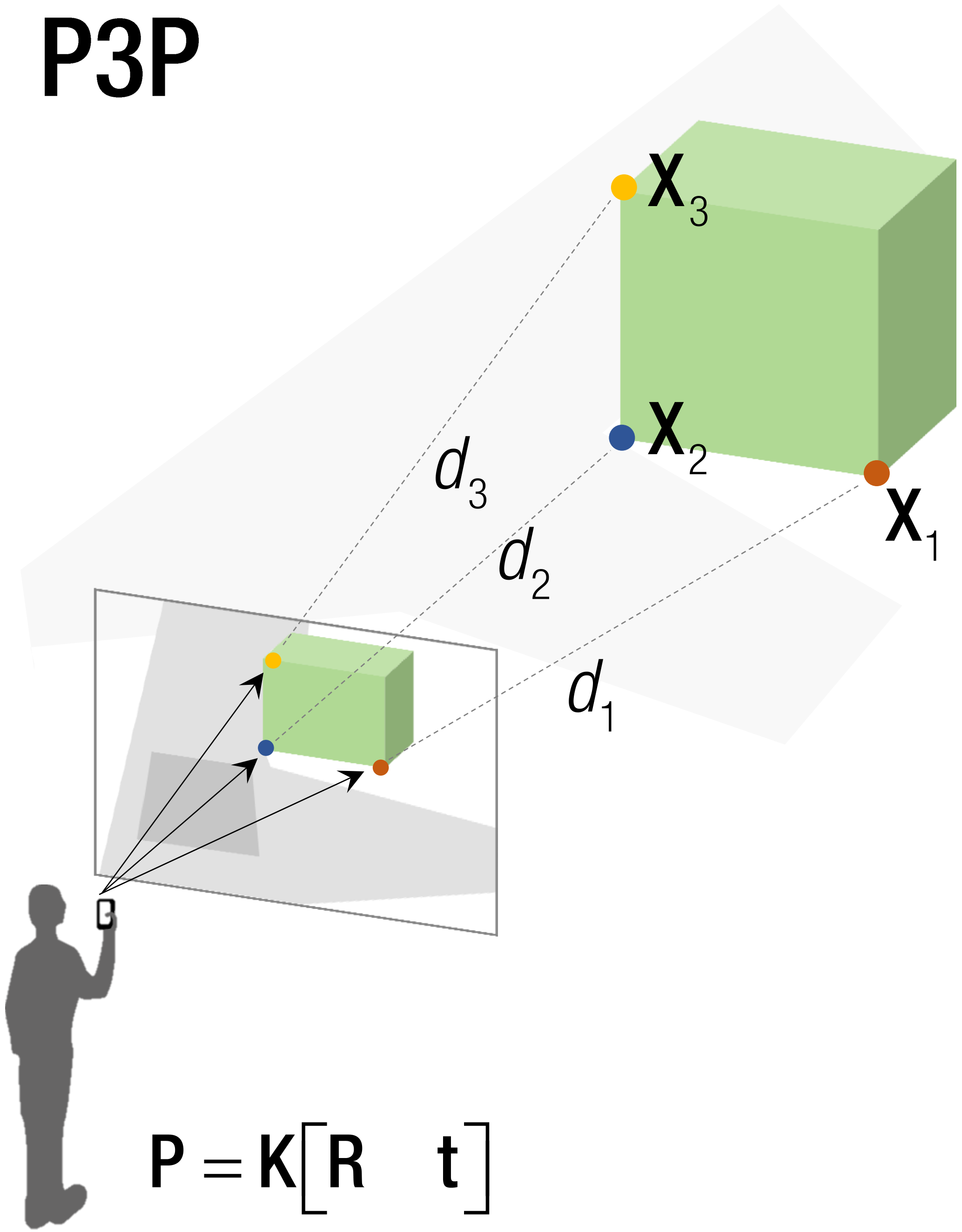
$$\mathbf{c}_{23} = \frac{\mathbf{p}_3 + \mathbf{p}_2}{2} \pm r_{23} \cos\theta_{23} \frac{\mathbf{p}_3 - \mathbf{p}_2}{\|\mathbf{p}_3 - \mathbf{p}_2\|}$$

where $r_{23} = \dfrac{\|\mathbf{p}_3 - \mathbf{p}_2\|}{2\sin\theta_{23}}$

4 combinations of circle centers

→ 4 solutions except for $\mathbf{p}_2$ ($\mathbf{p}_2$ is counted four times.).

P3P



$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

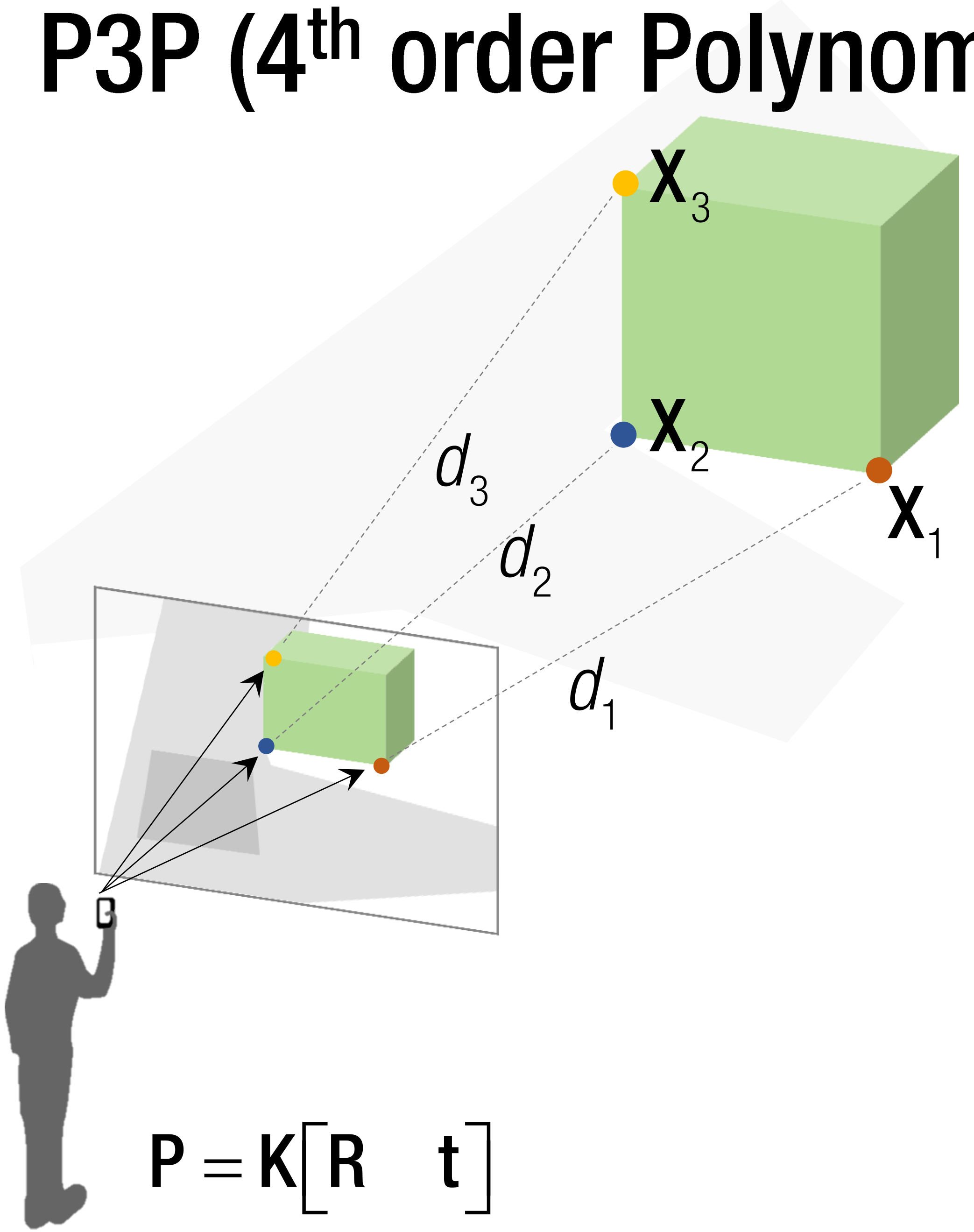
3 equations

The number of possible solutions: 8 = 2x2x2

$d_1 > 0$ $d_2 > 0$ $d_3 > 0$: 4 = 2x2x2/2

→ requires additional fourth point to verify the solution.

P3P (4th order Polynomial)



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$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

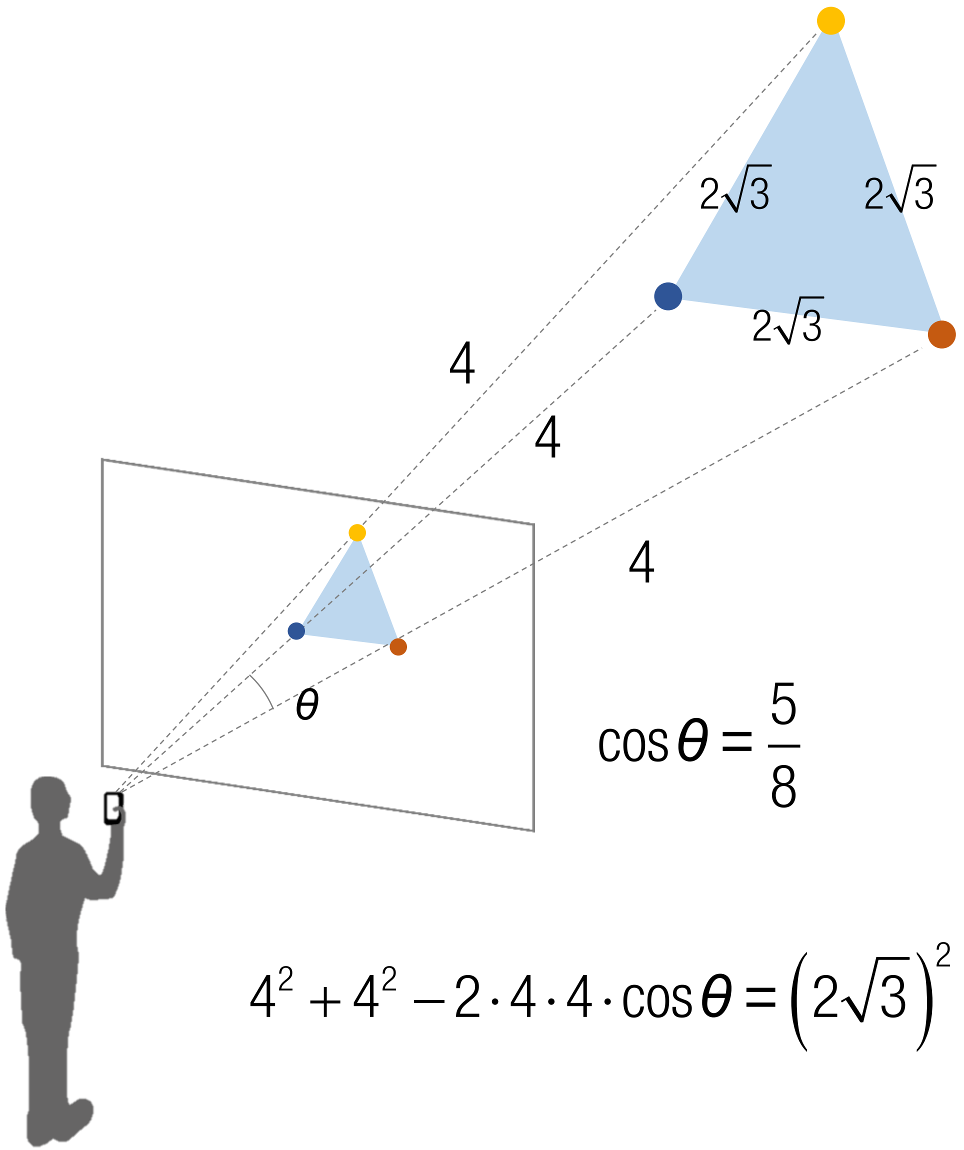
3 equations

4th order polynomial:

$$\rightarrow a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$$

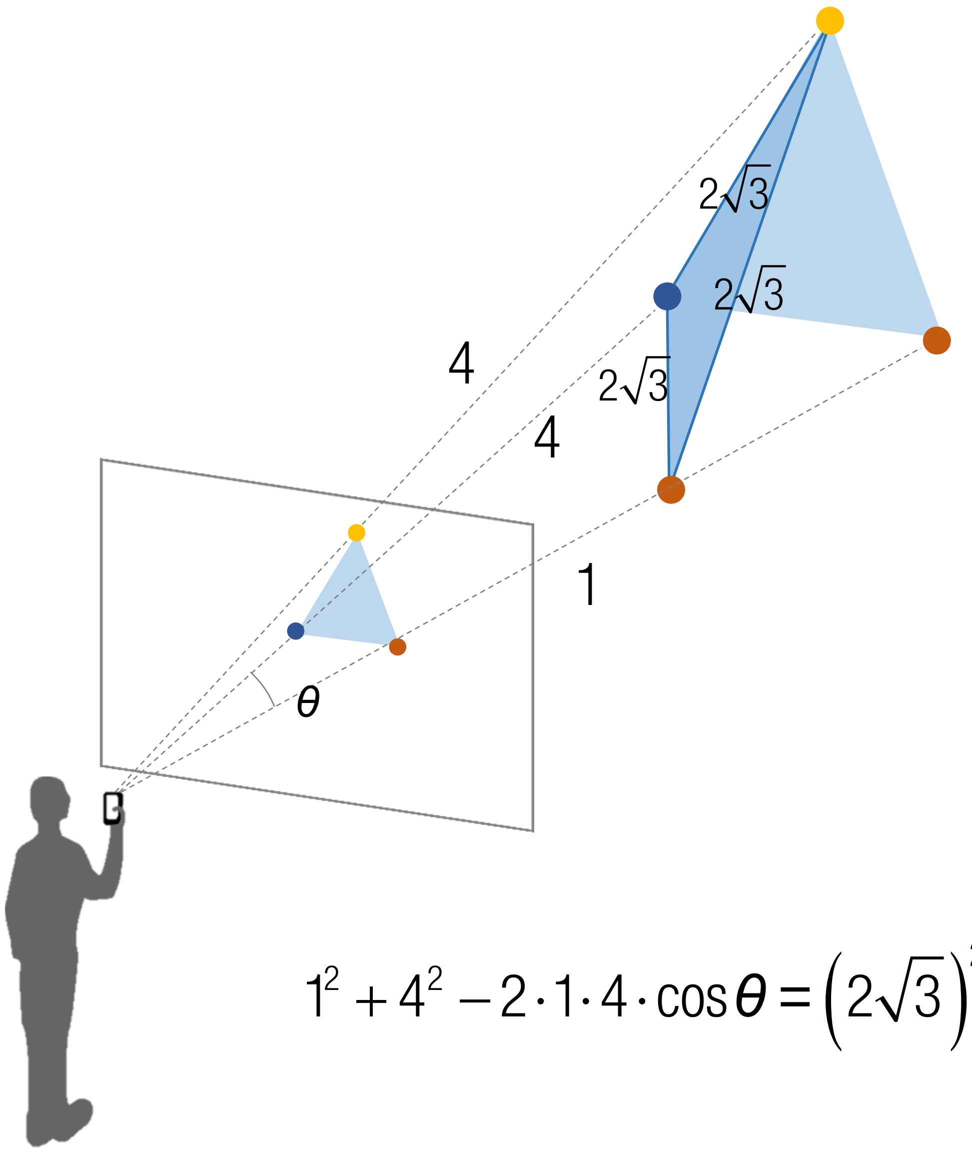
Closed form solutions exist.

Four Solution Example



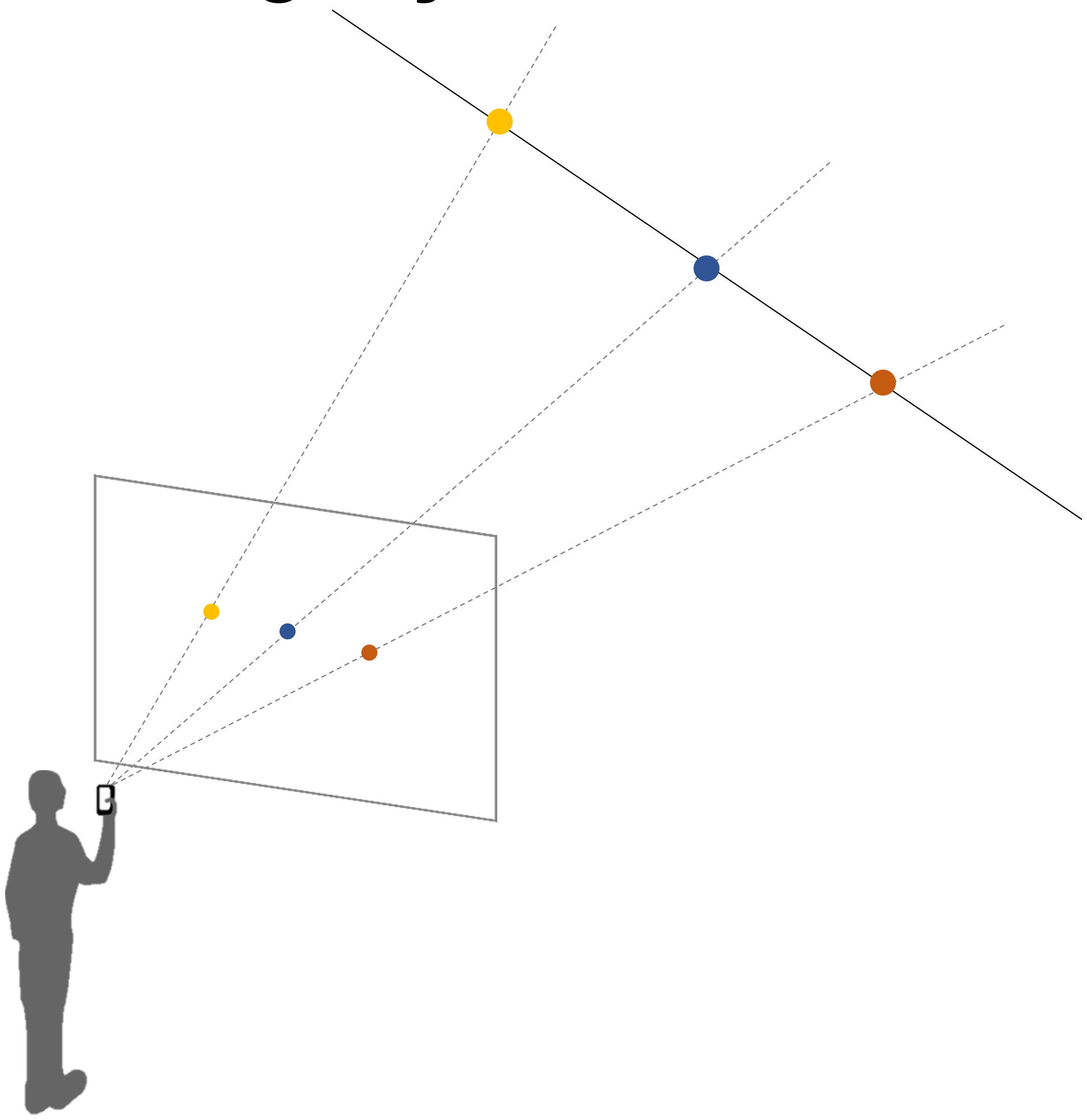
$$\cos\theta = \frac{5}{8}$$

$$4^2 + 4^2 - 2 \cdot 4 \cdot 4 \cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

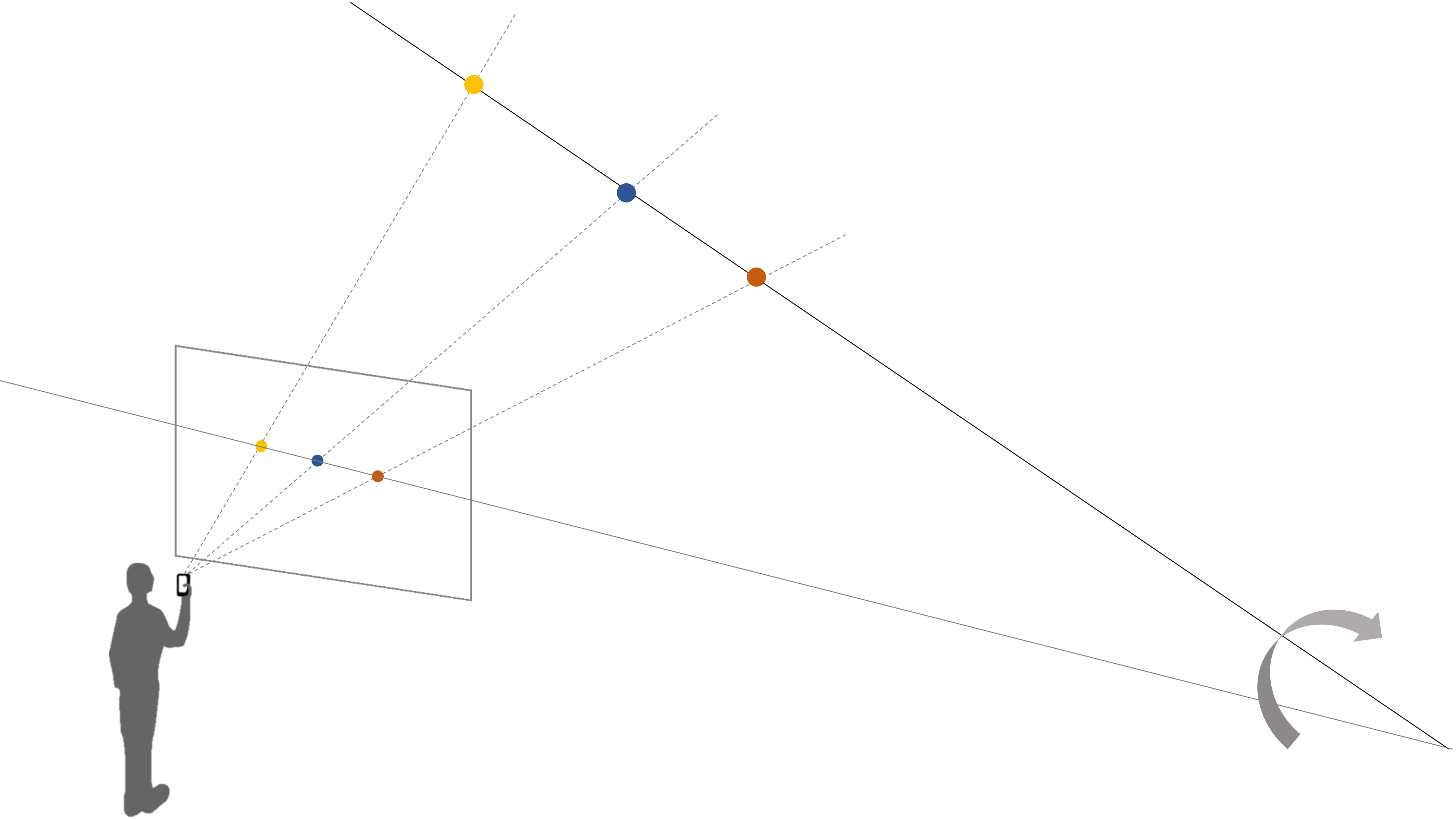


$$1^2 + 4^2 - 2 \cdot 1 \cdot 4 \cdot \cos\theta = \left(2\sqrt{3}\right)^2$$

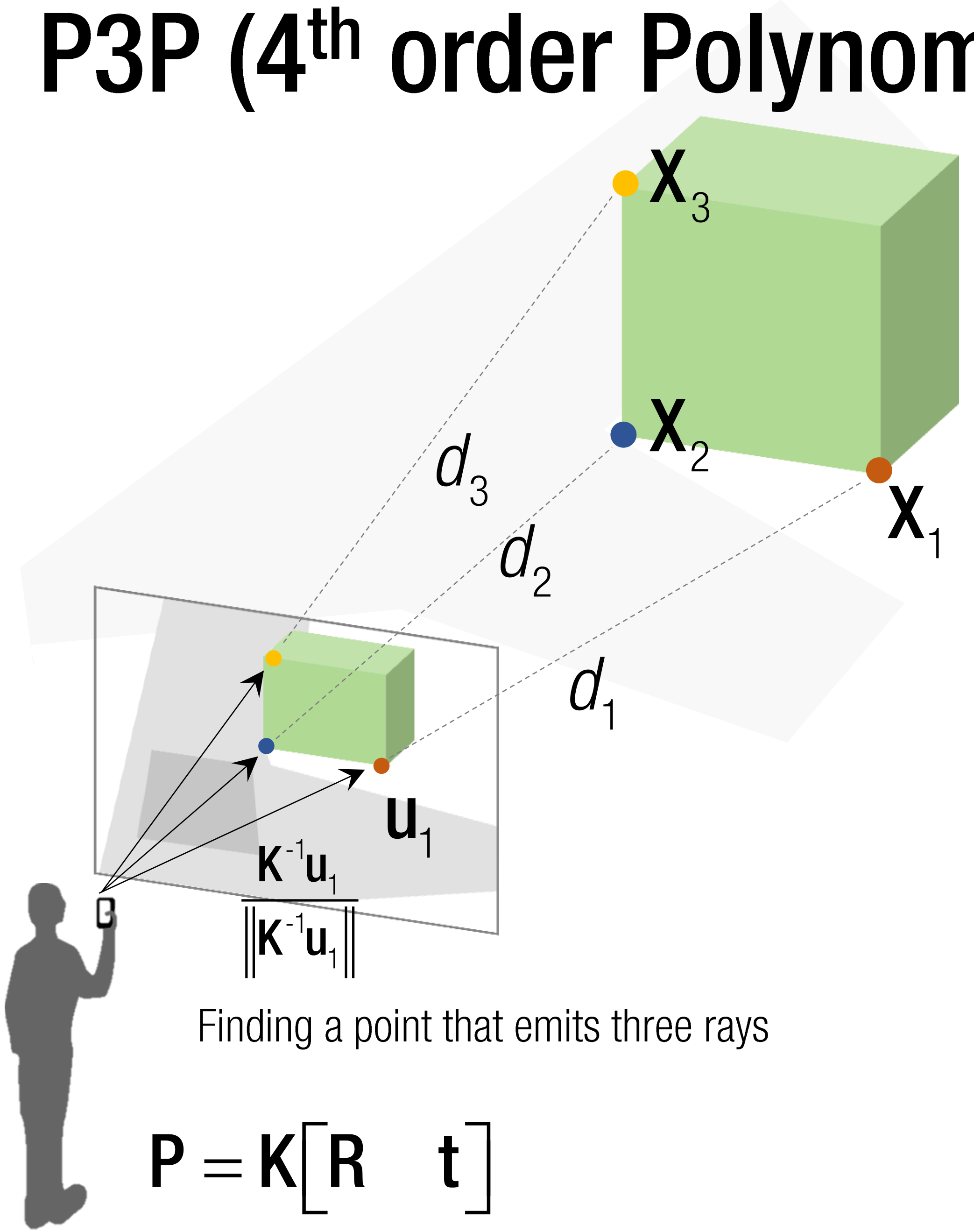
Ambiguity



Ambiguity (Colinear points)



P3P (4th order Polynomial)



Finding a point that emits three rays

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

2nd Cosine law:

$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{12} = p_{12}^2$$
$$d_3^2 + d_1^2 - 2d_3d_1\cos\theta_{31} = p_{31}^2$$
$$d_1^2 + d_2^2 - 2d_1d_2\cos\theta_{23} = p_{23}^2$$

3 equations

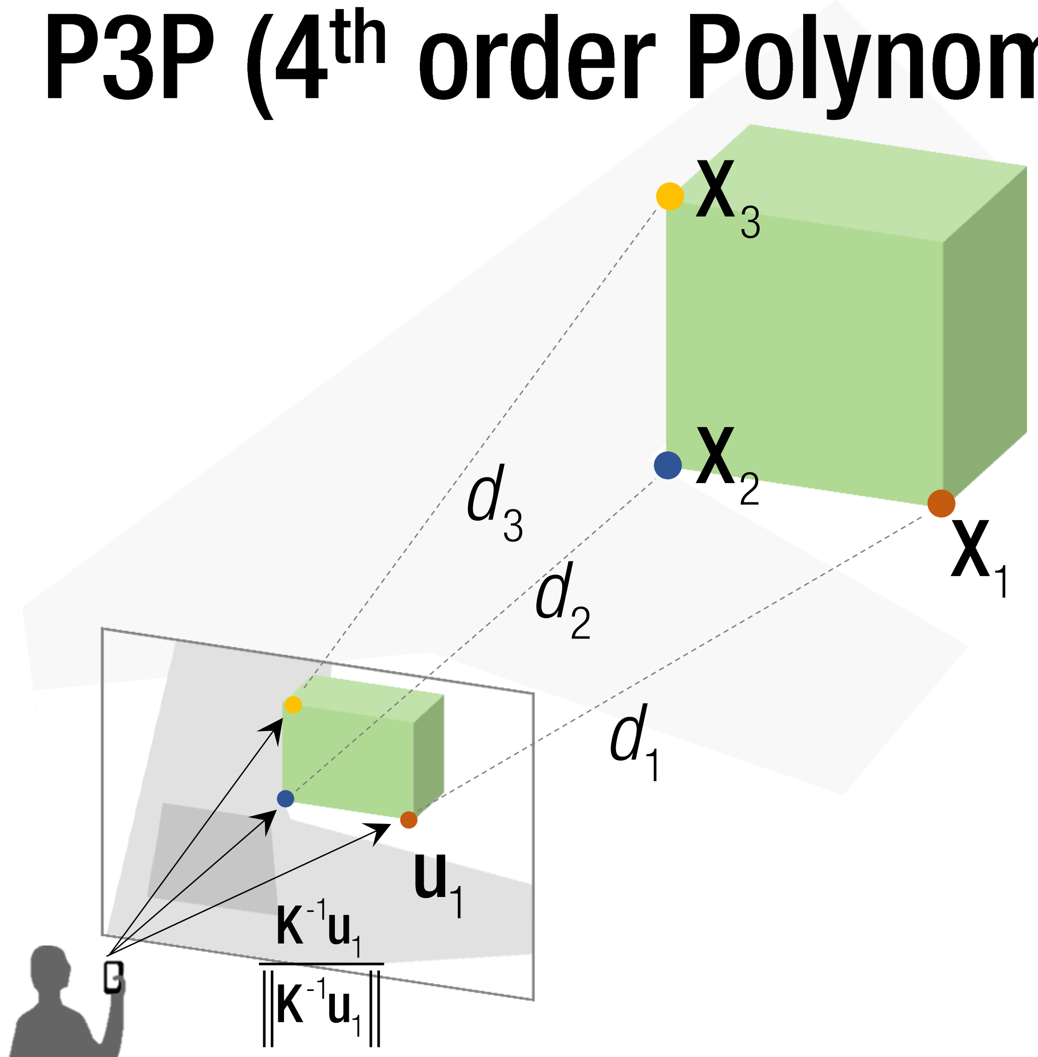
4th order polynomial:

→ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

→ Compute **t** using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

P3P (4^{th} order Polynomial)



Finding a point that emits three rays

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3 equations

4^{th} order polynomial:

→ $a_4y^4 + a_3y^3 + a_2y^2 + a_1y + a_0 = 0$

Closed form solutions exist.

→ Compute **t** using $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$, d_1, d_2, and d_3.

→ $\begin{bmatrix}\tilde{\mathbf{X}}_1 & \tilde{\mathbf{X}}_2 & \tilde{\mathbf{X}}_3\end{bmatrix} = \mathbf{R}\begin{bmatrix}\mathbf{X}_1 & \mathbf{X}_2 & \mathbf{X}_3\end{bmatrix}$

Rotation matrix computation

where $\tilde{\mathbf{X}}_1 = d_1\frac{\mathbf{K}^{-1}\mathbf{u}_1}{\|\mathbf{K}^{-1}\mathbf{u}_1\|}$ $\tilde{\mathbf{X}}_2 = d_2\frac{\mathbf{K}^{-1}\mathbf{u}_2}{\|\mathbf{K}^{-1}\mathbf{u}_2\|}$ $\tilde{\mathbf{X}}_3 = d_3\frac{\mathbf{K}^{-1}\mathbf{u}_3}{\|\mathbf{K}^{-1}\mathbf{u}_3\|}$