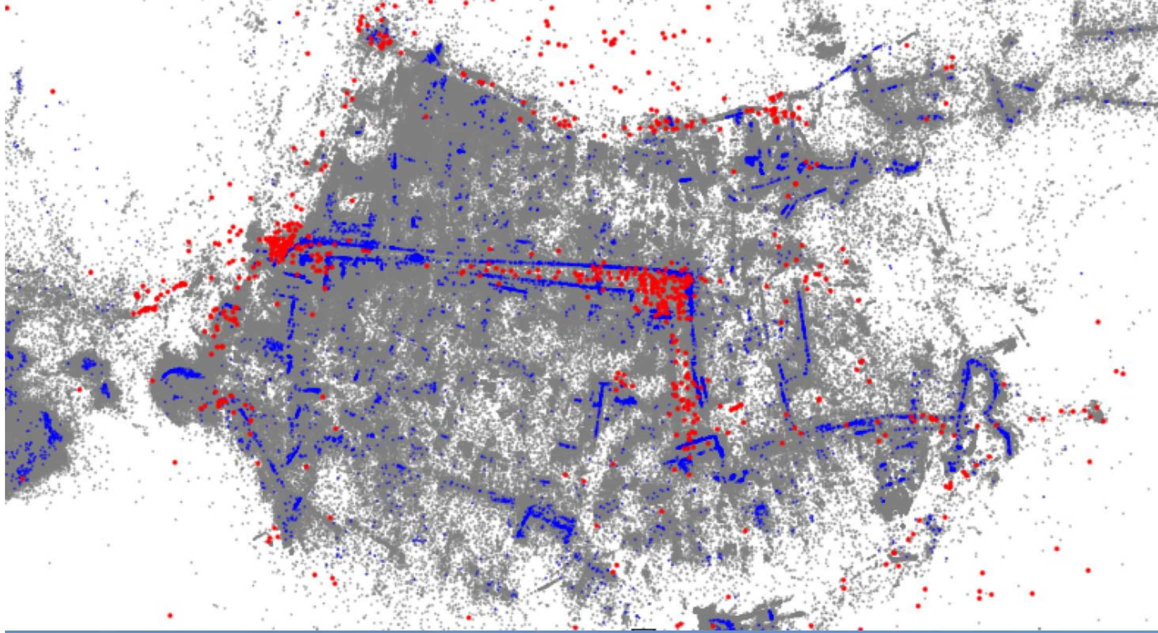
Structure from Motion: A Factorization Approach



Announcement

- HW #4 and 5 are graded.

- HW #6 is out
  - Two deadlines: May 3 (Problem 2,3,4,5) and May 10 (Problem 6 with full reconstruction)
  - 5980 students: < 5 image reconstruction
  - 8980 students: <10 image reconstruction

- Course Evaluation Today

Longuet-Higgins: Epipolar Geometry

A computer algorithm for reconstructing a scene from two projections

H. C. Longuet-Higgins

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A simple algorithm for computing the three-dimensiona ture of a scene from a correlated pair of perspective proj is described here, when the spatial relationship between t projections is unknown. This problem is relevant not o photographic surveying[1] but also to binocular vision[2], wh non-visual information available to the observer abo

Carlos Tomasi

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Shape and Motion from Image Streams under Orthography: a Factorization Method

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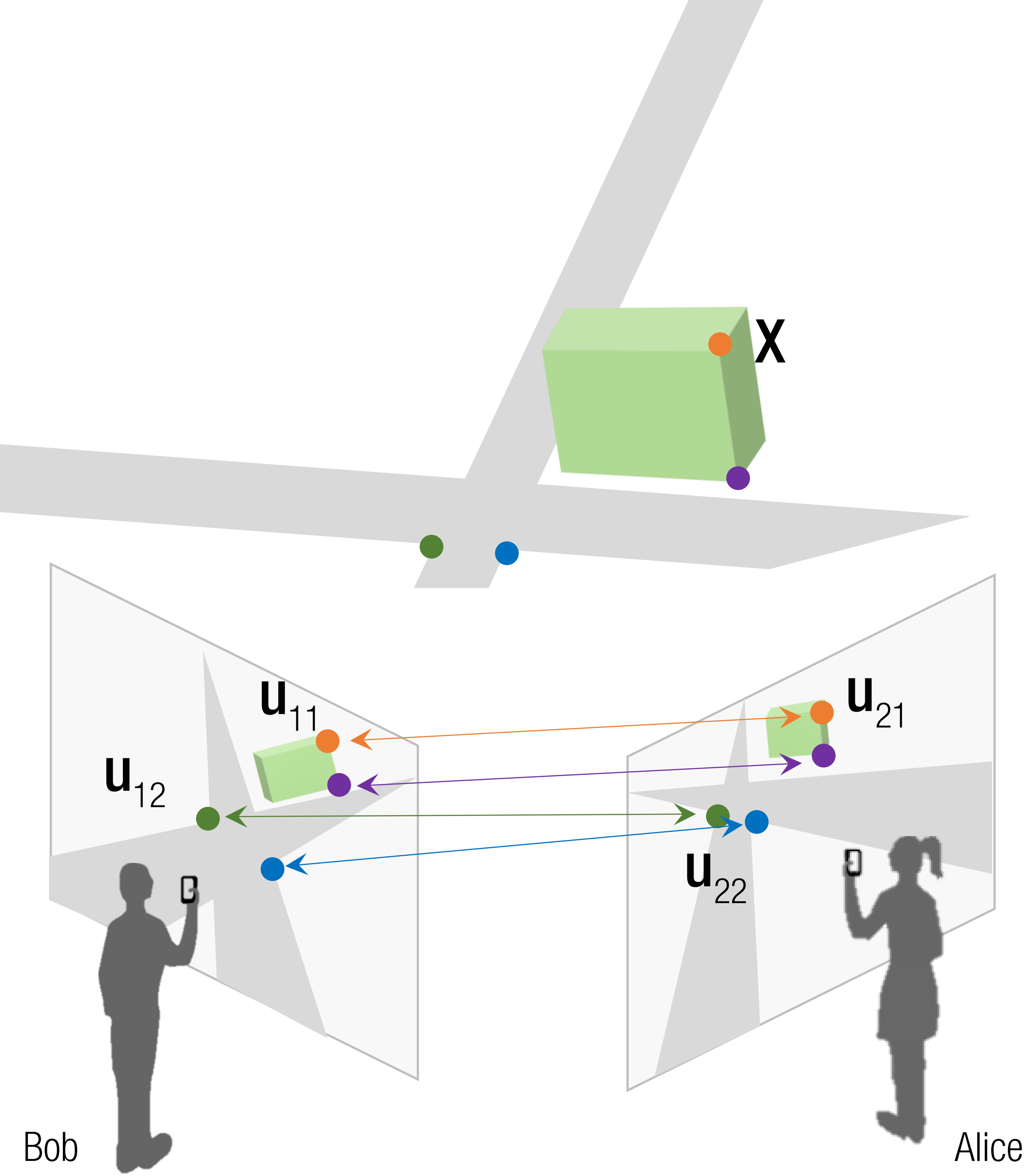
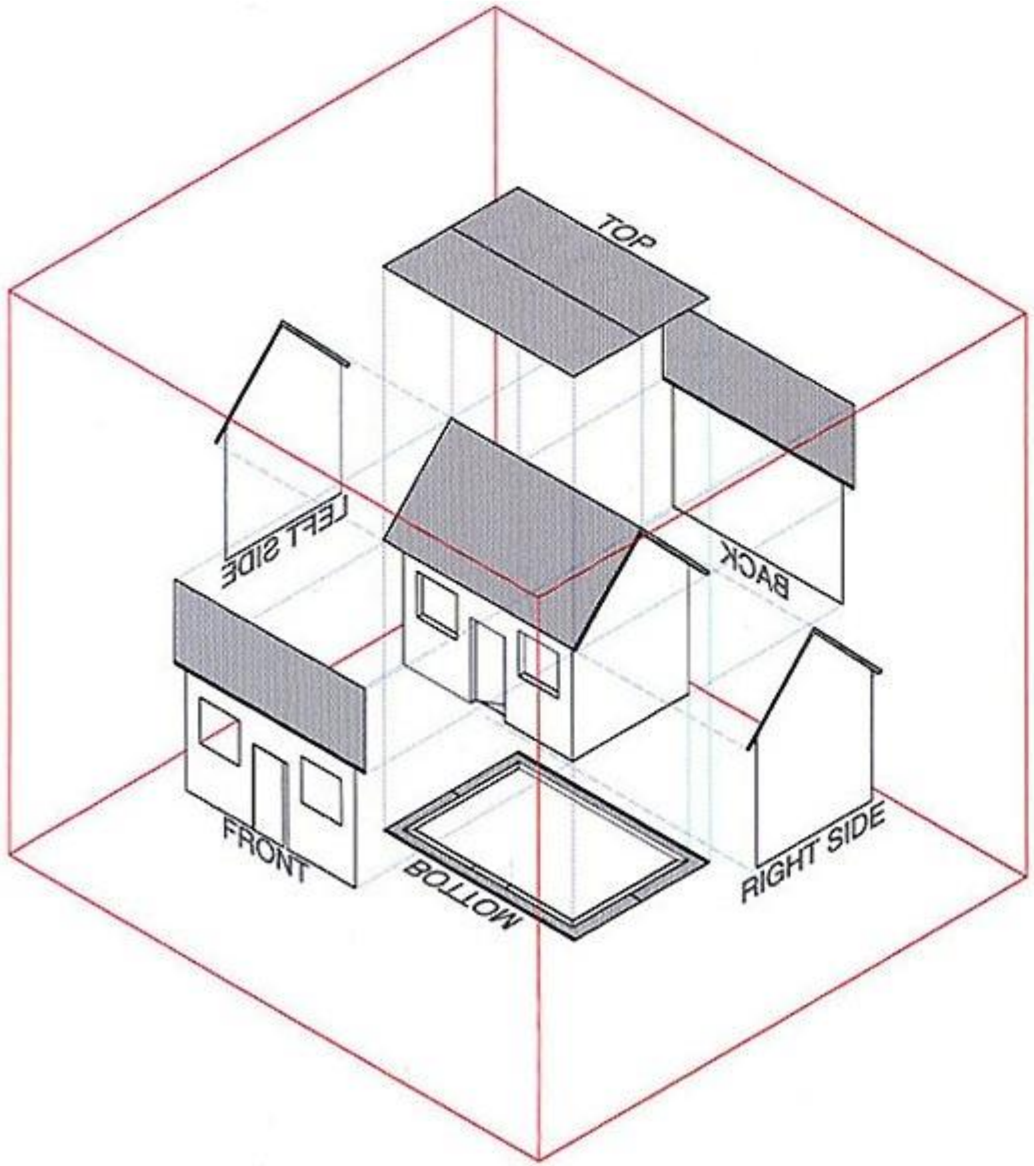
Tomasi-Kanade Factorization



$$\lambda_{11}\mathbf{u}_{11} = \mathbf{R}_1\mathbf{X}_1 + \mathbf{t}_1 \qquad \lambda_{21}\mathbf{u}_{11} = \mathbf{R}_2\mathbf{X}_2 + \mathbf{t}_2$$

$$\lambda_{12}\mathbf{u}_{12} = \mathbf{R}_1\mathbf{X}_2 + \mathbf{t}_1 \qquad \lambda_{22}\mathbf{u}_{22} = \mathbf{R}_2\mathbf{X}_2 + \mathbf{t}_2$$

Orthographic Camera



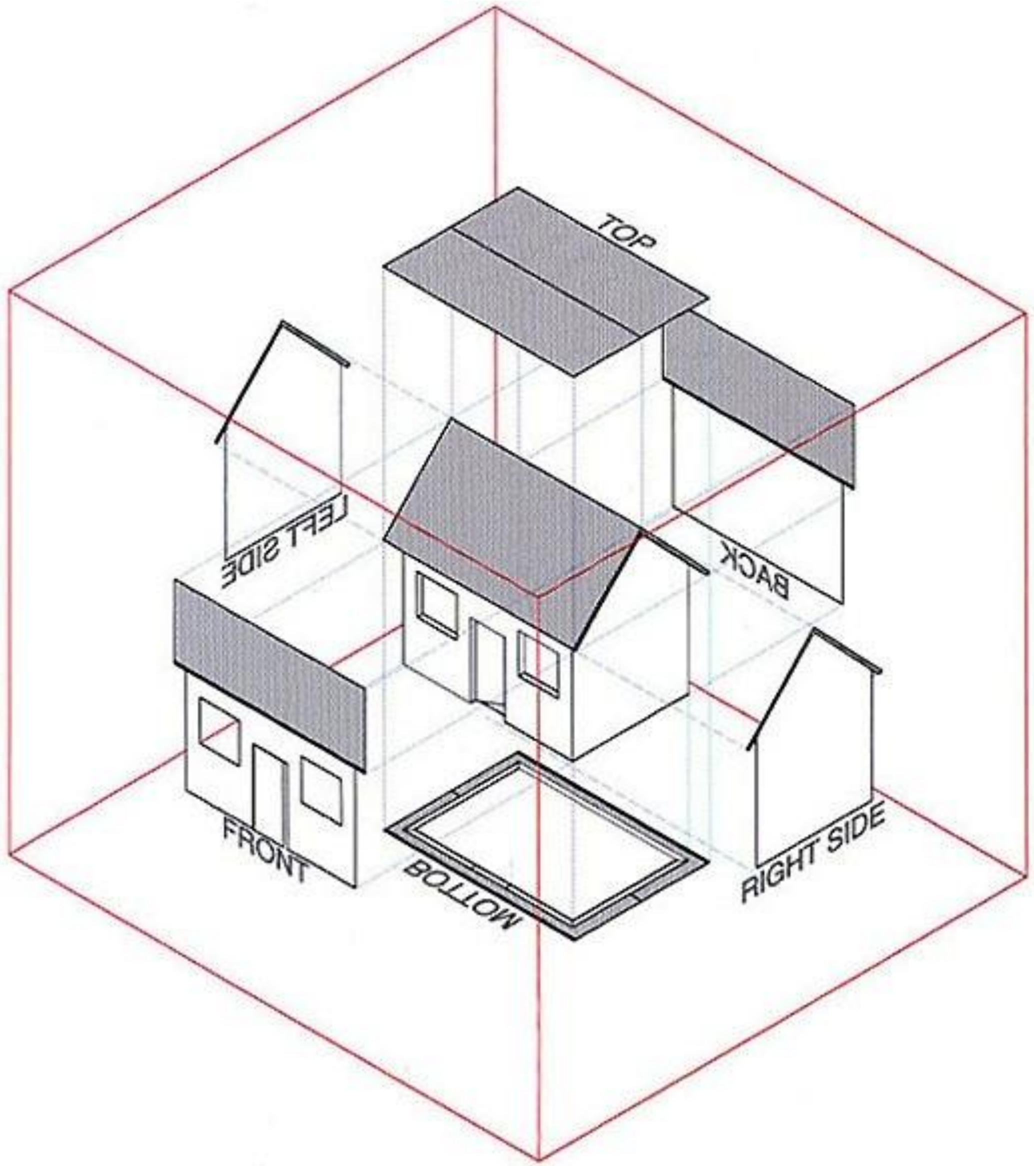
Affine camera:

$$\mathbf{P}_A = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & d \end{bmatrix} = d \begin{bmatrix} f/d & & p_x \\ & f/d & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

Orthographic Camera



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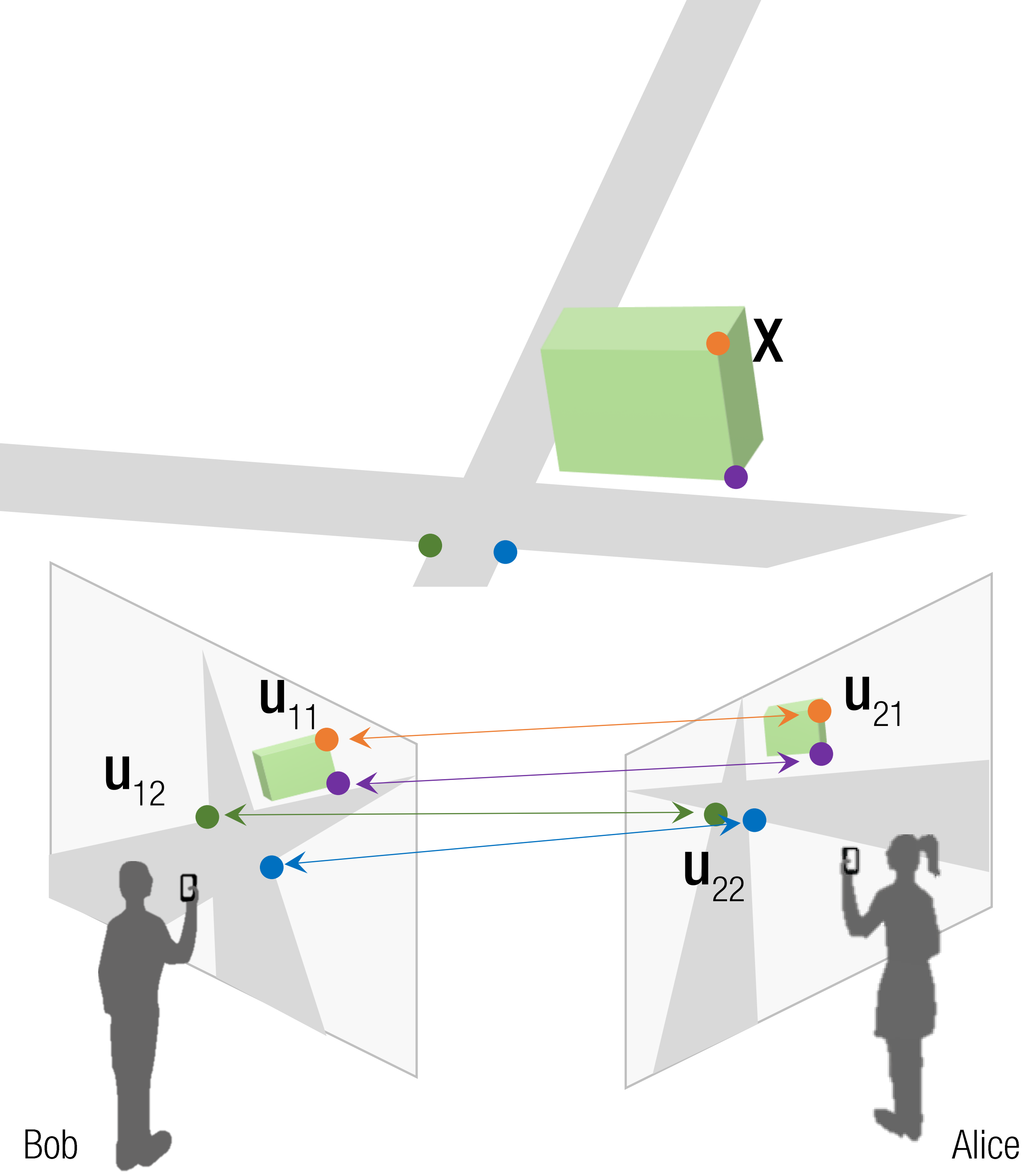
Orthographic camera:

$$f = 1 \quad p_X = p_Y = 0$$

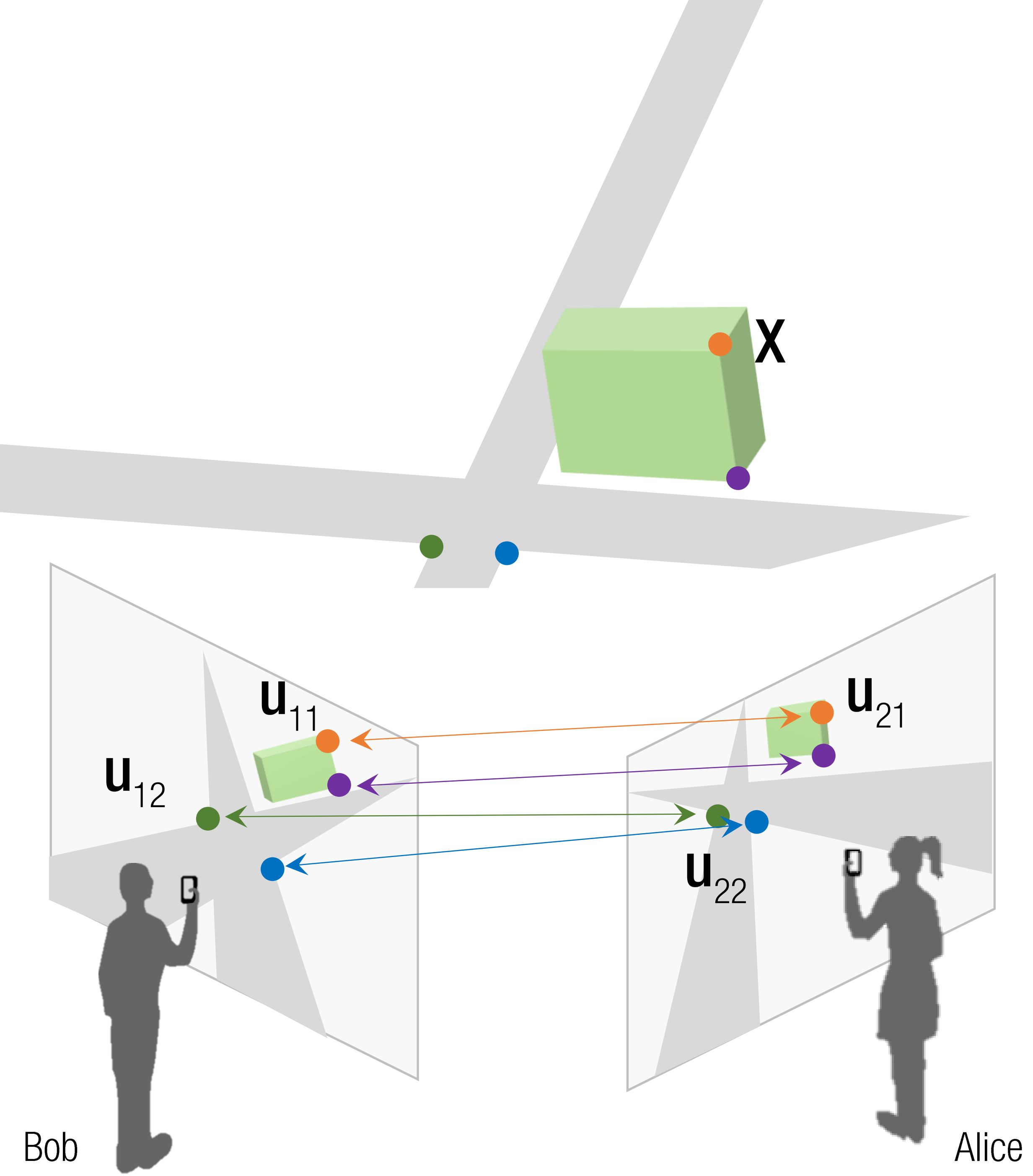
$$\mathbf{P}_0 = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & 0 \\ r_{y1} & r_{y2} & r_{y3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} & t_x \\ r_{y1} & r_{y2} & r_{y3} & t_y \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



$$\lambda_{11}\mathbf{u}_{11} = \mathbf{R}_1\mathbf{X}_1 + \mathbf{t}_1 \qquad \lambda_{21}\mathbf{u}_{11} = \mathbf{R}_2\mathbf{X}_2 + \mathbf{t}_2$$

$$\lambda_{12}\mathbf{u}_{12} = \mathbf{R}_1\mathbf{X}_2 + \mathbf{t}_1 \qquad \lambda_{22}\mathbf{u}_{22} = \mathbf{R}_2\mathbf{X}_2 + \mathbf{t}_2$$

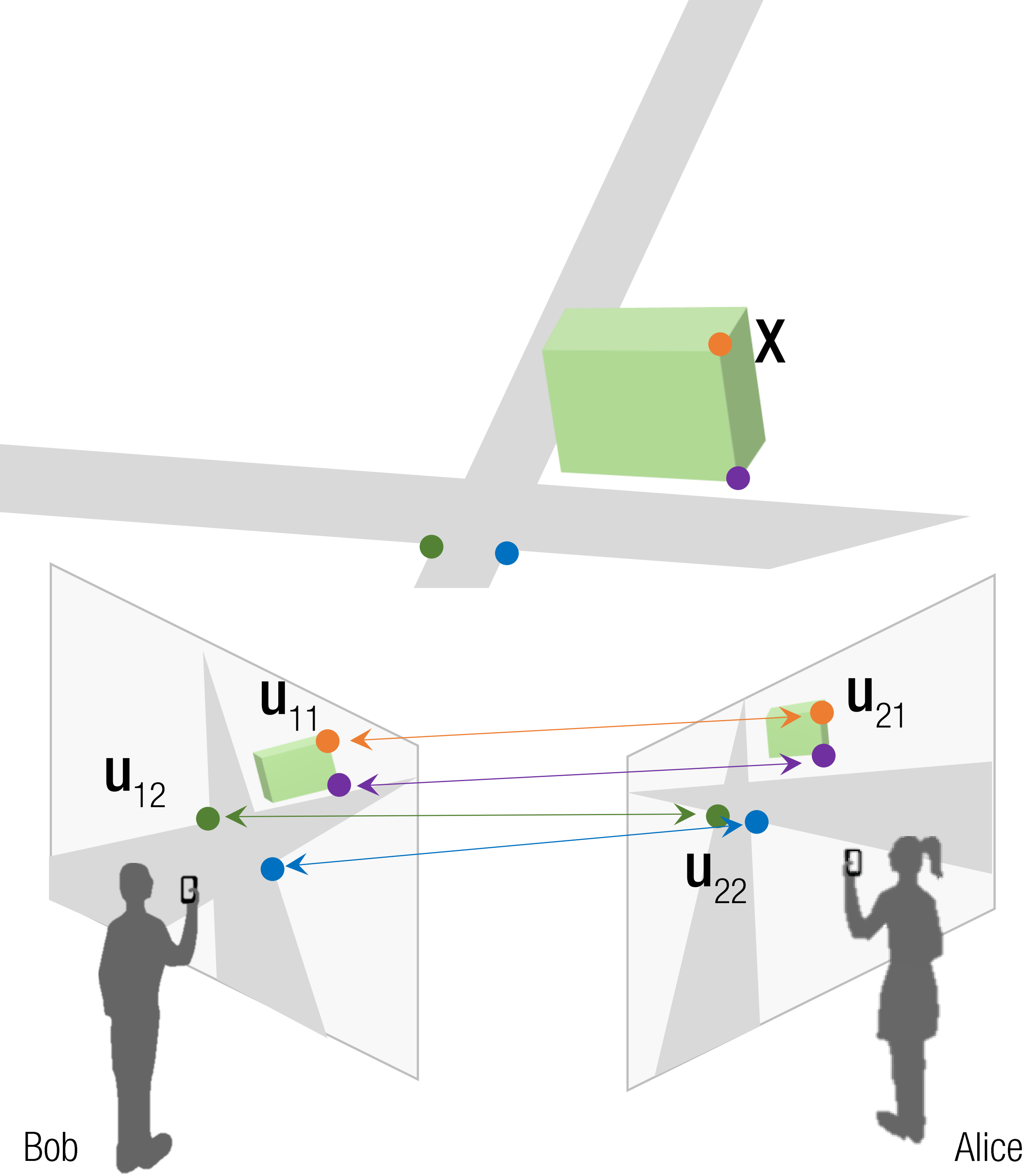


Orthographic projection:

$$\mathbf{u}_{11} = \hat{\mathbf{R}}_1 \mathbf{X}_1 + \hat{\mathbf{t}}_1 \qquad \mathbf{u}_{11} = \hat{\mathbf{R}}_2 \mathbf{X}_2 + \hat{\mathbf{t}}_2$$

$$\mathbf{u}_{12} = \hat{\mathbf{R}}_1 \mathbf{X}_2 + \hat{\mathbf{t}}_1 \qquad \mathbf{u}_{22} = \hat{\mathbf{R}}_2 \mathbf{X}_2 + \hat{\mathbf{t}}_2$$

where $\hat{\mathbf{R}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \end{bmatrix} \quad \hat{\mathbf{t}} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$



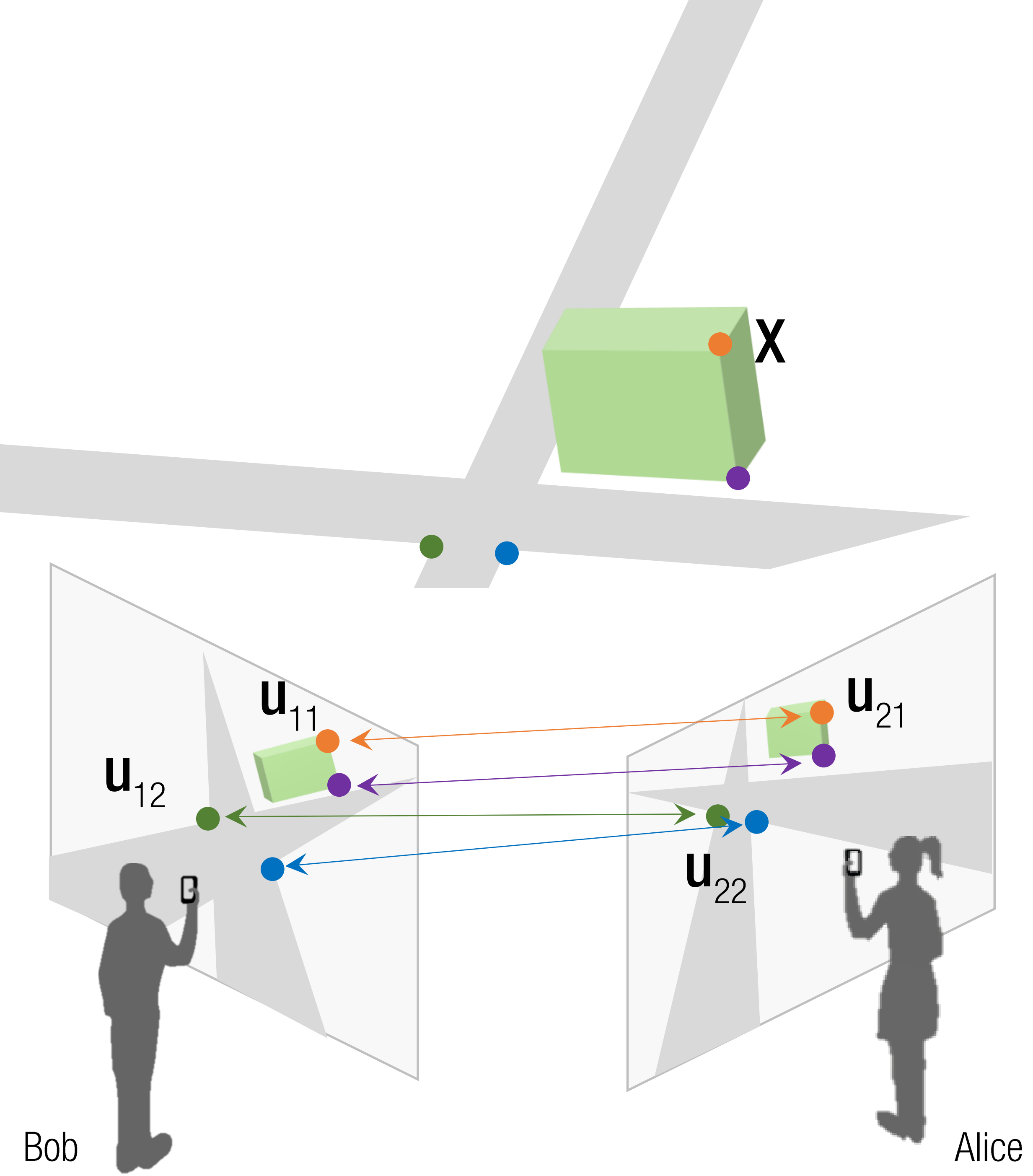
Orthographic projection:

$$\tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_1 \tilde{\mathbf{X}}_1 \qquad \tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_2 \tilde{\mathbf{X}}_2$$

$$\tilde{\mathbf{u}}_{12} = \hat{\mathbf{R}}_1 \tilde{\mathbf{X}}_2 \qquad \tilde{\mathbf{u}}_{22} = \hat{\mathbf{R}}_2 \tilde{\mathbf{X}}_2$$

where $\hat{\mathbf{R}} = \begin{bmatrix} r_{x1} & r_{x2} & r_{x3} \\ r_{y1} & r_{y2} & r_{y3} \end{bmatrix} \quad \hat{\mathbf{t}} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

$$\tilde{\mathbf{X}}_j = \mathbf{X}_j - \frac{1}{P}\sum_j^P \mathbf{X}_j \longrightarrow \tilde{\mathbf{u}}_{ij} = \mathbf{u}_{ij} - \frac{1}{P}\sum_j^P \mathbf{u}_{ij}$$

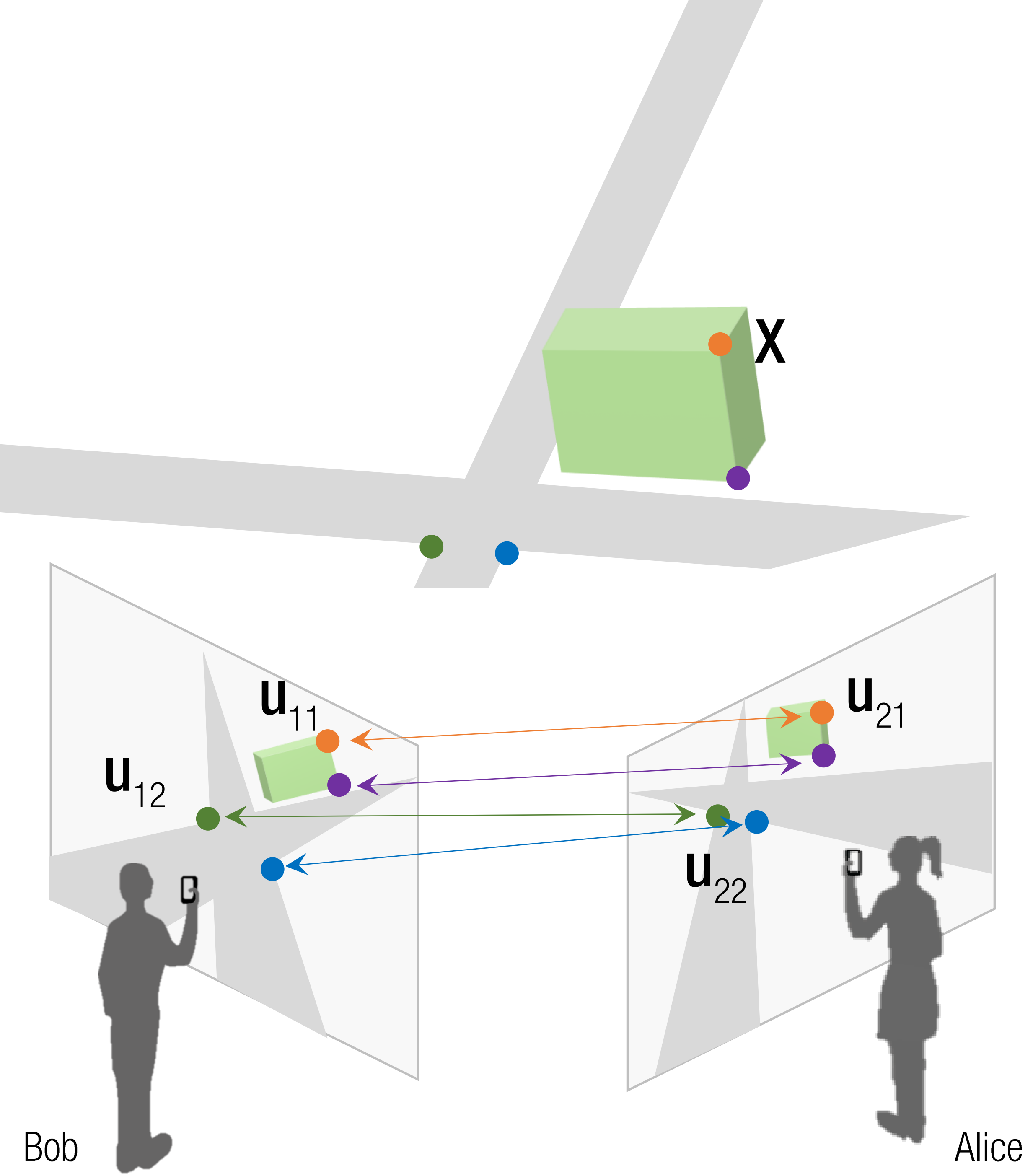


Orthographic projection:

$$\tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_1 \tilde{\mathbf{X}}_1 \qquad \tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_2 \tilde{\mathbf{X}}_2$$

$$\tilde{\mathbf{u}}_{12} = \hat{\mathbf{R}}_1 \tilde{\mathbf{X}}_2 \qquad \tilde{\mathbf{u}}_{22} = \hat{\mathbf{R}}_2 \tilde{\mathbf{X}}_2$$

$$\begin{bmatrix} \tilde{\mathbf{u}}_{11} & \cdots & \tilde{\mathbf{u}}_{1P} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{u}}_{F1} & \cdots & \tilde{\mathbf{u}}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{R}}_1 \\ \vdots \\ \hat{\mathbf{R}}_F \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_1 & \cdots & \tilde{\mathbf{X}}_P \end{bmatrix}$$



Orthographic projection:

$$\tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_1 \mathbf{X}_1 \qquad \tilde{\mathbf{u}}_{11} = \hat{\mathbf{R}}_2 \mathbf{X}_2$$

$$\tilde{\mathbf{u}}_{12} = \hat{\mathbf{R}}_1 \mathbf{X}_2 \qquad \tilde{\mathbf{u}}_{22} = \hat{\mathbf{R}}_2 \mathbf{X}_2$$

$$\begin{bmatrix} \tilde{\mathbf{u}}_{11} & \cdots & \tilde{\mathbf{u}}_{1P} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{u}}_{F1} & \cdots & \tilde{\mathbf{u}}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{R}}_1 \\ \vdots \\ \hat{\mathbf{R}}_F \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_1 & \cdots & \tilde{\mathbf{X}}_P \end{bmatrix}$$

2*F*x*P* — Knowns

2*F*x3 and 3x*P* — Unknowns

$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \mathbf{W} & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{2F\text{x}P} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ \mathbf{M} \\ \hat{R}_F \end{bmatrix}}_{2F\text{x}3} \underbrace{\begin{bmatrix} \tilde{X}_1 & \mathbf{S} \cdots & \tilde{X}_P \end{bmatrix}}_{3\text{x}P}$$

Measurement matrix   Motion mtx   Shape mtx

$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \mathbf{W} & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{2F\text{x}P} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ \mathbf{M} \\ \hat{R}_F \end{bmatrix}}_{2F\text{x}3} \underbrace{\begin{bmatrix} \tilde{X}_1 & \mathbf{S} & \tilde{X}_P \end{bmatrix}}_{3\text{x}P}$$

Measurement matrix | Motion mtx | Shape mtx

rank(   ) = 3

$$\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \mathbf{W} & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{R}_1 \\ \mathbf{M} \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & \mathbf{S} & \tilde{X}_P \end{bmatrix}$$

$2F\text{x}P$	$2F\text{x}3$	$3\text{x}P$
Measurement matrix	Motion mtx	Shape mtx

rank( ) = 3

$$\mathbf{W} = \mathbf{UDV}^{\mathrm{T}} \xrightarrow{?} \begin{aligned} \mathbf{M} &= \mathbf{UD}^{1/2} \\ \mathbf{S} &= \mathbf{D}^{1/2}\mathbf{V}^{\mathrm{T}} \end{aligned}$$

$$\underbrace{\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \mathbf{W} & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix}}_{2F\text{x}P} = \underbrace{\begin{bmatrix} \hat{R}_1 \\ \mathbf{M} \\ \hat{R}_F \end{bmatrix}}_{2F\text{x}3} \underbrace{\begin{bmatrix} \tilde{X}_1 & \mathbf{S} & \tilde{X}_P \end{bmatrix}}_{3\text{x}P}$$

Measurement matrix | Motion mtx | Shape mtx

rank($\mathbf{W}$) = 3

$$\mathbf{W} = \mathbf{U}\mathbf{D}\mathbf{V}^T \xrightarrow{?} \quad \mathbf{M} = \mathbf{U}\mathbf{D}^{1/2}, \quad \mathbf{S} = \mathbf{D}^{1/2}\mathbf{V}^T$$

No. There exists infinite **M** and **S** combinations.

$$\mathbf{M} = \mathbf{U}\mathbf{D}^{1/2}\mathbf{Q}$$

$$\mathbf{S} = \mathbf{Q}^{-1}\mathbf{D}^{1/2}\mathbf{V}^T$$

Any nonsingular **Q** can produce solutions.

Is there any constraint?

$$\begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & \mathbf{W} & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \begin{bmatrix} \hat{R}_1 \\ \mathbf{M} \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & \mathbf{S} \cdots & \tilde{X}_P \end{bmatrix}$$

$2F$xP — Measurement matrix

$2F$x3 — Motion mtx

3xP — Shape mtx

rank( ) = 3

$$\mathbf{W} = \mathbf{UDV}^{\mathrm{T}}$$

$$\mathbf{M} = \mathbf{UD}^{1/2}\mathbf{Q}$$

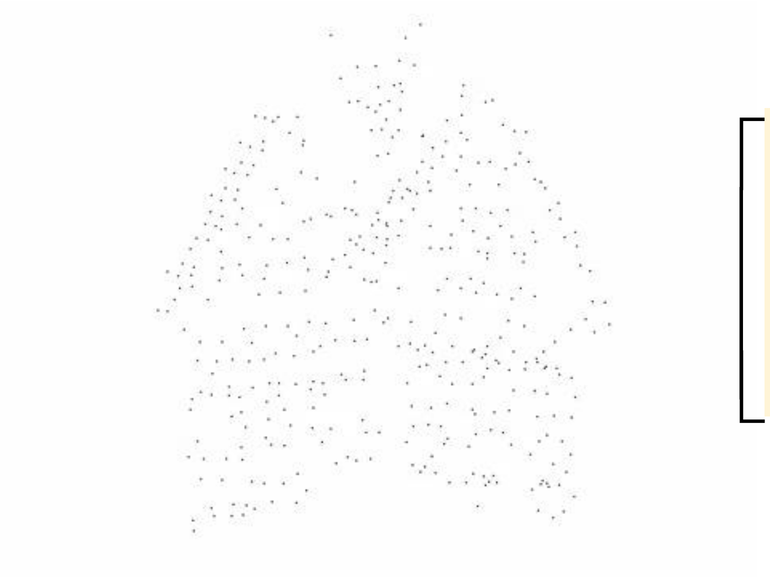
$$\mathbf{S} = \mathbf{Q}^{-1}\mathbf{D}^{1/2}\mathbf{V}^{\mathrm{T}}$$

Orthogonal constraint:

$$\mathbf{M} = \mathbf{UD}^{1/2}\mathbf{Q} = \begin{bmatrix} \mathbf{r}_{1x}^{\mathrm{T}} \\ \mathbf{r}_{1y}^{\mathrm{T}} \\ \\ \mathbf{r}_{Fx}^{\mathrm{T}} \\ \mathbf{r}_{Fy}^{\mathrm{T}} \end{bmatrix}$$

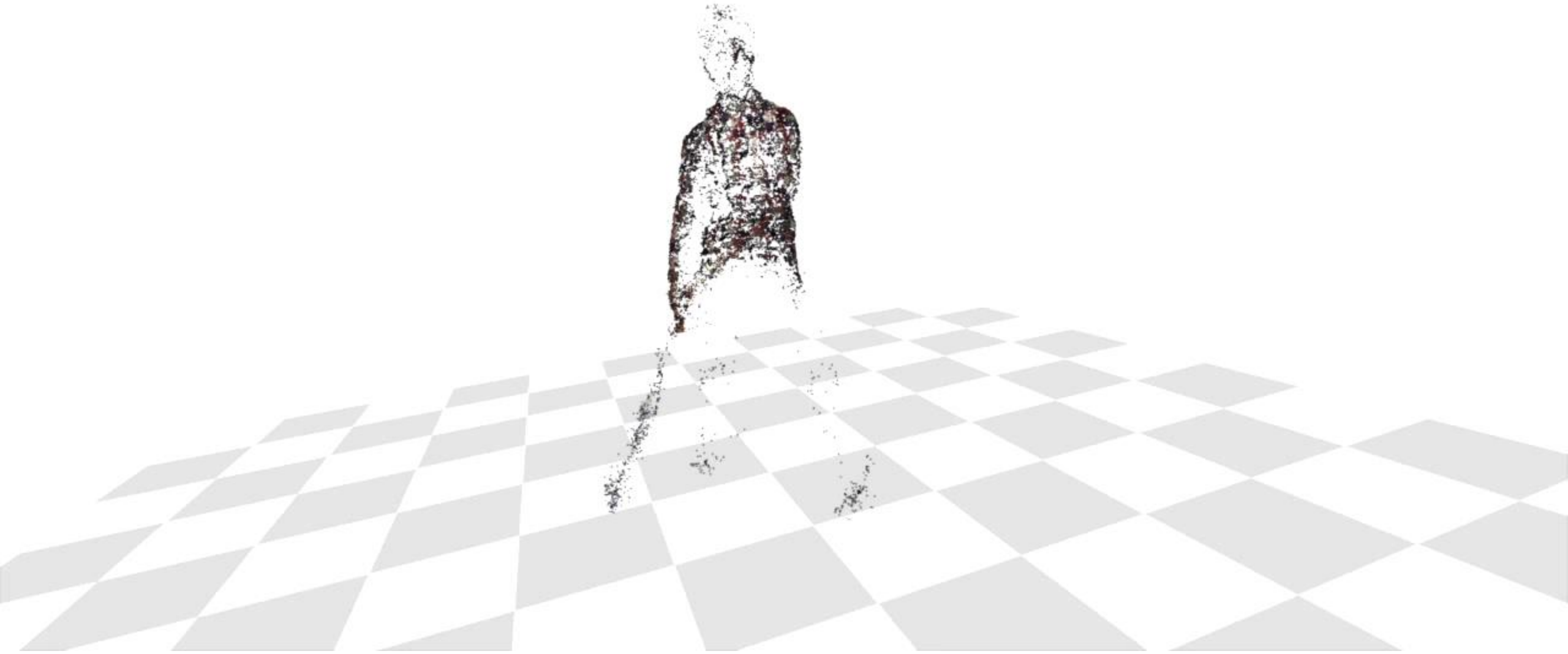
$$\longrightarrow \mathbf{r}_{1x}^{\mathrm{T}}\mathbf{r}_{1x} = 1, \quad \mathbf{r}_{1y}^{\mathrm{T}}\mathbf{r}_{1y} = 1, \quad \mathbf{r}_{1x}^{\mathrm{T}}\mathbf{r}_{1y} = 0$$

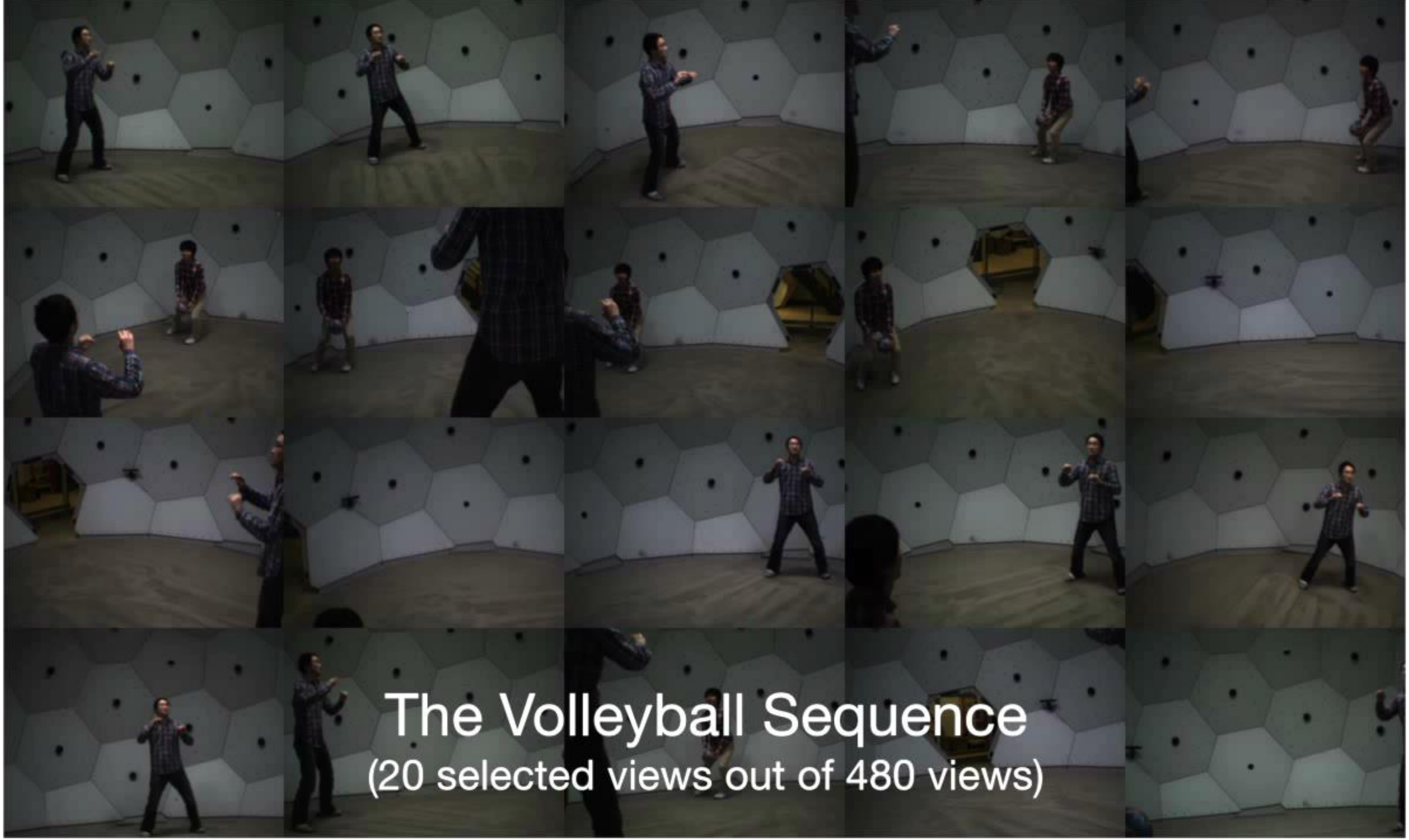
# of unknowns: 9 (**Q**: 3x3)
# of equations: 3F

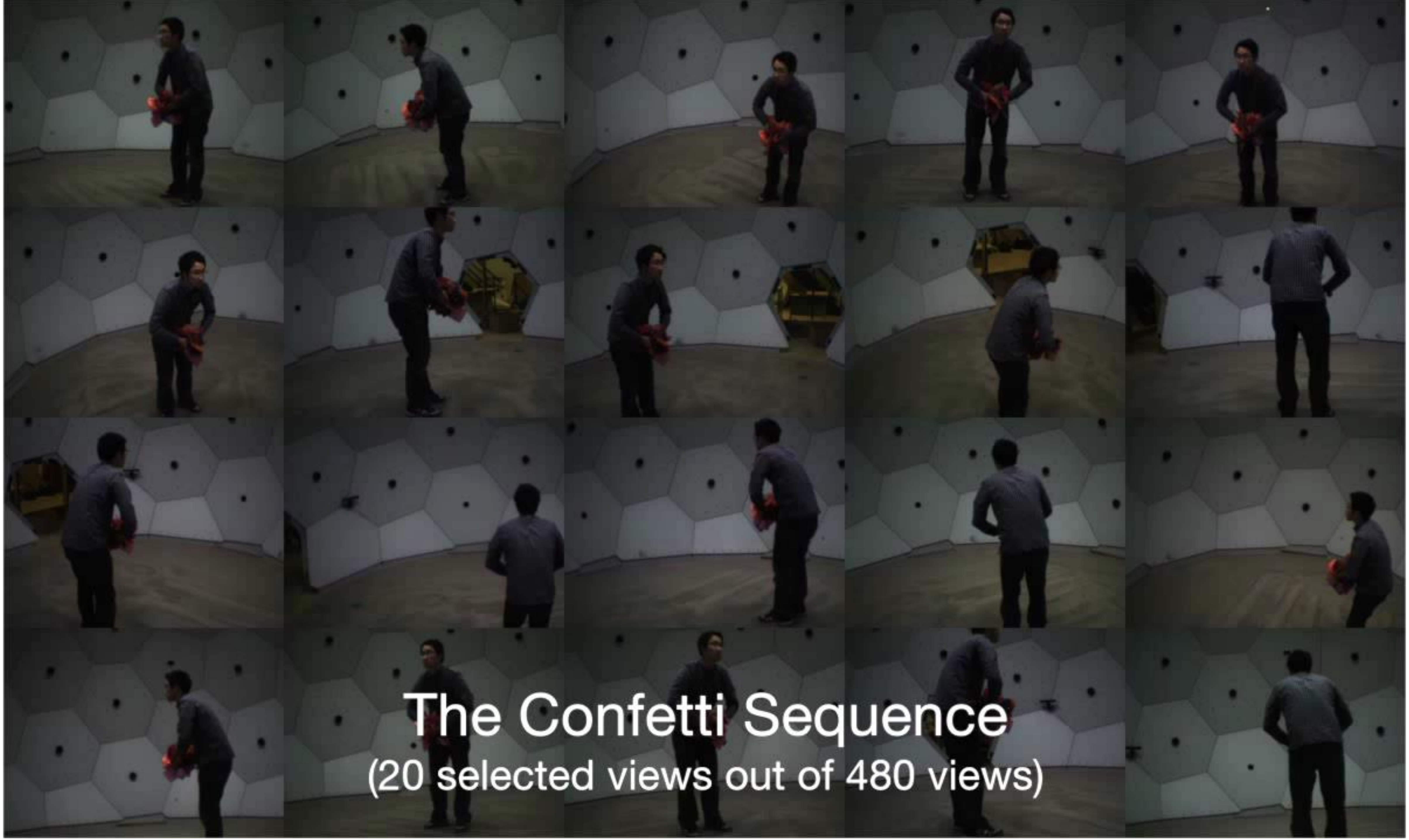


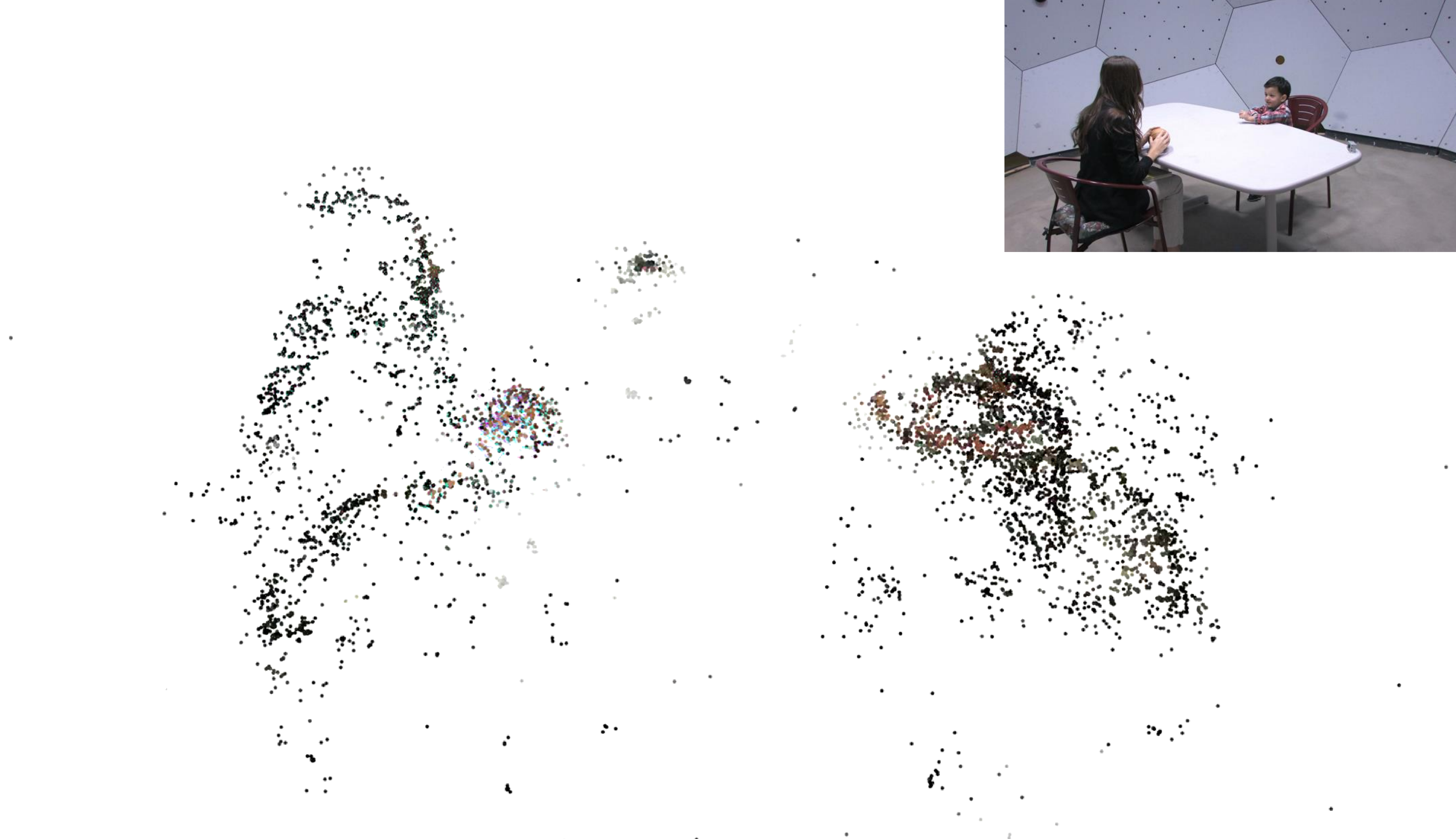
$$\mathbf{W} = \begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1P} \\ \vdots & & \vdots \\ \tilde{u}_{F1} & \cdots & \tilde{u}_{FP} \end{bmatrix} = \mathbf{M}\,\mathbf{S} = \begin{bmatrix} \hat{R}_1 \\ \vdots \\ \hat{R}_F \end{bmatrix} \begin{bmatrix} \tilde{X}_1 & \cdots & \tilde{X}_P \end{bmatrix}$$

Large scale reconstruction



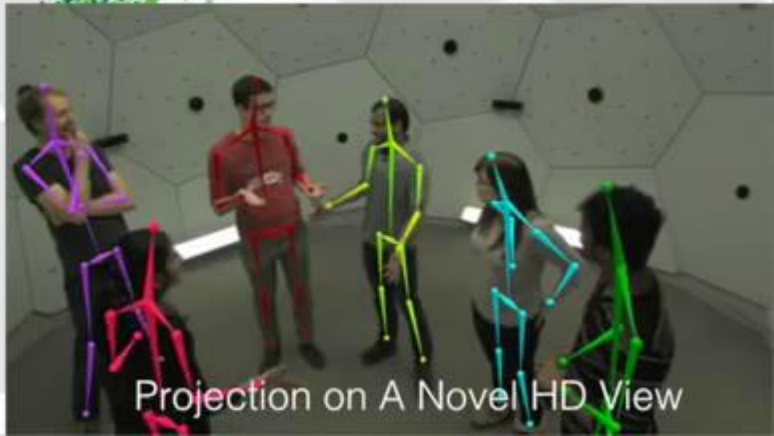

The Volleyball Sequence
(20 selected views out of 480 views)

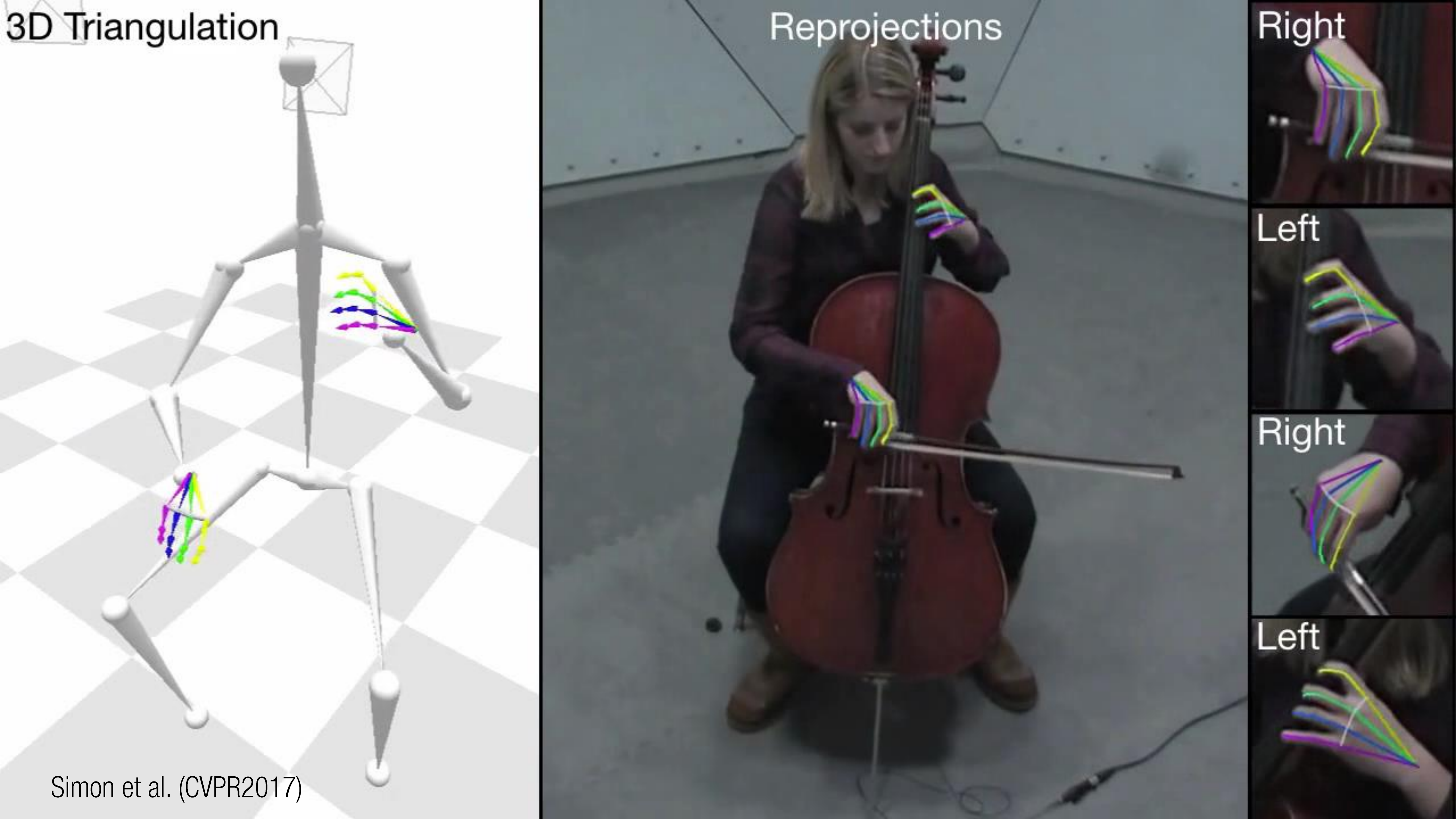

The Confetti Sequence
(20 selected views out of 480 views)



Joo et al. (ICCV2015)

3D Motion Reconstruction (Trajectory Stream Association)




3D Triangulation
Reprojections
Right
Left
Right
Left
Simon et al. (CVPR2017)


@Shepherd Laboratory (UMN)
System design:
20 Pix/mm^2 @100 fps

Field trip to Shepherd Laboratory