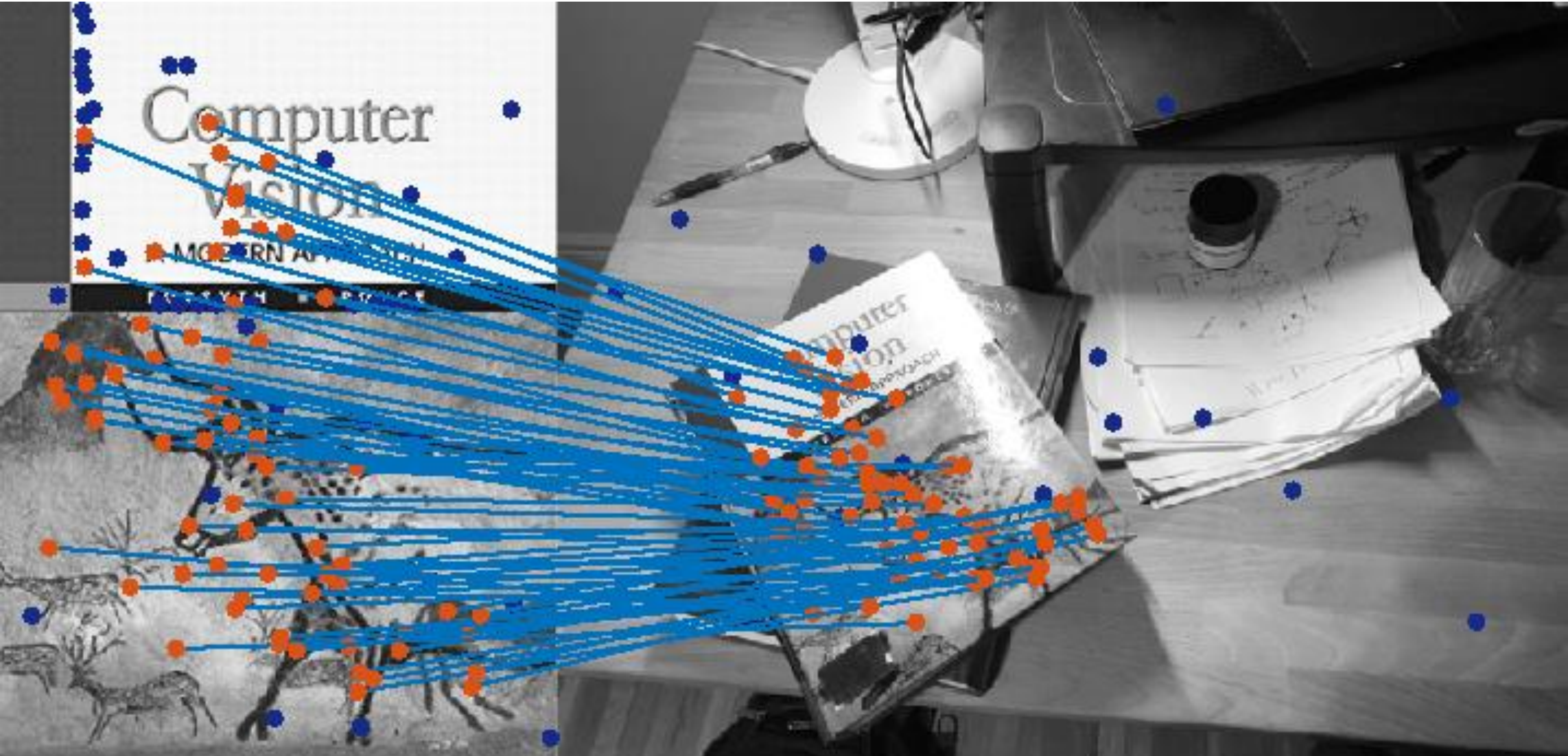
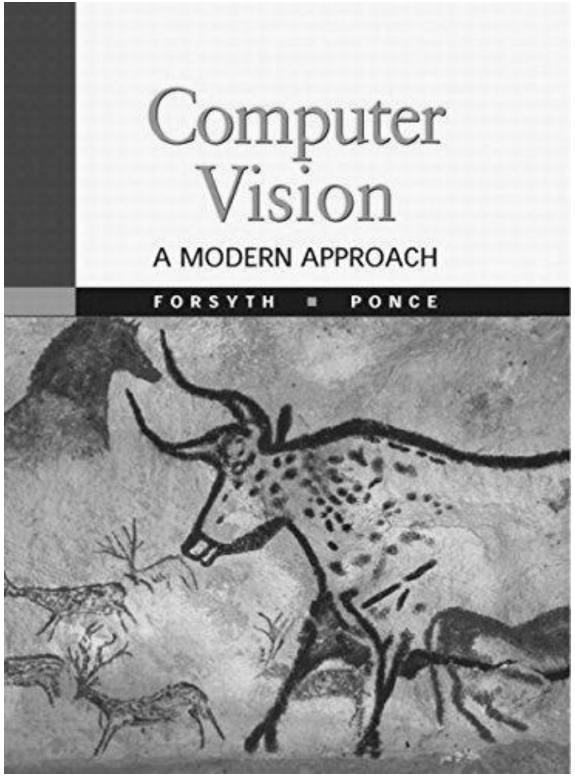
Image Warping

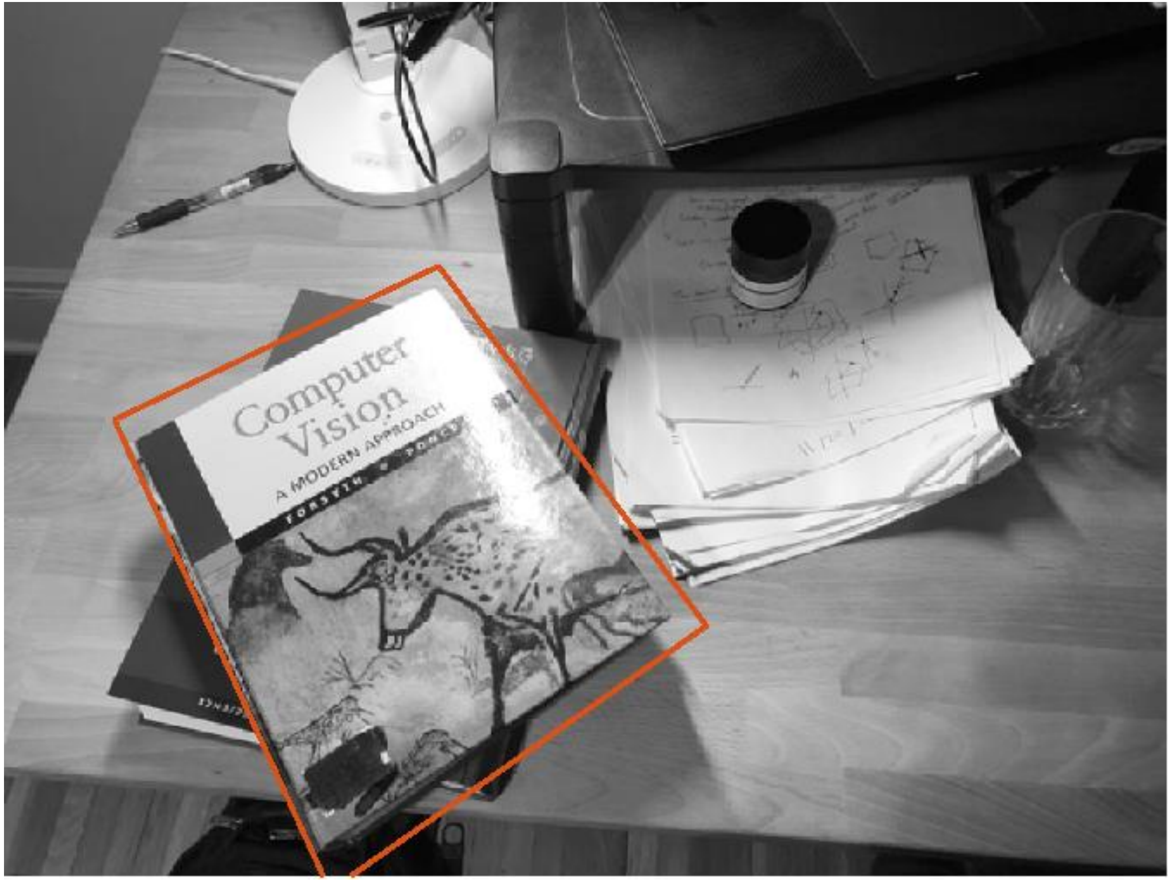
Hyun Soo Park

RECALL: ROBUST FILTERING

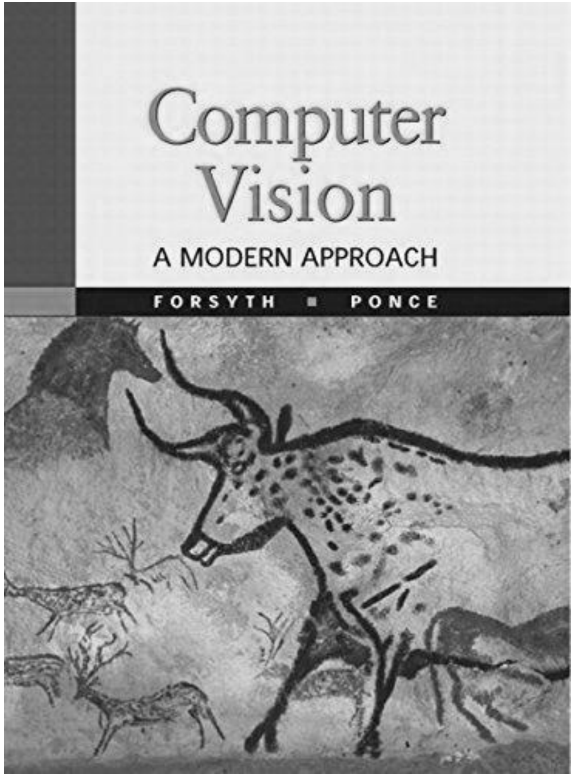


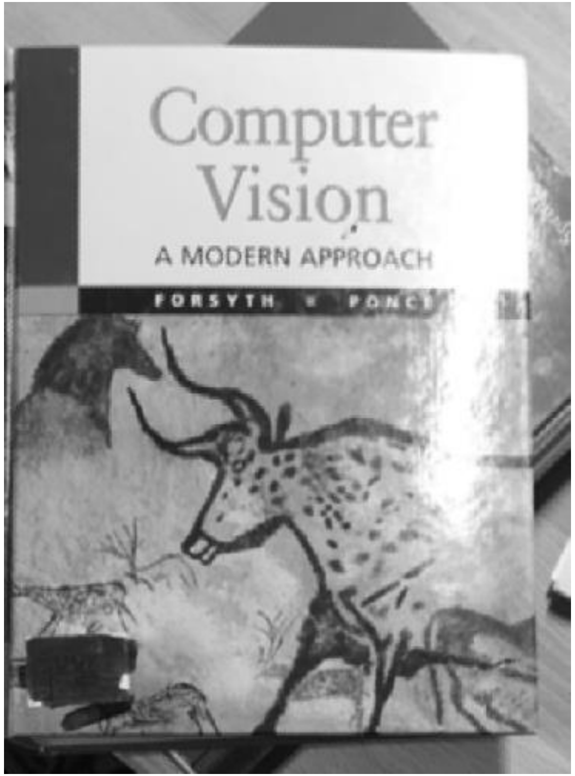
Recall: Parametric Model



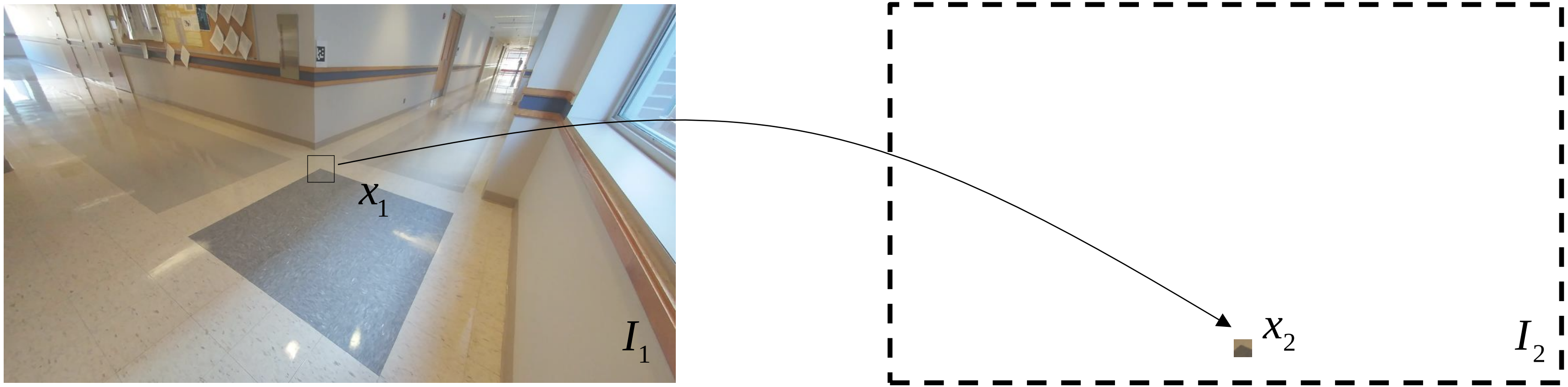


RECALL: IMAGE WARPING





Pixel Transport



Source image

Target image

$$x_2 = f(x_1)$$

Coordinate transform

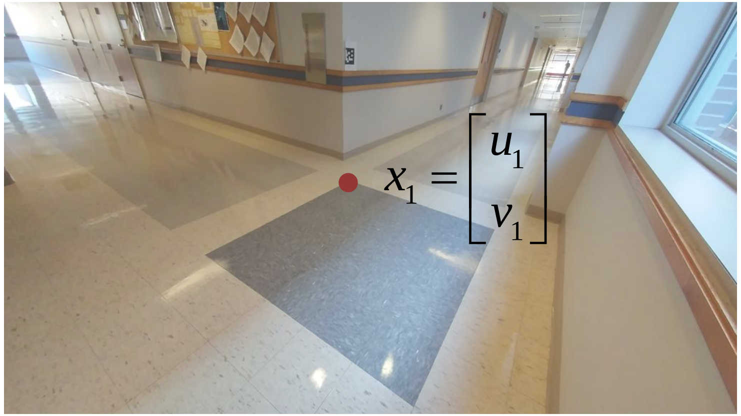
Pixel Transport

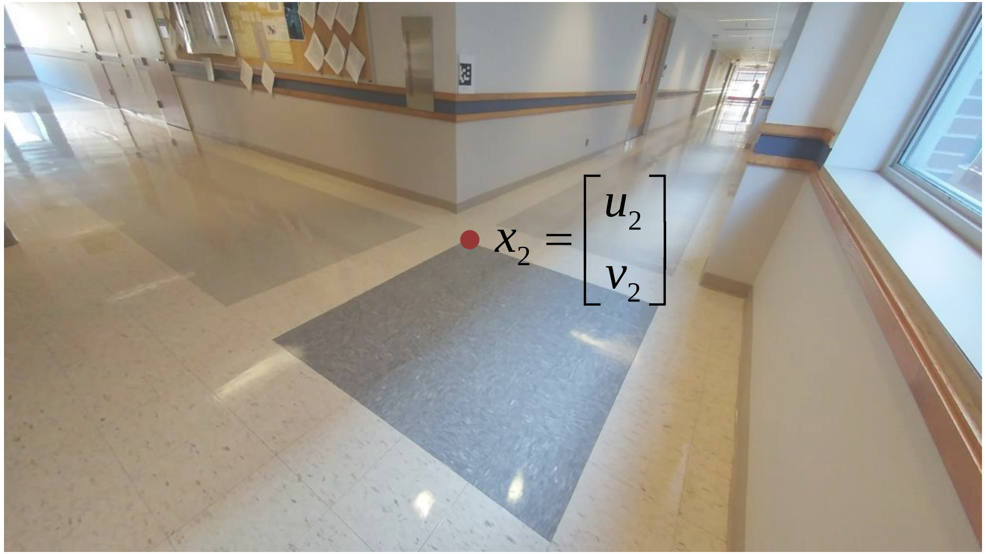
Source image

Target image

$$\underbrace{x_2 = f(x_1)}_{\text{Coordinate transform}} \qquad I_2(x_2) = I(f(x_1))$$

Uniform Scaling

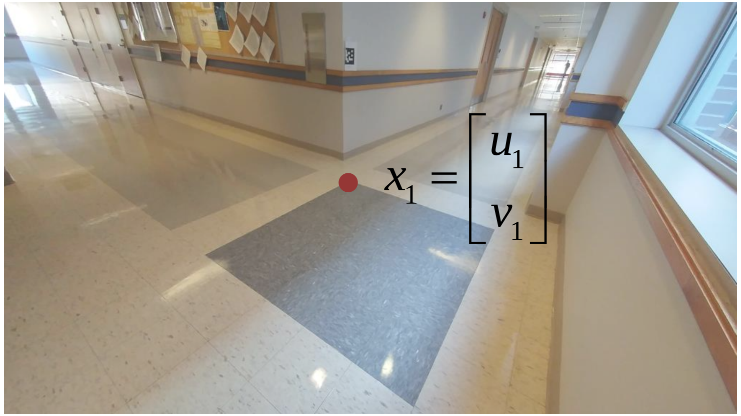


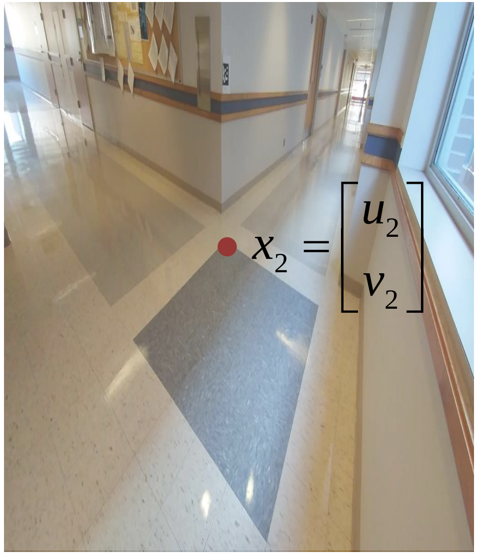


$$I_2(x_2) = I(f(x_1))$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= su_1 \\ v_2 &= sv_1 \end{aligned}$$

Non-uniform Scaling (Aspect Ratio Change)

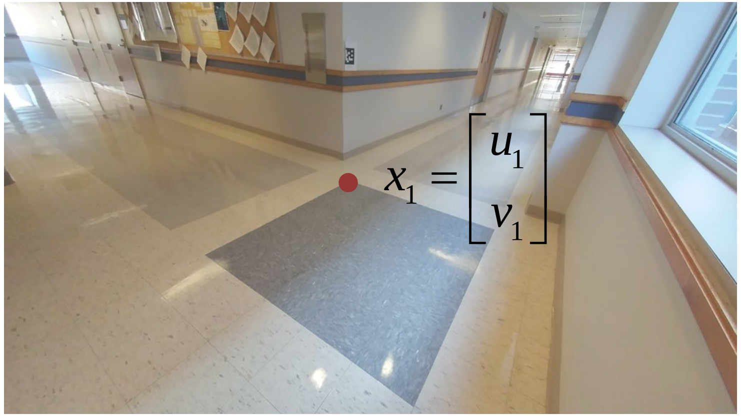


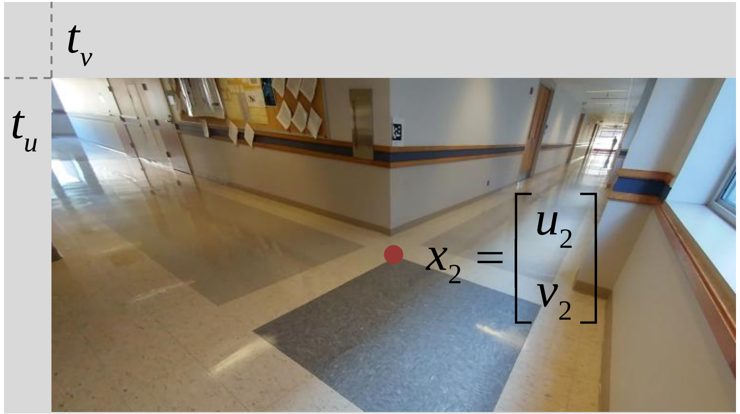


$$I_2(x_2) = I(f(x_1))$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= s_u u_1 \\ v_2 &= s_v v_1 \end{aligned}$$

TRANSLATION

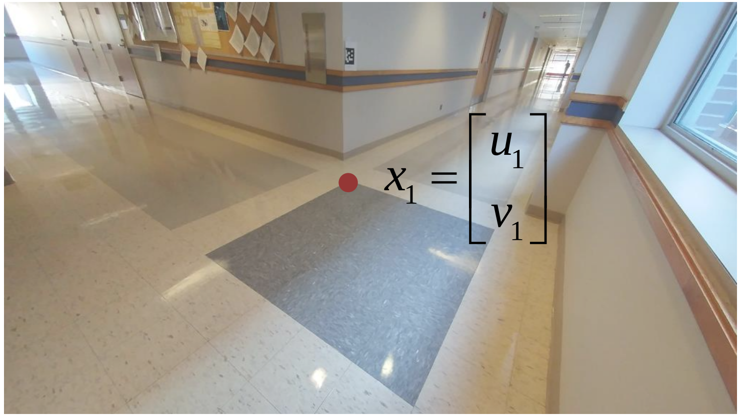


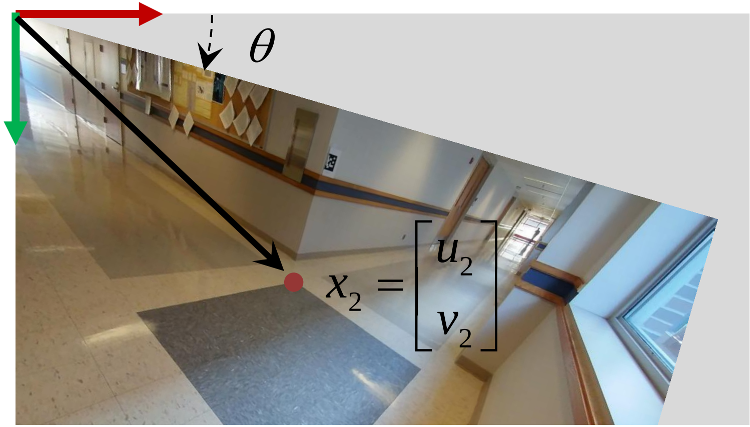


$$I_2(x_2) = I(f(x_1))$$

$$x_2 = f(x_1) \longrightarrow \begin{aligned} u_2 &= u_1 + t_u \\ v_2 &= v_1 + t_v \end{aligned}$$

Rotation

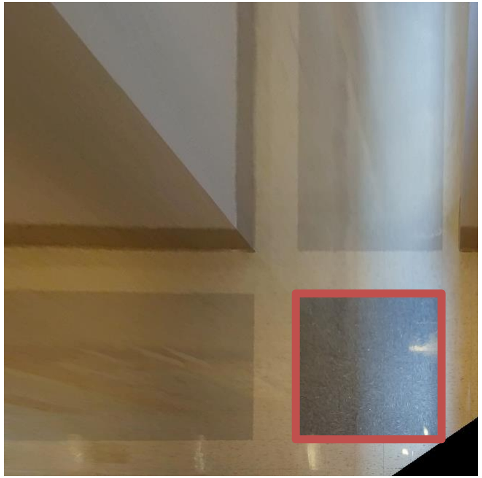


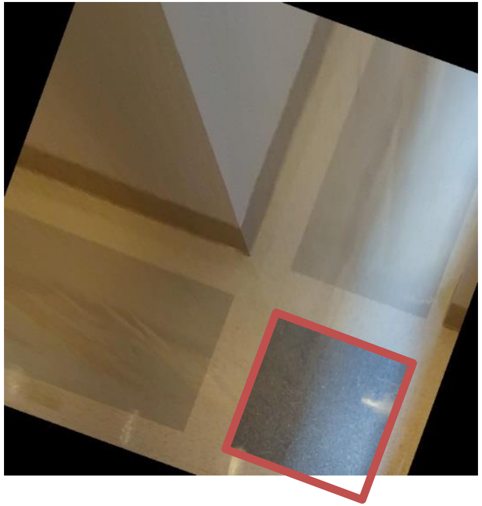


$$I_2(x_2) = I(f(x_1))$$

$$x_2 = f(x_1) \longrightarrow \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix}$$

Euclidean Transform SE(3) ~ Translation+Rotation





Invariant properties

- Length
- Angle
- Area

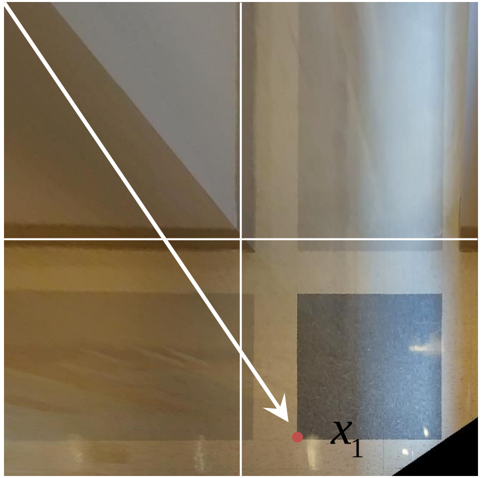
Degree of freedom

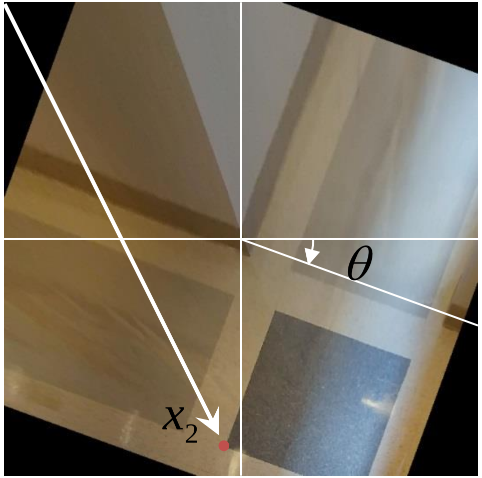
3 (2 translation+1 rotation)

$$I_2(x_2) = I(f(x_1))$$

$$x_2 = f(x_1) \longrightarrow \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_u \\ \sin\theta & \cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

EXAMPLE OF EUCLIDEAN TRANSFORM

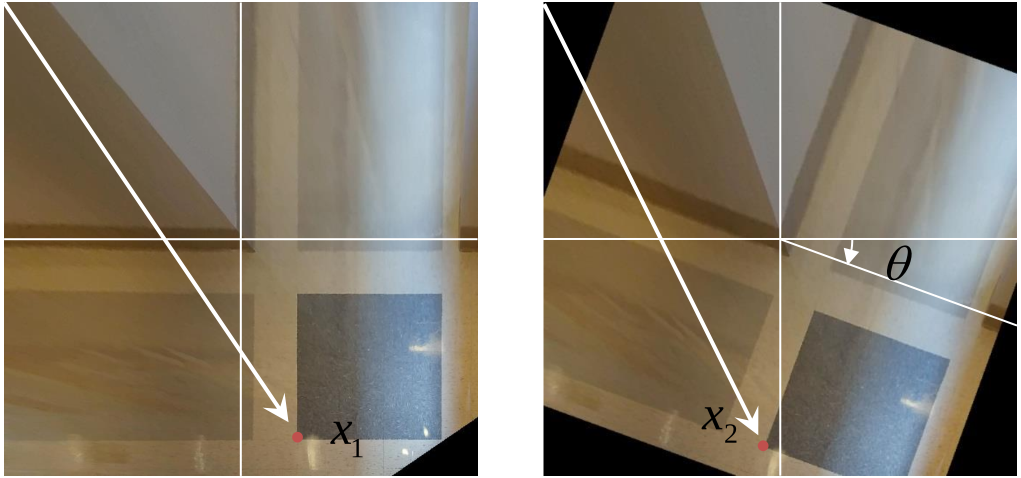




Rotate about the image center

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_u \\ \sin\theta & \cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

EXAMPLE OF EUCLIDEAN TRANSFORM



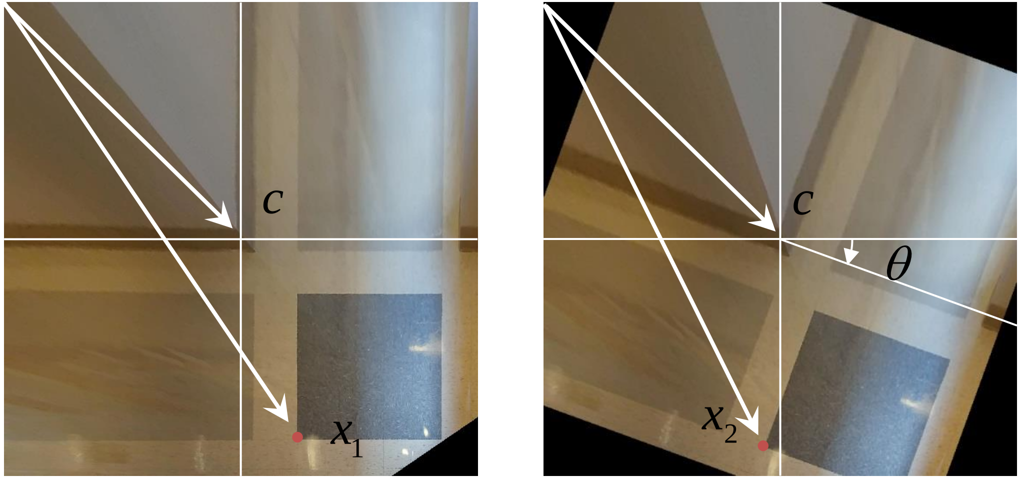
Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$$

$t = ?$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM



c : image center

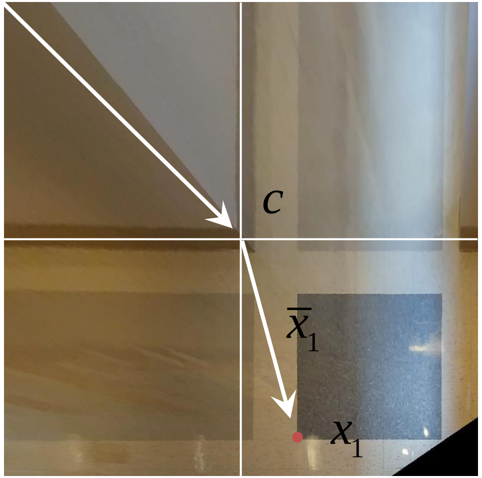
Rotate about the image center

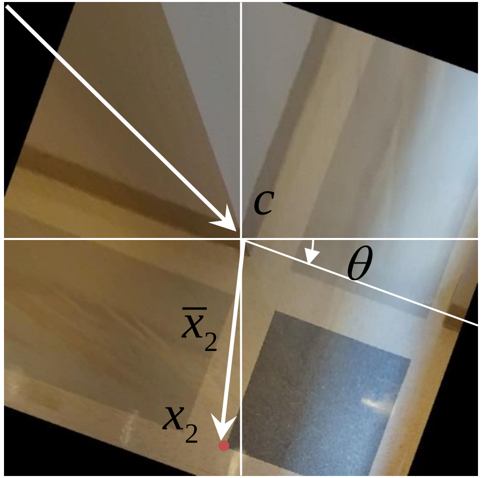
$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$$

$t = ?$

$$x_2 = Rx_1 + t$$

EXAMPLE OF EUCLIDEAN TRANSFORM





c : image center

Rotate about the image center

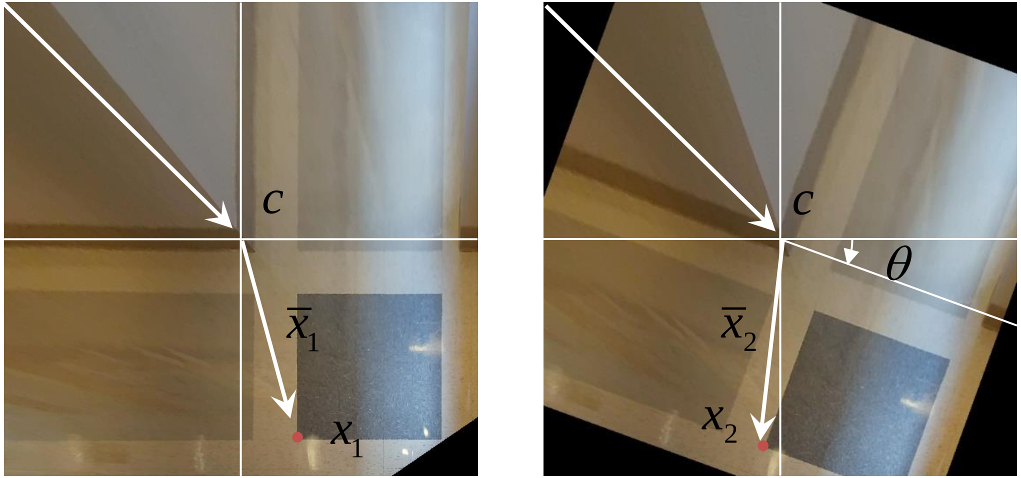
$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

$$x_2 = R x_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

Example of Euclidean Transform



c : image center

Rotate about the image center

$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix} \qquad t = ?$$

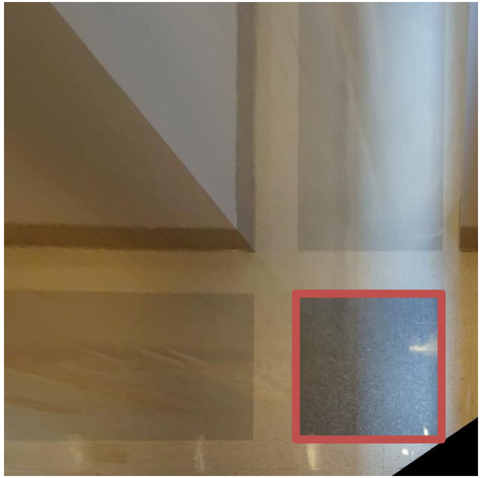
$$x_2 = R x_1 + t$$

$$x_1 = c + \bar{x}_1$$

$$x_2 = c + \bar{x}_2$$

$$\longrightarrow t = -Rc + c$$

Similarity Transform ~ Euclidean + Uniform Scaling



$$\begin{bmatrix} x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$$

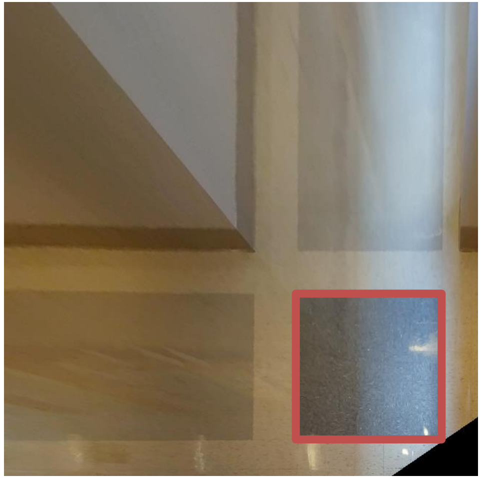
Invariant properties

- Length ratio
- Angle

Degree of freedom

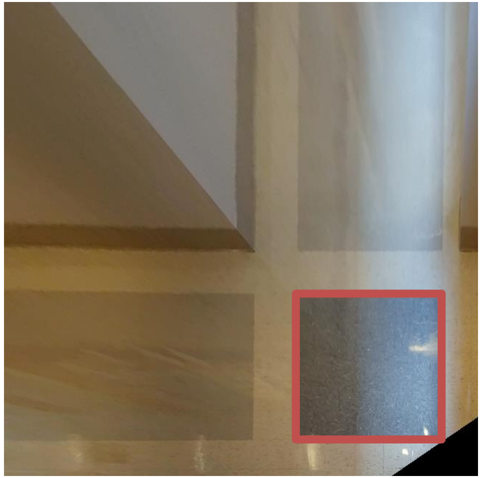
4 (2 translation+1 rotation+1 scale)

Similarity Transform ~ Euclidean + Uniform Scaling



$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_u \\ s\sin\theta & s\cos\theta & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

AFFINE TRANSFORM



$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

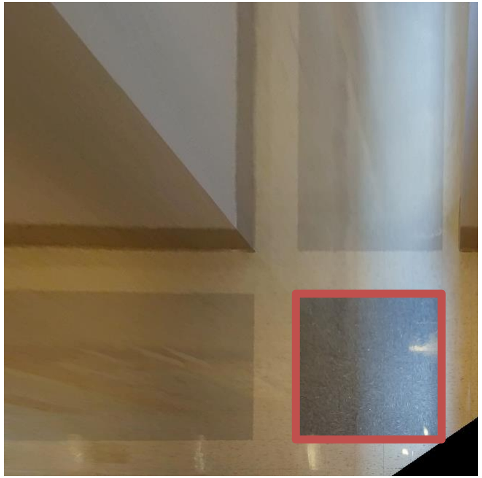
Invariant properties

- Parallelism
- Ratio of area
- Ratio of length

Degree of freedom

6

Perspective Transform ~ Homography



$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

Perspective Transform ~ Homography

$$\lambda \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

$$u_2 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

$$v_2 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + h_{33}}$$

: General form of plane to plane linear mapping

Perspective Transform ~ Homography

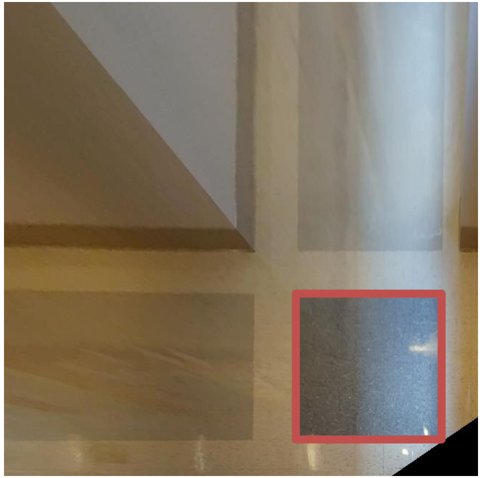
$$\lambda\begin{bmatrix}u_2\\v_2\\1\end{bmatrix}=\begin{bmatrix}h_{11}&h_{12}&h_{13}\\h_{21}&h_{22}&h_{23}\\h_{31}&h_{32}&1\end{bmatrix}\begin{bmatrix}u_1\\v_1\\1\end{bmatrix}$$

$$u_2=\frac{h_{11}u_1+h_{12}v_1+h_{13}}{h_{31}u_1+h_{32}v_1+1}$$

$$v_2=\frac{h_{21}u_1+h_{22}v_1+h_{23}}{h_{31}u_1+h_{32}v_1+1}$$

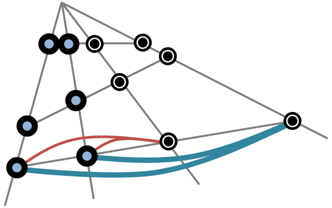
: General form of plane to plane linear mapping

PERSPECTIVE TRANSFORM ~ HOMOGRAPHY



Invariant properties

- Cross ratio



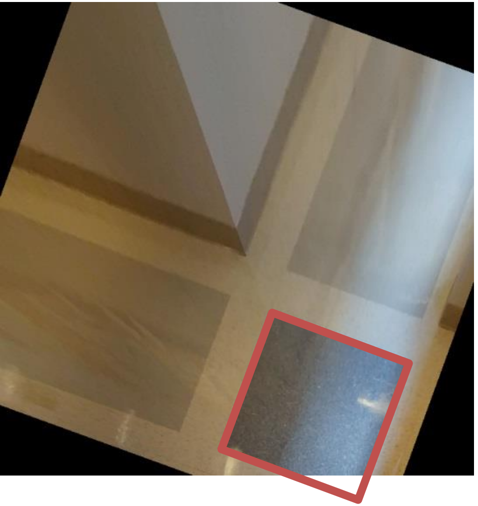
- Concurrency

- Colinearity

Degree of freedom

8 (9 variables – 1 scale)

Hierachy of Transformations



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Similarity (4 dof)

- Length ratio
- Angle

$$\begin{bmatrix} \alpha\cos\theta & -\alpha\sin\theta & t_x \\ \alpha\sin\theta & \alpha\cos\theta & t_y \\ & & 1 \end{bmatrix}$$

Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

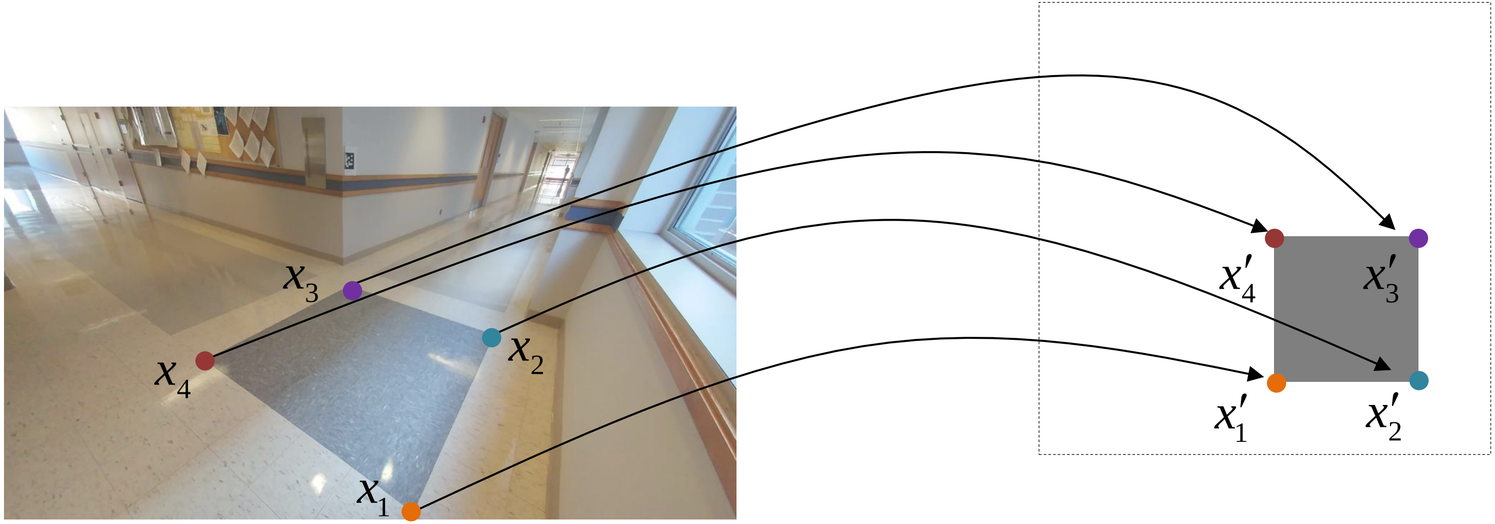
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

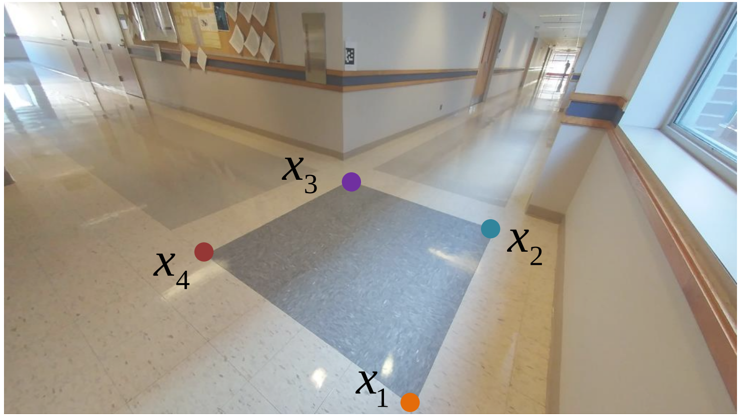
Homography Computation

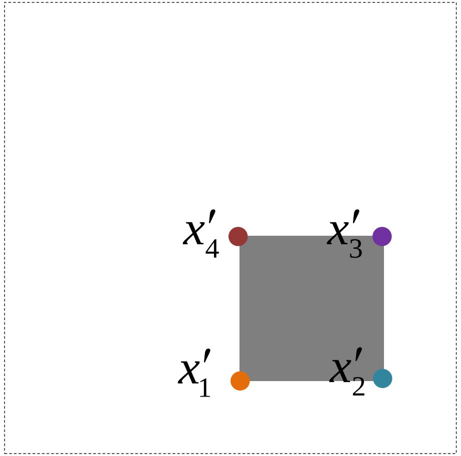


$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

The image can be rectified as if it is seen from top view.

Homography Computation



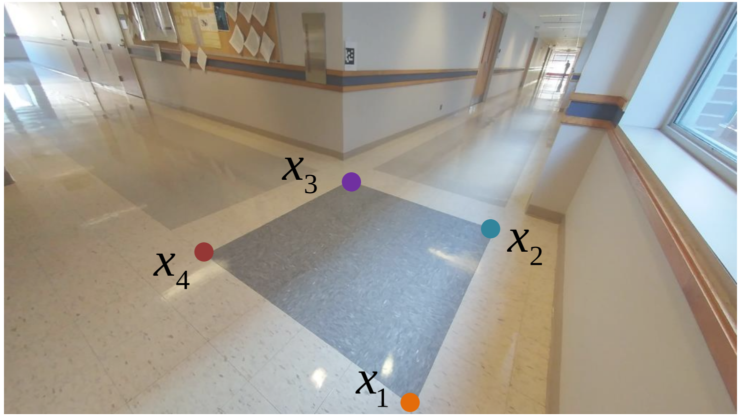


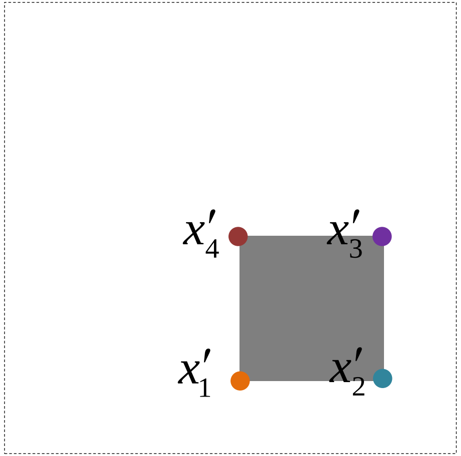
$$u'_1 = \frac{h_{11}u_1 + h_{12}v_1 + h_{13}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$v'_1 = \frac{h_{21}u_1 + h_{22}v_1 + h_{23}}{h_{31}u_1 + h_{32}v_1 + 1}$$

$$\longrightarrow \quad \begin{aligned} h_{11}u_1 + h_{12}v_1 + h_{13} - h_{31}u_1u'_1 - h_{32}v_1u'_1 - u'_1 = 0 \\ h_{21}u_1 + h_{22}v_1 + h_{23} - h_{31}u_1v'_1 - h_{32}v_1v'_1 - v'_1 = 0 \end{aligned}$$

Homography Computation



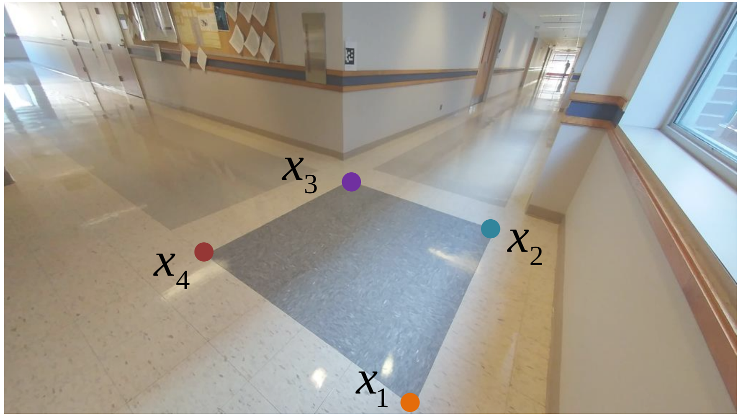


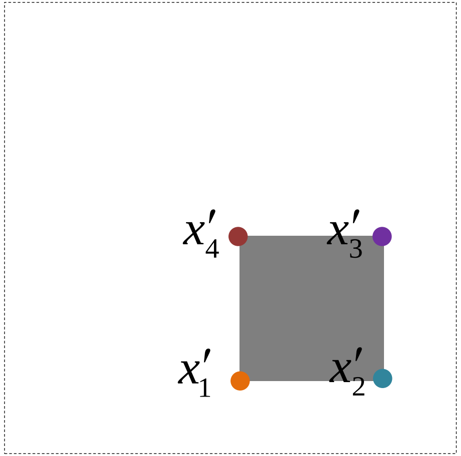
$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u' & -v_1u' \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v' & -v_1v' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} u'_1 \\ v'_1 \end{bmatrix}$$

# of equations: 2
# of unknows: 8

How many correspondences are needed?

Homography Computation

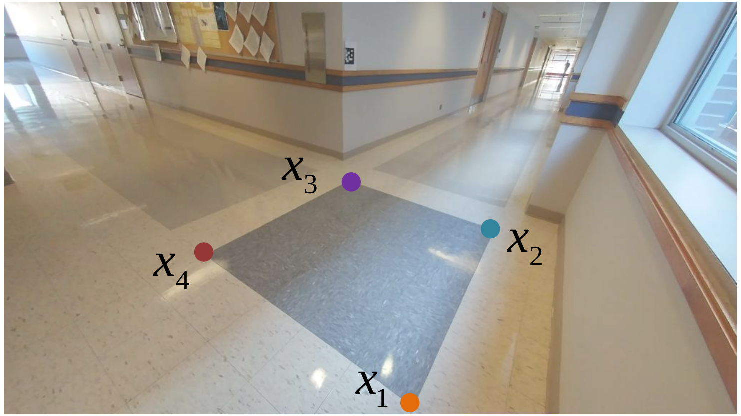


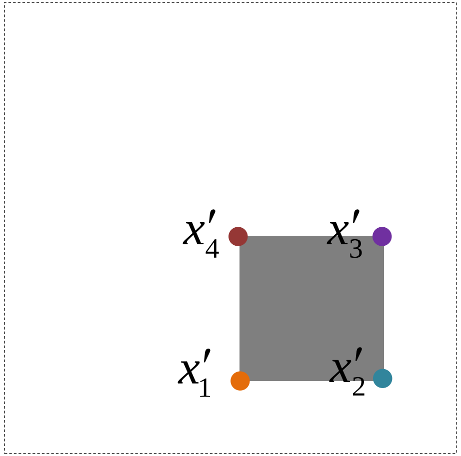


$$\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ & & & & \vdots & & & \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = \begin{bmatrix} u'_1 \\ v'_1 \\ \vdots \\ u'_4 \\ v'_4 \end{bmatrix}$$

\# of equations: 2
\# of unknows: 8

Homography Computation

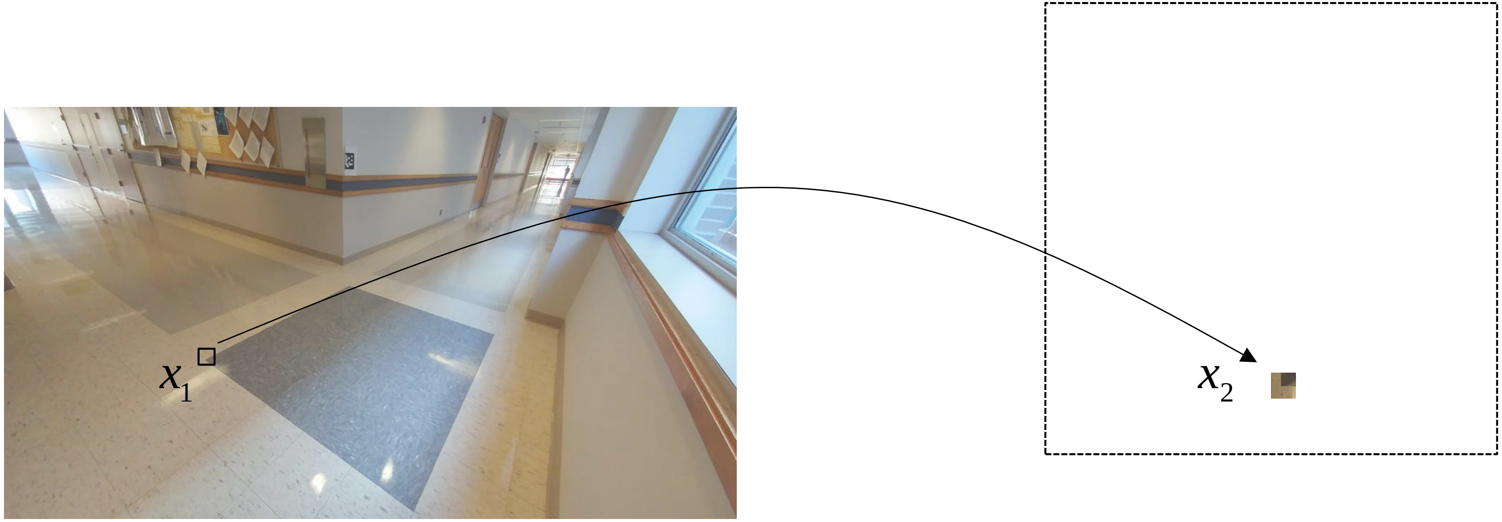




$$\underbrace{\begin{bmatrix} u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\ 0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\ & & & & \vdots & & & \\ u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\ 0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ \vdots \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} u'_1 \\ v'_1 \\ \vdots \\ u'_4 \\ v'_4 \end{bmatrix}}_{b}$$

$$Ax = b \longrightarrow x = \left(A^T A\right)^{-1} A^T b$$

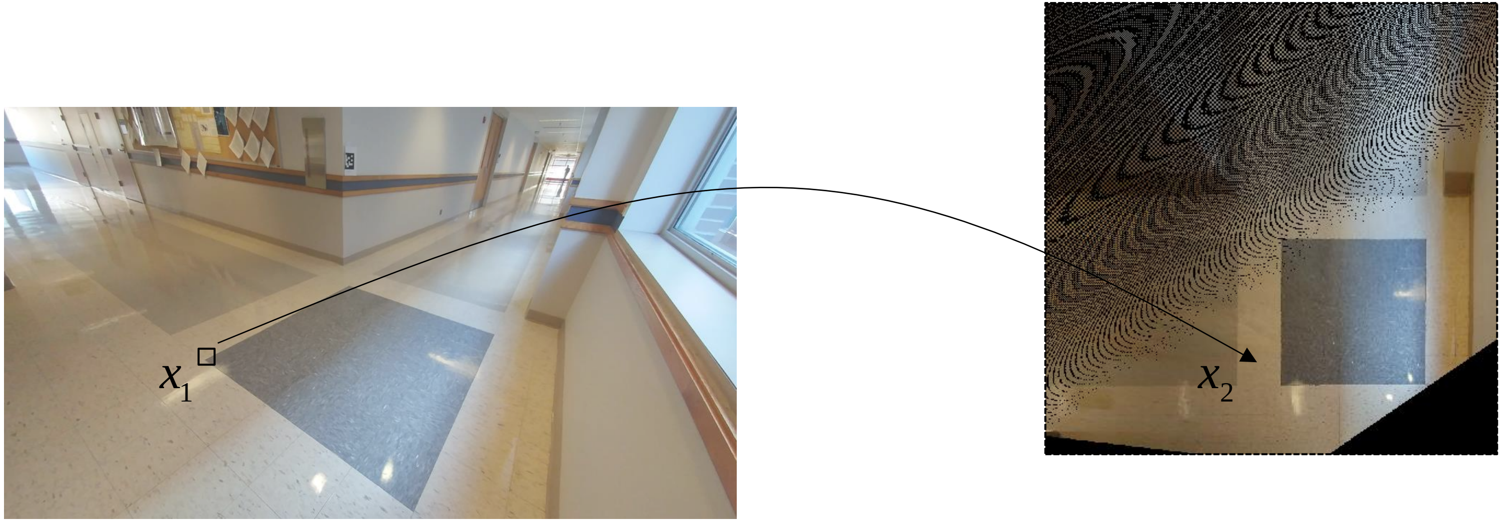
Image Warping



$$x_2 = f(x_1) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING



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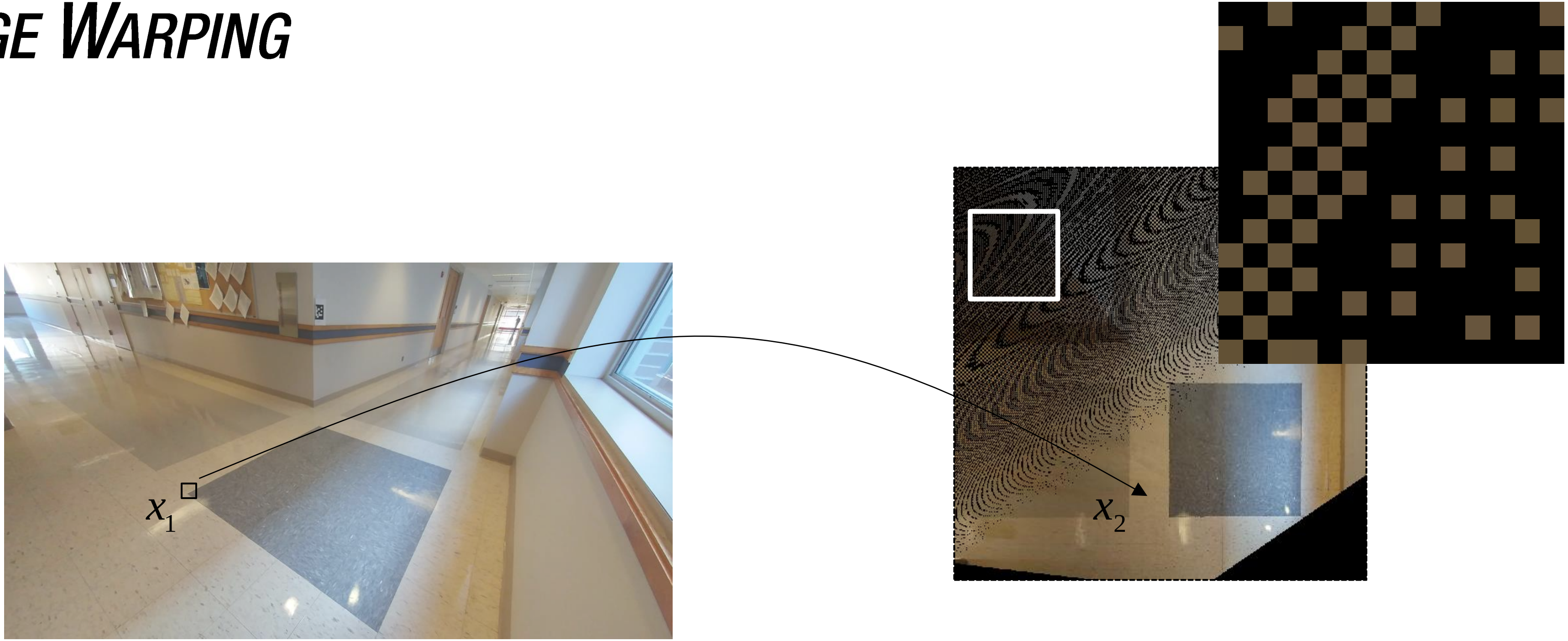

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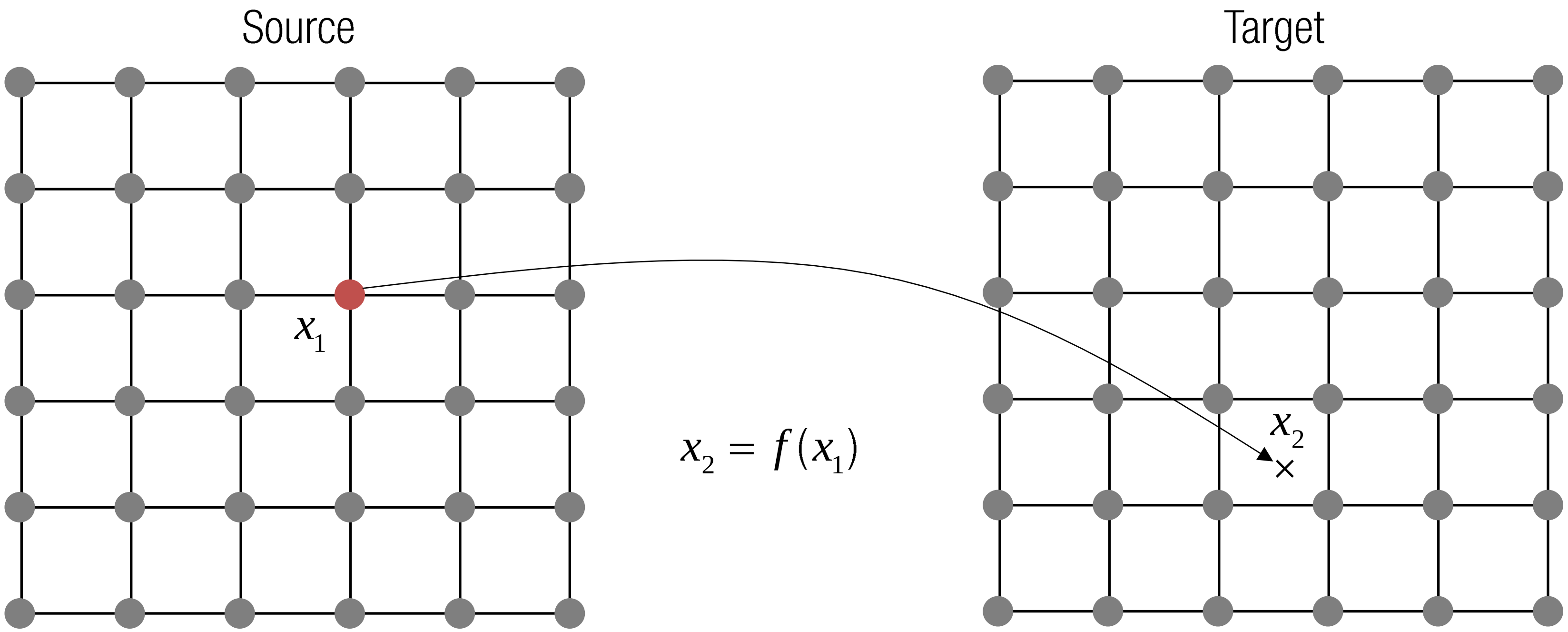

$$x_2 = f(x_1)$$

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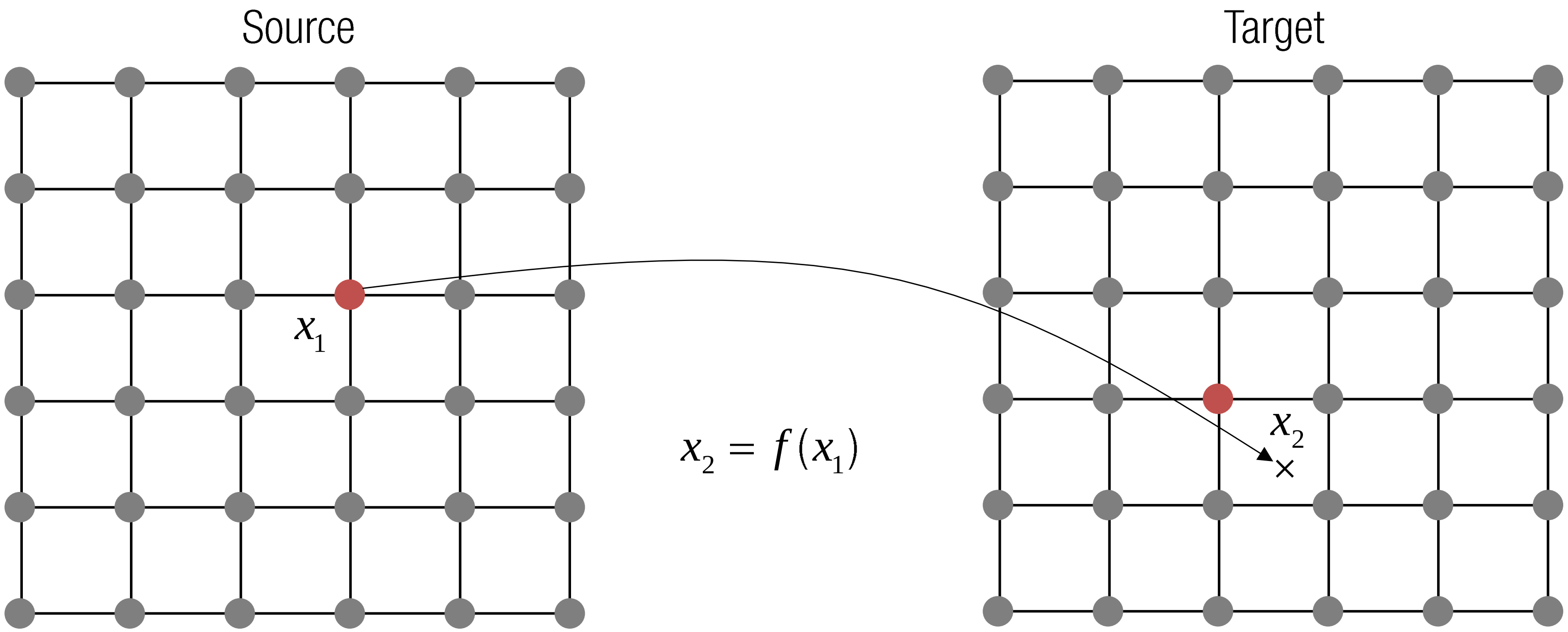
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

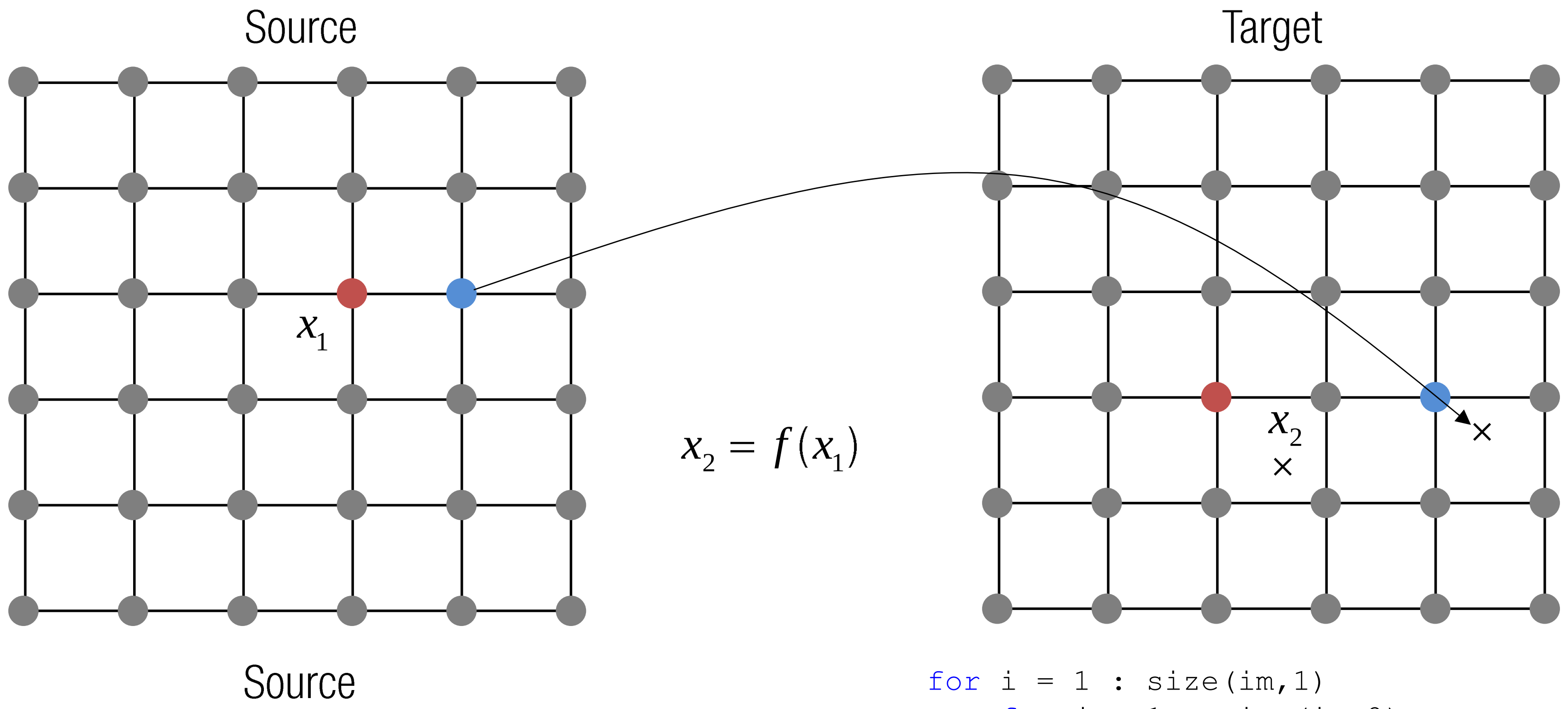
IMAGE WARPING



Source

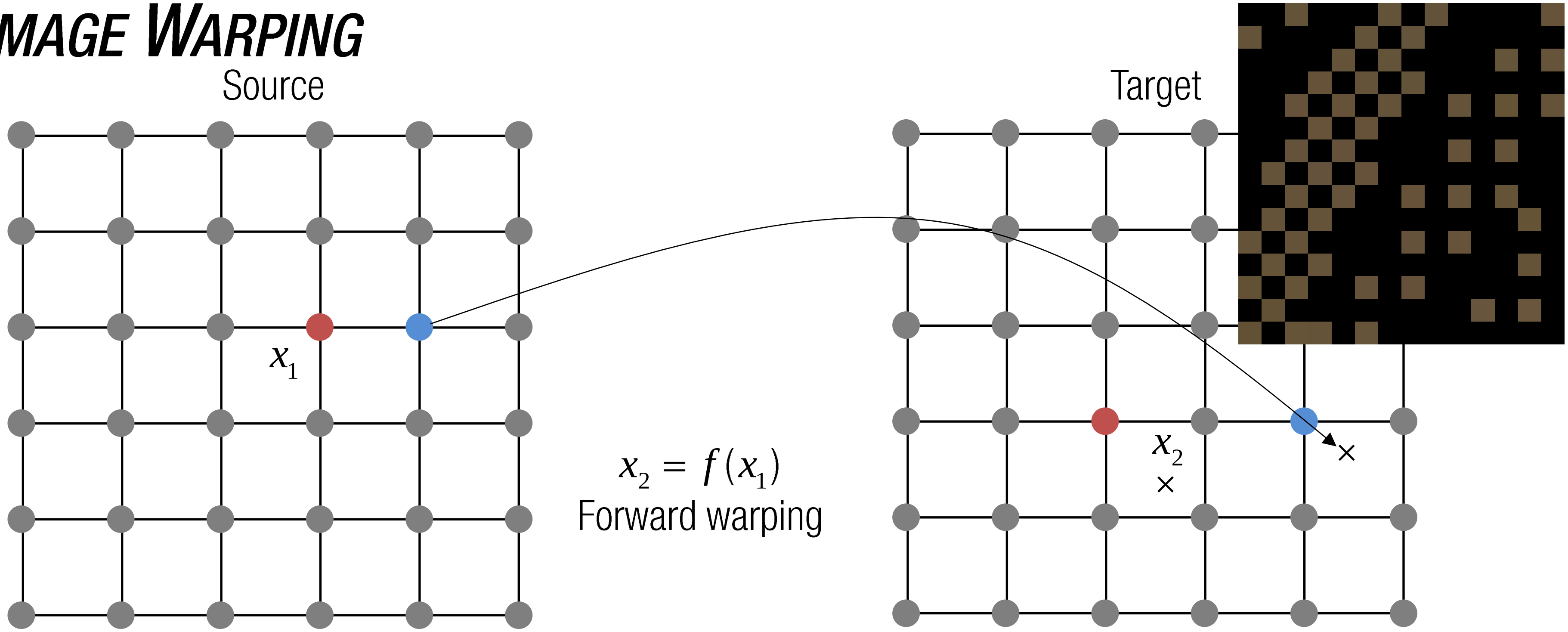
```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING



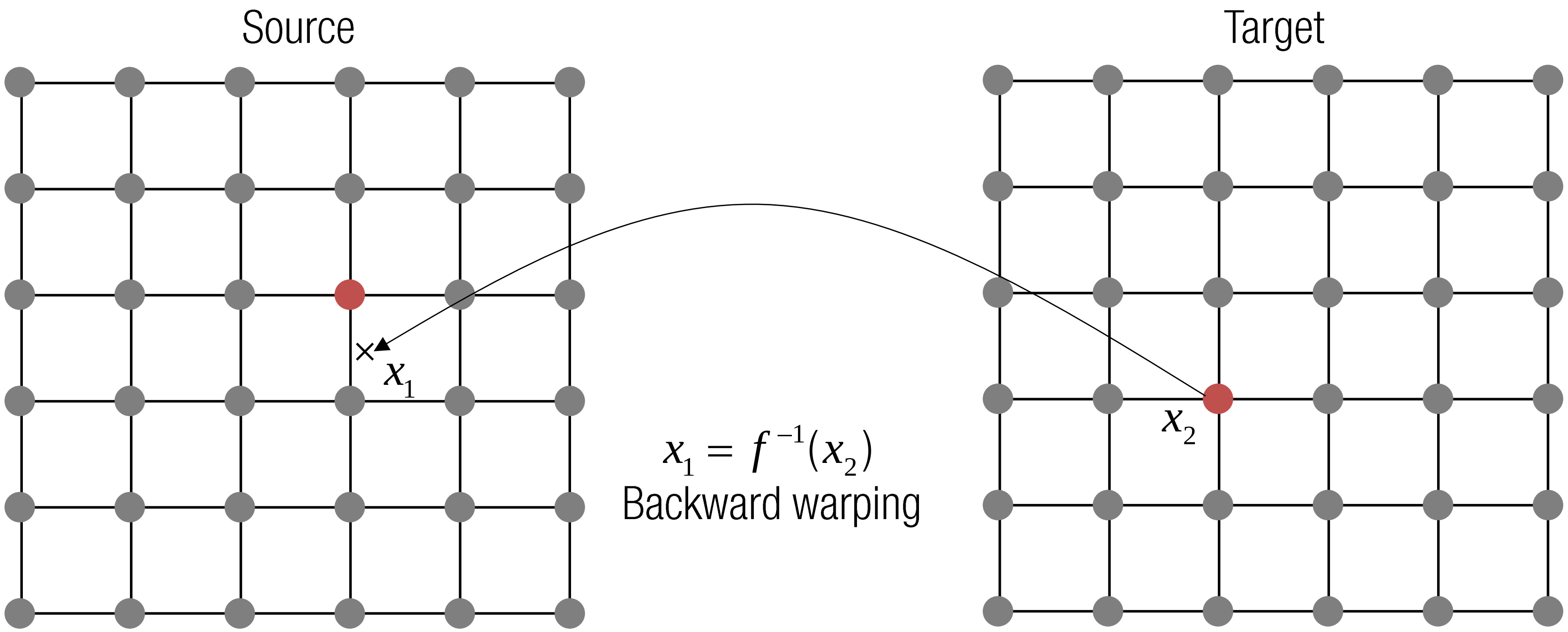

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING




```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

Inverse Warping

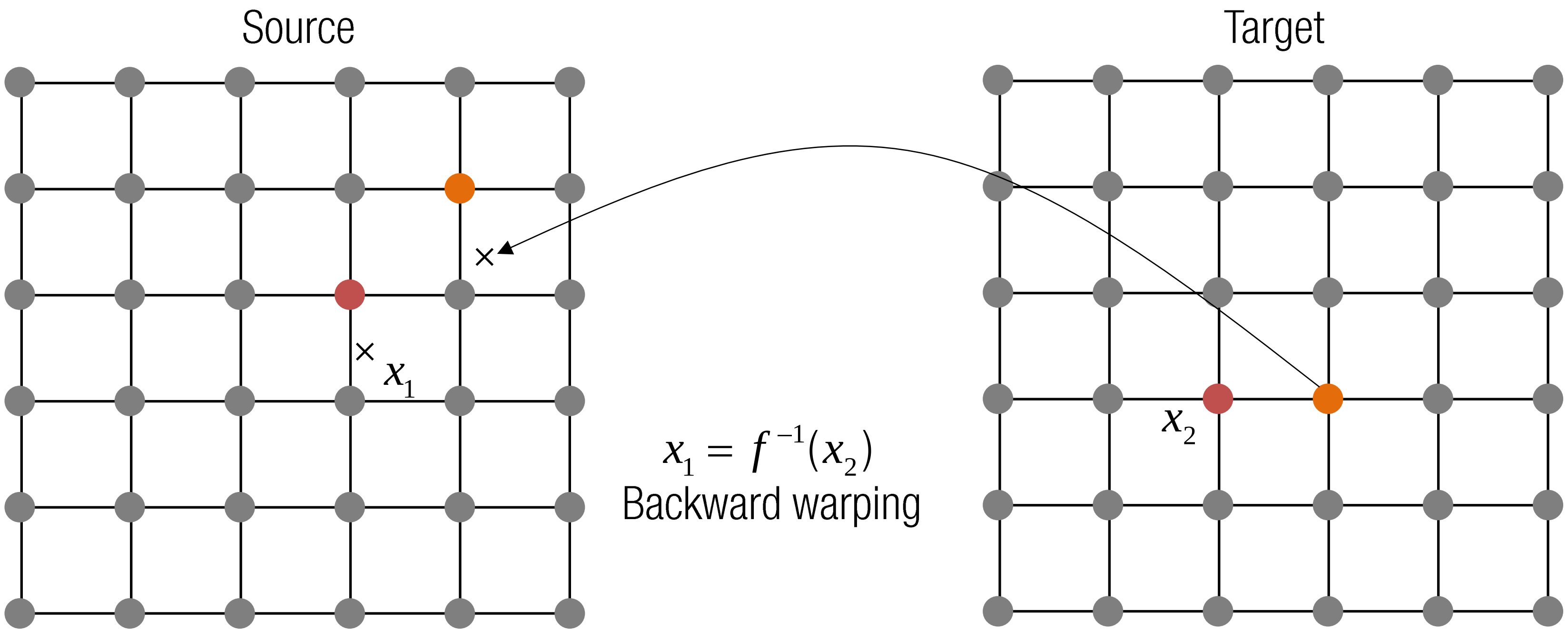


$$x_1 = f^{-1}(x_2)$$

Backward warping

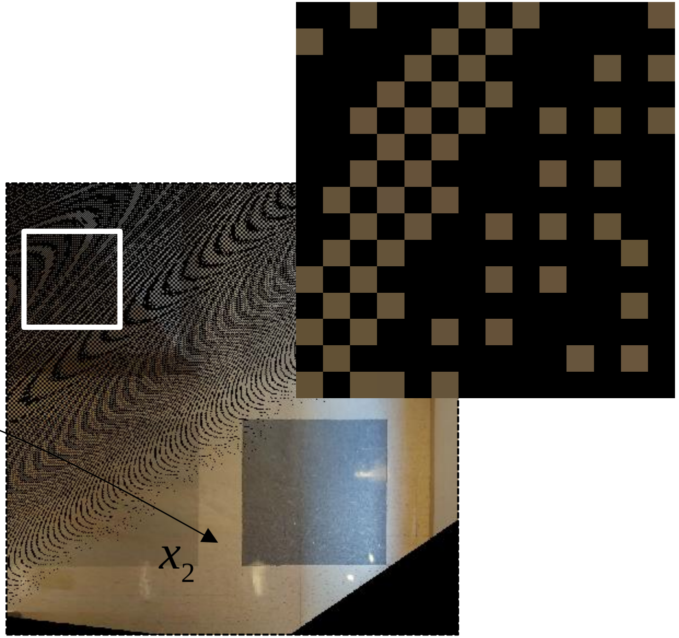
```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

INVERSE WARPING




```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING



Forward warping

$$x_2 = f(x_1) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x1 = [j, i];
        x2 = floor(mapping(x1));
        I2(x2) = I1(x1);
    end
end
```

Image Warping

Backward warping

$$x_1 = f^{-1}(x_2) \qquad I_2(x_2) = I_1(f(x_1))$$

```
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        x2 = [j, i];
        x1 = floor(inverse_mapping(x2));
        I2(x2) = I1(x1);
    end
end
```

IMAGE WARPING

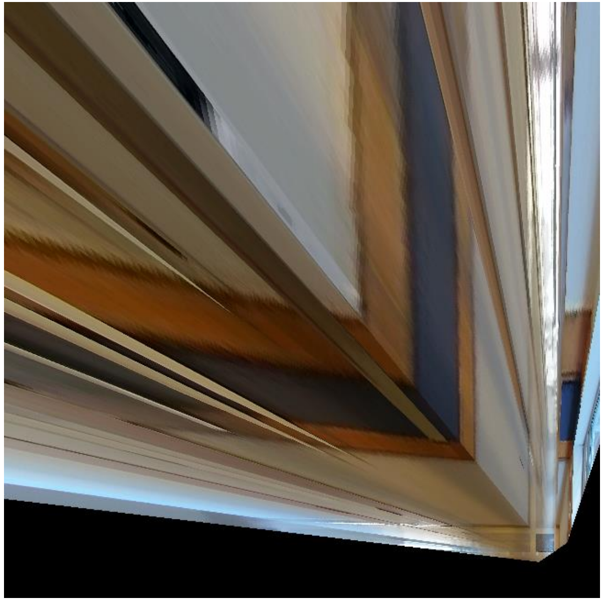


IMAGE WARPING

Image Warping

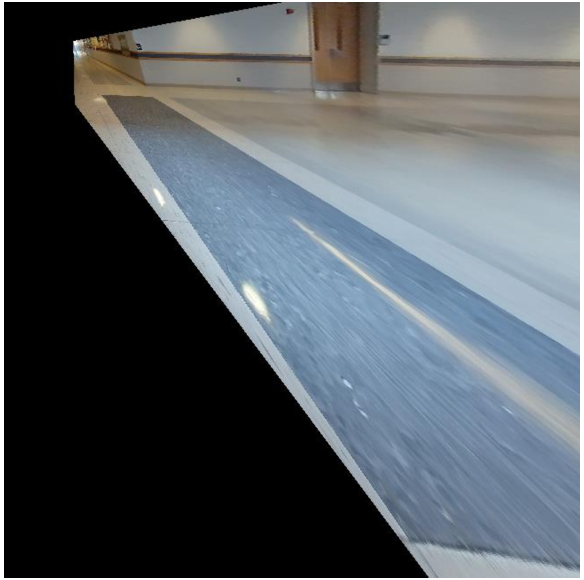
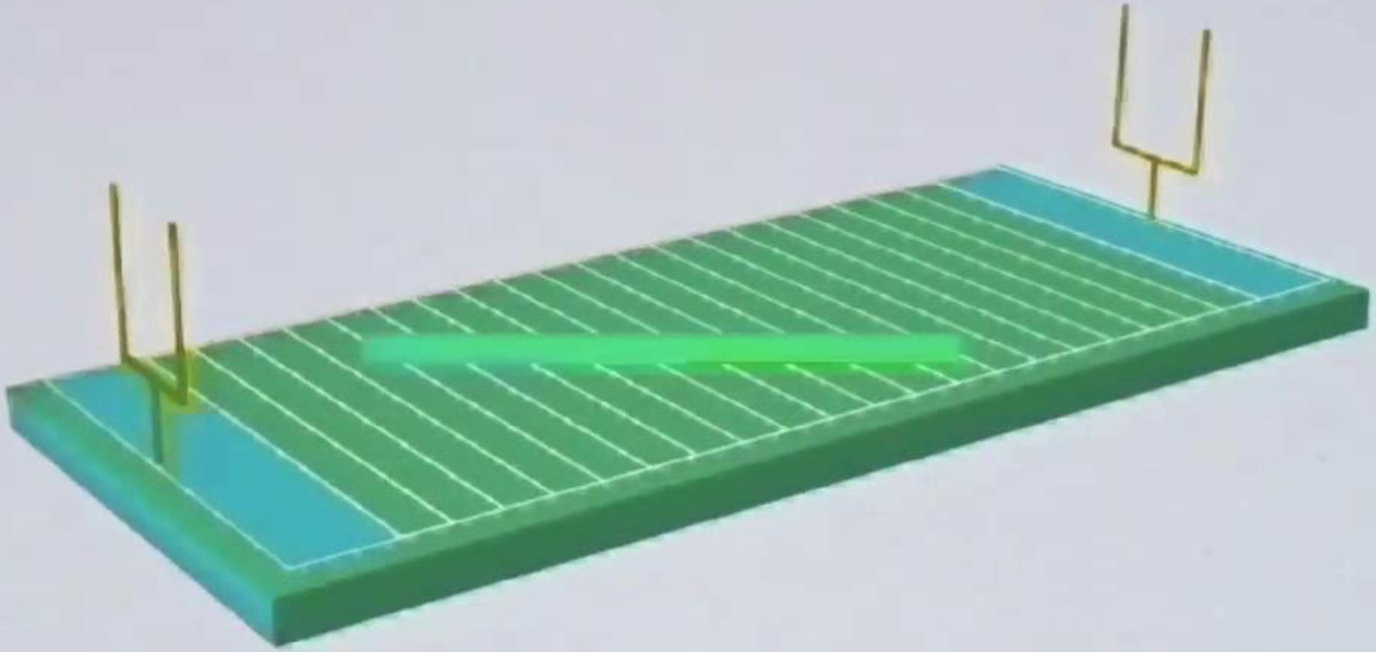


IMAGE WARPING

Virtual Advertisement




50
40
30
20
10
freeD

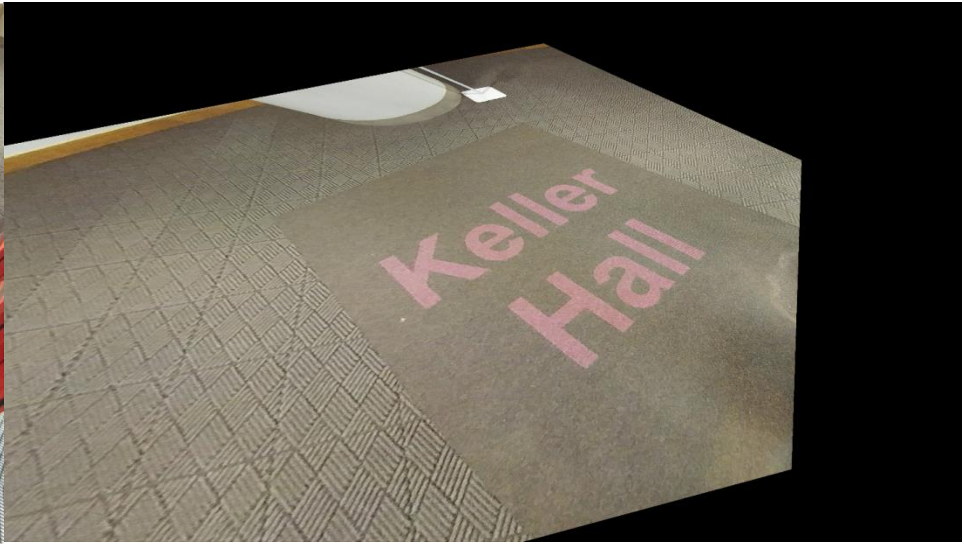
Image Warping

Keller entrance left

Keller entrance right

Image Warping

Keller entrance left



Right image to left

Image Warping

Image Warping

Left image to right

Right image to left

Image Warping

$$\lambda \mathbf{u} = \mathbf{H}\mathbf{v}$$

Lind Hall Left

Lind Hall Right

Euclidean Transform (Translation)

Homography


BURGER KING
WOMEN
E5


WOMEN
E5