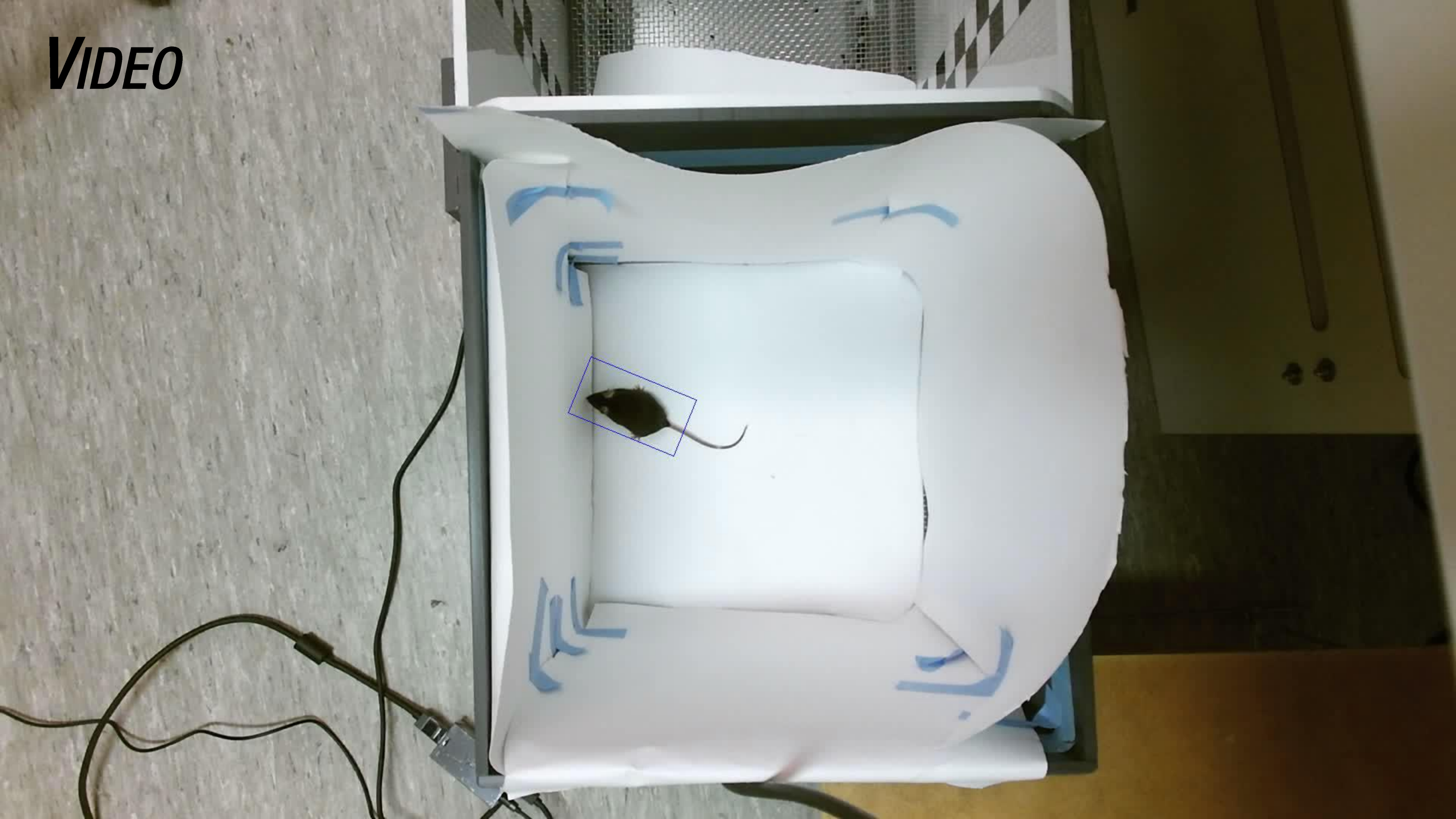
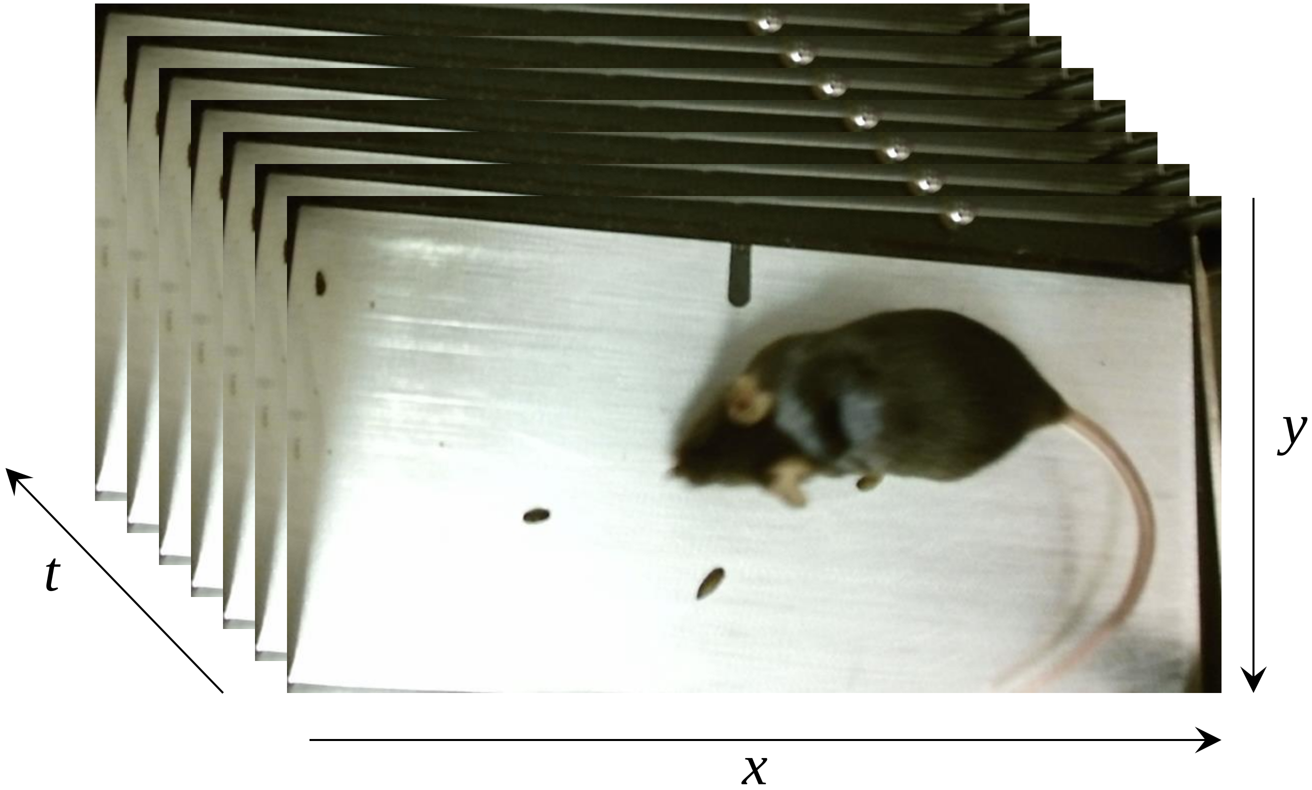
Patch Tracking

Hyun Soo Park


VIDEO

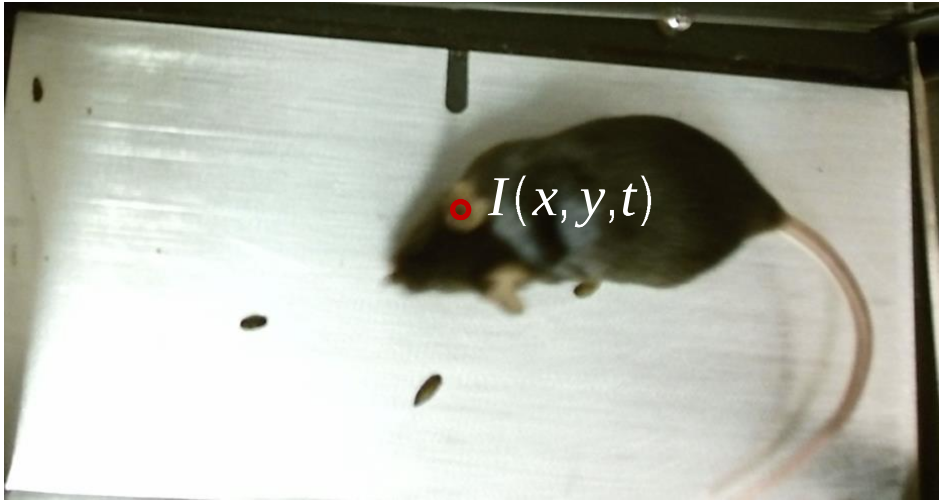
Continous Video Representation



Video as a function over spacetime volume

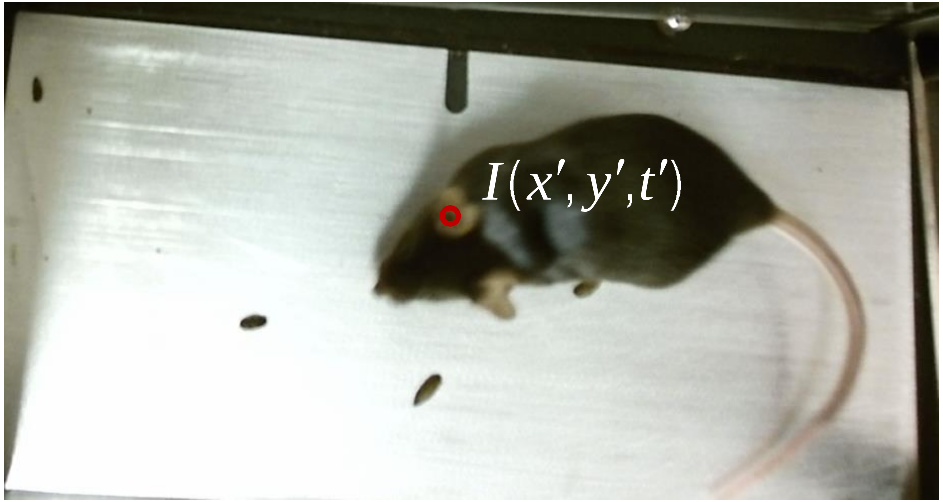
$$I(x, y, t)$$

Local Point Tracking



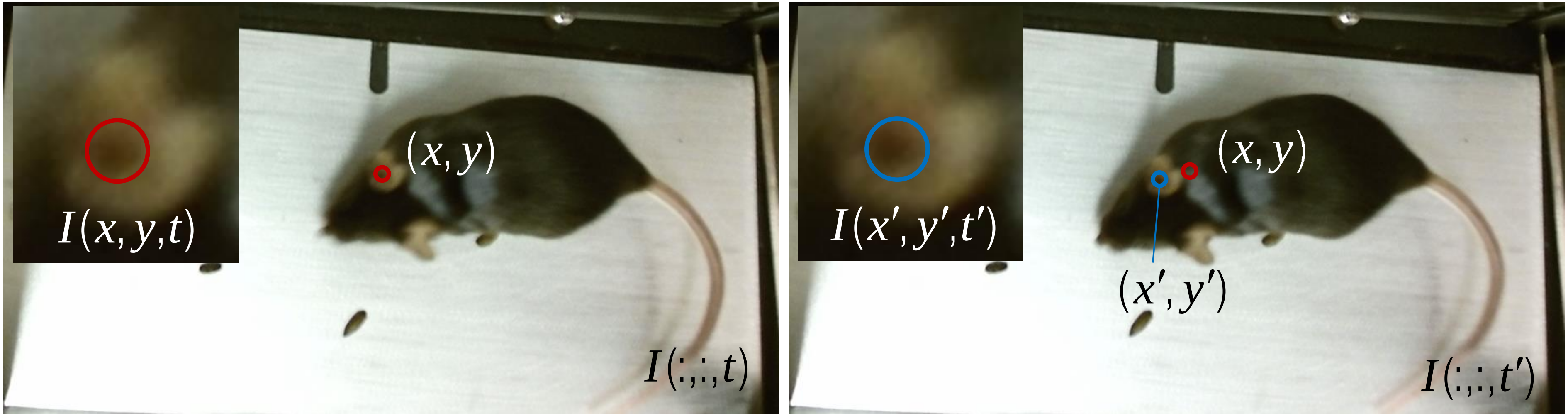
$$I(x,y,t)$$

Local Point Tracking



$$I(x', y', t')$$

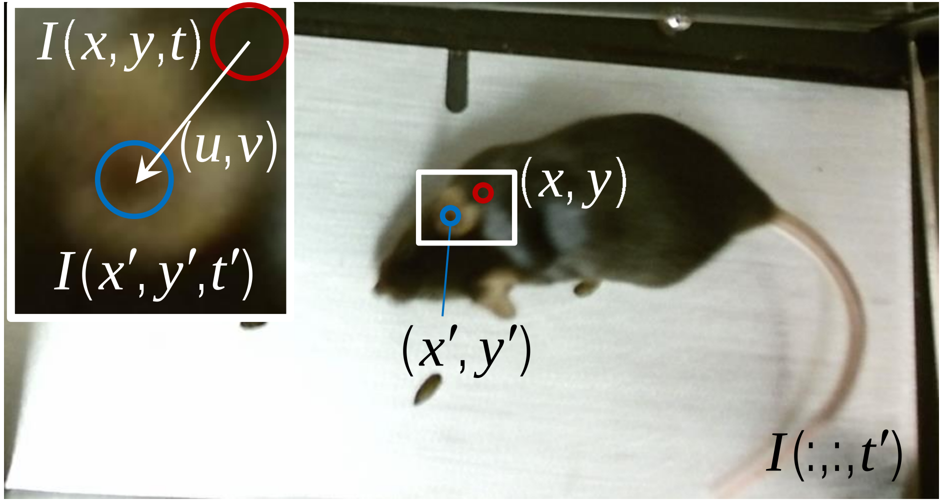
Assumption 1: Brightness Constancy



Brightness constancy: $I(x,y,t) \approx I(x',y',t')$

e.g., RGB value must be similar.

Assumption 2: Small Motion



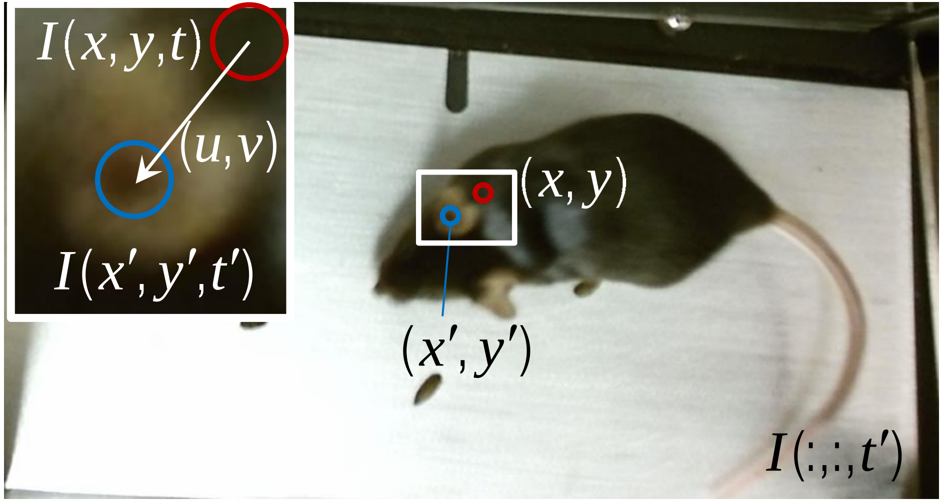
First order approximation must hold.

$$x' \approx x + u\delta t$$

$$y' \approx y + v\delta t$$

$$t' \approx t + \delta t$$

Assumption 2: Small Motion



First order approximation must hold.

$$x' \approx x + u\delta t$$

$$y' \approx y + v\delta t$$

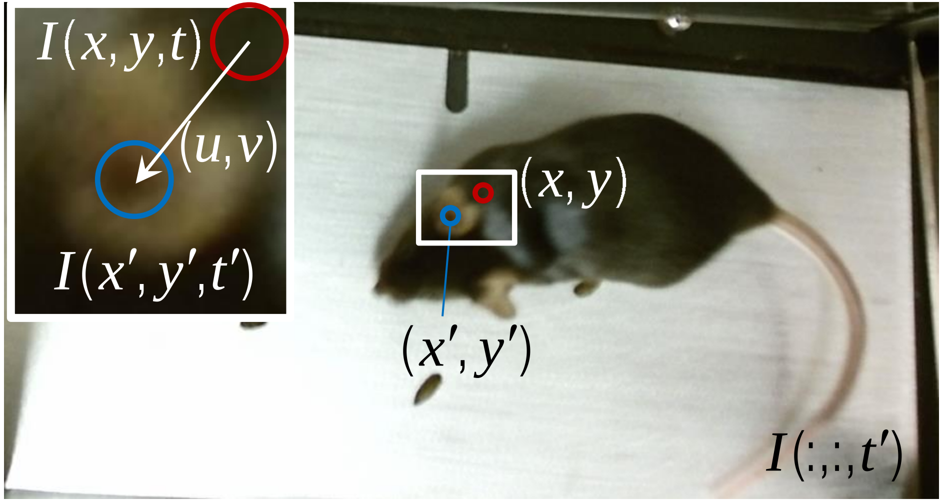
$$t' \approx t + \delta t$$

First order Taylor expansion:

$$I(x',y',t') \approx I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

Optical Flow



First order approximation must hold.

$$x' \approx x + u\delta t$$

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$$t' \approx t + \delta t$$

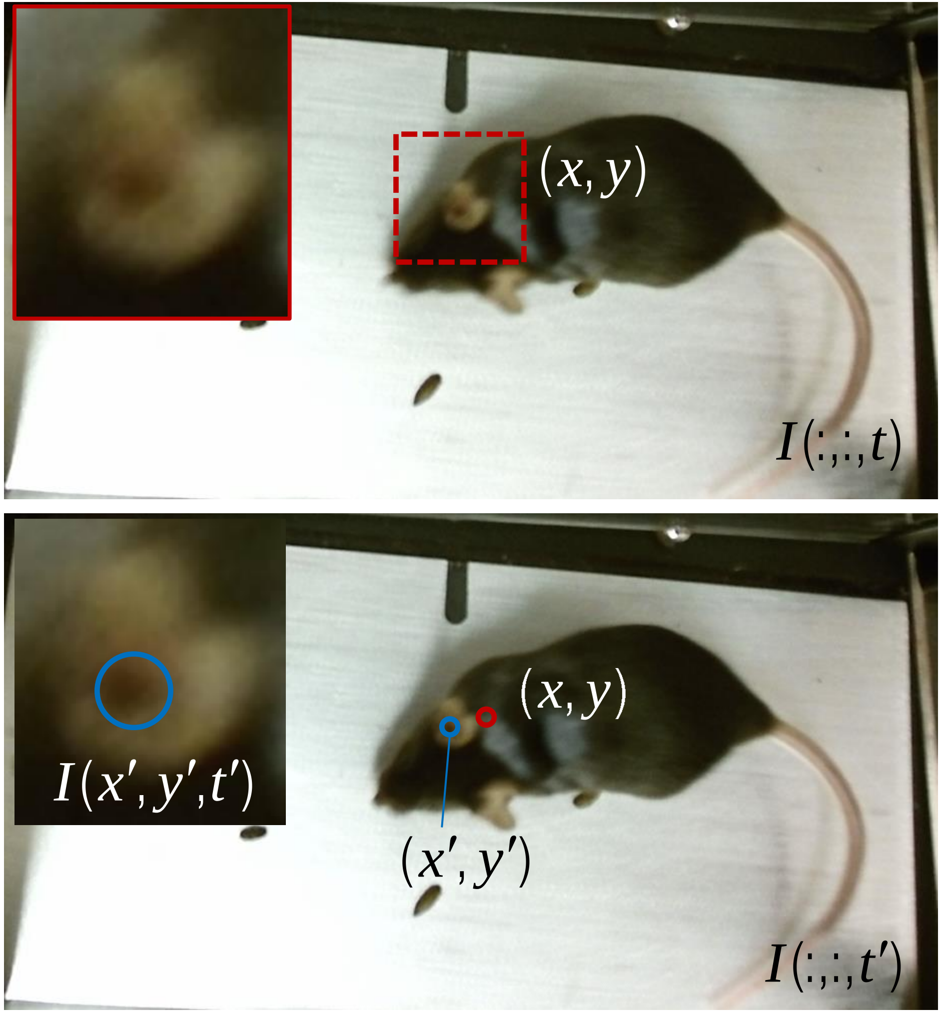
First order Taylor expansion:

$$I(x',y',t') \approx I(x+u\delta t, y+v\delta t, t+\delta t)$$

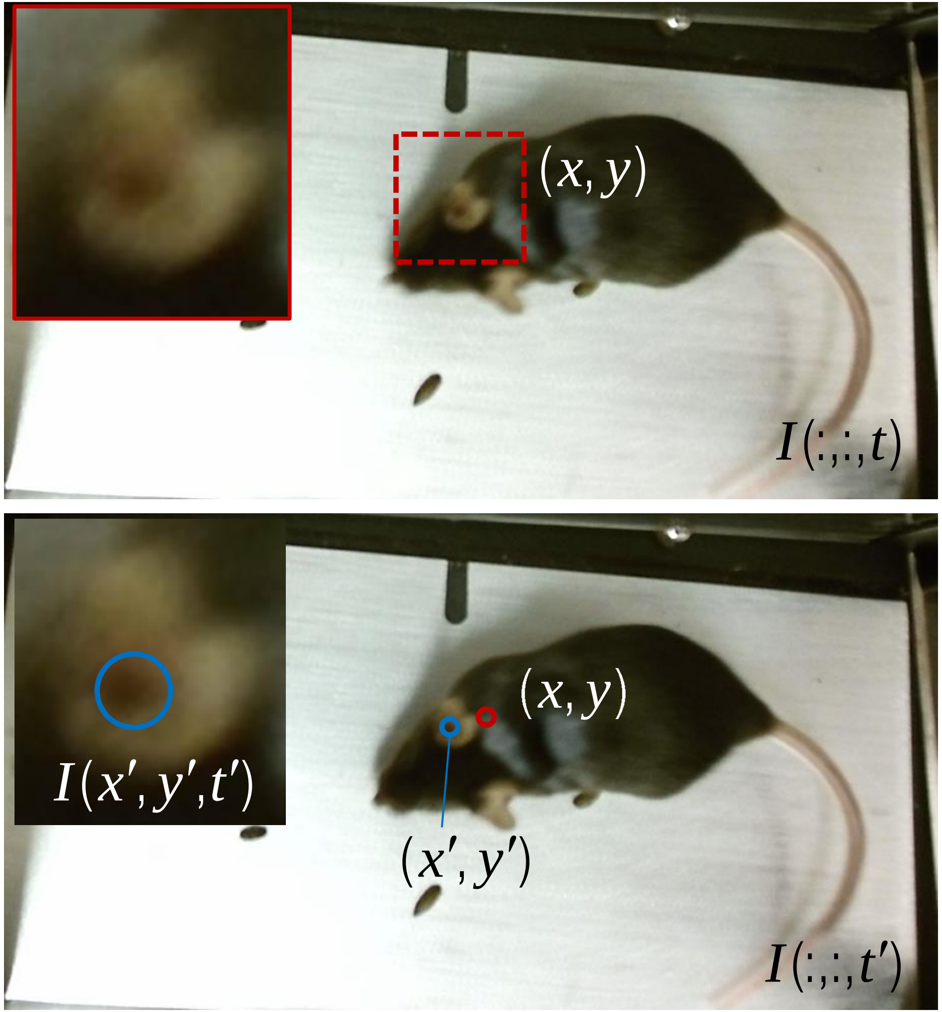
$$\approx I(x,y,t) + \frac{\partial I}{\partial x}u\delta t + \frac{\partial I}{\partial y}v\delta t + \frac{\partial I}{\partial t}\delta t$$

$$\approx \underline{I(x,y,t)}$$

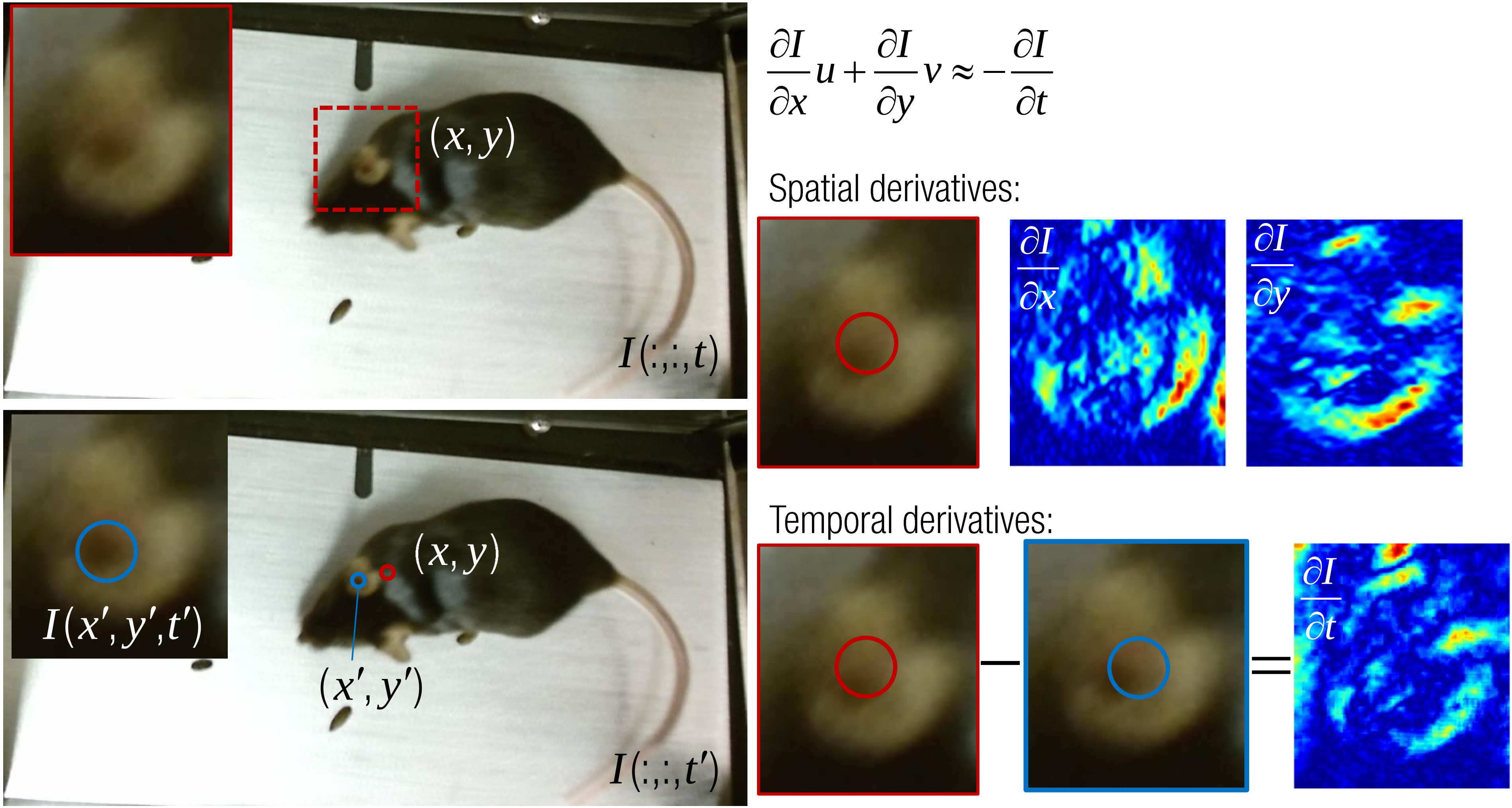
Brightness constancy



$$\frac{\partial I}{\partial x}u\delta t+\frac{\partial I}{\partial y}v\delta t+\frac{\partial I}{\partial t}\delta t\approx 0$$

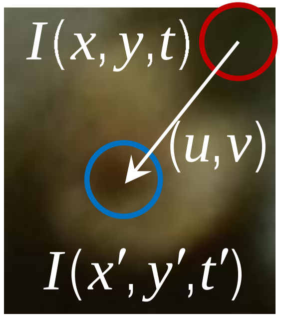


$$\frac{\partial I}{\partial x}u+\frac{\partial I}{\partial y}v\approx-\frac{\partial I}{\partial t}$$



(x, y)
I(:,:,t)
I(x′,y′,t′)
(x, y)
(x′, y′)
I(:,:,t′)
$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$
Spatial derivatives:
$\frac{\partial I}{\partial x}$
$\frac{\partial I}{\partial y}$
Temporal derivatives:
$\frac{\partial I}{\partial t}$

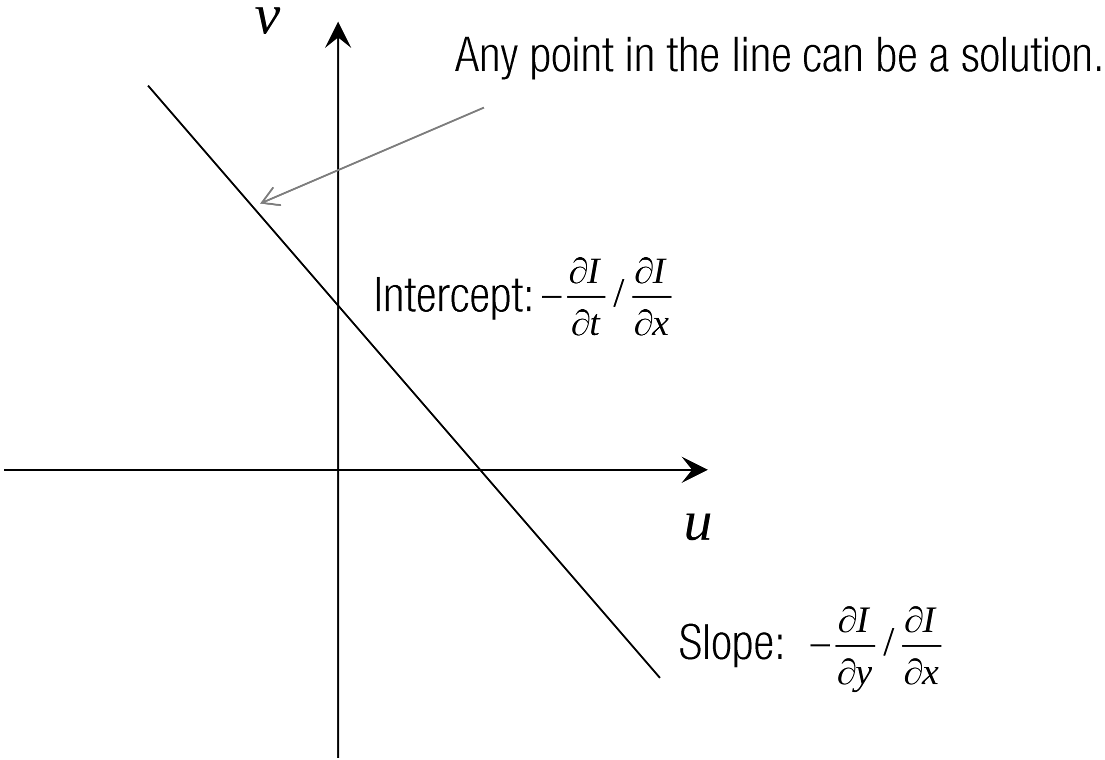
Ambiguity For Single Pixel Tracking



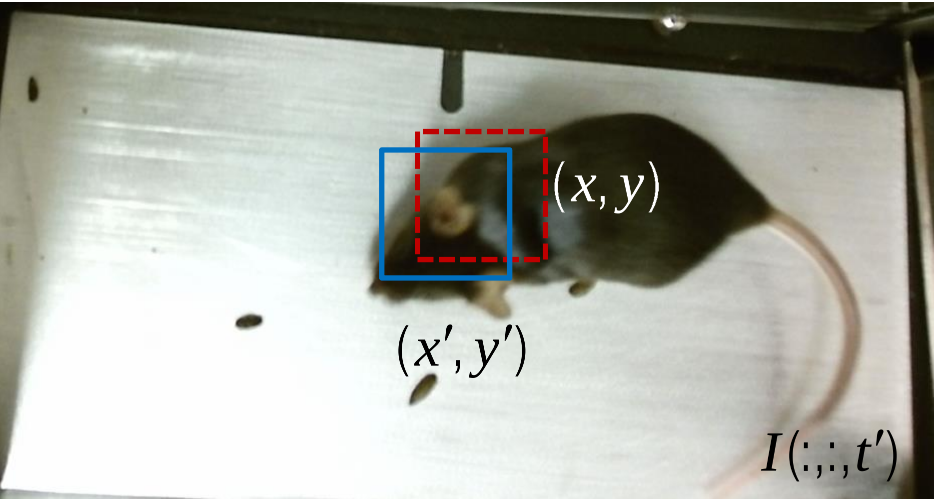
$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$$

# of unknowns: 2

# of equations: 1



Local Patch Tracking



Neighboring pixels move similarly.

$\{I(x_i, y_i, t)\}_{(x_i, y_i) \in N(x,y)}$

$\{I(x'_i, y'_i, t)\}_{(x'_i, y'_i) \in N(x',y')}$

$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1$$

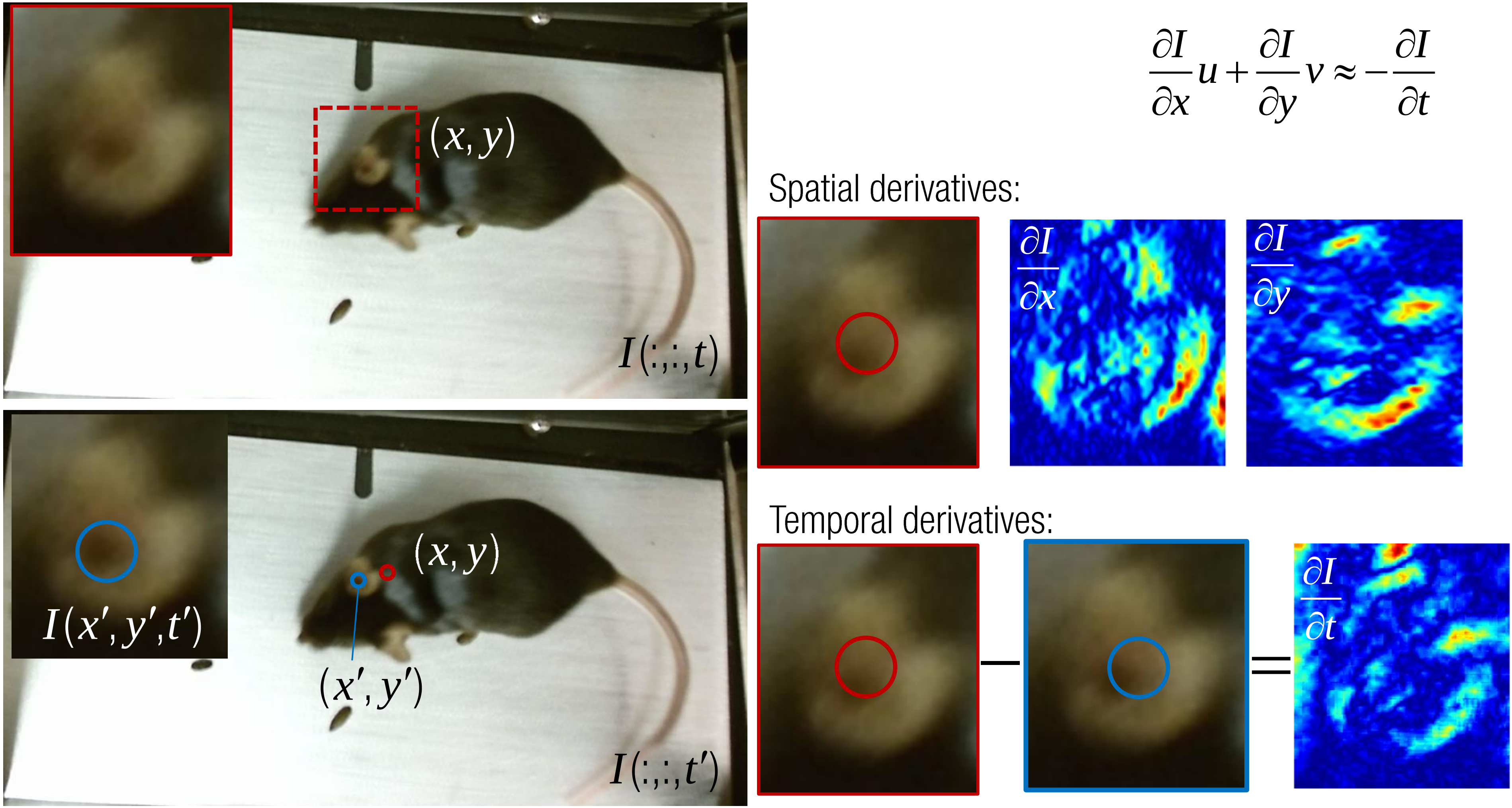
$$\vdots$$

$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n$$

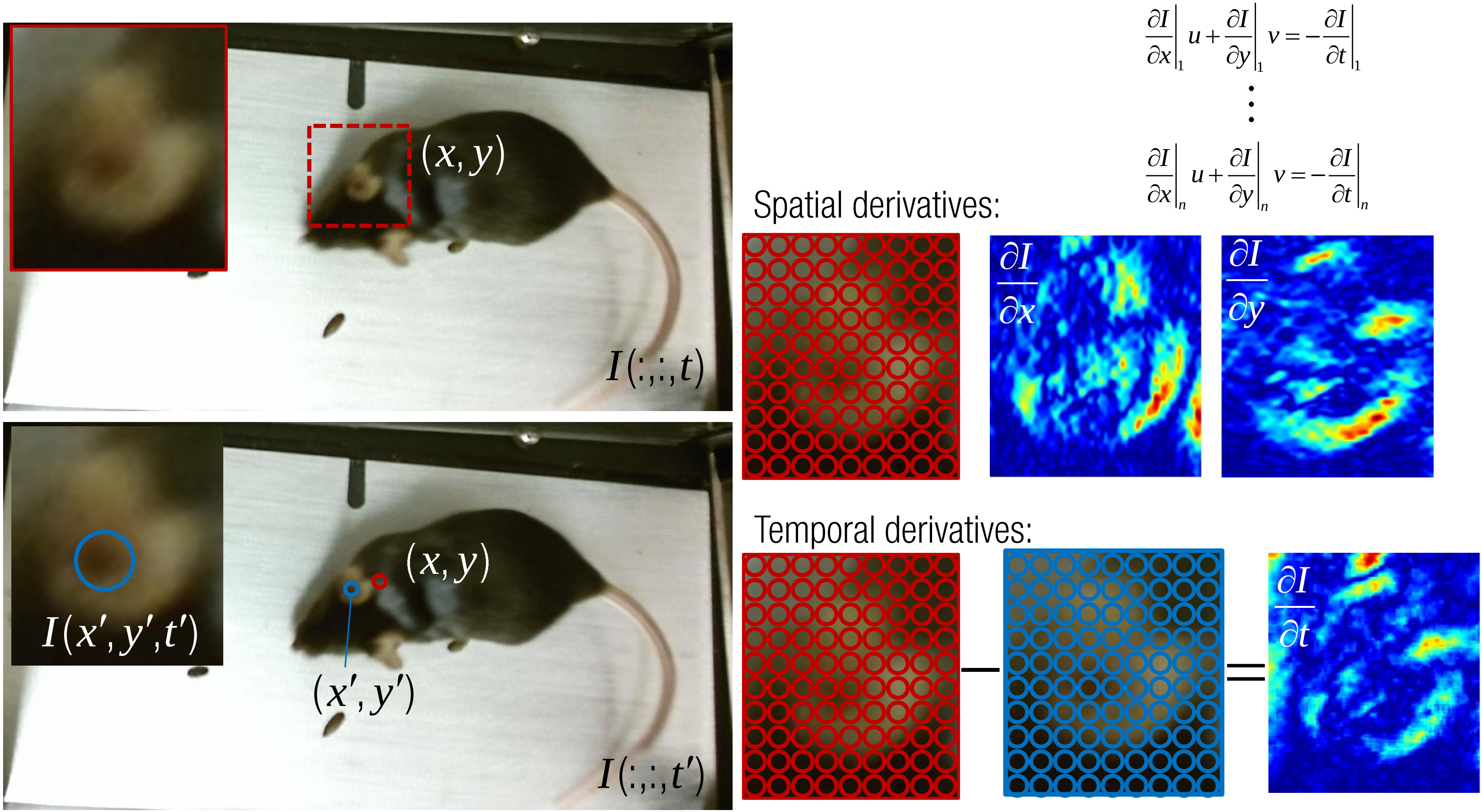
# of unknowns: 2

# of equations: # of pixels in the local patch

Can be solved via linear least squares.



(x, y)
$I(:,:,t)$
$I(x',y',t')$
(x, y)
(x', y')
$I(:,:,t')$
$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx -\frac{\partial I}{\partial t}$
Spatial derivatives:
$\frac{\partial I}{\partial x}$
$\frac{\partial I}{\partial y}$
Temporal derivatives:
$\frac{\partial I}{\partial t}$



(x, y)
$I(:,:,t)$
$I(x',y',t')$
(x, y)
(x',y')
$I(:,:,t')$
$\frac{\partial I}{\partial x}\Big|_1 u + \frac{\partial I}{\partial y}\Big|_1 v = -\frac{\partial I}{\partial t}\Big|_1$
$\vdots$
$\frac{\partial I}{\partial x}\Big|_n u + \frac{\partial I}{\partial y}\Big|_n v = -\frac{\partial I}{\partial t}\Big|_n$
Spatial derivatives:
$\frac{\partial I}{\partial x}$
$\frac{\partial I}{\partial y}$
Temporal derivatives:
$\frac{\partial I}{\partial t}$

Local Patch Tracking

$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1$$

$$\vdots$$

$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n$$

$$\longrightarrow$$

$$\left.I_x\right|_1 u + \left.I_y\right|_1 v = -\left.I_t\right|_1$$

$$\vdots$$

$$\left.I_x\right|_n u + \left.I_y\right|_n v = -\left.I_t\right|_n$$

$$\longrightarrow$$

$$\underbrace{\begin{bmatrix} \left.I_x\right|_1 & \left.I_y\right|_1 \\ \vdots & \vdots \\ \left.I_x\right|_n & \left.I_y\right|_n \end{bmatrix}}_{\text{nx2}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\text{Unknowns}} = -\underbrace{\begin{bmatrix} \left.I_t\right|_1 \\ \vdots \\ \left.I_t\right|_n \end{bmatrix}}_{\text{nx1}}$$

Local Patch Tracking

$$\left.\frac{\partial I}{\partial x}\right|_1 u + \left.\frac{\partial I}{\partial y}\right|_1 v = -\left.\frac{\partial I}{\partial t}\right|_1 \quad\longrightarrow\quad I_x|_1\, u + I_y|_1\, v = -I_t|_1$$

$$\vdots \qquad\qquad\qquad \vdots$$

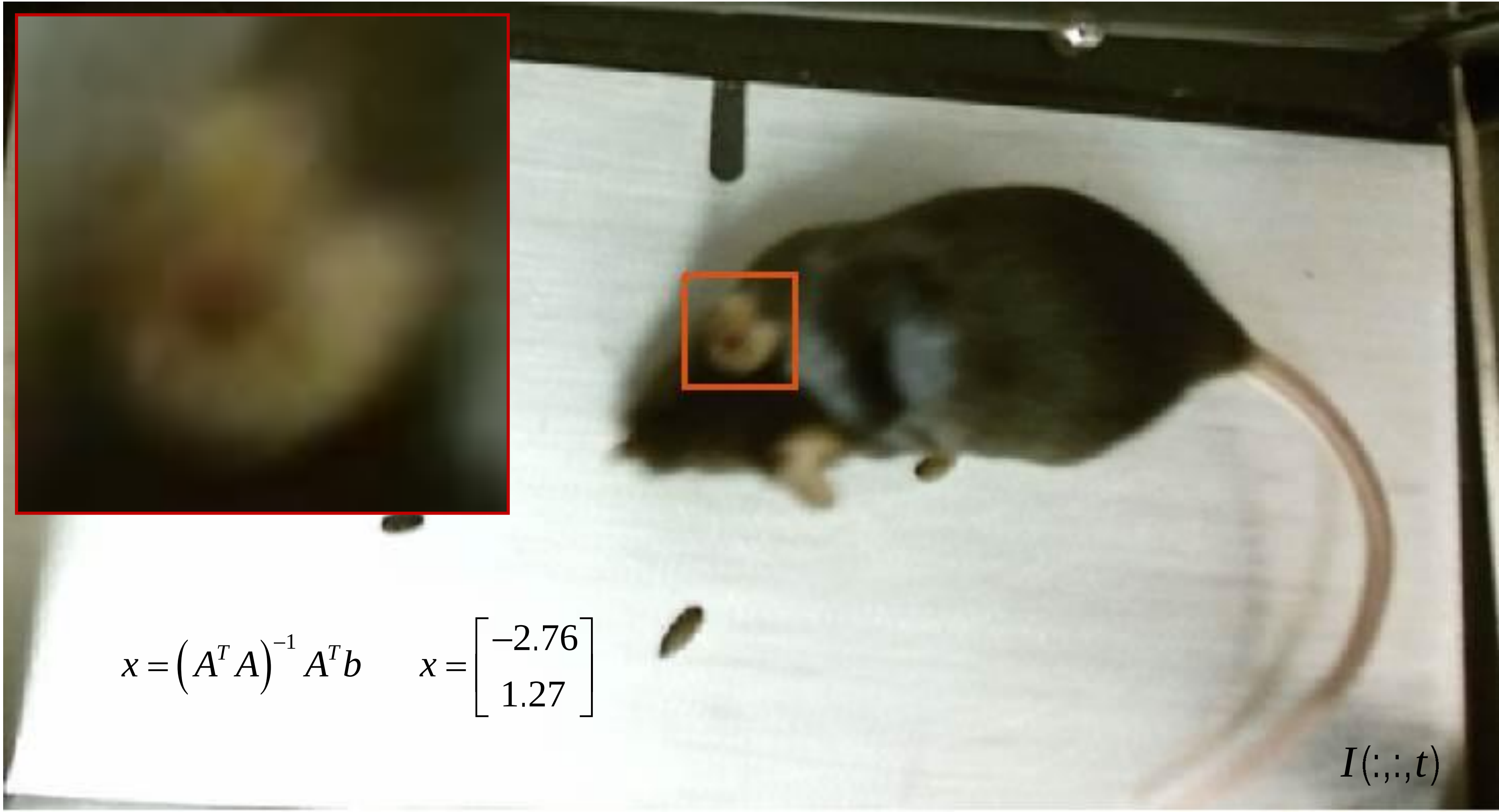
$$\left.\frac{\partial I}{\partial x}\right|_n u + \left.\frac{\partial I}{\partial y}\right|_n v = -\left.\frac{\partial I}{\partial t}\right|_n \quad\longrightarrow\quad I_x|_n\, u + I_y|_n\, v = -I_t|_n$$

$$\longrightarrow \quad \underbrace{\begin{bmatrix} I_x|_1 & I_y|_1 \\ \vdots & \vdots \\ I_x|_n & I_y|_n \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{x} = - \underbrace{\begin{bmatrix} I_t|_1 \\ \vdots \\ I_t|_n \end{bmatrix}}_{b}$$

nx2 — Unknowns — nx1

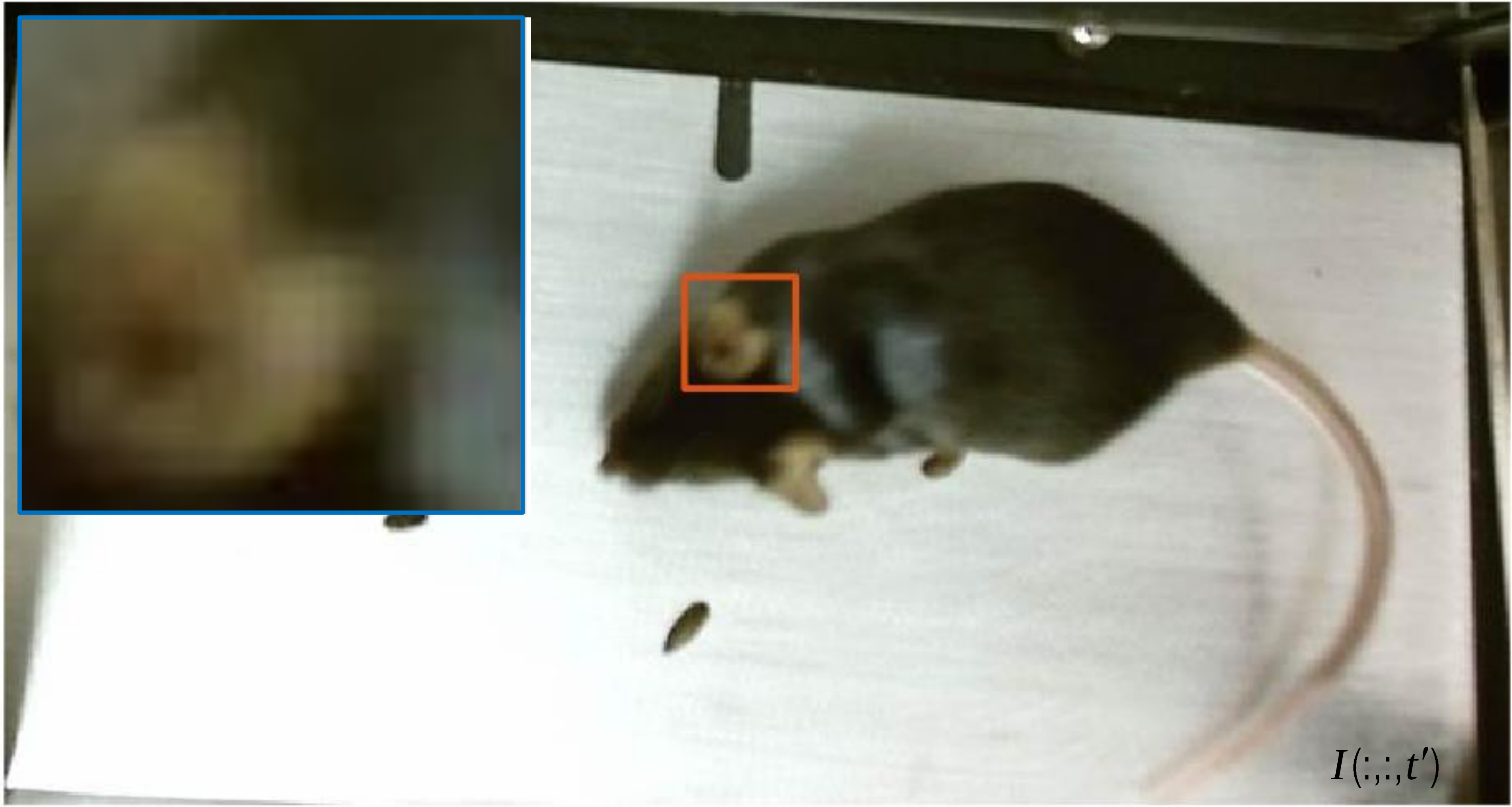
$$x = \left(A^T A\right)^{-1} A^T b$$

Least squares solution

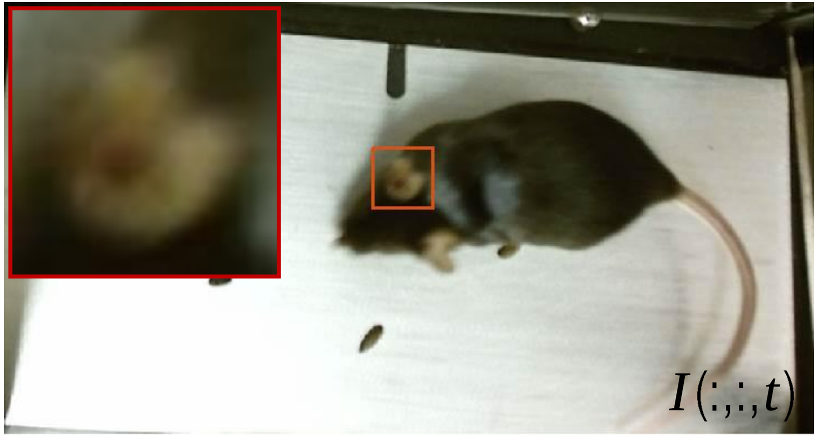


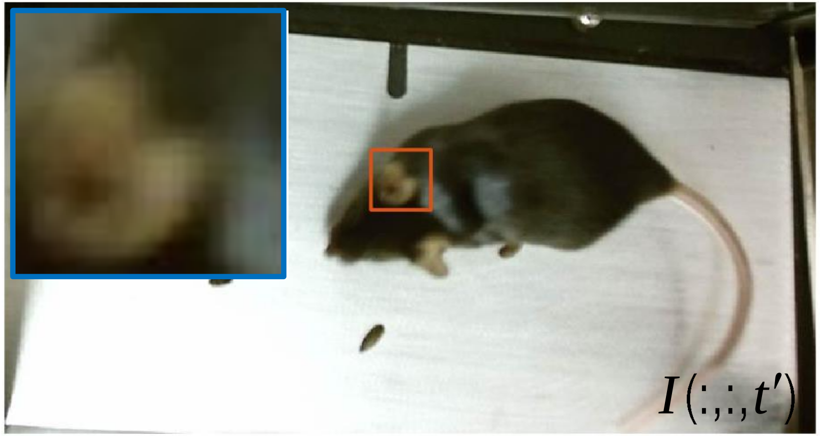
$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

$$I(:,:,t)$$


$I(:,:,t')$

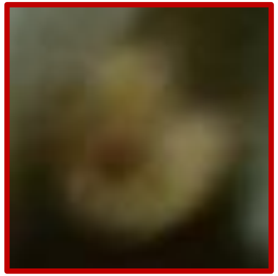
Local Patch Tracking





NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$



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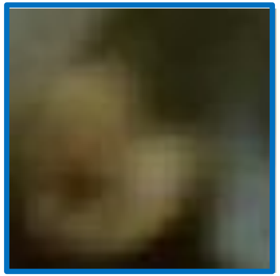

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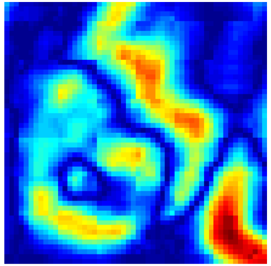
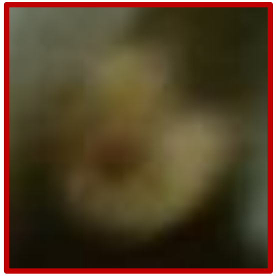
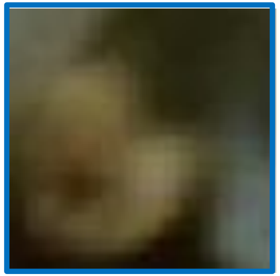
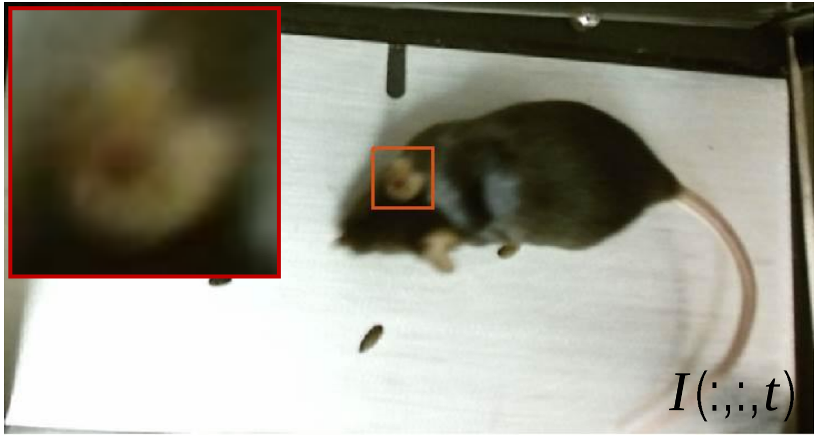


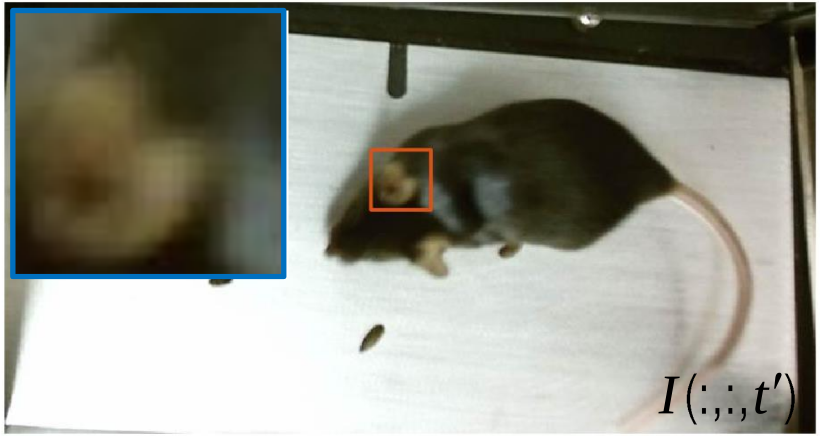
SSD: 3.85

Corr [ , ] = 0.92

Why is not perfect?

Local Patch Tracking





NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

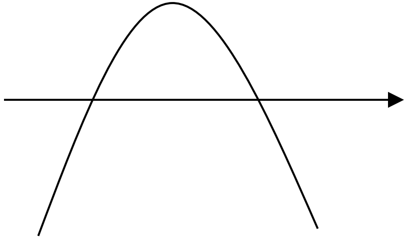
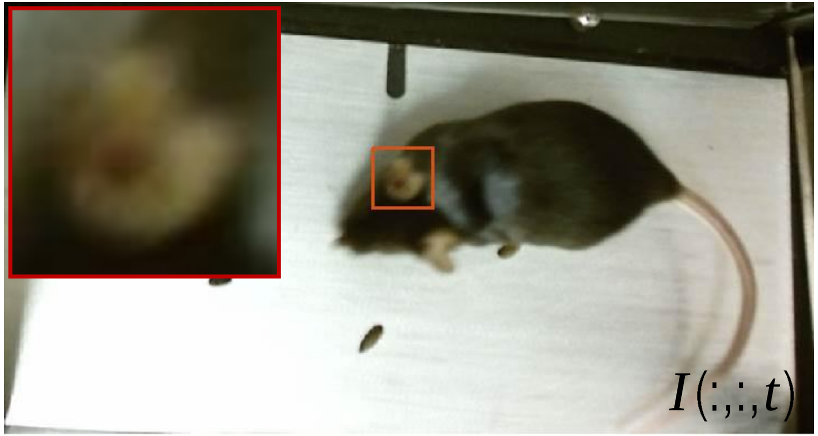
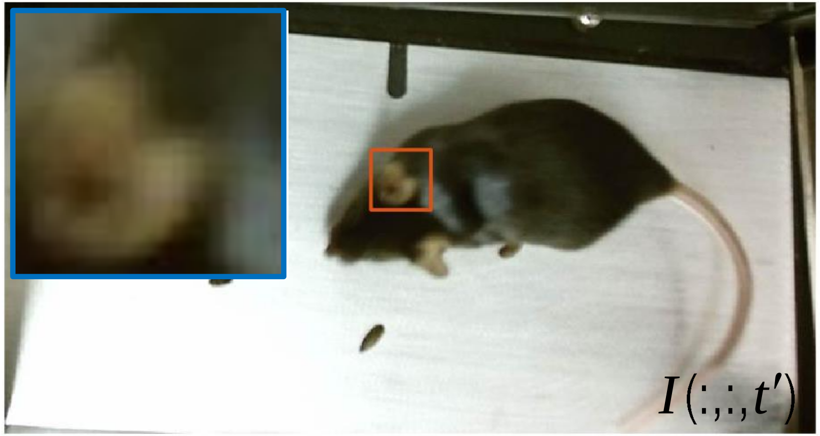


Image is nonlinear function!

Local Patch Tracking





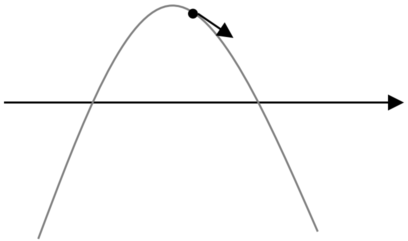
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

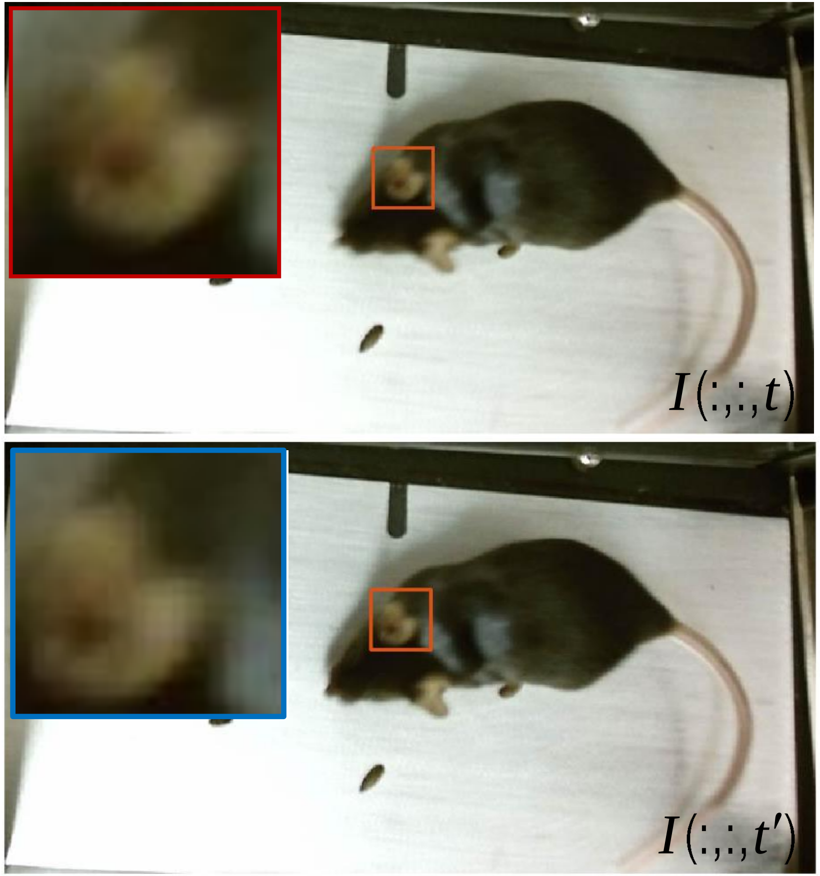
$$\approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



1.Linearize

Image is nonlinear function!

LOCAL PATCH TRACKING



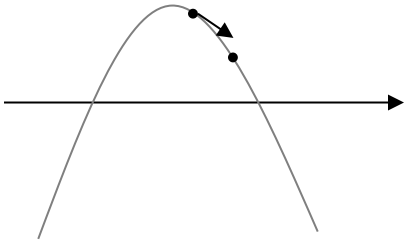
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$

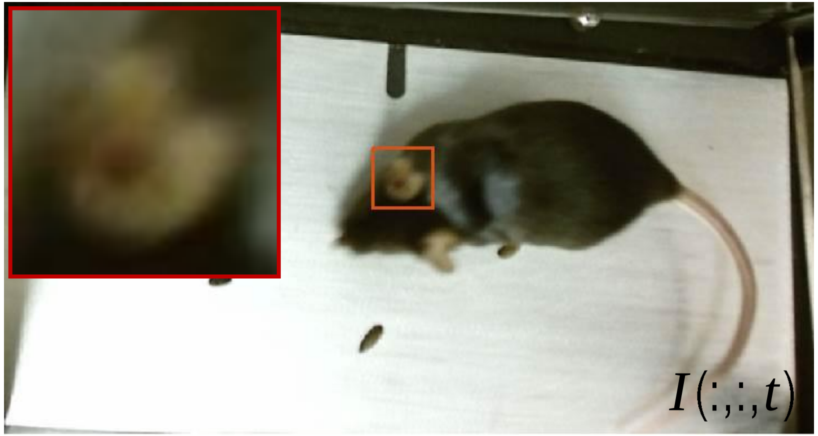


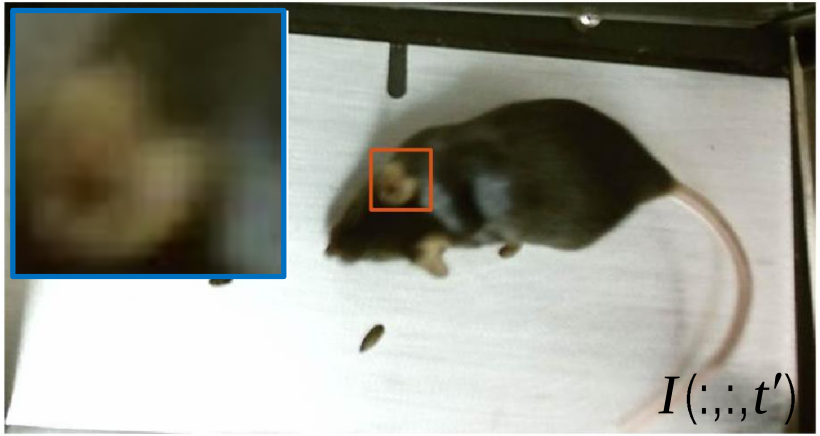
1.Linearize

2.Move $x \leftarrow x + (u,v)$

Image is nonlinear function!

Local Patch Tracking





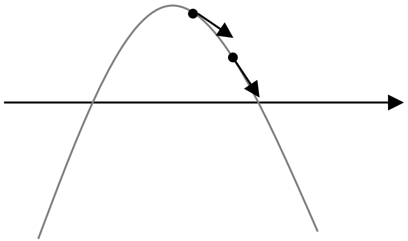
NCC: 0.923520; SSD: 3.854872

$$x = \left(A^T A\right)^{-1} A^T b \qquad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

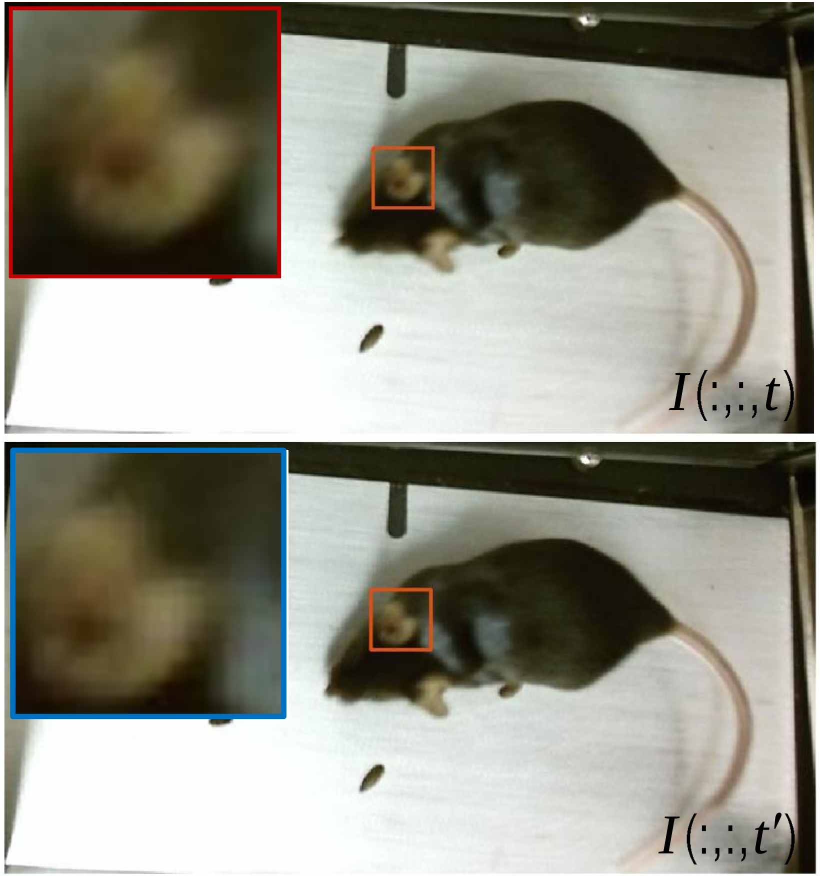
$$\approx I(x,y,t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t}\delta t$$

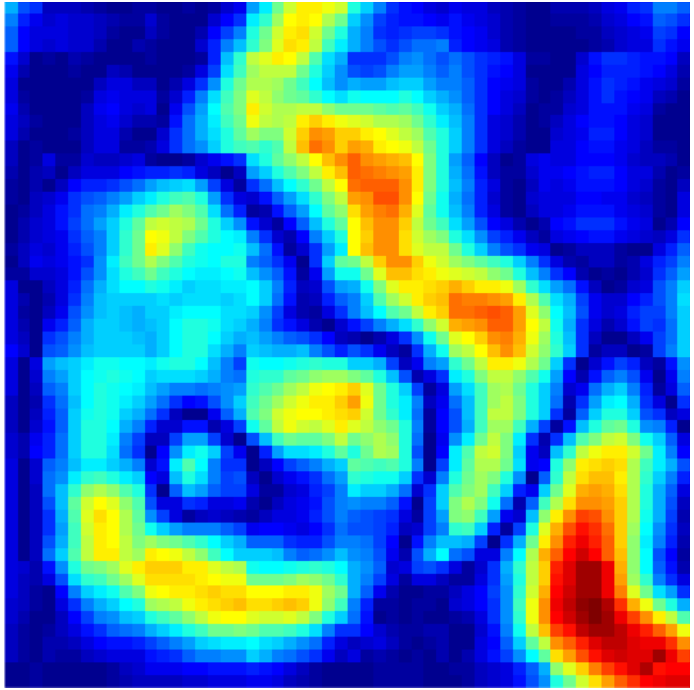


1. Linearize
2. Move $x \leftarrow x + (u,v)$
3. Goto 1

Image is nonlinear function!

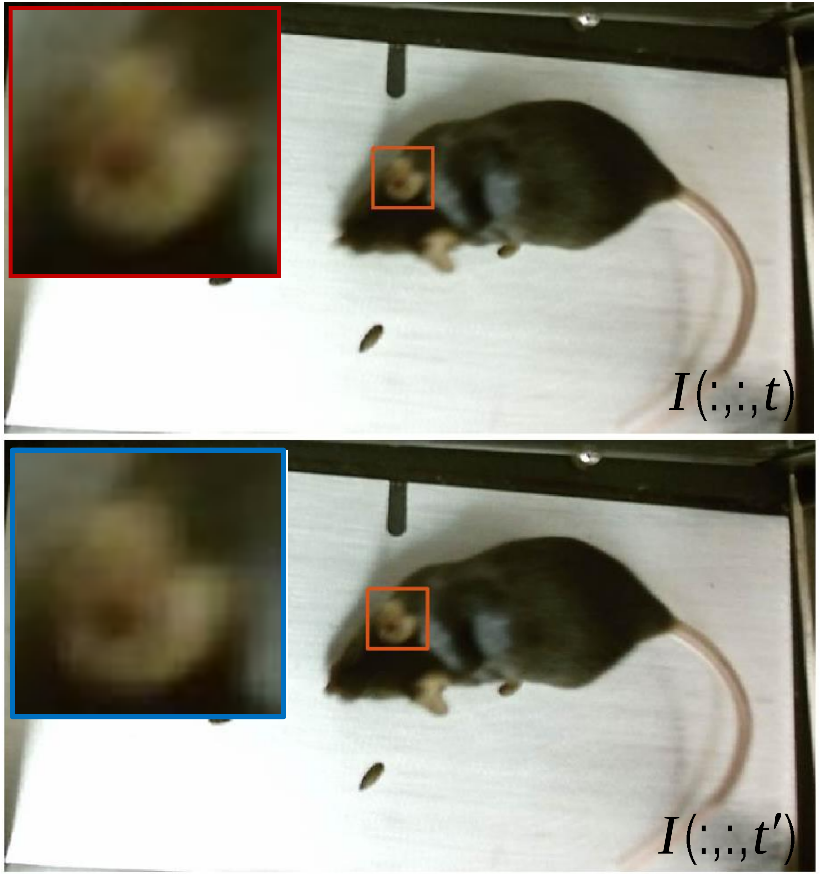
1ST ITERATION

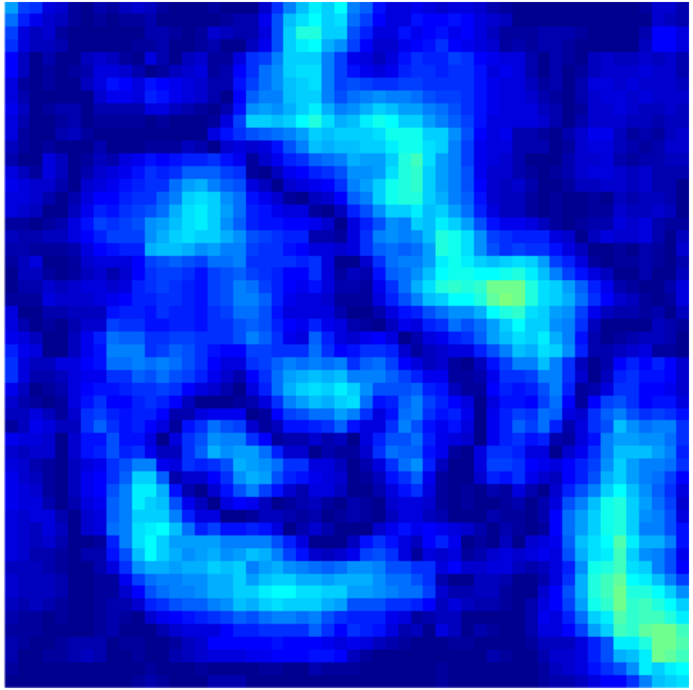




NCC: 0.923520; SSD: 3.854872

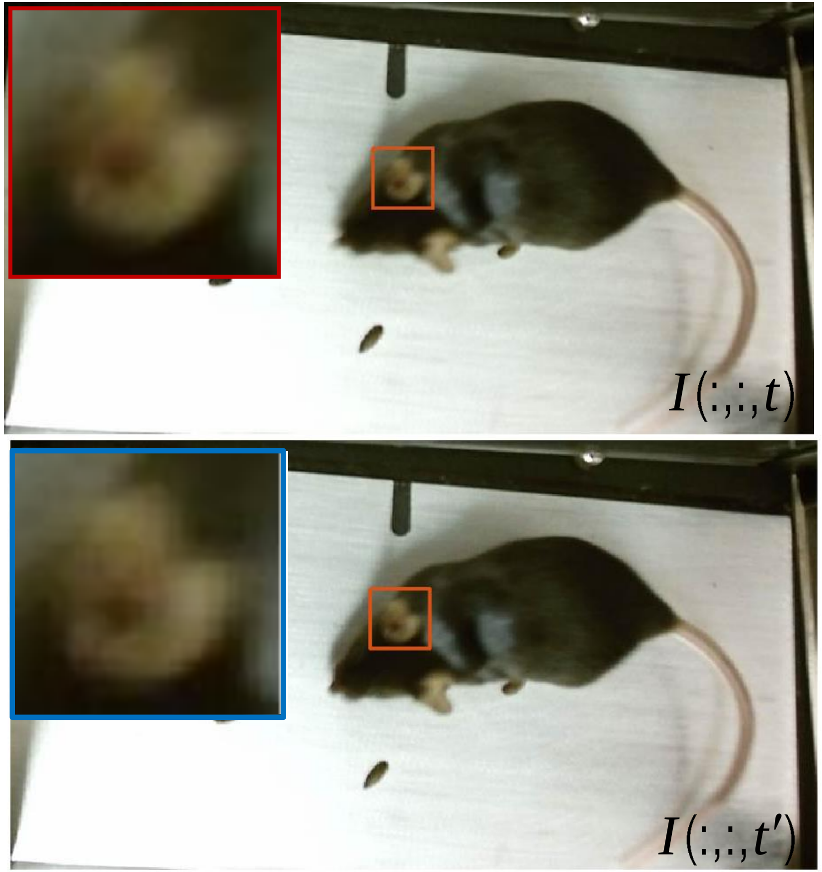
3RD ITERATION

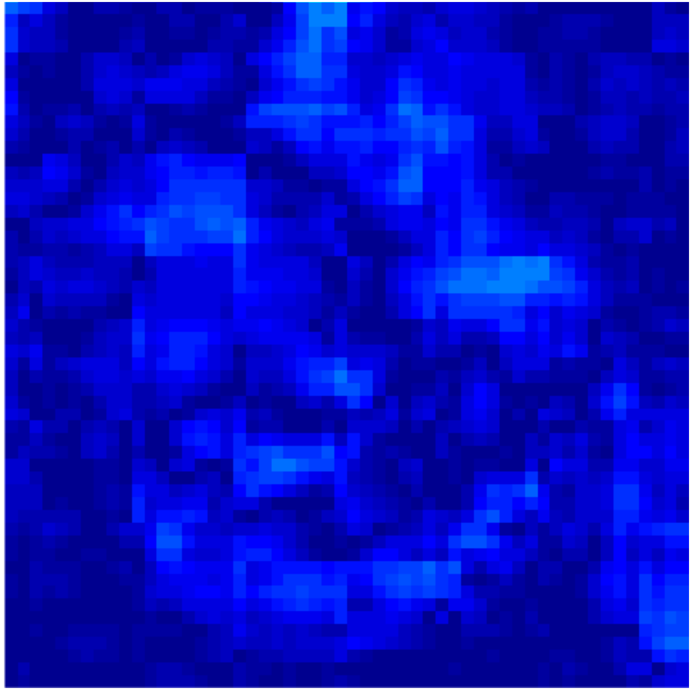




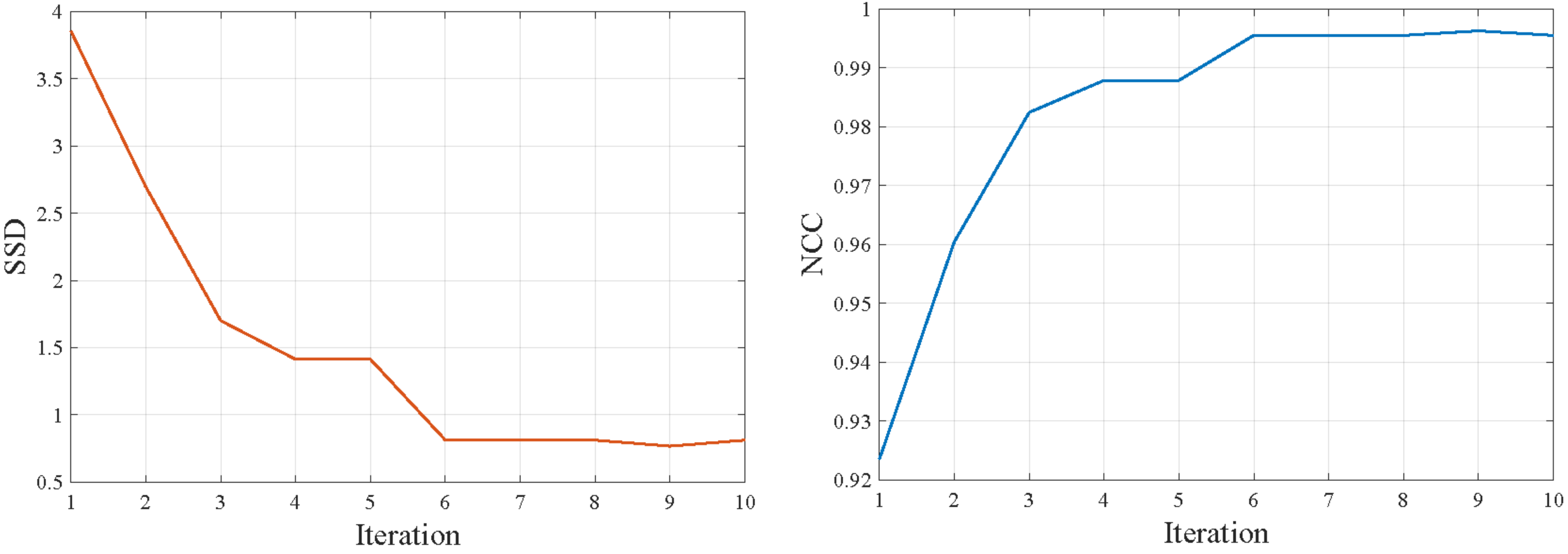
NCC: 0.982435; SSD: 1.700352

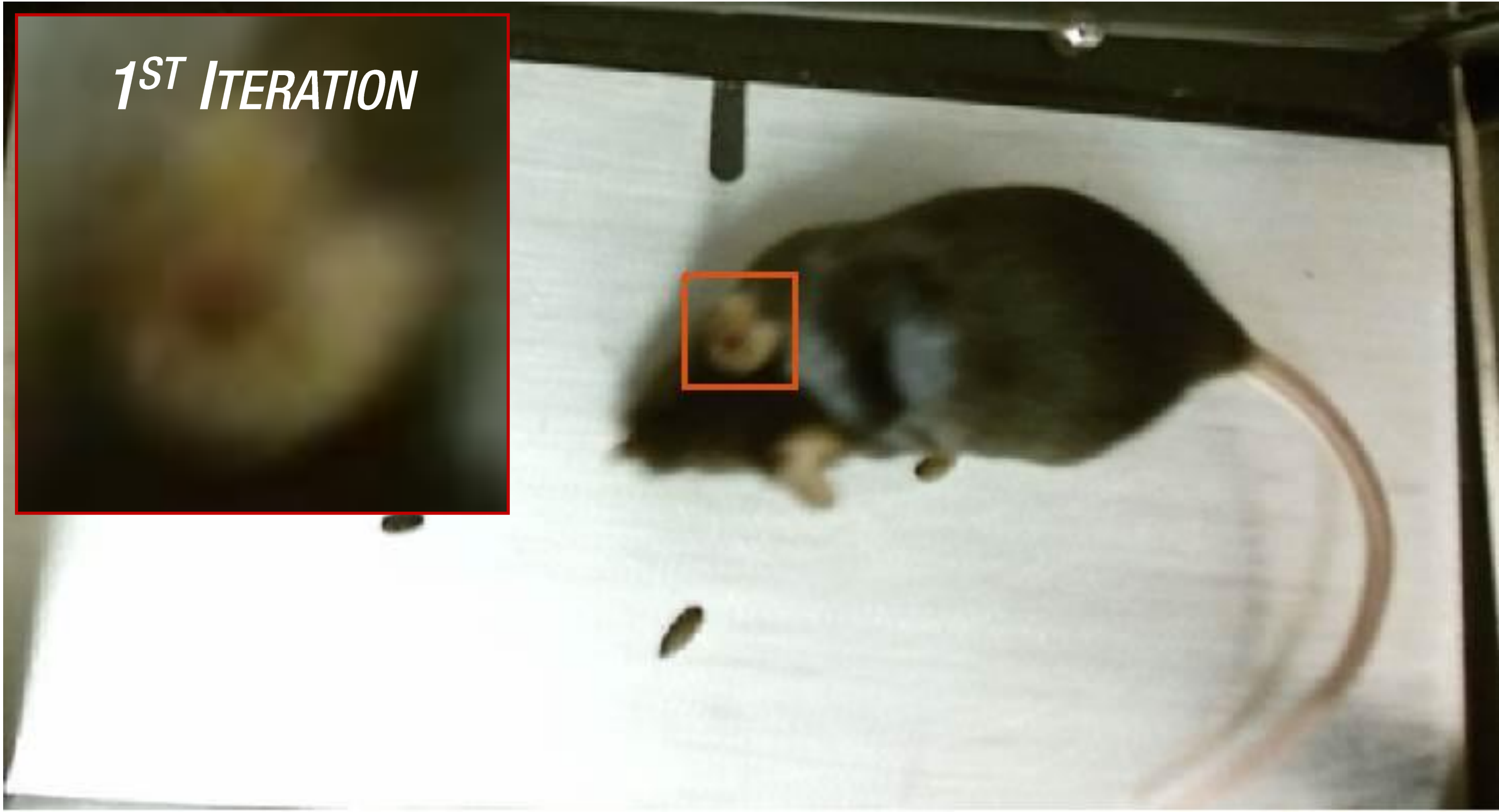
6^{TH} ITERATION

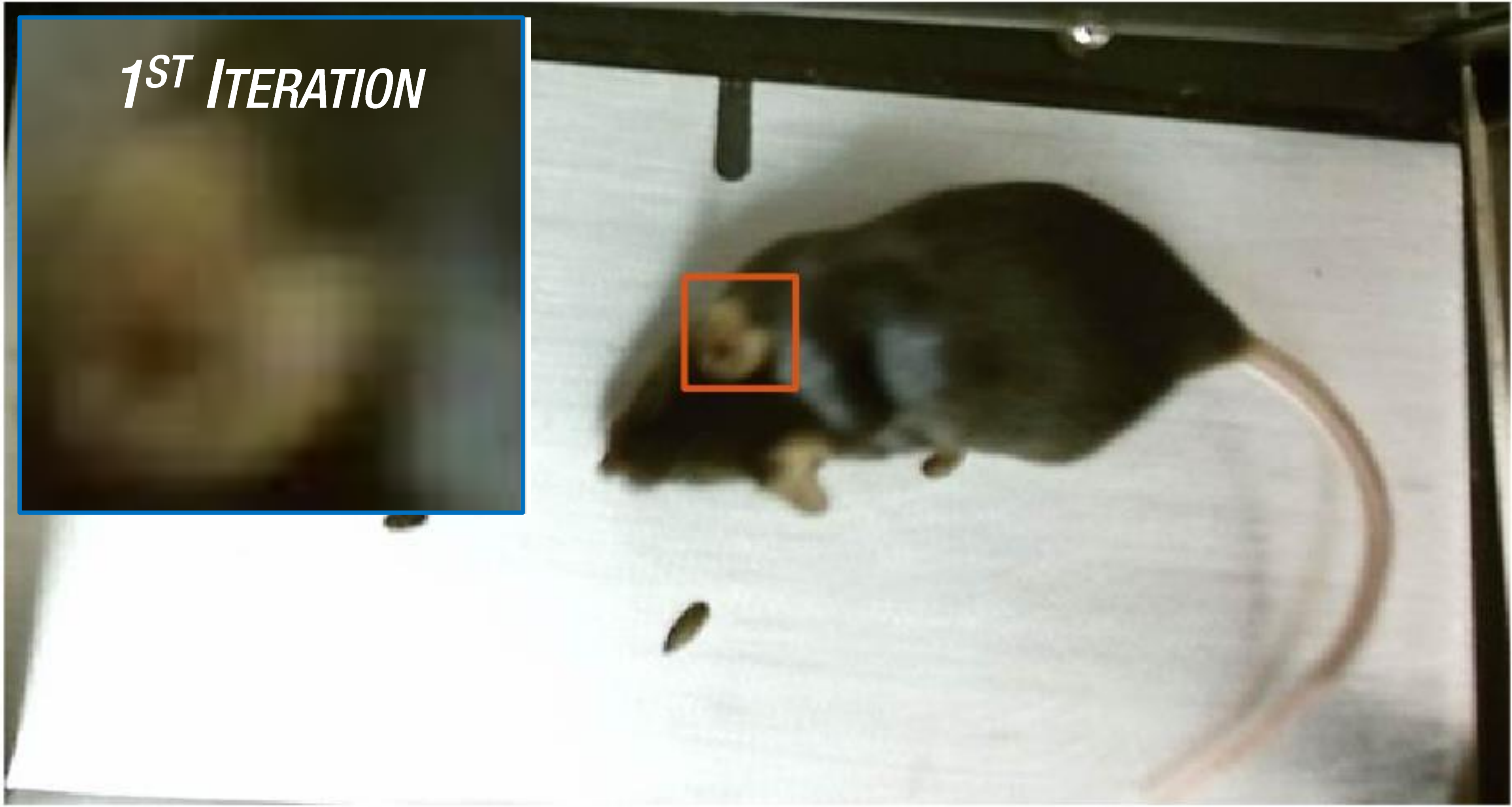


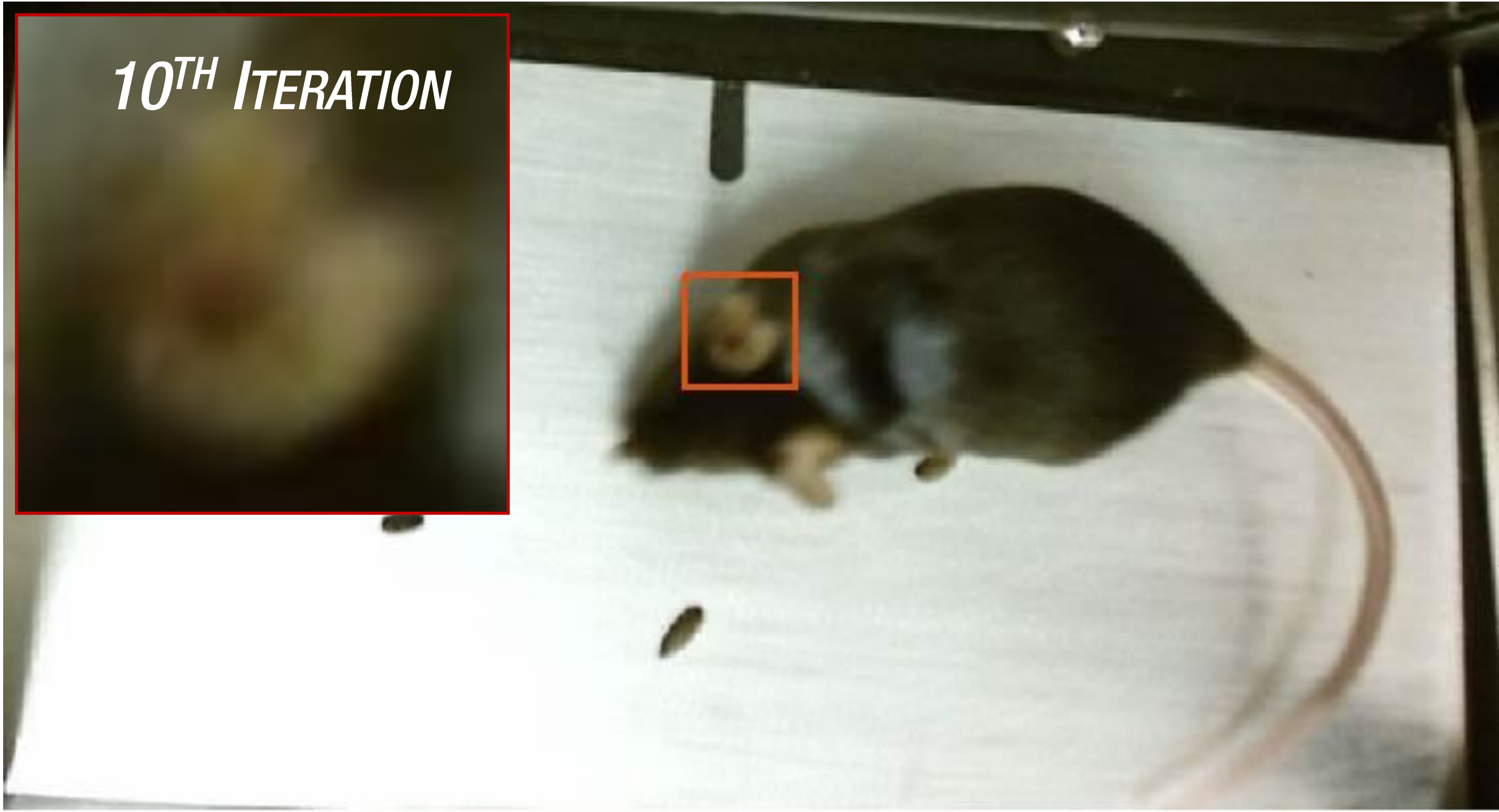


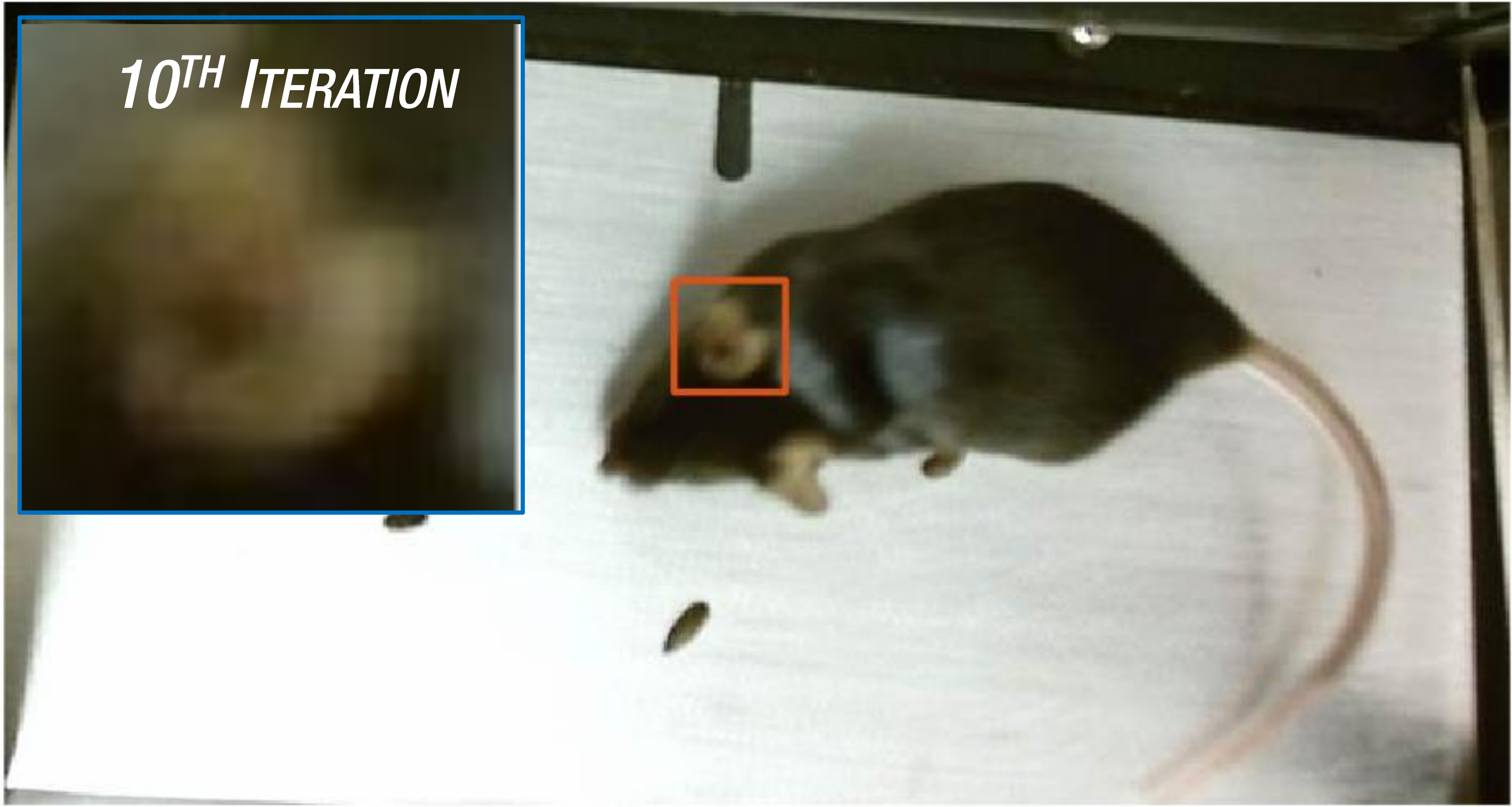
NCC: 0.995528; SSD: 0.811369


SSD
Iteration
NCC
Iteration
4
3.5
3
2.5
2
1.5
1
0.5
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
1 2 3 4 5 6 7 8 9 10


1ST ITERATION


1ST ITERATION


10TH ITERATION


10TH ITERATION